Robotics: Fundamentals

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University of Pennsylvania

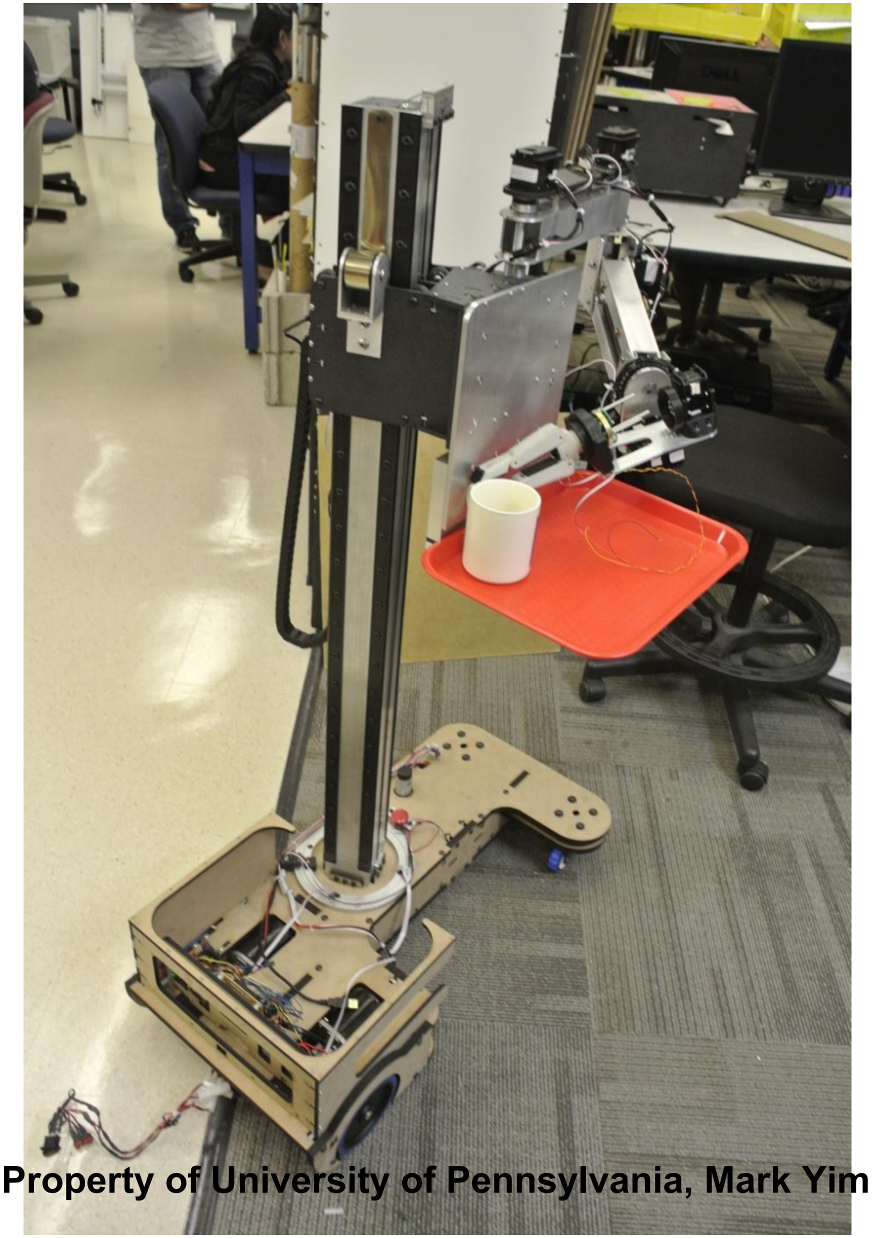
Week 6: Inverse Kinematics

Robotics: Fundamentals

Video 6.1

Mark Yim

Inverse Kinematics



Inverse Kinematics Definition

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{R} & \mathbf{P} \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_{0n}(q_1 \cdots q_n) = \boldsymbol{H}$$

$$\boldsymbol{T}_{0n}(q_1 \cdots q_n) = \boldsymbol{A}_1 \cdots \boldsymbol{A}_n$$

$$T_{ij}(q_1 \cdots q_n) = H_{ij}$$

Where, T_{ij}, H_{ij} refer to the 12 nontrivial entries of $\boldsymbol{T}_{0n}$, and $\boldsymbol{H}$

Stanford Arm Transform

$$H = \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$r_{11} = c_1[c_2(c_4c_5c_6 - s_4c_6) - s_2s_5c_6] - s_1(s_4c_5c_6 + c_4s_6)$$
$$r_{21} = s_1[c_2(c_4c_5c_6 - s_4c_6) - s_2s_5c_6] + c_1(s_4c_5c_6 + c_4s_6)$$
$$r_{31} = -s_2(c_4c_5c_6 - s_4s_6) - c_2s_5c_6$$
$$r_{12} = c_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] - s_1(-s_4c_5c_6 + c_4s_6)$$
$$r_{22} = s_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] + c_1(-s_4c_5c_6 + c_4s_6)$$
$$r_{32} = s_2(c_4c_5c_6 + s_4s_6) + c_2s_5c_6$$
$$r_{13} = c_1(c_2c_4s_5 + s_2c_5) - s_1s_4s_5$$
$$r_{23} = s_1(c_2c_4s_5 + s_2c_5) + c_1s_4s_5$$
$$r_{33} = -s_2c_4s_5 + c_2c_5$$
$$P_x = c_1s_2d_3 - s_1d_2 + d_6(c_1c_2c_4s_5 + c_1s_2c_5 - s_1s_4s_5)$$
$$P_y = s_1s_2d_3 + c_1d_2 + d_6(s_1c_2c_4s_5 + s_1s_2c_5 + c_1s_4s_5)$$
$$P_z = c_2d_3 + d_6(c_2c_5 - s_2c_4s_5)$$

12 non-linear eqns, 7 unknowns

Approaches

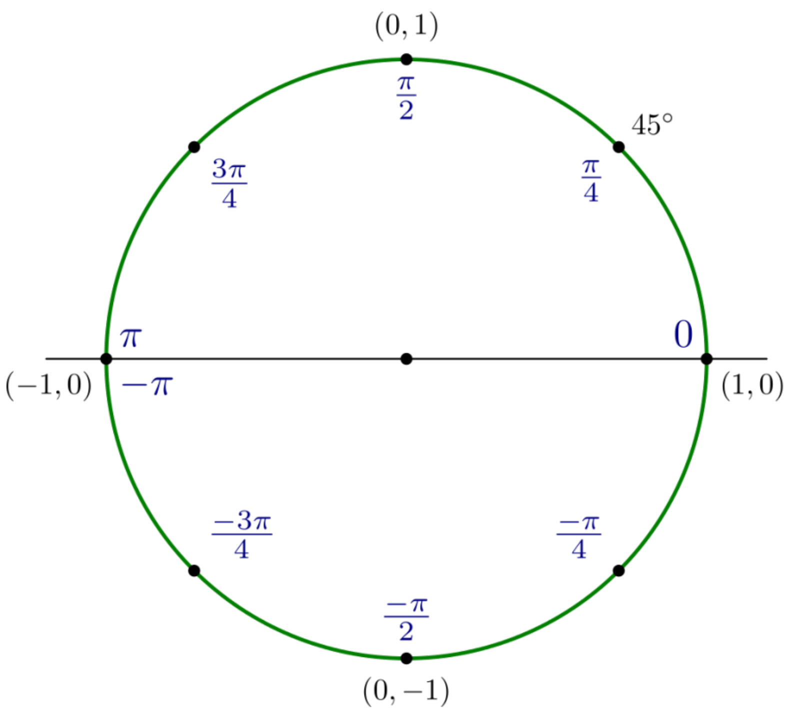
Closed form

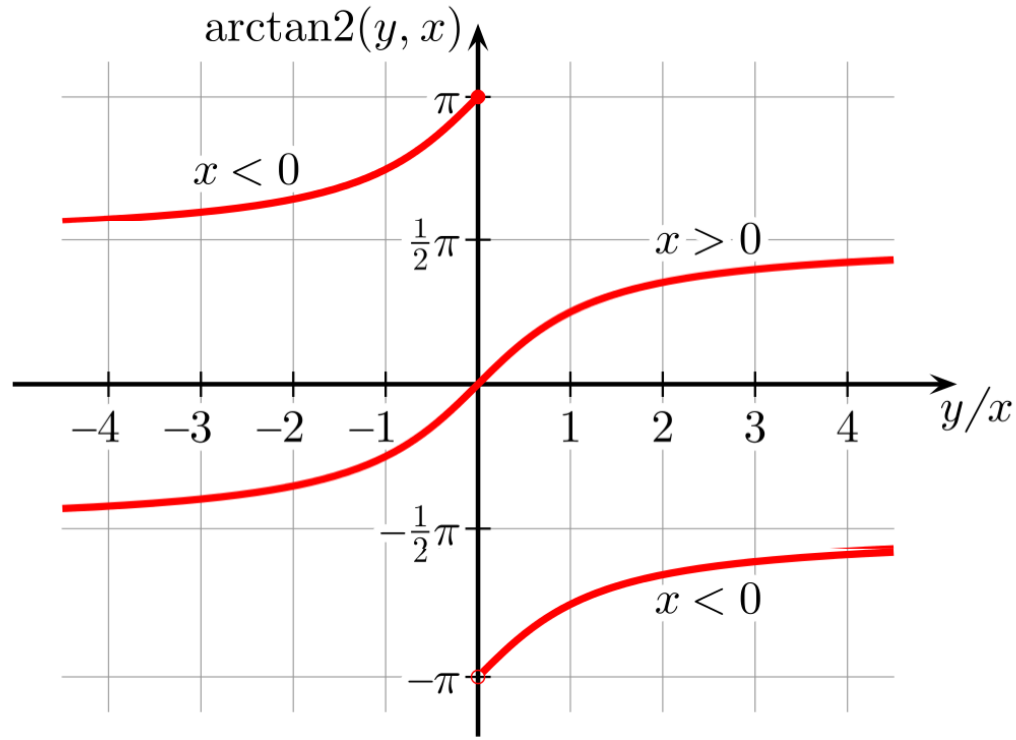
- $q_k = f_k(h_{11,} \cdots, h_{34}), k = 1, \cdots, n$
- Geometric approach
- *IKFast* (open source C++ code)

Approximate Solutions

- Iterative optimization
- Jacobian Inverse

atan(y/x) vs atan2(y,x)





By Qef (self-made using design of the bitmap version) [CC0]

By Geek3 (Own work) [GFDL]

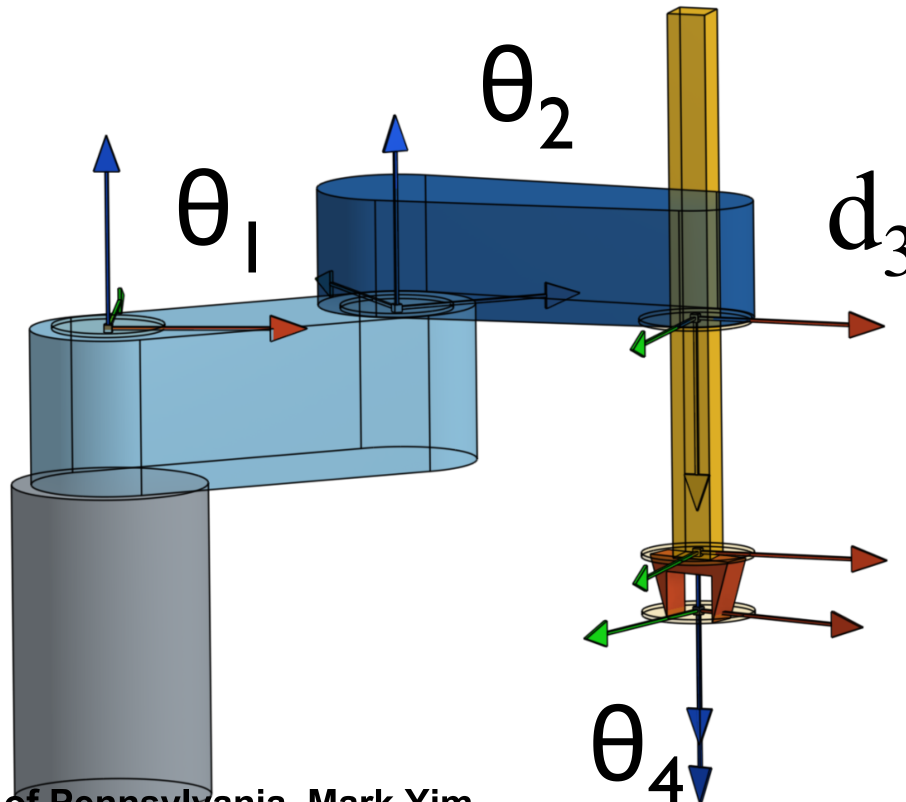
Robotics: Fundamentals

Video 6.2
Mark Yim

SCARA IK

$$\begin{bmatrix} \boldsymbol{c_{12}c_4} + \boldsymbol{s_{12}s_4} & -\boldsymbol{c_{12}s_4} + \boldsymbol{s_{12}c_4} & 0 & a_1\boldsymbol{c_1} + a_2\boldsymbol{c_{12}} \\ \boldsymbol{s_{12}c_4} - \boldsymbol{c_{12}s_4} & -\boldsymbol{s_{12}s_4} - \boldsymbol{c_{12}c_4} & 0 & a_1\boldsymbol{s_1} + a_2\boldsymbol{s_{12}} \\ 0 & 0 & -1 & -\boldsymbol{d_3} - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



SCARA IK

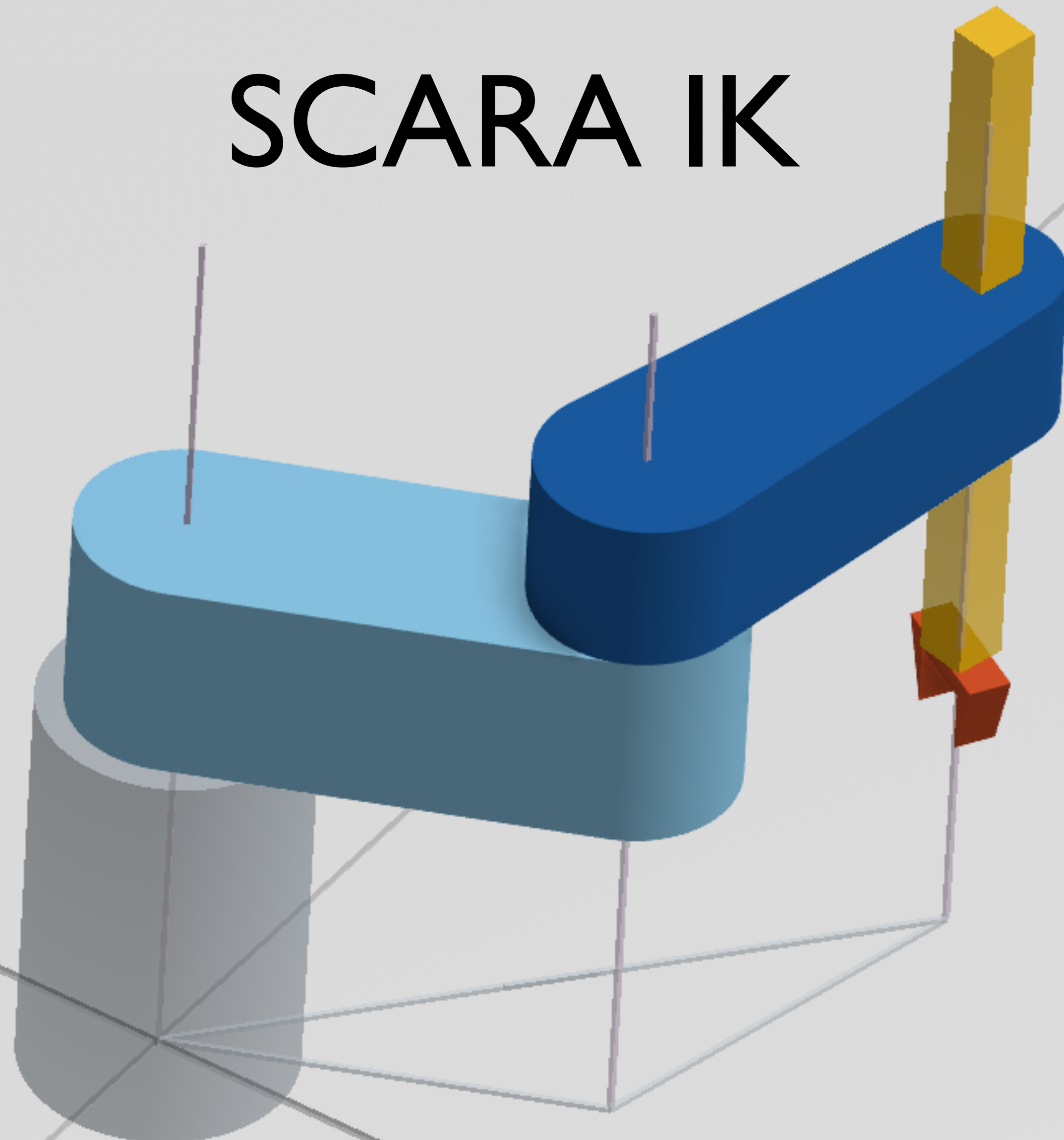
$$\begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} c_\psi & s_\psi & 0 \\ s_\psi & -s_\psi & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$angle = \operatorname{atan2}(y, x)$$

$$\psi = \operatorname{atan2}(r_{12}, r_{11})$$

SCARA IK



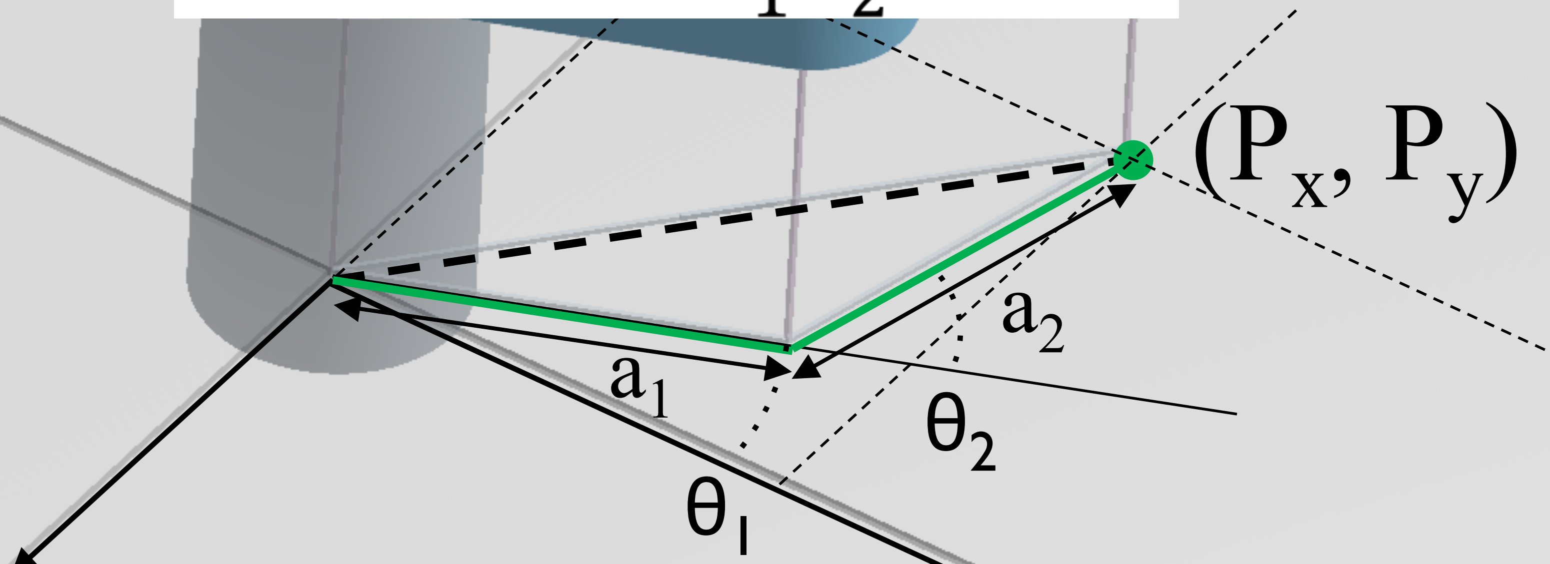
$$\theta_1 + \theta_2 - \theta_4 = \psi$$

SCARA IK

$$r^2 = a_1^2 + a_2^2 - 2a_1a_2\cos(\theta)$$

$$P_x^2 + P_y^2 = a_1^2 + a_2^2 + 2a_1a_2\,c_2$$

$$c_2 = \frac{P_x^2 + P_y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

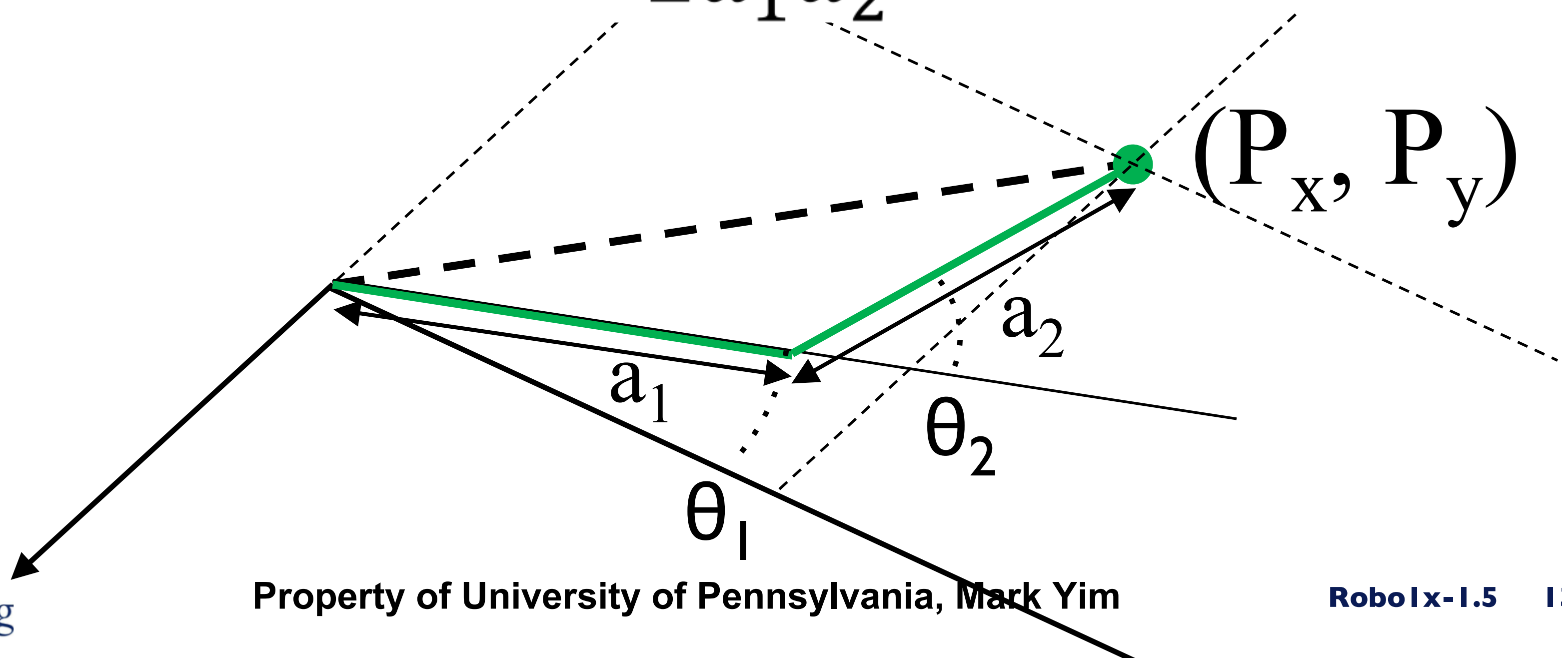


SCARA IK θ_2

$$r^2 = a_1^2 + a_2^2 - 2a_1a_2\cos(\theta_2)$$

$$P_x^2 + P_y^2 = a_1^2 + a_2^2 - 2a_1a_2\,c_2$$

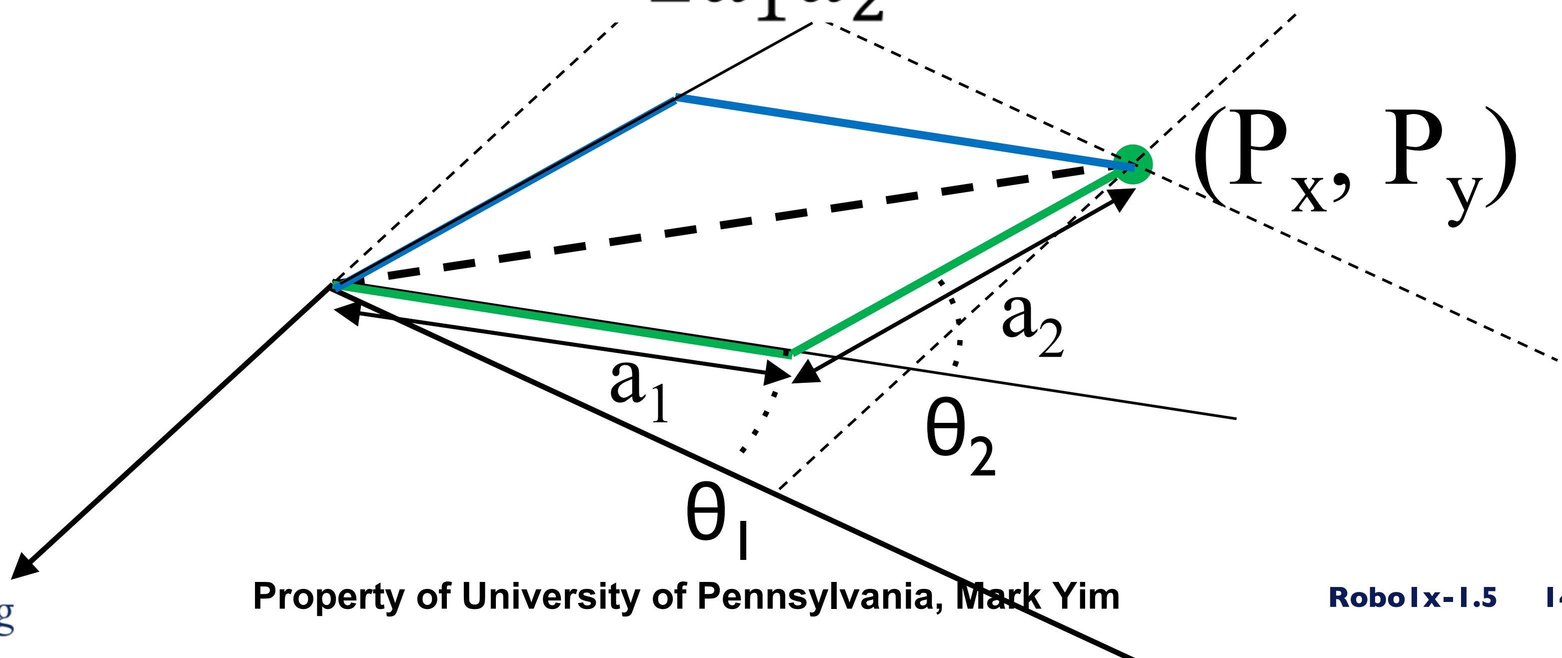
$$c_2 = \frac{P_x^2 + P_y^2 - (a_1^2 + a_2^2)}{2a_1a_2}$$



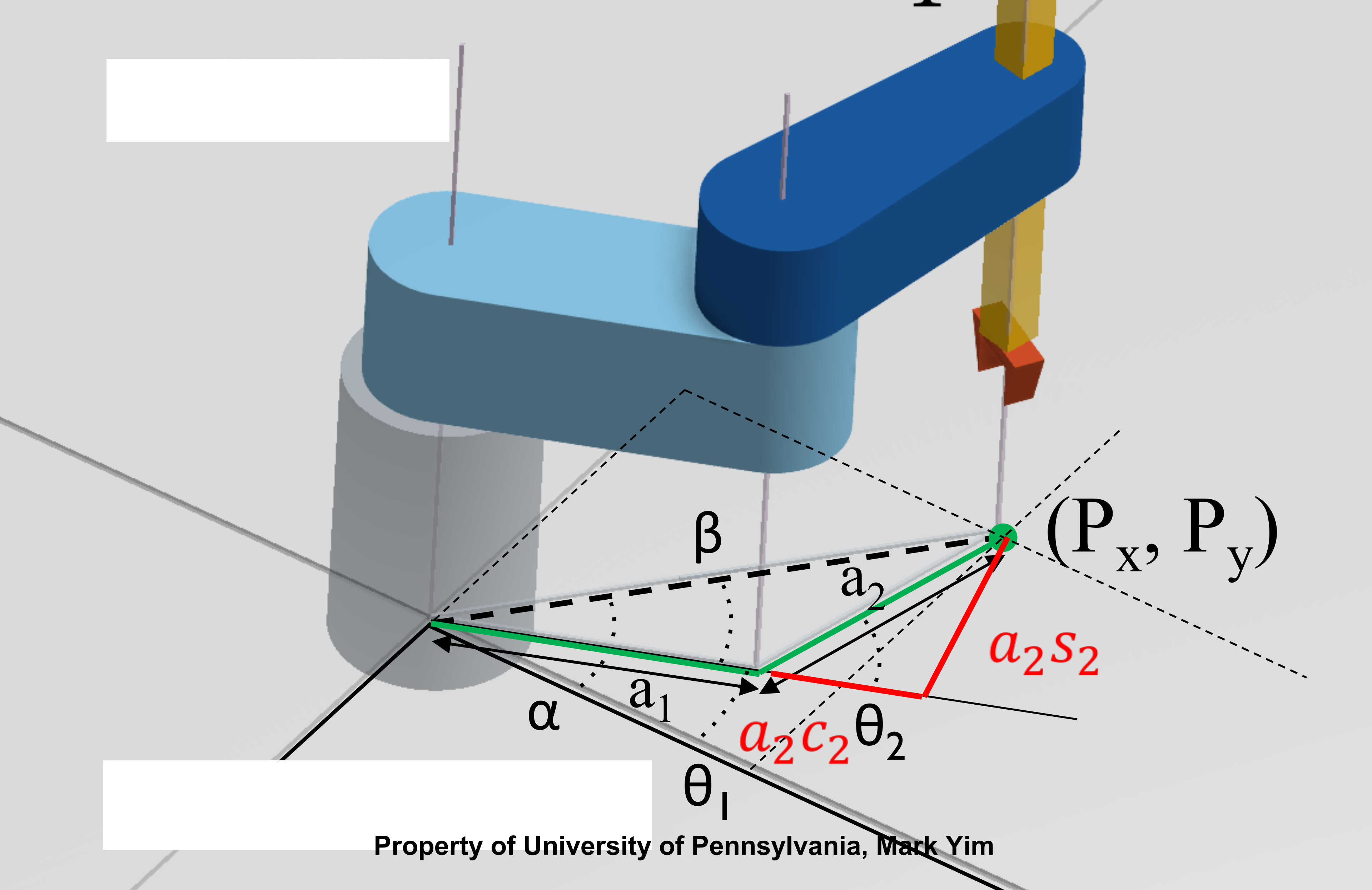
SCARA IK θ_2

$$\boxed{\theta_2 = \pm\cos^{-1}\left(\frac{P_x^2 + P_y^2 - (a_1^2 + a_2^2)}{2a_1 a_2}\right)}$$

$$c_2 = \frac{P_x^2 + P_y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

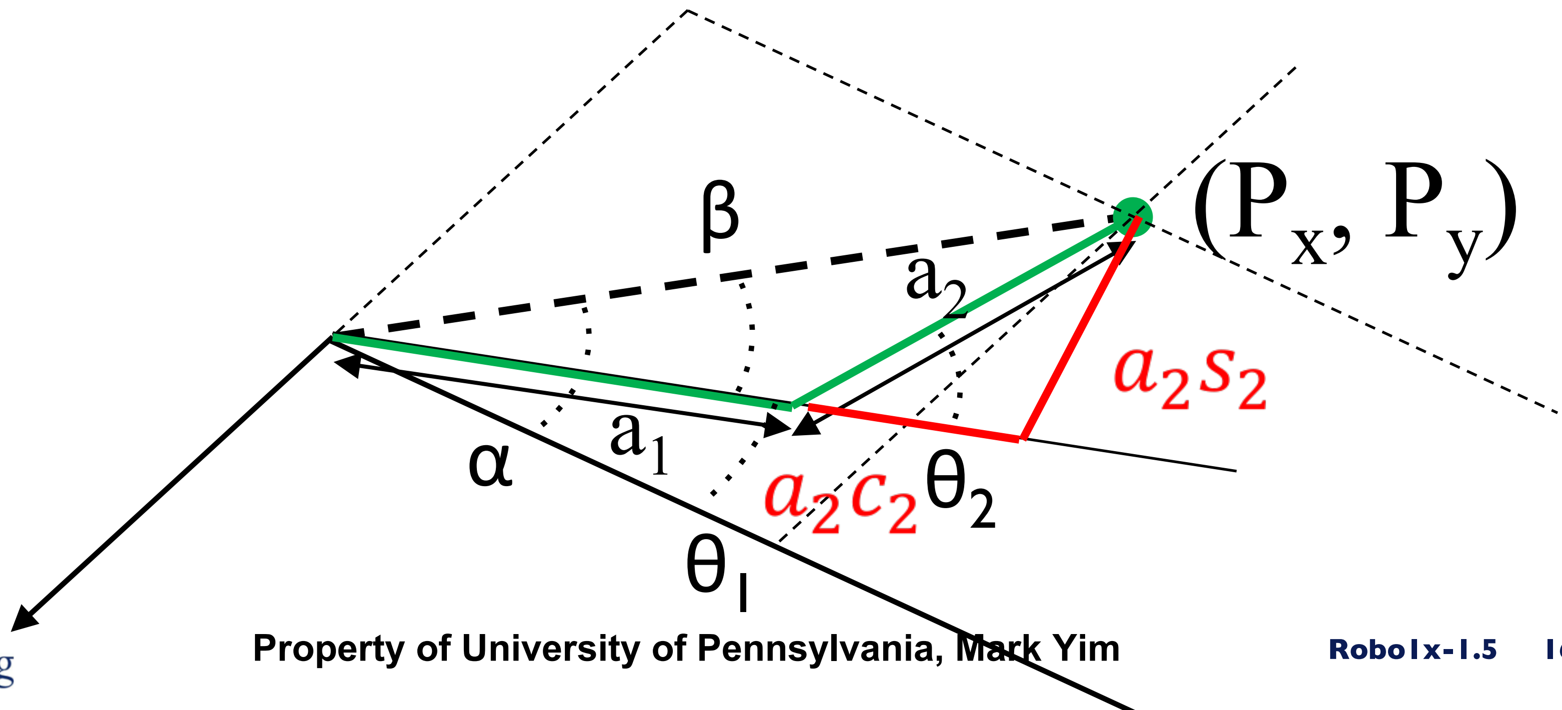


SCARA IK θ_1

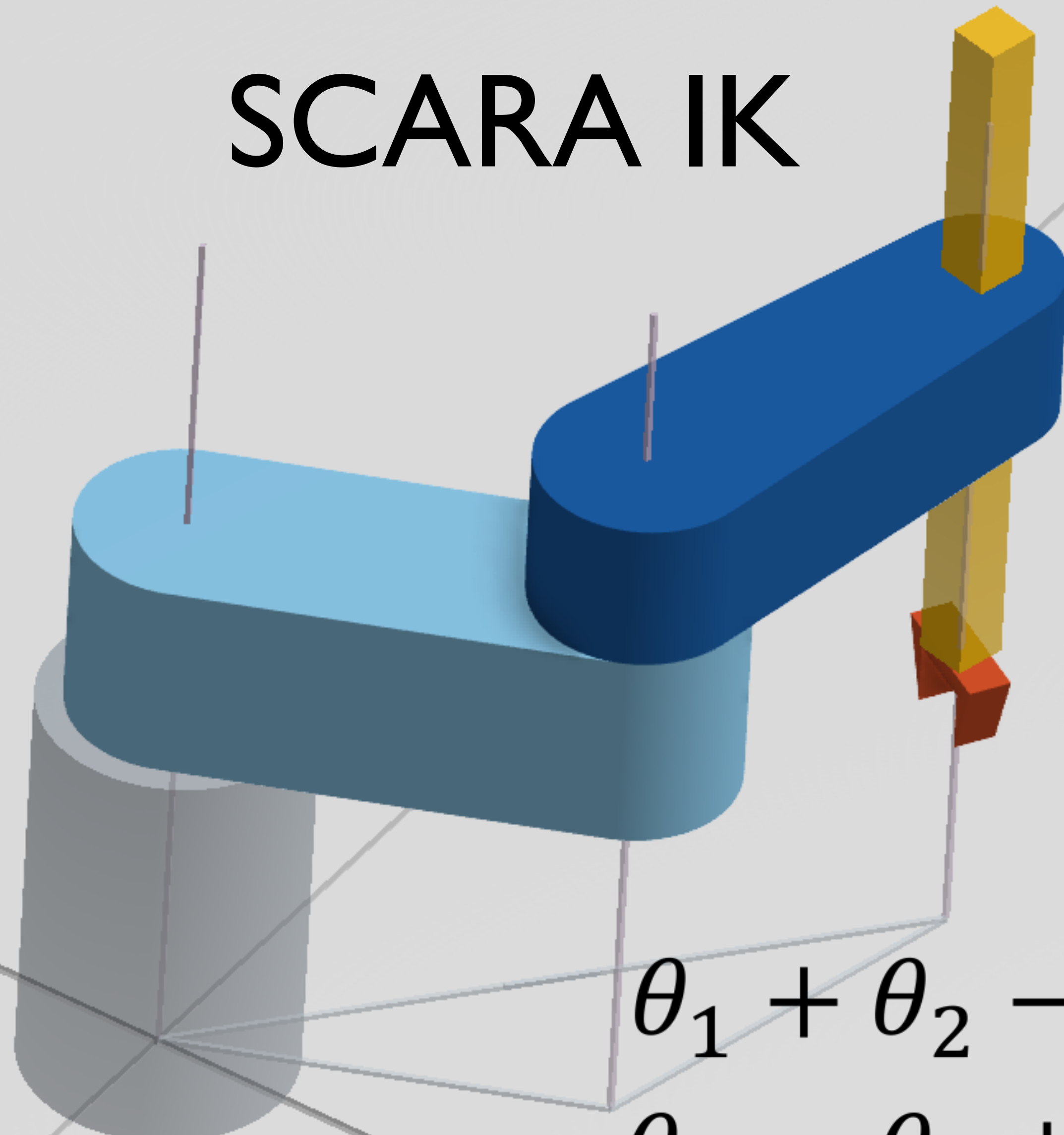


SCARA IK θ_1

$$\boxed{\theta_1 = \operatorname{atan2}(P_y, P_x) - \operatorname{atan2}(a_2 s_2,\ a_1 + a_2 c_2)}$$



SCARA IK

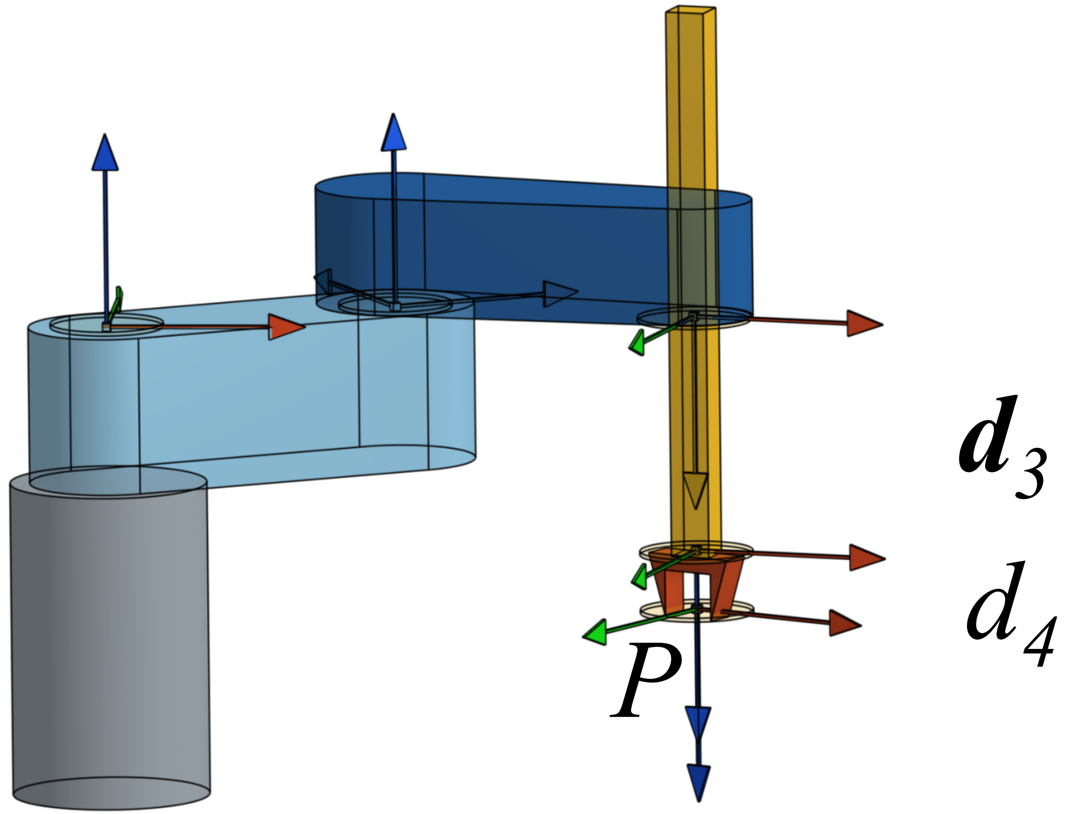


$$\theta_1 + \theta_2 - \theta_4 = \psi$$

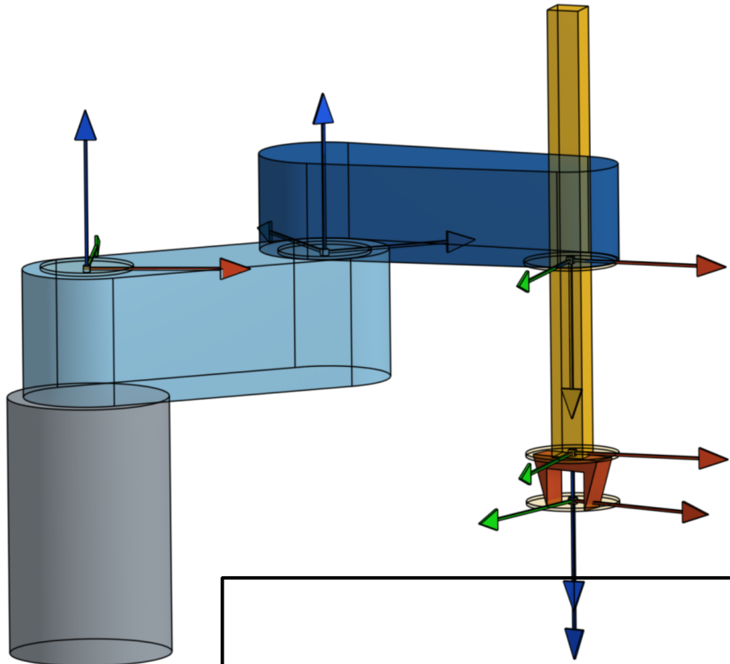
$$\theta_4 = \theta_1 + \theta_2 - \psi$$

$$\boxed{\theta_4 = \theta_1 + \theta_2 - \operatorname{atan}(r_{12}, r_{11})}$$

SCARA IK d_3



$$\boldsymbol{d}_3 = P_z + d_4$$



SCARA IK

$$\theta_2 = \pm\cos^{-1}\left(\frac{P_x^2 + P_y^2 - (a_1^2+a_2^2)}{2a_1a_2}\right)$$

$$\theta_1 = \operatorname{atan2}(P_y, P_x) - \operatorname{atan2}(a_2 s_2,\, a_1 + a_2 c_2)$$

$$\theta_4 = \theta_1 + \theta_2 - \operatorname{atan}(r_{12}, r_{11})$$

$$\boldsymbol{d}_3 = P_z + d_4$$

SCARA IK

$$\begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\boxed{\theta_2 = \pm\cos^{-1}\left(\frac{P_x^2 + P_y^2 - (a_1^2 + a_2^2)}{2a_1a_2}\right)}$$

$$\sqrt{P_x^2 + P_y^2} = \sqrt{(a_1 + a_2)^2}$$

$$P_x^2 + P_y^2 = a_1^2 + 2a_1a_2 + a_2^2$$

Workspace

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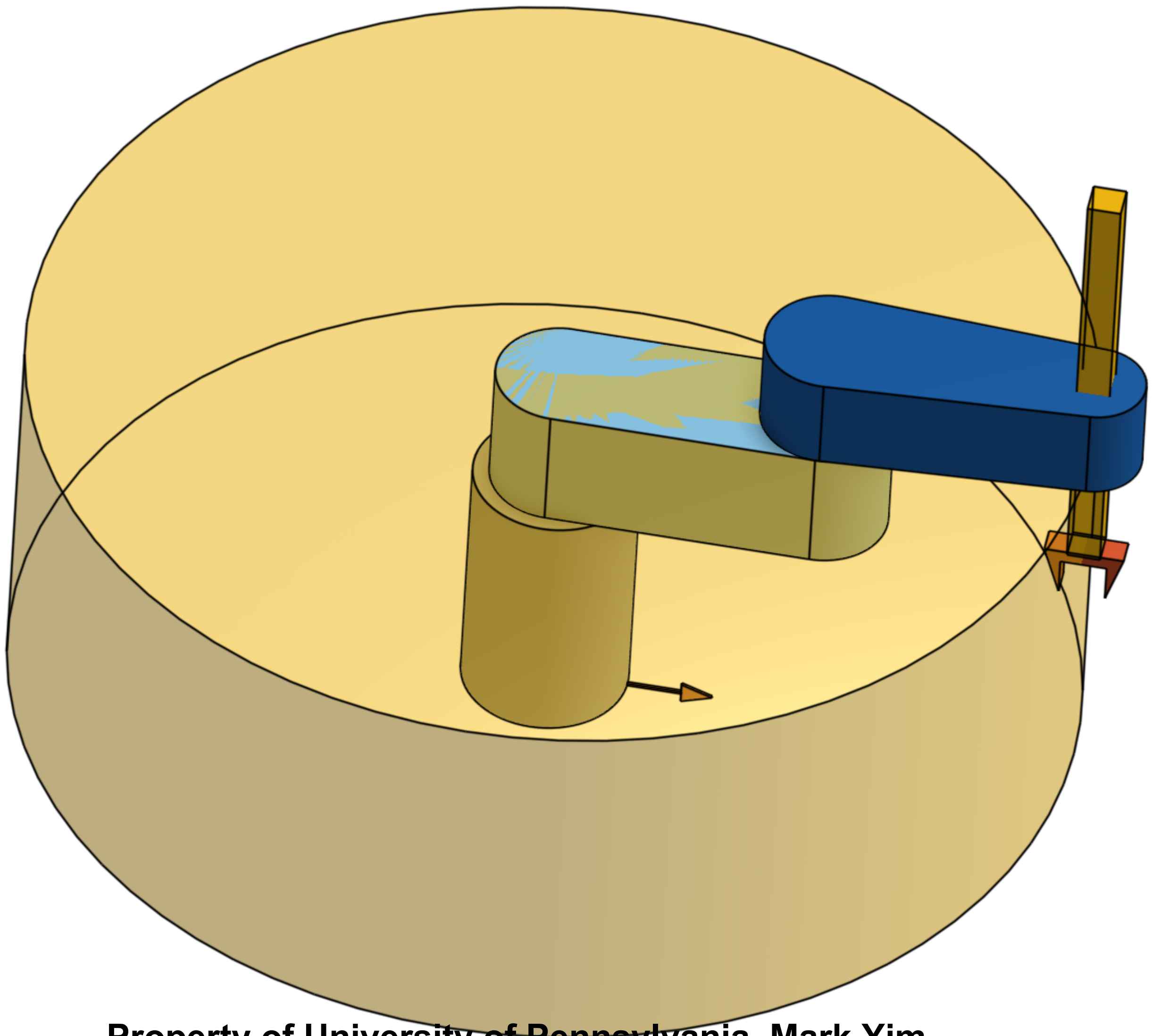

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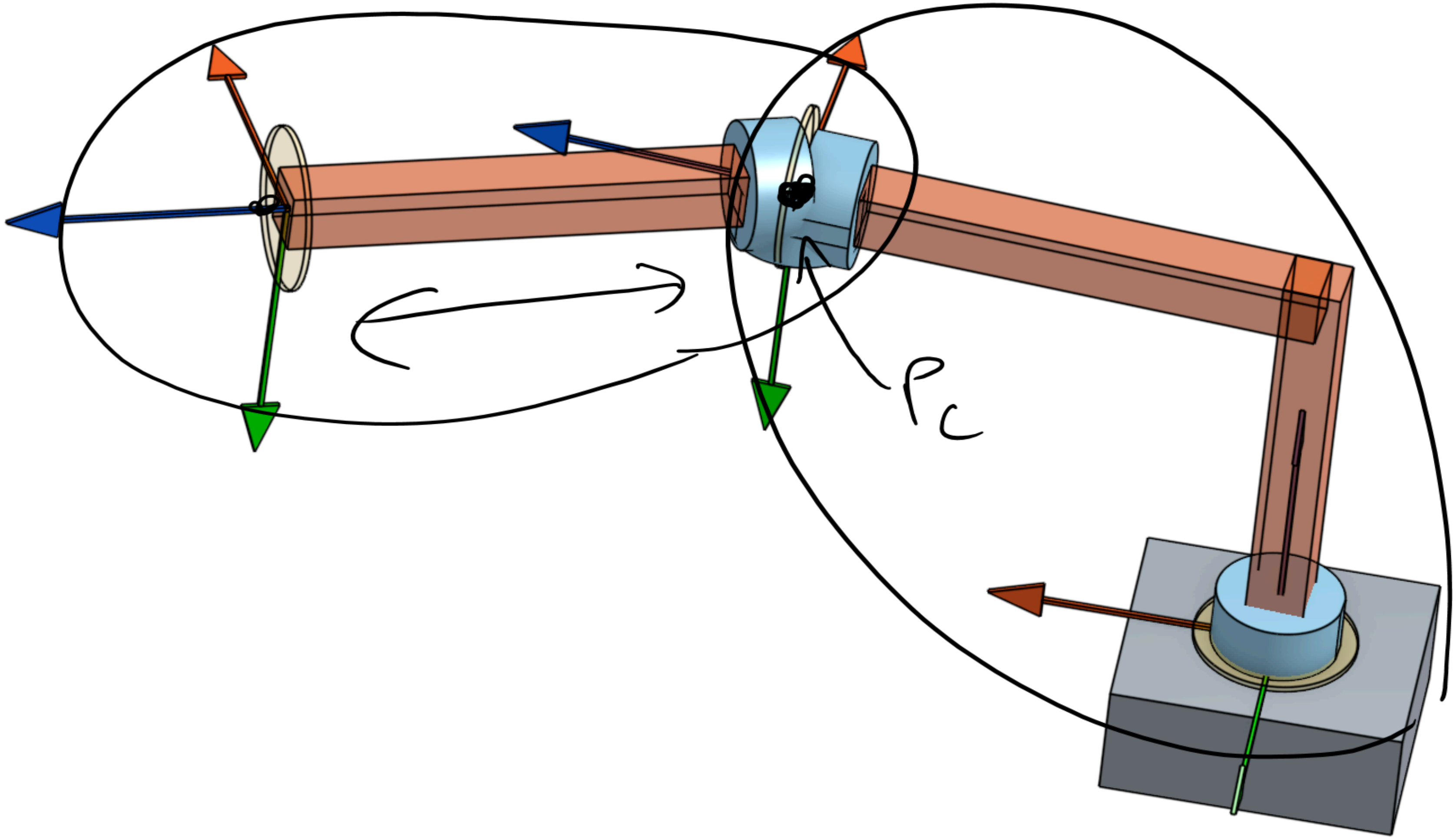
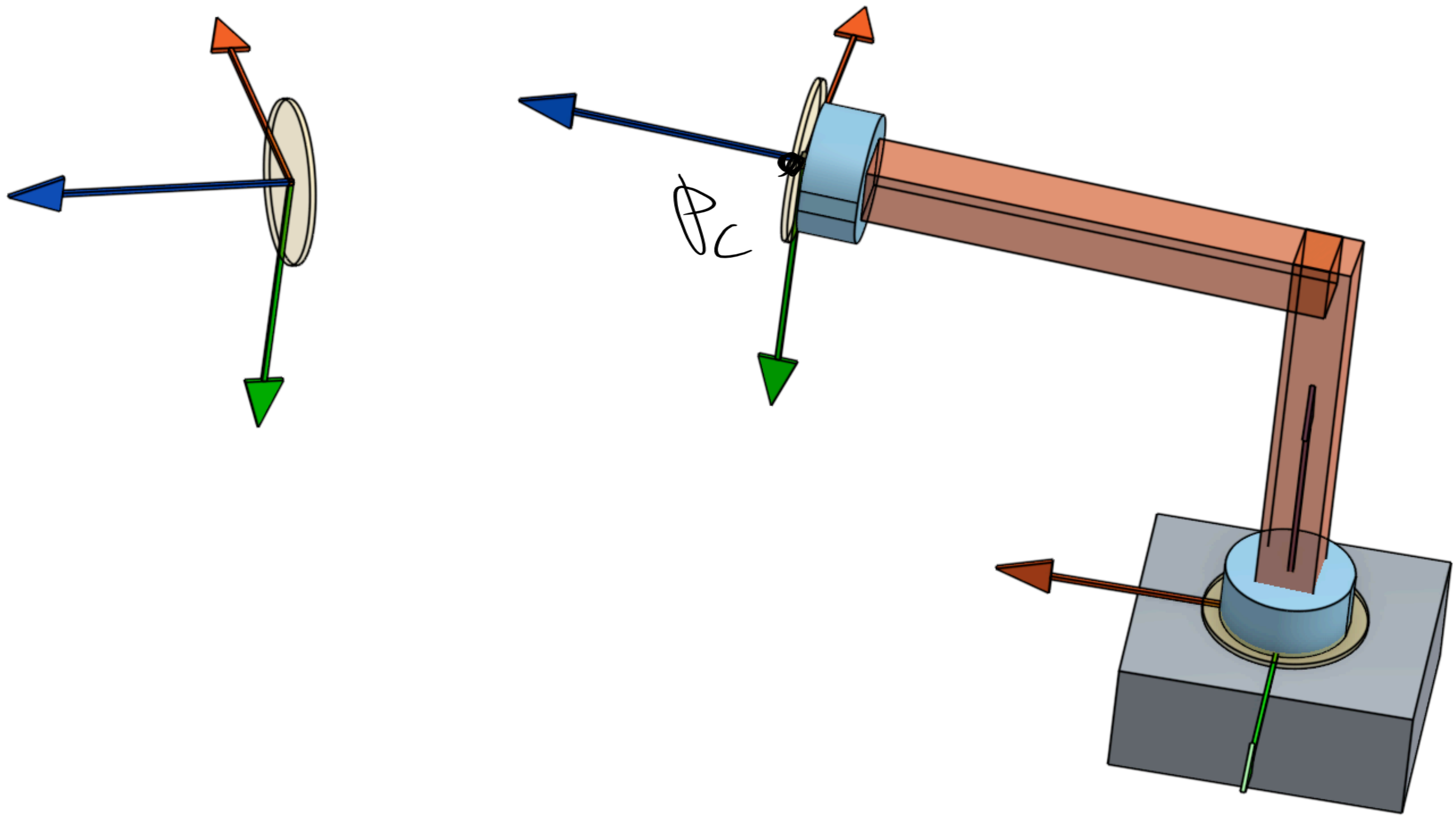
Video 6.3

Mark Yim

Kinematic Decoupling

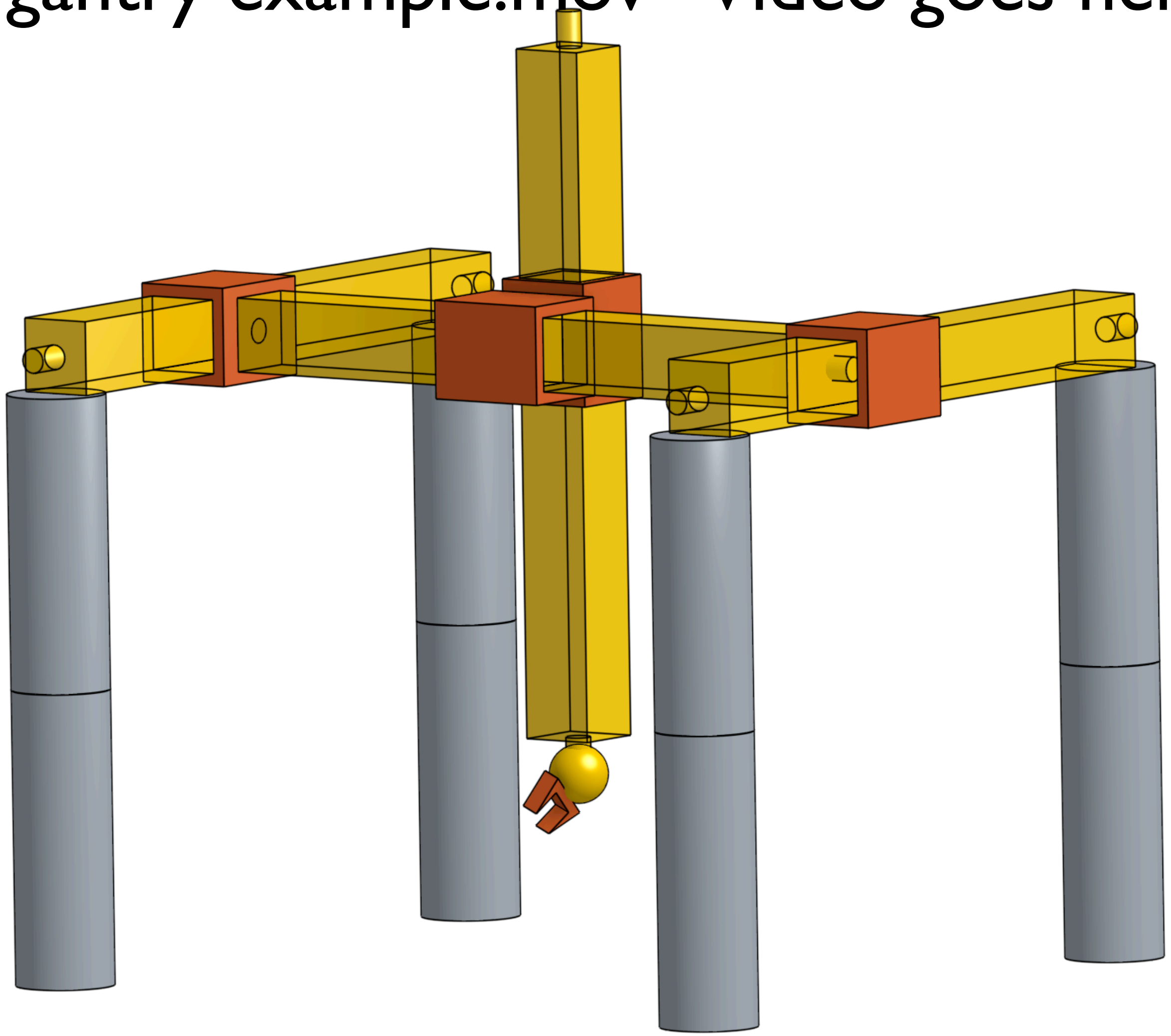


Kinematic Decoupling

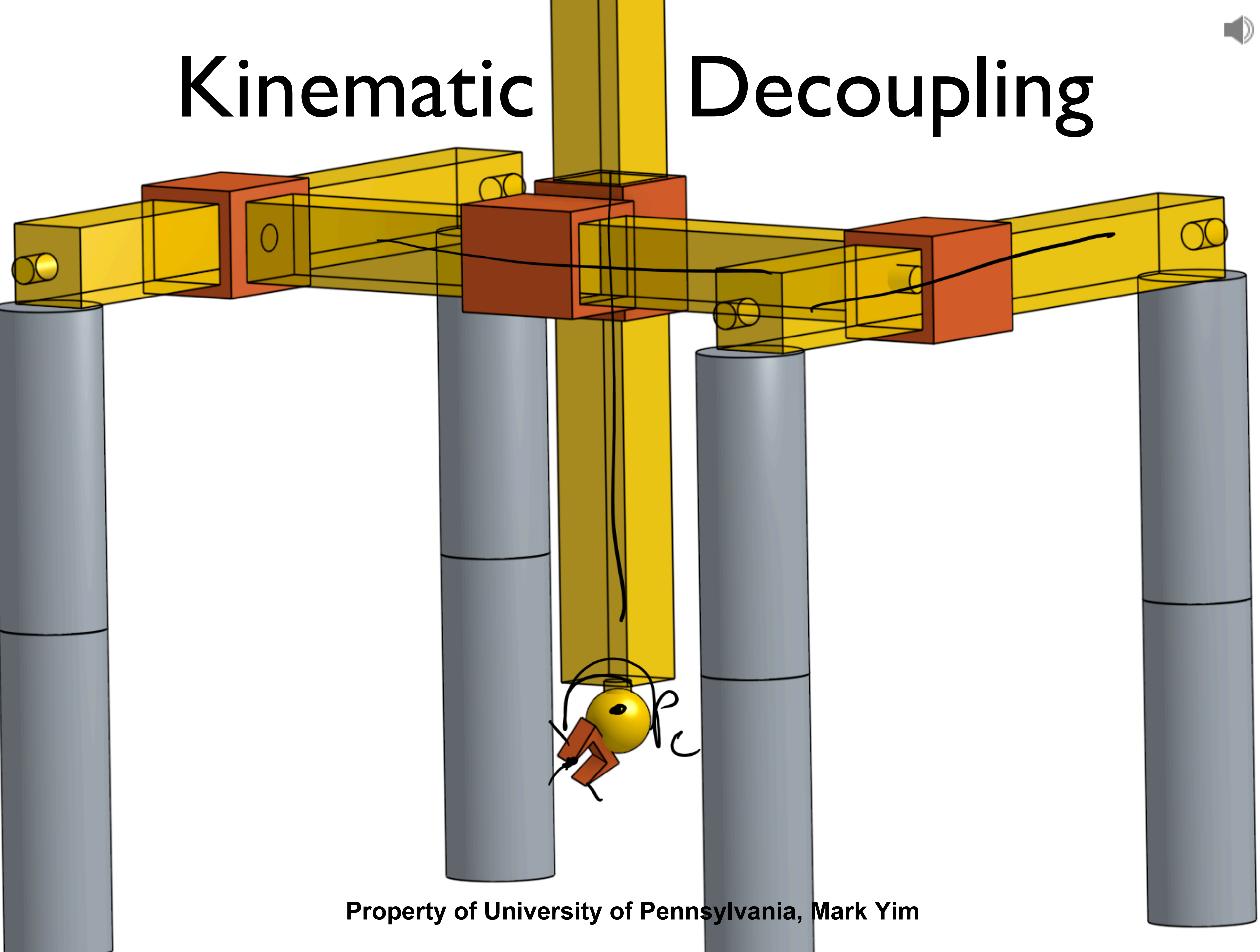


Kinematic Decoupling

- “gantry example.mov” Video goes here

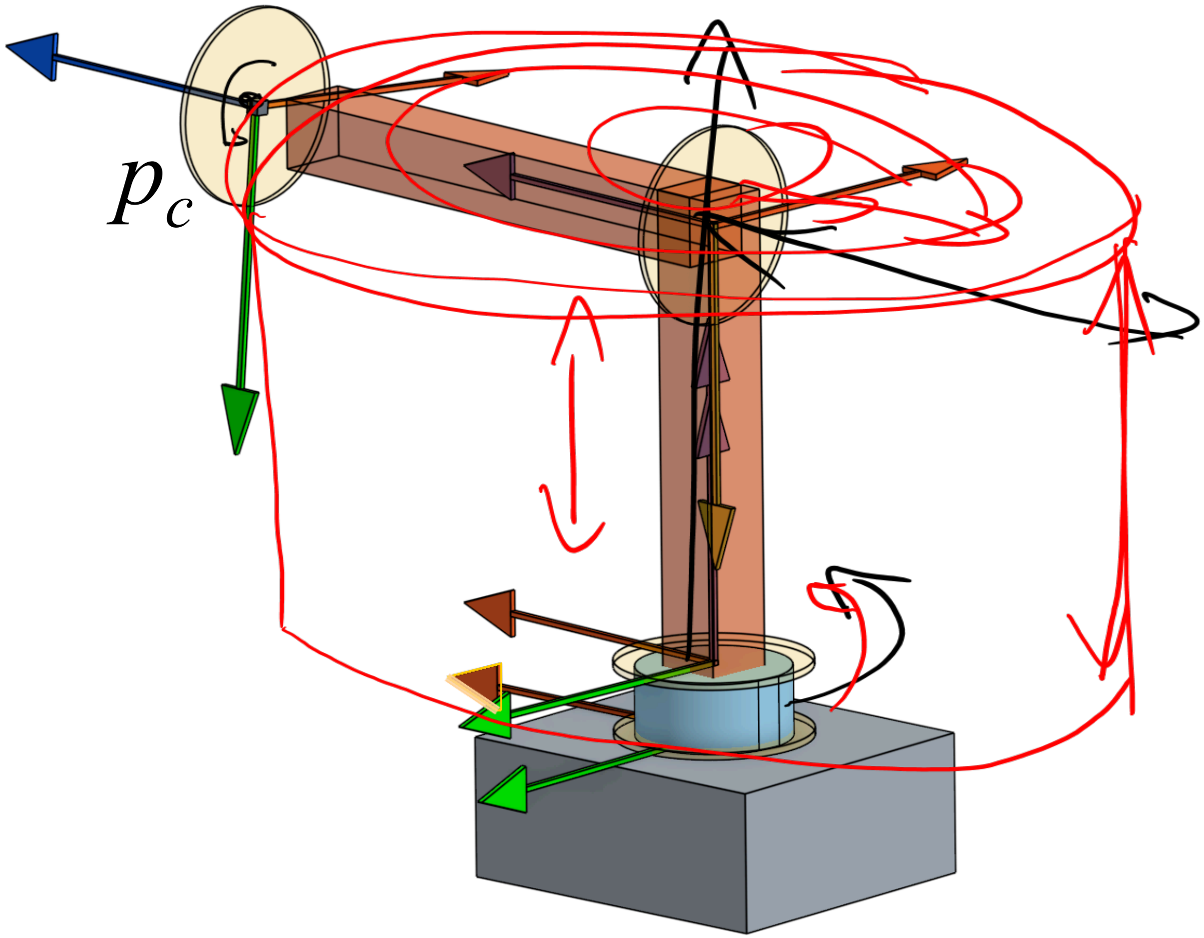


Kinematic Decoupling



Property of University of Pennsylvania, Mark Yim

Inverse Position: Cylindrical robot case

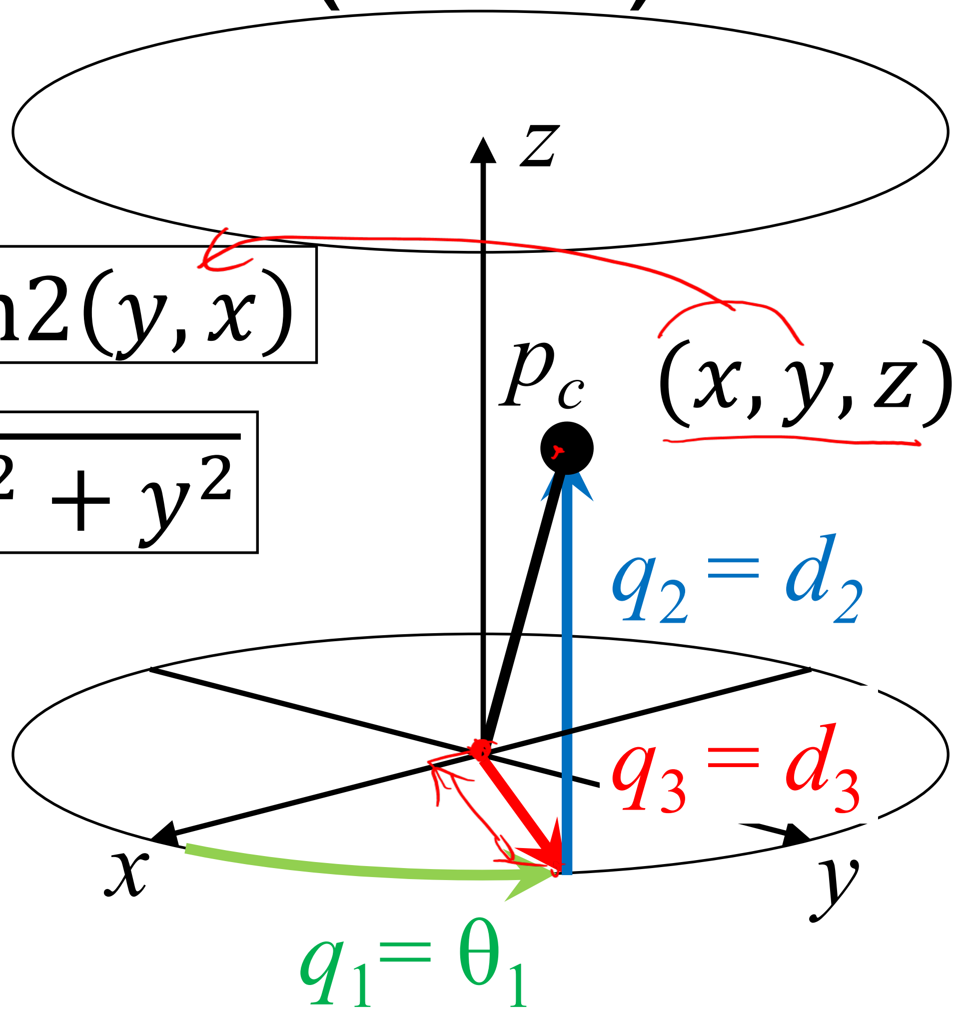


Cylindrical coordinates (RPP)

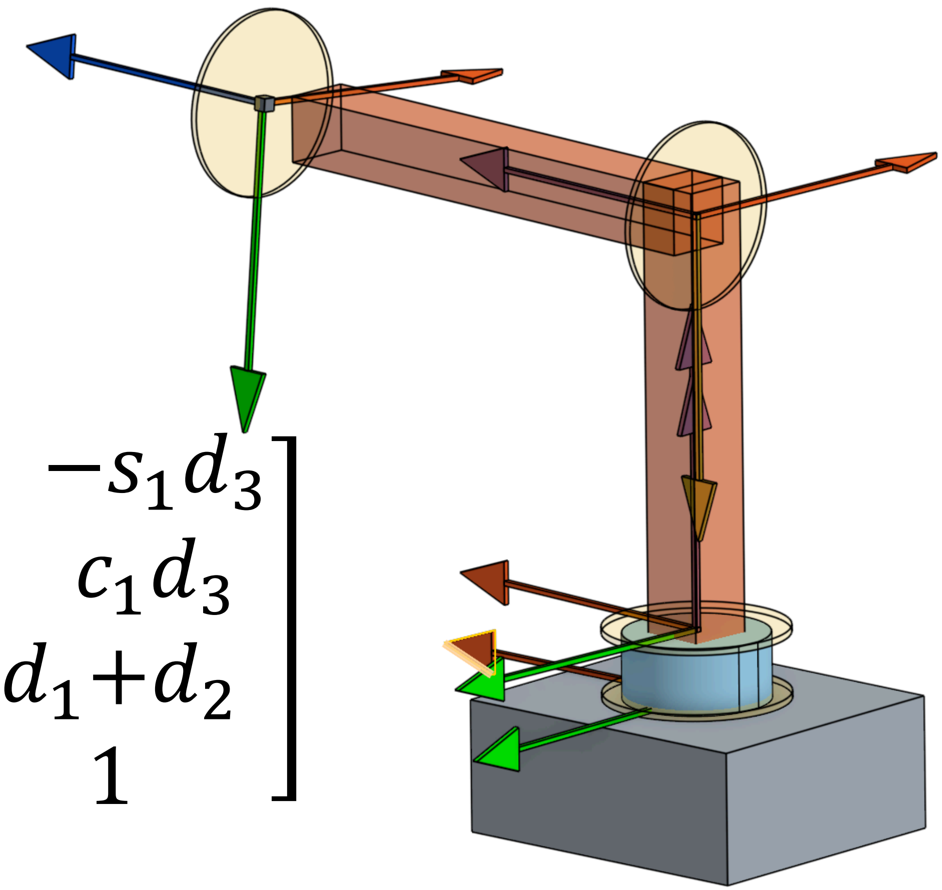
$$\boxed{\theta_1 = \operatorname{atan2}(y, x)}$$

$$\boxed{d_2 = \sqrt{x^2 + y^2}}$$

$$\boxed{d_3 = z}$$



Wrist Center Rotation



$$T_{03} = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1st part of SCARA (RRP)