Robotics: Fundamentals

Prof. Mark Yim
University of Pennsylvania

Week 5: Degrees of Freedom

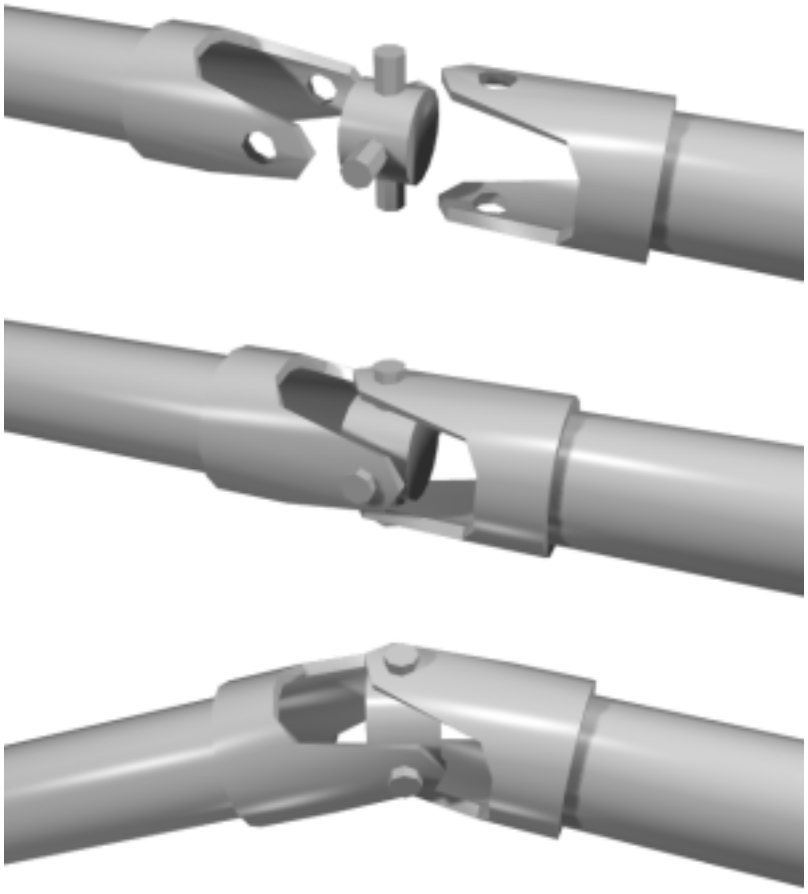
The Goal

- Understanding the position and orientation of robot links.
- Computing end-effector positions from joint angles. Computing joint angles from end effector positions.

Robotics: Fundamentals

Video 5.4
Mark Yim

Universal Joint Example



Degrees of Freedom

Grublers Criterion

$$F = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i$$

Where F = number of DOF

n = number of links

j = number of joints

λ = number of DOF in the space

f_i = number of DOF permitted by joint j_i

Degrees of Freedom

Constraint formulation

$$F = \lambda(n-1) - \sum_{i=1}^{j} C_i$$

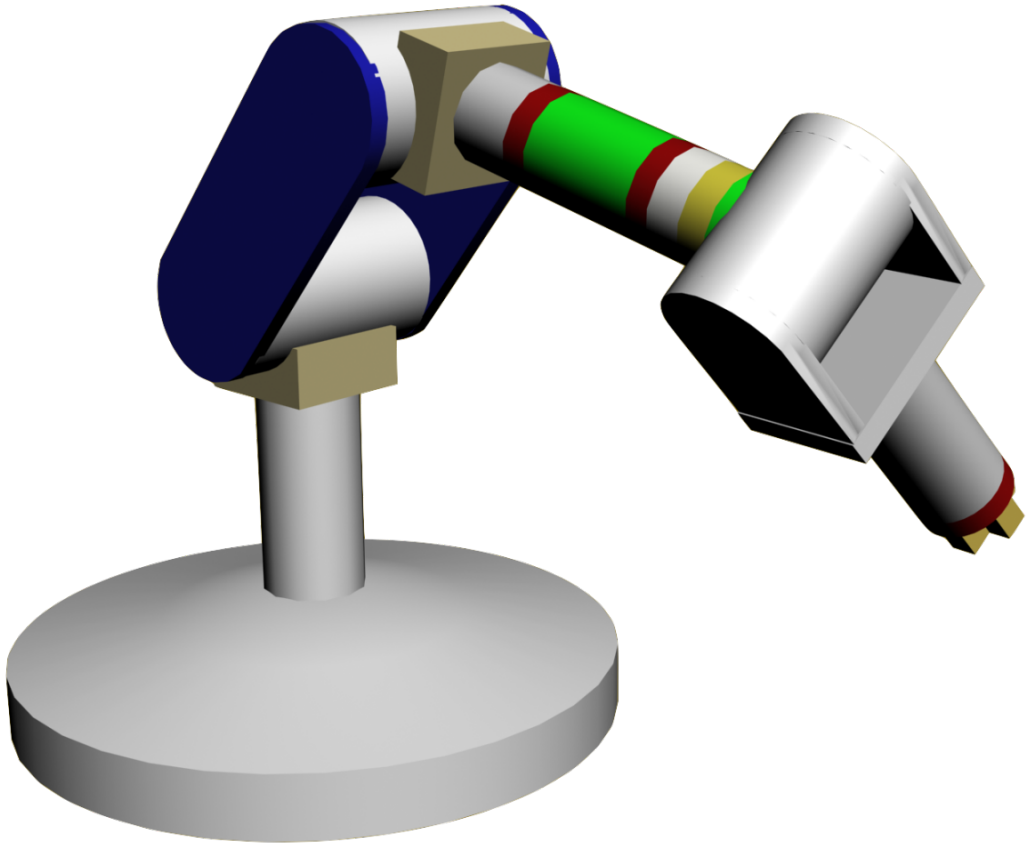
Where F = number of DOF

n = number of links

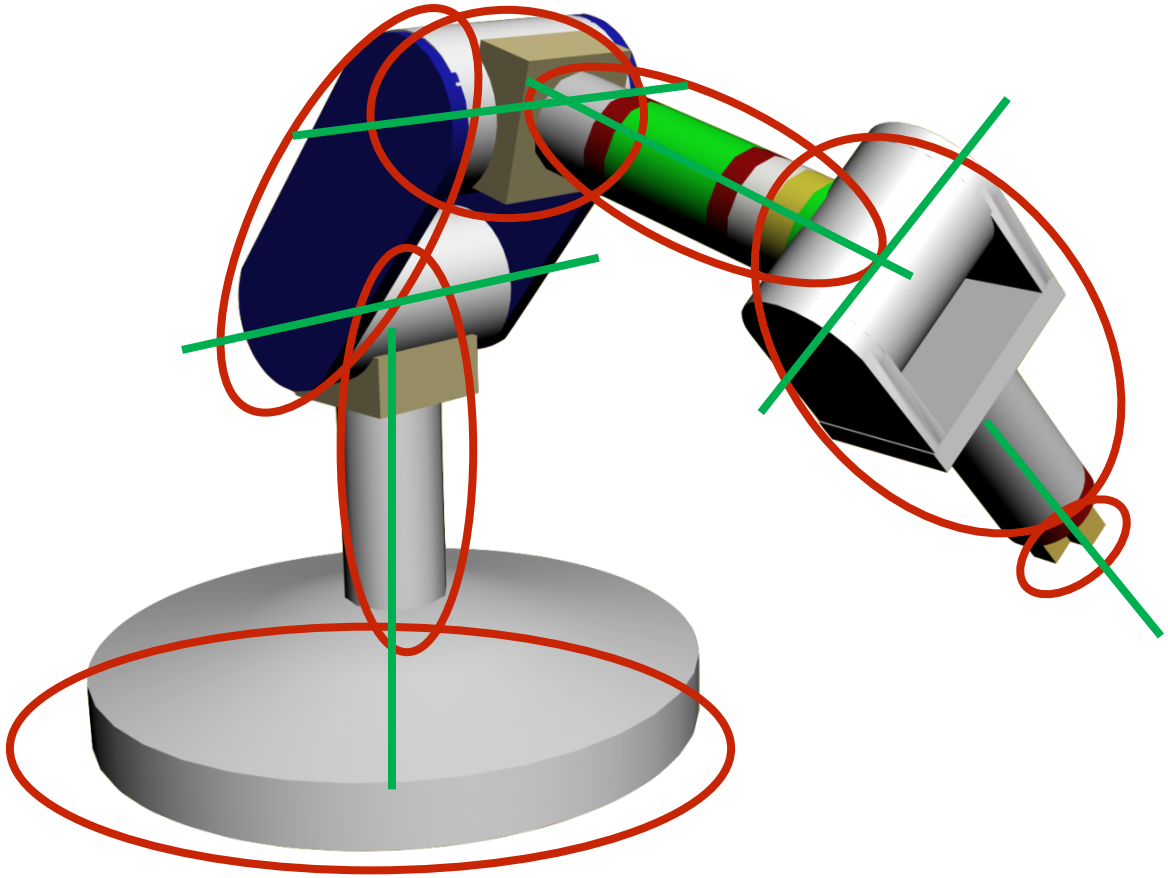
j = number of joints

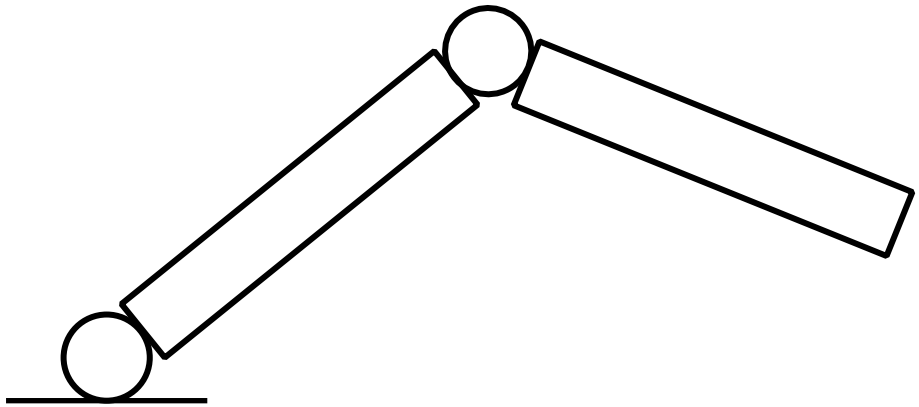
λ = number of DOF in the space

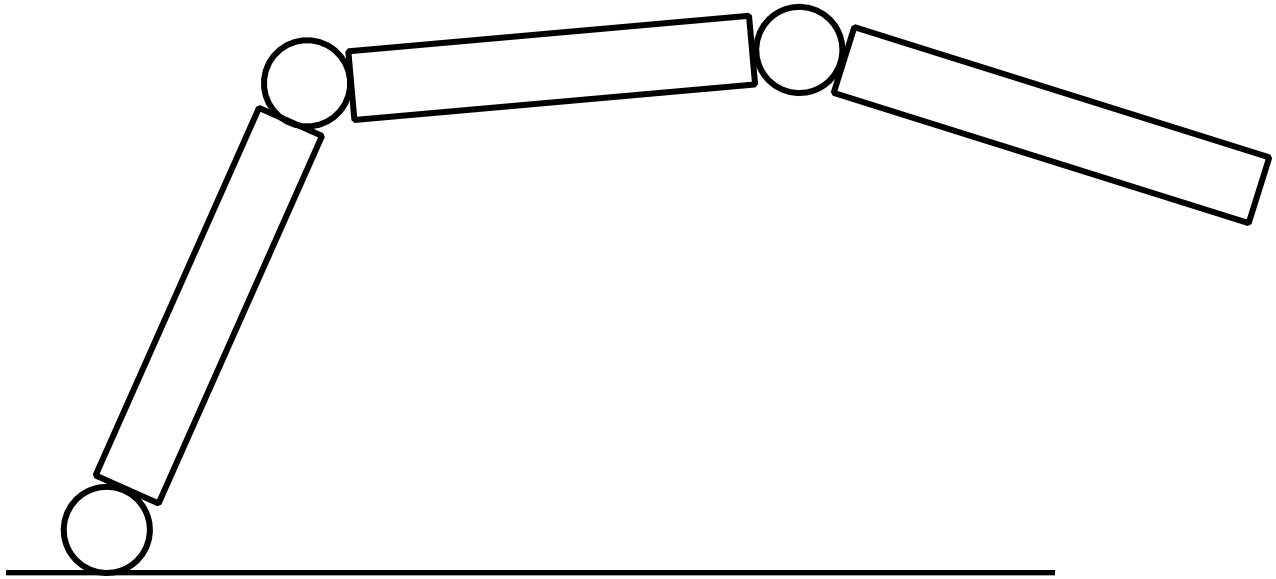
C_i= number of DOF constrained by joint j_i



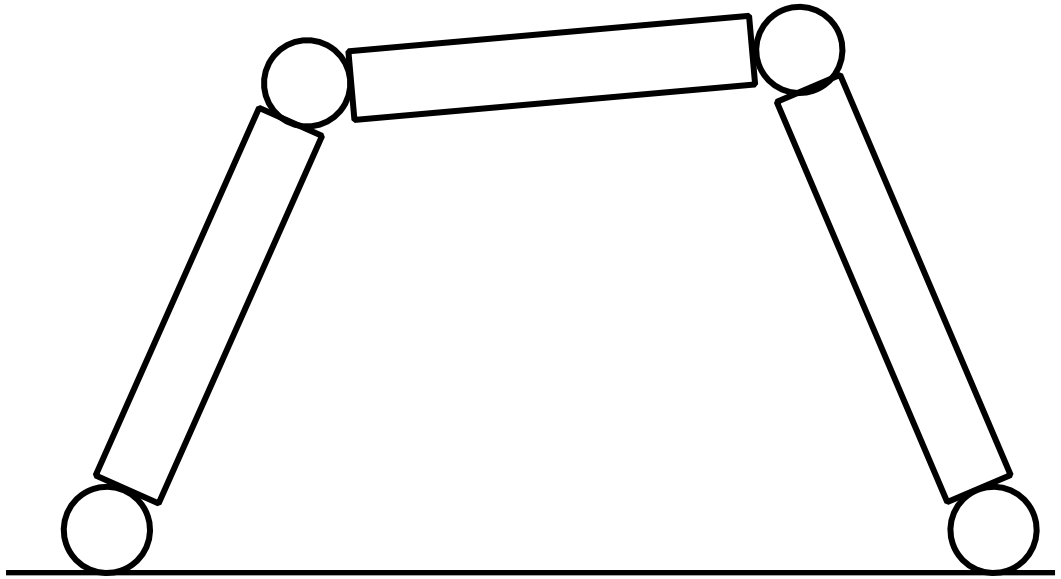
7 links: Base, spine, shoulder, shoulder twist, forearm, gripper
6 joints between the 7 links, all revolute
6(7-1) – 6(5) = 6 degrees of freedom



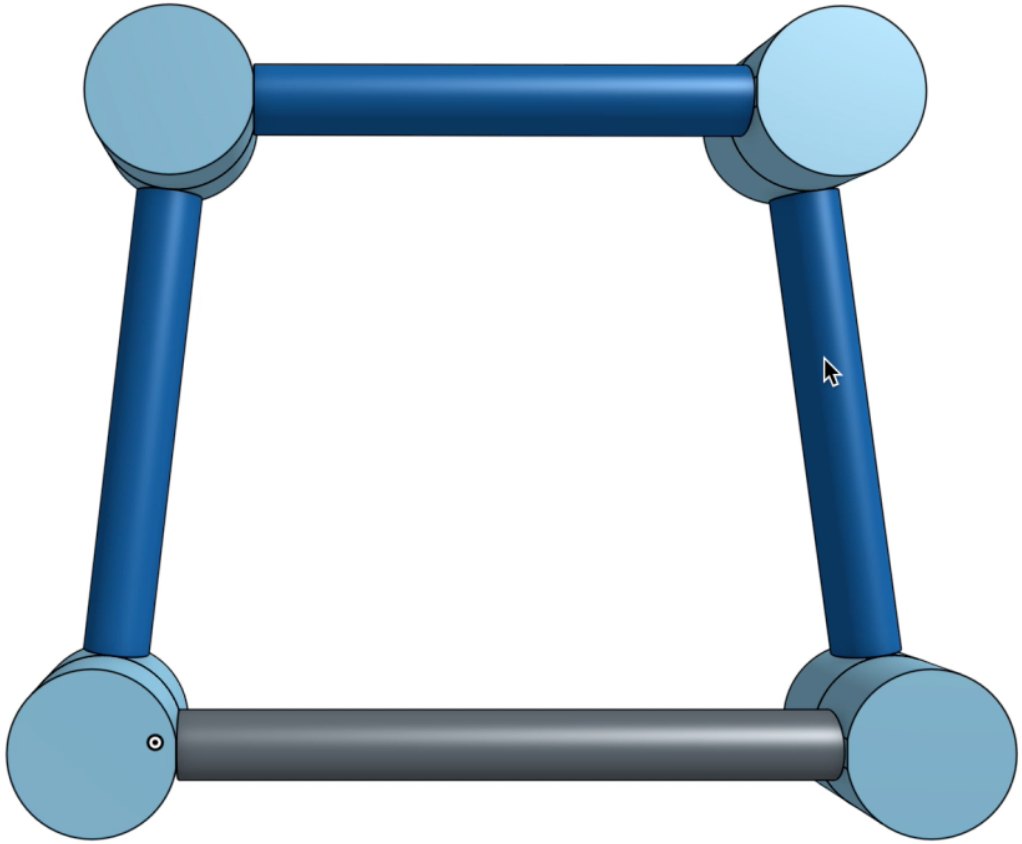


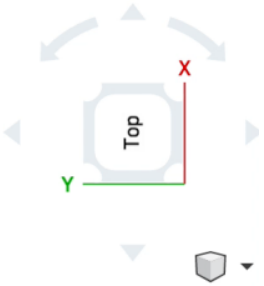


4-bar linkage

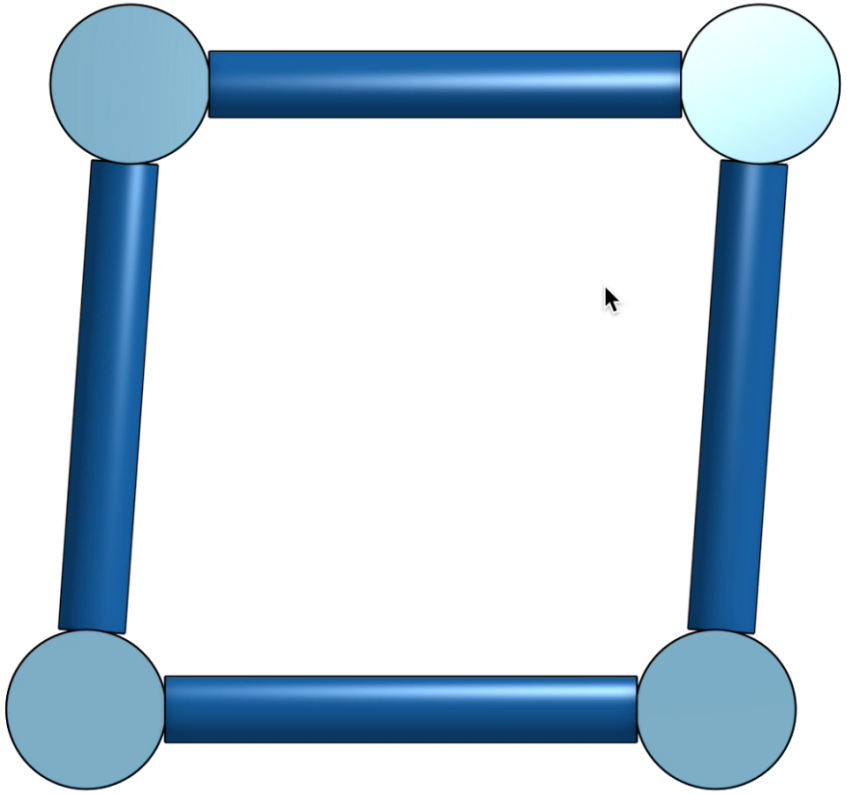


Four-bar linkage

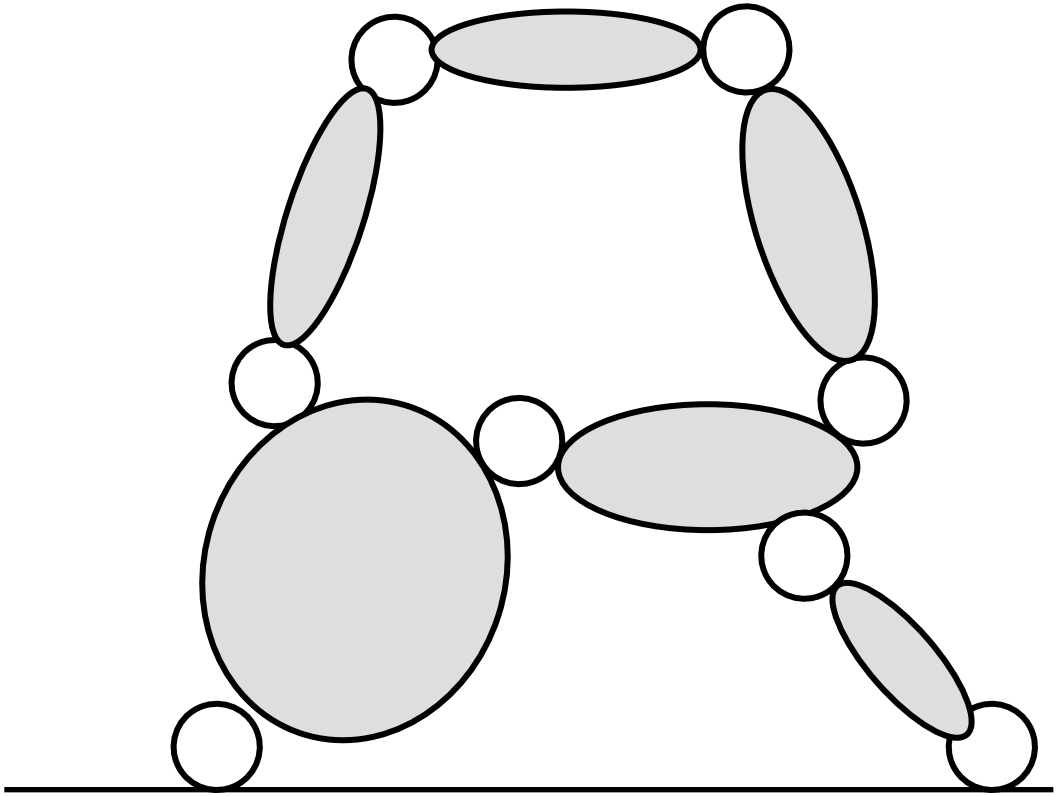




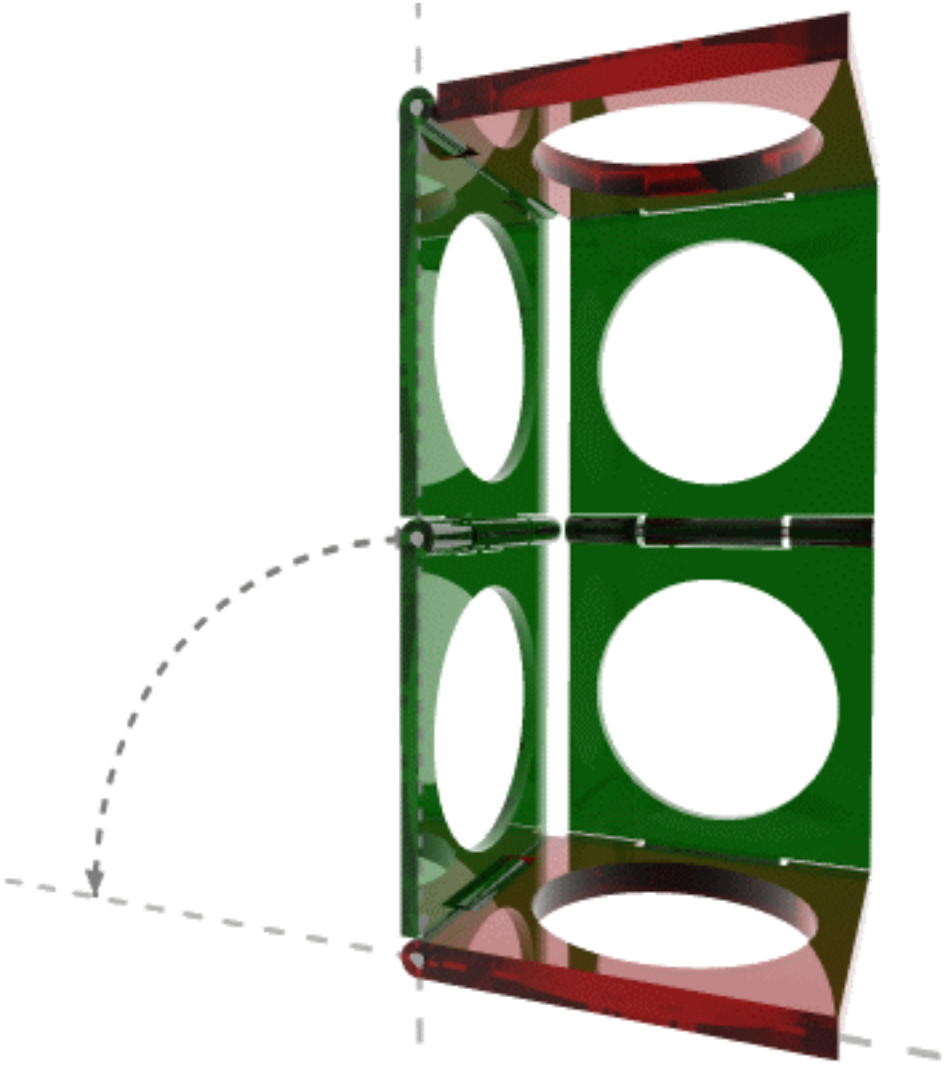
Parallel Four-bar



More complex linkage



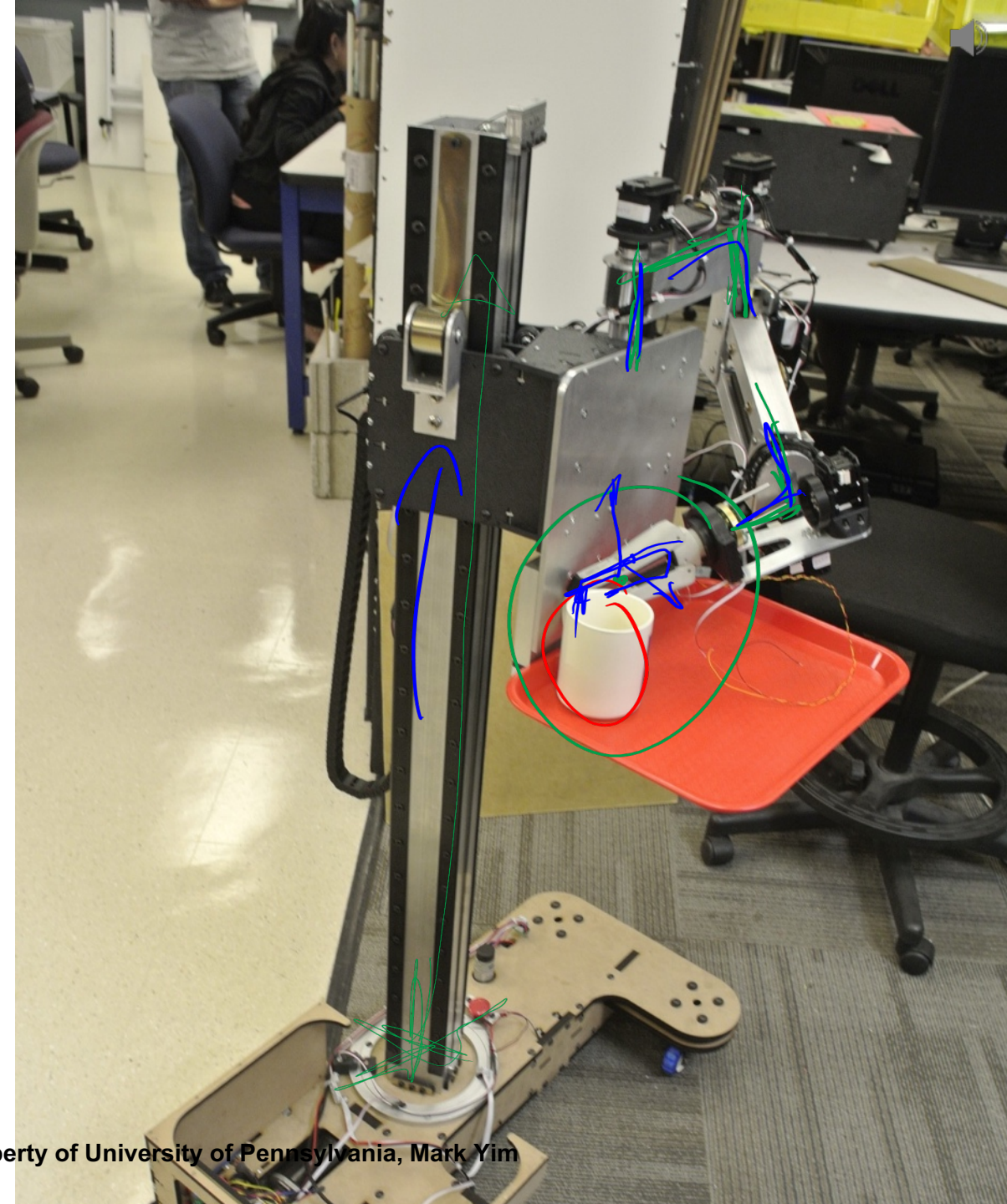
Pathological Exceptions



Robotics: Fundamentals

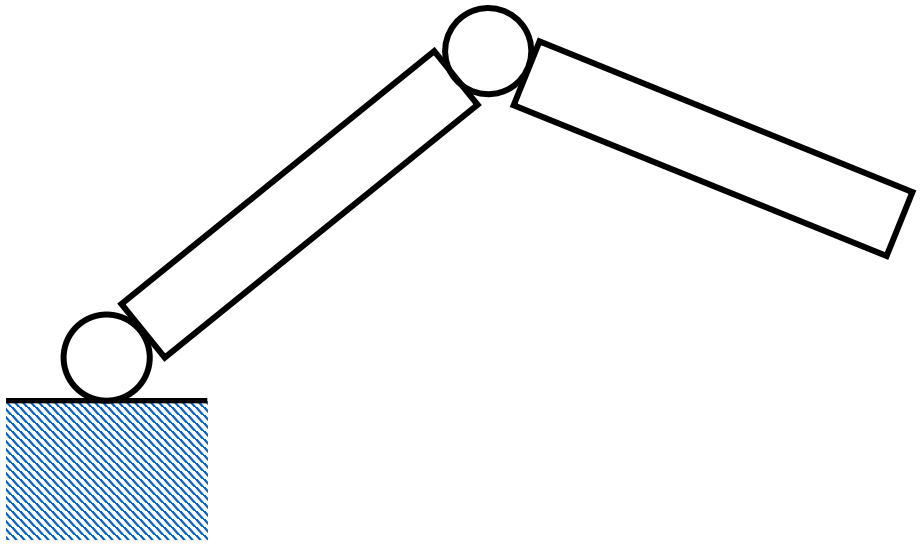
Video 5.5
Mark Yim

Forward Kinematics

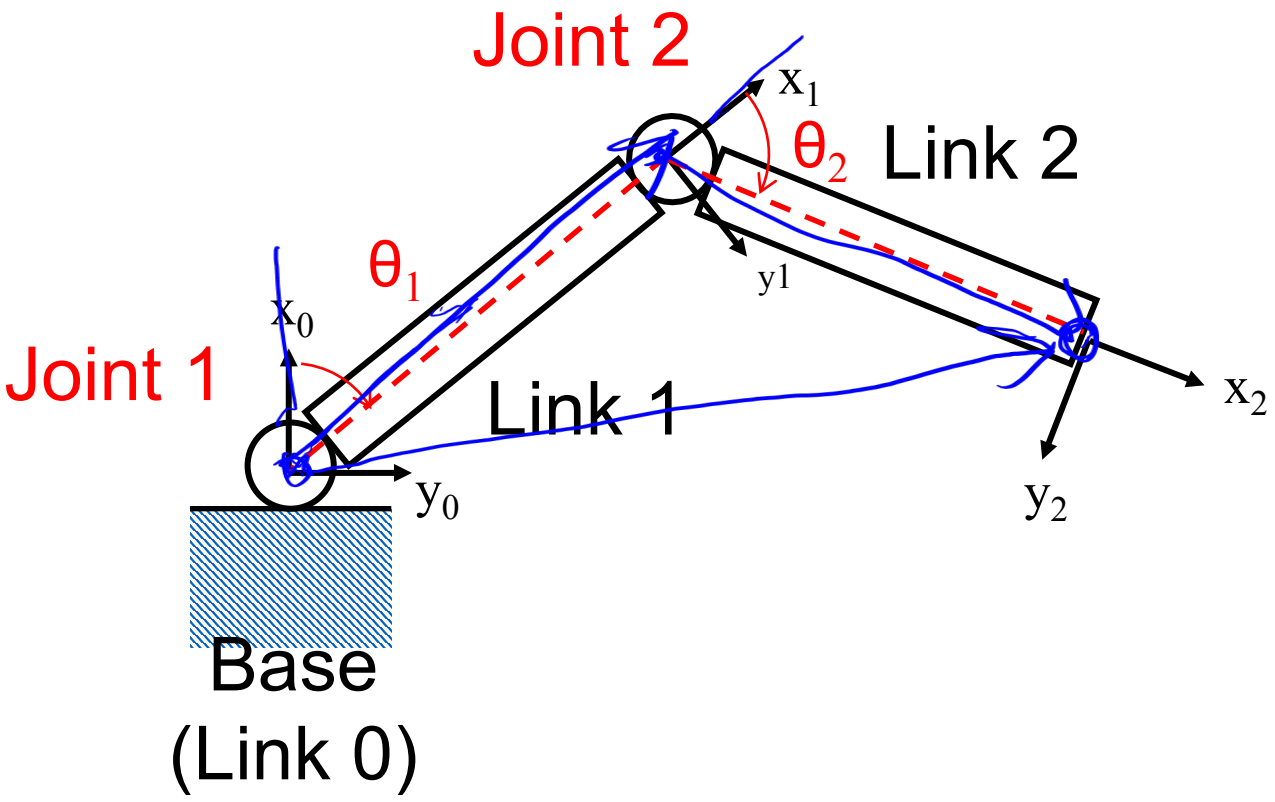


Penn Engineering

Labeling Conventions

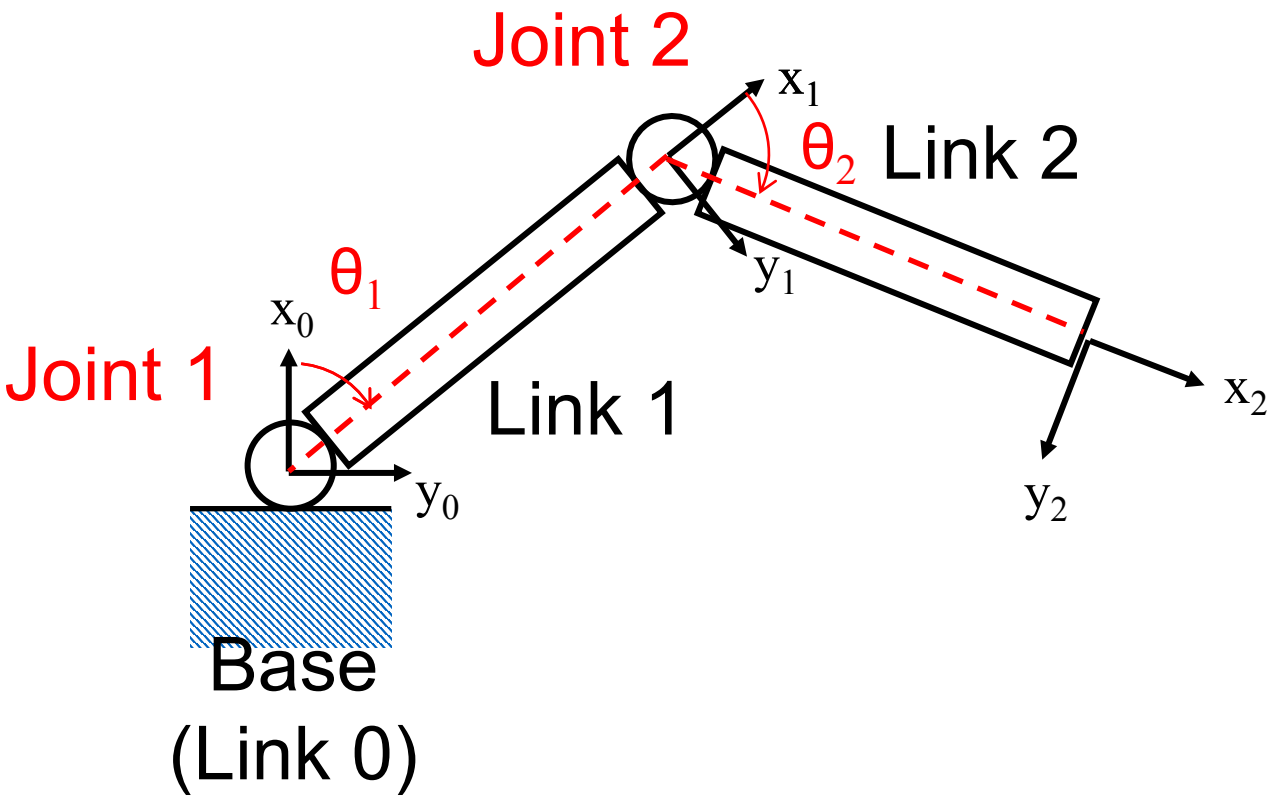


Labeling Conventions



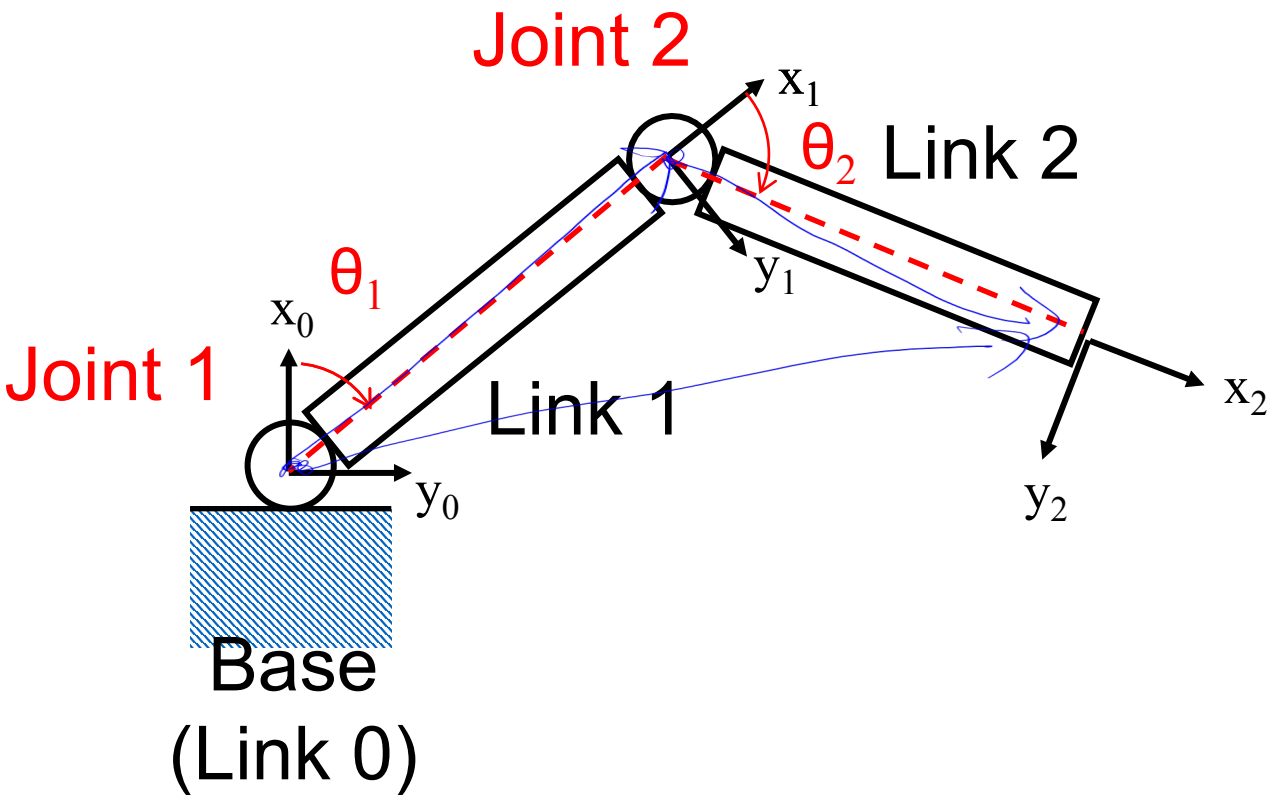
Planar Forward Kinematics

$$A_1 = \begin{bmatrix} R_{01} & \boldsymbol{d}_{01} \\ 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} R_{12} & \boldsymbol{d}_{12} \\ 0 & 1 \end{bmatrix}$$

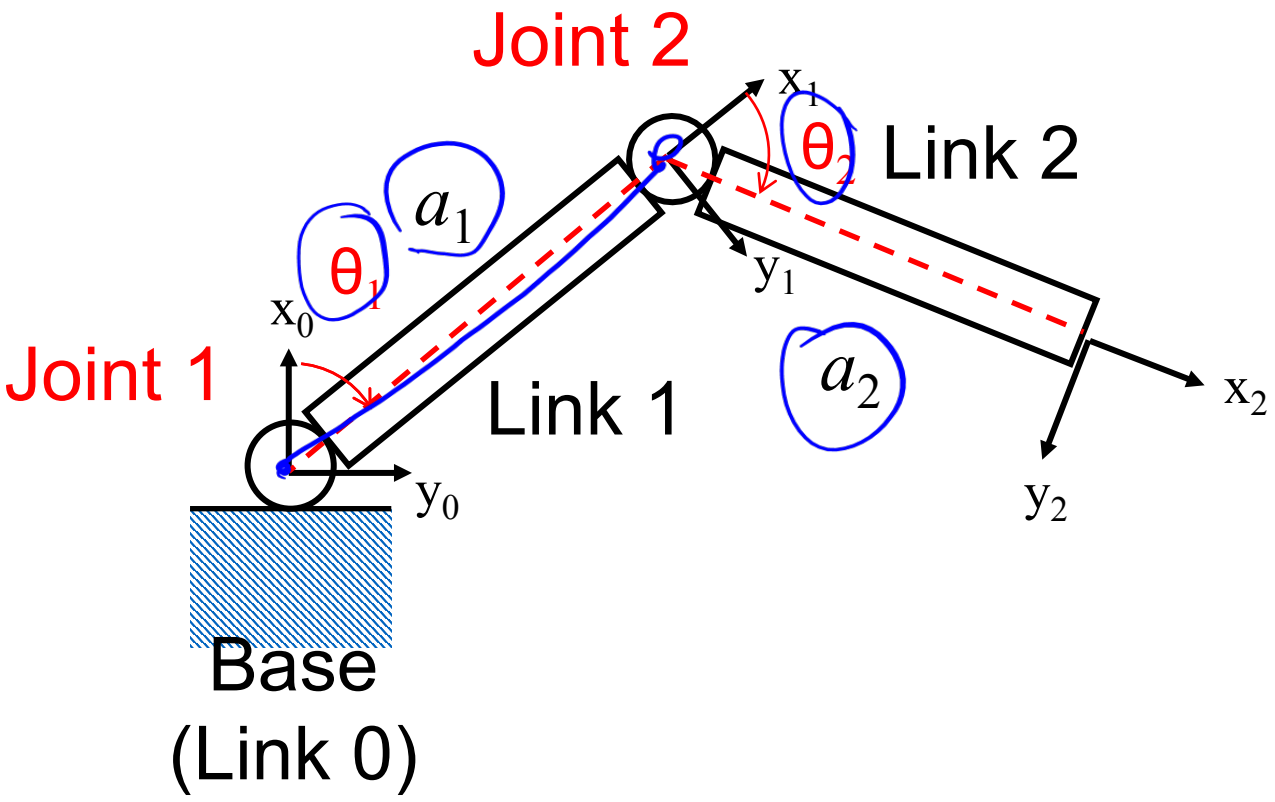


Planar Forward Kinematics

$$A_1 = \begin{bmatrix} R_{01} & \boldsymbol{d}_{01} \\ 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} R_{12} & \boldsymbol{d}_{12} \\ 0 & 1 \end{bmatrix}$$



Planar Forward Kinematics

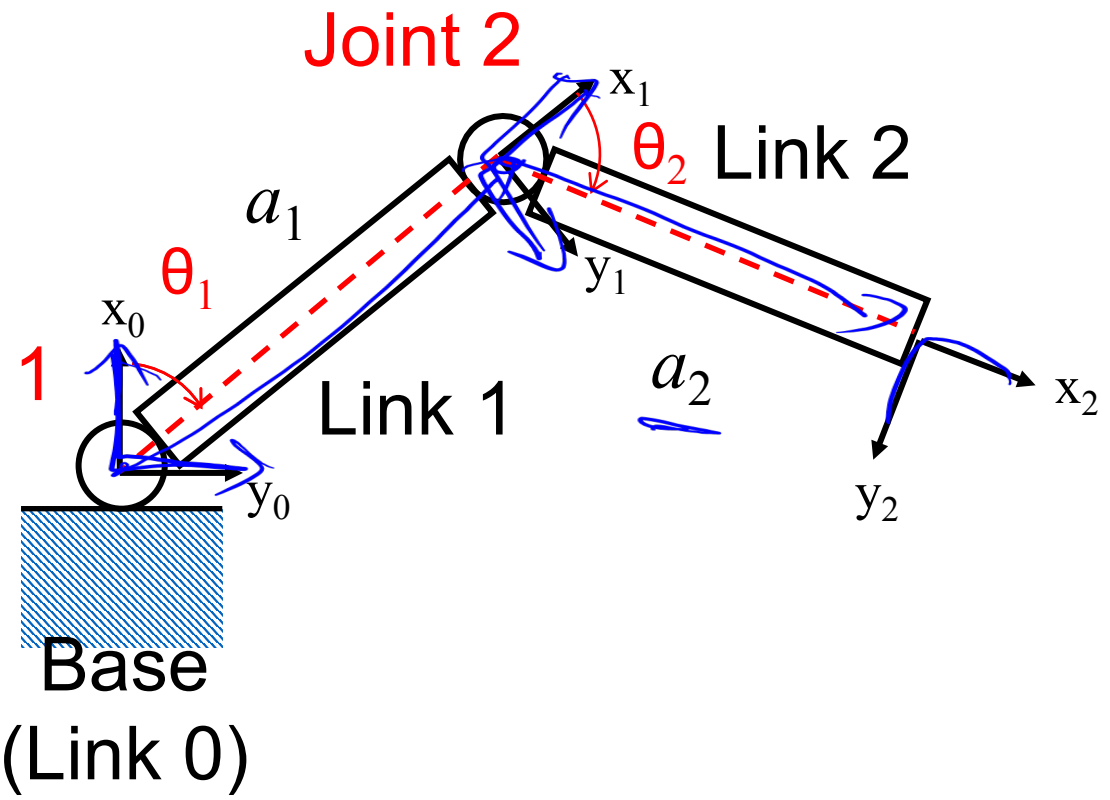


Planar Forward Kinematics

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

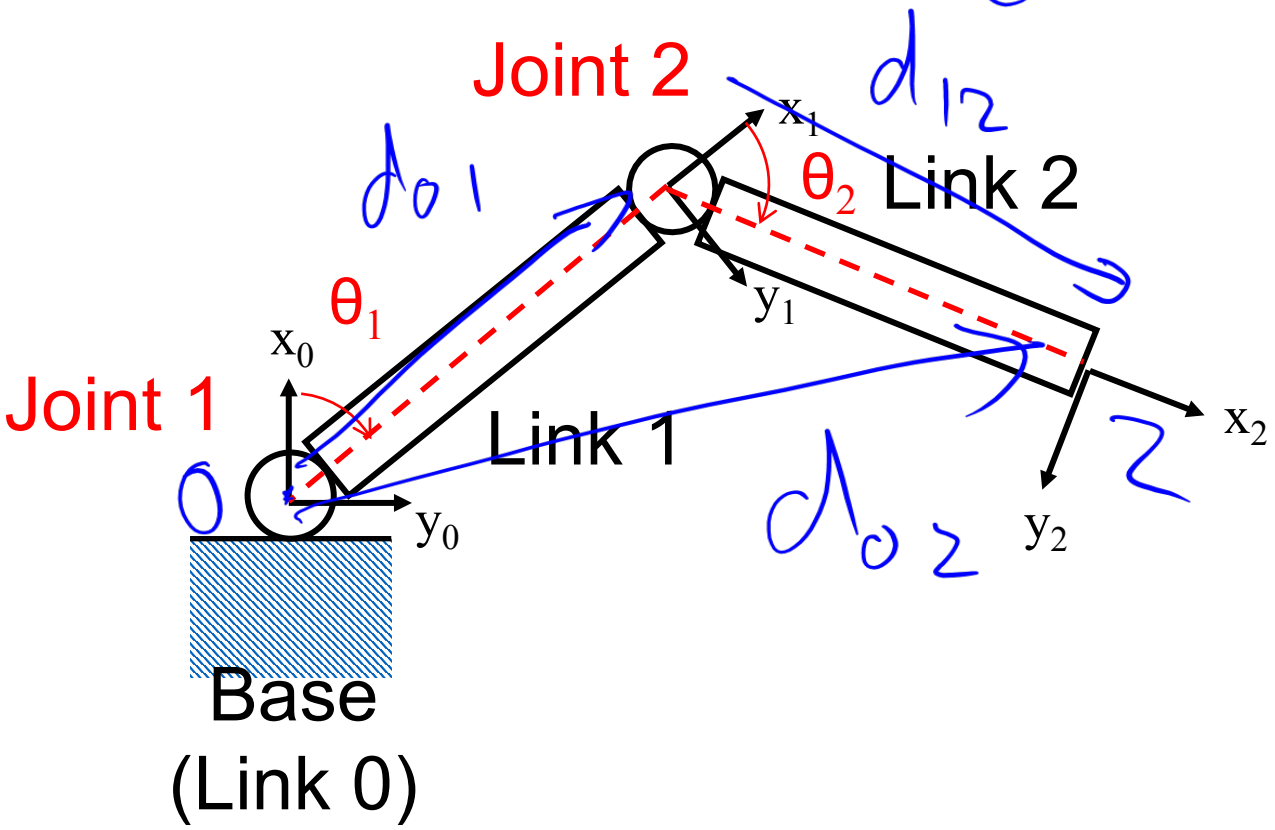
$c_1 = \cos(\theta_1)$

$s_1 = \sin(\theta_1)$

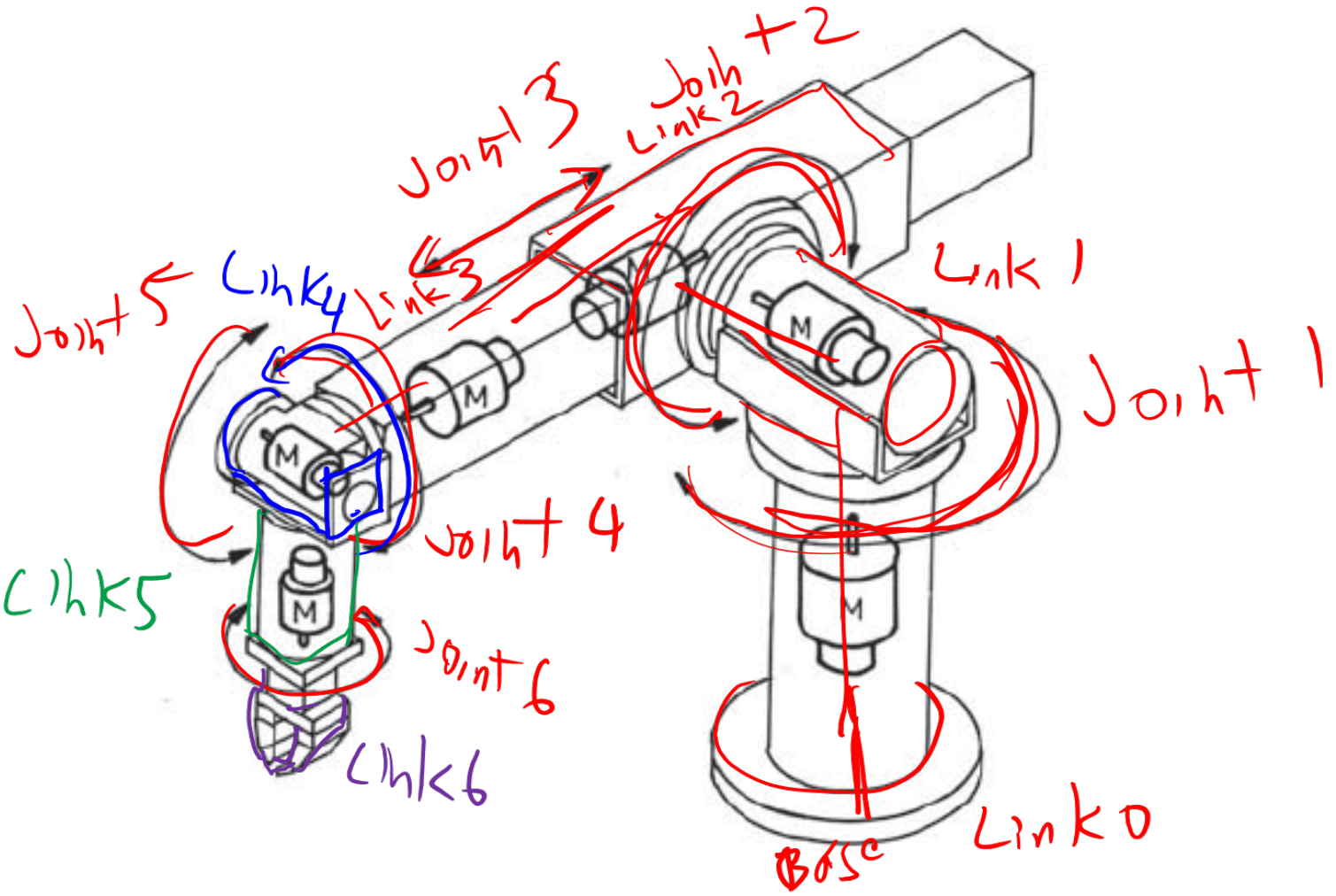


Planar Forward Kinematics

$$T_{02} = A_1 A_2 = \begin{bmatrix} R_{02} & \boldsymbol{d}_{02} \\ 0 & 1 \end{bmatrix}$$



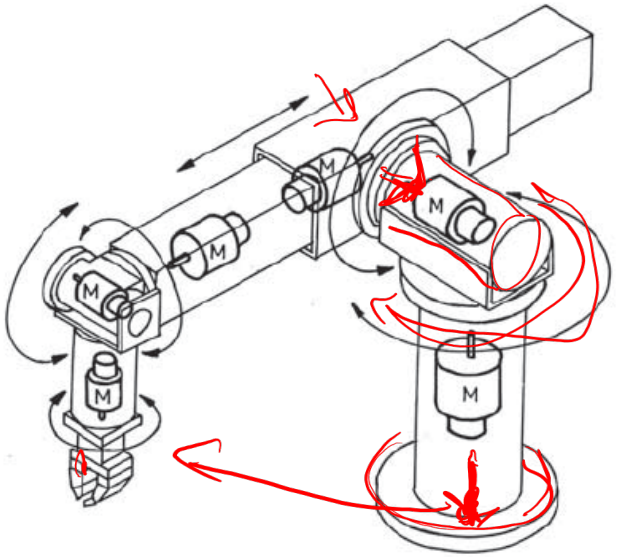
3D 6DOF Links/Joints



RRPRRR → 2RP3R

3D 6DOF Transformation

$$T_{06} = A_1 A_2 A_3 A_4 A_5 A_6 = \begin{bmatrix} R_{06} & \boldsymbol{d}_{06} \\ 0 & 1 \end{bmatrix}$$



Robotics: Fundamentals

Video 5.6a
Mark Yim

Denavit-Hartenberg Convention

4 parameters for each link/joint i

- a_i is **link length** of link i
- α_i is **link twist** of link i
- d_i is the **link/joint offset** of link/joint i
- θ_i is the **joint angle** of joint i

Fixed

Joint variable q_i

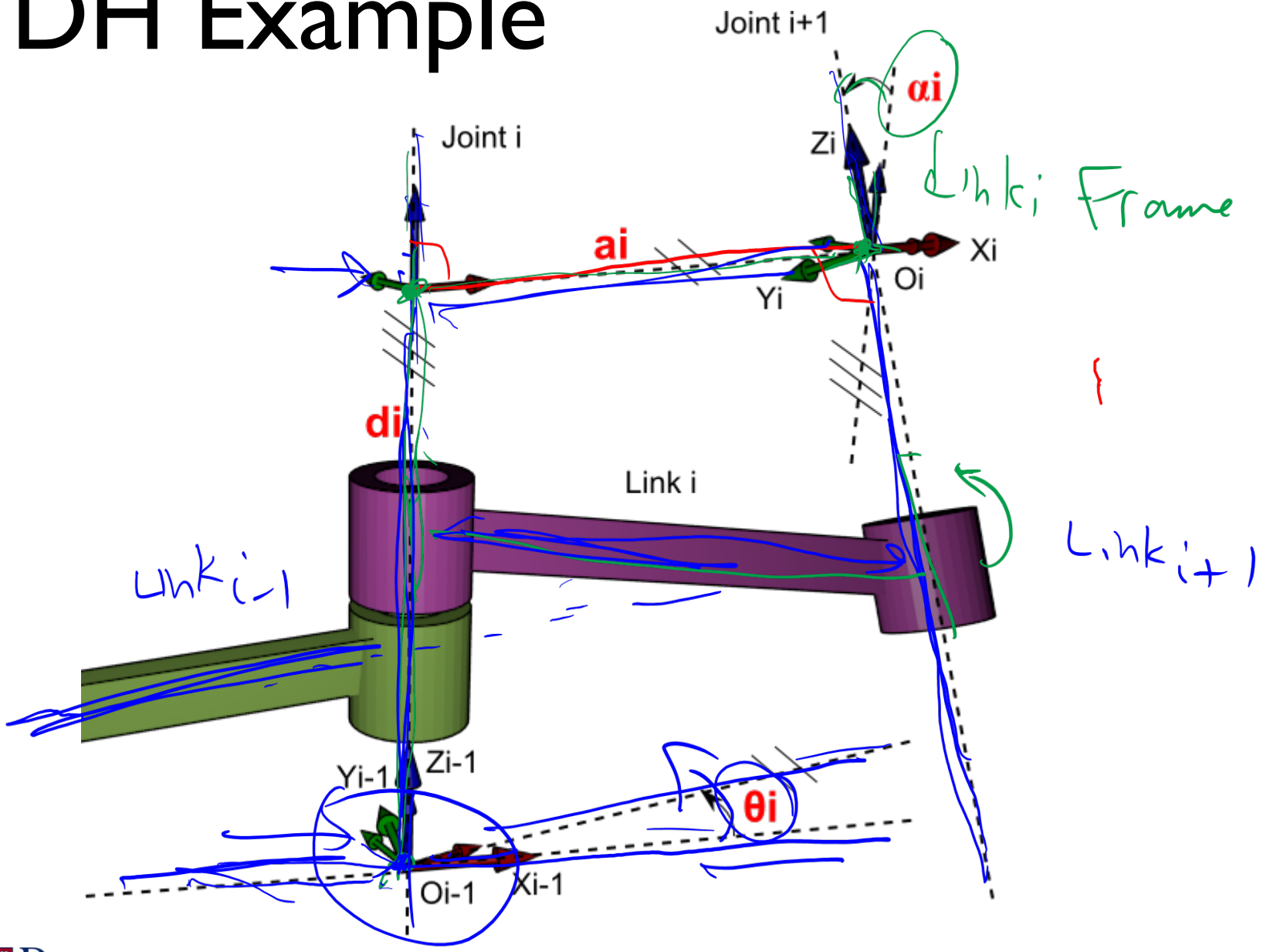
Prismatic / Revolute Variables

Denavit-Hartenberg Convention

4 parameters for each link/joint i

- a_i is **link length** of link i
- α_i is **link twist** of link i
- d_i is the **link/joint offset** of link/joint i
- θ_i is the **joint angle** of joint i

DH Example



DH Link Transformation

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha}$$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha & 0 \\ 0 & s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

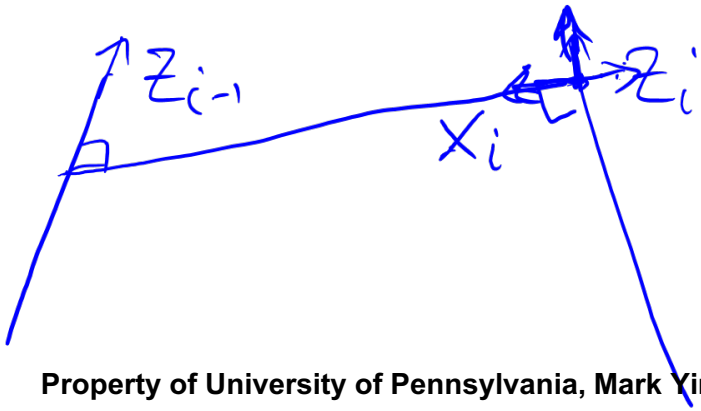
DH Frame Placement Rules

[DH1] The axis Z_{i-1} is the joint axis for joint i

- Axis of revolution for revolute joint
- Axis of translation for prismatic

[DH2] The axis X_i is perpendicular to the axis Z_{i-1}

[DH3] The axis X_i intersects the axis Z_{i-1}



DH Parameters

- a_i is distance between Z_i and Z_{i-1} along X_i
- α_i is the angle between Z_i and Z_{i-1} about X_i
- d_i is distance between X_i and X_{i-1} along Z_{i-1}
- θ_i is the angle between X_i and X_{i-1} about Z_{i-1}

DH Process

1. Label z_i axes
2. Set base frame and end effector frame x_0 and y_0 as arbitrary
3. For $i=1, \ldots n-1$,
   A. Find common normal between z_i and z_{i-1} (z_i and z_{i-1} parallel is a special case)
   B. Establish x_i on this normal
   C. Establish y_i perpendicular to x_i and z_i to form a right handed coordinate frame
4. Create a table of all link parameters a_i d_i α_i θ_i
5. Form homogeneous transformation A_i for each link
6. Form $T_{0n} = A_1 \cdots A_n$

Special Case

- If z_i and z_{i-1} are parallel:
- Choose any d. Other parameters are the same as before