Robotics: Fundamentals

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Week 7: Manipulator Jacobian

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Robotics: Fundamentals

Video 7.1

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Jacobian Matrices

- Definition
- If $\mathbf{p}$ is a point in $\mathbb{R}^n$ and $\mathbf{f}$ is differentiable at $\mathbf{p}$, then its derivative is given by $\mathbf{J_f}(\mathbf{p})$. In this case, the linear map described by $\mathbf{J_f}(\mathbf{p})$ is the best linear approximation of $\mathbf{f}$ near the point $\mathbf{p}$
- Linear approximation at the point of interest

Kinematics

Joint space

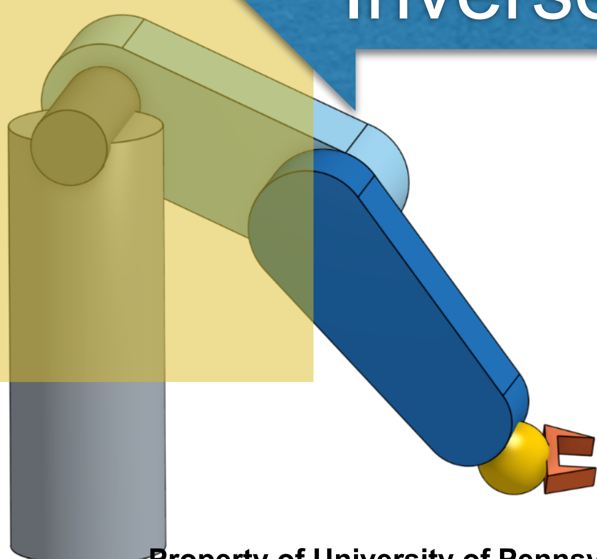
$$\boldsymbol{q} = [q_1, q_2, \cdots q_n]^T$$

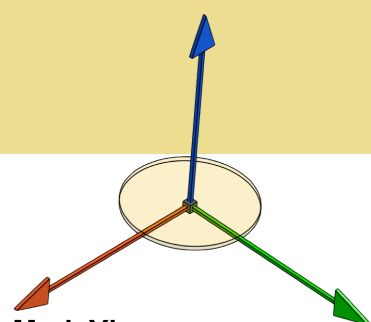
Forward

Inverse

Cartesian space

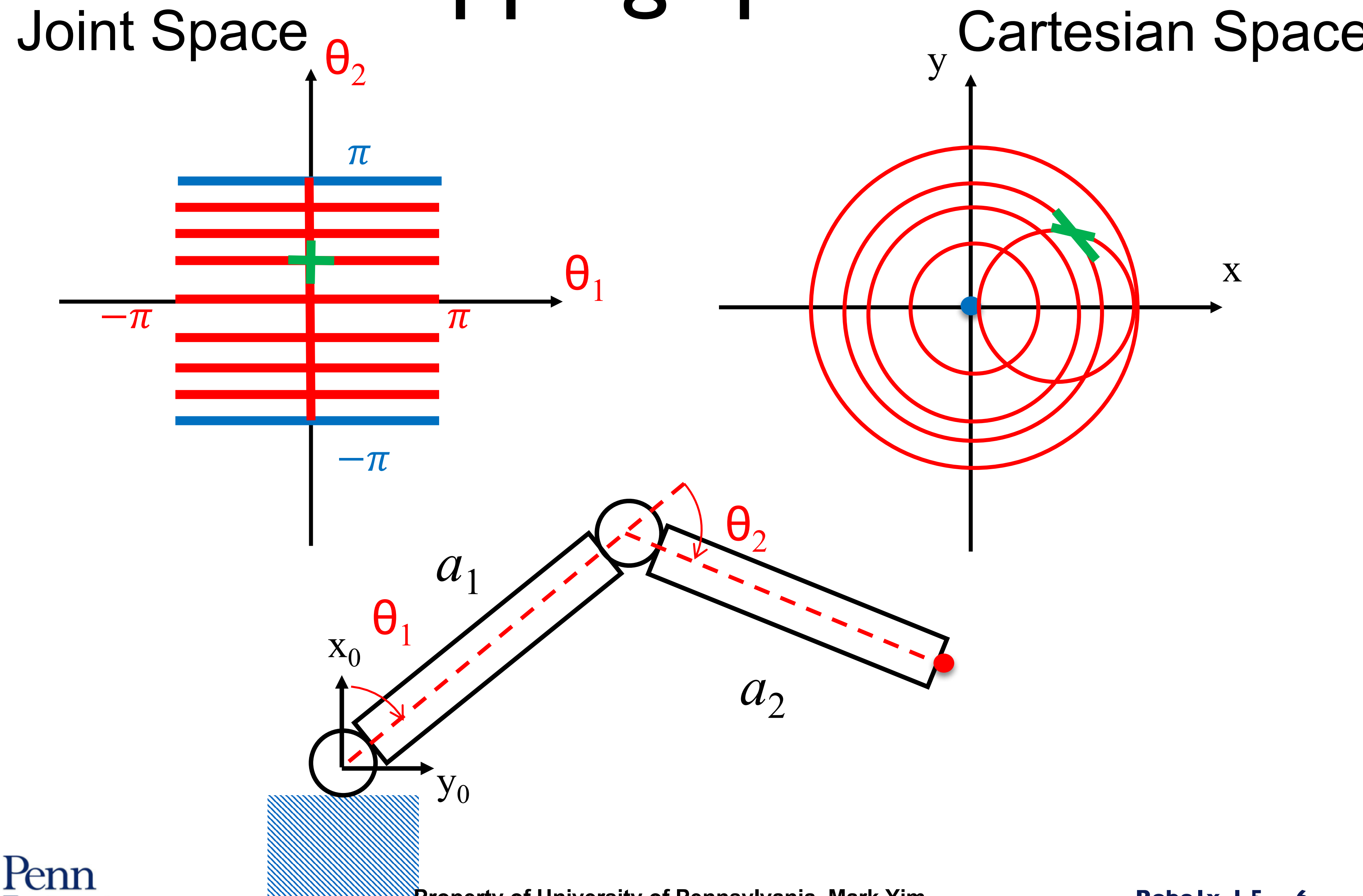
$$\boldsymbol{H} = \begin{bmatrix} [\boldsymbol{R}] & [\boldsymbol{P}] \\ [\boldsymbol{0}] & 1 \end{bmatrix}$$





Insert smallmotion.mov

Mapping spaces



Manipulator Jacobian

$$\dot{x} = f_{x1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{xn}(\boldsymbol{q})\dot{q}_n$$

$$\dot{y} = f_{y1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{yn}(\boldsymbol{q})\dot{q}_n$$

$$\dot{z} = f_{z1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{zn}(\boldsymbol{q})\dot{q}_n$$

$$\dot{\phi} = f_{\phi 1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{\phi n}(\boldsymbol{q})\dot{q}_n$$

$$\dot{\theta} = f_{\theta 1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{\theta n}(\boldsymbol{q})\dot{q}_n$$

$$\dot{\psi} = f_{\psi 1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{\psi n}(\boldsymbol{q})\dot{q}_n$$

$$\xi = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \mathbf{J}(\mathbf{q})\,\dot{\boldsymbol{q}}$$

Jacobian Matrix

$$\mathbf{f}\colon \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$\mathbf{q} \in \mathbb{R}^n \qquad \mathbf{f}(\mathbf{q}) \in \mathbb{R}^m$$

$$\mathbf{J} = \left[\frac{\partial \mathbf{f}}{\partial q_1} \quad \cdots \quad \frac{\delta \mathbf{f}}{\partial q_n}\right] = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \cdots & \frac{\partial f_1}{\partial q_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial q_1} & \cdots & \frac{\partial f_m}{\partial q_n} \end{bmatrix}$$

$$\mathbf{J}_{ij} = \frac{\partial f_i}{\partial q_j}$$

Manipulator Jacobian

$$\left.\begin{aligned}
\dot{x} &= f_{x1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{xn}(\boldsymbol{q})\dot{q}_n \\
\dot{y} &= f_{y1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{yn}(\boldsymbol{q})\dot{q}_n \\
\dot{z} &= f_{z1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{zn}(\boldsymbol{q})\dot{q}_n
\end{aligned}\right\} \boldsymbol{v} = \mathbf{J}_v\, \dot{\boldsymbol{q}}$$

$$\left.\begin{aligned}
\dot{\phi} &= f_{\phi 1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{\phi n}(\boldsymbol{q})\dot{q}_n \\
\dot{\theta} &= f_{\theta 1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{\theta n}(\boldsymbol{q})\dot{q}_n \\
\dot{\psi} &= f_{\psi 1}(\boldsymbol{q})\dot{q}_1 + \cdots + f_{\psi n}(\boldsymbol{q})\dot{q}_n
\end{aligned}\right\} \boldsymbol{\omega} = \mathbf{J}_\omega\, \dot{\boldsymbol{q}}$$

$$\xi = \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} = \mathbf{J}(\mathbf{q})\, \dot{q}$$

Robotics: Fundamentals

Video 7.2

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Position Jacobian

$$x(t) = f\left(q(t)_{1,} q(t)_2 \dots q(t)_n\right)$$

$$\frac{\partial x}{\partial t} = \sum_{i=1}^{n} \frac{\partial x}{\partial q_i} \frac{\partial q_i}{\partial t}$$

$$\mathbf{J}_v = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \cdots & \frac{\partial y}{\partial q_n} \\ \frac{\partial z}{\partial q_1} & \cdots & \frac{\partial z}{\partial q_n} \end{bmatrix} \qquad \boldsymbol{v} = \mathbf{J}_v \, \dot{\boldsymbol{q}}$$

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Position Jacobian

$$\boldsymbol{T}_{\boldsymbol{0n}} = \boldsymbol{A}_0 \ldots \boldsymbol{A}_n = \begin{bmatrix} [\boldsymbol{R}] & [\boldsymbol{P}] \\ [\mathbf{0}] & 1 \end{bmatrix}$$

$$\mathbf{J}_v = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \cdots & \frac{\partial y}{\partial q_n} \\ \frac{\partial z}{\partial q_1} & \cdots & \frac{\partial z}{\partial q_n} \end{bmatrix}$$

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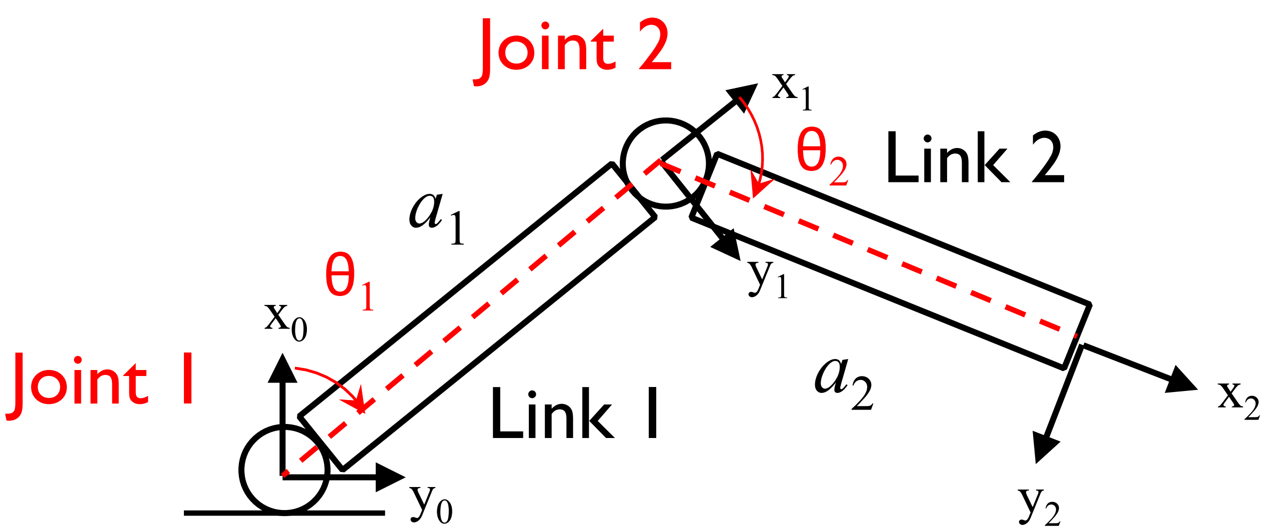
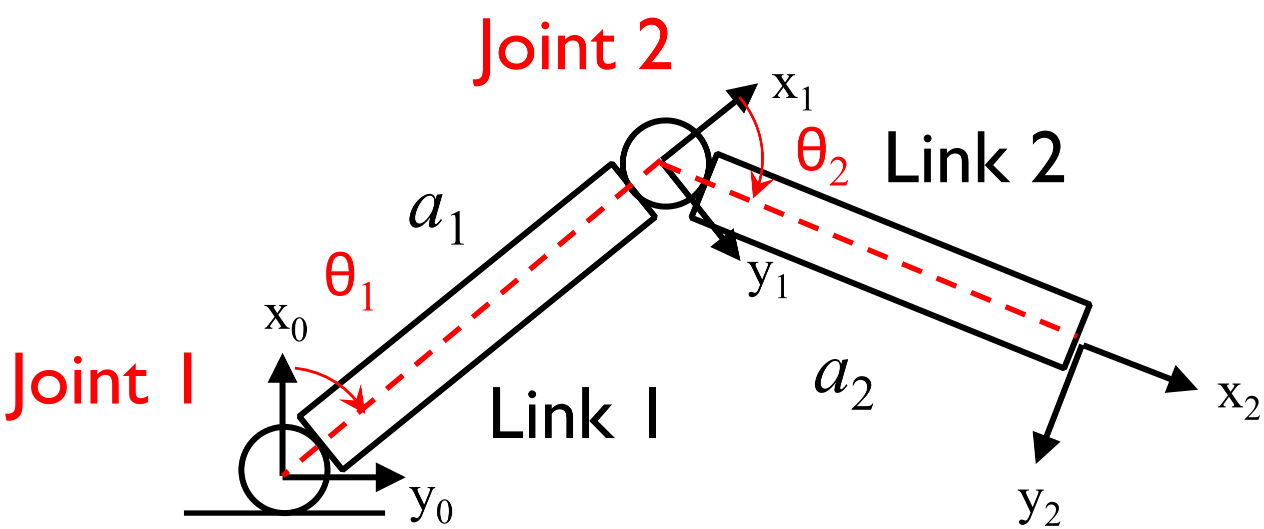
Position Jacobian

$$\boldsymbol{P} = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix} \qquad \boldsymbol{T_{0n}} = \begin{bmatrix} [\boldsymbol{R}] & [\boldsymbol{P}] \\ [\boldsymbol{0}] & 1 \end{bmatrix}$$



Position Jacobian

$$\boldsymbol{P} = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix} \quad \frac{\partial x}{\partial q_1} = -a_1 s_1 - a_2 s_{12} \quad \frac{\partial x}{\partial q_2} = -a_2 s_{12}$$



Position Jacobian

$$\boldsymbol{P} = \begin{bmatrix} a_1c_1 + a_2c_{12} \\ a_1s_1 + a_2s_{12} \\ 0 \end{bmatrix}$$

$$\frac{\partial x}{\partial q_1} = -a_1s_1 - a_2s_{12} \qquad \frac{\partial x}{\partial q_2} = -a_2s_{12}$$

$$\frac{\partial y}{\partial q_1} = a_1c_1 + a_2c_{12} \qquad \frac{\partial y}{\partial q_2} = a_2c_{12}$$

$$\frac{\partial z}{\partial q_1} = 0 \qquad \frac{\partial z}{\partial q_2} = 0$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a_1s_1 - a_2s_{12} & -a_2s_{12} \\ a_1c_1 + a_1c_{12} & a_2c_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\boldsymbol{v} = \mathbf{J}_v\, \dot{\boldsymbol{q}}$$

$\mathbf{J}_v$ in a serial chain

$$\boldsymbol{v} = \dot{\boldsymbol{P}} = \mathbf{J}_v\,\dot{\boldsymbol{q}}$$

$$\dot{\boldsymbol{P}}_{0n} = \sum_{i=1}^{n} \frac{\partial \boldsymbol{P}_{0n}}{\partial q_i}\dot{q}_i$$

$$\mathbf{J}_{v_i} = \frac{\partial \boldsymbol{P}_{0n}}{\partial q_i}$$

$$\mathbf{J}_v = \left[[\mathbf{J}_{v_1}] \quad \cdots \quad [\mathbf{J}_{v_n}] \right]$$

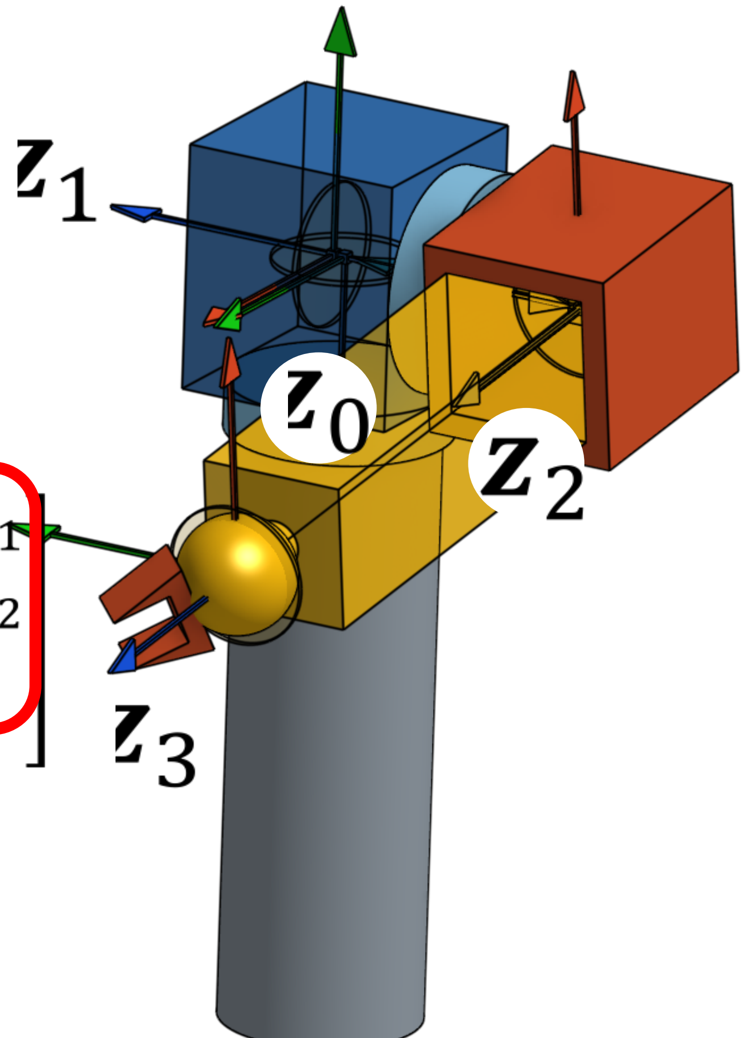
Prismatic Joints

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0

$$\mathbf{J}_{v_i} = \frac{\partial \boldsymbol{P}_{0n}}{\partial q_i}$$

$$\dot{\boldsymbol{P}}_{03} = \frac{\partial \boldsymbol{P}_{01}}{\partial q_1}\dot{q}_1 + \frac{\partial \boldsymbol{P}_{02}}{\partial q_2}\dot{q}_2 + \frac{\partial \boldsymbol{P}_{03}}{\partial q_3}\dot{q}_3$$

$$\boldsymbol{T}_{03} = \begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & d_3c_1s_2 - d_2s_1 \\ c_2s_1 & c_1 & s_1s_2 & d_3s_1s_2 + c_1d_2 \\ -s_2 & 0 & c_2 & c_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\dot{\boldsymbol{P}}_{03} = \begin{bmatrix} c_1s_2 \\ s_1s_2 \\ c_2 \end{bmatrix} \dot{d}_3$$

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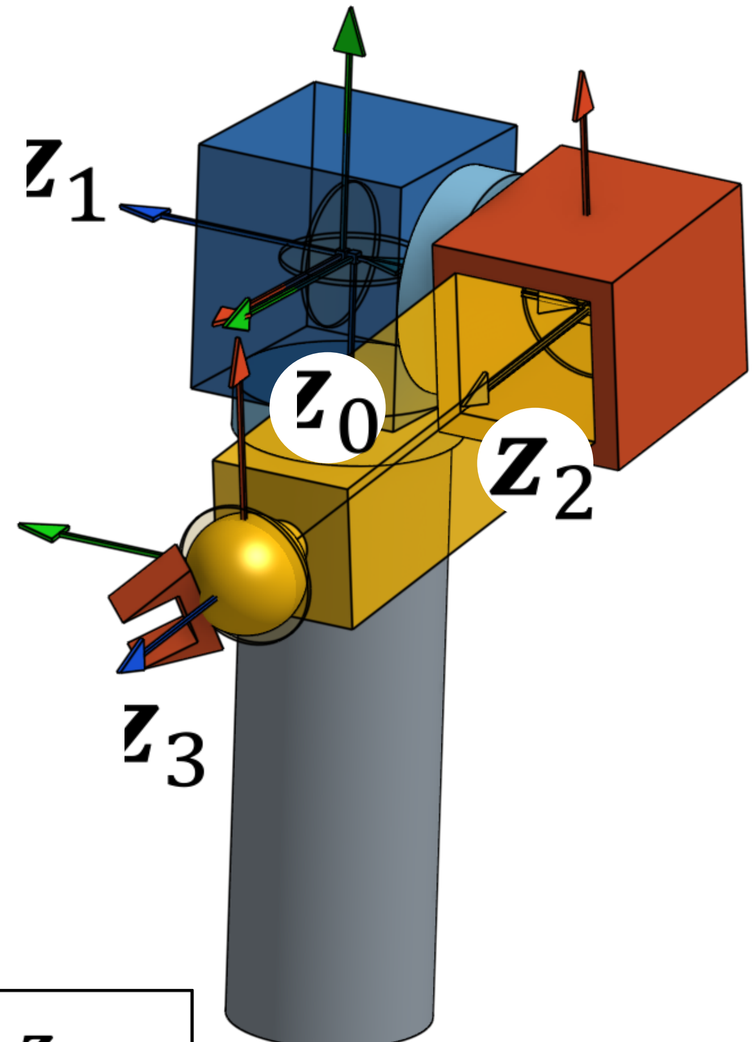
Prismatic Joints

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0

$$\dot{\boldsymbol{P}}_{03} = \frac{\partial \boldsymbol{P}_{01}}{\partial q_1}\dot{q}_1 + \frac{\partial \boldsymbol{P}_{02}}{\partial q_2}\dot{q}_2 + \frac{\partial \boldsymbol{P}_{03}}{\partial q_3}\dot{q}_3$$

$$\dot{\boldsymbol{P}}_{03} = \boldsymbol{z}_2\dot{q}_3 \qquad \mathbf{J}_{v_3} = \boldsymbol{z}_2$$



$$\dot{\boldsymbol{P}}_{03} = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix} \dot{d}_3$$

$$\boxed{\mathbf{J}_{v_i} = \boldsymbol{z}_{i-1}}$$

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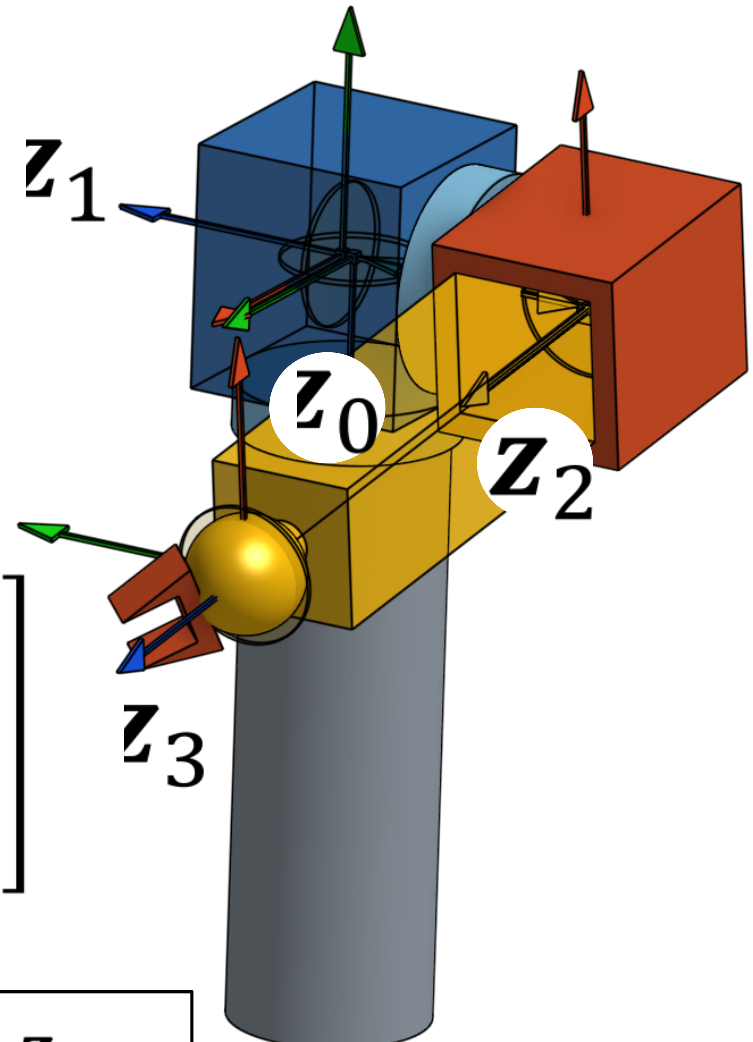
Prismatic Joints

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0

$$\dot{\boldsymbol{P}}_{03} = \frac{\partial \boldsymbol{P}_{01}}{\partial q_1}\dot{q}_1 + \frac{\partial \boldsymbol{P}_{02}}{\partial q_2}\dot{q}_2 + \frac{\partial \boldsymbol{P}_{03}}{\partial q_3}\dot{q}_3$$

$$\dot{\boldsymbol{P}}_{03} = \boldsymbol{z}_2\dot{q}_3 \qquad \mathbf{J}_{v_3} = \boldsymbol{z}_2$$

$$T_{02} = \begin{bmatrix} c_1 s_2 & -s_1 & c_1 s_2 & -d_2 s_1 \\ c_2 s_1 & c_1 & s_1 s_2 & d_2 c_1 \\ -s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\dot{\boldsymbol{P}}_{03} = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix} \dot{d}_3$$

$$\boxed{\mathbf{J}_{v_i} = \boldsymbol{z}_{i-1}}$$

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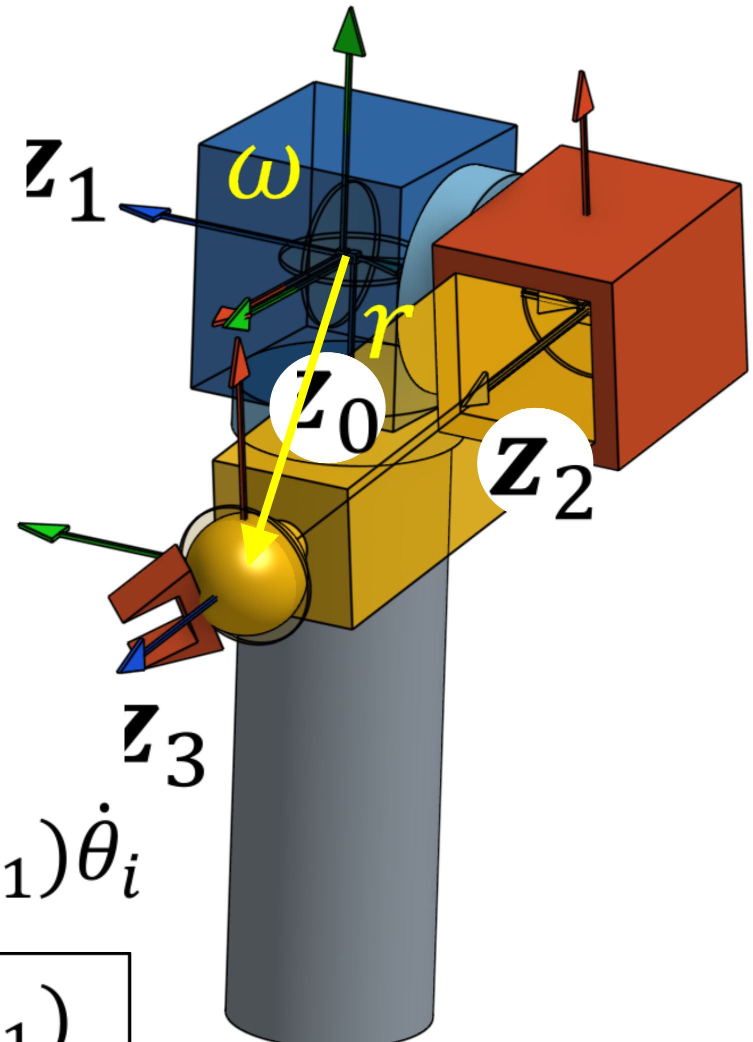
Revolute Joints

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0



$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$$

$$\boldsymbol{\omega} = \dot{\theta}_i \mathbf{z}_{i-1}$$

$$\boldsymbol{r} = (\boldsymbol{P}_n - \boldsymbol{P}_{i-1})$$

$$\boldsymbol{v} = \mathbf{z}_{i-1} \times (\boldsymbol{P}_n - \boldsymbol{P}_{i-1}) \dot{\theta}_i$$

$$\boxed{\mathbf{J}_{\boldsymbol{v}_i} = \mathbf{z}_{i-1} \times (\boldsymbol{P}_n - \boldsymbol{P}_{i-1})}$$

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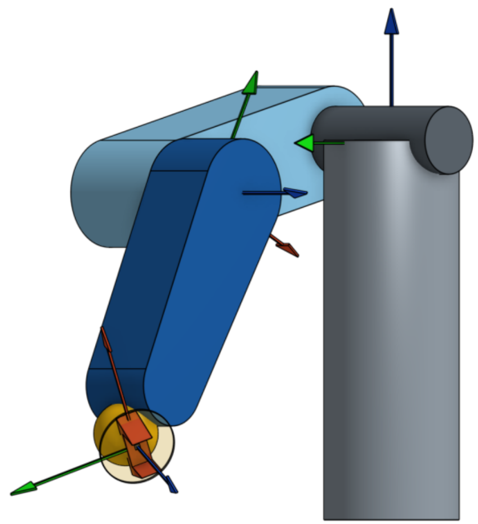
Orientation Jacobian

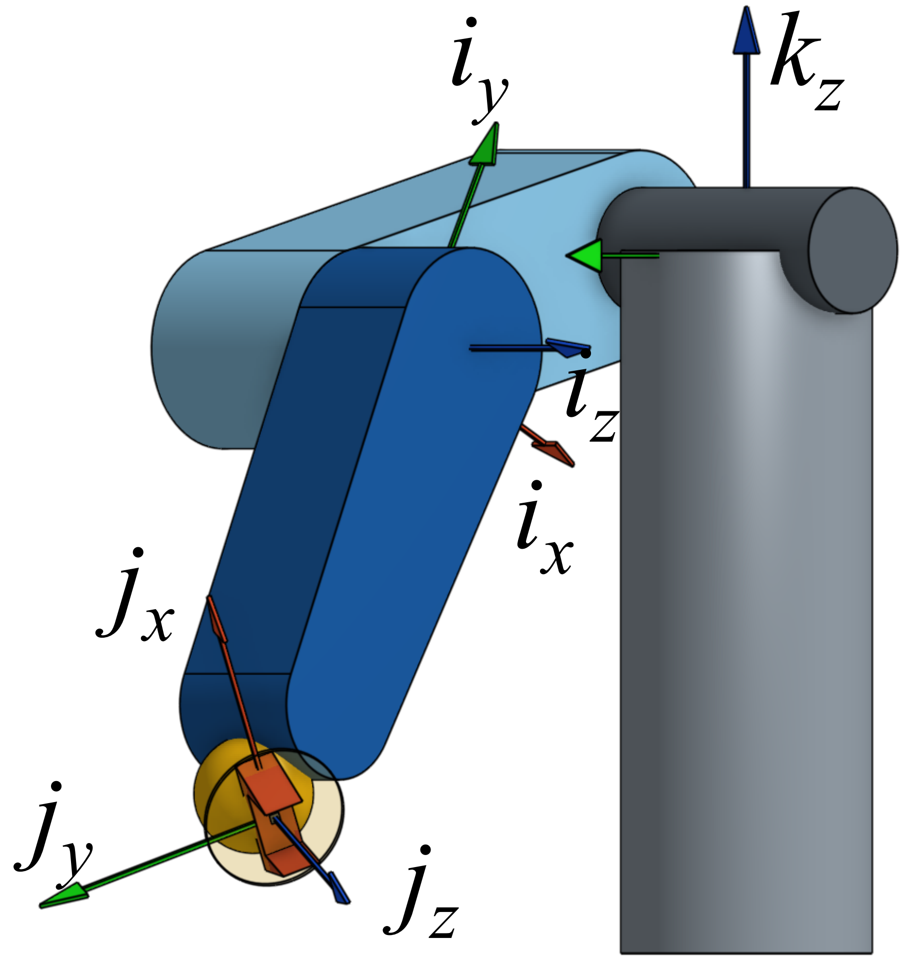
$$\frac{\partial x}{\partial t} = \sum_{i=1}^{n} \frac{\partial x}{\partial q_i} \frac{\partial q_i}{\partial t}$$



Angular Velocity

$$\omega = \mathbf{J}_\omega(q)\dot{\boldsymbol{q}}$$

$$\omega_{ij}^{k}$$



ω is the angular velocity of *j* with respect to *i* expressed in frame *k*

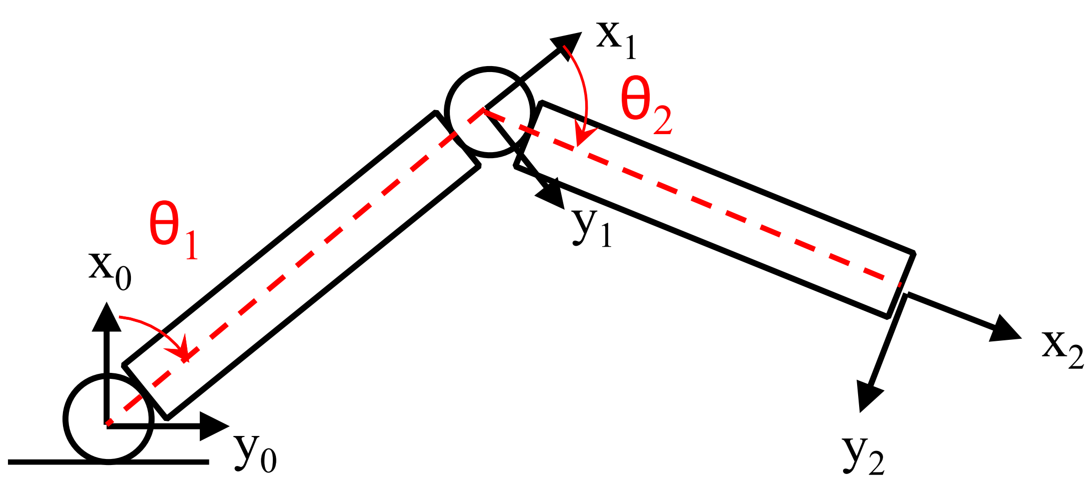
Orientation Jacobian

$$\omega_{01}^{0} = 0\hat{x}_0 + 0\hat{y}_0 + \dot{\theta}_1 \hat{z}_0$$

$$\omega_{12}^{1} = 0\hat{x}_1 + 0\hat{y}_1 + \dot{\theta}_2 \hat{z}_1$$

$$\omega_{12}^{0} = \boldsymbol{R}_{01} \omega_{12}^{1}$$

$$\omega_{02}^{0} = \omega_{01}^{0} + \boldsymbol{R}_{01} \omega_{12}^{1} = (\dot{\theta}_1 + \dot{\theta}_1)\hat{z}_0$$



Orientation Jacobian

$$\omega_{01}^{0} = 0\hat{x}_0 + 0\hat{y}_0 + \dot{\theta}_1\hat{z}_0$$

$$\omega_{12}^{1} = 0\hat{x}_1 + 0\hat{y}_1 + \dot{\theta}_2\hat{z}_1$$

$$\omega_{12}^{0} = \boldsymbol{R}_{01}\omega_{12}^{1}$$

$$\omega_{02}^{0} = \omega_{01}^{0} + \boldsymbol{R}_{01}\omega_{12}^{1} = (\dot{\theta}_1 + \dot{\theta}_1)\hat{z}_0$$

$$\omega_{0n}^{0} = \sum_{i=1}^{n} \boldsymbol{R}_{0(i-1)}\omega_{i-1\,i}^{i-1}$$

for revolute:

$$\omega_{0n}^{0} = \sum_{i=1}^{n} \hat{z}_{i-1}\dot{\theta}_i$$

Orientation Jacobian

for prismatic: $$\mathbf{J}_\omega = 0$$

$$\rho_i = \begin{matrix} 0 \text{ if } i \text{ is prismatic} \\ 1 \text{ if } i \text{ is revolute} \end{matrix}$$

$$\boxed{\mathbf{J}_\omega = [[\rho_1 \hat{\mathbf{z}}_0] \quad \cdots \quad [\rho_n \hat{\mathbf{z}}_{n-1}]]}$$

$$\omega_{0n}^0 = \sum_{i=1}^{n} \hat{z}_{i-1} \dot{\theta}_i$$

Combining Linear and Angular Velocity

$$\mathbf{J} = [[\mathbf{J}_1][\mathbf{J}_2] \quad \cdots \quad [\mathbf{J}_n]]$$

If Revolute: $$\mathbf{J}_i = \begin{bmatrix} \hat{\mathbf{z}}_{i-1} \times (\boldsymbol{P}_n - \boldsymbol{P}_{i-1}) \\ \hat{\mathbf{z}}_{i-1} \end{bmatrix}$$

If Prismatic: $$\mathbf{J}_i = \begin{bmatrix} \hat{\mathbf{z}}_{i-1} \\ 0 \end{bmatrix}$$

Robotics: Fundamentals

Video 7.4

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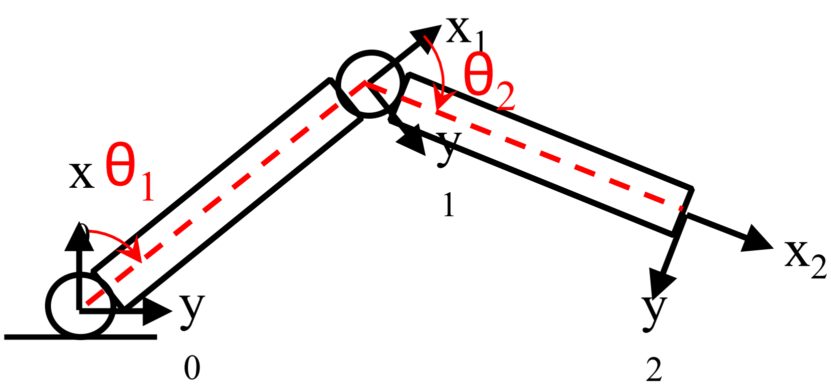
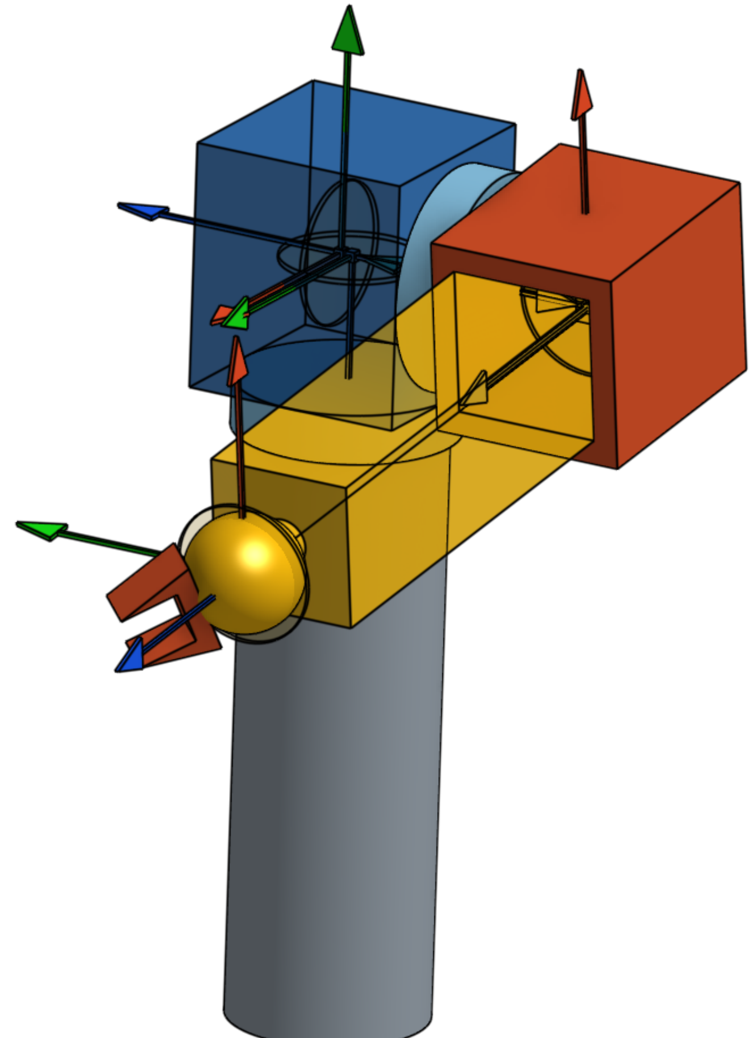
Jacobian Examples





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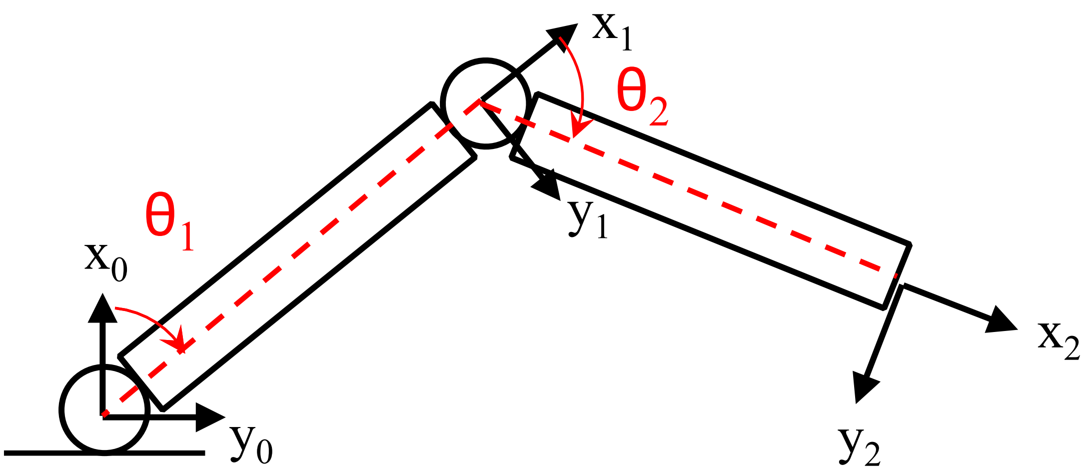
2 Link Arm Jacobian

$$\mathbf{J}(\boldsymbol{q}) = \begin{bmatrix} \hat{\boldsymbol{z}}_0 \times (\boldsymbol{P}_2 - \boldsymbol{P}_0) & \hat{\boldsymbol{z}}_1 \times (\boldsymbol{P}_2 - \boldsymbol{P}_1) \\ \hat{\boldsymbol{z}}_0 & \hat{\boldsymbol{z}}_1 \end{bmatrix}$$

If Revolute: $$\mathbf{J}_i = \begin{bmatrix} \hat{\boldsymbol{z}}_{i-1} \times (\boldsymbol{P}_n - \boldsymbol{P}_{i-1}) \\ \hat{\boldsymbol{z}}_{i-1} \end{bmatrix}$$

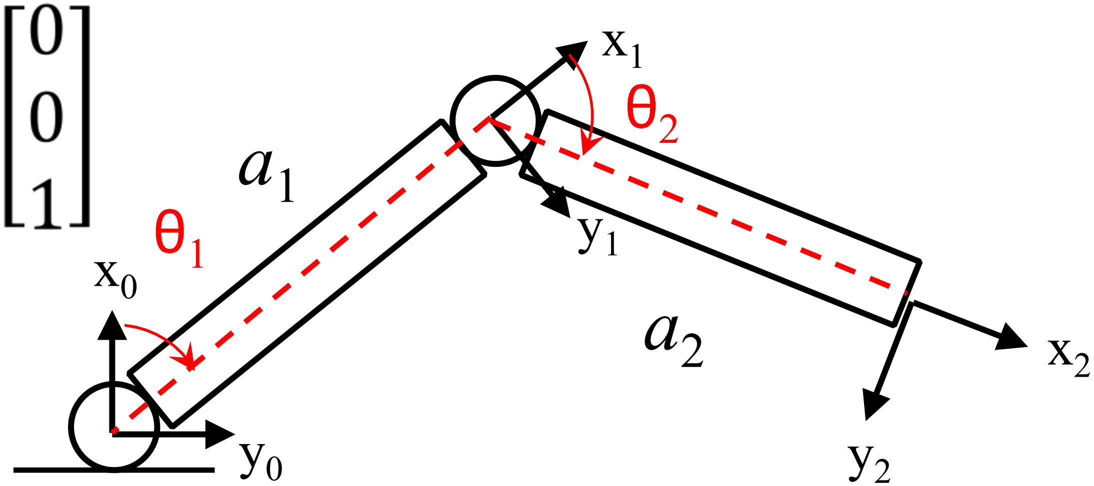


2 Link Arm Jacobian

$$\mathbf{J}(\boldsymbol{q}) = \begin{bmatrix} \hat{\mathbf{z}}_0 \times (\boldsymbol{P}_2 - \boldsymbol{P}_0) & \hat{\mathbf{z}}_1 \times (\boldsymbol{P}_2 - \boldsymbol{P}_1) \\ \hat{\mathbf{z}}_0 & \hat{\mathbf{z}}_1 \end{bmatrix}$$

$$\boldsymbol{P}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{P}_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix} \quad \boldsymbol{P}_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

$$\hat{\mathbf{z}}_0 = \hat{\mathbf{z}}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



2 Link Arm Jacobian

$$\mathbf{J}(\boldsymbol{q}) = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_2 c_{12} \\ a_2 s_{12} \\ 0 \end{bmatrix} \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\mathbf{J}(\boldsymbol{q}) = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

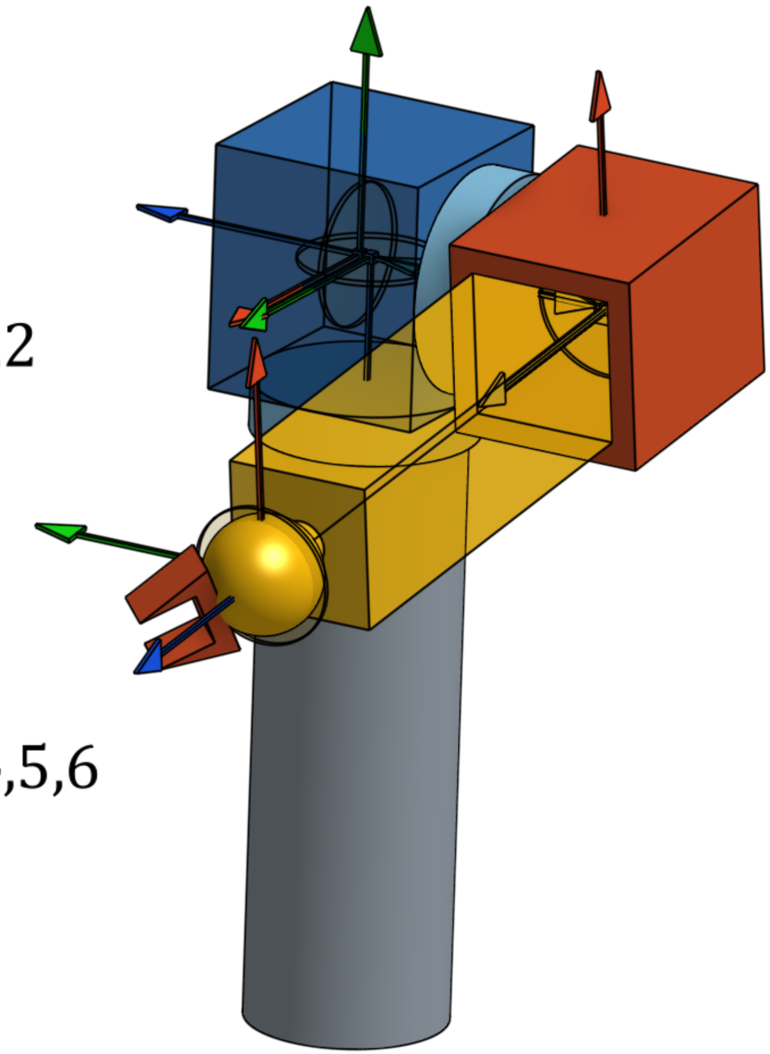
Stanford Arm Jacobian

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0

$$\mathbf{J}_i = \begin{bmatrix} \hat{\mathbf{z}}_{i-1} \times (\boldsymbol{P}_6 - \boldsymbol{P}_{i-1}) \\ \hat{\mathbf{z}}_{i-1} \end{bmatrix}, \qquad i = 1,2$$

$$\mathbf{J}_3 = \begin{bmatrix} \hat{\mathbf{z}}_2 \\ 0 \end{bmatrix}$$

$$\mathbf{J}_i = \begin{bmatrix} \hat{\mathbf{z}}_{i-1} \times (\boldsymbol{P}_6 - \boldsymbol{P}_{i-1}) \\ \hat{\mathbf{z}}_{i-1} \end{bmatrix}, \qquad i = 4,5,6$$



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Stanford Arm

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\boldsymbol{\theta_1}}$
2	0	90	d_2	$\underline{\boldsymbol{\theta_2}}$
3	0	0	$\underline{\boldsymbol{d_3}}$	0
4	0	-90	0	$\underline{\boldsymbol{\theta_4}}$
5	0	90	0	$\underline{\boldsymbol{\theta_5}}$
6	0	0	d_6	$\underline{\boldsymbol{\theta_6}}$

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Stanford Arm MATLAB

```
T01 = a1
T02 = T01*a2
T03 = T02*a3
T04 = T03*a4
T05 = T04*a5
T06 = T05*a6

Z0 = [0;0;1]; P0 = [0;0;0]
Z1 = T01(1:3, 3)
Z2 = T02(1:3, 3)
Z3 = T03(1:3, 3)
Z4 = T04(1:3, 3)
Z5 = T05(1:3, 3)
Z6 = T06(1:3, 3)
P1 = T01(1:3, 4)
P2 = T02(1:3, 4)
P3 = T03(1:3, 4)
P4 = T04(1:3, 4)
P5 = T05(1:3, 4)
P6 = T06(1:3, 4)

Jp1 = cross(Z0,P6-P0)
Jo1 = Z0
Jp2 = cross(Z1,P6-P1)
Jo2 = Z1
Jp3 = Z2
Jo3 = [0;0;0]
Jp4 = cross(Z3,P6-P3)
Jo4 = Z3
Jp5 = cross(Z4,P6-P4)
Jo5 = Z4
Jp6 = cross(Z5,P6-P5)
Jo6 = Z5

J = [Jp1 Jp2 Jp3 Jp4 Jp5 Jp6 ; Jo1 Jo2 Jo3 Jo4 Jo5 Jo6]
```

Results

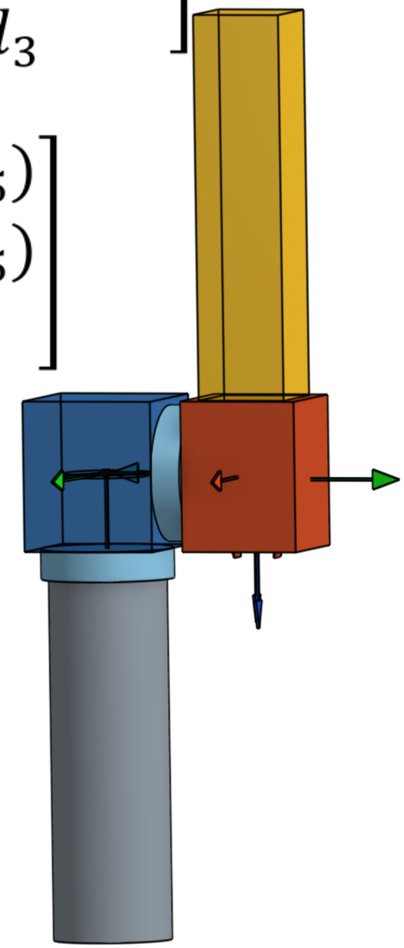
$$p_0 = p_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; p_2 = \begin{bmatrix} -s_1 d_2 \\ c_1 d_2 \\ 0 \end{bmatrix} \; p_3 = p_4 = p_5 = \begin{bmatrix} c_1 s_2 d_3 - s_1 d_2 \\ s_1 s_2 d_3 + c_1 d_2 \\ c_2 d_3 \end{bmatrix}$$

$$p_6 = \begin{bmatrix} c_1 s_2 d_3 - s_1 d_2 + d_6(c_1 c_2 c_4 s_5 + c_1 c_5 s_2 - s_1 s_4 s_5) \\ s_1 s_2 d_3 + c_1 d_2 + d_6(s_1 c_2 c_4 s_5 + s_1 c_5 s_2 + c_1 s_4 s_5) \\ c_1 d_3 + d_6(c_2 c_5 - s_2 c_4 s_5) \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \; z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}$$

$$z_2 = z_3 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix} \; z_4 = \begin{bmatrix} -c_1 c_2 s_4 - s_1 c_4 \\ -s_1 c_2 s_4 + c_1 c_4 \\ s_2 s_4 \end{bmatrix}$$

$$z_5 = \begin{bmatrix} c_1 c_2 c_4 s_5 - s_1 s_4 s_5 + c_1 s_2 c_5 \\ s_1 c_2 c_4 s_5 + c_1 s_4 s_5 + s_1 s_2 c_5 \\ -s_2 c_4 s_5 + c_2 c_5 \end{bmatrix}$$



Results

```
J11 - c1*d2 - d6*(s5*(c1*s4 + c2*c4*s1) + c5*s1*s2) - d3*s1*s2
J12 c1*(c2*d3 + d6*(c2*c5 - c4*s2*s5))
J13 c1*s2
J14 d6*s1*s2*(c2*c5 - c4*s2*s5) - c2*d6*(s5*(c1*s4 + c2*c4*s1) + c5*s1*s2)
J15 d6*(c1*c4 - c2*s1*s4)*(c2*c5 - c4*s2*s5) - d6*s2*s4*(s5*(c1*s4 + c2*c4*s1) + c5*s1*s2)
J16 0

J21 c1*d3*s2 - d6*(s5*(s1*s4 - c1*c2*c4) - c1*c5*s2) - d2*s1
J22 s1*(c2*d3 + d6*(c2*c5 - c4*s2*s5))
J23 s1*s2
J24 - c2*d6*(s5*(s1*s4 - c1*c2*c4) - c1*c5*s2) - c1*d6*s2*(c2*c5 - c4*s2*s5)
J25 d6*(c4*s1 + c1*c2*s4)*(c2*c5 - c4*s2*s5) - d6*s2*s4*(s5*(s1*s4 - c1*c2*c4) - c1*c5*s2)
J26 0

J31 0
J32 c1*(d2*s1 + d6*(s5*(s1*s4 - c1*c2*c4) - c1*c5*s2) - c1*d3*s2) - s1*(c1*d2 + d6*(s5*(c1*s4
    + c2*c4*s1) + c5*s1*s2) + d3*s1*s2)
J33 c2
J34 c1*d6*s2*(s5*(c1*s4 + c2*c4*s1) + c5*s1*s2) + d6*s1*s2*(s5*(s1*s4 - c1*c2*c4) - c1*c5*s2)
J35 d6*(c1*c4 - c2*s1*s4)*(s5*(s1*s4 - c1*c2*c4) - c1*c5*s2) - d6*(s5*(c1*s4 + c2*c4*s1) +
    + c5*s1*s2)*(c4*s1 + c1*c2*s4)
J36 0
```

```
J41 0
J42 -s1
J43 0
J44 c1*s2
J45 - c4*s1 - c1*c2*s4
J46 c1*c5*s2 - s5*(s1*s4 - c1*c2*c4)
```

```
J51 0
J52 c1
J53 0
J54 s1*s2
J55 c1*c4 - c2*s1*s4
J56 s5*(c1*s4 + c2*c4*s1) + c5*s1*s2
```

```
J61 1
J62 0
J63 0
J64 c2
J65 s2*s4
J66 c2*c5 - c4*s2*s5
```