Assignment 4, Chapter 2 (**due on 6 Oct**): P21; P24; P27; P30;P33; P36

2.21 Derive the normalization constant c_n in (2.5.28) by deriving the orthogonality relationship (2.5.29) using generating functions. Start by working out the integral

$$I = \int_{-\infty}^{\infty} g(x,t)g(x,s)e^{-x^2}dx,$$

and then consider the integral again with the generating functions in terms of series with Hermite polynomials.

2.24 Consider a particle in one dimension bound to a fixed center by a δ-function potential of the form

$$V(x) = -\nu_0\delta(x), \quad (\nu_0 \text{ real and positive}).$$

Find the wave function and the binding energy of the ground state. Are there excited bound states?

2.27 Derive an expression for the density of free-particle states in *two* dimensions, normalized with periodic boundary conditions inside a box of side length L. Your answer should be written as a function of k (or E) times $dEd\phi$, where ϕ is the polar angle that characterizes the momentum direction in two dimensions.

2.30 Using spherical coordinates, obtain an expression for $\mathbf{j}$ for the ground and excited states of the hydrogen atom. Show, in particular, that for $m_l \neq 0$ states, there is a circulating flux in the sense that $\mathbf{j}$ is in the direction of increasing or decreasing ϕ, depending on whether m_l is positive or negative.

2.33 The propagator in momentum space analogous to (2.6.26) is given by $\langle \mathbf{p}'', t | \mathbf{p}', t_0 \rangle$. Derive an explicit expression for $\langle \mathbf{p}'', t | \mathbf{p}', t_0 \rangle$ for the free-particle case.

2.36 Show that the wave-mechanical approach to the gravity-induced problem discussed in Section 2.7 also leads to phase-difference expression (2.7.17).