Assignment 7, Chapter 4 (**due on 3 Nov**): P3; P6; P9; P12

4.3 A quantum-mechanical state Ψ is known to be a simultaneous eigenstate of two Hermitian operators A and B that *anticommute*:

$$AB + BA = 0.$$

What can you say about the eigenvalues of A and B for state Ψ? Illustrate your point using the parity operator (which can be chosen to satisfy $\pi = \pi^{-1} = \pi^\dagger$) and the momentum operator.

4.6 Consider a symmetric rectangular double-well potential:

$$V = \begin{cases} \infty & \text{for } |x| > a+b; \\ 0 & \text{for } a < |x| < a+b; \\ V_0 > 0 & \text{for } |x| < a. \end{cases}$$

Assuming that V_0 is very high compared to the quantized energies of low-lying states, obtain an approximate expression for the energy splitting between the two lowest-lying states.

4.9 Let $\phi(\mathbf{p}')$ be the momentum-space wave function for state $|\alpha\rangle$—that is, $\phi(\mathbf{p}') = \langle \mathbf{p}'|\alpha\rangle$. Is the momentum-space wave function for the time-reversed state $\Theta|\alpha\rangle$ given by $\phi(\mathbf{p}')$, by $\phi(-\mathbf{p}')$, by $\phi^*(\mathbf{p}')$, or by $\phi^*(-\mathbf{p}')$? Justify your answer.

4.12 The Hamiltonian for a spin 1 system is given by

$$H = AS_z^2 + B(S_x^2 - S_y^2).$$

Solve this problem *exactly* to find the normalized energy eigenstates and eigenvalues. (A spin-dependent Hamiltonian of this kind actually appears in crystal physics.) Is this Hamiltonian invariant under time reversal? How do the normalized eigenstates you obtained transform under time reversal?