Assignment 2, Chapter 1 (due on 24 Sept.): P21; P24; P27; P30;P33

1.21 Evaluate the x-p uncertainty product $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle$ for a one-dimensional particle confined between two rigid walls,

$$V = \begin{cases} 0 & \text{for } 0 < x < a, \\ \infty & \text{otherwise.} \end{cases}$$

Do this for both the ground and excited states.

1.24 **(a)** Prove that $(1/\sqrt{2})(1+i\sigma_x)$ acting on a two-component spinor can be regarded as the matrix representation of the rotation operator about the x-axis by angle $-\pi/2$. (The minus sign signifies that the rotation is clockwise.)

(b) Construct the matrix representation of S_z when the eigenkets of S_y are used as base vectors.

1.27 **(a)** Suppose that $f(A)$ is a function of a Hermitian operator A with the property $A|a'\rangle = a'|a'\rangle$. Evaluate $\langle b''|f(A)|b'\rangle$ when the transformation matrix from the a' basis to the b' basis is known.

(b) Using the continuum analogue of the result obtained in (a), evaluate

$$\langle \mathbf{p}''|F(r)|\mathbf{p}'\rangle.$$

Simplify your expression as far as you can. Note that r is $\sqrt{x^2+y^2+z^2}$, where x, y, and z are *operators*.

1.30 The translation operator for a finite (spatial) displacement is given by

$$\mathcal{J}(\mathbf{l}) = \exp\left(\frac{-i\mathbf{p}\cdot\mathbf{l}}{\hbar}\right),$$

where $\mathbf{p}$ is the momentum *operator*.

(a) Evaluate

$$[x_i, \mathcal{J}(\mathbf{l})].$$

(b) Using (a) (or otherwise), demonstrate how the expectation value $\langle\mathbf{x}\rangle$ changes under translation.

1.33 **(a)** Prove the following:

i. $\langle p'|x|\alpha\rangle = i\hbar \dfrac{\partial}{\partial p'}\langle p'|\alpha\rangle$,

ii. $\langle \beta|x|\alpha\rangle = \displaystyle\int dp' \phi_\beta^*(p') i\hbar \frac{\partial}{\partial p'}\phi_\alpha(p')$,

where $\phi_\alpha(p') = \langle p'|\alpha\rangle$ and $\phi_\beta(p') = \langle p'|\beta\rangle$ are momentum-space wave functions.

(b) What is the physical significance of

$$\exp\left(\frac{ix\Xi}{\hbar}\right),$$

where x is the position operator and Ξ is some number with the dimension of momentum? Justify your answer.