Chapter 1 (**Sep 15, 2022, 11:59 PM**):
Sakurai: P1.3; P1.6; P1.9; P1.12;P1.15; P1.18

1.3 Show that the determinant of a 2×2 matrix $\boldsymbol{\sigma} \cdot \mathbf{a}$ is invariant under

$$\boldsymbol{\sigma} \cdot \mathbf{a} \to \boldsymbol{\sigma} \cdot \mathbf{a}' \equiv \exp\left(\frac{i\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}\phi}{2}\right)\boldsymbol{\sigma} \cdot \mathbf{a}\exp\left(\frac{-i\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}\phi}{2}\right).$$

Find a'_k in terms of a_k when $\hat{\mathbf{n}}$ is in the positive z-direction, and interpret your result.

1.6 Suppose $|i\rangle$ and $|j\rangle$ are eigenkets of some Hermitian operator A. Under what condition can we conclude that $|i\rangle + |j\rangle$ is also an eigenket of A? Justify your answer.

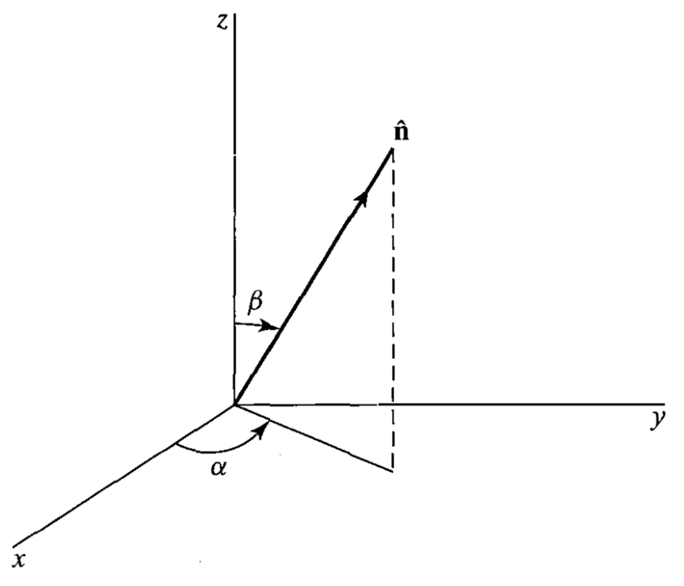
1.9 Construct $|\mathbf{S} \cdot \hat{\mathbf{n}}; +\rangle$ such that

$$\mathbf{S} \cdot \hat{\mathbf{n}} |\mathbf{S} \cdot \hat{\mathbf{n}}; +\rangle = \left(\frac{\hbar}{2}\right) |\mathbf{S} \cdot \hat{\mathbf{n}}; +\rangle,$$

where $\hat{\mathbf{n}}$ is characterized by the angles shown in the accompanying figure. Express your answer as a linear combination of $|+\rangle$ and $|-\rangle$. [*Note:* The answer is

$$\cos\left(\frac{\beta}{2}\right)|+\rangle + \sin\left(\frac{\beta}{2}\right)e^{i\alpha}|-\rangle.$$

But do not just verify that this answer satisfies the above eigenvalue equation. Rather, treat the problem as a straightforward eigenvalue problem. Also, do not use rotation operators, which we will introduce later in this book.]



1.12 A spin $\frac{1}{2}$ system is known to be in an eigenstate of $\mathbf{S}\cdot\hat{\mathbf{n}}$ with eigenvalue $\hbar/2$, where $\hat{\mathbf{n}}$ is a unit vector lying in the xz-plane that makes an angle γ with the positive z-axis.

(a) Suppose S_x is measured. What is the probability of getting $+\hbar/2$?

(b) Evaluate the dispersion in S_x—that is,

$$\langle (S_x - \langle S_x \rangle)^2 \rangle.$$

(For your own peace of mind, check your answers for the special cases $\gamma = 0$, $\pi/2$, and π.)

1.15 Let A and B be observables. Suppose the simultaneous eigenkets of A and B $\{|a', b'\rangle\}$ form a *complete* orthonormal set of base kets. Can we always conclude that

$$[A, B] = 0?$$

If your answer is yes, prove the assertion. If your answer is no, give a counterexample.

1.18 **(a)** The simplest way to derive the Schwarz inequality goes as follows. First, observe

$$(\langle\alpha| + \lambda^*\langle\beta|) \cdot (|\alpha\rangle + \lambda|\beta\rangle) \geq 0$$

for any complex number λ; then choose λ in such a way that the preceding inequality reduces to the Schwarz inequality.

(b) Show that the equality sign in the generalized uncertainty relation holds if the state in question satisfies

$$\Delta A|\alpha\rangle = \lambda \Delta B|\alpha\rangle$$

with λ purely *imaginary*.

(c) Explicit calculations using the usual rules of wave mechanics show that the wave function for a Gaussian wave packet given by

$$\langle x'|\alpha\rangle = (2\pi d^2)^{-1/4} \exp\left[\frac{i\langle p\rangle x'}{\hbar} - \frac{(x' - \langle x\rangle)^2}{4d^2}\right]$$

satisfies the minimum uncertainty relation

$$\sqrt{\langle(\Delta x)^2\rangle}\sqrt{\langle(\Delta p)^2\rangle} = \frac{\hbar}{2}.$$

Prove that the requirement

$$\langle x'|\Delta x|\alpha\rangle = (\text{imaginary number})\langle x'|\Delta p|\alpha\rangle$$

is indeed satisfied for such a Gaussian wave packet, in agreement with (b).