Assignment 6, Chapter 3 (**due on 22 Oct**): P21; P24; P27

3.21 The goal of this problem is to determine degenerate eigenstates of the three-dimensional isotropic harmonic oscillator written as eigenstates of $\mathbf{L}^2$ and L_z, in terms of the Cartesian eigenstates $|n_x n_y n_z\rangle$.

(a) Show that the angular-momentum operators are given by

$$L_i = i\hbar\varepsilon_{ijk}a_j a_k^\dagger$$

$$\mathbf{L}^2 = \hbar^2\left[N(N+1) - a_k^\dagger a_k^\dagger a_j a_j\right],$$

where summation is implied over repeated indices, ε_{ijk} is the totally antisymmetric symbol, and $N \equiv a_j^\dagger a_j$ counts the total number of quanta.

(b) Use these relations to express the states $|qlm\rangle = |01m\rangle$, $m = 0, \pm 1$, in terms of the three eigenstates $|n_x n_y n_z\rangle$ that are degenerate in energy. Write down the representation of your answer in coordinate space, and check that the angular and radial dependences are correct.

(c) Repeat for $|qlm\rangle = |200\rangle$.

(d) Repeat for $|qlm\rangle = |02m\rangle$, with $m = 0$, 1, and 2.

3.24 We are to add angular momenta $j_1 = 1$ and $j_2 = 1$ to form $j = 2$, 1, and 0 states. Using either the ladder operator method or the recursion relation, express all (nine) $\{j,m\}$ eigenkets in terms of $|j_1 j_2; m_1 m_2\rangle$. Write your answer as

$$|j = 1, m = 1\rangle = \frac{1}{\sqrt{2}}|+,0\rangle - \frac{1}{\sqrt{2}}|0,+\rangle, \ldots,$$

where $+$ and 0 stand for $m_{1,2} = 1, 0$, respectively.

3.27 Express the matrix element $\langle\alpha_2\beta_2\gamma_2|J_3^2|\alpha_1\beta_1\gamma_1\rangle$ in terms of a series in

$$\mathcal{D}^j_{mn}(\alpha\beta\gamma) = \langle\alpha\beta\gamma|jmn\rangle.$$