Assignment 9, Chapter 5 (**due on 17 Nov**): P24; P27; P30;P33;P36;P39

5.24 Consider a particle bound in a simple harmonic-oscillator potential. Initially ($t < 0$), it is in the ground state. At $t = 0$ a perturbation of the form

$$H'(x,t) = Ax^2 e^{-t/\tau}$$

is switched on. Using time-dependent perturbation theory, calculate the probability that after a sufficiently long time ($t \gg \tau$), the system will have made a transition to a given excited state. Consider all final states.

5.27 Consider a particle in one dimension moving under the influence of some time-independent potential. The energy levels and the corresponding eigenfunctions for this problem are assumed to be known. We now subject the particle to a traveling pulse represented by a time-dependent potential,

$$V(t) = A\delta(x - ct).$$

(a) Suppose that at $t = -\infty$ the particle is known to be in the ground state whose energy eigenfunction is $\langle x|i\rangle = u_i(x)$. Obtain the probability for finding the system in some excited state with energy eigenfunction $\langle x|f\rangle = u_f(x)$ at $t = +\infty$.

(b) Interpret your result in (a) physically by regarding the δ-function pulse as a superposition of harmonic perturbations; recall

$$\delta(x - ct) = \frac{1}{2\pi c}\int_{-\infty}^{\infty} d\omega e^{i\omega[(x/c)-t]}.$$

Emphasize the role played by energy conservation, which holds even quantum-mechanically as long as the perturbation has been on for a very long time.

5.30 Consider a two-level system with $E_1 < E_2$. There is a time-dependent potential that connects the two levels as follows:

$$V_{11} = V_{22} = 0, \quad V_{12} = \gamma e^{i\omega t}, \quad V_{21} = \gamma e^{-i\omega t} \quad (\gamma \text{ real}).$$

At $t = 0$, it is known that only the lower level is populated—that is, $c_1(0) = 1$, $c_2(0) = 0$.

(a) Find $|c_1(t)|^2$ and $|c_2(t)|^2$ for $t > 0$ by *exactly* solving the coupled differential equation

$$i\hbar \dot{c}_k = \sum_{n=1}^{2} V_{kn}(t) e^{i\omega_{kn} t} c_n, \quad (k = 1,2).$$

(b) Do the same problem using time-dependent perturbation theory to lowest nonvanishing order. Compare the two approaches for small values of γ. Treat the following two cases separately: (i) ω very different from ω_{21} and (ii) ω close to ω_{21}.

Answer for (a): (Rabi's formula)

$$|c_2(t)|^2 = \frac{\gamma^2/\hbar^2}{\gamma^2/\hbar^2 + (\omega - \omega_{21})^2/4} \sin^2 \left\{ \left[\frac{\gamma^2}{\hbar^2} + \frac{(\omega - \omega_{21})^2}{4} \right]^{1/2} t \right\},$$

$$|c_1(t)|^2 = 1 - |c_2(t)|^2.$$

5.33 Repeat Problem 5.32, but with the atomic hydrogen Hamiltonian

$$H = A\mathbf{S}_1 \cdot \mathbf{S}_2 + \left(\frac{eB}{m_e c} \right) \mathbf{S}_1 \cdot \mathbf{B},$$

where in the hyperfine term, $A\mathbf{S}_1 \cdot \mathbf{S}_2$, $\mathbf{S}_1$ is the electron spin and $\mathbf{S}_2$ is the proton spin. [Note that the problem here has less symmetry than the positronium case].

Please do 5.33.

Problem 5.32 listed below is for your reference.

5.32 **(a)** Consider the positronium problem you solved in Chapter 3, Problem 3.4. In the presence of a uniform and static magnetic field B along the z-axis, the Hamiltonian is given by

$$H = A\mathbf{S}_1 \cdot \mathbf{S}_2 + \left(\frac{eB}{m_e c}\right)(S_{1z} - S_{2z}).$$

Solve this problem to obtain the energy levels of all four states *using degenerate time-independent perturbation theory* (instead of diagonalizing the Hamiltonian matrix). Regard the first and second terms in the expression for H as H_0 and V, respectively. Compare your results with the exact expressions

$$E = -\left(\frac{\hbar^2 A}{4}\right)\left[1 \pm 2\sqrt{1 + 4\left(\frac{eB}{m_e c\hbar A}\right)^2}\right] \quad \text{for} \begin{cases} \text{singlet } m = 0 \\ \text{triplet } m = 0 \end{cases}$$

$$E = \frac{\hbar^2 A}{4} \qquad \text{for triplet } m = \pm 1,$$

where *triplet* (*singlet*) $m = 0$ stands for the state that becomes a pure triplet (singlet) with $m = 0$ as $B \to 0$.

(b) We now attempt to cause transitions (via stimulated emission and absorption) between the two $m = 0$ states by introducing an oscillating magnetic field of the "right" frequency. Should we orient the magnetic field along the z-axis or along the x- (or y-) axis? Justify your choice. (The original static field is assumed to be along the z-axis throughout.)

(c) Calculate the eigenvectors to first order.

5.36 Show that $\mathbf{A}_n(\mathbf{R})$ defined in (5.6.23) is a purely real quantity.

5.39 A particle of mass m constrained to move in one dimension is confined within $0 < x < L$ by an infinite-wall potential

$$V = \infty \quad \text{for } x < 0, x > L,$$
$$V = 0 \quad \text{for } 0 \leq x \leq L.$$

Obtain an expression for the density of states (that is, the number of states per unit energy interval) for *high* energies as a function of E. (Check your dimension!)