CHAPTER 1

What Is Systems Science?

The aim of science is not things themselves, . . . but the relations between things; outside those relations there is no reality knowable.
—HENRI POINCARÉ

An inevitable prerequisite for this book, as implied by its title, is a presupposition that systems science is a legitimate field of scientific inquiry. It is self-evident that I, as the author of this book, consider this presupposition valid. Otherwise, clearly, I would not conceive of writing the book in the first place.

I must admit at the outset that my affirmative view regarding the legitimacy of systems science is not fully shared by everyone within the scientific community. It seems, however, that this view of legitimacy is slowly but steadily becoming predominant. It is my hope that this book, whose purpose is to characterize the essence and spirit of systems science, will have a positive influence in this regard.

What is *systems science*? This question, which I have been asked on countless occasions, can basically be answered either in terms of activities associated with systems science or in terms of the domain of its inquiry. The most natural answers to the question are, almost inevitably, the following definitions:

1. Systems science is what systems scientists do when they claim they do science.
2. Systems science is that field of scientific inquiry whose objects of study are systems.

Without further explanation, these definitions are clearly of little use.

Definition (1) is meaningful but somewhat impractical. It is meaningful since systems scientists do, indeed, exist. I, for example, claim to be one of them, and so do colleagues at my department and other departments of systems science. Hence, the meaning of systems science could, in principle, be determined by observing and analyzing our scientific activities. This strategy, however, involves some inherent practical difficulties. First, systems scientists are still a rather rare species among all scientists and, consequently, they are relatively hard to find. Second, scientific activities of scientists who are officially labeled as systems scientists vary from person to person, and, moreover, some of these activities are clearly associated with other, well-established areas of science. Third, the strategy would require a massive data collection and extensive and sophisticated data analysis.

For all the reasons mentioned, and possibly some additional ones, it is virtually impossible to utilize definition (1) in an operational way for our purpose. Therefore, let me concentrate on definition (2). To be made operational, this definition requires that some broad and generally acceptable characterization of the concept of a *system* be established.

The term "system" is unquestionably one of the most widely used terms not only in science, but in other areas of human endeavor as well. It is a highly overworked term, which enjoys different meanings under different circumstances and for different people. However, when separated from its specific connotations and uses, the term "system" is almost never explicitly defined. To elaborate on this point, let me quote from a highly relevant paper by Rosen [**1986**]:

> Let us begin by observing that the word "system" is almost never used by itself; it is generally accompanied by an adjective or other modifier: physical system; biological system; social system; economic system; axiom system; religious system; and even "general" system. This usage suggests that, when confronted by a system of any kind, certain of its properties are subsumed under the adjective, and other properties are subsumed under the "system," while still others may depend essentially on both. The adjective describes what is special or particular; i.e., it refers to the specific "thinghood" of the system; the "system" describes those properties which are independent of this specific "thinghood."
>
> This observation immediately suggests a close parallel between the concept of *system* and the development of the mathematical concept of a set. Given any specific aggregate of things; e.g., five oranges, three sticks, five fingers, there are some properties of the aggregate which depend on the specific nature of the things of which the aggregate is compared. There are others which are totally independent of this and depend only on the "setness" of the aggregate. The most prominent of these is what we call the *cardinality* of the aggregate.
>
> It should now be clear that *systemhood* is related to thinghood in much the same way as setness is related to thinghood. Likewise, what we generally call *system properties* are related to systemhood in the same way as cardinality is related to setness. But systemhood is different from both setness and from thinghood: it is an *independent category*.

To begin our search for a meaningful definition of the term "system" from a broad perspective, let us consult a standard dictionary. We are likely to find that a system is "a set or arrangement of things so related or connected as to form a unity or organic whole" (*Webster's New World Dictionary*), although different dictionaries may contain stylistic variations of this particular formulation. It follows from this *common-sense definition* that the term "system" stands, in general, for a set of some things and a relation among the things. Formally, we have

$$\mathbf{S} = (T, R), \tag{1.1}$$

where **S**, T, R denote, respectively, a *system,* a *set of things* distinguished within **S**, and a *relation* (or, possibly, a set of relations) defined on T. Clearly, the thinghood and systemhood properties of **S** reside in T and R, respectively.

The common-sense definition of a system, expressed by Eq. (1.1), is rather primitive. This, paradoxically, is its weakness as well as its strength. The definition is weak because it is too general and, consequently, of little pragmatic value. It is strong because it encompasses all other, more specific definitions of systems. In this regard, this most general definition of systems provides us with a criterion by which we can determine whether any given object is a system or not: an object is a system if and only if it can be described in a form that conforms to Eq. (1.1).

For example, a collection of books is not a system, only a set. However, when we organize the books in some way, the collection becomes a system. When we order them, for instance, by authors' names, we obtain a system since any ordering of a set is a relation defined on the set. We may, of course, order the books in various other ways (by publication dates, by their size, etc.), which result in different systems. We may also partition the books by various criteria (subjects, publishers, languages, etc.) and obtain thus additional systems since every partition of a set emerges from a particular equivalence relation defined on the set. Observe now that a relation defined on a particular set of books, say the ordering by publication dates, may be applied not only to other sets of books, but also to sets whose elements are not books. For example, members of a human population may be ordered by their dates of birth.

These simple examples illustrate that the same set may play a role in different systems; these systems are distinguished from each other by different relations on the set. Similarly, the same relation, when applied to different sets, may play a role in different systems. In this case, the systems are distinguished by their sets or, in other words, by their thinghood properties.

Once we have the capability of distinguishing objects that are systems from those that are not, the proposed definition of systems science—*a science whose objects of study are systems*—becomes operational. Observe, however, that the term "system" is used in this definition without any adjective or other modifier. This indicates, according to the distinction between thinghood and systemhood, that systems science focuses on the study of systemhood properties of systems rather than their thinghood properties. Taking this essential aspect of systems science into consideration, the following, more specific definition of systems science emerges:

Systems science is a science whose domain of inquiry consists of those properties of systems and associated problems that emanate from the general notion of systemhood.

The principal purpose of this book is to elaborate on this conception of systems science. It is argued throughout the book that systems science, like any other science, contains a *body of knowledge* regarding its domain, a *methodology* for acquisition of new knowledge and for dealing with relevant problems within the domain, and a *metamethodology,* by which methods and their relationship to problems are characterized and critically examined. However, in spite of these parallels with classical areas of science, systems science is fundamentally different from science in the traditional sense. The difference can best be explained in terms of the notions of thinghood and systemhood.

It is a truism that classical science has been far more concerned with thinghood than systemhood. In fact, the many disciplines and specializations that have evolved in science during the last five centuries or so reflect predominantly the differences between the things studied rather than the differences in their ways of being organized. This evolution is still ongoing. Since at least the beginning of the 20th century, however, it has increasingly been recognized that studying the ways in which things can be, or can become, organized is equally meaningful and may, under some circumstances, be even more significant than studying the things themselves. From this recognition, a new kind of science eventually emerged, a science that is predominantly concerned with systemhood rather than thinghood. This new science is, of course, systems science.

Since disciplines of classical science are largely thinghood-oriented, the systemhood orientation of systems science does not make it a new discipline of classical science. With its orientation so fundamentally different from the orientation of classical science, systems science transcends all the disciplinary boundaries of classical science. From the standpoint of systems science, these boundaries are totally irrelevant, superficial, and even counterproductive. Yet, they are significant in classical science, where they reflect fundamental differences, for example, differences in measuring instruments and techniques. In other words, the disciplinary boundaries of classical science are thinghood-dependent but systemhood-independent. If systems science becomes divided into special disciplines in the future, the boundaries between these disciplines will inevitably be systemhood-dependent but thinghood-independent.

Classical science, with all its disciplines, and systems science, with all its prospective disciplines, thus provide us with two distinct perspectives from which scientific inquiry can be approached. These perspectives are complementary. Either of them can be employed without the other only to some extent. In most problems of scientific inquiry, the two perspectives must be applied in concert.

It may be argued that traditional scientific inquiries are almost never totally devoid of issues involving systemhood. This is true, but these issues are handled in classical science in an opportunistic, ad hoc fashion. There is no place in classical science for a comprehensive and thorough study of the various properties of systemhood. The systems perspective is thus suppressed within the confines of classical science in the sense that it cannot develop its full potential. It was liberated only through the emergence of systems science. While the systems perspective was not essential when science dealt with simple systems, its significance increases with the growing complexity of systems of our current interest and challenge.

From the standpoint of the disciplinary classification of classical science, systems science is clearly cross-disciplinary. There are at least three important implications of this fact. First, systems science knowledge and methodology are directly applicable in virtually all disciplines of classical science. Second, systems science has the flexibility to study systemhood properties of systems and the associated problems that include aspects derived from any number of different disciplines and specializations of classical science. Such multidisciplinary systems and problems can thus be studied as wholes rather than collections of the disciplinary subsystems and subproblems. Third, the cross-disciplinary orientation of systems science has a unifying influence on classical science, increasingly fractured into countless number of narrow specializations, by offering unifying principles that transcend its self-imposed boundaries.

Classical science and systems science may be viewed as complementary dimensions of modern science. As is argued later (Sec. 3.4), the emergence and evolution of systems science and its integration with classical science into genuine two-dimensional science are perhaps the most significant features of science in the information (or postindustrial) society.

CHAPTER 2

More about Systems

What is a system? As any poet knows, a system is a way of looking at the world.
—GERALD M. WEINBERG

2.1. Common-Sense Definition

The common-sense definition, as expressed by Eq. (1.1), looks overly simple:

$$\mathbf{S} = (T, R)$$

— a relation defined on T (systemhood)
— a set of certain things (thinghood)
— a system

Its simplicity, however, is only on the surface. That is, the definition is simple in its form, but it contains symbols, T and R, that are extremely rich in content. Indeed, T stands for any imaginable set of things of any kind, and R stands for any conceivable relation defined on T. To appreciate the range of possible meanings of these symbols, let us explore some examples.

Symbol T may stand for a single set with arbitrary elements, finite or infinite, but can also represent, for example, a power set (the set of all subsets of another set), any subset of the power set, or an arbitrary family of distinct sets. The content of symbol R is even richer. For each set T, with its special characteristics, the symbol stands for every relation that can be defined on the set.

To introduce the concept of a relation, we need first to introduce the underlying concept of a Cartesian product of sets. To do that, let us assume that the symbol T in Eq. (1.1) stands for the family of sets $A_1, A_2, \ldots, A_n$. That is,

$$T = \{A_1, A_2, \ldots, A_n\}$$

In this case, the *Cartesian product* of sets in this family, which is usually denoted by the symbol

$$A_1 \times A_2 \times \ldots \times A_n,$$

is the set of all possible ordered n-tuples formed by selecting the first component from set A_1, the second component from A_2, etc., and the last component from set A_n. Formally,

$$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n\}.$$

When n sets are involved in a Cartesian product, as in this case, we refer to it as *an n-dimensional Cartesian product.* For convenience, we often write

$$\times_{i=1}^{n} A_i$$

instead of $A_1 \times A_2 \times \ldots \times A_n$. Moreover, when $A_i = A$ for all $i = 1, 2, \ldots, n$, we often denote the Cartesian product by the symbol A^n.

As an example, let $T = \{A_x, A_y\}$, where A_x and A_y are given closed intervals of real numbers on Cartesian coordinates x and y, respectively, in the two-dimensional Euclidean space (xy-plane). When, for example, $A_x = [1, 4]$ and $A_y = [1, 3]$, the Cartesian product $A_x \times A_y$ consists of all points in the rectangle shown in Fig. 2.1.

Cartesian products provide a basis for defining relations. A *relation*, in general, is a subset of some Cartesian product of given sets. This means that many distinct relations can be defined on the same Cartesian product. Each Cartesian product characterizes a unique form of relations based on it. The number of distinct

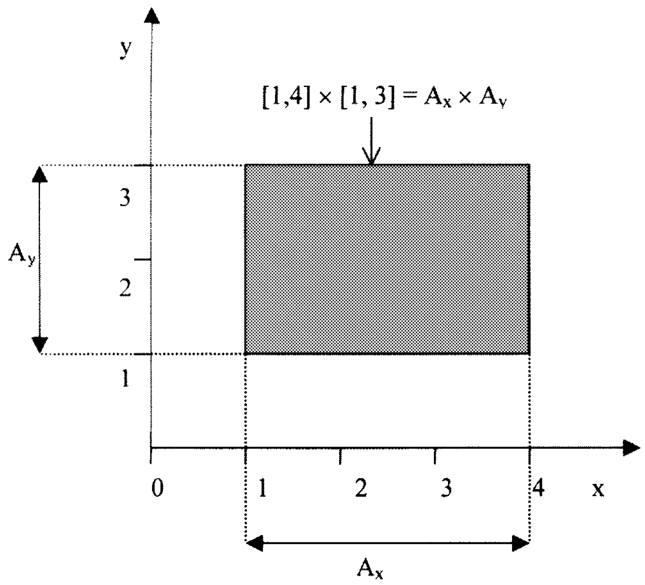

Figure 2.1. Geometric representation of Cartesian product.

relational forms, each based upon a particular Cartesian product, increases with the increasing number of distinct sets subsumed under the symbol T in Eq. (1.1). To illustrate the tremendous variety of relational forms, let us consider a few examples.

When T stands for a single set, say set A, the obvious relational forms are:

$$R \subseteq A^2 (= A \times A),$$

$$R \subseteq A^2 (= A \times A \times A),$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$R \subseteq A^n (= \underbrace{A \times A \times \ldots \times A}_{n\text{-times}}).$$

Relations of these forms are called, respectively, binary, ternary, . . . , n-ary relations on A. Examples of additional forms are:

$$R \subseteq (A \times A) \times A,$$

$$R \subseteq A \times (A \times A),$$

$$R \subseteq (A \times A) \times (A \times A).$$

All these forms define binary relations in which one or two sets, as designated in parentheses, are Cartesian products of $(A \times A)$. Similarly, the forms

$$R \subseteq (A \times A) \times (A \times A) \times (A \times A)$$

$$R \subseteq (A \times A \times A) \times (A \times A \times A)$$

define, respectively, ternary relations on $A \times A$ and binary relations on $A \times A \times A$.

When T consists of a family of two sets, $T = \{A, B\}$, the number of possible relational forms further increases. A few examples are:

$$R \subseteq A \times B,$$

$$R \subseteq (A \times A) \times B,$$

$$R \subseteq (\mathrm{A} \times B) \times (A \times B),$$

$$R \subseteq (A \times A \times A) \times B,$$

$$R \subseteq (A \times A \times A) \times (B \times B),$$

$$R \subseteq (A \times B) \times (A \times B) \times (A \times B).$$

It is now easy to see, I trust, how rapidly the number of possible relational forms increases with the increasing number of distinct sets in T, illustrating thus the tremendous richness of the systemhood symbol R. The fact that we discuss the meaning of this symbol solely in terms of mathematical relations is no shortcoming. The well-defined concept of a mathematical relation (as a subset of some Cartesian product) is sufficiently general to encompass the whole set of kindred concepts that pertain to systemhood, such as interaction, interconnection, coupling, linkage, cohesion, constraint, interdependence, function, organization, structure, association, correlation, pattern, etc.

Let us address now the issue of calculating the number of possible relations for each given relational form. First, it is easy to see that the number depends on both the Cartesian product and the sets employed in it. If the sets are infinite, as in the example illustrated in Fig. 2.1, the number of possible relations is also infinite (any subset of points in the rectangle in Fig. 2.1 is a relation). If the sets are finite, the number of possible relations is determined by the number of elements in each of the sets employed in the given Cartesian product. The number of elements in a finite set A is usually denoted by the symbol $|A|$ and it is called the *cardinality* of A. It is easy to see that the cardinality of a Cartesian product of finite sets is the arithmetic product of the cardinalities of the sets employed in it. For example,

$$|A \times B| = |A| \cdot |B|,$$

$$|A_1 \times A_2 \times \ldots \times A_n| = |A_1| \cdot |A_2| \ldots |A_n|,$$

$$|A^2| = |A|^2,\ |A^3| = |A|^3, \text{ etc.}$$

If some of the sets in the Cartesian product are themselves Cartesian products, the calculation remains the same. For example,

$$|(A \times B) \times (C \times D)| = |A \times B| \cdot |C \times D| = |A| \cdot |B| \cdot |C| \cdot |D|.$$

The set of all relations that can be defined on a given Cartesian product of finite sets is the *power set* (the set of all subsets) of the Cartesian product. To define a particular relation, two choices are available for each element (n-tuple) of the Cartesian product: the element is either included or not included in the relation. Let $|C|$ denote the cardinality of the given Cartesian product C. Then, the total number of choices and, therefore, the total number of relations on C, $\#R(C)$, is obtained by the arithmetic product

$$\#R(C) = \underbrace{2 \cdot 2 \ldots 2}_{|C|\text{-times}} = 2^{|C|}.$$

Thus, for example,

$$\#R(A \times B) = 2^{|A| \cdot |B|},$$

$$\#R(A^n) = 2^{|A|^n}.$$

When some sets in the Cartesian product are power sets of some other finite sets, their cardinalities must be properly calculated. Denoting the power set of A by $\mathcal{P}(A)$,we can apply the preceding argument to derive the formula

$$|\mathcal{P}(A)| = 2^{|A|}.$$

Similarly, for the power set of the power set of A, $\mathcal{P}(\mathcal{P}(A))$, we have

$$|\mathcal{P}(\mathcal{P}(A))| = 2^{2^{|A|}}.$$

Thus, for example,

$$|\mathcal{P}(A) \times \mathcal{P}(B)| = 2^{|A|} \cdot 2^{|B|} = 2^{|A|+|B|},$$

$$|A \times \mathcal{P}(B) \times \mathcal{P}(\mathcal{P}(C)) \times D| = |A| \cdot 2^{|B|}\, 2^{2^{|C|}} \cdot |D|$$

$$= |A| \cdot 2^{|B|+2^{|C|}} \cdot |D|.$$

2.2. More about Relations

To appreciate the richness of the concept of a relation, let us examine in more detail the simplest possible relations. These are binary relations of the form

$$R \subseteq T \times T$$

where T is a single set of things. In any relation of this form, things in T are related to themselves according to some given criterion c. Let R_c denote a relation based on criterion c. Then $(x, y) \in R_c$ if and only if thing x is related to thing y according to the given criterion c. Let us illustrate this general definition by the following examples:

- T = a set of people.
  Person x is related to person y iff x is *equal to* y in terms of a given characteristic (age, income, education, occupation, sex, citizenship, height, weight, name, employer, job performance, etc.). Relations of this kind are called *equivalence relations*.

- T = a set of English words.
  A word x is related to word y iff x is a *synonym of* y. Relations of this kind are called *compatibility relations.*
- T = a set of countries.
  Country x is related to country y iff it is *smaller than or equal to* it in terms of both the geographic area and the number of inhabitants. Relations of this kind are called *partial orderings.*
- T = a set of numbers.
  Number x is related to number y iff x is *smaller than* y. Relations of this kind are called *strict orderings* (also called linear, or total orderings).

The most fundamental classification of relations $R \subseteq T \times T$ (into equivalence relations, various ordering relations, etc.) is based on the following properties:

- R is *reflexive* iff $(x, x) \in R$ for each $x \in T$.
- R is *antireflexive* iff $(x, x) \notin R$ for each $x \in T$.
- R is *symmetric* iff, for every x and y in T, whenever $(x, y) \in R$, then also $(y, x) \in R$.
- R is *antisymmetric* if and only if, for every x and y in T, whenever $(x, y) \in R$ and $(y, x) \in R$, then $x = y$.
- R is *transitive* iff, for any three elements x, y, z in T, whenever $(x, y) \in R$ and $(y, x) \in R$, then also $(x, z) \in R$.

Various combinations of these properties lead to distinct classes of relations. The following classes of relations are the most common.

- *Equivalence relations*: reflective, symmetric, and transitive.
- *Compatibility relations*: reflective and symmetric.
- *Partial orderings*: reflexive, antisymmetric, and transitive.
- *Strict orderings*: antireflexive, antisymmetric, and transitive.

To illustrate these classes of relations, let us consider the set of students who took together the same course. They are listed by their first names (all distinct from each other) in Table 2.1 together with four characteristics: the grade of each student in the course, major field of study, age, and the status as either full-time or part-time student. Let T denote this set of students. Choosing one or more of the four characteristics, we may define a particular equivalence relation on T. We consider students who are not distinguished by the chosen characteristics as equivalent in terms of these characteristics. Thus, for example, Alan and Bob are equivalent in terms of their age and full-time status, but are not equivalent in terms of their grades and their major fields of study. On the other hand, Debby and George are equivalent in terms of each of the characteristics or any combination of them.

The equivalence relation $R_g \subseteq T \times T$ defined in terms of grades is expressed by the matrix in Table 2.2. Entries in this matrix are values of the characteristic

Table 2.1. Set of Students with Four Characteristics

Student	Grade	Major	Age	Full-time/ part-time
Alan	B	Biology	19	Full-time
Bob	C	Physics	19	Full-time
Cliff	C	Mathematics	20	Part-time
Debby	A	Mathematics	19	Full-time
George	A	Mathematics	19	Full-time
Jane	A	Business	21	Part-time
Lisa	B	Chemistry	21	Part-time
Mary	C	Biology	19	Full-time
Nancy	B	Biology	19	Full-time
Paul	B	Business	21	Part-time

function of this equivalence relation: 1 indicates that the associated pair of students are equivalent in terms of their grades, 0 means that they are not equivalent.

The same equivalence may also be expressed, perhaps more vividly, by the diagram in Fig. 2.2a, in which nodes (circles) represent elements of T (students) and their connections represent pairs that are contained in R_g. For convenience, the diagram is simplified. The connection from each node to itself (as required by

Table 2.2. Equivalence Relation R_g Defined on the Set of Students Listed in Table 2.1 with Respect to Their Grades

R_g	A	B	C	D	G	J	L	M	N	P
A	1	0	0	0	0	0	1	0	1	1
B	0	1	1	0	0	0	0	1	0	0
C	0	1	1	0	0	0	0	1	0	0
D	0	0	0	1	1	1	0	0	0	0
G	0	0	0	1	1	1	0	0	0	0
J	0	0	0	1	1	1	0	0	0	0
L	1	0	0	0	0	0	1	0	1	1
M	0	1	1	0	0	0	0	1	0	0
N	1	0	0	0	0	0	1	0	1	1
P	1	0	0	0	0	0	1	0	1	1

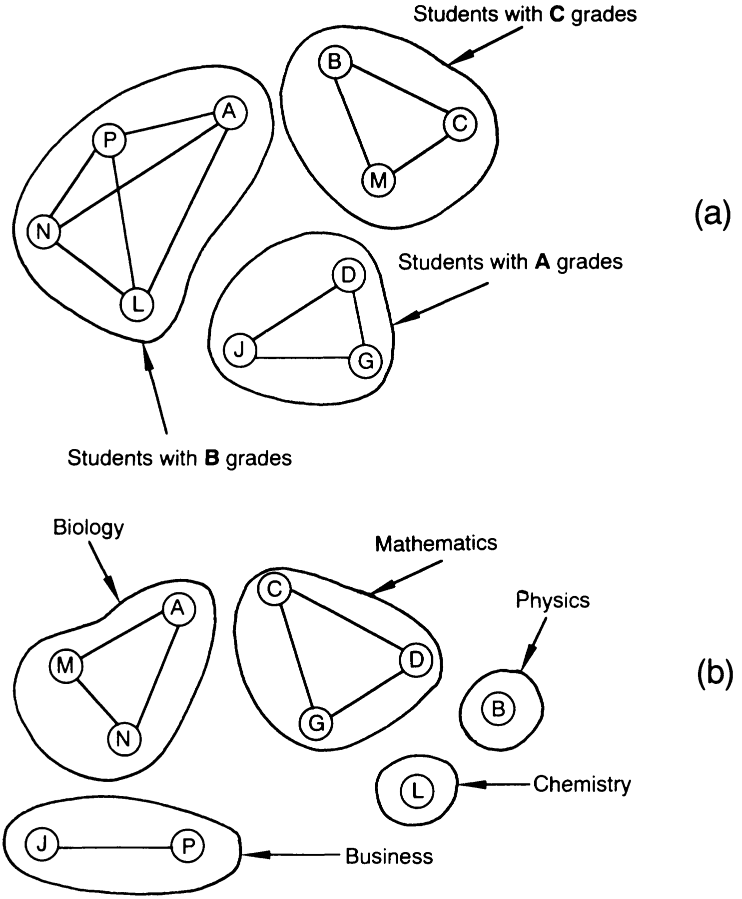

Figure 2.2. Equivalence relations defined on the set of students listed in Table 2.1. (a) R_g; (b) R_m.

reflexivity) is omitted and connections between nodes are not directed and represent thus both directions (as required by symmetry).

A diagram of another equivalence relation defined on the set of students, one based on their major field of study, R_m, is shown in Fig. 2.2b.

We can see that both equivalence relations expressed in Fig. 2.2 (R_g and R_m) partition in distinct ways the set of students into subsets such that students in each subset are related to each other while they are not related to students in any of the other subsets. These subsets are called *equivalence classes*. Each of them consists of students who are equivalent in terms of the relevant characteristics. Thus, for example, students with A grades form an equivalence class based on R_g, while students majoring in biology form an equivalence class based on R_m. It is easy to see that each equivalence relation partitions the set on which it is defined into equivalence classes.

Consider now a relation defined on the set of students by the following criterion: student x is related to student y iff x does not differ from y in more than two of the four chosen characteristics. Let R_{2C} denote this relation. Since the relation is obviously reflexive and symmetric, it can be represented by the simple diagram in Fig. 2.3. We can see from the diagram that R_{2C} is not transitive. For example, $(N, M) \in R_{2C}$ and $(M, B) \in R_{2C}$, but $(N, B) \notin R_{2C}$. Hence, R_{2C} is a compatibility relation. In this case students are classified into six *compatibility classes* shown in Fig. 2.3. These classes overlap, contrary to equivalence classes induced by an equivalence relation.

Different types of relations can be defined on the set of students by either of the following criteria: x is related to y iff x is younger than y; or x is related to y iff x has a lower grade than y. It is easy to check that both relations are antireflexive, antisymmetric, and transitive. Hence, they are *strict orderings*. Consider one additional criterion: x is related to y iff the grade of x is lower than or equal to the grade of y. The relation based on this criterion is clearly reflexive, antisymmetric, and transitive and, hence, it is a *partial ordering*.

When we deal with infinite sets, such as those illustrated in Fig. 2.1, each relation is again defined in terms of a chosen criterion by which it is decided for each element (a point in this case) of the relevant Cartesian product whether it is or it is not included in the relation. Examples of six relations on the Cartesian product $A_x \times A_y$ (as defined in Fig. 2.1) are shown in Fig. 2.4. For each relation, the criterion upon which it is based is specified in the figure. The criterion for relation

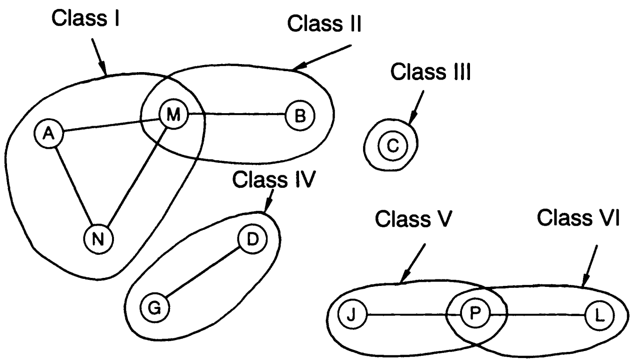

Figure 2.3. Compatibility relation R_{2C} defined on the set of students in Table 2.1 by the following criterion: two students are compatible if and only if they do not differ in more than two of the four characteristics.

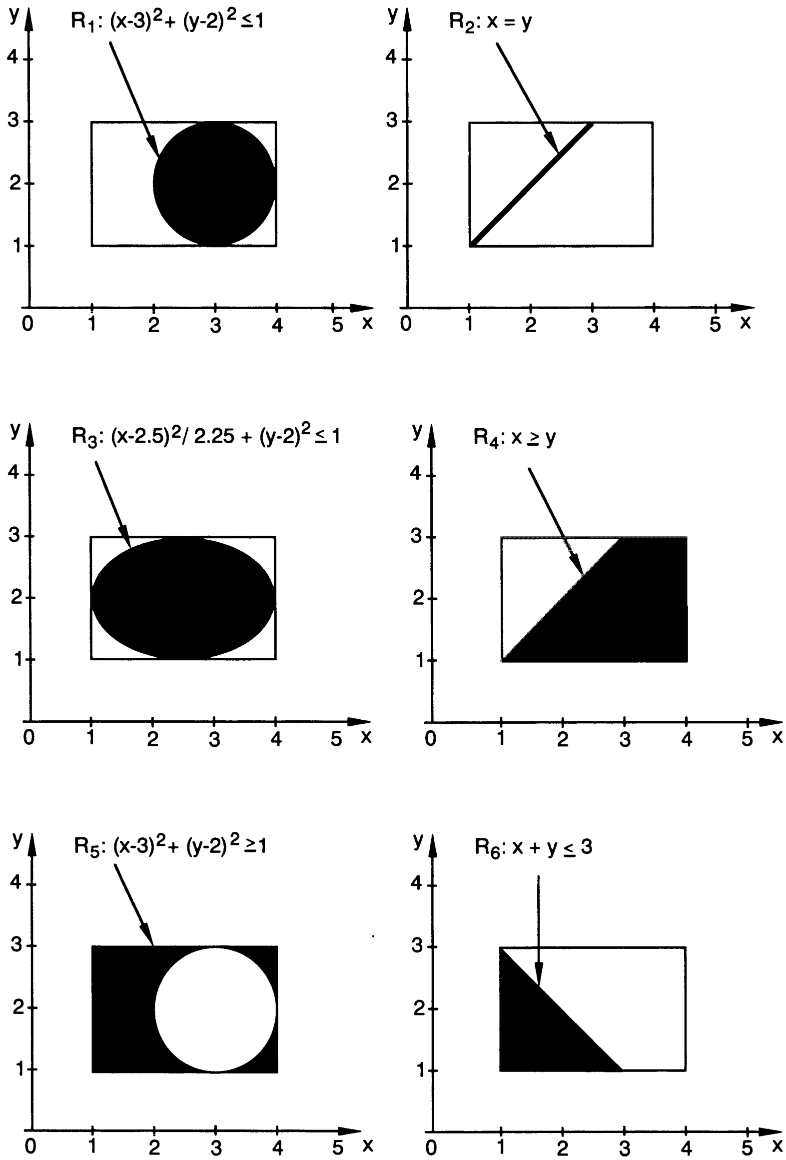

Figure 2.4. Examples of relations on $A_x \times A_y$ defined in Fig. 2.1.

R_1, for example, can be expressed as follows: a point $x \in A_x$ is related to point $y \in A_y$ iff the inequality

$$(x-3)^2 + (y-2)^2 \leq 1$$

holds. Criteria for $R_2 - R_6$ can be expressed in a similar way.

To close this section, let us illustrate how the common-sense definition of systems can be used to determine if a given object is a system. As an example, let us consider a discrete random variable, a variable that takes values in the set $V = \{1, 2, \ldots, n\}$ (for some integer $n \geq 2$) with probabilities $p(v)$ for all $v \in V$. Given a random variable, is it a system? The answer is affirmative if we can express the random variable in terms of the common-sense definition of systems. It is obvious that this can be done. Any random variable can be expressed as a system of the form

$$S = (T = \{V, [0, 1]\}, R \subseteq V \times [0, 1]).$$

Consider now a finite-state machine. It consists of

- a set of input states X
- a set of output states Y
- a set of internal states Z
- an output function f such that $y = f(x, z)$, where $x \in X$, $y \in Y$, and $z \in Z$
- a state-transition function (next-state function) g such that $z' = g(x, z)$, where $x \in X$, $z \in Z$, and $z' \in Z$.

This can be expressed in the form

$$S = (T = \{X, Y, Z\}, R \subseteq X \times Z \times Y \times Z).$$

Hence, any finite-state machine is a system.

2.3. Constructivism versus Realism

Although the common-sense conception of systems allows us to recognize a system, when one is presented to us, it does not help us to construct it. Whence do systems arise? To address this question, let me begin with some relevant thoughts offered by Brian Gaines [1979]:

> *Definition:* A system is what is distinguished as a system. At first sight this looks to be a nonstatement. Systems are whatever we like to distinguish as systems. Has anything been said? Is there any possible foundation here for a systems science? I want to answer both these questions affirmatively and show that this definition is full of content and rich in its interpretation. Let me first answer one obvious objection to the definition above and turn it to my advantage. You may ask, "What is peculiarly systemic about this definition?

Could I not equally well apply it to all other objects I might wish to define?" i.e., "A *rabbit* is what is distinguished as a rabbit." "Ah, but," I shall reply, "my definition is adequate to define a system but yours is not adequate to define a rabbit." In this lies the essence of systems theory: that to distinguish some entity as being a system is a necessary and sufficient criterion for its being a system, and this is uniquely true for systems. Whereas to distinguish some entity as being anything else is a necessary criterion to its being that something but not a sufficient one.

More poetically we may say that the concept of a system stands at the supremum of the hierarchy of being. That sounds like a very important place to be. Perhaps it is. But when we realize that getting there is achieved through the rather negative virtue of not having any further distinguishing characteristics, then it is not so impressive a qualification. I believe this definition of a system as being that which uniquely is defined by making a distinction explains many of the virtues, and the vices, of systems theory. The power of the concept is its sheer generality; and we emphasize this naked lack of qualification in the term *general systems theory*, rather than attempt to obfuscate the matter by giving it some respectable covering term such as *mathematical* systems theory. The weakness, and paradoxically the prime strength, of the concept is in its failure to require further distinctions.

It is a weakness when we fail to recognize the significance of those further distinctions to the subject matter in hand. It is a strength when those further distinctions are themselves unnecessary to the argument and only serve to obscure a general truth through a covering of specialist jargon. No wonder general systems theory is subject to extremes of vilification and praise. Who is to decide in a particular case whether the distinction between the baby and the bath water is relevant to the debate?

What then of some of the characteristics that we do associate with the notion of a system, some form of coherence and some degree of complexity? The *Oxford English Dictionary* states that a *system* is "a group, set or aggregate of things, natural or artificial, forming a connected or complex whole." I would argue that any other such characteristics arise out of the process of which making a distinction is often a part, and are some form of post hoc rationalization of the distinction we have made. One set of things is treated as distinct from another and it is that which gives them their coherence; it is that also which increases their complexity by giving them one more characteristic than they had before—that they have now been distinguished. Distinguish the words on this page that contain an "e" from those which do not. You now have a "system" and you can study it and rationalize why you made that distinction, how you can explain it, why it is a useful one. However, none of your postdistinction rationalizations and studies of the "coherency" and "complexity" of the system you have distinguished is intrinsically necessary to it being a "system." They are just activities that naturally follow on from making a distinction when we take note that we have done it and want to "explain" to ourselves, or others, why.

The point made by Gaines in this interesting discussion is that we should not expect that systems can be discovered, ready made for us. Instead, we should recognize that systems originate with us, human beings. We construct them by making appropriate distinctions, be they made in the real world by our perceptual capabilities or conceived in the world of ideas by our mental capabilities.

These sentiments are echoed and articulated with remarkable clarity by Goguen and Varela [**1979**]:

> A *distinction* splits the world into two parts, "that" and "this," or "environment" and "system," or "us" and "them," etc. One of the most fundamental of all human activities is the making of distinctions. Certainly, it is the most fundamental act of system theory, the very act of defining the system presently of interest, of distinguishing it from its environment.
>
> The world does not present itself to us neatly divided into systems, subsystems, environments, and so on. These are divisions which we make ourselves, for various purposes, often subsumed under the general purpose evoked by saying "for convenience." It is evident that different people find it convenient to divide the world in different ways, and even one person will be interested in different systems at different times, for example, now a cell, with the rest of the world its environment, and later the postal system, or the economic system, or the atmospheric system.
>
> The established scientific disciplines have, of course, developed different preferred ways of dividing the world into environment and system, in line with their different purposes, and have also developed different methodologies and terminologies consistent with their motivation.

All these considerations are extremely important for proper understanding of the nature of systems, at least as I conceive them, and consequently, as they are viewed in this book. According to this view, *systems do not exist in the real world independent of the human mind.* They are created by the acts of making distinctions in the real world or, possibly, in the world of ideas. Every act must be made by some agent, and, of course, the agent is in this case the human mind, with its perceptual and mental capabilities.

The view just stated is usually referred to as the *constructivist view* of reality and knowledge, or *constructivism.** The most visible contemporary proponent of this view, particularly well recognized within the systems science and cognitive science communities, is Ernst von Glasersfeld. To illuminate the constructivist view

*The founder of constructivism is generally considered Giambattista Vico (1668–1744), an Italian philosopher. The principal ideas from which constructivism emerged are presented in his early work, *On the Most Ancient Wisdom of the Italians* (Cornell University Press, Ithaca, New York, 1988), which was originally published in Italian in 1710. Perhaps the most visible contributor to constructivism in this century is Jean Piaget. Basic ideas of constructivism are well overviewed in some writings by Glasersfeld [1987, **1990**, 1995]. Arguments supporting constructivism, primarily of biological nature, are well presented by Maturana and Varela [1987].

a little more, let me use a short quotation from his many writings on constructivism [Glasersfeld, 1987]:

> Quite generally, our knowledge is useful, relevant, viable, or however we want to call the positive end of the scale of evaluation, if it stands up to experience and enables us to make predictions and to bring about or avoid, as the case may be, certain phenomena (i.e., appearance, events, experiences). If knowledge does not serve that purpose, it becomes questionable, unreliable, useless, and is eventually devaluated as superstition. That is to say, from the pragmatic point of view, we consider ideas, theories, and "laws of nature" as structures which are constantly exposed to our experiential world (from which we derived them), and they either hold up or they do not. Any cognitive structure that serves its purpose in our time, therefore, proves no more and no less than just that—namely, given the circumstances we have experienced (and determined *by* experiencing them), it has done what was expected of it. Logically, that gives us no clue as to how the "objective" world might be; it merely means that we know *one* viable way to a goal that we have chosen under specific circumstances in our experiential world. It tells us nothing—and cannot tell us anything—about how many other ways there might be, or how that experience which we consider the goal might be connected to a world *beyond* our experience. The only aspect of that "real" world that actually enters into the realm of experience, are its constraints. . . .
>
> Radical constructivism, thus, is *radical* because it breaks with convention and develops a theory of knowledge in which knowledge does not reflect an "objective" ontological reality, but exclusively an ordering and organization of a world constituted by our experience.

To avoid any confusion, let me emphasize that the constructivist view does not imply that the existence of the real world independent of the human mind is necessarily denied. This is a different issue, on which constructivism remains neutral. The constructivist view, at least from my perspective, is not an ontological view (concerned with the existence and ultimate nature of reality), but an epistemological view (concerned with the origin, structure, acquisition, and validity of knowledge).

The essence of constructivism is well captured by the following four quotes. The first is due to Giambattista Vico, the founder of constructivism:

> God is the artificer of Nature,
> man the god of artifacts.

The second quote is from a book by Stephane Leduc [1911]:

> Classes, divisions, and separations are all artificial,
> made not by nature but by man.

The third quote is due to Humberto R. Maturana, an important contributor to modern systems thinking:

> I maintain that all there is is that which the observer brings forth in his or her distinctions. We do not distinguish what is, but what we distinguish is.

The last quote is from a book by Maturana and Varela [**1987**], in which they bring forth convincing biological arguments in support of constructivism:

> All doing is knowing, and
> all knowing is doing.

Although the constructivist view may not be typical in classical science (at least not yet), it is undoubtedly the predominant view in contemporary systems science. Indeed, most writings on various aspects of systems science are either openly supportive of the view or, at least, compatible with it. My commitment to the constructivist view in this book reflects, therefore, not only my personal conviction, but also the mainstream of contemporary systems science.

The basic position regarding systems that I take in this book can thus be summarized as follows: Every system is a construction based upon some world of experiences, and these, in turn, are expressed in terms of purposeful distinctions made either in the real world or in the world of ideas.

Given some world of experiences, systems may be constructed in many different ways. Each construction employs some distinctions as primitives and others for characterizing a relation among the primitives. The former distinctions represent thinghood, while the latter represent systemhood of the system constructed.

The chosen primitives may be not only simple distinctions, but also sets of distinctions or even systems constructed previously. To illustrate this point, let all words printed on this page be taken as primitives and those that are verbs be distinguished from those that are not. Clearly, the primitives themselves are in this case rather complex systems, based upon many distinctions (visual, grammatical, semantic), but all these finer distinctions clustered around each word are taken for granted and left in the background when we choose to employ the words as primitives in a larger system. We consider the words as things whose recognition is assumed, and focus on the extra distinction by which verbs are distinguished from other words. This distinction imposes an equivalence relation on the set of words, which partitions the set into two equivalence classes, the class of all verbs and the class of all nonverbs on this page.

Constructivism is one of two opposing views about the nature of systems. The other view is usually referred to as *realism.*

According to realism, each system that is obtained by applying correctly the principles and methods of science *represents* some aspect of the real world. This representation is only approximate, due to limited resolution of our senses and measuring instruments, but the relation comprising the system is the *homomorphic image* (see Sec. 5.2) of its counterpart in the real world. When we use more refined instruments, the homomorphic mapping between entities of the system of concern and those of its real-world counterpart (the corresponding "real system") becomes also more refined, and the system becomes a better representation of its real-world counterpart.

Realism thus assumes the existence of systems in the real world, which are usually referred to as "real systems." It claims that any system obtained by sound scientific inquiry is an approximate (simplified) representation of a "real system" via an appropriate homomorphic mapping.

According to constructivism, all systems are artificial abstractions. They are not made by nature and presented to us to be discovered, but we construct them by our perceptual and mental capabilities with the domain of our experiences. The concept of a system that requires correspondence to real world is illusory because there is no way of checking such correspondence. We have no access to the real world except through experience.

The constructivist position liberates us from the commitment (inherent in realism) of viewing systems we deal with as models of "real systems." This commitment is vacuous since we have no access to the original system in this modeling relationship—the presumed "real system." Hence, we could define the homomorphic mapping between the two systems (assuming the existence of the "real system") only if we were omnipotent. Then, however, we would not have to worry about epistemology at all. This issue is discussed in more detail in Chapter 5.

2.4. Classification of Systems

Although much more could be said about the common-sense conception of systems (see, e.g., a thorough discussion by Marchal [1975]), I believe that enough has already been said here for our purpose. To make the concept of system useful, the common-sense definition must be refined in the sense that specific classes of ordered pairs (T, R), relevant to recognized problems, must be introduced. This can be done in one of two fundamentally different ways:

a. By restricting T to certain kinds of things;
b. By restricting R to certain kinds of relations.

Although the two types of restrictions are independent of each other, they can be combined.

Restrictions of type (a) are exemplified by the traditional classification of science into disciplines and specializations, each focusing on the study of certain kinds of things (physical, chemical, biological, economic, social, etc.) without committing to any particular kind of relations. Since different kinds of things are based on different types of distinctions, they require the use of different senses or measuring instruments and techniques. Hence, this classification is essentially experimentally based.

Restrictions of type (b) lead to fundamentally different classes of systems, each characterized by special kinds of relations, with no commitment to any particular kind of things on which the relations are defined. Since systems characterized by different types of relations require different theoretical treatment, this classification, which is fundamental to systems science, is predominantly theoretically based.

A prerequisite for classifying systems by their systemhood properties is a conceptual framework within which these properties can properly be codified. Each framework determines the scope of systems conceived. It captures some basic categories of systems, each of which characterizes a certain type of knowledge representation, and provides a basis for further classification of systems within each category. To establish firm foundations of systems science, a comprehensive framework is needed to capture the full scope of systemhood properties.

The issue of how to form conceptual frameworks for codifying systemhood properties, and the associated issue of how to classify systems by their systemhood properties are addressed in Chap. 4. Without waiting for the full discussion of these issues, however, we are able to introduce one rather important systemhood-dependent and thinghood-independent classification of systems right now, using only the common-sense definition of systems. This classification is based upon an important equivalence relation defined on the set of all systems of interest, say all systems captured by the common-sense definition. According to this relation, which is called an *isomorphic relation*, two systems are considered equivalent if their systemhood properties are totally preserved under some suitable transformation from the set of things of one system into the set of things of the other system.

To illustrate the notion of isomorphic systems, let us consider two systems, $\mathbf{S}_1 = (T_1, R_1)$ and $\mathbf{S}_2 = (T_2, R_2)$. Assume, for the sake of simplicity, that T_1, T_2 are single sets and $R_1 \subset T_1 \times T_1$, $R_2 \subset T_2 \times T_2$. Then $\mathbf{S}_1$ and $\mathbf{S}_2$ are called *isomorphic systems* if and only if there exists a transformation from T_1 to T_2 expressed in this case by a bijective (one-to-one) function h: $T_1 \to T_2$, under which things that are related in R_1 are also related in R_2 and, conversely, things that are related in R_2 are also related in R_1. Formally, systems $\mathbf{S}_1$ and $\mathbf{S}_2$ are isomorphic if and only if, for all $(x_1, x_2) \in T_1 \times T_1$,

$$(x_1, x_2) \in R_1 \text{ implies } [h(x_1), h(x_2)] \in R_2$$

and, for all $(y_1, y_2) \in R_2$,

$$(y_1, y_2) \in R_2 \text{ implies } [h^{-1}(y_1), h^{-1}(y_2)] \in R_1,$$

where h^{-1} denotes the inverse of function f. This definition is illustrated in Fig. 2.5. It must be properly extended when T_1, T_2 are families of sets and R_1, R_2 are not binary but n-dimensional relations with $n > 2$.

The notion of isomorphic systems imposes a binary relation on $\mathcal{S} \times \mathcal{S}$, where $\mathcal{S}$ denotes the set of all systems. Two systems, $\mathbf{S}_1$ and $\mathbf{S}_2$, are related by this relation (the pairs $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_2$, $\mathbf{S}_1$ are contained in the relation) if and only if they are isomorphic.

It is easy to verify that the *isomorphic relation* (or *isomorphism*) is reflexive, symmetric, and transitive. Consequently, it is an equivalence relation defined on the set of all systems (or on its arbitrary subset), which partitions the set into equivalence classes. These are the smallest classes of systems that can be distinguished from the standpoint of systemhood. In fact, we may view each of these equivalent classes as being characterized by a unique relation, defined on some particular set of things, which is freely interpreted in terms of other sets of things within the class. This unique relation may be taken as a canonical representative of the class.

Which set of things should be chosen for these canonical representations? Although the choice is arbitrary, in principle it is essential that the same selection criteria be used for all isomorphic classes. Otherwise, the representatives would not be compatible and, consequently, it would be methodologically difficult to deal with them. Therefore, it is advisable to define the representatives as systems whose

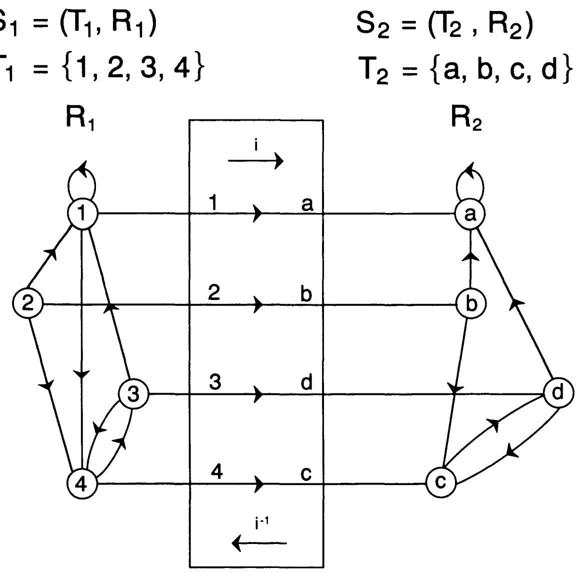

Figure 2.5. An example of isomorphic systems.

sets of things consist of some standard symbols that are abstract (interpretation-free), such as integers or real numbers, and whose relations are described in some convenient standard form.

Since the canonical representatives of isomorphic equivalence classes of systems are devoid of any interpretation, it is reasonable to call them general systems. Hence, *a general system is a standard and interpretation-free system chosen to represent a particular equivalence class of isomorphic systems.*

Observe that, according to this definition, each isomorphic equivalence class may potentially contain an infinite number of systems, owing to the unlimited variety of thinghood, but it contains only one general system. The number of isomorphic equivalence classes and, thus, the number of general systems, may also be potentially infinite owing to the unlimited variety of systemhood.

Each isomorphic equivalence class of systems contains not only a general system and its various interpreted systems, but also other abstract systems, different from the general system. The isomorphic transformations between the general system and the other abstract systems (described by the bijective function h) are rather trivial. They are just arbitrary replacements of one set of abstract symbols with another set. It seems appropriate to call these transformations *relabelings.*

The transformations between a general system and its various interpreted systems are by far not trivial since they involve different types and distinctions made in the real world, and these, in turn, are subject to different constraints of the real world. The isomorphic transformation from an interpreted system into the corresponding general system, which may be called an *abstraction*, is always possible.

The inverse transformation, which may be called an *interpretation*, is not guaranteed and must be properly justified in each case. Indeed, relations among things based upon distinctions made in the real world cannot be arbitrary, but must reflect genuine constraints of the real world, as represented in our world of experiences. Hence, each interpreted system determines uniquely its representative general systems by the isomorphic transformation, but not the other way around.

The independence (or orthogonality) of the two ways of classifying systems, by thinghood and by systemhood, is visually expressed in Fig. 2.6. This figure also illustrates the role of general systems and their connection to other abstracted systems and to interpreted systems.

The two dimensions of science, which reflect the two-dimensional classification of systems symbolized by Fig. 2.6, are complementary. When combined in scientific inquiries, they are more powerful than either of them alone. The traditional perspective of classical science provides a meaning and context to each inquiry. The perspective of systems science, on the other hand, provides a means for dealing with any desirable system, regardless of whether or not it is restricted to a particular discipline of classical science.

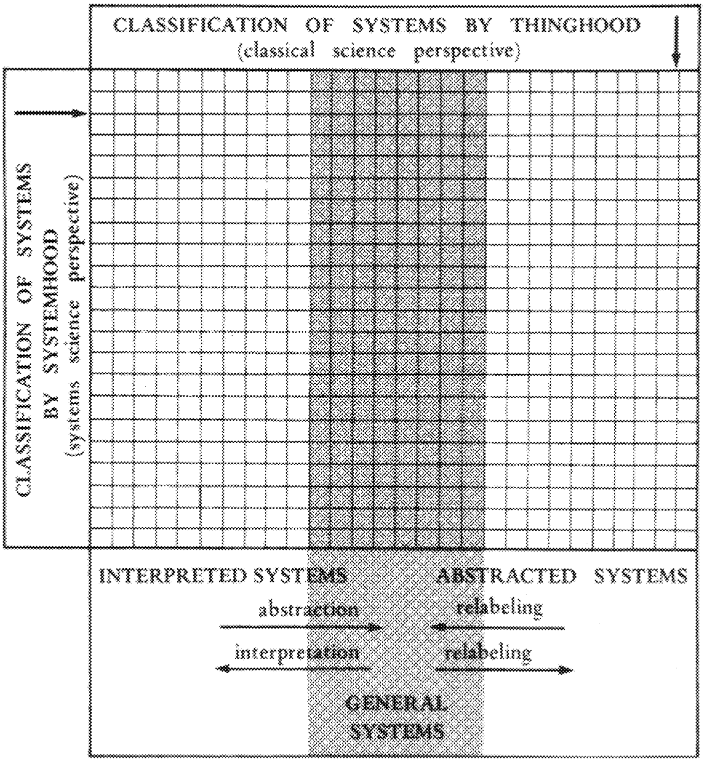

Figure 2.6. Two ways of classifying systems and the role of general systems.

Exercises

2.1. How many possible relations can be defined on each of the following Cartesian products of finite sets?

(a) $A \times B^2 \times C^3$;

(b) $(A \times B \times C)^2$

(c) $\mathcal{P}(A) \times (\mathcal{P}(B))^2 \times (\mathcal{P}(B))^3$;

(d) $A^2 \times \mathcal{P}(B) \times C \times \mathcal{P}(D)$;

(e) $\mathcal{P}(\mathcal{P}\_\mathcal{P}(A))) \times \mathcal{P}(\mathcal{P}(B))$;

(f) $(A \times B) \times (B \times C) \times (D \times E \times F)$;

(g) $(A \times B) \times \mathcal{P}(\mathcal{P}(C)) \times D^2$.

2.2. For each of the following relations, determine whether or not it is reflexive, antireflexive, symmetric, antisymmetric, or transitive.

(a) $R \subseteq \mathcal{P}(X) \times \mathcal{P}(X)$: $(A, B) \in R$ iff $A \subseteq B$ for all $A, B \in \mathcal{P}(X)$;

(b) $R \subseteq C \times C$, where C denotes the set of courses in a graduate program: $(a, b) \in R$ iff course a is a prerequisite of course b;

(c) $R_n \subseteq \mathbb{N} \times \mathbb{N}$, where $\mathbb{N}$ denotes the set of all natural numbers: $(a, b) \in R$ iff the remainders obtained by dividing a and b by n are the same, where n is some specific natural number greater than 1;

(d) $R \subseteq W \times W$, where W denotes the set of all English words: $(a, b) \in R$ iff a is a synonym of b;

(e) $R \subseteq F \times F$, where F denotes the set of all five-letter English words: $(a, b) \in R$ iff a differs from b in at most one position;

(f) $R \subseteq A \times A$: $(a, b) \in R$ iff $f(a) = f(b)$, where f is a function of the form

$$f\colon A \to A;$$

(g) $R \subseteq T \times T$, where T denotes all persons included in a family tree: $(a, b) \in R$ iff a is an ancestor of b;

(h) $R = \{(0, 0), (0, 1), (1, 0), (1, 1), (1, 2), (2, 2), (0, 2), (3, 3)\}$;

(j) $R = \{(0, 0), (0, 3), (1, 1), (2, 2), (1, 0), (0, 1), (3,1), (3, 3), (3,0)\}$.

2.3. A finite Markov chain is a system based on a finite set of states, A, in which the next state $a' \in A$ is determined by the present state $a \in A$ via conditional probability $p(a'|a)$, where

$$\sum_{a' \in A} p(a'|a) = 1$$

for each $a \in A$. Express the Markov chain based on state set A and conditional probabilities $p(a'|a)$ for all $a \in A$ and $a' \in A$ in terms of the common-sense definition of systems, $S = (T, R)$.

2.4. Consider an archive that contains a set of documents, D, each of which is characterized by a set of relevant index terms (keywords) taken from a set of all index terms employed, I. Describe the archive as a system $S = (T, R)$.

2.5. The Shannon (probabilistic) finite-state machine is a quadruple

$$M = (X, Y, Z, P),$$

where X, Y, Z are finite sets of input states, output states, and internal states, respectively, and P is a set of conditional probabilities

$$p(y_t, z_{t+1}|x_t, z_t),$$

where x_t, y_t, z_t denote, respectively, input, output, and internal states at time t, and z_{t+1} denotes an internal state at time $t + 1$. Show that the Shannon machine is a system since it can be expressed in terms of the common-sense definition

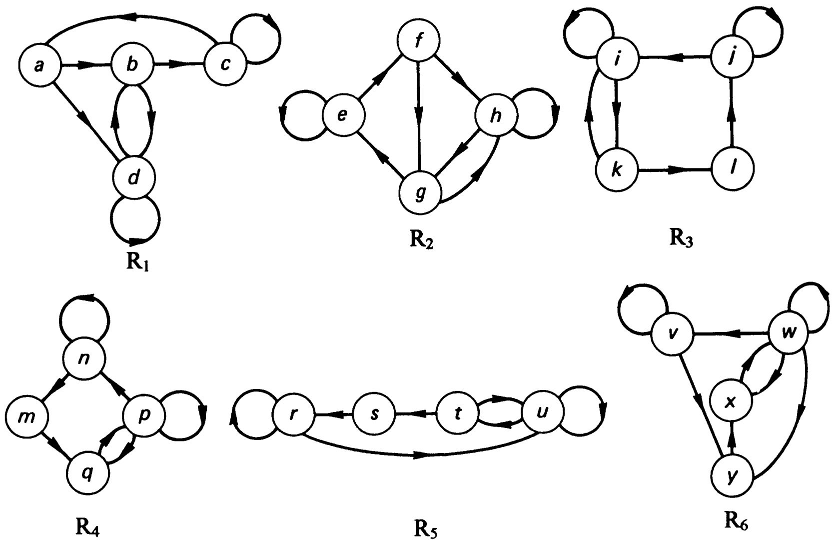

Figure 2.7. Relations in Exercise 2.8.

$$S = (T, R).$$

2.6. Given two systems S_1 and S_2 of the form

$$S_1 = (\{A_1, A_2, A_3\}, R_1 \subseteq A_1 \times A_2 \times A_3),$$

$$S_2 = (\{B_1, B_2, B_3\}, R_2 \subseteq B_1 \times B_2 \times B_3),$$

define the conditions under which these systems are isomorphic.

2.7. Show that the system based on the set of all letters of the English alphabet and their alphabetical order and the system based on the set $(10, 11, \ldots, 35\}$ with the usual numerical ordering are isomorphic.

2.8. Determine which pairs of relations defined by their diagrams in Fig. 2.7 are isomorphic and specify for each pair the function h under which the relations are isomorphic.