Key Mathematical Symbols Guide for Systems Science and Discrete Mathematics

Prepared for Independent Study

Contents

1   Set Theory and Logic (Foundational)

1.1   Basic Set Notation

- $\in$: "is an element of"

- $\notin$: "is not an element of"

- $\subseteq$: "is a subset of"

- $\subset$: "is a proper subset of"

- $\cup$: union

- $\cap$: intersection

- $\setminus$: set difference

- $\emptyset$: empty set

- $|A|$: cardinality (size) of set A

1.2   Set Construction

- $\{x : P(x)\}$: set builder notation

- $\{a, b, c\}$: set enumeration

- $[n] = \{1, 2, \ldots, n\}$: set of first n positive integers

1.3   Power Set

- 2^X : power set of X (set of all subsets)

1.4 Logic Symbols

- $\forall$: universal quantifier ("for all")

- $\exists$: existential quantifier ("there exists")

- $\land$: logical AND

- $\lor$: logical OR

- $\neg$: logical NOT

- $\Rightarrow$: implication ("if...then")

- $\Leftrightarrow$: equivalence ("if and only if")

2 Relations and Functions

2.1 Basic Relation Notation

- $R \subseteq A \times B$: relation R from A to B

- aRb : a is related to b under relation R

- $\langle a, b \rangle$: ordered pair

2.2 Function Notation

- $f : A \to B$: function from A to B

- $f(x)$: value of function f at x

2.3 Special Relations

- $\leq$: less than or equal to

- $<$: less than

- $\geq$: greater than or equal to

- $>$: greater than

- $=$: equal to

- $\neq$: not equal to

- $\approx$: approximately equal to

- $\sim$: equivalence relation

3 Graph Theory

3.1 Basic Graph Notation

- $G = (V, E)$: graph G with vertex set V and edge set E

- $\{u, v\}$: undirected edge between vertices u and v

- (u, v) : directed edge from u to v

3.2 Graph Properties

- $\deg(v)$: degree of vertex v

- $|V|$: number of vertices

- $|E|$: number of edges

3.3 Special Graphs

- K_n : complete graph on n vertices

- C_n : cycle graph on n vertices

- P_n : path graph on n vertices

4 Linear Algebra

4.1 Vector Notation

- $\vec{x} = \langle x_1, x_2, \ldots, x_n \rangle$: vector

- $\|\vec{x}\|$: norm (length) of vector $\vec{x}$

4.2 Matrix Notation

- $A = [a_{ij}]$: matrix A with elements a_{ij}

- A^T : transpose of matrix A

- A^{-1} : inverse of matrix A

4.3 Matrix Operations

- $A + B$: matrix addition

- AB : matrix multiplication

- $\det(A)$: determinant of matrix A

5 Order Theory

5.1 Partial Orders

- $\leq$: partial order relation

- $<$: strict partial order relation

5.2 Lattice Notation

- $\vee$: join (least upper bound)

- $\wedge$: meet (greatest lower bound)

- $\top$: top element

- $\bot$: bottom element

6 Advanced Set Theory and Algebra

6.1 Cartesian Product

- $A \times B$: Cartesian product of sets A and B

6.2 Boolean Algebra

- $\oplus$: exclusive OR (XOR)

- $\odot$: equivalence (XNOR)

6.3 Group Theory

- e : identity element

- a^{-1} : inverse of element a

7 Systems and Network Theory

7.1 System Notation

- $S = (X, Y, f)$: system with input set X, output set Y, and function f

7.2 Network Flow

- $c(u, v)$: capacity of edge (u, v)

- $f(u, v)$: flow on edge (u, v)

7.3 Hypernetworks

- $H = (V, E)$: hypergraph with vertex set V and hyperedge set E

8 Blockchain-Specific Notation

8.1 Blockchain Elements

- B : set of all blocks

- T : set of all transactions

- A : set of all addresses

8.2 Blockchain Functions

- $\text{prev}(b)$: previous block of block b

- $\text{tx}(b)$: set of transactions in block b

- $\text{sender}(t)$: sender of transaction t

- $\text{receiver}(t)$: receiver of transaction t