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Robert Rosen and Relational System Theory: An Overview

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ROBERT ROSEN AND RELATIONAL SYSTEM THEORY: AN OVERVIEW

by

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ABSTRACT

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Relational system theory is the science of organization and function. It is the study of how systems are organized which is based on their functions and the relations between their functions. The science was originally developed by Nicolas Rashevsky, and further developed by Rashevsky's student Robert Rosen, and continues to be developed by Rosen's student A. H. Louie amongst others. Due to its revolutionary character, it is often misunderstood, and to some, controversial. We will mainly be focusing on Rosen's contributions to this science. The formal and conceptual setting for Rosen's relational system theory is category theory. Rosen was the first to apply category theory to scientific problems, outside of pure mathematics, and the first to think about science from the point of view of category theory. We will provide an overview of Rosen's theory of modeling, complexity, anticipation, and organism. We will present the foundations of this science and the philosophical motivations behind it along with conceptual clarification and historical context. The purpose of this dissertation is to present Rosen's ideas to a wider audience.

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CONTENTS

For Maki, Machi, Cosmo

Introduction

It has been almost sixty-two years since the first publishing of C. P. Snow's Rede Lecture, *The Two Cultures*. Since then, not much seems to have changed. Perhaps things have even gotten worse. In the lecture Snow argues that the intellectual life of modern society has splintered into two different groups. There are the scientists; in particular the physical scientists. And then there are the humanities; in particular, what Snow calls the "literary intellectuals'' and these, he says, are the most influential in our "traditional" culture. The problem is, as he states it, "that neither the scientific system of mental development, nor the traditional, is adequate for our potentialities, for the work we have in front of us, for the world in which we ought to live."[1]

The problem is exacerbated, Snow argues, since neither of the two cultures has the means or desire to communicate with the another. We are each ignorant of what the other does. We are even hostile towards one another; you may recall the *science wars* that began in the 1990s. Sometimes, Snow is a rather harsh critic of those of us in the humanities:

> A good many times I have been present at gatherings of people who, by the standards of the traditional culture, are thought highly educated and who have with considerable gusto been expressing their incredulity at the illiteracy of the scientists. Once or twice I have been provoked and have asked the company how many of them could describe the Second Law of Thermodynamics. The response was cold: it was also negative. Yet I was asking something which is the scientific equivalent of: *Have you read a work of Shakespeare*? I now believe that if I had asked an even simpler question – such as, What do you mean by mass, or acceleration, which is the scientific equivalent of saying, Can you read? – not more than one in ten of the highly educated would have felt that I was speaking the same language. So the great edifice of modern physics goes up, and the majority of the cleverest people in

[1] Snow, *The Two Cultures*, 64.

> the western world have about as much insight into it as their Neolithic ancestors would have had.[2]

Snow even goes so far to call us in the humanities *intellectual Luddites*.

I too have come to find the problem of the two cultures unsettling. In my philosophical studies, as a graduate student, I was forced to come to terms with the other culture, the natural sciences. I felt I had no choice in the matter.

I entered the Comparative Literature program at the Graduate Center to study continental philosophy and the human condition as it manifests itself in the arts and literature. I was excited. In my first semester, I took a course titled History of Literary Theory and Criticism III with Dr. Giancarlo Lombardi. One of the works Dr. Lombardi assigned was Gilles Deleuze's and Felix Guattari's *A Thousand Plateaus, Capitalism and Schizophrenia*. At the time I found it intriguing so I decided to study it and more of the works of Deleuze. In Deleuze's *Difference and Repetition*, there is a passage in which he asserts that Alfred North Whitehead's *Process and Reality* is "one of the greatest books of modern philosophy."[3] This was quite a claim. I had never heard of it. I was in the library at the Graduate Center when I read the passage. I immediately ran down to a bookstore in Union Square and luckily, they had it, so I Picked up a copy, went back to the library, and began to read. After I finished, I was not sure if I understood one lick of it. It was formidable. I would later learn that some say it is the most difficult work in philosophy, but I found it fascinating. This was the beginning of my intense study of the philosophy of Whitehead which lasted for some years.

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[2] Ibid, 14 – 15.
[3] Deleuze, *Difference and Repetition*, 284 - 5.

The more I studied Whitehead the more I realized there was a problem that I was going to have to face in order to continue. Whitehead did not start doing philosophy proper until he was in his sixties. By trade, he was a mathematician and a theoretical physicist. He tells us that his entire metaphysics is based upon his understanding and interpretation of the implications of the mathematical physics of late 19th and early 20th centuries in which he was so well-versed. Unfortunately, I knew absolutely nothing about mathematical physics whatsoever, but I realized I would have to learn; at least to some extent. The problem got worse when I came across an essay by the physicist and philosopher Abner Shimony, "Quantum Physics and the Philosophy of Whitehead". In the essay, Shimony argues that there are certain properties of quantum systems (or better, the lack thereof) that call into question the very foundations of Whitehead's metaphysics. If this were true, it would be a devastating blow.

I began reading popular books on quantum physics. Not a good place to start. But then, shortly thereafter, I got a stroke of luck. I met a physics student in the lobby of the Graduate Center, Yonatan Ben, or Benben as he is called. I explained to him my problem and he was graciously willing to help. Beben had just started reading a book, *Philosophical Consequences of Quantum Theory*, *Reflections on Bell's Theorem*. It was a collection of papers that were presented at a conference which was held at the University of Notre Dame in October of 1987. He asked if I would like to read it with him and we could meet after reading each paper to discuss and he would take me through the physics. We started meeting in the science lab at the Graduate Center, right around the hall from the office for the Department of Comparative Literature. During one of our meetings, there was another physics student, Amol Deshmukh, who heard our discussion and became very intrigued.

Bell's theorem is concerned with what physicists call *foundational problems*. In general, physics courses do not usually concern themselves with such problems. After the three of us talked for some time, Amol suggested that we form a group and meet every week to continue to study foundational problems in physics in greater detail. And we did. We started meeting once a week, every Sunday, at three o'clock, in the science lab. We would pick papers, books, or chapters from books beforehand and then discuss them the next week. Shortly thereafter, another physics student, William Mayer became interested and joined the group.

At some point, I can't quite remember exactly when, I came across a book, *Quantum Expectations, Essays in Honour of David Bohm*. The physicists and I had studied a good bit of Bohm's work and he had become one of my favorite thinkers. I thought there might be some good papers in the book for us and I turned out to be right; we ended up studying at least five of the papers in the book.

I had read all of the papers in the book to see which ones would be of interest to the physicists and there was one, that at first, I did not share. It was an essay by Robert Rosen. Who was Robert Rosen? Never heard of him. The paper is titled "Some Epistemological Issues in Physics and Biology". As I read it, I couldn't believe my eyes. It contains the most revolutionary and original ideas I had ever come across. I could see an entirely new way of thinking and with it, an entirely new world. It is truism that physics is the science that concerns itself with the general study of nature. Biology, when compared to physics, is a marginal science since it concerns itself only with a very small portion of nature, the living. More, a lot of scientists, even a lot of biologists, would like to believe that biological phenomena can and will be eventually understood entirely though the means of contemporary physics. In the essay Rosen argues it is in fact the other way around: it is biology that is the more general science and it is contemporary physics that is marginal. "How could that be?" you might ask, "That makes no sense!", you might say. There's the rub.

"Who is this guy?". "How many more papers did he write about this stuff?". "Did he write books?". "If he wrote books that would be grand." After finishing the paper, I immediately did an internet search. Unfortunately, I found that Rosen passed in 1998 due to complications with diabetes. I found that Rosen did write and edit books, and that he had published hundreds of papers and essays. I decided to first read the last book he wrote before he passed, *Life Itself, A Comprehensive Inquiry Into the Nature, Origin, and Fabrication of Life*. It was everything I could have asked for and more. Reading it at first was brutal. It still is. I could follow some of the arguments and I could glimpse at some of its profundity. But the science, the math, a lot of really *abstract math*, a lot of *category theory*, I just did not know and could not follow. Not even the physicists knew this math. I was at a loss.

Earlier on, before coming across Rosen's work, I was walking to the science lab to talk with the physicists. In passing, coming towards me down the hallway, was Dr Rohit Parikh. I had the good fortune of having discussions with Dr. Parikh from time to time. Dr Parikh is a professor of philosophy, mathematics, and computer science at the Graduate Center and Brooklyn College. He was with a colleague of his that I did not know, Dr. Noson Yanofsky. Dr. Parikh kindly stopped me to introduce us. Dr. Yanofsky is a professor in the Department of Computer Science, both at the Graduate Center and Brooklyn College. In my hand was a copy of *Speakable and Unspeakable in Quantum Mechanics*, a collection of papers of the physicist J. S. Bell who I mentioned above. The physicists and I were studying Bell's papers. When he saw the book, Dr. Yanofsky remarked "that is not an easy read." "Yes", I said. Dr. Yanofsky then told me that he had recently written a book, *The Outer Limits of Reason, What Science, Mathematics, and Logic Cannot Tell Us*. In it, he said, was a chapter on quantum theory and Bell's theorem. He asked me to have a look if I like and to email him if I had any thoughts or questions. I did. His book became a favorite of mine and I use it often to teach my students.

Another stroke of luck: it turns out that Dr. Yanofsky is a category theorist. After struggling so much with *Life Itself*, I turned to Dr. Yanofsky. I asked him, how, God willing, could I learn category theory? "Here", he responded, "you start here". He showed me a copy of *Conceptual Mathematics, A first introduction to categories*, by William Lawvere and Stephen Shanuel. And so began my study of mathematics. Category theory is hard but it is very beautiful and philosophically rich. Outside of pure mathematics, it now has applications in physics, biology, computer science, even linguistics.

Gradually, I asked the physicists, if, from time to time, we could study some of Rosen's work together. It turned out that even for them, Rosen's work was difficult to understand. Nonetheless they were astonished. Even the way Rosen treats familiar concepts in physics is novel and takes some getting used to. Eventually, Benben and Amol graduated and moved on. William and I kept meeting. Together, we devoted some year and a half to the study of Rosen's work exclusively. We decided that even though I was a comparative literature major it would be best, if possible, to write my dissertation on Rosen and present his ideas to a more general audience. Then William graduated and moved on too. The meetings were over.

The physicists and I had formed the first, and I pray not the last, intellectual community of which I was proud and honored to be a member. Snow, and both scientists and my colleagues in the humanities alike may find it odd that they so willingly included me, someone so ignorant, in their studies of some of the most important discoveries and deepest problems in theoretical physics. All I can tell you is that I am grateful and that they taught me more than I could ever have dreamed. It is a beautiful thing.

Rosen's work is not well known, even in his own field of study, biology. It appears that not one course has ever been taught on his work exclusively in any biology department in the world. There are, however, a small number of scientists who do continue his work. But amongst those scientists who are aware of his work, there are also those who vehemently despise it. Part III of Rosen's *Essays on Life Itself*, published posthumously, is mainly a tribute to the great mathematician and natural philosopher René Thom. In it he writes:

> The appearance of René Thom's *Stabilité Structurelle et Morphogenèse* in 1972 (translated into English in 1976) was a watershed event in mathematics, in theoretical biology, and for the philosophy of science generally. The questions he raised in that book, both directly and by implication, were deeply disquieting to most practicing, empirical scientists, making their dogmatic slumbers untenable. The predictable responses took several forms: (1) outrage and indignation; (2) violent but irrelevant counterattacks; (3) pretense that Thom did not exist, and hence is ideas did not need be addressed at all; and (4) a distortion of his views into a benign soporific that would enable them to sleep again.[4]

I would argue that if you replace Thom's name with Rosen's and *Stabilité Structurelle et Morphogenèse* with "Rosen's work" you have an autobiography.

Rosen's first two published books were textbooks, *Optimality Principles in Biology* and *Dynamical System Theory in Biology*. He intended the latter to be the first of two volumes. The reviews for it were harsh. In response, he vowed to never publish another

[4] Rosen, *Essays on Life Itself*, 145.

textbook again. To be fair, *Dynamical System Theory in Biology* is full of errors, most unfortunately in the math. Rosen is a terrible proofreader and in many of his subsequent works there are errors abound. Nonetheless, the insights in both textbooks were either underappreciated or ignored.

All of Rosen's subsequent books are scientific monographs. For political reasons, and its controversial content, it took Rosen six years to find a publisher for his second monograph, *Anticipatory Systems, Philosophical, Mathematical and Methodological Foundations*:

> There were many people who, I should say, the ideas of functionality and ideas of anticipation raised hackles because, again, it was easy for them to feel it was a step away from science as they understood it. They felt that science was simplifying – dealing only with simple systems. And any attempt to complexity or any attempt to invoke a larger context – several larger contexts, in fact – was a step away from science that they wanted, or that they were seeking.[5]

Rosen was once accused of trying to find answers to questions that nobody wanted to ask. The most important question that motivates his entire life's work is the *Schrödinger question*, "What is life?". Or more, "What distinguishes a living system from a non-living one?", "What are the defining characteristics of a natural system for us to perceive it as being alive?". And as we shall see, he gave definitive answers.

Rosen tells us that as he got better at defending himself and putting forth his point of view fewer and fewer people would talk to him. In my studies of his work, I noticed a growing frustration with the orthodox scientific community overtime. From the late 1950s up until the 1980s, the presentation of his ideas is clear and congenial. From then on, while still

[5] Rosen, "The following is a transcript of a videotaped interview of Dr. Robert Rosen," 5.

optimistic about the direction and possibilities his ideas could take, the presentation is sometimes cryptic, and much more critical. In the opening paragraph for the preface of *Life Itself*, Rosen writes:

> For whom is this book intended? I do not know. The book itself has no pragmatic purpose for which I am aware. It is thus for anyone who wants to claim it.[6]

As said above, Rosen's work is difficult to understand. A good amount of his critics and commentators both misunderstand and mispresent his ideas. Most do also not take the pains to study enough of his work in the detailed manner it requires. This is even true for some of his more congenial commentators. To give just one example, I recall one such commentator lamenting that Rosen had not considered the relationship between his model for a living cell and what in mathematics are known as *adjoint functors*. Had the commentator read Rosen's most comprehensive essay on the mathematics involved in the model, "Some Relational Cell Models: The Metabolism—Repair Systems", on page 241 they would have seen:

> the two functors
>
> $$H(A, H(B, C)), H(AxB, C)$$
>
> mentioned above are also naturally equivalent (the proof is somewhat tedious, though not difficult). Indeed, the above natural equivalence, which indicates a deep relation between the Cartesian product and sets of mappings is the motivation for the important theory of *adjoint functors*, which may have important biological applications.[7]

Frustrating to say the least, but I have seen much worse.

[6] Rosen, *Life Itself*, xiii.
[7] Rosen, *Foundations of Mathematical Biology*, *Volume II*, 241.

In the future, it is my hope to make explicit, as much as possible, the philosophical implications of Rosen's work. His work offers vital alternative approaches and new insights to many philosophical problems. But before anyone including myself can do this, his work must be thoroughly understood. This is the purpose of this dissertation, to present some of Rosen's work in the clearest possible manner to a wider audience. Unfortunately, I could find no effective way for presenting his work in some kind of *linear* order; I am not sure there is one.

Unfortunately, there is some math. I have tried to avoid it as much as possible and when necessary to keep it as simple as possible, perhaps to the point of over-simplification; and all of this probably to the annoyance of the mathematicians. To them, I must apologize.

Chapter 1. Category Theory

1.1 An Historical Note

Rosen wanted to be a biologist even before he knew what the word meant:

> Among my earliest memories are walks though the wild and overgrown vacant lots which dotted the asphalt Brooklyn landscape into which I was born. Under every rock was a new and thrilling universe of living things. From these experiences was born an eternal passion, a lust, to understand why these things, in their separate ways, were alive, while the rock was not. The rocks were themselves mildly interesting, but in a bland, impersonal way; it was life which was the compelling challenge to me. If I could find out what life was, I would know what the rocks were, but as it even then seemed to me, not the other way around.[1]

At an early age Rosen began reading anything he could find dealing with life and the living. As a student at Stuyvesant High School, Rosen took courses in analytical and organic chemistry. His studies in chemistry led him to the study of physics. And then, as he puts it, "fatefully", his studies in physics led him to study mathematics, the language in which physics is expressed:

> I decided that henceforth, I must become proficient enough in that mathematical language to understand, to the root, what realities were being, or indeed could be, expressed through it. […] Up to that point, I had only the most perfunctory interest in the sciences of the inanimate; these were the rocks again, and not the life. Suddenly, it now seemed a matter of urgent necessity to master these things. To facilitate acquiring such a mastery, it seemed the most natural thing in the world to change my major. So I blithely shifted out of biology and into mathematics. It felt perfectly right to do so, and I regarded it as the merest tactical device in the service of the unchanging strategy I was groping for.[2]

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[1] Rosen, *Anticipatory Systems*, 422.
[2] Ibid, 423

Rosen decided to immerse himself entirely into the study of pure mathematics and mathematical physics with the instinct that therein must lie the secret to understand life and the living:

> I quickly came to recognize that my instincts had been correct; that the mathematical universe had much of value to offer me, which could not be acquired any other way. I saw that mathematical thought, though nominally garbed in syllogistic dress, was really about patterns; you had to learn to see the patterns though the garb. That was what they called "mathematical maturity". I learned that it was from patterns that the insights and theorems really sprang, and I learned to focus on the former rather than the latter.[3]

Rosen obtain a bachelor's degree in mathematics from Brooklyn College and within just a year, a master's degree in mathematics from Columbia University. Although, he initially entered the University of Chicago, to obtain a PhD in mathematics, he quickly changed his major and obtained a PhD in mathematical biology under the tutelage of Nicholas Rashevsky which we will have more to say about in Chapter 3.

It was at Columbia University that Rosen was first introduced to category theory. Category Theory was developed by Sanders Mac Lane and Samuel Eilenberg. The theory was first presented in their 1945 paper "General Theory of Natural Equivalences" which we will consider in a bit more detail below. Rosen took a course on abstract algebra with Eilenberg at Columbia University which turned out to really be a course on category theory in disguise. Rosen continued to study category theory with Mac Lane at the University of Chicago and it was at that time that Rosen made the revolutionary discovery that category theory "expressed in a purely mathematical realm the patterns of relations, between objects and

[3] Ibid, 423 – 424.

models, and between one model and another, which I was trying to find in the realm of the living."[4]

Rosen was the first to apply category theory to the natural sciences, outside of pure mathematics, beginning with his second published paper in 1959, "The representation of biological systems from the standpoint of the theory of categories". We will consider the content of this paper and its subsequent development in Chapter 6. More, as we shall see over and again, he was the first to *think* about science, or perhaps even better, *rethink* science, from the point of view of categories. Rosen recalls:

> I believe I was the first to suggest that category theory had important roles to play in the sciences. The realization that this was so actually began dawning on me a couple of years earlier. I remember well my suggesting this, with a great deal of diffidence, to Samuel Eilenberg, from whom I first learned category theory. This was in 1958, when Eilenberg was in residence for a quarter at the University of Chicago. I will never forget his two-word response: "Oh, no." At this point, I realized there would be no help from this quarter, and that I was on my own.[5]

Many years later, in Rosen's last book, *Life Itself*, he writes: "I do not believe that anyone has yet fathomed all the things category theory can do, even after decades of work. It can of course, be employed as a tool, for talking coherently about specific referents. It can be studied as a thing in itself. It can be used as a kind of transducer, to move ideas and methods from one part of mathematics to another. But each of these, important as they may be, is only a part, and preoccupation with a part obscures the whole."[6]

[4] Ibid, 428.
[5] Rosen, "On Models and Modeling," 372.
[6] Rosen, *Life Itself*, 151.

1.2 Category Theory: A General Theory of Models

For Rosen, category theory is, first and foremost, a general theory of modeling in and for mathematics or what Rosen calls *modeling relations*; category theory provides the means by which mathematics can *model itself*: category theory is "among other things, the natural habitat for discussing, not only specific modeling relations that have historically arisen in mathematics, but also the enterprise of modeling itself. It is thus unique among formal systems in its inherent reflexive characteristics, which as I said, approach those of a natural language, unique to the point that many mathematicians, even today, do not consider it mathematics at all."[7] This self-reflexive character of category theory, which is so striking, makes it, as Robert Geroch says, "the mathematics of mathematics."[8]

Informally, a category can be thought of as a kind of "mathematical universe" which is comprised of two kinds of entities: *objects* and *arrows* both of which are left unspecified. The objects are usually denoted, A, B, C, etc., or a, a'', b, b'', etc. The arrows are usually denoted, f, g, h, etc., or by the letters of the Greek alphabet, α, β, γ, etc. Every arrow has an object called its *domain* and an object called its *codomain*. For example, given an arrow f, from A to B, usually written, $f: A \to B$, the domain of the arrow f is A and the codomain of the arrow f is B. The arrows in a category can be thought of as ways of *transforming* or *translating* things in their domains into things in their codomains or as a general representation of *processes*.

[7] Rosen, *Life Itself*, 153
[8] Geroch, *Mathematical Physics*, 3

Most importantly, arrows in a category can be *composed*. Given two arrows f: A → B, and g: B → C, for which the codomain of the arrow f is the same as the domain of the arrow g, one can compose the arrow g with the arrow f and obtain a *new arrow* from A to C, usually written, g ∘ f: A → C. The symbol "∘" means *after* or *following*; so, g ∘ f means the arrow g *after* or *following* the arrow f.

$$\begin{tikzcd}
A \arrow[r, "f"] \arrow[dr, "g \circ f"'] & B \arrow[d, "g"] \\
& C
\end{tikzcd}$$

Every object in a category has an identity arrow, 1_A: A → A. An identity arrow can be thought of as a kind of "do nothing arrow"; no transformation or process has taken place.

A category satisfies two axioms. The first is the *associative law* for the composition of arrows: given the three arrows f: A → B, g: B → C, and h: C → D, we have

$$h \circ (g \circ f) = (h \circ g) \circ f$$

This axiom states that composing g with f before h is the same as composing h with g before f.

$$
\begin{tikzcd}
A \arrow[r, "f"] \arrow[dr, "g \circ f"'] & B \arrow[d, "g"] \arrow[dr, "h \circ g"] & \\
& C \arrow[r, "h"'] & D
\end{tikzcd}
$$

The second is the *identity law*: given an arrow f: A → B, if we compose it with the identity arrow for A and the identity for B, we have

$$f \circ 1_A = f = 1_B \circ f$$

This axioms states that composing an arrow, f, with the identity arrows for either its domain or codomain, does not "do anything" to f. It is analogous to multiplying any number save zero by the number 1.

Anything that satisfies the above conditions forms a category and it turns out that a vast amount of mathematical structures are categories. *Category theory* is the study of how different categories and their mathematical structures may be compared and the ways in which these structures may be related to one another, some of the means by which we now turn.

The first important concept for establishing a modeling relation between categories is called a *functor*.[9] Category theorists say that a functor is an arrow that is *structure*

[9] There are two kinds of functors, *covariant* and *contravariant*, but for our purposes, we need not bother with the distinction.

preserving between categories. Roughly, in mathematics, the term *structure* means the kinds of relations defined on an object. Rosen, would prefer to say that a functor preserves *entailment relations* between categories.[10] Given two categories, C and D, a functor F: C → D, from a category C to a category D, sends the objects and arrows in C to the objects and arrows in D; it does so in such a way that it preserves domains and codomains between arrows, identity arrows, and the composition of arrows of C in D; if arrows transform or translate things in their domains to things in their codomains, functors do the same except now from one entire category into another in a structure or entailment preserving way. Whenever it is possible to establish a functor between some category C and another category D, we can say, using Rosen's terminology, that the category D becomes a *model* for the category C; this is of the upmost importance because it means that one can learn about the properties of the category C by studying them (to the extent to which they are preserved) in D. The origins of the theory provide a good illustration.[11]

As we mentioned in Section1.1, category theory was developed by Sanders Mac Lane and Samuel Eilenberg. They originally developed the theory for the purpose of unifying two different areas of mathematics: geometry and algebra. More specifically, topological spaces are geometric objects on which some notion of continuity can be defined. Generally, these spaces are very complex objects and difficult to characterize and classify directly. However, the structure of these spaces can be translated or transformed into algebraic structures in which the properties of the spaces become easier to study through algebraic means. In this way, functors establish a modeling relation between topological spaces and algebraic

[10] We will have much more to say on entailment relations as we proceed.
[11] For more detailed examples of the modeling relation in mathematics see *Anticipatory Systems*, Section 3.1

structures for which the algebraic structures become models for the spaces. The modeling relation makes it possible to know the structure of a topological space through the collection of all of its algebraic models.[12] The point being that in order to do this, Mac Lane and Eilenberg needed to develop a language that was neither inherently geometric nor inherently algebraic and abstract enough to deal with both kinds of structures. As Yanofsky observes, this is where category theory gets its real power: by being about nothing it is about everything.[13]

The second ingredient for establishing modeling relations is called a *natural transformation*. A natural transformation preserves structure or entailments *between functors*; If F and G are two functors from a category C to a category D, F: C → D, G: C → D, a natural transformation α: F → G, is a rule that assigns to every object a in C an arrow α F(a) → G(a) called the *component* of α at a. This assignment must further satisfy the following *naturality condition*: for every arrow f: a → a' in C the following diagram commutes in D.

$$\begin{tikzcd} F(a) \arrow[r, "\alpha_a"] \arrow[d, "F(f)"'] & G(a) \arrow[d, "G(f)"] \\ F(a') \arrow[r, "\alpha_{a'}"'] & G(a') \end{tikzcd}$$

[12] For more details, see *Fundamentals of Measurement*, 126 -127.
[13] Yanofsky, *Computer Science for the Working Category Theorist*, 2.

A natural transformation can be interpreted as a *model of models*; it allows one to compare different models of a category C in D. It was one of the main purposes of Mac Lane's and Eilenberg's paper to make explicit and rigorously define this notion of *naturality*.

Before leaving this section, there is a very important aspect of category theory we must point out. We said above that a category is comprised of two entities: objects and the arrows between them. However, in category theory, there is *no absolute distinction between objects and arrows*; or in philosophical parlance, no absolute distinction between *relata* and *relations*. For one thing, the objects of a category correspond exactly to their identity arrows. This makes it possible to dispense altogether with the objects in the definition of a category we gave above and instead treat them as their identity arrows; i.e., there is an "arrows only" definition of a category.[14] Additionally, it turns out that categories and functors also form a category in which *the objects are entire categories and the arrows are now functors between them*; this is the *categories of categories*. It also turns out the functors and the natural transformations from a category in which *the objects are now functors and the arrows are now natural transformations between them*; this is the *functor category*; and this is just the beginning of a process *that can be iterated indefinitely as a hierarchical structure, one category after another, without limit*, and so on forever! Thus, at each level in the hierarchy what are arrows at one level become the objects in the next higher level; i.e., in this hierarchy, the distinction between objects and arrows is *context dependent*. We believe Yanofosky jokingly put it best:

> Between objects there are morphisms. Between morphisms, there are morphism. Between those morphisms, there are other morphisms. Between

[14] Mac Lane, *Categories for the Working Mathematician*, 9.

> those morphisms, there are other morphisms. Between those morphisms, there are other morphisms. Between those morphisms, there are other morphisms. Between those morphisms, there are other morphisms. Between those morphisms, there are other morphisms. Between those morphisms, there are other morphisms. Between those morphisms, there are other morphisms. Between those morphisms, there are other morphisms. Between those morphisms, there are other morphisms. Between those morphisms, there are other morphisms. Between those morphisms, there are other morphisms.…[15]

This indefinite hierarchy of levels is what gives category theory its reflexive character and in good part, the capacity for mathematics to model itself.

There is another way in which the distinction between objects and arrows is blurred which will become very important and is embodied in what is called a *hom-set*. A hom-set is a set or collection of arrows from one object to another.[16] Following Rosen, for any two objects A and B we will usually denote their hom-set H(A, B) with f ϵ H(A, B), where ϵ (epsilon) is the symbol that represents "member" or "element of"; i.e., the arrow f is an element of the hom-set H(A, B). At one extreme the hom-set could just be empty (save for the identity arrows required for all sets of the form H(A, A), H(B, B), etc.). At the other extreme, the hom-set could be the set of all arrows from the object A to the object B; or, the hom-set could be any number of the arrows between these two extremes. When a hom-set contains all of the arrows from A to B we will sometimes write B^A instead of H(A, B). Since hom-sets are objects they can be the domains and codomains of other arrows. For example, g: X → H(A, B), where the hom-set H(A, B) is now the codomain for the arrow g.

[15] Private conversation.
[16] Formally, H(A, B) = {f : f is an arrow f: A → B in the category C}.

We now turn to Rosen's extension of the modeling relation in category theory to theoretical science.

Chapter 2. The Modeling Relation in Science

The modeling relation is the point of departure for Rosen's entire scientific enterprise. We should note at the outset that in diagrammatic form, the modeling relation looks deceptively simple. As we shall see though, it is exceedingly rich in conceptual content and will play a key role in everything that follows.

2.1 Natural Law

There are, according to Rosen, two basic beliefs that are the minimum requirements to do science. These two beliefs constitute what Rosen calls *Natural Law*. The first is the belief that there are causal relations between the events and their qualities that we perceive in the external world; it must be presumed by the scientist that some of the events in the external world are not entirely whimsical, chaotic, or arbitrary, but obey definite laws or relations: "without a belief in the causal order, there could be no science and, very probably, no sanity."[1] The second belief is that these causal relations are, at least in part, capable of being grasped and understood by the human mind and can be articulated and expressed in language, and in particular, mathematics:

> Since mathematics, in the broadest sense, is the study of implication [entailment] relations in formal systems, or the art of extracting conclusions from premises, it follows that mathematics is integrally involved in the study of natural law.[2]

[1] Rosen, *Theoretical Biology and Complexity*, 179.
[2] Ibid, 180.

2.2 Formal Systems

Both mathematics and science are predicated on dualisms. As Rosen notes, they are impossible to avoid, since every mode of discrimination creates one.[3] The fundamental dualism in mathematics (or languages in general) is that between *syntax* and *semantics,* or more, in mathematics and logic, between *syntactic truth* and *semantic truth*. Syntax is what a language is *in and of itself*, independent of its meanings or referents. In any mathematical system there are propositions that are true purely in terms of their form which results from the way in which the mathematical system is "put together". Syntactically true propositions then, are true because of their *form*, independent of any referent or what is meaningfully asserted by the propositions. Semantics *is what a language is about* and pertains *to* a language's meanings or its referents (or as Rosen' puts it, semantics *means* "non-syntactic"). Semantically true propositions then, depend on their referents or what is meaningfully asserted by the propositions. In general, a *mathematical system* can have both. However, following Rosen, we are going to call a *formal system* an object or "sublanguage" (extracted from the mathematical world) that is specified only and entirely through its syntax.[4]

Formal systems are comprised of two things: *axioms* and *rules of inference*.[5] Axioms are a formal system's "initial propositions" and are assumed to be "true" without proof; they are like the rules of a game.[6] The rules of inference provide the means for entailing new propositions from given ones.[7] Roughly, to say that some proposition P (syntactically)

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[3] Rosen, *Life Itself*, 140.

[4] For Rosen the term *system* is a primitive; it is left undefined. Following A. H. Louie, we say that a system is "a collection of material or immaterial things that comprises one's object of study." *More Than Life Itself*, 82.

[5] Rosen sometimes uses the term *production rules* instead of *rules of inference* as in the passage below.

[6] For example, recall the two axioms for category theory from Section 1.2.

[7] For example, *modus ponens*.

entails a new proposition Q means that the truth of Q must follow as a necessary consequence of the truth of P. A syntactic entailment relation is usually denoted $P \vdash Q$, where $\vdash$ is the symbol (turnstile) that represents "syntactically entails." Thus, a formal system can be considered as a set of propositions (theorems) all of which are derived from its axioms and by the successive application of its axioms and its rules of inference. For now, following Rosen, we will call the kinds of syntactical entailments available in a formal system *inferential entailments*.[8] We will provide a more general representation of entailments (whether syntactic or semantic) in terms of objects and arrows, i.e., in a category theoretic context, in subsequent chapters.[9]

There is another dualism inherent formal systems between its rules of inference and the propositions they (along with the axioms) generate:

> Syntax involves its own inherent dualism between *proposition* and *production rules*. From a syntactical point of view, divorced from any external referents, propositions in the language are generally not *about* anything and described entirely in terms of conventional vehicles: letters, words, sentences, and so forth. The production rules are themselves propositions, but they do have referents, namely, the other propositions *in the language*. Their role is essentially a dynamic one, to enable the construction of new propositions from given ones, or the analysis of given propositions from simpler ones.[10]

We will have more to say about this in Chapter 4, but for now, we should note that, historically, there has been a strong motivation for the study and treatment of languages in terms of pure syntax. The reason is that syntax is considered to *objective* and hence amenable to scientific study. Semantics on the other hand is inherently subjective, the host

[8] Rosen sometimes uses the term *implication relations*, as quoted in Section 2.1.
[9] For an intriguing overview of mathematical and formal systems see *Anticipatory Systems*, Section 2.2.
[10] Rosen, *Life Itself*, 43.

of obscurities and ambiguities. Thus, the more syntax a language has (and the less semantics), the better a language it is presumed to be.

2.3 Natural Systems

The first dualism in science is that between the *internal world of the self* and the *external world of events and their qualities*.[11] Science requires the belief that at least some of our percepts (by which Rosen means our *basic units of awareness of sensory perceptions*) are not entirely generated by us but by events and their qualities in the external world. An event and its qualities in the external world are what Rosen calls a *natural system* to distinguish it from what we have called a *formal system* in Section 2.1. The qualities of natural systems that we believe are responsible for the generation of our associated percepts are what Rosen calls *observables*. Natural systems and their observables are the basis for all scientific inquiry.

But more, in accordance with Natural Law, science is the attempt to discover *relations* in and between natural systems and their observables. Rosen calls these relations in and between natural systems *causal entailments* to distinguish them from what we have called *inferential entailments* in and between formal systems. But as Rosen points out, there is a problem: it does not seem as if the causal relations in natural systems *are themselves percepts*. This makes it plausible to believe that one of the primary functions of the mind is to establish and organize relations between percepts. The mind then behaves *as if the relations it establishes between percepts are themselves percepts*; the mind *imputes* the

[11] Rosen sometimes, especially in *Life Itself*, likes to use the term *ambiance* for the external world.

relations it establishes between percepts to natural systems in the external world. These imputations are hypothetical models for how the external world is organized. Thus, the "perception" of causal relations is *subjective* in the sense that "we must admit that such [causal] relations reflect the properties of the active mind as much as they do the percepts which the mind organizes."[12] This observance allows Rosen to further characterize his notion of a natural system: a natural system *is a apart of the world or a set of qualities to which definite relations can be imputed*. Note that a natural system is not the same as a *material system*. A material system is *ontological*, something presumed to exist in the external world. A natural system is *epistemological* since the relations imputed to it are intrinsically involved in its conception.

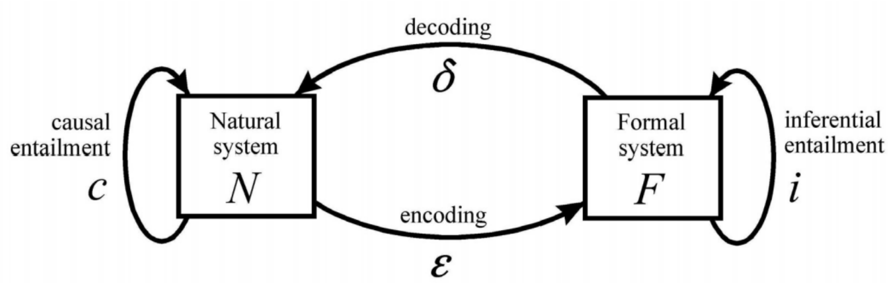
The second basic dualism of science is predicated on the first: a natural system is something that the mind *extracts* from the external world (which as Rosen points out, is analogous to the way in which the mind extracts a formal system from the world of mathematics). This extraction creates another dualism between a natural system and its *environment*. This distinction Rosen argues, is also imputed to the external world by the mind and serves as one of the ways in which the mind organizes and manages percepts. But before leaving this section, a forewarning for very important considerations in Section 7.1:

> The partition of the ambience into system and environment, and even more, the imputation of that partition to the ambience itself as an inherent property thereof, is a basic though fateful step for science […] Indeed it is precisely at this point that, as we shall see, fundamental trouble begins to creep in.[13]

[12] Rosen, *Anticipatory Systems*, 46.
[13] Rosen, *Life Itself*, 42.

2.4 Natural Law and the Modeling Relation

Both mathematics and science, are, in their own ways, concerned with systems of entailment, inferential and causal. Furthermore, Rosen argues that theoretical science is concerned with the construction of modeling relations which relate these two different kinds of entailments. A modeling relation between a formal system and its inferential entailments and a natural system and its causal entailments can be represented by the following diagram:



[2.4.1]

The left-hand box of the diagram represents a natural system in the external world. The arrow labeled *c* represents causal entailments in a natural system. The right-hand box represents a formal system in the mathematical or internal world of the self. The arrow *i* represents the inferential entailments in a formal system which are *imputed* to natural systems. The two arrows ε and δ represent the way in which a congruence relation is established between the external world of natural systems and their causal relations and the internal world of formal systems and inferential entailments. The arrow ε represents the *encoding* of qualities (observables) of a natural system into a formal system. The arrow δ represents the *decoding* of the inferential structure of a formal system that symbolically represents the causal structure of the natural system with which it is associated. Decodings represent interpretations, or more, *predictions* that can be made about the causal entailments of a natural system derived from the encodings and the propositions (theorems) that are generated by the inferential entailments in the formal system. The essence of the modeling relation is captured by the arrows ε and δ. They establish a *dictionary* which allows one to translate back and forth between natural and formal systems and causal and inferential entailments. Thus, a modeling relation is established between a natural system and a formal system whenever (1)

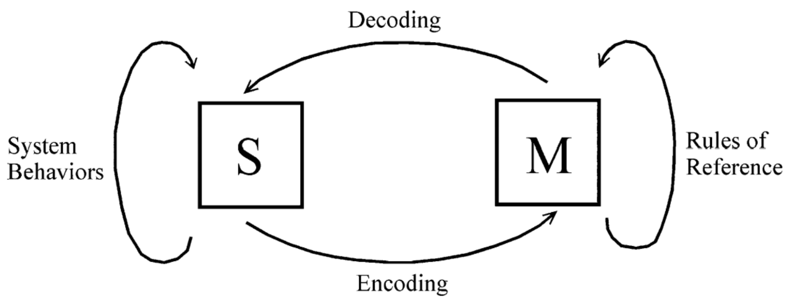
$$i = \varepsilon(c)$$

which represents an encoding of causal entailments into inferential entailments of a formal system, and (2) the diagram [2.4.1] *commutes*:

$$c = \delta \circ i \circ \varepsilon$$

Rosen calls the formal system on the left a *model* of the natural system on the right. Conversely, Rosen calls any natural system on the right a *realization* of a formal system on the left. In this way, the task of theoretical science is to determine what kind of formal systems can model certain kinds of natural systems; or again, conversely, what natural systems may realize certain kinds of formal systems as models.

All in all, we have:



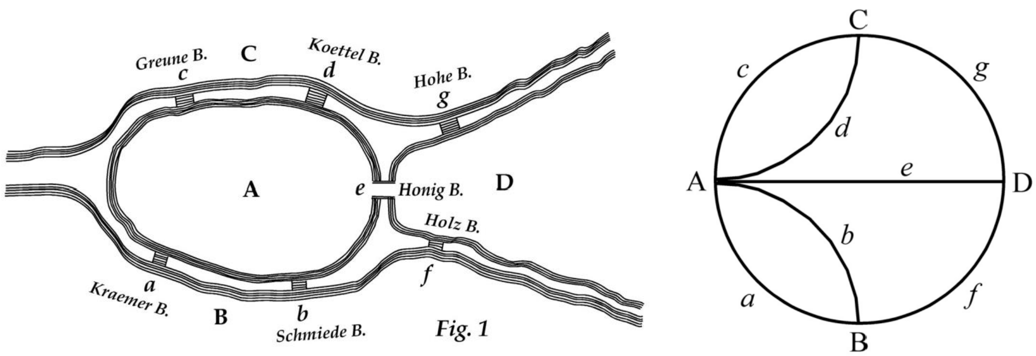
[2.4.2]

A simple but elegant illustration of a modeling relation is what is known as *the problem of the Seven Bridges of Königsberg* which was solved by the mathematician Leonhard Euler in 1735.

The problem was that in the town of Königsberg in Prussia there was an island, *Kneiphof*, with the two branches of the river *Pregel*, flowing around it, and seven bridges crossing the two river branches. Euler put the problem thus:

> The question is whether a person can plan a walk in such a way that he will cross each of the bridges once but not more than once. I was told that while some denied the possibility of doing this and others where in doubt, there were none who maintained that it was actually possible. On the basis of the above I formulated the following general problem for myself: Given any configuration of the river and the branches into which it may divide, as well as any number of bridges, to determine whether or not it is possible to cross each bridge exactly once.[14]

[14] Louie, *More than Life Itself*, 134.



Natural System ⇆ Model

[2.4.3]

In order the solve the problem, Euler developed what would become an entirely new branch of mathematics now known as *graph theory* or *network topology*. Roughly, a graph consists of two things: *vertices* and *edges* where the edges represent connections between vertices. Euler *encoded* the four land masses, labeled A, B, C, D into the vertices of the graph and the seven bridges labeled a, b, c, d, e, f, g into the edges of the graph as depicted in diagram [2.4.3]. Thus, the graph on the right-hand side of the diagram [2.4.3] is a *model* of the natural system on the left-hand side. Intuitively, now the problem of deciding whether or not it is possible to cross each bridge exactly once now becomes a *formal problem* (via the encodings) of whether or not one could traverse the graph in one continuous trace of the edges, passing along each edge exactly once. Euler proved that this was impossible for the graph. This allowed him to *predict* (decode) that it is impossible to take such a walk.[15]

[15] For more detailed examples of the modeling relation in the sciences, see *Anticipatory Systems*, Sections 3.3 and 3.4.

We can now see that Rosen's formulation of the modeling relation which represents what he considers to be the essence of theoretical science is an extension of ideas and concepts arising from category theory which we presented in Section 1.2. Natural systems are treated as if they are objects and their causal entailments are represented as arrows between the objects themselves.[16] The encoding and decoding arrows are analogous to functors or entailment preserving arrows between categories. The difference, of course, between functors and encoding - decoding arrows is that the former are entirely formal entities while the latter are not; and indeed, cannot be: the encoding and decoding arrows are the means for effectively moving back and forth between two entirely different worlds: causality in the informal external world and inferences in the formal world of mathematics or the internal world of the self.

A most important feature of the encoding and decoding arrows is that they *do not represent entailments nor are they entailed*; neither by causality in the external world, nor formally in the internal world of the self.

> Modeling, [...] is the art of bringing entailment structures into congruence. *It is indeed an art, just as surely as poetry, music, and painting are*.[17] Indeed, the essence of art is that, at root, it rests on the unentailed, on the intuitive leap. I have stressed repeatedly that the encodings and decodings on which modeling relations depend are themselves unentailed. Thus, theoretical scientists must be more artists than craftsmen; Natural Law assures them only that their art is not in vain, but it in itself provides not the slightest clue how to go about it.[18]

And

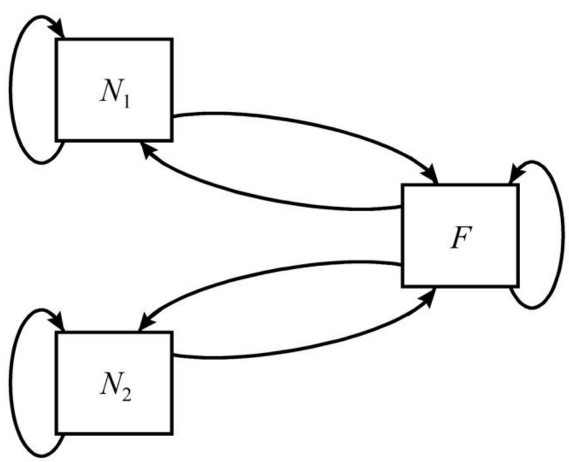
[16] In category theory, an arrow between an object and itself is called an *endomorphism*.
[17] Emphasis ours.
[18] Rosen, *Life Itself*, 152

> The modeling relation is intimately tied up with the notion of prediction. Natural Law, as embodied in modeling relations, thus equips us to look into the future of things; insofar as the future is entailed by the present, and insofar as entailment structure itself is captured in a congruent model, we can actually, in a sense, pull the future of our natural system into the present. *The benevolence of Natural Law lies in assuring us that such miracles are open to us, but it does not extend to telling us how to accomplish them; it us forever to discover the keys, the encodings and decodings, by which they can be brought to pass*.[19]

The modeling relation then, is *the art of science*. Encodings and decodings "manifest what Einstein called 'free creations of the human mind,' on which science depends."[20]

2.5 Analogies, Alternative Models, and Metaphors

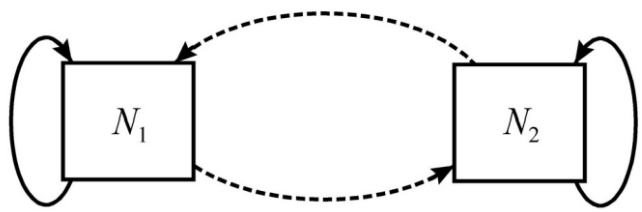
The modeling relation provides the means for the comparison of the causal entailments between two or more natural systems. If two natural systems N_1 and N_2, are realizations of the same formal system F, then, in terms of the modeling relation we have:



[2.5.1]

[19] Ibid, 64. Emphasis ours.
[20] Rosen, *Essays on Life Itself*, 159

The diagram [2.5.1] represents the situation where the two natural systems, N_1 and N_2, encode into the same formal system F. If such a relation holds between the natural systems N_1 and N_2, and the formal system F, it then becomes possible to establish encoding and decoding arrows *between the natural systems, N_1 and N_2, themselves*. This relation between two or more natural systems encoding into the same formal system is what Rosen calls an *analogy* between two or more natural systems. When an analogy can be established between two or more natural systems it means that one can learn about one natural system from studying the others and conversely:



[2.5.2]

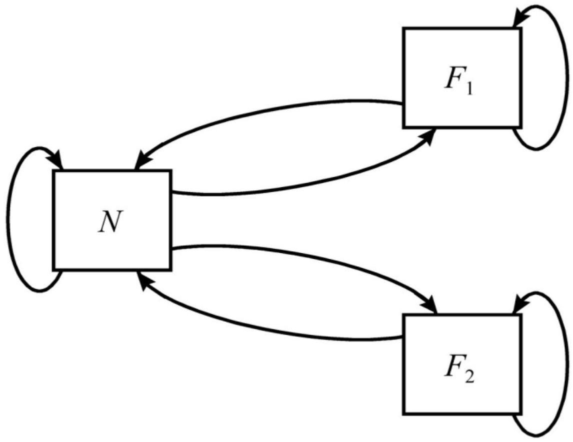
Scale models that are used in engineering are a good example of analogies. One of Rosen's favorite is William Rowan Hamilton's *optico-mechanical analogy* in physics.

Rosen points out that analogies between natural systems are of the most profound importance. After giving the examples just mentioned Rosen writes:

> Still more dramatic, perhaps, are the vistas opened by analogy for the further elucidation of causal entailments in natural systems that are, in conventional physical terms, of entirely different structures: organisms and "machines", organisms and social systems. This is another way of seeing […] that reduction to a common set of material constituents is not the only way, or even a good way, of comparing natural systems.[21]

[21] Rosen, *Life Itself*, 63.

The compliments of analogies are *alternative models*. This kind of situation arises when the same natural system, N, has encoding arrows into, and decoding arrows out of, two or more formal systems, F_1 and F_2.



[2.5.3]

While an analogy between two or more natural systems automatically guarantees the existence of encoding and decoding arrows between them, this is not the case with alternative models; there may or may not be any. The problem then becomes whether or not there is any way to establish encoding or decoding arrows (functors) between the two formalisms F_1 and F_2. Notice that if there are any arrows that can be established between the formalisms F_1 and F_2, then from the point of view of the natural system for which they serve as models, these arrows would be analogous to a natural transformation between functors.

Analogies and alternative models bear heavily on the problem of *reductionism* in science and philosophy. As Rosen alluded above, analogies establish an entirely non-reductionist approach to the modeling of natural systems. And, from the point of view of the modeling relation, reductionism is an entirely formal affair; it would mean that in the situation depicted in diagram [2.5.3] a model F1 of some natural system N could be *reduced to* or *embedded* into another formal system F2 through a functor; that is, F2 is "bigger" than F1 in the sense that the former captures all of the inferential structure of the latter and more. In the most extreme case, reductionism would require that every model of every natural system could be reduced to one and only one model or formal system; there would have to be a *largest model*; a *theory of everything*.[22]

Rosen calls a *metaphor* a modeling relation in which the only arrow between a natural system and formal system is the decoding arrow δ; metaphors do not explicitly involve the encoding of natural systems into formal ones. Rosen likens a metaphor to a "crystal ball" or a "one-way mirror" that allows one to study natural systems and interpret or make predictions (decodings) about them in a way in which only "half of the work" is involved. Metaphors, like analogies "have in fact been of profound importance in the history of science and in many areas continue to play a major role; in fact, they constitute what there is of theory in these areas."[23]For an important example, let us consider what Rosen calls the *open system metaphor*.

Traditionally, physics and in particular thermodynamics, has concerned itself with the behaviors of *closed* or *isolated systems*. A closed system is one which is exchanging energy with its environment, but not matter. An isolated system is one which is neither exchanging energy nor matter with its environment. Closed and isolated systems are governed by the second law of thermodynamics which roughly states that an isolated or closed system will proceed from orderly sates to disorderly states (eventually to a state of

[22] We will further discuss the problem of a largest model in Sections 4.3 and 4.5.
[23] Rosen, *Life Itself*, 65.

equilibrium; i.e., total disorder) and never the other way around; that is, never from disorderly states to orderly states. An *open system* is one which is exchanging both energy and matter with its environment "to which the Second Law of Thermodynamics is not directly applicable."[24]

Organisms, for example, are definitely open systems, exchanging both matter and energy with their environment. It was the general system theorist Ludwig von Bertalanffy who was the first to apply the open system metaphor to explain what appeared to be some of the odd, counterintuitive properties and behaviors of organisms (from the point of view of traditional physics in terms of closed or isolated systems).

As Rosen points out: "open systems share many interesting aspects of behavior simply by virtue of the fact that they are open."[25]One of the properties of open systems is that they have what are called *steady states* or *attractors*. These are states that open systems will tend towards regardless of any changes in the systems' initial states or conditions. Unlike closed or isolated systems, open systems have "the stubborn tendency [...] to reach the same final state despite experimental interventions such as amputations, randomizations, or hybridizations."[26] In 1891, the embryologist Hans Driesch discovered that if the first four cells of the embryo of a sea urchin are divided in two, both still develop into whole sea urchins (i.e., from a more "disorderly" state to a more "orderly" state). This convinced Driesch that no physical explanation of this kind of phenomena was possible; not even in principle. He called this kind of behavior *equifinal*. This would certainly be the case in terms

[24] Rosen, *Anticipatory Systems*, 231.
[25] Rosen, *Fundamentals of Measurement*, 125.
[26] Rosen, "Sixth Annual Ludwig von Bertalanffy Memorial Lecture," 241.

of the properties of closed or isolated systems. However, von Bertalanffy, without any specific encodings of sea urchins, was able to show that equifinality is just an instantiation of open systems tending towards their steady states; again, without any specific encodings, the developmental processes of organisms could be explained in terms of steady states.

Since we are on the subject, we should take the time to point out two other important aspects of open system theory. The first, is that formally, open system theory is more general than isolated or closed system theory. As von Bertalanffy notes:

> The intuitive choice of the open system as a general system model [metaphor] was a correct one. Not only from the physical viewpoint is the "open system" the more general case (because closed systems can always be obtained from open ones by equating transport variable to zero); it is also the general case mathematically because the system of simultaneous differential equations (equations of motion) used for description in dynamical system theory is the general form from which the description of closed systems derives by the introduction of additional constraints (e.g., conservation of mass in a closed chemical system).[27]

The second is that even though it is a more general theory, unfortunately, as Rosen observes

> I would say that, today, there is no satisfactory "physics" of open systems, primarily because people persist in thinking of closed systems as fundamental, and of open ones simply as closed ones conically perturbed. [28] [29]

Metaphors, as Rosen points out, exist, on their own, in the world of mathematics. Recall from Section 1.3 that a functor establishes a modeling relation between categories. Consider a functor F, from the category C to D, F: C → D, where the category D is a model of the category C. If we forget the functor F and the source category C, then the image of C in

[27] Bertalanffy, "The History and Status of General Systems Theory," 412.
[28] Rosen, *Anticipatory Systems*, 427.
[29] For a good overview of open system theory, see *Anticipatory Systems*, Chapter 5

the target category D becomes a metaphor for the category C.[30] More, the totality of all models of a natural system S form a category, what Rosen calls *C(S)*, for each natural system S. The totality of all models for a natural system is what Rosen calls its *epistemology*, and furthermore that:[31]

> Natural Law asserts that all we can know about [a system] S is embodied in its models, i.e., in what we have called C(S). Thus, we must build our knowledge of S from the formal structure of C(S). Thus C(S) itself becomes a kind of metaphor for S, for each individual S. And moreover, the totality of all these C(S), must be a kind of metaphor for the whole external world itself.[32]

However, the fact that metaphors do not require specific encodings for the natural systems they may represent is not without some controversy:

> To proceed metaphorically [...] is, of course, not an unreasonable thing to do. It is also clear, however, why experimentalists find such metaphors troubling and why they occupy an anomalous position in what passes nowadays for philosophy of science. For by giving up encoding, we also give up *verifiability* in any precise sense. Thus, experimentalists interested specifically in, say, a developing sea urchin, derive no tangible help from a metaphor. They need something to verify, couched in terms of some specific observation, or experiment they can perform. That is to say, they need precisely what is missing in the metaphor; they need encodings. Hence the indifference, if not active hostility, manifested by empiricists to theory couched in metaphorical terms. Metaphor is immune to such verification; insofar as science is identified with verification, as it is currently fashionable to do, metaphor is not even science. Nevertheless, it is clear that metaphor can embody a great deal of truth. And as with all crystal balls, it does have the irresistible attraction of offering something for free.[33]

Rosen ends the section in *Life Itself* devoted to metaphors with this remark:

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[30] Note that if there are two or more functorial images of C in D then a natural transformation establishes a modeling relation between those functors.
[31] Recall the distinction we made between a natural system and a material system in Section 2.3.
[32] Rosen, *Life Itself*, 180. For more on the category of models see *Life Itself*, Chapter 6. We are omitting Rosen's presentation of analytic and synthetic models.
[33] Ibid, 66.

> In some sense, it is precisely the unique metaphorical aspects of Category Theory that generates qualms in many mathematicians regarding it, which run quite parallel to those of any empiricist."[34]

We now turn to relational system theory and its models.

[34] Ibid.