GAME AI

with Jeff Wilson, PhD

Ballistic Projectiles

Introduction

Ballistic Projectiles

- Sometimes agents need to fire projectiles at targets

Ballistic Projectiles

- It's easy to simulate Newtonian Physics with particle and rigid body dynamics

Ballistic Projectiles

- It's generally easy to simulate Newtonian Physics with particle dynamics and rigid body dynamics
- But solving for intercepts is not always easy…

Intercept

- Recall Steering Behavior Pursue
- An approximate intercept was calculated by estimating how much time it would take for the agent to get in the neighborhood of the target
- Using that time estimate, the target's movement could then be extrapolated assuming a constant velocity

Intercept

- Pursue's intercept time is only estimated because it is going to be updated again the next simulation frame and doesn't need to be precise
- However, with a ballistic projectile, there is only one chance to get the trajectory correct
- Therefore, more computational effort is needed for an accurate trajectory

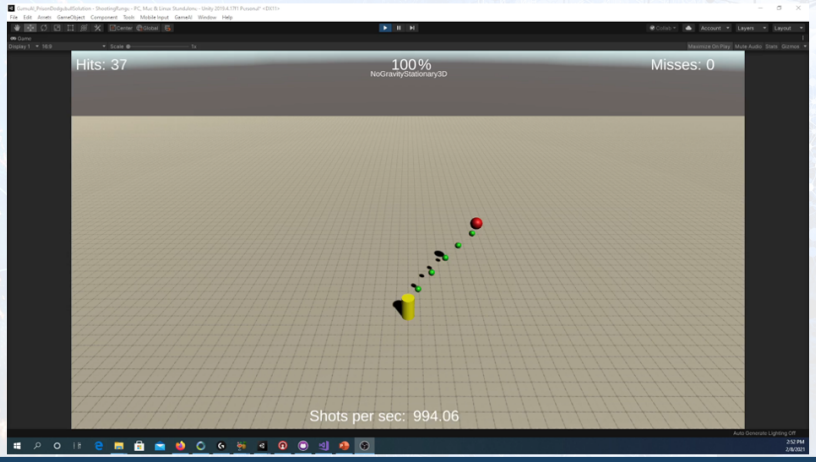
Intercept

- Projectiles like grenades or footballs aren't the only game objects that need a precise velocity solved
- Consider a game character like Rhino in Spider-Man
- Rhino might charge Spider-Man but not be able to change direction while charging
- If Spider-Man is running, Rhino's AI needs to calculate a precise intercept

Intercept

- Sometimes it's straightforward to solve an intercept
- For instance, if the target does not move at all and the projectile is constant velocity (not affected by gravity or drag) then only a relative position vector is needed to get launch direction (scale it by desired speed for full velocity)

Simple Intercept Trajectory



Ballistic Projectiles

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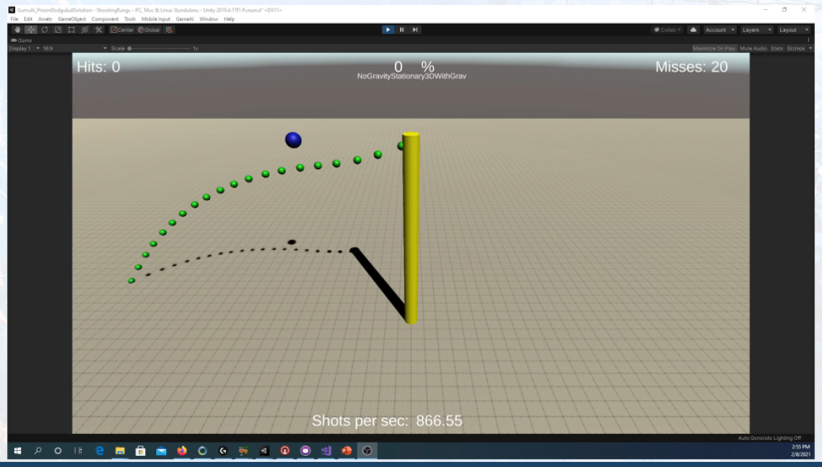
No Gravity

Intercept

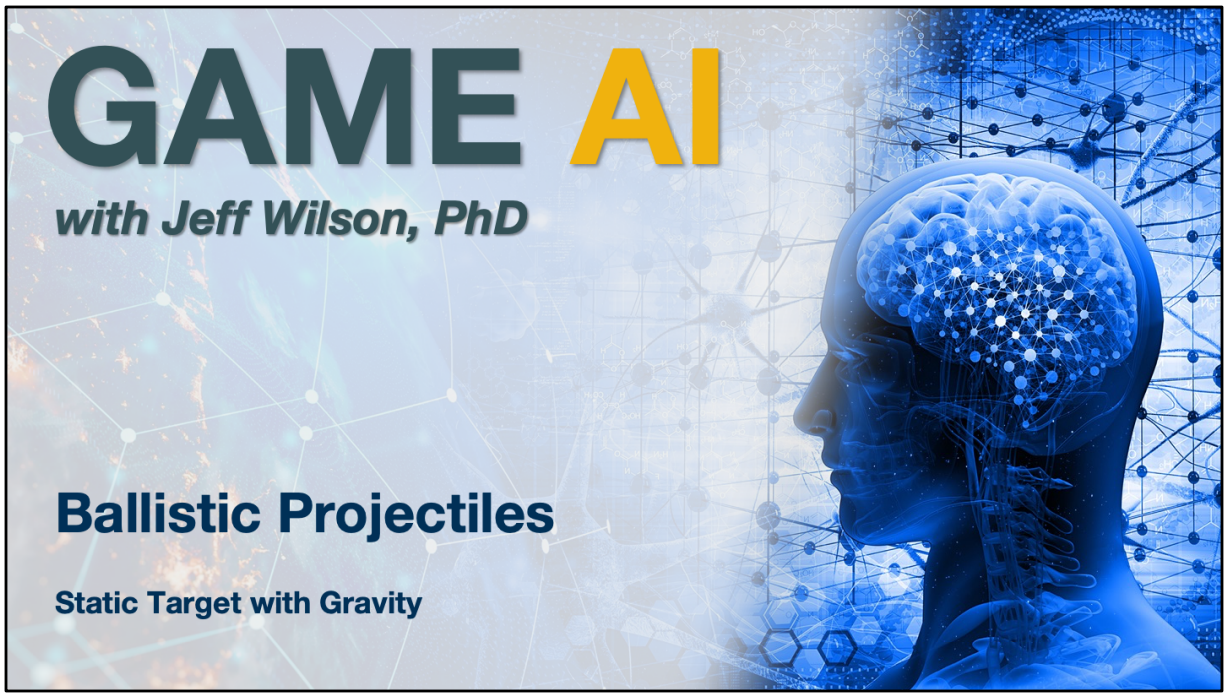
- But what if gravity affects the projectile? Or the target is moving?
- The more degrees of freedom allowed, the more complicated the system of equations needed solving gets
- Eventually, the equations can get so complicated that they cannot be directly solved, or become overly complicated or error prone

Simple Intercept Trajectory Fail



Methods

- There are two general methods for solving ballistic projectile launch velocities:
- Directly Solve – Formulate a system of equations that represents the problem and directly solve for the unknown variables
- Iteratively Solve – Estimate a reasonable solution, then refine parameters until the solution is close enough to the desired outcome.
- To iteratively solve, the movements of projectile and target must be able to be modeled according to the defined parameters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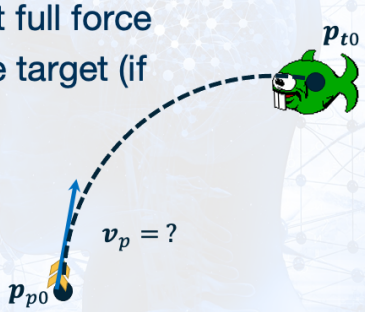
with Jeff Wilson, PhD
Ballistic Projectiles
Static Target with Gravity

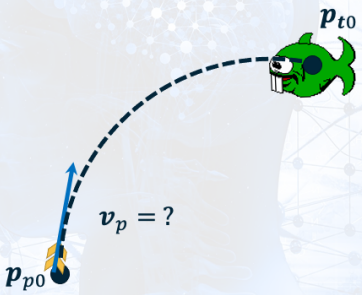
Ballistic Trajectory for Static Target

- Assume 3D positions of projectile and target
- Assume projectile is affected by gravity (but no drag)
- Assume projectile will always be launched at full force
- What is the direction of the velocity to hit the target (if possible)?



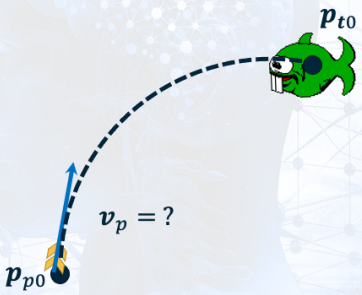
Ballistic Trajectory for Static Target

- $\boldsymbol{p}_t - \textit{Stationary Target Position}$
- $\boldsymbol{p}_{p0} - \textit{Initial Projectile Position}$
- $\boldsymbol{u}_{p0} - \textit{Initial (Launch) Projectile Direction}$
- $s_p - \textit{Initial (Launch)Projectile Speed}$
- $\boldsymbol{v}_p - \textit{Initial (Launch) Projectile Velocity}$
- $\boldsymbol{g}_p - \textit{Gravity Affecting Projectile}$
- $t_c - \textit{Time of collision}$
- Solving for $\boldsymbol{u}_{p0}$ because a full speed solution is desired. We only need the direction.



Ballistic Trajectory for Static Target

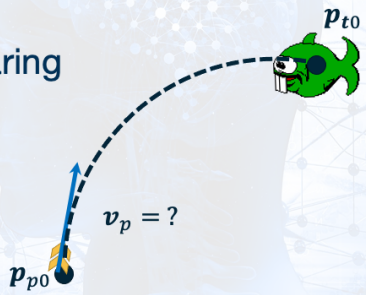
- From Particle Dynamics:
- $\boldsymbol{p}_t = \boldsymbol{p}_{p0} + \boldsymbol{u}_{p0} s_p t_c + \frac{1}{2}\boldsymbol{g}_p {t_c}^2$
- This represents three equations (3D):
- $p_{tx} = p_{p0x} + u_{p0x} s_p t_c + \frac{1}{2} g_{px} {t_c}^2$
- $p_{ty} = p_{p0y} + u_{p0y} s_p t_c + \frac{1}{2} g_{py} {t_c}^2$
- $p_{tz} = p_{p0z} + u_{p0z} s_p t_c + \frac{1}{2} g_{pz} {t_c}^2$



Unknowns are u_{p0x}, u_{p0y}, u_{p0z}, and t_c
So where is the fourth equation?

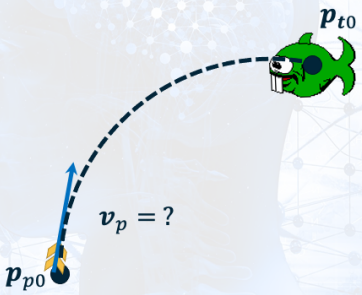
Ballistic Trajectory for Static Target

- A fourth equation can be added:
- $\|\boldsymbol{u}_{p0}\| = 1$
- $1 = {u_{p0x}}^2 + {u_{p0y}}^2 + {u_{p0z}}^2$
- Note that we got rid of Square Root by squaring both sides



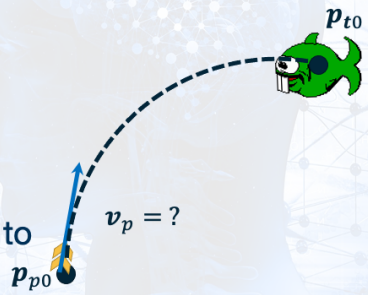
Ballistic Trajectory for Static Target

- With:
- $\boldsymbol{p}_t = \boldsymbol{p}_{p0} + \boldsymbol{u}_{p0} s_p t_c + \frac{1}{2}\boldsymbol{g}_p {t_c}^2$
- $\boldsymbol{u}_{p0}$ can be isolated
- $\boldsymbol{u}_{p0} = \frac{\boldsymbol{p}_t - \boldsymbol{p}_{p0} - \frac{1}{2}\boldsymbol{g}_p {t_c}^2}{s_p t_c}$
- Let $\Delta = \boldsymbol{p}_t - \boldsymbol{p}_{p0}$ so:
- $\boldsymbol{u}_{p0} = \frac{\Delta - \frac{1}{2}\boldsymbol{g}_p {t_c}^2}{s_p t_c}$



Ballistic Trajectory for Static Target

- Plug each dimension of $\boldsymbol{u}_{p0} = \frac{\Delta - \frac{1}{2}\boldsymbol{g}_p t_c^2}{s_p t_c}$ into:
- $1 = \|\boldsymbol{u}_{p0}\| = u_{p0x}^2 + u_{p0y}^2 + u_{p0z}^2$ giving terms:
- $u_{p0x}^2 = \left[\frac{\Delta_x - \frac{1}{2}g_{px}t_c^2}{s_p t_c}\right]^2$
- $u_{p0x}^2 = \frac{\Delta_x^2 - g_{px}\Delta_x t_c^2 + \frac{1}{4}g_{px}^2 t_c^4}{s_p^2 t_c^2}$
- Similarly solve for u_{p0y}^2 and u_{p0z}^2
- Note that vectors are losing dimensionality due to the magnitude operation



Ballistic Projectiles

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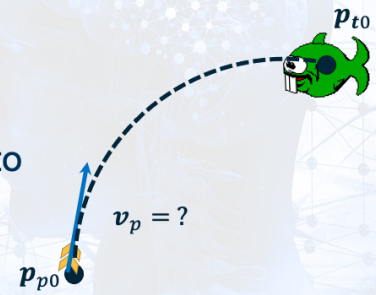
Only showing the x term

Ballistic Trajectory for Static Target

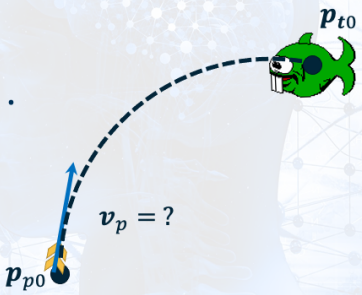
- Now:
- $1 = \frac{{\Delta_x}^2 - g_{px}\Delta_x {t_c}^2 + \frac{1}{4}{g_{px}}^2 {t_c}^4}{{s_p}^2 {t_c}^2} + \cdots$
- Move the denominator out:
- ${s_p}^2 {t_c}^2 = {\Delta_x}^2 - g_{px}{t_c}^2 + \frac{1}{4}{g_{px}}^2 {t_c}^4 + \cdots$
- We could stay in vector form, but we need to understand some vector operations…



Ballistic Trajectory for Static Target

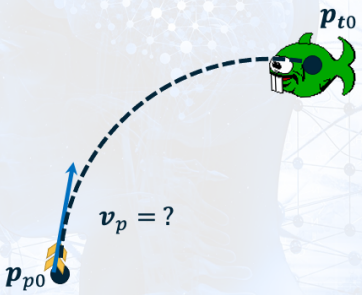
- We can see that multiplying two different Euclidean vectors (standard basis) together can only have non-zero terms along the Inner Product (similar to Dot Product):
- $\boldsymbol{r} \cdot \boldsymbol{s} = (r_0\hat{\boldsymbol{\imath}} + r_1\hat{\boldsymbol{\jmath}} + r_2\hat{\boldsymbol{k}})(s_0\hat{\boldsymbol{\imath}} + s_1\hat{\boldsymbol{\jmath}} + s_2\hat{\boldsymbol{k}})$
- $= (r_0\hat{\boldsymbol{\imath}} \cdot s_0\hat{\boldsymbol{\imath}}) + (r_1\hat{\boldsymbol{\jmath}} \cdot s_1\hat{\boldsymbol{\jmath}}) + (r_2\hat{\boldsymbol{k}} \cdot s_2\hat{\boldsymbol{k}}) + (r_0\hat{\boldsymbol{\imath}} \cdot s_2\hat{\boldsymbol{k}}) + (s_0\hat{\boldsymbol{\imath}} \cdot r_1\hat{\boldsymbol{k}}) + (r_1\hat{\boldsymbol{\jmath}} \cdot s_2\hat{\boldsymbol{k}}) + (s_1\hat{\boldsymbol{\jmath}} \cdot r_2\hat{\boldsymbol{k}}) + (r_0\hat{\boldsymbol{\imath}} \cdot s_1\hat{\boldsymbol{\jmath}}) + (s_0\hat{\boldsymbol{\imath}} \cdot r_1\hat{\boldsymbol{\jmath}})$
- $= r_0s_0\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\imath}} + r_1s_1\hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\jmath}} + r_2s_2\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}} = r_0s_0\hat{\boldsymbol{\imath}} + r_1s_1\hat{\boldsymbol{\jmath}} + r_2s_2\,\hat{\boldsymbol{k}}$
- $\boldsymbol{r} \cdot \boldsymbol{s} = r_0s_0 + r_1s_1 + r_2s_2$ (Dot Product)

The dot product of a vector with itself is the square of the vector's magnitude. Applied to unit vectors, the dot product is always 1.



Ballistic Trajectory for Static Target

- For Euclidean Vectors: $\boldsymbol{r} - is\ a\ 3D\ Vector$
- $\boldsymbol{r} \cdot \boldsymbol{r} = (r_0\hat{\boldsymbol{\imath}} + r_1\hat{\boldsymbol{\jmath}} + r_2\hat{\boldsymbol{k}})(r_0\hat{\boldsymbol{\imath}} + r_1\hat{\boldsymbol{\jmath}} + r_2\hat{\boldsymbol{k}})$
- $= (r_0\hat{\boldsymbol{\imath}} \cdot r_0\hat{\boldsymbol{\imath}}) + (r_1\hat{\boldsymbol{\jmath}} \cdot r_1\hat{\boldsymbol{\jmath}}) + (r_2\hat{\boldsymbol{k}} \cdot r_2\hat{\boldsymbol{k}}) + 2(r_0\hat{\boldsymbol{\imath}} \cdot r_2\hat{\boldsymbol{k}}) + 2(r_1\hat{\boldsymbol{\jmath}} \cdot r_2\hat{\boldsymbol{k}}) + 2(r_0\hat{\boldsymbol{\imath}} \cdot r_1\hat{\boldsymbol{\jmath}})$
- $= (r_0\hat{\boldsymbol{\imath}} \cdot r_0\hat{\boldsymbol{\imath}}) + (r_1\hat{\boldsymbol{\jmath}} \cdot r_1\hat{\boldsymbol{\jmath}}) + (r_2\hat{\boldsymbol{k}} \cdot r_2\hat{\boldsymbol{k}})$



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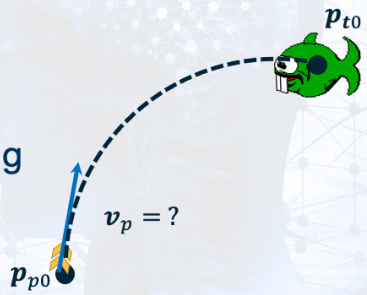
Basis vectors are orthonormal unit vectors

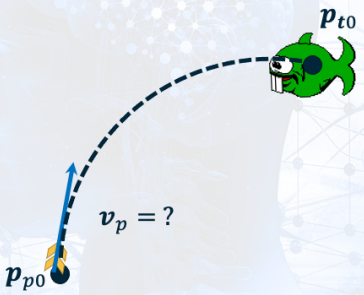
Ballistic Trajectory for Static Target

- So, $\boldsymbol{r} \cdot \boldsymbol{r} = {r_0}^2 \hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\imath}} + {r_1}^2 \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\jmath}} + {r_2}^2 \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}} = {r_0}^2 \hat{\boldsymbol{\imath}} + {r_1}^2 \hat{\boldsymbol{\jmath}} + {r_2}^2 \hat{\boldsymbol{k}} = {r_0}^2 \hat{\boldsymbol{\imath}} + {r_1}^2 \hat{\boldsymbol{\jmath}} + {r_2}^2 \hat{\boldsymbol{k}} = {r_0}^2 + {r_1}^2 + {r_2}^2$
- But we know the magnitude of $\boldsymbol{r}$ is:
- $\|\boldsymbol{r}\| = \sqrt{{r_0}^2 + {r_1}^2 + {r_2}^2}$
- So, $\boldsymbol{r} \cdot \boldsymbol{r} = \|\boldsymbol{r}\|^2 = {r_0}^2 + {r_1}^2 + {r_2}^2$
- Now we can handle multiplying and squaring vectors in our system of equations



Ballistic Trajectory for Static Target

- Now we can go back and simplify:
- $s_p{}^2 t_c{}^2 = \Delta_x{}^2 - g_{px}\Delta_x t_c{}^2 + \frac{1}{4} g_{px}{}^2 t_c{}^4 + \cdots$
- To:
- $s_p{}^2 t_c{}^2 = \|\Delta\|^2 - \boldsymbol{g}_p \cdot \Delta t_c{}^2 + \frac{1}{4}\|\boldsymbol{g}_p\|^2 t_c{}^4$
- And solve for 0:
- $0 = \|\Delta\|^2 - \boldsymbol{g}_p \cdot \Delta t_c{}^2 - s_p{}^2 t_c{}^2 + \frac{1}{4}\|\boldsymbol{g}_p\|^2 t_c{}^4$

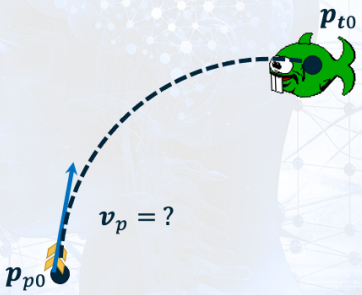


Ballistic Trajectory for Static Target

- Simplify:
- $0 = \frac{1}{4}\|\boldsymbol{g}_p\|^2 {t_c}^4 - (\boldsymbol{g}_p \cdot \Delta + {s_p}^2){t_c}^2 + \|\Delta\|^2$
- Quadratic formula is now in play solving for ${t_c}^2$
- ${t_c}^2 = \frac{\boldsymbol{g}_p \cdot \Delta + {s_p}^2 \pm \sqrt{(\boldsymbol{g}_p \cdot \Delta + {s_p}^2)^2 - \|\boldsymbol{g}_p\|^2 \|\Delta\|^2}}{\frac{1}{2}\|\boldsymbol{g}_p\|^2}$
- Then:
- $t_c = \sqrt{\frac{\boldsymbol{g}_p \cdot \Delta + {s_p}^2 \pm \sqrt{(\boldsymbol{g}_p \cdot \Delta + {s_p}^2)^2 - \|\boldsymbol{g}_p\|^2 \|\Delta\|^2}}{\frac{1}{2}\|\boldsymbol{g}_p\|^2}}$

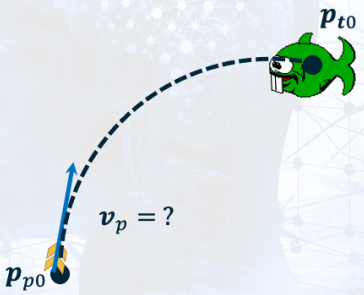
Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$when\ ax^2 + bx + c = 0$$



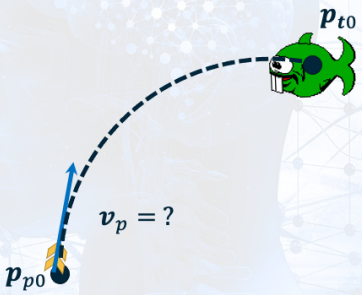
Ballistic Trajectory for Static Target

- If inner radicand is less than zero, there is no solution
- If either candidate t_c is negative, discard it.
- If no positive values are left, there is no solution.
- If both are positive, take the smaller one.
- Otherwise, return the one valid solution.
- In code implementation, do not repeat the same computation more than once. Cache common expressions in variables to reuse.

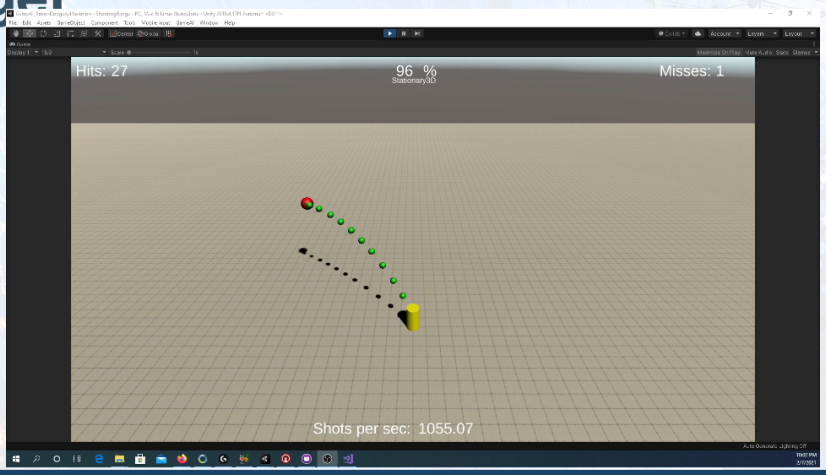


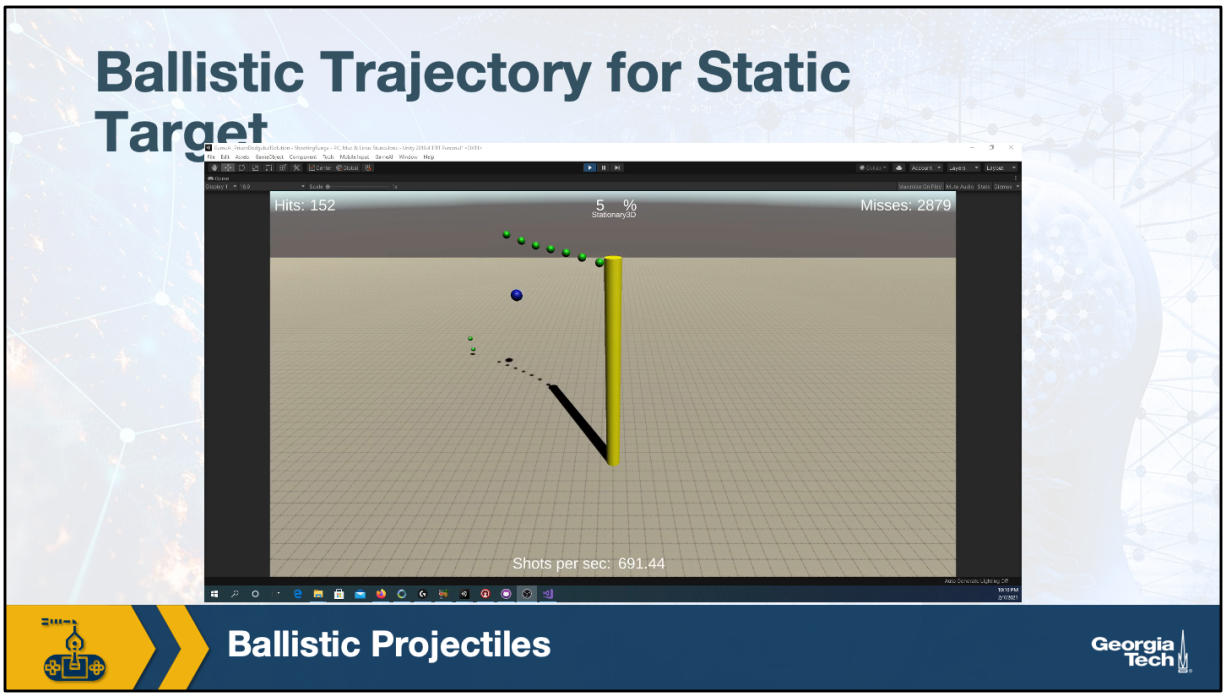
Ballistic Trajectory for Static Target

- Using this previously defined projectile motion equation, solve for $\boldsymbol{u}_{p0}$ with the found t_c and return the result:
- $\boldsymbol{p}_t = \boldsymbol{p}_{p0} + \boldsymbol{u}_{p0} s_p t_c + \frac{1}{2} \boldsymbol{g}_p {t_c}^2$
- $\boldsymbol{u}_{p0} = \frac{\boldsymbol{p}_t - \boldsymbol{p}_{p0} - \frac{1}{2} \boldsymbol{g}_p {t_c}^2}{s_p t_c}$
- $\boldsymbol{u}_{p0} = \frac{2\Delta - \boldsymbol{g}_p {t_c}^2}{2 s_p t_c}$



Ballistic Trajectory for Static Target

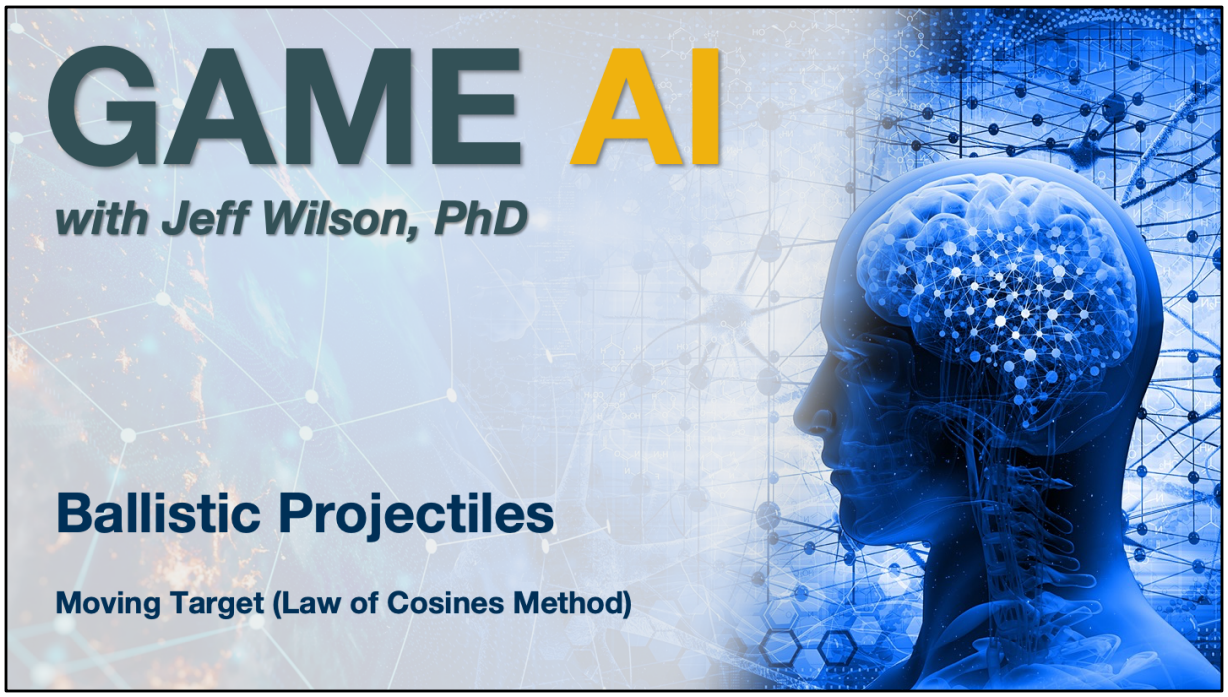




Can’t hit a fast moving target though

Ballistic Trajectory for Static Target

- Static Target Approach
- Pros:
  - Solves directly for a 3D target position with gravity affecting projectile
- Cons:
  - Does not handle moving targets!


GAME AI
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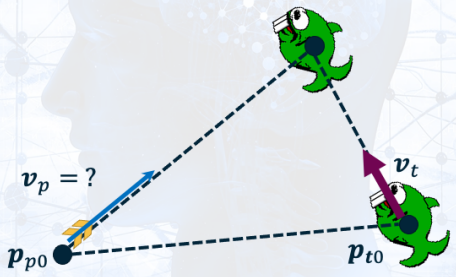
Ballistic Projectiles
Moving Target (Law of Cosines Method)

Moving Target

- The static target solution works well for that purpose, but what about moving targets?
- We might consider an iterative solution that leverages the static target method
- But first, let's consider solving for a moving target

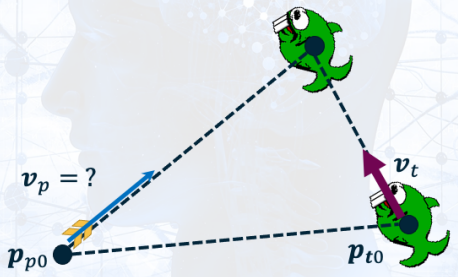
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- Assume top-down (XZ plane) with 2D vectors
  - This is necessary to reduce complexity so that the equations are still solvable
- Target and Projectile move with constant velocity
- But for projectile, only the speed is known
- We will also see how to adapt the 2D solution to 3D, possibly with gravity



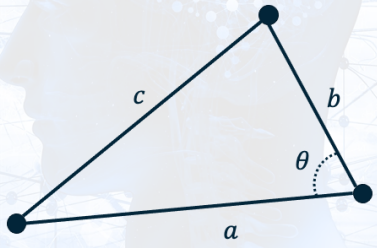
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- $\boldsymbol{p}_{t0} -$ *Initial Target Position*
- $\boldsymbol{p}_{p0} -$ *Initial Projectile Position*
- $\boldsymbol{v}_t -$ *Target Velocity*
- $s_t -$ *Target Speed*
- $s_p -$ *Projectile Speed*
- $\boldsymbol{v}_p -$ *Projectile Velocity*
- $t_c -$ *Time of collision*



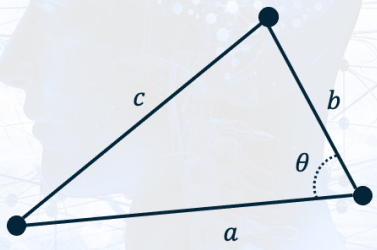
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- **Law of Cosines:**
- $a^2 + b^2 - 2ab\cos\theta = c^2$



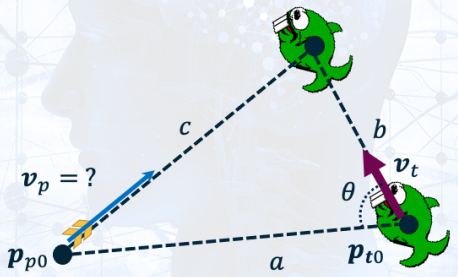
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- **Law of Cosines:**
- $a^2 + b^2 - 2ab\cos\theta = c^2$
- **Dot Product:**
- $\boldsymbol{a} \cdot \boldsymbol{b} = \|\boldsymbol{a}\|\|\boldsymbol{b}\|\cos\theta$
- $\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{b}} = \cos\theta$



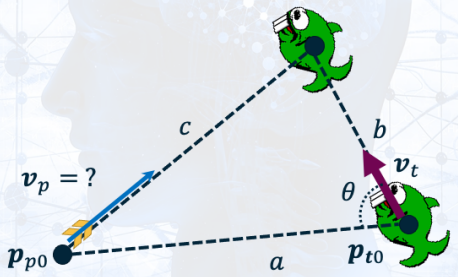
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- $\boldsymbol{a} = \boldsymbol{p}_{p0} - \boldsymbol{p}_{t0}$
- $a = \|\boldsymbol{a}\|$
- $s_t = \|\boldsymbol{v}_t\|$
- $\cos\theta = \hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{v}}_t$



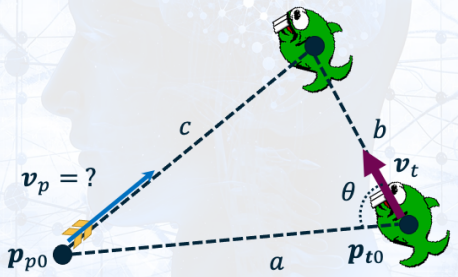
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- **Displacement by Velocity:**
- $\boldsymbol{d} = \boldsymbol{v}t$
- $\|\boldsymbol{d}\| = \|\boldsymbol{v}t\|$
- $d = \|\boldsymbol{v}t\|$
- $d = \|\boldsymbol{v}\|t$
- $b = s_t t_c$
- $c = s_p t_c$



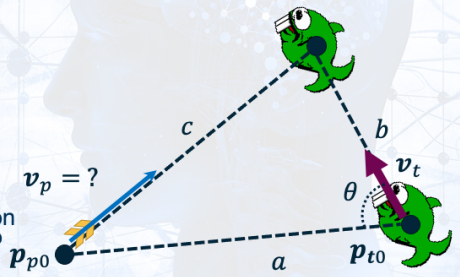
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- Plug $a, b, \& c$ into Law of Cosines
- $a^2 + (s_t t)^2 - 2as_t t\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{b}} = (s_p t)^2$
- Simplify
- ${s_p}^2 t^2 - {s_t}^2 t^2 + 2as_t t\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{b}} - a^2 = 0$
- $({s_p}^2 - {s_t}^2)t^2 + (2as_t\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{b}})t - a^2 = 0$



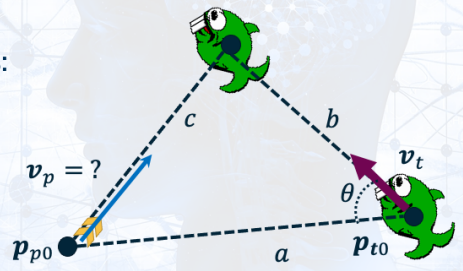
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- $(s_p{}^2 - s_t{}^2)t_c{}^2 + (2as_t\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{b}})t_c - a^2 = 0$
- Apply Quadratic Formula
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *when* $ax^2 + bx + c = 0$
- $t_c = \frac{-(2as_t\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{b}}) \pm \sqrt{(2as_t\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{b}})^2 + 4(s_p{}^2 - s_t{}^2)a^2}}{2(s_p{}^2 - s_t{}^2)}$
- Test radicand for negative, and denominator for zero. Only use a positive t. User smaller of two valid t's.
- In code implementation, do not repeat the same computation more than once. Cache common expressions in variables to reuse.



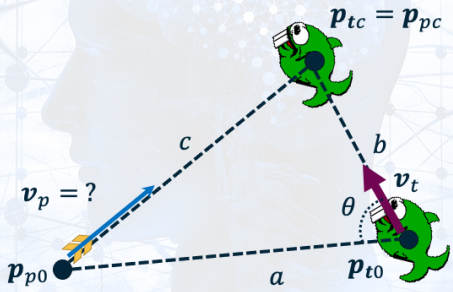
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- If denominator is zero, a solution may still be possible
- The zero occurs if s_p and s_t are the same
- If that is the case, we know that c and b of Law of Cosines equation are the same, so:
- $a^2 + b^2 - 2ab\cos\theta = b^2$ and solve for b:
- $b = \frac{a}{2\cos\theta}$ and then plug in known trajectory terms:
- $s_t t_c = \frac{\|\boldsymbol{a}\|}{2\hat{\boldsymbol{a}}\cdot\hat{\boldsymbol{v}}_t}$, where $\mathbf{a} = \boldsymbol{p}_{p0} - \boldsymbol{p}_{t0}$ and solve for t_c:
- $t_c = \frac{\|\boldsymbol{a}\|}{2s_t\hat{\boldsymbol{a}}\cdot\hat{\boldsymbol{v}}_t}$, and only valid if $\hat{\boldsymbol{a}}\cdot\hat{\boldsymbol{v}}_t > 0$



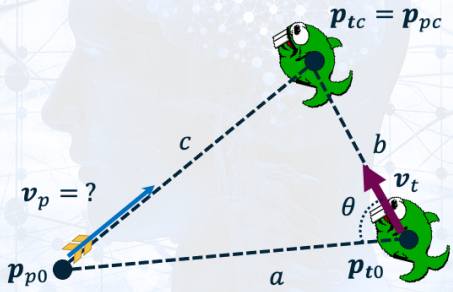
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- Now have time(s) of collision, still need to solve $\boldsymbol{v_p}$
- $\boldsymbol{p_{tc}}$ $-$ *Target position at collision*
- $\boldsymbol{p_{pc}}$ $-$ *Projectile position at collision*
- $\boldsymbol{p_{tc}} = \boldsymbol{p_{pc}}$
- $\boldsymbol{p_{tc}} = \boldsymbol{p_{t0}} + \boldsymbol{v_t} * t_c$ (particle dynamics)
- $\boldsymbol{v_p} = \frac{(\boldsymbol{p_{pc}} - \boldsymbol{p_{p0}})}{t_c} = \frac{\left((\boldsymbol{p_{t0}} + \boldsymbol{v_t} * t_c) - \boldsymbol{p_{p0}}\right)}{t_c}$
- **Finally:** $\boldsymbol{v_p} = \frac{(\boldsymbol{p_{t0}} - \boldsymbol{p_{p0}})}{t_c} + \boldsymbol{v_t}$



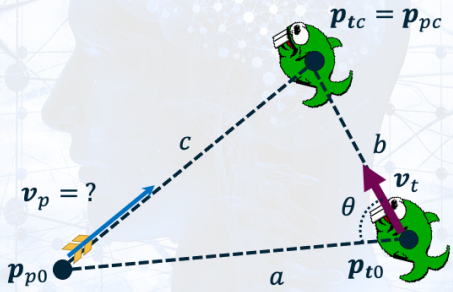
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- If the game is actually 3D, then $\boldsymbol{v}_p$ can be solved with 3D vectors and still get a collision
- However, $\|\boldsymbol{v}_p\| \geq S_p$ (y component adds speed)
- This may or may not be acceptable for gameplay
- To be addressed later…



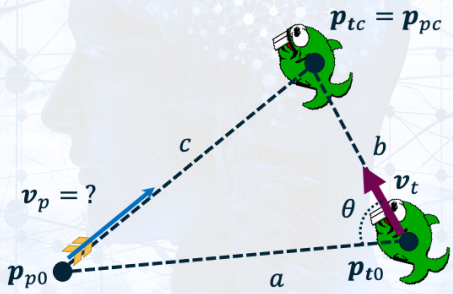
Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- What if the projectile is lobbed in 3D with the effects of gravity?
- Particle Dynamics: $\boldsymbol{p} = \boldsymbol{p}_0 + \boldsymbol{v}_0 t + \frac{1}{2}\boldsymbol{a}t^2$
- $\boldsymbol{a}_p$ - Gravity vector (with sign)
- $\boldsymbol{p}_{pc} = \boldsymbol{p}_{p0} + \boldsymbol{v}_p t_c + \frac{1}{2}\boldsymbol{a}_p {t_c}^2$
- $\boldsymbol{p}_{t0} + \boldsymbol{v}_t * t_c = \boldsymbol{p}_{p0} + \boldsymbol{v}_p t_c + \frac{1}{2}\boldsymbol{a}_p {t_c}^2$
- $\boldsymbol{v}_p = \frac{\boldsymbol{p}_{t0} - \boldsymbol{p}_{p0}}{t_c} + \boldsymbol{v}_t - \frac{1}{2}\boldsymbol{a}_p t_c$

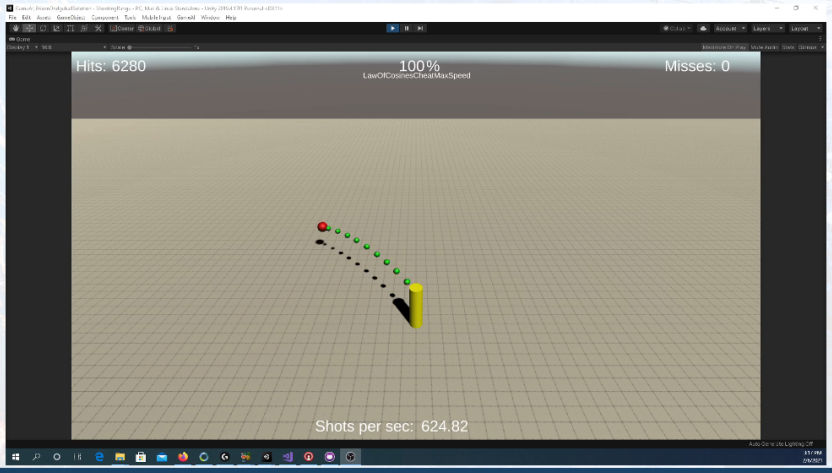


Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

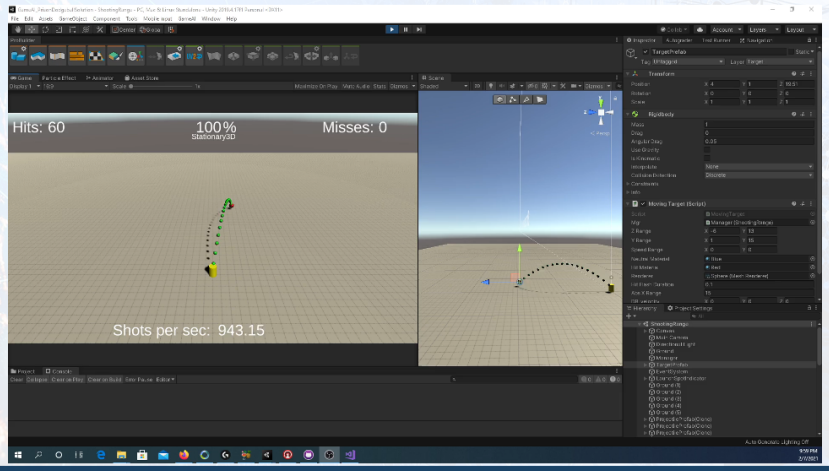
- Note that the lobbed projectile has the same problem with $\|\boldsymbol{v}_p\| \geq S_p$ (y component adds speed)
- Again, this may or may not be acceptable for gameplay
- Could separately enforce max intercept time or distance to intercept



Law of Cosines Approach

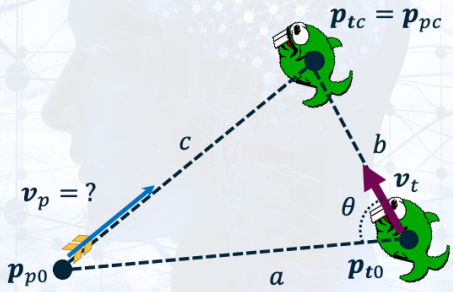


Law of Cosines Cheat Problem



Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- Law of Cosines Approach
- Pros:
  - Solves directly for a moving target in 2D
  - Can be made to work in 3D with limitations
- Cons:
  - Does not guarantee a max speed in 3D
  - Not ideal for extreme height differences or target moving significantly in the *y* direction
  - Not ideal for flight sims or space combat

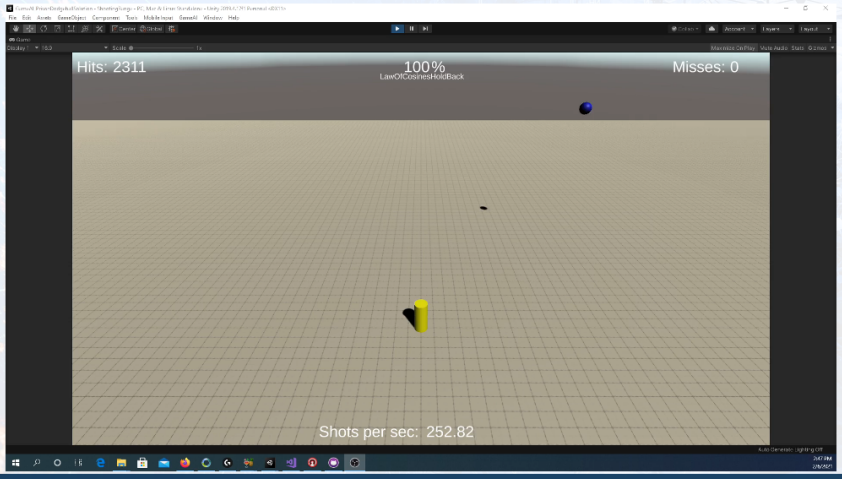


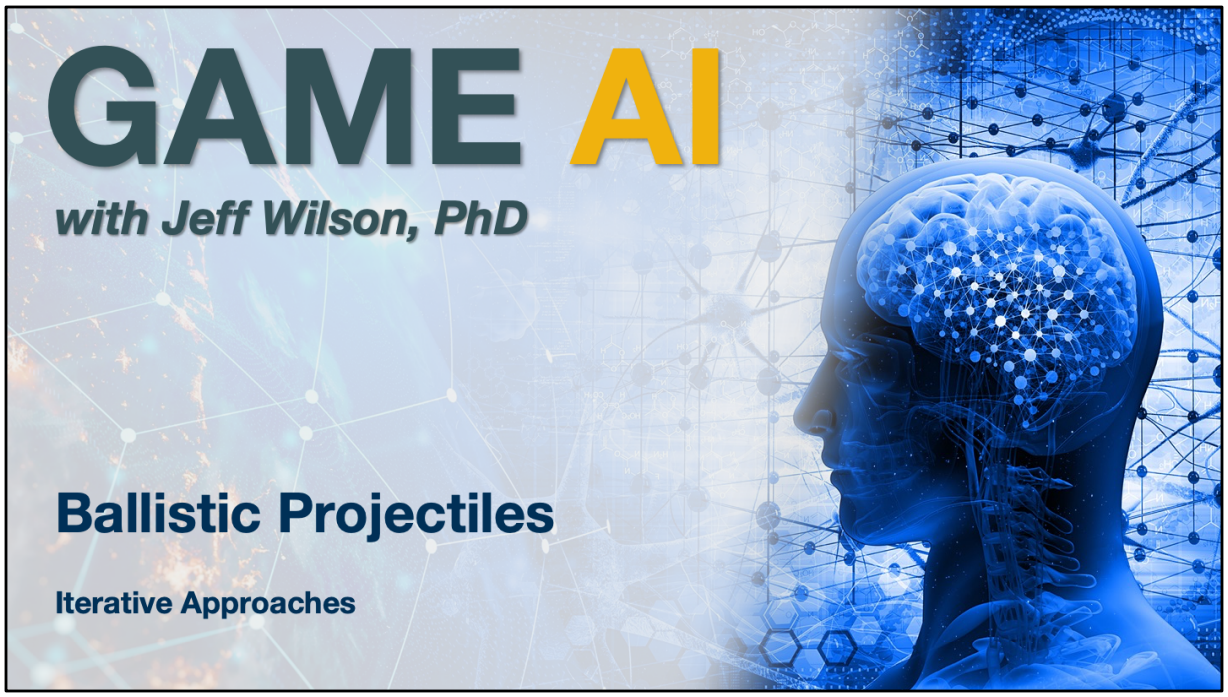
Could probably still be used in a flight/space sim if the algorithm selected the best axis aligned plane to use in any give circumstance

Ballistic Trajectory Intercept for Constant Velocity Projectile and Target

- If Law of Cosines is used but the projectile speed limit must be imposed on a 3D or 3D Lobbed launch, then one approach is to replace S_p with $S_{holdback}$, which is reduced by a Holdback percentage of S_p (like 90%)
- This leaves a fraction of total speed that can be applied to the y direction
- If solved successfully, check to see if $\|\boldsymbol{v}_p\| > S_p$
- If it is, fail to launch (or launch with capped speed knowing the projectile will not hit)
- Pros:
  - Often works when initial projectile position and target stay at close to the same elevation
  - Tendency to screen low probability launches (e.g., close to 45° launch angle)
- Cons:
  - Can fail to find a solution when one is possible
  - Can result in a projectile traveling at less than full speed

Law of Cosines Holdback




GAME AI
with Jeff Wilson, PhD
Ballistic Projectiles
Iterative Approaches

Ballistic Projectile: What about accelerating targets, or other advanced scenarios?

- Can sometimes solve directly but often results in challenging high order polynomials
- Usually best to use an iterative approach

Ballistic Trajectory: Simple Iterative Approach for Static Target Solver

- The method for solving a ballistic trajectory for a static target can be used to aim at moving targets
- An iterative method can leverage the static target solver

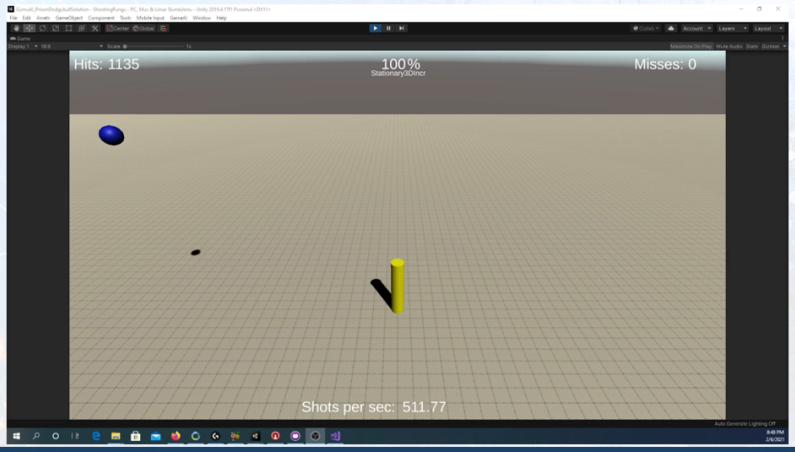
Ballistic Trajectory: Simple Iterative Approach for Static Target Solver

1. Assume target is not moving
2. Solve for t_n projectile intercept of target in step 1.
   - Perhaps use static 3D target method
3. Plug in t_0 for target's kinematic eqn (must be solvable):
   - $\boldsymbol{p}_{tn} = \boldsymbol{p}_{t0} + \boldsymbol{v}_{t0} t_n + \frac{1}{2} \boldsymbol{a}_t {t_n}^2$ (possible equation)
4. Now go to 2. but replace target position with $\boldsymbol{p}_{tn}$
5. Repeat until a terminating condition (like depth count or minimal change in positions, etc.)

Ballistic Trajectory: Simple Iterative Approach for Static Target Solver

- Cap the number of iterations (around 3 to 6)
- Preserve previous valid to return if future iterations fail to refine
- Fail immediately if first attempt fails
- Can perform a sanity check on final intercept time by plugging into projectile and target kinematic equations to determine if the iterative estimate is within some minimum distance
- This has added bonus of allowing for shots that are likely to get close enough to a target game object but not necessary hit the center

Static 3D Target with Iterative Refinement



Ballistic Projectiles

Georgia Tech

Ballistic Trajectory: Simple Iterative Approach for Static Target Solver

- **Pros**
  - Easy to implement
- **Cons**
  - May not be able to refine to a stable solution, especially around the point of closest approach of the target, or with higher derivatives of motion
  - Tends to favor shooting behind target. Not good for dramatic affect (e.g., FPS player won't see enemy shots missing)
  - Target that is initially unreachable, but will be reachable in the future, creates implementation difficulties
  - Doesn't work if kinematic movement of projectile and/or target cannot be directly evaluated (drag and other scenarios)

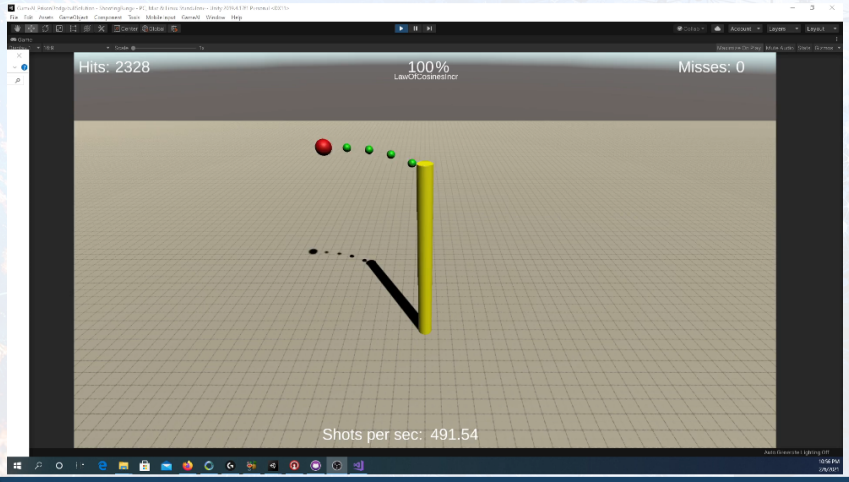
2D Law of Cosines Iterative Algorithm

- An iterative algorithm is also possible with Law of Cosines to enforce a max speed
- Start with $S_{holdback} = S_p$ This sets the target speed in the *XZ* plane
- If a solution is found, test for $\|\boldsymbol{v}_p\| > S_p$ (overall speed too fast?)
- If too fast, try again with an $S_{holdback}$ adjusted to be halfway closer to length of optimal launch (45 degrees)
- Otherwise, check if $\|\boldsymbol{v}_p\| \geq (1-\varepsilon)S_p$ AND $S_{holdback} \geq S_{XZAtOptimalAngle}$
- If it is, it's good enough. Immediately exit with result
- If too slow, try again with: $S_{holdback} = S_{prevHB} + \frac{1}{2}(S_p - S_{prevHB})$
- If the angle is high, try again with an $S_{holdback}$ adjusted to be halfway closer to length of optimal launch (45 degrees)

2D Law of Cosines Iterative Algorithm

- Cap the number of iterations (around 3 to 6)
- Preserve previous best valid result (fastest in *XZ* plane and within overall speed limit) to return if future iterations fail to refine
- Fail immediately if first attempt fails
- *XZ* vector length for optimal (45 deg.) launch can be solved with Pythagorean Theorem for 45-45-90 triangle

Law of Cosines Iterative



Ballistic Projectiles

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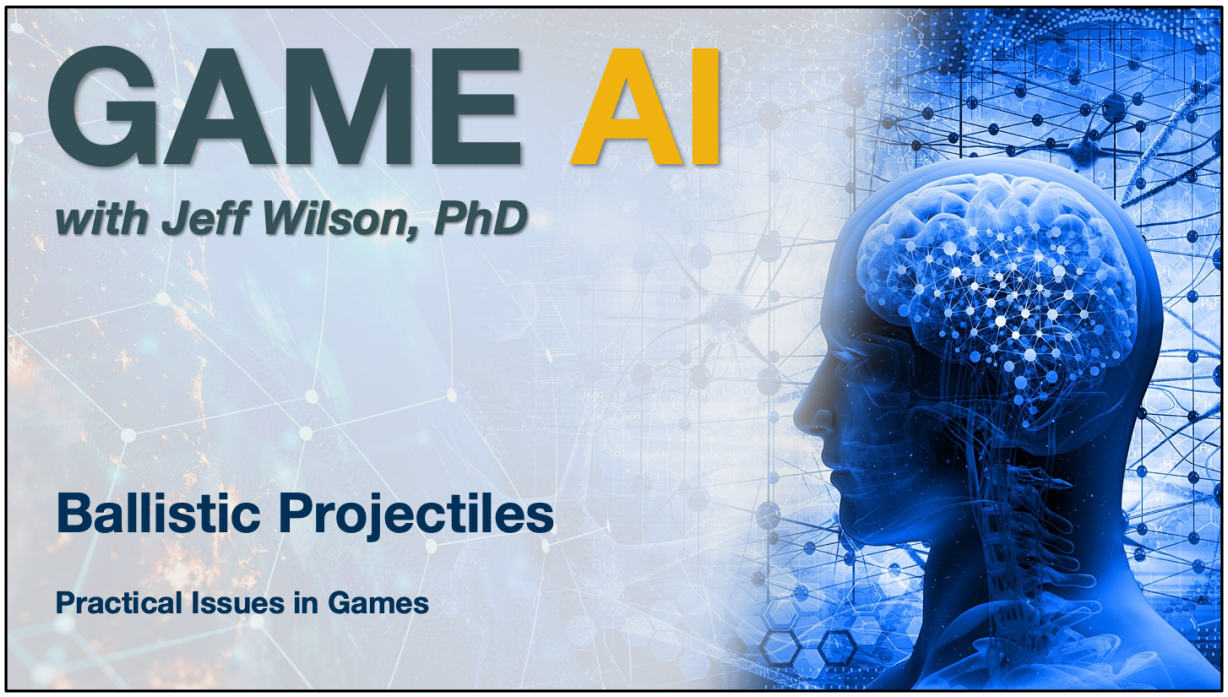
2D Law of Cosines Iterative Algorithm

- **Pros:**
  - Finds exact solutions for moving targets
- **Cons:**
  - May miss solutions near the limits
  - May not optimize shot for maximum speed and lowest angle (soonest intercept)
  - May not be able to refine to a stable solution, especially around the point of closest approach of the target, or with higher derivatives of motion
  - Target that is initially unreachable, but will be reachable in the future, creates implementation difficulties
  - Doesn't work if kinematic movement of projectile and/or target cannot be directly evaluated (drag and other scenarios)

Other Iterative Approaches

- Sometimes projectile and/or target movement can only be modeled incrementally (position/state at current timestep dependent on position/state at previous timestep)
- In this case, an incremental algorithm will need to increment through a simplified simulation of projectiles with candidate trajectories and determine the point of closest approach to the target
- Millington, *Artificial Intelligence for Games* discusses further


GAME AI
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Ballistic Projectiles
Practical Issues in Games

Ballistic trajectory: with animated characters

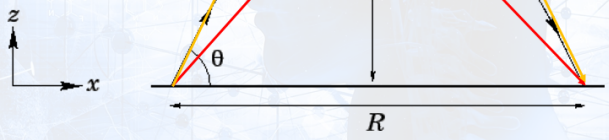
- Animation delay from projectile launch (is especially pronounced with throwing motion)
- Must make prediction based on when throw will actually happen.
- Predict throw from predicted future character position and relative launch position (e.g., where is the hand at throw release point?)

Ballistic Trajectory: Resource Management

- Agent maybe should not waste valuable ammo. Or commit to long animation and be unable to throw at a better time
- Implement heuristic for throw decision. Don't throw if conditions are bad
- Consider:
  - Is target at top speed? No? Probably accelerating. Just wait a bit…
  - Target turning? Yes? Wait till going straight…
  - Is target predicted to hit edge of navmesh? Will likely turn. Lay off throwing…
  - Is projectile predicted to hit something sooner than target intercept time? Don't throw…
  - Do you know target's goal? Anticipate arrival at that location

Ballistic Trajectory: Predicting lobbed projectile collisions

- Break parabola into line segments
- Raycast for each segment
- Can projectile bounce?
  - Maybe adjust aim lower and skip it, but consider friction slowing projectile down
- What if agent can control projectile speed and angle?



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Example

- Check out the Minion Targeting Video posted separately