MONASH University

MONASH BUSINESS SCHOOL

Tidy Time Series & Forecasting in R

6. Introduction to forecasting

robjhyndman.com/workshop2020

Outline

Outline

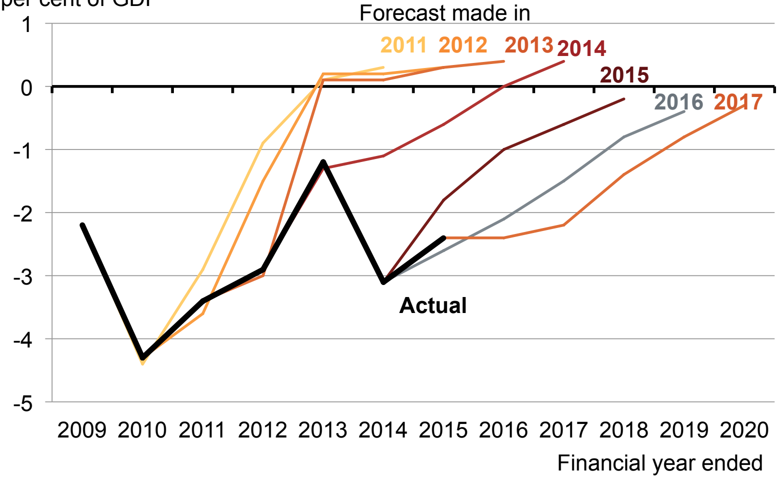
1. What can we forecast?
2. The statistical forecasting perspective
3. Benchmark methods
4. Lab Session 11
5. Residual diagnostics
6. Lab Session 12
7. Forecast accuracy measures
8. Lab Session 13

Forecasting is difficult

Commonwealth plans to drift back to surplus show the triumph of experience over hope

GRATTAN Institute

Actual and forecast Commonwealth underlying cash balance
per cent of GDP



Forecasting is difficult

A Timeline of Very Bad Future Predictions

1800

"Rail travel at high speed is not possible, because passengers, unable to breathe, would die of asphyxia."

Dr. Dionysys Larder, Professor of Natural Philosophy & Astronomy, University College London

1859

"Drill for oil? You mean drill into the ground to try and find oil? You're crazy!"

Associates of Edwin L. Drake refusing his suggestion to drill for oil in 1859 (Later that year, Drake succeeded in drilling the first oil well.)

1876

"This telephone has too many shortcomings to be seriously considered as a means of communication."

Western Union internal memo

1880

"Everyone acquainted with the subject will recognize it as a conspicuous failure."

Henry Morton, president of the Stevens Institute of Technology, on Edison's light bulb

1902

"Flight by machines heavier than air is unpractical and insignificant, if not utterly impossible."

Simon Newcomb, Canadian-American astronomer and mathematician, 18 months before the Wright Brothers' flight at Kittyhawk

1903

"The horse is here to stay, but the automobile is only a novelty, a fad."

The president of the Michigan Savings Bank, advising Henry Ford's lawyer not to invest in the Ford Motor Company

1916

"The idea that cavalry will be replaced by these iron coaches is absurd. It is little short of treasonous."

Comment of Aide-de-camp to Field Marshal Haig, at tank demonstration

1916

"The cinema is little more than a fad. It's canned drama. What audiences really want to see is flesh and blood on the stage."

Charlie Chaplin, actor, producer, director, and studio founder

1921

"The wireless music box has no imaginable commercial value. Who would pay for a message sent to no one in particular?"

Associates of commercial radio and television pioneer, David Sarnoff, responding to his call for investment in the radio

1946

"Television won't last because people will soon get tired of staring at a plywood box every night."

Darryl Zanuck, movie producer, 20th Century Fox

1977

"There is no reason for any individual to have a computer in his home."

Ken Olson, president, chairman and founder of Digital Equipment Corporation

1995

"The truth is no online database will replace your daily newspaper…"

Clifford Stoll, Newsweek article entitled ***The Internet? Bah!***

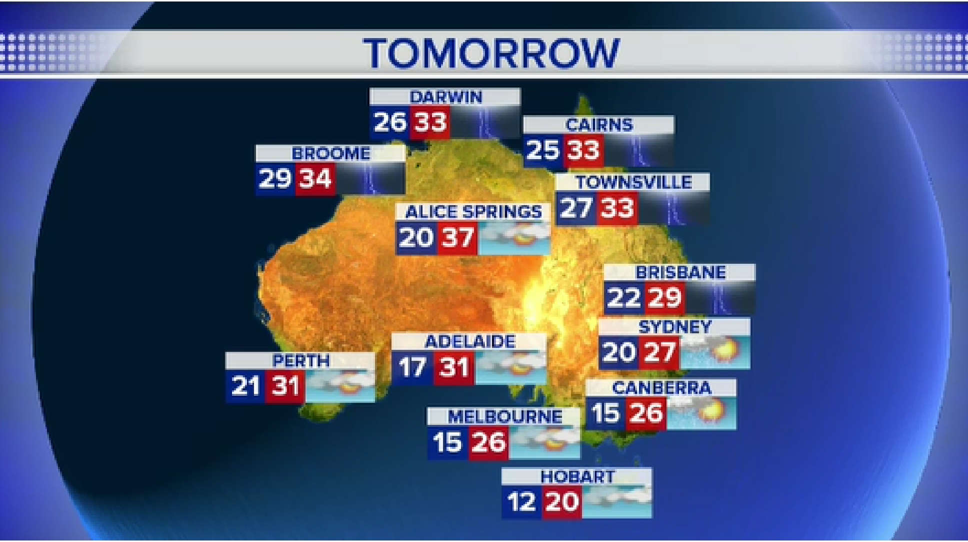
What can we forecast?

What can we forecast?

What can we forecast?

What can we forecast?

What can we forecast?



What can we forecast?

What can we forecast?

Which is easiest to forecast?

1. daily electricity demand in 3 days time
2. timing of next Halley's comet appearance
3. time of sunrise this day next year
4. Google stock price tomorrow
5. Google stock price in 6 months time
6. maximum temperature tomorrow
7. exchange rate of $US/AUS next week
8. total sales of drugs in Australian pharmacies next month

Which is easiest to forecast?

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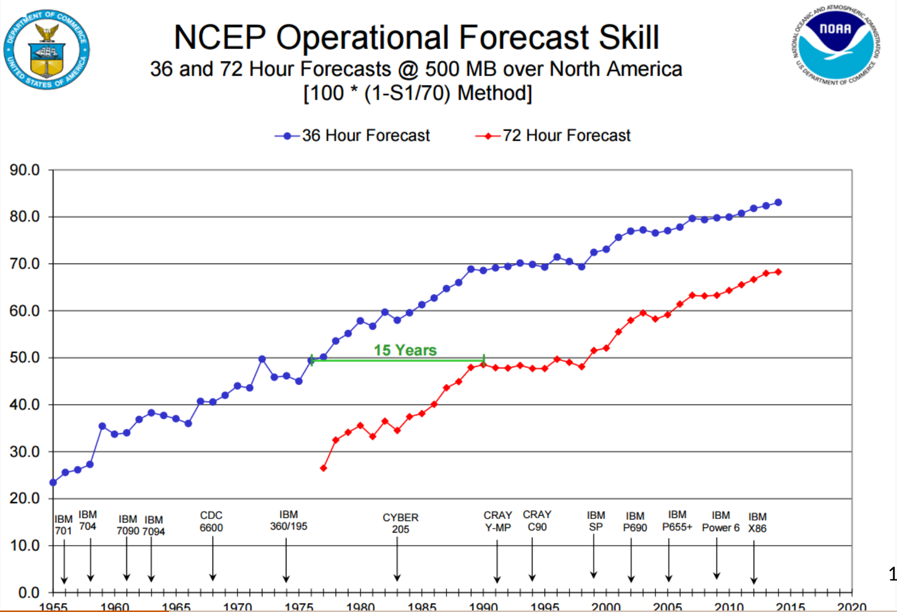
- how do we measure "easiest"?
- what makes something easy/difficult to forecast?

Factors affecting forecastability

Something is easier to forecast if:

- we have a good understanding of the factors that contribute to it
- there is lots of data available;
- the forecasts cannot affect the thing we are trying to forecast.
- there is relatively low natural/unexplainable random variation.
- the future is somewhat similar to the past

Improving forecasts



CASE STUDY 1: Paperware company

Problem: Want forecasts of each of hundreds of items. Series can be stationary, trended or seasonal. They currently have a large forecasting program written in-house but it doesn't seem to produce sensible forecasts. They want me to tell them what is wrong and fix it.

Additional information

- Program written in COBOL making numerical calculations limited. It is not possible to do any optimisation.
- Their programmer has little experience in numerical computing.
- They employ no statisticians and want

CASE STUDY 1: Paperware company

Methods currently used

- **A** 12 month average
- **C** 6 month average
- **E** straight line regression over last 12 months
- **G** straight line regression over last 6 months
- **H** average slope between last year's and this year's values. (Equivalent to differencing at lag 12 and taking mean.)
- **I** Same as H except over 6 months.
- **K** I couldn't understand the explanation.

CASE STUDY 2: PBS

CASE STUDY 2: PBS

The Pharmaceutical Benefits Scheme (PBS) is the Australian government drugs subsidy scheme.

- Many drugs bought from pharmacies are subsidised to allow more equitable access to modern drugs.
- The cost to government is determined by the number and types of drugs purchased. Currently nearly 1% of GDP.
- The total cost is budgeted based on forecasts of drug usage.

CASE STUDY 2: PBS

ABC News Online
AUSTRALIAN BROADCASTING CORPORATION

NewsRadio
Streaming audio news
LISTEN: WMP | Real

Select a Topic from the list below

Click "Refresh" or "Reload" on your browser for the latest edition.

This Bulletin: **Wed, May 30 2001 6:22 PM AEST**

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- Business
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- Arts
- Sci Tech
- Indigenous
- Weather
- Rural
- Local News
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Search

SPECIALS

- Federal Election

POLITICS

Opp demands drug price restriction after PBS budget blow-out

The Federal Opposition has called for tighter controls on drug prices after the Pharmaceutical Benefits Scheme (PBS) budget blew out by almost $800 million.

The money was spent on two new drugs including the controversial anti-smoking aid Zyban, which dropped in price from $220 to $22 after it was listed on the PBS.

the Public Record
For full election coverage

FEATURES

Public Record
Federal Election 2001

For a fresh perspective on the federal election, reach into ABC Online's campaign weblog, The Poll Vault.

Audio News Online

CASE STUDY 2: PBS

- In 2001: $4.5 billion budget, under-forecasted by $800 million.
- Thousands of products. Seasonal demand.
- Subject to covert marketing, volatile products, uncontrollable expenditure.
- Although monthly data available for 10 years, data are aggregated to annual values, and only the first three years are used in estimating the forecasts.
- All forecasts being done with the `FORECAST` function in MS-Excel!

CASE STUDY 3: Car fleet company

Client: One of Australia's largest car fleet companies

Problem: how to forecast resale value of vehicles? How should this affect leasing and sales policies?

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Additional information

- They can provide a large amount of data on previous vehicles and their eventual resale values.
- The resale values are currently estimated by a group of specialists. They see me as a threat and do not cooperate.

CASE STUDY 4: Airline

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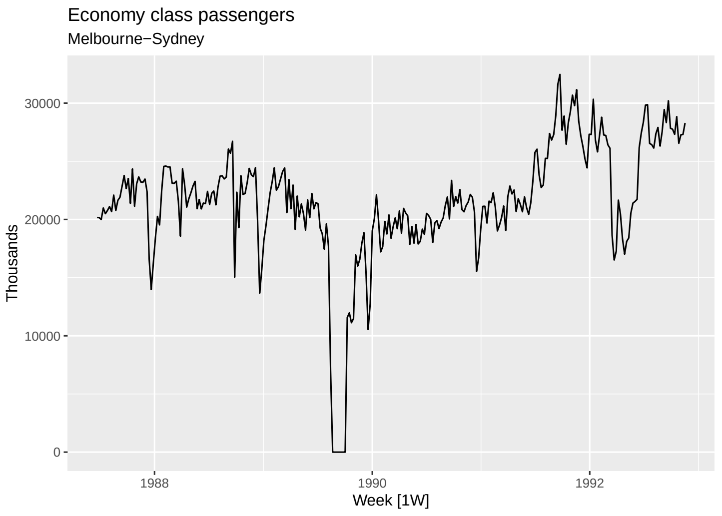


CASE STUDY 4: Airline

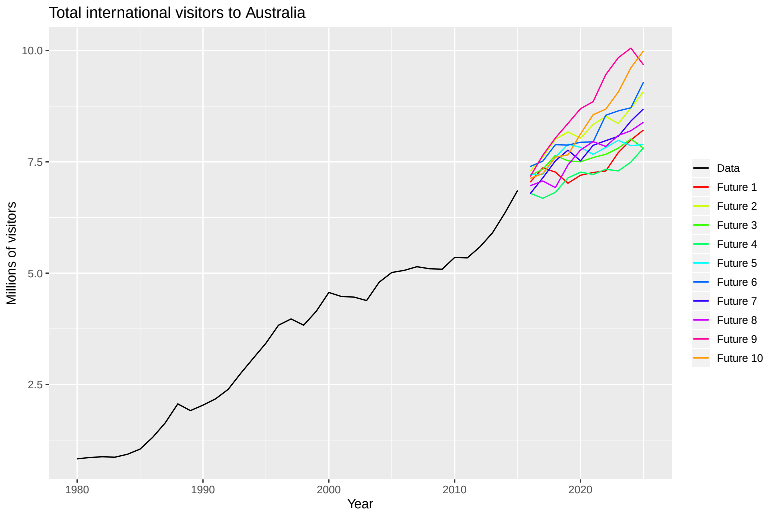
Problem: how to forecast passenger traffic on major routes?

Additional information

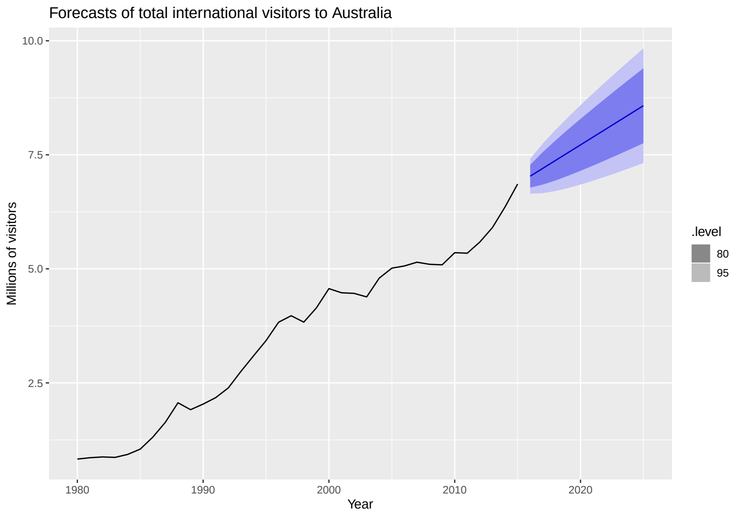
- They can provide a large amount of data on previous routes.
- Traffic is affected by school holidays, special events such as the Grand Prix, advertising campaigns, competition behaviour, etc.
- They have a highly capable team of people who are able to do most of the computing.

Outline

Sample futures



Forecast intervals

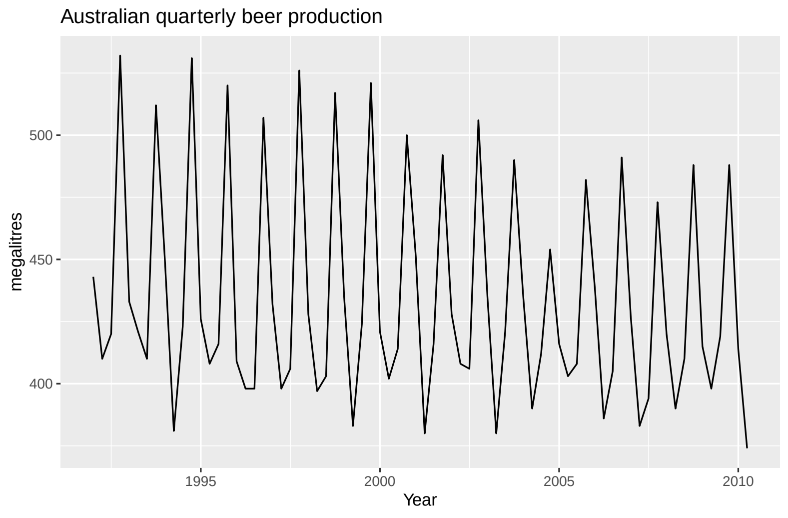


Statistical forecasting

- Thing to be forecast: a random variable, y_t.
- Forecast distribution: If $\mathcal{I}$ is all observations, then $y_t|\mathcal{I}$ means "the random variable y_t given what we know in $\mathcal{I}$".
- The "point forecast" is the mean (or median) of $y_t|\mathcal{I}$
- The "forecast variance" is $\text{var}[y_t|\mathcal{I}]$
- A prediction interval or "interval forecast" is a range of values of y_t with high probability.
- With time series, $y_{t|t-1} = y_t|\{y_1, y_2, \dots, y_{t-1}\}$.
- $\hat{y}_{T+h|T} = \text{E}[y_{T+h}|y_1, \dots, y_T]$ (an h-step forecast taking account of all observations up to time T).

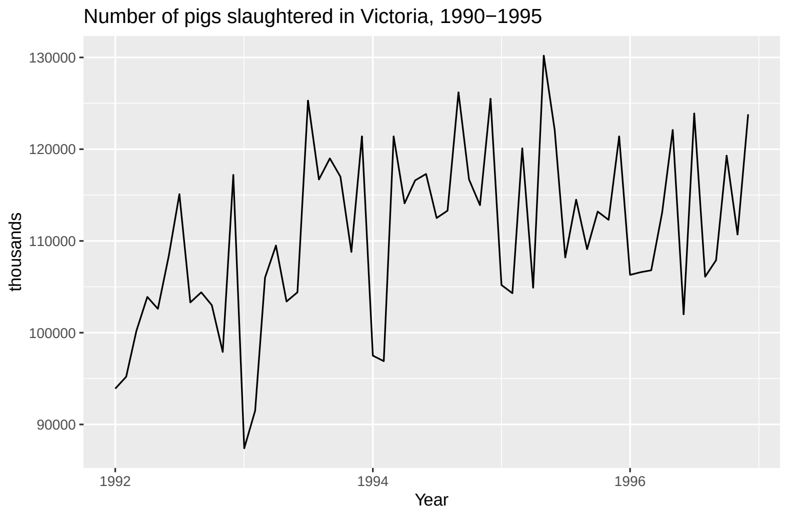
Outline

Some simple forecasting methods



How would you forecast these series?

Some simple forecasting methods



How would you forecast these series?

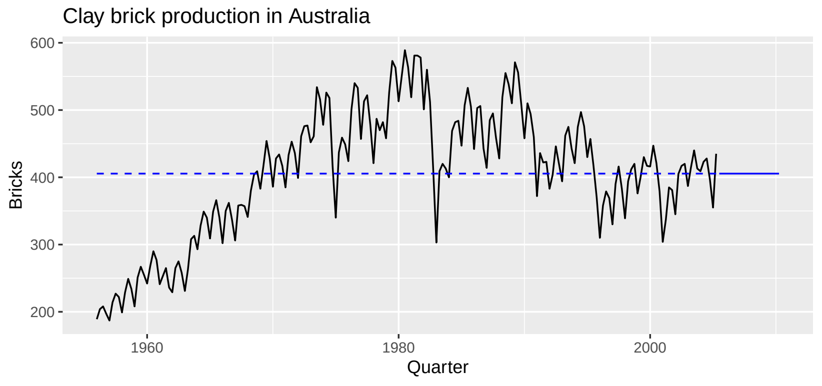
Some simple forecasting methods

How would you forecast these series?

Some simple forecasting methods

MEAN(y): Average method

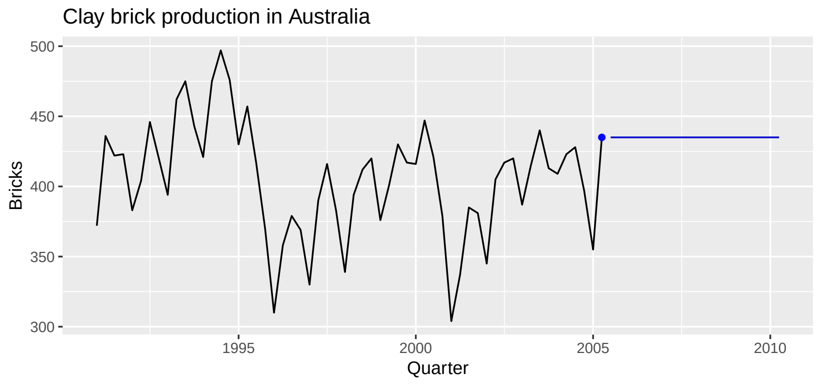
- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$



Some simple forecasting methods

`NAIVE(y)`: Naïve method

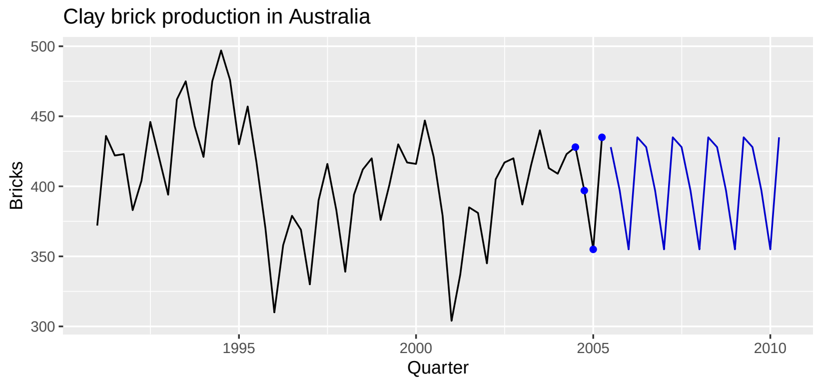
- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.



Some simple forecasting methods

`SNAIVE(y ~ lag(m))`: Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h-1)/m$.



Some simple forecasting methods

`RW(y ~ drift())`: Drift method

- Forecasts equal to last value plus average change.
- Forecasts:

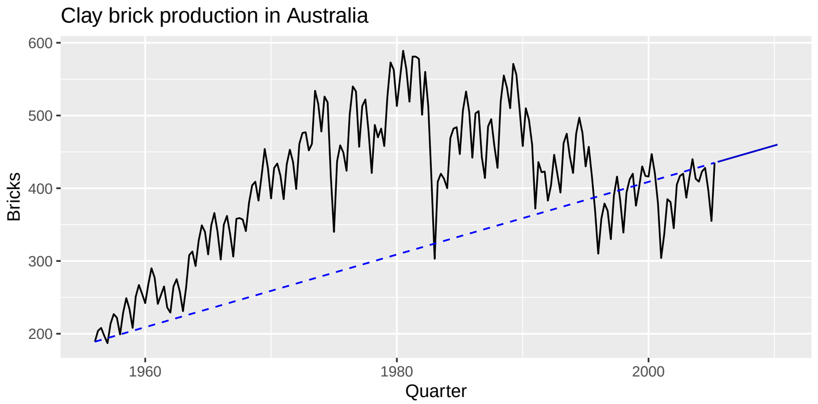
$$\hat{y}_{T+h|T} = y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})$$

$$= y_T + \frac{h}{T-1}(y_T - y_1).$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods

Drift method



Model fitting

The `model()` function trains models to data.

```
# Fit the models
beer_fit <- aus_production %>%
  model(
    Mean = MEAN(Beer),
    Naïve = NAIVE(Beer),
    Seasonal naïve = SNAIVE(Beer),
    Drift = RW(Beer ~ drift())
  )
beer_fit
```

```
## # A mable: 1 x 4
##   Mean    Naïve   Seasonal naïve Drift
##   <model> <model> <model>          <model>
## 1 <MEAN>  <NAIVE> <SNAIVE>         <RW w/ drift>
```

A `mable` is a model table, each cell corresponds to a fitted model.

Producing forecasts

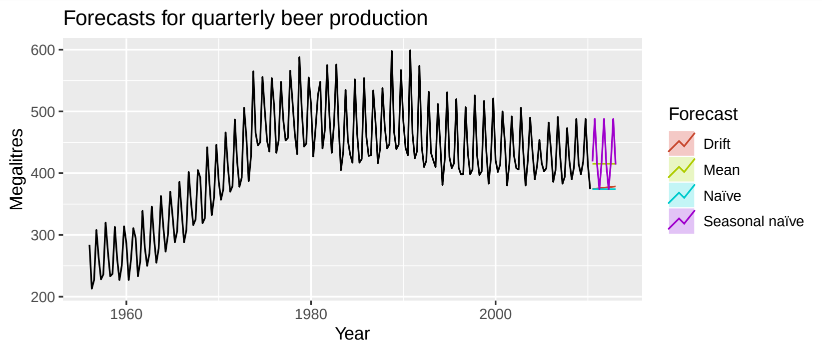
```r
beer_fc <- beer_fit %>%
  forecast(h = 11)
```

```
## # A fable: 44 x 4 [1Q]
## # Key:     .model [4]
##   .model Quarter  Beer .distribution
##   <chr>    <qtr> <dbl> <dist>
## 1 Mean   2010 Q3  415. N(415, 7409)
## 2 Mean   2010 Q4  415. N(415, 7409)
## 3 Mean   2011 Q1  415. N(415, 7409)
## 4 Mean   2011 Q2  415. N(415, 7409)
## # ... with 40 more rows
```

A `fable` is a forecast table with point forecasts and distributions.

Visualising forecasts

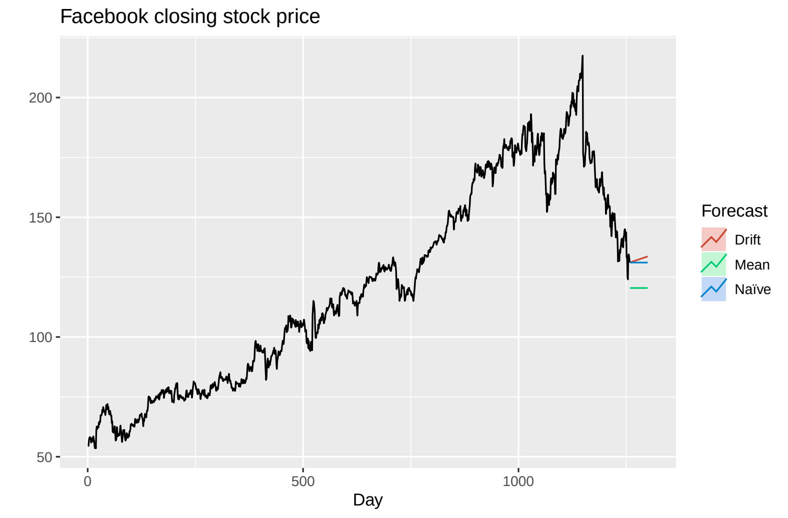
```
beer_fc %>%
  autoplot(aus_production, level = NULL) +
  ggtitle("Forecasts for quarterly beer production") +
  xlab("Year") + ylab("Megalitres") +
  guides(colour=guide_legend(title="Forecast"))
```



Facebook closing stock price

```
# Extract training data
fb_stock <- gafa_stock %>%
  filter(Symbol == "FB") %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index=trading_day, regular=TRUE)
# Specify, estimate and forecast
fb_stock %>%
  model(
    Mean = MEAN(Close),
    Naïve = NAIVE(Close),
    Drift = RW(Close ~ drift())
  ) %>%
  forecast(h=42) %>%
  autoplot(fb_stock, level = NULL) +
  ggtitle("Facebook closing stock price") +
    xlab("Day") + ylab("") +
    guides(colour=guide_legend(title="Forecast"))
```

Facebook closing stock price



Outline

Lab Session 11

- Produce forecasts using an appropriate benchmark method for household wealth (`hh_budget`). Plot the results using `autoplot()`.
- Produce forecasts using an appropriate benchmark method for Australian takeaway food turnover (`aus_retail`). Plot the results using `autoplot()`.

Outline

Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations $y_1, \dots, y_t$.
- We call these "fitted values".
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

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Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

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2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

Facebook closing stock price

```
fb_stock %>%
  autoplot(Close)
```

Facebook closing stock price

Naïve forecast:

$$\hat{y}_{t|t-1} = y_{t-1}$$

Facebook closing stock price

Naïve forecast:

$$\hat{y}_{t|t-1} = y_{t-1}$$

$$e_t = y_t - \hat{y}_{t|t-1} = y_t - y_{t-1}$$

Facebook closing stock price

Naïve forecast:

$$\hat{y}_{t|t-1} = y_{t-1}$$

$$e_t = y_t - \hat{y}_{t|t-1} = y_t - y_{t-1}$$

Note: e_t are one-step-forecast residuals

Facebook closing stock price

```r
fit <- fb_stock %>% model(NAIVE(Close))
augment(fit)
```

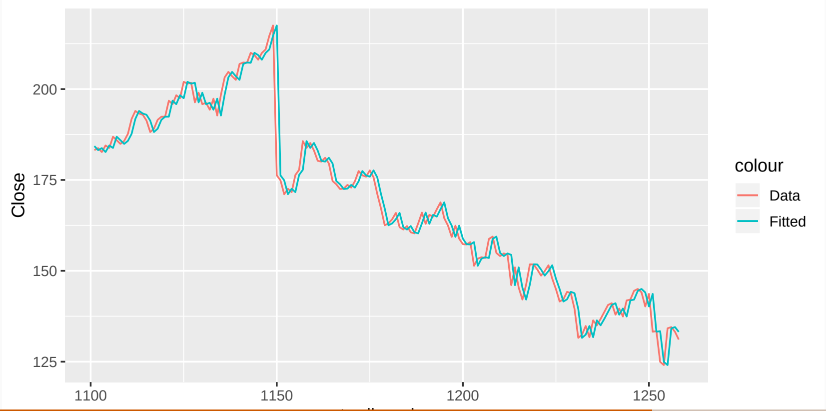
```
## # A tsibble: 1,258 x 6 [1]
## # Key:       Symbol, .model [1]
##    Symbol .model       trading_day Close .fitted .resid
##    <chr>  <chr>              <int> <dbl>   <dbl>  <dbl>
##  1 FB     NAIVE(Close)           1  54.7    NA   NA
##  2 FB     NAIVE(Close)           2  54.6    54.7 -0.150
##  3 FB     NAIVE(Close)           3  57.2    54.6  2.64
##  4 FB     NAIVE(Close)           4  57.9    57.2  0.720
##  5 FB     NAIVE(Close)           5  58.2    57.9  0.310
##  6 FB     NAIVE(Close)           6  57.2    58.2 -1.01
##  7 FB     NAIVE(Close)           7  57.9    57.2  0.720
##  8 FB     NAIVE(Close)           8  55.9    57.9 -2.03
##  9 FB     NAIVE(Close)           9  57.7    55.9  1.83
## 10 FB     NAIVE(Close)          10  57.6    57.7 -0.140
## # ... with 1,248 more rows
```

Facebook closing stock price

```
augment(fit) %>%
  ggplot(aes(x = trading_day)) +
    geom_line(aes(y = Close, colour = "Data")) +
    geom_line(aes(y = .fitted, colour = "Fitted"))
```

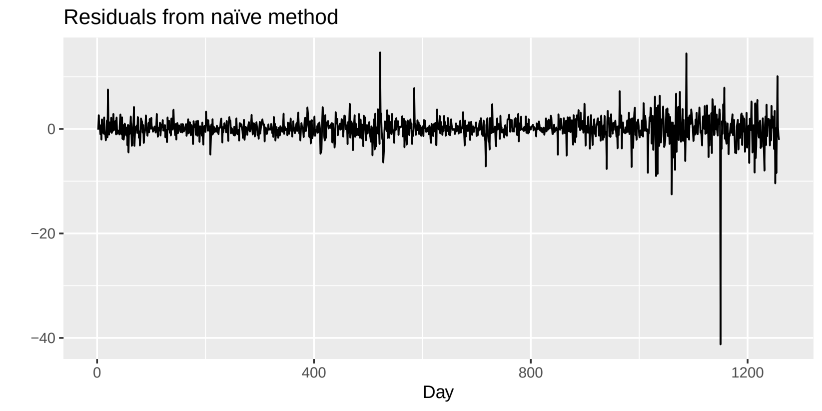
Facebook closing stock price

```r
augment(fit) %>%
  filter(trading_day > 1100) %>%
  ggplot(aes(x = trading_day)) +
    geom_line(aes(y = Close, colour = "Data")) +
    geom_line(aes(y = .fitted, colour = "Fitted"))
```



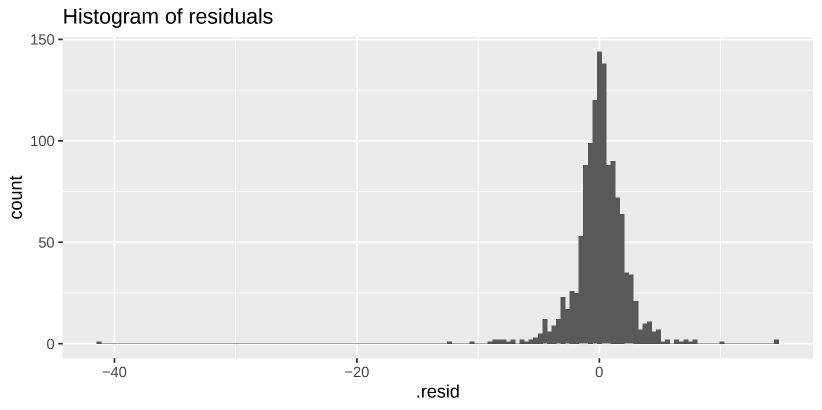
Facebook closing stock price

```r
augment(fit) %>%
  autoplot(.resid) + xlab("Day") + ylab("") +
    ggtitle("Residuals from naïve method")
```



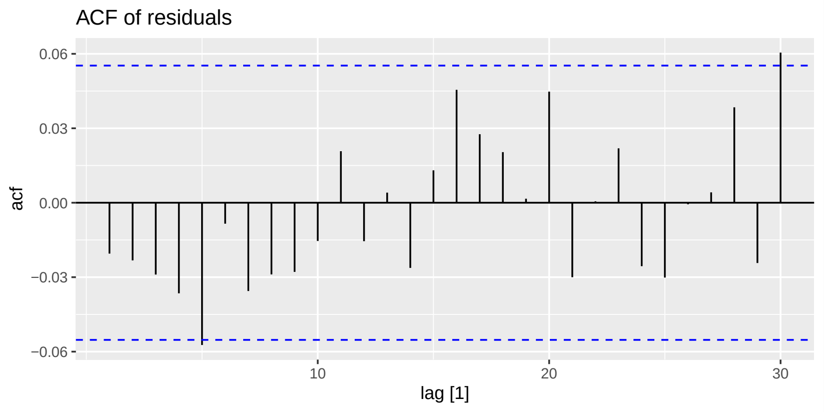
Facebook closing stock price

```r
augment(fit) %>%
  ggplot(aes(x = .resid)) +
    geom_histogram(bins = 150) +
    ggtitle("Histogram of residuals")
```



Facebook closing stock price

```r
augment(fit) %>% ACF(.resid) %>%
  autoplot() + ggtitle("ACF of residuals")
```

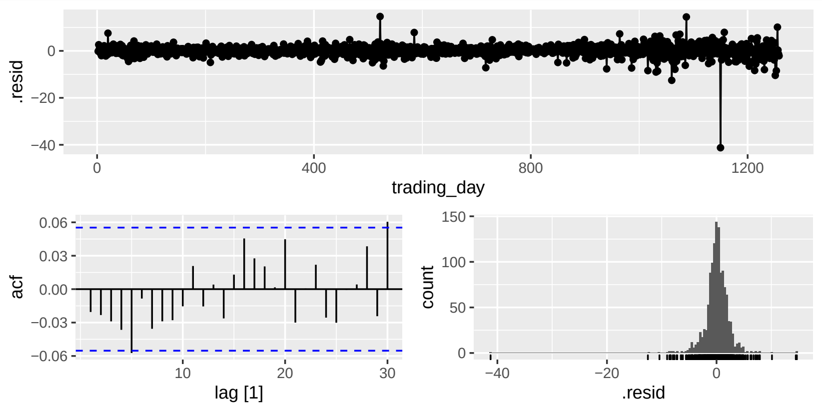


ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Combined diagnostic graph

```r
fit %>% gg_tsresiduals()
```



Ljung-Box test

Test whether *whole set* of r_k values is significantly different from zero set.

$$Q = T(T+2)\sum_{k=1}^{h}(T-k)^{-1}r_k^2$$

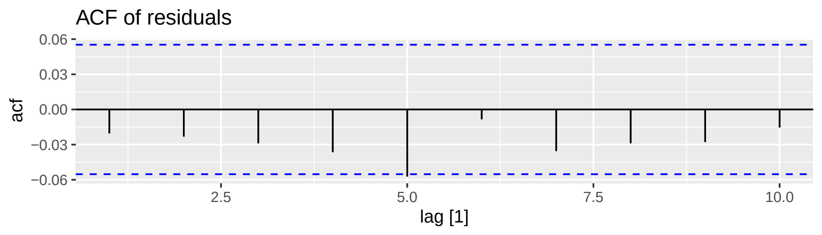
where h = max lag and T = # observations.

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (+ or $-$), Q will be **large**.
- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- If data are WN, $Q \sim \chi^2$ with $(h - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.

Ljung-Box test

$$Q = T(T+2)\sum_{k=1}^{h}(T-k)^{-1}r_k^2$$

where h = max lag and T = # observations.




```r
# lag=h and dof=K
ljung_box(augment(fit)$.resid, lag = 10, dof = 0)
```

```
##   lb_stat lb_pvalue
##    12.136     0.276
```

Outline

Lab Session 12

Compute seasonal naïve forecasts for quarterly Australian beer production from 1992.

```
recent <- aus_production %>% filter(year(Quarter) >= 1992)
fit <- recent %>% model(SNAIVE(Beer))
fit %>% forecast() %>% autoplot(recent)
```

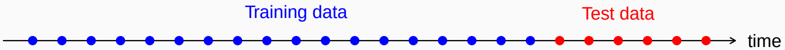
Test if the residuals are white noise.

```
Box.test(augment(fit)$.resid, lag=10, fitdf=0, type="Lj")
fit %>% gg_tsresiduals()
```

What do you conclude?

Outline

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- Forecast accuracy is based only on the test set.

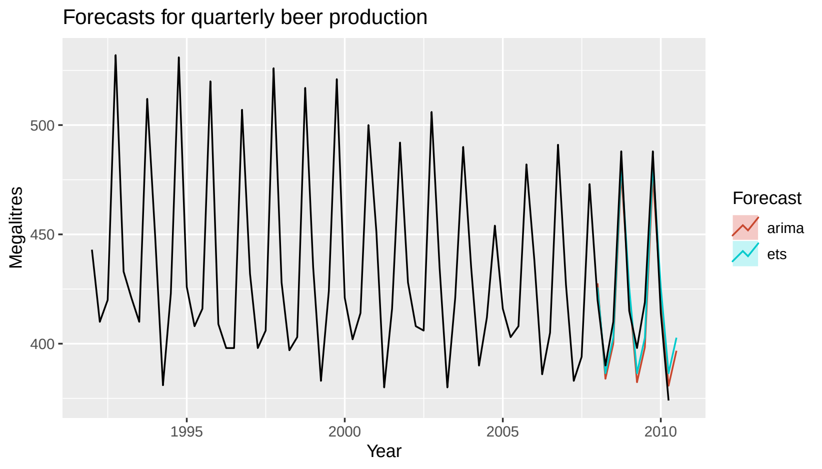
Forecast errors

Forecast "error": the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

Measures of forecast accuracy



Measures of forecast accuracy

$$\begin{aligned} y_{T+h} &= (T+h)\text{th observation, } h = 1, \dots, H \\ \hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\ e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T} \end{aligned}$$

$$\begin{aligned} \text{MAE} &= \text{mean}(|e_{T+h}|) \\ \text{MSE} &= \text{mean}(e^2_{T+h}) \qquad\qquad \text{RMSE} = \sqrt{\text{mean}(e^2_{T+h})} \\ \text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|) \end{aligned}$$

Measures of forecast accuracy

$$
\begin{aligned}
y_{T+h} &= (T+h)\text{th observation, } h = 1, \dots, H \\
\hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\
e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T}
\end{aligned}
$$

$$
\begin{aligned}
\text{MAE} &= \text{mean}(|e_{T+h}|) \\
\text{MSE} &= \text{mean}(e_{T+h}^2) \qquad\qquad \text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)} \\
\text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|)
\end{aligned}
$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T-1)^{-1}\sum_{t=2}^{T}|y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

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Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = (T-m)^{-1} \sum_{t=m+1}^{T} |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

Measures of forecast accuracy

```
recent_production <- aus_production %>%
  filter(year(Quarter) >= 1992)
train <- recent_production %>% filter(year(Quarter) <= 2007)
beer_fit <- train %>%
  model(
    ets = ETS(Beer),
    arima = ARIMA(Beer)
  )
beer_fc <- forecast(beer_fit, h="4 years")
accuracy(beer_fc, aus_production)
```

```
## # A tibble: 2 x 9
##   .model .type    ME  RMSE   MAE   MPE  MAPE  MASE  ACF1
##   <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 arima  Test  4.18  11.2  10.4  0.940  2.47 0.657 0.145
## 2 ets    Test  0.854  9.80  8.99 0.151  2.18 0.568 0.207
```

Outline

Lab Session 13

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