Quiz 4

1. (3 points) Find the eigenvalues λ_i of the following matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}.$$

Find an eigenvector ξ_i for each eigenvalue λ_i . Write down a general solution to

$$\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$$

using the fundamental set of solutions given by

$$\mathbf{x}^1 = \xi_1 e^{\lambda_1 t}, \quad \mathbf{x}^2 = \xi_2 e^{\lambda_2 t}.$$

Hint: the eigenvalues are real and distinct. You are not asked to prove that the solutions have $W \neq 0$.

2. Write as a system of ODEs the following higher order equations

(a) (2 points)
$$y'' + 3y = \cos(t)$$

(b) (1 point) y''' + y'' + y' + y = 0