

Solutions Quiz 1a

1. First order, linear.
2. All methods are appropriate
3. Using e.g. separation:

$$\begin{aligned}y'/y = -\cos(x)/\sin(x) &\Rightarrow \ln(y) = -\int \cos(x)/\sin(x)dx + C \\&\Rightarrow \ln(y) = -\ln(\sin(x)) + C \Rightarrow y = \frac{C}{\sin(x)}\end{aligned}$$

Using the method for exact equations:

$$\begin{aligned}M = y \cos x &\Rightarrow \partial_y M = \cos(x) \\N = \sin x &\Rightarrow \partial_x N = \cos(x) \\ \phi &= \int M dx + C_1(y) \Rightarrow \phi = y \sin x + C_1 \\ \phi &= \int N dy + C_2(x) \Rightarrow \phi = y \sin x + C_2 \\ \Rightarrow C_1 = C_2 = 0. \phi &= \text{Const. is solution} \Leftrightarrow y \sin x = C.\end{aligned}$$

4. $y' = e^{-y}$ has solution $y = \ln(x)$. $y' = y(y+3)$ has the trivial solution $y = 0$. $y' = \cos^2 y + \sin^2 y$ has solutions $y = x$.
5. Use the integrating factor method to find

$$x = \frac{-\cos(t) + C}{t}$$