Euclid's The Elements

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Due Sept 6 2019

Book I: Definitions

1. Point: That which has no part
2. Line: Something has length but no width
3. Straight line: A line that lies evenly with points on it.
4. Line segment: A line that is bounded by 2 points.
5. Surface: Something that has length and width but no height.
6. Plane surface: A surface that lies evenly having straight lines as a part of it.
7. Rectilinear: When the lines containing an angle are straight
8. Rectilinear Angle: Inclination of 2 lines that meet one another in a plane but do not form a straight line.
9. Right Angle / Perpendicular Lines: When a straight line standing on a straight line makes the adjacent angles equal to one another, those 2 angles are right, and the straight-line standing is called the perpendicular of the line it is standing on.
10. Obtuse Angle: An angle greater than a right angle
11. Acute Angle: An angle less than a right angle.
12. Boundary: something that is a limit or extreme
13. Figure: something contained by any boundaries.
14. Circle: A plane that is contained by one line such that if an imaginary line were to be drawn from one point, all the line segments from the point to the line will be of equal length.
15. Center of a Circle: the point in circle from which the boundary lines are of equal distance.
16. Radius of a Circle: A straight line drawn from the point on a circle to the boundary lines.
17. Diameter of a Circle: A straight line drawn from one boundary end of a circle to another going through the center point.
18. Semicircle: A figure contained by taking the diameter of a circle and one side of the circumference that it connects to. Half of a circle.
19. Rectilinear Figure / Polygon: Figure that is contained by straight lines.
20. Triangle: Polygon contained by 3 straight lines.
21. Equilateral Triangle: Triangle contained by 3 straight lines of equal length.
22. Isosceles Triangle: Triangle contained by 2 lines of equal length and a 3rd line connecting the other two.
23. Scalene Triangle: Triangle that has 3 sides of unequal length.
24. Right Triangle: Triangle that has a right angle as a part of it.
25. Obtuse Triangle: Triangle which has an obtuse angle as a part of it.
26. Acute Triangle: Triangle which has all 3 angles acute.
27. Quadrilateral: Polygon being contained by 4 straight lines.
28. Square: Quadrilateral that has all 4 sides of equal length and every angle is a right angle.
29. Rectangle: Quadrilateral that has all 4 angles as right angles.
30. Rhombus: Quadrilateral that has all sides of equal length but not all angles are right.
31. Parallelogram: Quadrilateral that has opposing sides parallel.

32. Trapezoid: Any other quadrilateral.
33. Parallel Lines: if a straight line falling on two straight lines makes alternate angles equal, then the 2 lines being crossed are considered parallel.
34. Vertical Angles: The pair of opposite angles made when 2 straight lines intersect.
35. Alternate Interior Angles: Pair of angles on the inner side of each of the 2 lines but on opposite sides of the transversal.
36. Alternate Exterior Angles: Pair of angles on the outer side of each of the two lines but on the opposite sides of the transversal.
37. Corresponding Angles: Angles in matching corners when two lines are crossed by another line.

Book I: Postulates

1. Let it be that you should be able to draw a straight line from any point to any point. You would need 2 points in order to be able to draw a straight line from one to the other. This produces a line segment.
2. To produce a finite straight line continuously in a straight line. You would need to already have a line segment put to use this postulate. If you have a line segment you can always extend it to be longer (continuous). The result would be a ray.
3. To draw a circle with any center and radius. You would already need a center and a radius to use the postulate. Then you would use the radius line segment with the center point to draw the circle. The result is a circle.
4. All right angles are equal to one another. This one is just a statement the if you have 2 right angles, they will always be equal to each other. 2 right angles result in 2 equal angles.
5. If 1 straight line falling on 2 straight lines results in 2 acute interior angles, The 2 straight lines if going indefinitely will eventually cross. You would need 3 lines that create 2 acute angles in order to have this postulate.

Book I: Common Notions

1. Things equal to the same thing are also equal to one another.
    a. If A = C AND B = C, Then A is equal to B.
2. If equal things are added to equal things, then the wholes are equal.
    a. If A = B AND C = D, Then A + C = B + D.
3. If equal things are subtracted from equal things, then the remainders are equal.
    a. If A + B AND C = D, Then C – A = D – B.
4. Things coinciding with one another are equal to one another.
    a. If the coordinates of A are (0,2), and the coordinates of B are (0,2), Then A = B.
5. The whole is greater than the part.
    a. If you have a circle C, and you slice A out of C, C as the whole will still always be greater than A.

Book I: Propositions that are Constructions

Number	Name (if applicable)	IF statement (what you must have)	THEN statement (what you will get)
[I,1]	making an equilateral triangle	Straight line Segment AB. Draw a circle with center A and radius AB. Draw a circle with center B and radius AB. Have the point of intersection be Point C. Draw a straight-line AC. Draw a straight-line BC.	Equilateral Triangle ABC.
[I,2]	copying a line segment	Straight Line Segment BC. Point A to copy it to. Draw a straight-line AB. Use [I,1] to draw equilateral triangle DAB. Extend the Line DA and the line BD creating AE and BF which are rays. Use B as a center and BC as a radius to draw a circle. Label the intersection with BF as G. Use the Radius of DG with D as the center and draw a circle. Label the intersection of the circle with AE as point L.	Line segment AL which is a copy of line segment BC.
[I,9]	angle bisection	Angle BAC Take an arbitrary point D on AB to create AD	construction: two congruent angles Angle BAF and CAF which are equal. AF is the bisection.

		Take the length of AD and use it to create AE along the line AC Use DE to construct an equilateral triangle DEF. Draw a line AF to complete the bisect.	
[I,10]	line bisection	line segment AB Use AB to construct an equilateral triangle ABC. Bisect the angle created by ACB. Call the intersection of the bisector with AB Point D. Your line CD would be the bisector for your line AB.	Line CD bisecting line AB.
[I,11]	dropping a perpendicular to a point on the line	line segment AB and a point C on the line Mark an arbitrary point D on AC. Make CE equal to AC on the segment CB. Using DE create an equilateral triangle AFB. Your line CF is your line that is perpendicular to AB.	Line CF perpendicular to AB.
[I,12]	dropping a perpendicular to a point off the line	Line Segment AB. Point C. Pick arbitrary point D on opposite side of where you want the perpendicular. Draw a circle from point C with radius D. Labeling the	Line CH Perpendicular to AB

		intersections with AB as E and G. Bisect the Line EG and create point H. Line CH is the perpendicular line to AB.	
[I,22]	copying a triangle	3 Straight lines A, B, and C to create a triangle from. Set a ray from DE, D is set E is continuous. Along the line DE, create a segment DF equal to A, FG equal to B, and GH equal to C. Draw a circle with center F and radius FD. Draw a circle with center G and radius GH. Mark the intersection of the 2 circles as point K. Draw a line FK. Your final copies triangle is FGK.	You get a final triangle FGK which is equal to the triangle with lengths A, B, and B.
[I,23]	copying an angle	Angle ABC to copy. Take 2 arbitrary points on the angle lines to create a triangle DBE. Using that triangle, USE I22 to reconstruct the triangle and in the end getting the necessary angle.	construction: a second angle congruent to the first
[I,31]	dropping a parallel	Line AB and point C to copy parallel to. Take arbitrary point D on the line AB. Draw a line CD.	

		Create an angle DCE equal to CDB. Extend EA to create EF and EF will be parallel to AC	

Book I: Propositions

Number	Name (if applicable)	IF statement (what you must have)	THEN statement (what you will get)
[I,4]	SAS	If 2 triangles have 2 sides equal respectively and the angle contained by the straight sides are also equal respectively.	The triangles are equal and all remaining angles and sides equal each other respectively.
[I,5]		isosceles triangle	two congruent angles
[I,6]		triangle with 2 congruent angles	isosceles triangle
[I,8]	SSS	If 2 triangles have all 3 sides equal respectively	The triangles are equal.
[I,13]		If a straight line stands on another straight line.	Then you have either 2 right angles, or 2 angles whose sums will equal that of 2 right angles.
[I,14]		If with any straight line and a point, two straight lines not on the same side sun up to 2 right angles.	You have two straight lines that are in a straight line with one another.
[I,15]		If two straight lines cut each other.	Then they make vertical angles that are equal to each other.
[I,16]		If with any triangle, one of the sides is produced.	Then the exterior angle is greater than either of the interior and opposite angles.
[I,17]		If you have any triangle	The sum of 2 of the angles will always be less than the sum of 2 right angles.
[I,18]		If you have any triangle.	The angle opposite of the longest side will always be the greatest angle in the triangle.
[I,19]		If you have any triangle.	The side opposite of the greatest angle will always be the longest side.

[I,20]		If you have any triangle	The sum of any of the 2 sides will always be greater than the length of the 3rd side.
[I,26]	ASA	If you have 2 triangles that have 2 angles respectively that are equal to each other, and the sides between the 2 angles are also equal to each other respectively.	Then you have congruent triangles.
	AAS	If you have 2 triangles that have 2 angles that are congruent respectively, and one of the sides not between the 2 angles are respectively congruent.	Then you have congruent triangles.
	RASS	If you have 2 triangles that each have right angles and one leg and hypotenuse are also respectively congruent.	Then you have 2 congruent triangles.
[I,27]		If a straight line falling on 2 straight lines makes equal alternate angles.	Then the straight lines are parallel.
[I,28]		If a straight line falling on 2 straight lines makes the exterior angle equal to the interior and opposite angle, or the sum of the interior angles equals 2 right angles.	Then you have 2 lines that are parallel.
[I,29]		If you have a straight line falling on 2 parallel straight lines.	Then you have the alternate angles congruent, exterior angles are congruent to the interior and opposite angle, and the sum of the interior angles are equal to 2 right angles.
[I,30]		If you have straight lines that are both parallel to the same straight line.	Then the 2 lines are also parallel to each other.
[I,32]		If in any triangle one of the sides is produced,	Then the exterior angle equals the sum of the two interior and opposite angles, and the sum of the three interior angles of the triangle equals two right angles.
[I,33]		If you have straight lines that join the ends of equal	Then the 2 lines are equal and parallel to each other.

		and parallel straight lines in the same direction.	
[I,34]		If you have a parallelogrammic area.	Then the opposite sides and angles are equal, and the diameter will bisect the areas.
[I,35]		If you have 2 parallelograms which have the same base and their parallels equal each other.	Then they are both equal to each other.
[I,36]		If you have 2 parallelograms that are on equal bases and in the same parallels.	Then the two parallelograms equal each other.
[I,37]		If you have 2 triangles which are on the same base and in the same parallels.	Then the 2 triangles are equal.
[I,38]		If you have 2 triangles that are on equal bases and in the same parallels.	Then the 2 triangles equal each other.
[I,39]		If you have 2 equal triangles which are on the same base and side.	Then they are also on the same parallels.
[I,47]	Pythagorean Theorem	If you have a right angled triangle.	The squares on the side opposite the right angle equals the sum of the squares on the sides containing the right triangles

Book III: Propositions that are Constructions

Number	IF statement (what you must have)	THEN statement (what you will get)
[III,1]	A Circle	Choose 2 Arbitrary points A and B on the circumference and draw line segment AB.

		Bisect the segment AB and mark the two point of intersection with the circle points C and E. Bisect line segment CE to get point F. F is your center.
[III,17]	You have circle with center E and you want to draw a straight line from point A touching the circle. TANGENT	From center E draw a circle with radius A. Draw segment AE and mark the intersection with the circle D. Draw the right angle of AE from point D. Mark the intersection of the outer circle as point F. Draw FE. Mark the intersection of the inner section as point B. Line AB will be the tangent.

Book III: Propositions

Number	IF statement (what you must have)	THEN statement (what you will get)
[III,2]	If you have two random points on the circumference of a circle.	The straight line joining the points will fall within the circle.
[III,3]	If you have 1 straight line going through the center and bisecting a straight line not in the center. If it also cuts the segment at a right angle.	It also bisects the segment that isn't crossing the center.
[III,4]	If in a circle you have two lines that do not pass through the center but intersect each other.	Then they do not bisect each other.
[III,5]	If 2 circles intersect each other.	Then they do not have the same center.
[III,6]	If 2 circles touch each other at any point.	Then they do not have the same center.
[III,12]	If 2 circles touch one another at one point without crossing.	Then the straight line joining their centers passes through the point of contact.
[III,16]	If you have a straight line drawn at a right angle from the end of a diameter of a circle.	Then the line will always fall outside the circle
[III,18]	If a straight line is tangent to a circle and another line	The straight line joined will be perpendicular to the tangent.

	is joined from the center to the intersection.	
[III,19]	If a straight line is tangent to a circle and from the intersection you draw a straight line at a right angle from the tangent.	The center of the circle will be on the line.
[III,20]	If you have a circle with an two points to create an angle from.	The angle at the center is double the angle at the circumference.
[III,21]	In a circle with a line segment.	The angles int the segment will equal one another.
[III,31]	If you have a circle where a right angle from the diameter, and take the intersection of bisector to the circle.	Then the segments from that points to the end of the diameter will also be right angles.

Book IV: Definitions

1. Inscribe: the largest object that you could fit inside of the figure.
2. Circumscribe: The smallest object you could fit outside of the figure.

Book IV: Propositions that are Constructions

Number	IF statement (what you must have)	THEN statement (what you will get)
[IV,3]	If you want to circumscribe a equiangular triangle using triangle DEF around a circle ABC.	Extend EF from the triangle in 2 directions to points E going to G and F going to H Take the center K of the circle and drab a radius KB at random. On the line KB construct an angle BKA equal to the angle of DEG and angle BKC equaling the angle DFH. Using points ABC draw The tangents at all 3 vertices. LAM, MBN, and NCL using the intersections. The resulting triangle LMN is your equiangular triangle.
[IV,4]	If you want to inscribe a circle about a triangle ABC	Bisect 2 of the angles and label the intersection D.

		Draw perpendicular lines to all 3 sides from point D, label the intersections E, F, and G. Your inscribed circle has center D and radius of either E, F, or G. All points fall on the circle.
[IV,5]	You have a triangle ABC and you want to circumscribe a circle around the triangle.	Bisect the lines AB and AC, label the points on the 2 sides, D and E. Mark the intersection as F. Draw perpendicular lines from the bisectors and label the intersection F. Join FB, FC, and FA Since AD is congruent to DB and DF is at a right angle the base AF is congruent to FB and FC. Your circumscribed circle is with center F and radius FA, FB, or FC since they are congruent.
[IV,6]	If you want to inscribe a square inside a given circle.	Draw to diameters AC and BD at right angles. Connect AB, BC, CD, DA to get your inscribed square.
[IV,7]	To circumscribe a square around a circle.	Draw two diameters at right angles. Draw the tangents at each of the 4 points. Connect the points to get your circumscribed square.
[IV,8]	If you want to inscribe a circle inside a square ABCD.	Bisect all sides of the square labeling the points EFKH. Draw the lines perpendicular on the midpoints. Mark the intersection G. Your inscribed circle has center G and radius GE.
[IV,9]	If you want to circumscribe a circle about a given square ABCD.	Draw AC and BD. Mark the intersection E.

		Your circumscribed circle has center E and radius EA.
[IV,10]	To get an isosceles triangle having each angle at the base double the remaining one.	Start with a line AB. Construct point C where rectangle AB by BC is equal to the square on CA. Construct a circle with center A and radius AB. Construct a line BD with the length of AC from point B to the circumference of the circle. ABD is your isosceles triangle.
[IV,11]	To inscribe a pentagon in a circle.	Inscribe triangle ABC in the circle. From point D inscribe a triangle DCE congruent to ABC. Pentagon ADBCE is your inscribed pentagon.