CSCI 6342: Linear Algebra: A Computational Approach. Assignment 1

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1 Pen and Paper Problems

1. Let $z_1 = a_1 + ib_1$. Let $z_2 = a_2 + ib_2$. Therefore, $\overline{z_1} = a_1 - ib_1$ and $\overline{z_2} = a_2 - ib_2$.

   (a) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

$$\overline{(a_1 + ib_1) + (a_2 + ib_2)} = a_1 - ib_1 + a_2 - ib_2$$
$$\overline{(a_1 + a_2) + i(b_1 + b_2)} = a_1 - ib_1 + a_2 - ib_2$$
$$(a_1 + a_2) - i(b_1 + b_2) = a_1 - ib_1 + a_2 - ib_2$$
$$(a_1 + a_2) - ib_1 - ib_2 = a_1 - ib_1 + a_2 - ib_2$$
$$a_1 - ib_1 + a_2 - ib_2 = a_1 - ib_1 + a_2 - ib_2$$

   (b) $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

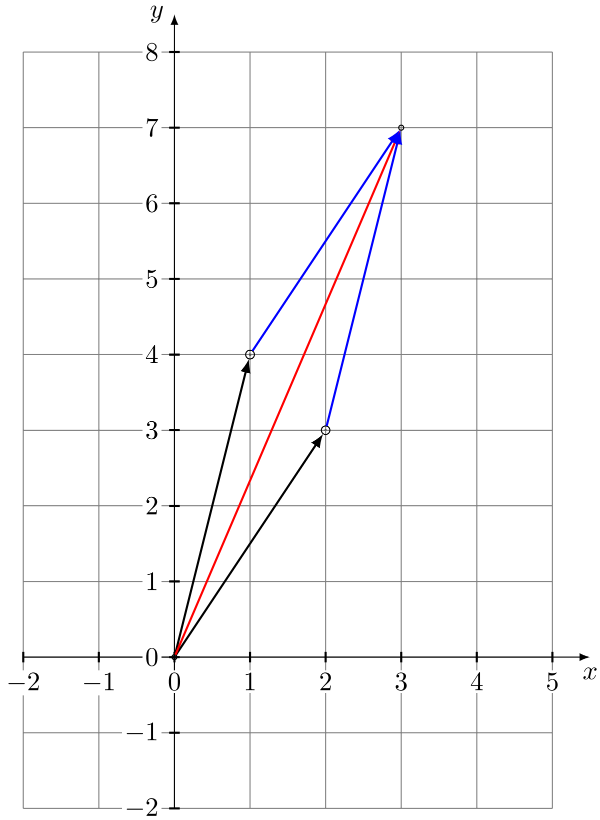
$$\overline{(a_1 + ib_1) - (a_2 + ib_2)} = a_1 - ib_1 - a_2 - ib_2$$
$$\overline{(a_1 - a_2) + i(b_1 - b_2)} = a_1 - ib_1 + a_2 - ib_2$$
$$(a_1 - a_2) - i(b_1 - b_2) = a_1 - ib_1 + a_2 - ib_2$$
$$(a_1 - a_2) - ib_1 - ib_2 = a_1 - ib_1 + a_2 - ib_2$$
$$a_1 - ib_1 - a_2 - ib_2 = a_1 - ib_1 + a_2 - ib_2$$

   (c) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

$$\overline{(a_1 + ib_1)(a_2 + ib_2)} = \overline{(a_1 + ib_1)(a_2 + ib_2)}$$
$$\overline{a_1 a_2 + ia_1 b_2 + ib_1 a_2 - b_1 b_2} = (a_1 - ib_1)(a_2 - ib_2)$$
$$a_1 a_2 - ia_1 b_2 - ib_1 a_2 - b_1 b_2 = a_1 a_2 - ia_1 b_2 - ib_1 a_2 - b_1 b_2$$

2. Geometric Proof for $|\mathbf{u}+\mathbf{v}| \leq |\mathbf{u}|+|\mathbf{v}|$.

   Let $\mathbf{u}+\mathbf{v}=\mathbf{z}$. $\mathbf{u}=(1,4)$. $\mathbf{v}=(2,3)$. Black lines are the original vectors. The red line is the vector for $\mathbf{z}$.

   The vector $\mathbf{z}$ bisects the parallelogram drawn in the visual creating two triangles. The two legs of the respective triangles represent $\mathbf{u}$ and $\mathbf{v}$. Since we know that the shortest route between two points is a straight line, we know that $|\mathbf{z}|$ is the shortest length between the origin and the end point. The norms (or the lengths) of the two vectors added together will therefore at least be greater than or equal to the norm of $\mathbf{z}$ as they create the aforementioned triangle with $\mathbf{z}$ as the base.

3. Since the expression, $|\mathbf{u}+\mathbf{v}| \leq |\mathbf{u}|+|\mathbf{v}|$ applies to vectors, we can reasonably come to the conclusion that it implies $|z_1+z_2| \leq |z_1|+|z_2|$. This is the case because we can conceptualize complex numbers as two-dimensional vectors since they have a real component and an imaginary component. Graphing imaginary numbers looks the same as graphing vectors when you use the set of real numbers and the set of imaginary numbers as your axises for the real and imaginary components of the complex number. Finding the magnitude of an absolute number is essentially the same as finding the norm of the vector especially since the formulas are the same.

4. $|\mathbf{u} \cdot \mathbf{v}| = ||\mathbf{u}||\mathbf{v}| \cos \theta|$ The range of $\cos \theta$ consists of all real numbers in [-1,1]. The inclusion of the absolute value around the expression makes the range of values to [0,1]. Therefore, $|\mathbf{u} \cdot \mathbf{v}| \leq |\mathbf{u}||\mathbf{v}|$.