Problem 1:

a. **Define the Decision variables.**
   X = no. of Collegiate produced per week
   Y = no. of Mini produced per week

b. **Objective Function** (what quantity of each type of backpack to be produced per week)

   Z(Maximizing profit) = 32X + 24Y
   where,
   i. profit per unit of Collegiate(X) is $ 32.
   ii. profit per unit of Mini(Y) is $ 24.

c. **Constraints**
   i. 3X + 2Y <= 5000 (3 Sq.ft of nylon sheet per unit of Collegiate(x) and 2 sq.ft per unit of Mini(y) with in the total capacity of 5000 sq.ft of rip resistant nylon fabric)

   ii. X <= 1000 (quantity of collegiate(x) should be less than or equal to 1000 units)
   Y <= 1200 (quantity of Mini(y) should be less than or equal to 1200 units)

   iii. 45X + 40Y <= 84000 (45 minutes per labor produce of collegiate and 40 minutes per labor produce of Mini is less than or equal to 84000 minutes (35 laborers * 40 hours * 60 minutes) of labor per week)

d. **Mathematical Formulation:**
   Z = 32X + 24Y
   where:
   X <= 1000 & Y <= 1200, (which are non-negative constraints)

   and is subject to time constraints:
   45X + 40Y <= 84000 (35 laborers * 40 hours * 60 minutes) [in minutes]
   Or
   (3/4) X + (2/3) Y <= 1400 (35 laborers * 40 hours) [in hours]

   (45 minutes = 3/4 hours)
   (40 minutes = 2/3 hours)

Problem 2:

a. **Decision variables:**

Let X be the large size product.
Let Y be the medium size product.
Let Z be the small size product.

Decision variables for all three plants:

Plant	Large	Medium	Small
Plant- 1	X_1	Y_1	Z_1
Plant- 2	X_2	Y_2	Z_2
Plant- 3	X_3	Y_3	Z_3

b. **Formulation of Linear programming Model**

Objective Function: (How much of each of the Sizes should be produced by each of the plants to maximize profit)

Z (Maximizing profit) = $420 (X_1 + X_2 + X_3) + 360 (Y_1 + Y_2 + Y_3) + 300 (Z_1 + Z_2 + Z_3)$

Subject to Constraints:

Excess Production Capacity Constraints:

$X_1 + Y_1 + Z_1 <= 750$ (Excess Production capacity of Plant 1 is 750 units per day)

$X_2 + Y_2 + Z_2 <= 900$ (Excess Production Capacity of Plant 2 is 900 units per day)

$X_3 + Y_3 + Z_3 <= 450$ (Excess Production Capacity of Plant 3 is 450 units per day)

Storage Capacity Constraints:

$20X_1 + 15Y_1 + 12Z_1 <= 13000$ (Storage capacity of Plant 1 is 13000 Square feet)

$20X_2 + 15Y_2 + 12Z_2 <= 12000$ (Storage Capacity of Plant 2 is 12000 Square feet)

$20X_3 + 15Y_3 + 12Z_3 <= 5000$ (Storage Capacity of Plant 3 is 5000 Square feet)

Sales forecast Constraints:

$X_1 + X_2 + X_3 <= 900$ (900 units from plant 1 to be sold per day)

$Y_1 + Y_2 + Y_3 <= 1200$ (1200 units from plant 2 to be sold per day)

$Z_1 + Z_2 + Z_3 <= 750$ (750 units from plant 3 to be sold per day)

Where $X_x, Y_x, Z_x >= 0$

At each plant, as a result of the excess capacity, employees will need to be laid off. So, the management decided that the plants should use the same percentage of the excess capacity to produce the new product.

Therefore, the percentage of excess capacity used should be:
$(X_1 + Y_1 + Z_1) / 750 = (X_2 + Y_2 + Z_2) / 900 = (X_3 + Y_3 + Z_3) / 450$

Where $X_1, X_2, X_3, Y_1, Y_2, Y_3, Z_1, Z_2, Z_3 >= 0$ (non-negative constraints)