Group 1: Tennis

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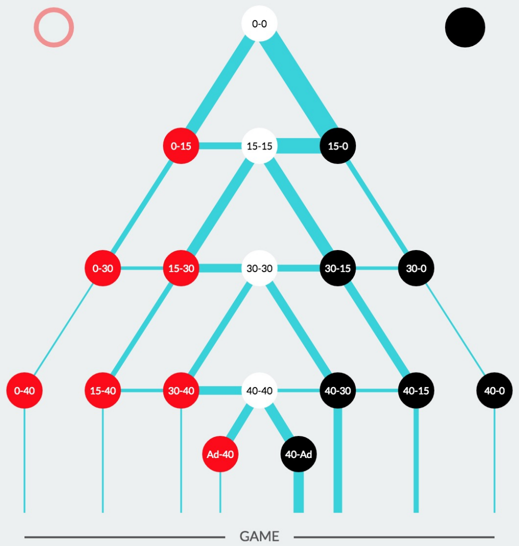
Project

- Project based off a paper by Paul K. Newton and Joseph B. Keller called Probability of Winning at Tennis 1. Theory and Data
- Paper discussed how the probability of winning a game, a set, and a match in tennis are computed based on each player's probability of winning a point on a serve
- Based on other papers the conclusion was made to treat serves in tennis as independent identically distributed (iid) random variables
- It is noted that points in tennis are not iid for most purposes this is not a bad assumption as the divergence from iid is small
- Both two out of three and three out of five set matches were considered aloowing for 13-point tiebreaker in each set if needed

The Game of Tennis

- Point: Smallest unit of measurement (Love-15-30-40-game)
- Game: A game is one when a player reaches 4 points with at least a 2 point advantage
- Set: A set consists of 6 games and is won by the player who reaches 6 games first
- Advantage Set: If a game score of 6-6 is reached and advantage set rules are used, a player can only win a set with a 2 game lead
- Matches: Best of 3 set (for women) or 5 sets (for men)
- Tie-break game: If a game score of 6-6 is reached and tie-break set rules are used. In a tie-break game, a player/team must reach 7 points with a two point advantage to win

Game of Tennis



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Game

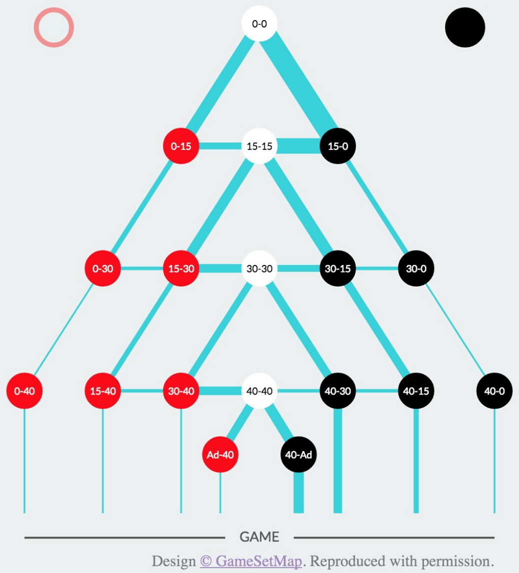

Figure 1:

Values

Things we need to know:

- Empirical probability of winning a rally on serve
- Empirical probability of winning a game on serve

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Things we need to know:

- Empirical probability of winning a rally on serve
- Empirical probability of winning a game on serve

Things we need to calculate:

- Theoretical probability of winning a game on serve
- Theoretical probability of winning a set
- Theoretical probability of winning the tie-breaker
- Theoretical probability of winning a match

Theoretical Probabilities: Rally and Game

$$\text{P(A Winning a Rally} \mid \text{A Served)} = p_A^R = \frac{\text{Points Won On Serve}}{\text{Points Served}}$$

Theoretical Probabilities: Rally and Game

$$\text{P(A Winning a Rally} \mid \text{A Served)} = p_A^R = \frac{\text{Points Won On Serve}}{\text{Points Served}}$$

$$\text{P(A Winning a Game} \mid \text{A Served)}$$
$$= p_A^G = (p_A^R)^4[1 + 4q_A^R + 10(q_A^R)^2] + 20(p_A^R q_A^R)^3 (p_A^R)^2 [1 - 2p_A^R q_A^R]^{-1}$$

- Where Probability A loses a rally given A served:

$$q_A^R = 1 - \frac{\text{Points Won On Serve}}{\text{Points Served}}$$

Theoretical Probabilities: Set

$$\text{P(A Winnning a Set} \mid \text{A Served)}$$

$$= p_A^S = \sum_{j=0}^{4} p_A^S(6, j) + p_A^S(7, 5) + p_A^S(6, 6)p_A^T$$

Theoretical Probabilities: Set

$$P(\text{A Winnning a Set} \mid \text{A Served})$$

$$= p_A^S = \sum_{j=0}^{4} p_A^S(6,j) + p_A^S(7,5) + p_A^S(6,6)p_A^T$$

Where, $p_A^S(i,j)$ is defined recursivly as:

$p_A^S(0,0) = 1$, $p_A^S(i,j) = 0$ if i<0 or j<0.

- if i+j -1 is even: $p_A^S(i,j) = p_A^S(i-1,j)p_A^G + p_A^S(i,j-1)q_A^G$
  - omit i-1 term if j=6 and i<6
  - omit j-1 term if i=6 and j<6
- if i+j -1 is odd: $p_A^S(i,j) = p_A^S(i-1,j)q_B^G + p_A^S(i,j-1)p_B^G$
  - omit i-1 term if j=6 and i<6
  - omit j-1 term if i=6 and j<6

Theoretical Probabilities: Tie Breaker

$$\text{P(A Winning the Tie Breaker} \mid \text{A Served Initially)}$$
$$= p_A^T = \sum_{j=0}^{5} p_A^T(7,j) + p_A^T(6,6) \sum_{n=1}^{\infty} p_A^T(n+2,n)$$

Theoretical Probabilities: Tie Breaker

$$\begin{aligned} &\text{P(A Winning the Tie Breaker} \mid \text{A Served Initially)} \\ &= p_A^T = \sum_{j=0}^{5} p_A^T(7,j) + p_A^T(6,6) \sum_{n=1}^{\infty} p_A^T(n+2,n) \end{aligned}$$

Where, $p_A^T(i,j)$ is defined recursivly as:

$p_A^T(0,0) = 1$, $p_A^T(i,j) = 0$ if i<0 or j<0.

- if $i + j - 1 \mod 4 \equiv 0$ or 3:
  $p_A^S(i,j) = p_A^T(i-1,j)p_A^R + p_A^T(i,j-1)q_A^R$
  - omit i-1 term if j=7 and i<7
  - omit j-1 term if i=7 and j<7
- if $i + j - 1 \mod 4 \equiv 1$ or 2:
  $p_A^T(i,j) = p_A^T(i-1,j)q_B^R + p_A^T(i,j-1)p_B^R$
  - omit i-1 term if j=7 and i<7
  - omit j-1 term if i=7 and j<7

Theoretical Probabilities: Match

Women's (Best of 3):

$$P(\text{A Winning the Match} \mid \text{A Served First})$$

$$= p_A^M = (p_A^S)^2 + 2(p_A^S)^2 p_B^S$$

Theoretical Probabilities: Match

Women's (Best of 3):

$$\text{P(A Winning the Match} \mid \text{A Served First)}$$

$$= p_A^M = (p_A^S)^2 + 2(p_A^S)^2 p_B^S$$

Men's (Best of 5):

$$\text{P(A Winning the Match} \mid \text{A Served First)}$$

$$= p_A^M = (p_A^S)^3 + 3(p_A^S)^3 p_B^S + 6(p_A^S)^3 (p_B^S)^2$$

Where:

- $p_A^S =$ Probabilty A wins a set given A served
- $p_B^S =$ Probabilty B wins a set given B served

Data being used

Data used in paper:

- Men and Women semifinalists in the 2002 U.S Open tournament
- Men and Women semifinalists in the 2002 Wimbledon tournament

Data used in our simulation:

- Men and Women semifinalists in the 2019 Wibledon tournament

Results of Game

Table 1: Data for the Womens Semifinalists in the 2002 U.S. Open Tournament

	P(Win a Rally)	Empirical P(Win a Game)	Paper P(Win a Game)	Our P(Win a Game)
S. Williams	0.69	0.71	0.89	0.89
V. Williams	0.63	0.80	0.79	0.79
L. Davenport	0.65	0.85	0.83	0.83
A. Mauresmo	0.63	0.77	0.79	0.79

Table 2: Data for the Mens Semifinalists in the 2002 U.S. Open Tournament

	P(Win a Rally)	Empirical P(Win a Game)	Paper P(Win a Game)	Our P(Win a Game)
P. Sampras	0.73	0.95	0.93	0.93
A. Agassi	0.66	0.87	0.85	0.85
L. Hewitt	0.67	0.85	0.86	0.86
S. Schalken	0.68	0.90	0.88	0.88