2016302580320-任思远-第七次作业

5.1

(1)

$$
\begin{aligned}
S &= \frac{1}{n}\sum_{k}(X_k - M)(X_k - M)^T \\
&= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{3}{16} \end{pmatrix}
\end{aligned}
$$

(2)

$$
\begin{aligned}
\lambda_1 = \frac{1}{2}, x_1 = (1,0)^T \\
\lambda_2 = \frac{3}{16}, x_2 = (0,1)^T
\end{aligned}
$$

(3)

采用 K-L 变换：

- 求自相关矩阵：

$$
\begin{aligned}
R &= \frac{1}{n}\sum_{k} X_k X_k^T \\
&= \begin{pmatrix} \frac{19}{2} & \frac{27}{4} \\ \frac{27}{4} & \frac{21}{4} \end{pmatrix}
\end{aligned}
$$

- 求 R 特征值，选择前 1 个，并计算特征向量：

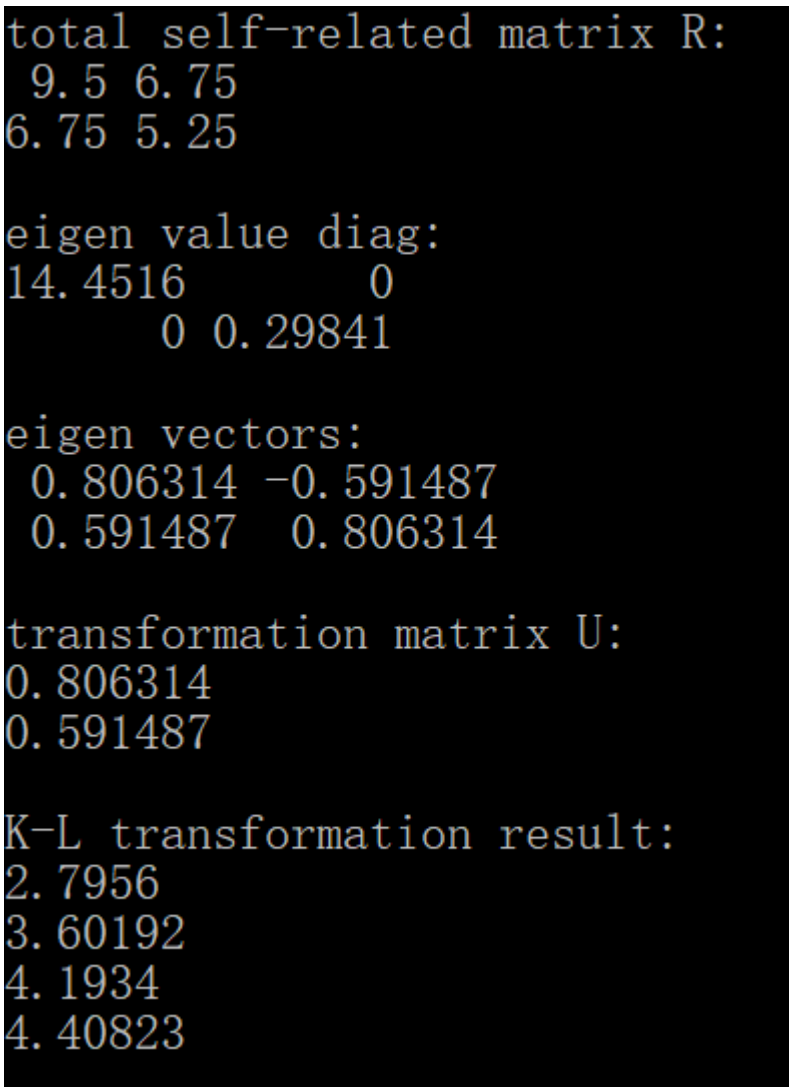
$$\begin{aligned}\lambda_1 &= 14.45, \quad x_1 = (0.8, 0.6)^T \\ \lambda_2 &= 0.2985\end{aligned}$$

变换矩阵 $U = x_1 = (0.8, 0.6)^T$

- 变换：$X^* = U^T X$

$$\begin{aligned}X_1^* &= 2.8 \\ X_2^* &= 3.6 \\ X_3^* &= 4.2 \\ X_4^* &= 4.4\end{aligned}$$

(C++)使用Eigen库实现K-L变换




```cpp
#include <iostream>
#include <vector>
#include <utility>
#include <algorithm>
#include <type_traits>
#include <Eigen/Dense>
#include <Eigen/Eigenvalues>
namespace eigen = Eigen;

#define OUTPUT_INTERMEDIATE_RESULT

template<
    std::size_t N,
    typename = std::enable_if_t<(N > 0)>
```

```
        > std::vector<eigen::MatrixXd> // return Matrix: (dx 1) - Vector
    KL_transformation(
        const std::vector<eigen::Matrix<double, N, 1>>& vectors,
        const std::size_t dimension)
{
    assert((dimension > 0) && (dimension < N));

    // 1. calculate total self-related matrix R
    eigen::Matrix<double, N, N> R = eigen::Matrix<double, N, N>::Constant(0.0);
    for (auto const& v : vectors)
        R += v * v.transpose();
    R /= vectors.size();
#ifdef OUTPUT_INTERMEDIATE_RESULT
    std::cout << "total self-related matrix R:" << std::endl;
    std::cout << R << std::endl << std::endl;
#endif // OUTPUT_INTERMEDIATE_RESULT

    // 2. calculate the greater dimension eigenvalues and eigenvectors
    eigen::EigenSolver<eigen::Matrix<double, N, N>> _es{ R };
    eigen::Matrix<double, N, N> _diag = _es.pseudoEigenvalueMatrix();
    eigen::Matrix<double, N, N> _eigen_vectors = _es.pseudoEigenvectors();
#ifdef OUTPUT_INTERMEDIATE_RESULT
    std::cout << "eigen value diag:" << std::endl;
    std::cout << _diag << std::endl << std::endl;
    std::cout << "eigen vectors:" << std::endl;
    std::cout << _eigen_vectors << std::endl << std::endl;
```

```
41  #endif // OUTPUT_INTERMEDIATE_RESULT
42      // sort
43      std::vector<std::pair<double, std::size_t>>
    _eigenValues{ N };
44      for (std::size_t i = 0; i < N; i++)
45          _eigenValues[i] = std::pair<double, std::size_t>
    (_diag(i, i), i);
46      std::sort(_eigenValues.begin(), _eigenValues.end(),
47          [](const std::pair<double, std::size_t>& lhs, const
    std::pair<double, std::size_t>& rhs)
48      { return lhs.first >= rhs.first; });
49
50      // 3. construct transformation matrix U
51      eigen::MatrixXd U{ N, dimension };
52      for (std::size_t i = 0; i < dimension; i++)
53          U.block(0, i, N, 1) = _eigen_vectors.block(0,
    _eigenValues[i].second, N, 1);
54  #ifdef OUTPUT_INTERMEDIATE_RESULT
55      std::cout << "transformation matrix U:" << std::endl;
56      std::cout << U << std::endl << std::endl;
57  #endif // OUTPUT_INTERMEDIATE_RESULT
58
59      // 4. K-L transformation
60      const auto U_T = U.transpose();
61      std::vector<eigen::MatrixXd> transformation_result;
62      transformation_result.reserve(vectors.size());
63      for (const eigen::Matrix<double, N, 1>& v : vectors)
64          transformation_result.push_back(U_T * v);
65
66      return transformation_result;
67  }
68
69  void test_KL()
70  {
```

```
    constexpr std::size_t N = 2;
    constexpr std::size_t dimension = 1;
    std::vector<eigen::Matrix<double, N, 1>> vectors;
    vectors.reserve(4);

    {
        eigen::Matrix<double, N, 1> v;
        v(0, 0) = 2; v(1, 0) = 2; vectors.push_back(v);
        v(0, 0) = 3; v(1, 0) = 2; vectors.push_back(v);
        v(0, 0) = 3; v(1, 0) = 3; vectors.push_back(v);
        v(0, 0) = 4; v(1, 0) = 2; vectors.push_back(v);
    }

    std::vector<eigen::MatrixXd> transformation_result = KL_transformation<N>(vectors, dimension);

    std::cout << "K-L transformation result:" << std::endl;
    for (auto const& v : transformation_result)
        std::cout << v << std::endl;
}


int main()
{
    test_KL();
    std::getchar();
    return 0;
}
```