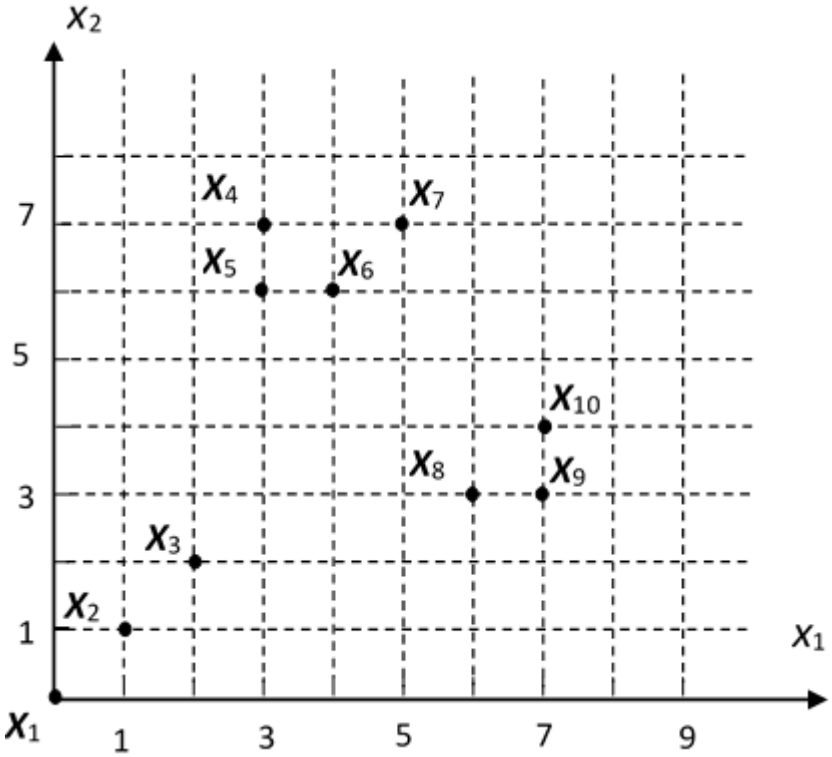
20163025803 20 任思远 第二次作业

2.1



1. 选择 X_1 作为 Z_1。
2. 选择距离最远的 X_7 作为 Z_2。
3. 记 $M_i = min\{||X_i - Z_1||, |X_i - Z_2|\}$，经计算：

$$
\begin{aligned}
M_2 &= \sqrt{2} \\
M_3 &= 2\sqrt{2} \\
M_4 &= 2 \\
M_5 &= \sqrt{5} \\
M_6 &= \sqrt{2} \\
M_8 &= \sqrt{17} \\
M_9 &= \sqrt{20} \\
M_{10} &= \sqrt{13}
\end{aligned}
$$

$M_9 > \theta||Z_1 - Z_2||$，选择 M_9 作为 Z_3，再无满足的点，结束聚类中心的选择。

4. 划分：

$\{X_1, X_2, X_3\}$, $\{X_7, X_4, X_5, X_6\}$, $\{X_9, X_8, X_{10}\}$

2.2

k-means算法流程：

input：样本集 $D = \{x_1, x_2, \ldots, x_n\}$，聚类簇数 k.

output：聚类划分的集合。

算法流程：

1. 从 D 中随机选择 k 个作为初始均值向量 $\{\mu_1, \mu_2, \ldots, \mu_k\}$.
2. loop until 均值向量不再变化（或变化不大，或其他近似条件）

令 $C_i = \varnothing, \quad i = 1, \ldots, k$ 表示每个聚类的集合，

$for\ j = 1, \ldots n:$

计算每个向量和均值向量的距离
$d_{ji} = dist(x_j, \mu_i), \quad i = 1, \ldots, k$

选择最近的簇，并加入 $\lambda_j = argmin_i\{d_{ji}\}, \quad C_j \cup = x_j$

重新计算均值向量：

$for\ j = 1, \ldots, k:$

$\mu_j = \frac{1}{|C_j|} \sum_{x \in C_j} x$

3. 退出 2 中的 loop 后，得到的 $\{C_i\} \quad i = 1, \ldots, k$ 即为聚类划分。

C++ 实现 k-means C++代码太长了，这里放不下。

Python 实现 k-means Python代码：

```python
import numpy as np
from functools import reduce


def k_means(cluster_num: int, dist, *data_set):
    size = len(data_set)
    assert size >= cluster_num > 0
    s = set()
    while len(s) != cluster_num:
        s.add(np.random.random_integers(0, size - 1))

```

```
    mean_vecs = []  # mean vectors in each cluster, initialized with random vector selected in data set.
    for num in s:
        mean_vecs.append(data_set[num])

    C = []  # clusters
    for i in range(cluster_num):
        C.append([])

    isChanged = True

    '''
    Divide each vector into one of the sets,
    and calculate the new mean_vecs.

    Loop until mean_vecs have no change.
    '''
    while isChanged:
        isChanged = False
        for c in C:
            c.clear()

        # for each vector in data set, divide it into one of the clusters.
        for vec in data_set:
            d = dist(vec, mean_vecs[0])
            label = 0
            for l in range(cluster_num):
                cur_dist = dist(vec, mean_vecs[l])
                if cur_dist < d:
                    d = cur_dist
```

```
                    label = l
            C[label].append(vec)

        # update mean_vecs, and set up `isChanged` flag if needed
        for label in range(cluster_num):
            assert len(C[label]) > 0
            mean_vec = reduce(lambda x, y: x + y, C[label]) / len(C[label])
            if dist(mean_vec, mean_vecs[label]) > 10e-2:
                mean_vecs[label] = mean_vec
                isChanged = True

    return C
```