Simulated Annealing for Makespan Scheduling

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Overview

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- **Simulated Annealing**
  - Cooling Schedule
  - Initial Temperature

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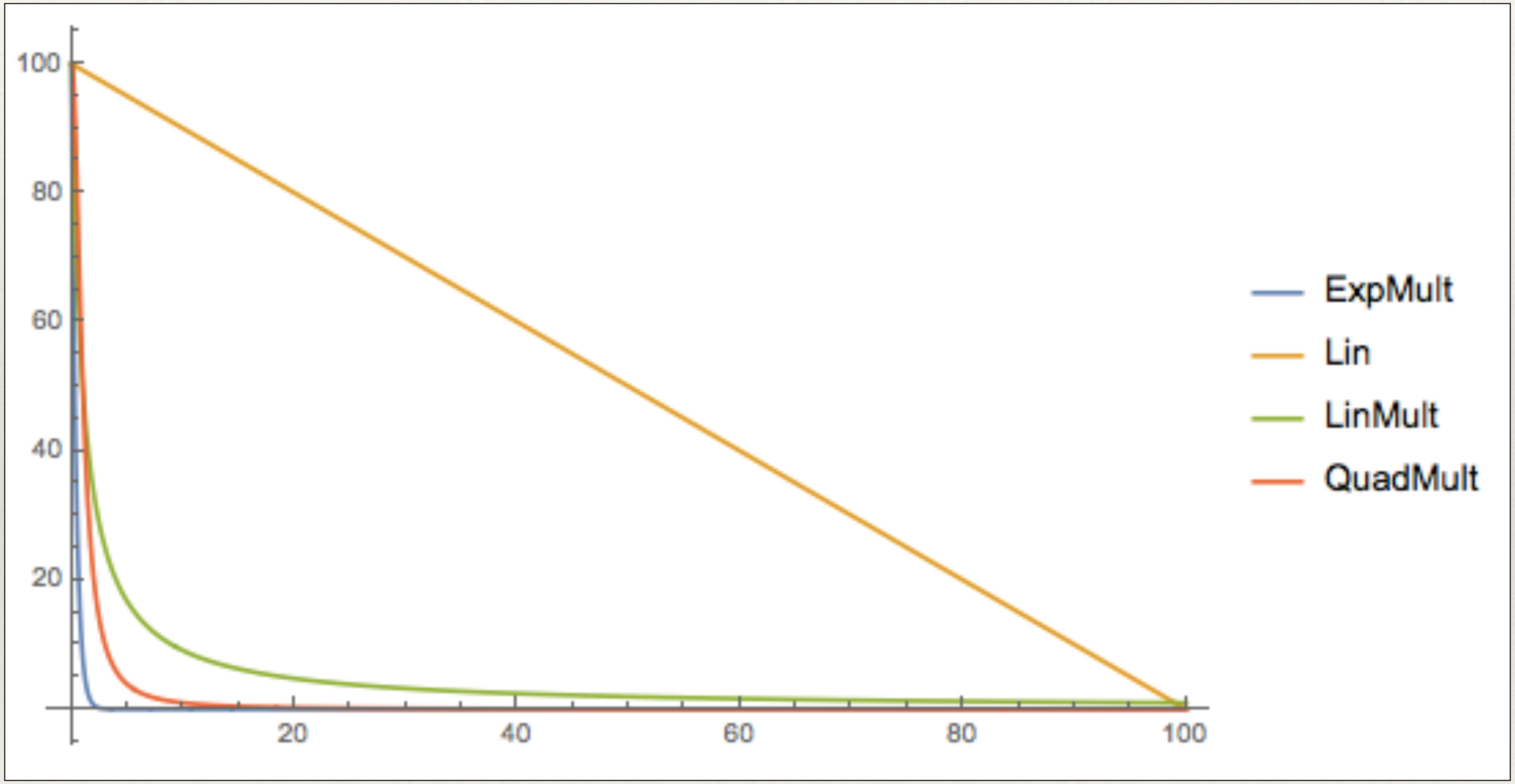
- Free parameters:
  1. Neighbourhood: 2-exchange “jump”
  2. Cooling schedule
  3. Initial temperature value

Simulated Annealing: Cooling Schedules

- **4 different cooling schedules considered:**

1. Exponential multiplicative: $f_1(T_0, I) = T_0 \cdot \mu^I$
2. Simple exponential: $f_2(T_0, I) = T_0 - I$
3. Linear multiplicative: $f_3(T_0, I) = \frac{1}{1+I} \cdot T_0$
4. Quadratic multiplicative: $f_4(T_0, I) = \frac{1}{1+I^2} \cdot T_0$

Simulated Annealing: Cooling Schedules



Simulated Annealing: Initial Temperature

- Good choice of initial temperature depends largely on instance
- Algorithm by Ben-Ameur [1]:
  - Generates a temperature so that the probability of accepting a cost increase is equal to a specified value

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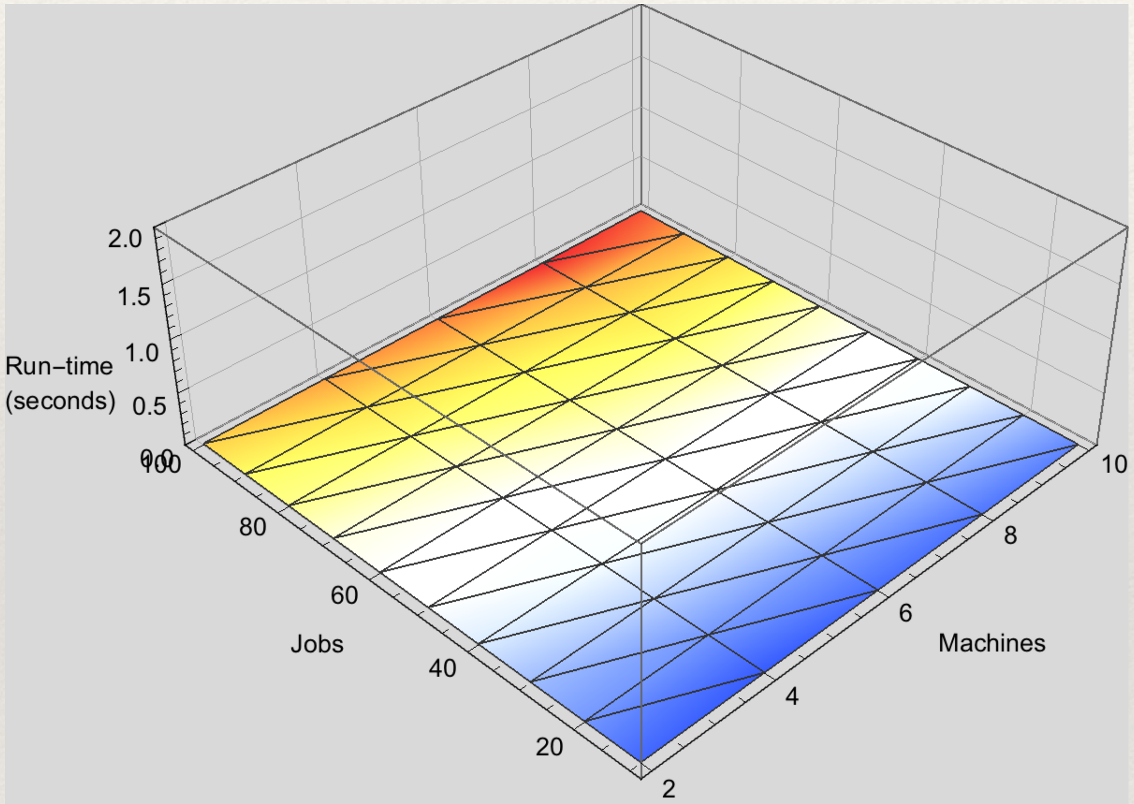
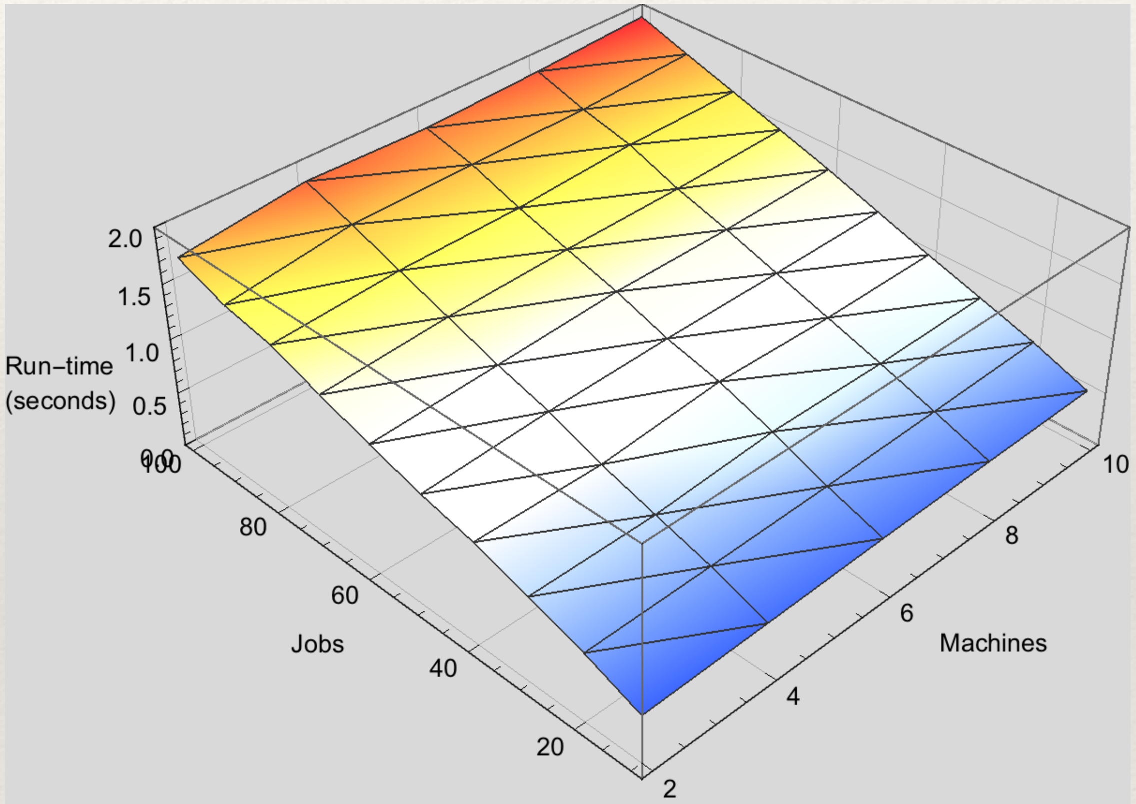


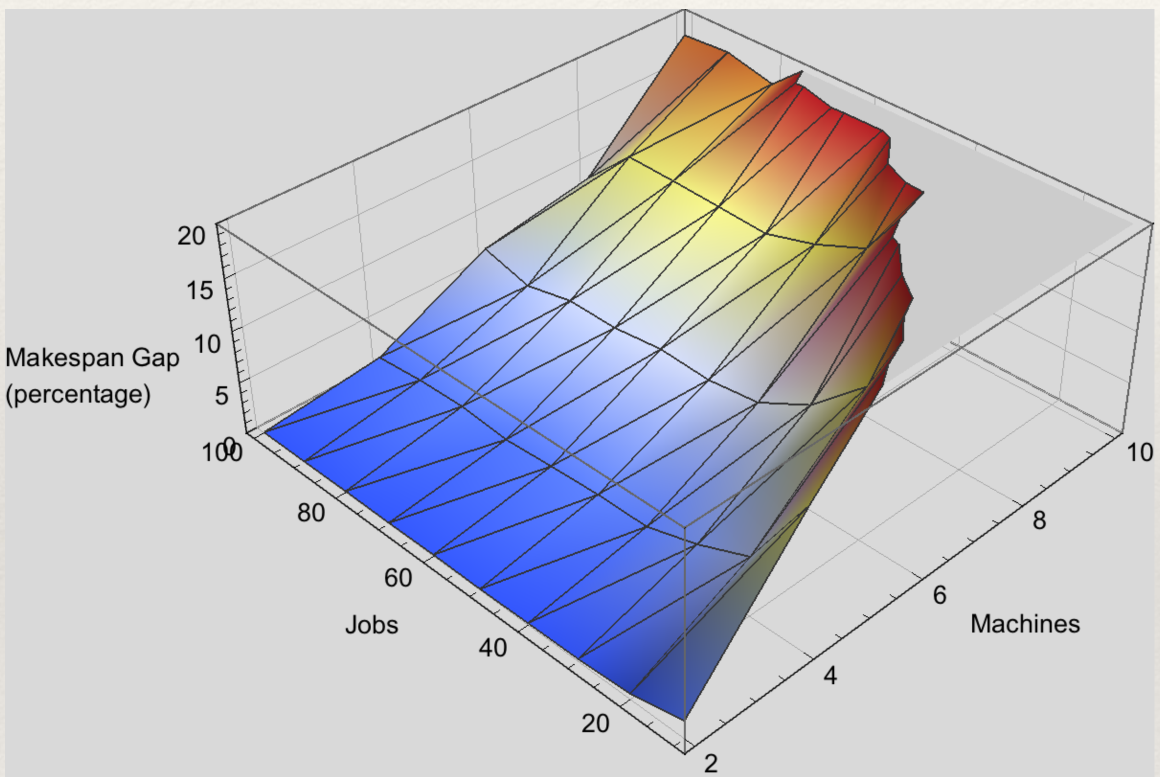
Linear Multiplicative Cooling



Quadratic Multiplicative Cooling

Run time

Experiments: Cooling Schedules



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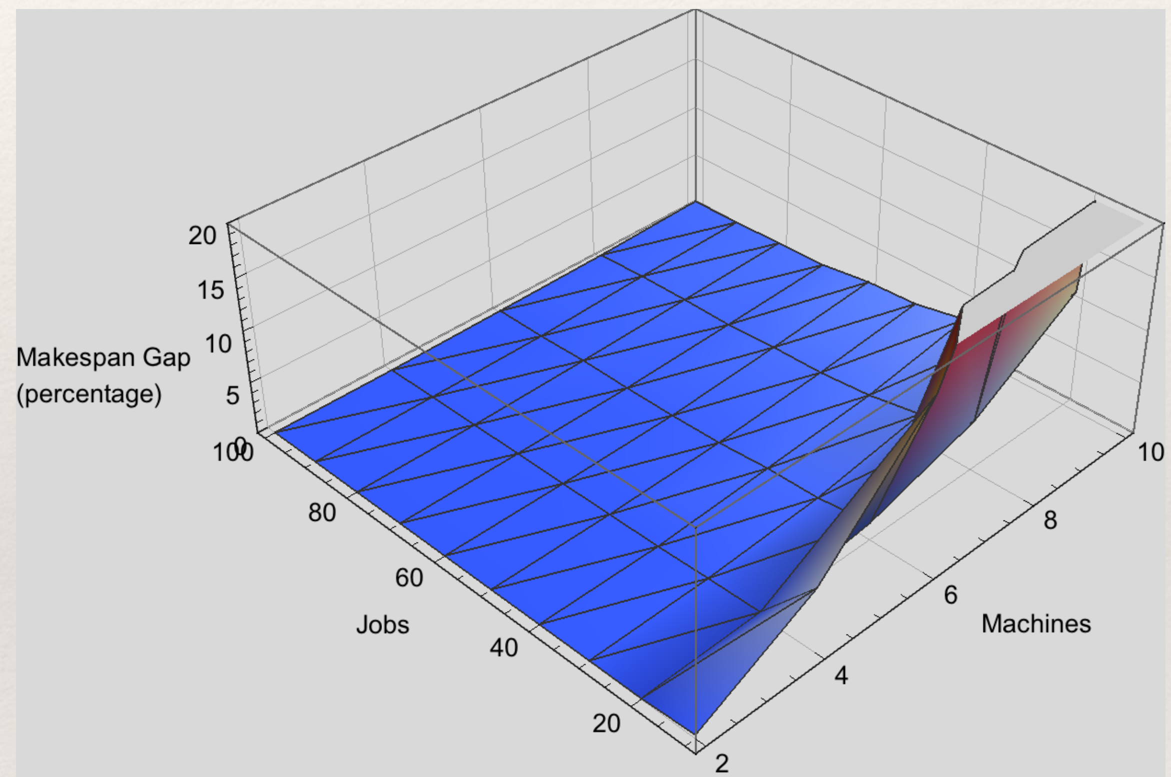

Quadratic Multiplicative Cooling

Makespan Gap

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- Conclusion: choose initial temperature to be 1.5 times the maximum processing time

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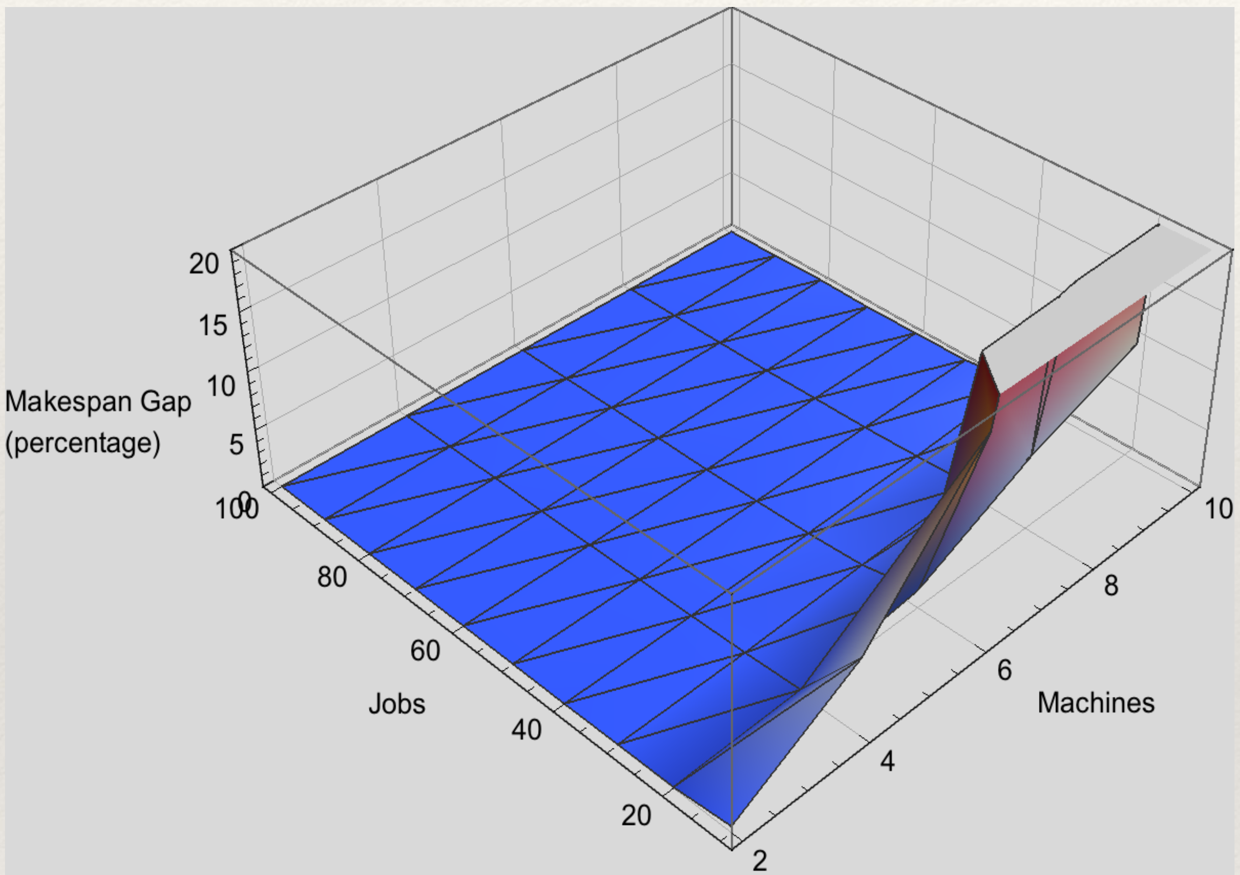

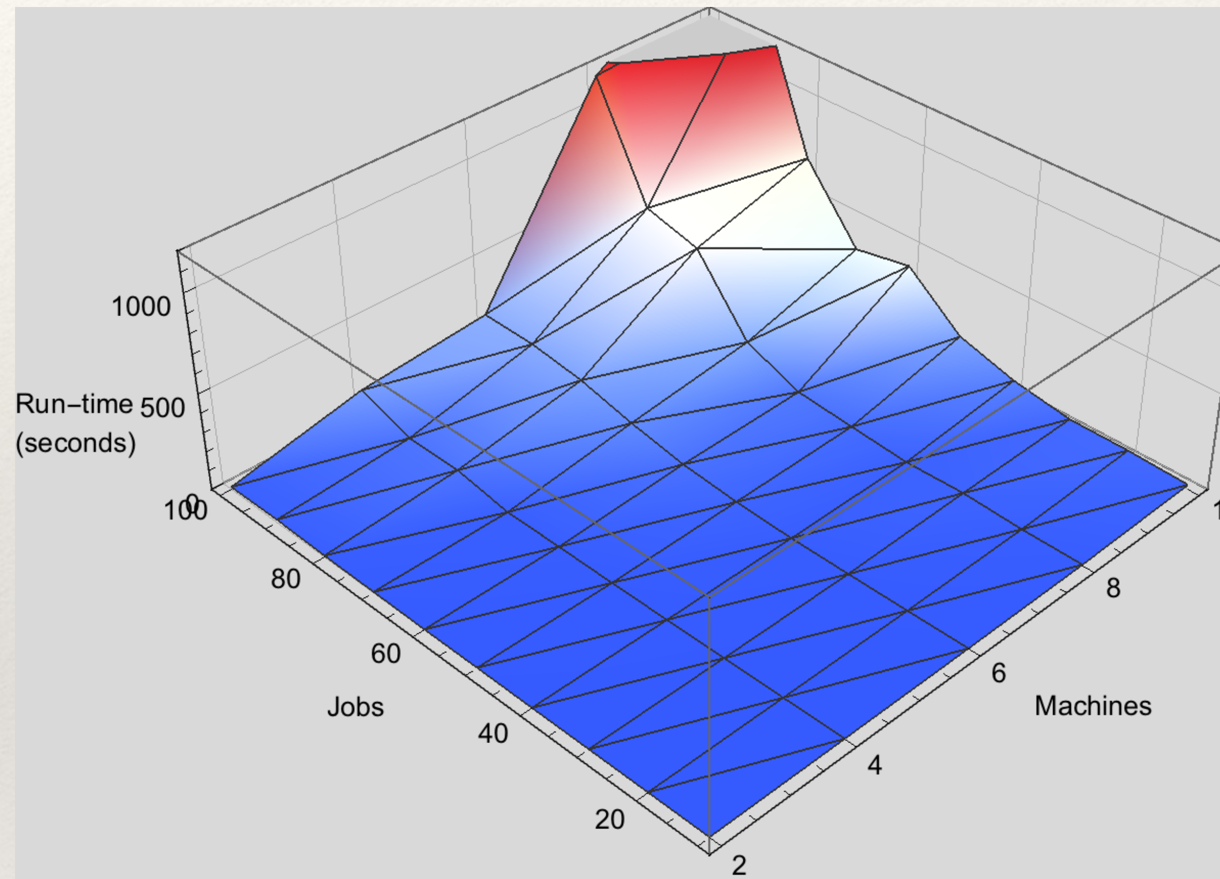

GLS Results

(Initial Solution: GMS)

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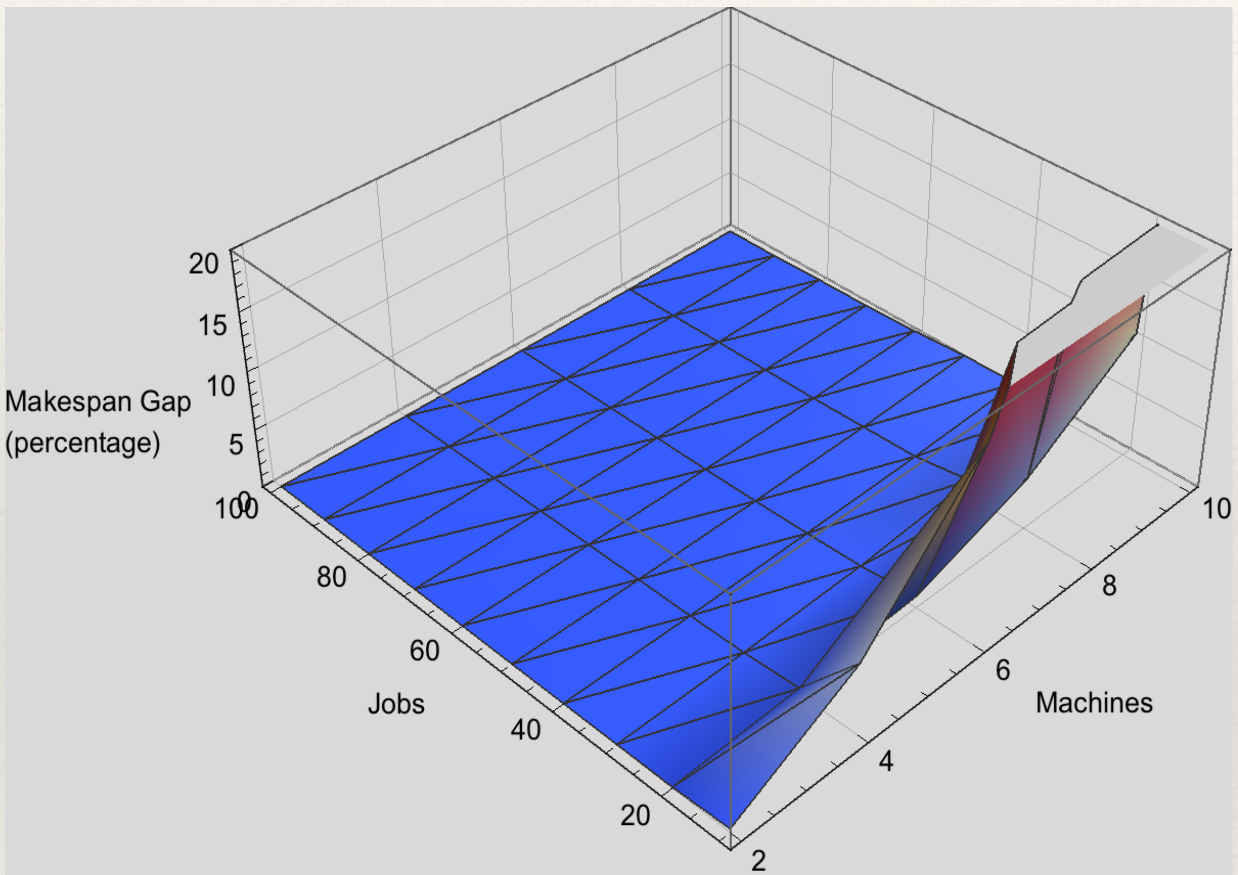
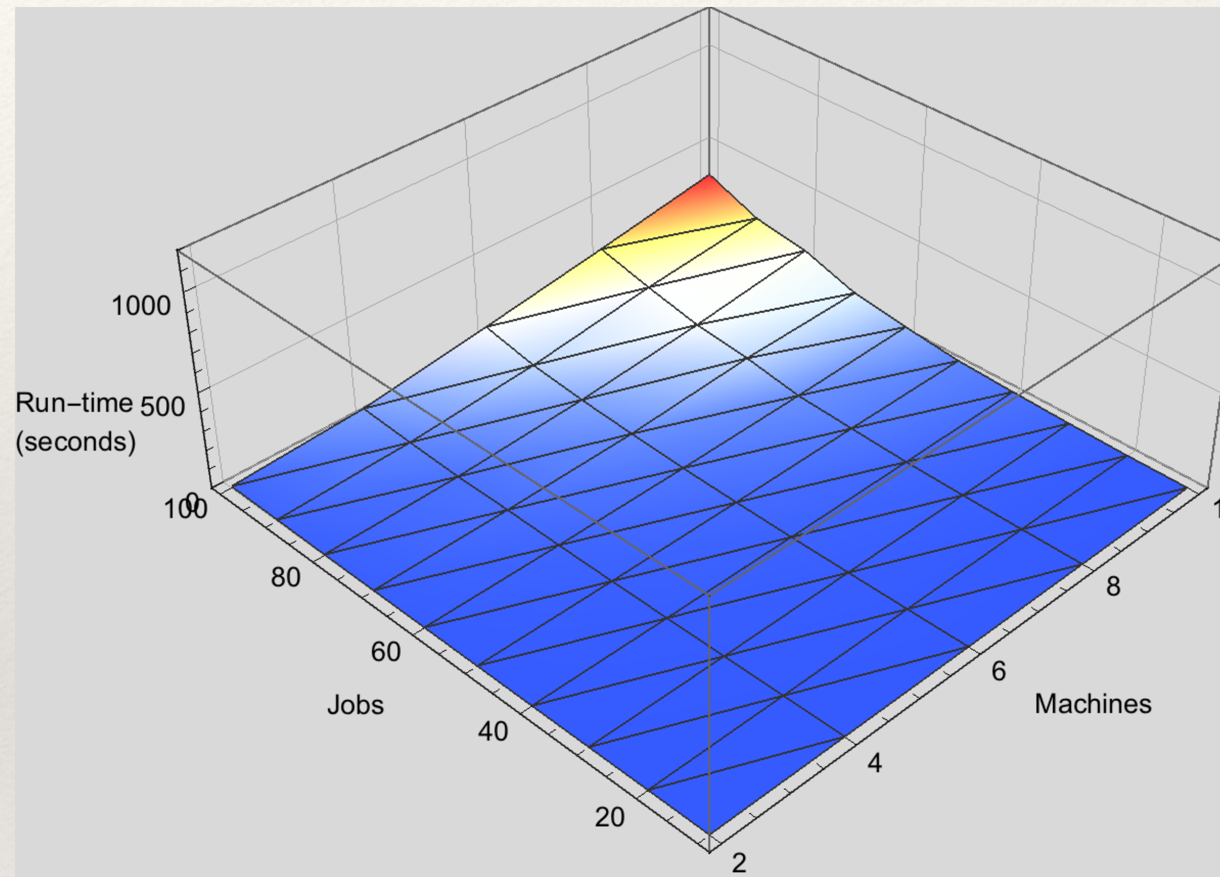
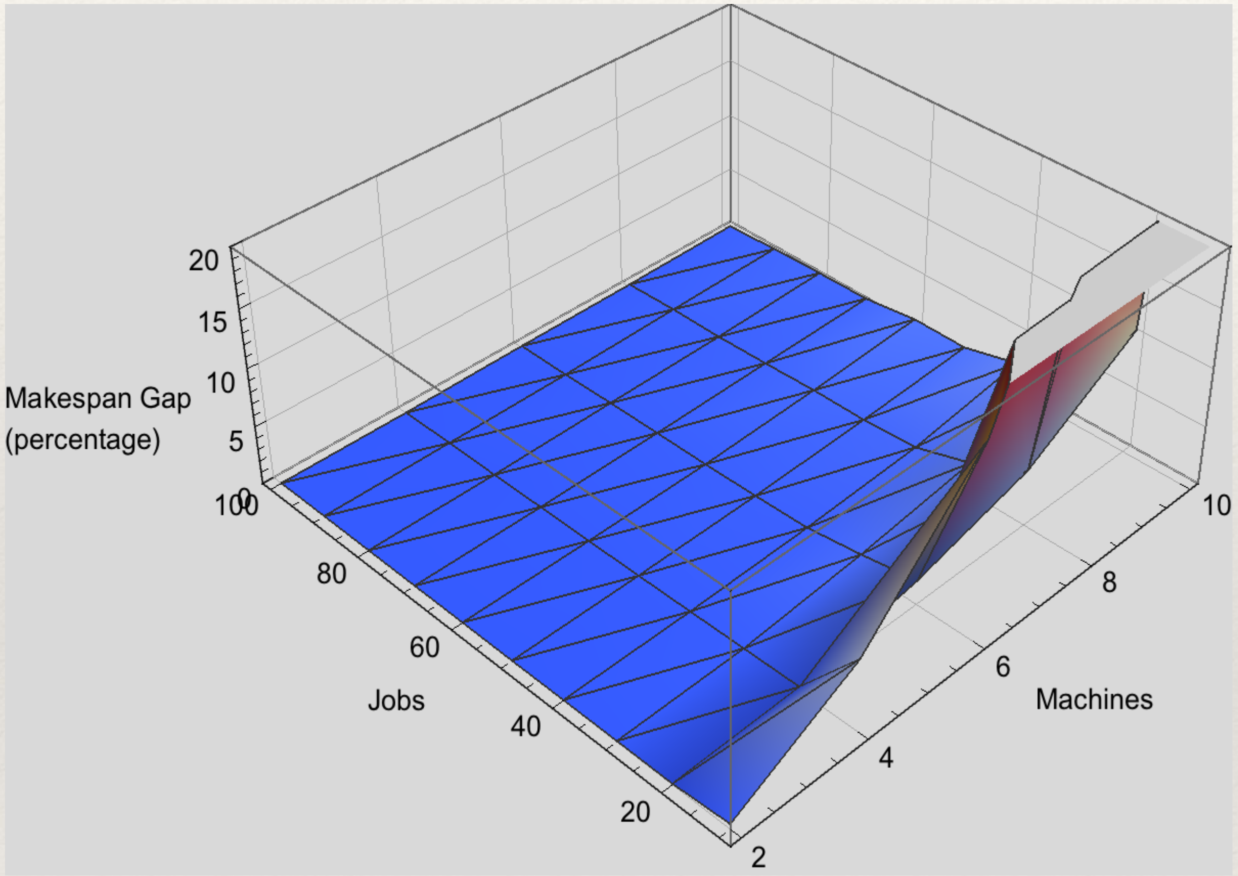


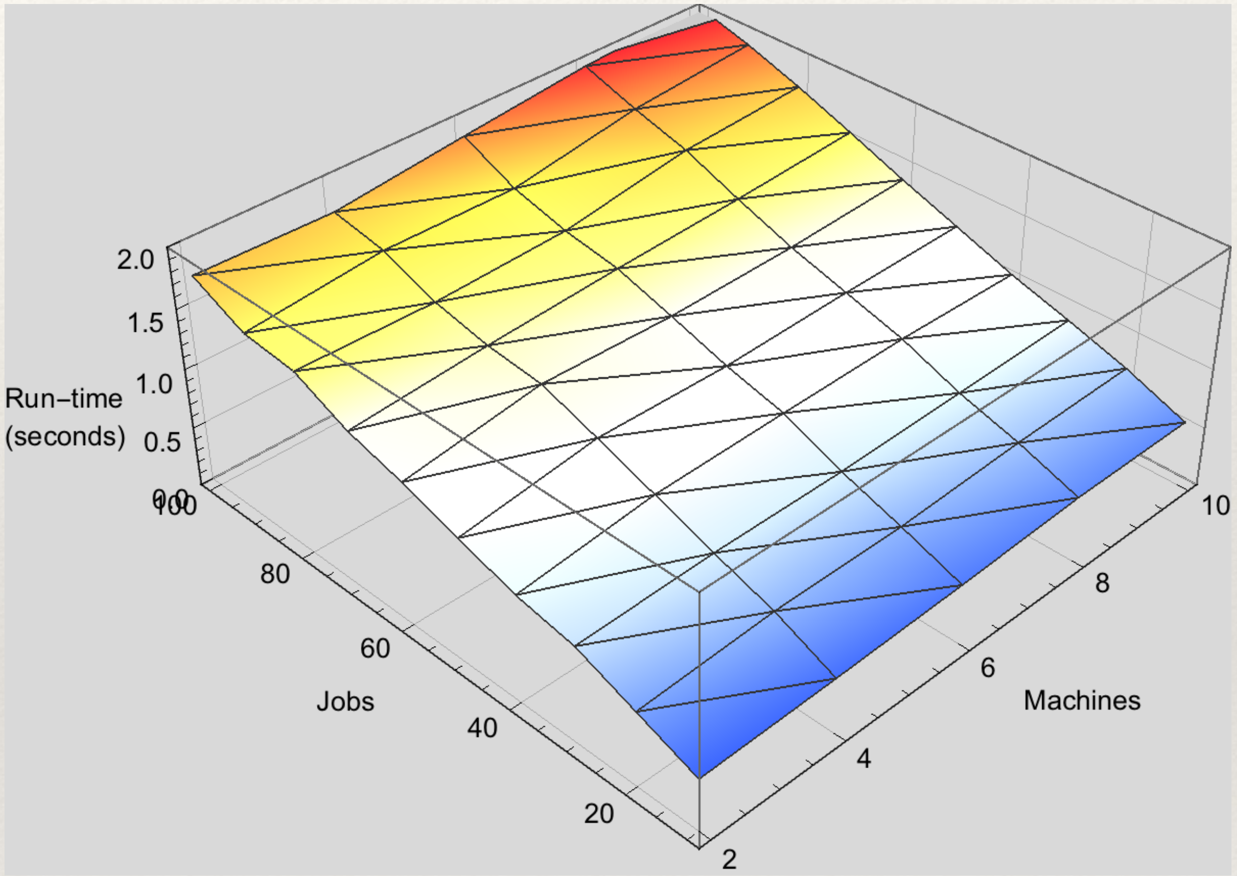


VDS Results

(Initial Solution: GMS)

Experiments: SA Results





SA Results

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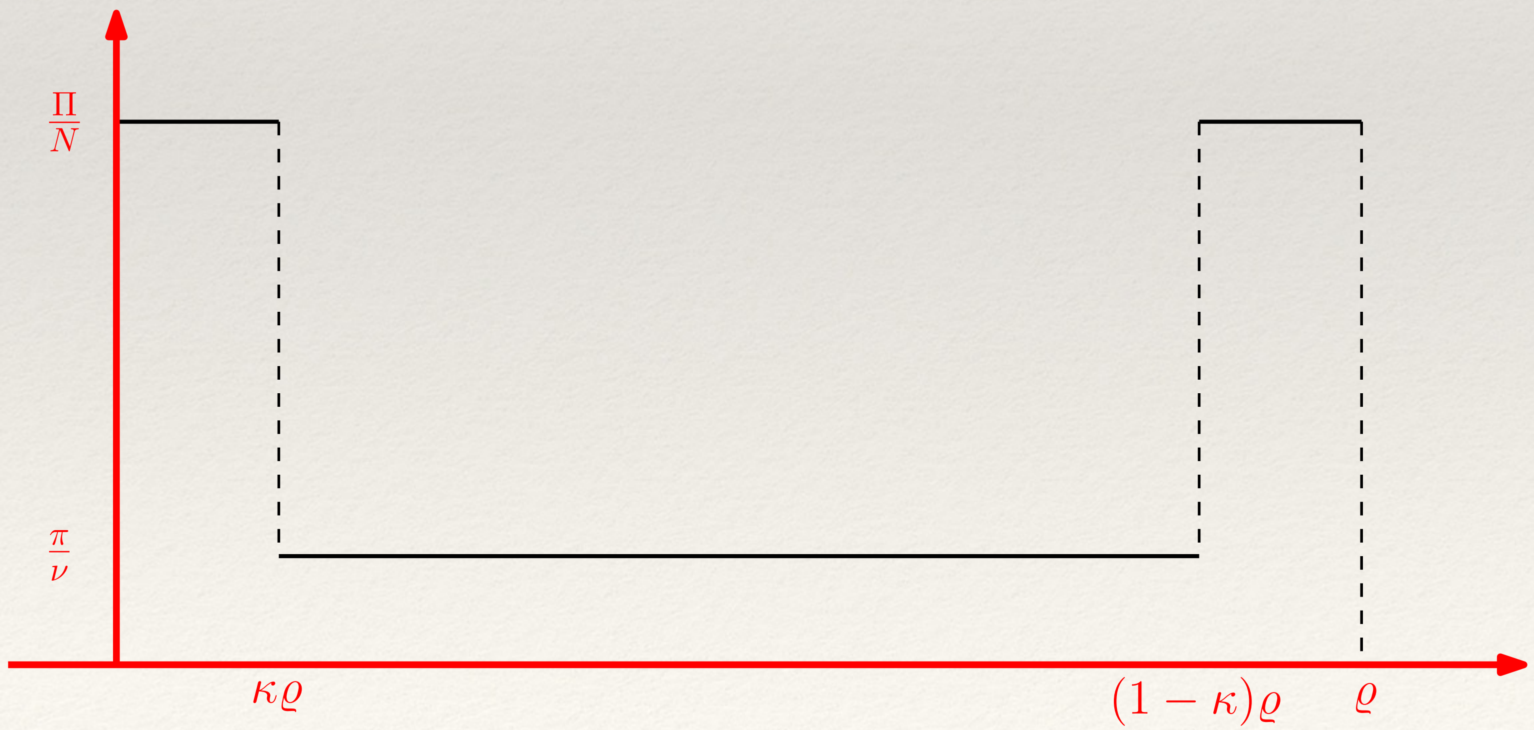
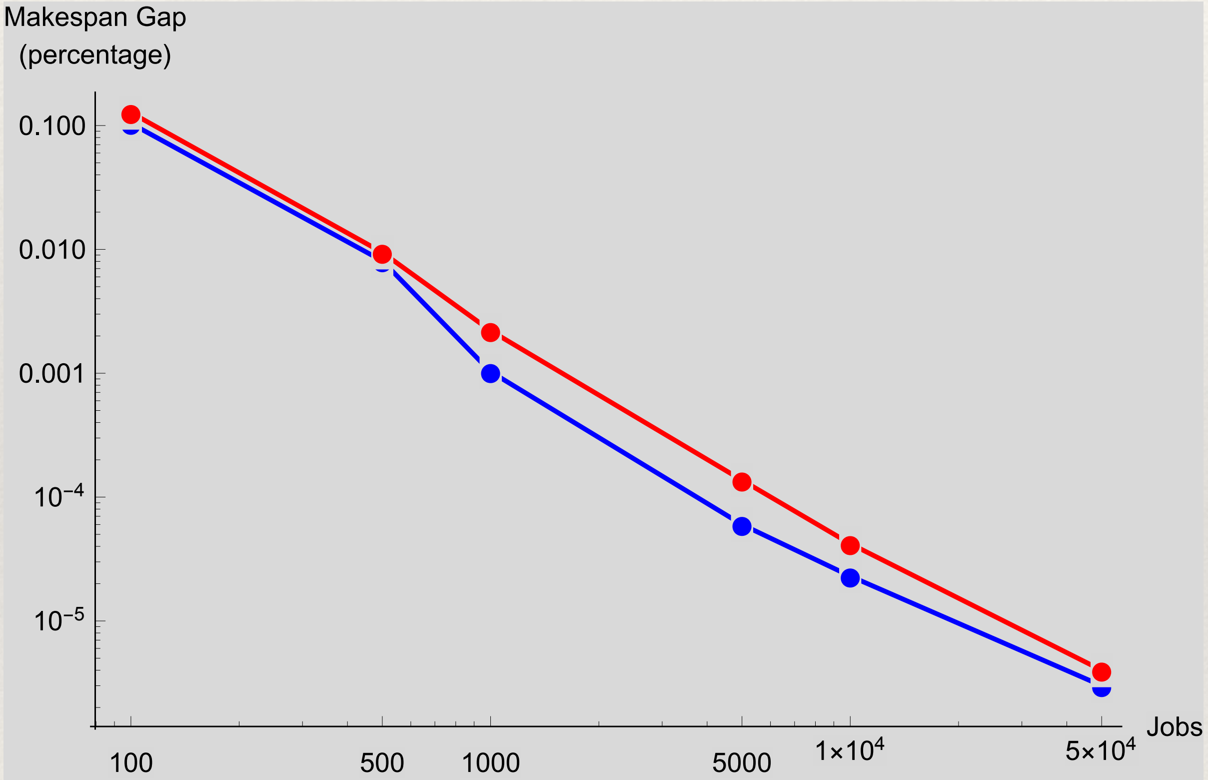
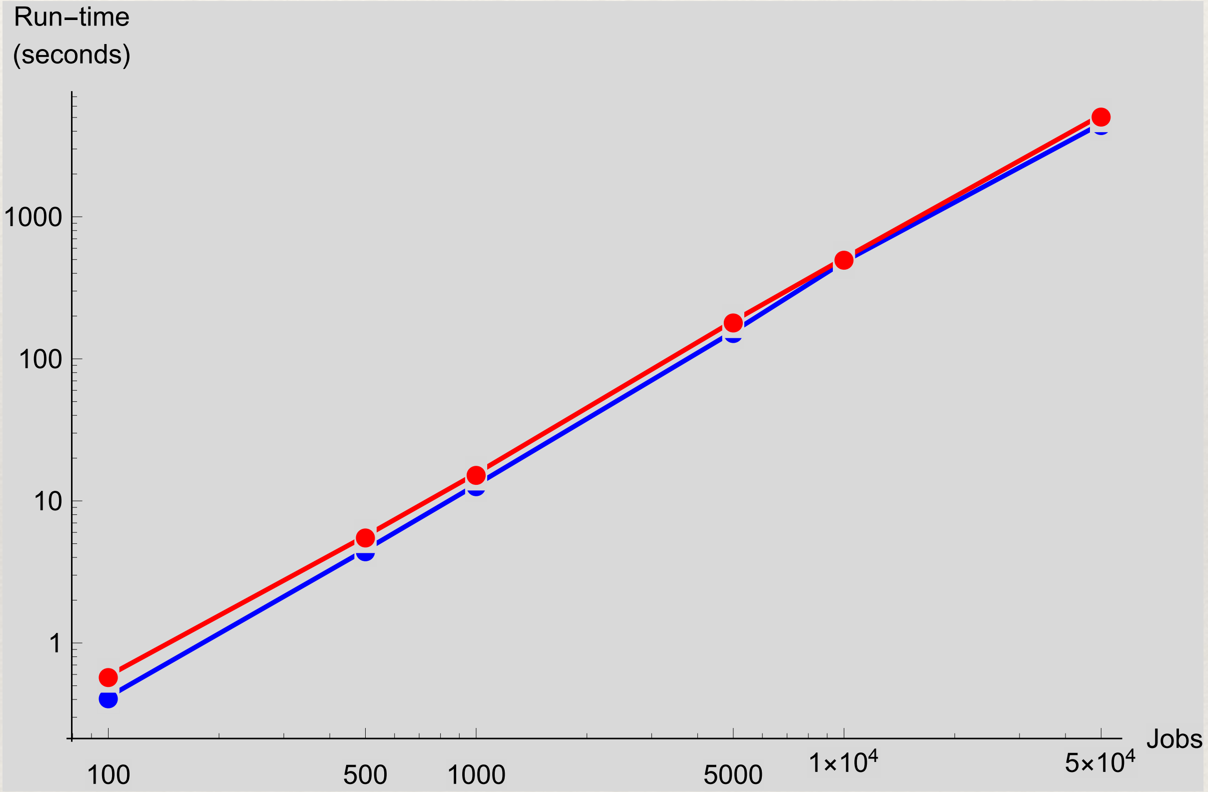


Experiments: Performance Limitations

- Increase size of instance: 500, 1000, 5000, 10000 jobs…





Uniform Distribution

Poor Distribution

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- Constraint Programming (CP) model to generate an *exact* solution
- Runtime of CP model varies drastically, especially for larger instances

Experiments: Solution Quality Measure

m \ n	10	20	30	40	50	60	70	80	90	100
2	0.002	0	0	0	0	0	0	0	0	0
4	0.000	0.003	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0
6	0	0.013	0.007	0.004	0.003	0.002	0.001	0.001	0.001	0.001
8	0	0.014	0.019	0.012	0.007	0.004	0.004	0.003	0.003	0.002
10	0	0.003	0.042	0.048	0.017	0.008	0.006	0.007	0.002	0.001

The average ratio minus 1 of the SA solution to the optimal solution (by CP), for instances that were solved to optimality within 5 minutes.

Summary

- We applied Simulated Annealing to the Makespan Scheduling problem
- Different cooling schedules and an algorithm for initial temperature were investigated
- Simulated Annealing found to be much faster than GLS, VDS and CP

Thank you!

References

1. Ben-Ameur, W. (2004). *Computing the initial temperature of simulated annealing*. *Computational Optimization and Applications*, 29(3):369–385.