Simulated Annealing for Makespan Scheduling

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Overview

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  - Cooling Schedule
  - Initial Temperature

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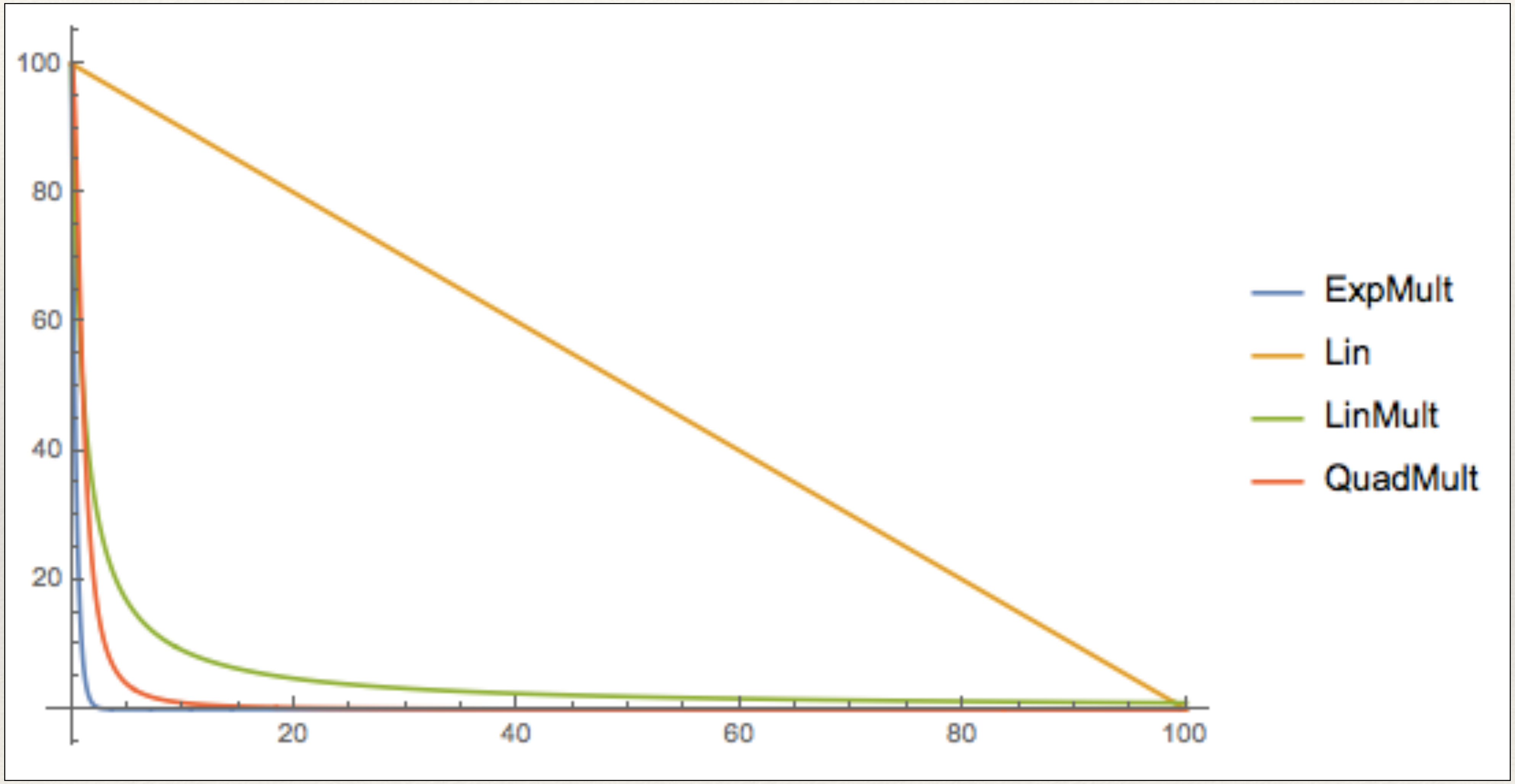
- Free parameters:
    1. Neighbourhood: 2-exchange “jump”
    2. Cooling schedule
    3. Initial temperature value

Simulated Annealing: Cooling Schedules

- **4 different cooling schedules considered:**

1. Exponential multiplicative: $f_1(T_0, I) = T_0 \cdot \mu^I$
2. Simple exponential: $f_2(T_0, I) = T_0 - I$
3. Linear multiplicative: $f_3(T_0, I) = \frac{1}{1+I} \cdot T_0$
4. Quadratic multiplicative: $f_4(T_0, I) = \frac{1}{1+I^2} \cdot T_0$

Simulated Annealing: Cooling Schedules



Simulated Annealing: Initial Temperature

- Good choice of initial temperature depends largely on instance
- Algorithm by Ben-Ameur [1]:
  - Generates a temperature so that the probability of accepting a cost increase is equal to a specified value

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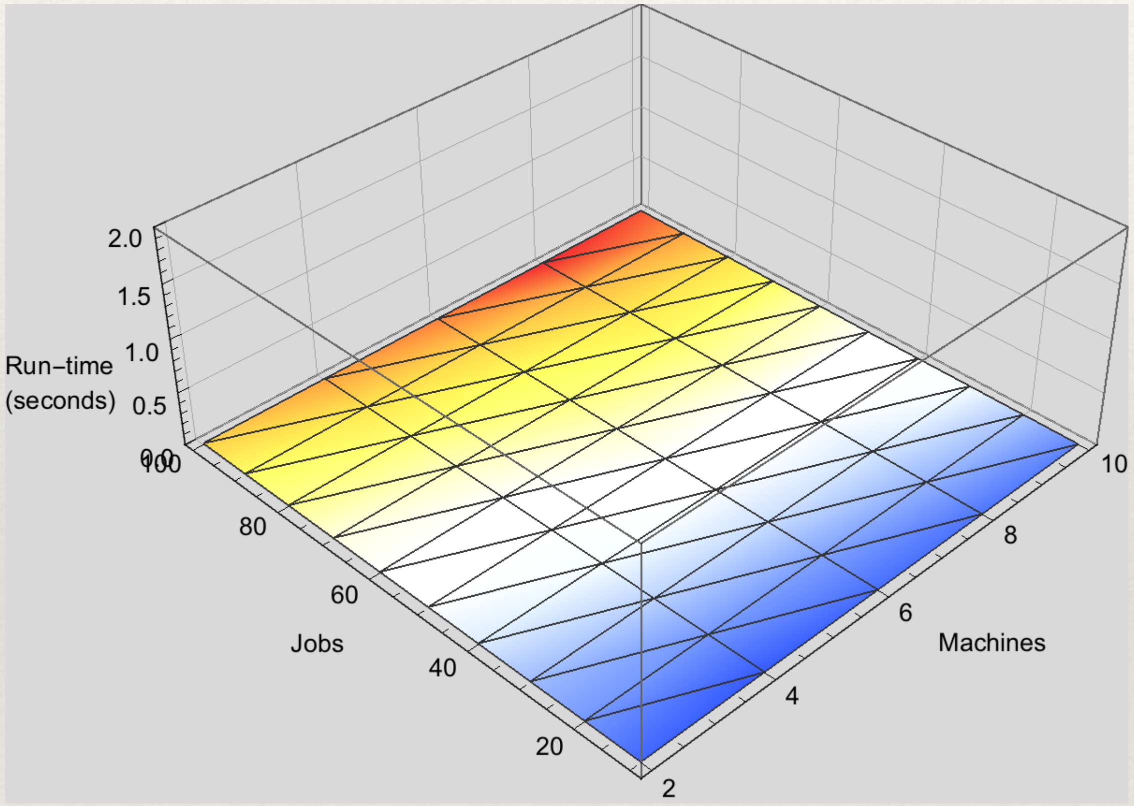
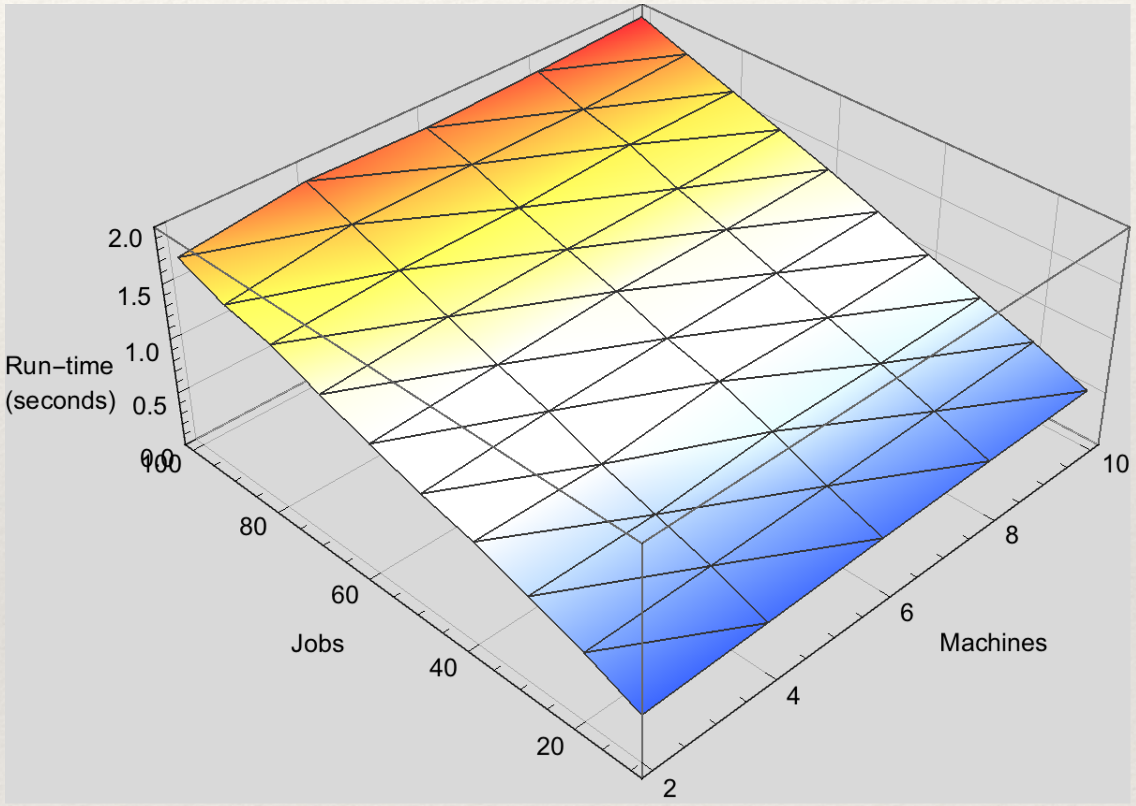


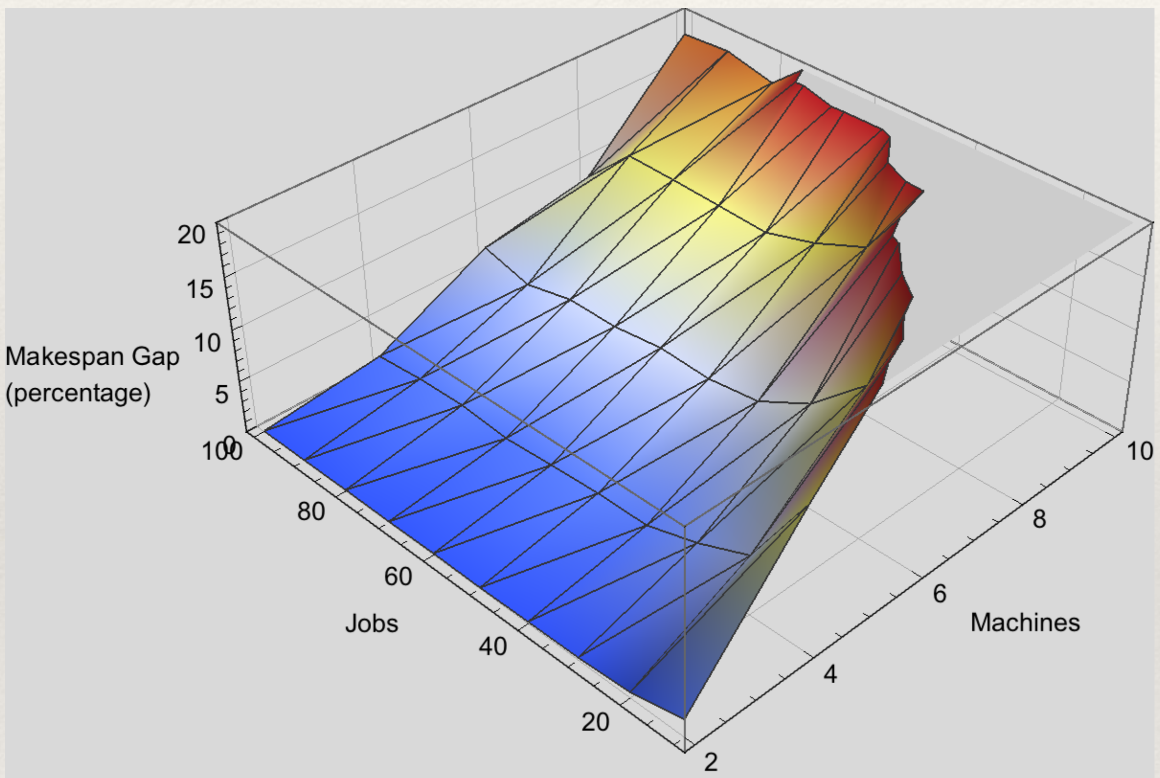
Linear Multiplicative Cooling



Quadratic Multiplicative Cooling

Run time

Experiments: Cooling Schedules



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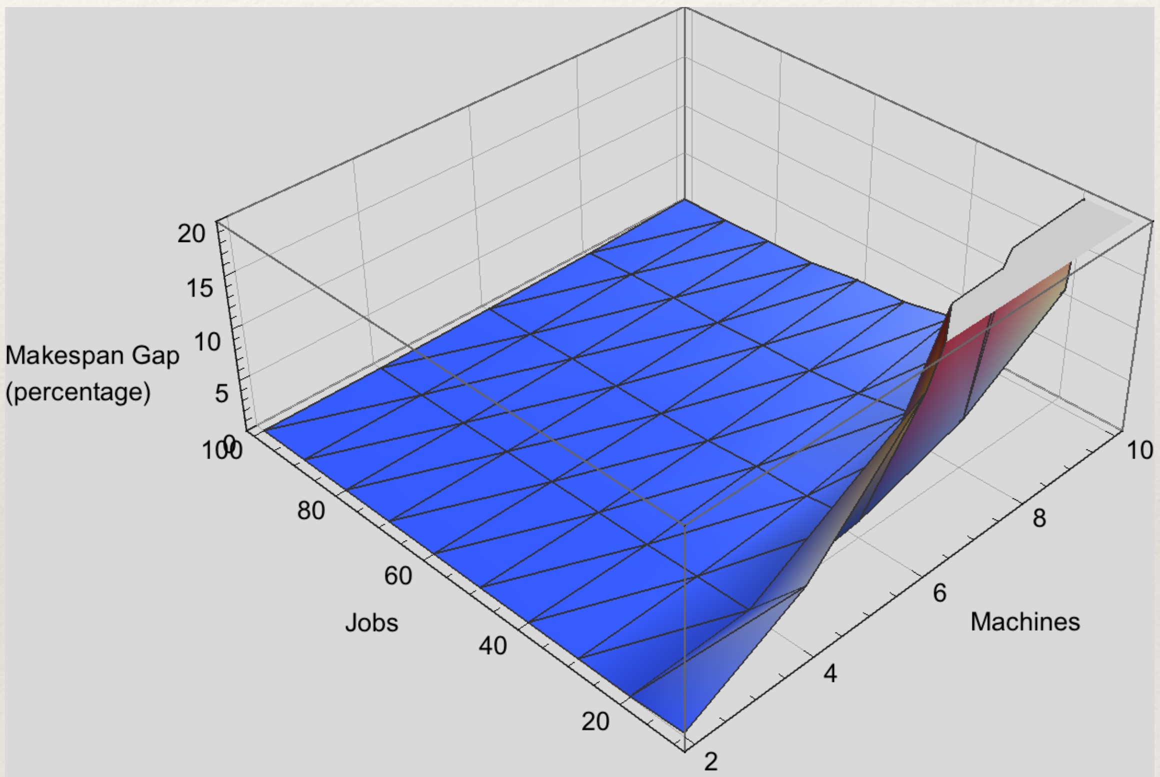

Quadratic Multiplicative Cooling

Makespan Gap

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- Conclusion: choose initial temperature to be 1.5 times the maximum processing time

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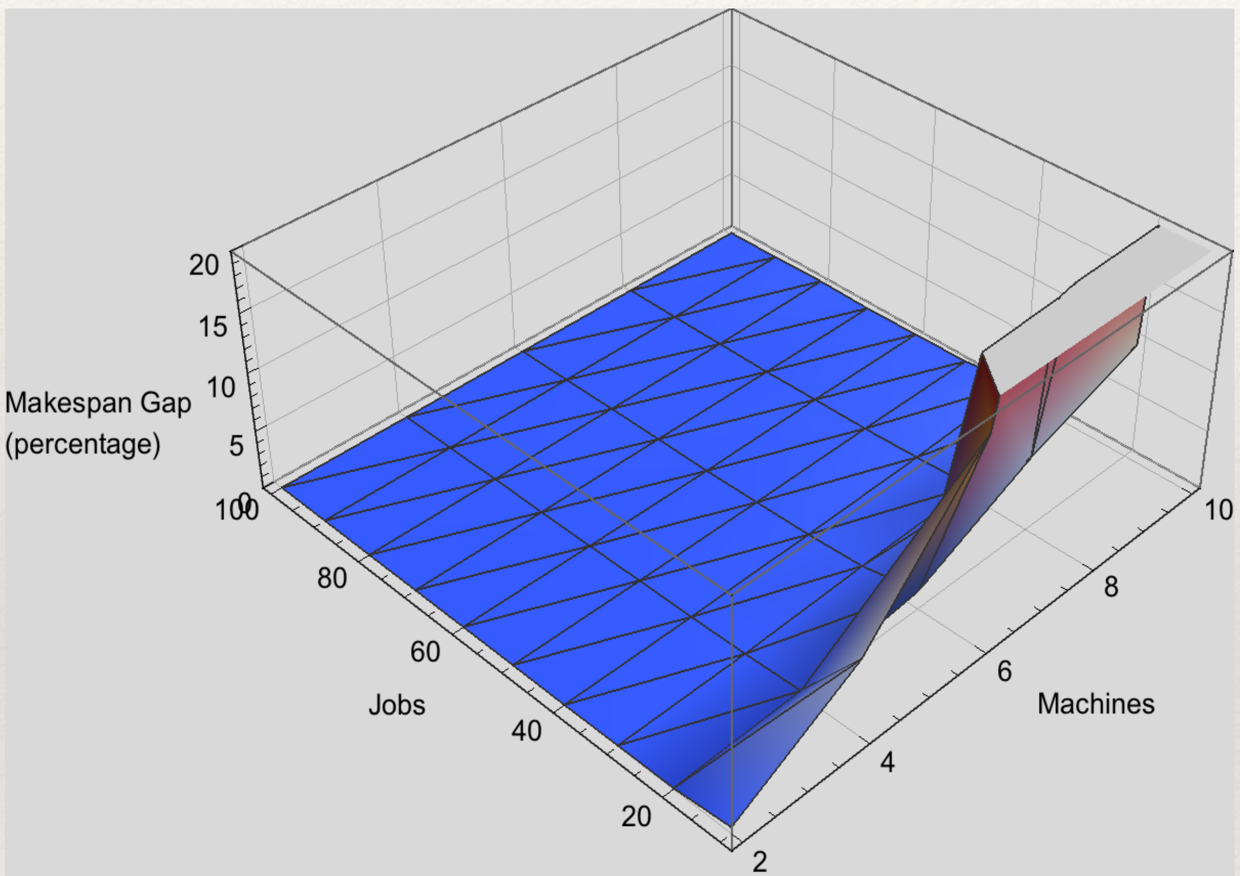
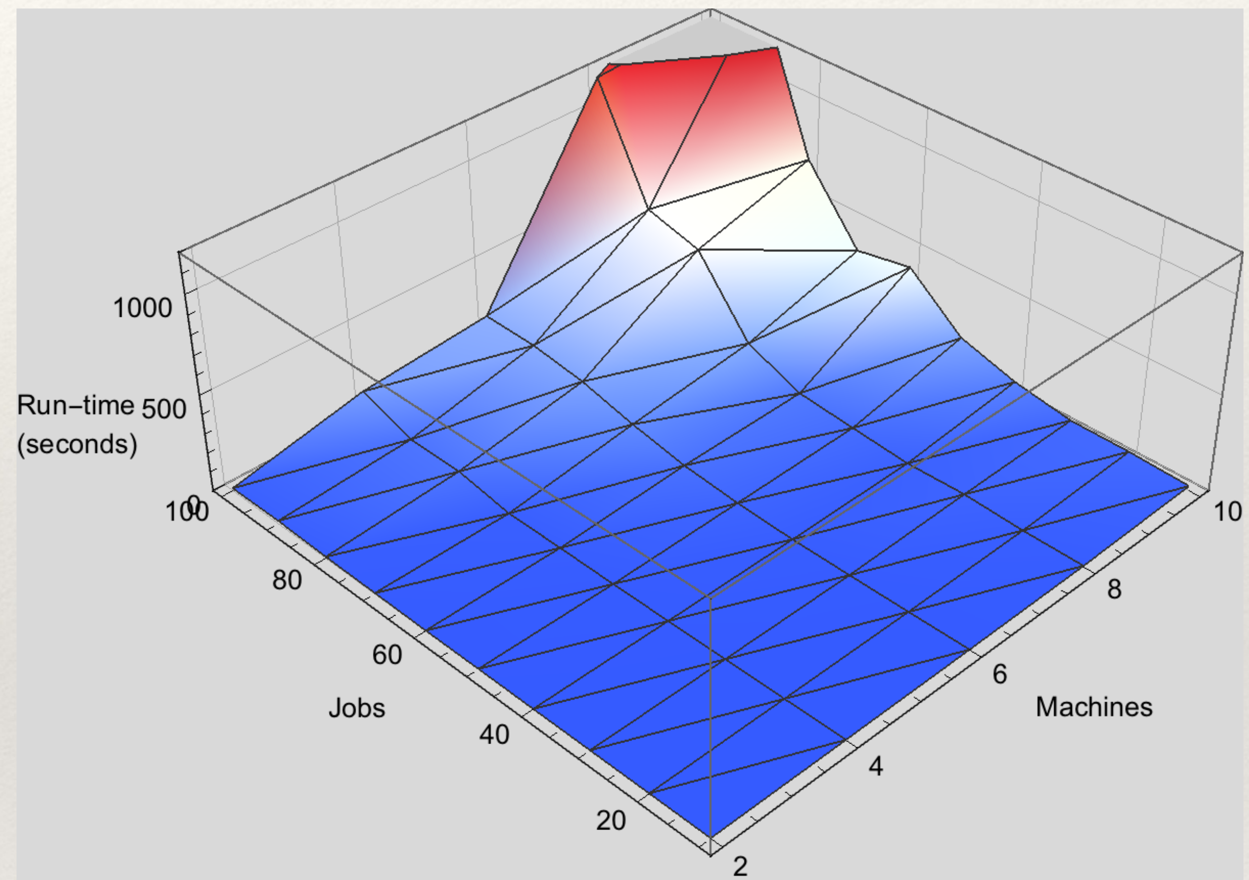
Experiments: SA Comparison to GLS





GLS Results

(Initial Solution: GMS)

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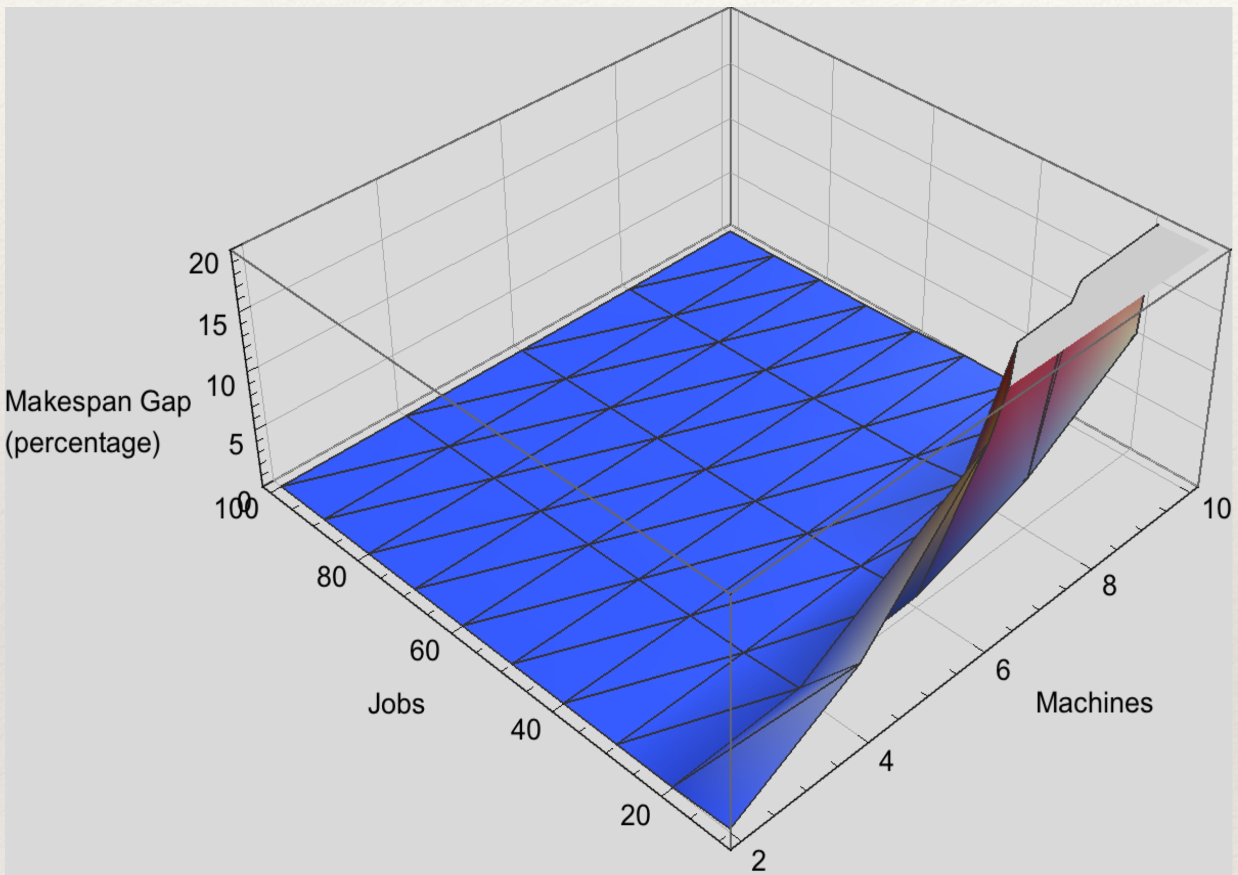

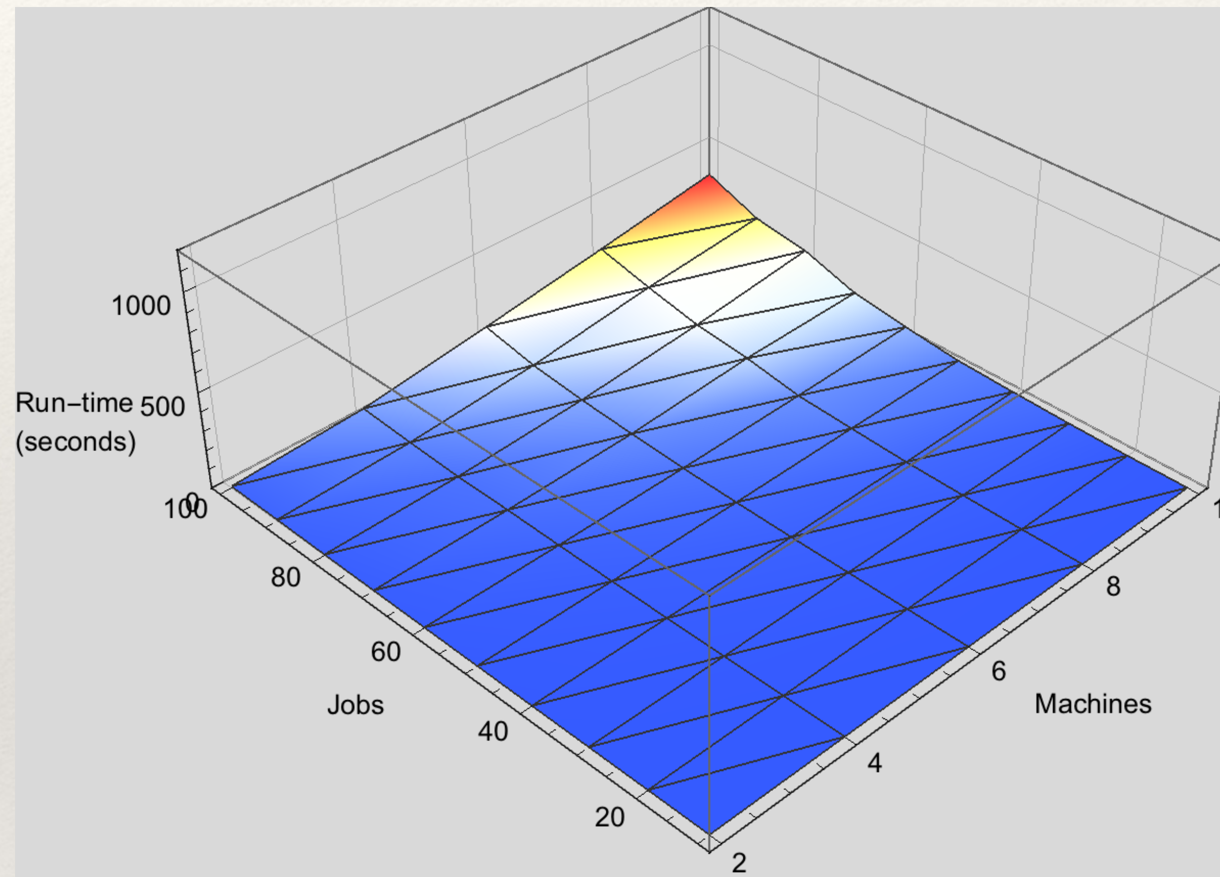

VDS Results

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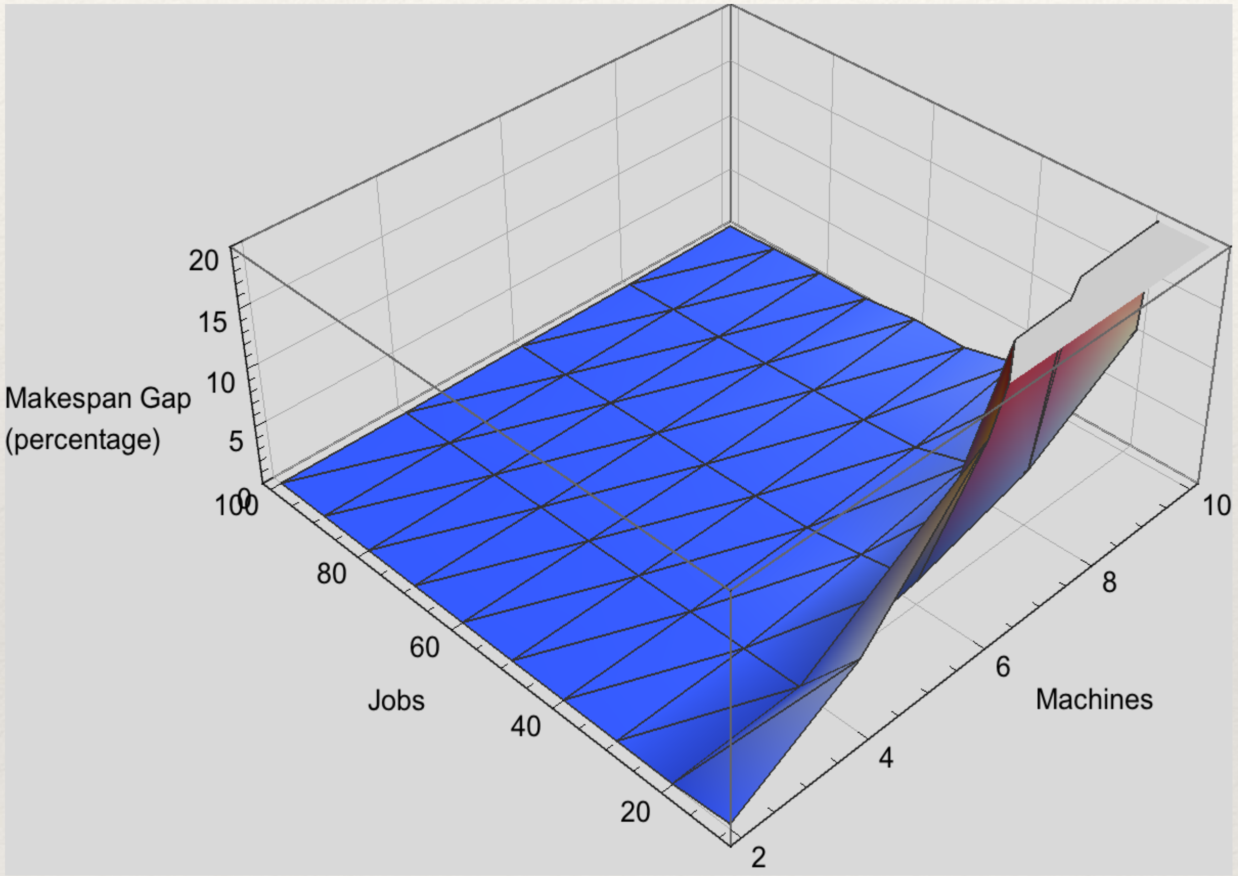
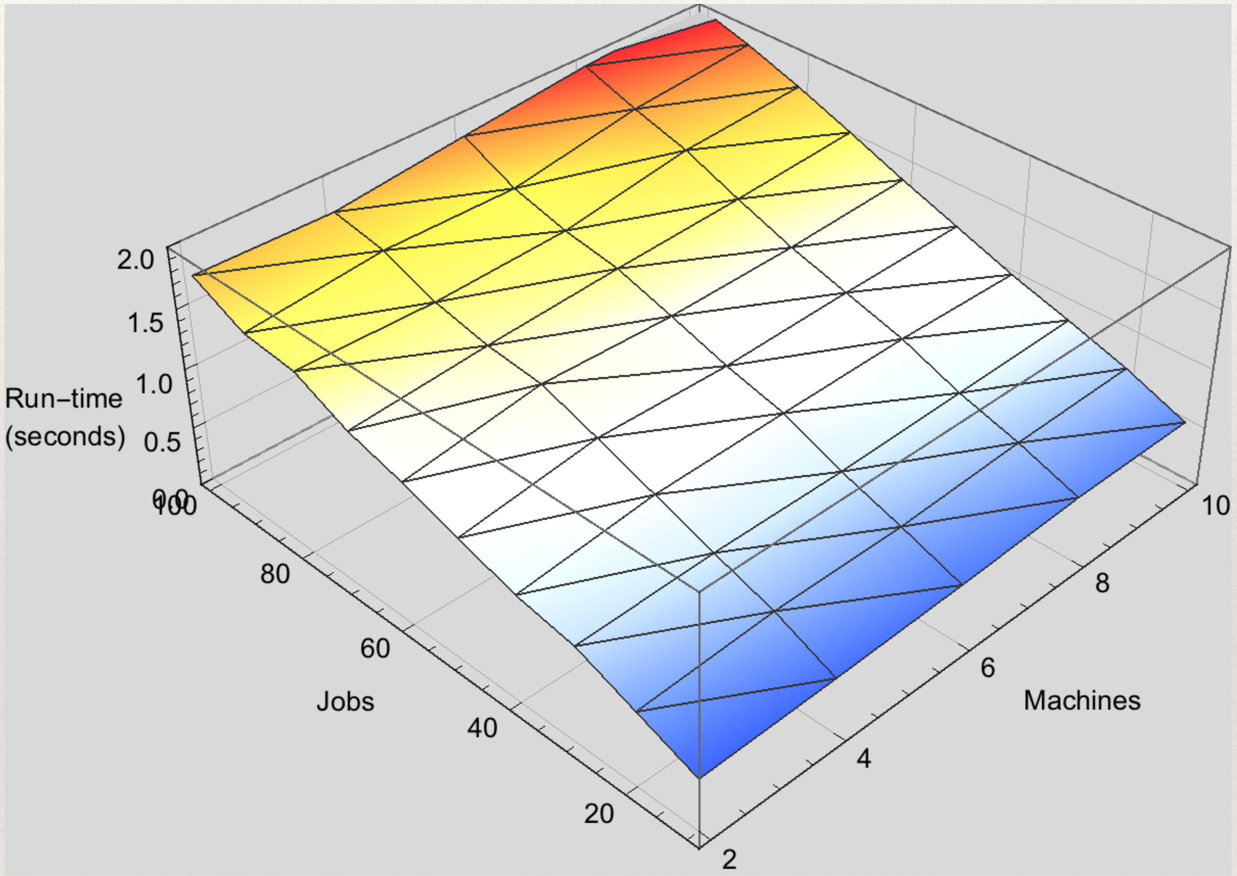
Experiments: SA Results





SA Results

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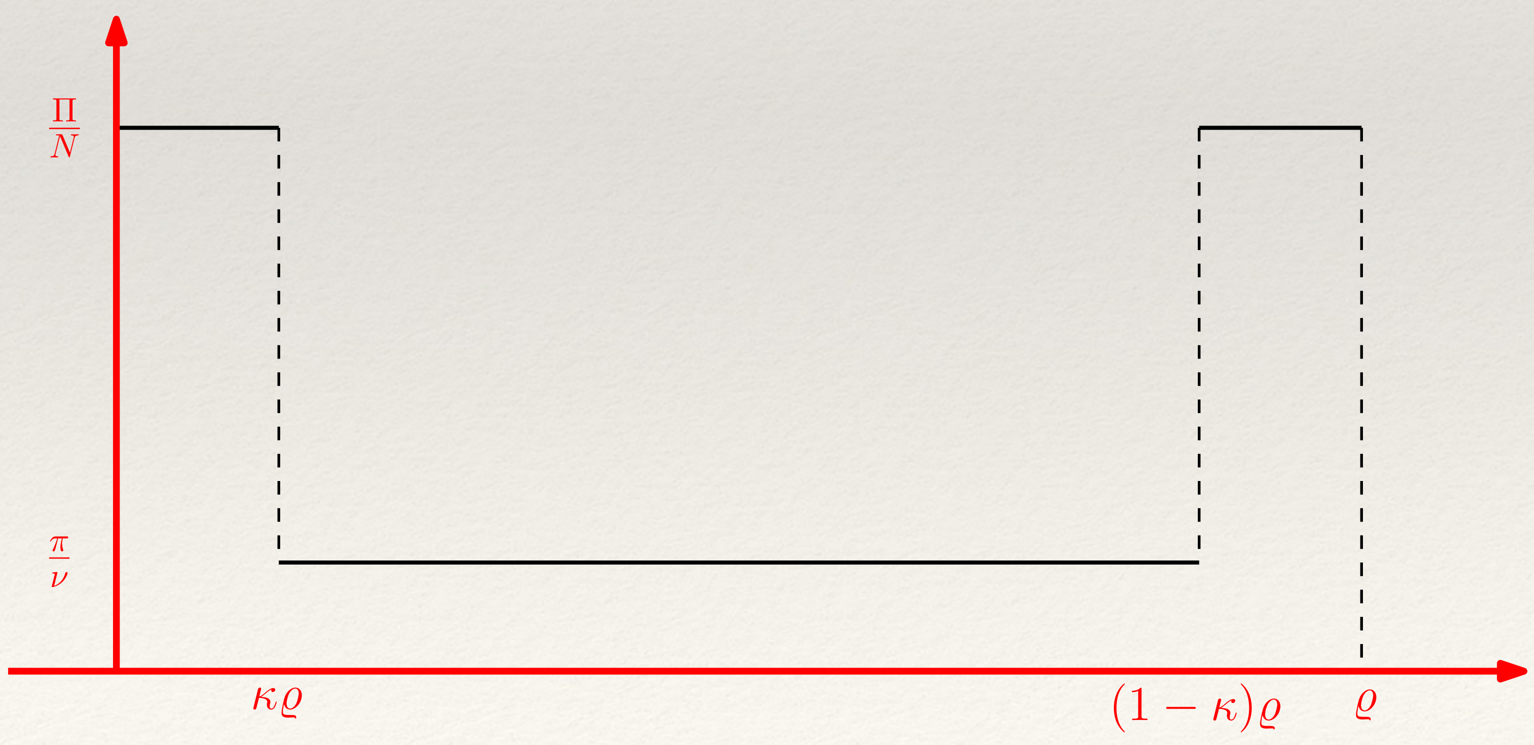
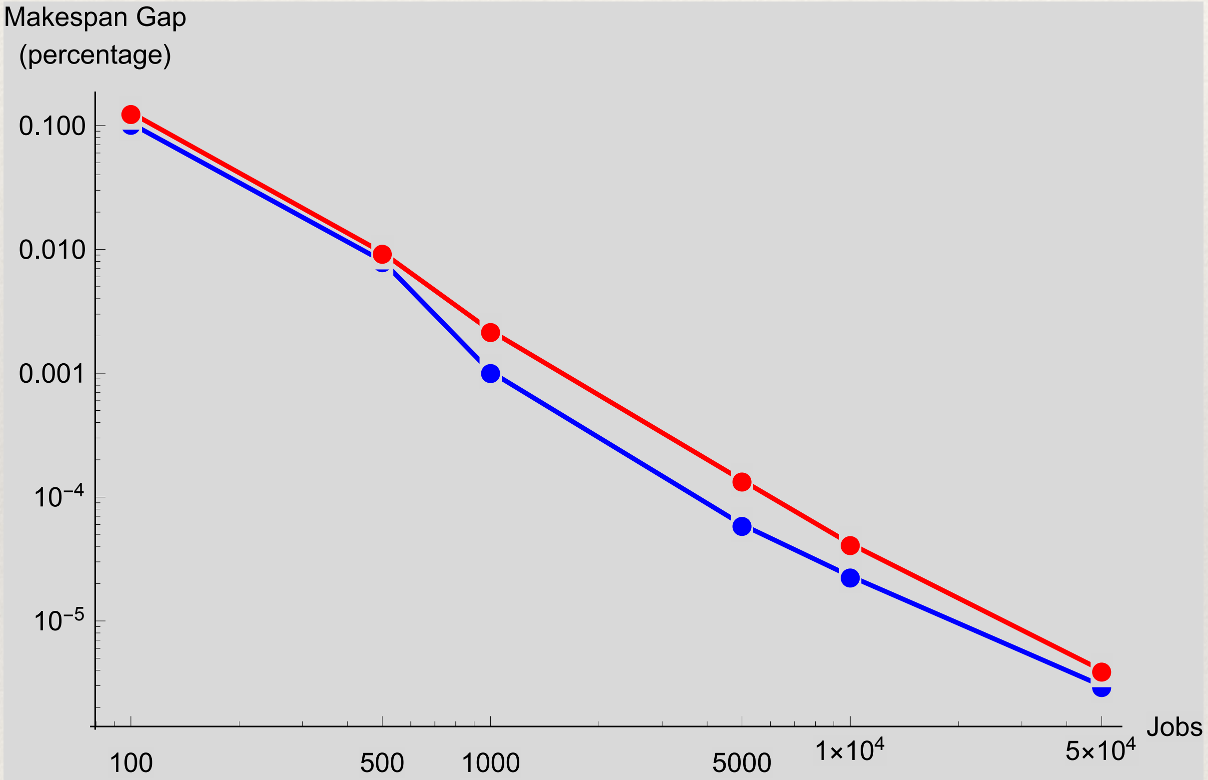
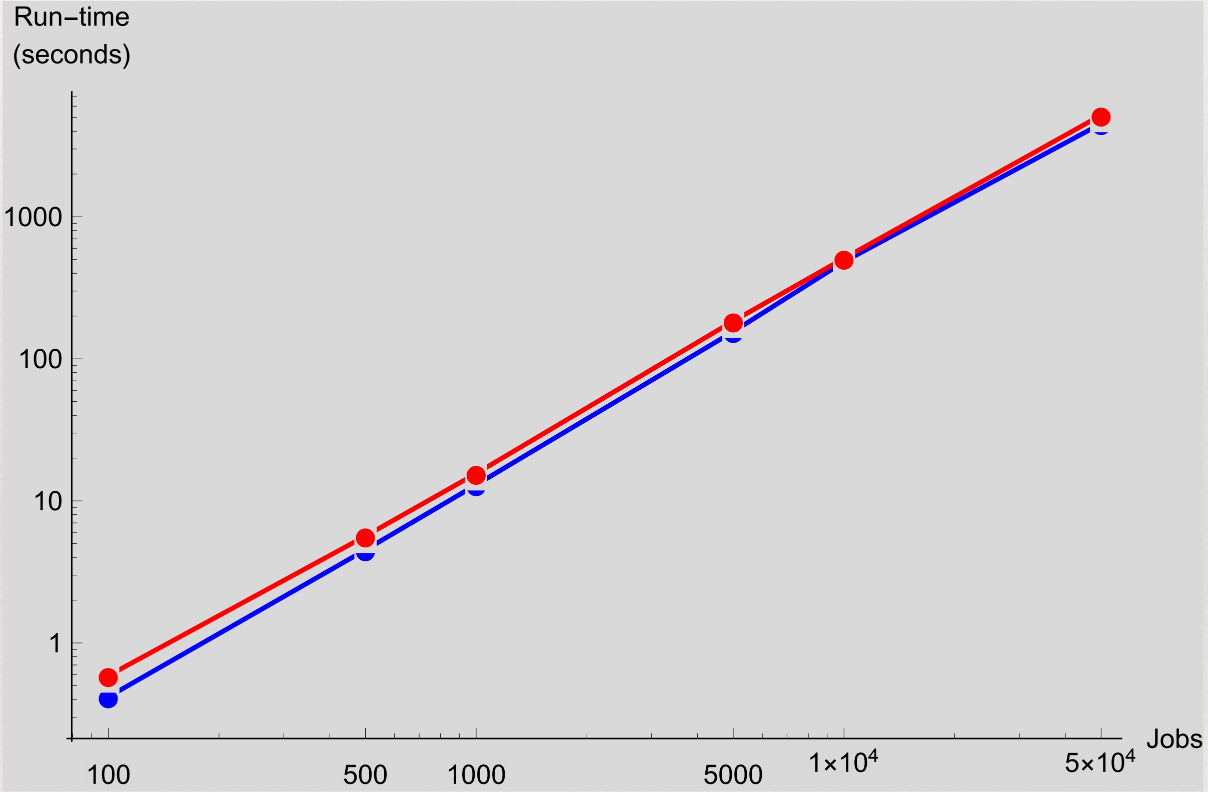


Experiments: Performance Limitations

- Increase size of instance: 500, 1000, 5000, 10000 jobs…





Uniform Distribution

Poor Distribution

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- Fractional lower bound is not a good measure of solution quality when n and m are comparable
- Constraint Programming (CP) model to generate an <u>*optimal*</u> solution
- Runtime of CP model varies drastically, especially for larger instances

Experiments: Solution Quality Measure

m \ n	10	20	30	40	50	60	70	80	90	100
2	0.24	0	0	0	0	0	0	0	0	0
4	0.04	0.28	0.12	0.05	0.04	0.03	0.02	0.01	0.02	0
6	0	1.32	0.66	0.43	0.28	0.22	0.09	0.13	0.10	0.08
8	0	1.37	1.85	1.16	0.74	0.42	0.39	0.25	0.26	0.18
10	0	0.33	4.20	3.30	1.74	0.79	0.63	0.69	0.22	0.10

The optimality gap (in %) of the SA solution to the optimal solution (by CP), for instances that were solved to optimality within 5 minutes.

Summary

- We applied Simulated Annealing to the Makespan Scheduling problem
- Investigated different cooling schedules, an algorithm for initial temperature, performance limitations
- Simulated Annealing found to be much faster than GLS, VDS and CP

Thank you!

References

1. Ben-Ameur, W. (2004). *Computing the initial temperature of simulated annealing*. *Computational Optimization and Applications*, 29(3):369–385.