Homework 7

Due on: Wednesday, March 11

Problem 1

Consider the map $\varphi(A) = L$ from an element A in $SL(2,\mathbb{C})$ to an element L of the proper orthochronous part of $O(3,1)$,

$$\varphi(A) = A^\dagger(x^\mu \sigma_\mu)A = \sigma_\mu L^\mu{}_\nu x^\nu \ , \tag{1.1}$$

where $\sigma_0 = \mathbb{I}$ and σ^k are the Pauli matrices. What is the covering group? What is the kernel? Write down the isomorphism which follows from this map. Is the covering group simply connected? Is the proper orthochronous part of the Lorentz group connected? Is it simply connected? Is it compact?

Problem 2

Determine the shadow of $SL(2,\mathbb{R})$. First determine the generator of $SL(2,\mathbb{R})$, then evaluate the group elements in the exponential representations. What is the shadow of $SU(2)$, of $SO(3)$?

Problem 3

Show that the Lie algebras of $SU(1,1)$ and $SL(2,\mathbb{R})$ are isomorphic.