Homework 9

Due on: Monday, March 30

Problem 1

Properties of SU(4). The weight diagrams and root diagram of $SU(3)$ can be depicted in two dimensions, but for $SU(4)$ they become three-dimensional. One can still "see" them geometrically. We now study them.

(a) Write down the Cartan generators. Deduce the weights of the defining representation. Show that they form a tetrahedron.

(b) Why are the sides of the tetrahedron equal to the roots? What are the positive roots? Which of them are simple roots? They form a cone, sketch this cone.

(c) Determine the fundamental weights. Identify the representation of which they are the highest weights. Write down the corresponding tensors.

(d) Explain that the antisymmetric tensor $t^{\mu\nu\rho}$ transforms the same way as $t^*_\mu = (t^\mu)^*$. Explain that $\epsilon^{\mu\nu\rho\sigma}$ is an invariant tensor of $\mathfrak{su}(4) = \mathrm{Lie}(SU(4))$.

(e) Consider the representation which corresponds to the reducible tensor $u^\mu v^\nu w_\rho$. Decompose it into irreps, and give the Young tableaux for these irreps, with their dimensions indicated below the tableaux. (In the notes the same is done for $SU(3)$, you should do it for $SU(4)$.)