Homework 11

Due on: Monday, April 13

Problem 1

Denote the Weyl reflections associated with the simple roots $\vec{\boldsymbol{\alpha}}^{I}$ and $\vec{\boldsymbol{\alpha}}^{II}$ of $SO(5)$ by σ_I and σ_{II}. Draw in a picture the hypersurfaces (lines) orthogonal to $\vec{\boldsymbol{\alpha}}^{I}$ and $\vec{\boldsymbol{\alpha}}^{II}$ with dashed lines, and in the same picture the two fundamental weights $\vec{\boldsymbol{\mu}}^{I}_{FW}$ and $\vec{\boldsymbol{\mu}}^{II}_{FW}$. How do σ_I and σ_{II} act on $\vec{\boldsymbol{\mu}}^{I}_{FW}$ and $\vec{\boldsymbol{\mu}}^{II}_{FW}$?

If a vector $\vec{\boldsymbol{v}}$ is written as $\lambda_I \vec{\boldsymbol{\mu}}^{I}_{FW} + \lambda_{II} \vec{\boldsymbol{\mu}}^{II}_{FW}$, how does the vector $(\lambda_I, \lambda_{II})$ transform under σ_I and σ_{II}?

The group elements σ_I and σ_{II} generate the Weyl group W. What is the meaning of the transformations $\sigma_I \sigma_{II}$ and $\sigma_{II} \sigma_I$? How many elements does W have and how many are reflections and how many rotations? Identify this finite group.

Problem 2

In the notes we matched irreps of $SU(4)$ with irreps of $SO(6)$ by requiring that their dimensions agree. Some matches were left open. What are the Young tableaux for the following irreps:

$$SU(4) : \underline{\mathbf{15}} \quad , \quad \underline{\mathbf{20}}$$
$$SO(6) : \underline{\mathbf{10}} \quad \text{(hint: (anti)self-duality)}$$
$$SO(5) : \underline{\mathbf{10}} \quad , \quad \underline{\mathbf{30}} \; .$$

You may use the section in the notes where formulas for the dimensions of irreps of $SU(N)$, $SO(2N)$and $SO(2N+1)$ are given.

Problem 3

Decompose the following products of irreps into sums of irreps, and write down the dimensions under the irreps

(a) $SO(5) \quad : \quad \ydiagram{2} \otimes \ydiagram{1} =$

(b) $SO(6) \quad : \quad \ydiagram{1,1} \otimes \ydiagram{1} =$