Homework 4

Due on: February 17.

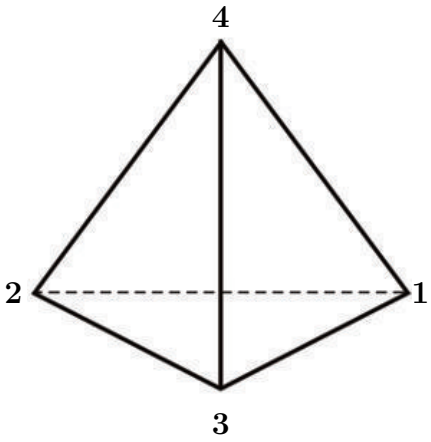
Problem 1

Give an example of a finite group that is semi-simple but not simple.

Problem 2

The Dirac group in 2 Euclidean dimensions (as defined in the notes) has order 8. In the table of section 5 of the notes we found 5 groups of order 8. To which of these 5 groups is the Dirac group isomorphic?

Problem 3



Construct a 3-dimensional matrix representation of S_4 (the group of isometries of a tetrahedron). Next construct a 4-dimensional representation of S_4. By construct we mean: give the matrices for all group elements. To save time, only construct the matrices for the following four group elements: (12), (23), (132), (123). If we read products of cycles from left to right, we get (12)(23) = (132). Then check the relation

$$M_{\text{passive}}(12)M_{\text{passive}}(23) = M_{\text{passive}}(132)\ . \tag{3.1}$$

If, however, we read from right to left we get

$$(12)(23) = (312) = (123)\ . \tag{3.2}$$

Now verify that

$$M_{\text{active}}(12)M_{\text{active}}(23) = M_{\text{active}}(123)\ . \tag{3.3}$$

The matrices M_{passive} rotate the frame axes, while the matrices M_{active} rotate vectors in the frames.