Homework 13

Due on: Monday, April 27

Problem 1.

Show that the property of an irrep of a Lie algebra being real, pseudoreal or complex is preserved by similarity transformations.

Problem 2.

What are the reality properties of the irreps of $USp(2N)$? What can you say about the reality properties of irreps of $SU(N)$? Use the isomorphisms $USp(2) \simeq SU(2)$, $USp(4) \simeq SO(5)$ and $SU(4) \simeq SO(6)$ to check your results.

Problem 3.

Derive the CG series for the following direct products of the spinorial irreps

$$\begin{aligned} SO(5): &\quad \times \otimes \times \\ SO(6): &\quad \times \otimes \times \text{ and } \times \otimes + \,. \end{aligned} \tag{3.1}$$

First show that the product of two spinors ψ^α and χ^β of $SO(2N+1)$ can be expanded in terms of the complete set of Dirac matrices as

$$\psi^\alpha \chi^\beta = \sum_k c_k \left(\gamma^{\mu_1 \cdots \mu_k} C^{-1}\right)^{\alpha\beta} \left(\psi^T C \gamma^{\mu_1 \cdots \mu_k} \chi\right). \tag{3.2}$$

Problem 4.

We found that the spinorial irreps of all $SO(2N+1)$ were either real or pseudoreal, but never complex. Was that an accident? We claim that for complex spinorial irreps of $SO(2N)$ the s and c spinorial irreps are similar to the complex conjugate of each other. Is that an accident?