PHY 611 - Homework 7

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SREDNICKI 88.2

In the standard model, we take neutrinos to be left-handed Weyl spinors. To quote Srednicki from pg. 458:

Eq. (75.8) shows us that there are only two kinds of particles associated with this field ... $b_-^\dagger(\mathbf{p})$ creates a particle with charge +1 and helicity -1/2, and $d_+^\dagger(\mathbf{p})$ creates a particle with charge -1 and helicity +1/2.

Modifying this for neutrinos (which have zero charge), we still find that there must a particle with negative helicity, and its antiparticle has positive helicity.

SREDNICKI 88.3

The Lagrangian we want to consider is

$$\mathcal{L} = i\ell_I^{i\dagger}\bar{\sigma}^\mu \left(D_\mu \ell_I\right)_i + i\bar{e}_I^\dagger \bar{\sigma}^\mu D_\mu \bar{e}_I - \left(\varepsilon^{ij}\varphi_i \ell_{jI} y_{IJ}\bar{e}_J + \text{h.c.}\right)$$

$$= i\nu_I^\dagger \bar{\sigma}^\mu D_\mu \nu_I + ie_I^\dagger \bar{\sigma}^\mu D_\mu e_I + i\bar{e}_I^\dagger \bar{\sigma}^\mu D_\mu \bar{e}_I - \left(\frac{1}{\sqrt{2}}(v+H)e_I y_{IJ}\bar{e}_J + \text{h.c.}\right)$$

This is invariant under the transformation

$$\nu_I \to e^{i\alpha_I}\nu_I \qquad e_I \to e^{i\alpha_I}e_I \qquad \bar{e}_I \to e^{-i\alpha_I}\bar{e}_I$$

If we take α_I to be different for each of the three possible I's, then this is a $U(1)\times U(1)\times U(1)$ symmetry. Since each part of this transformation should act only on its corresponding subspace, each generation will only be charged under its own $U(1)$ transformation. This means that $\mathcal{E}_e$ and $\mathcal{N}_e$ have an electron number of 1, $\mathcal{E}_\mu$ and $\mathcal{N}_\mu$ have a muon number of 1, $\mathcal{E}_\tau$ and $\mathcal{N}_\tau$ have a tau number of 1, and all other possibilities are zero.

SREDNICKI 89.4

For the $SU(3)$ field, equations (89.1-3) tell us that we have couplings to q, which is in the $\mathbf{3}$, and $\bar{u}$ and $\bar{d}$, which are in the $\bar{\mathbf{3}}$. We can therefore use eq. (73.41) with $T(A)=3$, $T(3)=T(\bar{3})=1/2$, and $n_f=3$:

$$\beta_3 = -\left[\frac{11}{3}\times 3 - \frac{4}{3}\times 3\times 3\times\frac{1}{2}\right]\frac{g_3^3}{16\pi^2} = \frac{9g_3^3}{16\pi^2}$$

For the $SU(2)$ field, equations (87.1) and (88.1) tell us that we have couplings to the fields q, ℓ, and φ, all of which are in the $\mathbf{2}$ of $SU(2)$. From equations (73.41) and (66.27) we find the beta function

$$\beta_2 = -\left[\frac{11}{3}T(A) - \frac{4n}{3}T(R)\right]\frac{g_2^3}{16\pi^2} + T(R')\frac{g_2^3}{48\pi^2}$$

Using $T(A)=2$, $T(2)=1/2$, and $n_f=3$ gives

$$\beta_2 = \left[-\frac{11}{3} \times 2 + \frac{4}{3} \times \frac{1}{2} \times 2 \times 3\right] \frac{g_2^3}{16\pi^2} + \frac{1}{2}\frac{g_2^3}{48\pi^2} = -\frac{19g_2^3}{96\pi^2}$$

The $U(1)$ couples to the Higgs by eq. (87.1), all the leptons by (88.32) and all the quarks by (89.27). The beta function is

$$\beta_1 = \left[-\frac{11}{3}T(A) + \frac{4}{3} \times 3 \times \left(T(-1/2) + T(1) + T(1/6) + T(-2/3) + T(1/3)\right)\right] \frac{g_1^3}{16\pi^2} + T(-1/2)\frac{g_1^3}{48\pi^2}$$

Using $U(1) \simeq SO(2)$ and the fact that for $SO(N)$, $T(A) = 2N - 4$, we see that $T(A) = 0$. This gives us

$$\beta_1 = 4\left[\frac{1}{2} + 1 - \frac{1}{6} + \frac{2}{3} - \frac{1}{3}\right] \frac{g_1^3}{16\pi^2} + \frac{g_1^3}{96\pi^2} = \frac{41g_1^3}{96\pi^2}$$

SREDNICKI 91.1

As we saw in problem 88.3, conservation of lepton number corresponds to a $U(1) \times U(1) \times U(1)$ symmetry. It is clear that the mass term

$$\mathcal{L}_{mass} = -\frac{1}{2}M(\bar{\nu}\bar{\nu} + \bar{\nu}^\dagger\bar{\nu}^\dagger)$$

does not respect the $U(1)$ symmetry $\bar{\nu} \to e^{-i\alpha}\bar{\nu}$.