Homework 1

Due on: Wednesday, September 4

Problem 1

We discussed in class the following set of relations:

$$\begin{array}{lcl} X^0 \text{ independent} & : & L(X^\mu, P_\mu, e) \xrightarrow{P_\mu} L(X^\mu, e) \xrightarrow{e} L(X^\mu) \\ & & \Big\downarrow{\scriptstyle X^0=ct} \qquad\qquad \Big\downarrow{\scriptstyle X^0=ct} \qquad\qquad \Big\downarrow{\scriptstyle X^0=ct} \\ X^0 = c\,t & : & L(\vec{\boldsymbol{X}}, P_\mu, e) \xrightarrow{P_\mu} L(\vec{\boldsymbol{X}}, e) \xrightarrow{e} L(\vec{\boldsymbol{X}}) \end{array}$$

(a) Write down the explicit expressions for these six Lagrangians.

(b) Show that the Lagrangians in the upper row have a gauge invariance, and specify the transformation rules in each of the three cases.

(c) Show that by choosing the gauge choice $X^0 = ct$ one recovers the three Lagrangians in the lower row.

(d) In the scheme above, in the line on top, we first eliminate P_μ and then e. Can one also first eliminate e and then P_μ?