Homework 3

Due on: Monday, February 24

Problem 1

We study background gauge invariance of $\mathcal{L}_{qu} = \mathcal{L}_{cl} + \mathcal{L}_{fix} + \mathcal{L}_{ghost}$. We discussed Weyl invariance in class. Now analyze the invariance of $\mathcal{L}_{qu}$ under background local Lorentz transformations. How do all fields X^μ, $\psi^{\mu A}$, $e_{\alpha,qu}^{\ \ a}$, $e_{\alpha,back}^{\ \ a}$, $\chi_{\alpha,qu}^{\ \ A}$, $\chi_{\alpha,back}^{\ \ A}$, $d_a^{\ \alpha}$, $b_a^{\ \alpha}$, c^α, c^{ab}, c_W, $\Delta_A^{\ \alpha}$, $\beta_A^{\ \alpha}$, γ^A, γ^A_{sc} transform? Is $\mathcal{L}_{qu}$ invariant, or only invariant up to a total derivative?

Hint: Pay attention to terms with derivatives of λ_L, and study how they are canceled.

Problem 2

We are going to derive the fermionic field ψ_+ in the z-plane, and construct the stress tensor $T^{(\psi)}(z)$.

(a) Write the infinitesimal conformal transformation of $\psi_+(t,\sigma)$ as a sum of infinitesimal Einstein (E), local Lorentz (L), and Weyl (W) transformations, as discussed in class for the antighost $b_a^{\ \alpha}$. First write down how $\psi = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} = \begin{pmatrix} \psi_- \\ -\psi_+ \end{pmatrix}$ transforms under infinitesimal E, L and W gauge transformations.

(b) Next integrate these infinitesimal transformations to obtain the finite conformal transformations which relates $\psi_+(t+\sigma)$ to $\psi(z)$.

(c) Now we use

$$\delta_{conf}\psi_+(\sigma^+) = [C, \psi_+(\sigma^+)] \quad \text{with} \quad C = \frac{i}{\hbar}\int_0^{2\pi} \xi^+(t+\sigma)T_{++}(t+\sigma)d\sigma \tag{2.1}$$

where $T_{++}(t+\sigma)$ is the stress tensor on the worldsheet for $\psi_+(t+\sigma)$. Transform to the z-plane, using such relations as

$$\int_0^{2\pi} d\sigma = \oint \frac{dz}{iz}\,; \quad \xi^+ = \frac{\xi^z}{iz} \tag{2.2}$$

and also the relation $\psi_+(-i\tau+\sigma) = z^\alpha \psi(z)$ where α is a real constant you should derive. This should give you $T^{(\psi)}(z)$.