PHY 622 - Homework 7

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PROBLEM 1

The superconformal transformations for ψ are

$$\delta\psi^{+\mu} = -2\left(\partial_- X^\mu\right)\epsilon^- + F^\mu\epsilon^+ \qquad \delta\psi^{-\mu} = 2\left(\partial_+ X^\mu\right)\epsilon^+ + F^\mu\epsilon^-$$

and the light cone gauge is $\psi^+_\pm = 0$. Raising the spinor indices gives $-\psi^{+-} = \psi^{++} = 0$. The boundary conditions and field equations allow us to set $F = 0$, so if we choose the parameters $\epsilon^\pm$ such that

$$2\left(\partial_- X^+\right)\epsilon^- = \psi^{++} \qquad -2\left(\partial_+ X^+\right)\epsilon^+ = \psi^{+-}$$

then we find the desired gauge.

PROBLEM 2

Part (a)

The states at the next-to-lowest level in the NS-NS sector are of the form

$$b^i_{-1/2}|\Omega\rangle \otimes \tilde{b}^j_{-1/2}|\Omega\rangle$$

which obey the mass formula

$$\alpha' M^2 = 2\left[\frac{1}{2} + \frac{1}{2} - a - \tilde{a}\right]$$

The indices i, j take the values $1, \ldots, 8$, which tells us that these states are of the form $\mathbf{8}_v \otimes \mathbf{8}_v = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_v$, which we have previously seen correspond to a scalar, a graviton, and an antisymmetric tensor. We see that we want this to be a massless state, so

$$2\left[1 - a - \tilde{a}\right] = 0 \implies a + \tilde{a} = 1$$

Part (b)

The lowest lying states allowed by level matching are

$$\xcancel{b^i_{-1/2}|\Omega\rangle_L \otimes |\Omega, a\rangle_R} \qquad b^i_{-1/2}|\Omega\rangle_L \otimes |\Omega, \bar{a}\rangle_R$$

which have $N = 1/2$, $\tilde{N} = 0$. The next states allowed by level matching are

$$b^i_{-1/2}b^j_{-1/2}b^k_{-1/2}|\Omega\rangle_L \otimes d^\ell_{-1}|\Omega, a\rangle_R \qquad \xcancel{b^i_{-1/2}\alpha^j_{-1}|\Omega\rangle_L \otimes \alpha^\ell_{-1}|\Omega, a\rangle_R}$$

$$b^i_{-1/2}\alpha^j_{-1}|\Omega\rangle_L \otimes d^\ell_{-1}|\Omega, a\rangle_R \qquad \xcancel{b^i_{-1/2}b^j_{-1/2}b^k_{-1/2}|\Omega\rangle_L \otimes \alpha^\ell_{-1}|\Omega, a\rangle_R}$$

$$\cancel{b^i_{-1/2}b^j_{-1/2}b^k_{-1/2}|\Omega\rangle_L \otimes d^\ell_{-1}|\Omega,\bar a\rangle_R} \qquad b^i_{-1/2}\alpha^j_{-1}|\Omega\rangle_L \otimes \alpha^\ell_{-1}|\Omega,\bar a\rangle_R$$

$$\cancel{b^i_{-1/2}\alpha^j_{-1}|\Omega\rangle_L \otimes d^\ell_{-1}|\Omega,\bar a\rangle_R} \qquad b^i_{-1/2}b^j_{-1/2}b^k_{-1/2}|\Omega\rangle_L \otimes \alpha^\ell_{-1}|\Omega,\bar a\rangle_R$$

$$b^i_{-3/2}|\Omega\rangle_L \otimes d^\ell_{-1}|\Omega,a\rangle_R \qquad \cancel{b^i_{-3/2}|\Omega\rangle_L \otimes \alpha^\ell_{-1}|\Omega,a\rangle_R}$$

$$\cancel{b^i_{-3/2}|\Omega\rangle_L \otimes d^\ell_{-1}|\Omega,\bar a\rangle_R} \qquad b^i_{-3/2}|\Omega\rangle_L \otimes \alpha^\ell_{-1}|\Omega,\bar a\rangle_R$$

which have $N = 3/2$, $\tilde N = 1$. Π^-_{NS} eliminates all states with an even number of b operators, and Π^-_R eliminates states with $\Gamma = +1$.

Part (c)

The projection operator $\Pi^+_R \otimes \Pi^-_R$ restricts us to states with $\Gamma \otimes \tilde\Gamma = (+1, -1)$. At the lowest level ($N = \tilde N = 0$), the states are

$$\cancel{|a\rangle_L \otimes |b\rangle_R} \qquad \cancel{|\bar a\rangle_L \otimes |b\rangle_R} \qquad |a\rangle_L \otimes |\bar b\rangle_R \qquad \cancel{|\bar a\rangle_L \otimes |\bar b\rangle_R}$$

At the second lowest level ($N = \tilde N = 1$), the states are

$$\cancel{d^i_{-1}|a\rangle_L \otimes d^j_{-1}|b\rangle_R} \qquad \cancel{d^i_{-1}|a\rangle_L \otimes \alpha^j_{-1}|b\rangle_R} \qquad \alpha^i_{-1}|a\rangle_L \otimes d^j_{-1}|b\rangle_R \qquad \cancel{\alpha^i_{-1}|a\rangle_L \otimes \alpha^j_{-1}|b\rangle_R}$$

$$d^i_{-1}|\bar a\rangle_L \otimes d^j_{-1}|b\rangle_R \qquad \cancel{d^i_{-1}|\bar a\rangle_L \otimes \alpha^j_{-1}|b\rangle_R} \qquad \cancel{\alpha^i_{-1}|\bar a\rangle_L \otimes d^j_{-1}|b\rangle_R} \qquad \cancel{\alpha^i_{-1}|\bar a\rangle_L \otimes \alpha^j_{-1}|b\rangle_R}$$

$$\cancel{d^i_{-1}|a\rangle_L \otimes d^j_{-1}|\bar b\rangle_R} \qquad \cancel{d^i_{-1}|a\rangle_L \otimes \alpha^j_{-1}|\bar b\rangle_R} \qquad \cancel{\alpha^i_{-1}|a\rangle_L \otimes d^j_{-1}|\bar b\rangle_R} \qquad \alpha^i_{-1}|a\rangle_L \otimes \alpha^j_{-1}|\bar b\rangle_R$$

$$\cancel{d^i_{-1}|\bar a\rangle_L \otimes d^j_{-1}|\bar b\rangle_R} \qquad d^i_{-1}|\bar a\rangle_L \otimes \alpha^j_{-1}|\bar b\rangle_R \qquad \cancel{\alpha^i_{-1}|\bar a\rangle_L \otimes d^j_{-1}|\bar b\rangle_R} \qquad \cancel{\alpha^i_{-1}|\bar a\rangle_L \otimes}\alpha^j_{-1}|\bar b\rangle_R$$