Homework 5

Due on: Wednesday, September 25

Problem 1

We have seen that for open strings in flat d–dimensional spacetime one can choose two types of boundary conditions (b.c.) both at $\sigma = 0$ and at $\sigma = \pi$, and for any given value of the index μ of X^μ: either N b.c. which read $\partial_\sigma X^\mu = 0$, or D b.c. which read $X^\mu = \alpha^\mu$. In the text we discussed the N b.c. in detail; here we work out the corresponding results for the other cases. There are clearly 4^d cases for open strings in d spacetime dimensions.

To explain the physical ideas behind D b.c., we consider an example. Let $d = 10$, and at $\sigma = 0$ the X^μ satisfy $\partial_\sigma X^\mu = 0$ for $\mu = 0, 1, 3, 4$ and $X^\mu = \alpha^\mu$ for $\mu = 2, 5, \cdots 9$. Further, at $\sigma = \pi$ $\partial_\sigma X^\mu = 0$ for $\mu = 0, 1, 6, 7, 8$ and $X^\mu = \beta^\mu$ for $\mu = 2, 3, 4, 5, 9$. Since the endpoint at $\sigma = 0$ is fixed in the $\mu = 2, 5, \cdots, 9$ direction, we can introduce a $p = 3$ brane D_1 spanned by X^0, X^1, X^3, X^4 (this is called a 3–brane because at any time X^0 it is $p = 3$–dimensional). The string originates then at D_1. Similarly, at $\sigma = \pi$ X^μ is fixed for $\mu = 2, \cdots, 5, 9$, and the $p = 4$ brane D_2 is spanned by X^0, X^1, X^6, X^7, X^8. The string ends then at the D_2.

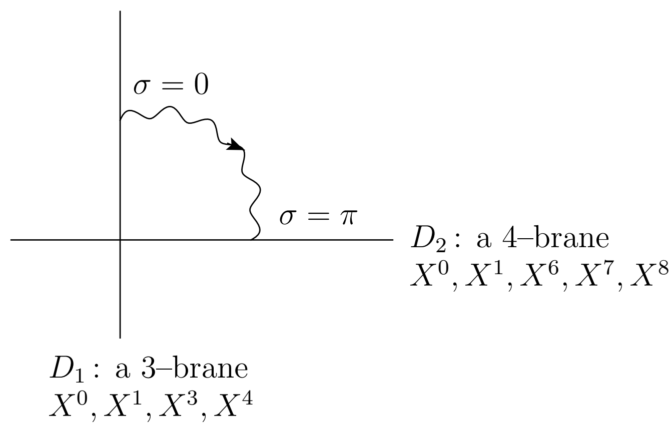

Figure 1: The endpoint with $\sigma = 0$ lies on D_1 (it can not move in the $\mu = 2, 5, \cdots, 9$ directions), and the endpoint with $\sigma = \pi$ lies on D_2 (it can not move in the $\mu = 2, \cdots, 5, 9$ directions).

It is clear from the figure that the boundary conditions for the various values of μ are of the following type:

$X^0:$ NN	$X^3:$ ND	$X^6:$ DN	$X^9:$ DD
$X^1:$ NN	$X^4:$ ND	$X^7:$ DN	
$X^2:$ DD	$X^5:$ DD	$X^8:$ DN	

(a) Write down the mode expansion for the three new cases DD, ND and DN. *Hint:* The field equation allows terms of the form $a + bt + c\sigma + de^{-in(t+\sigma)} + fe^{-in(t-\sigma)}$.

(b) Quantize the string for each of these cases. *Hint:* Decompose X^{μ_0} into a background part that depends on α^μ and a quantum part, and quantize the latter.