Homework 1

Due on: Monday, February 10

Problem 1

Using canonical quantization for the following action

$$S = (-2)\int \left[ik(b_{++\,++})(\partial_{=}c^{++}) + \kappa(\beta_{+++})(\partial_{=}i\gamma^{+}) + ik(b_{=}{}^{++})(\partial_{++}c^{=}) + \kappa(\beta_{-}{}^{++})(\partial_{++}i\gamma^{-})\right] d\sigma dt \quad (1.1)$$

with real b, c, γ but imaginary β, show that $k = \kappa = -\frac{1}{2\pi}$. The prefactor (-2) is due to $b_{++}{}^{=} = -2b_{++\,++}$ and $\beta_{+}{}^{=} = -2\beta_{+++}$. Define

$$\begin{aligned} b_{++\,++} &= \sum b_n e^{-in(t+\sigma)}\,; \qquad \beta_{+++} = \sum \beta_n e^{-in(t+\sigma)} \\ c^{++} &= \sum c_n e^{-in(t+\sigma)}\,; \qquad \gamma^{+} = \sum \gamma_n e^{-in(t+\sigma)} \end{aligned} \quad (1.2)$$

and impose $\{c_m, b_n\} = \hbar\delta_{m+n,0}$ and $[\gamma_m, \beta_n] = \hbar\delta_{m+n,0}$ with $c_m^\dagger = c_{-m}$, $b_n^\dagger = b_{-n}$, $\gamma_m^\dagger = \gamma_{-m}$ but $\beta_m^\dagger = -\beta_{-m}$.

Problem 2

Show that $\bar{\psi}\cdot\rho^\alpha D_\alpha(\omega)\psi = \bar{\psi}\cdot\rho^\alpha\partial_\alpha\psi$ for Majorana spinors ψ.

Problem 3

Show that $\mathcal{L}_{cl}$ in the quantum action after integrating out fields and chosing the gauges discussed in class reduces to

$$\mathcal{L}_{cl} = T\left[2\partial_{++}X\cdot\partial_{=}X + \frac{i}{2}\left(\psi_{+}\cdot\partial_{=}\psi_{+} + \psi_{-}\cdot\partial_{++}\psi_{-}\right)\right] \quad (3.1)$$

where $T = \frac{1}{\pi l^2}$, $\eta^{++=} = -2$, $\partial_{++} = \frac{1}{2}(\partial_t + \partial_\sigma)$, $\partial_{=} = \frac{1}{2}(\partial_t - \partial_\sigma)$ and $\psi = \begin{pmatrix}\psi^{+}\\ \psi^{-}\end{pmatrix} = \begin{pmatrix}\psi_{-}\\ -\psi_{+}\end{pmatrix}$.

Derive the anticommutators

$$\{\psi_{+}^{\mu}(\sigma,t), \psi_{+}^{\nu}(\sigma',t)\} = \{\psi^{\mu}(\sigma,t), \psi^{\nu}(\sigma',t)\} = 2l^2\pi\hbar\eta^{\mu\nu}\delta(\sigma-\sigma')\,. \quad (3.2)$$