Introduction to Supersymmetry and Supergravity

Homework 1

January 23, 2019
Due: February 1

1.

(a) Calculate the following (anti) commutators:

$$\{\Gamma^{\mu_1\cdots\mu_m}, \Gamma^{\nu_1\cdots\nu_n}\} \qquad \text{and} \qquad [\Gamma^{\mu_1\cdots\mu_m}, \Gamma^{\nu_1\cdots\nu_n}]$$

for $(m, n) = (1, n)$ and $(3, 2)$.

(b) Determine c_n and d_n in the following products:

$$\Gamma^{\mu_1\cdots\mu_n}\Gamma_\mu\Gamma_{\mu_1\cdots\mu_n} = c_n\Gamma_\mu\ , \qquad \text{and} \qquad \Gamma^\mu\Gamma_{\mu_1\cdots\mu_n}\Gamma_\mu = d_n\Gamma_{\mu_1\cdots\mu_n}$$

2.

(a) Determine $c_1, ..., c_5$ in

$$\Gamma^\mu\Gamma^{\rho\sigma}\Gamma_{\mu\nu} = c_1\Gamma_\nu{}^{\rho\sigma} + c_2\delta_\nu^{[\rho}\Gamma^{\sigma]}\ , \quad \text{and} \quad \Gamma_{[\mu}\Gamma^{\rho\sigma}\Gamma_{\nu]} = c_3\Gamma_{\mu\nu}{}^{\rho\sigma} + c_4\delta_{[\mu}^{[\rho}\Gamma_{\nu]}{}^{\sigma]} + c_5\delta_{[\mu}^{[\rho}\delta_{\nu]}^{\sigma]}$$

(b) Given that ψ_a is an anticommuting vector-spinor, and χ is a spinor, which one of the following expressions vanish?

$$\bar{\psi}_a\Gamma^c\psi_b\ , \qquad \bar{\psi}_a\Gamma^{abc}\psi_b\ , \qquad \bar{\psi}_c\Gamma^{a_1\ldots a_5}\psi_c\ , \qquad \bar{\psi}_a\Gamma^a\chi - \bar{\chi}\Gamma^a\psi_a$$

3.

(a) Show that $(\Gamma^\mu)_\alpha{}^\beta$ is invariant under Lorentz transformations.

(b) Show that $\bar{\psi}\psi$, where $\bar{\psi} \equiv \psi^\dagger A$, is invariant under Lorentz transformations.

(c) Show that $B^{-1}\psi$ also transforms as a spinor under Lorentz transformations, just as ψ does.

4. Starting from the Dirac equation $(i\Gamma^\mu\partial_\mu - m)\psi = 0$, write the Dirac spinor as sum of Weyl spinors as $\psi = \psi_L + \psi_R$, and determine the field equations for ψ_L and ψ_R separately, using the van der Wardeen symbols. Show that if ψ_L or ψ_R is vanishing, then the the mass m must vanish as well.

5.

Find a representation for the Γ^μ, A, B and C matrices in $(1,4)$ and $(1,5)$ dimensions. In the latter case, let Γ^7 be diagonal, and determine also the chirally projected Γ matrices in this case.

6.

(a) Write down all possible generators on the right hand side of the anti-commutator $\{Q_\alpha, Q_\beta\}$, in $(1,3)$ and $(1,7)$ dimensions. Take Q_α to be Majorana in $(1,3)$ and pseudo-Majorana in $(1,7)$ dimensions.

(b) Repeat the exercise for $\{Q^i_\alpha, Q^j_\beta\}$ in $(1,5)$ dimensions, where $i,j = 1,2$ are the $Sp(1)$ doublet indices, and Q^i_α is symplectic Majorana-Weyl. Use chirally projected Γ-matrices so as to avoid the chiral projection matrices in the algebra.