**Introduction to Supersymmetry and Supergravity**

**Homework 4**

March 20, 2019
Due: April 1

**1.** Consider the following action for a sigma model in $D = 1$ with $N = 2$ supersymmetry:

$$I = \int dt\, d^2\theta\, \frac{1}{2}\, \epsilon_{AB}\, D_A\Phi^i\, D_B\Phi^j\, g_{ij}(\Phi)\ ,$$

where $D_A = \frac{\partial}{\partial\theta^A} + i\theta_A\partial_t$ and

$$\Phi^i = \phi^i + \theta^A\psi^{iA} + \frac{i}{2}\epsilon^{AB}\theta^A\theta^B F^i\ , \quad A = 1,2, \quad i = 1, ..., n.$$

$\phi^i$ is real and $g_{ij}(\phi)$ is the metric on the sigma model manifold. Calculate the component form of the action and supersymmetry transformations. Eliminating the auxiliary field $F^i$, show that the action takes the form:

$$I = \int dt \left(\frac{1}{2}\partial_t\phi^i\partial_t\phi^j g_{ij}(\phi) + \frac{i}{2}\psi^{iA}D_t\psi^{jA}g_{ij}(\phi) + \frac{1}{4}R_{ikj\,l}\,\psi^{i1}\psi^{j2}\psi^{k1}\psi^{\ell\,2}\right)\ ,$$

where $D_t\psi^{jA} = \partial_t\psi^{jA} + \Gamma^j_{mn}\partial_t\phi^m\psi^{nA}$ and the supersymmetry transformations take the form

$$\delta\phi^i = \epsilon^A\psi^{iA}\ , \quad \delta\psi^{iA} = i\partial_t\phi^i\epsilon^A - \Gamma^i_{jk}\delta\phi^j\psi^k\ .$$

**2.** A bosonic Wess-Zumino action in one dimension is given by

$$I = \int d\tau\ \partial_\tau\phi^i A_i\ ,$$

where $A_i(\phi)$ is a vector field in the target space and $\phi^i$ $(i = 1, ..., n)$ are real.

(a) Supersymmetrize this action with respect to the following transformations

$$\delta\phi^i = \epsilon\psi^i\ , \qquad \delta\psi^i = \partial_\tau\phi^i\epsilon\ ,$$

where $\epsilon$ is a constant anti-commuting transformation parameter, and $\psi^i$ are one component real fermionic variables that are the superpartners of $\phi^i$. Show that the supersymmetry transformations form a closed algebra. (Note that $\partial_\tau\phi^i\partial_i = \partial_\tau$, where $\partial_i := \partial/\partial\phi^i$.)

(b) Show that in order the action constructed in (a) to be invariant under the infinitesimal transformations

$$\delta\phi^i = \beta^{(r)}V^{i(r)}(\phi)\ , \qquad \delta\psi^i = \beta^{(r)}\partial_j V^{i(r)}\psi^j\ ,$$

where $\beta^{(r)}$ are constant parameters and $V^{i(r)}(\phi)$ are a set of Killing vectors of the scalar manifold forming the Lie algebra of a group $G$, there must exist functions $C^r(\phi)$ which satisfy the condition $V^{j(r)}F_{ji} = \partial_i C^{(r)}$ , where $F_{ij} = (\partial_i A_j - \partial_j A_i)$.