PHYS 653 - Homework 1

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PROBLEM 1

Throughout this problem, I use the identity

$$\Gamma_{\mu_1\ldots\mu_m}\Gamma^{\nu_1\ldots\nu_n} = \Gamma_{\mu_1\ldots\mu_m}{}^{\nu_1\ldots\nu_n} + mn\Gamma_{[\mu_1\ldots\mu_{m-1}}{}^{[\nu_2\ldots\nu_n}\eta^{\nu_1]}{}_{\mu_m]} + \frac{nm(n-1)(m-1)}{2!}\Gamma_{[\mu_1\ldots\mu_{m-2}}{}^{[\nu_3\ldots\nu_n} + \eta_{\mu_{m-1}}{}^{\nu_2}\eta^{\nu_1]}{}_{\mu_m]} + \ldots \tag{1}$$

Part (a)

$$\{\Gamma_\mu, \Gamma^{\nu_1\ldots\nu_n}\} = \Gamma_\mu\Gamma^{\nu_1\ldots\nu_n} + \Gamma^{\nu_1\ldots\nu_n}\Gamma_\mu = \Gamma_\mu{}^{\nu_1\ldots\nu_n} + \Gamma^{\nu_1\ldots\nu_n}{}_\mu + n\left(\Gamma^{[\nu_2\ldots\nu_n}\eta_\mu^{\nu_1]} + \Gamma^{[\nu_1\ldots\nu_{n-1}}\eta_\mu^{\nu_n]}\right) \tag{2}$$

Now we want to permute the indices to make the terms resemble each other. In the second term, notice that when we permute μ through n indices, since Γ is totally antisymmetric, this will pick up a minus sign when n is odd, resulting in the cancellation of the first two terms. Similarly, the third and fourth terms cancel when n is even. This gives us the result

$$\{\Gamma_\mu, \Gamma^{\nu_1\ldots\nu_n}\} = \left(1 + (-1)^n\right)\Gamma_\mu{}^{\nu_1\ldots\nu_n} + n\left(1 - (-1)^n\right)\Gamma^{[\nu_1\ldots\nu_{n-1}}\eta_\mu^{\nu_n]} \tag{3}$$

When we calculate the commutator of these quantities, we flip the signs between the first and second terms and between the third and fourth terms. This results in

$$[\Gamma_\mu, \Gamma^{\nu_1\ldots\nu_n}] = \left(1 - (-1)^n\right)\Gamma_\mu{}^{\nu_1\ldots\nu_n} - n\left(1 + (-1)^n\right)\Gamma^{[\nu_1\ldots\nu_{n-1}}\eta_\mu^{\nu_n]} \tag{4}$$

$$\{\Gamma_{abc}, \Gamma^{cd}\} = \Gamma_{abc}{}^{de} + 6\Gamma_{[ab}{}^{[e}\eta_{c]}^{d]} + 6\Gamma_{[a}\eta_b^{[e}\eta_{c]}^{d]} + \Gamma^{de}{}_{abc} + 6\Gamma^{[d}{}_{[bc}\eta_{a]}^{e]} + 6\Gamma_{[c}\eta_b^{[d}\eta_{a]}^{e]} \tag{5}$$

After considering the antisymmetrization, we find that the second and fifth terms cancel. This gives

$$\{\Gamma_{abc}, \Gamma^{de}\} = 2\Gamma_{abc}{}^{de} + 12\Gamma_{[c}\eta_b^{[d}\eta_{a]}^{e]} \tag{6}$$

When calculating the commutator, the sign change results in the others remaining, which gives the result

$$[\Gamma_{abc}, \Gamma^{de}] = 12\Gamma_{[ab}{}^{[e}\eta_{c]}^{d]} \tag{7}$$

Part (b)

$$\Gamma^{\mu_1\ldots\mu_n}\Gamma_\mu\Gamma_{\mu_1\ldots\mu_n} = \Gamma^{\mu_1\ldots\mu_n}\left(\Gamma_{\mu_1\ldots\mu_n}\Gamma_\mu + \left[\Gamma_\mu, \Gamma^{\mu_1\ldots\mu_n}\right]\right) \tag{8}$$

$$= \Gamma^{\mu_1\ldots\mu_n}\Gamma_{\mu_1\ldots\mu_n}\Gamma_\mu + \Gamma^{\mu_1\ldots\mu_n}\left[\left(1 - (-1)^n\right)\Gamma_\mu{}^{\nu_1\ldots\nu_n} - n\left(1 + (-1)^n\right)\Gamma_{[\mu_1\ldots\mu_{n-1}}\eta_{\mu_n]\mu}\right] \tag{9}$$

$$= \left(\Gamma_\mu + \left(1-(-1)^n\right)\right)\Gamma^{\mu_1\ldots\mu_n}\Gamma_{\mu_1\ldots\mu_n} - n\left(1+(-1)^n\right)\Gamma^{\mu_1\ldots\mu_n}\Gamma_{[\mu_1\ldots\mu_{n-1}}\eta_{\mu_n]\mu} \tag{10}$$

$$= \left(\Gamma_\mu + \left(1-(-1)^n\right)\right)(-1)^{n(n-1)/2}\eta^{[\mu_1}_{[\mu_1}\ldots\eta^{\mu_n]}_{\mu_n]} - n\left(1+(-1)^n\right)\Gamma^{\mu_1\ldots\mu_n}\Gamma_{[\mu_1\ldots\mu_{n-1}}\eta_{\mu_n]\mu} \tag{11}$$

The second term vanishes by symmetry, and with some help from a computer to simplify the mess, we find

$$= \left[(-1)^{n(n-1)/2}\frac{1}{n!}\sum_{k=1}^{n}\left(A_k D^k\right)\right]\Gamma_\mu \tag{12}$$

where A_k is the k^{th} signed Stirling number of the first kind.

$$\Gamma^\mu\Gamma_{\mu_1\ldots\mu_n}\Gamma_\mu = \Gamma^\mu\left(\Gamma_\mu\Gamma_{\mu_1\ldots\mu_n} - [\Gamma_\mu,\Gamma_{\mu_1\ldots\mu_n}]\right) \tag{13}$$

$$= D\Gamma_{\mu_1\ldots\mu_n} - \Gamma^\mu\left[\left(1-(-1)^n\right)\Gamma_{\mu_1\ldots\mu_n} - n\left(1+(-1)^n\right)\Gamma_{[\mu_1\ldots\mu_{n-1}}\eta_{\mu_n]\mu}\right] \tag{14}$$

$$= D\Gamma_{\mu_1\ldots\mu_n} - \left(1-(-1)^n\right)\Gamma_{[\mu_1\ldots\mu_n}\eta^\mu_{\mu]} + n\left(1+(-1)^n\right)\Gamma^\mu\Gamma_{[\mu_1\ldots\mu_{n-1}}\eta_{\mu_n]\mu} \tag{15}$$

$$= D\Gamma_{\mu_1\ldots\mu_n} - \left(1-(-1)^n\right)\left(D-n(-1)^n\right)\Gamma_{\mu_1\ldots\mu_n} + n\left(1+(-1)^n\right)\Gamma_{\mu_1\ldots\mu_n} \tag{16}$$

$$= (-1)^n(D-2n)\Gamma_{\mu_1\ldots\mu_n} \tag{17}$$

PROBLEM 2

Part (a)

$$\Gamma^\mu\Gamma^{\rho\sigma}\Gamma_{\mu\nu} = \Gamma^\mu\left(\Gamma^{\rho\sigma}{}_{\mu\nu} + 4\Gamma^{[\rho}{}_{[\nu}\eta^{\sigma]}{}_{\mu]} + 2\eta^{[\rho}{}_{[\nu}\eta^{\sigma]}{}_{\mu]}\right) \tag{18}$$

$$= \Gamma^\mu\Gamma_{\rho\sigma\mu\nu} + \Gamma^\sigma\Gamma^\rho{}_\nu - \Gamma^\mu\Gamma^\rho{}_\mu\eta^\sigma_\nu - \Gamma^\rho\Gamma^\sigma{}_\nu + \Gamma^\mu\Gamma^\sigma{}_\mu\eta^\rho_\nu + \Gamma^\sigma\eta^\rho_\nu - \Gamma^\rho\eta^\sigma_\nu \tag{19}$$

$$= 4\Gamma_{[\sigma\mu\nu}\eta^\mu_{\rho]} + 2\Gamma^\sigma{}_{\rho\nu} - (3-d)\Gamma_\rho\eta^\sigma_\nu + (3-d)\Gamma_\sigma\eta^\rho_\nu \tag{20}$$

$$= (d-5)\Gamma_\nu{}^{\rho\sigma} + (d-3)\Gamma^{[\rho}\delta^{\sigma]}_\nu \tag{21}$$

$$\Gamma_{[\mu}\Gamma^{\rho\sigma}\Gamma_{\nu]} = \frac{1}{2}\left(\Gamma_\mu{}^{\rho\sigma} + 2\Gamma^{[\sigma}\eta^{\rho]}_\mu\right)\Gamma_\nu - \frac{1}{2}\Gamma_\nu\left(\Gamma^{\rho\sigma}{}_\mu + 2\Gamma^{[\rho}\eta^{\sigma]}_\mu\right) \tag{22}$$

$$= \frac{1}{2}\left(\Gamma^{\mu\rho\sigma}{}_{\nu} + 3\Gamma^{[\mu\rho}\eta^{\sigma]}_{\nu} + 2\Gamma^{[\sigma}\eta^{\rho]}_{\mu}\Gamma_{\nu}\right) - \frac{1}{2}\left(\Gamma_{\nu}{}^{\rho\sigma\mu} + 3\Gamma^{[\sigma\mu}\eta^{\rho]}_{\nu} + 2\Gamma_{\nu}\Gamma^{[\rho}\eta^{\sigma]}_{\mu}\right) \tag{23}$$

The terms with coefficient 3/2 cancel and we get

$$= \Gamma_{\mu}{}^{\rho\sigma}{}_{\nu} + \frac{1}{2}\left(\eta^{\mu}_{\rho}\eta^{\sigma}_{\nu} - \eta^{\mu}_{\sigma}\eta^{\rho}_{\nu} - \eta^{\nu}_{\rho}\eta^{\sigma}_{\mu} + \eta^{\sigma}_{\nu}\eta^{\rho}_{\mu}\right) = \Gamma_{\mu\nu}{}^{\rho\sigma} + 2\eta^{[\rho}_{[\mu}\eta^{\sigma]}_{\nu]} \tag{24}$$

Part (b)

Here, I use the Majorana flip conventions of Freedman and Van Proeyen the values t_i.

$$\bar{\psi}_a\Gamma^{abc}\psi_b = t_3\bar{\psi}_b\Gamma^{abc}\psi_a = -t_3\bar{\psi}_a\Gamma^{abc}\psi_b = \epsilon\eta\bar{\psi}_a\Gamma^{abc}\psi_b \tag{25}$$

so this vanishes when $\epsilon\eta = -1$.

$$\bar{\psi}_c\Gamma^{a_1\ldots a_5}\psi_c = t_1\bar{\psi}_c\Gamma^{a_1\ldots a_5}\psi_c = -\epsilon\eta\bar{\psi}_c\Gamma^{a_1\ldots a_5}\psi_c \tag{26}$$

so this vanishes when $\epsilon\eta = +1$.

$$\bar{\psi}_a\Gamma^{a}\chi - \bar{\chi}\Gamma^{a}\psi_a = t_1\bar{\chi}\Gamma^{a}\psi_a - \bar{\chi}\Gamma^{a}\psi_a = -\epsilon\eta\bar{\chi}\Gamma^{a}\psi_a - \bar{\chi}\Gamma^{a}\psi_a \tag{27}$$

so this vanishes when $\epsilon\eta = -1$.

The first expression does not have the necessary symmetry properties to vanish.

PROBLEM 3

Part (a)

Considering $(\Gamma^{\mu})_{\alpha}{}^{\beta}$, we see that with two spinor indices and a Lorentz index, it must transform as

$$\delta\Gamma^{\mu} = \frac{1}{2}\Lambda^{\rho\sigma}\left(M_{\rho\sigma}\right)^{\mu\nu}\Gamma_{\nu} + \frac{1}{2}\Lambda^{\rho\sigma}\Gamma_{\mu}\Gamma_{\rho\sigma} - \frac{1}{2}\Lambda^{\rho\sigma}\Gamma_{\rho\sigma}\Gamma_{\mu} \tag{28}$$

But with the results from Question 1, we can rewrite this as

$$\frac{1}{2}\Lambda^{\rho\sigma}\left(M_{\rho\sigma}\right)^{\mu\nu}\Gamma_{\nu} + \frac{1}{2}\Lambda^{\rho\sigma}\Gamma_{\mu}\Gamma_{\rho\sigma} - \frac{1}{2}\Lambda^{\rho\sigma}\Gamma_{\mu}\Gamma_{\rho\sigma} - 4\Lambda^{\rho\sigma}\Gamma_{[\rho}\eta_{\sigma]\mu} \tag{29}$$

$$= \frac{1}{2}\Lambda^{\rho\sigma}\left(M_{\rho\sigma}\right)^{\mu\nu}\Gamma_{\nu} \tag{30}$$

which is the standard Lorentz transformation.

Part (b)

$$\delta(\bar{\psi}\psi) = (\delta\bar{\psi})\psi + \bar{\psi}\delta\psi = \left(-\frac{1}{4}\bar{\psi}\Gamma^{\mu\nu}\Lambda_{\mu\nu}\right)\psi + \bar{\psi}\left(\frac{1}{4}\Gamma^{\mu\nu}\Lambda_{\mu\nu}\psi\right) = 0 \tag{31}$$

Part (c)

B^{-1} does not change under a Lorentz transformation, so

$$\delta(B^{-1}\psi) = (\delta B^{-1})\psi + B^{-1}\delta\psi = B^{-1}\delta\psi = \frac{1}{4}\Gamma^{\mu\nu}\Lambda_{\mu\nu}B^{-1}\psi \tag{32}$$

PROBLEM 4

When our gamma matrices are chirally projected, we can write (in the Weyl representation)

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{33}$$

Then, the Dirac equation becomes

$$i\begin{pmatrix} -m & \sigma^\mu\partial_\mu \\ \bar{\sigma}^\mu\partial_\mu & -m \end{pmatrix}\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0 \tag{34}$$

which splits into two equations:

$$i\sigma^\mu\partial_\mu\psi_R - m\psi_L = 0 \qquad i\bar{\sigma}^\mu\partial_\mu\psi_L - m\psi_R = 0 \tag{35}$$

Here, we see that if one of ψ_L or ψ_R vanishes, then for the entire left side of each equation to vanish, either the other ψ must vanish, or m must vanish. Substituting ψ_L into ψ_R (and vice-versa), we find

$$(\bar{\sigma}^\mu\sigma^\nu\partial_\mu\partial_\nu + m^2)\psi_R = (\partial^2 + m^2)\psi_R = 0 \tag{36}$$

$$(\bar{\sigma}^\mu\sigma^\nu\partial_\mu\partial_\nu + m^2)\psi_L = (\partial^2 + m^2)\psi_L = 0 \tag{37}$$

PROBLEM 5

As in Freedman and Van Proeyen, we can construct a representation for Lorentzian gamma matrices as follows:

$$\begin{aligned}
\gamma^0 &= i\sigma_1 \otimes \mathbb{1} \otimes \dots \\
\gamma^1 &= \sigma_2 \otimes \mathbb{1} \otimes \dots \\
\gamma^2 &= \sigma_3 \otimes \sigma_1 \otimes \dots \\
\gamma^3 &= \sigma_3 \otimes \sigma_2 \otimes \dots \\
&\vdots
\end{aligned} \tag{38}$$

We create the highest-rank element from a product of all other elements (in even dimensions):

$$\gamma^{D+1} = (-1)^{(s-t)/4}\gamma^0\gamma^1\dots\gamma^{D-1}$$

We define the matrix A as the product of all "timelike" gammas. Such matrices are anti-hermitian. We also define an invertible matrix B that satisfies

$$\gamma_\mu^* = \eta B \gamma_\mu B^{-1} \tag{39}$$

and a matrix C that satisfies

$$\gamma_\mu^T = (-1)^t \eta C \gamma_\mu C^{-1} \tag{40}$$

Alternatively, C can be defined as $C = BA$.
In $D = 4+1$, this gives us

$$\begin{aligned}
\gamma^0 &= i\sigma_1 \otimes \mathbb{1}_2 \\
\gamma^1 &= \sigma_2 \otimes \mathbb{1}_2 \\
\gamma^2 &= \sigma_3 \otimes \sigma_1 \\
\gamma^3 &= \sigma_3 \otimes \sigma_2 \\
\gamma^4 &= \sigma_3 \otimes \sigma_3
\end{aligned} \tag{41}$$

$$A = \gamma^0 \tag{42}$$

After a side calculation, (one possible) C is found to be $C = \gamma^0\gamma^2\gamma^4$. Using $C = BA$, we find $B = -\gamma^0\gamma^2\gamma^4\gamma^0$.

In $D = 5+1$, we have

$$\begin{aligned}
\gamma^0 &= i\sigma_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \\
\gamma^1 &= \sigma_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \\
\gamma^2 &= \sigma_3 \otimes \sigma_1 \otimes \mathbb{1}_2 \\
\gamma^3 &= \sigma_3 \otimes \sigma_2 \otimes \mathbb{1}_2 \\
\gamma^4 &= \sigma_3 \otimes \sigma_3 \otimes \sigma_1 \\
\gamma^5 &= \sigma_3 \otimes \sigma_3 \otimes \sigma_2
\end{aligned} \tag{43}$$

$$\gamma^7 = -i\sigma_1\sigma_2 \otimes \sigma_1\sigma_2 \otimes \sigma_1\sigma_2 = -\sigma_3 \otimes \sigma_3 \otimes \sigma_3$$

Using this construction, γ^7 is diagonal as required, as it is a product of diagonal sub-matrices. A side calculation gives $A = \gamma^0$, $C = \gamma^0\gamma^2\gamma^4$ and $B = -\gamma^0\gamma^2\gamma^4\gamma^0$.
The chirally projected gammas are defined as

$$\gamma^\mu_- = \frac{1}{2}(\mathbb{1} - \gamma^7)\gamma^\mu \frac{1}{2}(\mathbb{1} + \gamma^7) \qquad \gamma^\mu_+ = \frac{1}{2}(\mathbb{1} + \gamma^7)\gamma^\mu \frac{1}{2}(\mathbb{1} - \gamma^7) \tag{44}$$

A computer calculation shows that

$$\gamma_-^0 = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&i&0&0\\ 0&0&0&0&0&0&i&0\\ 0&0&0&0&0&0&0&0\\ i&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&i&0&0&0&0 \end{pmatrix} \quad \gamma_-^1 = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&-i&0&0\\ 0&0&0&0&0&0&-i&0\\ 0&0&0&0&0&0&0&0\\ i&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&i&0&0&0&0 \end{pmatrix} \quad \gamma_-^2 = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&-1&0&0 \end{pmatrix}$$

$$\gamma_-^3 = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&-i&0&0&0&0\\ i&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&i&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&-i&0&0 \end{pmatrix} \quad \gamma_-^4 = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0 \end{pmatrix} \quad \gamma_-^5 = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ i&0&0&0&0&0&0&0\\ 0&0&0&i&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&i&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&i&0 \end{pmatrix}$$

$$\gamma_+^0 = \begin{pmatrix} 0&0&0&0&i&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&i\\ 0&0&0&0&0&0&0&0\\ 0&i&0&0&0&0&0&0\\ 0&0&i&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \gamma_+^1 = \begin{pmatrix} 0&0&0&0&-i&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&-i\\ 0&0&0&0&0&0&0&0\\ 0&i&0&0&0&0&0&0\\ 0&0&i&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \gamma_+^2 = \begin{pmatrix} 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$\gamma_+^3 = \begin{pmatrix} 0&0&-i&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&i&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&i\\ 0&0&0&0&-i&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \gamma_+^4 = \begin{pmatrix} 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \gamma_+^5 = \begin{pmatrix} 0&-i&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&-i&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&-i&0&0&0\\ 0&0&0&0&0&0&0&-i\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

PROBLEM 6

Part (a)

First, we need to determine the symmetry properties of n-rank gamma matrices in $(1,3)$ and $(1,7)$ dimensions from

$$\left(\Gamma_{\mu_1 \ldots \mu_n} C^{-1}\right)^T = \epsilon \eta^{t+n} (-1)^{(t-n)(t-n+1)/2} \Gamma_{\mu_1 \ldots \mu_n} C^{-1} \tag{45}$$

For a Majorana spinor in $D = (1,3)$, $\epsilon = \eta = 1$, and for a pseudo-Majorana spinor in $D = (1,7)$, $\epsilon = +1$, $\eta = -1$.

D \ Rank	1	2	3	4	5	6	7	8
$(1,3)$	S	S	A	A	x	x	x	x
$(1,7)$	S	A	A	S	S	A	A	S

In $D=(1,3)$, the given anticommutator is then (up to potential normalization factors)

$$\{Q_\alpha, Q_\beta\} = \left(\Gamma^\mu C^{-1}\right)_{\alpha\beta} P_\mu + \left(\Gamma^{\mu\nu} C^{-1}\right)_{\alpha\beta} A_{\mu\nu} \tag{46}$$

In $D=(1,7)$,

$$\{Q_\alpha, Q_\beta\} = \left(\Gamma^\mu C^{-1}\right)_{\alpha\beta} P_\mu + \left(\Gamma^{\mu\nu\rho\sigma} C^{-1}\right)_{\alpha\beta} A_{\mu\nu\rho\sigma} + \left(\Gamma^{\mu\nu\rho\sigma\lambda} C^{-1}\right)_{\alpha\beta} B_{\mu\nu\rho\sigma\lambda} \tag{47}$$

Part (b)

For $D=(1,5)$ symplectic Majorana spinors, we have $\epsilon=-1$, $\eta=+1$. This gives

Rank	1	2	3	4	5	6
Parity	A	A	S	S	A	A

Since we have the antisymmetric matrix Ω_{ij}, we want to use the antisymmetric entries.
This gives

$$\{Q^i_\alpha, Q^j_\beta\} = 2\Omega^{ij}\sigma^\mu_{\alpha\beta} + \sigma^{\mu\nu}_{\alpha\beta}A^{ij}_{\mu\nu} + \sigma^{\mu\nu\rho\sigma\lambda}_{\alpha\beta}B^{ij}_{\mu\nu\rho\sigma\lambda} + \sigma^{\mu\nu\rho\sigma\lambda\delta}_{\alpha\beta}C^{ij}_{\mu\nu\rho\sigma\lambda\delta} \tag{48}$$