Introduction to Supersymmetry and Supergravity

Homework 1

February 5, 2019
Due: February 15

1.

(a) Give the definition of the classical Lie algebras $SO(N)$, $USp(2N)$ and $SU(N)$.

(b) Specify the invariant tensors of these algebras.

(c) Specify the dimensions of the first three lowest lying irreps for each one of this algebras and represent them in terms of tensor/spinor fields, specifying all conditions on them such as reality, and when applicable, the symmetry/trace conditions.

2. Construct the smallest **massless** representation of the $\mathbf{D = 4, N = 6}$ Poincaré superalgebra. Indicate the little group content of the representation. Which Lorentz covariant fields do they correspond to?

3. Construct the smallest **massive** representation of the $\mathbf{D = 4, N = 4}$ Poincaré superalgebra **with central charges** $z_1 = z_2 = m$. (This is known as the *short multiplet*). Indicate the little group content of the representation. Which Lorentz covariant fields do they correspond to?

(*Hints:* Determine the representation content of the active creation operators under the $U(1) \times USp(4)$ subgroup of the little group. Build up the states, and then recombine them into irreps of the full little group).

4.

(a) Construct the smallest **massless** representation of the $\mathbf{D = 5, N = 8}$ Poincaré superalgebra. Indicate the little group content of the representation. Which Lorentz covariant fields do they correspond to?

(b) Taking the Clifford vacuum to be in the $(3,1)$ representation of the little group $SO(3) \times USp(8)$, determine the resulting multiplet. Which Lorentz covariant fields do they correspond to?

(*Hints:* The supercharges are symplectic Majorana, and in the 8 of $USp(8)$. Determine the representation content of the active creation operators under the $U(1) \times USp(8)$ subgroup of the little group. Build up the states, and then recombine them into irreps of the full little group).

5. Using the decompositions of the tensor products of the $SO(8)$ irreps $(8_+, 8_-, 8_v)$, determine smallest massless representations of the $(1,1)$ and $(2,0)$ Poincare superalgebras in $D = 10$. Specify the Lorentz covariant fields the multiplet of states belong to.