Homework 7

Sunday, 25 April 2021 10:13

2) Let P be a finite process.
Let $P \xrightarrow{\alpha} P'$

By induction on length of the inference for $P \xrightarrow{\alpha} P'$

IND HYP: $Act_{P'} \subseteq Act_P$

BASE CASE

$P = \sum_{i \in I} \alpha_i . P_i$, $Act_P = \bigcup_{i \in I} \{\alpha_i\} \cup \bigcup_{i \in I} Act_{P_i}$

For every P_i, $Act_{P_i} \subseteq Act_P \implies Act_{P'} \subseteq Act_P$ ✓

IND STEP:

2 CASES:

1) $P = Q \setminus a$, $Q \xrightarrow{\alpha} Q'$ and $\alpha \notin \{a, \bar{a}\}$

By IND HYP: $Act_{Q'} \subseteq Act_Q$

If we remove the restricted action a, for both sets, we still have that:

$$\underbrace{Act_{Q'} \setminus \{a, \bar{a}\}}_{Act_{P'}} \subseteq \underbrace{Act_Q \setminus \{a, \bar{a}\}}_{Act_P}$$ ✓

2) Let define $Act_{H_1|H_2} = Act_{H_1} \cup Act_{H_2}$

$P = P_1 | P_2$, here we have 4 cases to consider:

a) $P = P_1 | P_2$, $P_1 \xrightarrow{\alpha} P_1' \implies P' = P_1' | P_2$

By IND HYP: $Act_{P_1'} \subseteq Act_{P_1}$

It's also true that:

$$\underbrace{Act_{P_1'} \cup Act_{P_2}}_{Act_{P'}} \subseteq \underbrace{Act_{P_1} \cup Act_{P_2}}_{Act_P}$$ ✓

b) $P = P_1 | P_2$, $P_2 \xrightarrow{\alpha} P_2' \implies P' = P_1 | P_2'$

By IND HYP: $Act_{P_2'} \subseteq Act_{P_2}$

It's also true that:

$$\underbrace{Act_{P_1} \cup Act_{P_2'}}_{Act_{P'}} \subseteq \underbrace{Act_{P_1} \cup Act_{P_2}}_{Act_P}$$ ✓

c) $P = P_1 | P_2$, $P_1 \xrightarrow[①]{a} P_1'$ and $P_2 \xrightarrow[②]{\bar{a}} P_2' \implies P' = P_1' | P_2'$

By HYP IND: ① $Act_{P_1'} \subseteq Act_{P_1}$

By HYP IND: ② $Act_{P_2'} \subseteq Act_{P_2}$

$$\underbrace{Act_{P_1'} \cup Act_{P_2'}}_{Act_{P'}} \subseteq \underbrace{Act_{P_1} \cup Act_{P_2}}_{Act_P}$$ ✓

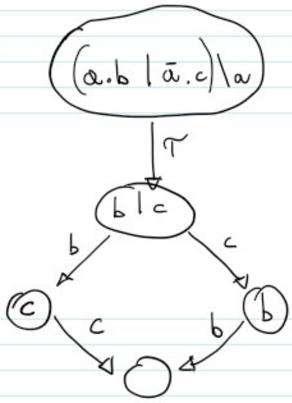
d) $P = P_1 | P_2$, $P_1 \xrightarrow[①]{\bar{a}} P_1'$ and $P_2 \xrightarrow[②]{a} P_2' \implies P' = P_1' | P_2'$

By HYP IND: ① $Act_{P_1'} \subseteq Act_{P_1}$

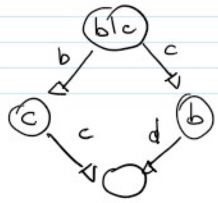
By HYP IND: ② $Act_{P_2'} \subseteq Act_{P_2}$

$$\underbrace{Act_{P_1'} \cup Act_{P_2'}}_{Act_{P'}} \subseteq \underbrace{Act_{P_1} \cup Act_{P_2}}_{Act_P}$$ ✓

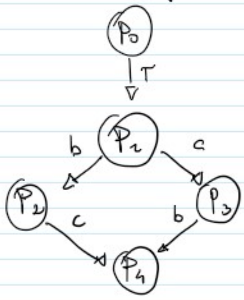
1) T =

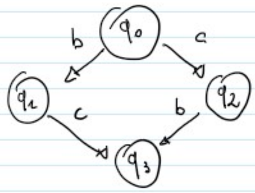


T' =



Relabeling:

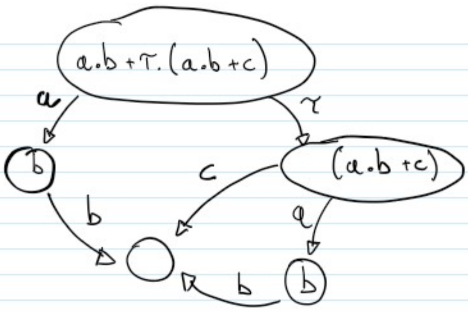




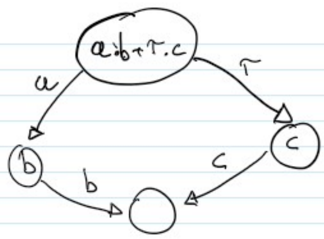
$S = \{(p_0,q_0),(p_1,q_0),(p_2,q_1),(p_3,q_2),(p_4,q_3)\}$

$S^{-1} = \{(q_0,p_0),(q_0,p_1),\underbrace{(q_1,p_2)}_{\stackrel{\hat{b}}{\Longrightarrow}},\ \underbrace{(q_2,p_3)}_{\stackrel{\hat{c}}{\Longrightarrow}},(q_3,p_4)\}$

4) T =



T' =



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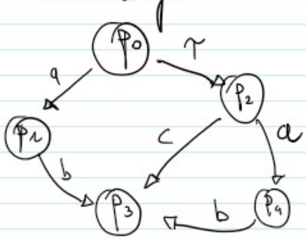

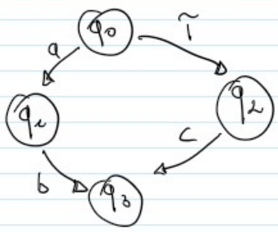

$S = \{(p_0,q_0),(p_2,q_0),(p_1,q_1),(p_3,q_3),(p_4,q_1)\}$

$S^{-1} = \{(q_0,p_0),(q_2,p_2),$

This isn't a weak bisimulation, the action $q_0 \xrightarrow{\tau} q_2$ has to be "replied" by T', BUT $(p_i,q_2) \notin S$!!

3) We have to show that $p \approx q$ and $q \approx r \implies p \approx r$ for all p, q, r

Let us consider the following relation:

$$S = \{ (x,z) : \exists y \text{ s.t. } (x,y) \in S_1 \land (y,z) \in S_2 \}$$

Where S_1 and S_2 are weakly bisim.

Let $(x,z) \in S$ and $x \xrightarrow{a} x'$

Since S_1 is weak bisim. there exists $y \stackrel{\hat{a}}{\Longrightarrow} y'$ s.t. $(x',y) \in S_1$

Since S_2 is weak bisim. there exists $z \stackrel{\hat{a}}{\Longrightarrow} z'$ s.t. $(y',z') \in S_2$

$x \xrightarrow{a} x'$ we found $z \stackrel{\hat{a}}{\Longrightarrow} z'$ s.t. $(x',z') \in S$ because there exists a y' s.t. $(x',y') \in S_1$ and $(y',z') \in S_2$