Homework 8

Friday, 7 May 2021 12:39

1)

$(a.0 \mid \bar{a}.0)\backslash a \sim \tau.0$

$\vdash (a.0 \mid \bar{a}.0)\backslash a =$

$\quad ((a.(0 \mid \bar{a}.0) + \bar{a}.\ a.0 \mid 0) + \tau.(0 \mid 0))\backslash a$ (Ax for parallel)

$\vdash (a.(0 \mid \bar{a}.0))\backslash a + (\bar{a}.(a.0 \mid 0)\backslash a) + \tau(0 \mid 0)\backslash a$ (Ax.2 for restriction)

$\vdash 0 + 0 + \tau.(0 \mid 0)$ (Ax.3. for restriction)

$\vdash \tau.\ (0 \mid 0)$ (x2 Ax.1 for sum)

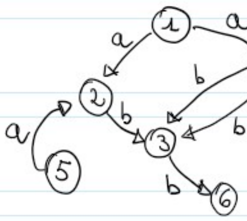
$\vdash (0 \mid 0) = 0 + 0$ (Ax for parallel)

$\vdash 0 + 0 = 0$ (Ax.1 for sum)

$\vdash \tau.0$ (By lemma 3.4, since $(0 \mid 0) \sim 0$)

2)

Assigned value	State
1	$a.b.b + a.(b \mid b)$
2	$b.b$
3	b
4	$b \mid b$
5	$a.b.b$
6	0



$P = \{\{1,2,3,4,5,6\}\}$

Choose a and $B = \{1,2,3,4,5,6\}$

$tgt_a(1) = tgt_a(5) = \{B\}$; $tgt_a(2) = tgt_a(3) = tgt_a(4) = tgt_a(6) = \emptyset$

$P = \{\{1,5\}, \{2,3,4,6\}\}$

choose b and $B = \{2,3,4,6\}$

$tgt_b(2) = tgt_b(3) = tgt_b(4) = \{B\}$; $tgt_6 = \emptyset$

$P = \{\{1,5\}, \{2,3,4\}, \{6\}\}$

choose b and $B = \{1,5\}$

$tgt_b(1) = tgt_b(5) = \emptyset$

We end with $P = \{\{1,5\}, \{2,3,4\}, \{6\}\}$

1 and 5 are in the same block $\Rightarrow a.b.b + a.(b \mid b) \sim a.b.b$

3)

$P_1 = \tau.b + \tau.(a+b)$ $\qquad P_2 = \tau.b + \tau.a$

$\varphi_1 = \Diamond\tau(\Diamond a\, TT \wedge \Diamond b\, TT)$ $\qquad \varphi_2 = \Diamond\tau\Box b\, FF$

$P_1 \models \varphi_1$ $\qquad P_1 \not\models \varphi_2$

$P_2 \not\models \varphi_1$ $\qquad P_2 \models \varphi_2$

4)

Recall that S is a weak simulation iff $\forall (p,q) \in S\ \forall p \xrightarrow{\alpha} p'\ \exists q'$ s.t. $q \stackrel{\hat{\alpha}}{\Longrightarrow} q'$ and $(p,q) \in S$

P is weakly simulated by Q if there exist a weak simulation S s.t. $(P,Q) \in S$

Prove that:

$$\mathcal{L}^w(P) \subseteq \mathcal{L}^w(Q) \Longrightarrow P \text{ is weakly simulated by } Q$$

Let define $R \triangleq \{ (P,Q) : \mathcal{L}^w(P) \subseteq \mathcal{L}^w(Q) \}$ weak simulation

$$\varphi \triangleq \Diamond a \varphi' \text{ where } \varphi' \triangleq \bigwedge_{\varphi'' \in \mathcal{L}^w(P')} \varphi''$$

By construction $P \models \varphi$, hence $Q \models \varphi$ $(\mathcal{L}^w(P) \subseteq \mathcal{L}^w(Q))$

then $\exists Q' : Q \stackrel{\hat{a}}{\Longrightarrow} Q'$ s.t. $Q' \models \varphi'$. Hence, $\forall \varphi'' \in \mathcal{L}^w(P), \varphi'' \in \mathcal{L}^w(Q) \Rightarrow \mathcal{L}^w(P') \subseteq \mathcal{L}^w(Q') \Longrightarrow P'$ is weakly simulated by Q'

$(P', Q') \in R$