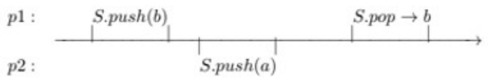
Homework 4

Saturday, 20 March 2021 11:47

Exercise 1. Let S be a stack and consider the history $\hat{H}$:



Recall

- A complete history $\hat{H}$ is linearizable if $\exists \hat{S}$ sequential s.t.

1) $\forall i .\ \hat{S}|X_i \in semantics(X_i)$
2) $\forall p_j .\ \hat{H}|p_j = \hat{S}|p_j$
3) $\rightarrow_H \subseteq \rightarrow_S$

- A complete history $\hat{H}$ is sequentially consistent if $\exists \hat{S}$ sequential s.t.

1) $\forall i .\ \hat{S}|X_i \in semantics(X_i)$
2) $\forall p_j .\ \hat{H}|p_j = \hat{S}|p_j$
3) $\rightarrow_{proc} \subseteq \rightarrow_S$

$\hat{H}$ is sequentially consistent:

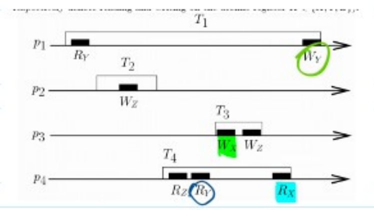
$\hat{S} = S.push(a) <_S S.push(b) <_S S.pop \rightarrow b$

$\hat{H}$ is not linearizable, since we have to ensure that:

$ret[S.push(b)] <_H inv[S.push(a)] <_H res[S.push(a)] <_H inv[S.pop() \rightarrow b]$ property ③

but it leads to conflict with ① (the stack follows a LIFO)

Exercise 2)



1) Is this scenario opaque? The presence of T_4 makes this history not opaque

T_4 has to be aborted because its R_X happens after W_X (by T_3)

read-pref $(T_4) = \{R_Z, R_Y\}$

It's not opaque: $T_1 < T_2 <$ read-prefix $(T_4) < T_3$

R_Y after W_Y

It is virtual consistent:

- total ord. on the commited tran's in $T_1 < T_2 < T_3$
- partial order in $T_2 <$ read-prefix (T_4)

Exercise 3)

Show that every linearizable history is also sequentially consistent.

Recall

- A complete history $\hat{H}$ is linearizable if $\exists\, \hat{S}$ sequential s.t.

1) $\forall i .\ \hat{S}|X_i \in semantics(X_i)$

2) $\forall p_j .\ \hat{H}|p_j = \hat{S}|p_j$

3) $\xrightarrow{}_H \subseteq \xrightarrow{}_S$

- A complete history $\hat{H}$ is sequentially consistent if $\exists\, \hat{S}$ sequential s.t.

1) $\forall i .\ \hat{S}|X_i \in semantics(X_i)$

2) $\forall p_j .\ \hat{H}|p_j = \hat{S}|p_j$

3) $\xrightarrow{}_{proc} \subseteq \xrightarrow{}_S$

Proof by the way of contradiction

1

$\hat{S}$ is the linearization of $\hat{H}$ and $\hat{S}$ is not the sequential consistency of $\hat{H}$

The only point which is different is 3) (otherwise it's a trivial contradiction)

That means: $\xrightarrow{}_{proc} \cancel{\subseteq} \xrightarrow{}_S \Longrightarrow$ there exists some $\xrightarrow{}_j \subseteq \xrightarrow{}_{proc}$ and $\xrightarrow{}_j \cancel{\subseteq} \xrightarrow{}_S \supseteq \xrightarrow{}_H$ by 3)

$\hat{H}|p_j = (\forall t .\ op_{p_j,t} \xrightarrow{}_j op_{p_j,t+1})$

the total order given by p_j

but $\xrightarrow{}_j \subseteq \xrightarrow{}_H$ by construction

this is a contradiction

To sum up:

$$\xrightarrow{}_{proc} \subseteq \xrightarrow{}_H \subseteq \xrightarrow{}_S \checkmark$$

Exercise 4)

$\hat{H}' = \hat{H}.\text{ret}[op(res) \text{ from } X \text{ to } p]$

We know that $\hat{S}$ is the linearization of $\hat{H}$

Let take $\hat{S} = event_1 <_S event_2 <_S inv[op(arg) \text{ on } X \text{ by } p] <_S event_n \ldots$

and we modify it, with the linearization $\hat{S}'$ of $\hat{H}'$

$\hat{S}' = event_1 <_{S'} event_2 <_{S'} inv[op(arg) \text{ on } X \text{ by } p] <_{S'} ret[op(res) \text{ from } X \text{ to } p] <_{S'} event_n \ldots$