Homework 9

Saturday, 22 May 2021 10:25

$$\dfrac{\dfrac{\text{S-CON}\quad [\mathit{True}_a \mid (\bar{a}\langle b,c\rangle.(b().P \mid c().Q)] \longmapsto \bar{b}\langle\rangle \mid b().P \mid c().Q}{\text{S-EXT}\quad (\nu b,c)[\bar{a}\langle b,c\rangle.(b().P \mid c().Q) \mid \mathit{True}_a] \longmapsto (\nu b)[b().P \mid c().Q \mid \bar{b}\langle\rangle]}}{\mathit{Test}(P;Q) \mid \mathit{True}_a \longmapsto (\nu b)[b().P \mid c().Q \mid \bar{b}\langle\rangle]}$$

$$\begin{tikzcd}
 & \mathit{Test}_a \mid \mathit{True}_a \mid \mathit{False}_a \arrow[dl, "\tau"'] \arrow[dr, "\tau"] & \\
(\nu b,c)\,(b() \mid c() \mid \bar{b}\langle\rangle) \mid \mathit{False}_a \arrow[d, "\tau"] & & (\nu b,c)\,(b() \mid c() \mid \bar{c}\langle\rangle) \mid \mathit{True}_a \arrow[d, "\tau"] \\
(\nu b,c)\,(c()) \mid \mathit{False}_a & & (\nu b,c)\,(b()) \mid \mathit{True}_a
\end{tikzcd}$$

2)

$T_w = [\,]$

$T_b = [[\,]]$

$T_x = [[[\,]]]$

$T_y = [[\,]]$

$T_a = [[[[\,]]], [[\,]]]$

$P = a(x,y).\,Q$

$Q = (\nu b)\,R$

$R = (\bar{x}\langle b\rangle \mid\, !y(w).\bar{b}\langle w\rangle)$

$$\dfrac{\Gamma(y) = T_y = [T_w] \qquad \dfrac{\Gamma(b) = T_b \qquad [\underline{T_w}] \qquad \Gamma(w) = T_w \qquad \Gamma \vdash 0}{\Gamma, w : T_w \vdash \bar{b}\langle w\rangle}}{\dfrac{\Gamma \vdash y(w).\bar{b}\langle w\rangle}{\Gamma \vdash\, !y(w).\bar{b}\langle w\rangle}}$$

$$\dfrac{\Gamma(a) = T_a \qquad \dfrac{\dfrac{\dfrac{\Gamma(x) = T_x = [T_b] \quad \Gamma(b) = T_b \qquad \Gamma \vdash 0}{\Gamma \vdash \bar{x}\langle b\rangle}}{\exists T_b : \Gamma, b : T_b \vdash R}}{\Gamma, x : T_x, y : T_y \vdash Q}}{\Gamma \vdash P}$$

3)

$$fn(0) = \emptyset$$
$$fn(P_1 | P_2) = fn(P_1) \cup fn(P_2)$$
$$fn(\bar{a}\langle b\rangle.P) = \{a, b\} \cup fn(P)$$
$$fn(a(x)P) = \{a\} \cup (fn(P) / \{x\})$$
$$fn(\nu a\, P) = fn(P) / \{a\}$$
$$fn(!P) = fn(P)$$
$$fn([a=a]P) = fn(P)$$

Proof that if $P \xrightarrow{\alpha} P$ then it may be possible that $fn(P') \not\subseteq fn(P)$

Let $\underbrace{(\nu b)\, \bar{a}\langle b\rangle . b(x)}_{P} \longrightarrow \underbrace{b(x)}_{P'}$

$$b \notin fn(P) \text{ and } b \in fn(P')$$

4)

$$\text{E-MISMATCH}\ \frac{}{a(\theta).P \mid \bar{a}\langle p\rangle.Q \uparrow} (\theta \in V \text{ and } p \in N) \text{ or } (p \in V \text{ and } \theta \in N)$$

$$\text{E-RES}\ \frac{P\uparrow}{(\nu a)P\uparrow} \qquad \text{E-PAR}\ \frac{P\uparrow}{P \mid Q\uparrow} \qquad \text{E-STRUCT}\ \frac{P\uparrow \quad P \equiv Q}{Q\uparrow}$$