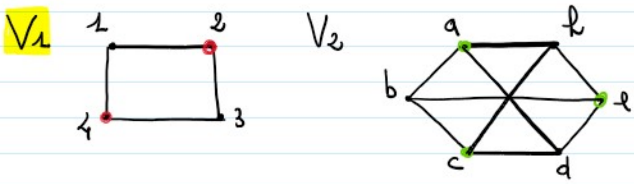
TaielloRiccardo1914000

domenica 12 aprile 2020 16:01

1.1)

Moove	Spoiler	Duplicator
1	Pick **a** in **V2**	Pick 2 in **V1**
2	Pick **c** in **V2**	Pick 4 in **V1**
3	Pick **e** in **V2**	LOSE



Duplicator doesn't win because after three mooves, because there aren't three distinct vertex in V1 s.t. there isn't an edge between them.

The following first order sentence of rank-3 witnesses the Spoiler's win:

$$\exists x_1 \exists x_2 \exists x_3 \left[x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge x_2 \neq x_3 \wedge \neg E(x_1,x_2) \wedge \neg E(x_1,x_3) \wedge \neg E(x_2,x_3) \right]$$

1.2)

$$-1 \quad \exists x_1 \exists x_2 \exists x_3 \left[x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge x_2 \neq x_3 \wedge \neg E(x_1,x_2) \wedge \neg E(x_1,x_3) \wedge \neg E(x_2,x_3) \right] \begin{cases} \to \not\models V_1 \\ \to \models V_2 \end{cases}$$

$$-2 \quad \forall x_1 \exists x_2 \exists x_3 \left[x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge x_2 \neq x_3 \wedge \neg E(x_1,x_2) \wedge \neg E(x_1,x_3) \wedge \neg E(x_2,x_3) \right] \begin{cases} \to \not\models V_1 \\ \to \models V_2 \end{cases}$$

---

3)

G is a complete graph

$$\forall x \forall y \left[x \neq y \to E(x,y) \right]$$

G has two isolated nodes

$$\exists x \exists y \left[x \neq y \wedge \neg E(x,y) \wedge \forall z \left(z \neq x \wedge z \neq y \to \neg E(x,z) \wedge \neg E(y,z) \right) \wedge \exists k \left(z \neq k \wedge k \neq y \wedge x \neq k \wedge E(z,k) \right) \right]$$

G has degree at least 2

$$\forall x \exists y \exists z \left[x \neq y \wedge x \neq z \wedge y \neq z \to \left(E(x,z) \wedge E(x,y) \right) \right]$$

G contains a path of length 4.

$$\exists x \exists y \exists z \exists k \exists w \left[x \neq y \wedge x \neq z \wedge x \neq k \wedge x \neq w \wedge y \neq z \wedge y \neq k \wedge y \neq w \wedge z \neq k \wedge z \neq w \wedge k \neq w \wedge E(x,y) \wedge E(y,z) \wedge E(z,k) \wedge E(k,w) \right]$$

7) Let **V** the set of vertices of the graph, let **S** a proper subset of **V**.

$$\exists S \left[\exists x \exists y \left(\underbrace{x \in S}_{1} \wedge \underbrace{y \notin S}_{2} \wedge \underbrace{\forall u \forall v \left(E(u,v) \Rightarrow u \in S \Leftrightarrow v \in S \right)}_{3} \right) \right]$$

1 There exists an element in **S, we ensure that S is not empty**

2 There exists an element in **V-S, we ensure that S has less element rather than V**

3 This ensure that there aren't a paths, which goes from an element of **S** to an element of **V-S**

8) Let **f** an injective function, which indentifies a number of vertices of the graph s.t. the first **k** vertices form a **k-clique.**

$$\exists f \,.\, \forall x \forall y \left(x \neq y \wedge f(x) < c_k \wedge f(x) < c_k \right) \to E(x,y)]$$