Homework 4 Taiello 1914000

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4) Consider the unprovable sentence G, for some theory $T \supseteq HA$ satisfying $T \vdash G \leftrightarrow \forall x \neg Proof_T(x, code(G))$

Prove that if T is consistent then $T + \{\neg G\}$ is consistent

Recall the definition of ω-inconsistent:
A theory T is called ω-inconsistent iff for some fml $F(x)$
$T \vdash \exists x F(x)$ but for every $n \in \mathbb{N}$ $T \vdash \neg F(n)$

We know that:

- $T \vdash G \Leftrightarrow \underbrace{\forall x \neg Proof_T(x, code(G))}_{\neg \exists x Proof_T(x, code(G))}$
- G is unprovable, then $\neg\neg \exists x Proof_T(x, code(G))$ ① is provable (T is consistent)

Proof

Let $T + \{\neg G\}$, T doesn't contain G, then it's consistent

Of course $\exists x Proof_T(x, code(G))$ ① is a theorem of T

But for any particular $n \in \mathbb{N}$ $\neg Proof_T(n, code(G))$ is correct and therefore for any extension of HA including T must hold.

5)

Recall Rosser's sentence

$$E \leftrightarrow [\forall y (F(\underbrace{code(E)}_{①}, y)) \to \exists z (z < y \wedge H(\underbrace{code(E)}_{②}, z))]$$

① Let $F(x,y)$ be a formula that represents in T the following relation:
$R(a,b) \Longleftrightarrow a$ is the code of a formula and b the code of the proof in T

② Let $H(x,y)$ be a formula that represents in T the following relation:
$S(a,b) \Leftrightarrow a$ is the code of a formula with single free variable and b is code of the proof of the negated formula in T.

if $T \supseteq HA$ consistent, then $T \nvdash E$ and $T \nvdash \neg E$

Proof $T \nvdash \neg E$

BWOC Suppose $T \vdash \neg E$ $\exists y (F(code(E), -)) \wedge \forall z (z \leq y \to \neg H(code(E), z))$

For some n $H(code(E), n)$, for every $z \leq y$ $H(code(E), z)$ must be false.

Therefore $n > y$, we can entail that for some $k < n$ $F(code(E), k)$, this is

Proof $T \nvdash E$

BWOC Suppose $T \vdash E$, then for some n $F(code(E), n)$ and for some $k \leq n$ $H(code(E), k)$ this implies $T \vdash \neg E$ CONTRADICTION

Part 2

3) Prove that the formula $\Box E \leftrightarrow \neg\Diamond\neg E$ is valid (for all frames $(X, \leq)$ and for all assig

$\Rightarrow$ ① Assume that $\Box E$ is true at node $x \in X$

② Suppose bwoc that $\Diamond \neg E$ is true at x

① $x \vDash_\alpha \Box E$: for all nodes $x' \leq x$ $x' \vDash_\alpha E$

② $x \vDash_\alpha \Diamond \neg E$: for some node x': $x \leq x'$ $x' \vDash_\alpha \neg E$

CONTRADICTION

$\Leftarrow$

① Assume that $\neg \Diamond \neg E$ is true at node $x \in X$

② Suppose that $\neg \Box E$ is true at node $x \in X$

① $x \vDash_\alpha \neg \Diamond \neg E$

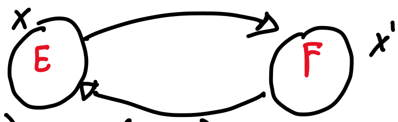
$x \nvDash_\alpha \Diamond \neg E$: meaning that for some x': $x \leq x'$ $x' \vDash E$

② $x \nvDash_\alpha \Box E$: meaning that for all x': $x \leq x'$ $x' \nvDash E$

CONTRADICTION

6) Let a frame $M=(X,R)$ $X=\{x,x'\}$ $R=\{(x,x'),(x',x)\}$

Let an assignment $x \rightarrow E$, $x' \rightarrow F$

$M=$ 

$M \nvDash (\Diamond E \wedge \Diamond F) \rightarrow \Diamond(E \wedge F)$