

Seismic Project: From Signal Capture to Population Warning

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May 28, 2020

Abstract

The main purpose of this project is to develop a MATLAB application that could be used for methodically dispatching rescue to earthquake-struck area and issuing fast warning to population concerning a possible incoming tsunami. To this end, the students will use their signal processing, their coding skills, their common sense and an ounce of intuition.

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1 Organization and project evaluation

This project is divided in 7 sessions of 2h40. The assessment relies on a personal grade depending on the **student's behavior** (assiduity and reliability during the sessions), as well as two **reports on the project**.

This year, two reports have to be done. The first one is for the Language Resource Center (CREL) and science teachers, the second one is for the science teachers only. The instructions for the **first** report are given below:

- A first version of your report is to be sent at the Language Resource Center (CREL) **on May 15th before 6PM**. A corrector will be assigned, and each group will be informed of the corrector's email address for sending the report.
- Professors from the CREL will give feedback to each group.
- The report will then be revised based on feedback. This 2nd draft (of the first report) will be due one week after feedback is given by the English teacher.
- This report will focus on the parts of the subject before Section 4.1 (including Section 4.1). This report should be structured as follows:
 1. **Introduction:** in this section you have to present the project, using your **own words** (do not rewrite the subject!). The reader should understand the goal and the context of the project.
 2. **Seismic data visualization:** in this section you will answer Section 3. A reader who has a signal processing background should understand some key properties of seismic signals and how he can relate these properties to geological phenomena.
 3. **Automatic P-waves detection:** in this section you will answer Section 4.1. The reader should understand how the seismic data can be used to detect P-waves automatically. In particular, you will explain the how, what and why of the different signal processing steps.
 4. **Numerical results on automatic P-waves detection:** in this section you will also present some results about the method that has been presented before. This section is also related to the Section 4.1.
 5. **Conclusion:** in this section, the reader must understand the main results of your work, and you should provide some perspective about your work.
- The report should be typed using L^AT_EX or Word, in plain page format with a font size of 11 points. The target number of pages is 6 with **10 pages the absolute maximum**.
- If you are using Word, all equations should be typed with the specific editor, and should be numbered if they are cited. All figures should have a number, a title and labeled axes (with units).
- If you are using L^AT_EX, use a proper spell checker (LanguageTool for instance) and be aware that the one of Overleaf is not that good.

Your **final report** is an **extended version** of the CREL report containing the same sections except that the following parts should be inserted before the conclusion section:

1. **Hypocenter localization:** in this section you will answer Section 4.2 about hypocenter localization. The reader should understand how the P-Wave detection step can be further processed to locate the hypocenter of an earthquake. You will also present numerical results as well as

the main limitations of the proposed method and eventually how you manage to bypass those limitations.

2. **Measuring the intensity of the earthquake:** in this section you will answer Section 5. The reader should understand the process done automatically computing the magnitude of the earthquake as well as its intensity. You will also show how every feature computed during the project can be displayed to unequivocally warn the population in case of intense earthquake or tsunami.

2 Introduction

The main purpose of this project is to implement signal processing algorithms using MATLAB to detect seismic activity and analyze this activity in order to warn the population of a Tsunami risk or dispatch rescue to area that experienced potentially damaging violent shaking. It has been shown that warning the population of incoming tsunami saves lives, to the condition of being alerting enough. It is then crucial to ensure speed of issuance and accuracy of predicted height. Before manipulating data and algorithms, a proper introduction to the phenomena which are dealt with is given.

2.1 Earthquake and tsunami phenomena

An earthquake is a geological event which is the result of a rupture between two rock systems. This rupture starts at a given point -the hypocenter- and spreads along a plane which separates the rock systems: the fault. The orthogonal projection of the hypocenter at the surface of the Earth is called the epicenter.

As one can see on the figure 1, the slip amount between the two rock systems represented by arrows is heterogeneous along the rupture. The shaking is potentially the greatest when close to a large slip zone, which is often assimilated to the hypocenter. This approximation is false for medium to great earthquakes which magnitude exceeds 5.5, because when the surface increases, one can be far from the hypocenter but close to the rupture zone, therefore experiencing strong shaking.

You can watch the following video to better understand the real time unfolding of a Magnitude 7-class earthquake : <https://youtu.be/m5xM-CTDL00?t=60>

In this video, the area which is rupturing (where the rock are moving past to each other) is colored in white, the ruptured zone left behind is in yellow and the surface seismic waves in red.

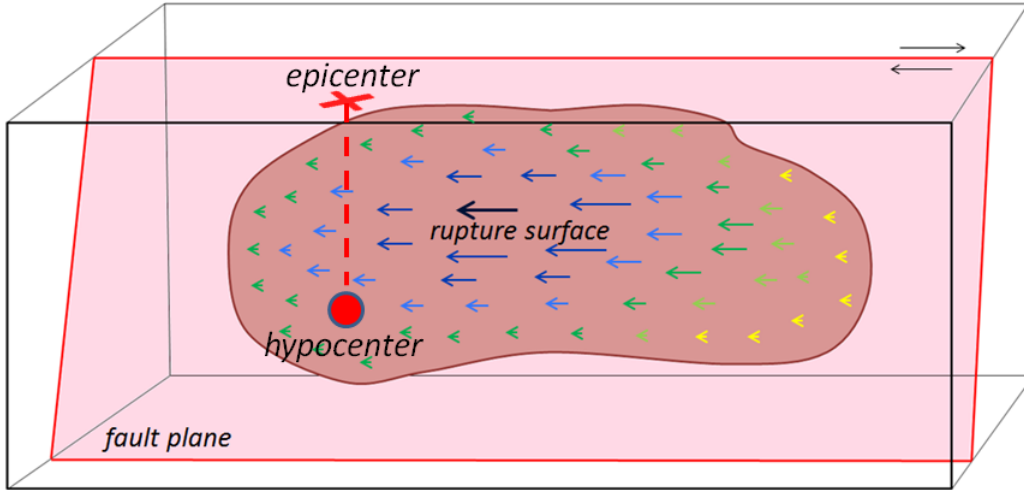


Figure 1: 3D representation of an earthquake rupture

Given the solid and elastic properties of rock at a macroscopic scale, this rupture releases energy under the form of heat (about 66%), the remaining being several type of mechanical waves (around 33%) known as seismic (or body) waves, radiated in all directions. These seismic waves travel inside the Earth and split into two categories :

- The P-Waves which travel the fastest, around $v_P = 7.3 \text{ km.s}^{-1}$ near the surface. As seen on 2, these are compression waves, like sound, and deform the rock in the direction of propagation at a rather high frequency (10Hz). Also responsible of the thunder noise happening during an earthquake, they moderately shake the ground mainly vertically.
- The S-Waves are slower (4km/s near the surface) but carry more energy than the P-Waves. They can be destructive as they hit the ground, because the motions induced are mainly horizontal, something against which buildings are vulnerable. Lower frequencies (around the hertz or lower) are met and can emphasize with resonance frequency of buildings which explains why they can be very damaging. S waves also create surface waves such as Love and Rayleigh waves as they hit the ground above the rupture area. These two are responsible for ground rolling motion but are out of the scope of this subject.

P and S waves have a velocity depending on the rock density which varies according to depth. This creates rays pattern which are curved inside the Earth due to progressive refraction. In the remaining of the subject, given the fact that we are at close range of each earthquake, we make the assumption of a constant velocity model with straight rays.

You can find a short introductory video on the wave propagation patterns at this link https://www.iris.edu/hq/inclass/animation/3component_seismogram_records_seismicwave_motion.

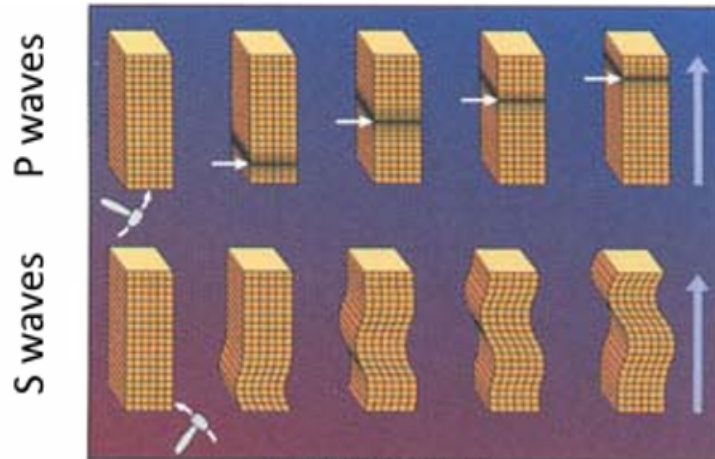


Figure 2: P and S waves propagation patterns

When an earthquake happens undersea and the mean slip vector (which would be the mean of all the arrow of figure 1) has a vertical component as well as in a potential shallow rupture zone case, there is a danger of tsunami. Indeed, as seen in figure 3, the vertical displacement between the two rock blocks lifts up a considerable amount of water. The tsunami travels at approximately 800 km.h^{-1} in deep water, decelerating at around 50 km.h^{-1} but gaining in height while approaching the coast. Its destructive power lies in the fact that contrary to classic wind-induced waves, the energy it carries is distributed on the whole water height and not close to the surface. Also, its wavelength exceeds dozens of kilometers. This results in a series of complete flood of the coast, the first one usually not being the greatest.

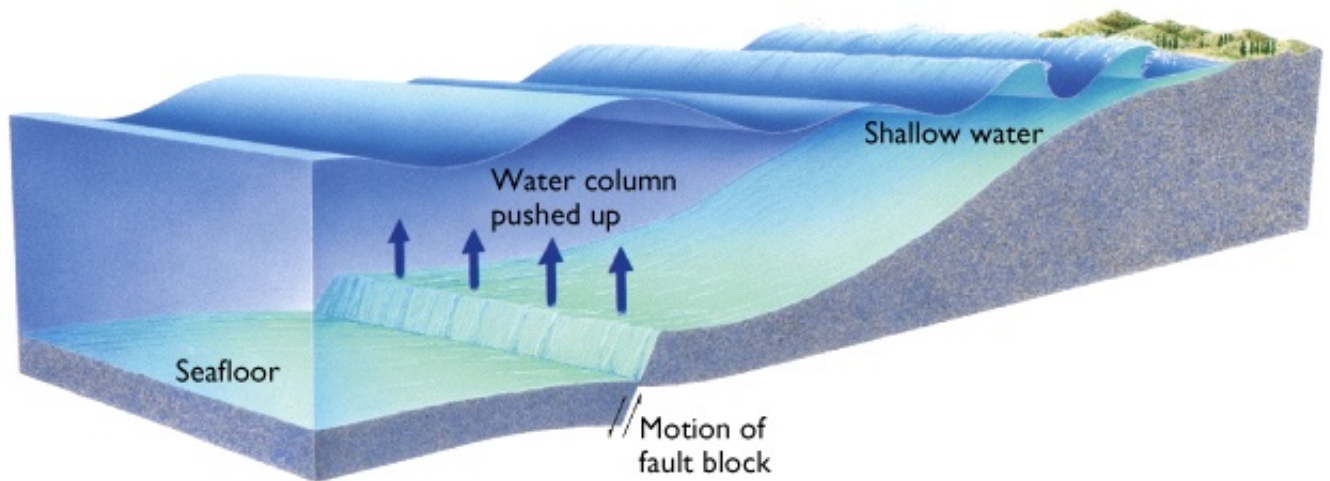


Figure 3: Tsunami generation mechanism

This is for these reasons that Tsunami Warning have been developed around the world. The goal is to provide accurate warning as fast as possible. For example, the Japan Meteorological Agency is capable of providing a Tsunami Warning 3 minutes after the earthquake occurrence.

2.2 Goal of this project

The main purpose of this project is to implement signal processing algorithms using MATLAB to detect seismic activity and analyze this activity in order to warn the population of a Tsunami risk or of a large magnitude seismic activity. During this project you are going to

- detect automatically the occurrence of an earthquake,
- locate automatically the hypocenter of an earthquake in space and time,
- evaluate automatically the magnitude of an earthquake.

A tool for warning people in case of a tsunami hazard is the conceived from these points.

2.3 Signal acquisition

Seismic activity is monitored thanks to seismographs that records ground motion in different Japanese cities. Each station records a three dimensional acceleration signal $\mathbf{a}(t) = [a_x(t) \ a_y(t) \ a_z(t)]$ at a sampling frequency of $F_s = 100Hz$ where

- a_x is the acceleration in the North-South direction,
- a_y is the acceleration in the East-West direction,
- a_z is the acceleration in the Up-Down direction.

In the MATLAB data, $\mathbf{a}(t)$ is given as a $3 \times N$ matrix where the first column represents $a_x(t)$ the second represents $a_y(t)$ and the third $a_z(t)$.

2.4 Signal Database

A database containing different earthquake recordings from various stations in Japan is provided. It can be downloaded at <https://rtajan.github.io/ts114/>. Each .mat file can be loaded into the MATLAB workspace using the command `load`, and is composed of a vector of structure called *list*. More specifically, the s^{th} element of this vector (`list(s)`) is itself a structure containing the following fields:

- `list(s).sismo` contains a three dimensional acceleration signal in cm/s^2 . The three components of this field represent acceleration in the North-South direction, the East-West direction and the Up-Down direction.
- `list(s).id` contains the id of the station (as well as its name in Japanese).
- `list(s).lon` is the longitude of the station.
- `list(s).lat` is the latitude of the station.
- `list(s).offset` is the time offset (in seconds) of the first sample of the signal (this offset is given compared to a common reference for all signals).

3 Signal Visualization

The goal of this section is to display seismic signals for highlighting their main physical properties. The goal for an engineer is to understand how these properties can be exploited to automatic process seismic in order to extract features such as the earthquake distance or position. In a report, figures are here to highlight some key concepts, hence they must be presented, they must be labeled and have a caption indicating what should be observed in the figure.

3.1 Time display

MATLAB. Under MATLAB, load the signals given in the file entitled *recording_1.mat* and display the time evolution of the 5th recording. On the same figure, highlight regions of this signal containing seismic noise (when there is no earthquake-induced shaking), the P-wave, and the S-wave.

Report task. In the section dedicated to the seismic signaling, use the previous MATLAB task to illustrate the following key points presented in introduction about seismic waves

- S-waves are more powerful than P-waves,
- P-waves are mainly vertical,
- S-waves are mainly horizontal.

A suggestion to answer this task is to reinforce the to your report and comment those figures:

- A figure of the complete signal that you can annotate.
- A "zoom" focusing on the very beginning of the earthquake.
- A "zoom" focusing on the S-wave.

3.2 Time/Frequency display

As seismic signals are non-stationary, we will then use a spectrogram to display their frequency content. Indeed, seismic activity varies slowly (compared to the sampling frequency of 100Hz), since the P-wave comes first then the S-Wave. Considering the Fourier transform (or Discrete Fourier Transform) will provide the frequency content of the signal, however it does not show well the variation of the signal through time. Consequently, we analyze $x(t)$ signal using a sliding window. In particular, computing the Discrete Fourier Transform (DFT) of each window captures the frequency content of the windowed signal and how it changes with time. This operation is called **Short Term Fourier Transform** (STFT). Mathematically, the STFT is expressed as follows

$$X(m, \nu) = \sum_{n=0}^{L-1} x_n w_{n-m} e^{-j2\pi\nu n}. \quad (1)$$

The modulus squared of this STFT is called the spectrogram

$$S_x(m, \nu) = \frac{1}{N} |X(m, \nu)|^2. \quad (2)$$

To compute spectrograms, you may use the MATLAB function `spectrogram`, before doing so **read the documentation carefully** !

MATLAB. Under MATLAB, load the signals given in the file entitled *recording_1.mat* and display the spectrogram of the 5th recording. On the same figure, highlight regions of this signal containing seismic noise (when there is no earthquake), the P-wave, and the S-wave. **At first consider you can consider a rectangular window of 1 second (100 samples) with an overlap of 99 samples.** From this first spectrogram, you can now play with the different parameters, for example:

- you can change the type of the window: Hamming, Hann, Blackman, Welsh...
- you can change the duration of the window (0.5s to 5s or more)

Report task. Choose a spectrogram in which the P-Wave and the S-wave can easily be observed and comment this spectrogram in your report.

MATLAB. After computing the spectrogram, this task is divided into two sub-tasks :

1. Compute the energy contained in each window of the spectrogram (you may use the Parseval's identity). Plot this energy as function of time.
2. Compare the spectrum of a window of signal containing the P-wave only with another window containing the S-wave.

Report task. From the previous tasks, illustrate the fact that S-waves are much more powerful than P-waves and observe the frequency content of both type of waves.

4 P-Wave detection and epicenter localization

4.1 P-Wave detection

For station s , the P-wave can be automatically detected using the algorithm described hereafter. For each sample n in the signal of station s , compute the following two signals

$$STA^{(s)}(n) = \frac{1}{N_l} \sum_{i=n}^{n+N_l-1} \left| a_z^{(s)}(i) \right|^2 \quad (3)$$

$$LTA^{(s)}(n) = \frac{1}{N_L} \sum_{i=n-N_L}^{n-1} \left| a_z^{(s)}(i) \right|^2 \quad (4)$$

where N_l and N_L are integers respectively equivalent to duration $T_l = 1$ and $T_L = 5$ seconds. The start of the P-Wave is then located by finding the **smallest** n such that the ratio of STA and LTA is above a threshold Γ , that is

$$n_P^{(s)} = \inf \left\{ n : \frac{STA^{(s)}(n)}{LTA^{(s)}(n)} > \Gamma \right\} \quad (5)$$

A good value for Γ can be $\Gamma = 2.5$ but feel free to try other values. The time in seconds at which the station s can detect a P-wave is then obtained by converting the result of equation into seconds (5) :

$$t_P^{(s)} = t_O^{(s)} + \frac{n_P^{(s)} - 1}{F_e} \quad (6)$$

where $t_O^{(s)}$ is the timing offset.

MATLAB. Implement a function `sta_lta` that takes as input $a_z^{(s)}(t)$, F_e , T_l , T_L , Γ , $t_O^{(s)}$ and returns $t_P^{(s)}$.

Report task. Explain the STA/LTA algorithm with your own words. In particular, you should lay emphasis on

- why this algorithm works, what is the rationale behind this method
- what signal processing steps did you use to implement this algorithm (filters, FFT, ...)
- what will happen if Γ is changed

(Hint) Here if we deliberately remove illustrations about this method; do not hesitate to add some (draw the algorithm as signal processing chain, plot some signals...).

4.2 Estimation of the hypocenter localization

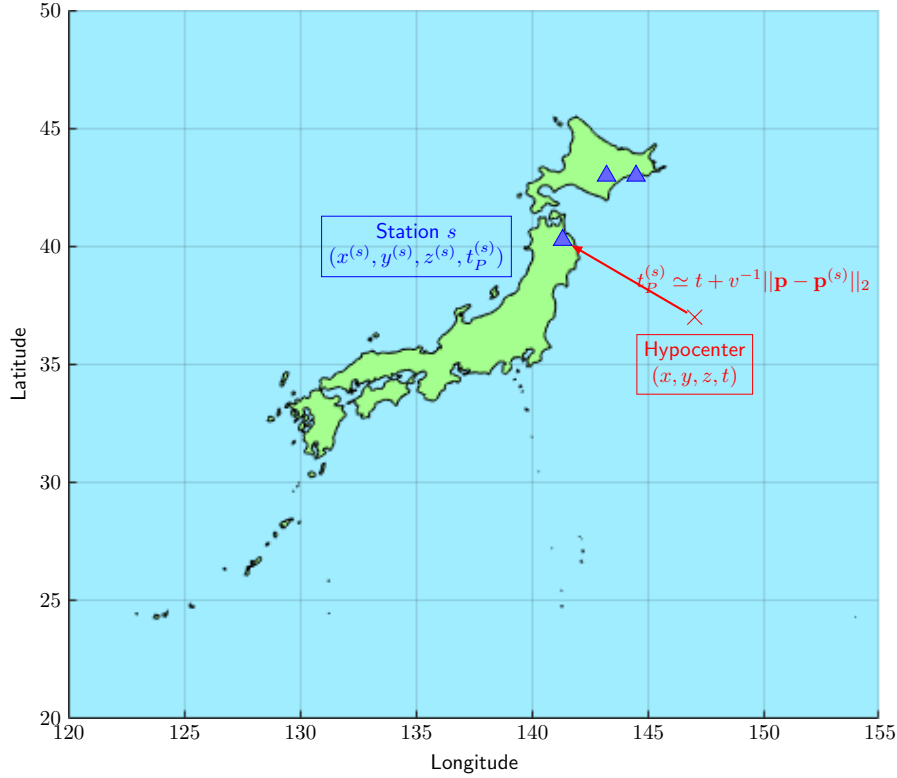


Figure 4: Example of hypocenter estimation

The algorithm for locating the hypocenter is the following :

1. For every station s in the database, compute the time of arrival of the P-wave, let t_P^s be this time of arrival.
2. For every station s in the database, compute its position for its longitude and latitude. Let the vector $\mathbf{p}^{(s)} = (\mathbf{x}^{(s)}, t_P^s) = (x^{(s)}, y^{(s)}, z^{(s)}, t_P^s)$ be this position in space and time. We will assume $z^{(s)} = 0$.

Let $\mathbf{p} = (\mathbf{x}, t)$ be the position (in space and time) of the hypocenter at the start of the earthquake. To locate the hypocenter, we will use a simple model for the P-waves: we will assume that they travel through the earth at a speed of $v = 7300\text{m/s}$. Under this assumption, the time at which the P-wave reaches station s , t_P^s can be expressed as

$$t_P^{(s)} \simeq t + v^{-1} \|\mathbf{x} - \mathbf{x}^{(s)}\|_2 \quad (7)$$

3. The hypocenter is then located by finding \mathbf{p} that minimize the following cost function

$$J(\mathbf{p}) = \sum_{s=1}^S \underbrace{\left(t + v^{-1} \|\mathbf{x} - \mathbf{x}^{(s)}\|_2 - t_P^{(s)} \right)^2}_{(\Phi^{(s)}(\mathbf{p}))^2} \quad (8)$$

This function will be minimized using a damped Gauss-Newton procedure.

MATLAB. 1. Write a MATLAB function `phi` that takes as inputs the vector \mathbf{p} and from the reference positions $(\mathbf{p}^{(s)})_s$ and returns the vector

$$\Phi(\mathbf{p}) = \left(\Phi^{(1)}(\mathbf{p}), \dots, \Phi^{(S)}(\mathbf{p}) \right)$$

2. Write a MATLAB function `Jphi` that computes the Jacobian matrix $J_{\Phi}(\mathbf{p})$ from an input vector \mathbf{p} and from the reference positions $(\mathbf{p}^{(s)})_s$.
3. Using points (a) and (b), write a function `gauss_newton` that computes the hypocenter position iteratively

$$\mathbf{p}_{n+1} = \mathbf{p}_n - \alpha (J_{\Phi}^T J_{\Phi})^{-1} J_{\Phi}^T \Phi(\mathbf{p}_n)^T$$

where α is a damping factor (usually set to 1) and $J_{\Phi} = J_{\Phi}(\mathbf{p}_n)$.

Report task. First take \mathbf{p}_0 corresponding to a point with longitude 146° , latitude 36° , altitude -10km and initial time 0s . Comment the figure containing on the same graph the cost function $J(\mathbf{p}_n)$ as a function of n for $\alpha = 0.01$ and 0.1 . Also, compare the final values you will find with the ground truth :

- Latitude: 38.2°
- Longitude: 141.92°
- Time: 3.4s
- Depth: 66km
- Local Magnitude: 7.2

Report task. For $\alpha = 0.1$, comment the result obtained for the following initial points :

- \mathbf{p}_0 of longitude 130° , latitude 25° , altitude -10km and initial time 0s .
- \mathbf{p}_0 of longitude 146° , latitude 36° , altitude 10km and initial time 0s

Propose a way to solve these problems.

5 Estimation of the earthquake impact

Several tools exist to assess the potential earthquake impact on a populated area among which one can find those described in the following subsections.

5.1 JMA seismic intensity

Seismic intensity scales have been designed to characterize damage and human perception of shaking at the ground surface (different from magnitude). One earthquake generates an uneven spatial intensity distribution. Generally the closer you are from a ruptured area, the higher is the seismic intensity. However local soil condition can amplify the shaking and thus intensity. Among widely used seismic intensity scales can be found the Modified Mercalli Intensity (MMI), EMS98 (used in Europe), etc. The Japan Meteorological Agency (JMA) developed its own seismic intensity scale (also called the Shindo scale). It categorizes the local seismic intensity into 10 levels which, for human perception, can be summarized as follows:

- 0 ($I_{JMA} < 0.5$): Imperceptible to people, but recorded by seismometers

- 1 ($0.5 \leq I_{JMA} < 1.5$): Felt slightly by some people keeping quiet in buildings.
- 2 ($1.5 \leq I_{JMA} < 2.5$): Felt by many people keeping quiet in buildings. Some people may be awoken.
- 3 ($2.5 \leq I_{JMA} < 3.5$): Felt by most people in buildings. Felt by some people walking. Many people are awoken.
- 4 ($3.5 \leq I_{JMA} < 4.5$): Most people are startled. Felt by most people walking. Most people are awoken.
- 5 Lower ($4.5 \leq I_{JMA} < 5.0$): Many people are frightened and feel the need to hold onto something stable.
- 5 Upper ($5.0 \leq I_{JMA} < 5.5$): Many people find it hard to move; walking is difficult without holding onto something stable.
- 6 Lower ($5.5 \leq I_{JMA} < 6.0$): It is difficult to remain standing.
- 6 Upper ($6.0 \leq I_{JMA} < 6.5$): It is impossible to remain standing or move without crawling. People may be thrown through the air.
- 7 ($I_{JMA} \geq 6.5$): same as 6 Upper for human perception.

You can find more details about this seismic scale at this address: <https://www.jma.go.jp/jma/en/Activities/inttable.html>.

This seismic intensity scale is computed directly from the norm of a filtered acceleration vector defined as

$$a(t) = \|\mathbf{a}_f(t)\|_2$$

First, each component of $\mathbf{a}_f(t)$ is filtered by means of a bandpass filter whose coefficients are provided here https://rtajan.github.io/assets/cours/TS114/jma_filter.mat then the norm is taken. The output of this process $a(t)$ is further processed to compute the JMA instrumental seismic intensity scale I_{JMA} . These steps are detailed in the section *JMA instrumental seismic intensity* of [1].

MATLAB. After reading the corresponding part of [1], write a code to compute the JMA seismic intensity scale for a given station.

Report task. Detail the signal processing steps algorithm that you have developed for computing the JMA seismic intensity scale and present the results for your algorithm on the map of Japan. On this map, each station should be drawn using a color indicating the JMA intensity scale. Your final map should look like this: <https://www.jma.go.jp/en/quake/20200518030442395-18120046.html>.

5.2 Magnitude

The magnitude of an earthquake is an indicator which purpose is to compare earthquakes between them but also gives hints about how wide can the ruptured area be. Several definitions of magnitude exist among which we can find the two followings.

5.2.1 The Local Magnitude M_L

Introduced by Charles Francis Richter in 1935 as the Richter Scale and updated thereafter, it has the advantage of providing a quick estimation of the earthquake size. It is usually used for Early Warning. It relies on making the assumption that the earthquake is a point and the rupture happens instantaneously, therefore linking the energy released with the maximum displacement amplitude and an attenuation formula.

The local magnitude M_L is obtained by the empirical mean of the magnitude estimations at each seismic stations $m_l^{(s)}$ inside a 200km radius from the hypocenter. By noting \mathcal{S} the subset of station for which $\|\mathbf{x} - \mathbf{x}^{(s)}\|_2 \leq 200km$

$$M_L = \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} m_l^{(s)} \quad (9)$$

At a given station s , the magnitude estimation is described as :

$$m_l^{(s)} = 1.1235 \times [\log_{10}(\Delta_{max}) + \log_{10}(D_s) + 1.9 \times 10^{-3} D_s - 5 \times 10^{-3} d_H - 0.02] \quad (10)$$

Where:

- Δ_{max} is the maximum modulus of high-pass filtered displacement, at the $0.5Hz$ cut-off frequency, in μm . You can design the high pass filter using the following MATLAB command `[a,b]=butter(2,0.5/Fe,'high')`, a and b as numerator and denominator for the `filter` function.
- D_s is the distance between the station and the hypocenter in km.
- d_H is the hypocenter depth in km.

MATLAB. Write a MATLAB code to compute the local magnitude M_L of the earthquake. Compare it with the value given in 4.2.

5.2.2 The Moment Magnitude M_W

Brought by Thomas Hanks and Hiroo Kanamori in 1979, it is globally adopted as the definition for significant earthquakes ($M_W \geq 6.0$), meaning that is incorrect to refer to it as "Richter Magnitude". It evolves logarithmically with the total amount of seismic energy released by the fault rupture, therefore having a physical meaning. Its computation requires heavy calculations (which takes at least 7 min on supercomputers), therefore unsuitable for Early Warning. It can be explained by the fact that it mathematically reconstructs the rupture surface and displacement distribution from the inversion of the actual seismic waves, However, since it is the best estimator even for sizable earthquakes, its value is required for the issuance of Tsunami Warning.

5.3 Tsunami alert

As seen in introductory part, Tsunami Warnings are critical to alert people about incoming tsunami. They generally split into several levels of warning, depending on the expected height h and thus population expected response:

- Tsunami Advisory if $50cm \leq h < 1m$. The population shall leave beaches and ports.
- Tsunami Warning if $1m \leq h < 3m$. The population shall evacuate the coast and riversides to designated areas.

- Major Tsunami Warning if $3m \leq h$. The population shall evacuate the coast and riversides to even higher designated areas as fast as possible or in a concrete building higher than 3 stories.

Modern methods use the rupture process modeling to compute the distribution and orientation of fault rupture thus estimating the displaced amount of water. This requires heavy computation even for supercomputers. To avoid that, common practice is to compute the local magnitude M_L and convert it into moment magnitude, using the following empirical formula:

$$M_W = M_L - 0.171 \quad (11)$$

Then, this magnitude is used to predict the wave heights using spheres centered on the hypocenter. The radius in kilometers of the spheres corresponding to each warning level heights are given by:

$$\begin{aligned} r_{0.5} &= 4.45 \times 10^{M_W - 5.85} \\ r_1 &= 3.78 \times 10^{M_W - 6.21} \\ r_3 &= 4.05 \times 10^{M_W - 6.59} \end{aligned} \quad (12)$$

To display a warning radius on a map, you shall use the radius of the circle described by the intersection between the warning sphere and the Earth's sphere.

Note that the intersection exists only if the sphere radius is greater than the depth d of its center. In this case, the intersection radius \bar{r}_h is:

$$\bar{r}_h = R \arccos \left(\frac{R^2 + (R - d)^2 - r_h^2}{2R(R - d)} \right) \quad (13)$$

Where:

- R is the Earth radius (6371 km)
- d is the hypocenter depth in km (positive value)
- r_h is the warning sphere radius in km for the wave height h

MATLAB. Implement a function `tsunami_warning` which takes the hypocenter 3-D Cartesian coordinates and local magnitude M_L as input and returns the intersection radius of each warning level ($\bar{r}_{0.5}, \bar{r}_1$ and \bar{r}_3). You can output zero values as default if the warning sphere radius is smaller than the hypocenter depth.

Report task. Plot the epicenter and warning circles for each warning level. Which actions should the population take?

6 Bonus

MATLAB. Transform your main script to a function `population_warning` which takes a station dataset as input then outputs the earthquake latitude, longitude, depth, local magnitude, relative time of occurrence as well as figures of intensity map and tsunami warning circles.

Report task. Run the function on the other provided datasets. Which observations can you make on the fourth earthquake? Its real moment magnitude M_W was as high as 9.1. Discuss on why the algorithm could have underestimated it and which impact it can have on populations.

References

- [1] Khosrow T Shabestari and Fumio Yamazaki. A proposal of instrumental seismic intensity scale compatible with mmi evaluated from three-component acceleration records. *Earthquake Spectra*, 17(4):711–723, 2001.
- [2] Thomas C. Hanks and Hiroo Kanamori. A moment magnitude scale. *Journal of Geophysical Research: Solid Earth*, 84(B5):2348–2350, 1979.