TS345
-
Codage pour la 5G

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Plan

1. Codes Linéaires (binaires) en blocs
2. LDPC

MAP-bit (2)

MAP-bit

- Le **décodeur MAP-bit** encodage systématique :

$$\Psi^{(j)}_{MAP-bit}(\mathbf{y}) = \underset{x_j \in \{0,1\}}{\operatorname{argmax}} \mathbb{P}(X_j = x_j | \mathbf{Y} = \mathbf{y})$$

- Le **décodeur MAP-bit** encodage systématique (2) :

$$\begin{aligned}\Psi^{(j)}_{MAP-bit}(\mathbf{y}) &= \underset{x'_j \in \{0,1\}}{\operatorname{argmax}} \sum_{\substack{\mathbf{x} \in \mathbb{F}_2^n \\ \text{avec } x_j = x'_j}} \mathbb{P}(\mathbf{Y} = \mathbf{y} | \mathbf{X} = \mathbf{x}) \mathbb{1}(\mathbf{x}H^T = \mathbf{0}) \\ &= \underset{x'_j \in \{0,1\}}{\operatorname{argmax}} \sum_{\substack{\mathbf{x} \in \mathbb{F}_2^n \\ \text{avec } x_j = x'_j}} \prod_{i=0}^{n-1} \mathbb{P}(Y_i = y_i | X_i = x_i) \mathbb{1}(\mathbf{x}H^T = \mathbf{0})\end{aligned}$$

Sans structure sur $\mathcal{C}$, ce décodeur est aussi trop complexe !

Plan

1. Codes Linéaires (binaires) en blocs

2. LDPC
   - Définition
   - Graphe de Tanner associé à un code LDPC

Définition des codes LDPC

Définitions

1. Soit une matrice H

$$H = \begin{pmatrix} h_{0,0} & h_{0,1} & \dots & h_{0,n-1} \\ h_{1,0} & h_{1,1} & \dots & h_{1,n-1} \\ \vdots & \vdots & & \vdots \\ h_{m-1,0} & h_{m-1,1} & \dots & h_{m-1,n-1} \end{pmatrix}$$

   Densité de H : $\dfrac{\left|\{i,j : h_{i,j} = 1\}\right|}{m\,n}$

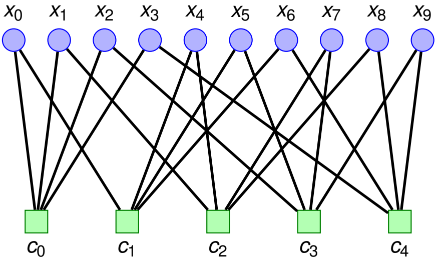
2. **Codes LDPC** : Codes possédant une matrice de parité H peu dense (creuse). Ordre de grandeur pour n grand ≤ 0.01.
3. **Codes réguliers** : poids des lignes constant r, poids des colonnes constant g
4. Rendement d'un code LDPC régulier : $R \geq 1 - \dfrac{m}{n} = 1 - \dfrac{g}{r}$
5. $R_d = 1 - \dfrac{g}{r}$ est appelé **rendement de construction** d'un code LDPC

Graphe de Tanner

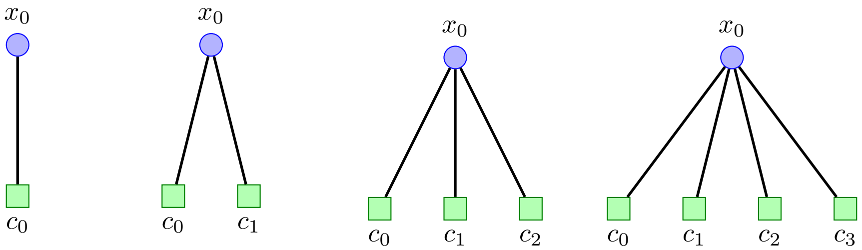
Le **graphe de Tanner** est un **graphe bipartite** avec :

1. n **nœuds de variables** représentant les variables x_j $j \in \{0, \dots n-1\}$
2. m **nœuds de parité (contrôle)** c_i $i \in \{0, \dots m-1\}$
3. Une arrête est dessinée entre nœud de variable x_j et le nœud de parité c_i ssi $h_{i,j} = 1$

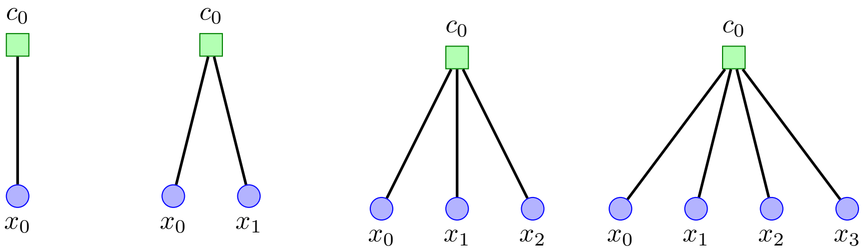
$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$



Degrés des nœuds de variable

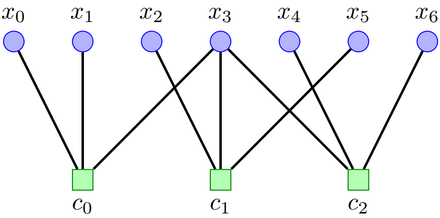


Degrés des nœuds de parité



Codes LDPC irréguliers

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

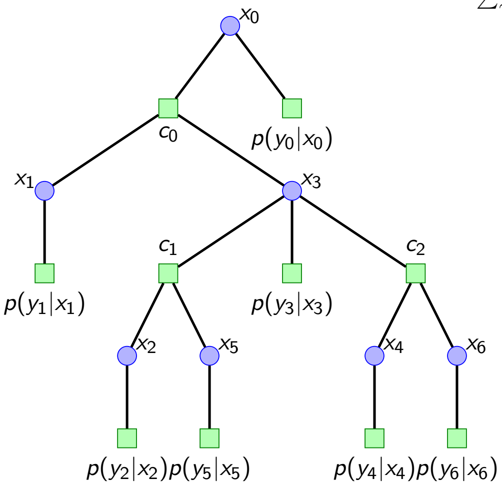


$$\lambda(x) = \sum_{d=1}^{d_v} \lambda_d x^{d-1}$$

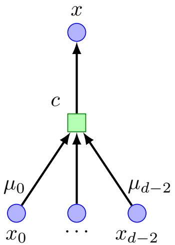
$$\rho(x) = \sum_{d=1}^{d_c} \rho_d x^{d-1}$$

$$R \geq 1 - \frac{\int_0^1 \rho(x)dx}{\int_0^1 \lambda(x)dx}$$

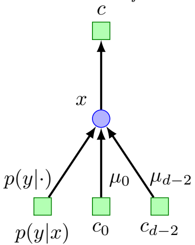
Algorithme somme-produit



$$\sum_{\sim x} \mathbb{1}(x + x_0 + \ldots + x_{d-2} = 0) \prod_0^{d-2} \mu_k(x)$$



$$\mu(x) = p(y|x) \prod_{f=0}^{d-2} \mu_f(x)$$



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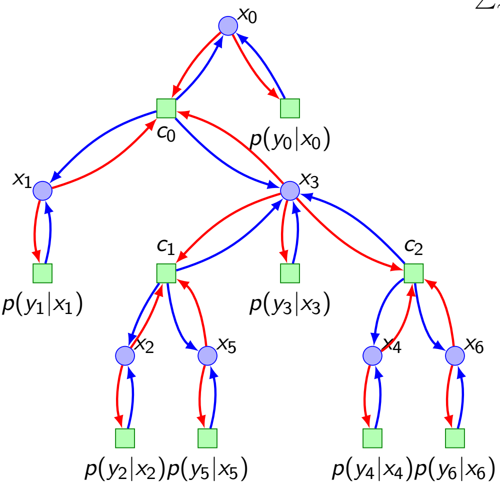

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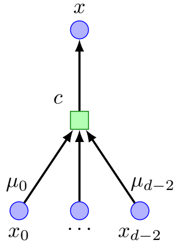

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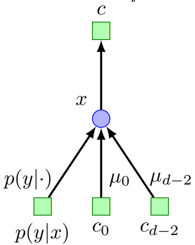

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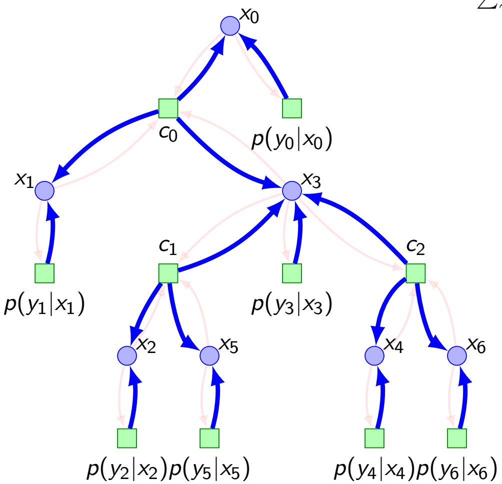

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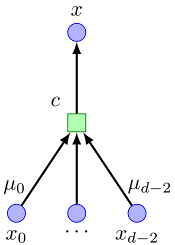

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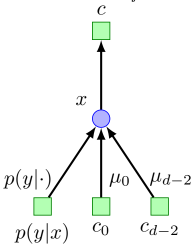

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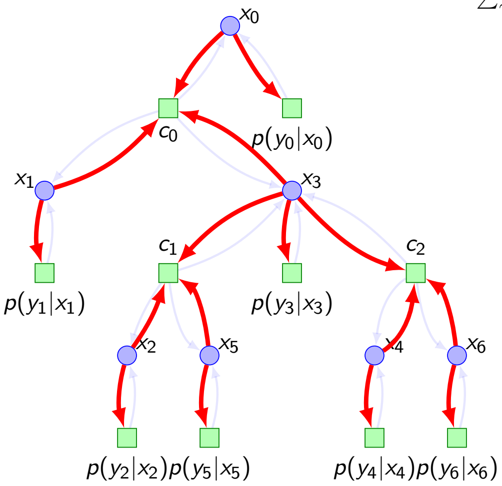

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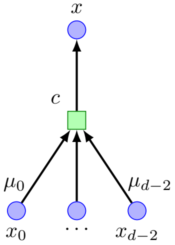

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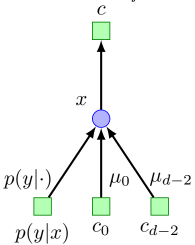

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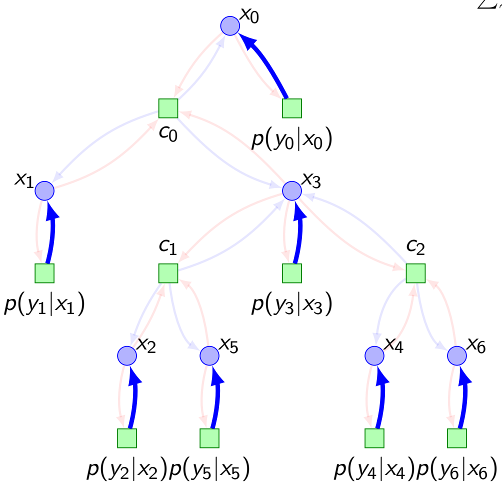

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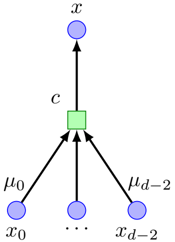

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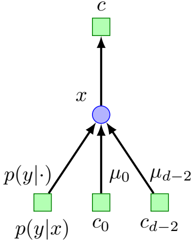

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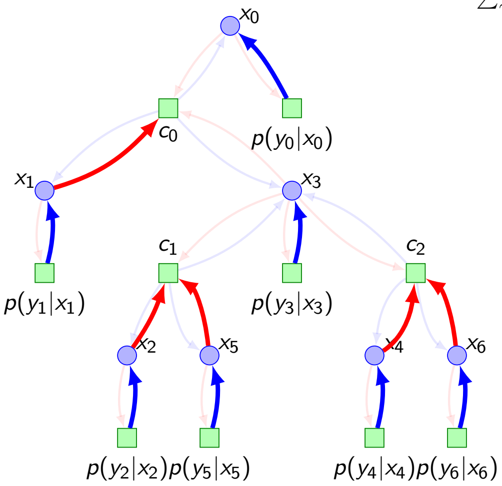

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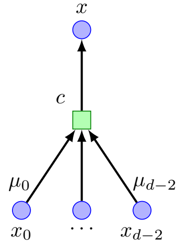

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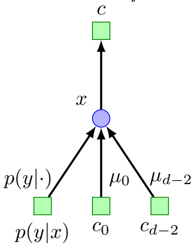

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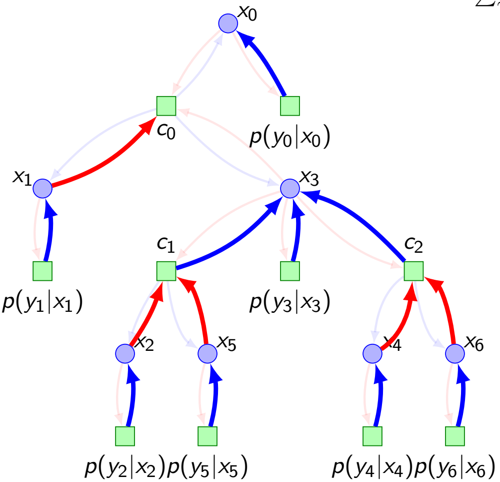

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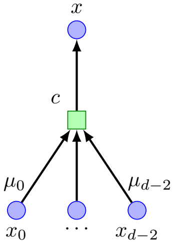

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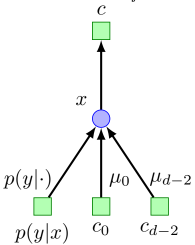

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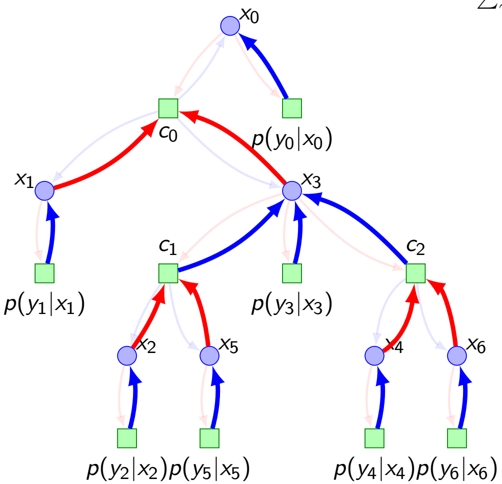

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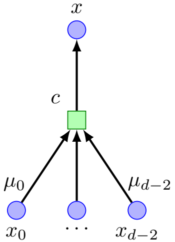

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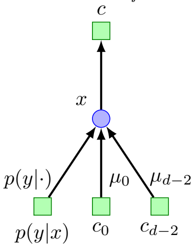

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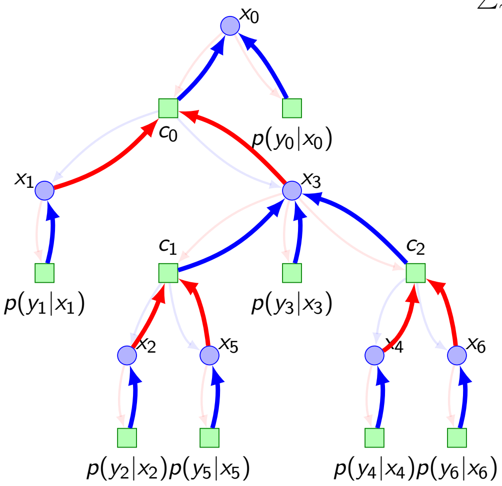

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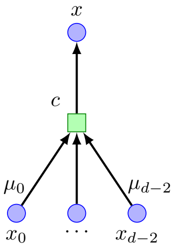

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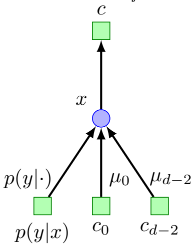

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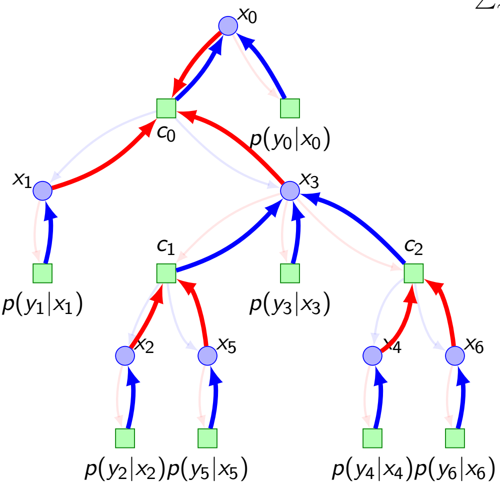

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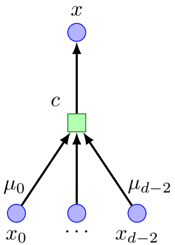

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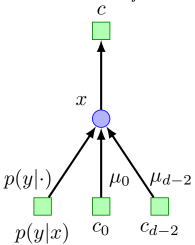

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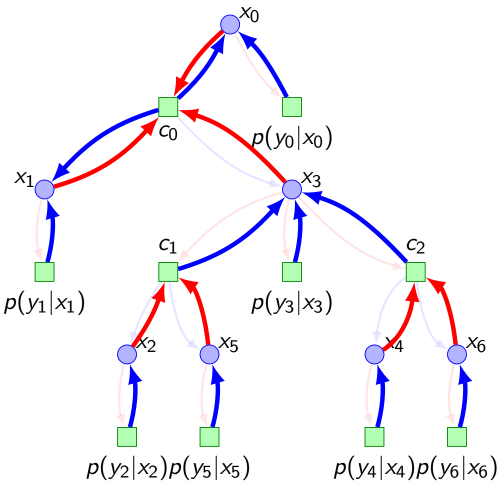

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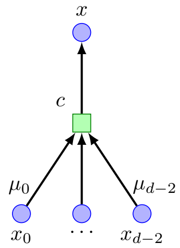

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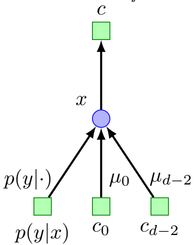

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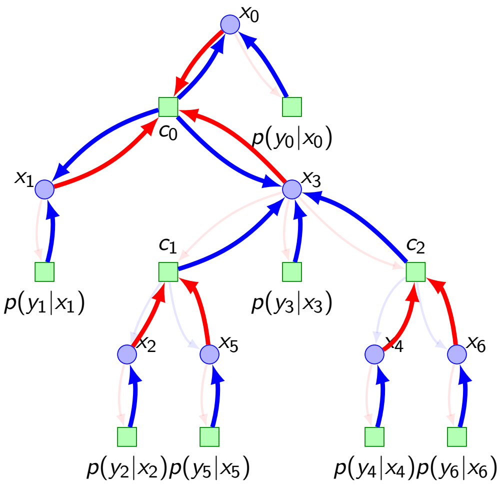

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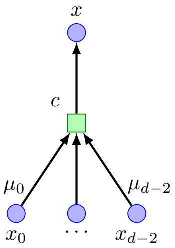

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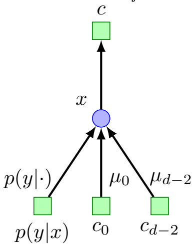
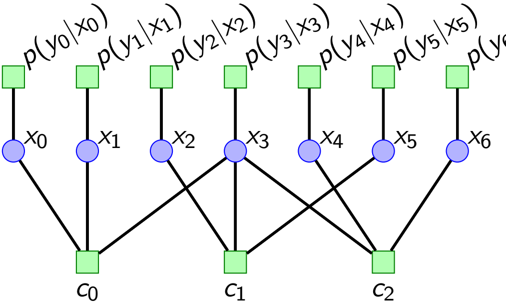


Ordonnancement : flooding

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$



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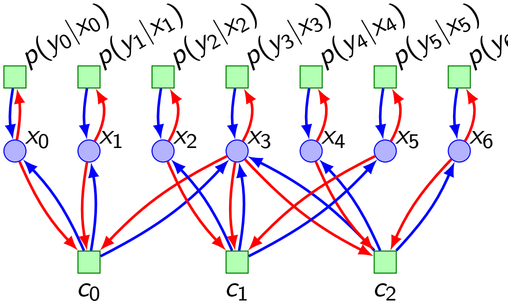

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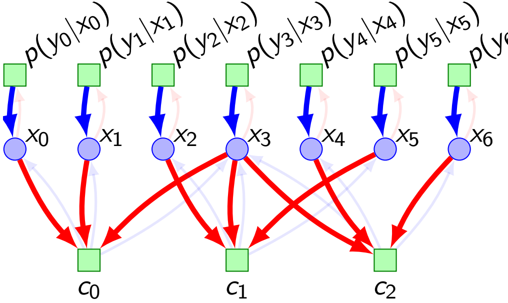

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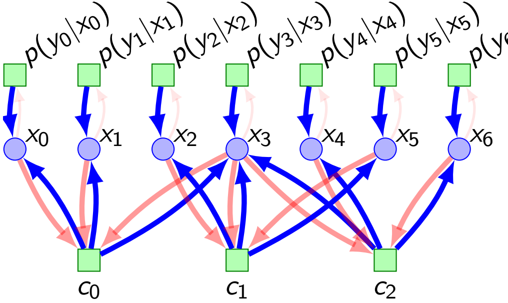
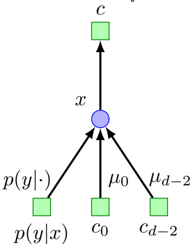


Conclusion sur l'algorithme somme-produit

Cet algorithme permet de calculer,

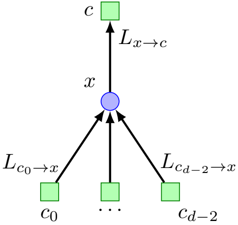
Calculer avec des LLR - Nœuds de variables

$$\mu(x) = p(y|x) \prod_{f=0}^{d-2} \mu_f(x)$$



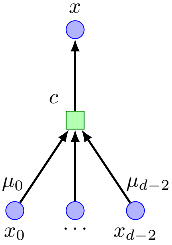
$\Rightarrow$

$$L_{x\to c} = \sum_{k=0}^{d-2} L_{c_k \to x}$$



Calculer avec des LLR - Nœuds de variables

$\sum_{\sim x} \mathbb{1}(x + x_0 + \ldots + x_{d-2} = 0) \prod_0^{d-2} \mu_k(x)$



$\Rightarrow$

$L_{c \to x} = 2\,\text{atanh}\left(\prod_{k=0}^{d-2} \tanh \frac{L_{x_k \to c}}{2}\right)$