MICA project: Matlab Interface for a Cardiac Analyst

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Abstract

The main purpose of this project is to develop a MATLAB application dedicated to cardiac analysts. This application will be able to display electrocardiograms, detect pathologies as well as additional information such as the heart rate or a frequency representation of the signals. Signal processing algorithms are developed to automatically compute the heart rate but also to extract features highlighting potential pathologies.

Contents

1 Organization and project evaluation

The duration of the project is 20h, divided in 5 sessions of 4h. The work must be carried out by groups of 2 students **maximum**. The assessment relies on a personal grade depending on the **student's behavior** (assiduity and reliability during the sessions), as well as a **report of the project**. The instructions for the report are given below :

- A first printed version is to be sent at the Language Resource Center (CREL) **on May 24th**. Professors from the CREL will give back their reviews **during group meetings on May 27th**.
- An electronic **pdf version** of the final report is to be sent by e-mail to the professor in charge of your group, **on May 28th, before midnight**. The MATLAB source code produced during the project must be attached to the e-mail, and **does not have to be included in the appendices of the report**.
- The report should be typed using LaTeXor Word, in plain page format with a font size of 11 points. The target number of pages is 10 with **15 pages the absolute maximum**.

  If you are using Word, all equations should be typed with the specific editor, and should be numbered if they are cited. All figures should have a number, a title and labeled axes (with units).

2 Introduction

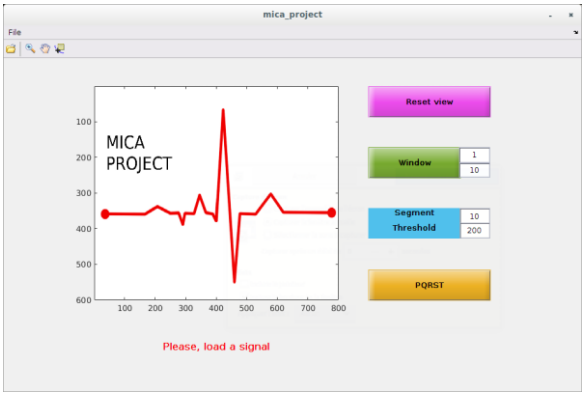
2.1 Presentation of the MICA project

The main purpose of this project is to implement basic signal processing algorithms using MATLAB to detect some cardiac pathologies, and develop a graphical user interface (GUI) to load, process and display ECG, in order to help clinicians through their diagnosis. A first version of the GUI interface exists and can be accessed by cloning the following repository on GITHUB:

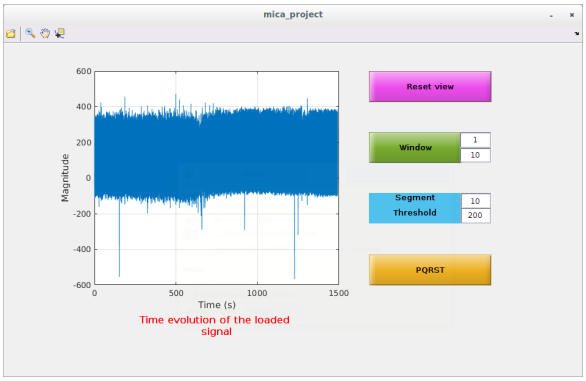
```
git clone https://gitlab.com/batalinux/MICA_project.git.
```

This repository contains the MATLAB code for the interface shown in Figure 1(a) as well as the MATLAB codes for

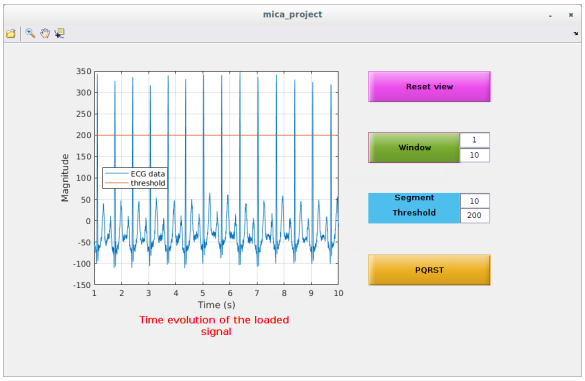
- loading ECG signals (see Figure 1(b)),
- displaying a windowed version of the signal and selecting a threshold (see Figure 1(c)),
- finding a QRS complex (see Section 2.2 for further details) using a very simple threshold method and labeling this signal with estimated positions of Q, R and S waves (see Figure 1(d)),
- computing and displaying the heart rate (see Figure 1(d)),
- performing unitary tests on previously defined functions.



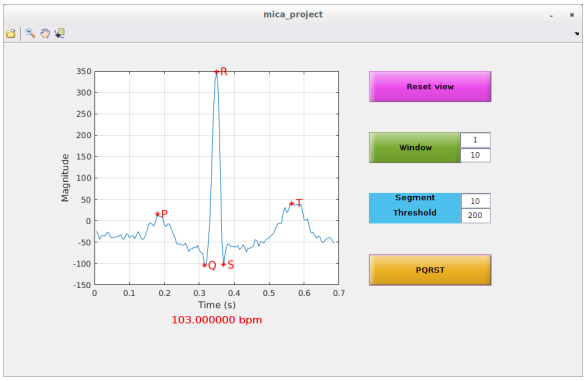
(a) Existing interface of the MICA project



(b) Same interface with a loaded ECG signal.



(c) Same interface with a windowed ECG signal.



(d) Same interface with a QRS complex and its labels.

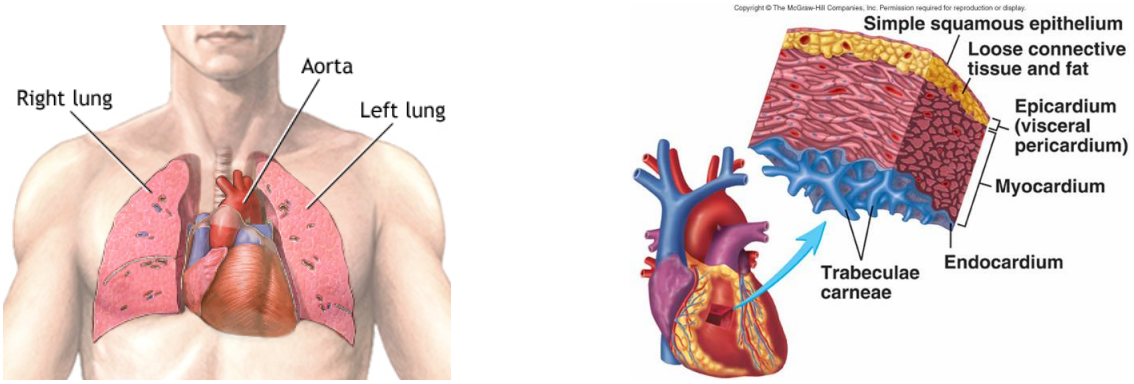
Figure 1: Screenshots of the MICA interface. (a) The interface when the program is started. (b) The interface with a loaded ECG. (c) The interface showing a Windowed version of the ECG and the chosen threshold. (d) The interface showing one QRS complex and its labels.

2.2 Cardiac anatomy and physiology

2.2.1 General presentation

The human body is the centre of an intense electrical activity arising, for example during brain cells communication, nerve impulses, and especially during cardiac activity. The blood pumping function of the heart is essentially provided by the contraction of the cardiac muscle triggered by an electrical signal.

Heart The heart is a vital organ of the human circulatory system located in the centre of the thoracic cavity (see Figure 2(a)). It is made up of four chambers, from the top to the bottom, namely the right atrium, left atrium, right ventricle and left ventricle. Collectively, the four chambers work in a coordinated manner to circulate blood in the body, supplying oxygen and nutrients to the tissues. These cavities are composed of layers named, from outside to inside, epicardium, myocardium and endocardium (see Figure 2(b)).



(a) Heart location [1]

(b) Herat muscle layers [2]

Figure 2: Human heart schematics

The blood flow is initiated and maintained by the heart (see Figure 3). The atria are responsible for the blood reception while the ventricles are known as blood ejection chambers; both atria and ventricles contract in coordination, first the atria contract together then the ventricles contract. The right side is responsible for receiving blood from the whole body and pumps it to the lungs for oxygenation. In parallel, left atrium and ventricle receive oxygenated blood to be distributed again through the body. Valves force the blood to flow in the right direction and improve the mechanical efficiency of the pump. The tricuspid valve is situated between the right atrium and ventricle and the mitral valve is localized between the left atrium and ventricle. The pulmonary valve obstructs the reverse flow of blood from the pulmonary arteries to the right ventricle, while the aortic valve does the same between the aorta and left ventricle.

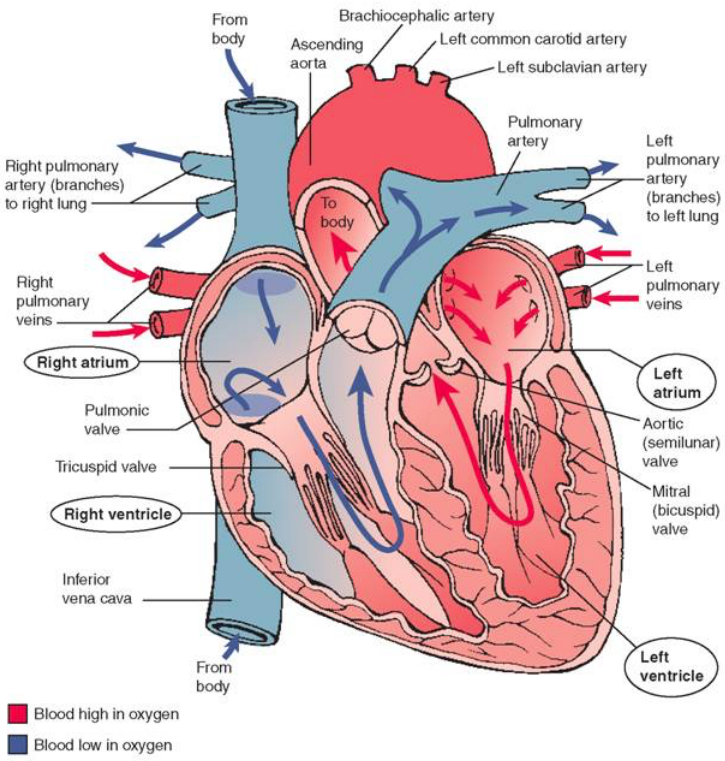


Figure 3: Cardiac anatomy and blood flow [3]

The efficiency of the mechanical activity of the heart is closely related to the synchrony of the muscle fibers when they contract. This synchronization is provided by the cardiac electrical activity. The electrical stimulation triggering cardiac contraction is spontaneously generated in a structure that works as the natural pacemaker of the heart called the sinoatrial (SA) node in the upper wall of the right atrium, (see Figure 4). The stimulus spreads throughout the atria to reach the atrioventricular (AV) node, in this tissue at the bottom of the right atrium the signal undergoes a delay due to the slow conduction velocity, and the reflection on the mechanical behaviour is a delay allowing to fill the ventricles with blood. After this delay, the electrical activation propagates into the bundle of His, which is divided into two branches located within the interventricular septum and leading to Purkinje fibers. These are very thick fibers which allow a very rapid spread causing an almost simultaneous contraction of the entire ventricle. Without any pathology, this normal contraction is named *sinus rhythm*.

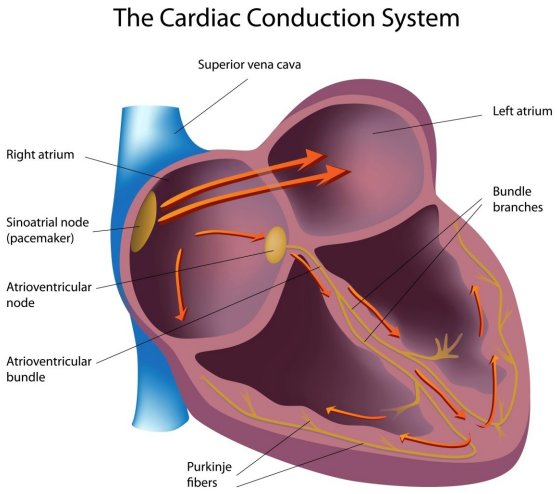
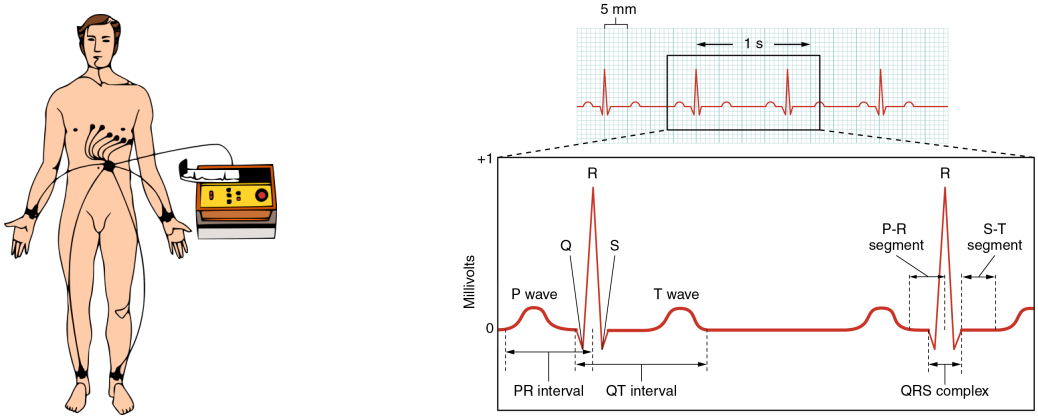


Figure 4: Human heart electrical conduction system [4]

2.2.2 Basics of the electrocardiogram

An electrocardiogram (ECG) records the heart's electrical activity (or the polarization changes) by using electrodes placed at specific locations of the body, to measure electric potential between several points, as shown in Figure 5(a).



(a) The ECG device [5]

(b) Normal sinus rhythm [6]

Figure 5: Electrocardiogram procedure

The typical shape of an ECG signal during a cycle of heart contraction is given in Figure 5(b) and is composed of five main variations called *waves*.

Waves [7] The propagation of the initial electrical stimulation through the right atrium to the AV node gives rise to an *atrial depolarization* called *P wave* (see Figure 6). While attaining the left and right ventricles, the electric field undergoes series of three polarization changes occurring in a short time, called the *QRS complex*. Finally, the re-polarization of the ventricles give birth to the *T wave*. These waves are a valuable marker to detect abnormalities in the heart cycle.

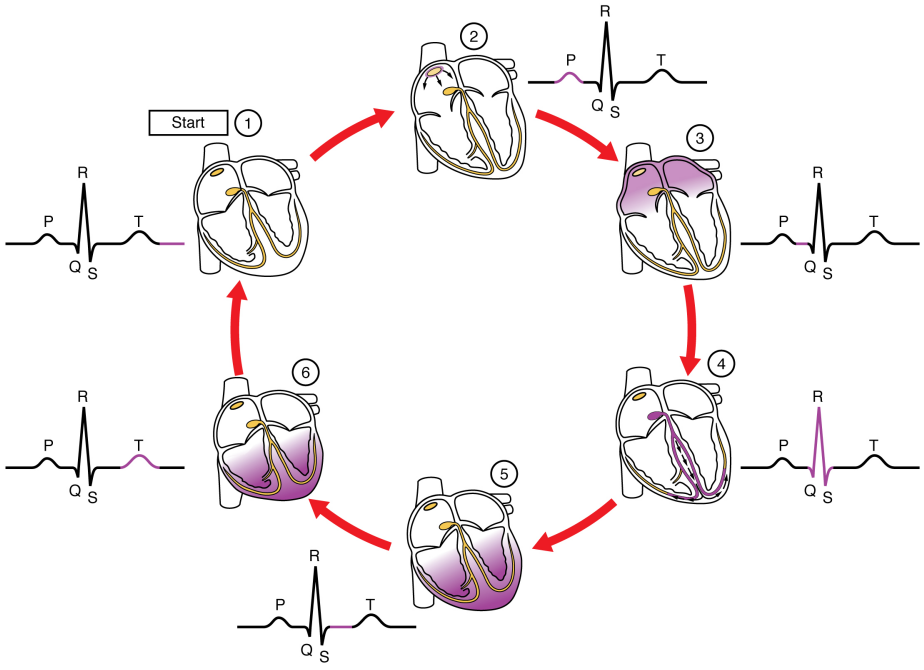


Figure 6: Electrical tracing of the cardiac signal [6]

(1)The sinoatrial (SA) node and the remainder of the conduction system are at rest. (2) The SA node initiates the action potential, which sweeps across the atria. (3) After reaching the atrioventricular node, there is a delay of approximately 100 ms that allows the atria to complete pumping blood before the impulse is transmitted to the atrioventricular bundle. (4) Following the delay, the impulse travels through the atrioventricular bundle and bundle branches to the Purkinje fibers, and also reaches the right papillary muscle via the moderator band. (5) The impulse spreads to the contractile fibers of the ventricle. (6) Ventricular contraction begins.

Time intervals The period between the different waves also provides important information about the heart electrical activity, and potential pathologies. The *PR interval*, measured from the beginning of the P wave to the beginning of the QRS complex (see Figure 7), represents the duration of propagation of the electrical wave inside the right atrium (more precisely from the SA node to the AV node). The *PR segment* represents the propagation of the electrical wave from the AV node to the right and the left bundle branches, which is characterized by an absence of contraction. The *QT interval* starts from the beginning of the QRS complex and finishes at the end of the T wave, and characterizes the duration of the ventricular contraction. The *ST segment*, starting from the end of the QRS complex and ending at the beginning of the T wave characterizes the time interval during which the ventricles are depolarized. Finally, the *R-R interval* representing the time interval between two recurrences of R waves can be used to measure the duration between two contraction cycles, and it is therefore used as a measure of the heart rate.

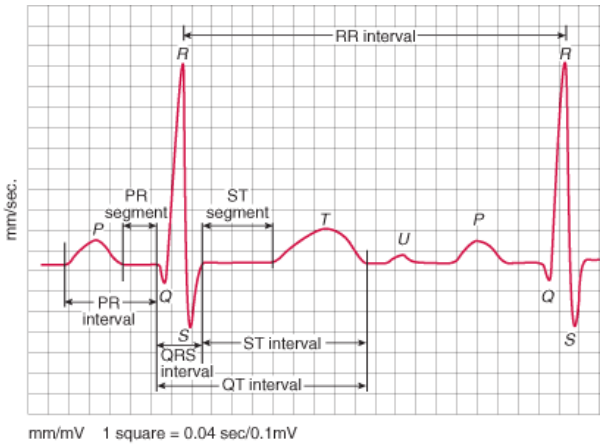
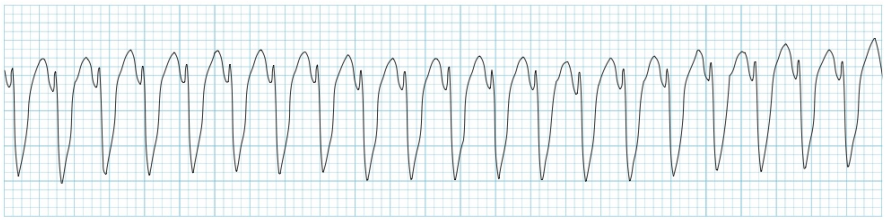


Figure 7: ECG segments schematic [8]

2.2.3 Pathologies

The analysis of the cardiac rhythm makes possible to detect abnormalities in the heart. Some of these pathologies are introduced below. *Arrhythmia* is related to abnormalities in the cardiac rhythm, whose normal value generally spreads from 60 to 100 bpm (beats per minute, for an adult) and affects millions of people worldwide. An example of irregularities is a faster heart rate ($>$ 100 bpm), a phenomenon known as *tachycardia*, or a slower heart rate ($<$ 60 bpm), which is known as *bradycardia*.

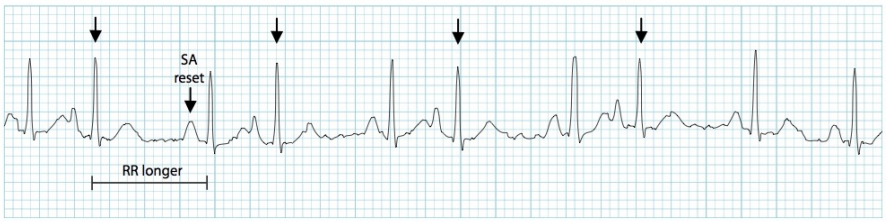
(a) Atrial tachycardia [9]



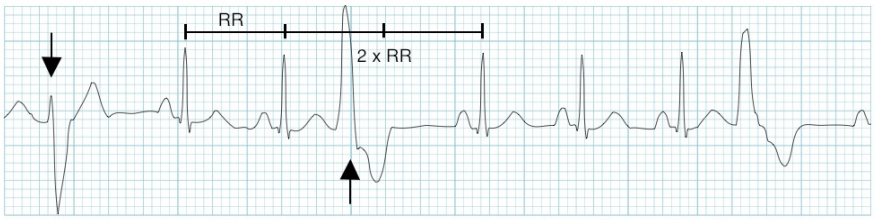
(b) Ventricular tachycardia [10]

Figure 8: Example of ECG with tachycardias (arrows)

In some cases, the electrical stimulation initiated originally from the SA node may start from the atrium or from the ventricles, leading respectively to a *premature atrial contraction (PCA)* or *premature ventricular contraction (PVC)*, that is an anticipated reset of the cardiac rhythm.



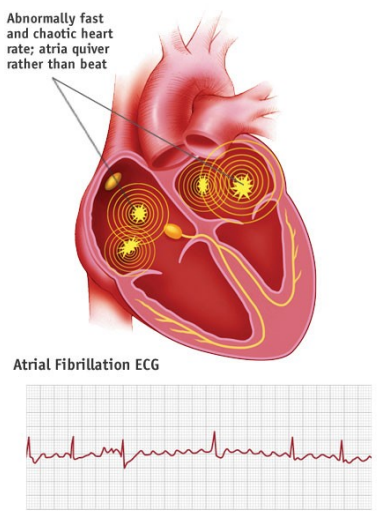
(a) Premature atrial contraction [11]



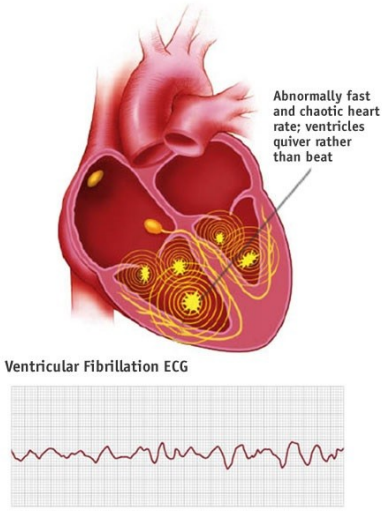
(b) Premature ventricular contraction [12]

Figure 9: Example of ECG with premature contractions (arrows)

Another frequent phenomenon, known as *fibrillation*, is defined by the anarchical disorganization of electrical activity in some parts of the heart. There are two types of fibrillation: the *atrial fibrillation* and the *ventricular fibrillation*. The atrial fibrillation occurs when the original electrical stimulation of the SA node is "overwhelmed" by the chaotic circulation of many small electric wavelets taking place in the atrium ; the consequences of the ECG is shown in Figure 10(a). Two major hypothesis can explain atrial fibrillation. The first one suggests that atrial fibrillation is entirely disorganized and that disorder is sustained by random propagation of multiple irregular waves. The second one argues that activity in AF is hierarchical in which disorder is driven by dominant regions of organized activity in the form of rotors or focal impulses. Ventricular fibrillation occurs in the ventricles and is characterized by a similar phenomenon, that is, an addition of many rapid and small depolarizations. Unlike the atrial fibrillation, the ventricular fibrillation is generally fatal within 5 minutes, unless a defibrillator is used ; the consequences of ventricular fibrillation on the ECG are shown in Figure 10(b).



(a) Atrial fibrillation, characterized by rapid oscillations and absence of P wave [13]



(b) Ventricular fibrillation characterized by an absence of P, T, and QRS complex waves [13]

Figure 10: Effects of fibrillation on the ECG

Note: If you want to understand easily, see the following video: `https://www.youtube.com/watch?v=TJR2AfxVHsM`.

3 Work organisation

3.1 Goals

As you may have remarked, several signals are provided with this project. Many differences can be observed between them as some are from normal behavior and others presenting pathologies.
Your missions are as follows :

- familiarisation with electrocardiogram signals and new methods for analysing signals
- improving the algorithm for automatically detecting the PQRST complexes. This will be done using the Pan and Tompkins algorithm presented in Section 5.
- designing algorithms for identifying different pathologies such as tachycardia, bradycardia, ectopic beats, fibrillation. Some help for performing this task is provided in Section 6.
- writing a report presenting the different techniques that you have developed as well as your application.
- **Bonus** : designing an original and user-friendly interface for cardiac analysts. This interface should accessible for cardiologists that does not possess any signal processing skills but also contain some advanced features. Furthermore, this interface should display the results of your algorithm in a way that it helps the clinician in his diagnosis of a potential pathology.

3.2 Organization of your report

Your report will have five imposed parts: an introduction, a technical part, a presentation of your application and of your results, a conclusion and a part containing references.

The first part should be the introduction where you have to present the project, using your **own words** (do not rewrite the subject!). The reader should understand the goal and the environment of the project. **Illustrate this section with spectrograms that illustrate the goals of the proposed application.** (Refer to *section 4*)

The second part will be a more the technical part. In this part, you will describe the Pan and Tompkins algorithm (with your own words) providing details on each step of this algorithm. (Refer to *section 5*)

By reading the second part, a signal processing specialist should understand how and why this algorithm works and should understand how you have used the results of this algorithm to identify the different pathologies.

The third part should provide enough results to show the different pathologies detected by your algorithms. **Here again provide enough visual insights (plots, spectrograms ...) for the reader to understand how your algorithms work.**

As a bonus and if you had enough time, a fourth part will dedicated to the presentation of your application. In this part you should explain how your application works, what are its possibilities and new features. By reading the fifth part, a clinician (not necessarily a signal processing expert) should understand how the proposed application is used and be convinced that it can assist him in his professional activity.

Finally, you will conclude this report by doing a synthesis of the previous part and by proposing some perspectives to the project. If some documents have been used during the project, they should be cited in a "References" section, at the end of the report.

4 Data visualization

ECG signals are intrinsically non stationary, indeed heart rate varies in time for example after making some efforts or because of some pathologies. Although the Fourier transform provides frequency content of the analyzed signal; it does not capture its localization in time. Consequently, we will analyze signals using a sliding window. In particular, computing the Discrete Fourier Transform (DFT) of each window captures the frequency content of the windowed signal and how it changes through time. This operation is called **Short Term Fourier Transform** (STFT). Mathematically, the STFT is expressed as

$$X(m,\nu) = \sum_{n=0}^{L-1} x_n w_{n-m} e^{-j2\pi\nu n}. \tag{1}$$

One can show that the computation of the STFT for $m = 0, d, 2d, \ldots$ can be performed following these three steps:

1. decompose the vector x (of length L) into $M = \left\lfloor \frac{L}{d} \right\rfloor$ frames of N samples. Each frame will be stored as columns. The first frame contains elements $(x_0, \ldots x_{N-1})^T$. The second frame contains elements $(x_d, \ldots x_{d+N-1})^T$ so that it overlaps the first frame on the $N-d$ last elements. Figure 11 illustrates this processing.

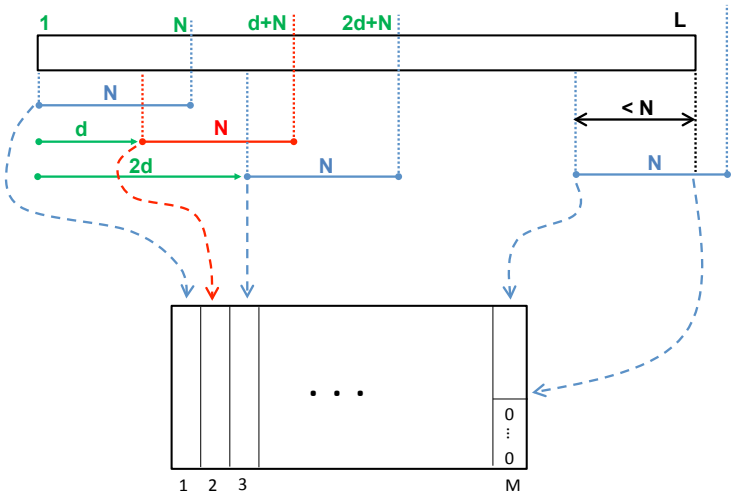


Figure 11: Framing of x_n.

2. multiply each column of the matrix created in step 1 by a window $w = (w_0, w_1, \ldots w_{N-1})$. The window signal w_n can be rectangular, triangular, Hamming, or Hanning, however in this work, we will only consider rectangular or Hamming window. Figure 12 illustrates this processing.

3. Compute the DFT on N_{fft} points of each row using `fft`. We recall that if x has a length N, `fft(x)` computes the DFT at the frequencies $\nu = (0, \frac{1}{N}, \ldots \frac{N+1}{N})$, where $\nu = \frac{f}{F_s}$. On the other hand, if x has a length N, `fft(x,Nfft)` computes the DFT at the frequencies $\nu = (0, \frac{1}{N_{fft}}, \ldots \frac{N_{fft}-1}{N_{fft}})$, where $\nu = \frac{f}{F_s}$

Create a function `stft` that performs the STFT of the signal x_n. This function will have the following header

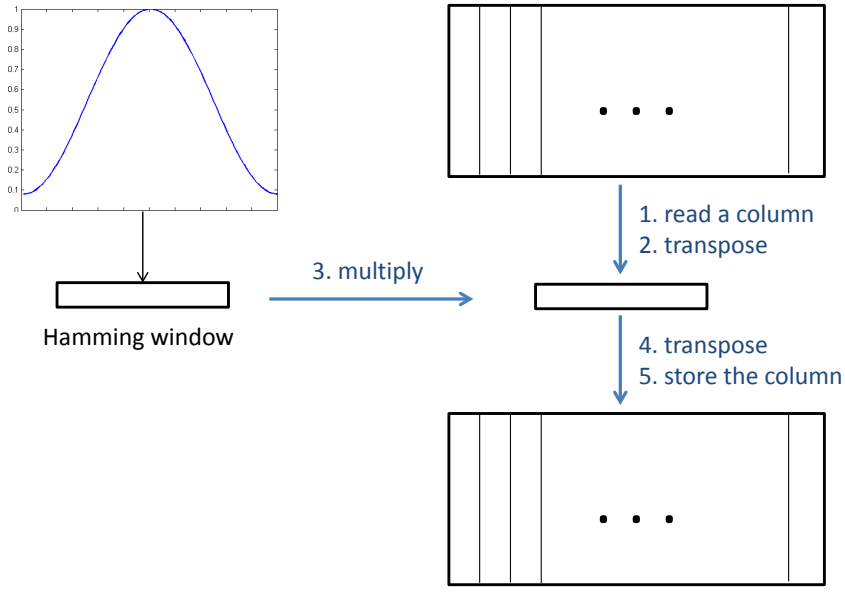


Figure 12: Windowing of x_n.

stft.m

```
function [X, f, t] = stft(x,w,d,N_fft,Fs)
% This function computes the stft for m = [0, d, 2d, 3d...]
% This function outputs are:
% -> X, which is  a matrix of n_fft lines and M columns
%    M is the number of elements of m
%    X(i,j) is the value of the spectrogram for time t(i) and frequency f(j)
% -> f, is a column vector of the frequencies (in Hz)
% -> t, is a row vector containing the times of the beginning of the windows
```

The modulus squared of the STFT is called the spectrogram

$$S_x(m,\nu) = \frac{1}{N}\left|X(m,\nu)\right|^2. \tag{2}$$

Create a function `spectro` that computes the spectrogram of x_n.

spectro.m

```
function [Sx, f, t] = spectro(x,w,d,N_fft,Fs)
% This function computes the spectrogram for m = [0, d, 2d, 3d...]
% This function outputs are:
% -> Sx, which is  a matrix of n_fft lines and
%                            M (number of elements of m) columns
%    Sx(i,j) is the value of the spectrogram for time t(i) and frequency f(j)
% -> f, is a column vector of the frequencies (in Hz)
% -> t, is a row vector containing the times of the beginning of the windows
```

Providing a good representation of signals is the first step toward finding new ways of identifying pathologies.

In your report, apply the spectrogram to *ecg_ normal_ 1.mat* and *ecg_ VF.mat* with windows of duration $8s$ to illustrate the aims of automatic detection of pathologies. To do so, give your analysis of the respective spectrogram obtained and discuss the difference observed.

5 Detection of PQRST complex

5.1 PQRST detection

We have seen in the previous section how to recognize the Q, R and S waves within a QRS complex as well as a P wave and a T wave. In this section, we aim at detecting these five waves automatically. To this end, several signal processing algorithms are studied and tested on different ECG.

Since the R wave is the part of the QRS complex with the maximum height, it can be detected more easily than the Q and the S waves. For this reason the first step is to focus on detecting R waves.

5.1.1 R wave detection

To detect R wave, we will use the Pan and Tompkins algorithm. This algorithm is named after its inventors J. Pan and W. Tompkins and can be found in [14]. The algorithm consists in the processing steps on the ECG signal presented in Figure 13.

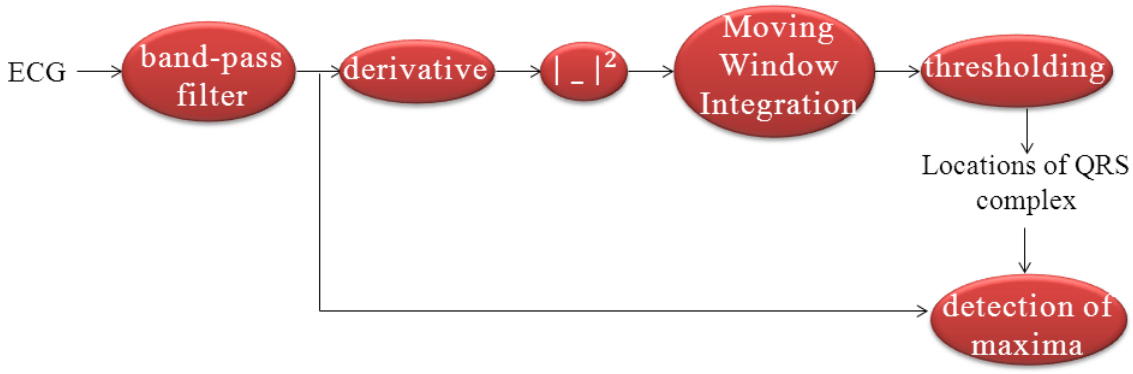


Figure 13: Pan and Tompkins algorithm

The band-pass filter is a combination of a low-pass filter and a high-pass filter. The transfer function of the low-pass filter is given by

$$H(z) = \frac{(1 - z^{-6})^2}{(1 - z^{-1})^2} \tag{3}$$

and the transfer function of the high-pass filter is given by

$$H(z) = \frac{(-1 + 32z^{-16} - 32z^{-17} + z^{-32})}{(1 - z^{-1})}. \tag{4}$$

After filtering, the signal is differentiated to provide the QRS complex slope information. A five-point differentiation filter is used, whose transfer function is:

$$H(z) = \frac{1}{8T_s}\left(-z^{-2} - 2z^{-1} + 2z^{1} + z^{2}\right) \tag{5}$$

The signal is then squared to intensify the local extrema. The next processing consists in a moving-window integration step. We denote s_{sq} the samples of the ECG after the squaring step and s_{MWI}

the output of the moving-window integration. Then :

$$s_{MWI}(n) = \frac{1}{N} \sum_{i=0}^{N-1} s_{sq}(n-i) \tag{6}$$

The width N of the window should be around the width of an average QRS complex. A step of thresholding is applied to the processed ECG to locate the QRS complex. Indeed, its temporal location is given by the rising slope of the integrated ECG. The last processing consists in detecting the maximum on the ECG signal which are located within the intervals found in the previous step.

Cautions : For your report your are asked to be precise on what you implement regarding your filters. For example, for the low-pass and high-pass filters, you will have to specify:

- its nature (low-pass, high-pass), and how does it impact the signal representation,
- its type (Finite Impulse Response (FIR), Infinite Impulse Response (IIR)),
- is it Causal or not ? If not explains the consequences on the implementation of the filter,
- its 3dB cut-off frequency,
- its group delay,
- if its phase is linear or not.

5.1.2 Q and S wave detection

Once the R waves are located, the Q and the S waves are defined as the first minimum before and after the R wave, respectively.

5.1.3 P and T wave detection

P and T waves can be detected by using the locations of the R waves. When considering an R-R interval, the T wave is assumed to be the highest peak between the first R peak and 0.7 times the R-R interval. We propose to detect the T wave using a three-step processing based on [15]. The first step consists in filtering the ECG signal with a differentiator, the transfer function of which is given by:

$$G_1(z) = 1 - z^{-6} \tag{7}$$

The second step is a low-pass filtering using the following transfer function :

$$G_2(z) = \frac{1 - z^{-8}}{1 - z^{-1}} \tag{8}$$

Finally, the detection of the T wave is achieved through detecting the signal crosses the level 0. The P wave is then the highest peak in the remaining section of the R-R interval.

6 Automatic identification of cardiac pathologies

In this section, we make use of the methods developed in the previous section to detect Q, R and S waves, and to detect classic cardiac pathologies.

6.1 Spectrogram analysis

As you have seen in *section 4*, this kind of processing may be the first step for identifying pathologies.

In this section, apply the spectrogram to *ecg_ normal_ 1.mat* and *ecg_ VF.mat* for windows of duration $4s$. Give your analysis of the respective spectrogram obtained and discuss the difference observed.

6.2 Tachycardia/Bradycardia

Basic arrhythmia pathologies, such as tachycardia, bradycardia, can be naively detected based on the inspection of the cardiac rhythm. Usually, bradycardia is declared when the cardiac rhythm falls under 60 bpm while tachycardia is characterized by a rhythm above 100 bpm. As stated in the introduction, cardiac rhythm is defined, for a normal ECG, as the duration of the R-R interval. We assume here that the detection of P, QRS and T waves has been done successfully.

The basic method for estimating the cardiac rhythm consists in computing the mean of all the occurrences of R-R intervals contained in the ECG signal considered. Assume that the ECG signal $s(t)$ contains N R peaks at times $t_0 < \ldots < t_N$. Define the n-th occurrence of the R-R interval as $\Delta_n = t_{n+1} - t_n$. Then the sample mean estimate of the R-R interval is given by

$$\overline{\Delta} = \frac{1}{N} \sum_{n=0}^{N-1} \Delta_n.$$

6.3 Other pathologies

6.3.1 Ectopic beat

Ectopic beats, caused by atrial or ventricular premature contraction, alter the duration of the R-R interval. More precisely, ectopic beats are characterized by the appearance of an early R wave, followed by a prolonged R-R interval until the next normal beat. This premature occurrence is usually detected by comparing to a certain threshold the difference of durations of the two R-R intervals respectively preceding and following the heartbeat studied; for example, for the test of n-th R peak, an ectopic beat is detected if the following inequality is verified

$$|\Delta_n - \Delta_{n-1}| \geq \epsilon,$$

where ϵ is a threshold value.

6.3.2 Fibrillation

Atrial fibrillation During atrial fibrillation, the process $(\Delta_n)_{n\geq 0}$ describing R-R intervals durations is usually modelled as white noise, and therefore exhibits in practice an almost flat spectral content [16]. In this situation, a statistical test can be computed to decide whether an ECG signal segment is under atrial fibrillation or not. This statistical test is based on the following feature

$$\hat{\gamma}_k = \frac{1}{N-k-1} \sum_{n=0}^{N-k-1} \left(\Delta_{n+k} - \overline{\Delta}\right)\left(\Delta_n - \overline{\Delta}\right),$$

which represents the sample auto-covariance function of the process $(\Delta_n)_{n\geq 0}$.

Ventricular fibrillation Episodes of ventricular fibrillation are usually characterized by a total absence of traditional P, Q, R, S and T waves, and by an ECG signal similar to a pure sine, with rapid oscillations between 240 to 600 bpm.

7 Bonus

As you may have remarked, the present version of the interface has many drawbacks and must be improved. Indeed, although it allows a clinician to display ECG signals, it requires him to have knowledge on signal processing (for example, the clinician as to choose a threshold and a window...)and it does not assist him for his diagnosis. Propose here new features allowing to analyse different types of pathologies. A work on the presentation of the GUI can be performed to have a new and more enjoyable interface for the user.

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