# Generative Models

2021. 8

Yongjin Jeong, KwangWoon University

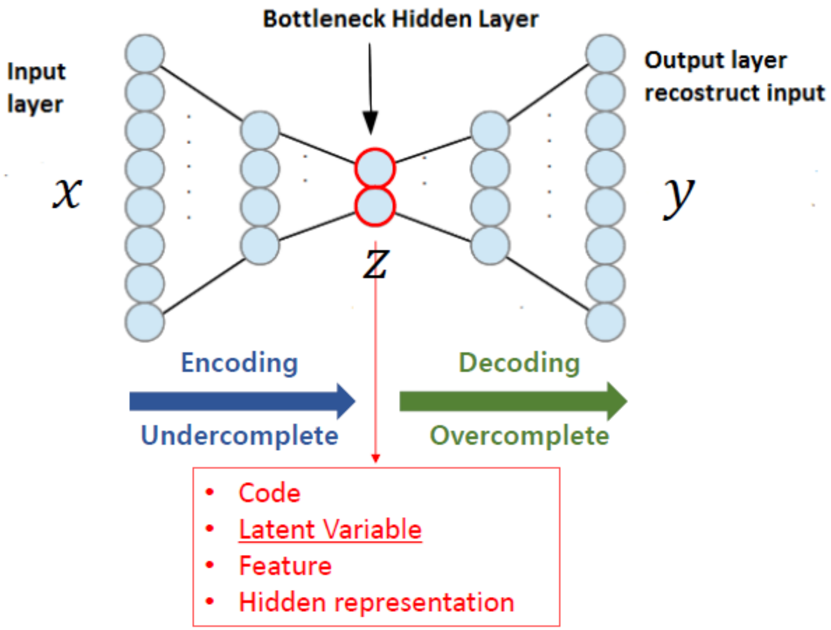
[참고] 본 자료에는 인터넷에서 다운받아 사용한 그림이나 수식들이 일부 있으니 다른 용도로 사용하거나 외부로 유출을 금해 주시기 바랍니다.

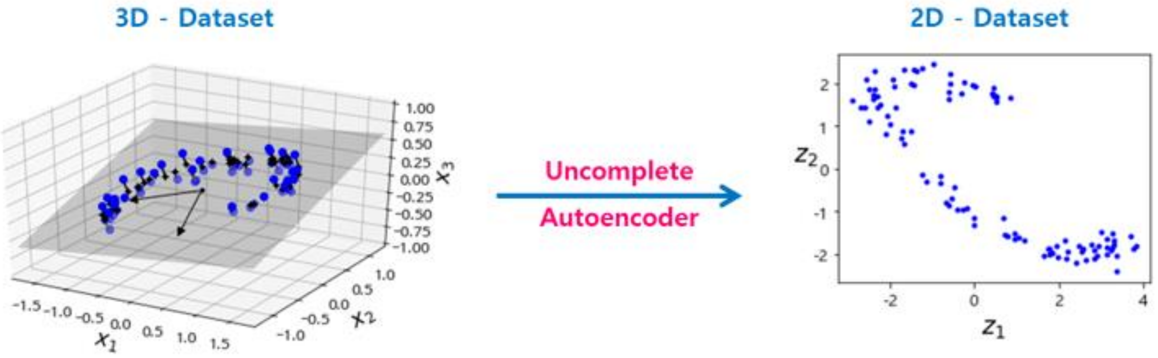
# Contents

- Autoencoders
- Variational Autoencoder (VAE)
- GANs

# Autoencoders

- ## Autoencoder
  - To learn a representation (encoding) for training data set (dimensionality reduction) by training the network to ignore signal "noise", in other words, feature extraction.
  - Focus on the encoder.



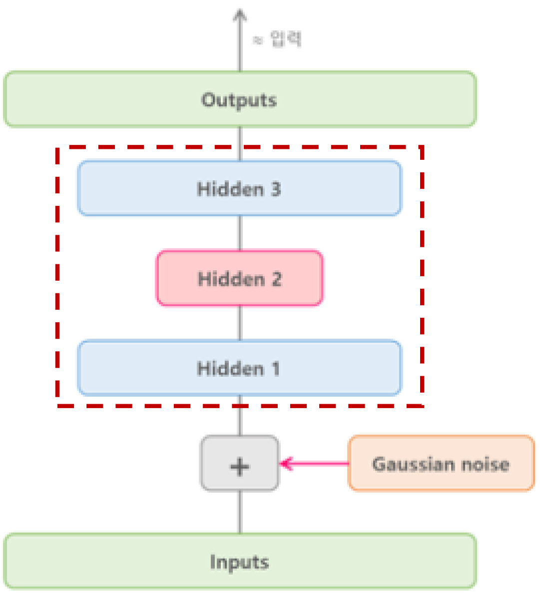

http://videolectures.net/deeplearning2015_vincent_autoencoders/?q=vincent%20autoencoder

# Autoencoders

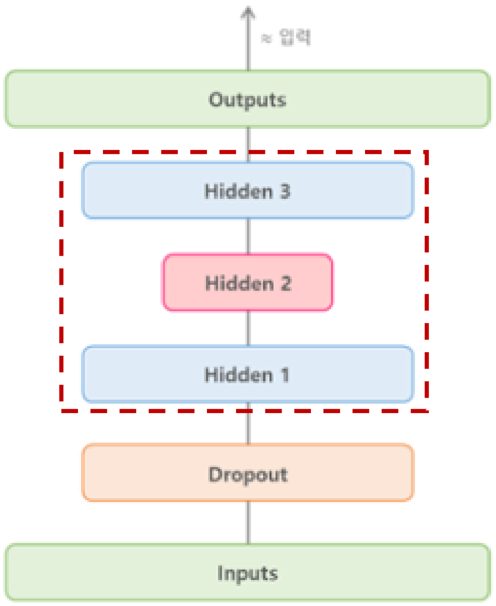
- **Applications of Autoencoders**
  - Dimensionality reduction
  - Anomaly detection
  - Image denoising
  - Image compression
  - Image generation

# Denoising Autoencoder

- **Denoising Autoencoder**
  - Add noise to the input and learn to reconstruct the original input without noise
  - Noise can be generated in random input units using <u>Gaussian Noise</u> or <u>Dropout</u>.




Denoising Autoencoder
With Gaussian noise



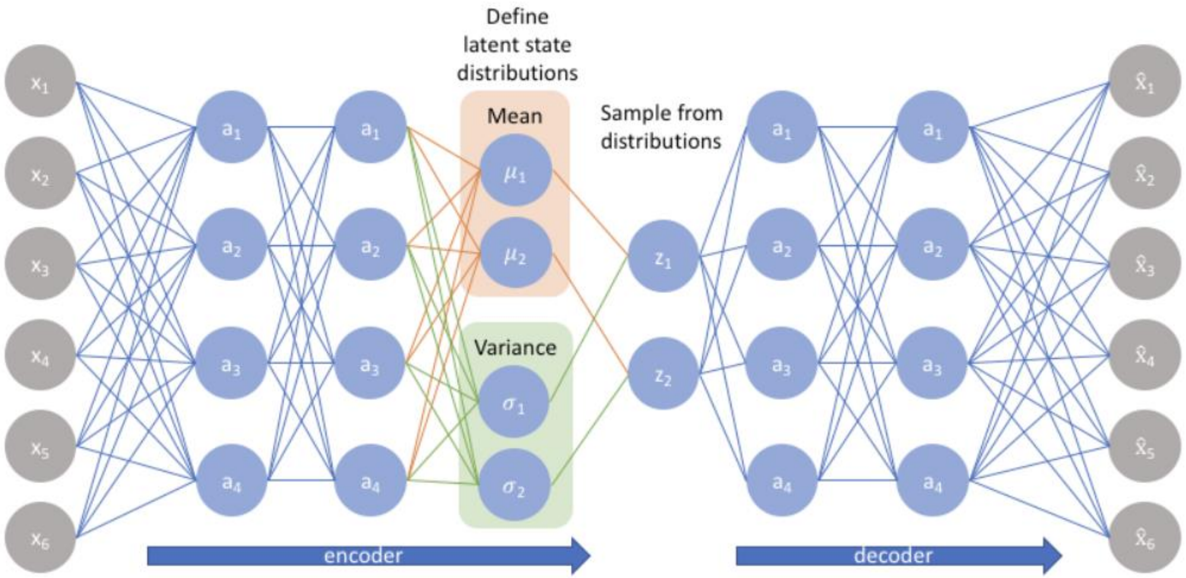

Denoising Autoencoder
With Dropout

# Stacked Autoencoder

- **Training One Autoencoder at a time**
  - Rather than training the whole stacked autoencoder in one go, it is much faster to train one shallow autoencoder at a time, then stack all of them into a single.

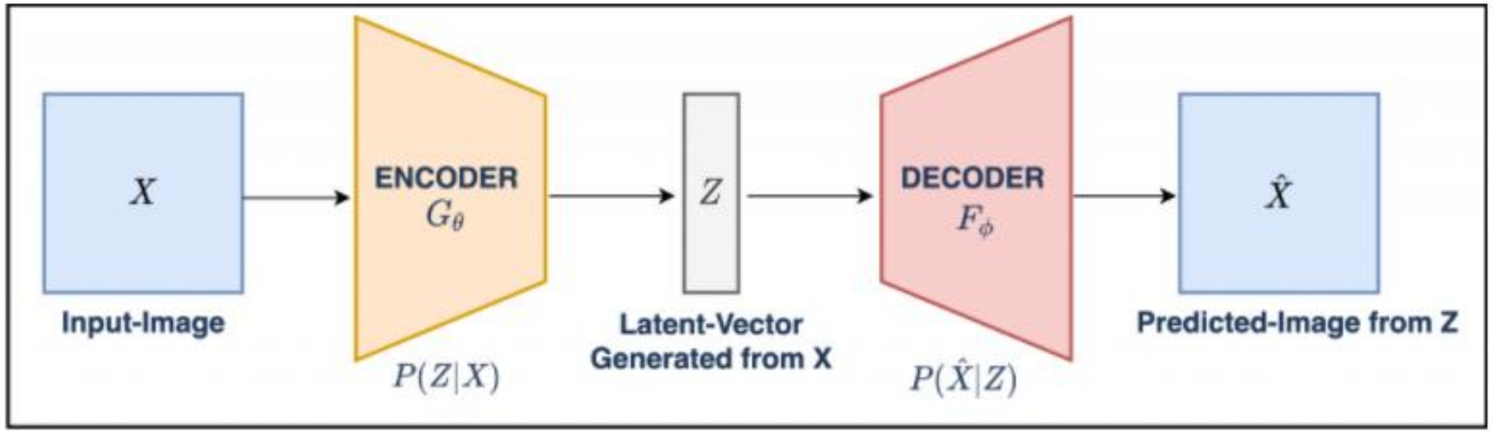
# Deep Learning

- **Variational Autoencoder (VAE)**
  - Provides a probabilistic manner for describing an observation in latent space
  - Encoder is formulated to describe a probability distribution for each latent attribute
  - Encoder is referred to as recognition model, and decoder is referred to as the generative model



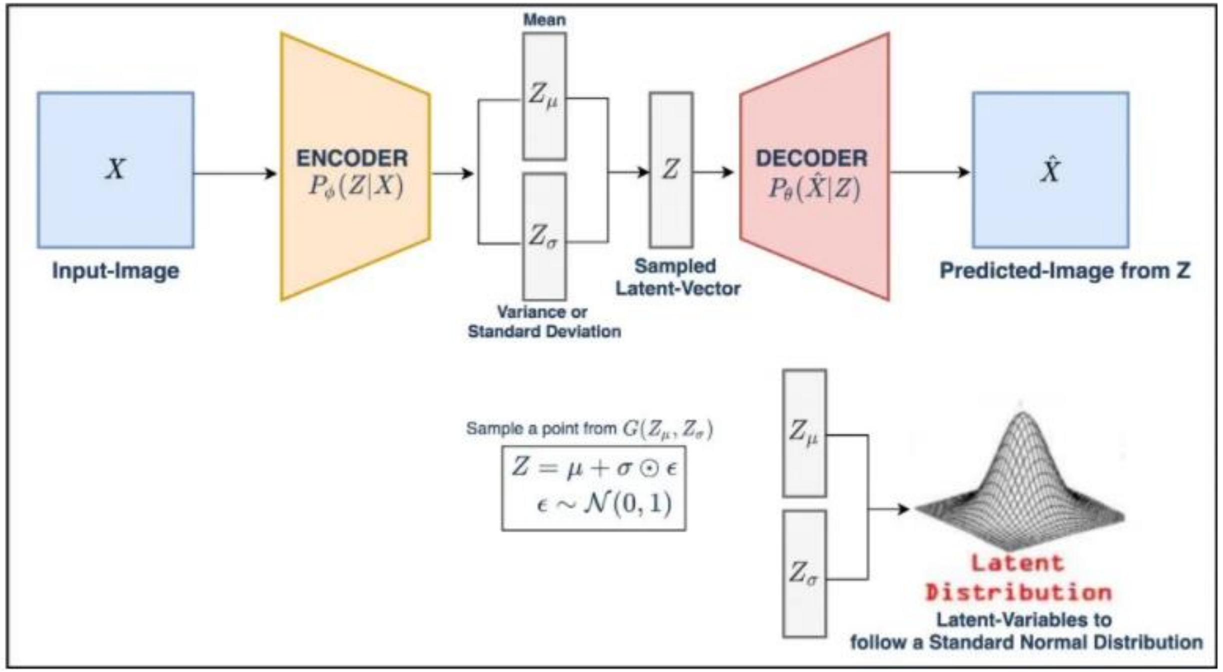
# Autoencoders

- **Auto-encoder**
  - **Reconstruction loss (MSE)** = $\frac{1}{N}\sum_{i=1}^{N}(X_i - \hat{X}_i)^2$ (N is number of images in a batch)
  - It is not good at generating new images because of its latent space structure:
    - The latent space was not continuous and did not allow easy interpolation.
    - Encoded vectors are grouped in clusters corresponding to different data classes, and there are huge gaps between the clusters.
    - While generating a new sample, the decoder often produces a gibberish output if the chosen point in the latent space did not contain any data

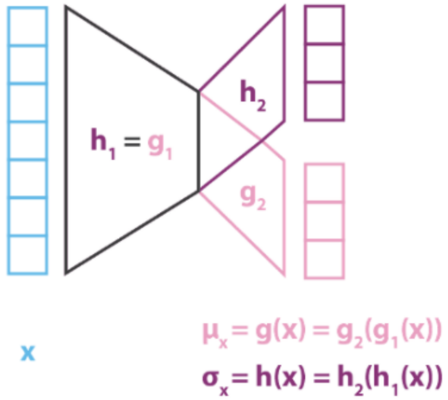


# Variational Autoencoder (VAE)

- A generative model (focus on the decoder)
- Instead of mapping the image on a point in space, the encoder of VAE maps the image onto a normal distribution.
- Sample a latent vector Z from two latent variables $Z_\mu$ and $Z_\sigma$ (also called a sampling-layer)
- We want the latent vector Z to follow a standard normal distribution. ( $Z_\mu$ and $Z_\sigma$ should be trained such that they are close to 0 and 1 respectively)

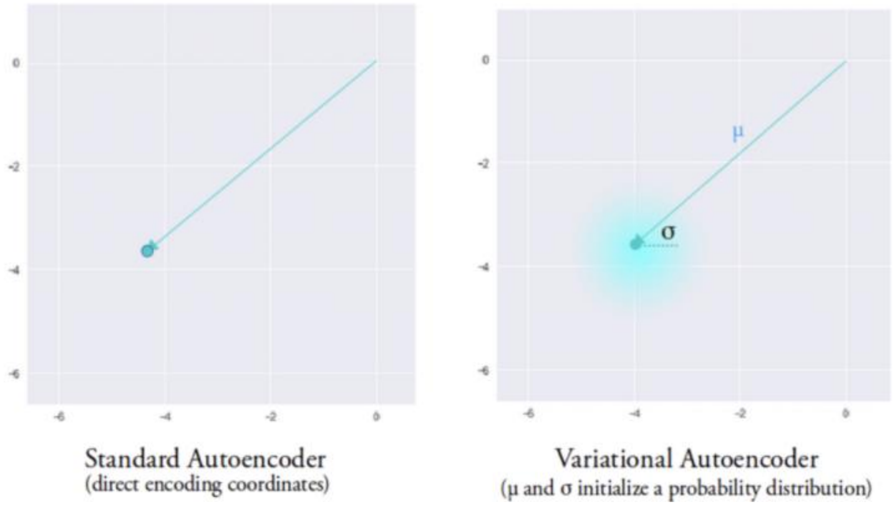



https://learnopencv.com/variational-autoencoder-in-tensorflow/



# Variational Autoencoder (VAE)

- **Discrete latent vector (Standard auto-encoder) and a standard normal distribution (VAE)**
  - Instead of a single point, the VAE covers a certain area: a sample from anywhere in the area will be very similar to the original input.




- **Why normal distribution?**
  - We assume that our dataset would inherently follow a distribution similar to the normal distribution.
  - Enforcing the latent variables to follow a normal distribution in VAE is very common and works the best.

# Variational Autoencoder (VAE)

- **Objective Function of VAE**
  - Unit Gaussian Distribution $\mathcal{N}(0,1)$, and
  - Minimize the reconstruction error $\frac{1}{N}\sum_{i=1}^{N}(X_i - \hat{X}_i)^2$.

- **VAE's total loss**
  - **Loss = Reconstruction_loss + KL_divergence_loss**

$$L_{MSE}(\theta,\phi) = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - f_\theta(g_\phi(x_i))\right)^2$$

$$L_{KL}[G(Z_\mu, Z_\sigma)\|\mathcal{N}(0,1)] = -0.5 * \sum_{i=1}^{N} 1 + log(Z_{\sigma_i}^2) - Z_{\mu_i}^2 - Z_{\sigma_i}^2$$

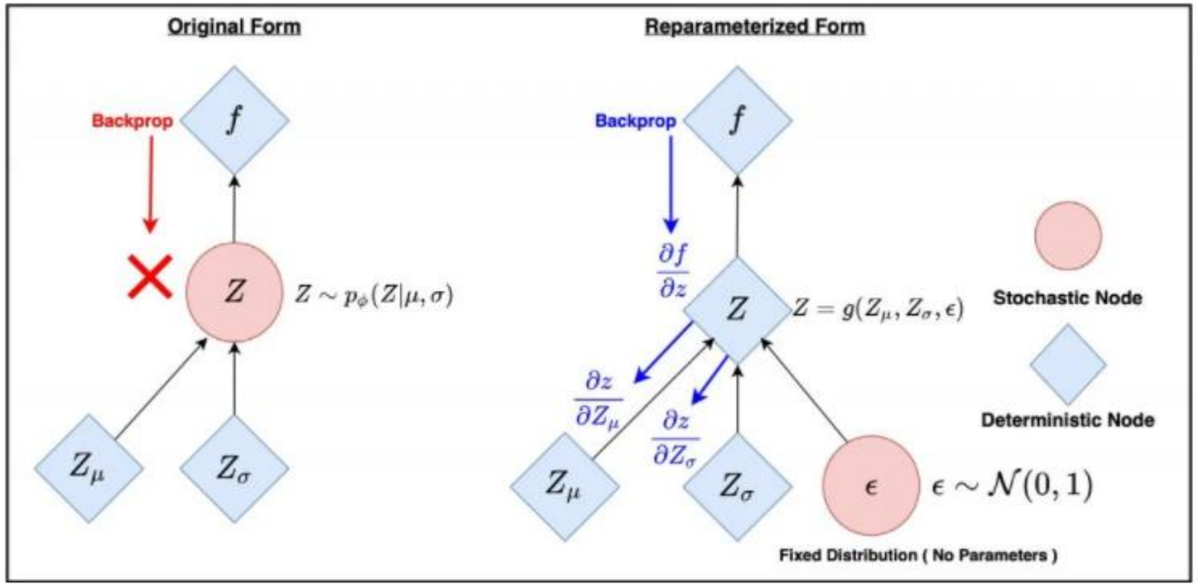
```
kl_loss =  -0.5 * numpy.sum(1 + numpy.log(Z_sigma ** 2) - numpy.square(Z_mean)
- numpy.exp(np.log(Z_sigma ** 2), axis = 1)

kl_loss =  -0.5 * numpy.sum(1 + numpy.log(Z_sigma ** 2) - numpy.square(Z_mean)
- Z_sigma ** 2, axis = 1)
```

# Variational Autoencoder (VAE)

- **Reparameterization Trick**
  - Sampling prevents backpropagation and their training.
  - Trick: convert the random node $\boldsymbol{Z}$ to a deterministic node ($\boldsymbol{Z}_{\mu}$ and $\boldsymbol{Z}_{\sigma}$ remain as the learnable parameters while still maintaining the stochasticity of the entire system via ε.)

$$\mathrm{Z} = \mathrm{Z}_{\mu} + Z_{\sigma}^{2} \odot \varepsilon$$

( Here, $\varepsilon \sim \mathcal{N}(0,1)$ and $\odot$ is element-wise multiplication. )



# Variational Autoencoder (VAE)

- KL Divergence
  - 두 확률 분포의 다른 정도 (= Relative Entropy)
  - 두 분포가 Gaussian 일 경우 간략하게 표현 가능

$$q_\theta(z|x_i) \rightarrow \frac{1}{\sqrt{2\pi\sigma_q^2}} \exp\left(-\frac{(x-\mu_q)^2}{2\sigma_q^2}\right)$$

$$p(z) \rightarrow \frac{1}{\sqrt{2\pi\sigma_p^2}} \exp\left(-\frac{(x-\mu_p)^2}{2\sigma_p^2}\right)$$

$$\sigma_p = 1 \text{ and } \mu_p = 0,$$

$$-D_{KL}(q_\theta(z|x_i)||p(z)) =$$

$$\int \frac{1}{\sqrt{2\pi\sigma_q^2}} \exp\left(-\frac{(x-\mu_q)^2}{2\sigma_q^2}\right) \log\left(\frac{\frac{1}{\sqrt{2\pi\sigma_p^2}} \exp\left(-\frac{(x-\mu_p)^2}{2\sigma_p^2}\right)}{\frac{1}{\sqrt{2\pi\sigma_q^2}} \exp\left(-\frac{(x-\mu_q)^2}{2\sigma_q^2}\right)}\right) dz$$

$$= \int \frac{1}{\sqrt{2\pi\sigma_q^2}} \exp\left(-\frac{(x-\mu_q)^2}{2\sigma_q^2}\right) \times$$

$$\left\{-\frac{1}{2}\log(2\pi) - \log(\sigma_p) - \frac{(x-\mu_p)^2}{2\sigma_p^2} + \frac{1}{2}\log(2\pi) + \log(\sigma_q) + \frac{(x-\mu_q)^2}{2\sigma_q^2}\right\} dz$$

# Variational Autoencoder (VAE)

- **VAE Loss function**

$$
\begin{aligned}
-D_{KL}(q_\theta(z|x_i)||p(z)) &= \log(\sigma_q) - \frac{\sigma_q^2 + \mu_q^2}{2} + \frac{1}{2} \\
&= \frac{1}{2}\log(\sigma_q^2) - \frac{\sigma_q^2 + \mu_q^2}{2} + \frac{1}{2} \\
&= \frac{1}{2}\left[1 + \log(\sigma_q^2) - \sigma_q^2 - \mu_q^2\right]
\end{aligned}
$$

$$
\mathcal{L} = -\sum_{j=1}^{J} \frac{1}{2}\left[1 + \log(\sigma_i^2) - \sigma_i^2 - \mu_i^2\right] - \frac{1}{L}\sum_{l} E_{\sim q_\theta(z|x_i)}\left[\log p(x_i|z^{(i,l)})\right]
$$

$$
(\theta^*, \phi^*) = argmin_{(\theta,\phi)}\mathcal{L}(\theta, \phi)
$$

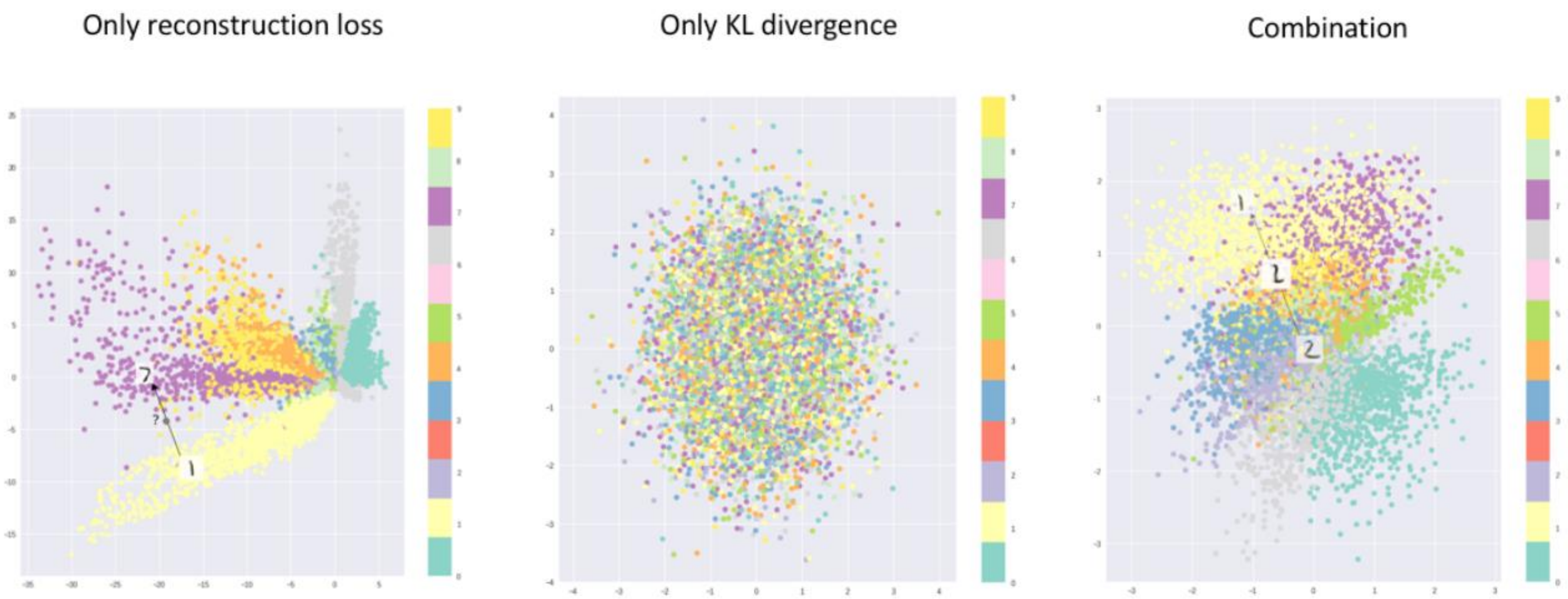
Cross entropy

L: number of samples
J: dim of latent vector z

Ref: https://arxiv.org/pdf/1907.08956.pdf

# Variational Autoencoder (VAE)

$$\mathcal{L}\left(x, \hat{x}\right) + \beta \sum_{j} KL\left(q_j\left(z|x\right) || N\left(0, 1\right)\right)$$



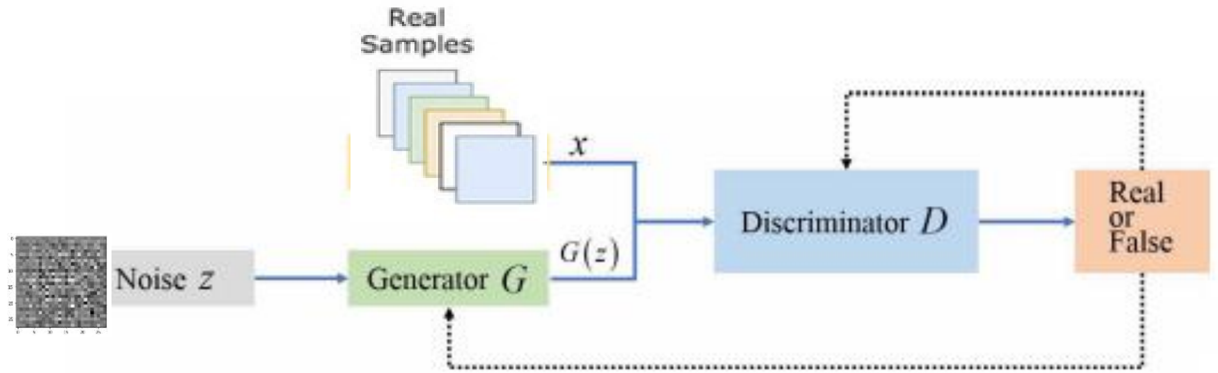

- Focus only on reconstruction loss, and allows the decoder be able to reproduce the original handwritten digits.
- there are areas in latent space which don't represent *any* of our observed data

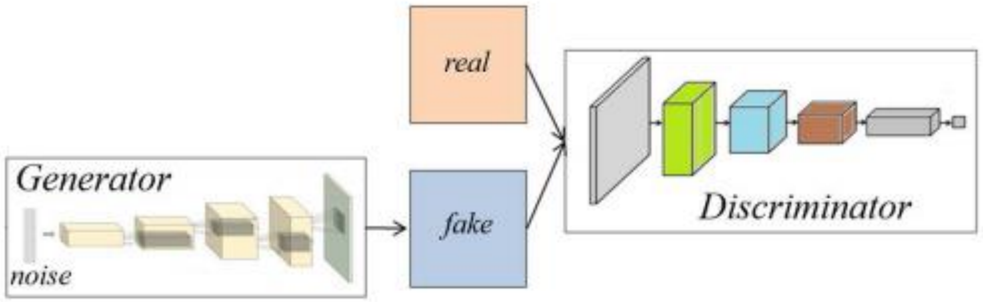
- Focus only on KL divergence loss, and describe every observation using the same Gaussian (e.g. same characteristics)
- failed to describe the original data

https://www.jeremyjordan.me/variational-autoencoders/

# Deep Learning

- **Generative Adversarial Network (GAN)**
    - Loss function: to make real(D(x)) close to 1, and fake(D(G(z)) close to 0
    - Discriminator: binary classifier (1: real, 0: fake)
    - Two separated optimizers




$$\min_G \max_D V(D, G) = \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}[\log D(\boldsymbol{x})] + \mathbb{E}_{z \sim p_{\text{noise}}}\left[\log\left(1 - D(G(z))\right)\right]$$

$$H_p(q) = -\frac{1}{N}\sum_{i=1}^{N} \underline{y_i \cdot log(p(y_i))} + \underline{(1 - y_i) \cdot log(1 - p(y_i))}$$

yi=1 의 loss function

yi=0 의 loss function

# GAN ?

- **Generative Adversarial Network (GAN)**

$$\min_G \max_D V(D, G) = \mathbb{E}_{x \sim P_{\text{data}}}[\log D(x)] + \mathbb{E}_{z \sim P_{\text{noise}}}\left[\log\left(1 - D(G(z))\right)\right]$$

```python
def discriminator_loss(real_output, fake_output):
    real_loss = cross_entropy(tf.ones_like(real_output), real_output)
    fake_loss = cross_entropy(tf.zeros_like(fake_output), fake_output)
    total_loss = real_loss + fake_loss
    return total_loss
```

true

pred

```python
def generator_loss(fake_output):
    return cross_entropy(tf.ones_like(fake_output), fake_output)
```

```python
generator_optimizer = tf.keras.optimizers.Adam(1e-4)
discriminator_optimizer = tf.keras.optimizers.Adam(1e-4)
```

[ref: https://www.tensorflow.org/tutorials/generative/dcgan?hl=ko]

# (Unconditional) GAN

$$\min_G \max_D V(D,G) = \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z})))]$$

Expectation

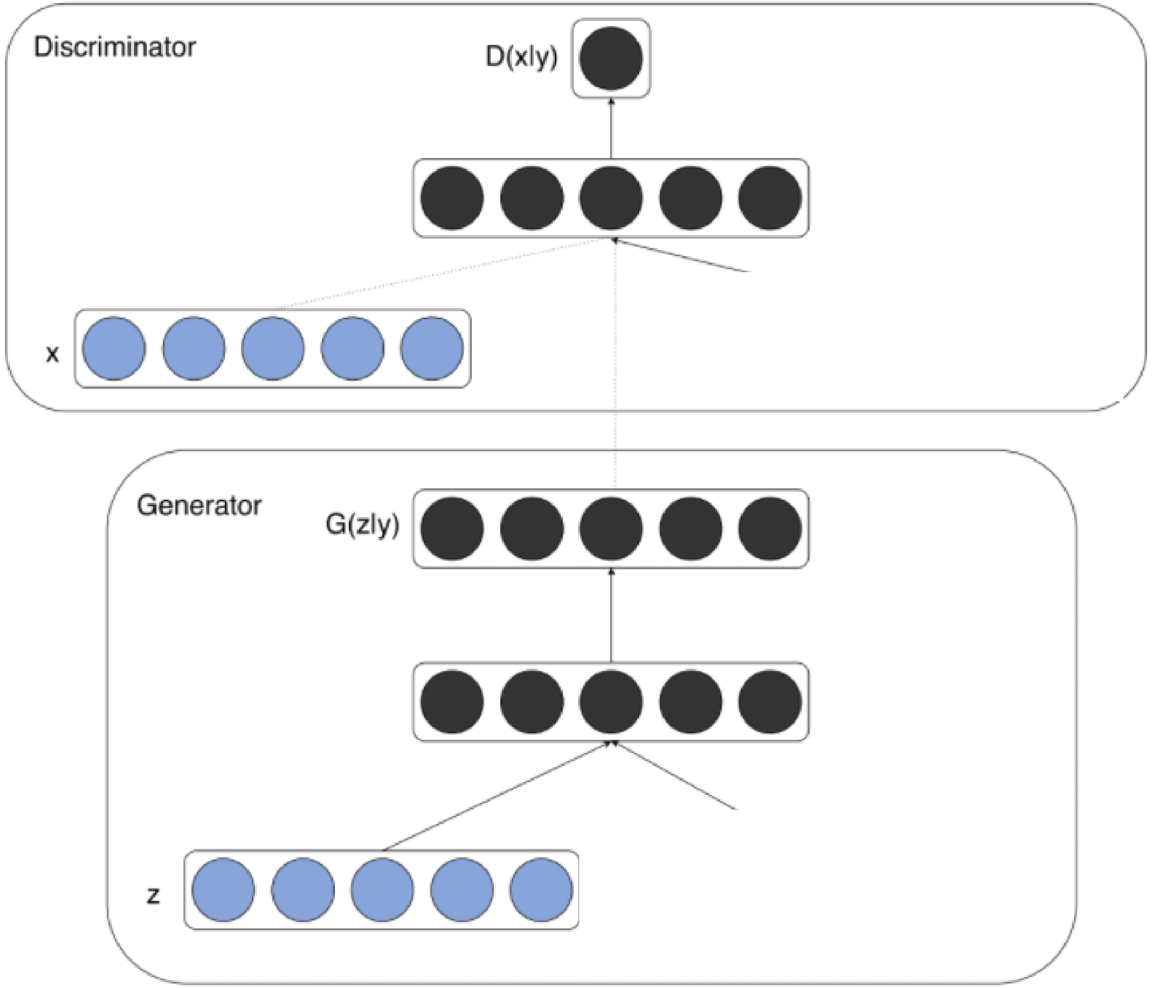
x is sampled from real data

Probability of D(real)

z is sampled from N(0, 1)

Probability of D(fake)

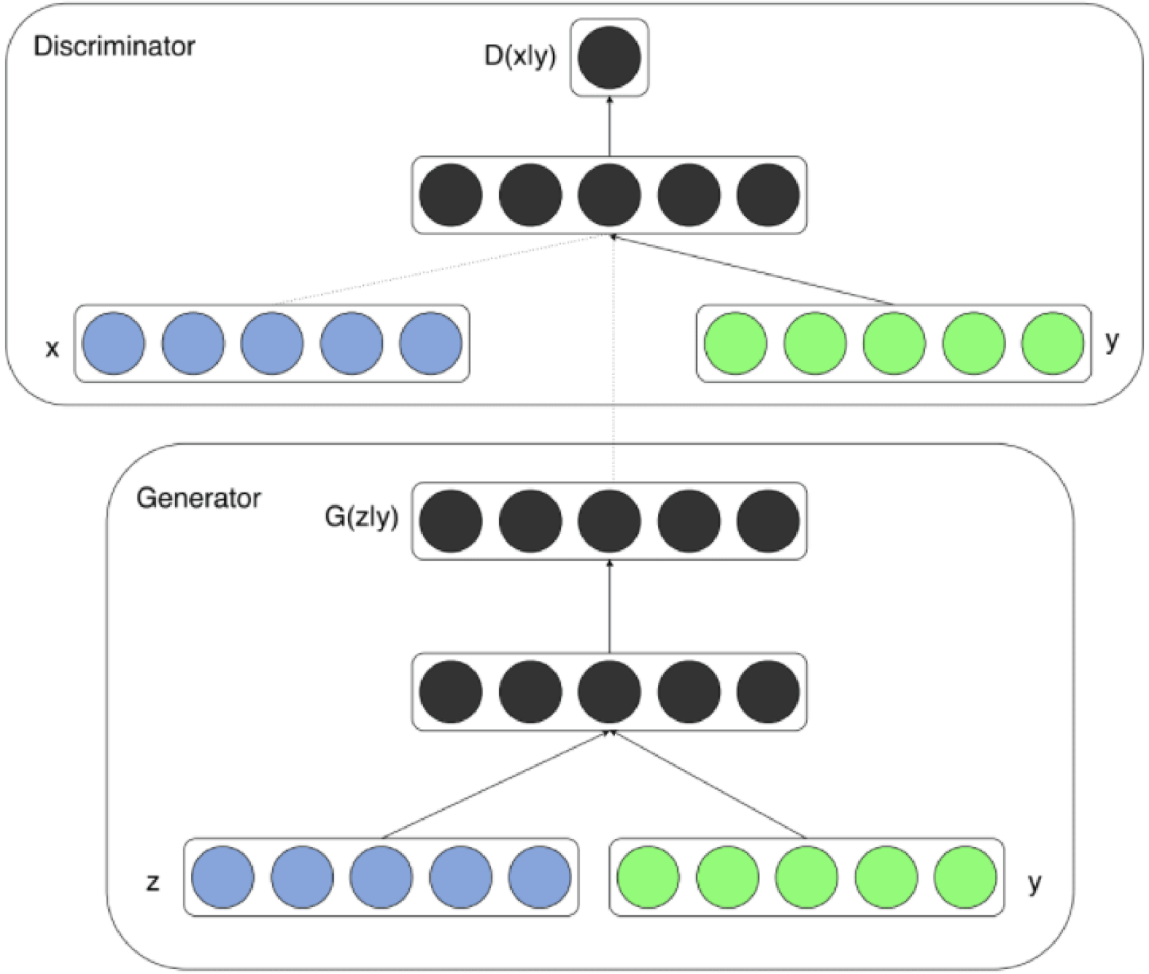
fake




Example of a Conditional Generator and a Conditional Discriminator in a Conditional Generative Adversarial Network.
Taken from Conditional Generative Adversarial Nets, 2014.

# Conditional GAN (cGAN)

$$min_G max_D V(D,G) = \mathbb{E}_{x\sim p_{data}(x)}[logD(x|y)] + \mathbb{E}_{x\sim p_z(z)}[log(1-D(G(z|y)))]$$




Example of a Conditional Generator and a Conditional Discriminator in a Conditional Generative Adversarial Network.
Taken from Conditional Generative Adversarial Nets, 2014.