INTRODUCTION TO LIE ALGEBRAS - EXERCISES

- 18. This is a continuation of Example 18 from the lectures. We define the G_{λ} 's and the form (-,-) on \mathfrak{t}^* by applying the procedure from the lectures to the trace form instead of the Killing form.
 - (i) Show that every $\lambda \in \mathfrak{t}^*$ can uniquely be written in the form $\sum_{i=1}^n \lambda_i \varepsilon_i$ with $\sum_{i=1}^{n} \lambda_i = 0.$ (ii) Describe G_{λ} and (-,-) in terms of the coordinates λ_i .

 - (iii) What are the squared lengths of the roots $\varepsilon_i \varepsilon_j$, $i \neq j$?
 - (iv) Describe the reflection $r_{\varepsilon_i-\varepsilon_j}$, $i \neq j$, in terms of the coordinates λ_k .