January 26, 2012 Lecturer Rudolf Tange Hilary Term 2012

Course 2318 2012

S h e e t 2

Due: see www.maths.tcd.ie/∼rtange/teaching/algebraic_geometry/algebraic_geometry.html

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Exercise 1

Show that the following set in the plane $\mathbb{R}^2$ is a quadric and determine its type:

(i) The set of points p which are at equal distances from a point and a line.

(ii) The set of points p with

$$d(p, a) + d(p, b) = C,$$

where a and b are given distinct points, $d(p, a)$ is the distance from p to a and C is a constant with $C > d(a, b)$.

(iii) The set of points p with

$$|d(p, a) - d(p, b)| = C,$$

where a and b are given distinct points, $d(p, a)$ is the distance from p to a and C is a constant with $0 \leq C < d(a, b)$.

Hint. Write the equation for the distances and eliminate square roots by rearranging the terms and repeatedly squaring both sides. In the case of (iii) you work with $\pm C$.

Exercise 2

Show by giving explicit transformations that any ellipse, hyperbola and parabola are projectively equivalent, i.e. can be transformed into each other.

Exercise 3

(i) Show that the set of lines through a fixed point p in the projective plane $\mathbb{P}^2_{\mathbb{C}}$ form a projective line.

(ii) Show that every projective curve intersects every line at least once. *Hint.* Use the lemma from the lectures about the zeros of a homogeneous form in X and Y.

(iii) Deduce from (i) and (ii) that every curve in $\mathbb{P}^2_{\mathbb{C}}$ has infinitely many points.