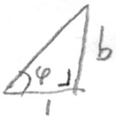
$\boxed{15}$ Rem: In questions 1, 2 and 3 I don't care too much about the arguments

(1) (i) $\boxed{3}$ The biggest domain for $\tan = \sin/\cos$ is $\mathbb{R} \setminus \{x \in \mathbb{R} \mid \cos(x) = 0\}$
$= \mathbb{R} \setminus \{\pi/2 + k\pi \mid k \in \mathbb{Z}\}$. The codomain is $\mathbb{R}$

(ii) $\tan(0) = \sin(0)/\cos(0) = 0/1 = 0$ and $\tan(\pi) = \sin(\pi)/\cos(\pi) = 0/{-1} = 0$.
So tan is not injective.
From the def of tan with rectangular triangles we deduce that tan is surjective: If $b \in \mathbb{R}^+$, $\tan(\varphi) = b$ where $\varphi$ is the angle




. Now use that $\tan(-x) = -\tan(x)$ and $\tan(0) = 0$

(iii) The restriction of tan to $(-\pi/2, \pi/2)$ is bijective. One can see this using the geometric arguments in (ii).

(2) $\boxed{3}$
- a divides b on $\mathbb{Z}$ is reflexive and transitive, but not symmetric
- $\neq$ on $\mathbb{C}$ is symmetric, but not reflexive and not transitive.
- "$\exists$ bijection between $A$ and $B$" on the subsets of a set $X$ is an equivalence relation (so it is reflexive, symmetric and transitive)

(3) $\boxed{4}$
- $\mathbb{R}^2$ with subtraction: not a semigroup: $1-(1-1) = 1-0 = 1$ and $(1-1)-1 = 0-1 = -1$
- nonnegative integers with minimum: this is a semigroup, but not a monoid: $(a \min b) \min c = \min\{a,b,c\} = a \min (b \min c)$, but if $e$ were a unit, the we would have $e \geq e \min a = a$ for all nonnegative integers. This is clearly impossible.
- The nonzero complex numbers form a group with multiplication
- The powerset $\mathcal{P}(S)$ of a set $S$ is a monoid with intersection as operation. The unit element is $S$. It is not a group if $S \neq \emptyset$.

(4) $\boxed{3}$ If $y \notin \text{range}(f)$, then $f^{-1}(\{y\}) = \emptyset$, so $|f^{-1}(\{y\})| = 0$.
So, after replacing $B$ by range$(f)$, we may assume that $f$ is surjective. Then $\{f^{-1}(\{y\}) \mid y \in B\}$ is a partition of $A$ from which the result immediately follows.

(5) $\boxed{2}$
- $(y^{-1}x^{-1})xy = y^{-1}y = e$ and $xy(y^{-1}x^{-1}) = xx^{-1} = e$, so $(xy)^{-1} = y^{-1}x^{-1}$.
- If $x^2 = x$, then $e = x^{-1}x = x^{-1}x^2 = x$.