$\boxed{15}$

(1) (i) $\boxed{6}$ $\boxed{3}$ If $g \in G$ has finite order $k \geq 1$, then the map $n \mapsto g^n : G \to G$ sends 0 and k to the unit 1 of G. So it is not injective.
Now assume the above map is not injective. Then there exist $k, l \geq 0$ such that $k \neq l$ and $g^k = g^l$. We may assume $k > l$. Then $g^l g^{k-l} = g^k = g^l = g^l \cdot 1$ and, cancelling g^l on both sides, we obtain $g^{k-l} = 1$. So g has finite order. If G is finite, then the given map can, of course, never be injective (see Section 7 from the lectures).

(ii) $\boxed{3}$ For $k, l \in \{0, \ldots, d-1\}$ we have $k + l < 2d$. If $k + l \geq d$, then
$g^{k+l} = g^d g^{k+l-d} = 1 \cdot g^{k+l-d} = g^{k+l-d}$ and $0 \leq k+l-d < d$.
In particular we have, for $k \in \{0, \ldots, d-1\}$, $g^k g^{d-k} = g^0 = 1$. So $(g^k)^{-1} = g^{d-k}$ and $g^{-1} = g^{d-1}$.
So $\{g^k \mid k \in \{0, \ldots, d-1\}\}$ is a subgroup of G.

(2) (i) $\boxed{5}$ $\boxed{2}$ $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix} = (1\,5)(2\,4)$

$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix} = (1\,5\,3)(2\,4)$

(ii) $\boxed{3}$ $\mathrm{sign}(\sigma) = 1$ (2 even length cycles) and $\mathrm{sign}(\tau) = -1$ (1 even length cycle)
$\sigma^2 = ((1\,5)(2\,4))^2 \underset{(15) \text{ and } (24) \text{ commute}}{=} (1\,5)^2 (2\,4)^2 = \mathrm{id}\ \mathrm{id} = \mathrm{id}$. Since $\sigma \neq \mathrm{id}$, it's order is 2
$\tau^6 \underset{(153) \text{ and } (24) \text{ commute}}{=} (1\,5\,3)^6 (2\,4)^6 = \mathrm{id}\ \mathrm{id} = \mathrm{id}$ and you can check that no smaller number does this. So the order is 6

(3) $\boxed{4}$ 1 of order 1: id
9 of order 2: $(1\,2), (1\,3), (1\,4), (2\,3), (2\,4), (3\,4), (1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3)$
8 of order 3: $(1\,2\,3), (1\,3\,2), (1\,2\,4), (1\,4\,2), (1\,3\,4), (1\,4\,3), (2\,3\,4), (2\,4\,3)$
6 of order 4: $(1\,2\,3\,4), (1\,2\,4\,3), (1\,3\,2\,4), (1\,3\,4\,2), (1\,4\,2\,3), (1\,4\,3\,2)$
$24 = 4!$ in total