

- 11.** For each of the following classical Lie algebras  $\mathfrak{g}$  over  $\mathbb{C}$  prove that the *centre* of  $\mathfrak{g}$  consists of the zero element only: if  $[Z, X] = 0$  for every  $X \in \mathfrak{g}$ , then  $Z = 0$ .
- (i)  $\mathfrak{g} = \mathfrak{sl}_n\mathbb{C}$ , the special linear Lie algebra with  $n = 3$ .
  - (ii)  $\mathfrak{g} = \mathfrak{so}_n\mathbb{C}$ , the orthogonal Lie algebra with  $n = 4$ .
- 12.** Let  $\mathfrak{g}$  be the *two-dimensional non-Abelian* Lie algebra over the field  $\mathbb{C}$ . It has two basis vectors  $E$  and  $F$  with the commutation relation  $[E, F] = E$ . Take any element  $X = aE + bF \in \mathfrak{g}$  with  $a, b \in \mathbb{C}$ . Compute the matrix of the linear operator  $\text{ad}_X : Y \mapsto [X, Y]$  in  $\mathfrak{g}$  relative to the basis  $E, F$ .