January 16, 2012 Lecturer Rudolf Tange Hilary Term 2012

Course 2318 2012

Sheet 1

Due: see www.maths.tcd.ie/~rtange/teaching/algebraic_geometry/algebraic_geometry.html

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Exercise 1

Which of the following sets can always be transformed into each other by a euclidean transformation?

(i) Two lines in the plane.

(ii) Two pairs of intersecting lines in the plane.

(iii) Two pairs of intersecting lines in the plane forming the same angles.

Justify your answer.

Exercise 2

Determine the type (in the classification: ellipse, hyperbola etc.) of the following quadric:

(i) $xy + x - 1 = 0$;

(ii) $x^2 - 2xy + y^2 + x - 1 = 0$;

(iii) $x^2 - 2xy + y^2 - 1 = 0$;

(iv) $x^2 + y^2 - x - y + 1 = 0$;

Exercise 3

In $\mathbb{R}^3$ consider two planes A and B given by $Z = 1$ and $X = 1$ respectively. Consider inhomogeneous coordinates $(x, y) = (X/Z, Y/Z)$ on A and $(y', z') = (Y/X, Z/X)$ on B. Let $L \subset A$ be the line given by $y = ax + b$.

(i) Describe by a homogeneous equation the projective line $\widetilde{L}$ in $\mathbb{P}^2$ induced by L, i.e. containing all the lines passing through 0 and intersecting L.

(ii) Give an equation for the line induced by $\widetilde{L}$ on the plane B using the coordinates (y', z').

(iii) What is the "infinity point" of $\widetilde{L}$, i.e. the point in $\mathbb{P}^2$ which does not correspond to any point of L?

(iv) For which a and b, the latter "infinity point" corresponds to a point of B?