

INTRODUCTION TO LIE ALGEBRAS – EXERCISES

4. For each of the following classical Lie algebras \mathfrak{g} over the field \mathbb{C} prove that the derived subalgebra \mathfrak{g}' of \mathfrak{g} coincides with \mathfrak{g} , i.e. $\mathfrak{g}' = \mathfrak{g}$ as vector spaces:
 - (i) $\mathfrak{g} = \mathfrak{sl}_n\mathbb{C}$, the special linear Lie algebra with $n = 3$.
 - (ii) $\mathfrak{g} = \mathfrak{so}_n\mathbb{C}$, the orthogonal Lie algebra with $n = 4$.
 - (iii) $\mathfrak{g} = \mathfrak{sp}_{2n}\mathbb{C}$, the symplectic Lie algebra with $n = 2$.
5. For $n \geq 1$ prove that the derived subalgebra \mathfrak{g}' of the general linear Lie algebra $\mathfrak{g} = \mathfrak{gl}_n\mathbb{C}$ coincides with the special linear Lie algebra $\mathfrak{sl}_n\mathbb{C}$, i.e. $\mathfrak{g}' = \mathfrak{sl}_n\mathbb{C}$ as vector subspaces of \mathfrak{g} . *Hint:* use an appropriate basis of the vector space \mathfrak{g} .