MA1214 Sheet 1

(1) (i) Give the (biggest) domain and a suitable codomain of the tangent function $\tan$.

   (ii) Is $\tan$ injective? Is it surjective?

   (iii) Now answer the same questions for the restriction of $\tan$ to the interval $(-\pi/2, \pi/2)$.

(2) State of each of the following binary relations whether it is reflexive, symmetric, and/or transitive.

- The relation "a divides b" on $\mathbb{Z}$.

- The relation $\neq$ on $\mathbb{C}$.

- The relation "there exists a bijection between A and B" on the set $\mathcal{P}(X)$ of subsets of a given set X ($\mathcal{P}(X)$ is called the *powerset* of X).

(3) State of each of these sets with a binary operation whether it is a group, a monoid but not a group, a semigroup but not a monoid, or not even a semigroup.

- $\mathbb{R}^2$ with subtraction.

- The set of nonegative integers with minimum as operation.

- The powerset $\mathcal{P}(S)$ of a set S with intersection as the operation.

- The set $\mathbb{C}^\times$ of nonzero complex numbers with multiplication.

(4) Let A and B be finite sets and let $f : A \to B$ be a map. Show that $|A| = \sum_{y \in B} |f^{-1}(y)|$.

(5) Prove that in any group $(xy)^{-1} = y^{-1}x^{-1}$ and show that the identity is the only element x that satisfies the equation $x^2 = x$.