

15 (1)(i) Recall that e_i has a one on the i th position and zeros elsewhere.

(2) Furthermore, the i th column of X is equal to $L_X(e_i)$.

Now X is upper triangular iff $\forall i \in \{1, \dots, n-1\}$ the i th column of X has zeros strictly below the i th position.

So X is upper triangular iff $\forall i \in \{1, \dots, n-1\} L_X(e_i) \in V_i$ iff

$$\forall i \in \{1, \dots, n-1\} L_X(V_i) \subseteq V_i.$$

(ii) Clearly $I \in B$. Furthermore, we have for $X, Y \in B$ and $i \in \{1, \dots, n-1\}$,

(2) $L_X(V_i) \subseteq V_i$ and $L_Y(V_i) \subseteq V_i$. So $L_{XY}(V_i) = L_X(L_Y(V_i)) \subseteq L_X(V_i) \subseteq V_i$

which means that $XY \in B$. Finally we have for $X \in B$ and $i \in \{1, \dots, n-1\}$

$$L_{X^{-1}}(V_i) = L_{X^{-1}}(L_X(V_i)) = V_i. \text{ So } X \in B.$$

(iii) $D^*D = \text{diag}(\bar{z}_1 z_1, \dots, \bar{z}_n z_n)$ and $\bar{z}_i z_i = |z_i|^2$.

(1) So D is unitary iff $\forall i \in \{1, \dots, n\} |z_i| = 1$.

(2) Reflections have determinant -1 , so they must occur on the list $-A$, A a rotation symmetry of the cube.

(5) Relative to a suitable basis A looks like $\begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix}$, where R

is a 2×2 rotation matrix. So $-A = \begin{bmatrix} -R & 0 \\ 0 & -1 \end{bmatrix}$. This is a reflection iff $-R = \text{id}$ iff $R = -\text{id}$, i.e. a rotation over π radians (180 degrees). Only the axes of type A_1 (4-fold, through the centres of opposite faces) and type A_2 (2-fold, through the centres of opposite edges) give such rotations.

Of course we get one reflection for each of these axes: the reflection plane is the plane orthogonal to the axis.

So the three axes of type A_1 produce the first three reflections and the six axes of type A_2 produce the other six.

(3)

(5)

	1	ρ	ρ^2	r	ρr	$\rho^2 r$
1	1	ρ	ρ^2	r	ρr	$\rho^2 r$
ρ	ρ	ρ^2	1	ρr	$\rho^2 r$	r
ρ^2	ρ^2	1	ρ	$\rho^2 r$	r	ρr
r	r	$\rho^2 r$	ρr	1	ρ^2	ρ
ρr	ρr	r	$\rho^2 r$	ρ	1	ρ^2
$\rho^2 r$	$\rho^2 r$	ρr	r	ρ^2	ρ	1

← only this row is not straightforward!