14. Let $\mathfrak{g}$ be any Lie algebra over the field $\mathbb{C}$. Show that the following are equivalent:

(i) the Lie algebra $\mathfrak{g}$ is solvable;

(ii) there exist ideals $\mathfrak{g}_0 \supset \mathfrak{g}_1 \supset \ldots \supset \mathfrak{g}_n$ *of* $\mathfrak{g}$ such that $\mathfrak{g}_0 = \mathfrak{g}$, $\mathfrak{g}_n = \{0\}$, and for every index $i = 0, \ldots, n-1$ the factor Lie algebra $\mathfrak{g}_i/\mathfrak{g}_{i+1}$ is Abelian.

15. Let $\mathfrak{g}$ be any Lie algebra over a field $\mathbb{F}$. Consider the image $\operatorname{ad}\mathfrak{g}$ of the homomorphism $\operatorname{ad} : \mathfrak{g} \to \operatorname{Der}(\mathfrak{g})$. This image is a Lie subalgebra in $\operatorname{Der}(\mathfrak{g})$, the Lie algebra of all derivations of $\mathfrak{g}$. Prove that the Lie algebra $\mathfrak{g}$ is solvable or nilpotent if and only if this holds for the Lie algebra $\operatorname{ad}\mathfrak{g}$.