MA1214 Sheet 2

(1). Let G be a group. The *order* of an element $g \in G$ is defined as the smallest (strictly) positive integer k such that $g^k = 1$ if it exists and ∞ otherwise. For convenience, we put $g^0 = 1$.

(i) Show that $g \in G$ has finite order if and only if the map $n \mapsto g^n : \mathbb{N} \to G$ is not injective. Deduce that if G is finite (i.e. G has finitely many elements) all its elements have finite order.

(ii) Let $g \in G$ have finite order d. Show that $\{g^k \mid k \in \{0, \ldots, d-1\}\}$ is closed under multiplication and is group with this multiplication. What is the inverse of g?

(2). Consider the elements $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix}$ of S_5.

(i) Write σ and τ in disjoint cycle notation.

(ii) Determine the order and the sign of σ and τ.

(3). Write all elements of S_4 into disjoint cycle notation and group them according to order.