MA1214 Sheet 6

(1) The quaternion group Q_8 has elements $\pm 1, \pm i, \pm j, \pm k$. The elements -1 and 1 (the unit) commute with everything, $(-1)^2 = 1$, $(-1)i = -i$, $(-1)(-i) = i$ and the same for j and k. Furthermore, $i^2 = j^2 = k^2 = ijk = -1$.

(i) Compute the products ij, ji, ik, ki, jk, kj.

(ii) Is Q_8 isomorphic to the dihedral group D_8? Explain your answer.

(2) Compute the Cayley table of $\mathbb{Z}_{12}^*$.

(3) Partition the following list into isomorphism classes.

(a) The Klein four group $V_4 = \{1, a, b, c\}$ where $a^2 = b^2 = c^2 = 1$.

(b) $\mathbb{Z}_4$.

(c) $\mathbb{Z}_{12}^*$.

(d) $\mathbb{Z}_2 \times \mathbb{Z}_2$.

(4) Use the extended Euclidean algorithm to find integers $r, s \in \mathbb{Z}$ with $r*125+s*23 = \gcd(125, 23)$. Deduce that 23 is invertible modulo 125 and find its inverse.