MA1214 Sheet 3

(1) Let n be a positive integer and let B be the set of invertible upper triangular $n \times n$ matrices with complex entries.

(i) Denote the linear map $\mathbb{C}^n \to \mathbb{C}^n$ corresponding to the $n \times n$ matrix X by L_X. If we identify $\mathbb{C}^n$ with column vectors, then L_X is given by $L_X(v) = Xv$, where the product Xv is the usual matrix multiplication. Let $(e_1, \ldots, e_n)$ be the standard basis of $\mathbb{C}^n$. For $i \in \{1, \ldots, n-1\}$ let V_i be the linear span of $\{e_1, \ldots, e_i\}$. Show that X is upper triangular if and only if $L_X(V_i) \subseteq V_i$ for all $i \in \{1, \ldots, n-1\}$.

(ii) Use (i) to show that B is a subgroup of $GL(n)$. *Hint.* You may use that if X is invertible, $L_X(V_i) \subseteq V_i$ implies $L_X(V_i) = V_i$.

(iii) Let $D = \mathrm{diag}(z_1, \ldots, z_n)$ be a diagonal matrix. What is the condition on the z_i for D to be unitary?

(2) Recall the 48 symmetries of the cube mentioned in the lectures. There are 24 rotations about three types of axes and there are 24 isometries of the form $-\rho$, where ρ is one of the 24 rotation symmetries. Consider the reflections in the planes through the centre of the cube which are parallel to a face of the cube. There are three such planes. Where on the list of 48 symmetries do the corresponding reflections occur? Answer the same question for the six planes passing through opposite edges.

(3) Consider the Dihedral group D_6 of symmetries of the triangle. Complete the Cayley table below, using the relation $r\rho r = \rho^{-1} = \rho^2$ (or: $r\rho = \rho^2 r$), and of course $\rho^3 = 1$ and $r^2 = 1$.

	1	ρ	ρ^2	r	ρr	$\rho^2 r$
1						
ρ						
ρ^2						
r						
ρr						
$\rho^2 r$						