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5 (i)

	2	1	2	6	
357	133	91	42	7	0
1	0	1	-1	3	
0	1	-2	3	-8	

So $\gcd(357, 133) = 7$ and $3 \times 357 - 8 \times 133 = 7$

(2)(i) [2] Recall that $p \in \mathbb{Z}$ is called prime if $p > 1$ and 1 and p are its only positive divisors. Now let $a, b \in \mathbb{Z}$ be nonzero and assume that $p \mid ab$, but $p \nmid a$. Since $\gcd(p, a)$ divides p, it is either 1 or p. But it also divides a and $p \nmid a$, so $\gcd(p, a) = 1$. So, by the given result, $p \mid b$.

(ii) [3] By induction on n we show that n can be written as a product of primes: It is clear for 1 (empty product), so assume $n > 1$. If n is prime, it is obvious; otherwise we can write $n = lm$, for certain integers l, m with $1 < l, m < n$. By the IH they can be written as a product of primes, but then, of course, n as well.
Uniqueness: If $n = p_1^{k_1} \cdots p_r^{k_r} = q_1^{l_1} \cdots q_s^{l_s}$ are two prime decompositions of n, then, by applying (i) repeatedly, p_1 must occur among the q_i. after renumbering the q_i we may assume $p_1 = q_1$. Then $n/p_1 = p_1^{k_1 - 1} p_2^{k_2} \cdots p_r^{k_r} = q_1^{l_1 - 1} q_2^{l_2} \cdots q_s^{l_s}$ and we can finish again by induction.

(iii) [2] We have $\mathbb{Z}a \cap \mathbb{Z}b = \operatorname{lcm}(a, b)\mathbb{Z}$.
So $a, b \mid m \Leftrightarrow m \in \mathbb{Z}a \cap \mathbb{Z}b \Leftrightarrow \operatorname{lcm}(a, b) \mid m$.

(iv) [3] If $a = p_1^{k_1} \cdots p_r^{k_r}$ and $b = p_1^{l_1} \cdots p_r^{l_r}$, the p_i distinct primes and the k_i and l_i integers ≥ 0. Then $\gcd(a, b) = \prod_{i=1}^{r} p_i^{\min(k_i, l_i)}$ and $\operatorname{lcm}(a, b) = \prod_{i=1}^{r} p_i^{\max(k_i, l_i)}$. Now $\min(k_i, l_i) + \max(k_i, l_i) = k_i + l_i$, so $\gcd(a, b) \cdot \operatorname{lcm}(a, b) = \prod_{i=1}^{r} p_i^{k_i + l_i} = ab$.