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(1)(i) $ijk = k^2$, so cancelling k on both sides gives $\underline{ij = k}$ (a)

$\boxed{3}$ Multiplying (a) on the left by i gives $\underline{ik = -j}$ (b)

Multiplying (a) on the right by j gives $\underline{kj = -i}$ (c)

Multiplying (b) on the right by k gives $\underline{jk = i}$ (d)

Multiplying (c) on the left by k gives $\underline{ki = j}$

Multiplying (d) on the left by j gives $\underline{ji = -k}$

(ii) Both groups have a centre consisting of 2 elements, so

$\boxed{3}$ their centre does not distinguish them. However, Q_8 has 6 elements of order 4 and 1 of order 2 and D_8 has 2 elements of order 4 (p and p^3) and 5 of order 2. So Q_8 and D_8 are not isomorphic.

(2)

$\boxed{3}$

	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

So all non unit elements in $\mathbb{Z}_{12}^\times$ have order 2

(3) $\{(a), (c), (d)\}$, $\{(b)\}$

$\boxed{3}$

(4)

$\boxed{3}$

$$\begin{array}{ccccc} & 5 & 2 & 3 & \\ 125 & 23 & 10 & 3 & 1 \\ 1 & 0 & 1 & -2 & 7 \\ 0 & 1 & -5 & 11 & -38 \end{array}$$

So $\gcd(125, 23) = 1 = 7 \times 125 - 38 \times 23$.

So 23 is invertible modulo 125 and its inverse is $-38 \equiv 87 \pmod{125}$