Statistical Inference: Course Project

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Part II: Tooth Growth

This paper will explore the `ToothGrowth` data in the R data sets package. In particular, the objective is to use confidence intervals and/or hypothesis tests to compare tooth growth by supp and dose.

Discussion of the Data and Experimental Design

The `?ToothGrowth` page gives insight into the data set, which originates from a study by C. I. Bliss in *The Statistics of Bioassay*, (1952). Three variables describe the length of odontoblasts (teeth) in each of 10 guinea pigs at each of three dose levels of Vitamin C (0.5, 1, and 2 mg) with each of two delivery methods (orange juice or ascorbic acid):

- **len:** Tooth length
- **supp:** Supplement type (VC or OJ)
- **dose:** Dose in milligrams

To begin the analysis, we load the data and run summaritive printouts:

```
library(datasets); data(ToothGrowth);
summary(ToothGrowth)
```

```
##       len          supp         dose
##  Min.   : 4.20   OJ:30   Min.   :0.500
##  1st Qu.:13.07   VC:30   1st Qu.:0.500
##  Median :19.25           Median :1.000
##  Mean   :18.81           Mean   :1.167
##  3rd Qu.:25.27           3rd Qu.:2.000
##  Max.   :33.90           Max.   :2.000
```

```
table(ToothGrowth$supp, ToothGrowth$dose)
```

```
##
##      0.5  1  2
##   OJ  10 10 10
##   VC  10 10 10
```

The data conforms to the expectations from reading the help file. In particular, we note that the sample sizes are equally distributed across the variables `supp` and `dose`. Although the help file mentions that there were only 10 guinea pigs, and that the data fits nicely into 10-sample subsets, there is no identifier variable for each guinea pig. As a result, I will assume throughout this paper that the data is not structured in a paired order, and accordingly conduct independent-group hypothesis tests. It is also assumed that data is independent and identically distributed (IID) *within groups*, and that the variance between these groups unequal (the more conservative assumption).

To begin the analysis, I plot the pairs of variables against one another:

```
pairs(ToothGrowth)
```
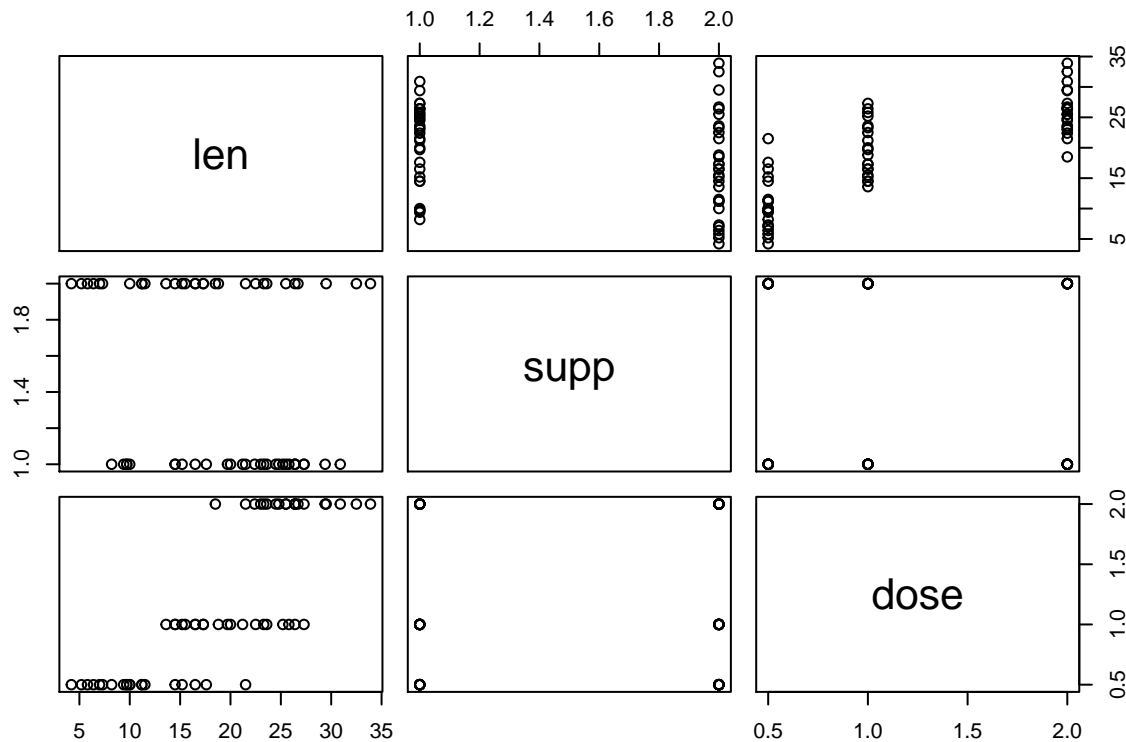

At first glance, it appears that there is a strong positive relationship between tooth length and dose. A potential relationship between the length and supplement type is less apparent.

Setting Up Our Tests

The first step is to set a decision rule, chose the appropriate tests, and state our null and alternate hypotheses. According to the assumptions made in the introductory paragraphs, my choice of test is an independent, two-sided t-test. Although the plot in the exploratory phase suggests that tooth growth increases directly proportionate to dosage, it would have been equally plausible and meaningful if the inverse was true (decreasing dosage $\implies$ increased length). So as not two bias the hypothesis test with a lower tolerance resulting from having already explored the data visually, I justify the two-sided alternative as the appropriate option.

I would normally set the family-wise tolerance at $\alpha = 0.05$, and accordingly make bonferroni corrections (i.e. divide α by the number of tests). However, given these adjustments were not covered in class, and that 95% confidence intervals are requested, I will take a different approach. Instead of testing each possible pair ($\binom{6}{2} = 15$ total), I will be more selective in which pairs to test, chosing only the tests that we would reasonably expect to have differences. For this, I choose to test only three hypotheses:

1. In the first test, I would like to test if there is a difference in the pair of $0.5mg$ and $2mg$ dose groups for the guinea pigs that were given orange juice. The null hypothesis is that $H_0 : \mu_{0.5mg} = \mu_{2mg}$ and the alternate hypothesis is $H_a : \mu_{0.5mg} \neq \mu_{2mg}$.

2. In the second test, I explore the same idea, but for the guinea pigs that were administered the ascorbic acid supplement. The null hypothesis is that $H_0 : \mu_{0.5mg} = \mu_{2mg}$ and the alternate hypothesis is $H_a : \mu_{0.5mg} \neq \mu_{2mg}$.

3. In the third test, I investigate whether the choice of supplement type affects tooth growth. In order to do this, we must control for dosage. There is no basis for pooling all dosage groups together since

this is a case control study, and that aggregating all groups together would undermine the assumption the IID data. I have randomly chosen the group with a dosage of $1mg$. The null hypothesis is that $H_0 : \mu_{OJ} = \mu_{VC}$ and the alternate hypothesis is $H_a : \mu_{OJ} \neq \mu_{VC}$.

Conclusions

We set up each vector of tooth growth by supplement and dose.

```
OJ1 = subset(ToothGrowth$len, ToothGrowth$supp == "OJ" & ToothGrowth$dose == 0.5)
OJ2 = subset(ToothGrowth$len, ToothGrowth$supp == "OJ" & ToothGrowth$dose == 1)
OJ3 = subset(ToothGrowth$len, ToothGrowth$supp == "OJ" & ToothGrowth$dose == 2)
VC1 = subset(ToothGrowth$len, ToothGrowth$supp == "VC" & ToothGrowth$dose == 0.5)
VC2 = subset(ToothGrowth$len, ToothGrowth$supp == "VC" & ToothGrowth$dose == 1)
VC3 = subset(ToothGrowth$len, ToothGrowth$supp == "VC" & ToothGrowth$dose == 2)
```

We use a t.test for each of the three hypotheses we wish to test:

```
t1 = t.test(OJ3, OJ1, alternative = "two.sided", var.equal = FALSE)
t2 = t.test(VC3, VC1, alternative = "two.sided", var.equal = FALSE)
t3 = t.test(OJ2, VC2, alternative = "two.sided", var.equal = FALSE)
cbind(t1 = t1$p.value, t2 = t2$p.value, t3 = t3$p.value)
```

```
##                 t1           t2          t3
## [1,] 1.323784e-06 4.681577e-08 0.001038376
```

The p-values of each test fall below the threshold of .05. We therefore reject the null hypothesis in all three cases and report that:

1. On average, guinea pigs that were given higher doses of vitamin C ($2mg$) through orange juice experienced greater tooth growth (M = 26.06) than guinea pigs given lower doses of $0.5mg$ through orange juice (M = 13.23). This difference was significant $t(14.668) = 7.817$, $p < .01$.

2. On average, guinea pigs that were given higher doses of vitamin C ($2mg$) through ascorbic acid delivery experienced greater tooth growth (M = 26.14) than guinea pigs given lower doses of $0.5mg$ through ascorbic acid delivery (M = 7.98). This difference was significant $t(14.327) = 10.3878$, $p < .01$.

3. On average, guinea pigs that were given $2mg$ of vitamin C experienced greater tooth growth when this was adminstered through orange juice (M = 22.70) compared with ascorbic acid delivery (M = 16.774). This difference was significant $t(15.358) = 4.0328$, $p < .01$.

The full results of each t-test are given in the appendix.

Finally, the Gosset t-intervals are given by:

```
rbind(t1 = t1$conf.int, t2 = t2$conf.int,  t3 = t3$conf.int)
```

```
##          [,1]      [,2]
## t1  9.324759 16.335241
## t2 14.418488 21.901512
## t3  2.802148  9.057852
```

None of the 95% intervals cross over zero. This makes sense, given that the 95th t-quantile *is* the rejection threshold when $\alpha = .05$.

Appendix A: Full Results of the t-Tests

The full printout of the t-test for each of our three tests is given below:

t1

```
##
##  Welch Two Sample t-test
##
## data:  OJ3 and OJ1
## t = 7.817, df = 14.668, p-value = 1.324e-06
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##   9.324759 16.335241
## sample estimates:
## mean of x mean of y
##     26.06     13.23
```

t2

```
##
##  Welch Two Sample t-test
##
## data:  VC3 and VC1
## t = 10.3878, df = 14.327, p-value = 4.682e-08
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  14.41849 21.90151
## sample estimates:
## mean of x mean of y
##     26.14      7.98
```

t3

```
##
##  Welch Two Sample t-test
##
## data:  OJ2 and VC2
## t = 4.0328, df = 15.358, p-value = 0.001038
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  2.802148 9.057852
## sample estimates:
## mean of x mean of y
##     22.70     16.77
```