

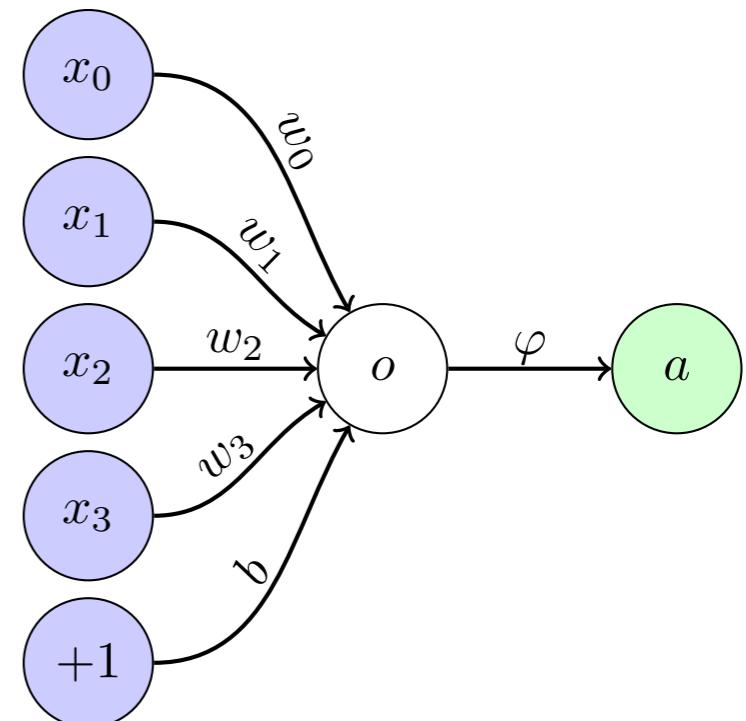
Deep Learning

2. Multi Layer Perceptrons

A course @EDHEC
by Romain Tavenard (Prof. @Univ. Rennes 2)

Limitations of the Perceptron

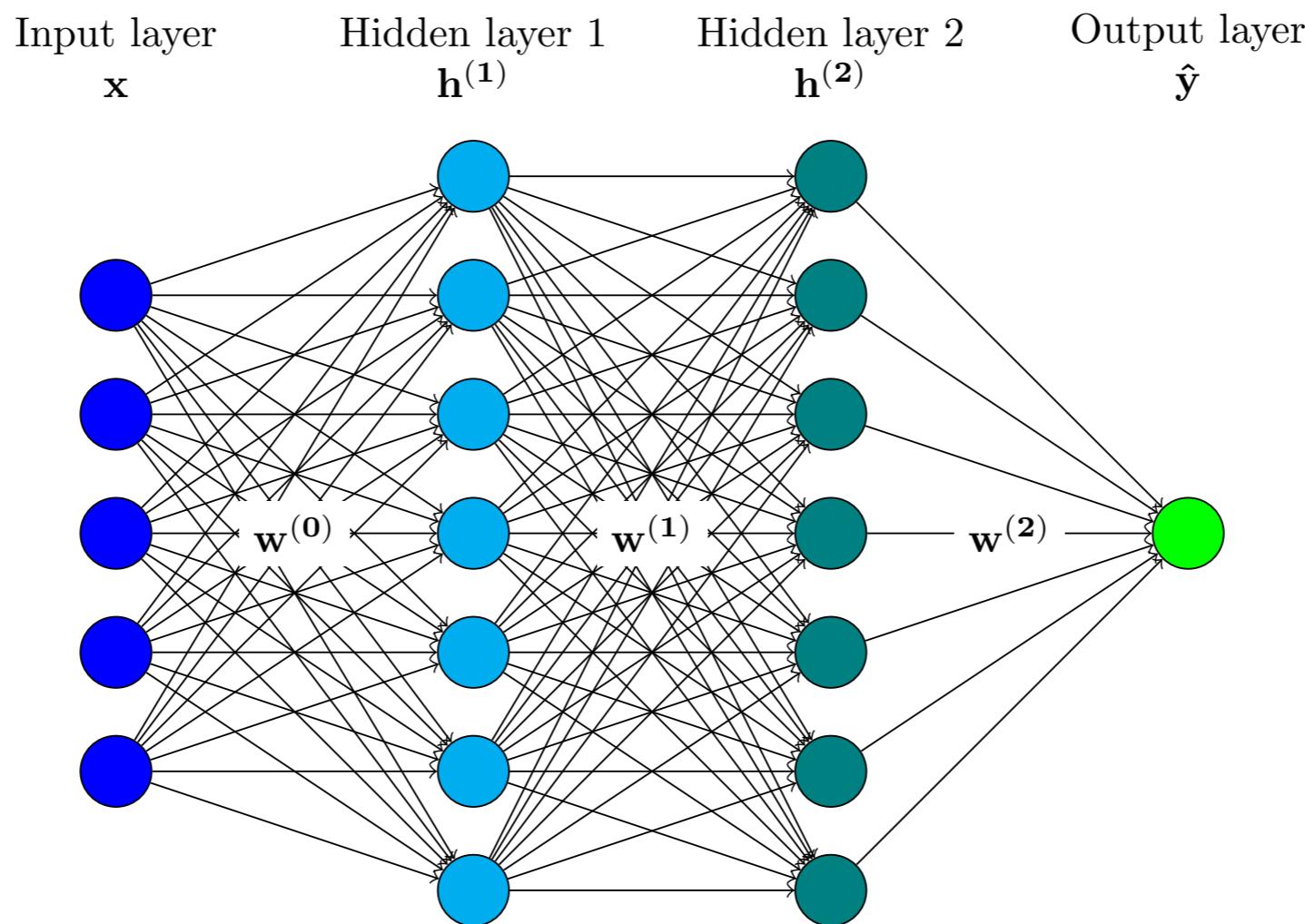
- Input: $x = \{x_0, \dots, x_D\}$
- Output: $a = \varphi(\sum_j w_j x_j + b)$
- Can do
 - Linear regression
 - Linear boundaries for classification
 - Single output



Multi-Layer Perceptron (MLP) model (Rumelhart, Hinton & Williams, 1985)

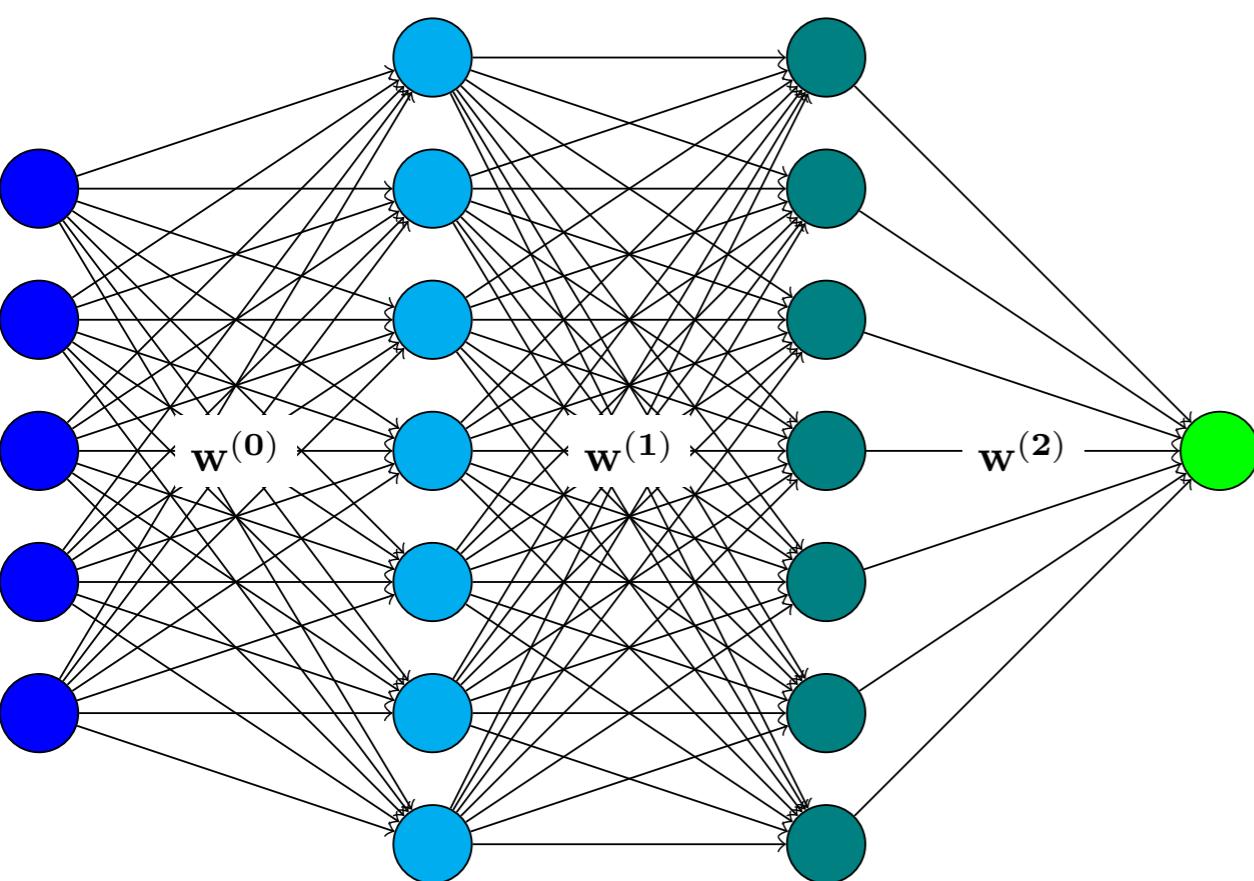
Definition

A Multilayer perceptron is an acyclic graph of neurons, where neurons are structured in successive layers, beginning by an input layer and finishing with an output layer.



Multi-Layer Perceptron (MLP) model (Rumelhart, Hinton & Williams, 1985)

Input layer Hidden layer 1 Hidden layer 2 Output layer



$$\hat{y} = \varphi_{\text{out}} \left(\sum_i w_i^{(2)} h_i^{(2)} + b^{(2)} \right)$$

$$\forall i, h_i^{(2)} = \varphi \left(\sum_j w_{ij}^{(1)} h_j^{(1)} + b_i^{(1)} \right)$$

$$\forall i, h_i^{(1)} = \varphi \left(\sum_j w_{ij}^{(0)} x_j + b_i^{(0)} \right)$$

Why introduce hidden layer(s)?

Universal approximation theorem (Cybenko, 1989)

- Under reasonable assumptions on the activation function to be used*
- For any continuous function on a compact g and any precision threshold ϵ
- There exists a 1-hidden-layer MLP with a finite number of neurons that can approximate g at level ϵ

*Non-constant, bounded, monotonically increasing, continuous
also holds for some unbounded functions like ReLU, cf. [Somoda & Murata, 2015]

Why introduce hidden layer(s)?

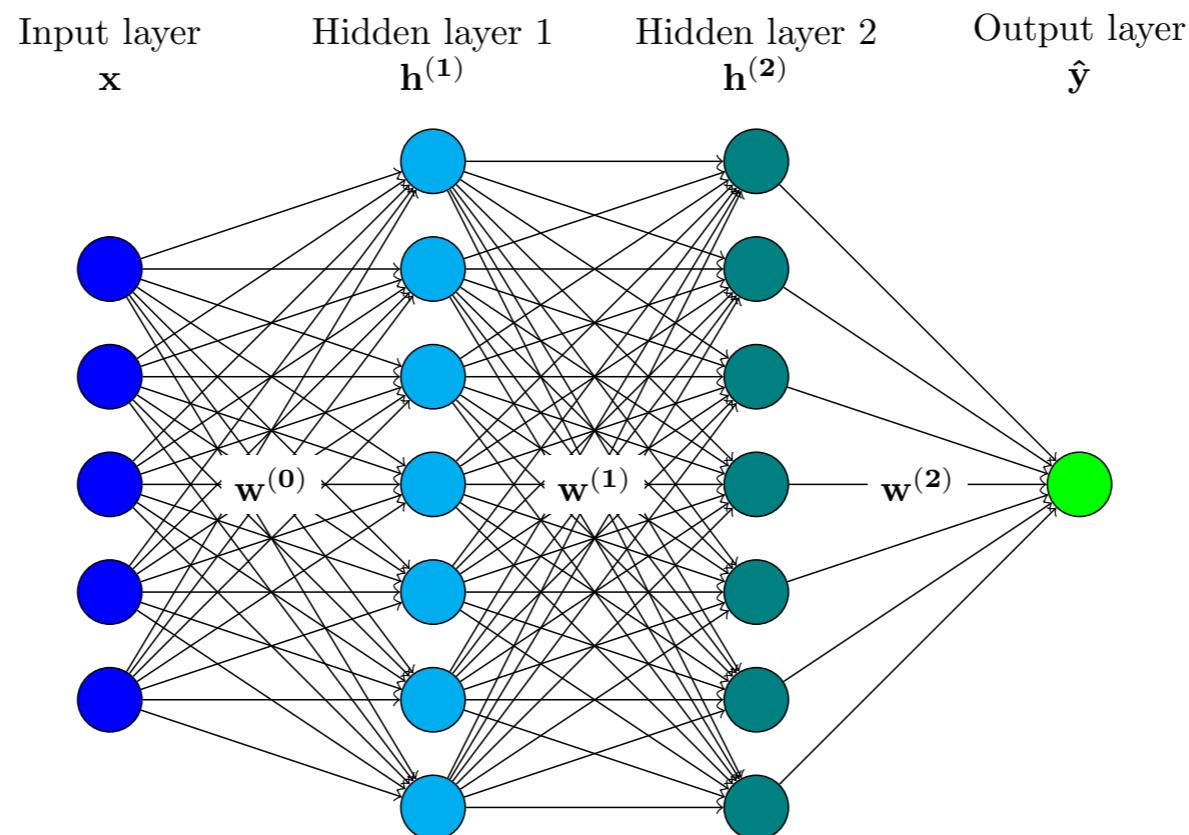
In practice

- Universal Approximation Theorem:
 - A single hidden layer is sufficient in theory
 - The number of neurons in this layer is not given
- In practice, for a given approximation level
 - Stacking more layers requires less parameters
 - ⚠ Very deep networks suffer from specific problems too (discussed later in the course)
 - 2-3 hidden layers is a good start for an MLP

Building blocks of neural networks

Input/Output layers

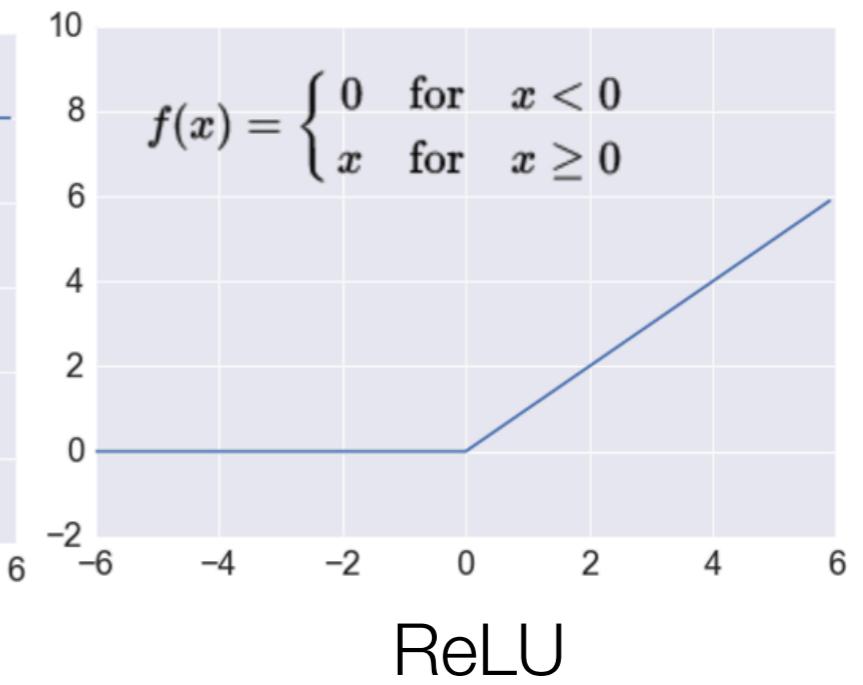
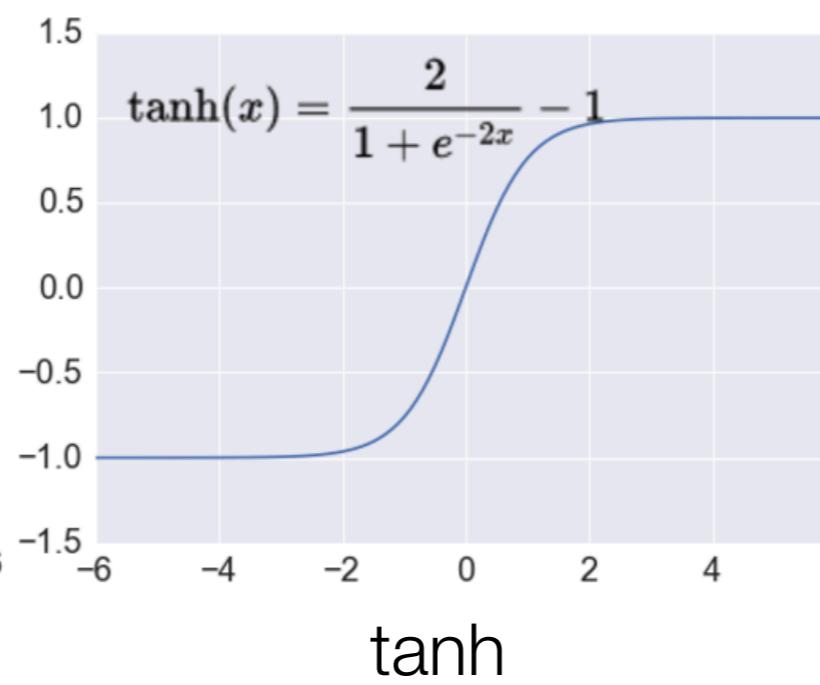
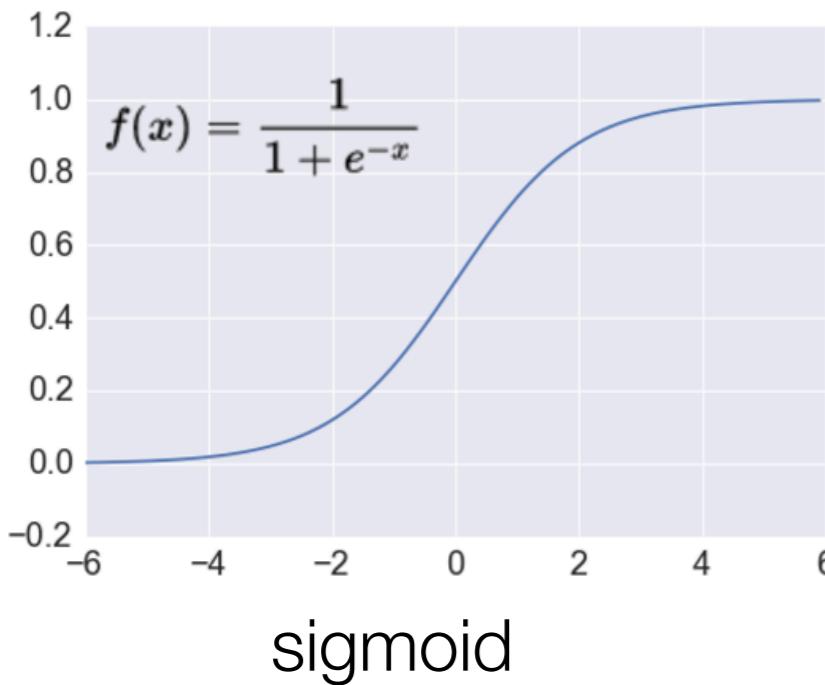
- Given a dataset (X, y)
- Constraints on model structure:
 - Input layer dimension is the number of features in X
 - Output layer has as many units as columns in y



Building blocks of neural networks

Activation functions

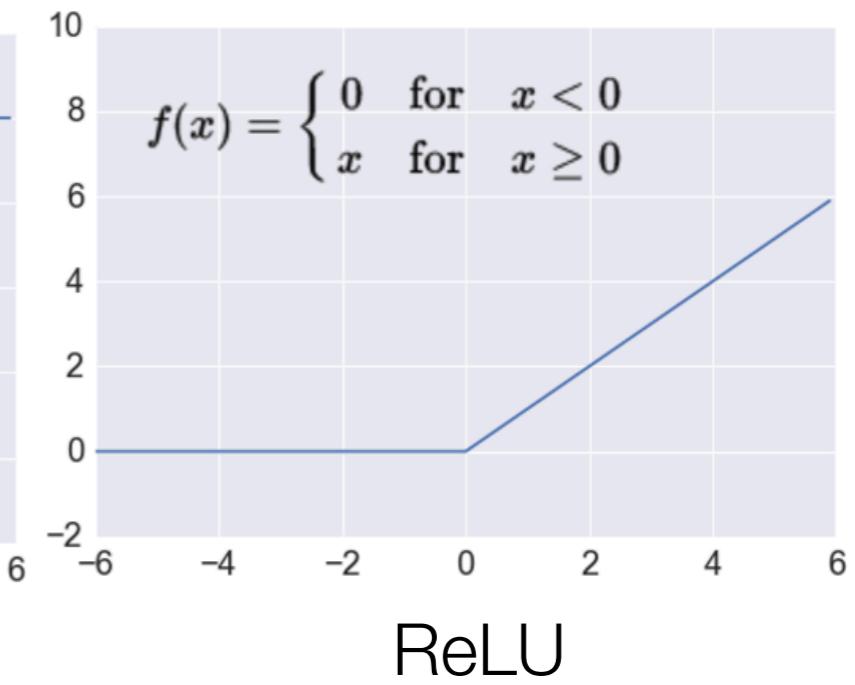
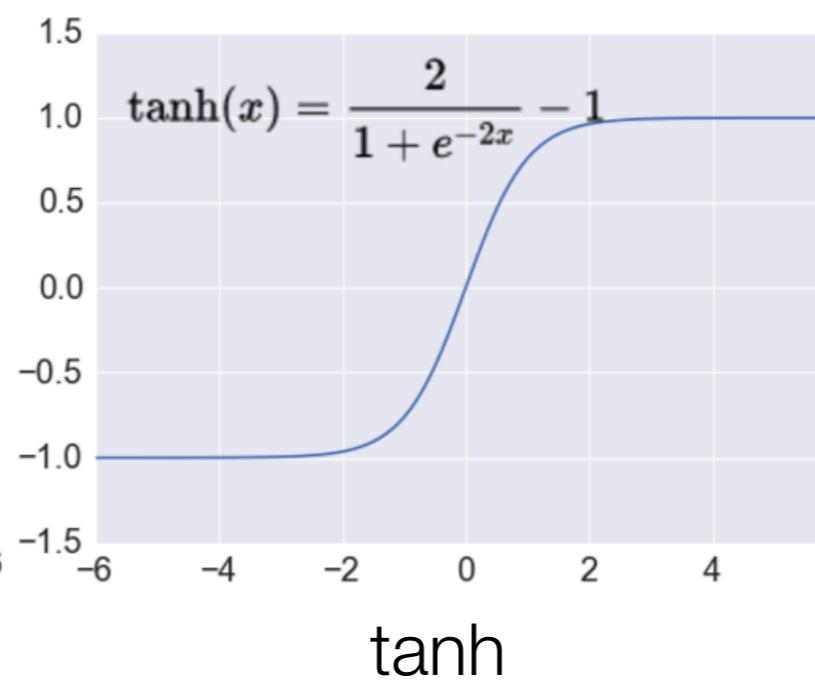
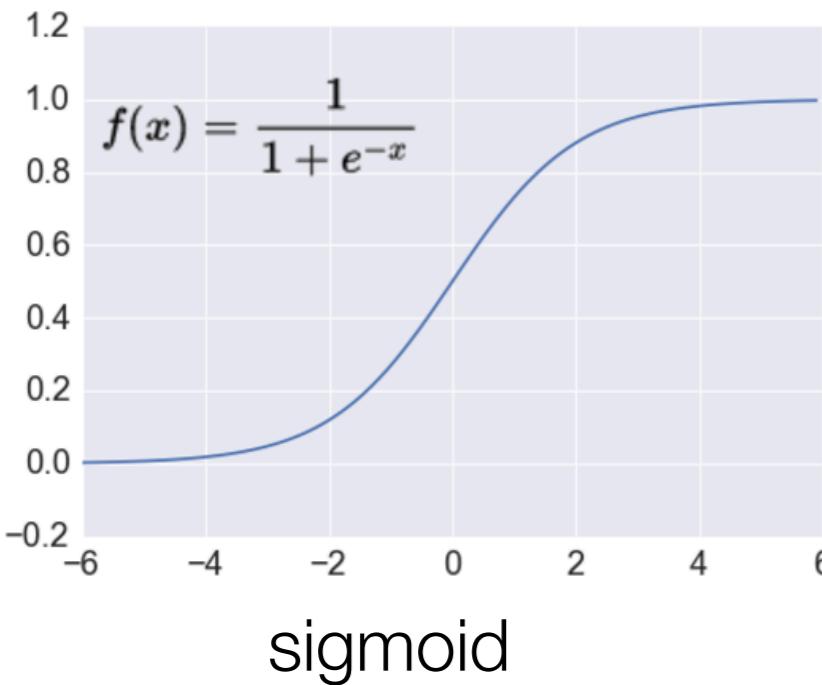
- Important features
 - φ should be differentiable almost everywhere
 - Non-linearities
 - Some linear regime
- Examples



Building blocks of neural networks

Activation functions: the reign of ReLU

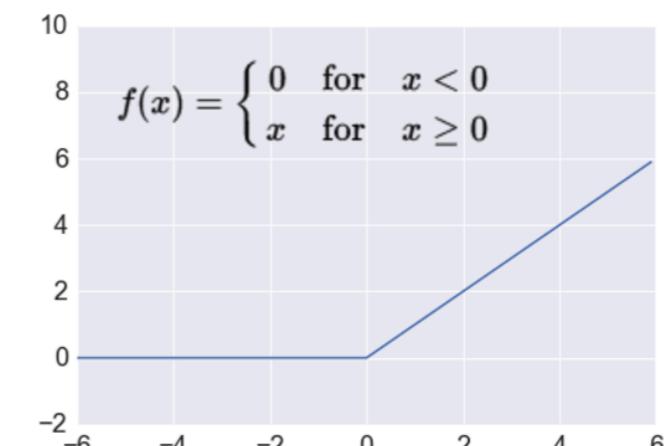
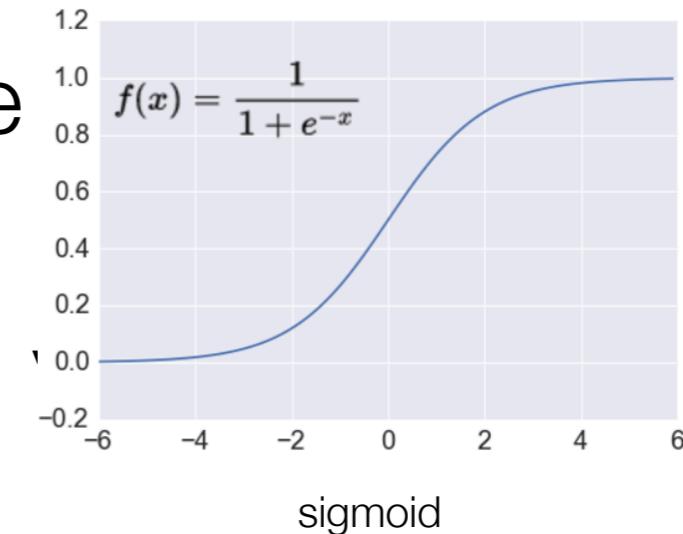
- ReLU has become the default choice **for internal layers** over time
- 2 main reasons:
 - cheap to compute (both ReLU and its derivative)
 - vanishing gradients phenomenon (more on that later)



Building blocks of neural networks

Activation functions: the case of the output layer

- Output activation functions drive the values:
 - identity ("linear" in keras): any real
 - ReLU:
any positive value
 - sigmoid:
any value in [0, 1]
 - softmax:
>0 and sums to 1
(across output neurons)



$$\text{soft-max}(o)_i = \frac{e^{o_i}}{\sum_j e^{o_j}}$$