Extension of Dynamic Time Warping (DTW) to Continuous-Time: Theoretical Foundations and Applications

Scientific Supervision

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Context

This internship aims to explore the extension of a time series comparison method, Dynamic Time Warping, from discrete to continuous-time domain. Dynamic Time Warping (DTW [2, 4]) has become a reference method for comparing time series data, thanks to its ability to account for temporal shifts between series.[1] However, DTW's standard formulation is limited to discrete-time data. An extension of this method, coined Generalized DTW (GDTW [1]) tackles the continuous-time setting, finding application as a loss in ML pipelines where the set of allowed time distortions need to be constrained [5]. It appears, however, that GDTW is not exactly the continuous-time extension of DTW, and that a more thorough study of the behaviour of DTW in the large-sampling-rate limit is required.

To address these limitations and provide a solid mathematical foundation for DTW in machine learning applications, this internship aims to explore an extension of DTW that is actually the limit of DTW as the sampling rate approaches infinity. We hypothesize that such an extension, formulated directly in continuous-time, could offer several advantages over existing methods:

1. **Mathematical Insights:** A continuous-time formulation may reveal nicer mathematical properties than DTW itself, providing deeper insights into its behavior and performance, particularly in high-sampling-rate regimes ;

2. **Better Understanding of Optimization Objective:** By studying the limiting case, we can gain a clearer understanding of what practitioners are optimizing when using DTW in machine learning applications with high sampling rates ;

[1] A gentle introduction to DTW is available in [3].

3. **Flexibility and Potential Superior Performance:** The proposed extension may prove more flexible than the baseline GDTW method, potentially leading to improved outcomes, especially on forecasting tasks.

Objectives

During this internship, the student will:

1. Refine a preliminary mathematical formulation of DTW in continuous-time ;
2. Mathematically prove that the discrete DTW tends towards the new formulation as the number of temporal samples approaches infinity and study the mathematical properties of the proposed similarity measure ;
3. Explore potential applications of this continuous-time extension in machine learning problems with discrete-time data.

Point 3 will occupy the majority of the internship time.

Required Skills

- Decent mathematical background
- Proficiency in Python
- Interest in machine learning methods and signal processing

References

[1] Dave Deriso and Stephen Boyd. A general optimization framework for dynamic time warping. *Optimization and Engineering*, 24(2):1411–1432, 2023.

[2] Hiroaki Sakoe and Seibi Chiba. Dynamic programming algorithm optimization for spoken word recognition. *IEEE transactions on acoustics, speech, and signal processing*, 26(1):43–49, 1978.

[3] Romain Tavenard. An introduction to dynamic time warping. `https://rtavenar.github.io/blog/dtw.html`, 2021.

[4] Taras K Vintsyuk. Speech discrimination by dynamic programming. *Cybernetics*, 4(1):52–57, 1968.

[5] Ming Xu, Sourav Garg, Michael Milford, and Stephen Gould. Deep declarative dynamic time warping for end-to-end learning of alignment paths. In *ICLR 2023*, 2023.