

Deep Learning for Time Series

Session 3b: Practical aspects of forecasting

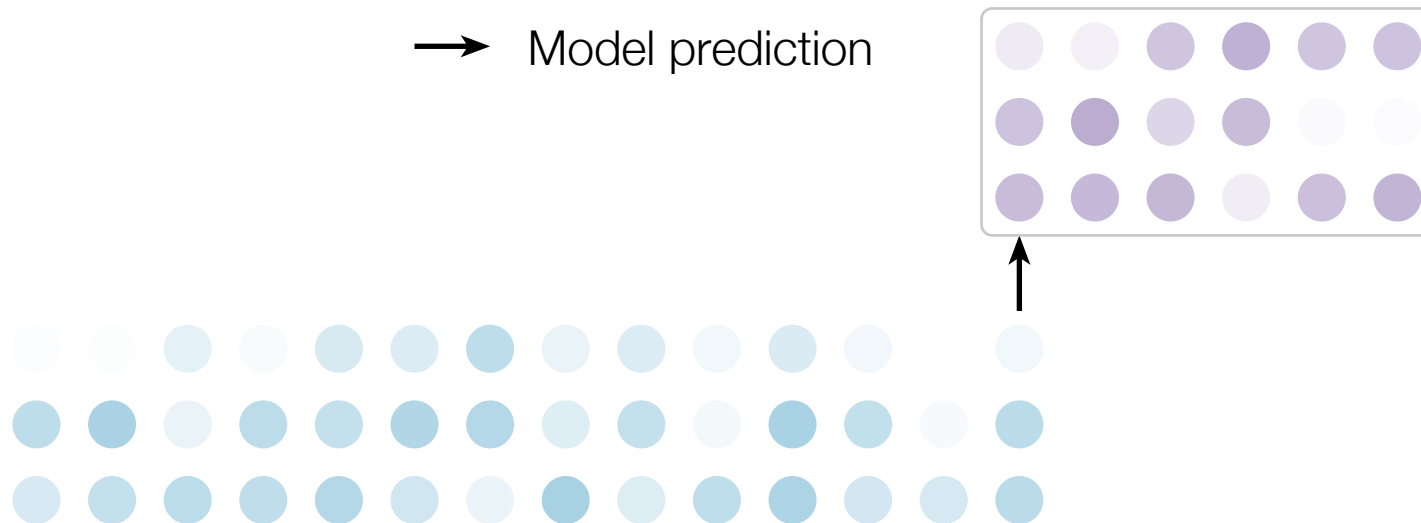
Romain Tavenard

Multi-step ahead forecasting

- Single-step forecasting: predict x_{t+1} given x_1, \dots, x_t
- Multi-step forecasting: predict x_{t+1}, \dots, x_{t+H} given x_1, \dots, x_t
- Two main approaches:
 1. Direct forecasting: predict all H steps at once
 2. Autoregressive forecasting: iteratively predict one step at a time

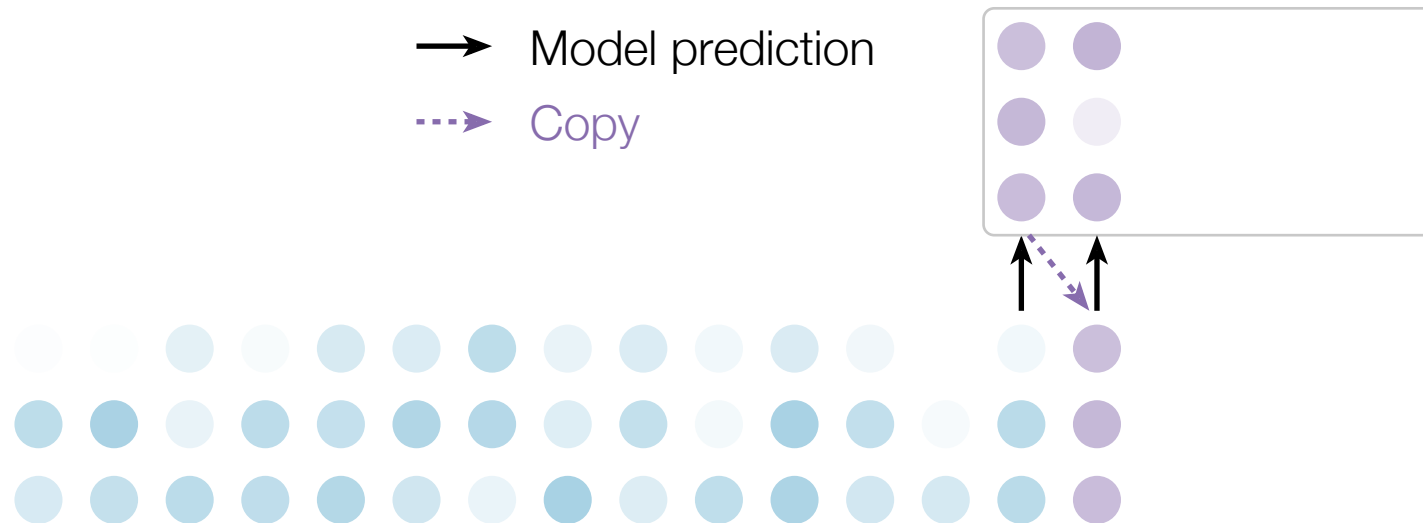
Direct forecasting

- Model outputs all H future values in one forward pass
- Pros: No error accumulation, faster inference
- Cons: Model must learn to predict all horizons simultaneously



Autoregressive forecasting

- Model predicts \hat{x}_{t+1} , then uses it to predict \hat{x}_{t+2} , etc.
- Pros: Reuses same model, naturally handles variable horizons
- Cons:
 - Error accumulation, **exposure bias** (train/test mismatch)
 - Model needs to predict all modalities



Strategy 1: Next-step training / AR inference

- **Training:** Train model to predict one step ahead

$$\mathcal{L} = \frac{1}{T} \sum_t \|x_{t+1} - \hat{x}_{t+1}\|^2$$

- **Inference:** Use model autoregressively for multi-step prediction
- Advantage: Simple training procedure
- Disadvantages:
 - Train/test mismatch: model sees ground truth during training but its own predictions during inference
 - Error accumulation: prediction errors compound over time
 - Model may not learn to handle its own prediction errors

Strategy 2: Curriculum learning

- Gradually increase the autoregressive window during training
- Start with short sequences, progressively increase to full horizon
- Typical training procedure:
 1. Epoch 1-10: predict 1 step ahead
 2. Epoch 11-20: predict 2 steps ahead (autoregressively)
 3. ... continue until reaching full horizon H
- Benefits:
 - Model learns to handle its own predictions progressively
 - Reduces exposure bias by gradually exposing model to autoregressive inference

Strategy 3: Scheduled sampling

- During training, randomly replace ground truth with model predictions
- Probability of using prediction increases over time
- Helps bridge train/test gap

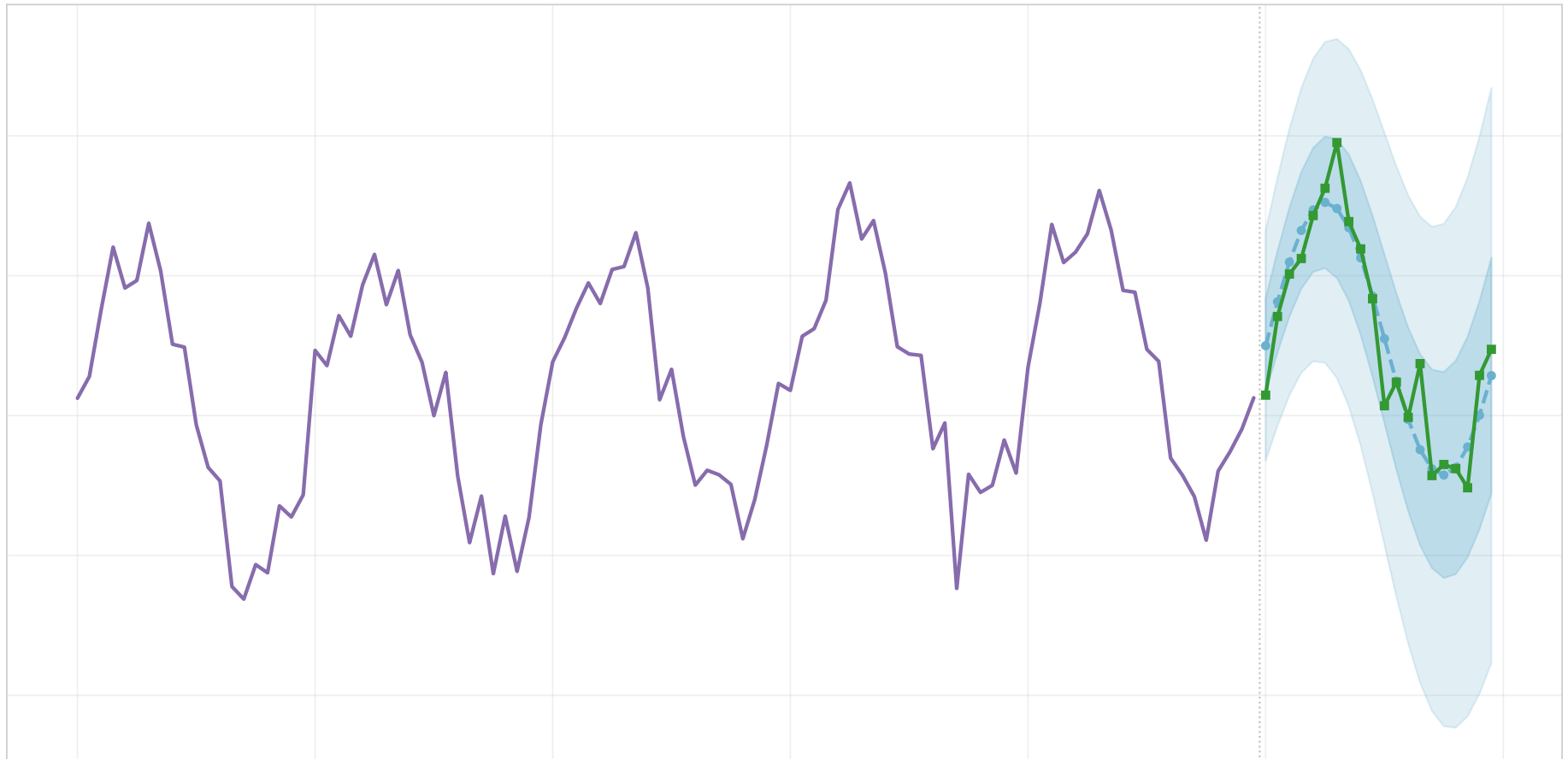
$$x_{t+1}^{\text{input}} = \begin{cases} x_{t+1} & \text{with probability } p \\ \hat{x}_{t+1} & \text{with probability } 1 - p \end{cases}$$

where p decreases from 1 to 0 during training

Probabilistic forecasting

- So far: deterministic models output single values \hat{x}_{t+h}
- Reality: future is uncertain
- **Probabilistic forecasting**: predict distribution

$$p(x_{t+h} \mid x_1, \dots, x_t)$$



- Quantify uncertainty: know when model is confident vs uncertain
- Better decision making: risk-aware planning
- Handle non-Gaussian error distributions: capture skewness

Approaches to probabilistic forecasting

1. Parametric distributions

- Model outputs parameters of a distribution (e.g., mean and variance for Gaussian)
- $\hat{\mu}_{t+h}, \hat{\sigma}_{t+h} = f(x_1, \dots, x_t)$
- Predict $x_{t+h} \sim \mathcal{N}(\hat{\mu}_{t+h}, \hat{\sigma}_{t+h}^2)$

2. Quantile regression

- Predict multiple quantiles (e.g., 10th, 50th, 90th percentiles)
- Captures uncertainty without distributional assumptions

3. Sample-based methods

- Generate multiple samples from the model
- Use ensemble or Monte Carlo methods

- Model outputs distribution parameters
- Training via maximum likelihood
- Example: Negative Gaussian log-likelihood

$$\begin{aligned}\ell &= -\log p(x_{t+h} \mid \hat{\mu}_{t+h}, \hat{\sigma}_{t+h}) \\ &= \frac{1}{2} \log(2\pi\hat{\sigma}_{t+h}^2) + \frac{(x_{t+h} - \hat{\mu}_{t+h})^2}{2\hat{\sigma}_{t+h}^2}\end{aligned}$$

- Model learns both mean prediction and uncertainty

- Predict multiple quantiles simultaneously
- Loss function: quantile loss (*aka* pinball loss)

$$\ell_{\tau}(x_{t+h}, \hat{x}_{t+h}) = \begin{cases} \tau \cdot (x_{t+h} - \hat{x}_{t+h}) & \text{if } \hat{x}_{t+h} \leq x_{t+h} \\ (\tau - 1) \cdot (x_{t+h} - \hat{x}_{t+h}) & \text{if } \hat{x}_{t+h} > x_{t+h} \end{cases}$$

- Common quantiles: $\tau \in \{0.1, 0.5, 0.9\}$
 - Median ($\tau = 0.5$) provides point forecast
 - Other quantiles provide uncertainty intervals

Calibration

- Predicted 90% intervals should contain true value ~90% of the time

Sharpness

- Among well-calibrated forecasts, prefer narrower intervals

Proper scoring rules

- Log-likelihood on held-out data
- Continuous Ranked Probability Score (CRPS)

Continuous Ranked Probability Score (CRPS)

TODO: add formula



- Multi-step forecasting strategies:
 - Direct: predict all steps at once
 - Autoregressive: iterative one-step predictions
- Training considerations:
 - Next-step training simple but suffers from exposure bias
 - Curriculum learning helps bridge train/test gap
- Probabilistic forecasting:
 - Captures uncertainty in predictions
 - Multiple approaches: parametric, quantile regression
 - Important for real-world decision making