Simplified Radar Equation

$$P_R = \underbrace{P_T\left(\frac{\lambda}{4\pi}\right)^2 G_T G_R}_{\substack{S\\ \text{Radar system}\\ \text{properties}}} \underbrace{\frac{1}{\left[2\left(h+\frac{z}{n}\right)\right]^2}}_{\substack{G\\ \text{Geometric}\\ \text{spreading loss}}} \underbrace{T^2}_{\substack{T\\ \text{Transmission}\\ \text{Loss}}} \overbrace{L^2}^{\substack{A\\ \text{Attenuation}}} \underbrace{B^2}_{\substack{B\\ \text{Birefringence}\\ \text{Loss}}} \overbrace{\rho\Gamma}^{\substack{R\\ \text{Reflectivity}}}$$

Complex Dielectric Constant of Ice

Controls wave speed and wavelength in the material

Controls wave absorption in the material (i.e. radar attenuation)

$$\epsilon_r = \epsilon_r' + j\epsilon_r'' \approx \underbrace{3.17 + j\frac{\sigma}{2\pi f_c \epsilon_0}}$$

Gradients in full complex permittivity control strength of radar reflections

Density – strong linear control on ϵ_r'

Chemical impurities – strong control on σ and therefore ϵ_r''

Liquid water content – strong control on both ϵ_r' and ϵ_r'' (but ϵ_r'' more sensitive)

Temperature – strong control on σ and therefore ϵ_r''

Ice Sheet Material Properties

Material	ϵ_r'	ϵ_r''
Air	1	0
Meteoric ice	3.17	0.0197
Seawater	77	870
Groundwater	80	112
Fresh water	80	0.16
Unfrozen till	18	14.76
Unfrozen bedrock	6.6	2.7
Frozen till	2.8	0.098
Frozen bedrock	2.7	0.059
Marine ice	3.43	0.17

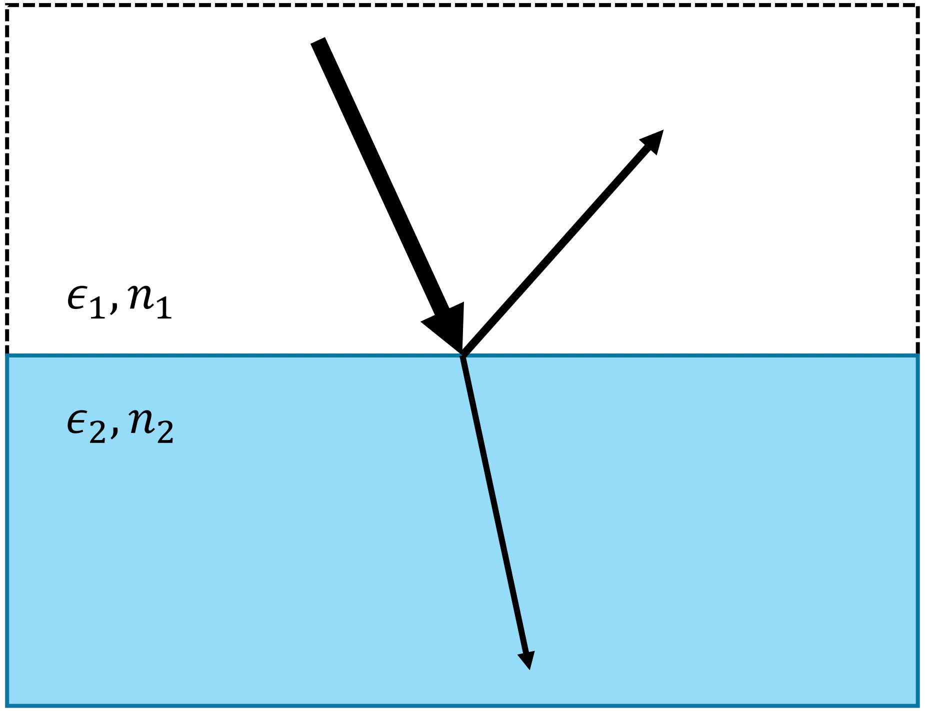
Peters et al (2005)

ϵ_r, Propagation Speed, and Reflectivity

$$n = \sqrt{\epsilon_r}$$

$$v = \frac{c}{n}$$

$$\Gamma = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2 = \left|\frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}\right|^2$$



Reflectivity Exercise

Calculate the radar reflectivity at an interface between meteoric ice and each of the other materials listed in the table below.

Material	ϵ_r'	ϵ_r''
Air	1	0
Meteoric ice	3.17	0.0197
Seawater	77	870
Groundwater	80	112
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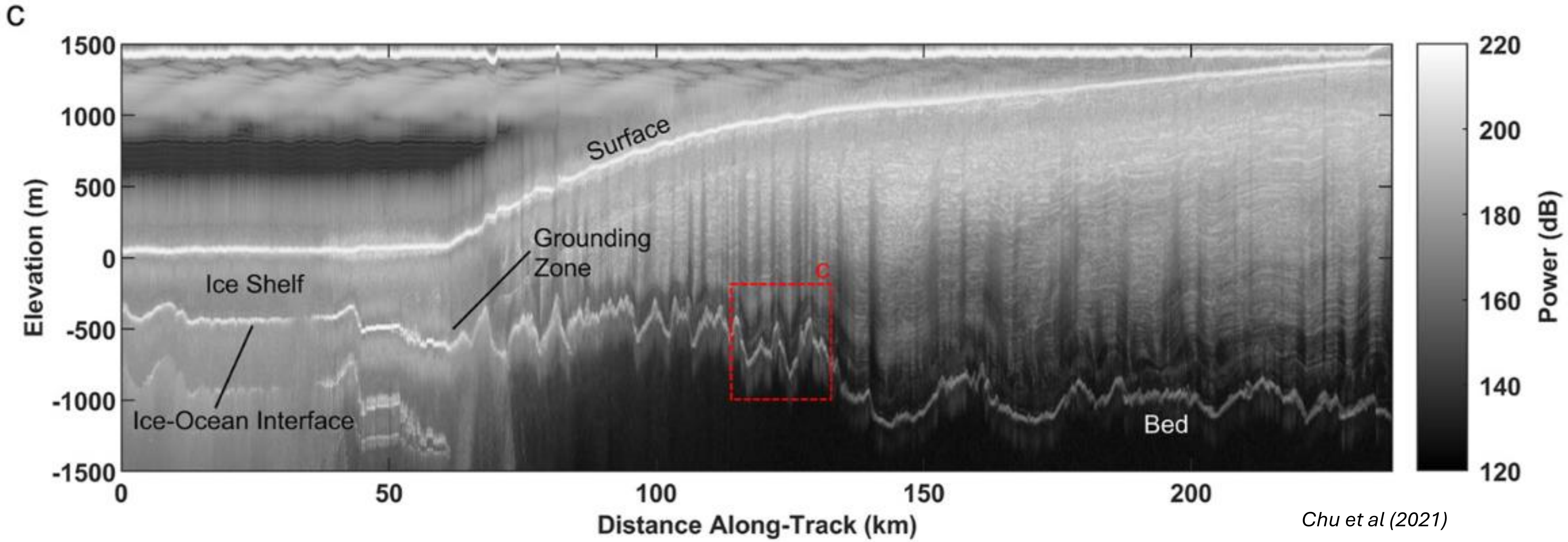
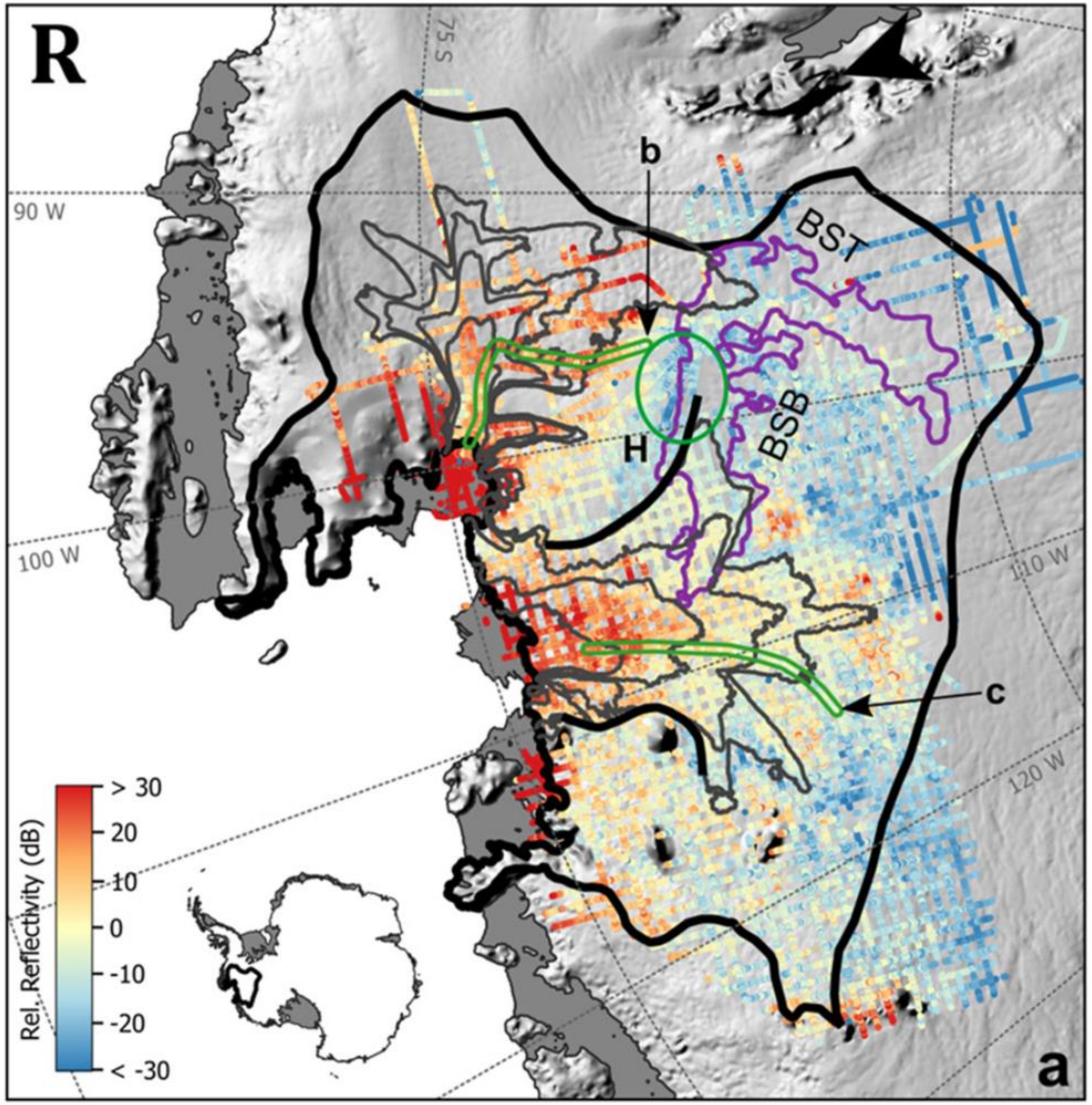
Peters et al (2005)

Reflectivity of the Basal Interface

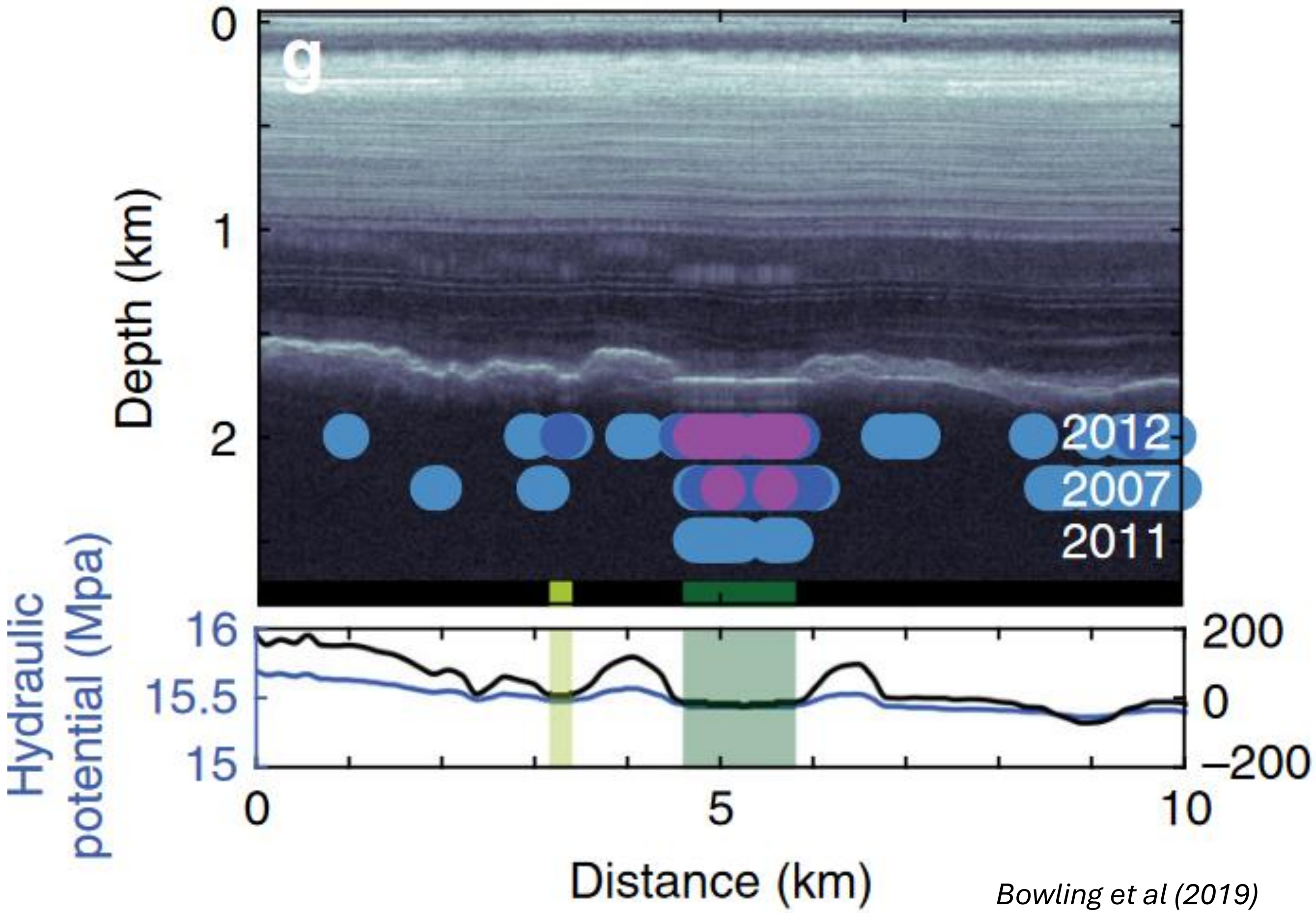


Chu et al (2021)

Reflectivity of the Basal Interface

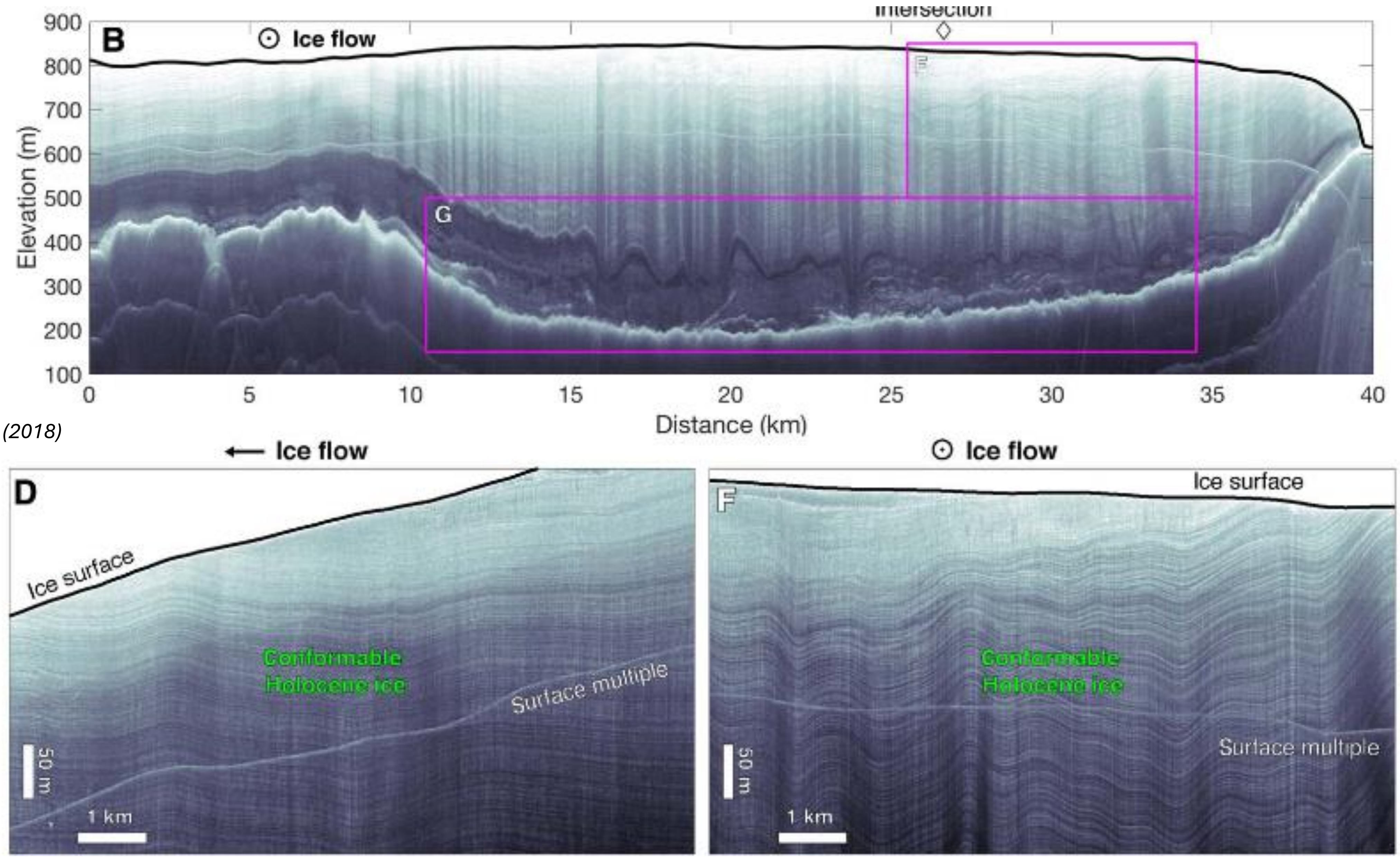


Chu et al (2021)



Bowling et al (2019)

Reflectivity of Englacial Layers



Kjaer et al (2018)

Reflectivity of Englacial Layers

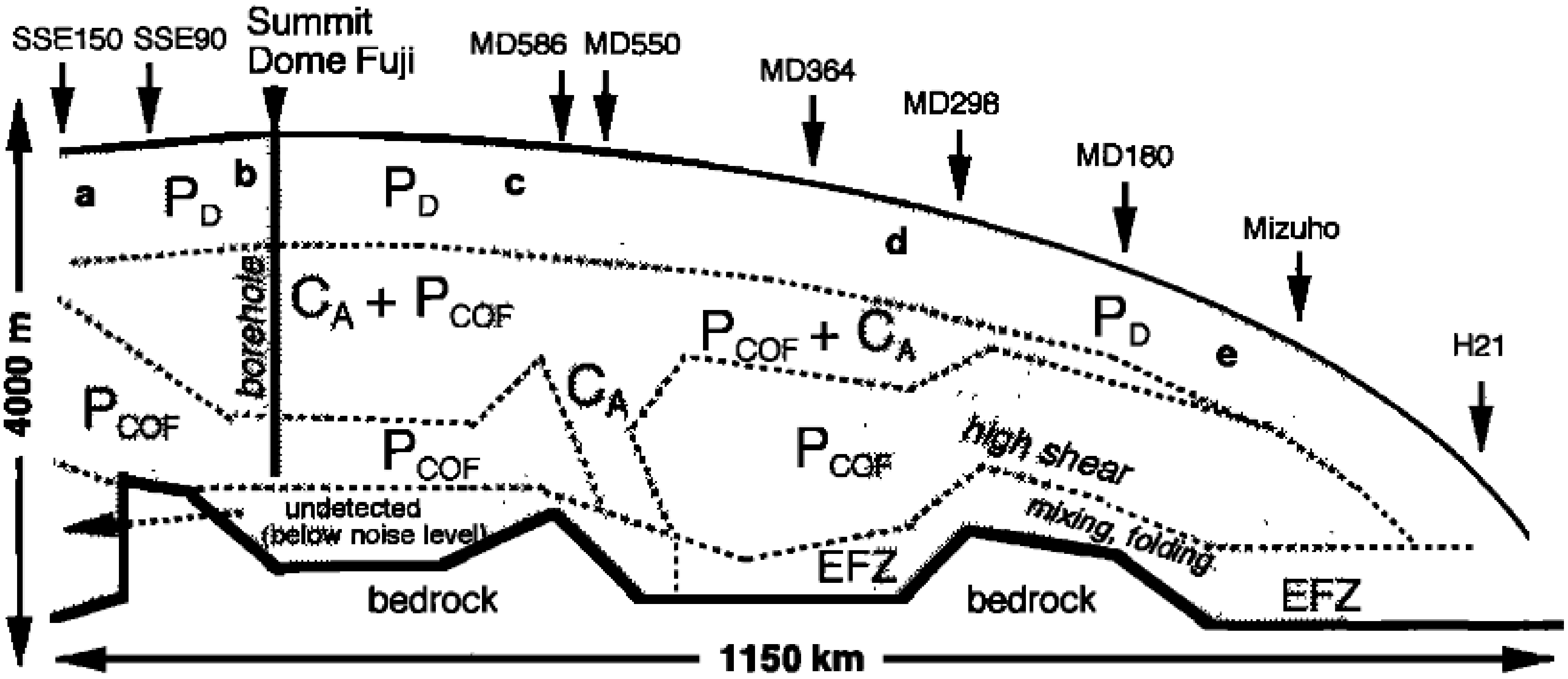
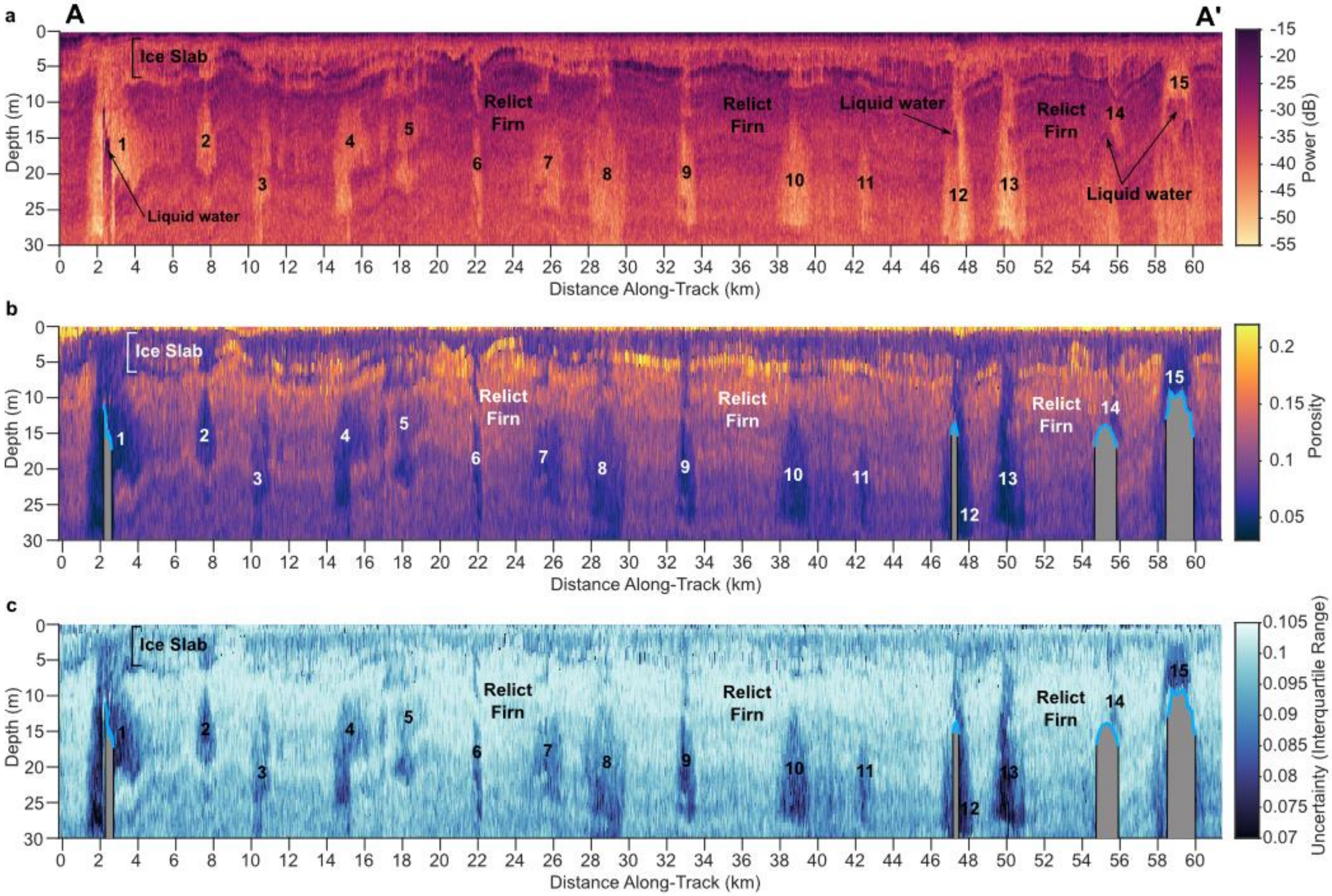


Figure 3. Schematic map interpretation of the dominant radio echo reflection mechanisms from the dome summit area to the coast. Sections a-e of Figures 1 and 2, and Plate 2 are shaded.

Fujita et al (1999)

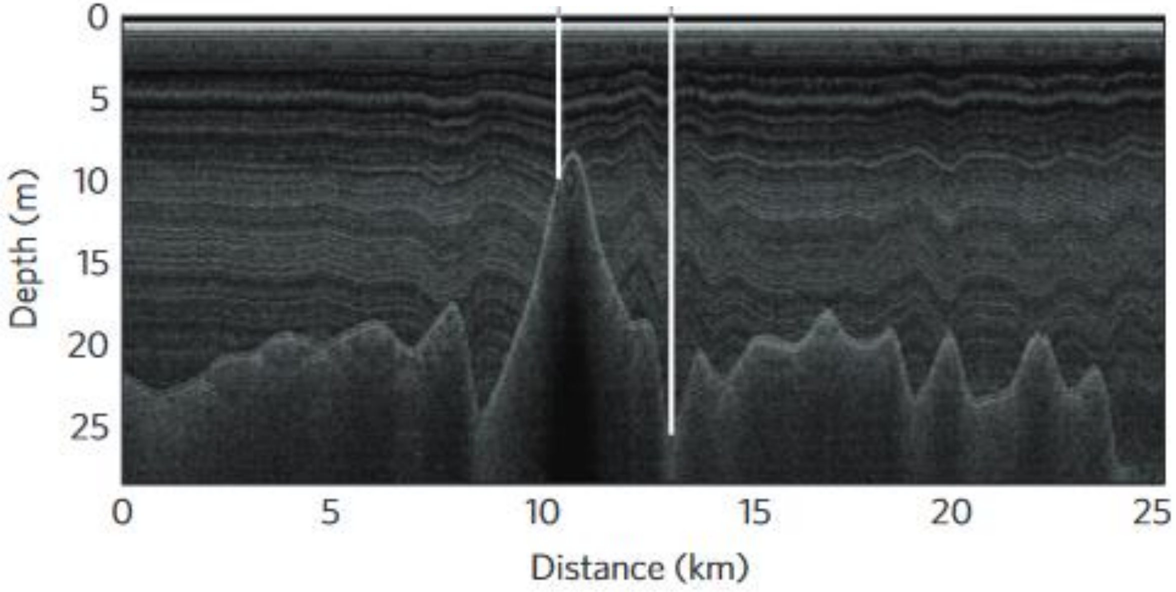
Reflectivity of Density Contrasts in the Firn

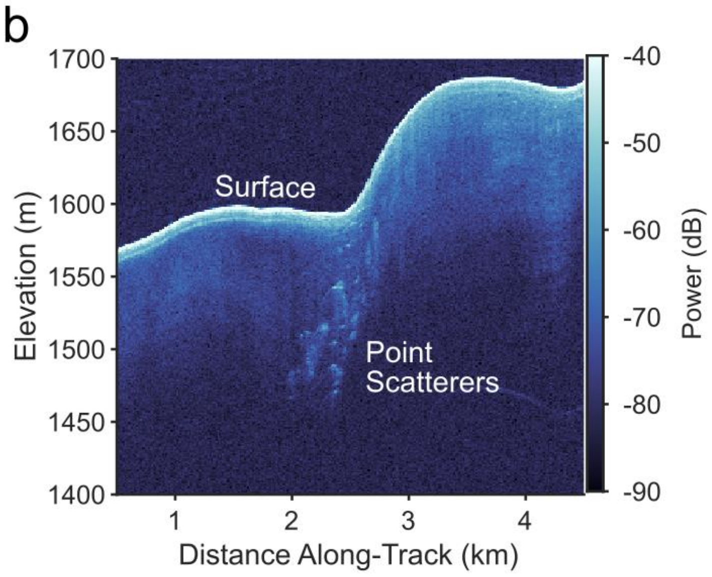


Culberg et al (2022)

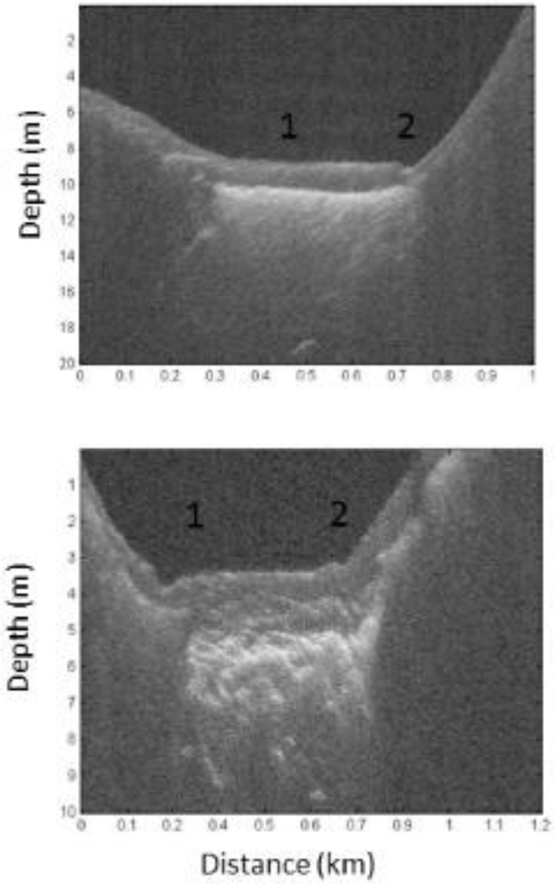
Englacial Water

Forster et al (2014)

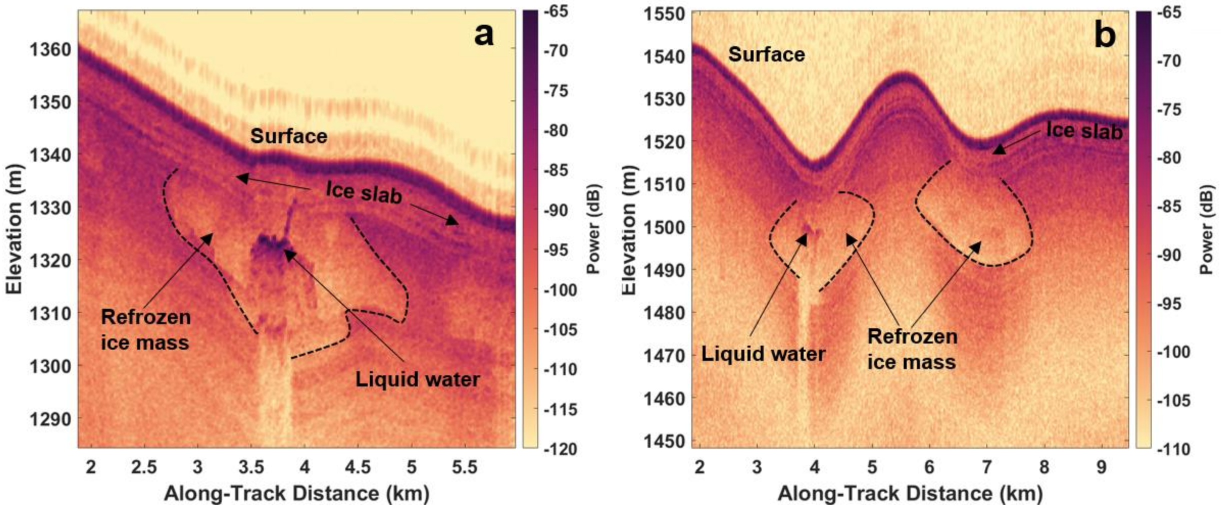




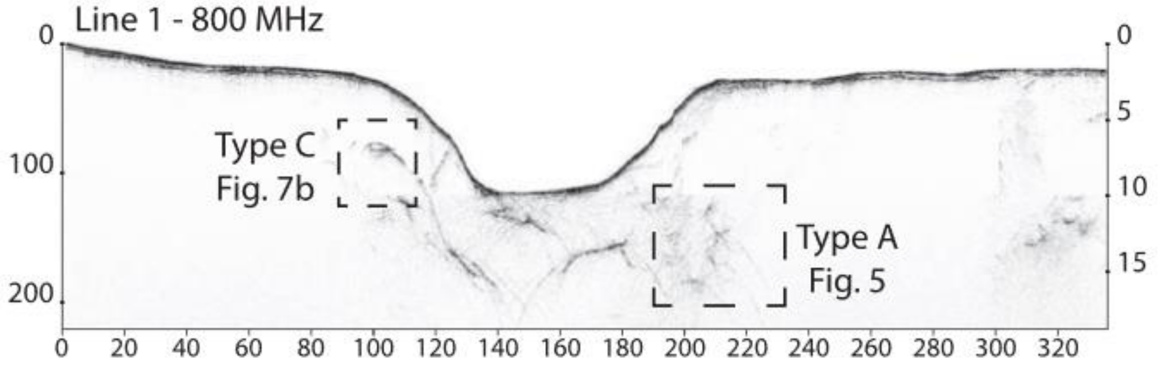
Culberg (2023)



Koenig et al (2015)



Culberg et al (2022)



Schaap et al (2019)

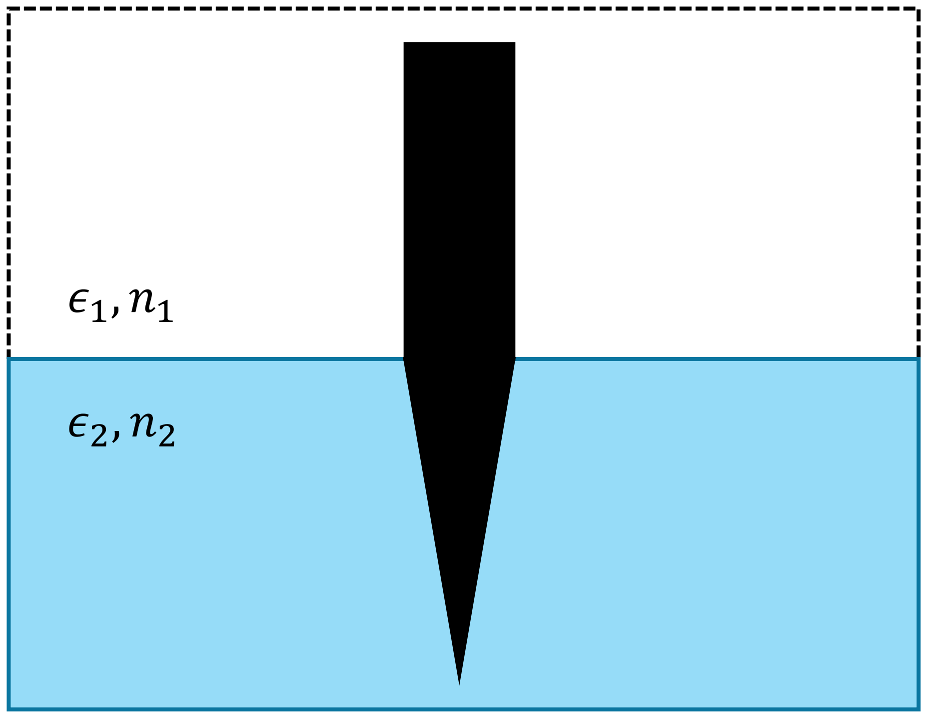
ϵ_r and Attenuation

$$A = e^{-2\alpha(2z)}$$

$$\alpha = 2\pi f \left[\frac{\mu_0 \epsilon_r' \epsilon_0}{2} \left[\sqrt{1 + \left(\frac{\epsilon_r''}{\epsilon_r'} \right)^2} - 1 \right] \right]^{\frac{1}{2}}$$

For ice sheets (low loss):

$$\alpha \approx \frac{\pi \epsilon_r''}{\lambda \sqrt{\epsilon_r'}} = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r'}} \left(\frac{\sigma}{2} \right)$$



ϵ_r and Attenuation

$$A_{tot} = e^{-2\alpha(2z)}$$

$$\alpha = 2\pi f \left[\frac{\mu_0 \epsilon_r' \epsilon_0}{2} \left[\sqrt{1 + \left(\frac{\epsilon_r''}{\epsilon_r'} \right)^2} - 1 \right] \right]^{\frac{1}{2}}$$

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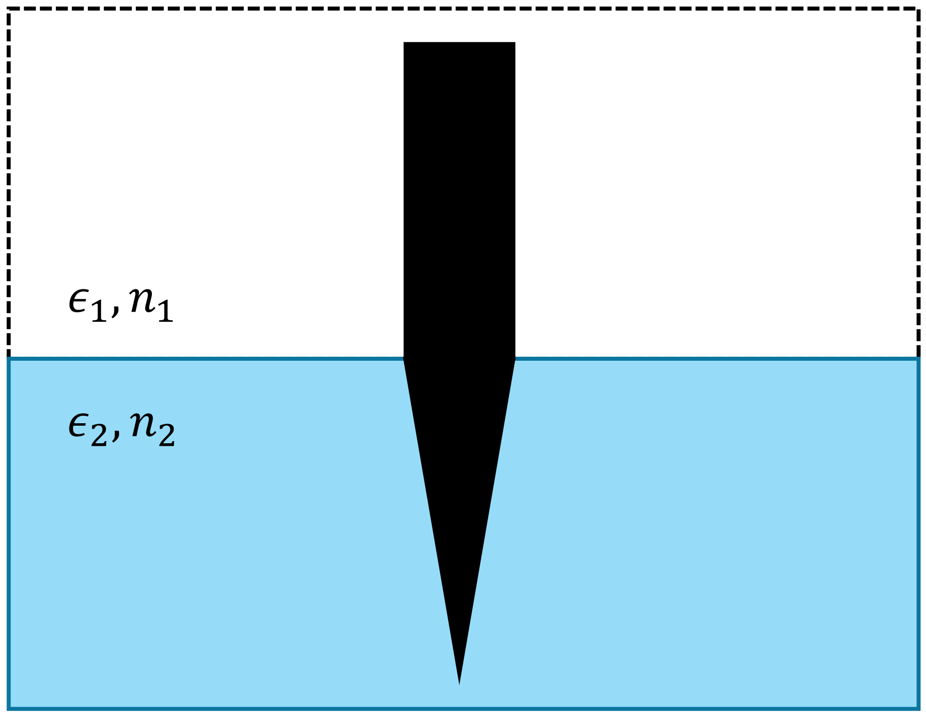
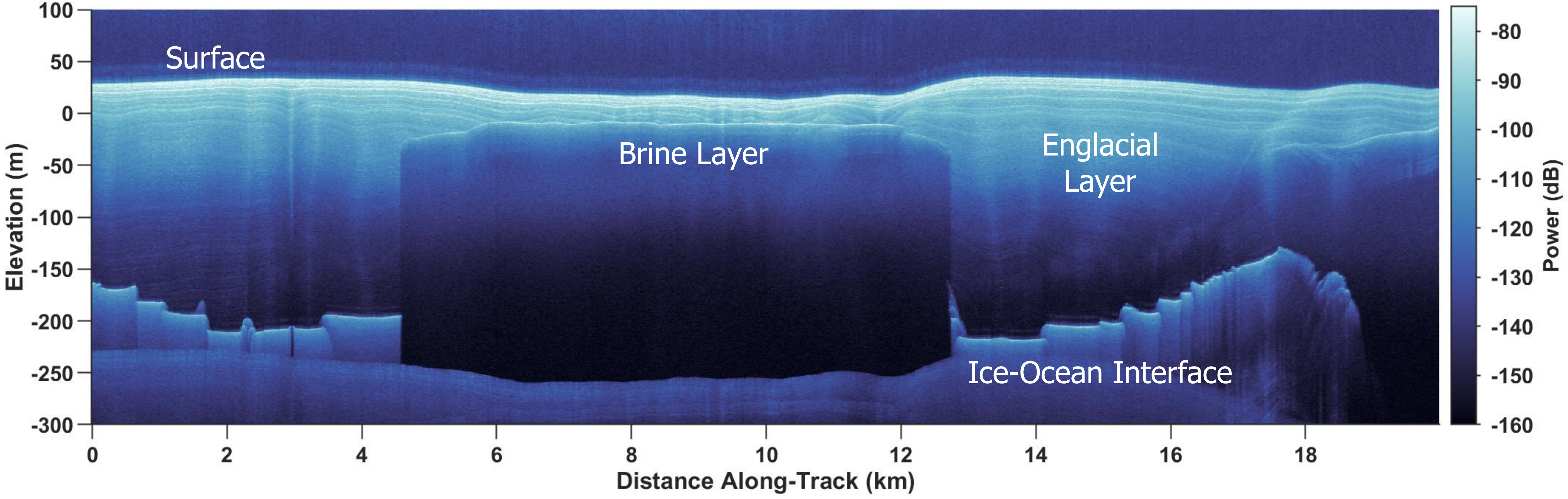


Visualizing Attenuation

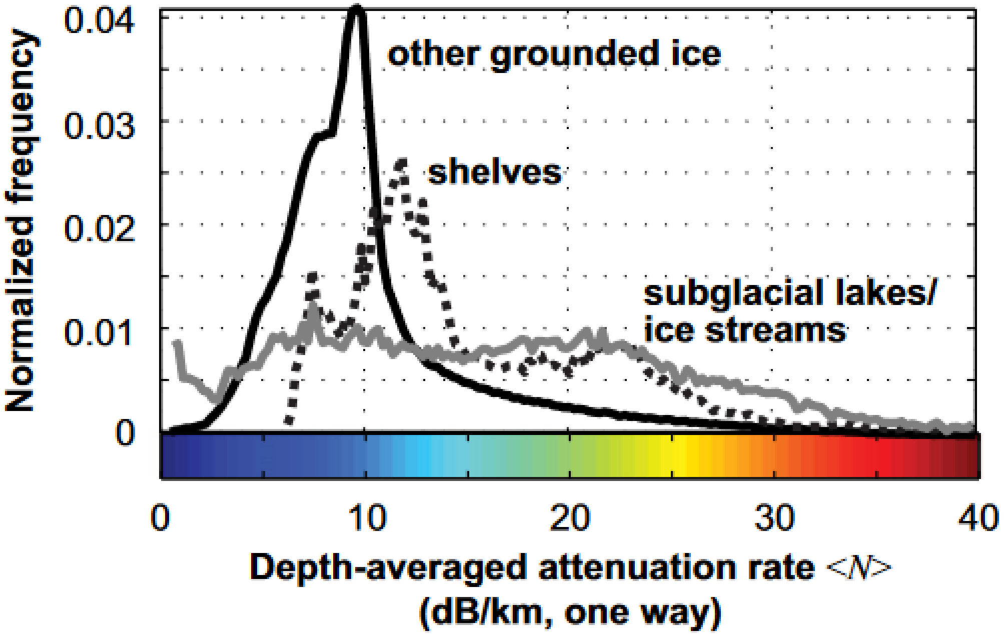


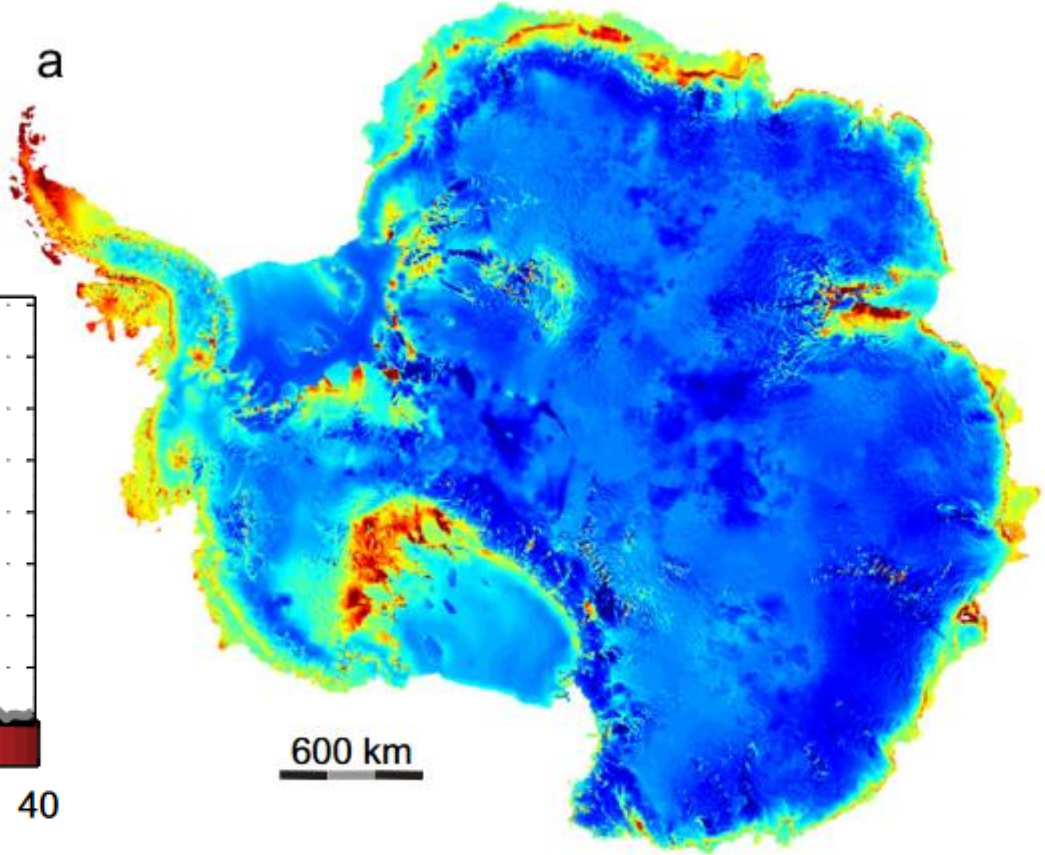
The Simple Version of Attenuation

Attenuation Rate (A) = dB of power lost per km traveled through the ice (units of dB/km)

$$A_{tot} = 2Az$$

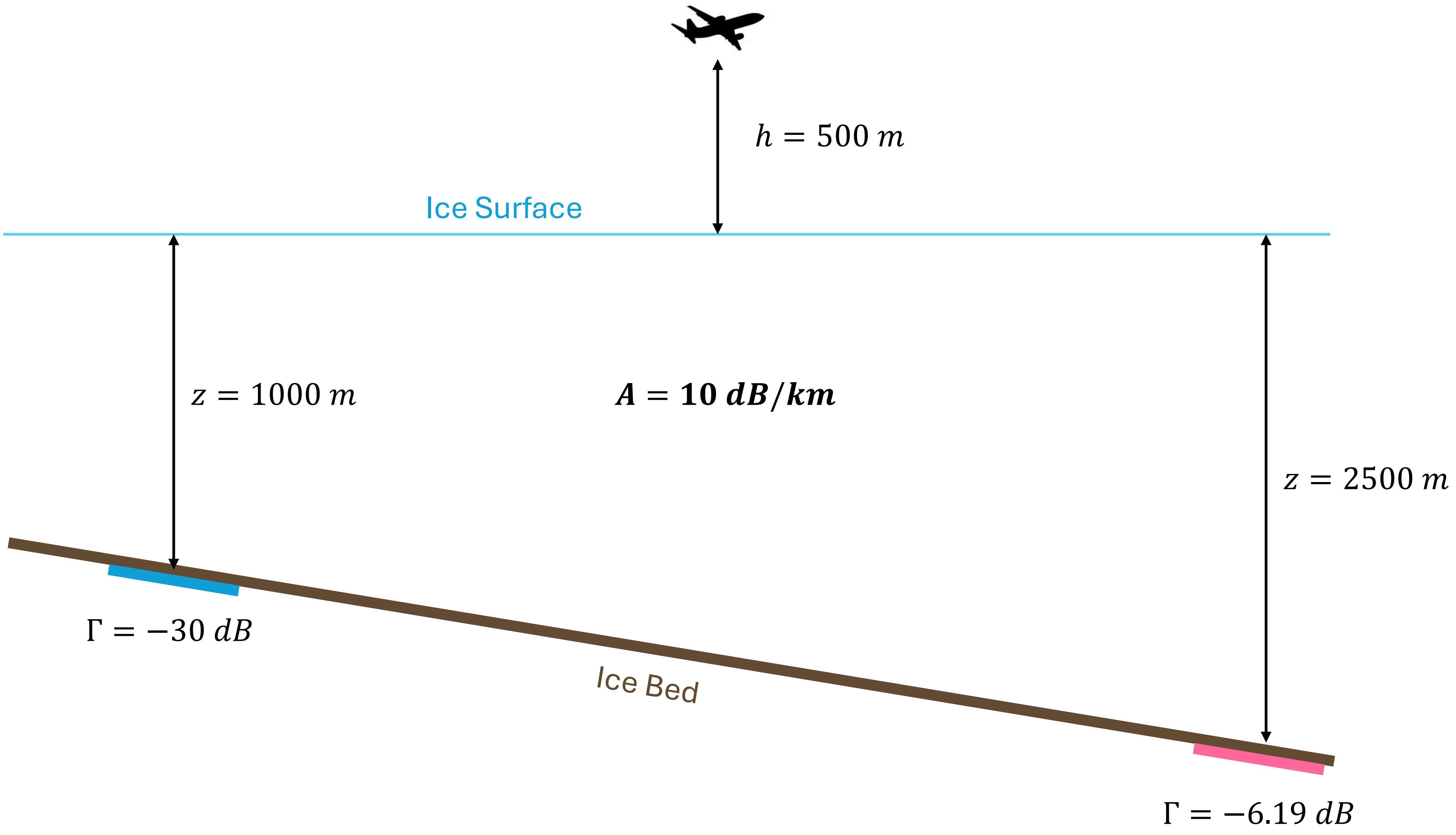
$A_{tot} =$ total power lost
$A =$ attenuation rate (dB/km)
$z =$ ice thickness (km)



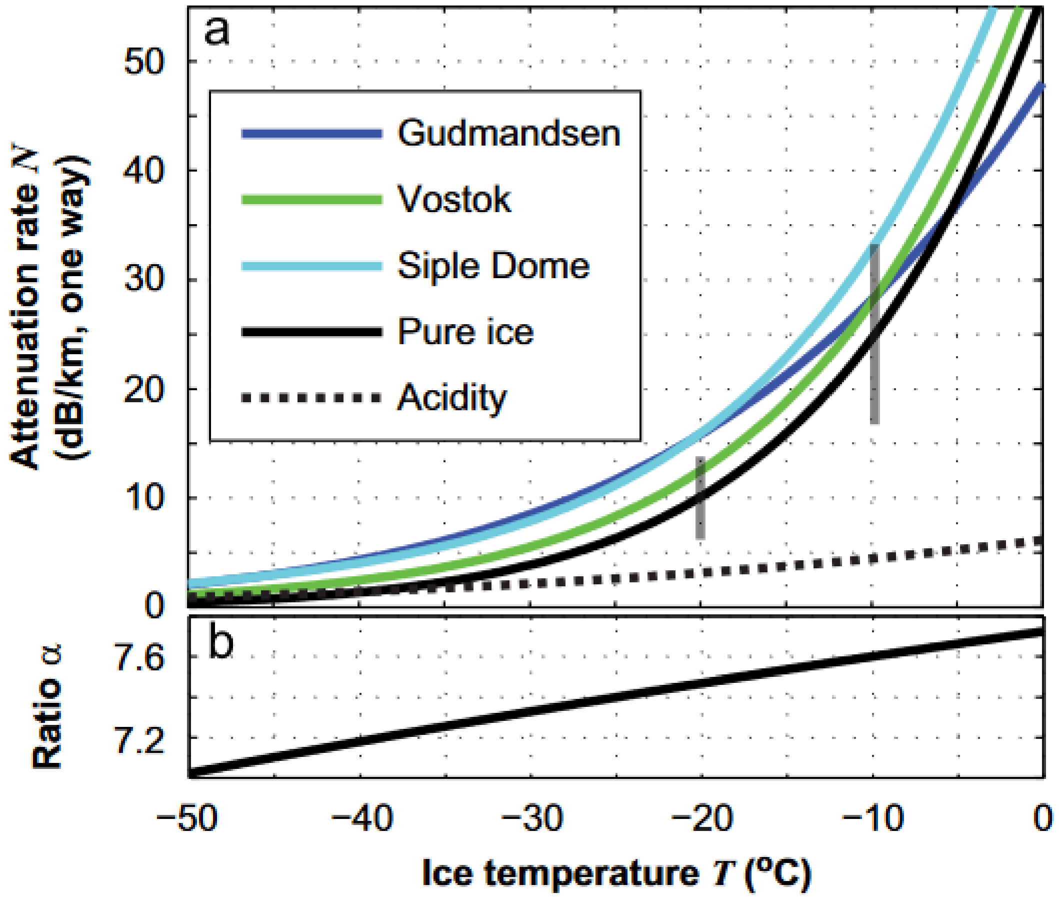


Matsuoka et al (2012)

Why is Attenuation Correction Important?



Calculating Attenuation Corrections



Matsuoka et al (2012)

$$\sigma = \sigma_{core} \exp\left[\frac{E}{R}\left(\frac{1}{T_{ref}} - \frac{1}{T}\right)\right]$$

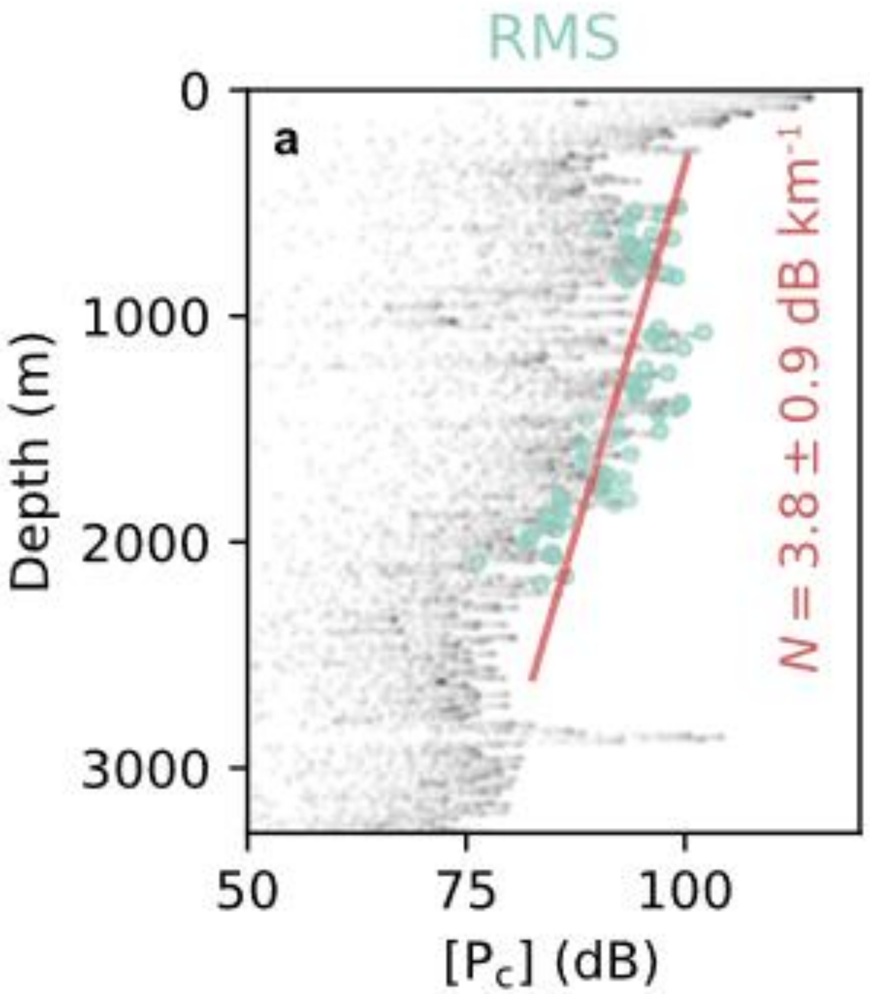
$$A = 10\log_{10}(e^{-2\alpha}) = 8.686\alpha = 8.686\sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r'}}\left(\frac{\sigma}{2}\right)$$

$$A_{tot} = 2Az$$

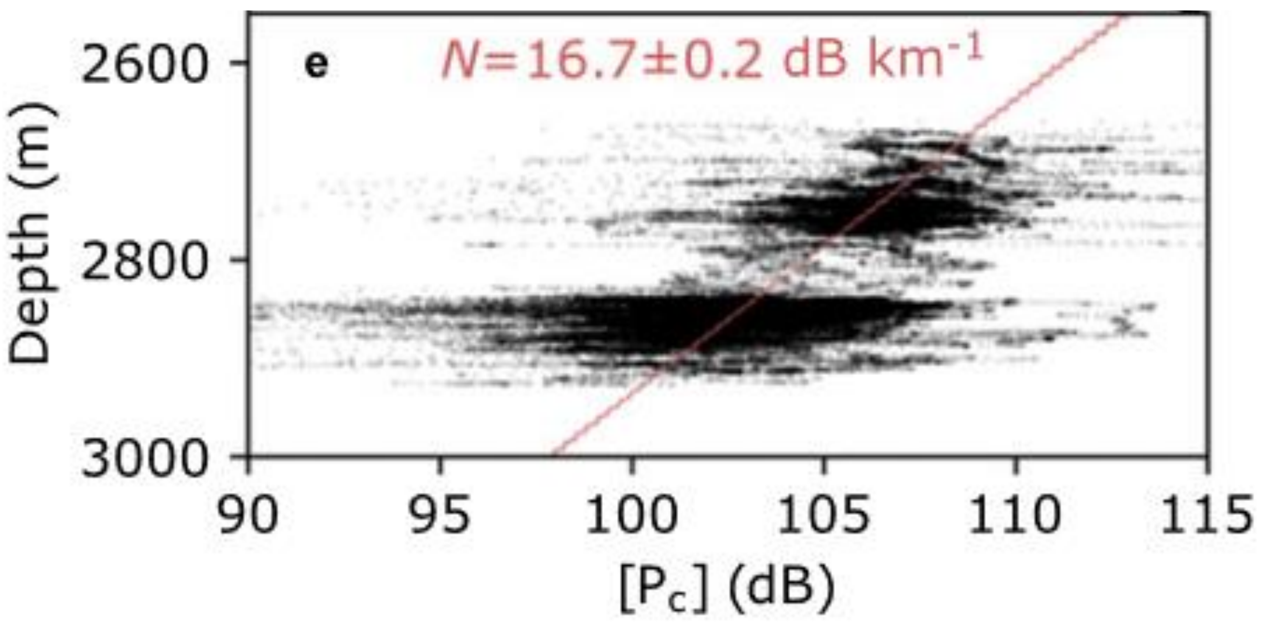
Zirizzotti et al (2014)

σ_{core} = conductivity measured from ice core
E = activation energy
R = gas constant (8.314 J/mol K)
T_{ref} = temperature of core when it was measured
T = true temperature at a given depth in the ice sheet
σ = corrected (true) conductivity
α = attenuation coefficient
A = attenuation rate (dB/m)
z = ice thickness
μ_0 = magnetic permeability of free space ($4\pi \times 10^{-7}\ Hm^{-1}$)
ϵ_0 = permittivity of free space ($8.85 \times 10^{-12}\ Fm^{-1}$)
ϵ_r' = real part of the ice permittivity (~ 3.17)

Estimating Attenuation from Data



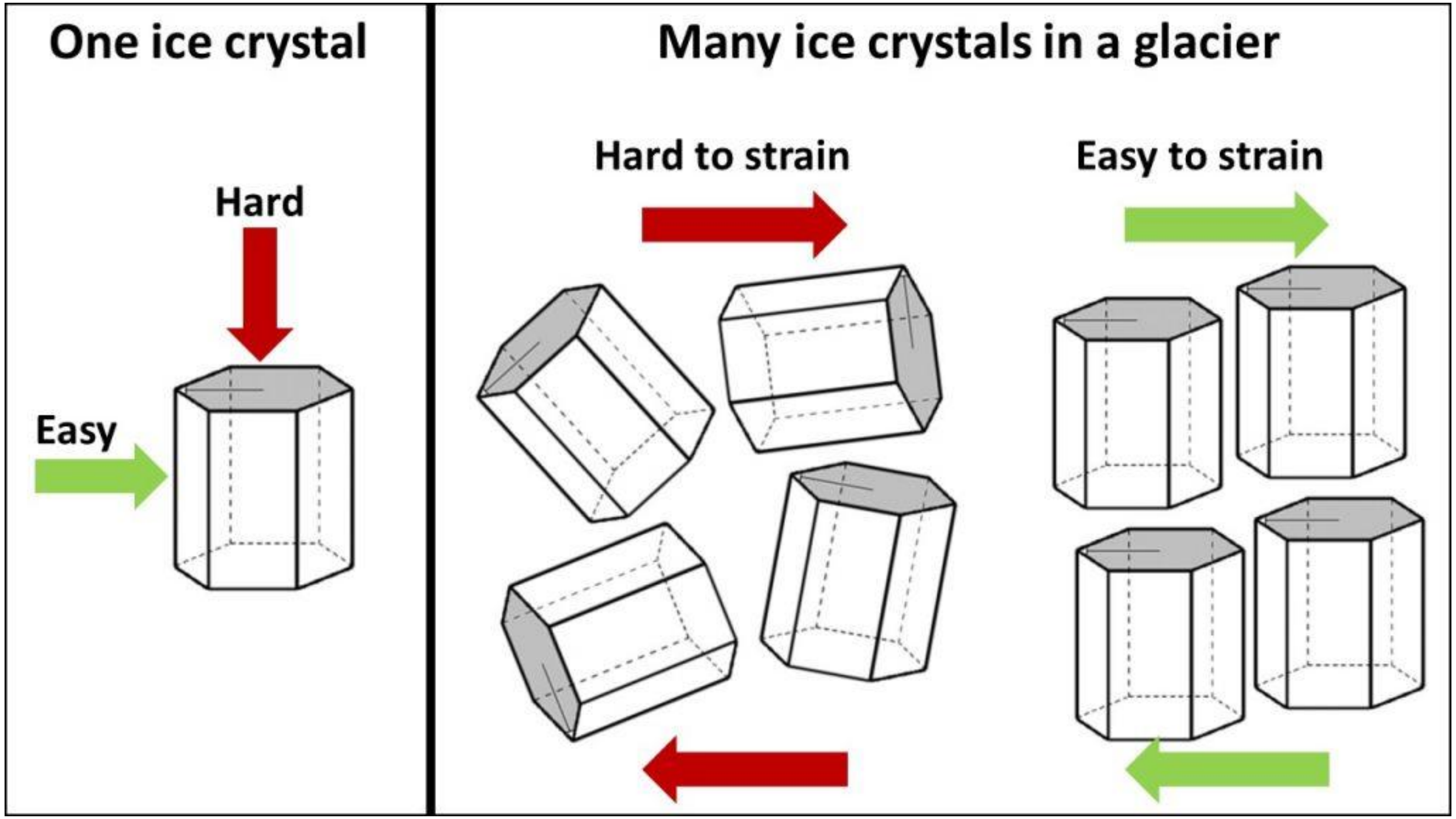
Trend in englacial layer power with depth

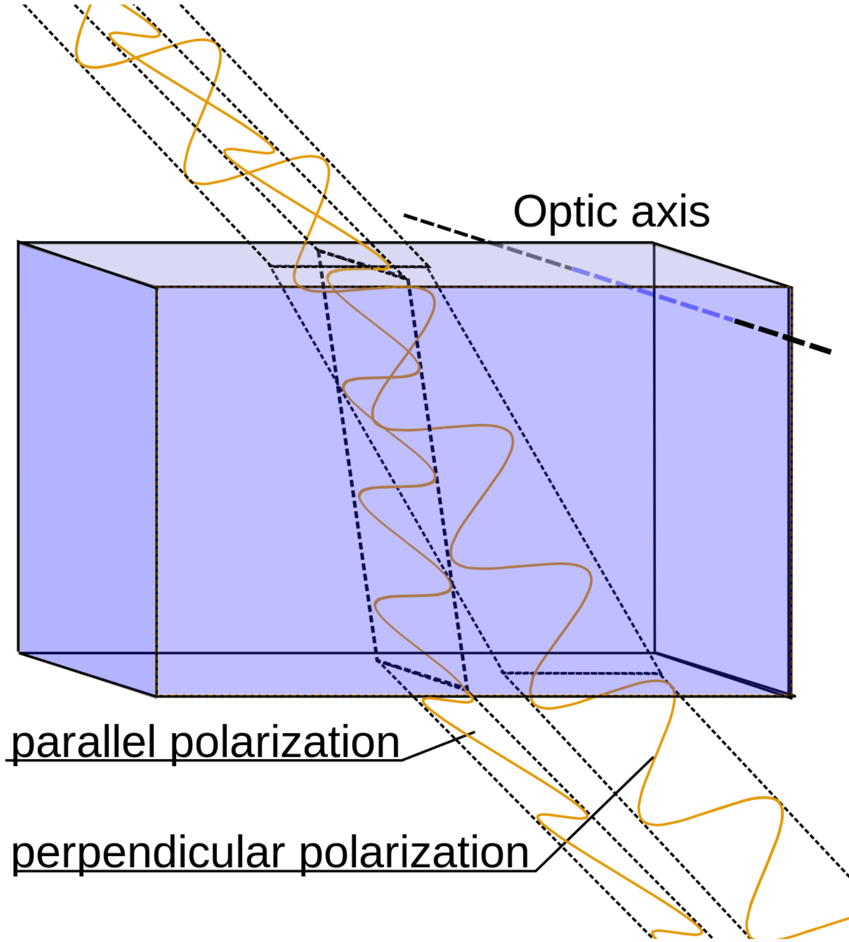


Trend in basal reflector power with depth

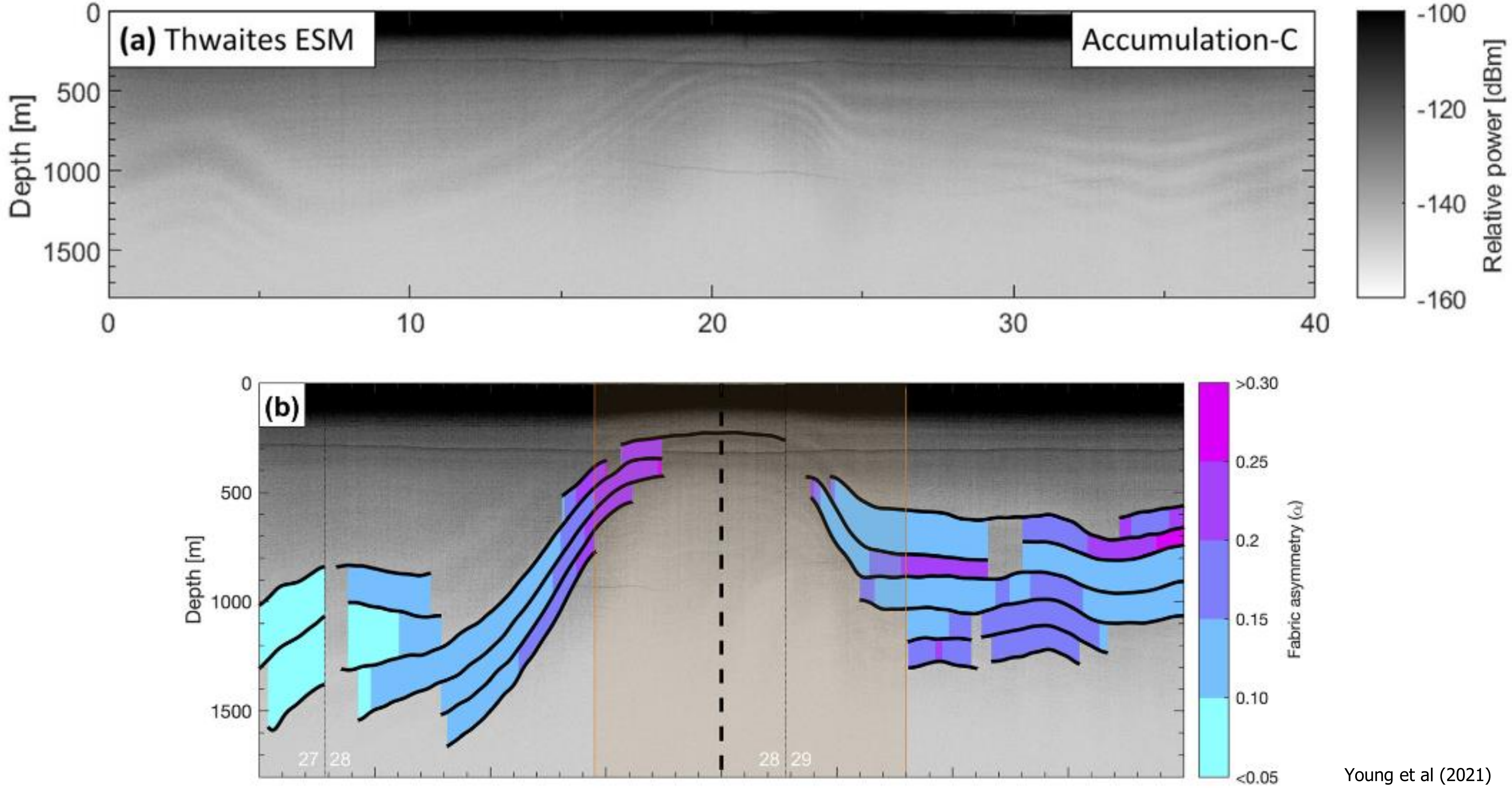
Hills et al (2020)

Birefringence & Crystal Orientation Fabric





Inferring Ice Fabric from Power Loss Patterns



Young et al (2021)