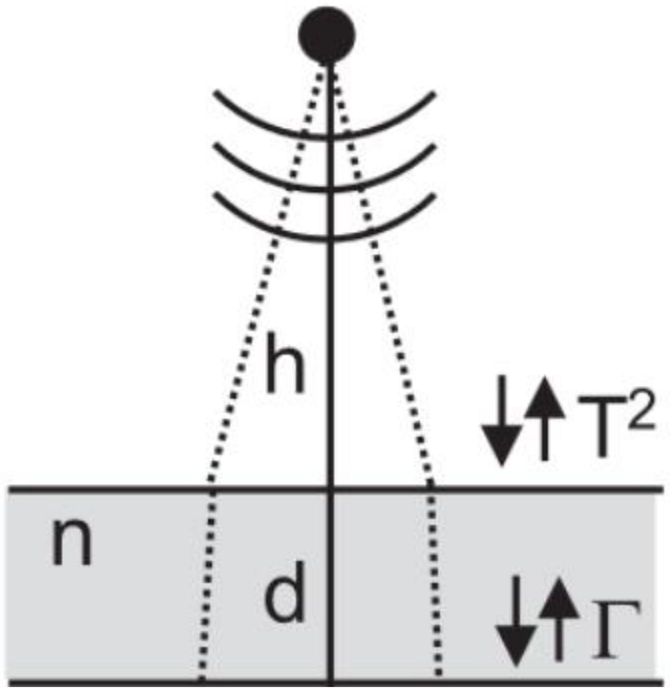
The Radar Equation – Smooth Surface

$$P_R = P_T\left(\frac{\lambda}{4\pi}\right)^2 G_T G_R \frac{1}{\left[2\left(h+\frac{z}{n}\right)\right]^2} T^2 L^2 B^2 \rho \Gamma$$

$P_T =$ transmit power
$\lambda =$ wavelength in air
$G_T =$ transmitting antenna gain
$G_R =$ receiving antenna gain
$h =$ radar clearance above surface
$z =$ depth to target below surface
$n =$ refractive index of subsurface
$T =$ surface transmission coefficient
$L =$ one-way attenuation through ice
$B =$ one-way birefringence loss
$\rho =$ roughness loss factor
$\Gamma =$ reflection coefficient of target

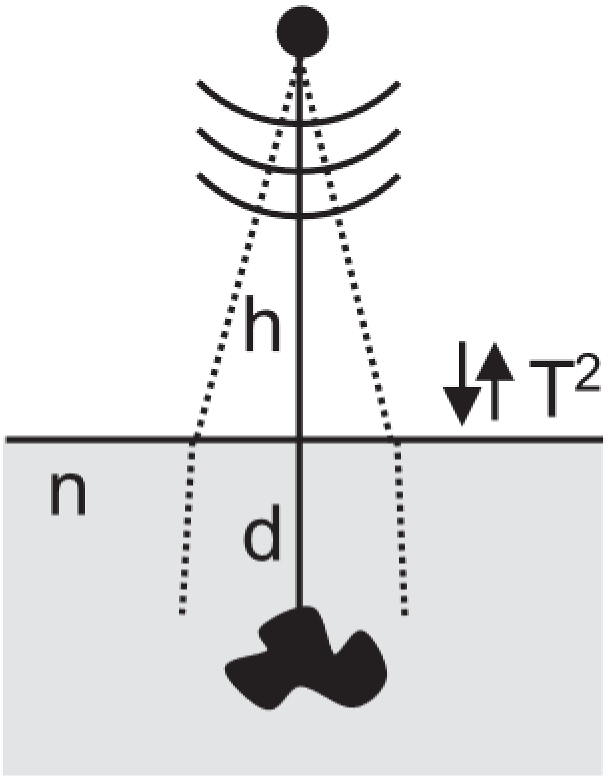


Haynes (2020)

The Radar Equation – Rough Surface

$$P_R = P_T \frac{\lambda^2}{(4\pi)^3} G_T G_R \frac{1}{\left(h + \frac{z}{n}\right)^4 n^2} T^2 L^2 B^2 \sigma^0 A$$

$P_T =$ transmit power
$\lambda =$ wavelength in air
$G_T =$ transmitting antenna gain
$G_R =$ receiving antenna gain
$h =$ radar clearance above surface
$z =$ depth to target below surface
$n =$ refractive index of subsurface
$T =$ surface transmission coefficient
$L =$ one-way attenuation through ice
$B =$ one-way birefringence loss
$\sigma^0 =$ normalized radar cross-section
$\mathrm{A} =$ area of target (max area is radar resolution)



Haynes (2020)

Simplified Radar Equation

$$P_R = \underbrace{P_T\left(\frac{\lambda}{4\pi}\right)^2 G_T G_R}_{\substack{S\\ \text{Radar system}\\ \text{properties}}} \underbrace{\frac{1}{\left[2\left(h+\frac{z}{n}\right)\right]^2}}_{\substack{G\\ \text{Geometric}\\ \text{spreading loss}}} T^2 L^2 B^2 \overbrace{\rho\Gamma}^{\substack{R\\ \text{Reflectivity}}}$$

A
Attenuation

T
Transmission Loss

B
Birefringence Loss

$$P_R = SGTABR$$

The Radar Equation – In dB

$$\boxed{\lfloor P \rceil_{dB} = 10\log_{10}\left(\frac{P}{1W}\right)}$$

$$\overbrace{\lfloor P_R \rceil_{dB}}^{\text{Received echo power}} = \underbrace{\lfloor S \rceil_{dB}}_{\text{Radar system properties (including transmit power)}} + \underbrace{\lfloor G \rceil_{dB}}_{\text{Geometric spreading loss}} + \underbrace{\lfloor T \rceil_{dB}}_{\text{Transmission Loss}} + \underbrace{\lfloor A \rceil_{dB}}_{\text{Englacial Attenuation}} + \underbrace{\lfloor B \rceil_{dB}}_{\text{Birefringence Loss}} + \underbrace{\lfloor R \rceil_{dB}}_{\text{Material Reflectivity}}$$

Theoretical Signal to Noise Ratio (SNR)

$$SNR = \frac{P_R}{N} \qquad N = kTBF_n$$

k = Boltzmann constant (1.38e-28 J/K)
T = electronics operating temperature (~290 K)
B = radar bandwidth
F_n = noise factor – additional noise in the receiver

$$\lfloor SNR \rfloor_{dB} = \lfloor P_R \rfloor_{dB} - \lfloor N \rfloor_{dB} + \underbrace{\lfloor G_R \rfloor_{dB}}_{\text{Range compression gain}} + \underbrace{\lfloor G_A \rfloor_{dB}}_{\text{Azimuth processing gain}}$$

The Radar Equation for Ice Sheet Analysis

We want to estimate the values for one of more these parameters because they tell us about physical properties of the ice sheet.

We measure this

$$\lfloor P_R \rceil_{dB} = \lfloor S \rceil_{dB} + \lfloor G \rceil_{dB} + \lfloor T \rceil_{dB} + \lfloor A \rceil_{dB} + \lfloor B \rceil_{dB} + \lfloor R \rceil_{dB}$$

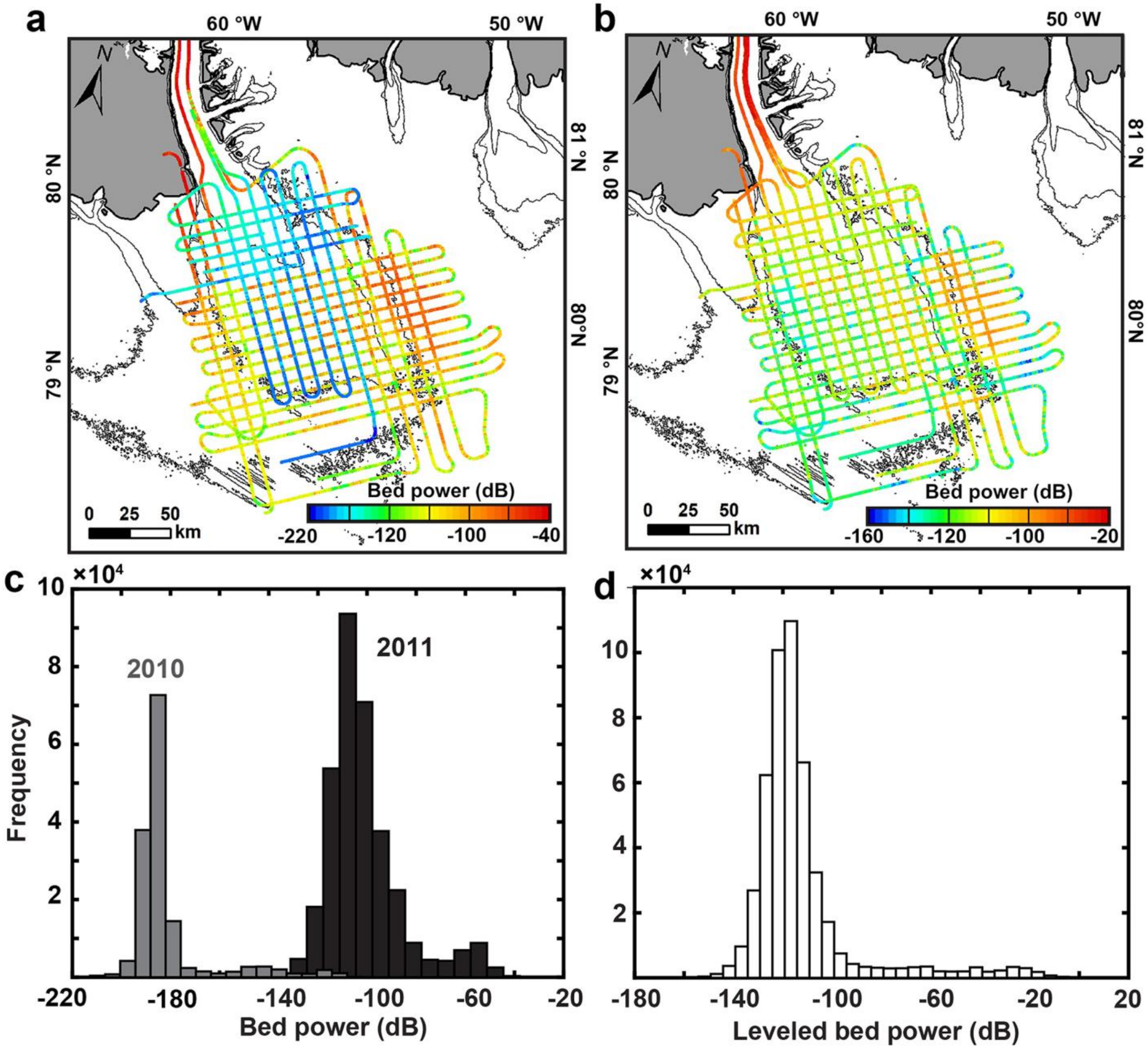
Find a way to empirically calibrate this out

Calculate this from known position of the aircraft

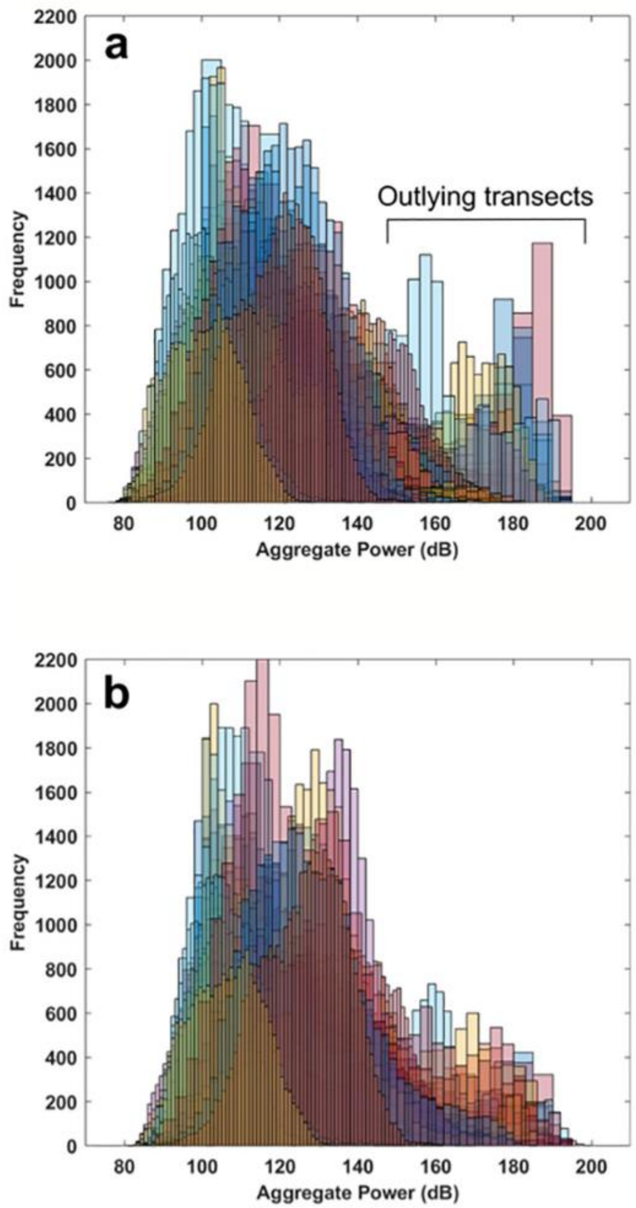
Calculate this from estimated surface density

Calibrating out System Properties

Chu et al (2018)



Chu et al (2021)



Geometric Spreading Correction

$$P_{corr} = P_R \left[2\left(h + \frac{z}{n}\right)\right]^2$$

OR

$$\lfloor P_{corr} \rfloor_{dB} = \lfloor P_R \rfloor_{dB} + 20\log_{10}\left[2\left(h + \frac{z}{n}\right)\right]$$

<u>RESAnalysisTools</u>: GeometricCorrection.py

```
corrected = GeometricPowerCorrection(surface, time, radar_data, n)
```

Surface Transmission Loss Correction

$$P_{corr} = \frac{P_R}{\left(1 - \left|\frac{n_{air} - n_{surf}}{n_{air} + n_{surf}}\right|^2\right)^2}$$

OR

$$\lfloor P_{corr} \rfloor_{dB} = \lfloor P_R \rfloor_{dB} - 20\log_{10}\left[1 - \left|\frac{n_{air} - n_{surf}}{n_{air} + n_{surf}}\right|^2\right]$$

Note: You only need to make this correction if you want to compared observed radar power to simulations OR you think there is a big spatial gradient in surface density across your study area.