Physics 30 Review of Physics 20 Essentials

Welcome to Physics 30. The Physics 30 course builds on a number of ideas and skills that you learned in Physics 20. Physics 30 requires that the following ideas and skills are well understood:

⇒ manipulation of equations with ease (**Essential**)
⇒ problem solving – proper selection of equations and identifying known and unknown quantities
⇒ adding and calculating vectors
⇒ understanding the dynamics of situations (i.e. application of Newton's laws)
⇒ uniform circular motion

I. Examples

Example 1

How fast will an object be travelling after falling for 7.0 s?

$\vec{v}_1 = 0$
$\vec{v}_2 = ?$
$\Delta t = 7.0 \text{ s}$
$\vec{a} = -9.81 \text{ m/s}^2$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\vec{v}_2 = \vec{a}\Delta t + \vec{v}_1$$

$$\vec{v}_2 = (-9.81 \tfrac{m}{s^2})(7.0 s)$$

$\vec{v}_2$ = **-69 m/s or 69 m/s down**

Example 2

A man standing on the roof of a building throws a stone downward at 20 m/s and the stone hits the ground after 5.0 s. How tall is the building?

$\vec{v}_1 = -20 \text{ m/s}$
$\vec{a} = -9.81 \text{ m/s}^2$
$\Delta t = 5.0 \text{ s}$
$\Delta d = ?$

$$\Delta\vec{d} = \vec{v}_1\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2$$

$$\Delta\vec{d} = -20 m/s(5.0 s) + \tfrac{1}{2}(-9.81 \tfrac{m}{s^2})(5.0 s)^2$$

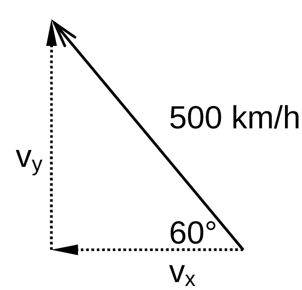
Δd = – 2.2 x 10^2 m or 2.2 x 10^2 m down

Example 3

What are the components of a plane flying at 500 km/h at 60° N of W?

v_y = 500 km/h sin 60° = **433 km/h north**

v_x = 500 km/h cos 60° = **250 km/h west**



Example 4

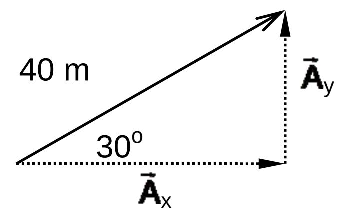
A man walks 40 m at 30° N of E, then 70 m at 60° S of E, and finally 20 m at 45° N of W. What is the displacement of the man?

Step 1.
Our first task is to find the north, south, east and west components of all the vectors.

Vector $\vec{A}$

The EAST component $\vec{A}_x = 40 \cos 30° = 34.64$ m (E)

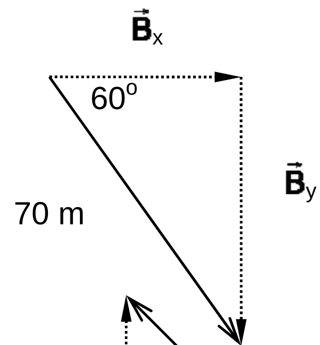
The NORTH component $\vec{A}_y = 40 \sin 30° = 20$ m (N)



Vector $\vec{B}$

the East component $\vec{B}_x = 70 \cos 60° = 35$ m (E)

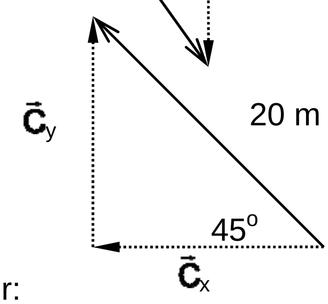
the South component $\vec{B}_y = 70 \sin 60° = 60.62$ m (S)



Vector $\vec{C}$

the West component $\vec{C}_x = 20 \cos 45° = 14.14$ m (W)

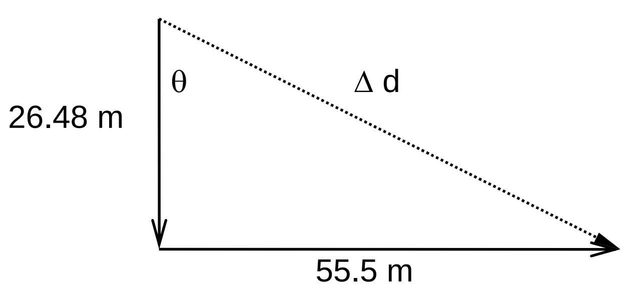
the North component $\vec{C}_y = 20 \sin 45° = 14.14$ m (N)



Step 2
Find the north-south and east-west components of the resultant vector:

(N - S) = 20 m (N) + 60.62 m (S) + 14.14 m (N) = 26.48 m south
(E - W) = 34.64 m (E) + 35 m (E) + 14.14 m (W) = 55.5 m east

Step 3
Drawing a sketch of the two vectors we get:



the angle can be found

$$\tan \theta = \frac{opp}{adj} = \frac{55.5 \text{ m}}{26.48 \text{ m}}$$

$\theta = 64.5°$ E of S

the displacement can be found by

$$\Delta d = \sqrt{55.5^2 + 26.48^2} = 61.5 \text{ m}$$

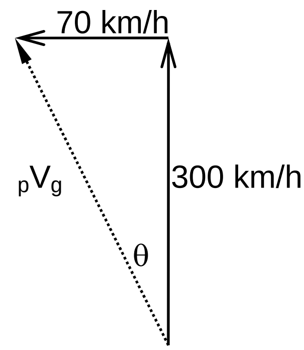
$\Delta\vec{d}$ **= 61.5 m [64.5° E of S]**

Example 5

An airplane flies North at 300 km/h but a wind is blowing west at 70 km/h

a) What is the velocity of the plane relative to the ground?
b) How far off course will the plane be after 3 hours?

a)



$\tan\theta$ = opp/adj = 70 km/h / 300 km/h

θ = 13.13° W of N

$${}_pV_g = \sqrt{(70\text{km / h})^2 + (300\text{km / h})^2}$$

${}_pV_g$ = 308.15 km/h

$\vec{{}_pV_g}$ **= 308 km/h at 13° W of N**

b) the wind is the direct cause for the plane going off course.

$d = v_1\Delta t + \cancel{\tfrac{1}{2} a \Delta t^2}$ (no acceleration)

d = 70 km/h (W) x 3 hours = **210 km (W)**

Example 6

A car traveling at 20 m/s goes around an unbanked curve in the road which has a radius of 122 m. What is the acceleration experienced by the car? What provided the centripetal force?

$a_c = \dfrac{v^2}{r} = \dfrac{(20\text{ m/s})^2}{122\text{ m}} = \mathbf{3.27\ m/s^2}$ (The force is provided by the friction between the tires and the road.)

Example 7

A 1.8 kg object is swung from the end of a 0.50 m string in a horizontal circle. If the time of revolution is 1.2 s, what is the tension in the string?

The centripetal force (F_c) is the tension force (F_T) in the string. Therefore

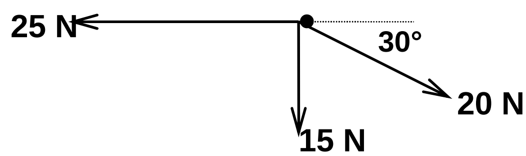
$$F_c = \frac{4\pi^2 mr}{T^2}$$

$$F_c = \frac{4\pi^2(1.8\text{kg})(0.50\text{m})}{(1.2\text{s})^2}$$

F_T = **25 N**

Example 8

A 2.0 kg object experiences a 15 N force pulling south, a 25 N force pulling west and a 20 N force pulling at 30° S of E. What is the acceleration experienced by the object?

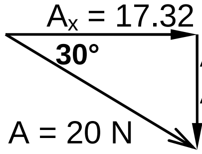


free body diagram

Using the methods for adding vectors using components we first solve for the net force.

$A_x = 20 \cos 30$

$A_x = 17.32$ N (E)

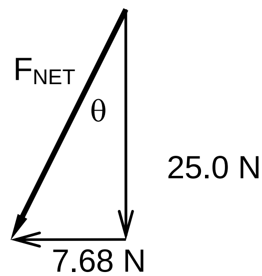


$A_y = 20 \sin 30$

$A_y = 10.0$ N (S)

(N-S) = 15 N (S) + 10.0 N (S) = 25.0 N (S)

(E-W) = 25 N (W) + 17.32 N (E) = 7.68 N (W)



$F = \sqrt{25^2 + 7.68^2}$

$F_{net} = 26.16$ N

$\tan\theta = 7.68 / 25$

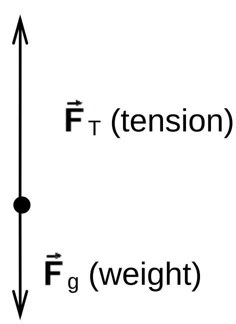
$\theta = 17.1^o$ W of S

Thus the acceleration becomes

$$\vec{a} = \frac{\vec{F}_{NET}}{m} = \frac{26.16 \text{ N } [17.1^o \text{ W of S}]}{2.0 \text{ kg}} = \mathbf{13.08\ m/s^2\ [17.1° \text{ W of S}]}$$

Example 9

What is the force required to accelerated a 50 kg object upward at 2.0 m/s^2?



Acceleration <u>upward</u> implies an unbalanced (net) force acting upward.

<u>Step 1</u>

$\vec{F}_g = m\,\vec{g} = 50 \text{ kg}(-9.81 \text{ m/s}^2)$

$\vec{F}_g = -490$ N

<u>Step 2</u>

$\vec{F}_{NET} = m\,\vec{a} = 50 \text{ kg}(+2.0 \text{ m/s}^2)$

$\vec{F}_{NET} = +100$ N

<u>Step 3</u>

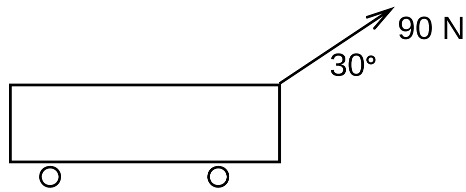
$\vec{F}_T + \vec{F}_g = \vec{F}_{NET}$

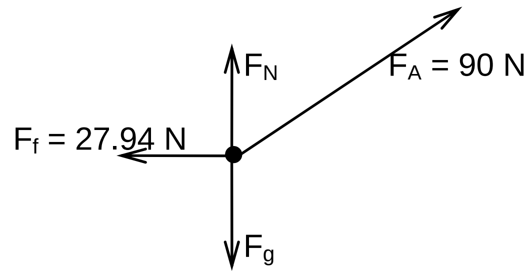
$\vec{F}_T + (-490 \text{ N}) = +100$ N

$\vec{F}_T =$ **+590 N** or **590 N upward**

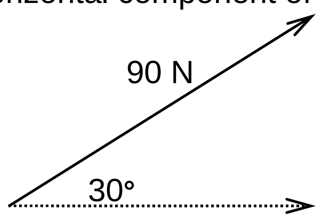
Example 10

A force of 90 N is applied to a wagon (mass 40 kg) at an angle of 30° to the horizontal. If the frictional force is 27.94 N, what is the resulting acceleration of the wagon?





Unless the applied force is large enough to pull the wagon off the ground, only the horizontal component of the force will cause the wagon to accelerate.



F_x = 90 N (cos 30)
F_x = 77.94 N

$$\vec{F}_{NET} = \vec{F}_x + \vec{F}_f$$
$$\vec{F}_{NET} = +77.94\text{ N} + (-27.94\text{ N})$$
$$\vec{F}_{NET} = 50.0\text{ N east}$$

$$\vec{a} = \frac{\vec{F}_{NET}}{m} = \frac{50.0\text{ N (E)}}{40\text{ kg}} = \mathbf{1.25\ m/s^2\ east}$$

Example 11

A 10 kg box is dragged over a horizontal surface by a force of 40 N. If the box moves with a constant speed of 0.50 m/s, what is the coefficient of kinetic friction for the surface?

The velocity is constant. $a = 0$ $F_{net} = 0$ $\therefore$ $F_A = F_f = 40$ N

The surface is horizontal: $\therefore$ $F_N = F_g = m\,g = 10\text{ kg}(9.81\text{ m/s}^2) = 98.1$ N

$$\mu = \frac{F_f}{F_N} = \frac{40\text{ N}}{98.1\text{ N}} = \mathbf{0.41}$$

Kinematics review

1. State the displacement of each of the following changes in position
   a) -8 km to -2 km
   b) +2 km to -8 km
   c) 0 km to +8 km
   d) -8 km to +20 km
   e) +8 km to -8 km
2. An electron travels at a uniform speed of 1.3×10^5 m/s. How much time is required for the electron to move a distance of 1.0 m? (7.7 μs)
3. A rally driver completes three consecutive sections of a straight rally course as follows: section 1 (10 km) in 7.50 min, section 2 (18 km) in 14.40 min, and section 3 (9.8 km) in 5.80 min. What was the average speed through the three sections? (1.4 km/min)
4. A car accelerates uniformly from 10 m/s to 30 m/s in 10 s. What is the acceleration? (2.0 m/s^2)
5. A ball thrown straight up in the air has an initial velocity of 40 m/s and reaches its maximum height in 4.0 s. What was the acceleration of the ball? (-10 m/s^2)
6. A ball is rolled up a slope with an initial speed of 6.0 m/s. The ball experiences an acceleration of 2.0 m/s^2 down the slope. What is its velocity after (a) 2.0 s, (b) 3.0 s and (c) 4.0 s? (+2.0 m/s, 0, -2.0 m/s)
7. An electron is accelerated uniformly from rest to a speed of 2.0×10^7 m/s. If the electron travelled 0.10 m while it was being accelerated, what was its acceleration? How long did the electron take to reach its final speed? (2.0×10^{15} m/s^2, 1.0×10^{-8} s)
8. A bullet is shot vertically into the air with an initial velocity of 500 m/s. Ignoring air resistance:
   a) How long does it take for the bullet to reach its maximum height? (51 s)
   b) How high does the bullet go? (1.3×10^4 m)
9. A balloon is ascending at the rate of 9.0 m/s and has reached a height of 80 m above the ground when the occupant releases a package. How long does the package take to hit the ground? (Hint: What is the initial velocity of the package relative to the ground?) (5.1 s)
10. A person drops a ball from a height of 20 m. What is the ball's final velocity and how long does it take to fall? (-19.8 m/s, 2.0 s)
11. A stone is thrown upward with an initial velocity of 11 m/s. Calculate the maximum height and the time the stone is in the air. (2.2 s, 6.2 m)
12. A stone is thrown vertically upward from a 117.82 m high cliff with an initial velocity of 19.62 m/s. How long will it take for the stone to hit the water below? (7.29 s)
13. A stone is thrown horizontally at 19.62 m/s from a 117.82 m high cliff. How long will it take for the stone to hit the water below? How far from the base of the cliff will the stone land? (4.90 s, 96.2 m)

Vectors review

1. The gondola ski lift at Keystone, Colorado is 2830 m long. On average, the ski lift rises 14.6^o above the horizontal. How high is the top of the ski lift relative to the base? (713 m)
2. A highway is planned between two towns in Saskatchewan, one of which lies 35.0 km south and 72.0 km west of the other. What is the shortest length of highway that can be built and what would be its bearing? (80.0 km @ 26^o S of W)
3. Three ropes are attached to a heavy box on a frictionless surface. One rope applies a 475 N force due east and the other applies a 315 N force due south. What force must be applied to the other rope to cancel the other forces? (570 N @ 34^o N of W)
4. A helicopter is travelling with a speed of 67.0 m/s @ 38.0^o south of east. What are the south and east components of the helicopter's velocity? (41.2 m/s south, 52.8 m/s east)
5. An airplane is being flown due east with respect to the air at 120 km/h. If a wind is blowing 70 km/h @ 30^o south of east relative to the ground, what is the airplane's velocity relative to the ground? (184 km/h @ 11^o S of E)
6. A force vector of 45.0 N @ 30.0^o north of east is added to an other force vector 75.0 N [N]. What is the resulting force? (105 N @ 22^o E of N)
7. A pilot flies from point A to B to C in two straight line segments. The displacement vector for the first leg is 243 km @ 50.0^o N of E and the second leg displacement vector is 57.0 km @ 20.0^o S of E. What is the resultant displacement vector? (268 km @ 38^o N of E)

Circular motion review

1. A 1.0 kg ball at the end of a 2.8 m rope rotates once every 0.98 s.
   a) What is the speed of the ball? (18 m/s)
   b) What is the tension in the rope? (1.2×10^2 N)
2. A car with a mass of 822 kg rounds an unbanked curve in the road at a speed of 28.0 m/s. If the radius of the curve is 105 m, what is the minimum frictional force required to keep the car on the road? (6.14×10^3 N)
3. A 1.7 kg object is swung from the end of a 0.60 m string in a horizontal circle. If the time of one revolution is 1.1 s, what is the tension in the string? (33 N)

Dynamics review

1. A car is at rest on a horizontal road surface. The driver presses down on the gas pedal and the car accelerates forward for a few seconds until it reaches the speed limit. The driver eases off of the gas pedal and the car moves at constant speed. Describe and explain the car's motion in terms of Newton's three laws.

2. A net force of 30.0 N south acts on a 10.0 kg object. What is the acceleration of the object? (3.00 m/s^2 south)

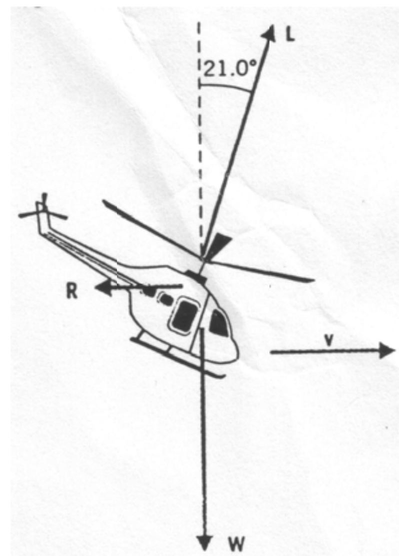
3. A 22 kg object accelerates uniformly from rest to a velocity of 2.5 m/s west in 8.7 s. What is the net force acting on the car during the acceleration? (-6.3 N)

4. What is the acceleration due to gravity near the surface of the moon if an object has a weight of 36.0 N and a mass of 22.0 kg? (1.64 m/s^2)

5. An object with a mass of 45.0 kg is pulled along a horizontal surface by a rope that makes an angle of 32.0^o with the horizontal. If the frictional force is 50.0 N and the tension in the rope is 95.0 N, what is the acceleration of the object? (0.679 m/s^2)

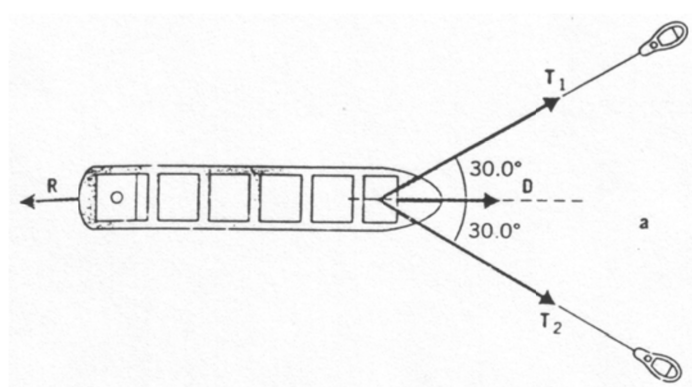
6. An object with a mass of 25 kg is pulled across a floor at constant speed with a pulling force of 68 N. What is the coefficient of friction on the floor surface? (0.28)

7. What is the tension in the cable of a 1.20 x 10^3 kg elevator that is
   a) accelerating downward at 1.05 m/s^2? (1.05 x 10^4 N)
   b) accelerating upward at 1.05 m/s^2? (1.30 x 10^4 N)
   c) moving downward at 1.05 m/s? (1.18 x 10^4 N)



8. The helicopter in the drawing to the right is moving horizontally with a constant velocity. The weight of the helicopter is 5.38 x 10^4 N and the lift force **L** generated by the rotating blade makes an angle of 21.0^o with respect to the vertical.
   a) What is the magnitude of the lift force? (5.76 x 10^4 N)
   b) What is the air resistance **R** that opposes the motion? (-2.07 x 10^4 N)

9. A supertanker (mass = 1.50 x 10^8 kg) is being towed by two tugboats as shown in the diagram. The tensions in the towing cables apply the force $\mathbf{T_1}$ and $\mathbf{T_2}$ at equal angles of 30.0^o with respect to the tanker's axis. In addition, the tanker's engines produce a forward driving force **D** of 7.50 x 10^4 N and the water applies an opposing force **R** of 4.00 x 10^4 N. The tanker moves forward with an acceleration of 2.00 x 10^{-3} m/s^2. Find the tensions in the towing cables. (1.53 x 10^5 N)



Physics 30 Lesson 1

Momentum and Conservation of Momentum in One Dimension

I. Physics principles

Students often ask me if Physics 30 is "harder" than Physics 20. This, of course, depends on the aptitudes, attitudes and work ethic of the individual student. However, there is one major difference between Physics 20 and Physics 30. Physics 20 was dominated by problem solving and the calculation of an answer. Physics 30 has a substantial problem-solving component, but it also requires that students learn and understand the Physics Principles that form the foundation of physics. (These principles are listed on your Physics Data Sheet and are reproduced to the right.) In other words, you will be required to explain how the physics principles are being applied to a particular problem – you will demonstrate that you know the theory behind the problem solving.

0 Uniform motion $(\vec{F}_{net} = 0)$

1 Accelerated motion $(\vec{F}_{net} \neq 0)$

2 Uniform circular motion $(\vec{F}_{net}$ is radially inward)

3 Work-energy theorem

4 Conservation of momentum

5 Conservation of energy

6 Conservation of mass-energy

7 Conservation of charge

8 Conservation of nucleons

9 Wave-particle duality

Some of these principles (0, 1, 2, 3 and 5) were taught in Physics 20 and we will see them again in different contexts in Physics 30. The remaining principles are what Physics 30 is all about. This lesson will introduce principle 4, the conservation of momentum.

II. Momentum

A very useful physical concept is **momentum**. The momentum ($\vec{p}$) of an object is defined as the product of its <u>mass</u> and <u>velocity</u>. Recall that velocity is a vector quantity that involves both a speed and a direction. Thus, **momentum is a vector quantity**.

$$\vec{p} = m\vec{v}$$

m	mass
$\vec{v}$	velocity
$\vec{p}$	momentum

There is no chosen unit for momentum like there is for force (N) or energy (J). The unit for momentum is a combination of the mass unit and velocity unit. Some examples are kg·m/s, kg·km/h, g·m/s, etc. (You may refer to Pearson pages 446 to 449 for a different discussion about momentum.)

Example 1

What is the momentum of a 1500 kg car travelling west at 5.0 m/s?

$$\vec{p} = m\vec{v}$$
$$\vec{p} = 1500\,\text{kg}(5.0\,{}^{\text{m}}\!/\!{}_{\text{s}}\ \text{west})$$
$$\boxed{\vec{p} = 7.5\times10^{3}\,{}^{\text{kg}\cdot\text{m}}\!/\!{}_{\text{s}}\ \text{west}}$$

Example 2

A micro-meteorite with a mass of 5.0 g has the same momentum as the car in *Example 1*. What is the velocity of the meteorite?

$$\vec{p} = m\vec{v}$$
$$\vec{v} = \frac{\vec{p}}{m}$$
$$\vec{v} = \frac{7.5\times10^{3}\,{}^{\text{kg}\cdot\text{m}}\!/\!{}_{\text{s}}}{0.0050\,\text{kg}}\ \text{west}$$
$$\boxed{\vec{v} = 1.5\times10^{6}\,{}^{\text{m}}\!/\!{}_{\text{s}}\ \text{west}}$$

A major misconception that people have is that **inertia** and **momentum** mean the same thing. In the examples above, both objects have the same momentum (mass x velocity) but the inertia (i.e. mass) of the car is substantially larger than the inertia of the meteorite. In other words, inertia refers to the mass of an object while momentum refers to the mass and motion of the object.

III. Systems

Before we can understand momentum more fully, we must also be aware of different kinds of **systems**. There are several types of systems:

- **Closed system:** no mass enters or leaves the system
- **Isolated system:** no external forces act on the system and no energy leaves the system.
- **Open system:** mass may enter or leave the system and external forces may influence the system.

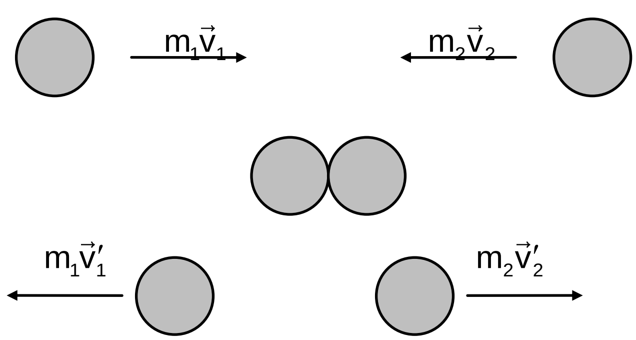
To illustrate the difference between these kinds of systems, consider a real-life collision between two cars:

- First, the collision of the cars is **not** an **isolated** system – i.e. they are not isolated from the Earth. Frictional forces between the Earth and the cars will cause the cars to slow down. If the cars collided on a very slippery surface where the frictional forces with the Earth were minimised, this would almost constitute an isolated system.
- Second, if the cars collide and all of the parts of the cars stay attached to the cars we have a **closed** system. However, if parts of the cars fly off we have an **open** system.

In our investigation of the **conservation of momentum** below we will be assuming **closed** and **isolated** systems.

IV. Conservation of Momentum

The real importance of the concept of momentum is that in any isolated closed system, the **total** momentum of the system is conserved (i.e. remains constant). For example, consider two objects (m_1 and m_2) that collide as shown below.



Before collision

During collision

After collision

Although the momentum of each individual object changes during the collision, the sum of the momenta before the collision ($m_1\vec{v}_1 + m_2\vec{v}_2$) and after the collision ($m_1\vec{v}_1' + m_2\vec{v}_2'$) are the same. The general statement for the **law of conservation of momentum** is:

The total momentum of an isolated system of objects remains constant.

In other words, the sum of the momenta before a collision or explosion ($\Sigma\vec{p}$) equals the sum of the momenta after a collision or explosion ($\Sigma\vec{p}'$).

$$\sum\vec{p} = \sum\vec{p}'$$

Note that the prime symbol is used to denote "after" the collision or explosion.

In <u>any</u> collision or explosion, the total momentum is <u>always</u> conserved. This principle proves to be very useful in predicting what will happen when objects collide or explode.

Actually, the principle of the Conservation of Momentum is a direct consequence of Newton's Third Law of Motion that we learned about in Physics 20. Recall that when any object exerts a force on another object, the second object will exert an equal and opposite force on the first object. Consider the collision between the two masses illustrated above. When they collide, mass 1 exerts a force on mass 2 ($\vec{F}_{1on2}$) resulting in an acceleration (change in velocity) of mass 2. According to Newton's 3rd Law, if object 1 exerts a force on object 2, object 2 will exert an equal and opposite reaction force on object 1 ($-\vec{F}_{2on1}$) resulting in a change in velocity of mass 2. Therefore

$$\vec{F}_{1on2} = -\vec{F}_{2on1}$$

If we apply Newton's Second Law of Motion $\vec{F} = m\vec{a} = \dfrac{m\Delta\vec{v}}{\Delta t}$

$$\vec{F}_{1on2} = -\vec{F}_{2on1}$$

$$\frac{m_2\Delta\vec{v}_2}{\cancel{\Delta t}} = \frac{-m_1\Delta\vec{v}_1}{\cancel{\Delta t}}$$

$$m_2\Delta\vec{v}_2 = -m_1\Delta\vec{v}_1$$

$\vec{F}_{1on2}$ causes a change in the momentum of 2, and $\vec{F}_{2on1}$ causes a change in momentum of object 1

the time of contact is the same for both objects $\therefore$ Δt cancels out

Notice that when the change in momentum of 1 and 2 are added together, the result is zero.

$$m_2\Delta\vec{v}_2 + m_1\Delta\vec{v}_1 = 0$$

This result indicates that while the momentum of each object changes, **the total change in momentum of the system (i.e. both objects) is zero**. This is also a statement of the principle of **conservation of momentum**.

V. Elastic and inelastic collisions

There are two basic types of collisions – elastic and inelastic – and we use the law of conservation of momentum to predict and calculate the results.

Elastic collisions are those where **both** momentum and kinetic energy are conserved. Purely elastic collisions are very hard to produce in the ordinary world because there is always some kinetic energy converted into heat, sound, deformation or some other form of energy. As we shall see in future lessons on the nature of the atom, collisions between subatomic particles are elastic collisions.

Inelastic collisions are those where momentum is conserved, but kinetic energy is not conserved. A **completely inelastic** collision is when the objects collide and stick together.

Example 3

A 2.00 kg object travelling east at 4.00 m/s collides with a 3.00 kg object travelling west at 1.50 m/s. If the 3.00 kg object ends up travelling east at 1.25 m/s, what is the final velocity of the 2.00 kg object?

Before | After

$\vec{v}_1 = +4.00$ m/s $\vec{v}_2 = -1.50$ m/s $\vec{v}_1' = ?$ $\vec{v}_2' = +1.25$ m/s

$m_1 = 2.00$ kg $m_2 = 3.00$ kg $m_1 = 2.00$ kg $m_2 = 3.00$ kg

When we apply the conservation of momentum principle we want the equation to reflect the context of the question. In this question there are two objects before the collision and two objects after the collision. Therefore there are two $m\vec{v}$ terms on the before side and two $m\vec{v}$ terms on the after side.

$$\sum \vec{p} = \sum \vec{p}'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{v}_1' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 - m_2\vec{v}_2'}{m_1}$$

Rearrange the equation, plug in the known values, and calculate the unknown.

$$\vec{v}_1' = \frac{2.00\,\text{kg}(+4.00\,\tfrac{m}{s}) + 3.00\,\text{kg}(-1.50\,\tfrac{m}{s}) - 3.00\,\text{kg}(+1.25\,\tfrac{m}{s})}{2.00\,\text{kg}}$$

$$\boxed{\vec{v}_1' = -0.13\,\tfrac{m}{s} \text{ or } 0.13\,\tfrac{m}{s}\text{ west}}$$

Example 4

A 30 kg object travelling at 45 m/s west collides with a 40 kg object at rest. If the objects stick together on contact, what is the resulting velocity of the combined masses?

Before | After

$\vec{v}_1 = -45$ m/s | $\vec{v}' = ?$

$m_2 = 40$ kg | $m_1 = 30$ kg | $m_{1+2} = 40$ kg + 30 kg

$$\sum \vec{p} = \sum \vec{p}'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_{1+2}\vec{v}'_{1+2}$$

$$\vec{v}'_{1+2} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_{1+2}}$$

$$\vec{v}'_{1+2} = \frac{30\,kg(-45\,{}^{m}\!/\!{}_{s}) + 0}{40\,kg + 30\,kg}$$

$$\boxed{\vec{v}'_{1+2} = -19\,{}^{m}\!/\!{}_{s} \text{ or } 19\,{}^{m}\!/\!{}_{s} \text{ west}}$$

In this question there are two objects before the collision and one object after the collision. Therefore there are two $m\vec{v}$ terms on the before side and one $m\vec{v}$ term on the after side.

This is an example of a **completely inelastic** collision.

Example 5

A 400 kg object travelling east at 50 m/s collides with a moving 50 kg object. The masses stick together after they collide and move at 27.8 m/s to the east. What was the initial velocity of the 50 kg object?

Before | After

$\vec{v}_1 = +\ 50$ m/s | $\vec{v}_2 = ?$ | $\vec{v}' = +\ 27.8$ m/s

$m_1 = 400$ kg | $m_2 = 50$ kg | $m_{1+2} = 400$ kg + 50 kg

$$\sum \vec{p} = \sum \vec{p}'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_{1+2}\vec{v}'_{1+2}$$

$$\vec{v}_2 = \frac{m_{1+2}\vec{v}'_{1+2} - m_1\vec{v}_1}{m_2}$$

$$\vec{v}_2 = \frac{(400\,kg + 50\,kg)(+27.8\,{}^{m}\!/\!{}_{s}) - 400\,kg(+50\,{}^{m}\!/\!{}_{s})}{50\,kg}$$

$$\boxed{\vec{v}_2 = 1.5 \times 10^2\,{}^{m}\!/\!{}_{s} \text{ west}}$$

Example 6

A 2.00 kg object travelling east at 4.00 m/s collides with a 3.00 kg object travelling west at 1.50 m/s. If the 3.00 kg object ends up travelling east at 2.10 m/s.

a. What is the final velocity of the 2.00 kg object?
b. Was the collision elastic?
c. How can momentum always be conserved while kinetic energy is not always conserved?

a. Use the conservation of momentum to calculate the final velocity of the 2.00 kg object.

Before | After

$\vec{v}_1 = +4.00$ m/s $\vec{v}_2 = -1.50$ m/s $\vec{v}_1' = ?$ $\vec{v}_2' = +2.10$ m/s

$m_1 = 2.00$ kg $m_2 = 3.00$ kg $m_1 = 2.00$ kg $m_2 = 3.00$ kg

$$\sum \vec{p} = \sum \vec{p}'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{v}_1' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 - m_2\vec{v}_2'}{m_1}$$

$$\vec{v}_1' = \frac{2.00\,\text{kg}(+4.00\,\text{m/s}) + 3.00\,\text{kg}(-1.50\,\text{m/s}) - 3.00\,\text{kg}(+2.10\,\text{m/s})}{2.00\,\text{kg}}$$

$$\boxed{\vec{v}_1' = -1.40\,\text{m/s} \text{ or } 1.40\,\text{m/s west}}$$

b. To determine if the collision was elastic, calculate the kinetic energy before the collision and after the collision and compare the values.

$$E_{ki} = E_{ki1} + E_{ki2}$$

$$E_{ki} = \tfrac{1}{2}m_1v_{i1}^2 + \tfrac{1}{2}m_2v_{i2}^2$$

$$E_{ki} = \tfrac{1}{2}(2.00\,\text{kg})(4.00\,\text{m/s})^2 + \tfrac{1}{2}(3.00\,\text{kg})(-1.50\,\text{m/s})^2$$

$$E_{ki} = 19.375\,\text{J}$$

$$E_{kf} = E_{kf1} + E_{kf2}$$

$$E_{kf} = \tfrac{1}{2}m_1v_{f1}^2 + \tfrac{1}{2}m_2v_{f2}^2$$

$$E_{kf} = \tfrac{1}{2}(2.00\,\text{kg})(-1.40\,\text{m/s})^2 + \tfrac{1}{2}(3.00\,\text{kg})(2.10\,\text{m/s})^2$$

$$E_{kf} = 8.575\,\text{J}$$

Since the final kinetic energy is less than the initial kinetic energy, the collision is **inelastic**.

c. Momentum in an isolated system is **always** conserved. This law is a direct result of Newton's third law of motion where the change in momentum of one object is equal and opposite the change in momentum of the other object. The combined change in momentum is zero.

Kinetic energy, on the other hand, can easily be transformed into heat, sound, and the deformation of objects. The Conservation of Energy (i.e. − the total amount of energy in an isolated system does not change, but energy can be transformed into other types) does not depend on Newton's Laws.

VI. Explosions

The primary difference between collisions and explosions is that prior to an explosion there is one object, and after the explosion there are several objects.

Example 7

A 5.0 kg rifle fires a 0.020 kg bullet to the east. If the muzzle speed of the bullet is 400 m/s, what is the recoil velocity of the rifle?

Note that initially the bullet and rifle are at rest. Therefore $\Sigma\vec{p} = 0$

$$\sum\vec{p} = \sum\vec{p}'$$

$$0 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{v}_1' = \frac{-m_2\vec{v}_2'}{m_1}$$

$$\vec{v}_1' = \frac{-0.020\,\text{kg}(+400\,\text{m/s})}{5.0\,\text{kg}}$$

$$\boxed{\vec{v}_1' = 1.6\,\text{m/s west}}$$

Example 8

A 4.00 kg bomb is rolling across a floor at 3.00 m/s to the east. After the bomb explodes a 1.50 kg piece has a velocity of 15.0 m/s east. What is the velocity of the other piece?

$$\sum\vec{p} = \sum\vec{p}'$$

$$m_4\vec{v}_4 = m_{1.5}\vec{v}_{1.5}' + m_{2.5}\vec{v}_{2.5}'$$

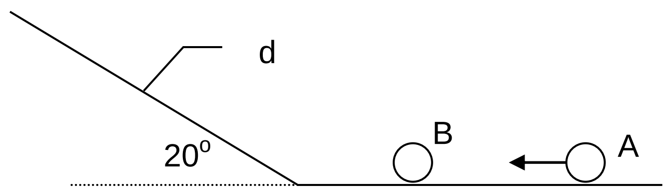
$$\vec{v}_{2.5}' = \frac{m_4\vec{v}_4 - m_{1.5}\vec{v}_{1.5}'}{m_{2.5}}$$

$$\vec{v}_{2.5}' = \frac{4.00\,\text{kg}(+3.00\,\text{m/s}) - 1.50\,\text{kg}(+15.0\,\text{m/s})}{2.00\,\text{kg}}$$

$$\boxed{\vec{v}_{2.5}' = 4.20\,\text{m/s west}}$$

Example 9

In the diagram below, ball A (mass = 5.0 kg) is traveling to the left at 12 m/s and collides with a stationary ball B (mass = 4.0 kg). After the collision ball A is moving at 0.50 m/s to the left. How high up the incline will ball B rise before coming to rest?



The first part of the problem involves a collision between ball A and ball B. Thus we use the conservation of momentum to find the final speed of ball B. Once we know the speed of ball B, we can use the conservation of energy to determine how high it rises on the incline.

Part A – Find v_B'

$$\sum \vec{p}_B = \sum \vec{p}_A$$
$$m_A \vec{v}_A = m_B \vec{v}_B' + m_A \vec{v}_A'$$
$$\vec{v}_B' = \frac{m_A \vec{v}_A - m_A \vec{v}_A'}{m_B}$$
$$\vec{v}_B' = \frac{5.0\text{kg}(-12\,\text{m/s}) - 5.0\text{kg}(-0.50\,\text{m/s})}{4.0\text{kg}}$$
$$\vec{v}_B' = 14.375\,\text{m/s}\ \text{left}$$

Part B – Find the height up the ramp
The kinetic energy of ball B after the collision is converted into potential energy (h).

$$E_k = E_p$$
$$\tfrac{1}{2}mv^2 = mgh$$
$$h = \frac{\tfrac{1}{2}v^2}{g}$$
$$h = \frac{\tfrac{1}{2}(14.375\,\text{m/s})^2}{9.81\,\text{m/s}^2}$$
$$h = 10.5\,\text{m}$$

$$\sin 20 = \frac{h}{d}$$
$$d = \frac{h}{\sin 20}$$
$$d = \frac{10.5\,\text{m}}{\sin 20}$$
$$d = \mathbf{31m}$$

VII. Practice Problems

1. A 0.25 kg ball travelling east at 4.5 m/s collides with a 0.30 kg steel ball travelling west at 5.0 m/s. After the collision the 0.30 kg ball is travelling east with a speed of 0.40 m/s. What is the final velocity of the 0.25 kg ball? (1.98 m/s west)

2. A freight train is being assembled in a switching yard. Boxcar #1 has a mass of 6.4×10^4 kg and moves with a velocity of +0.80 m/s. Boxcar #2, with a mass of 9.2×10^4 kg and a velocity of +1.2 m/s, overtakes Boxcar #1 and couples with it. Neglecting friction, find the common velocity of the two cars after they have coupled. (+1.0 m/s)

3. Starting from rest, two skaters “push off” against each other on smooth, level ice. One is a woman (m = 54 kg) and the other is a man (m = 88 kg). After pushing off, the woman moves away with a velocity of +2.5 m/s. Find the recoil velocity of the man. (–1.5 m/s)

VIII. Hand-in Assignment

1. Explain, in your own words, the difference between momentum and inertia.

2. What is the momentum of a 6.0 kg bowling ball with a velocity of 2.2 m/s [S]? (13.2 kg·m/s [S])

3. The momentum of a 75 g bullet is 9.00 kg•m/s [N]. What is the velocity of the bullet? (120 m/s [N])

4. A hockey puck has a momentum of 3.8 kg•m/s [E]. If its speed is 24 m/s what is the mass of the puck? (1.6×10^{-1} kg)

5. A jet flies west at 190 m/s.
   (a) What is the momentum of the jet if its total mass is 2250 kg? (4.28×10^5 kg·m/s [W])
   (b) What would be the momentum of the jet if the mass was 4 times its original value and the speed increased to 6 times its original value? (1.03×10^7 kg·m/s [W])

6. A 30.0 kg object moving to the right at a velocity of 2.00 m/s collides with a 20.0 kg object moving to the left with a speed of 6.00 m/s. If the 20.0 kg object rebounds to the right with a speed of 0.75 m/s, what is the final velocity of the 30.0 kg object? (2.50 m/s [left])

7. A 225 g ball with a velocity of 40.0 cm/s [right] collides with a 125 g ball moving with a velocity of 15.0 cm/s [right]. After the collision the velocity of the 125 g ball was 35.0 cm/s [right]. What was the velocity of the 225 g ball after the collision? Was the collision elastic or inelastic? (28.9 cm/s [right], inelastic)

8. A 925 kg car moving with a velocity of +20.0 m/s collides with a stationary truck of unknown mass. The vehicles lock together and move off with a velocity of +6.75 m/s. What was the mass of the truck? (1.82×10^3 kg)

9. In a football game, a receiver catches a ball while he is standing still. Before he can move, a tackler, running at 4.75 m/s, grabs him. The tackler holds onto the receiver and the two move off with a speed of 2.50 m/s. If the mass of the tackler is 125 kg, what is the mass of the receiver? (113 kg)

10. An arrow travelling at 45 m/s strikes and embeds itself in a 450 g apple which is initially at rest. They move off horizontally at 12 m/s after impact. What is the mass of the arrow? (164 g)

11. A 1.0×10^5 N truck moving at a velocity of 17 m/s north collides head on with a 1.0×10^4 N car moving at 29 m/s south. If they stick together, what is the final velocity of the car? (12.8 m/s north)

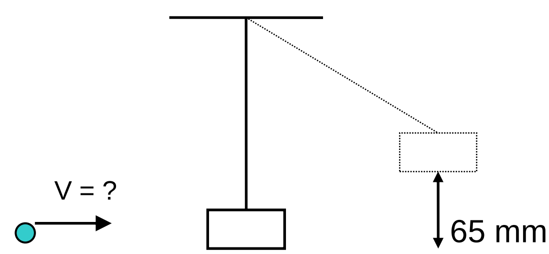
12. An astronaut is motionless in outer space. Upon command, the propulsion unit strapped to her back ejects gas with a velocity of +16 m/s causing the astronaut to recoil with a velocity of –0.55 m/s. After the gas is ejected, the astronaut has a mass of 160 kg. What is the mass of the ejected gas? (5.5 kg)

13. A two-stage rocket moves in space at a constant velocity of +4900 m/s. The two stages are then separated by a small explosive charge placed between them. Immediately after the explosion the velocity of the 1200 kg upper stage is +6000 m/s. What is the velocity of the 2400 kg lower stage after the explosion? (+4350 m/s)

14. In the James Bond movie *Diamonds are Forever* the lead female character fires a machine gun while standing on the edge of an off-shore drilling rig. As she fires the gun she is driven back over the edge into the sea. The mass of a bullet is 10 g and its velocity is 750 m/s. If her mass (including the gun) is 55 kg, what recoil velocity does she acquire in response to a single shot from a stationary position? (–0.14 m/s)

15. An atom of uranium disintegrates into two particles. One particle has a mass 60 times as great as the other. If the larger particle moves to the left with a speed of 2.3×10^4 m/s, with what velocity does the lighter particle move? ($+1.4 \times 10^6$ m/s)

16. A ballistic pendulum is a laboratory device which might be used to calculate the velocity of a bullet. A 5.0 g bullet is caught by a 500 g suspended mass. After impact, the two masses move together until they stop at a point 65 mm above the point of impact. What was the velocity of the bullet prior to impact? How much of the original kinetic energy of the bullet energy was converted into heat? (114 m/s, -32.2 J)



*17. A 70 kg boy sits in a 30 kg canoe at rest on the water. He holds two cannon balls, each of mass 10 kg. He picks them up and throws both together to the stern of the canoe. The two balls leave his hands with a velocity of 5.0 m/s *relative to the canoe*.
A 50 kg girl sits in a 50 kg canoe also at rest on the water. She also holds two 10 kg cannon balls. However she throws them over the stern of her canoe one at a time, each ball leaving her hands with a velocity of 5.0 m/s *relative to the canoe*. Assuming negligible friction between the water and the canoes (a poor assumption), calculate the final velocity for each canoe. (Hint: Describe the velocities relative to the water, not the canoes.) (0.83 m/s, 0.87 m/s)

*18. Two men, each with a mass of 100 kg, stand on a cart of mass 300 kg. The cart can roll with negligible friction along a north-south track and everything is initially at rest. One man runs toward the north and jumps off the cart with a speed of 5.00 m/s *relative to the cart*. After he has jumped, the second man runs south and jumps off the cart again with a speed of 5.0 m/s *relative to the cart*. Calculate the speed and direction of the cart after both men have jumped off. (0.25 m/s North)

*19. An argon atom with a mass of 6.680×10^{-26} kg travels at 17.00 m/s [right] and elastically strikes an oxygen atom with a mass of 2.672×10^{-26} kg dead centre travelling at 20.00 m/s [left]. What are the final velocities of each atom? (4.143 m/s [left], 32.86 m/s [right])

Momentum in One Dimension Activity

Getting started

This activity involves the use of an applet. Google **phet** from the University of Colorado. Find the **Collisions** applet and download/run it.

- Use the Introduction applet. We will use the Advanced one in the next lesson.
- Play with the applet for a while. Learn how to manually set the mass and velocity of each puck and to set the “elasticity” (e) value. When e = 1 both momentum and kinetic energy are conserved. When e = 0 the pucks stick together. When e has a value between 0 and 1, momentum is conserved but kinetic energy is not conserved.

Conservation of Momentum

Any object that is moving has velocity, and also momentum. But what happens if two or more objects collide? What happens to the velocity or the momentum of each object? Let's explore this question with the applet.

Perform three different collisions

- For each collision, make sure that each puck has a different mass.
- One collision should involve the red puck coming to a stop or continuing to move in its original direction.
- One collision should involve the red puck rebounding back in the opposite direction.
- One collision should involve the pucks sticking together.

and complete the following tables:

Collision 1							
Object	mass (kg)	$\vec{v}_{initial}$ (m/s)	$\vec{v}_{final}$ (m/s)	$\Delta\vec{v}$ (m/s)	$\vec{p}_{initial}$ (kg·m/s)	$\vec{p}_{final}$ (kg·m/s)	$\Delta\vec{p}$ (kg·m/s)
Red							
Green							

Collision 2							
Object	mass (kg)	$\vec{v}_{initial}$ (m/s)	$\vec{v}_{final}$ (m/s)	$\Delta\vec{v}$ (m/s)	$\vec{p}_{initial}$ (kg·m/s)	$\vec{p}_{final}$ (kg·m/s)	$\Delta\vec{p}$ (kg·m/s)
Red							
Green							

Collision 3							
Object	mass (kg)	$\vec{v}_{initial}$ (m/s)	$\vec{v}_{final}$ (m/s)	$\Delta\vec{v}$ (m/s)	$\vec{p}_{initial}$ (kg·m/s)	$\vec{p}_{final}$ (kg·m/s)	$\Delta\vec{p}$ (kg·m/s)
Red							
Green							

Now, look at the information you recorded and calculated in the previous question. What relationships exist between what happens to the red mass and the green mass?

a. Is there a relationship between the change in <u>velocity</u> of the red mass and the green mass? If yes, describe the relationship.

b. Is there a relationship between the change in <u>momentum</u> of the red mass and the green mass? If yes, describe the relationship.

c. You should see an interesting connection between the change in momentum of each mass. If the change in momentum of one object is exactly equal, but opposite to the change in momentum of another object, what does that indicate about the total momentum of the system?

If the total momentum of a system before a collision is equal to the total momentum of a system after a collision, then we say that **momentum is conserved**. For the three collisions verify that the total <u>initial</u> momentum is equal to the total <u>final</u> momentum:

Note: $\sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{initial green puck}} + \sum \vec{p}_{\text{initial blue puck}}$

$$\sum \vec{p}_{\text{final}} = \sum \vec{p}_{\text{final green puck}} + \sum \vec{p}_{\text{final blue puck}}$$

Collision #	total initial momentum $\sum \vec{p}_{\text{initial}}$ (kg·m/s)	total final momentum $\sum \vec{p}_{\text{final}}$ (kg·m/s)	Is momentum conserved? $\sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{final}}$
1			
2			
3			

Physics 30 <u>Lesson 2</u>

Conservation of Momentum in Two Dimensions

Refer to Pearson pages 487 to 499 for a discussion on two-dimensional momentum.

As we learned in the previous lesson, momentum has two major properties:
1. Momentum is always conserved.
2. Momentum is a vector quantity.

When we extend the idea into two (or more) dimensions, we must add momentum using **vector addition**. We can accomplish this by using one of two basic techniques:
- the component method
- the vector addition method

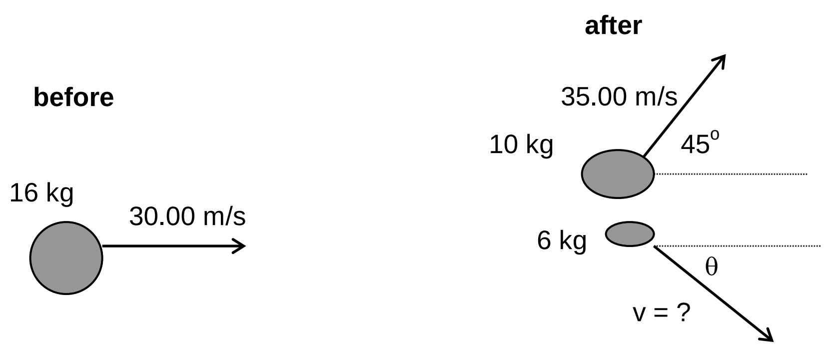
To learn how the methods work carefully consider and study the following examples.

I. Component method

The main principle used in the component method for solving two dimensional collision problems is that the momentum in the east-west direction and the momentum in the north-south direction are <u>independent</u> – we treat east-west momentum *separately* from north-south momentum. The component method uses the idea that momentum is conserved in the north-south direction and in the east-west direction at the same time. In effect, the component method does two one-dimensional problems simultaneously. The method is best explained via one or two examples.

Example 1

A 16.0 kg object traveling east at 30.00 m/s explodes into two pieces. The first part has a mass of 10.0 kg and it travels away at 35.00 m/s [45° N of E]. The second part has a mass of 6.0 kg. What is the velocity of the 6.0 kg mass?



a. Calculate the total momentum <u>before</u> the explosion in <u>each</u> direction.

(**x-direction**)

$\vec{p}_x = m\,\vec{v} = 16.0\text{ kg} \times 30.00\text{ m/s east}$

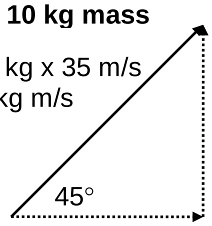
$\vec{p}_x = 16.0\text{ kg} \times 30.00\text{ m/s east}$

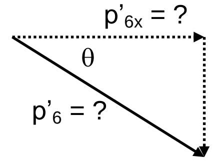
$\vec{p}_x = 480.0\text{ kg m/s east}$

(**y-direction**)

$\vec{p}_y = 0\text{ kg m/s}$ (no motion in vertical direction)

b. For the total momentum <u>after</u> the explosion calculate the components.





c. Apply the conservation of momentum principle to each direction

(**x direction**)

$\Sigma\vec{p}_x = \Sigma\vec{p}'_x$

$\vec{p}_x = \vec{p}'_{10x} + \vec{p}'_{6x}$

$\vec{p}'_{6x} = \vec{p}_x - \vec{p}'_{10x}$

$\vec{p}'_{6x}$ = (480.0 kg•m/s east) – (247.5 kg•m/s east)

$\vec{p}'_{6x}$ = 232.5 kg•m/s east

(**y direction**)

$\Sigma\vec{p}_y = \Sigma\vec{p}'_y$

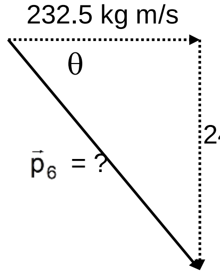
$0 = \vec{p}'_{10y} + \vec{p}'_{6y}$

$\vec{p}'_{6y} = -\vec{p}'_{10y}$

$\vec{p}'_{6y}$ = – 247.5 kg•m/s north

$\vec{p}'_{6y}$ = 247.5 kg•m/s south

d. Using the components we can calculate the momentum and velocity (remember to include direction) of the 6 kg mass



$$\vec{p}'_6 = \sqrt{235.5^2 + 247.5^2}$$

$$\vec{p}'_6 = 339.5\,\text{kg}\cdot\text{m/s}$$

$$\vec{v}'_6 = \frac{\vec{p}'_6}{m_6} = \frac{339.5\,\text{kg}\cdot\text{m/s}}{6\,\text{kg}}$$

$$\vec{v}'_6 = 56.6\,\text{m/s}$$

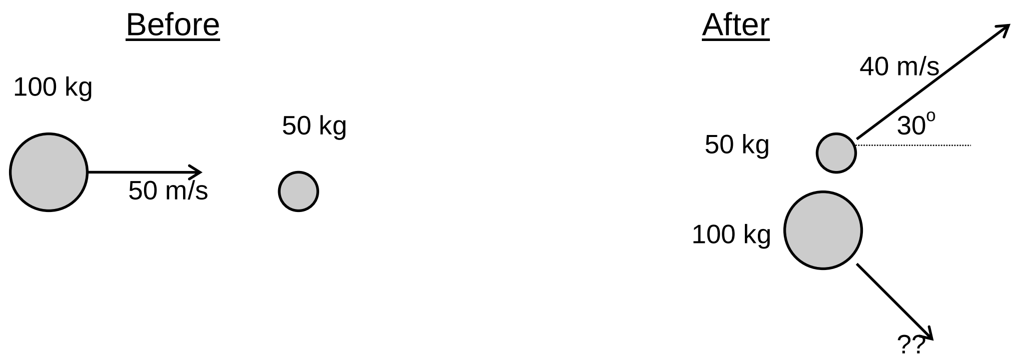
$$\theta = \tan^{-1}\left(\frac{247.5}{232.5}\right) = 46.8^o\ \text{S of E}$$

e. write the desired answer

$\vec{v}'_6$ = **56.6 m/s @ 46.8^o S of E**

A 100.0 kg object traveling east at 50.0 m/s collides with a 50.0 kg object at rest. If the 50.0 kg object travels away after the collision at 40.0 m/s at 30^o N of E, what is the velocity of the 100.0 kg object after the collision?

a. Draw Diagram



b. Find the Total Momentum Before the Collision

(x-direction)

$\vec{p}_x = m_1\vec{p}_{1x} + m_2\vec{p}_{21x}$

$\vec{p}_x$ = 100 kg x 50 m/s east + 0

$\vec{p}_x$ = 5000 kg m/s east

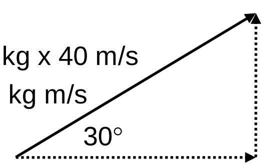
(y-direction)

$\vec{p}_y = 0$

c. Momentum after collision

50 kg mass

$\vec{p}'_{50}$ = 50.0 kg x 40 m/s
= 2000 kg m/s



$\vec{p}'_{50x}$ = 2000cos30
= 1732 kg m/s east

$\vec{p}'_{50y}$ = 2000sin30
= 1000 kg m/s north

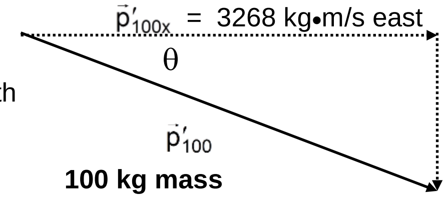
$\Sigma\vec{p}_x = \Sigma\vec{p}'_x$

$\vec{p}_x = \vec{p}'_{50x} + \vec{p}'_{100x}$

$\vec{p}'_{100x} = \vec{p}_x - \vec{p}'_{50x}$

$\vec{p}'_{100x}$ = (5000 east) – (1732 east)

$\vec{p}'_{100x}$ = 3268 kg•m/s east



100 kg mass

$\Sigma\vec{p}_y = \Sigma\vec{p}'_y$

$0 = \vec{p}'_{50y} + \vec{p}'_{100y}$

$\vec{p}'_{100y} = -\vec{p}'_{50y}$

$\vec{p}'_{100y}$ = – (1000 north)

$\vec{p}'_{100y}$ = 1000 kg•m/s south

$$\vec{p}'_{100} = \sqrt{1000^2 + 3268^2}$$
$$\vec{p}'_{100} = 3418 \text{ kg}\cdot\text{m/s}$$

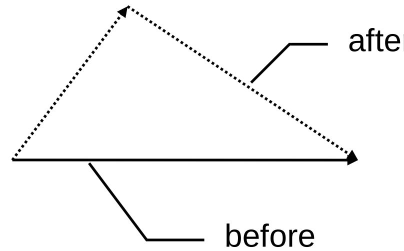
$$\vec{v}'_{100} = \frac{\vec{p}'_{100}}{m_{100}}$$
$$\vec{v}'_{100} = \frac{3418\,\text{kg}\cdot\text{m/s}}{100\,\text{kg}}$$
$$\vec{v}'_{100} = 34.18\,\text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{1000}{3268}\right)$$
$$\theta = 17.0^o\ \text{S of E}$$

$\vec{v}'_{100}$ = **34.18 m/s @ 17° S of E**

II. Vector addition method

In this method we add the momentum vectors using tip-to-tail vector addition. Using the resulting triangle, we apply the Cosine Law and the Sine Law to calculate the required values.



Cosine Law $\quad c^2 = a^2 + b^2 - 2ab\cos C$

Sine Law $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Again, the best way to understand how this method works is via an example or two. In *Example 3* we will do *Example 1* using the vector addition method.

Example 3

A 16.0 kg object traveling east at 30.00 m/s explodes into two pieces. The first part has a mass of 10.0 kg and it travels away at 35.00 m/s [45° N of E]. What is the velocity of the remaining 6.0 kg mass?

a. Calculate the known momentum vectors <u>before</u> and <u>after</u> the explosion.

$\vec{p}_{16}$ = 16.0 kg x 30.00 m/s east

$\vec{p}_{16}$ = 480.0 kg•m/s east

$\vec{p}\,'_{10}$ = 10.0 kg x 35.00 m/s @ 45° N of E

$\vec{p}\,'_{10}$ = 350 kg•m/s @ 45° N of E

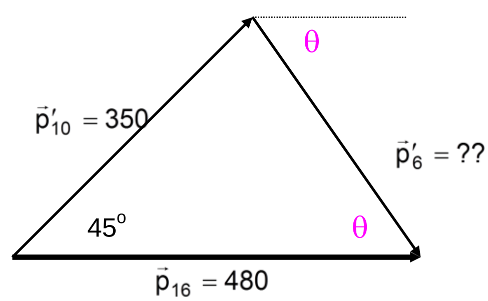
b. Momentum is conserved

$\Sigma\vec{p} = \Sigma\vec{p}\,'$

thus we can write the following <u>vector</u> equation:

$\vec{p}_{16} = \vec{p}\,'_{10} + \vec{p}\,'_{6}$

In other words the two vectors after the explosion add up to the before explosion vector. We create the following diagram.



c. We do not have a right triangle, ∴ calculate $\vec{p}\,'_6$ and $\vec{v}\,'_6$ using the Cosine Law

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\vec{p}\,'_6 = \sqrt{350^2 + 480^2 - 2(350)(480)\cos 45^0}$$

$$\vec{p}\,'_6 = 339.5\,\text{kg}\cdot\text{m/s}$$

$$\vec{v}\,'_6 = \frac{\vec{p}\,'_6}{m_6} = \frac{339.5\,\text{kg}\cdot\text{m/s}}{6\,\text{kg}}$$

$$\vec{v}\,'_6 = 56.6\,\text{m/s}$$

d. Find θ using the Sine Law. Note that the direction angle θ for $\vec{p}\,'_6$ outside the triangle is the same as the internal angle θ.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

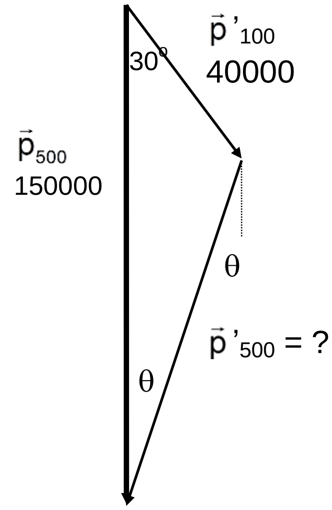
$$\frac{\sin\theta}{350} = \frac{\sin 45}{339.5}$$

$$\theta = 46.8^o$$

$$\boxed{\vec{v}\,'_6 = \mathbf{56.6\ m/s\ @\ 46.8^o\ S\ of\ E}}$$

Example 4

A 500 kg mass traveling south at 300 m/s collides with a 100 kg object at rest. If the 100 kg object ends up traveling at 400 m/s[30° E of S], what is the final velocity of the 500 kg object?



before $\vec{p}_{500}$ = 500 kg x 300 m/s = 150000 kg m/s [S]

after $\vec{p}'_{100}$ = 100 kg x 400 m/s = 40000 kg m/s [30° E of S]

$\vec{p}'_{500}$ = ?

conservation of momentum:

$\vec{p}_{500} = \vec{p}'_{100} + \vec{p}'_{500}$

calculate $\vec{p}'_{500}$ using the Cosine Law

$c^2 = a^2 + b^2 - 2\,a\,b\cos C$

$$\vec{p}'_{500} = \sqrt{150000^2 + 40000^2 - 2(150000)(40000)\cos 30^0}$$

$\vec{p}'_{500}$ = 117080 kg m/s

$$v'_{500} = \frac{\vec{p}'_{500}}{m} = \frac{117080\ \text{kg m/s}}{500\ \text{kg}} = 234\ \text{m/s}$$

calculate θ using the Sine Law

$$\frac{\sin\theta}{40000} = \frac{\sin 30^o}{117080}$$

$\theta = 10^o$

$\vec{v}'_{500}$ = 234 m/s [10° W of S]

Example 5

A 20 kg bomb is at rest. The bomb explodes into three pieces. A 2.50 kg piece moves south at 350 m/s and a 14.0 kg piece west at 95.0 m/s. What is the velocity of the other piece?

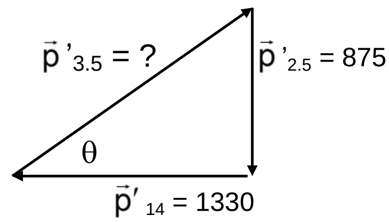
since the initial momentum is 0, the sum of the momentum vectors after the explosion will also add up to 0

initial $\vec{p}_{20} = 0$

final $\vec{p}\,'_{2.5}$ = 2.5 kg x 350 m/s = 875 kg•m/s south

$\vec{p}\,'_{14}$ = 14 kg x 95.0 m/s = 1330 kg•m/s west

$\vec{p}\,'_{3.5}$ = ?



Since we have a right triangle calculate $\vec{p}\,'_{3.5}$ using Pythagoras

$$\vec{p}'_{3.5} = \sqrt{1330^2 + 875^2}$$
$$\vec{p}'_{3.5} = 1592\ \text{kg}\cdot\text{m/s}$$

$$\vec{v}'_{3.5} = \frac{\vec{p}'_{3.5}}{m_{3.5}} = \frac{1592\,\text{kg}\cdot\text{m/s}}{3.5\,\text{kg}}$$
$$\vec{v}'_{3.5} = 455\,\text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{875}{1330}\right) = 33.3^{o}\ \text{N of E}$$

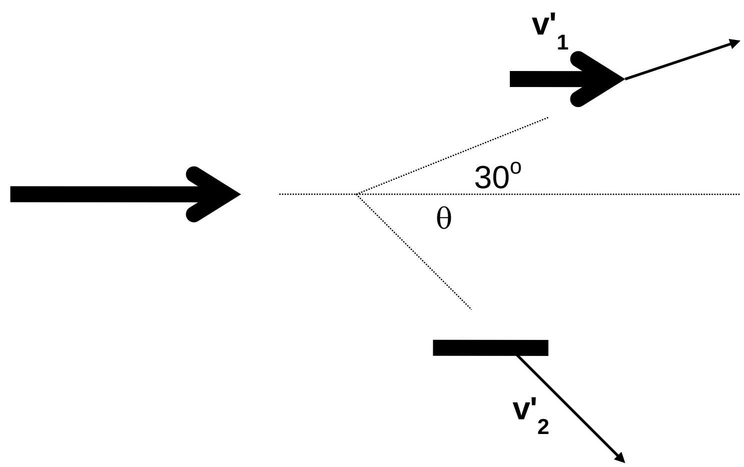
$\vec{v}'_{3.5}$ **= 455 m/s @ 33.3^{o} N of E**

III. Practice Problems

1. A 4.0 kg object is traveling south at 2.8 m/s when it collides with a 6.0 kg object traveling east at 3.0 m/s. If the two objects collide and stick together, what is the final velocity of the masses? (2.1 m/s [58^{o} E of S])

2. A 100 kg mass traveling west at 25 m/s collides with an 80 kg mass traveling east at 20 m/s. After an inelastic collision, the 100 kg mass moves away at 9.5 m/s at 28^o south of west. What is the final velocity of the 80 kg mass? (5.63 m/s [7.8^o W of N])

3. A Canada Day rocket (mass = 25.0 kg) is moving at a speed of 50.0 m/s to the right. The rocket suddenly breaks into two pieces (11.0 kg and 14.0 kg) and they fly away from each other as shown in the diagram. If the velocity of the first mass is 77.8 m/s @ 30^o to the horizontal, what is the velocity of the other piece? (47.5 m/s @ 40.1^o below the horizontal)



IV. Hand-in Assignment

1. A 1.4 x 10^3 kg car is westbound at 37.0 km/h when it collides with a 2.0 x 10^3 kg northbound truck traveling at 35.0 km/h. If the vehicles lock together upon collision, what is their final velocity? (25.6 km/h @ 53.5^o N of W)

2. A nucleus, initially at rest, decays radioactively. In the process, it emits an electron horizontally to the east with momentum 9.0 x 10^{-21} kg m/s and a neutrino to the south with momentum 4.8 x 10^{-21} kg m/s.

   A. In what direction does the residual nucleus move? (28^o N of W)
   B. What is the magnitude of its momentum? (1.0 x 10^{-20} kg m/s)
   C. If the mass of the residual nucleus is 3.6 x 10^{-25} kg, what is its recoil velocity? (2.8 x 10^4 m/s @ 28^o N of W)

3. A steel ball (mass 0.50 kg) moving with a speed of 2.0 m/s strikes a second ball (mass 0.30 kg) initially at rest. The glancing collision causes the first ball to be deflected by an angle of 30^o with a speed of 1.50 m/s. Determine the velocity of the second ball after the collision. (1.7 m/s @ 47^o)

4. A 3000 kg space capsule is travelling in outer space with a speed of 200 m/s. In an effort to alter its course it fires a 25.0 kg projectile perpendicular to its original direction of motion at a speed of 2000 m/s. What is the new velocity of the space capsule? (2.02 x 10^2 m/s @ 4.76^o from original line)

5. A 0.250 kg steel ball with a speed of 7.00 m/s collides with a stationary 0.100 kg steel ball. After the collision, the 0.100 kg ball has a velocity of 5.50 m/s at an angle of 56.0^o from the original line of action. What is the final velocity of the 0.250 kg ball? (6.05 m/s @ 17.5^o from the original line)

6. A 10.0 kg mass is traveling west at 20.0 m/s when it explodes into two pieces of 6.0 kg and 4.0 kg. If the final velocity of the 6.0 kg piece is 12.5 m/s at 40^o south of east, what is the final velocity of the 4.0 kg piece? (65.5 m/s @ 10.6^o N of W)

7. Two steel balls, each with a mass of 2.50 kg, collide. Prior to the collision, one of the balls was at rest. After the collision the speed of one ball is 3.00 m/s and the other has a speed of 4.00 m/s. If the angle between them after the collision is 90^o, what was the original speed of the moving ball? (5.00 m/s)

8. Two pieces of plasticene slide along a frictionless horizontal surface and collide, sticking together. One of the pieces has a mass of 0.20 kg and a velocity of 5.0 m/s at 30^o west of north. The other piece has a mass of 0.30 kg and is moving at 4.0 m/s at 45^o north of east. What is the velocity of the combined lump after they collide? (3.5 m/s @ 79^o N of E)

Momentum in Two Dimensions Activity

Getting started

This activity involves the use of an applet. Google **phet** from the University of Colorado. Find the **Collisions** applet and download/run it.

- Use the **Advanced** setting.
- Fool around with the applet. Have the red puck approach the green puck in the positive x direction. Start the green puck at rest.

.

Conservation of Momentum in Two Dimensions

Problem: Is momentum conserved in two dimensional collisions?

Procedure: Using the Applet choose one collision that you like and fill in the data table below.

Observations:

Object	mass (kg)	$\vec{v}_{initial}$ (m/s)	$\vec{v}_{final}$ (m/s)	scatter angle	$\vec{p}_{initial}$ (kg·m/s)	$\vec{p}_{final}$ (kg·m/s)
Red						
Green						

Analysis:

1. Calculate the momentum of each puck before and after the collision.

2. Using the **component method**, calculate the x and y components for each momentum vector. Compare the momentum of the system before and after the collision in both the x and y directions. Comment on how they compare.

3. Using the **vector addition method**, construct a <u>scale</u> drawn vector diagram (i.e. accurate vector lengths and directions) to compare the momentum of the system before and after the collision. (You may want to ask your teacher for a demonstration of how this is done.) Comment on how they compare.

4. Is momentum conserved in two dimensions? Discuss.

Physics 30 Lesson 3

Impulse and Change in Momentum

I. Impulse and change in momentum

According to Newton's 2nd Law of Motion (Physics Principle **1**) , to change the motion (i.e. momentum) of an object an unbalanced force must be applied. If, for example, we want to change the motion of a car we have to apply a force for a given time. Further, one could apply a large force for a short time or a smaller force for a longer time to effect the same change in velocity. Beginning with Newton's 2nd Law we can derive a useful equation that describes the relationship between force ($\vec{F}$), time (Δt), mass (m) and change in velocity ($\Delta \vec{v}$).

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\frac{\Delta \vec{v}}{\Delta t}$$

$$\boxed{\vec{F}\Delta t = m\Delta \vec{v}}$$

This equation is on your formula sheet.

Recall that $\boldsymbol{\Delta \vec{v}}$ means "the change in velocity" $\boldsymbol{\Delta \vec{v} = \vec{v}_f - \vec{v}_i}$

Do not mistake the **change** in velocity for the **final** velocity

The product of mass and change in velocity is the **change in momentum** ($\Delta \vec{p} = m\Delta \vec{v}$).

The product of force and time ($\vec{F}\Delta t$) is called the **impulse**. The impulse that acts on an object results in a change in the object's momentum. Since impulse is a combination of force and time, one can apply a large force for a short time or a small force for a long time or a medium force for a medium time to achieve the same change in momentum. For example, imagine a person jumping off a three story building. If the person landed on the ground on her back she would experience a very large force over a short stopping time. The force would be large enough to cause significant damage to the body. However, if she landed on a large piece of foam like they use for pole vaults her stopping time would be longer and the force acting on her would be far smaller. This is the same idea behind the use of elastic ropes for wall climbers, air bags in cars, and other safety devices.

Example 1

An average force of 17.0 N acts on an object for 0.025 s. What is the change in momentum?

$$\Delta \vec{p} = \vec{F}\Delta t$$

$$\Delta \vec{p} = 17N(0.025s)$$

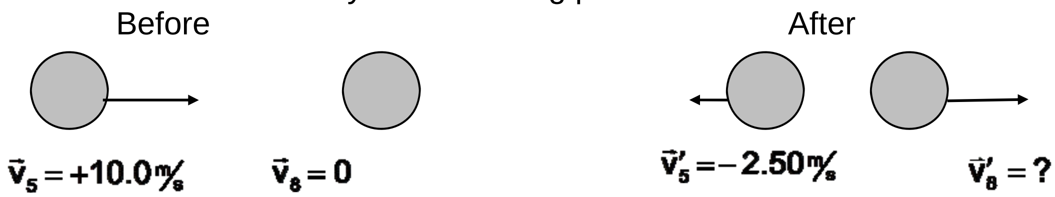
$$\boxed{\Delta \vec{p} = 0.425\ N \cdot s \text{ or } 0.425\ \tfrac{kg \cdot m}{s}}$$

Note that the unit N•s is equivalent to kg•m/s. Why? Impulse ($\boldsymbol{\vec{F}\Delta t = N \cdot s}$) is equal to change in momentum ($\boldsymbol{m\Delta \vec{v} = \tfrac{kg \cdot m}{s}}$).

Example 2

A 5.00 kg puck slides to the right at 10.0 m/s on a frictionless surface and collides with a stationary 8.00 kg puck. The 5.00 kg puck rebounds with a speed of 2.50 m/s.

A. What is the final velocity of the 8.00 kg puck?



$$\sum \vec{p} = \sum \vec{p}'$$
$$m_5\vec{v}_5 + m_8\vec{v}_8 = m_5\vec{v}'_5 + m_8\vec{v}'_8$$
$$\vec{v}'_8 = \frac{m_5\vec{v}_5 + m_8\vec{v}_8 - m_5\vec{v}'_5}{m_8}$$
$$\vec{v}'_8 = \frac{5.00\,\text{kg}(+10.0\,\text{m/s}) + 0 - 5.00\,\text{kg}(-2.50\,\text{m/s})}{8.00\,\text{kg}}$$
$$\boxed{\vec{v}'_8 = +7.8125\,\text{m/s} \text{ or } 7.8125\,\text{m/s east}}$$

B. What is the change in momentum of each puck?

$$\Delta\vec{p}_5 = m\Delta\vec{v} = m(\vec{v}_f - \vec{v}_i)$$
$$\Delta\vec{p}_5 = 5.00\text{kg}(-2.50\,\text{m/s} - (+10.0\,\text{m/s}))$$
$$\boxed{\Delta\vec{p}_5 = -62.5\,\text{kg·m/s}}$$

$$\Delta\vec{p}_8 = m\Delta\vec{v} = m(\vec{v}_f - \vec{v}_i)$$
$$\Delta\vec{p}_8 = 8.00\text{kg}(+7.8125\,\text{m/s} - 0)$$
$$\boxed{\Delta\vec{p}_8 = +62.5\,\text{kg·m/s}}$$

Note that the changes in momentum are the same value, but for one object it is positive and the other is negative. This is a consequence of the conservation of momentum – i.e. the total change in momentum is zero:

$$\Delta\vec{p} = \Delta\vec{p}_5 + \Delta\vec{p}_8$$
$$\Delta\vec{p} = (-62.5\,\text{kg·m/s}) + (+62.5\,\text{kg·m/s})$$
$$\Delta\vec{p} = 0$$

C. If the interaction lasted for 3.0 ms, what average force acted on each mass?

$$\vec{F}_8\Delta t = m\Delta\vec{v}$$
$$\vec{F}_8 = \frac{m\Delta\vec{v}}{\Delta t}$$
$$\vec{F}_8 = \frac{8.00\text{kg}(+7.8125\,\text{m/s} - 0)}{0.0030\,\text{s}}$$
$$\boxed{\vec{F}_8 = +20833\,\text{N}}$$

$$\vec{F}_5\Delta t = m\Delta\vec{v}$$
$$\vec{F}_5 = \frac{m\Delta\vec{v}}{\Delta t}$$
$$\vec{F}_5 = \frac{5.00\text{kg}(-2.50\,\text{m/s} - (+10.0\,\text{m/s}))}{0.0030\,\text{s}}$$
$$\boxed{\vec{F}_5 = -20833\,\text{N}}$$

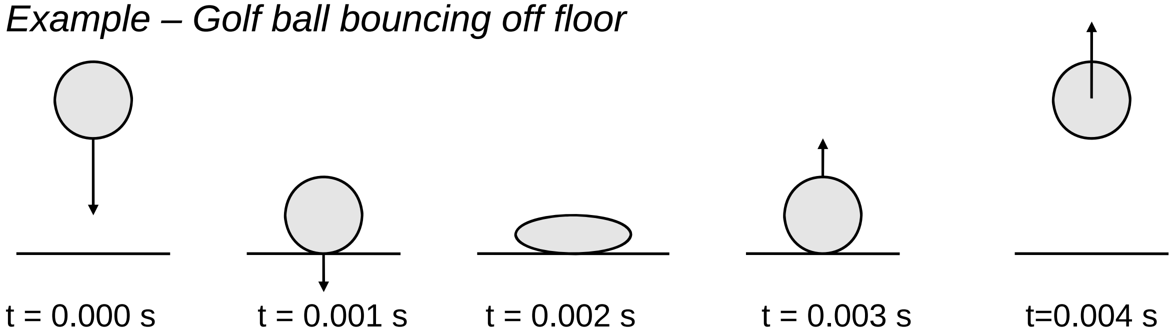
Note that the forces are equal and opposite – i.e. Newton's 3rd Law of Motion.

Refer to Pearson pages 454 to 467 for a discussion about impulse and change in momentum.

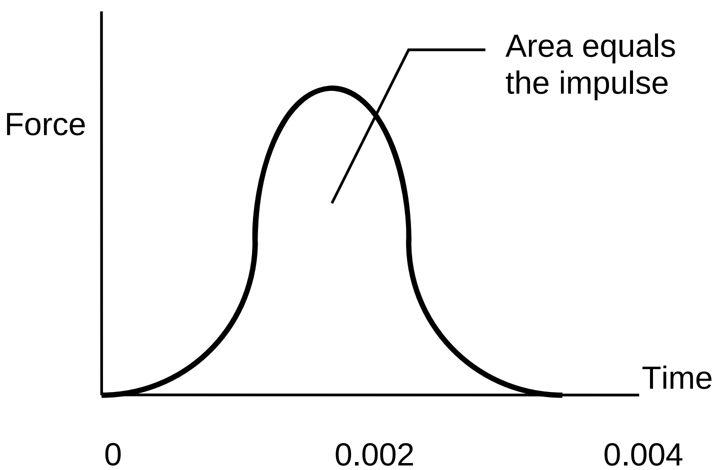
II. Analysis of Interactions Involving Impulse

In the real world, the change in momentum of an object is rarely due to a constant force that does not change over time. For example, consider a golf ball bouncing off of a floor. When the ball strikes the floor the force builds up over time and reaches a maximum when the ball is at its greatest compression. As the ball rebounds from the floor, the force decreases to zero over time.

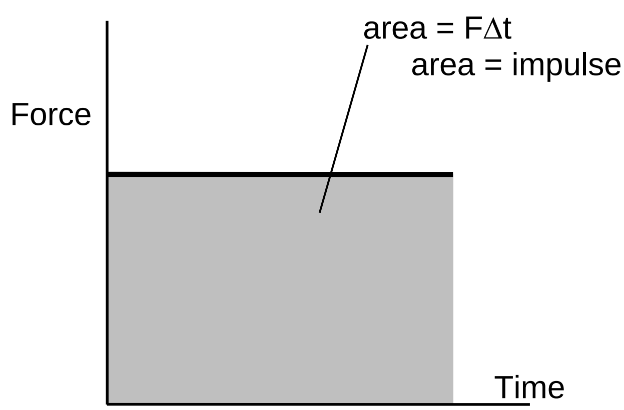
Example – Golf ball bouncing off floor



A force-time diagram showing the impulse acting on the ball will look something like the following:



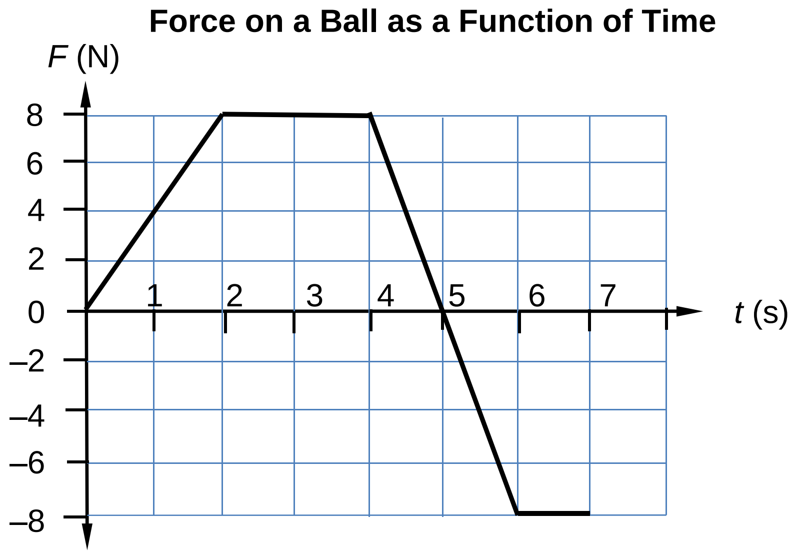
Force-Time Graph for Golf Ball



Force-Time for Constant Force
(for comparison)

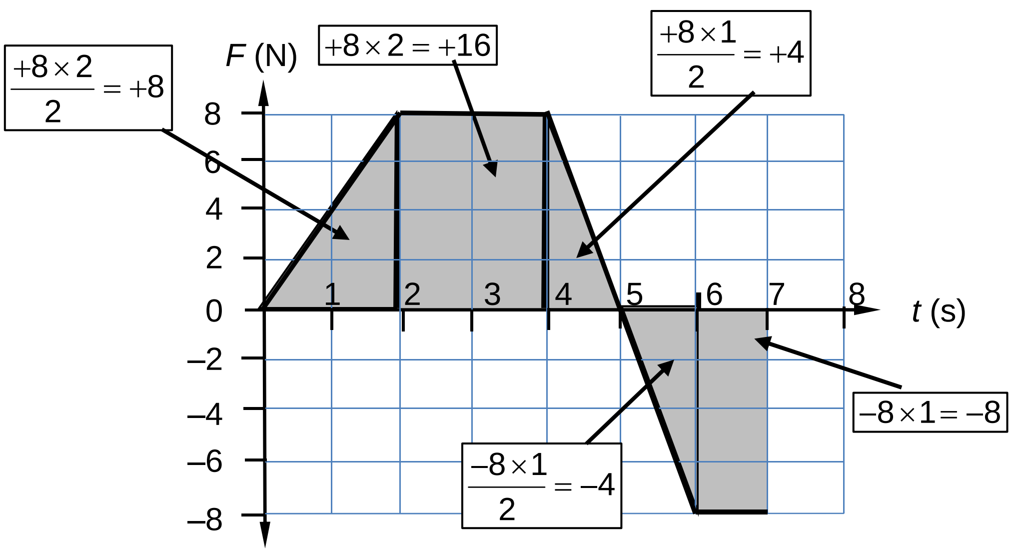
The impulse may be found by calculating the area under the force–time graph.

Example 3



Calculate the net impulse delivered to the ball from t = 0.0 s to t = 7.0 s.

The net impulse can be calculated as the area of the force-time graph.



Impulse = (+8)+(+16)+(+4)+(-4)+(-8)

= **+16 N•s**

Refer to Pearson pages 459 to 462 for a discussion about impulse and force-time graphs.

III. Practice problems

1. A 2.50 kg object is initially moving north at 5.00 m/s. If it is brought to a stop in 0.75 s:
   A. What is the impulse?
   B. What is the force required?

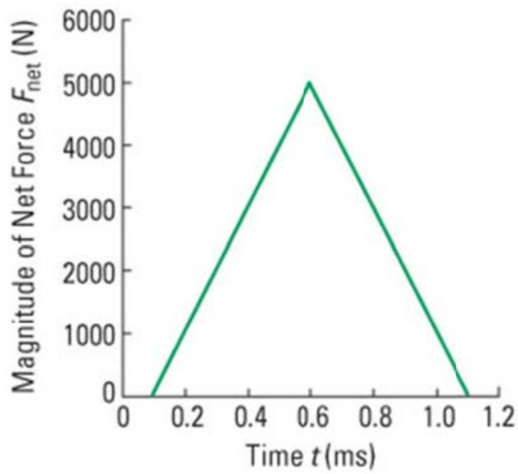
2. A 75 kg person falls from a height of 2.0 m. If the person lands on a bed, what is the change in momentum? What is the impulse? If the stopping time was 0.75 s what average force did the bed apply on the person?

IV. Hand-in Assignment

1. What quantities are used to calculate impulse? State the units of impulse.

2. Using the concept of impulse, explain how a karate expert can break a board.

3. From the graph to the right, what is the magnitude of the impulse provided to a 48 g tennis ball that is served due south? What is the velocity of the ball when the racquet and ball separate? (2.50 N·s [S], 52 m/s [S])



4. Whiplash occurs when a car is rear-ended and either there is no headrest or the headrest is not properly adjusted. The torso of the motorist is accelerated by the seat, but the head is jerked forward only by the neck, causing injury to the joints and soft tissue. What is the average net force on a motorist's neck if the torso is accelerated from 0 to 14.0 m/s [W] in 0.135 s? Assume that the force acting on the head (m = 5.40 kg) is the same magnitude as the force on the torso. (5.60 x10^2 N [W])

5. Using the principles of impulse and change in momentum, explain how the use of seat belts and headrests save lives and prevent injuries.

6. Two men pushing a stalled car generate a net force of +840 N for 5.0 s. What is the final momentum of the car? (+4.2 x 10^3 kg m/s)

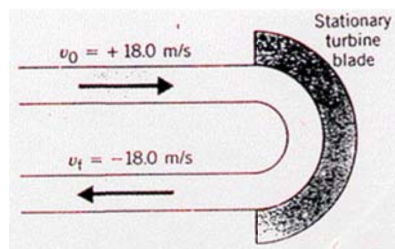
7. A woman drives a golf ball off the tee to a speed of 28 m/s. The mass of the ball is 45 g and the time of contact was 6.0 ms.
   A. What is the change in momentum of the ball? (+1.3 kg m/s)
   B. What is the impulse? (+1.3 N·s)
   C. What was the average force exerted by the club on the ball? (+210 N)
   D. If the angle of flight was initially 20^o from the horizontal, how far would the ball go before it landed? (51 m)

8. A child hits a ball with a force of 150 N. If the ball and bat are in contact for 0.12 s, what impulse does the ball receive? What is its change in momentum? If the mass of the ball is 750 g and the ball was initially moving toward the boy at 12.8 m/s, what is its final velocity? (+18 N·s, +18 kg m/s,+11.2 m/s)

9. A 300 g ball is struck by a bat with an impact that lasts 0.020 s. If the ball moves through the air towards the bat at 50 m/s and leaves at 100 m/s in the opposite direction, calculate the average force exerted by the bat on the ball? (–2.3 x 10^3 N)

10. An 8.0 g bullet travelling at 400 m/s goes through a stationary block of wood in 4.0 x 10^{-4} s, emerging at 100 m/s.
    A. What average force did the wood exert on the bullet? (–6.0 x 10^3 N)
    B. How thick is the wood? (1.0 x 10^{-1} m)

11. A stream of water strikes a stationary turbine blade (see drawing). The incident water stream has a velocity of +18.0 m/s and the exiting stream has a velocity of –18.0 m/s. The water strikes the blade at a rate of 25.0 kg/s. Find the net force acting on the water and on the blade. (–900 N,+900 N)



Physics 30

Lesson 4 Graphing for Profit and Pleasure

When scientists are trying to determine the relationship between variables they often turn to graphical analysis. In addition, scientists often use graphs that form a best-fit straight line from which they can calculate a slope from which, in turn, they can calculate a required value. This lesson is designed for you to learn how to plot graphs from which we can calculate a desired value.

I. Calculating slopes – the basics

Recall from your previous course work that the basic procedure for creating graphs is:

1. **Choose a suitable scale**. (Unfortunately students have often been taught to use the entire sheet of graph paper rather than using axis scales that are easy to use. In this course, always choose a scale that is easy to use. You do not need to use the entire sheet of graph paper.)
2. **Plot the points**.
3. **Draw a line-of-best-fit**. (Use a ruler to draw the line-of-best-fit. The line-of-best-fit is more important than the points that were used to make the line.)
4. **Choose two points on the line (not original data points)**.
5. **Calculate the slope of the line including units in the calculation**.

In addition, in previous course work you were taught to place the manipulated/ independent variable on the horizontal axis and the responding/dependent variable on the vertical axis. This is a good rule under some circumstances, however for our purposes the rule is too confining. We are more interested in relating the data to a known equation. **Therefore our choice of which variable is assigned to which axis is dependent on how the data relates to a known equation**. Unless you are told to do so, do not worry about which is the dependent or independent variable.

Perhaps the best way to see how this works is to carefully read the following example.

height (m)	energy (kJ)
0	0
5	0.12
10	0.25
15	0.39
20	0.49
25	0.60

The following data relating potential energy and height was obtained for an object. What is the mass of the object?

A quick survey of the data indicates a direct relationship between energy and height. The first step is to find the equation that relates what we are given (energy, height) with what we want to find (mass). From our formula sheet, the equation is

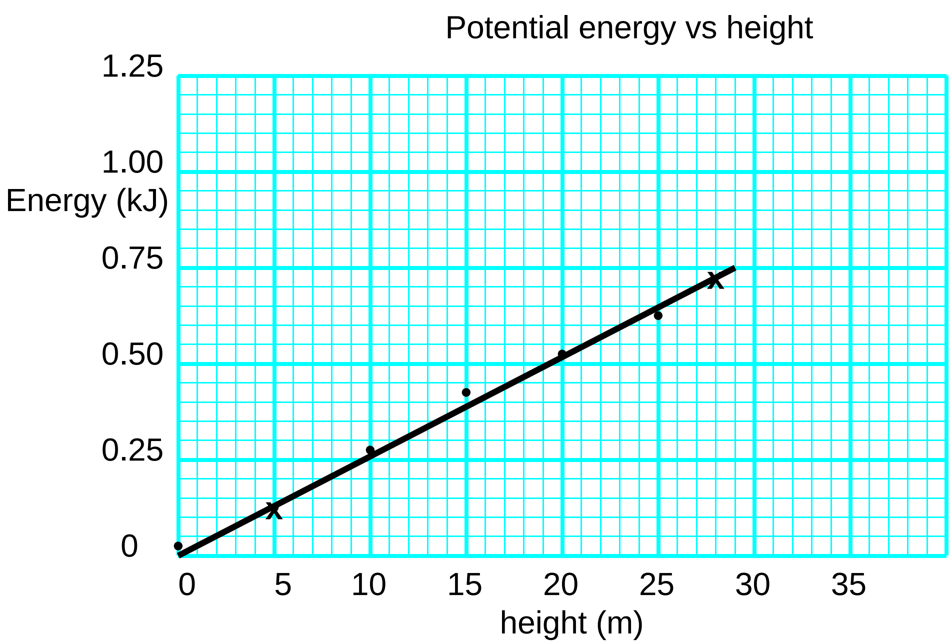
$$E = m\,g\,h$$

The second step is to rearrange the equation so that the given variables (E & h) calculate a slope:

$$m\,g = \frac{E}{h} \qquad \text{slope} = \frac{\text{rise}}{\text{run}}$$

rise (E) is the vertical axis, run (h) is the horizontal axis and the slope calculated from the graph equals (m g)

The third step is to plot the graph and calculate the slope



$$\textbf{slope} = \frac{\textbf{rise}}{\textbf{run}}$$

$$\textbf{slope} = \frac{\mathbf{(0.72 - 0.12) x 10^3\ J}}{\mathbf{28 m - 5 m}}$$

slope = 26.1 J/m

The final step is to calculate the desired value (m).

$$\text{slope} = m\,g$$

$$m = \frac{\text{slope}}{g} = \frac{26.1\ \text{J/m}}{9.81\ \text{m/s}^2} = \mathbf{2.7\ kg}$$

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Example 1 indicates the basic process for calculating a value from a given set of data. In the example there is a direct, linear relation between energy and height and, therefore, a simple graphical relationship is found. In the next two examples we shall see what to do if a direct relationship is not immediately apparent.

Example 2

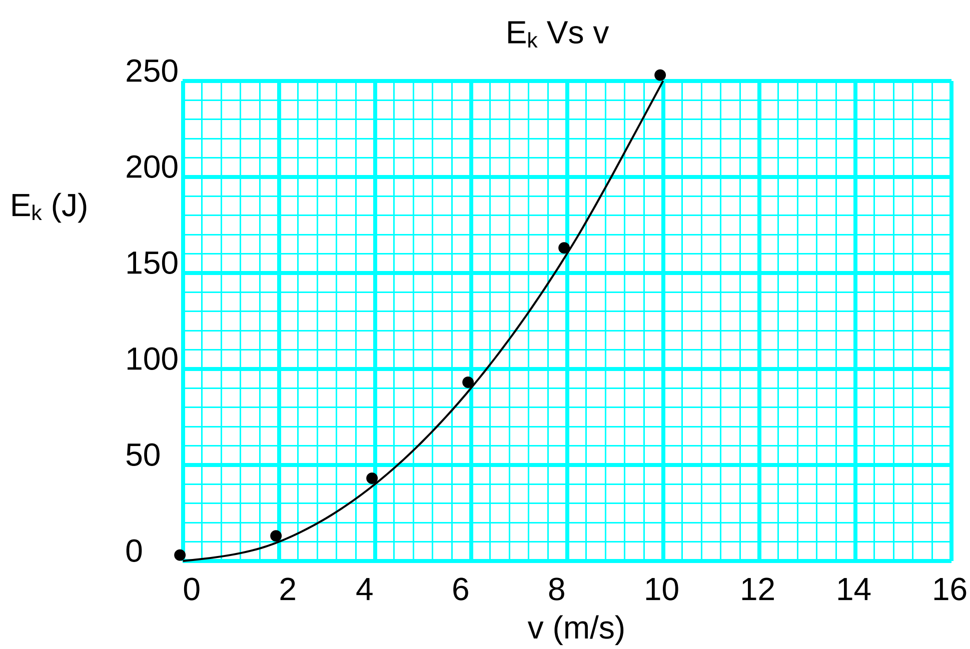
The following data relating kinetic energy and speed was obtained for an object. What is the mass of the object?

speed (m/s)	energy (J)
0	0
2	10
4	40
6	90
8	160
10	250

A quick survey of the data indicates that E_k increases at a different rate than the increase in v. This tells us that a direct relationship is unlikely. The first step is to find the equation that relates E_k, v and m

$$E_k = ½ \, m \, v^2$$

We note that E_k is related to v^2. A graph of E_k Vs v will not produce a line, rather it results in a curve. Normally we would simply recognize that a graph of E_k Vs v results in a curve and we would manipulate the data to produce a line. However, for the purpose of illustration let us plot a graph of E_k Vs v.

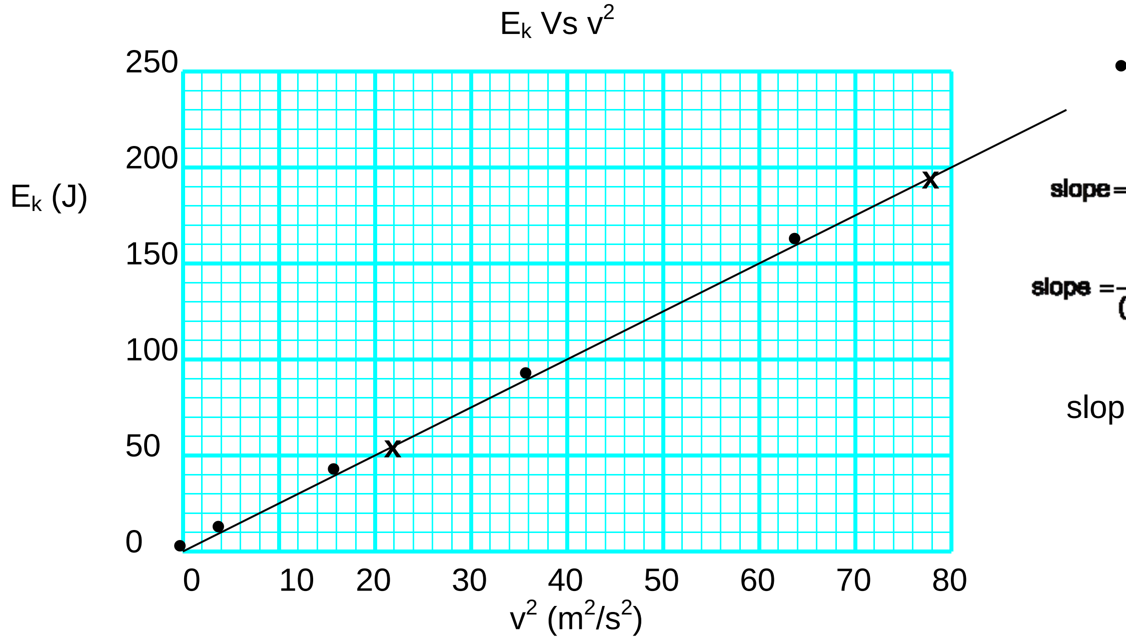


To produce a linear relationship we can use the equation to guide us. The next step is to rearrange our equation into its slope form.

$$\tfrac{1}{2}m = \frac{E_k}{v^2}$$ E_k is the vertical axis, v^2 is the horizontal axis and slope = ½ m

If we plot a graph of E_k Vs v^2 we will get a linear relationship. Adding a third column to the data table we calculate v^2 from v. We then plot the graph.

energy (J)	speed (m/s)	v^2 (m^2/s^2)
0	0	0
10	2	4
40	4	16
90	6	36
160	8	64
250	10	100



$$slope = \frac{rise}{run}$$

$$slope = \frac{(195 - 55)J}{(79 - 21)m^2/s^2}$$

slope = 2.4 kg

The final step is to calculate the desired value (m).

slope = ½ m
m = 2 x slope = 2 x 2.4 kg = **4.8 kg**

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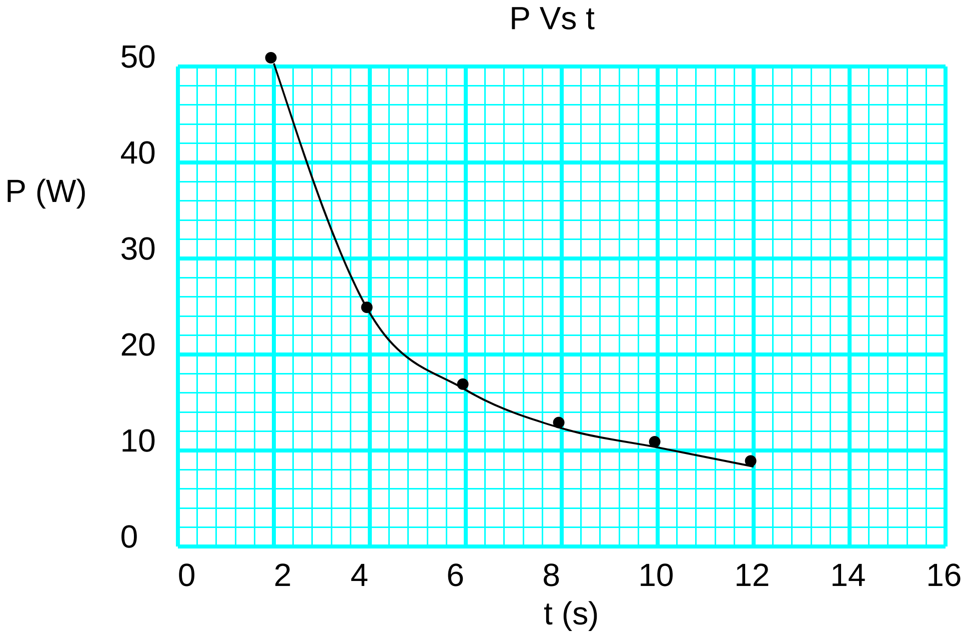
The following data relating power and time was obtained for an object. What is the work done on the object?

power (W)	time (s)
50	2
25	4
17	6
13	8
10	10
8	12

A quick survey of the data indicates that there is an inverse relationship between power and time (i.e. the greater the power, the less time is required). The first step is to find the equation that relates P, t and W.

$W = P\ t$

Since P and t are in an inverse relationship, a graph of P Vs t will not produce a line, rather it results in a curve. For the purpose of illustration let us plot a graph of P Vs t.



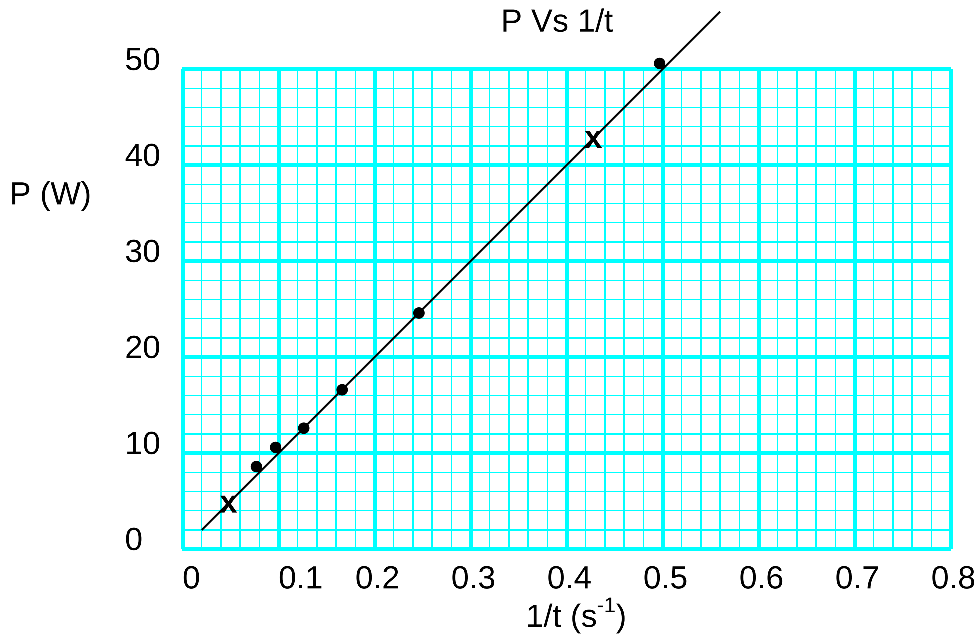
To produce a linear relationship from an inverse relationship we invert one of the variables and then graph the result. The equation becomes into its slope form.

$$W = \frac{P}{1/t}$$

P is the vertical axis, 1/t is the horizontal axis and slope = W

If we plot a graph of P Vs 1/t we will get a linear relationship. Adding a third column to the data table we calculate 1/t from t. We then plot the graph.

power (W)	time (s)	1/t (s^{-1})
50	2	0.50
25	4	0.25
17	6	0.17
13	8	0.13
10	10	0.10
8	12	0.08



$$slope = \frac{rise}{run}$$

$$slope = \frac{(43 - 5)W}{(0.43 - 0.05)s^{-1}}$$

slope = 100 W•s

The final step is to calculate the desired value (W).

slope = W
W = 100 W•s = **100 J**

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II. Practice problems

1. The following data was generated experimentally to determine the coefficient of friction for a horizontal surface. Using a suitable graphing technique find the coefficient of friction for the surface.

F_N (N)	F_f (N)
2.0	0.72
4.0	1.3
6.0	2.1
8.0	2.9
10.0	3.4
12.0	4.2

2. The following data was collected to calculate the constant (k) in the following equation:

$$\left|\vec{E}\right| = k\frac{q}{r^2}$$

where $\left|\vec{E}\right|$ is the electric field strength, q is the charge generating the field, and r is the distance from the charge. The charge used for this experiment was 2.0 x 10^{-3} C. Using a suitable graphing technique find the value of k.

distance - r (m)	$\left\|\vec{E}\right\|$ (x 10^7 N/C)
0.20	45
0.40	11
0.60	5.0
0.80	2.8
1.0	1.8

III. Hand-in assignment

1. The following data was generated experimentally using a uniform circular motion device. The radius was kept at a constant 1.25 m for every trial. Using a suitable graphing technique find the mass of the object used in the experiment.

force (N)	speed (m/s)
0.60	1.0
2.40	2.0
5.40	3.0
9.60	4.0
15.0	5.0
18.0	6.0

2. The following data was generated experimentally using a water ripple tank. Using a suitable graphing technique find the speed of the waves in the ripple tank.

frequency (Hz)	wavelength (m)
5	2.4
10	1.2
15	0.8
20	0.6
25	0.5
30	0.4

3. The following data was generated experimentally using a modified Cavendish balance. One of the masses was 20 kg and the distance between the masses was held at a constant 0.10 m. The force of gravity was measured as the size of the other mass varied. Using a suitable graphing technique find the value for the gravitational constant G. Calculate the percent error from the theoretical value for G on your formula sheet.

m_2 (kg)	F (x 10^{-6} N)
0	0
2	0.27
4	0.53
6	0.80
8	1.07
10	1.33
12	1.60
14	1.87
16	2.13
18	2.40
20	2.67

4. The following data was generated experimentally using a modified Cavendish balance. Both of the masses were 20 kg and the force was measured for different distances between the masses. Using a suitable graphing technique find the value for the gravitational constant G. Calculate the percent error from the theoretical value for G on your formula sheet.

r (m)	F (x 10^{-6} N)
0.02	66.7
0.03	29.6
0.04	16.7
0.05	10.7
0.06	7.41
0.07	5.44
0.08	4.17
0.09	3.29
0.10	2.67
0.11	2.20
0.12	1.85

Physics 30 Lesson 1 to 4 Review

1. An empty sled is sliding on frictionless ice when Susan drops vertically from a tree above onto the sled.
   a. When she lands, does the sled speed up, slow down, or keep the same speed? Explain.
   b. Later, Susan falls sideways off the sled. When she drops off, does the sled speed up, slow down, or keep the same speed? Explain.

2. Why can a batter hit a pitched baseball further than a ball tossed in the air by the batter?

3. A 12 kg hammer strikes a nail at a velocity of 8.5 m/s and comes to rest in a time interval of 8.0 ms.
   a. What is the impulse imparted to the nail? (102 kg·m/s)
   b. What is the average force acting on the nail? (12750 N)

4. A hockey player makes a slap shot, exerting a force of 30 N on the hockey puck for 0.16 s seconds.
   a. What impulse is given to the puck? (4.8 N·s)
   b. If the hockey puck has a mass of 0.115 kg and was at rest before the shot, what speed does it head towards the net? (42 m/s)

5. A constant friction force of 25 N acts on a 65 kg skier for 20 s. What is the skier's change in velocity? (–7.69 m/s)

6. For a top tennis player, a tennis ball may leave the racket on the serve with a speed of 55 m/s. If the ball has a mass of 0.060 kg and is in contact with the racket for about 4.0 ms, estimate the average force of the ball. Would this force be large enough to lift a 60 kg person? (825N, enough)

7. Water leaves a hose at a rate of 1.5 kg/s with a speed of 20 m/s and is aimed at the side of a car, which stops it. (That is, we ignore any splashing back). What is the force exerted by the water on the car? (–30N)

8. Is it possible for an object to receive a larger impulse from a small force than from a large force? Explain.

9. An object travelling east at 40 m/s and having a mass of 50 kg collides with an object with a mass of 40 kg and travelling east at 20 m/s. If they stick together on contact:
   a. What is the resultant velocity of the combined mass? (31.1 m/s east)
   b. What is the kinetic energy of the combined mass ? (43.5 kJ)

10. A 68 kg object travelling west at 45,750 m/s collides with a 56,975 kg object travelling east at 0.0078 m/s. If the 68 kg object ends up travelling east at 22,456 m/s what is the velocity of the 56,975 kg object? (81 m/s west)

11. A 95 kg halfback moving at 4.1 m/s on an apparent breakaway for a touchdown is tackled from behind. When he was tackled by an 85 kg cornerback running at 5.5 m/s in the same direction, what is their mutual speed immediately after the tackle? (4.76 m/s)

12. Calculate the recoil velocity of a 5.0 kg rifle that shoots a 0.020 kg bullet at a speed of 620 m/s? (–2.5 m/s)

13. A 10000 kg railroad car, A, travelling at a speed of 24.0 m/s strikes an identical car, B, at rest.
    a. If the cars lock together as a result of the collision, what is their common speed just after the collision? (12.0 m/s)
    b. Is the collision elastic or inelastic? If the collision is inelastic, calculate how much of the initial kinetic energy is transformed to thermal or other forms of energy. (1.44×10^6 J)

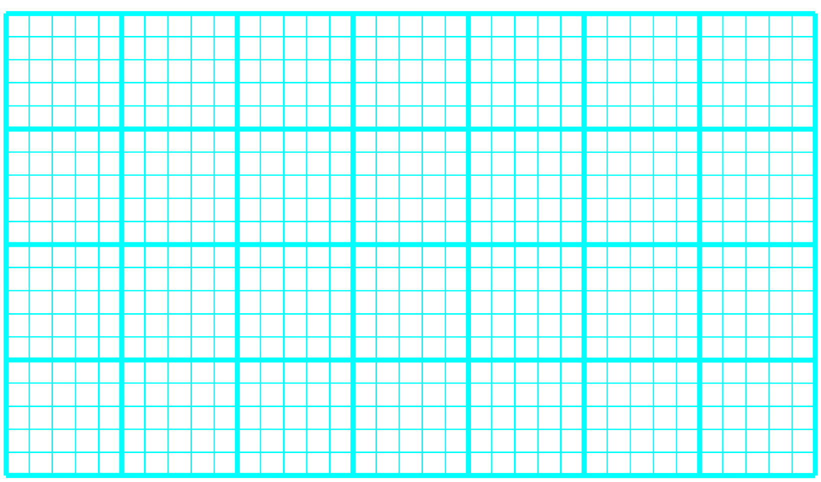
14. Billiard ball A of mass 0.400 kg moving with speed 1.80 m/s strikes ball B, initially at rest of mass 0.500 kg. As a result of the collision, ball A is deflected at an angle of 30.0˚ with a speed of 1.10 m/s. Find the resultant velocity of ball B. (0.808 m/s at 33˚ from original direction of motion)

15. A mass of 50 kg travelling north at 45 m/s collides with a mass of 60 kg travelling 50 m/s at 28˚ N of W. If they stick together on contact, what is the resulting velocity of the combined masses? (41.1 m/s at 36˚ W of N)

16. A 100 kg mass explodes into three parts. The first part travels away at 50 m/s straight north and has a mass of 20 kg. The second part travels away at 35 m/s straight west and has a mass of 50 kg. What is the resultant velocity of the third part? (66.7 m/s @ 30^{o} S of E)

17. The following data relates the mass and change in velocity for various objects having the same impulse over a 3.0 second time interval. Find the average force exerted on the objects during the impulse.

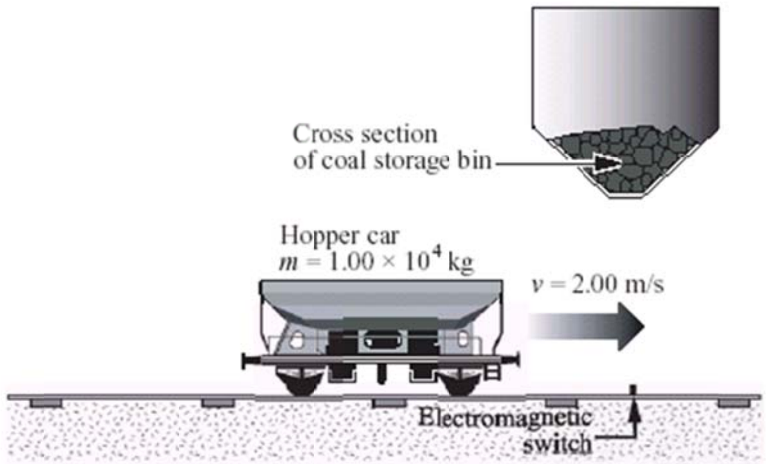
Mass (kg)	$\Delta \boldsymbol{v}$ (m/s)
37.5	20
50.0	15
62.5	12
75.3	10
93.8	8.0
125	6.0



Old diploma exam questions

Use the following information to answer the next question.

At a coal mine, a train engine bumps an empty hopper car that has a mass of 1.00×10^4 kg such that it rolls at a constant speed of 2.00 m/s under a coal storage bin. When the hopper car triggers an electromagnetic switch on the track below the storage bin, the bin drops a load of 1.20×10^4 kg of coal into the hopper car.



Numerical Response

1. The speed of the hopper car immediately after receiving the load of coal, expressed in scientific notation, is _____× 10^{-w} m/s.

(Record your **three-digit answer** in the numerical-response section on the answer sheet.)

1. Which of the following quantities are scalar quantities?

A. Acceleration and impulse

B. Acceleration and mass

C. Impulse and energy

D. Energy and mass

Use the following information to answer the next two questions.

When a motor vehicle slows down suddenly and the wheels are locked, the kinetic energy of the vehicle is transferred into heat energy. A skid mark is left on the road. Police can estimate the speed at which a vehicle was travelling before the brakes were applied b measuring the length of a skid mark *d* and applying the formula $v = \sqrt{2g\mu d}$, where $\mu = 0.750$ for a dry road surface.

After the brakes are applied and the wheels are locked, a 1.00×10^3 kg vehicle comes to a stop in 3.80 s. The vehicle leaves a 52.9 m skid mark.

Numerical Response

2. The estimated speed of the vehicle is calculated to be __________ m/s.

(Record your **three-digit answer** in the numerical-response section on the answer sheet.)

*Use your recorded answer from **Numerical Response 2** to answer **Numerical Response 3**.*

Numerical Response

3. The magnitude of the impulse necessary to stop the vehicle, expressed in scientific notation, is ____ $\times 10^w$ kg·m/s.

(Record your **three-digit answer** in the numerical-response section on the answer sheet.)

2. A car is travelling north at 60 km/h. It crashes into a truck, which is stationary. After the collision, the two vehicles lock together and travel off in the car's original direction. Which of the following statements about the collision is true?

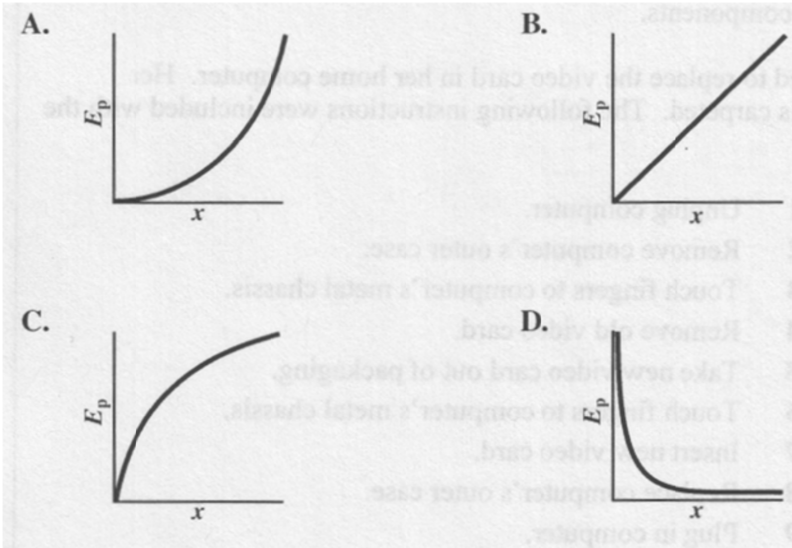
A. Both momentum and kinetic energy are conserved.

B. Neither momentum nor kinetic energy is conserved.

C. Momentum is conserved, but kinetic energy is not conserved.

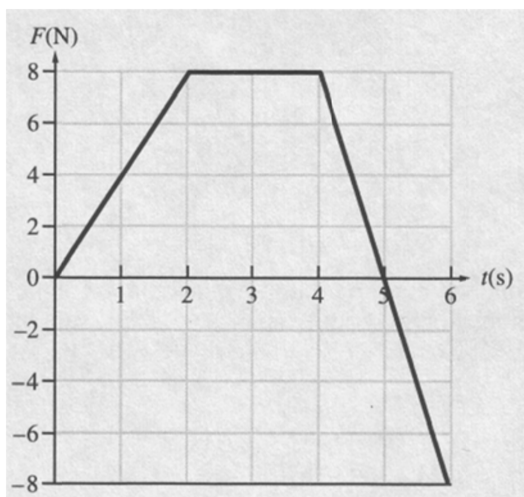
D. Kinetic energy is conserved, but momentum is not conserved.

3. The energy stored in a spring is given by $E_p = ½ kx^2$. Which of the following graphs shows the relationship between the energy stored in a spring, E_p, and the distance, x, that the spring is stretched?



Use the following information to answer the next question.

Force on a Ball as a Function of Time



4. The net impulse delivered to the ball from t = 0.0 s to t = 6.0 s is

A. 24 N·s

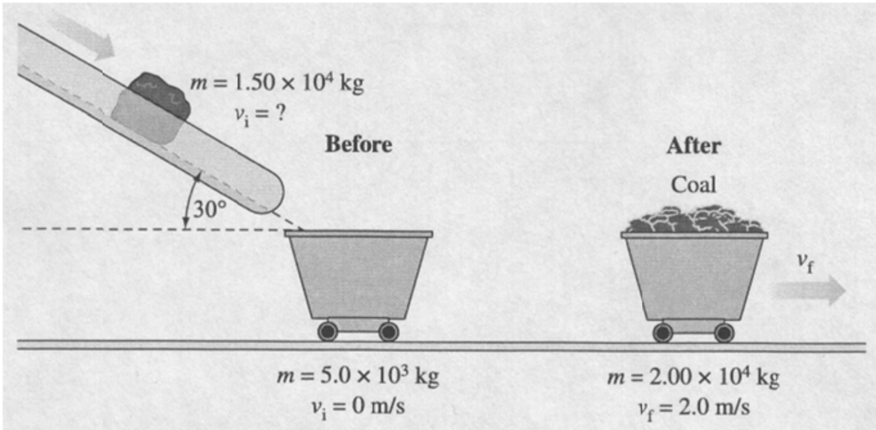
B. 28 N·s

C. 32 N·s

D. 48 N·s

Use the following information to answer the next question.

A coal chute angled at 30° to the horizontal releases 1.50×10^4 kg of coal to fill a stationary, empty 5.0×10^3 kg cart. The cart and coal move forward with a horizontal velocity of 2.0 m/s.



5. The speed of the coal along the chute is

A. 1.5 m/s

B. 2.7 m/s

C. 3.1 m/s

D. 5.3 m/s

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6. A space shuttle astronaut has a mass of 110 kg with her space suit on. She is on a space walk and picks up a full can of spray with a mass of 20 kg. Relative to the space shuttle, she is at rest. She then holds the can directly in front of her centre of mass to avoid rotation and releases 3.0 kg of spray at a speed of 15 m/s. Her speed, relative to the space shuttle, when she has stopped spraying is approximately

A. 0.35 m/s

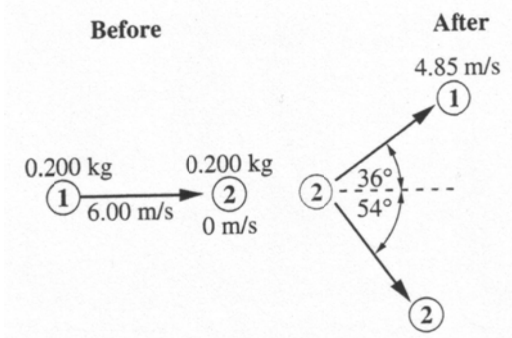
B. 0.41 m/s

C. 2.3 m/s

D. 2.5 m/s

Use the following information to answer the next question.

Two identical metal pucks were made to collide on a frictionless surface. Before the collision, puck 1 was moving at 6.00 m/s and puck 2 was stationary. After the collision, the pucks moved as shown in the diagram below.



7. The magnitude of the **momentum** of puck 2 after the collision was

   A. 1.33 kg·m/s

   B. 0.970 kg·m/s

   C. 0.705 kg·m/s

   D. 0.570 kg·m/s

8. Which of the following units are correct units for momentum?

   A. J·s

   B. N·m

   C. N·s

   D. N/J

Answers

Multiple choice	Numerical Response
1. D	1. 9.09
2. C	2. 27.9
3. A	3. 2.79
4. A	
5. C	
6. A	
7. C	
8. C	