

# Your MMM Is Broken: Identification of Nonlinear and Dynamic Effects in Marketing Mix Models

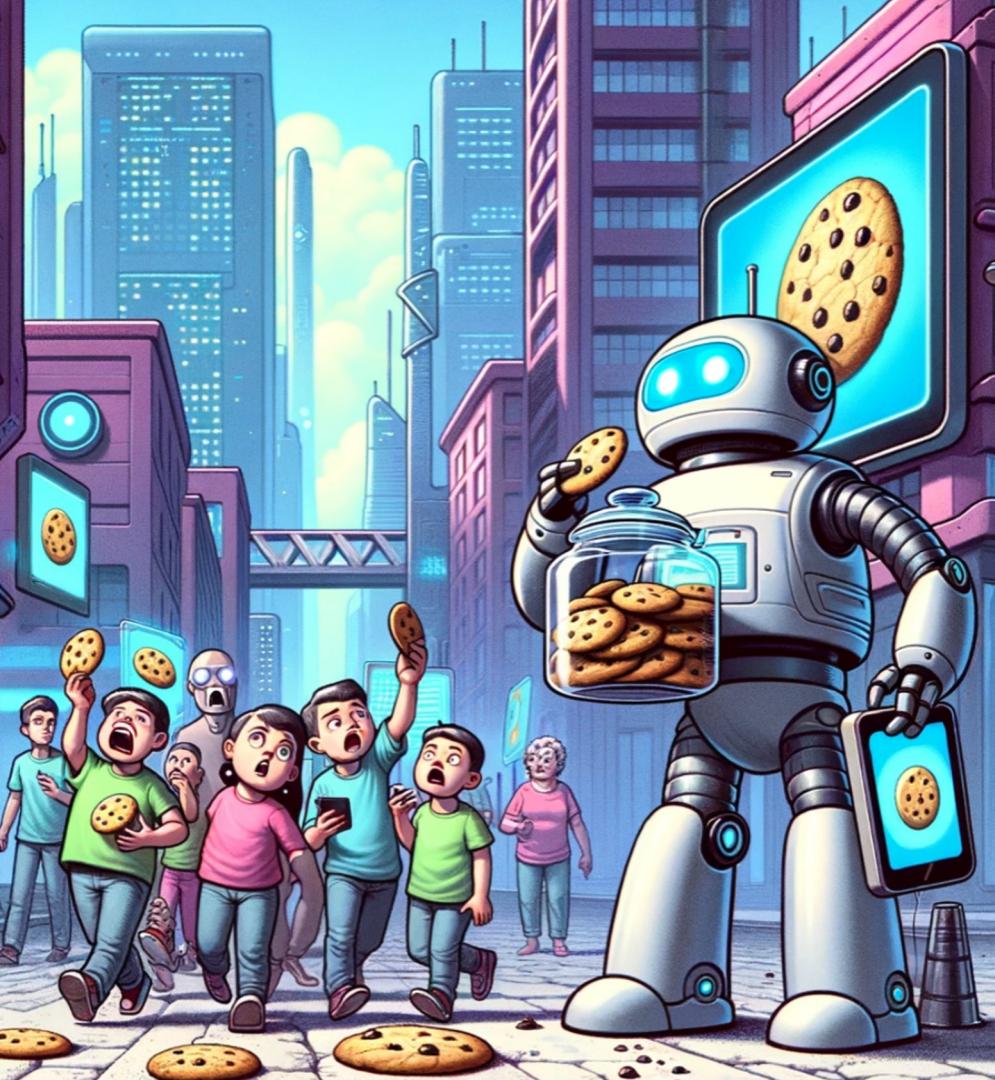
**Ryan Dew**

*The Wharton School  
University of Pennsylvania*

Joint work with Nicolas Padilla and Anya Shchetkina

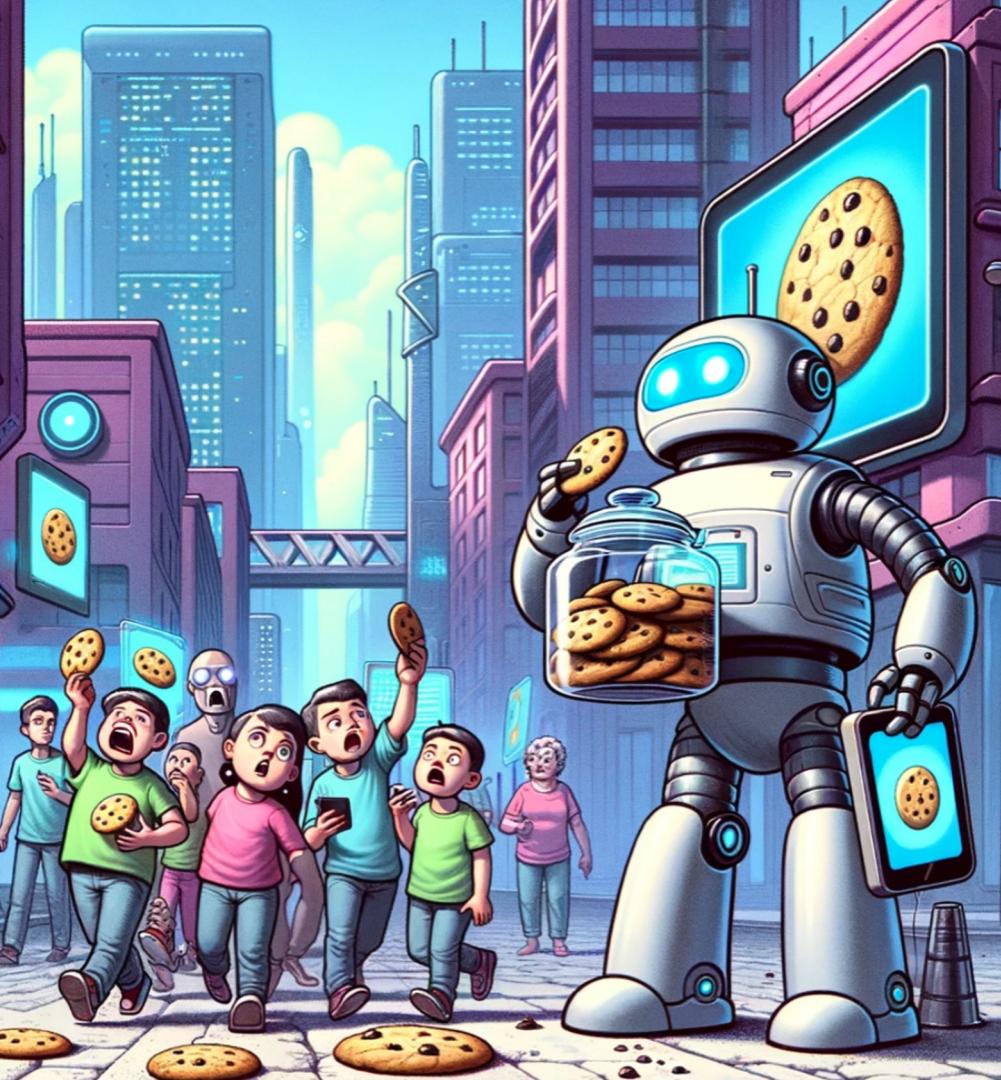
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- For the past 10+ years, individual-level trackability has been central to measuring marketing efficacy



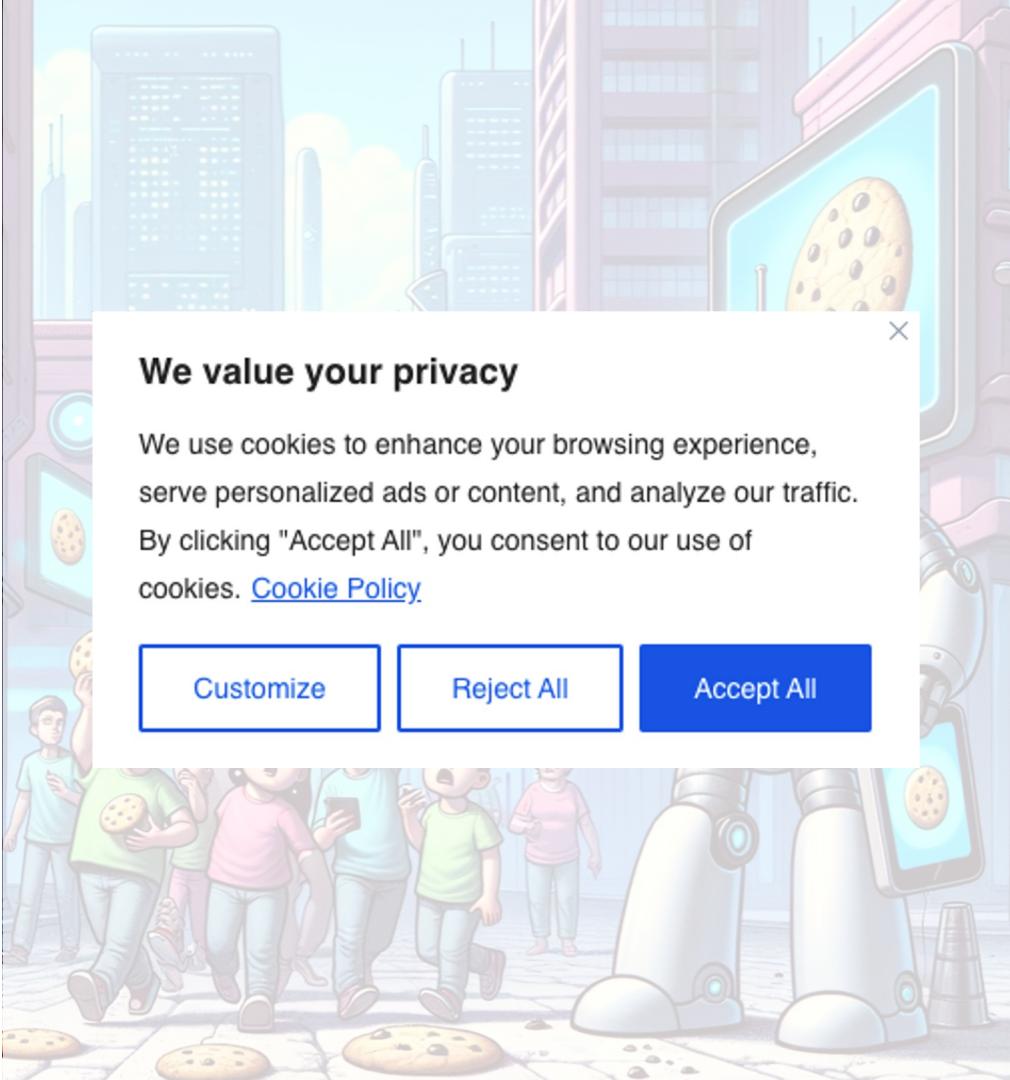
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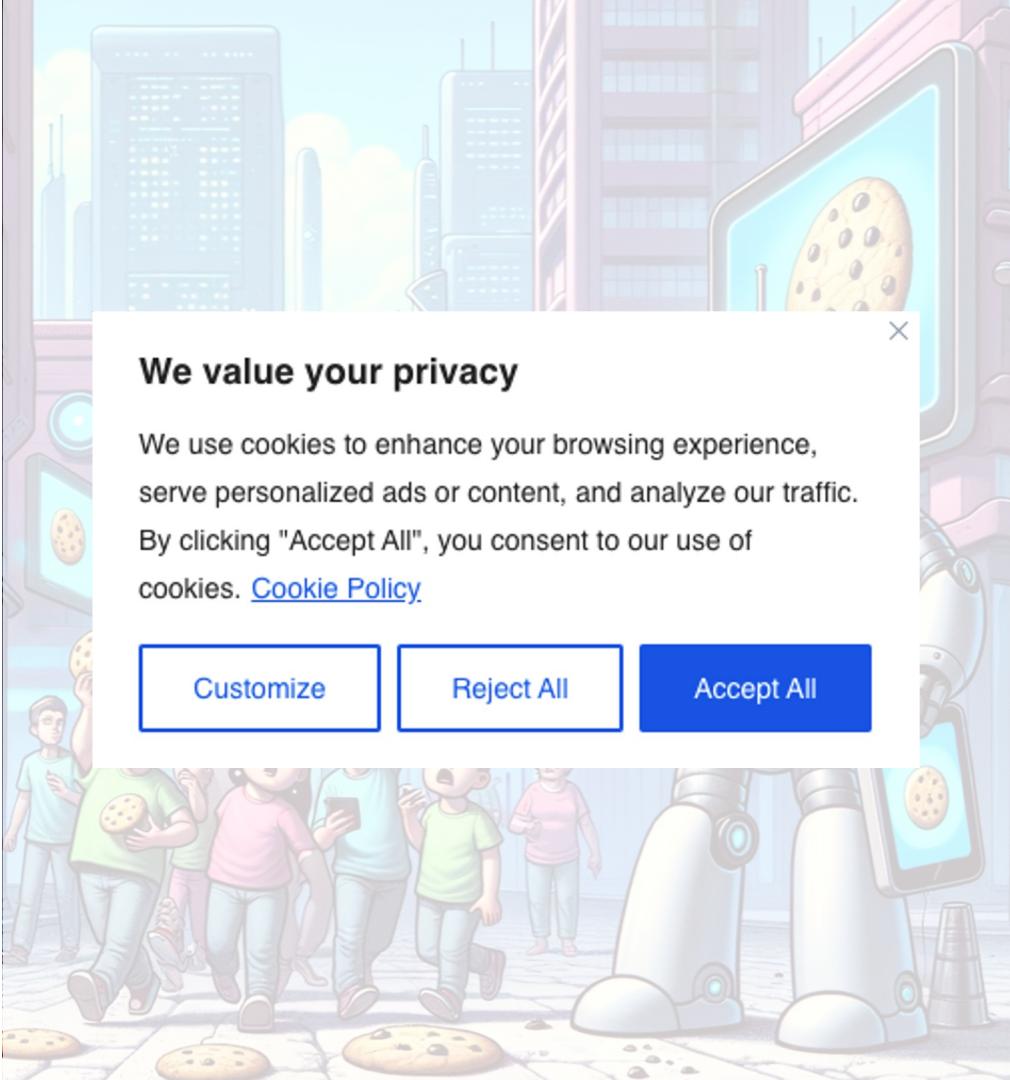
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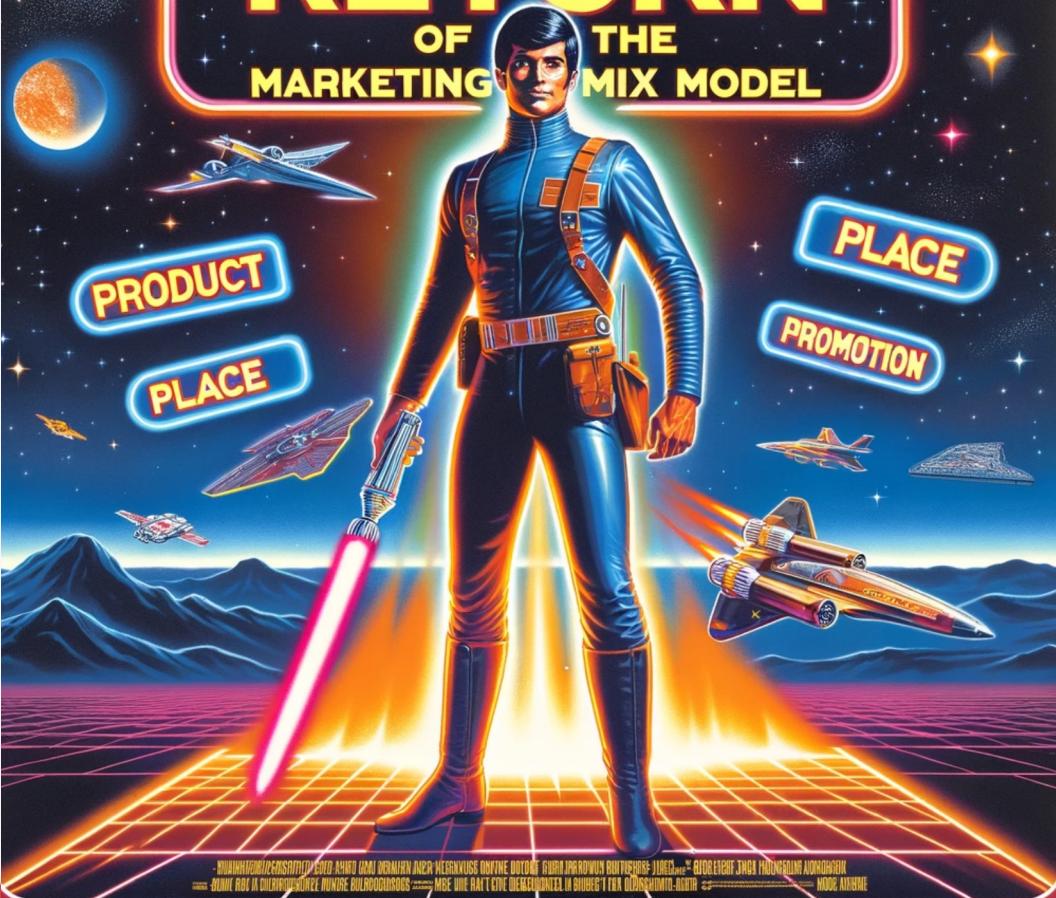


# The Cookie-free Future

- For the past 10+ years, individual-level trackability has been central to measuring marketing efficacy
- Now? Gone!
- Result: huge resurgence in interest in models based on **aggregate data**



# THE RETURN OF THE MARKETING MIX MODEL



- HUANHEDZERNSDIT - GOL AHIO UMI DERNEN AER - HEGRKUSS DUVNE BOTON - GURJAPROWUN RUTTUSCHF - JUREL - BODGELETH THOI YOUNGUNG NORGHEIN  
- BEUNE ROE IA DIERENGEDE NUNDE BULBOORHOOS - MEE UUE RAT' ETE DEMEONUNES IN BUBB'S FRA UDERSHOMME - KUH - MODE ARABIE

## Statistics &gt; Applications

[Submitted on 7 Jun 2021 ([v1](#)), last revised 5 Sep 2021 (this version, v3)]

# Bayesian Time Varying Coefficient Model with Applications to Marketing Mix Modeling

Edwin Ng, Zhishi Wang, Athena Dai

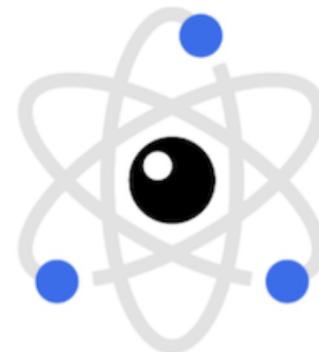
Both Bayesian and varying coefficient models are very useful tools in practice as they can be used to model parameter heterogeneity in a generalizable way. Motivated by the need of enhancing Marketing Mix Modeling at Uber, we propose a Bayesian Time Varying Coefficient model, equipped with a hierarchical Bayesian structure. This model is different from other time varying coefficient models in the sense that the coefficients are weighted over a set of local latent variables following certain probabilistic distributions. Stochastic Variational Inference is used to approximate the posteriors of latent variables and dynamic coefficients. The proposed model also helps address many challenges faced by traditional MMM approaches. We used simulations as well as real world marketing datasets to demonstrate our model superior performance in terms of both accuracy and interpretability.

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# Orbit

[Submitted on 7 Ju



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## Bayesian

Edwin Ng, Zhi

Both Bayesian and frequentist approaches have their own merits and generalizability. Robyn is built on a Bayesian framework, which provides a more principled model, equipped with prior distributions that can incorporate domain knowledge and coefficients are interpreted as posterior distributions. Robyn also provides methods to approximate the posterior distributions of the parameters, which are useful for traditional Marketing Mix Modeling (MMM) analysis. Robyn is designed to be user friendly in terms of both the interface and the underlying statistical models.

# Robyn

Robyn is an experimental, AI/ML-powered and open sourced Marketing Mix Modeling (MMM) package from Meta Marketing Science.

Getting Started

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## Bayesian

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Both Bayesian and frequentist methods have their own strengths and weaknesses. Robyn is designed to be a generalizable, user-friendly, and efficient package for Bayesian modeling. It provides a wide range of features, including support for various prior distributions, model selection, and model averaging. It also includes tools for generating posterior distributions, estimating model parameters, and performing hypothesis testing. Robyn is equipped with a variety of built-in functions for common statistical analyses, such as regression, classification, and clustering. It also allows users to define their own custom models and priors. Robyn is designed to be easy to use, even for those who are new to Bayesian statistics. It provides clear documentation and examples, and it includes a comprehensive set of resources for learning more about Bayesian modeling.



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Getting Started



PyMC-Marketing

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Both Bayesian and frequentist approaches have their own strengths and weaknesses. Bayesian methods are more generalizable and can incorporate prior knowledge, while frequentist methods are more robust to model misspecification. Bayesian methods also provide a natural way to handle uncertainty and make predictions based on the posterior distribution of parameters. In contrast, frequentist methods often rely on asymptotic approximations and can be less transparent about the assumptions underlying their results.



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Modeling

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Marketing

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## Note: Terminology

Throughout the talk,  
dynamic = time-varying

# What's new? Firepower



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Meridian

Wang et al. (2017), Jin et al. (2017), Sun et al. (2017), Zhang et al. (2023)

$$\begin{aligned}y_{g,t} = & \mu_t + \tau_g + \sum_{c=1}^C \gamma_{g,c} z_{g,t,c} \\& + \sum_{m=1}^M \beta_{g,m} HillAdstock \left( \left\{ x_{g,t-s,m} \right\}_{s=0}^L ; \alpha_m, ec_m, slope_m \right) \\& + \sum_{n=1}^N \beta_{g,n}^{(rf)} Adstock \left( \left\{ r_{g,t-s,n} \cdot Hill \left( f_{g,t-s,n}; ec_n^{(rf)}, slope_n^{(rf)} \right) \right\}_{s=0}^L ; \alpha_n^{(rf)} \right) \\& + \epsilon_{g,t}\end{aligned}$$

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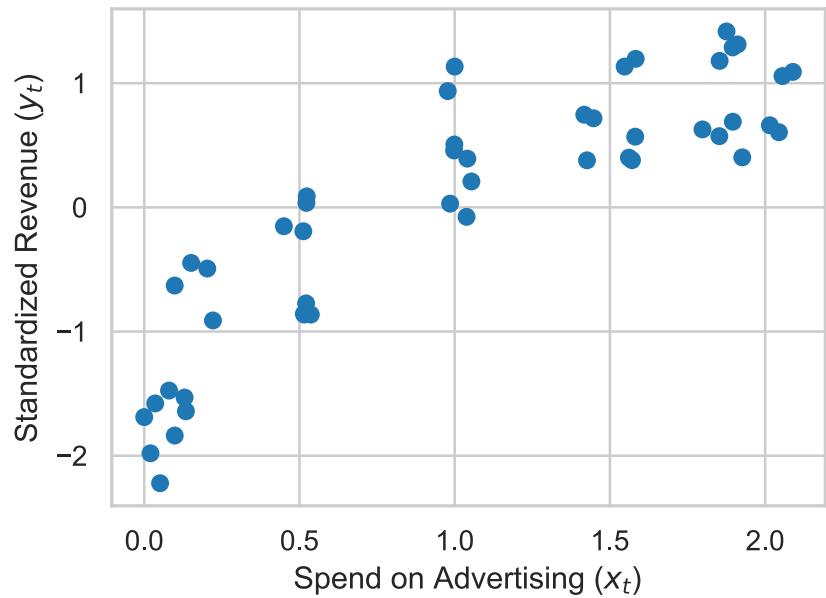
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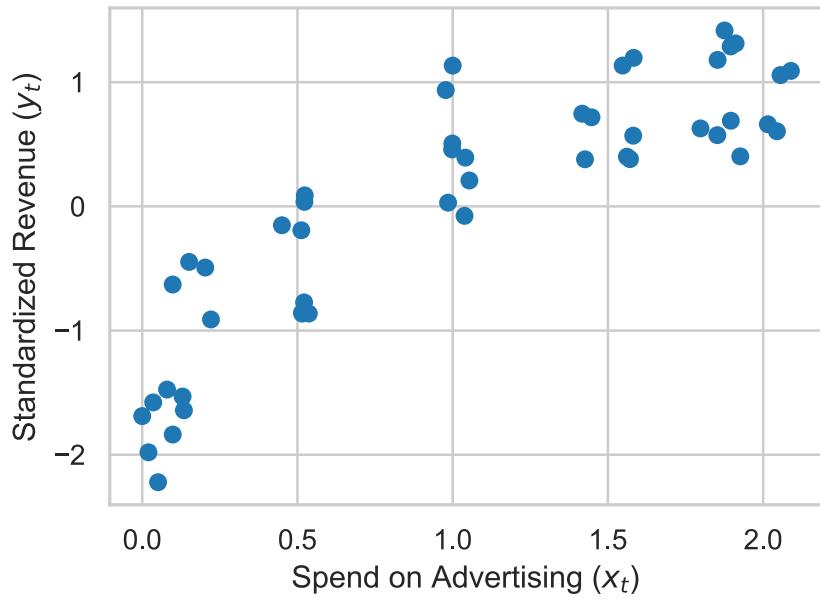
Ng et al., (2021)

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$$\ln(\hat{y}_t) = l_t + s_t + \sum_{p=1}^P \ln(x_{t,p}) \beta_{t,p}$$
$$\beta_{t,p} = \sum_j w_j(t) \cdot b_{j,p},$$
$$w_j(t) = k(t, t_j) / \sum_{i=1}^J k(t, t_i),$$

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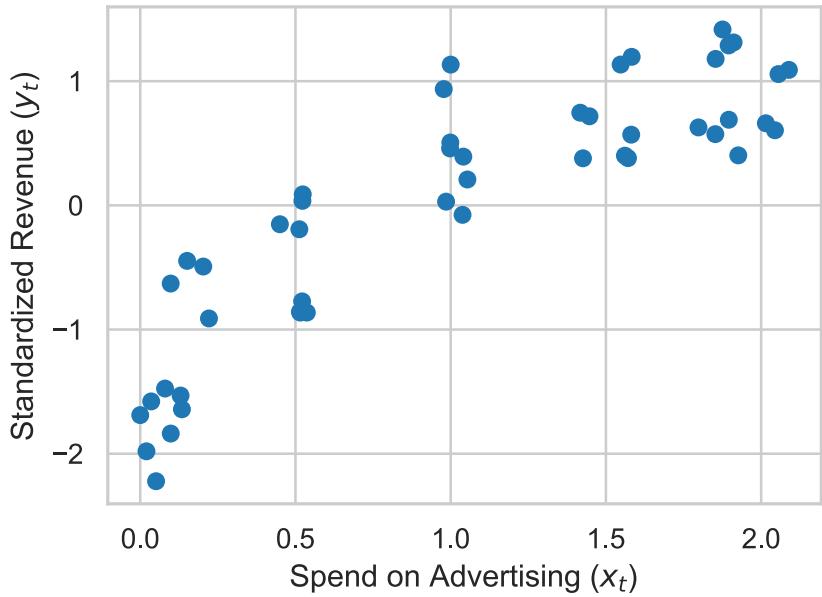




**Surely this is nonlinear,  
right...?**

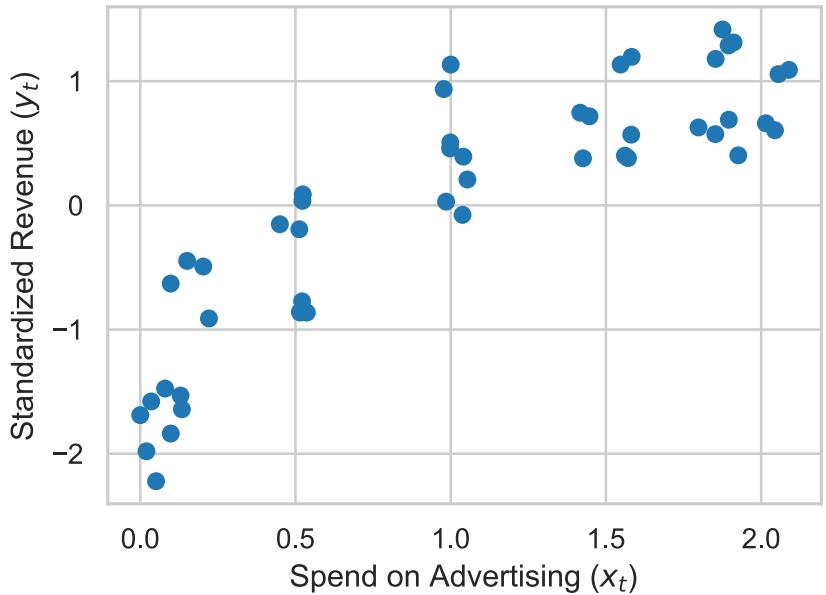
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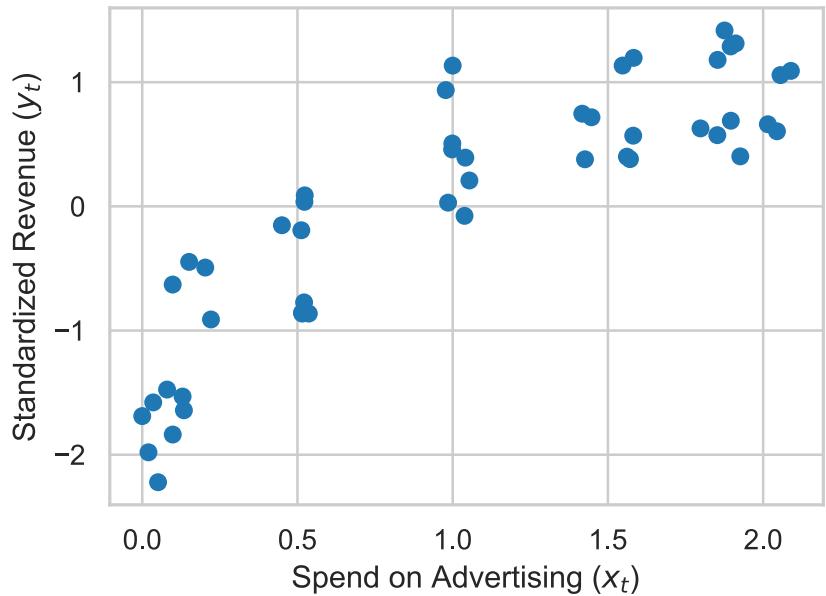


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3. Problems are exacerbated under common managerial practices, like **autoregressive decision-making**
4. Similarly fitting models can have **fundamentally different implications** in terms of optimal decision-making

# (A little) Math

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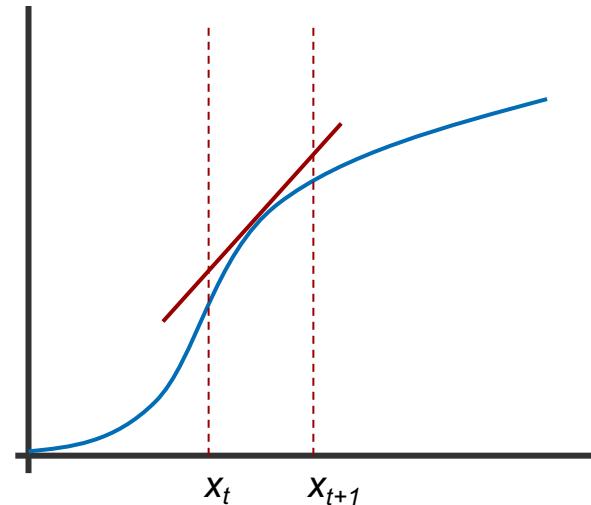
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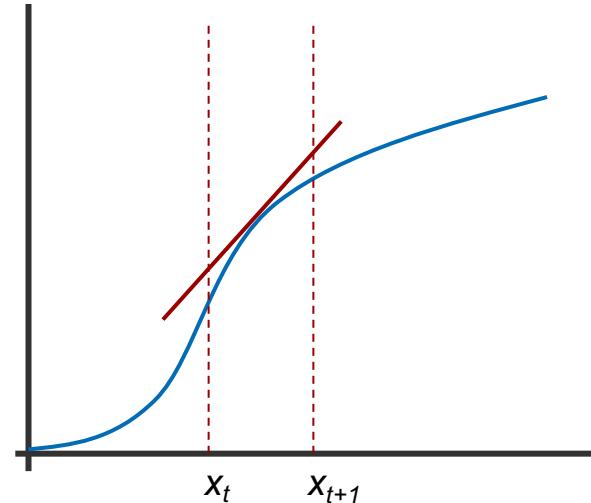


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- When will  $\beta_t$  be smooth (“forecastable”)? If  $f$  is **smooth** and  $x_t$  and  $x_{t+1}$  are **close together**!



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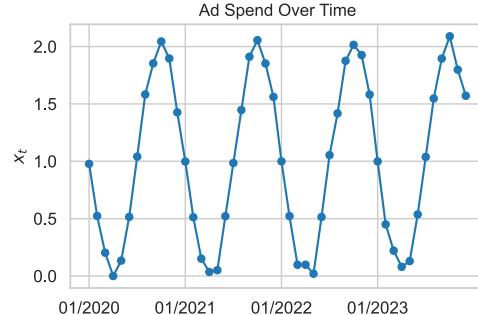
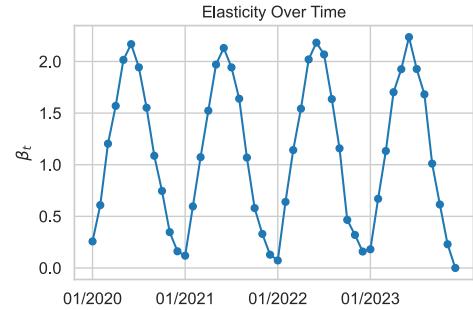
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# Simulations

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Four types of simulations:

1. Flexible nonlinear response

$$y_t = f(x_t) + \varepsilon_t$$

2. Dynamic coefficients, linear (or log-linear) model

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3. Nonlinear response, parametric hill function

$$y_t = \text{Hill}(x_t) + \varepsilon_t$$

4. Dynamic coefficients inherited from common parent

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## Gaussian processes

Two important levers:

- Smoothness
- Amplitude

# Primer: Gaussian Processes

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Lengthscale

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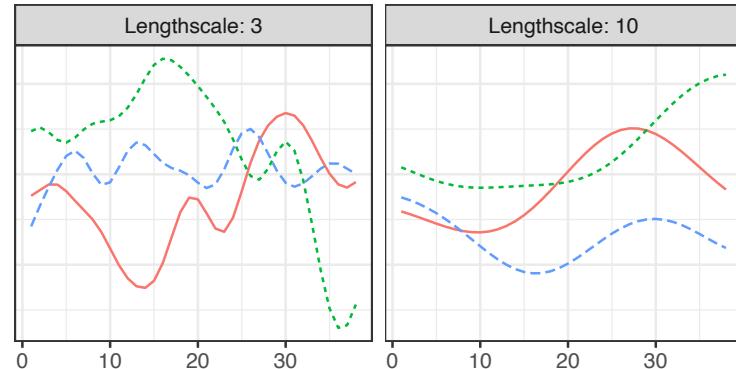


$$f(x_1, \dots, x_N) \sim \mathcal{N}(m(x_1, \dots, x_N), K), \text{ s.t. } K_{ij} = k(x_i, x_j)$$

Amplitude

$$k(x, x') = \eta^2 \exp \left\{ -\frac{(x - x')^2}{2\rho^2} \right\}$$

Lengthscale



# When does conflation *actually* happen?

Four types of simulations:

1. Flexible nonlinear response

$$y_t = \boxed{f(x_t)} + \varepsilon_t$$

2. Dynamic coefficients, linear (or log-linear) model

$$y_t = \boxed{\beta(t)x_t} + \varepsilon_t$$

3. Nonlinear response, parametric hill function

$$y_t = \text{Hill}(x_t) + \varepsilon_t$$

4. Dynamic coefficients inherited from common parent

$$y_t = \beta(t)x(t) + \varepsilon_t, (\beta(t), x(t)) \sim \text{Pa}(t)$$

## Gaussian processes

Two important levers:

- Smoothness
- Amplitude

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Four types of simulations:

1. Flexible nonlinear response

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3. Nonlinear response, parametric hill function

## Other manipulated features:

4. Dynamic coefficient
- Autoregressive coefficient in  $x$
  - Noise in  $x$ 's autoregressive process
  - Variance of the error term

## Gaussian processes

Two important levers:

- Smoothness
- Amplitude

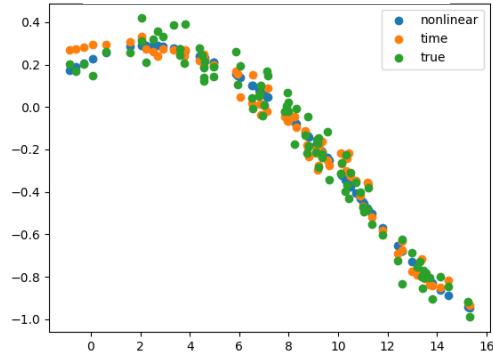
# Simulation Results

- For each simulation type, >300 settings, systematically varying the previously described factors, with 100 simulations per setting
- Fit both models (nonlinear and dynamic), measure conflation through validation RMSE

# Examples

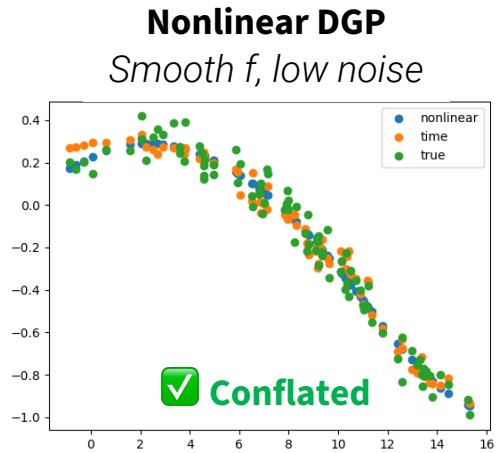
## Nonlinear DGP

*Smooth  $f$ , low noise*



# Examples

Data

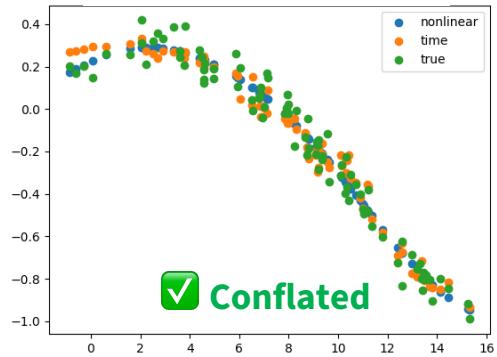


# Examples

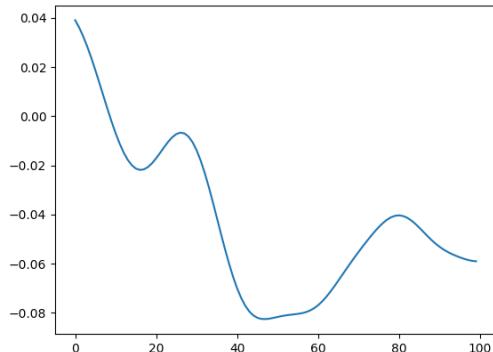
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Data

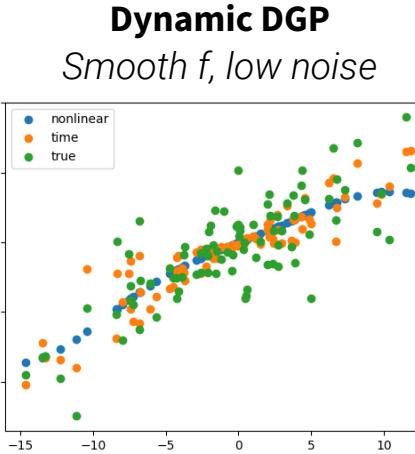
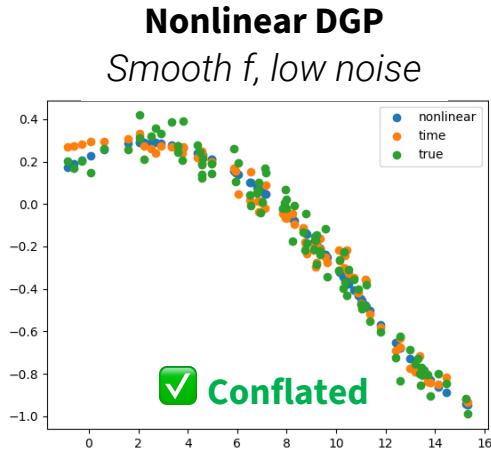


Implied  
 $\beta(t)$

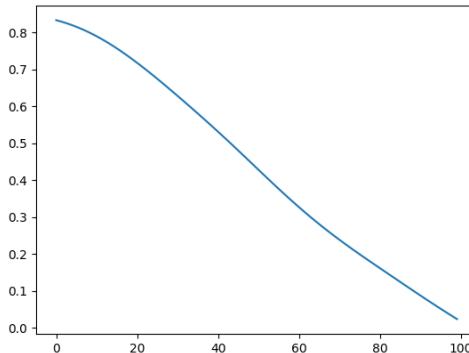
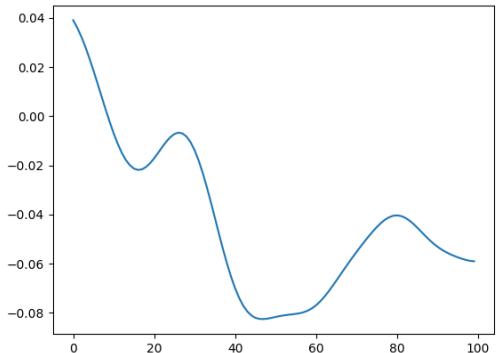


# Examples

Data

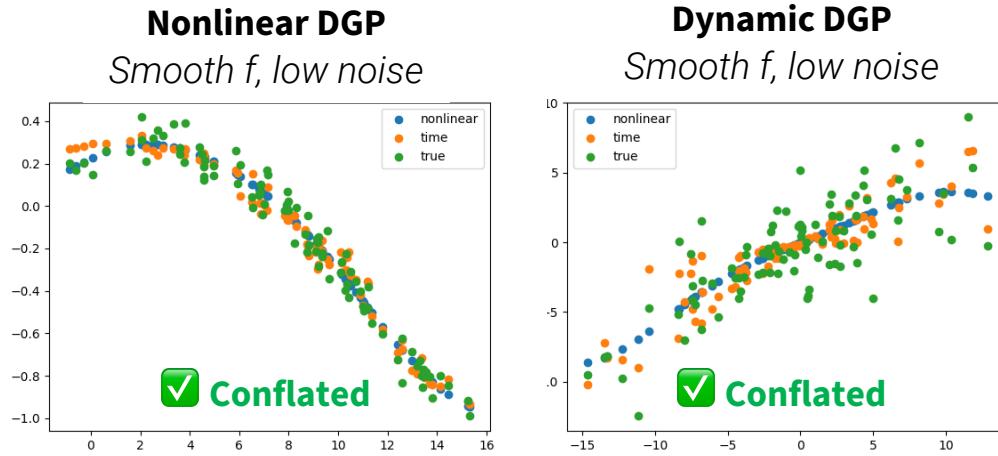


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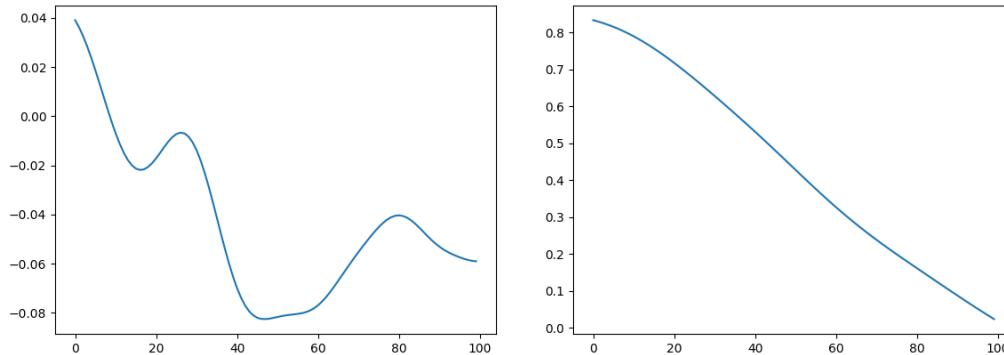


# Examples

Data

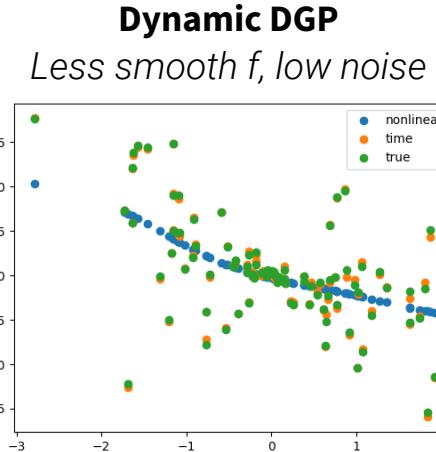
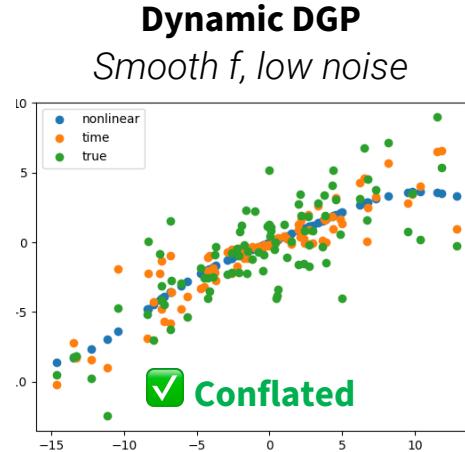
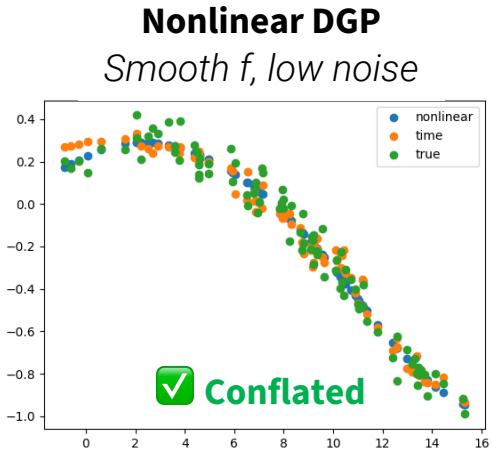


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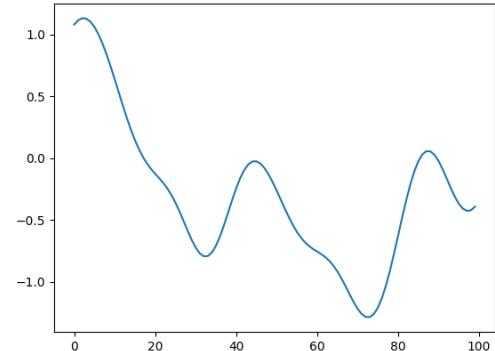
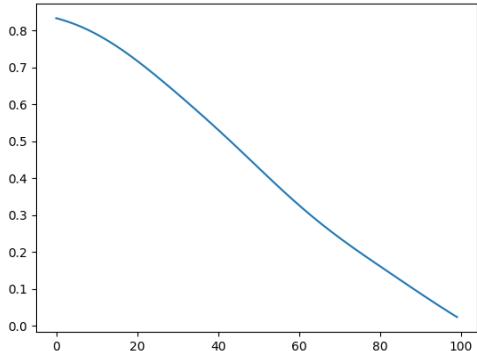
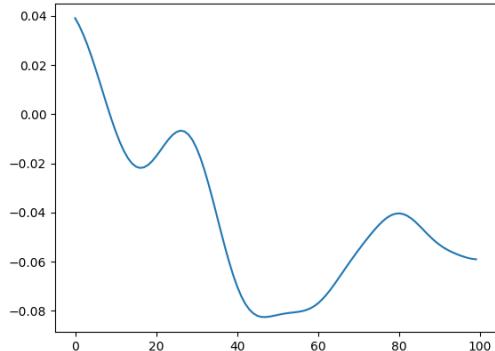


# Examples

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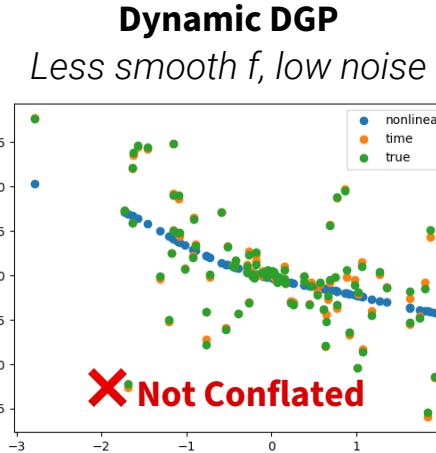
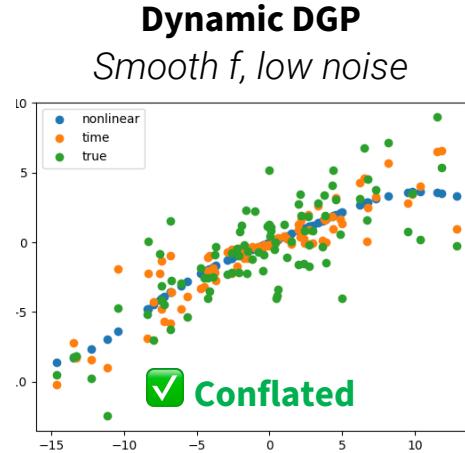
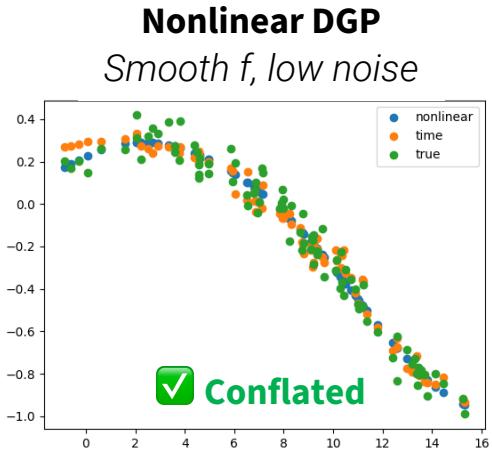


Implied  
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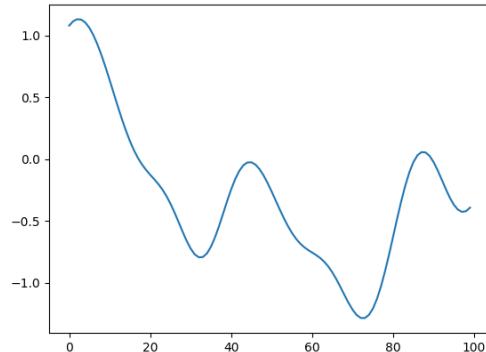
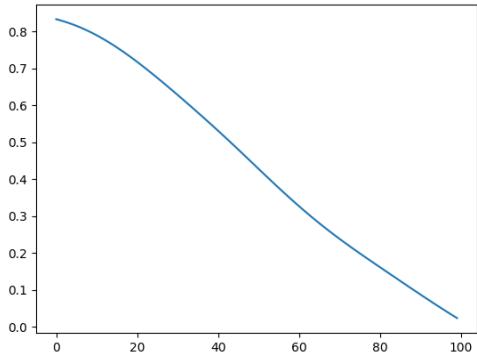
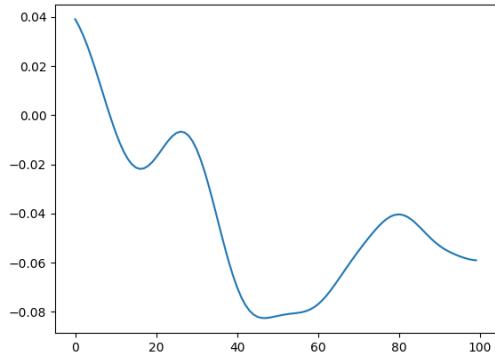


# Examples

Data



Implied  
 $\beta(t)$



# Simulation Results

- For each simulation type, >300 settings, systematically varying the previously described factors, with 100 simulations per setting
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## Main result: huge prevalence of conflation

- Under the nonlinear DGP, **82%** exhibited some conflation, with **23% exhibiting major conflation** (*defined as >25% of simulations conflated*)
- Under the time-varying DGP, **80%** exhibited some conflation, with **27% exhibiting major conflation**

# Diving Deeper

**DV:** % Conflated Simulations

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DV: % Conflated Simulations

Variable	Level	Nonlinear DGP		Dynamic DGP	
		Coef	$P(> t )$	Coef	$P(> t )$
Amplitude, $f$ :	Low	-	-	-	-
	Middle	-0.06	0.94	2.94	0.01
	High	-0.11	0.88	8.99	0.00
Smoothness, $f$ :	Low	-	-	-	-
	Middle	9.99	0.00	3.44	0.00
	High	19.43	0.00	8.10	0.00
AR coef, $x$ :	Low	-	-	-	-
	Middle	0.81	0.28	1.05	0.36
	High	2.66	0.00	4.77	0.00
Variance, $x$ :	Low	-	-	-	-
	Middle	-0.16	0.83	5.80	0.00
	High	0.07	0.92	11.66	0.00
Noise, $y$	Low	-	-	-	-
	Middle	7.60	0.00	6.69	0.00
	High	14.81	0.00	20.25	0.00
	Very High	24.02	0.00	40.19	0.00

**Table 1:** Simulation Results: DV = Percentage Conflation; Intercept omitted for clarity.

# Diving Deeper

**DV:** % Conflated Simulations

Conflation more likely with...

- Noisier data (i.e.,  $\epsilon_t$ )

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DV: % Conflated Simulations

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*Decisions are often autocorrelated!*

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## Even worse with AdStock

Under the nonlinear DGP with AdStock, **93%** exhibited some conflation,  
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(previously: 82% and 23%)

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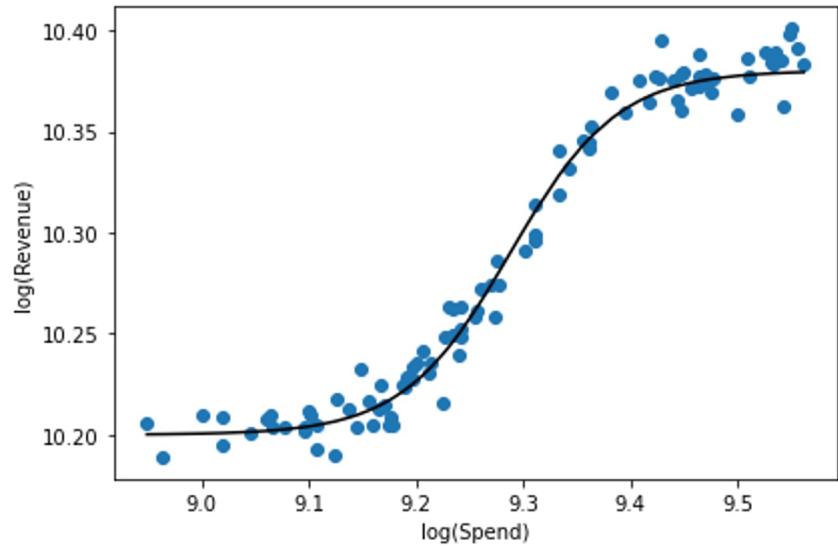
(previously: 82% and 23%)

Under the time-varying DGP with AdStock, **87%** exhibited some conflation,  
with **52% exhibiting major conflation**

(previously: 80% and 27%)

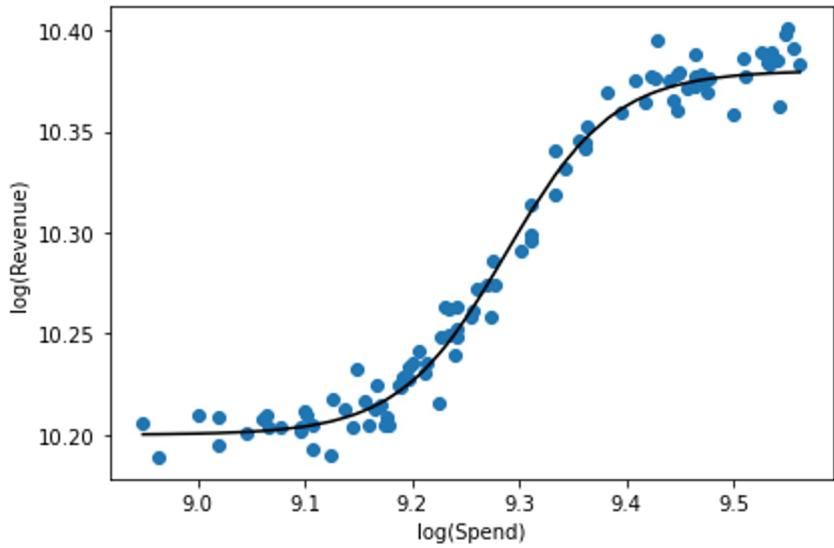
# Implications

# One last simulation...

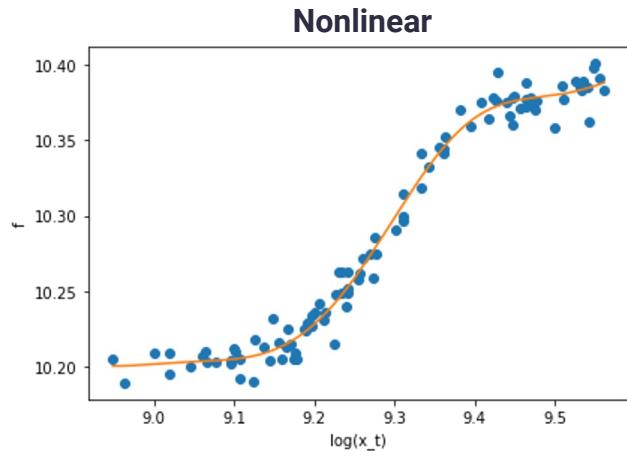


(or from roughly \$8,000 to \$14,000)

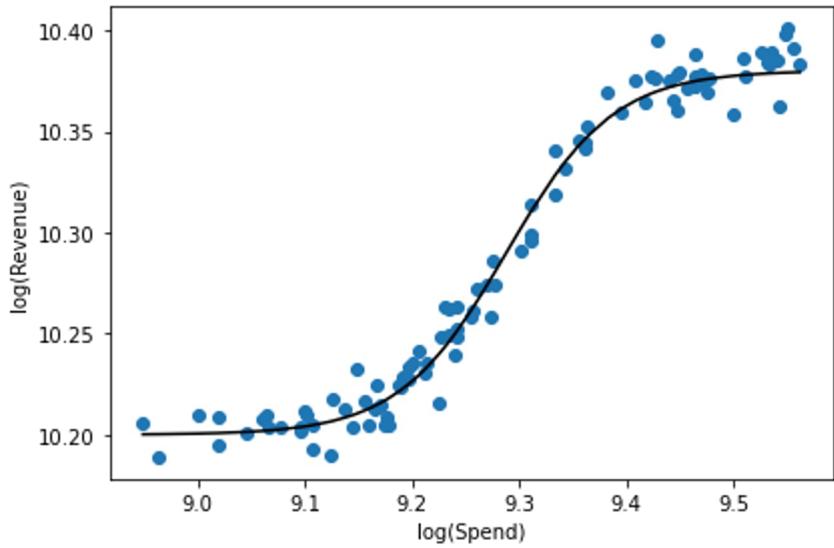
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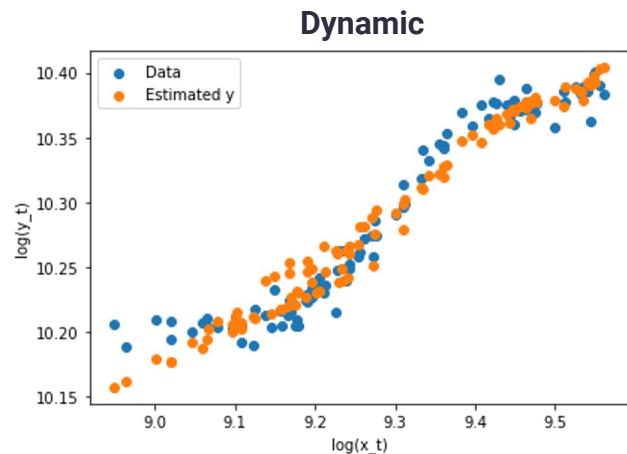
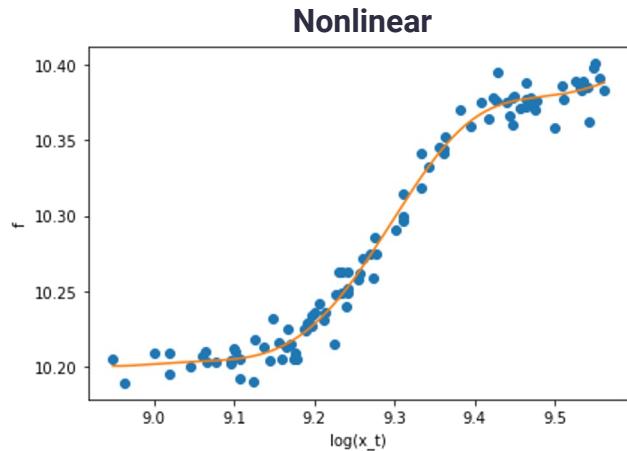
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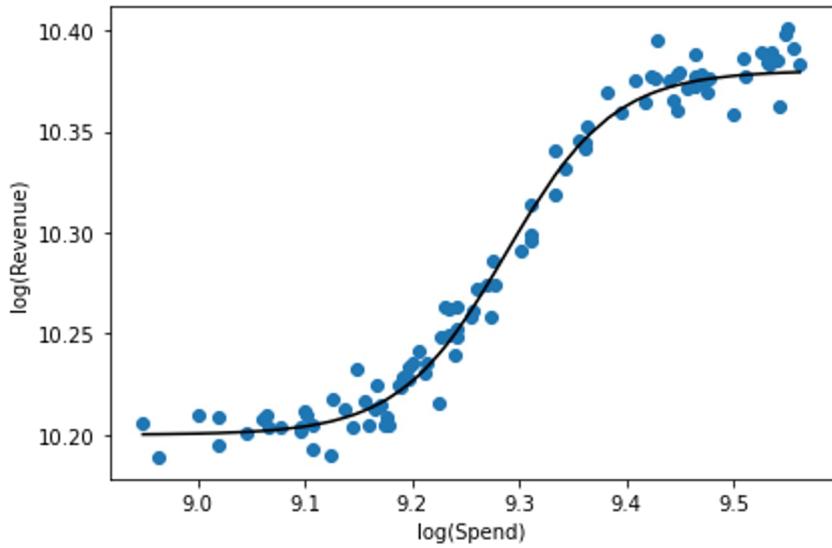
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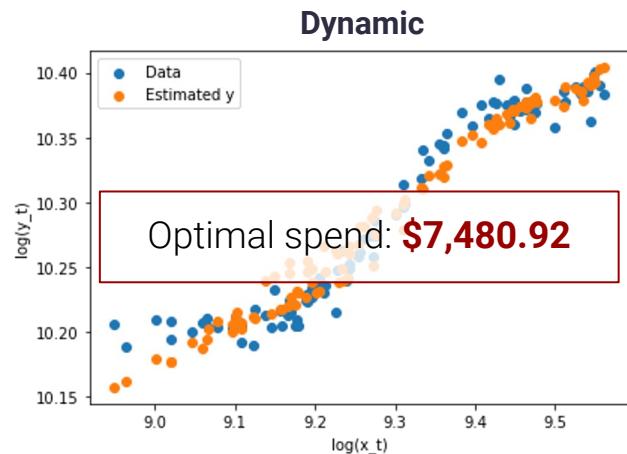
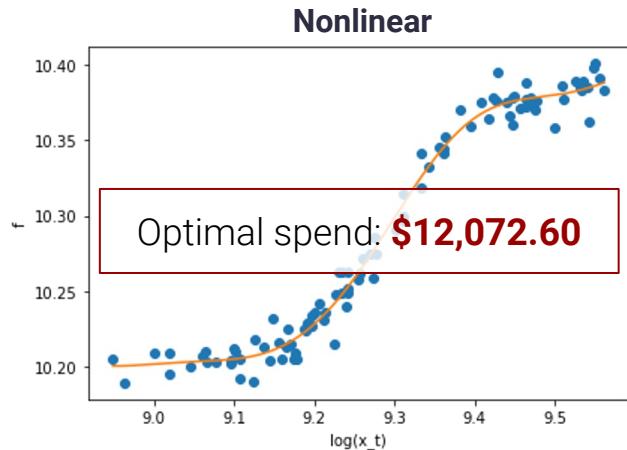
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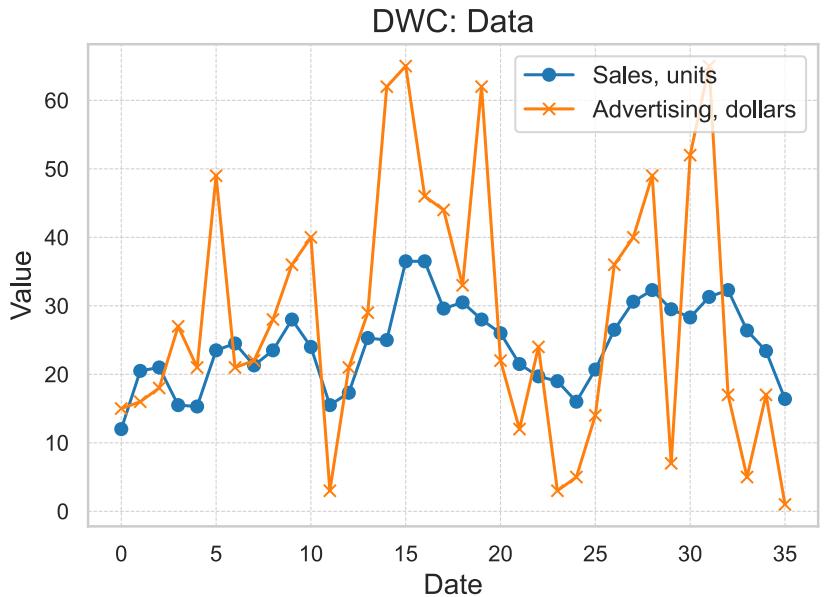
# Real Data: Classic and Modern

- Classic Application 1: Dietary Weight Control (DWC)  
(Bass and Clark, 1972)
- Classic Application 2: Lydia Pinkham (LP)  
(Palda, 1964)
- Modern Applications: MMM data constructed from Nielsen

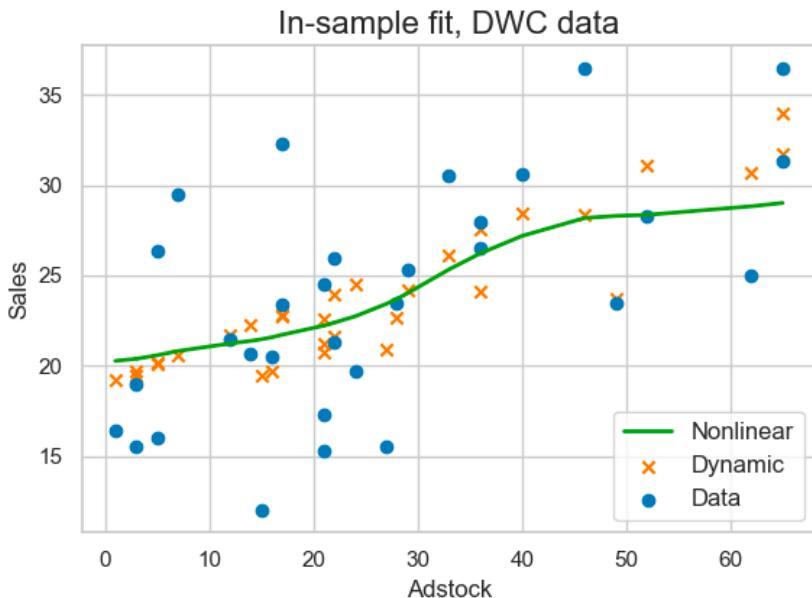
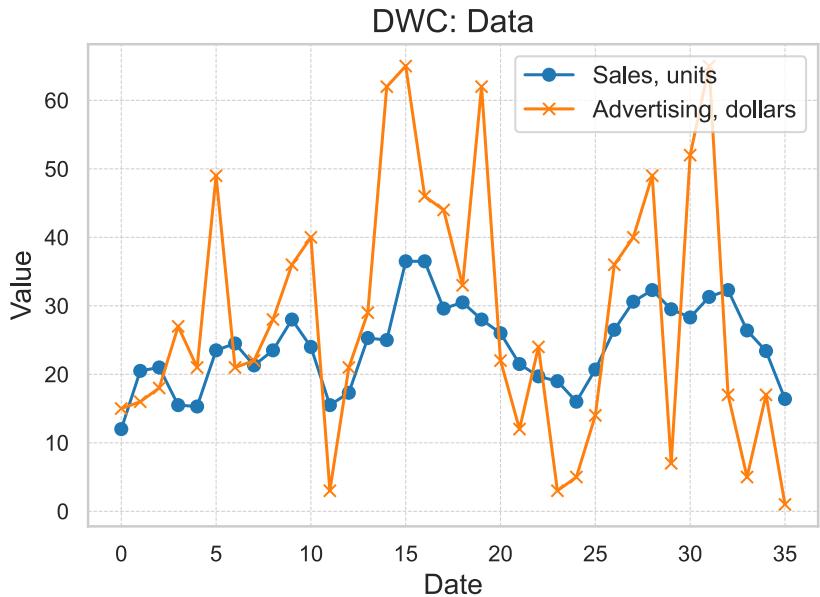
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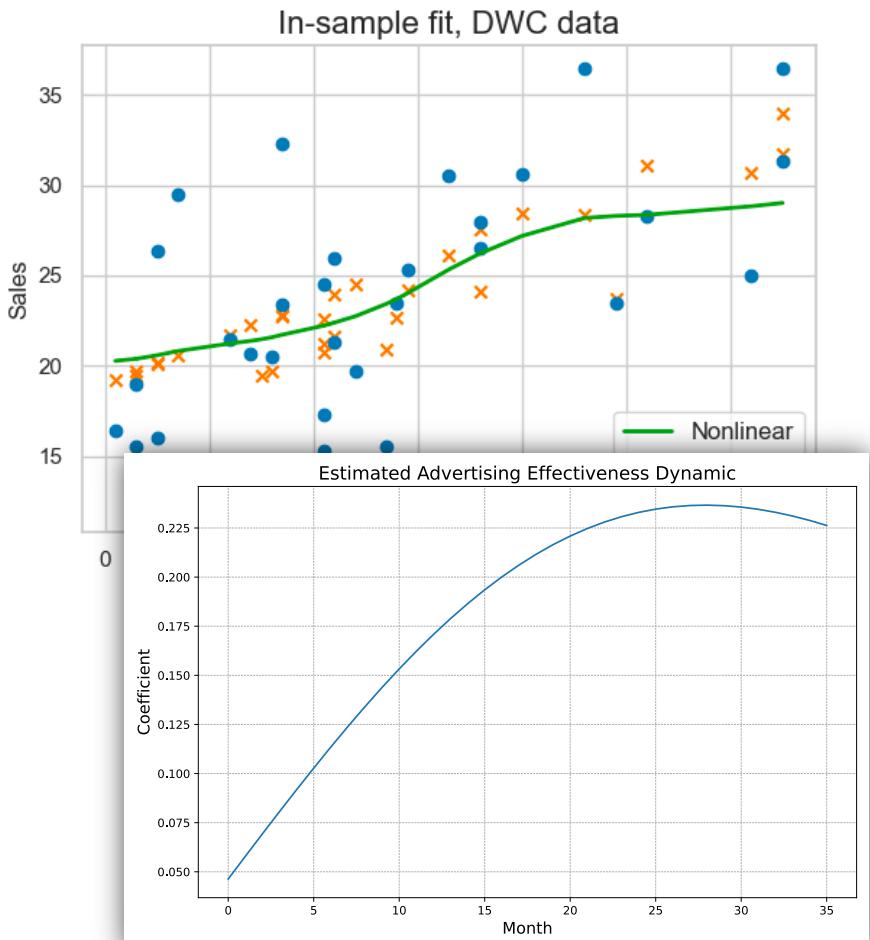
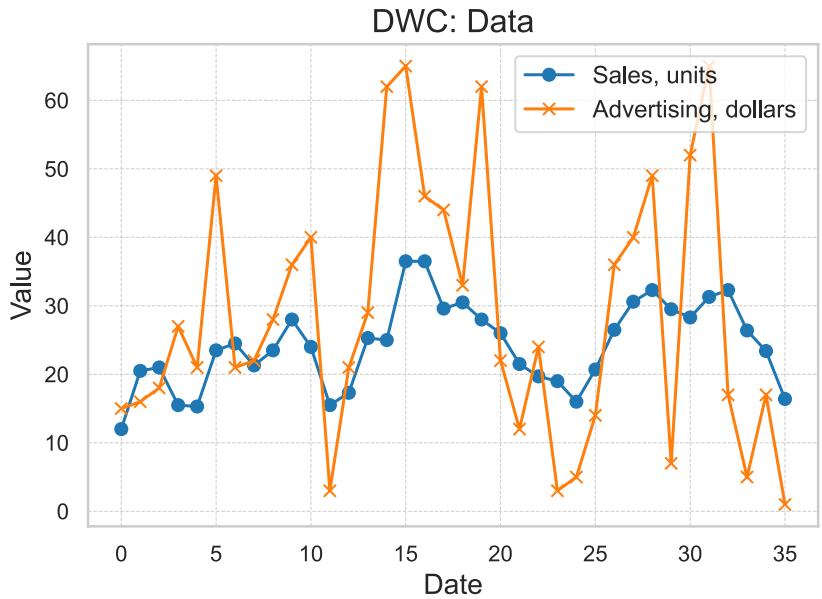
# Dietary Weight Control



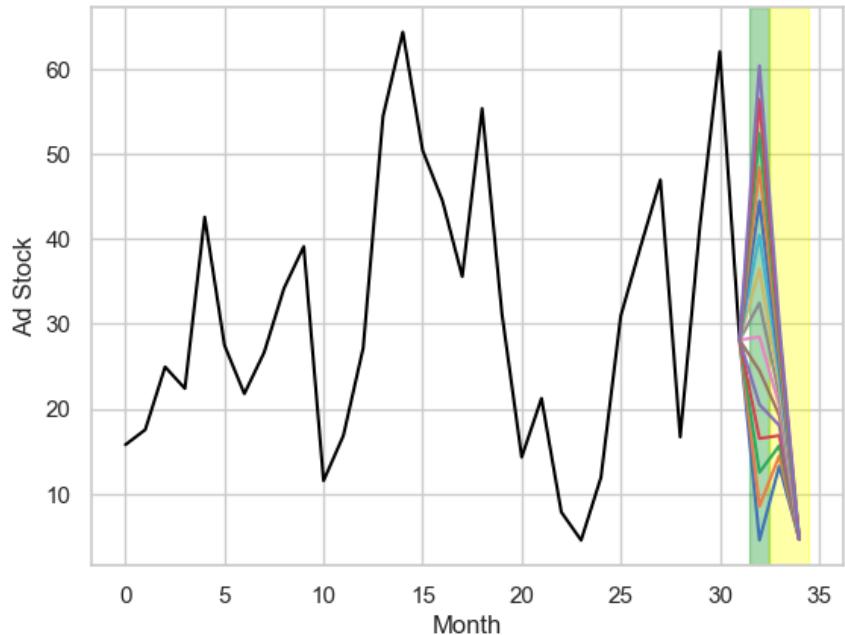
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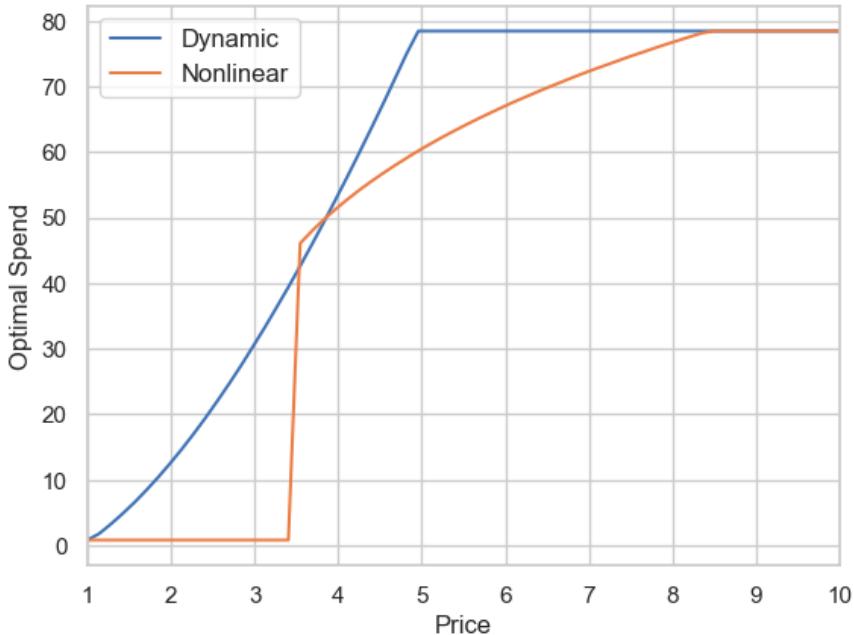
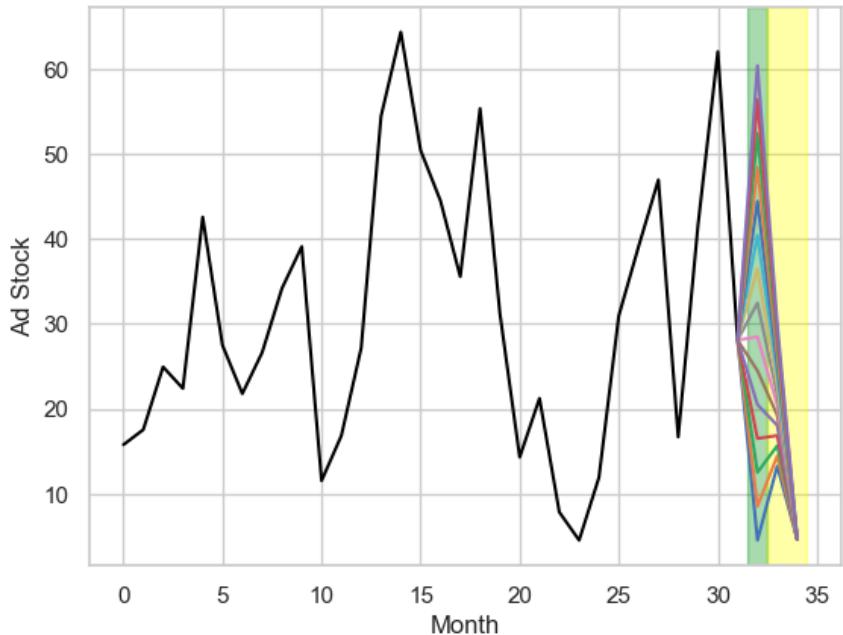
# Dietary Weight Control



# Ad Optimization a la Google

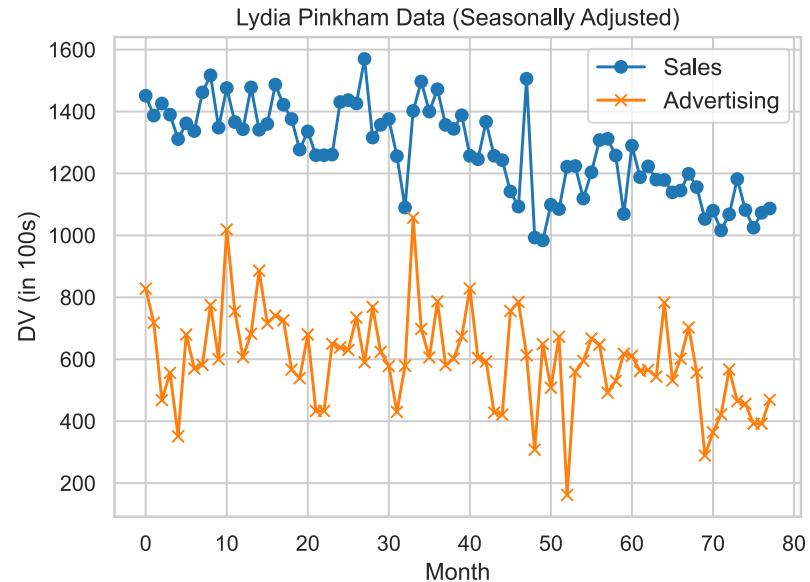


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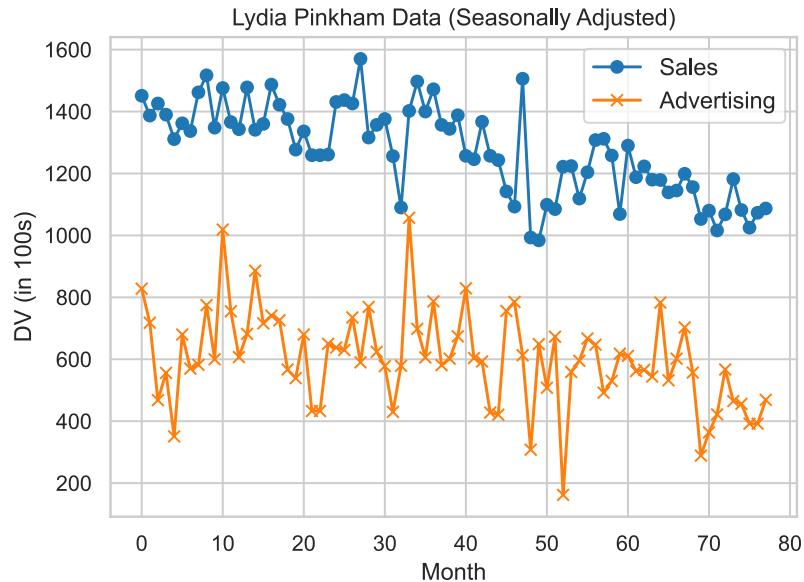
# Lydia Pinkham

Sales and advertising of Lydia Pinkham's herbal products, monthly, 1954-1960 ([Palda, 1964](#))



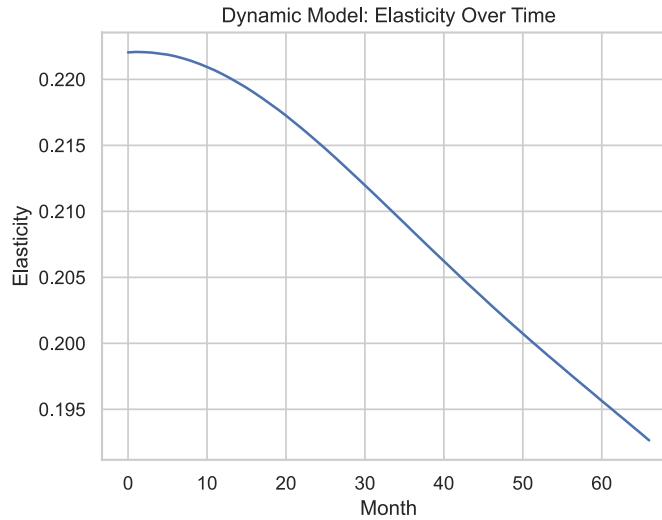
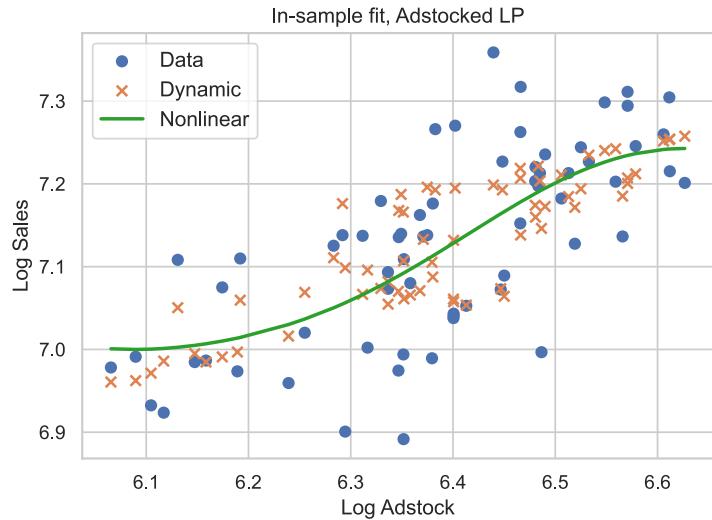
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Sales and advertising of Lydia Pinkham's herbal products, monthly, 1954-1960 (Palda, 1964)

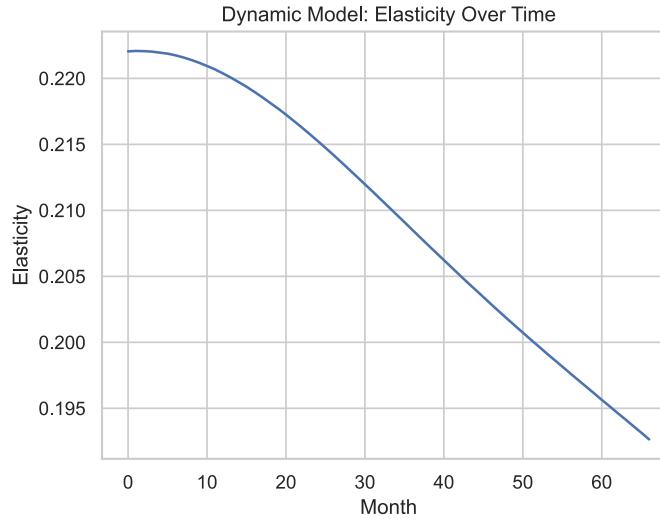
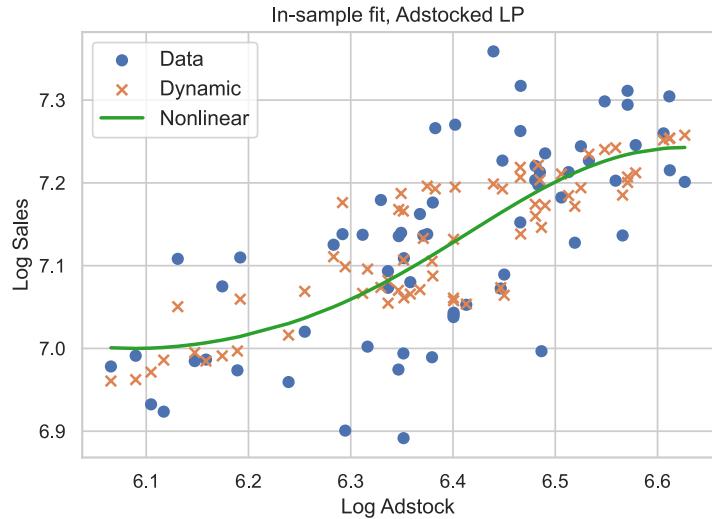


**Lydia Pinkham**  
150 Tablets  
★★★★★ 764  
50+ bought in past month  
\$21<sup>99</sup> (\$0.15/Count)  
✓prime Two-Day  
FREE delivery Thu, May 9  
Add to cart

# Again, conflated...



# Again, conflated...



and yield different optimal expenditures.

Under nonlinear:  
**\$89,883**

Under dynamic:  
**< \$78,333**

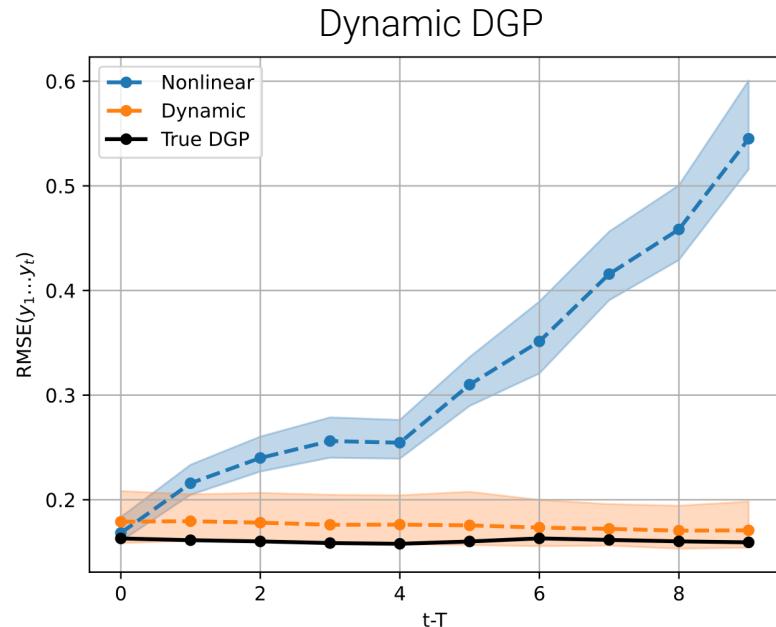
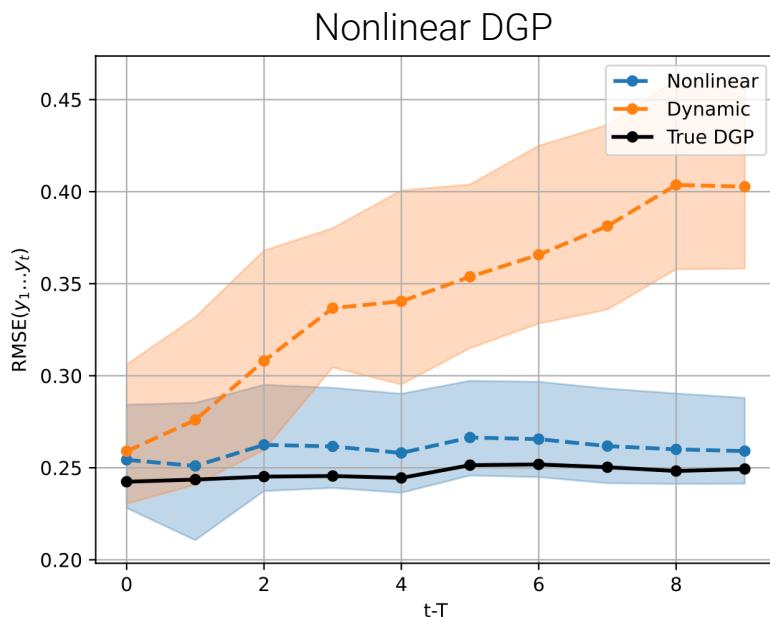
# Solutions

# Is MMM doomed?

- In short: **no!**
- Careful ad planning can help disentangle the two stories
- **“Bump-up”  
incrementality tests**

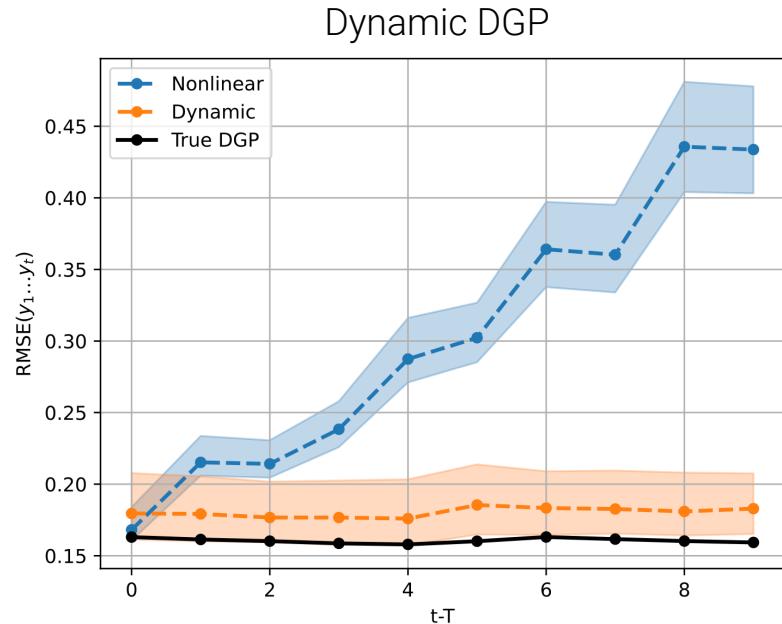
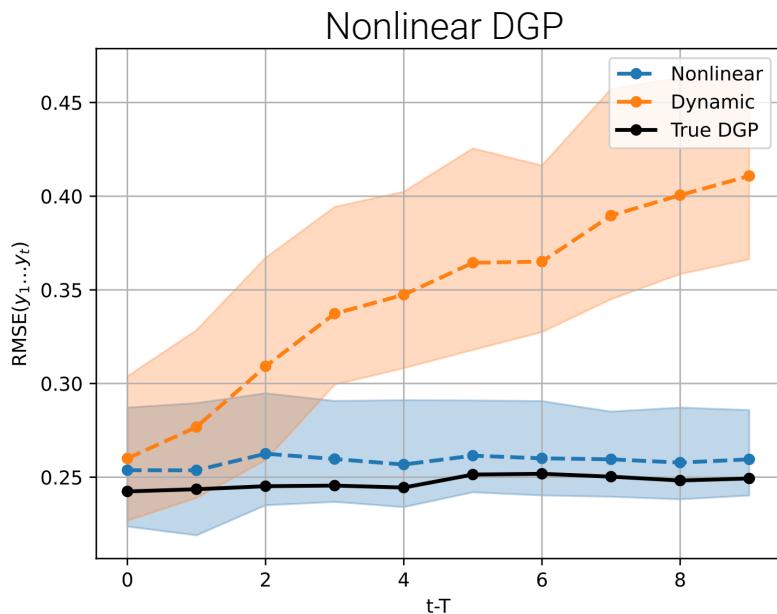
# Test 1: Maximal Separation

*Set spending next period to maximize the difference in predicted values across the two models*



## Test 2: See-saw

Rapidly changing spending across adjacent periods can break some of the common causes of conflation



# Conclusions

# Current MMM Practice Might Be Flawed!

- We show that, under many common spending patterns, **time-varying and nonlinear effects cannot be disentangled**, despite having different implications
- This problem is potentially **very widespread**: increasing complexity in models, widespread practice of “model refreshes” to capture changing markets
- Our work both introduces a **framework for estimating** these types of models, and provides **solutions for understanding and preventing** conflation

# Thanks!

Feedback or questions:  
*ryandew@wharton.upenn.edu*

Working paper soon:  
[www.rtdew.com](http://www.rtdew.com)



You

generate an image of a marketing mix model paradise with some very happy computers and a cameo from the reverend thomas bayes

