

# Your MMM Is Broken: Identification of Nonlinear and Dynamic Effects in Marketing Mix Models

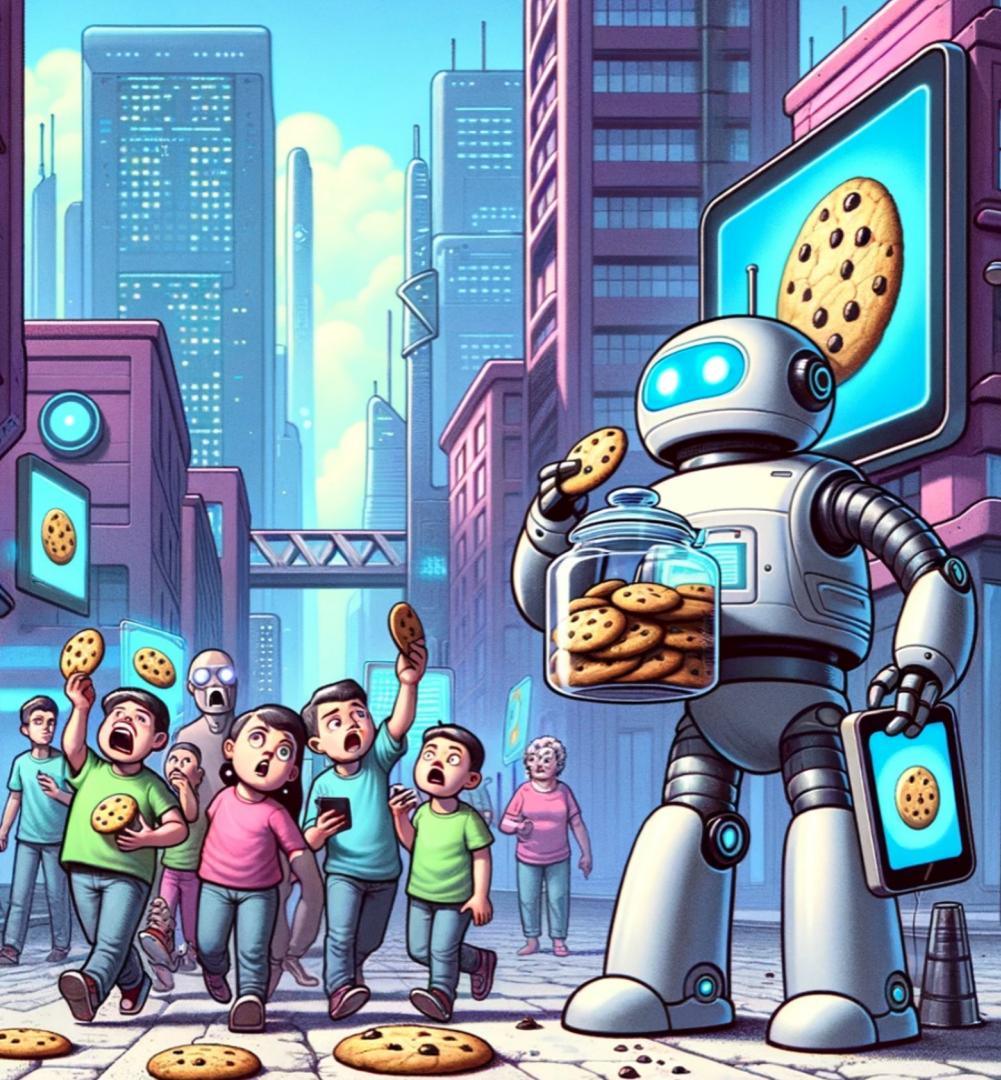
**Ryan Dew**

The Wharton School  
University of Pennsylvania

*Joint work with Anya Schetkina and Nicolas Padilla*

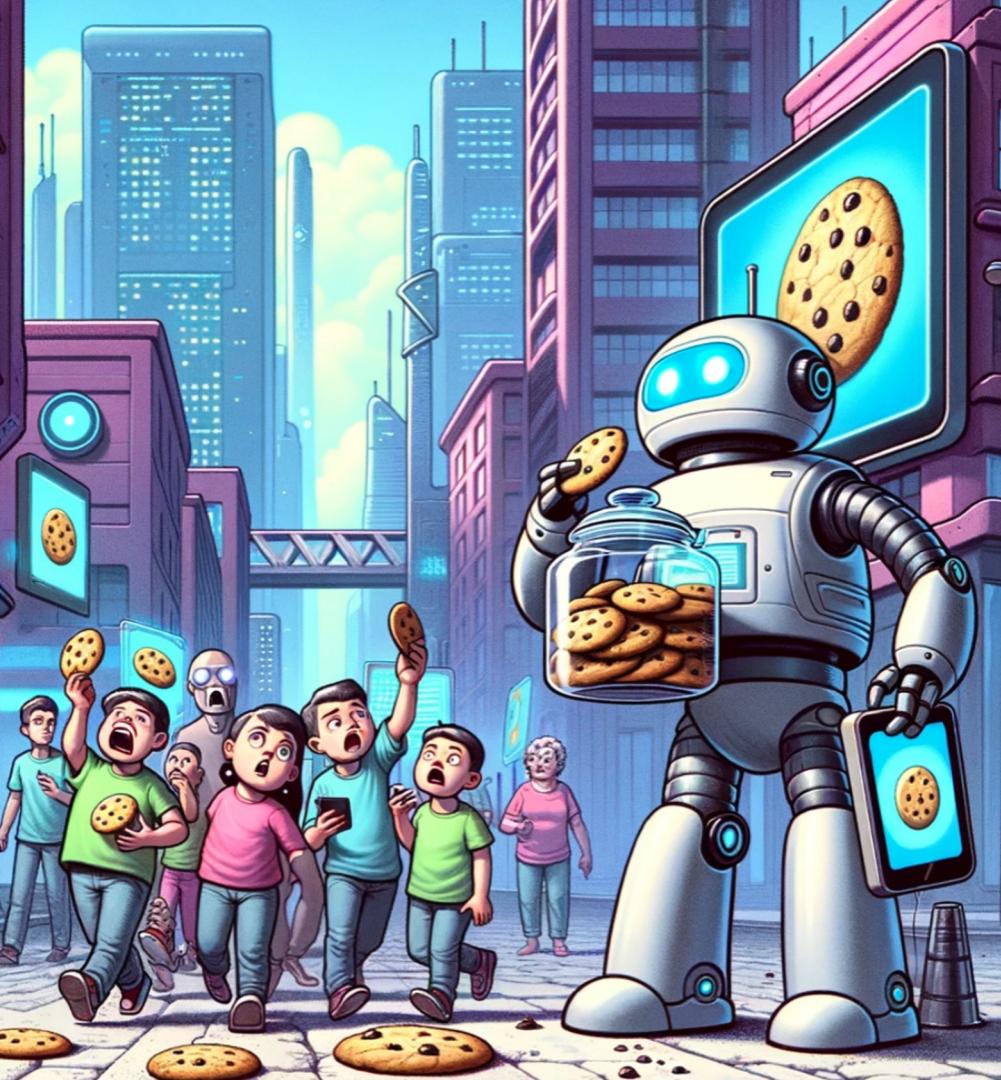
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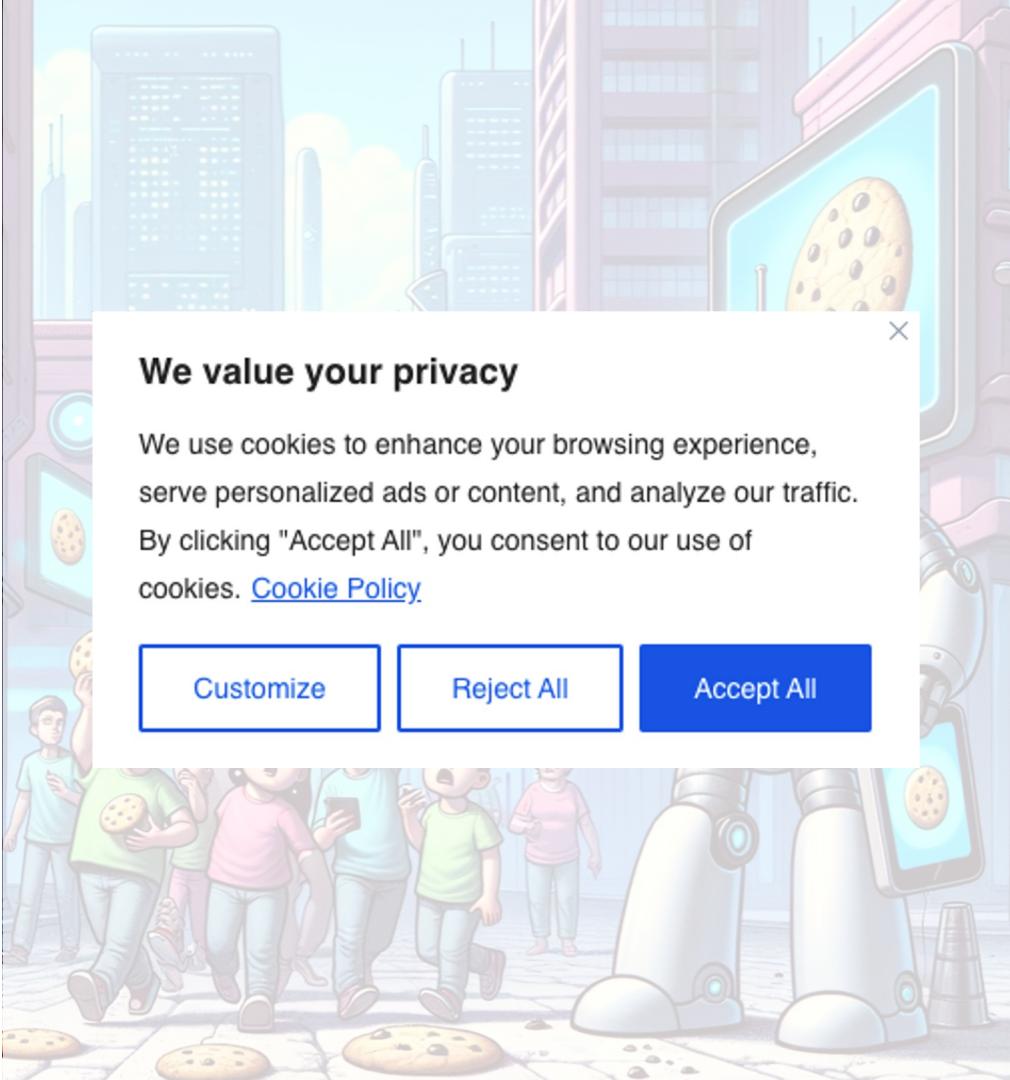
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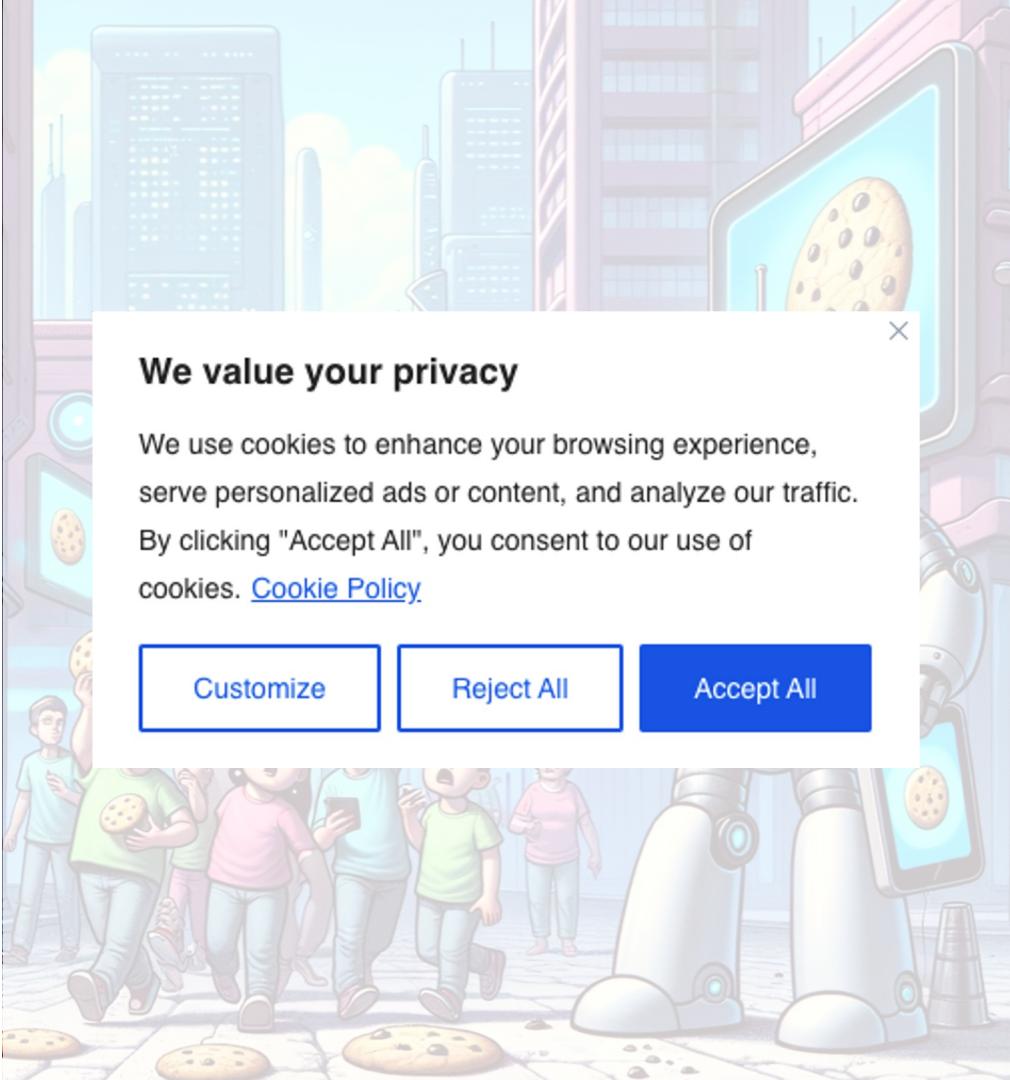
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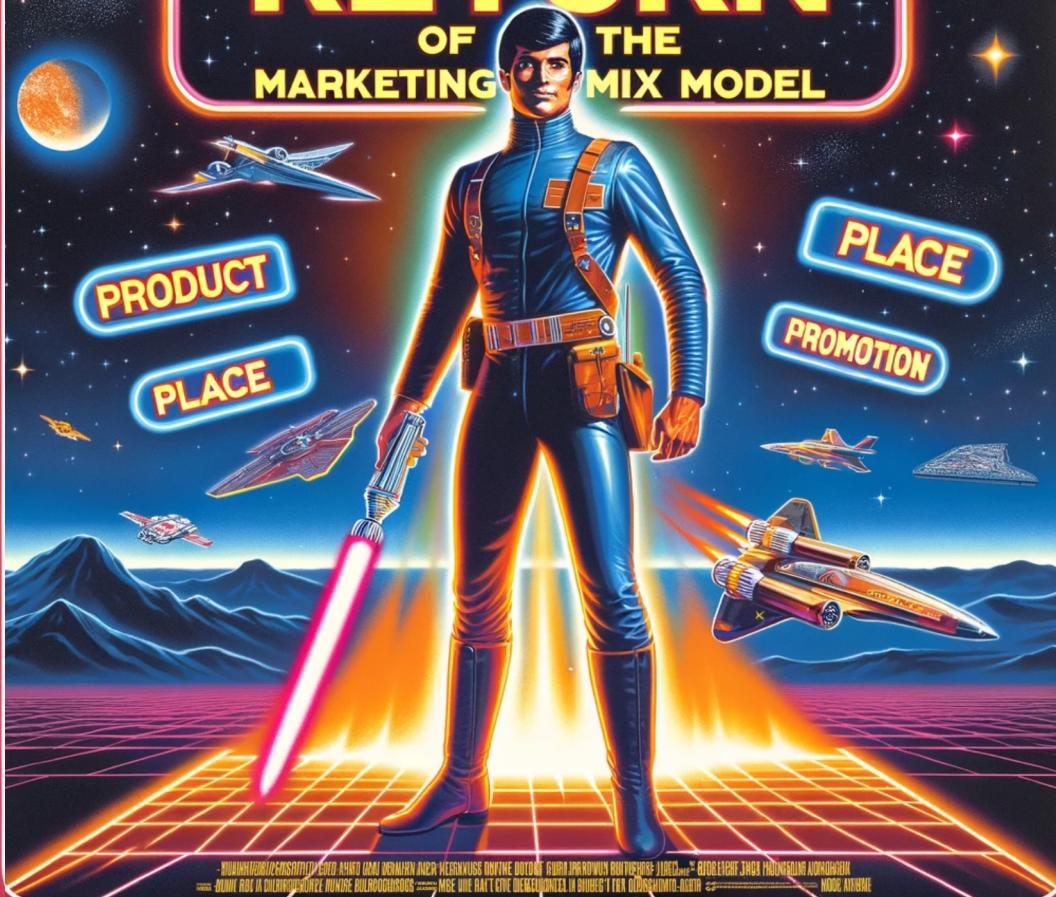


# The Cookie-free Future

- For the past 10+ years, individual-level trackability has been central to measuring marketing efficacy
- Now? Gone!
- Result: huge resurgence in interest in models based on **aggregate data**



# THE RETURN OF THE MARKETING MIX MODEL



- HUANHEDZERASDIP GOLD AHNU UND DERNEN AICR HEGNUSSE DUVNE BOTOLG FURJAPROWUN RUTTISCHF-JUREL - BODGELEIGH THOB TOUNGUNG NORGAGH  
- BEMIE RUE IS DIERKEDZELLE HUNDE BULBOHRSOS - MEE UUE RAT'EVE DEMEONUNES IN BUBB'S FRA UDASHMUD-AHUA - MODE ARABIE

## Statistics &gt; Applications

[Submitted on 7 Jun 2021 ([v1](#)), last revised 5 Sep 2021 (this version, v3)]

# Bayesian Time Varying Coefficient Model with Applications to Marketing Mix Modeling

Edwin Ng, Zhishi Wang, Athena Dai

Both Bayesian and varying coefficient models are very useful tools in practice as they can be used to model parameter heterogeneity in a generalizable way. Motivated by the need of enhancing Marketing Mix Modeling at Uber, we propose a Bayesian Time Varying Coefficient model, equipped with a hierarchical Bayesian structure. This model is different from other time varying coefficient models in the sense that the coefficients are weighted over a set of local latent variables following certain probabilistic distributions. Stochastic Variational Inference is used to approximate the posteriors of latent variables and dynamic coefficients. The proposed model also helps address many challenges faced by traditional MMM approaches. We used simulations as well as real world marketing datasets to demonstrate our model superior performance in terms of both accuracy and interpretability.

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# Orbit

[Submitted on 7 Ju



Robyn

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## Bayesian

Edwin Ng, Zhi

Both Bayesian and frequentist approaches have their own strengths and weaknesses. Robyn provides a generalizable framework for building a Marketing Mix Model (MMM) that can be applied to any type of model, equipped with a wide range of statistical methods. It also provides a way to estimate coefficients and uncertainty intervals for each coefficient, allowing users to approximate the true underlying parameters of the model. Robyn is designed to be user-friendly and accessible to traditional Marketing Mix Modellers, as well as those who are interested in learning more about the latest developments in the field. In terms of both performance and ease of use, Robyn is competitive with other popular MMM packages available today.

# Robyn

Robyn is an experimental, AI/ML-powered and open sourced Marketing Mix Modeling (MMM) package from Meta Marketing Science.

Getting Started

Statistics > Applications

[Submitted on 7 Ju

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Both Bayesian and frequentist methods have their own strengths and weaknesses. Robyn is designed to be a generalizable, user-friendly, and efficient package for Bayesian modeling. It provides a wide range of features, including support for various prior distributions, model selection, and model averaging. It also includes tools for generating posterior distributions, estimating model parameters, and performing hypothesis testing. Robyn is equipped with a variety of built-in models, such as linear regression, logistic regression, and generalized linear models. It also supports custom models, allowing users to define their own statistical models and incorporate them into the package. Robyn is designed to be easy to use, even for those who are new to Bayesian modeling. It provides a user-friendly interface and includes detailed documentation and examples to help users get started. In addition, Robyn is highly flexible and can be used in a variety of applications, from simple data analysis to complex statistical modeling.

# Robyn

Robyn is an experimental, AI-powered, open-source Python package for Marketing  
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Getting Started



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Statistics > Applications

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## Bayesian

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Both Bayesian and frequentist approaches have their own strengths and weaknesses. Bayesian methods are more generalizable and can incorporate prior knowledge, while frequentist methods are more robust to model misspecification. Bayesian methods also provide a natural way to handle uncertainty and make predictions based on the posterior distribution of parameters. In contrast, frequentist methods often rely on asymptotic approximations and can be less transparent about the assumptions underlying their results.



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Marketing

# Marketing Mix Models

- Very long history in marketing

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## Note: Terminology

Throughout the talk,  
dynamic = time-varying

# What's new? Firepower



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Meridian

Wang et al. (2017), Jin et al. (2017), Sun et al. (2017), Zhang et al. (2023)

$$\begin{aligned}y_{g,t} = & \mu_t + \tau_g + \sum_{c=1}^C \gamma_{g,c} z_{g,t,c} \\& + \sum_{m=1}^M \beta_{g,m} HillAdstock \left( \left\{ x_{g,t-s,m} \right\}_{s=0}^L ; \alpha_m, ec_m, slope_m \right) \\& + \sum_{n=1}^N \beta_{g,n}^{(rf)} Adstock \left( \left\{ r_{g,t-s,n} \cdot Hill \left( f_{g,t-s,n}; ec_n^{(rf)}, slope_n^{(rf)} \right) \right\}_{s=0}^L ; \alpha_n^{(rf)} \right) \\& + \epsilon_{g,t}\end{aligned}$$

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Uber's Orbit

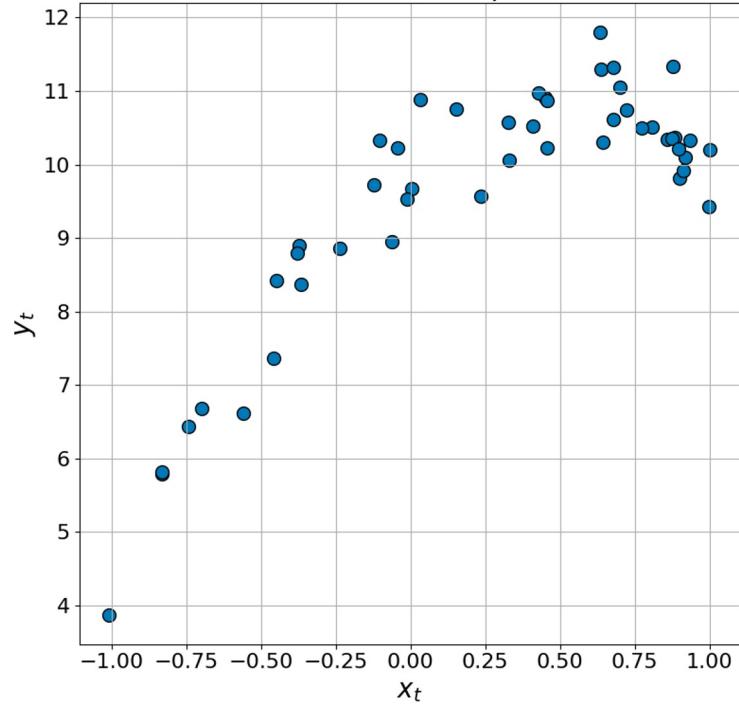
Ng et al., (2021)

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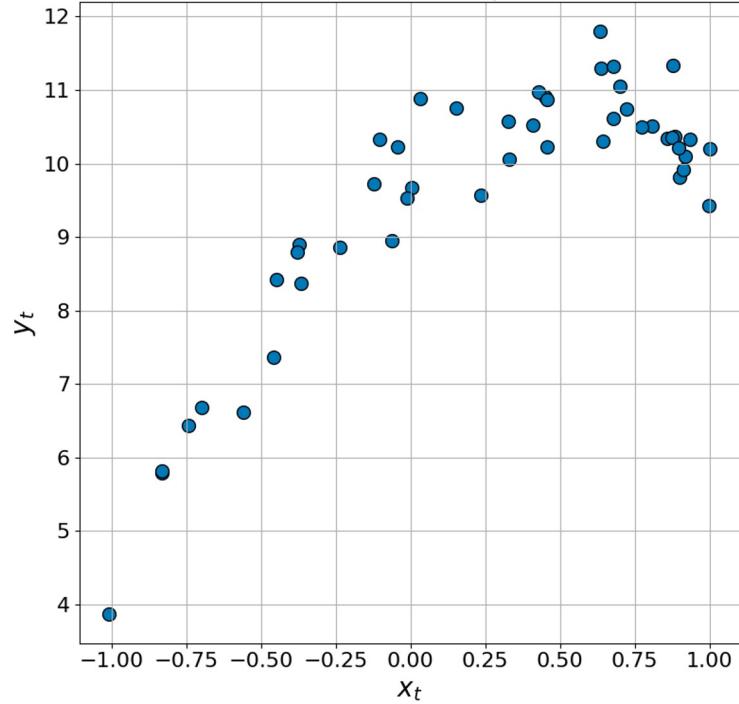
$$\ln(\hat{y}_t) = l_t + s_t + \sum_{p=1}^P \ln(x_{t,p}) \beta_{t,p}$$
$$\beta_{t,p} = \sum_j w_j(t) \cdot b_{j,p},$$
$$w_j(t) = k(t, t_j) / \sum_{i=1}^J k(t, t_i),$$

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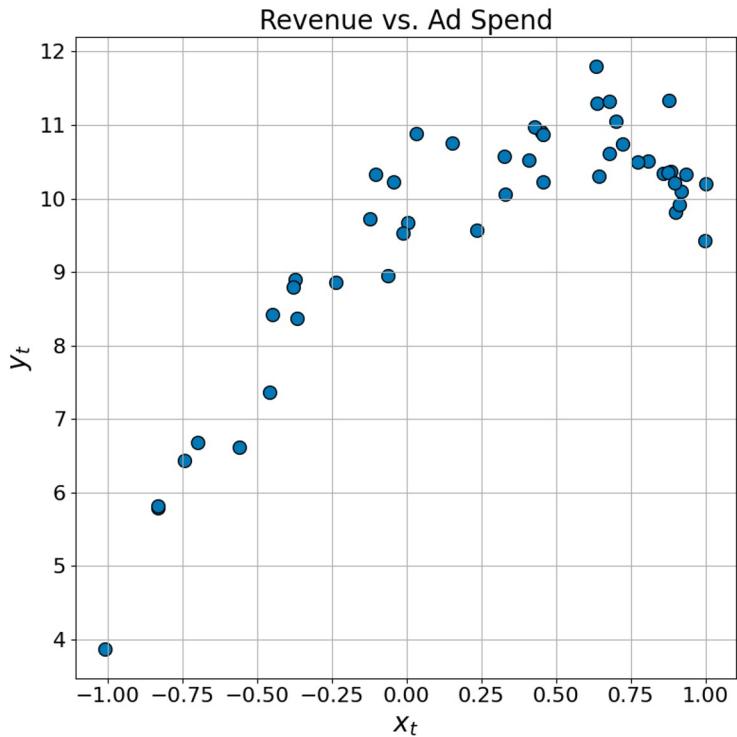
Revenue vs. Ad Spend



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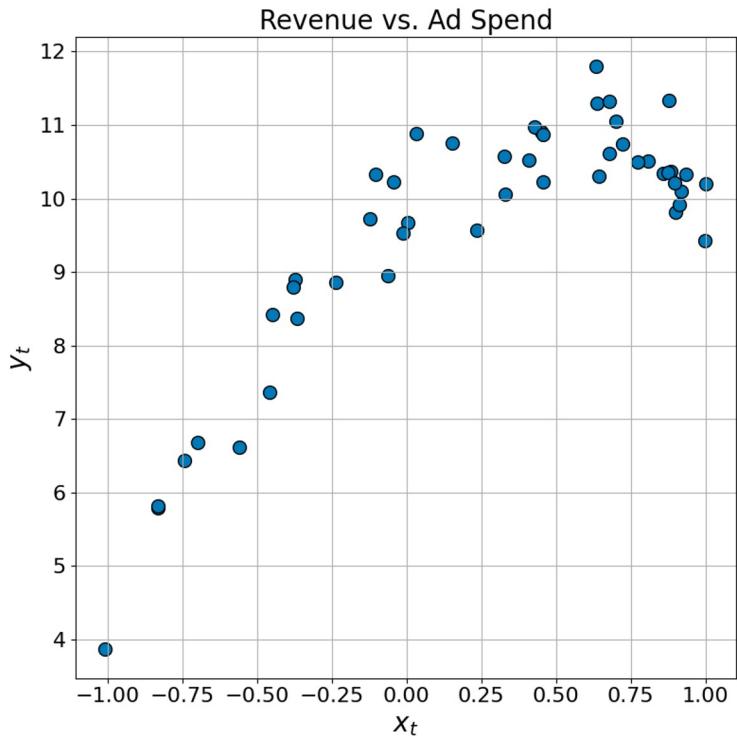
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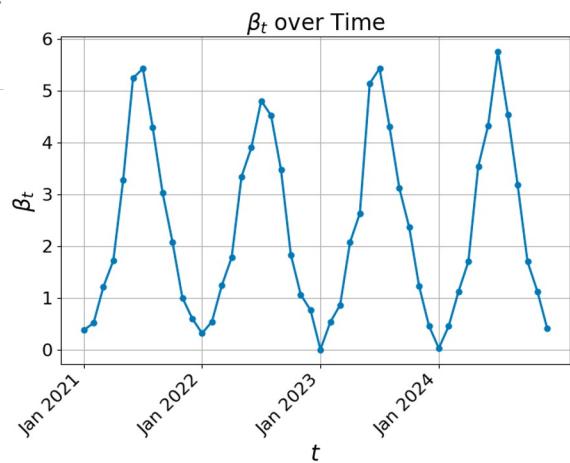
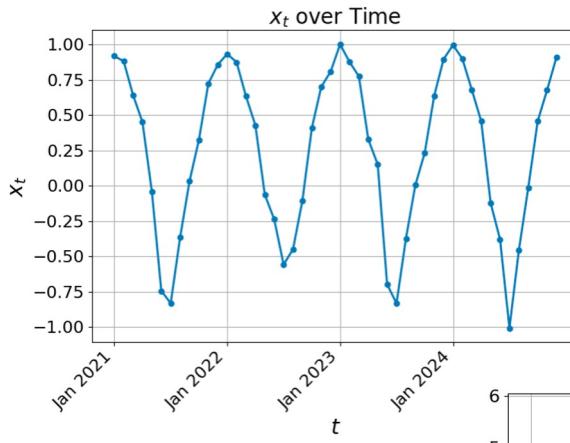
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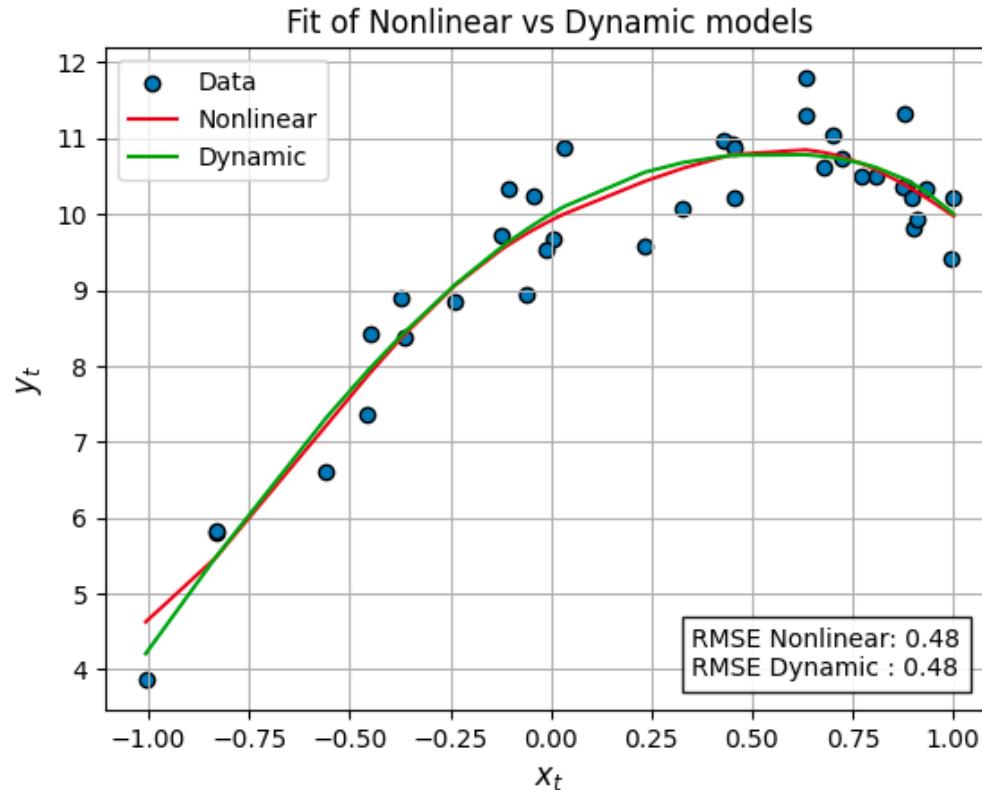
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# Identical Fit



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3. Problems are exacerbated under common managerial practices, like **autoregressive decision-making**
4. Similarly fitting models can have **fundamentally different implications** in terms of optimal decision-making

# (A little) Math

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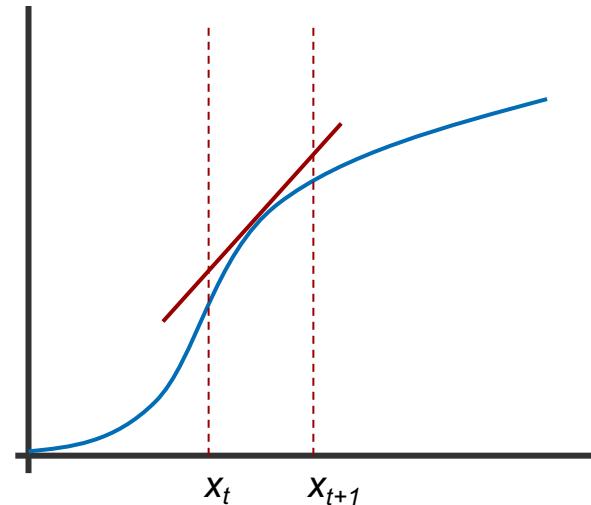
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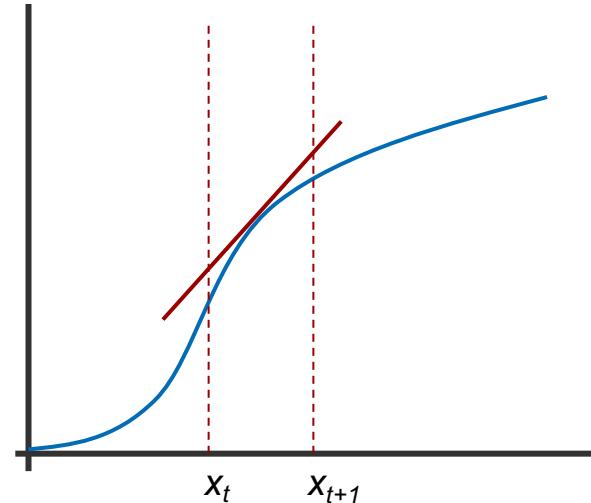


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- A more nuanced answer: in any local region of  $x$ , a local linear approximation will work pretty well
- When will  $\beta_t$  be smooth (“forecastable”)? If  $f$  is **smooth** and  $x_t$  and  $x_{t+1}$  are **close together**!



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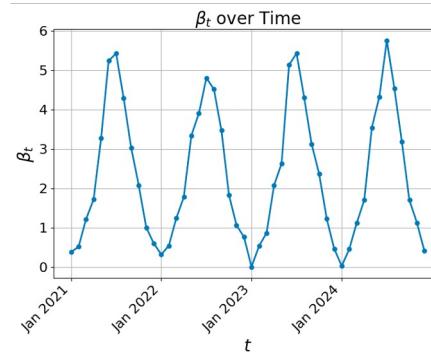
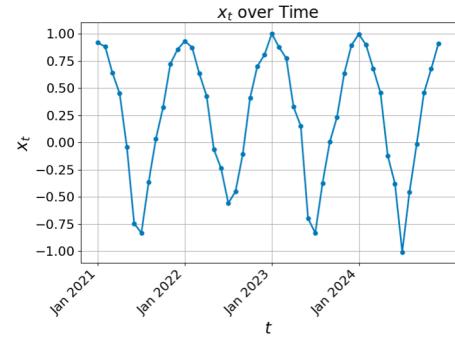


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# Simulations

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Four types of simulations:

1. Flexible nonlinear response

$$y_t = f(x_t) + \varepsilon_t$$

2. Dynamic coefficients, linear (or log-linear) model

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3. Nonlinear response, parametric hill function

$$y_t = \text{Hill}(x_t) + \varepsilon_t$$

4. Dynamic coefficients inherited from common parent

$$y_t = \beta(t)x(t) + \varepsilon_t, (\beta(t), x(t)) \sim \text{Pa}(t)$$

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## Gaussian processes

Two important levers:

- Smoothness
- Amplitude

# Primer: Gaussian Processes

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Amplitude

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Lengthscale

# Primer: Gaussian Processes

**Gaussian processes**: a Bayesian nonparametric approach to modeling unknown functions

$$f(x) \sim \mathcal{GP}(m(x), k(x, x'))$$

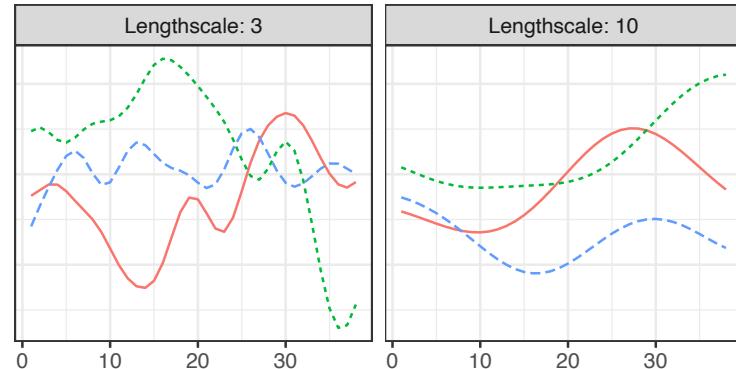


$$f(x_1, \dots, x_N) \sim \mathcal{N}(m(x_1, \dots, x_N), K), \text{ s.t. } K_{ij} = k(x_i, x_j)$$

Amplitude

$$k(x, x') = \eta^2 \exp \left\{ -\frac{(x - x')^2}{2\rho^2} \right\}$$

Lengthscale



# When does conflation *actually* happen?

Four types of simulations:

1. Flexible nonlinear response

$$y_t = \boxed{f(x_t)} + \varepsilon_t$$

2. Dynamic coefficients, linear (or log-linear) model

$$y_t = \boxed{\beta(t)x_t} + \varepsilon_t$$

3. Nonlinear response, parametric hill function

$$y_t = \text{Hill}(x_t) + \varepsilon_t$$

4. Dynamic coefficients inherited from common parent

$$y_t = \beta(t)x(t) + \varepsilon_t, (\beta(t), x(t)) \sim \text{Pa}(t)$$

## Gaussian processes

Two important levers:

- Smoothness
- Amplitude

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## Other manipulated features:

4. Dynamic coefficient
- Autoregressive coefficient in  $x$
  - Noise in  $x$ 's autoregressive process
  - Variance of the error term

## Gaussian processes

Two important levers:

- Smoothness
- Amplitude

# Simulation Results

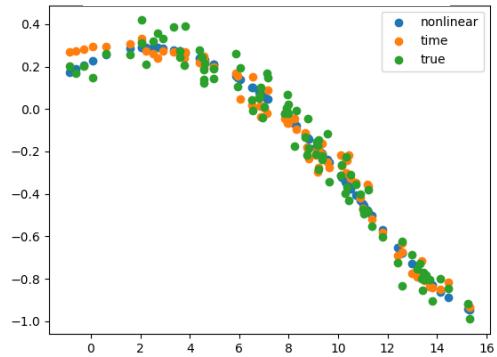
- For each simulation type, >300 settings, systematically varying the previously described factors, with 100 simulations per setting
- Fit both models (nonlinear and dynamic), measure conflation through validation RMSE

# Examples

## Nonlinear DGP

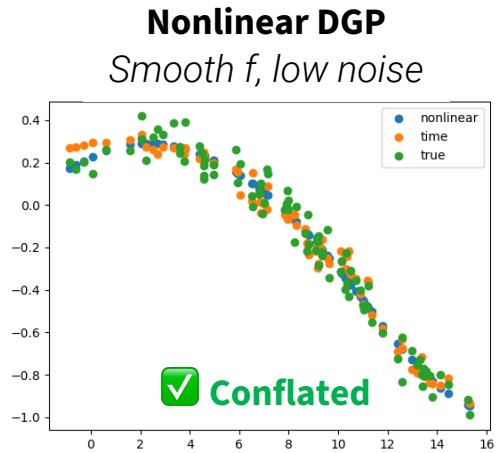
*Smooth  $f$ , low noise*

Data



# Examples

Data

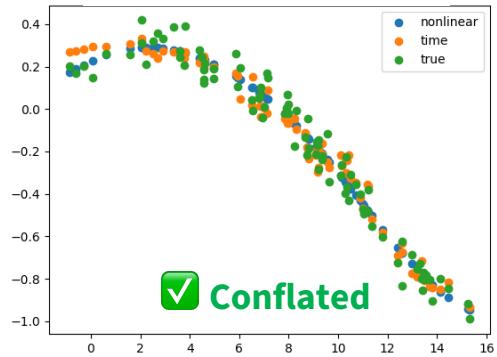


# Examples

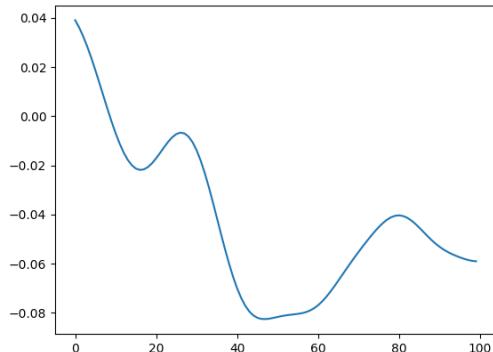
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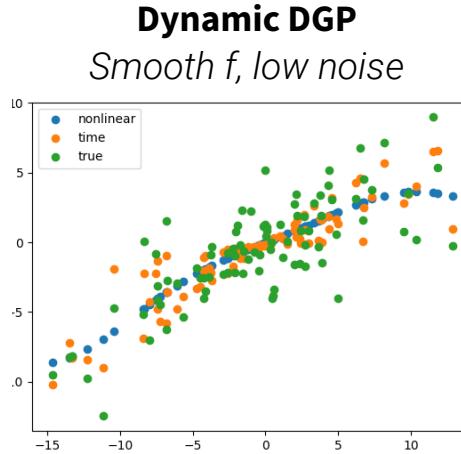
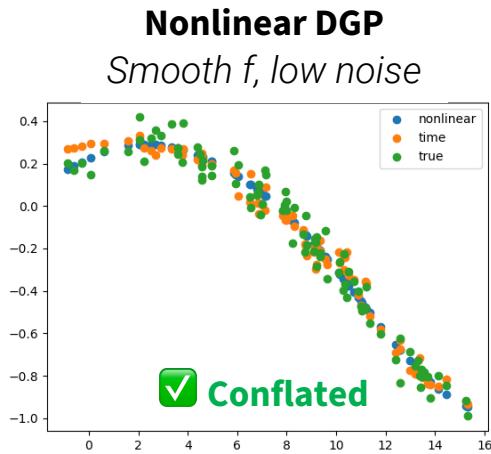


Implied  
 $\beta(t)$

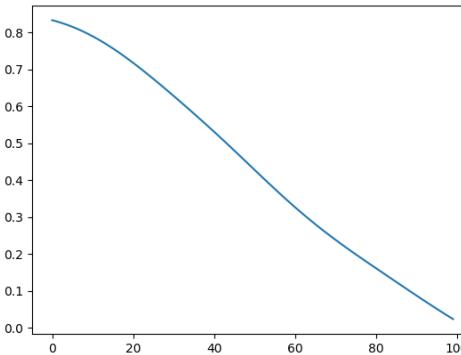
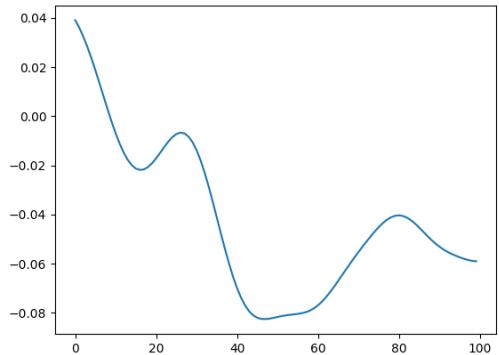


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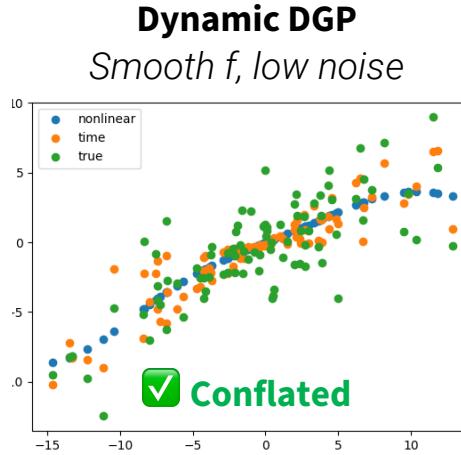
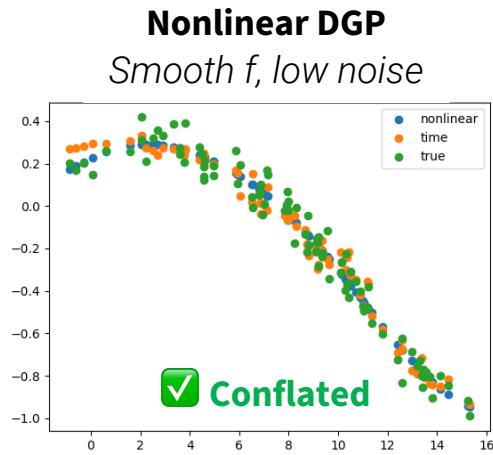


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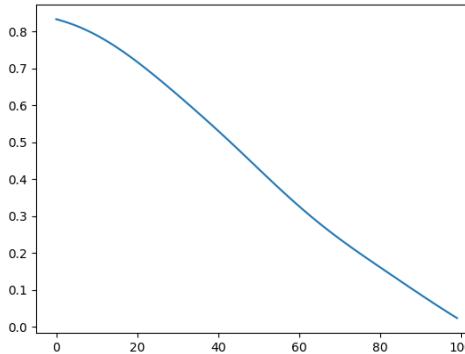
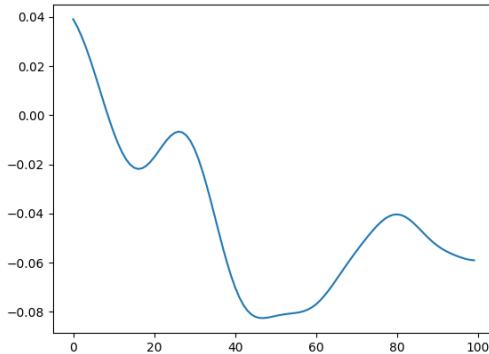


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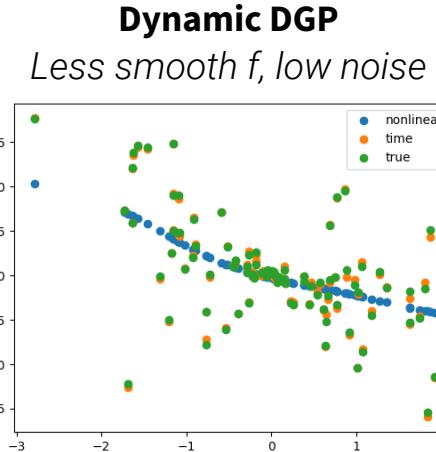
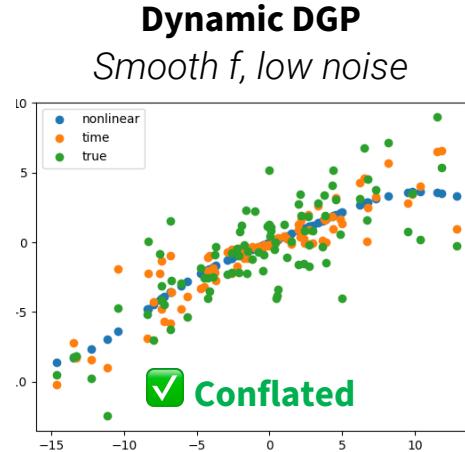
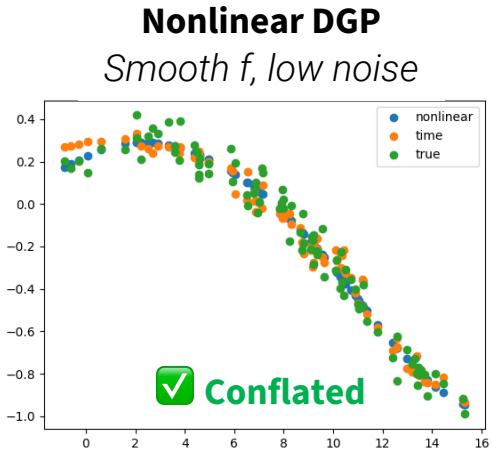


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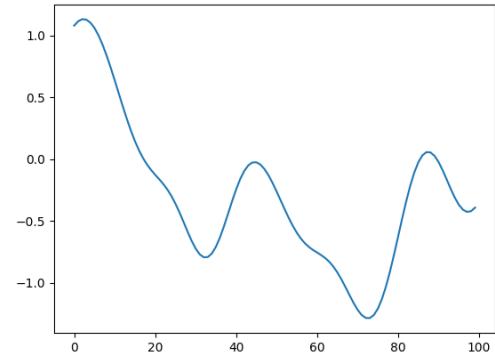
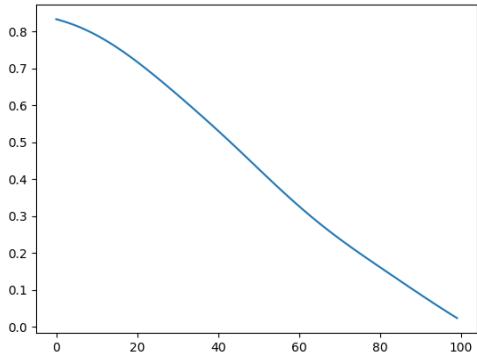
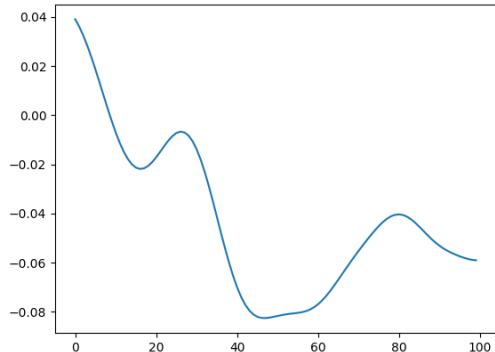


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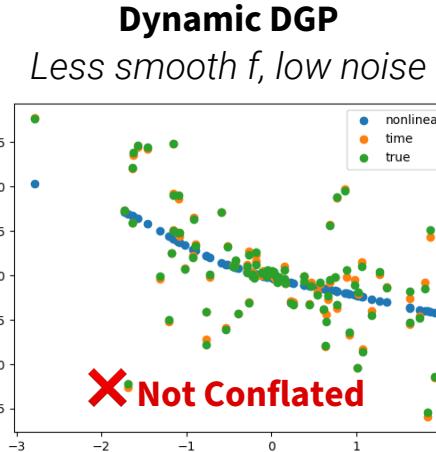
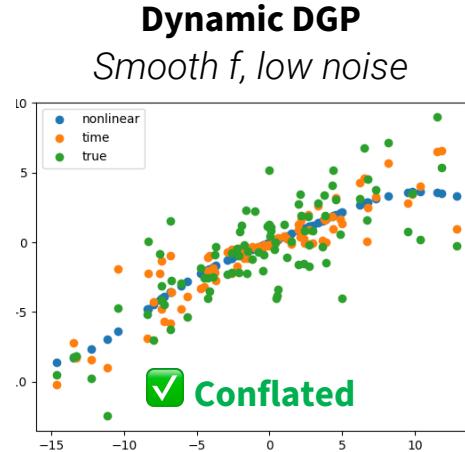
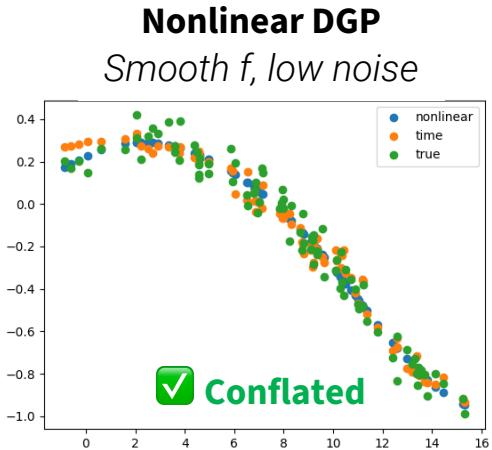


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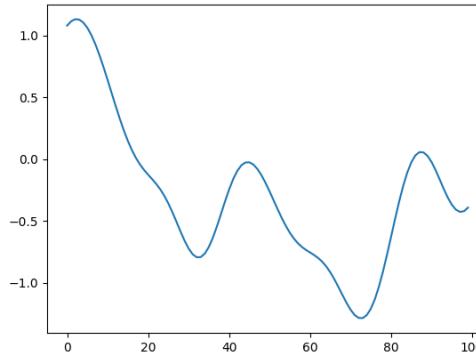
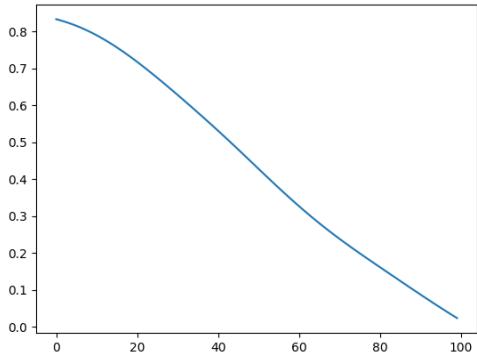
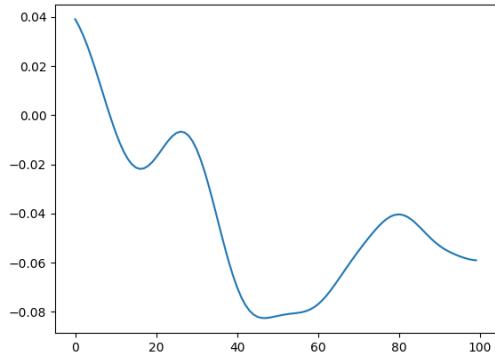


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# Simulation Results

- Systematically vary the previously described factors, with 100 simulations per setting
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## Main result: huge prevalence of conflation

- Under the nonlinear DGP, **82%** exhibited some conflation, with **23% exhibiting major conflation** (*defined as >25% of simulations conflated*)
- Under the time-varying DGP, **80%** exhibited some conflation, with **27% exhibiting major conflation**

# Diving Deeper

**DV:** % Conflated Simulations

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Variable	Level	Nonlinear DGP		Dynamic DGP	
		Coef	$P(> t )$	Coef	$P(> t )$
Amplitude, $f$ :	Low	-	-	-	-
	Middle	-0.06	0.94	2.94	0.01
	High	-0.11	0.88	8.99	0.00
Smoothness, $f$ :	Low	-	-	-	-
	Middle	9.99	0.00	3.44	0.00
	High	19.43	0.00	8.10	0.00
AR coef, $x$ :	Low	-	-	-	-
	Middle	0.81	0.28	1.05	0.36
	High	2.66	0.00	4.77	0.00
Variance, $x$ :	Low	-	-	-	-
	Middle	-0.16	0.83	5.80	0.00
	High	0.07	0.92	11.66	0.00
Noise, $y$	Low	-	-	-	-
	Middle	7.60	0.00	6.69	0.00
	High	14.81	0.00	20.25	0.00
	Very High	24.02	0.00	40.19	0.00

**Table 1:** Simulation Results: DV = Percentage Conflation; Intercept omitted for clarity.

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*Decisions are often autocorrelated!*

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## Even worse with AdStock

Under the nonlinear DGP with AdStock, **93%** exhibited some conflation,  
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(previously: 82% and 23%)

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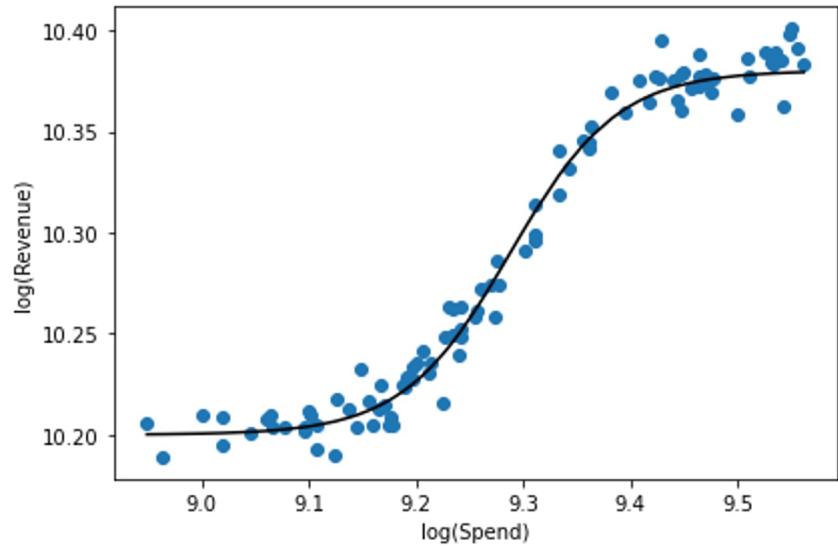
(previously: 82% and 23%)

Under the time-varying DGP with AdStock, **87%** exhibited some conflation,  
with **52% exhibiting major conflation**

(previously: 80% and 27%)

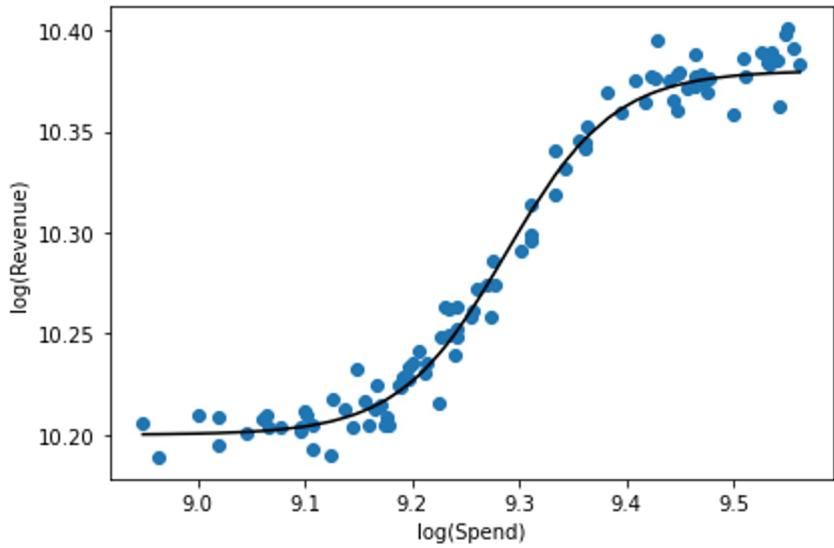
# Implications

# One last simulation...

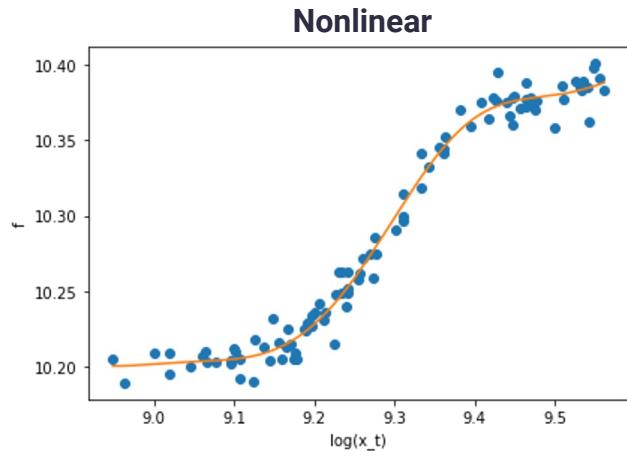


(or from roughly \$8,000 to \$14,000)

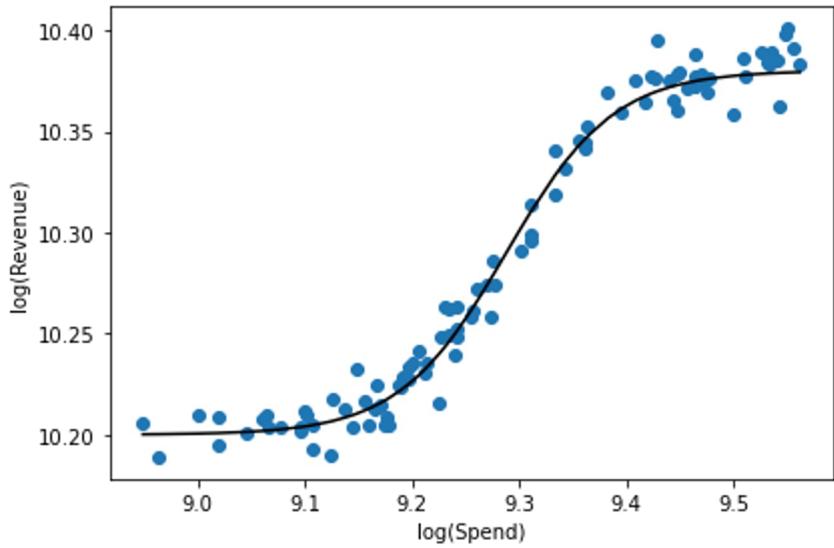
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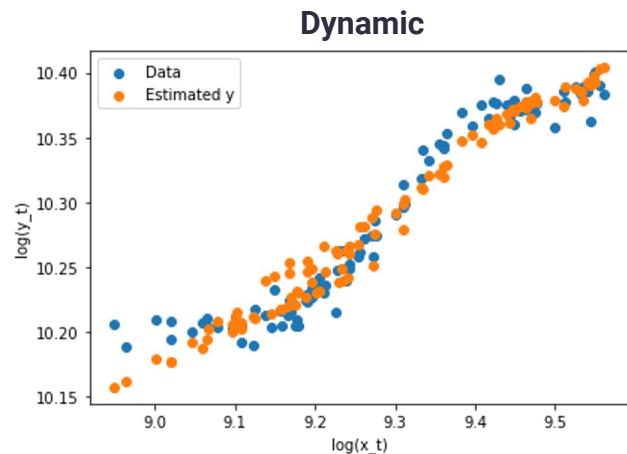
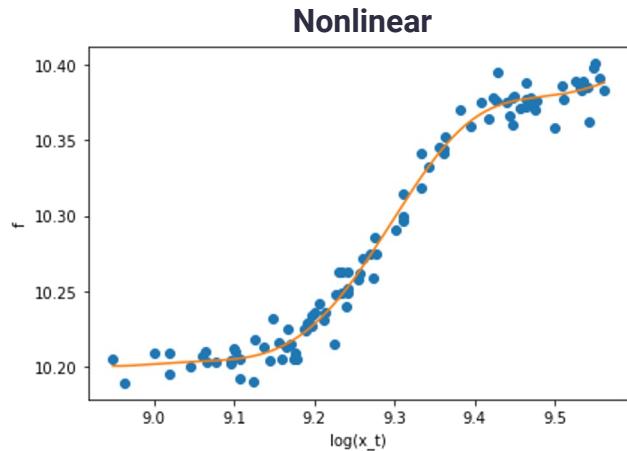
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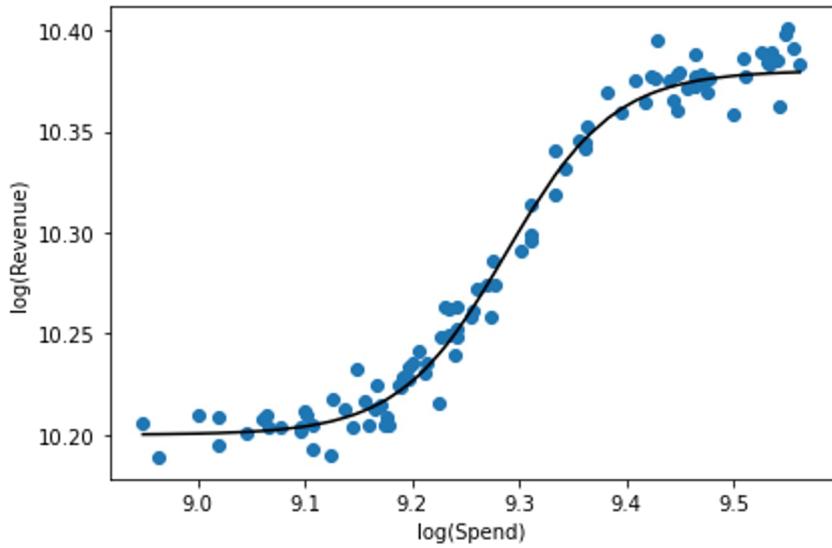
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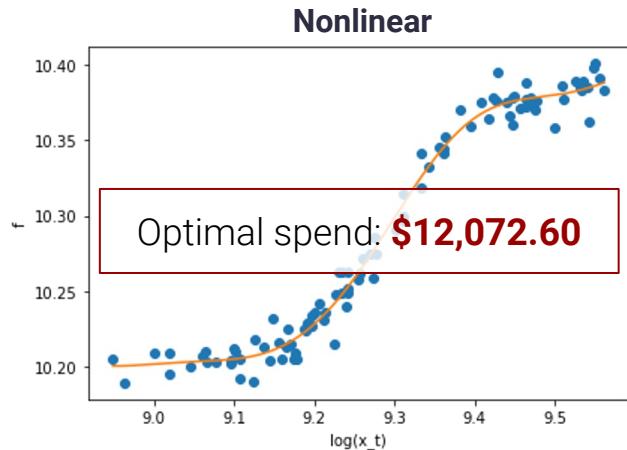
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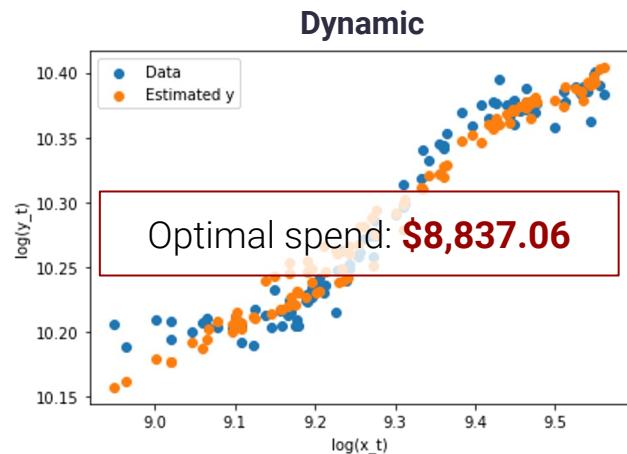
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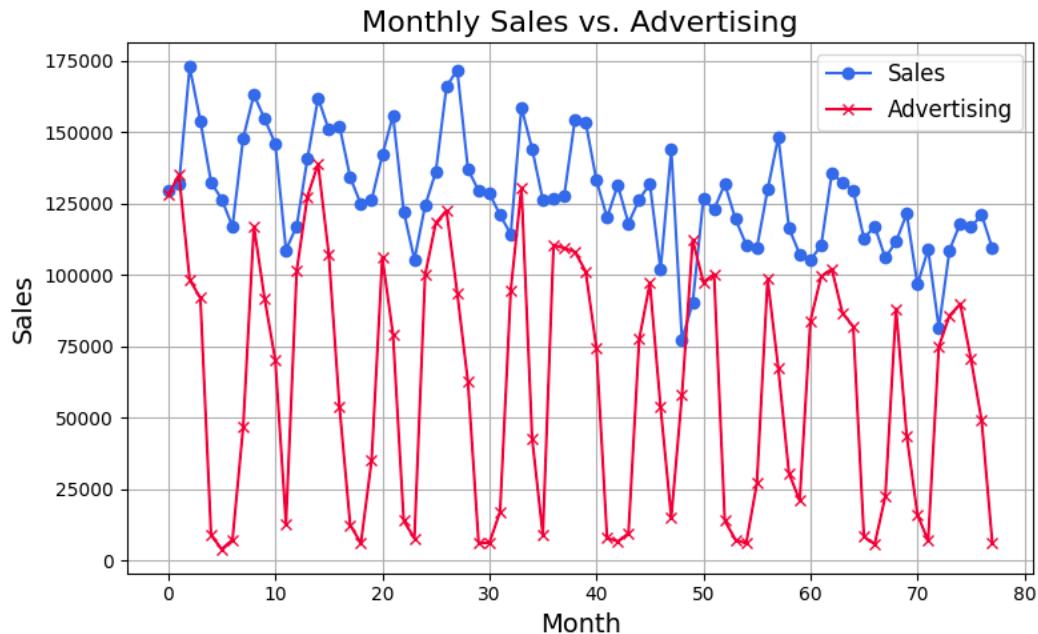
Optimal spend: **\$12,072.60**



Optimal spend: **\$8,837.06**

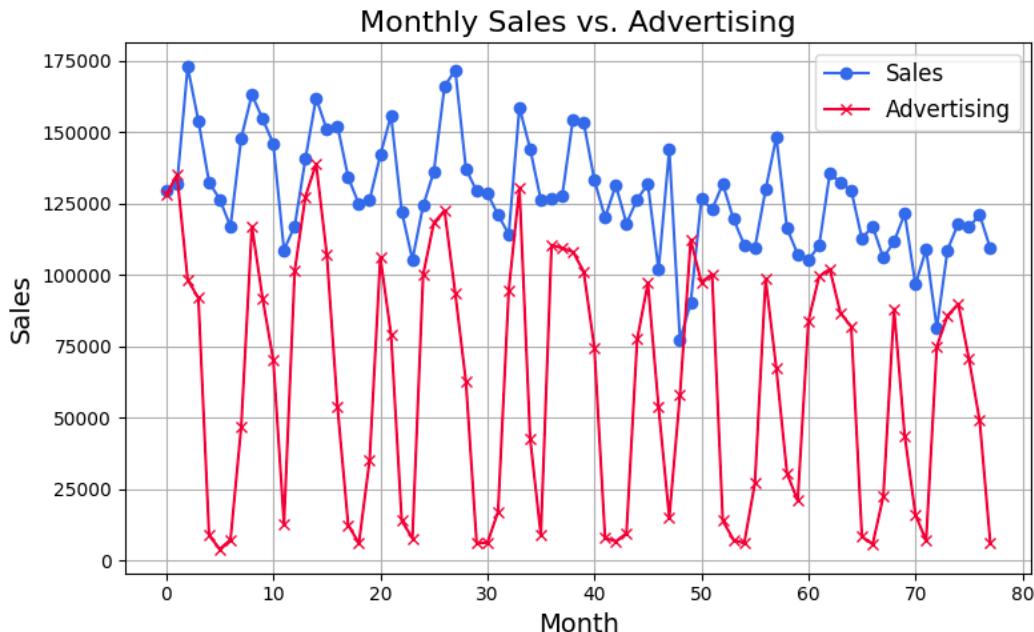
# Application: The Classic Lydia Pinkham Dataset

Sales and advertising of Lydia Pinkham's herbal products, monthly, 1954-1960 ([Palda, 1964](#))



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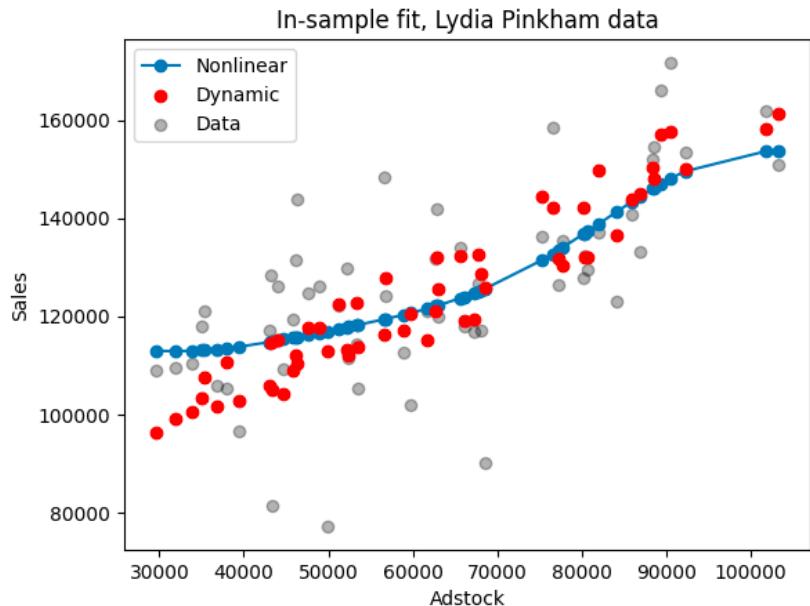
**Lydia Pinkham**  
150 Tablets  
★★★★★ 764 reviews  
50+ bought in past month  
\$21.99 (\$0.15/Count)  
✓prime Two-Day  
FREE delivery Thu, May 9  
Add to cart

# Conflation

Fit both nonlinear and dynamic models, using AdStock with carryover  
 $\alpha = 0.7$  and  $L = 13$  ([Bultez and Naert, 1979](#))

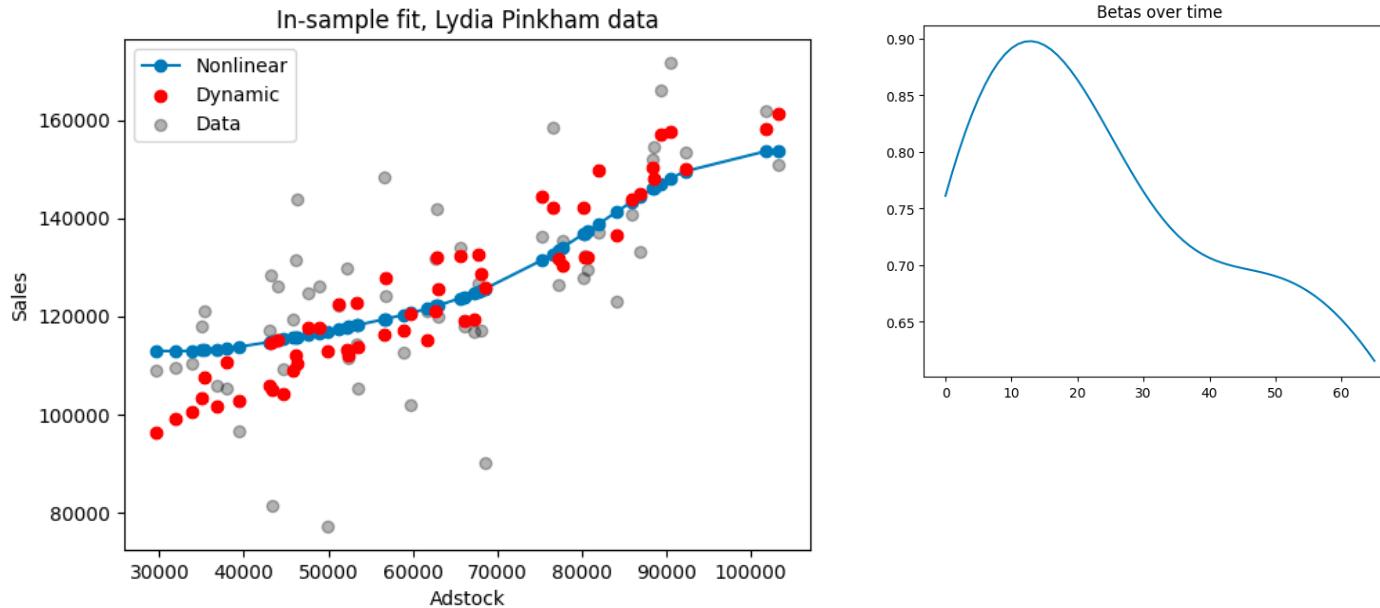
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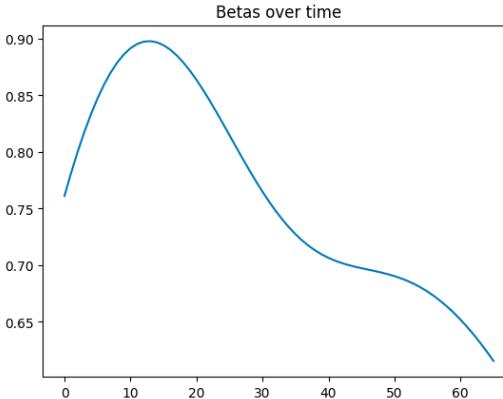
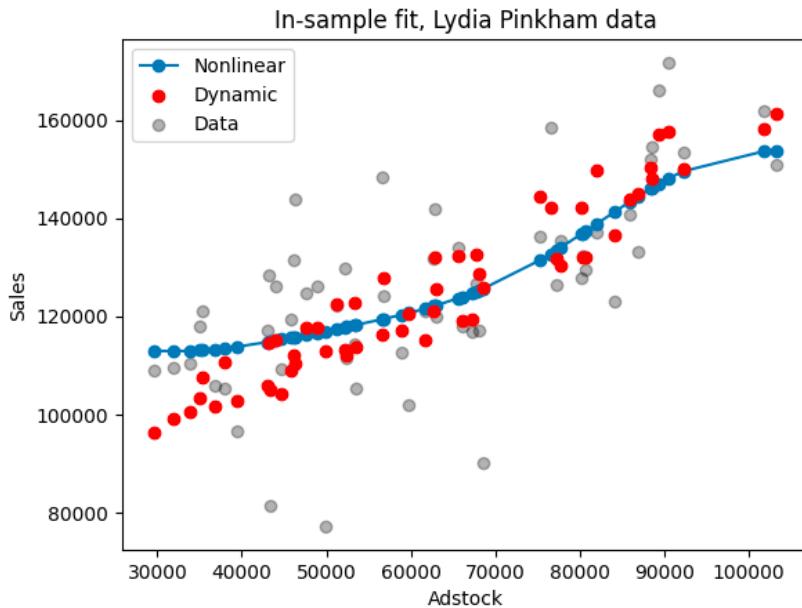
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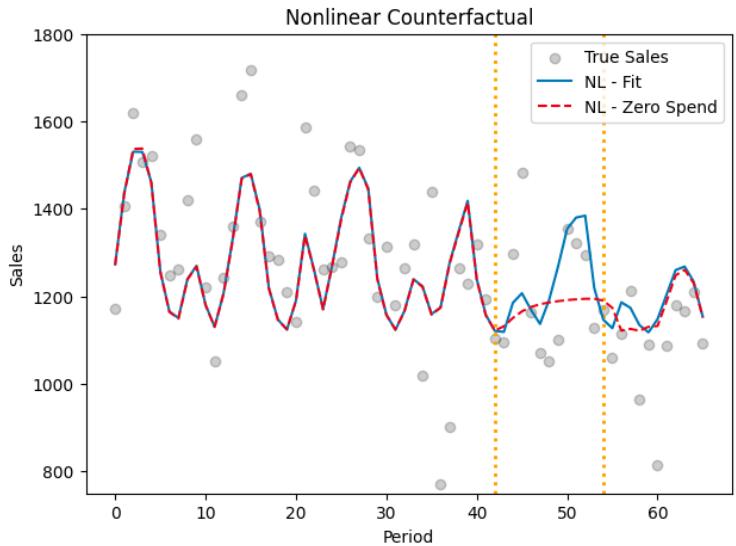
95% posterior predictive MSE intervals:  
**Nonlinear:** [126.80, 174.78]  
**Dynamic:** [ 93.31, 127.18]

# Returns from Advertising

$$\text{Return}[T_1, T_2] = \sum_{t>T} \hat{y}_t(x_{[T_1, T_2]} = x_{\text{True}}) - \hat{y}_t(x_{[T_1, T_2]} = 0)$$

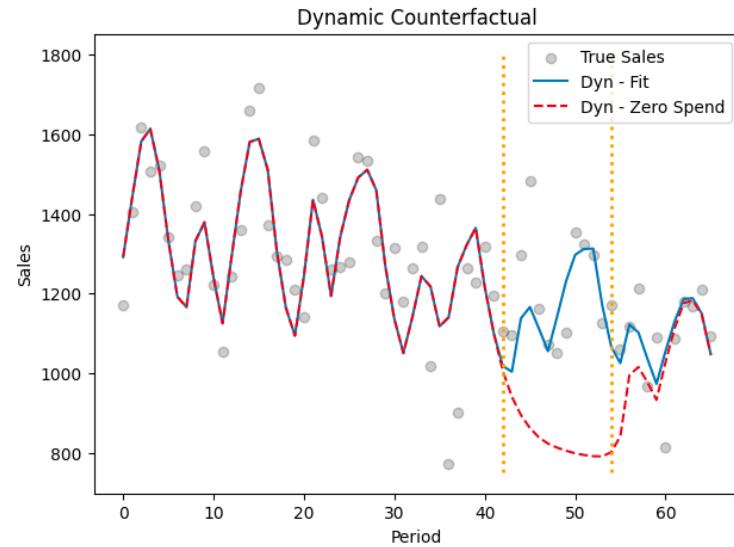
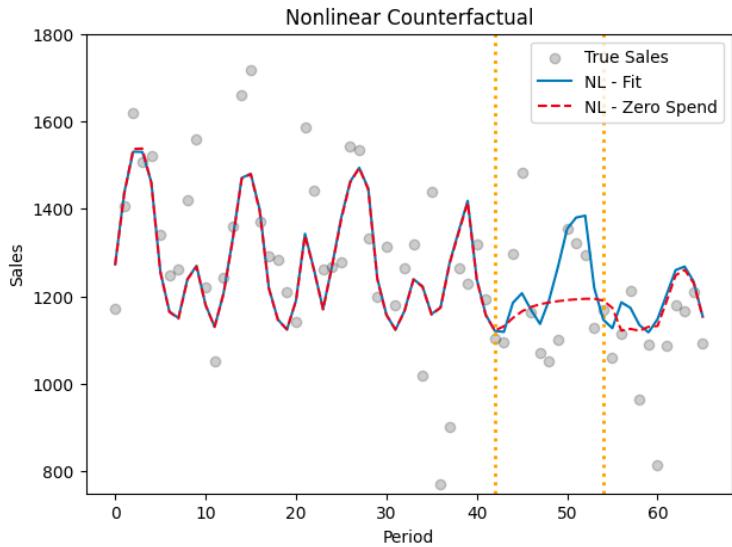
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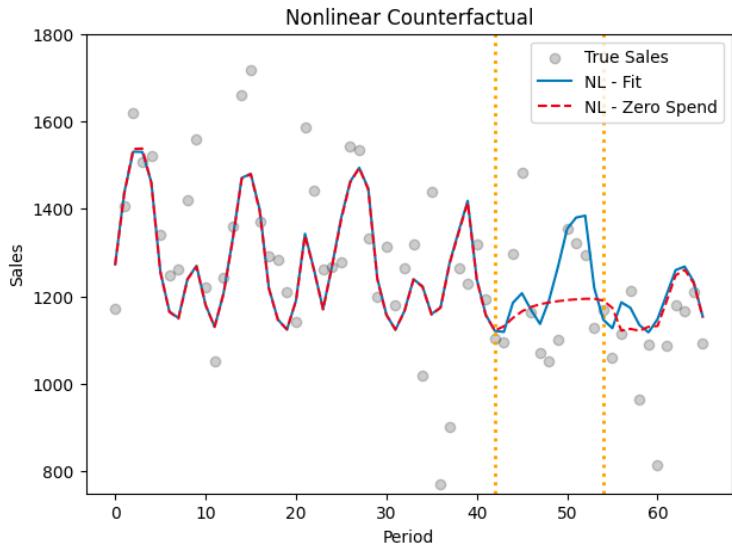
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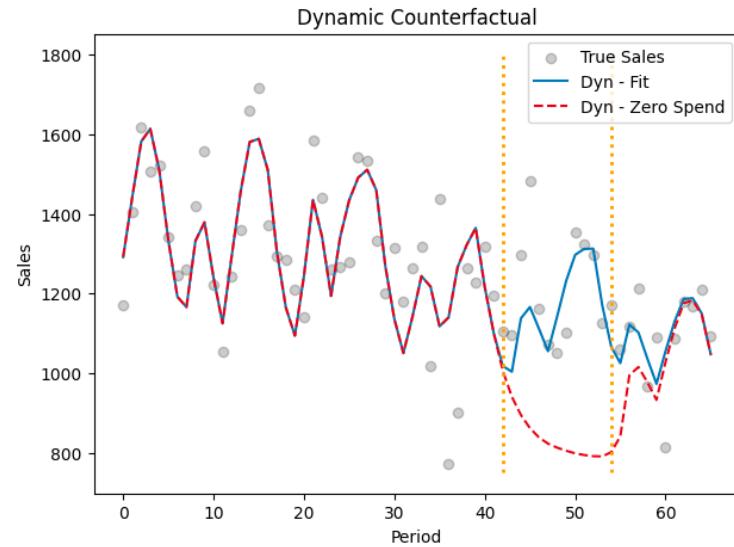


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Estimated revenue: **\$66,427**



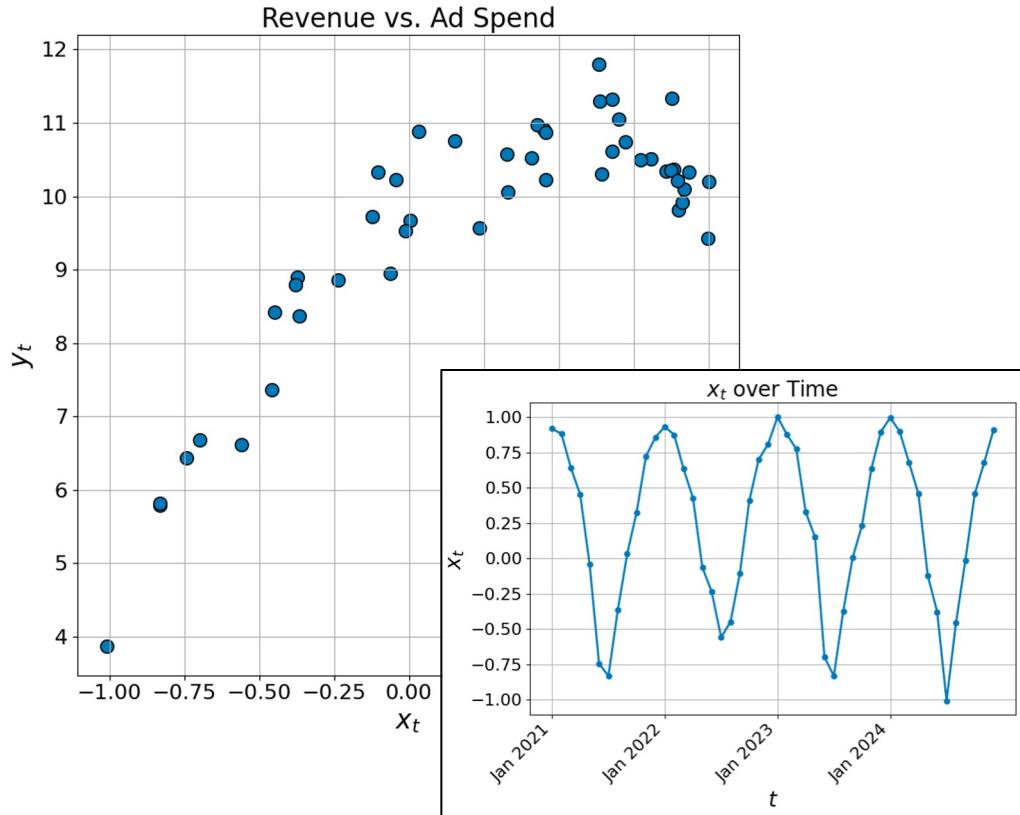
Estimated revenue: **\$468,832**

# Is MMM doomed?

- In short: **no!**
- Careful ad planning can help disentangle the two stories

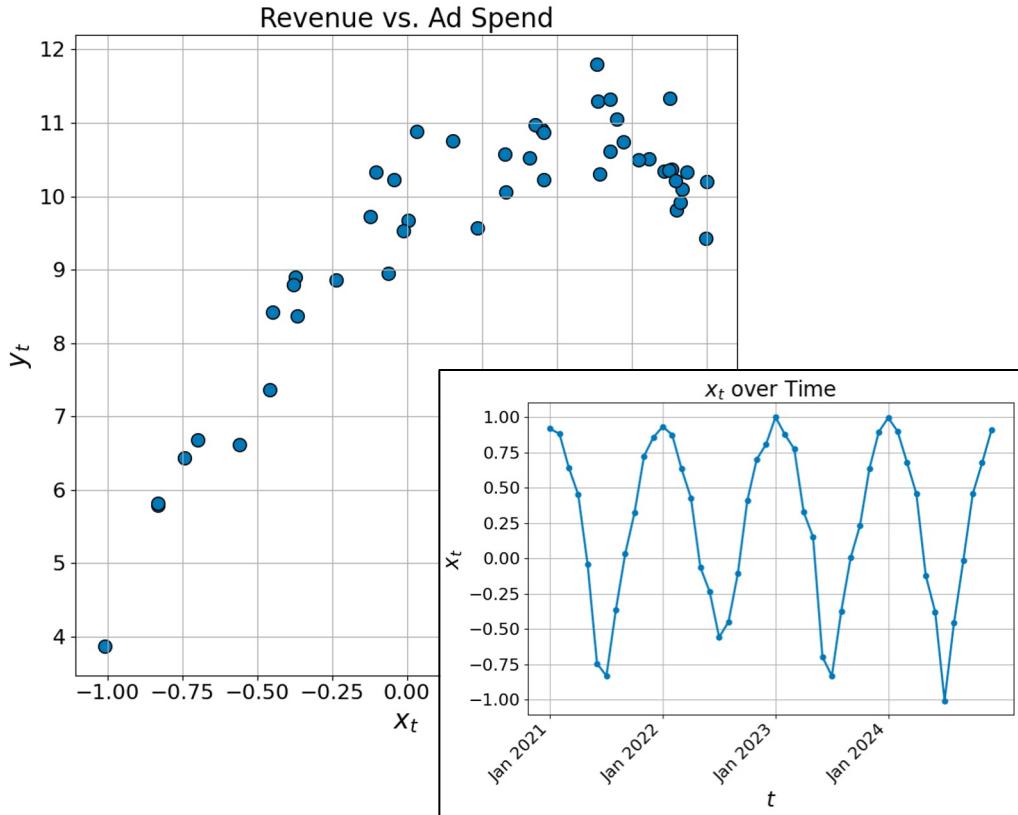
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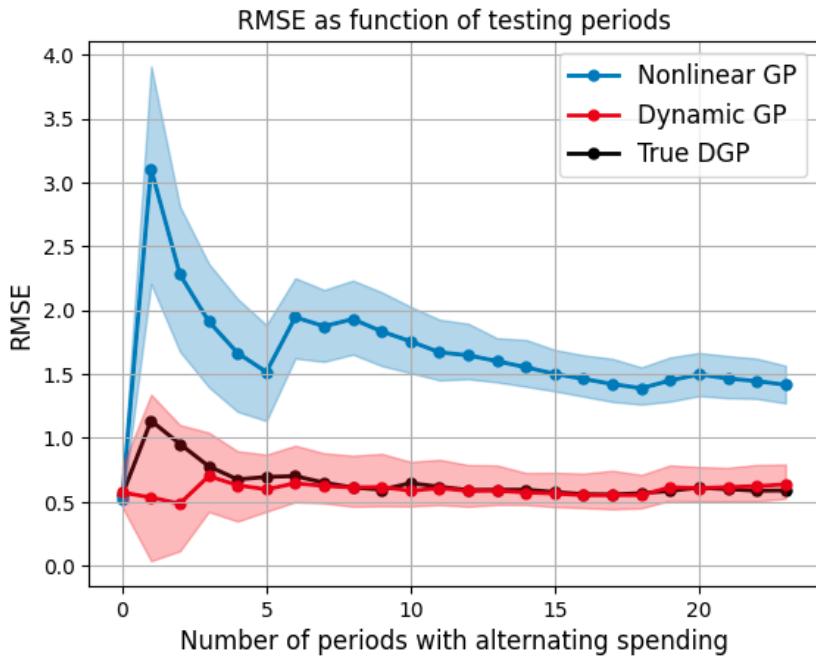
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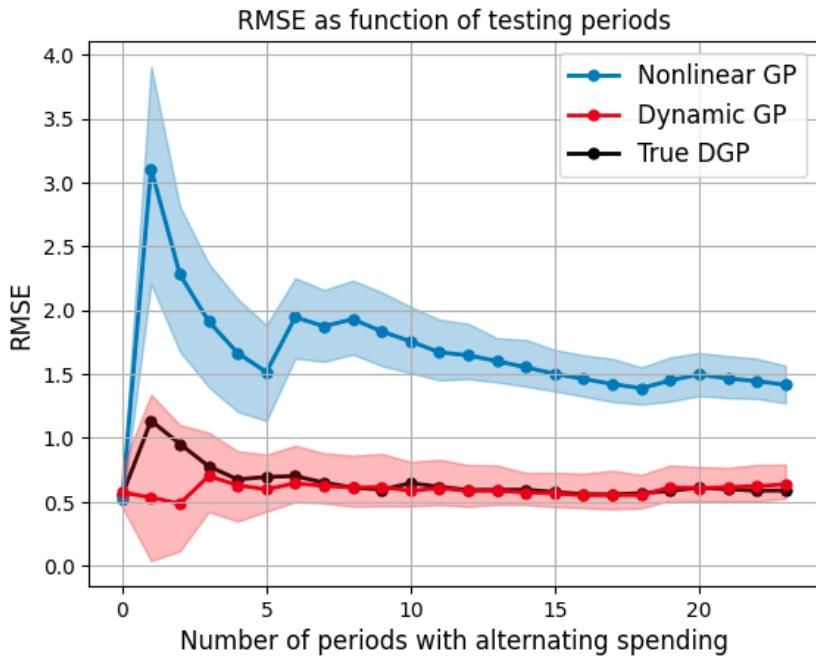
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**Ongoing work:** optimal test design for model inference



# Conclusions

# Current MMM Practice Might Be Flawed!

- We show that, under many common spending patterns, **time-varying and nonlinear effects cannot be disentangled**, despite having different implications
- This problem is potentially **very widespread**: increasing complexity in models, widespread practice of “model refreshes” to capture changing markets
- Our work both introduces a **framework for estimating** these types of models, and provides **solutions for understanding and preventing** conflation

# Thanks!

Feedback or questions:  
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Working paper soon:  
[www.rtdew.com](http://www.rtdew.com)



You

generate an image of a marketing mix model paradise with some very happy computers and a cameo from the reverend thomas bayes

