Your MMM Is Broken:
Identification of Nonlinear and Time-varying Effects in Marketing Mix Models

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The Wharton School
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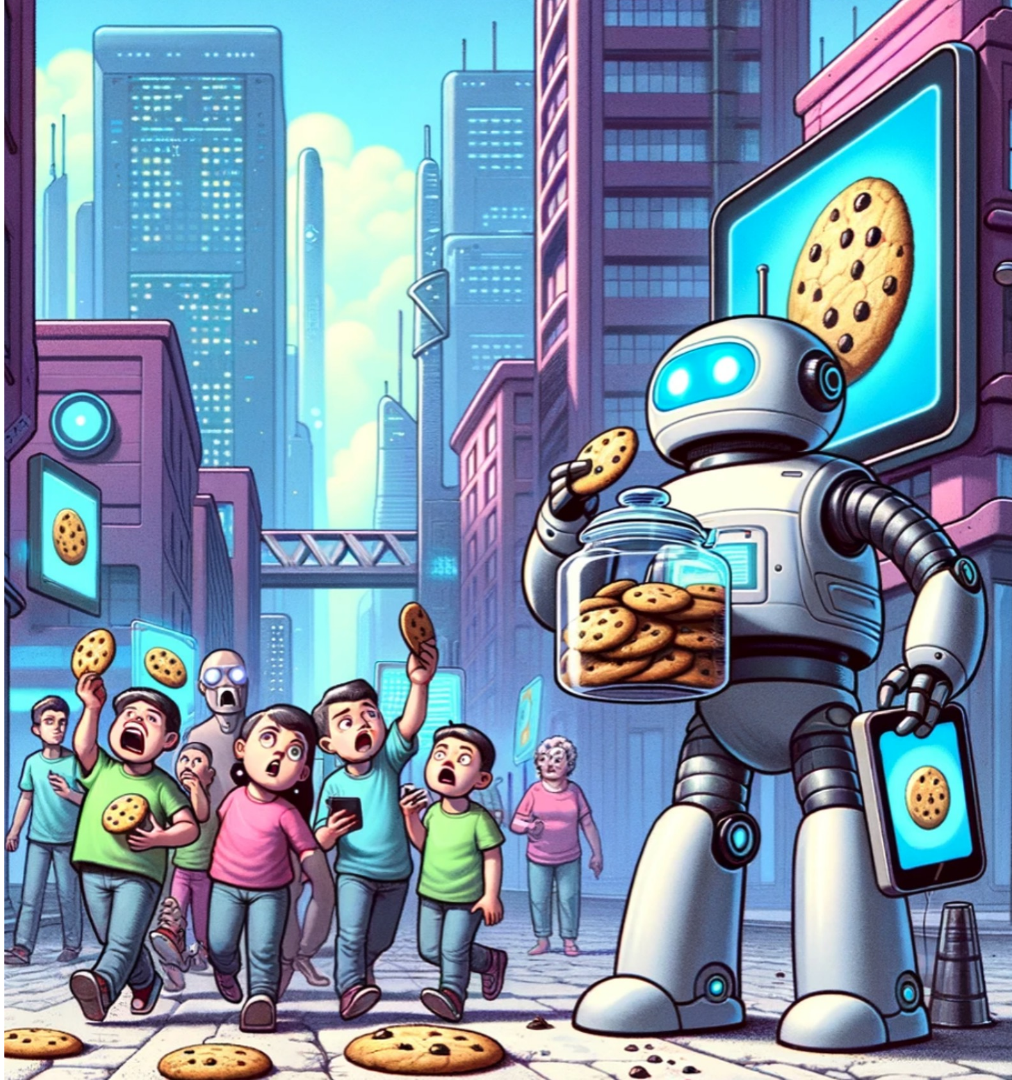
Joint work with Nicolas Padilla (LBS) and Anya Shchetkina (Wharton)

The Cookie-free Future

- For the past 10+ years, individual-level trackability has been central to measuring marketing efficacy



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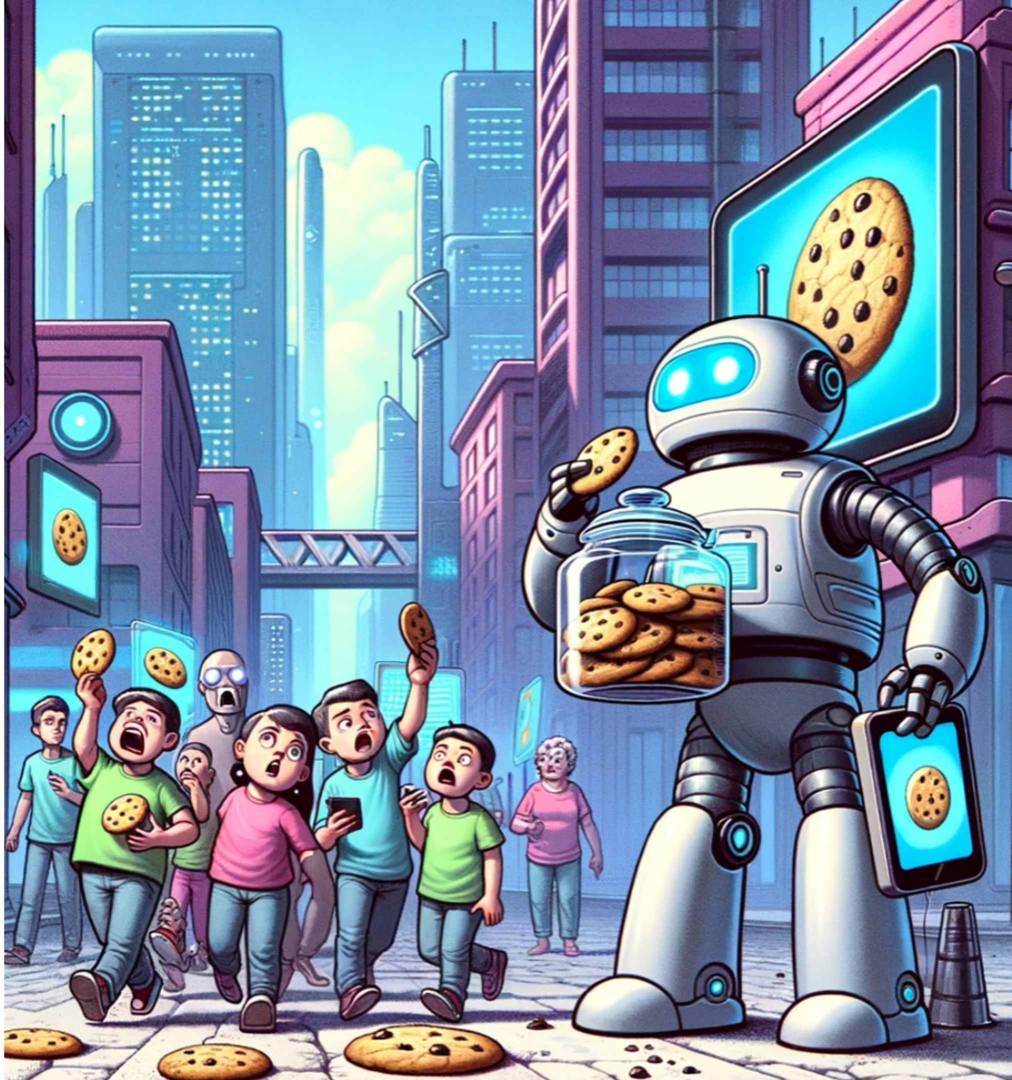

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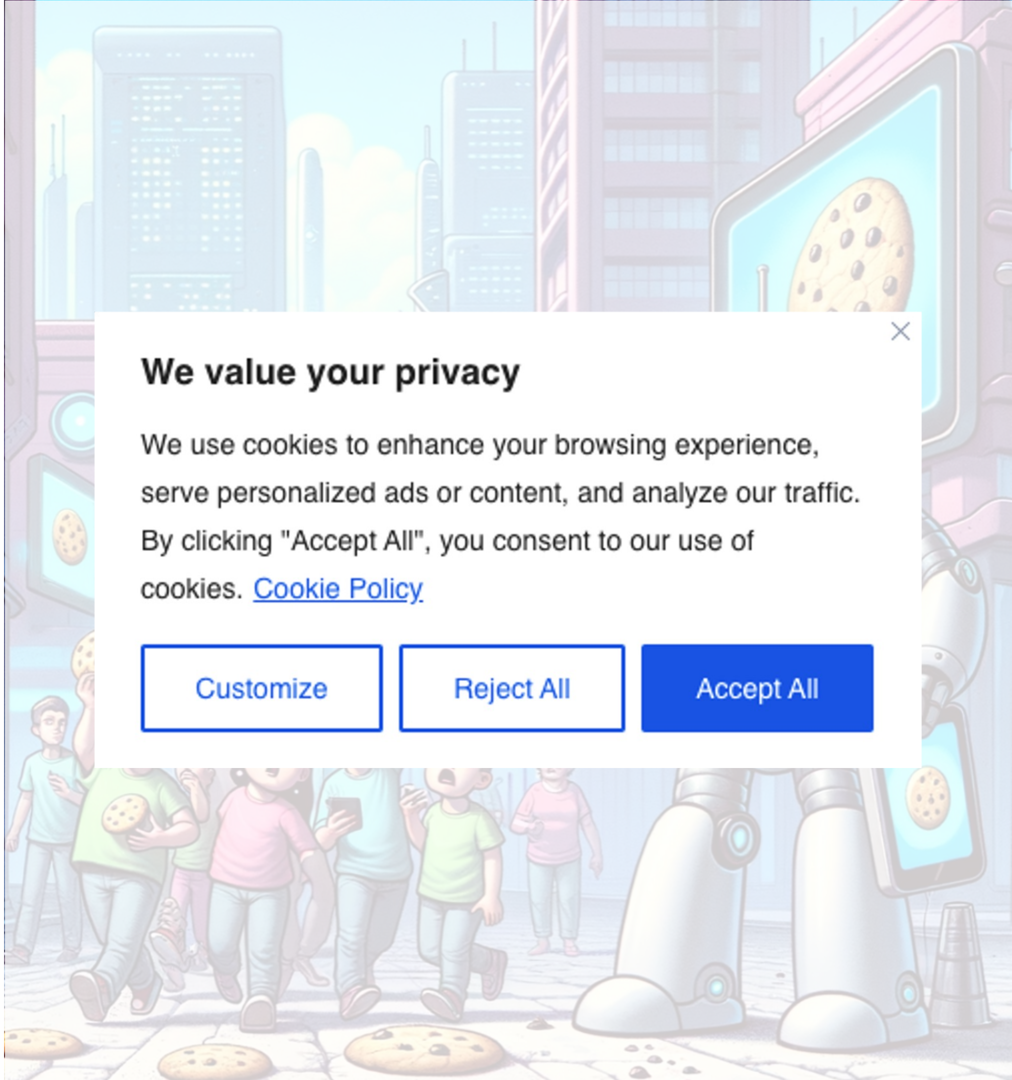

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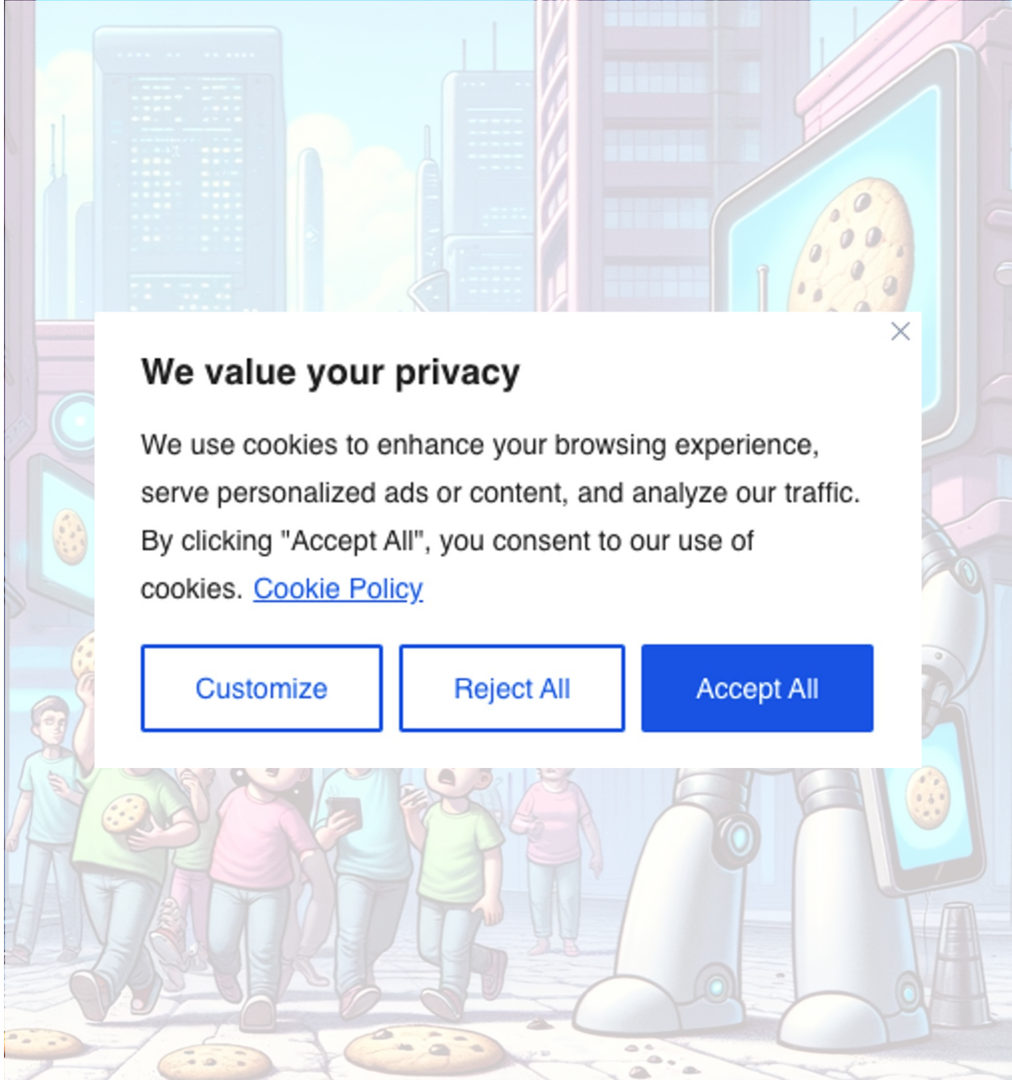
- Result: huge resurgence in interest in models based on **aggregate data**




THE
RETURN
OF THE
MARKETING MIX MODEL
PRODUCT
PLACE
PLACE
PROMOTION

[Submitted on 7 Jun 2021 (v1), last revised 5 Sep 2021 (this version, v3)]

Bayesian Time Varying Coefficient Model with Applications to Marketing Mix Modeling

Edwin Ng, Zhishi Wang, Athena Dai

Both Bayesian and varying coefficient models are very useful tools in practice as they can be used to model parameter heterogeneity in a generalizable way. Motivated by the need of enhancing Marketing Mix Modeling at Uber, we propose a Bayesian Time Varying Coefficient model, equipped with a hierarchical Bayesian structure. This model is different from other time varying coefficient models in the sense that the coefficients are weighted over a set of local latent variables following certain probabilistic distributions. Stochastic Variational Inference is used to approximate the posteriors of latent variables and dynamic coefficients. The proposed model also helps address many challenges faced by traditional MMM approaches. We used simulations as well as real world marketing datasets to demonstrate our model superior performance in terms of both accuracy and interpretability.

arXiv > stat > arXiv:2106.03322

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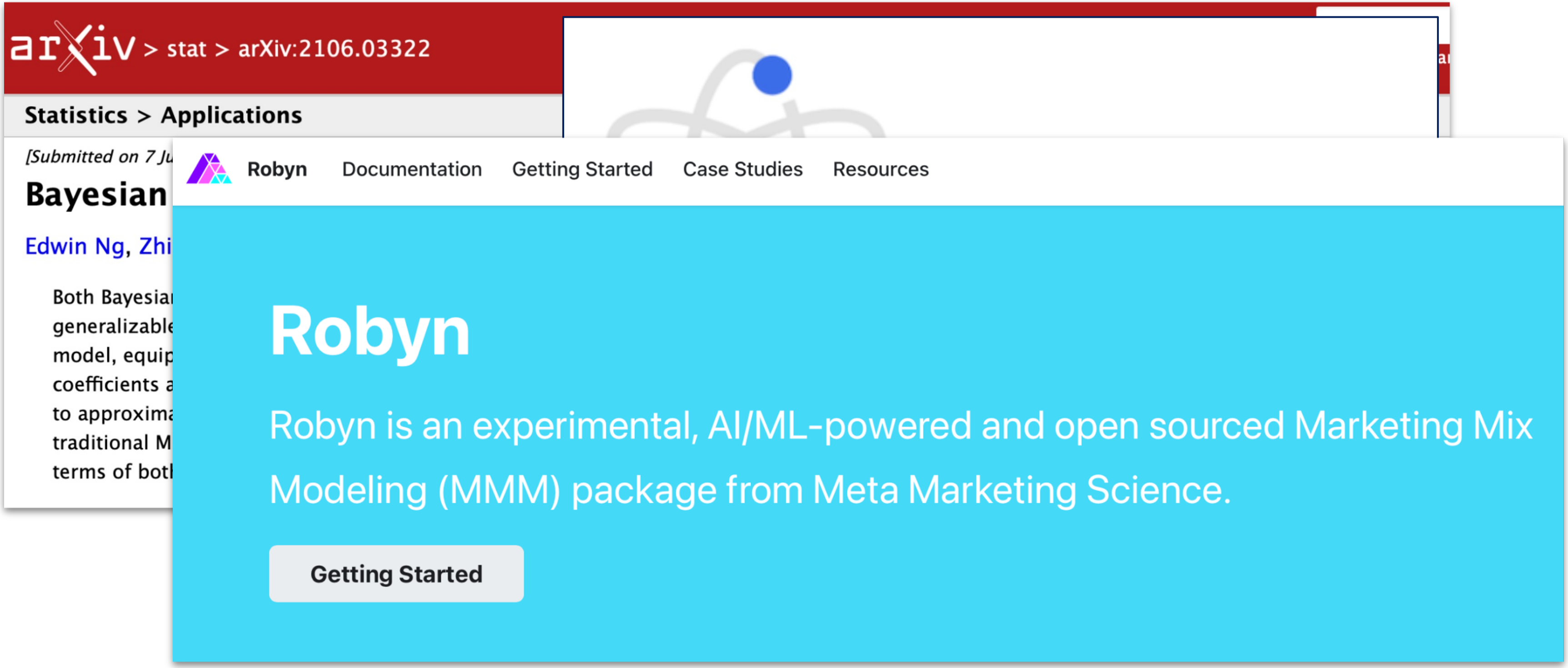

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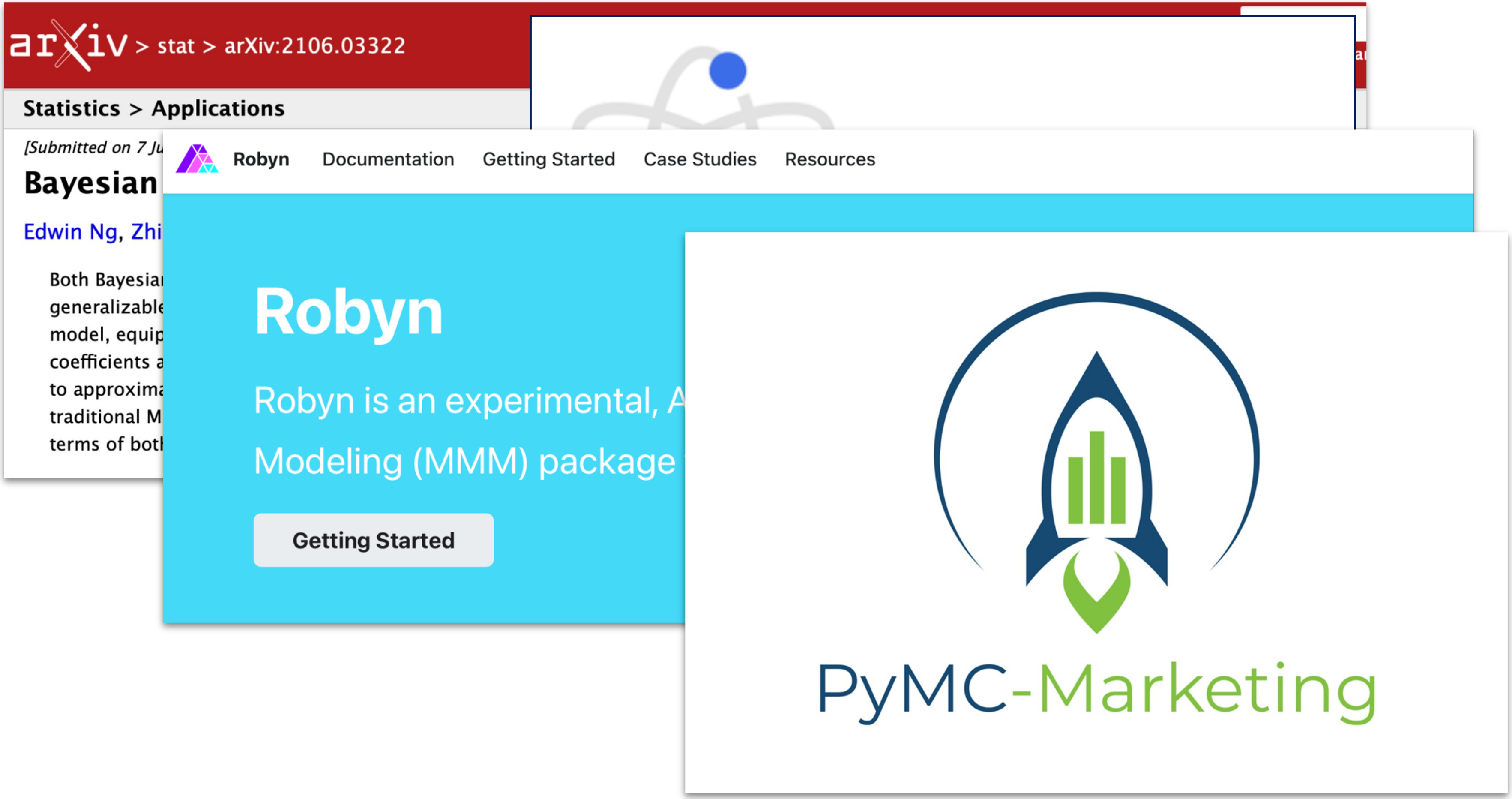
Getting Started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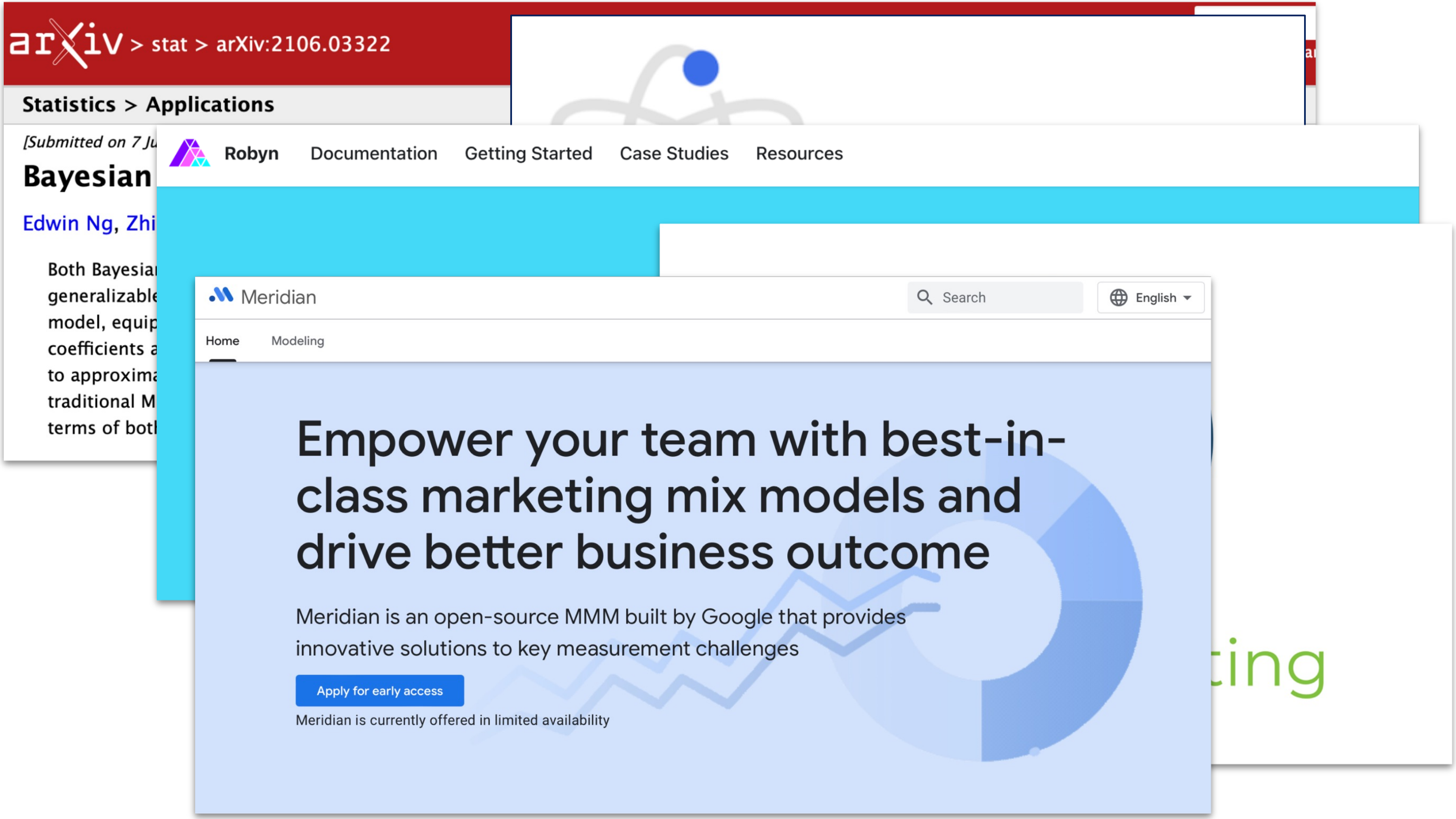

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Meridian
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Modeling
Empower your team with best-in-class marketing mix models and drive better business outcome
Meridian is an open-source MMM built by Google that provides innovative solutions to key measurement challenges
Apply for early access
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Note: Terminology

Throughout the talk, dynamic = time-varying

What’s new? Firepower

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Meridian

Wang et al. (2017), Jin et al. (2017), Sun et al. (2017), Zhang et al. (2023)

$$
\begin{aligned}
y_{g,t} = \mu_t + \tau_g + \sum_{c=1}^{C} \gamma_{g,c} z_{g,t,c} \\
+ \sum_{m=1}^{M} \beta_{g,m} HillAdstock\left(\left\{x_{g,t-s,m}\right\}_{s=0}^{L}\ ;\ \alpha_m, ec_m, slope_m\right) \\
+ \sum_{n=1}^{N} \beta_{g,n}^{(rf)} Adstock\left(\left\{r_{g,t-s,n} \cdot Hill\left(f_{g,t-s,n};\ ec_n^{(rf)},\ slope_n^{(rf)}\right)\right\}_{s=0}^{L}\ ;\ \alpha_n^{(rf)}\right) \\
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\end{aligned}
$$

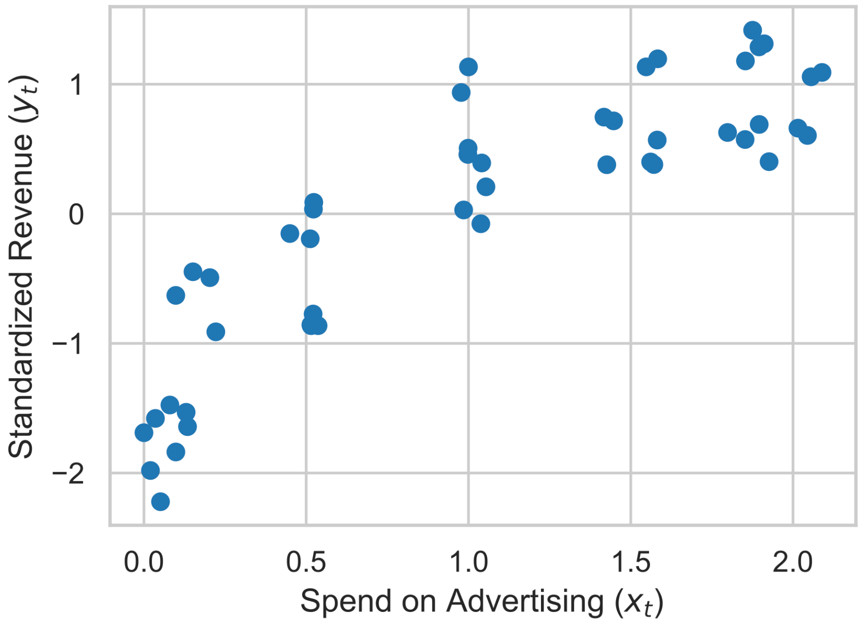
Uber's Orbit

Ng et al., (2021)

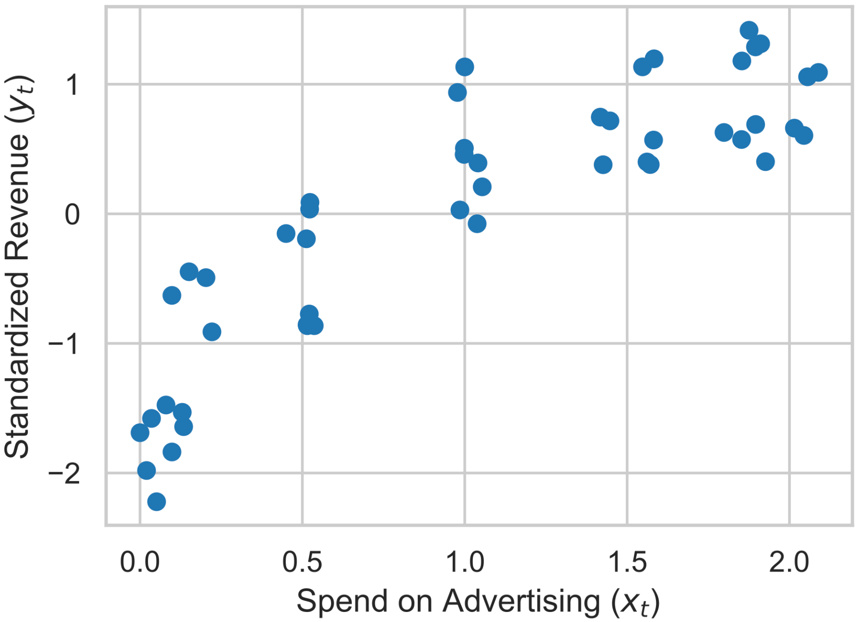
$$ln(\hat{y}_t) = l_t + s_t + \sum_{p=1}^{P} ln(x_{t,p})\beta_{t,p}$$

$$\beta_{t,p} = \sum_j w_j(t) \cdot b_{j,p},$$

$$w_j(t) = k(t, t_j) / \sum_{i=1}^{J} k(t, t_i),$$



1
0
−1
−2
0.0
0.5
1.0
1.5
2.0
Standardized Revenue (y_t)
Spend on Advertising (x_t)



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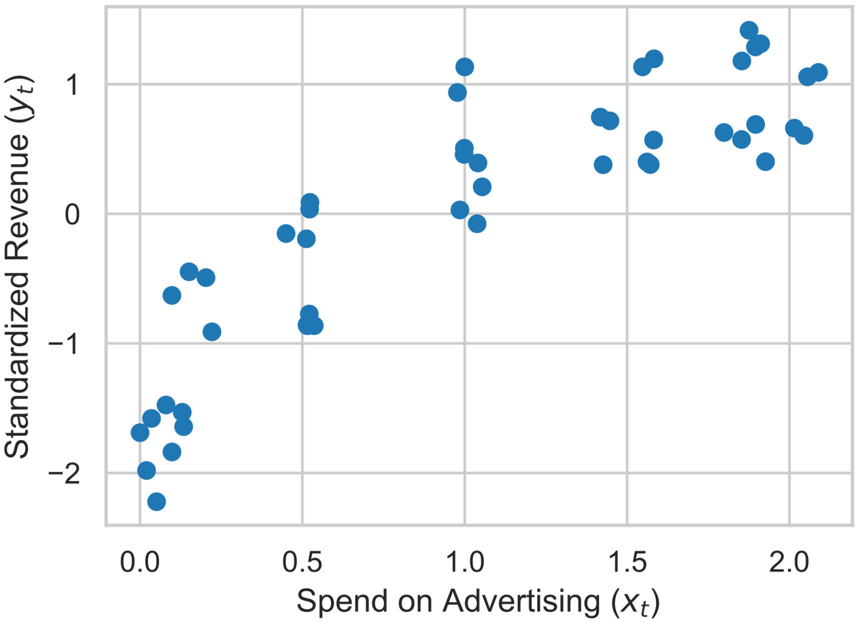

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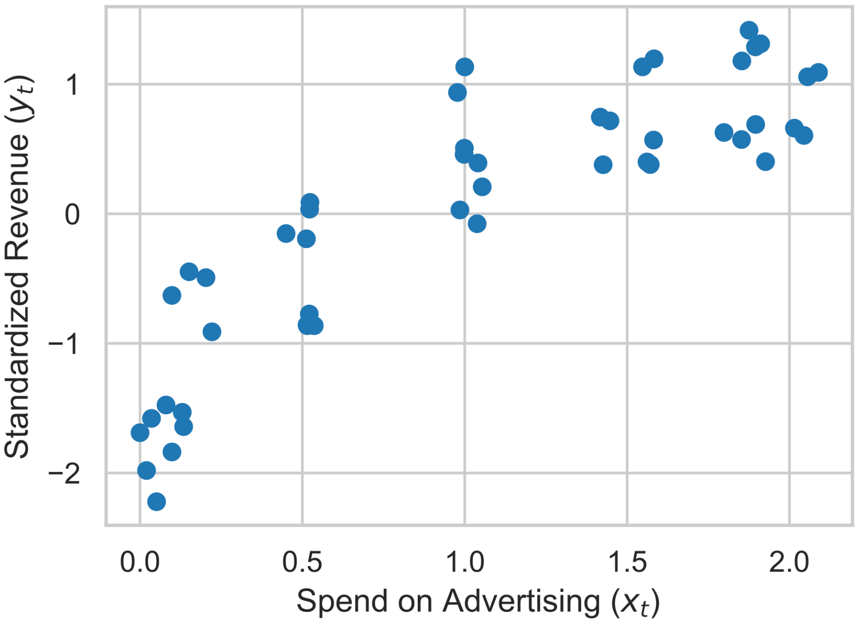

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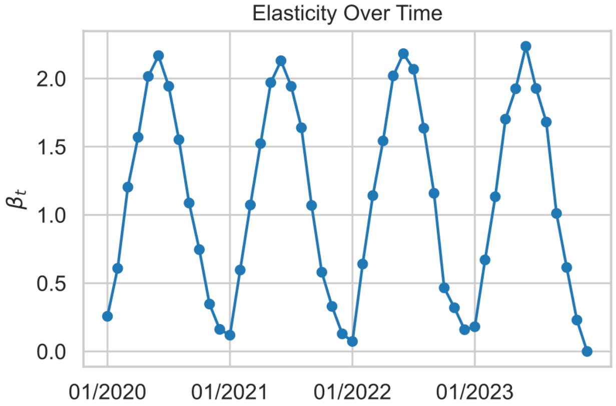

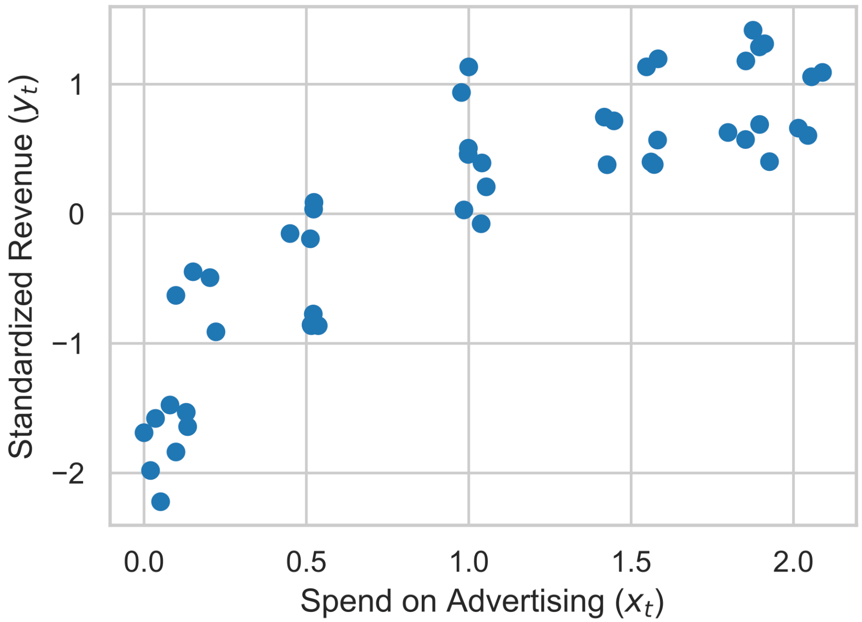

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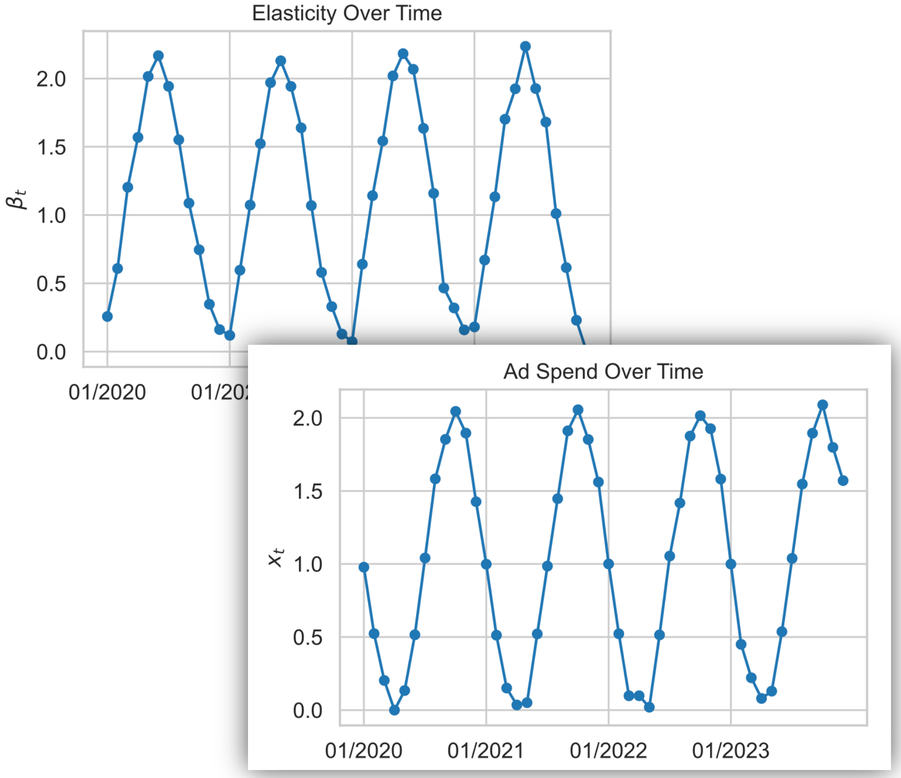

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3. Problems are exacerbated under common managerial practices, like **autoregressive decision-making**
4. Similarly fitting models can have **fundamentally different implications** in terms of optimal decision-making

(A little) Math

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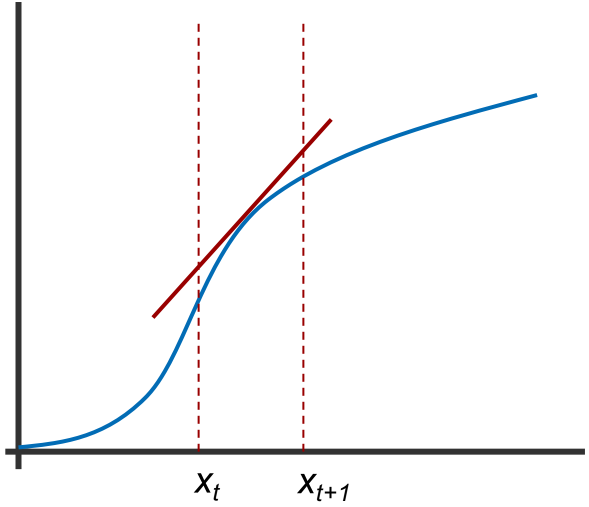

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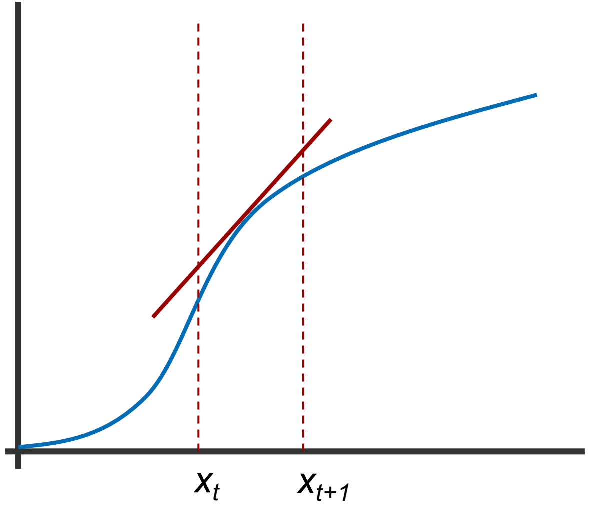
- When will β_t be smooth ("forecastable")? If *f* is smooth and x_t and x_{t+1} are close together!



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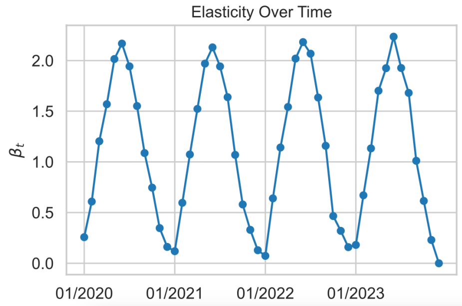

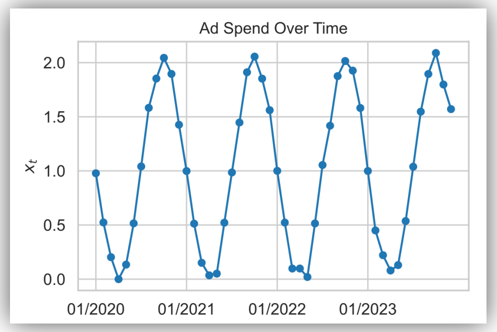

Simulations

When does conflation *actually* happen?

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Four types of simulations:

1. Flexible nonlinear response

$$y_t = f(x_t) + \varepsilon_t$$

2. Dynamic coefficients, linear (or log-linear) model

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3. Nonlinear response, parametric hill function

$$y_t = \mathrm{Hill}(x_t) + \varepsilon_t$$

4. Dynamic coefficients inherited from common parent

$$y_t = \beta(t)x(t) + \varepsilon_t, \;\; (\beta(t), x(t)) \sim \mathrm{Pa}(t)$$

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Gaussian processes

Two important levers:

- Smoothness
- Amplitude

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Gaussian processes: a Bayesian nonparametric approach to modeling unknown functions

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$$\Updownarrow$$

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$$k(x, x') = \eta^2 \exp\left\{-\frac{(x - x')^2}{2\rho^2}\right\}$$

Primer: Gaussian Processes

Gaussian processes: a Bayesian nonparametric approach to modeling unknown functions

$$f(x) \sim \mathcal{GP}(m(x), k(x, x'))$$

$$\Updownarrow$$

$$f(x_1, \ldots, x_N) \sim \mathcal{N}(m(x_1, \ldots, x_N), K), \text{ s.t. } K_{ij} = k(x_i, x_j)$$

Amplitude

$$k(x, x') = \boxed{\eta^2} \exp\left\{-\frac{(x - x')^2}{2\boxed{\rho^2}}\right\}$$

Lengthscale

Primer: Gaussian Processes

Gaussian processes: a Bayesian nonparametric approach to modeling unknown functions

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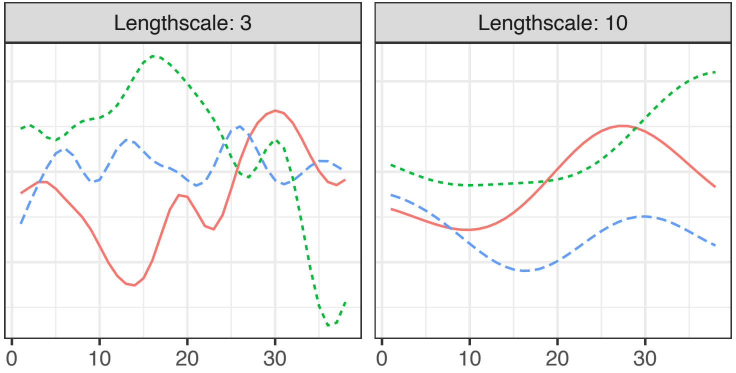
Lengthscale



When does conflation *actually* happen?

Four types of simulations:

1. Flexible nonlinear response

$$y_t = \boxed{f(x_t)} + \varepsilon_t$$

2. Dynamic coefficients, linear (or log-linear) model

$$y_t = \boxed{\beta(t)} x_t + \varepsilon_t$$

3. Nonlinear response, parametric hill function

$$y_t = \mathrm{Hill}(x_t) + \varepsilon_t$$

4. Dynamic coefficients inherited from common parent

$$y_t = \beta(t)x(t) + \varepsilon_t, \ (\beta(t), x(t)) \sim \mathrm{Pa}(t)$$

Gaussian processes

Two important levers:

- Smoothness
- Amplitude

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3. Nonlinear response, parametric hill function
4. Dynamic c

Gaussian processes
Two important levers:
- Smoothness
- Amplitude

Other manipulated features:
- Autoregressive coefficient in x
- Noise in x's autoregressive process
- Variance of the error term

Simulation Results

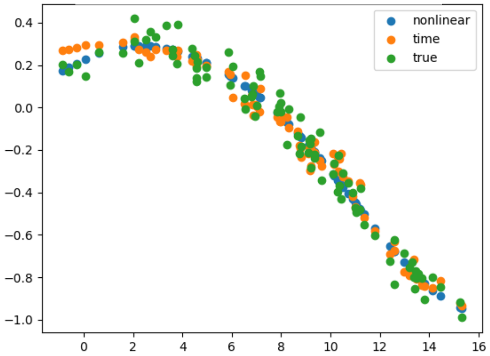
- For each simulation type, >300 settings, systematically varying the previously described factors, with 100 simulations per setting
- Fit both models (nonlinear and dynamic), measure conflation through validation RMSE

Examples

Nonlinear DGP

Smooth f, low noise

Data



Examples

Nonlinear DGP

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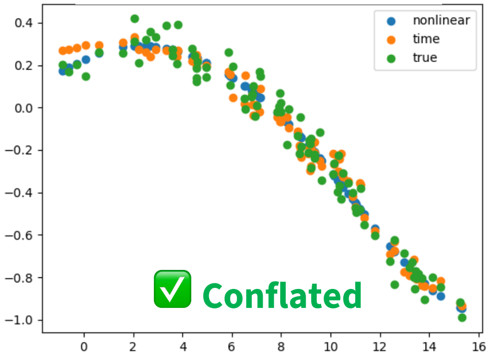
Data

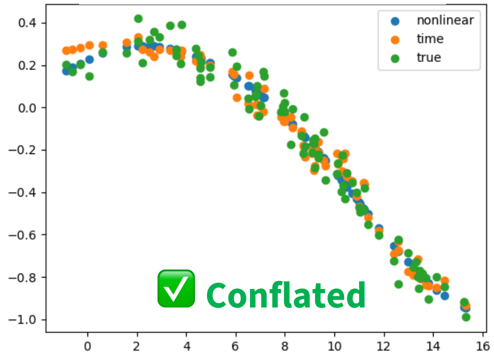


Examples

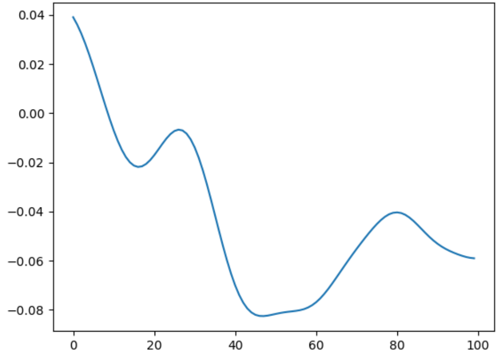
Nonlinear DGP

Smooth f, low noise

Data



Implied $\beta(t)$



Examples

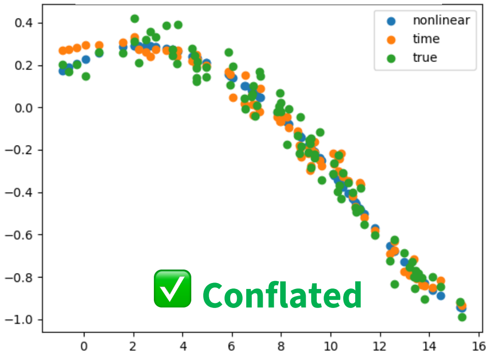
Nonlinear DGP

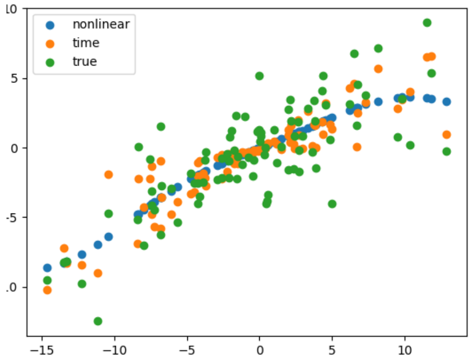
Smooth f, low noise

Dynamic DGP

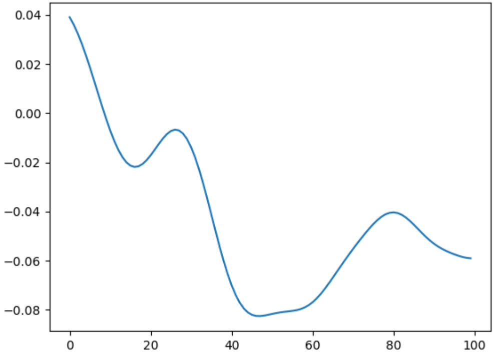
Smooth f, low noise

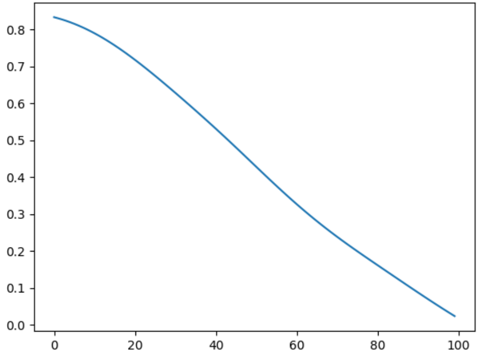
Data



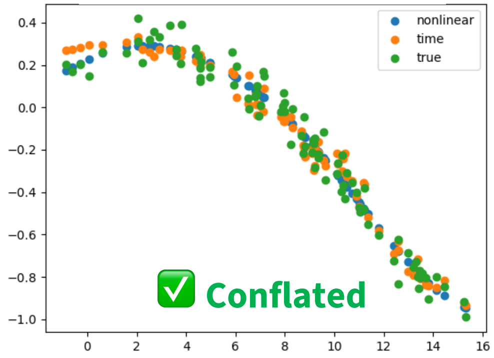
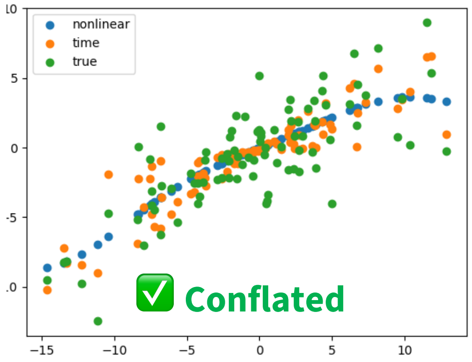
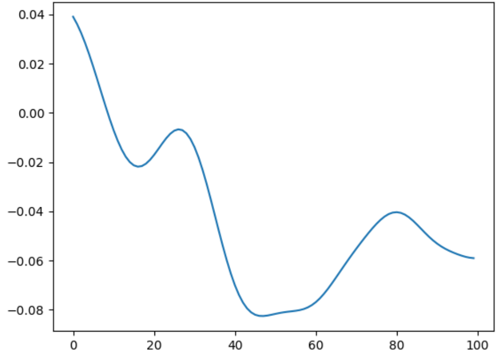
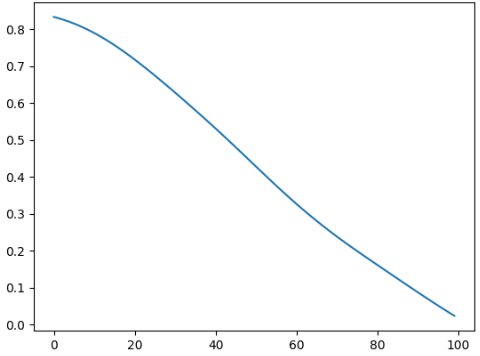


Implied
$\boldsymbol{\beta(t)}$

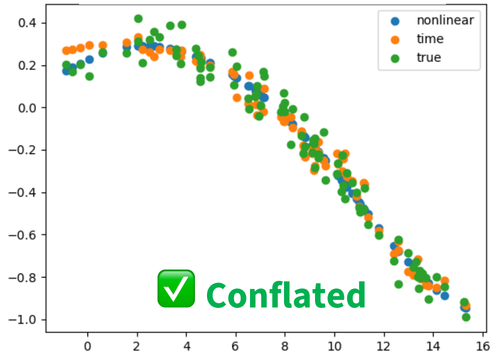
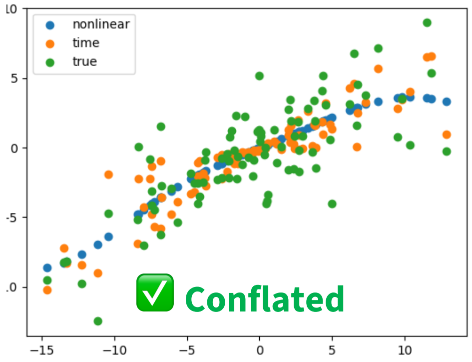
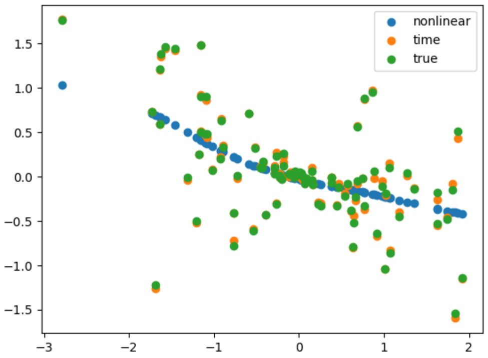
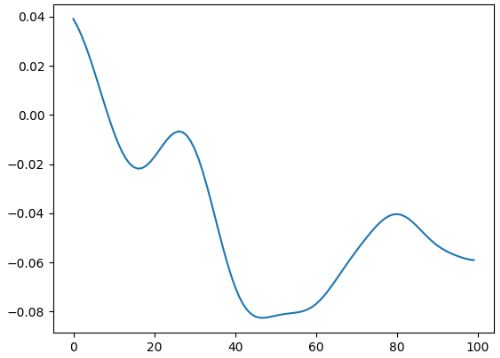
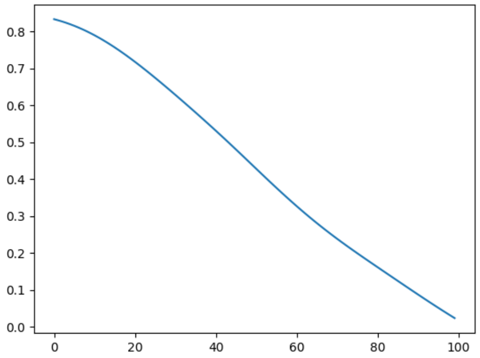
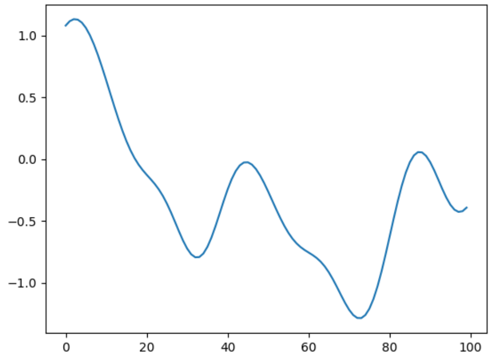




Examples

	Nonlinear DGP *Smooth f, low noise*	**Dynamic DGP** *Smooth f, low noise*
Data		
Implied $\boldsymbol{\beta(t)}$		

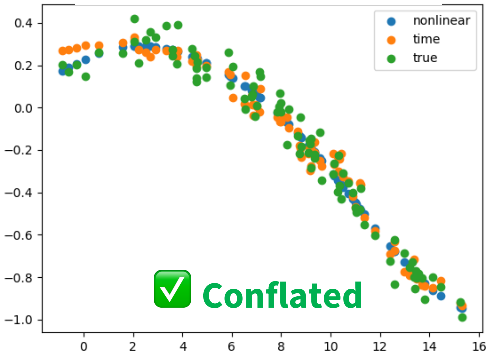
Examples

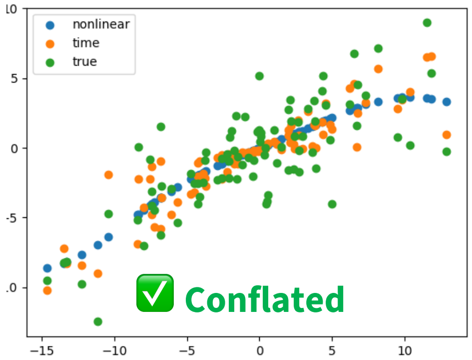
	Nonlinear DGP *Smooth f, low noise*	**Dynamic DGP** *Smooth f, low noise*	**Dynamic DGP** *Less smooth f, low noise*
Data			
Implied $\beta(t)$			

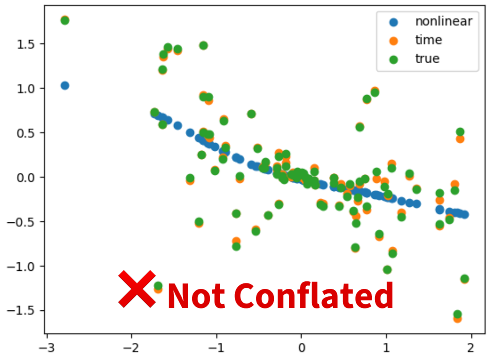
Examples

	Nonlinear DGP	**Dynamic DGP**	**Dynamic DGP**
	Smooth f, low noise	*Smooth f, low noise*	*Less smooth f, low noise*

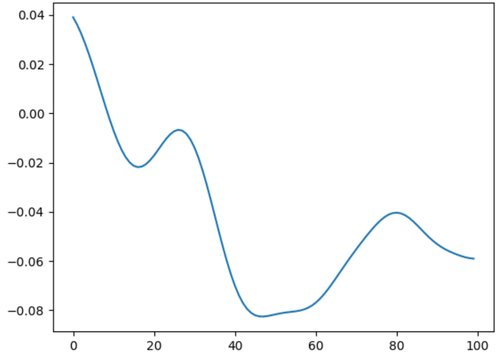
Data

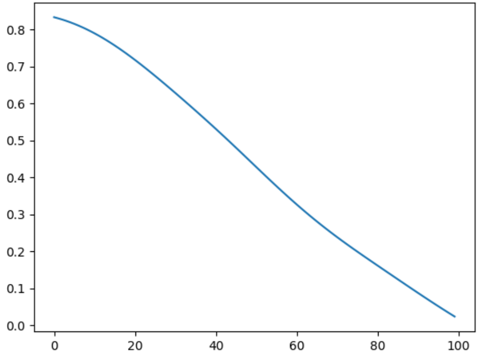


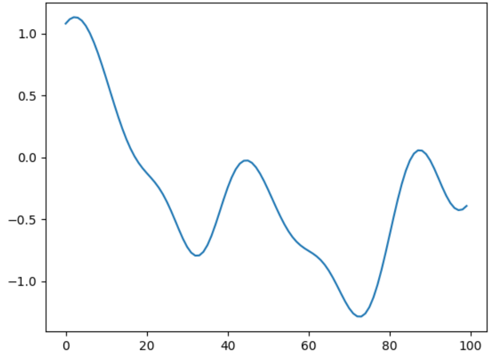




Implied $\boldsymbol{\beta(t)}$







Simulation

- Systematically vary the previously described factors, running 100 simulations per setting, and fit both models (nonlinear and dynamic)
- Any conflation = for at least one simulation of that setting, the wrong model had better validation RMSE
- Major conflation = conflation on over 25% of that setting's simulations

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	No Stock		High Stock	
True DGP	Any	Major	Any	Major
Nonlinear	81%	27%	85%	30%
Time-varying	91%	40%	99%	44%
Hill	83%	46%	92%	47%

Diving Deeper

DV: % Conflated Simulations

		Nonlinear DGP		Time-varying DGP	
Variable	Level	Coef	$P(>\|t\|)$	Coef	$P(>\|t\|)$
Amplitude, f:	Low (1)	-	-	-	
	Middle (2)	-0.24	0.65	0.23	0.62
	High (5)	-0.65	0.22	0.09	0.85
Smoothness, f:	Low (0.1)	-	-	-	
	Middle (0.5)	10.36	0.00	7.93	0.00
	High (1)	18.86	0.00	12.21	0.00
AR coef, x:	Low (0)	-	-	-	
	Middle (0.5)	-0.56	0.29	0.63	0.18
	High (1)	0.42	0.43	13.38	0.00
AR Variance, x:	Low (1)	-	-	-	
	Middle (5)	0.11	0.84	0.06	0.90
	High (10)	-0.47	0.37	0.36	0.44
Noise, y	Low (0.01)	-	-	-	
	Middle (0.1)	14.10	0.00	13.05	0.00
	High (0.2)	23.28	0.00	22.71	0.00
AdStock	Low (0)	-	-	-	
	Middle (0.3)	-0.14	0.79	0.83	0.08
	High (0.8)	1.59	0.00	2.49	0.00

Table 2: Simulation Results
DV = Percentage Conflation; Intercept omitted for clarity.

Diving Deeper

DV: % Conflated Simulations

Conflation more likely with…

- Noisier data (i.e., ϵ_t)

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Diving Deeper

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- Noisier data (i.e., ϵ_t)
- Smoother functions

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Diving Deeper

DV: % Conflated Simulations

Conflation more likely with…

- Noisier data (i.e., ϵ_t)
- Smoother functions
- High degree of autocorrelation

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Diving Deeper

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Conflation more likely with...

- Noisier data (i.e., ϵ_t)
- Smoother functions
- High degree of autocorrelation

Decisions are often autocorrelated!

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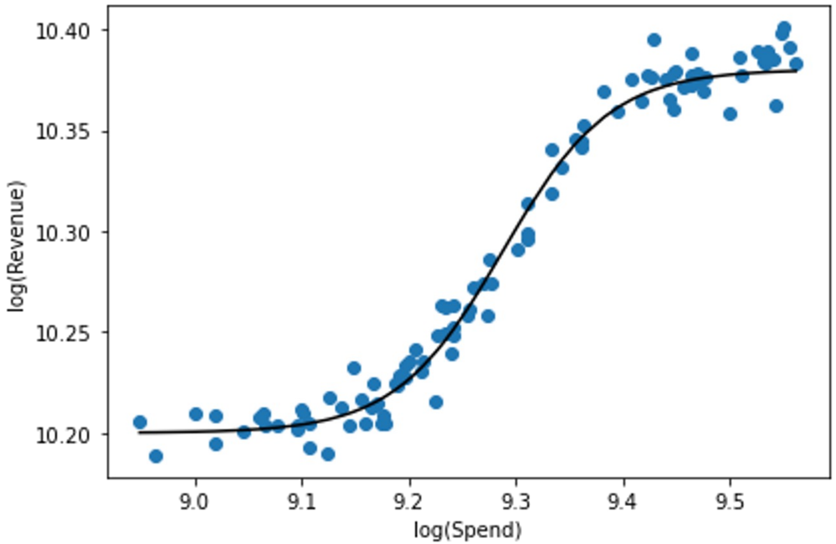
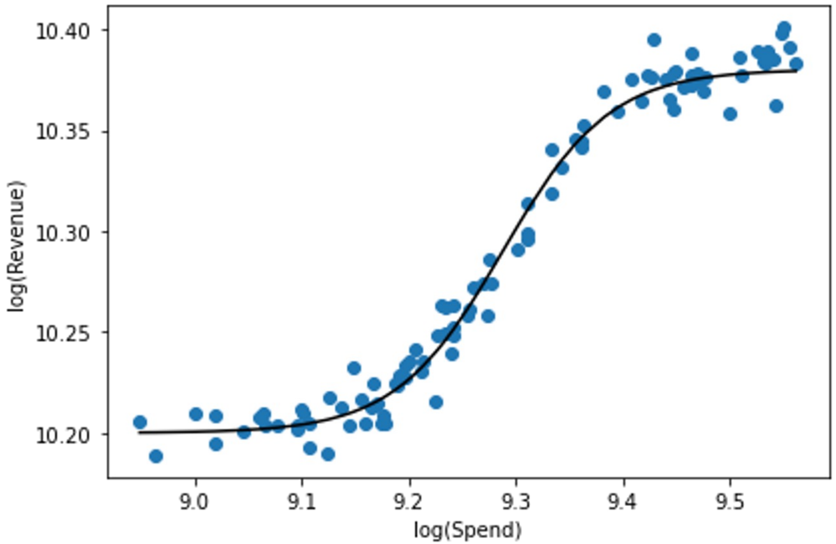
Implications

One last simulation…



(or from roughly $8,000 to $14,000)

One last simulation…



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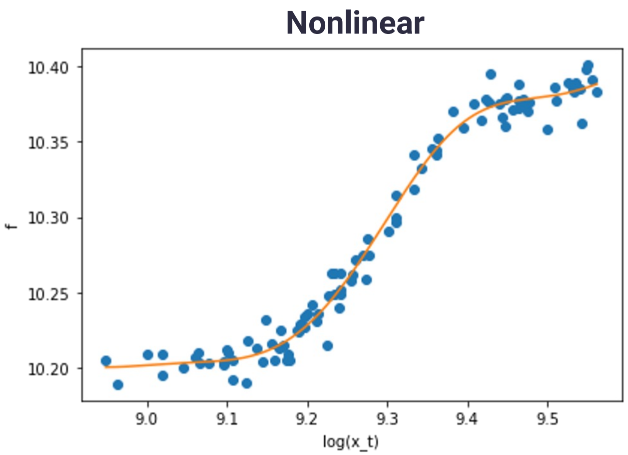

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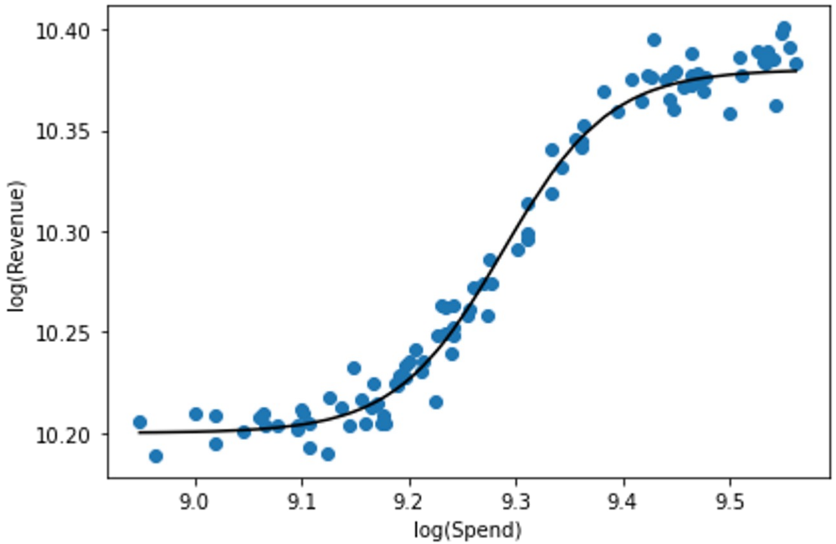

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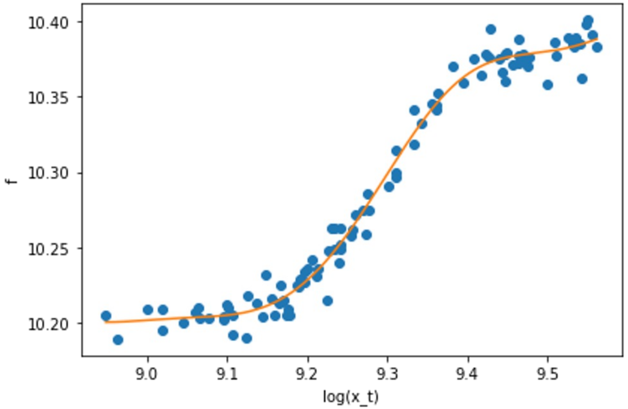
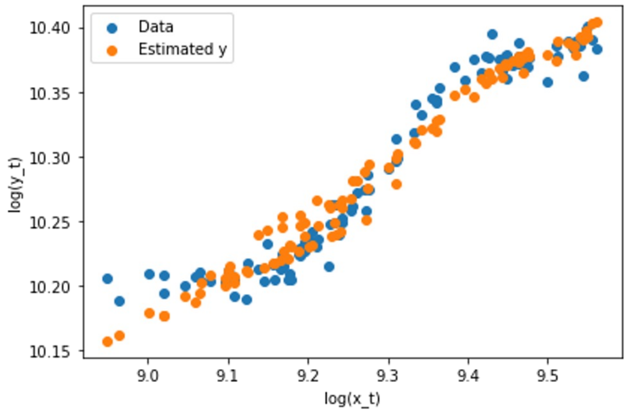
Nonlinear



Dynamic



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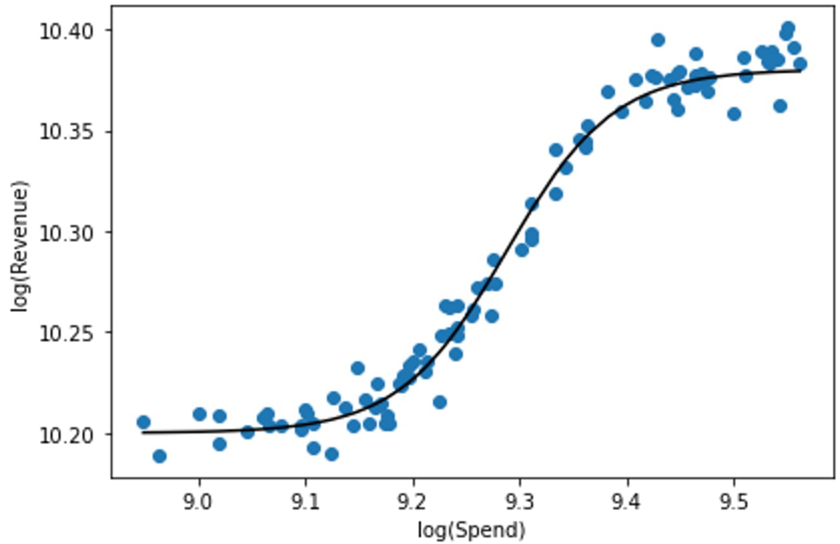

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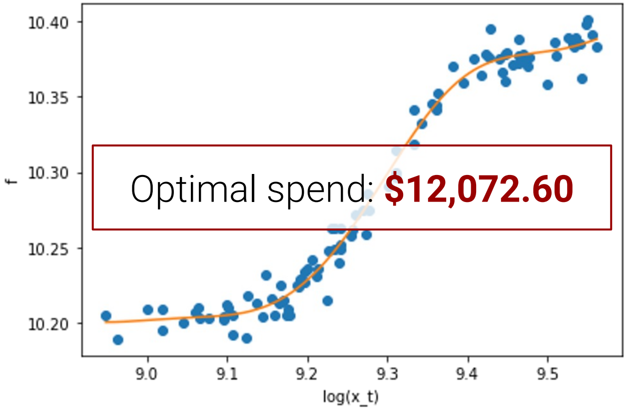
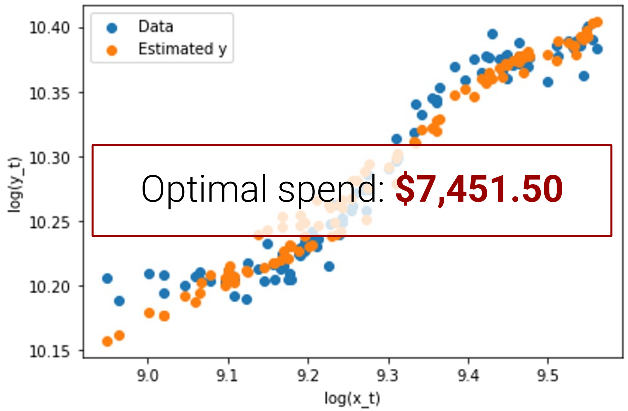
Nonlinear



Optimal spend: **$12,072.60**

Dynamic



Optimal spend: **$7,451.50**

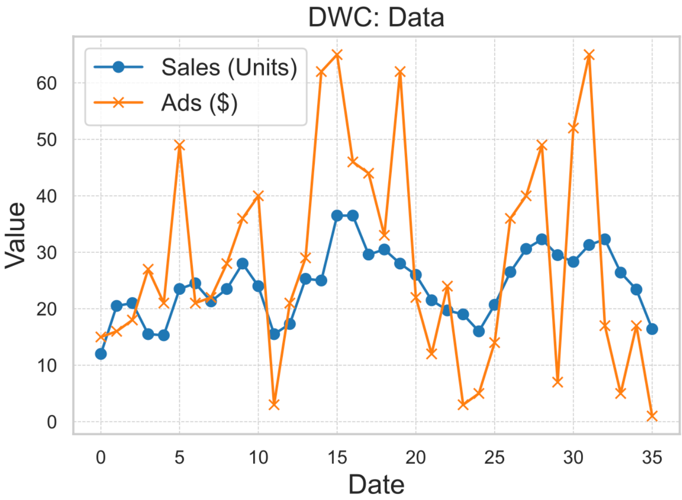
Real Data: Classic and Modern

- Classic Application 1: Dietary Weight Control (DWC)
  (Bass and Clark, 1972)

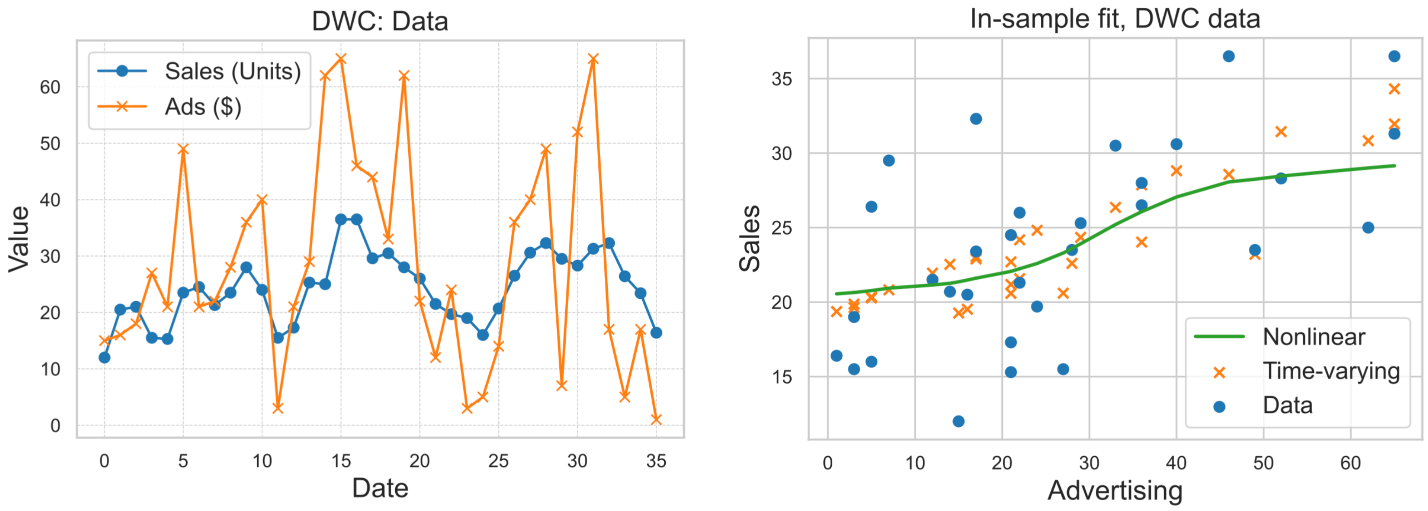
- Classic Application 2: Lydia Pinkham (LP)
  (Palda, 1964)

- Modern applications: MMM data from Nielsen

DWC

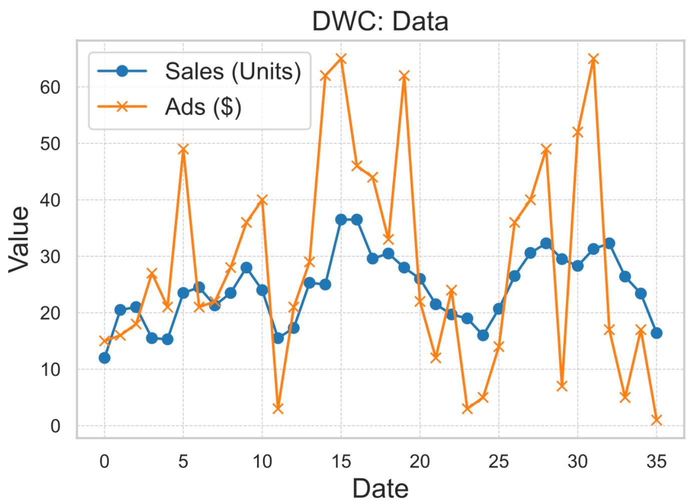

DWC: Data
Sales (Units)
Ads ($)
Value
Date
0
10
20
30
40
50
60
0
5
10
15
20
25
30
35

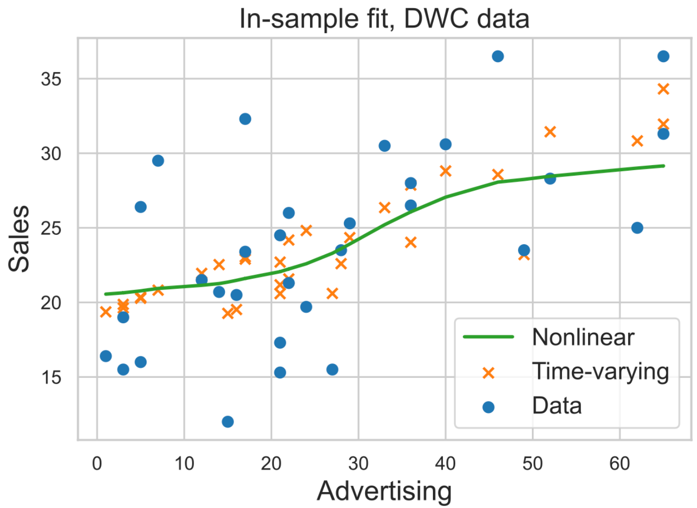
DWC



DWC: Data
Sales (Units)
Ads ($)
Value
Date
0
10
20
30
40
50
60
0
5
10
15
20
25
30
35
In-sample fit, DWC data
Nonlinear
Time-varying
Data
Sales
Advertising
15
20
25
30
35
0
10
20
30
40
50
60

DWC





	Nonlinear	Time-varying	Hill	Log Time-varying
Dietary Weight Control	3.23 [2.22, 5.58]	2.97 [1.72, 5.09]	3.10 [2.59, 4.44]	2.55 [1.50, 4.13]

Spend Optimization

- DWC data: sales volume, but no prices
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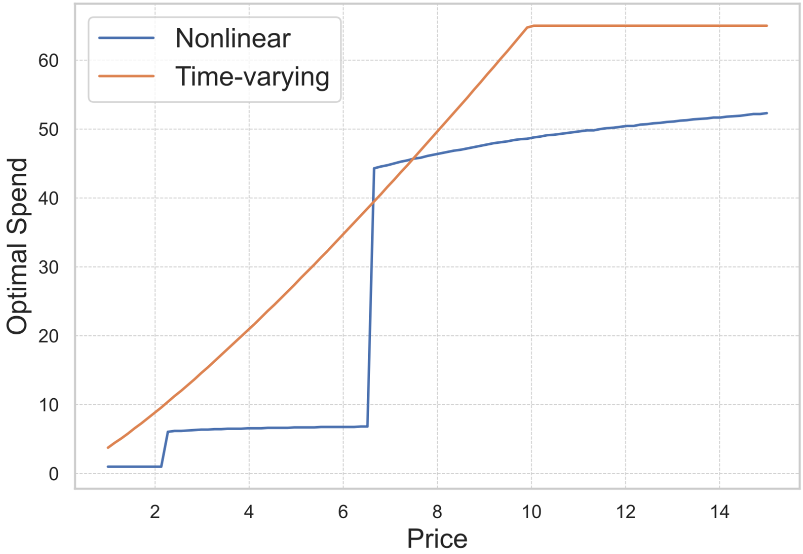
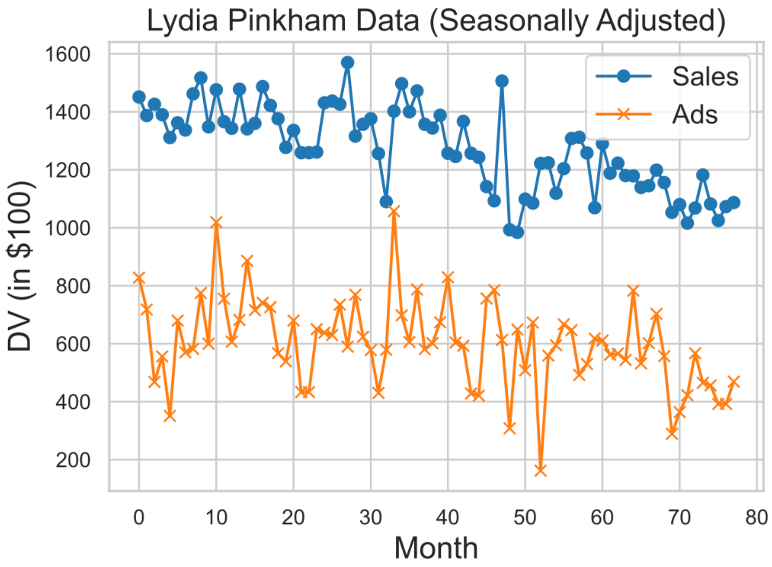
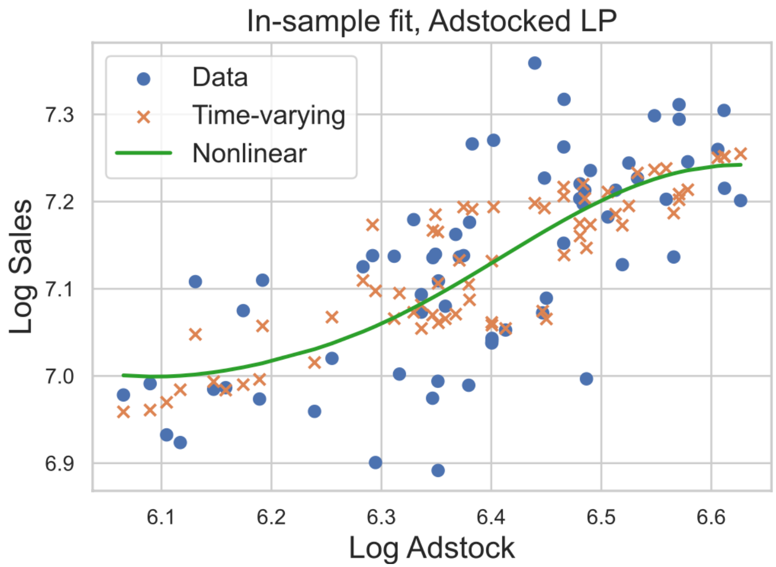


Lydia Pinkham





	Nonlinear	Time-varying	Hill	Log Time-varying
Lydia Pinkham	89.37	83.21	100.50	88.92
	[79.41, 104.88]	[75.44, 95.24]	[91.68, 114.97]	[83.23, 98.00]

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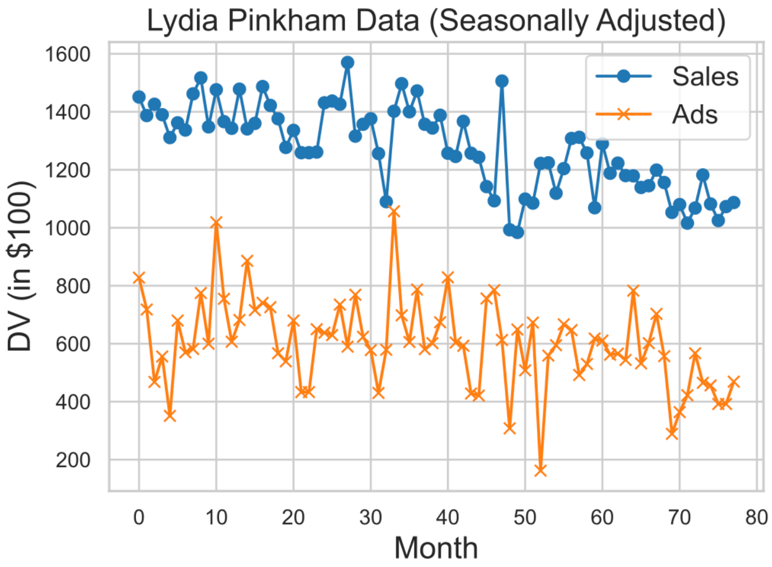

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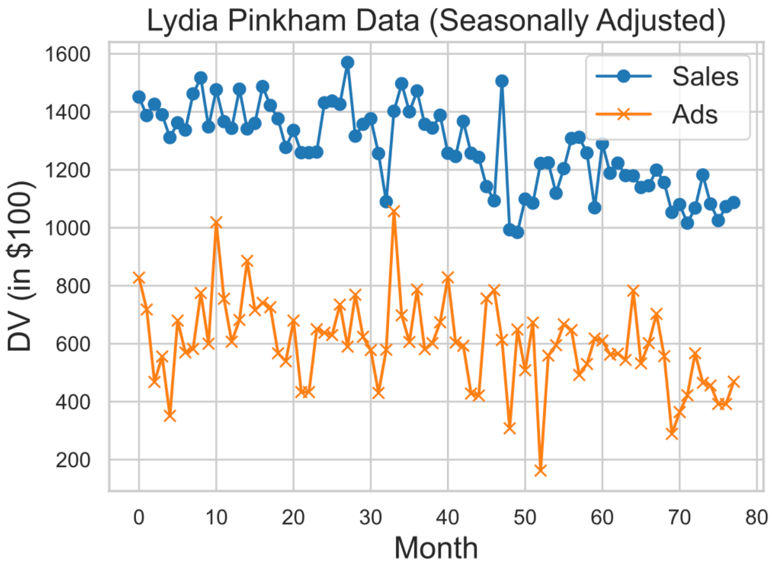

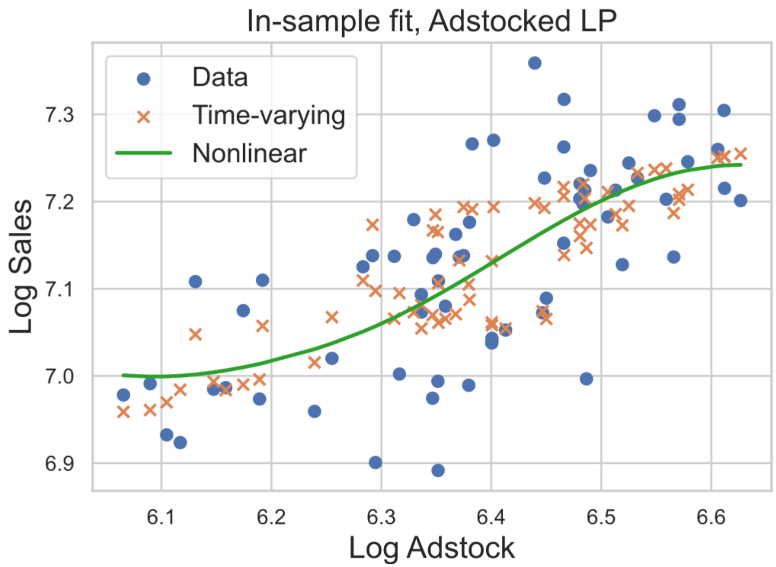

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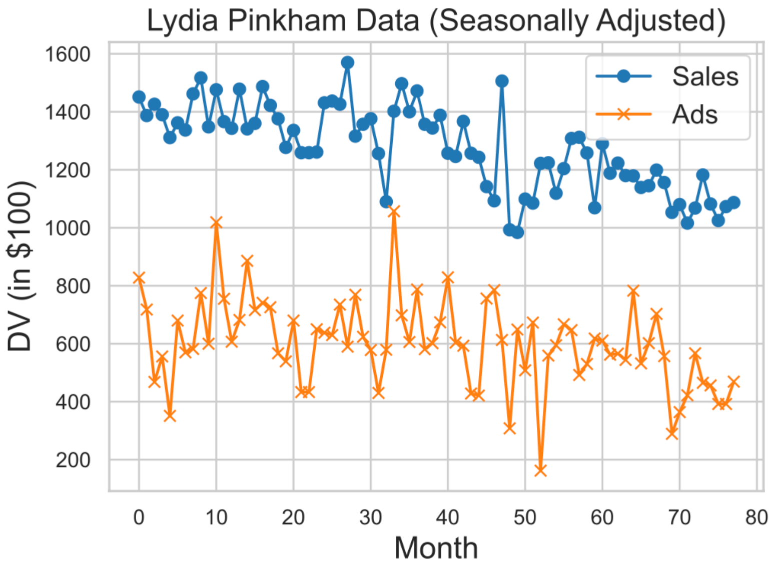

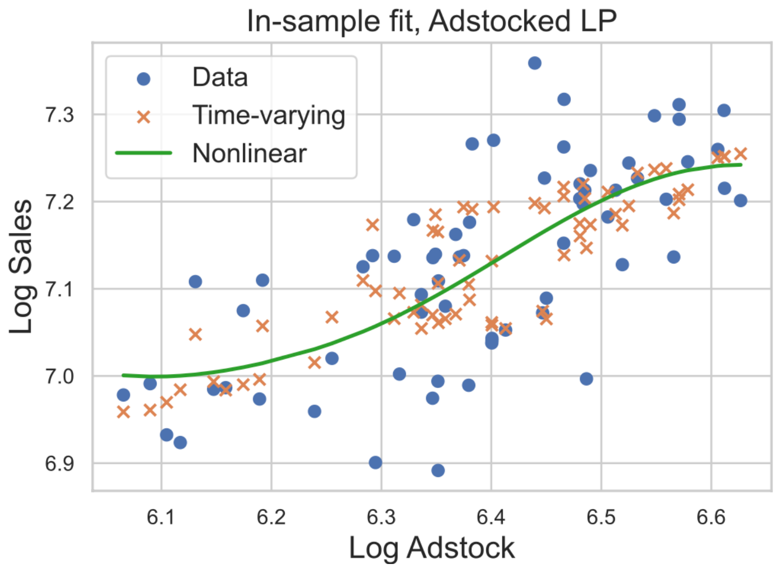

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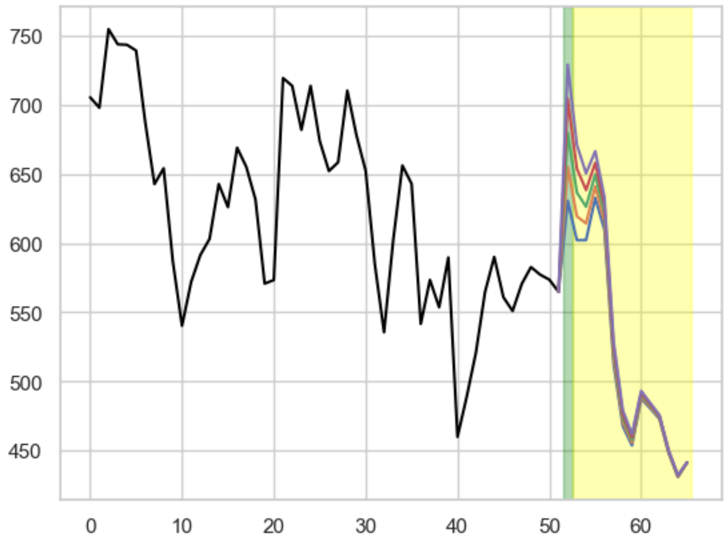
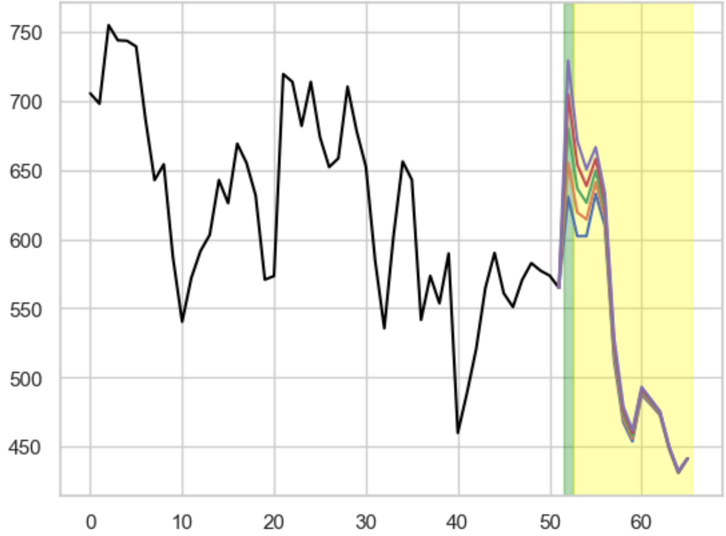


Spend Optimization with Carryover

- Trickier: spending today affects AdStock for many future periods
- Follow Google's practice: for *L* carryover periods, determine optimal spend in a test window, predict revenue *L* periods out
- For LP, optimal ad spend:
  - Nonlinear model: **$90,127**
  - Time-varying model: **$78,333**



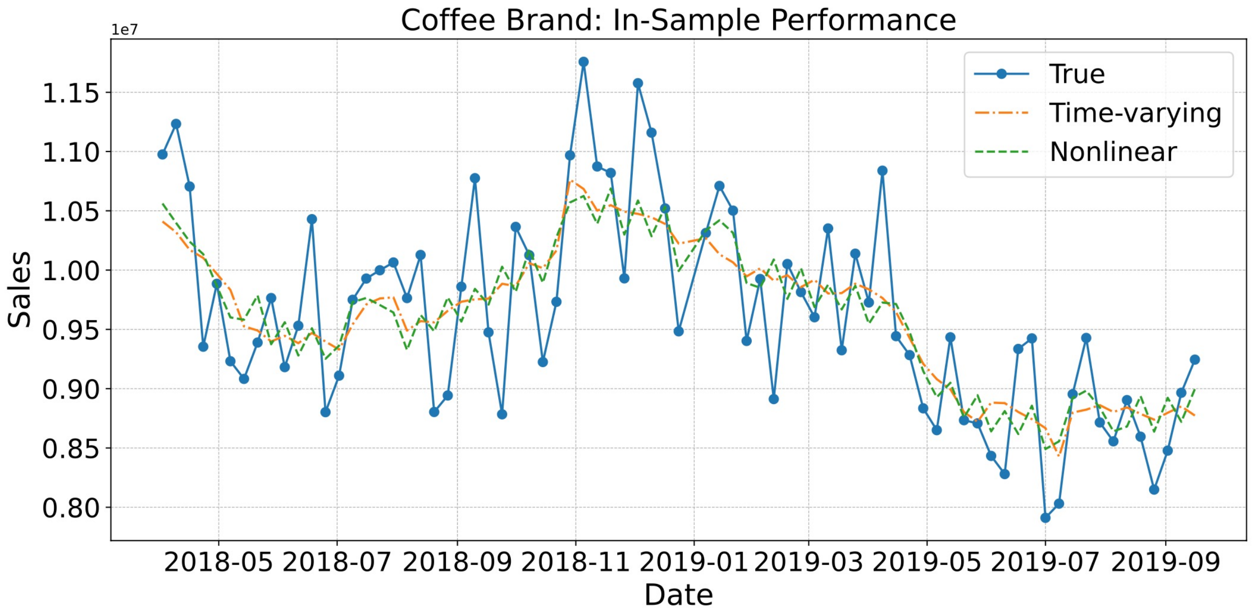
Modern MMM Data

- Again, clear conflation...

	Chocolate	Pet Food	Coffee	Beer
Nonlinear (GP)	368,766	48,438	722,076	443,817
	[207,634, 601,163]	[27,920, 83,249]	[509,527, 919,820]	[236,730, 911,732]
Time-varying (GP)	387,623	41,825	733,269	423,085
	[207,350, 671,246]	[24,631, 65,232]	[490,785, 994,303]	[207,914, 802,740]

Modern MMM Data

- Case study: Coffee



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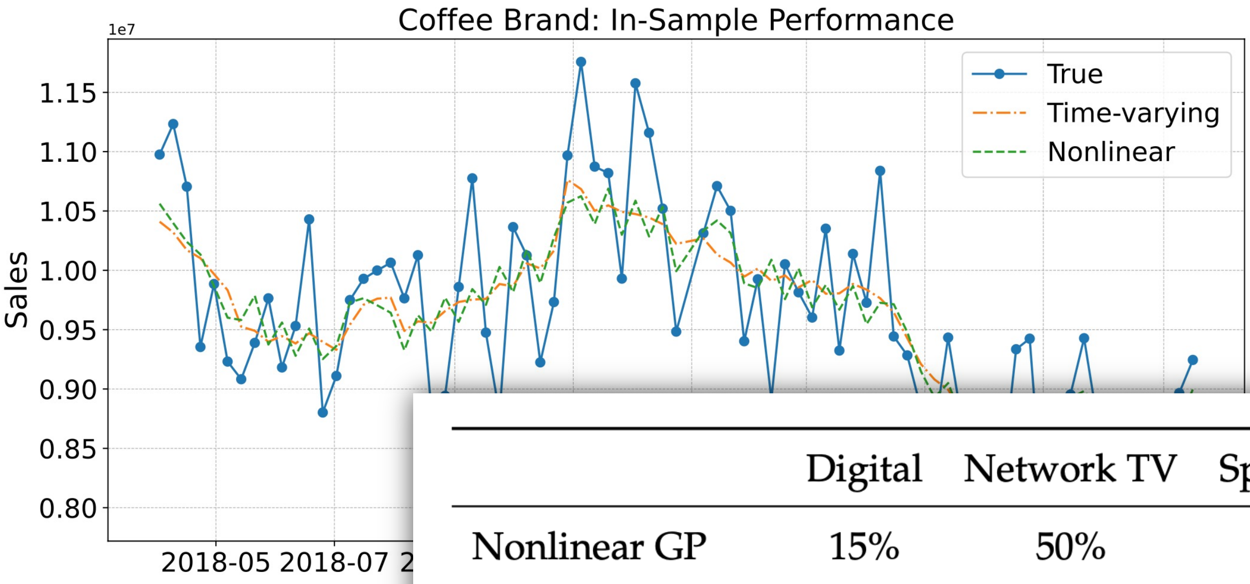

	Digital	Network TV	Spot TV	Conflation Cost
Nonlinear GP	15%	50%	35%	$227,000
Time-varying GP	15%	85%	0%	$61,000

So is MMM doomed?

Test 1: Maximal Separation

- A type of “bump up” incrementality test
- Intuition: set spending next period to maximize the difference in predicted values across the two models

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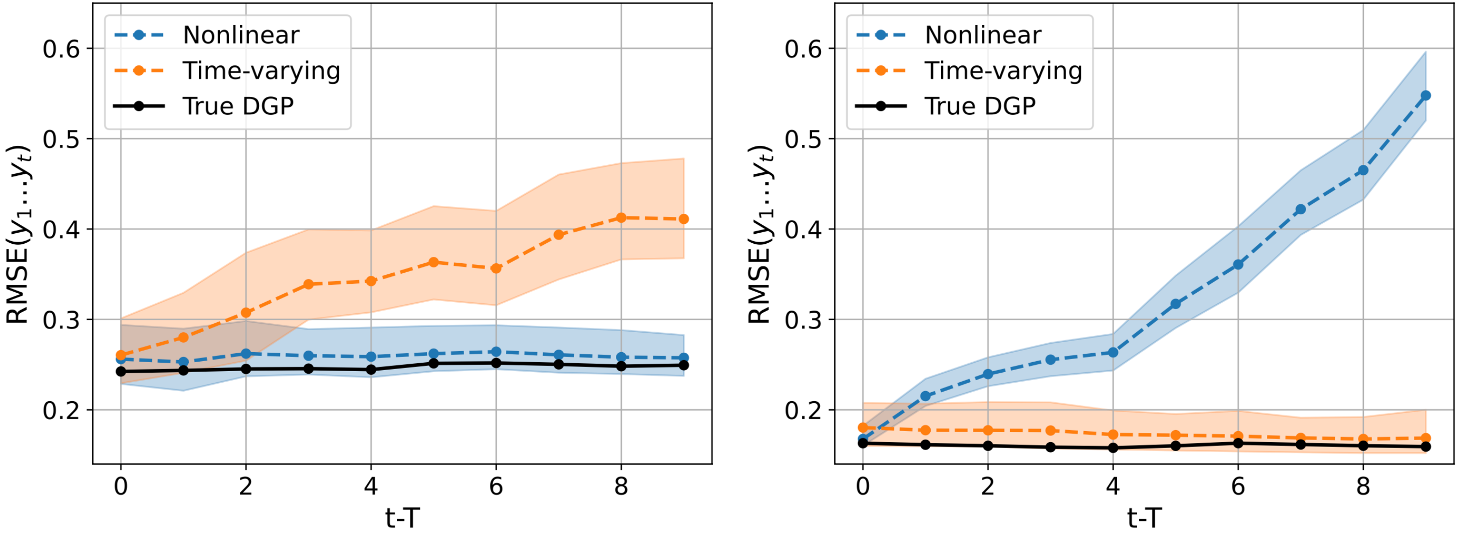

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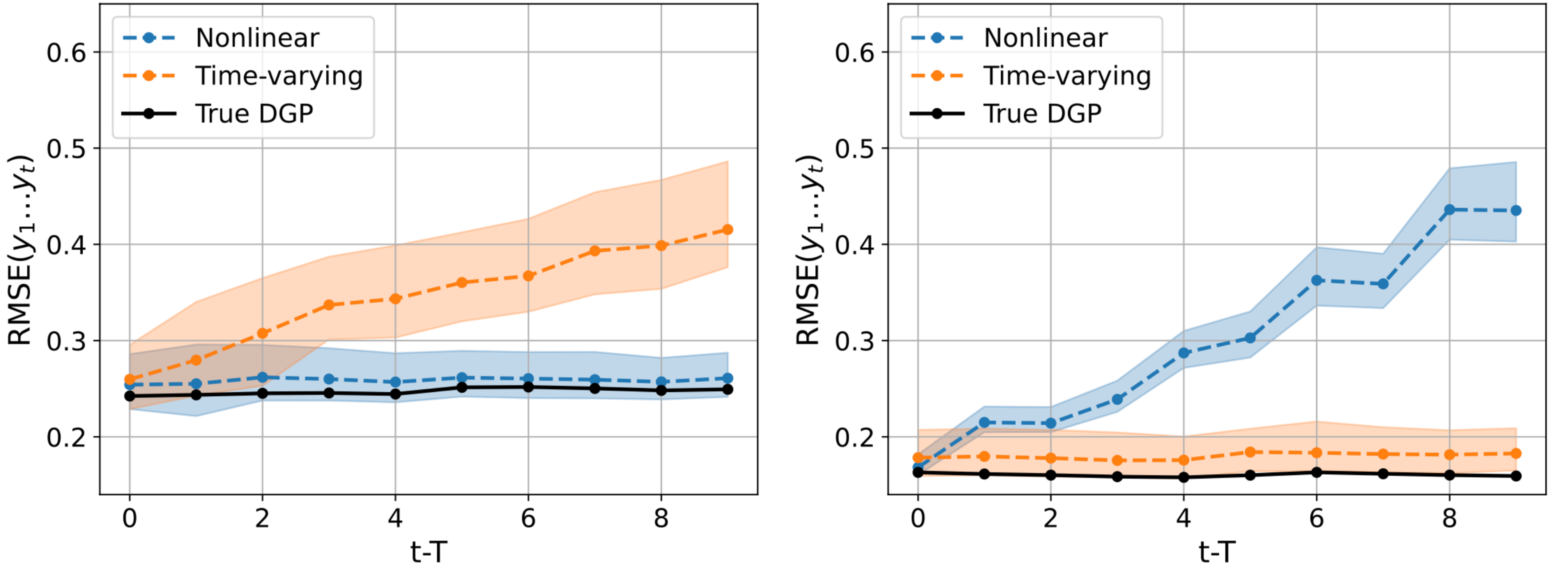

Conclusions

Current MMM Practice Might Be Flawed!

- We show that, under many common spending patterns, **time-varying and nonlinear effects cannot be disentangled**, *despite having different implications*
- This problem is potentially **very widespread**: increasing complexity in models, widespread practice of “model refreshes” to capture changing markets
- Our work both introduces a **framework for estimating** these types of models, and provides **solutions for understanding and preventing** conflation

Thanks!

Feedback or questions:

ryandew@wharton.upenn.edu

Working paper:

www.rtdew.com

You

generate an image of a marketing mix model paradise with some very happy computers and a cameo from the reverend thomas bayes