Quantum Algorithms via Semidefinite Programming

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Abstract

Let f be a Boolean function and x be a bitstring input to f. We consider the query model where an algorithm evaluates $f(x)$ by asking an oracle about the bits of x. The optimal quantum query complexity of f is the fewest queries any quantum algorithm takes to determine $f(x)$ over all inputs x. We determine the optimal quantum query complexity of f by solving a semidefinite programming (SDP) problem. This solution also provides a span program that evaluates f in optimal quantum query complexity on a quantum computer.

Query Complexity

$$f_1(x) = (x_1 \wedge (x_2 \vee \bar{x}_2)) \vee x_3$$
$$f_2(x) = x_1 \vee x_2 \vee x_3$$

x	$f_1(x)$	$f_2(x)$	*I(x)*	*Span?*
000	0	0	0,0,0	False
010	0	1	0,1,0	True
001	1	1	0,0,1	True

Span Programs

Do the input vectors I span to the target vector τ?

For OR, $I(x) = x$ and $\tau = 1$.

Problem

Given a Boolean function f…

1. What is the optimal quantum query complexity of f?

2. What algorithms meet the optimal quantum query complexity?

Semidefinite Program (SDP)[1]

Minimize

$$\max_{y \in D} \sum_{j \in [n]} \langle y, j | \mathbb{X} | y, j \rangle$$

subject to

$$\sum_{j \in [n] : y_j \neq z_j} \langle y, j | \mathbb{X} | z, j \rangle = 1$$

for all y, z s.t. $f(y) \neq f(z)$

Quantum Query Optimizer

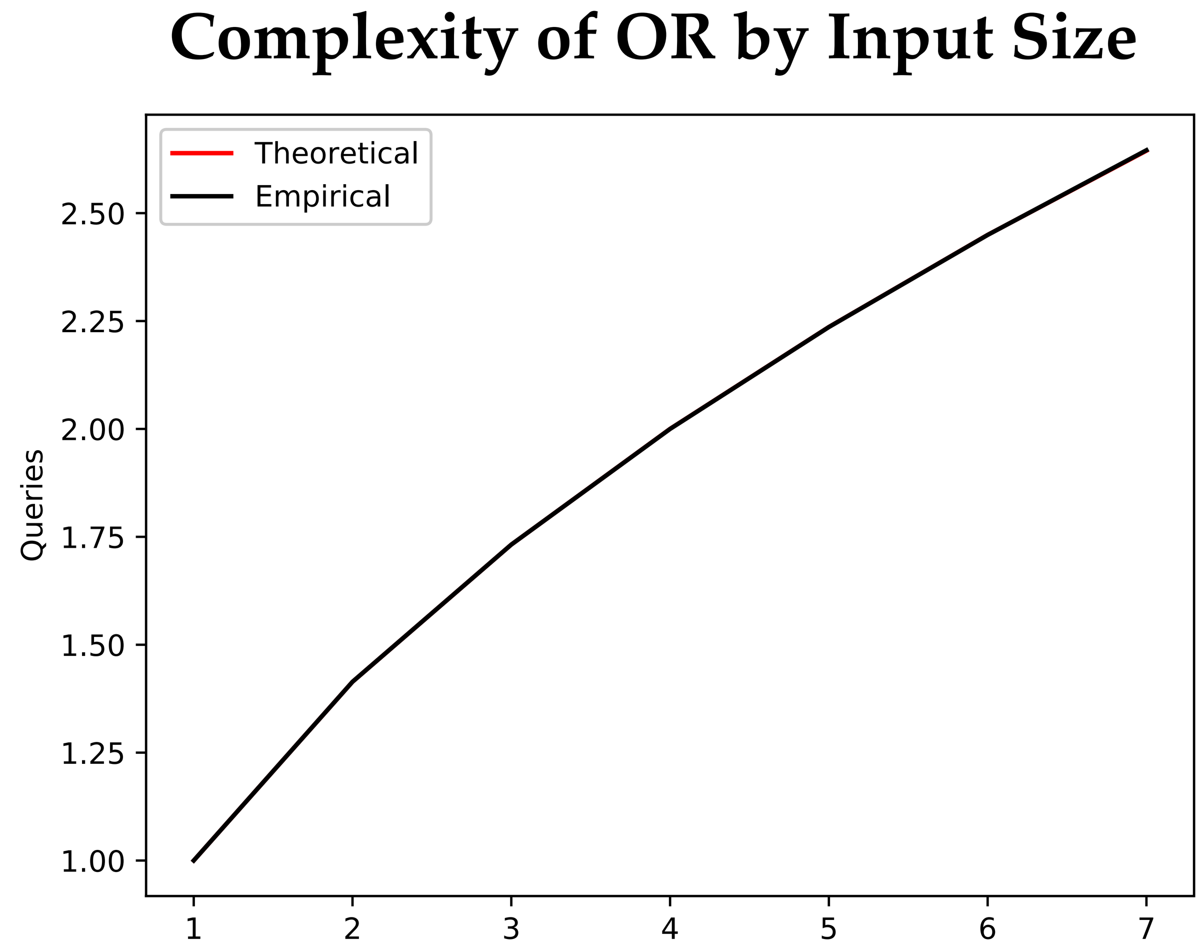
github.com/rtealw/QuantumQueryOptimizer

```
# Example 1
D = ['00', '01', '10', '11']
E = ['0', '1', '1', '1']
qqo.runSDP(D=D, E=E)

# Example 2
qqo.runSDPForN(
  getD=qqo.getDAll, getE=qqo.getEOR,
  n_end=2, n_start=2)
```

Results[2]

Complexity of OR by Input Size



Solution to SDP for 2-bit OR

$$\mathbb{X} = \left[\begin{array}{cc|cc|cc|cc}
0.7 & 0 & 0 & 0 & 1 & 0 & 0.5 & 0 \\
0 & 0.7 & 0 & 1 & 0 & 0 & 0 & 0.5 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \sqrt{2} & 0 & 0 & 0 & 0.7 \\
\hline
1 & 0 & 0 & 0 & \sqrt{2} & 0 & 0.7 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0.5 & 0 & 0 & 0 & 0.7 & 0 & 0.6 & 0 \\
0 & 0.5 & 0 & 0.7 & 0 & 0 & 0 & 0.6
\end{array}\right]$$

[1]Reichardt, Ben W. "Span programs and quantum query complexity: The general adversary bound is nearly tight for every boolean function." 50th Annual IEEE Symposium on Foundations of Computer Science. IEEE, 2009.

[2]Wen, Zaiwen, Donald Goldfarb, and Wotao Yin. "Alternating direction augmented Lagrangian methods for semidefinite programming." Mathematical Programming Computation 2.3-4 (2010): 203-230.