Applications of Graph Theory and Probability in the Board Game *Ticket to Ride*

R. Teal Witter & Alex Lyford

Middlebury College

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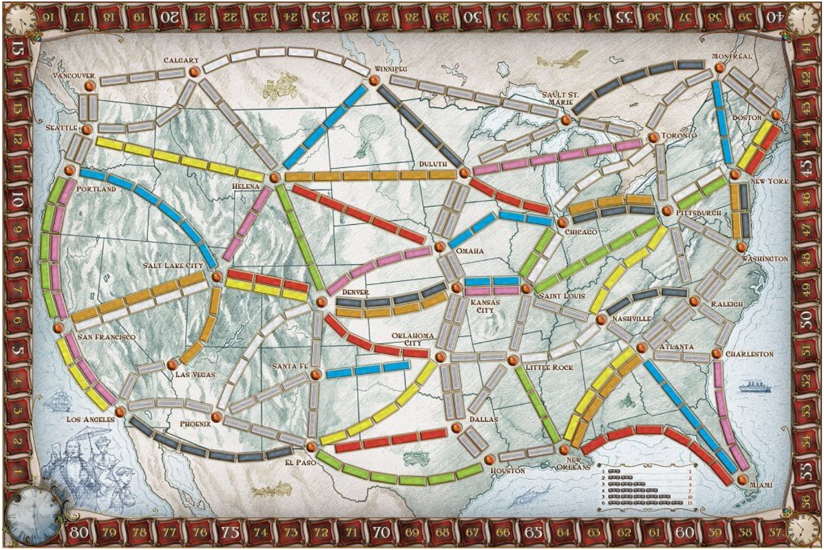
Ticket to Ride (USA)



Overview

Routes

- Long routes are overvalued ... and can be used to easily win.
- We can find a better route scoring scheme with indicator random variables.

Destination Tickets

- Players with some Destination Tickets perform better than players with others ... why?
- We use regression to identify the best Destination Tickets.

Current Route Values

Route Length	1	2	3	4	5	6
Points Scored	1	2	4	7	10	15
Points per Train	1.00	1.00	$1.\overline{33}$	1.75	2.00	2.50

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Arguments for

- Collecting many trains of the same color is hard

Arguments against

- Is it really *that* hard?
- Only one route can be claimed per turn
- Collecting multiple colors simultaneously helps

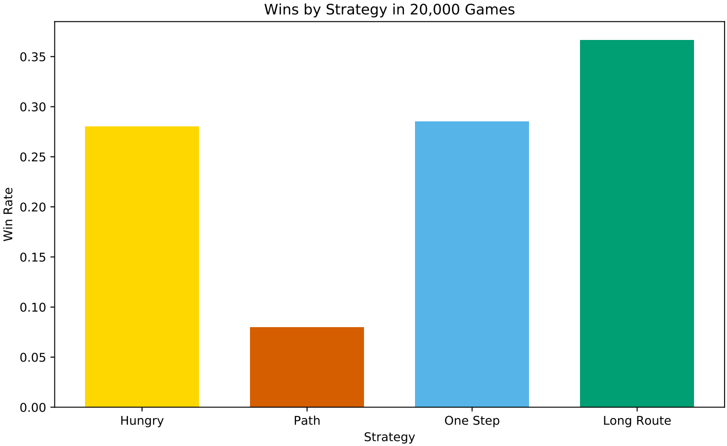
Games with routes of length at most k

For all games, all 45 trains will be collected over 23 turns.[2]

k	Composition	Points	Turns	Points per Turn
1	1 x 45	45	23 + 45	0.66
2	2 x 22, 1 x 1	45	23 + 23	0.98
3	3 x 15	60	23 + 15	1.58
4	4 x 11, 1 x 1	78	23 + 12	2.23
5	5 x 9	90	23 + 9	2.81
6	6 x 7, 3 x 1	109	23 + 8	3.52

[2] We ignore locomotives collected from the five face up cards.

Win Rate in Simulated Games



How *should* routes be valued?

Idea: value = expected time to collect [3]

[3] measured in number of cards rather than turns

Expected number to find k blue cards

Without loss of generality, our goal is to calculate the expected numbers to find k blue cards.

Expected number to find k blue cards

$$\begin{aligned} N_k &:= \text{number of cards until } k \text{ blue cards are found} \\ C &:= \text{all cards} \\ B &:= \text{blue cards} \\ x &\in C \setminus B \\ I_{x,k} &:= \begin{cases} 1 & \text{if } x \text{ appears before the } k^{th} \text{ blue card} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Then

$$N_k = k + \sum_{x \in C \setminus B} I_{x,k}$$

Expected number to find k blue cards

$$N_k = k + \sum_{x \in C \setminus B} I_{x,k}$$

Taking expectation,

$$\mathrm{E}[N_k] = k + (|C \setminus B|) \times \mathrm{E}[I_{x,k}]$$

Recall $\mathrm{E}[I_{x,k}] = 1 \times \mathrm{P}(I_{x,k} = 1) + 0 = \mathrm{P}(I_{x,k} = 1)$.

Then

$$\mathrm{E}[N_k] = k + (|C \setminus B|) \times \mathrm{P}(I_{x,k} = 1)$$

Expected number to find k blue cards

Think of the deck as non-blue cards separated by blue cards into $|B| + 1$ piles (possibly of size 0):

$$xxxbxbbxxxbxbb \ldots xbxxxb$$

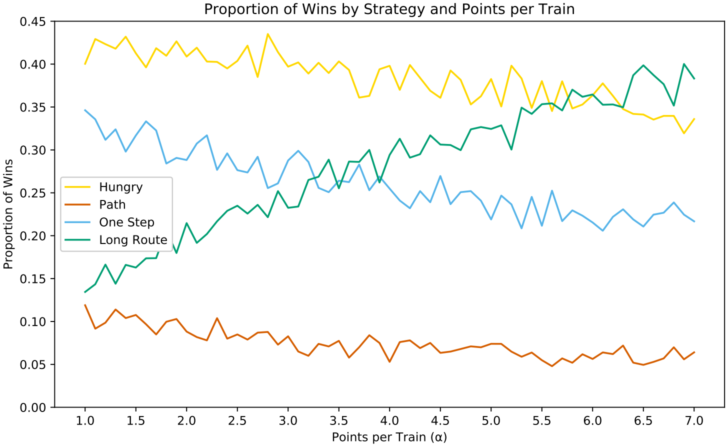
Then $\mathrm{P}(I_{x,k} = 1)$ is $k/(|B| + 1)$

Expected number to find k blue cards

$$\begin{aligned}
\mathrm{E}[N_k] &= k + (|C \setminus B|) \times \frac{k}{|B|+1} \\
&= \left(1 + \frac{110-12}{12+1}\right) k \\
&= \frac{111}{13} k
\end{aligned}$$

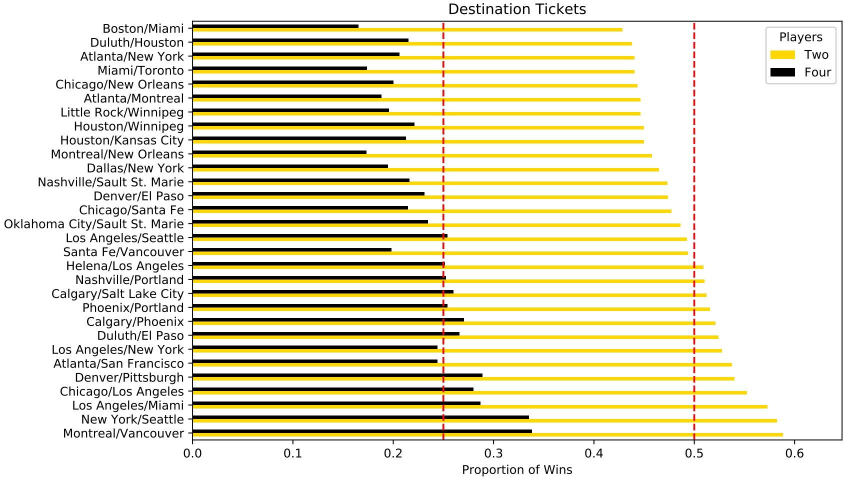
Thus our scoring should be linear!!

Choosing a scalar



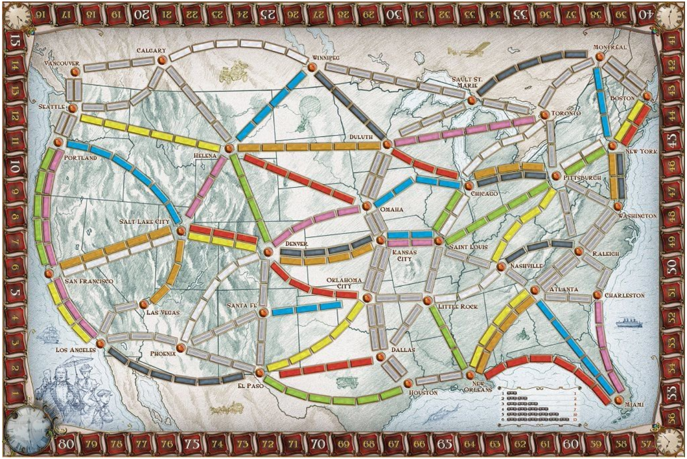
Perhaps somewhere between 3.5 and 5?

Destination Tickets and Wins



Best and Worst

Best: Montreal/Vancouver, New York/Seattle



Worst: Boston/Miami

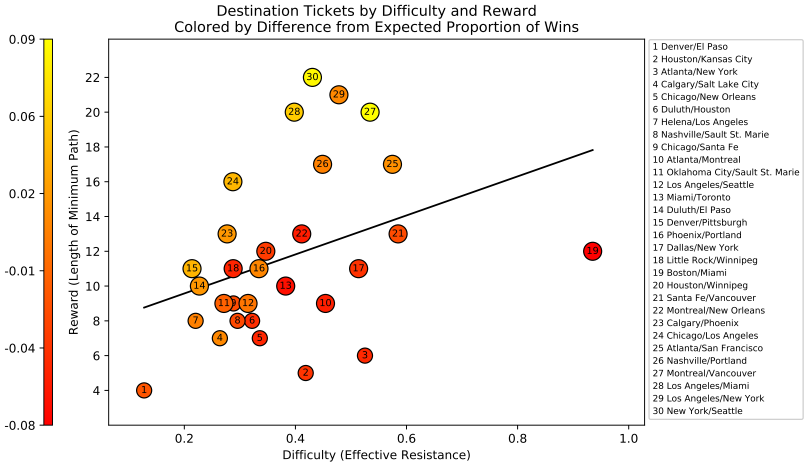
Effective Resistance?

A measure of connectivity between two nodes on a graph: electric flow from one node to another

$$\text{resistance} = \min_{\text{flows}} \sum_{\text{routes}} (\text{flow on route})^2$$

More, shorter paths $\rightarrow$ lower resistance

Effective Resistance!



Rankings by Minimum Path Length and Residual

Summary

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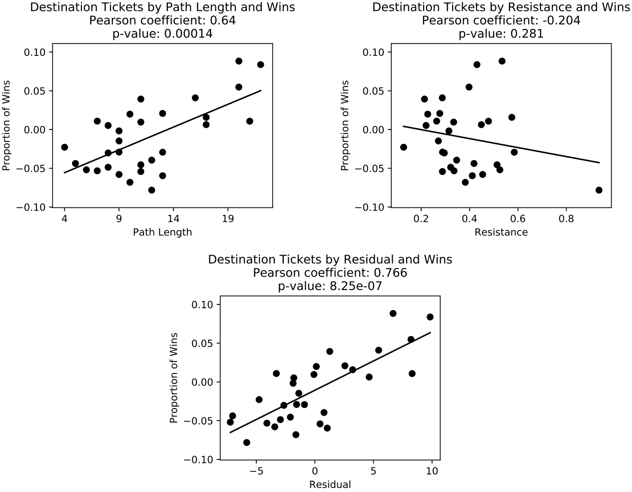
Thank you!

Questions?

Correlations with overall wins



References

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