Week 12

↳ Office Hours Monday + <u>Thursday</u> 12:30-2:00

↳ Midterm on 11/25, practice exams next week :)

↳ Project Proposal due 12/2

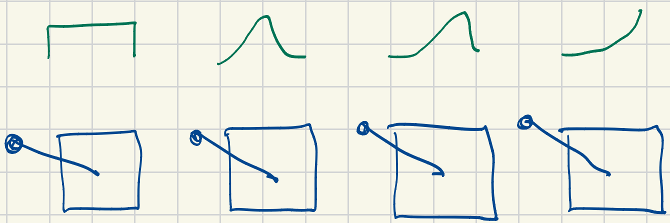
Reinforcement Learning

State, action, reward, ...

Key: Exploration vs. Exploitation

This Week: theoretical understanding

Simple model: multi-armed bandits



Hidden reward distribution

"slot machines"

Question:
What can we say about draws from these distributions?

Goal: Pull arms to maximize reward

↳ clinical trials

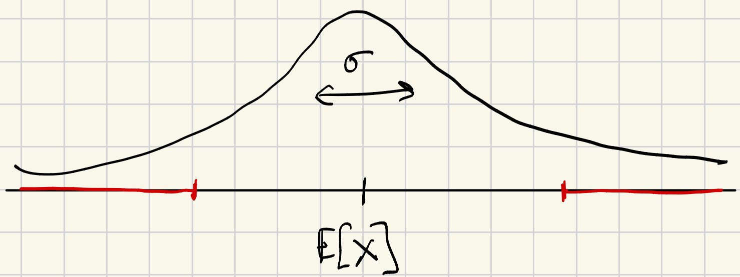
↳ recommendation systems

Random Variables

X r.v. $\qquad Pr(X=x)$ = prob X takes value x

$$\mathbb{E}[X] = \sum_x x \, Pr(X=x)$$

$$Var(X) = \mathbb{E}[(X-\mathbb{E}[X])^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = \sigma^2$$

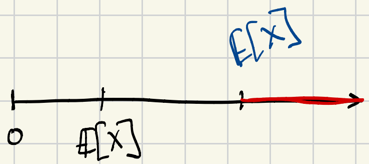


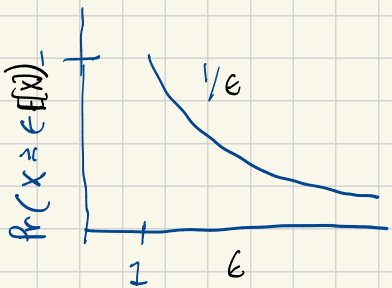
Goal: Bound probability of extreme values

Markov's Inequality

Consider non-negative X

For $\epsilon > 0$, $\Pr(X \geq \epsilon\, \mathbb{E}[X]) \leq \frac{1}{\epsilon}$





Proof:

$$\mathbb{E}[X] = \sum_{x} \Pr(X = x)\, x$$

$$= \sum_{x : x \geq \epsilon} \Pr(X = x)\, x + \sum_{x : x < \epsilon} \Pr(X = x)\, x$$

$$\geq \sum_{x : x \geq \epsilon} \Pr(X = x)\, \epsilon + \sum_{x : x < \epsilon} \Pr(X = x) \cdot 0$$

$$= \epsilon \sum_{x : x \geq \epsilon} \Pr(X = x)$$

$$= \epsilon \Pr(X \geq \epsilon)$$

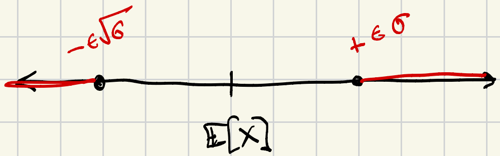
Q: Fix ϵ. Build a rv where $\Pr(X \geq \epsilon) = \frac{\mathbb{E}[X]}{\epsilon}$

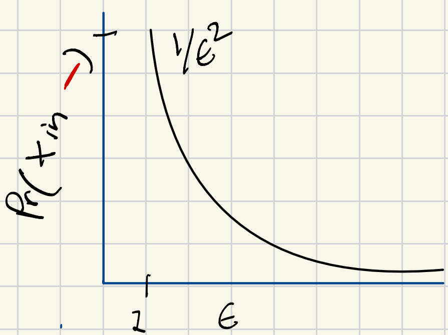
Chebyshev's Inequality

Idea: Use extra information for tighter bound

$\sigma^2 = Var(X)$. For $\epsilon > 0$,

$$\Pr(|X - \mathbb{E}[X]| \ge \epsilon\sigma) \le \frac{1}{\epsilon^2}$$





Proof: $Z = (X - \mathbb{E}[X])^2$

By Markov's,

$$\Pr(Z \ge \epsilon\,\mathbb{E}[Z]) \le \frac{1}{\epsilon}$$

$\Leftrightarrow$

$$\Pr\Big((X - \mathbb{E}[X])^2 \ge \epsilon\, \overbrace{\mathbb{E}[(X - \mathbb{E}[X])^2]}^{\sigma^2}\Big) \le \frac{1}{\epsilon}$$

$\Leftrightarrow$

$$\Pr(|X - \mathbb{E}[X]| \ge \sqrt{\epsilon}\,\sigma) \le \frac{1}{\epsilon}$$

$\Leftrightarrow$

$$\Pr(|X - \mathbb{E}[X]| \ge \epsilon\sigma) \le \frac{1}{\epsilon^2}$$

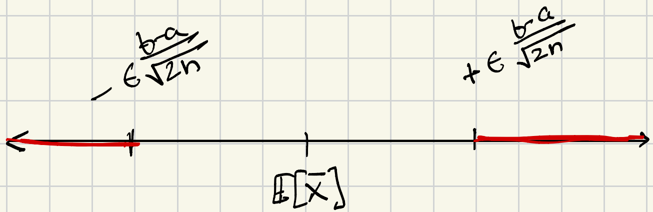
Hoeffding's Inequality

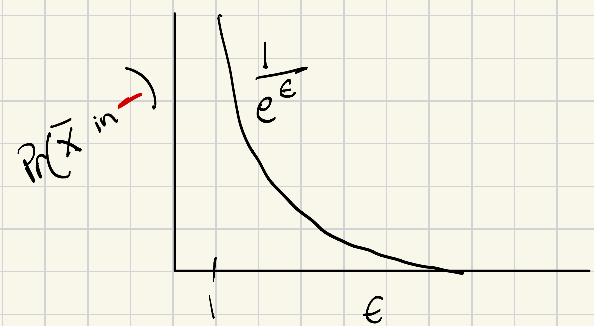
Idea: Formalize central limit theorem

Consider $X_1, \dots, X_n$ where $a < X_i < b$

Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. For $\epsilon > 0$,

$$\Pr\left(|\bar{X} - \mathbb{E}[\bar{X}]| \geq \epsilon \right) \leq 2 \exp\left(-\frac{2n\epsilon^2}{(b-a)^2} \right)$$





Union Bound

$$\Pr(E_1 \cup E_2 \cup \ldots \cup E_m) \leq \Pr(E_1) + \Pr(E_2) + \ldots + \Pr(E_m)$$