Randomized Algorithms 2026 !

This class:

- probability + linear algebra
- data science at scale
- beautiful math

challenging course!

Prereqs: linear algebra, probability, algorithms

Time Expectation: 12.5 hr/week

2.5	1	1	2	1	1	1
class	Problem Set			OH	discussion	"rock"

- Earlier = better
- More drawing = better
- More talking = better

www.rtealwitter.com/rads 2026

- discord for communication
- reading for each lecture
- these slides are online

Plan:

- Math Review
- Streaming and Sketching
- Linear Algebra and Spectral Methods

Course Goal: Use randomness to solve problems more efficiently

Probability

Random variable X

e.g., X = outcome of fair dice

For $x \in \{1,2,3,4,5,6\}$,

$$\Pr(X=x) = 1/6$$

What is X on average?

Expectation of X:

$$\mathbb{E}[X] = \sum_x x \Pr(X=x)$$

$$= \sum_{x=1}^{6} x \, 1/6$$

$$= 1/6\,(1+2+\ldots+6)$$

$$= 21/6$$

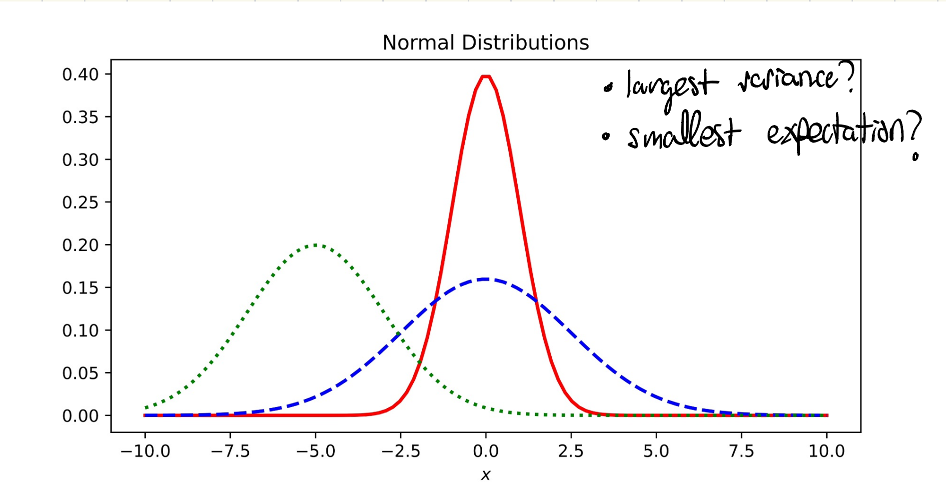
How close is X to its expectation?

Variance of X:

$$\mathrm{Var}(X) = \mathbb{E}[(X-\mathbb{E}[X])^2]$$

$$= \sum_x (x-\mathbb{E}[X])^2 \Pr(X=x)$$

$$=$$



- largest variance?
- smallest expectation?

Multiplying by Scalars

$$\mathbb{E}[\alpha X] = \sum_x \alpha x \Pr(X = x)$$

$$= \alpha \sum_x x \Pr(X = x)$$

$$= \alpha \mathbb{E}[X]$$

$$\mathrm{Var}(\alpha X) =$$

Independent Random Variables

A, B are events

A = dice shows 1 or 2

B = dice roll is odd

"and" "given"

$$\Pr(A \cap B) = \Pr(A)\Pr(B \mid A)$$
$$= \Pr(B)\Pr(A \mid B)$$

"if and only if"

$\Pr(B \mid A) = \Pr(B)$ iff A, B independent

$\Leftrightarrow$ $\Pr(A \cap B) = \Pr(A)\Pr(B)$ iff A, B independent

Example A, B indep?

Random Variables X, Y are indep

iff $\Pr(X = x \cap Y = y) = \Pr(X = x)\Pr(Y = y)$

for all possible outcomes x, y

Independent?

- Heads on 1st flip, Heads on 2nd
- Ace on 1st draw, Ace on 2nd
- Rain, carrying an umbrella

Linearity of Expectation

Thm: $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$

Proof: $\mathbb{E}[X+Y] = \sum_x \sum_y (x+y) \Pr(X=x \cap Y=y)$

$$= \sum_x \sum_y x \Pr(X=x \cap Y=y) + \sum_x \sum_y y \Pr(X=x \cap Y=y)$$

$$= \sum_x x \sum_y \Pr(X=x \cap Y=y) + \sum_y y \sum_x \Pr(Y=y \cap X=x)$$

$$= \sum_x x \Pr(X=x) + \sum_y y \Pr(Y=y)$$

$$= \mathbb{E}[X] + \mathbb{E}[Y]$$

Expected number of shared birthdays?

Useful Facts

If X, Y indep, $\mathbb{E}[XY] = \mathbb{E}[X]\,\mathbb{E}[Y]$

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

If X, Y indep, $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$