Benchmarking Estimators for Natural Experiments[1,2,3]

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Treatment Effect Estimation

Problem Setup: There are n *observations*, each with *covariates* $\boldsymbol{x}_i \in \mathbb{R}^d$ for $i \in [n]$. Each observation i receives the "treatment" with *propensity* $p_i \in (0,1)$ and the "control" otherwise. We then observe either the *treatment outcome* $y_i^{(1)}$ or the *control outcome* $y_i^{(0)}$, but not both.

Goal: Estimate the **average treatment effect**:

$$\tau = \frac{1}{n}\sum_{i\in[n]} y_i^{(1)} - y_i^{(0)}$$

A Novel Treatment Effect Dataset

Reach Out and Read Colorado (RORCO) is an early childhood literacy nonprofit that gives children books at pediatric visits.

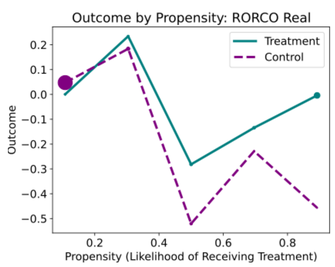
Dataset	Size	Variables	Treated %	BCE	Corr($\mathbf{y}^{(1)}$, $\mathbf{p}$)	Corr($\mathbf{y}^{(0)}$, $\mathbf{p}$)
JOBS	722	8	41.1	0.0856	0.0355	0.0541
TWINS	50820	40	49.4	0.499	-0.00311	-0.0036
IHDP	747	26	18.6	0.452	0.0967	0.0236
NEWS	5000	3	45.8	0.545	0.86	-0.565
ACIC 2016	4802	54	18.4	0.372	0.112	0.0383
ACIC 2017	4302	50	47.4	0.436	-0.269	-0.153
RORCO Real	4178	78	25.3	0.158	-0.000602	-0.0739
RORCO	21663	78	44.3	0.212	-0.986	-0.989

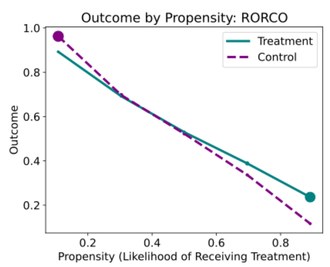
RORCO Real includes student covariates and standardized literacy scores while *RORCO* includes student covariates and synthetic outcomes designed by literacy experts.

Confoundedness

Challenge: RORCO prioritizes under-served children to maximize the positive impact of their work, causing

a) Confounding between outcomes and propensity and
b) Confounding between treatment effect and propensity.





Benchmark

Squared error on RORCO over 100 runs.

Method	Mean	1st Quartile	2nd Quartile	3rd Quartile	Time (s)
Regression Discontinuity	4.65e-03	2.72e-03	3.84e-03	5.52e-03	9.55e-04
Propensity Stratification	2.57e-03	1.52e-03	2.25e-03	3.29e-03	2.78e-03
Direct Difference	4.48e-01	3.57e-01	4.18e-01	5.79e-01	4.74e-04
Adjusted Direct	6.29e-03	5.25e-03	6.20e-03	7.14e-03	1.15e+01
Horvitz-Thompson	1.06e-02	4.29e-03	9.20e-03	1.44e-02	4.65e-04
TMLE	1.19e-01	7.21e-03	2.60e-02	7.43e-02	2.35e+01
Off-policy	3.17e-03	1.86e-03	2.86e-03	4.11e-03	1.14e+01
Double-Double	1.07e-05	1.06e-06	4.41e-06	1.45e-05	2.29e+01
Doubly Robust	9.98e-07	1.48e-07	5.42e-07	1.37e-06	9.89e+00
Direct Prediction	1.36e-02	3.60e-03	1.02e-02	1.94e-02	1.23e+01
SNet	2.57e-02	4.85e-03	1.21e-02	3.62e-02	3.49e+01
FlexTENet	1.15e-03	4.28e-05	1.09e-04	4.95e-04	1.56e+02
OffsetNet	1.10e-03	7.72e-04	9.90e-04	1.41e-03	1.30e+02
TNet	8.05e-04	6.39e-05	2.50e-04	4.37e-04	1.06e+02
TARNet	1.92e-04	2.70e-05	1.04e-04	2.38e-04	1.01e+02
DragonNet	2.18e-02	4.42e-03	1.71e-02	2.46e-02	6.88e+00
SNet3	1.80e-02	3.48e-03	9.80e-03	2.50e-02	2.36e+01
DRNet	5.00e-03	1.53e-04	6.01e-04	2.25e-03	1.14e+02
RANet	7.85e-04	3.67e-05	2.08e-04	7.06e-04	1.91e+02
PWNet	2.28e-01	7.02e-03	4.00e-02	2.82e-01	1.13e+02
RNet	2.96e-03	2.47e-03	2.84e-03	3.43e-03	5.83e+01
XNet	1.00e-03	3.08e-05	2.29e-04	9.26e-04	2.41e+02

Doubly Robust Estimators

Let functions $f^{(1)}, f^{(0)}: \mathbb{R}^d \to \mathbb{R}$ be learned predictions of the treatment and control outcomes, respectively.

$$\hat{\tau}(\mathbf{z}) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i^{(1)} - f^{(1)}(\mathbf{x}_i)}{p_i}\mathbb{1}_{z_i=1} - \frac{y_i^{(0)} - f^{(0)}(\mathbf{x}_i)}{1-p_i}\mathbb{1}_{z_i\neq 1} + f^{(1)}(\mathbf{x}_i) - f^{(0)}(\mathbf{x}_i)\right)$$

Doubly robust estimators are **asymptotically** unbiased if either:

a) The propensity scores are accurate or
b) The functions are accurate.

Non-asymptotic Analysis

Question: Why are doubly robust estimators so accurate in the finite setting?

We exactly analyze doubly robust estimators that learn functions $f^{(1)}, f^{(0)}: \mathbb{R}^d \to \mathbb{R}$ separately from the data they are applied to. For these estimators, the variance is

$$\mathrm{Var}[\hat{\tau}(\mathbf{z}) - \tau] = \frac{1}{n^2}\sum_{i=1}^{n}\mathbb{E}_{\mathbf{z},S_1,S_2}\left[\left((y_i^{(1)} - \hat{f}^{(1)}_{\mathbf{z},S(i)}(\mathbf{x}_i))\sqrt{\frac{1-p_i}{p_i}} + (y_i^{(0)} - \hat{f}^{(0)}_{\mathbf{z},S(i)}(\mathbf{x}_i))\sqrt{\frac{p_i}{1-p_i}}\right)^2\right]$$
$$+\frac{1}{n^2}\sum_{i\neq j}\mathbb{E}_{\mathbf{z},S_1,S_2}\left[\left(\hat{y}_i(\mathbf{z}^{(j\to 1)}) - \hat{y}_i(\mathbf{z}^{(j\to 0)})\right)\left(\hat{y}_j(\mathbf{z}^{(i\to 1)}) - \hat{y}_j(\mathbf{z}^{(i\to 0)})\right)\right].$$

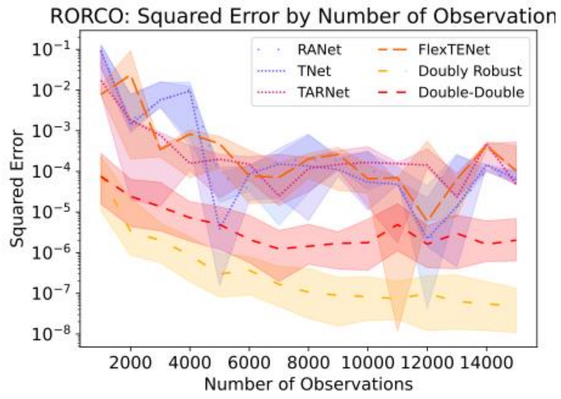
Double-Double Algorithm

Insight: What if we learn the functions to minimize an upper bound on the exact variance?

Method	Mean	1st Quartile	2nd Quartile	3rd Quartile	Time (s)
Doubly Robust	9.98e-07	1.48e-07	5.42e-07	1.37e-06	9.89e+00
DR + Weighting	4.02e-06	5.46e-07	2.62e-06	5.57e-06	9.81e+00
DR + 2x Weighting	3.80e-06	2.48e-07	9.71e-07	3.82e-06	9.80e+00
DR + Split	9.82e-05	3.27e-06	1.21e-05	3.65e-05	2.22e+01
DR + Split + Weight	1.12e-04	2.19e-06	1.03e-05	2.41e-05	2.22e+01
Double-Double	1.07e-05	1.06e-06	4.41e-06	1.45e-05	2.29e+01

Doubly robust estimators *without* the training split perform better but they effectively use twice as much data as doubly robust estimators *with* the training split.

Performance by Sample Size



Natural Experiments Package

```
import naturalexperiments as ne

X, y, z = ne.dataloaders['RORCO']()        # Load datasets

estimator = ne.methods['Double-Double']    # Load estimator

p = ne.estimate_propensity(X, z)           # Estimate propensity

estimated_effect = estimator(X, y, z, p, ne.train)
```

[1]The datasets and code are available at github.com/rtealwitter/naturalexperiments

[2]Our work is available on arXiv at arxiv.org/abs/2409.04500

[3]This work was supported by the National Science Foundation under Grant No. 2045590 and Graduate Research Fellowship Grant No. DGE-2234660