Data Analysis
Week 9: Generalised Linear Models

1 Introduction

In Weeks 3 and 6 we looked at modelling data using linear regression models were we had:

- a **continous response variable** y and
- one or more **explanatory variables** $x_1, x_2, \dots, x_p$, which were **numerical/categorical** variables.

Recall that for data $(y_i, x_i),\ i = 1, \dots, n$, where y is a continuous response variable, we can write a simple linear regression model as follows:

$$y_i = \alpha + \beta x_i + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2),$$

where

- y_i is the i^{th} observation of the continuous response variable;
- α is the **intercept** of the regression line;
- β is the **slope** of the regression line;
- x_i is the i^{th} observation of the explanatory variable; and
- ϵ_i is the i^{th} random component.

Thus, the full probability model for y_i given x_i $(y_i|x_i)$ can be written as

$$y_i|x_i \sim N(\alpha + \beta x_i, \sigma^2),$$

where the mean $\alpha + \beta x_i$ is given by the deterministic part of the model and the variance σ^2 by the random part. Hence we make the assumption that the outcomes y_i are normally distributed with mean $\alpha + \beta x_i$ and variance σ^2. However, what if our response variable y is not a continuous random variable?

1.1 Generalised linear models

The main objective this week is to introduce **Generalised Linear Models (GLMs)**, which extend the linear model framework to response variables that don't follow the normal distribution. GLMs can be used to model non-normal continuous response variables, but they are most frequently used to model binary, categorical or count data. Here we shall focus on binary/categorical response variables. The generalised linear model can be written as:

$$\begin{aligned} y_i &\sim f(g(\boldsymbol{\mu}_i)) \\ \boldsymbol{\mu}_i &= \mathbf{x}_i^\top \boldsymbol{\beta}, \end{aligned}$$

where the response y_i is predicted though the linear combination $\boldsymbol{\mu}_i$ of explanatory variables by the link function $g(\cdot)$, assuming some distribution $f(\cdot)$ for y_i, and $\mathbf{x}_i^\top$ is the i^{th} row of the design matrix $\boldsymbol{X}$. For example, the simple linear regression model above for a continuous response variable has the normal distribution distribution as $f(\cdot)$, with corresponding link function equal to the Identity function, that is, $g(\boldsymbol{\mu}_i) = \boldsymbol{\mu}_i$.

What if our response variable y is binary (e.g. yes/no, success/failure, alive/dead)? That is, the independent responses y_i can either be:

- **binary**, taking the value 1 (say success, with probability p_i) or 0 (failure, with probability $1 - p_i$) or

- **binomial**, where y_i is the number of successes in a given number of trials n_i, with the probability of success being p_i and the probability of failure being $1 - p_i$.

In both cases the distribution of y_i is assumed to be binomial, but in the first case it is $\text{Bin}(1, p_i)$ and in the second case it is $\text{Bin}(n_i, p_i)$. Hence, a binary response variable y_i has a binomial distribution with corresponding link function $g(\cdot)$ equal to the **logit link** function, that is

$$g(p_i) = \log\left(\frac{p_i}{1 - p_i}\right),$$

which is also referred to as the **log-odds** (since $p_i \ / \ 1 - p_i$ is an odds ratio). Why is such a transformation required when looking at a binary response variable? Well here we are interested in modelling the probability of success p_i, and as we know probabilities must be between 0 and 1 ($p_i \in [0, 1]$). So if we want to model the probability of success using a linear model we need to ensure that the probabilities obtained are between 0 and 1. However, if we just use the identity link function, such that

$$p_i = \mathbf{x}_i^\top \boldsymbol{\beta},$$

we would need to ensure that in some way $\mathbf{x}_i^\top \boldsymbol{\beta} \in [0, 1]$, that is, the linear combination of the explanatory variables and their corresponding regression coefficients was between 0 and 1. Hence some restrictions of some sort would need to be put in place to ensure this was the case. However, if we use the logit link function, such that

$$\log\left(\frac{p_i}{1 - p_i}\right) = \mathbf{x}_i^\top \boldsymbol{\beta},$$

no restrictions need to be in place on our estimates of the parameter vector $\boldsymbol{\beta}$, since the inverse of the logit link function will always gives us valid probabilities since

$$p_i = \frac{\exp\left(\mathbf{x}_i^\top \boldsymbol{\beta}\right)}{1 + \exp\left(\mathbf{x}_i^\top \boldsymbol{\beta}\right)} \quad \in [0, 1].$$

This linear regression model with a binary response variable is referred to as **logistic regression**. As such, when it comes to looking at binary response variables we shall be looking at odds ratios and probabilities of success/failure. The table below is a reminder of the distribution and link function used for the normal model we have previously looked at as well as the logistic regression model we shall be examining for the rest of this week.

Model	Random component	Systematic component	Link function
Normal	$y_i \overset{\text{indep}}{\sim} \text{N}(\mu_i, \sigma^2),$	$\boldsymbol{x}_i^\top \boldsymbol{\beta} = \beta_0 + \beta_1 x_i + \beta_2 x_i + \dots$	$g(\mu_i) = \mu_i$
Logistic	$y_i \overset{\text{indep}}{\sim} \text{Bin}(1, p_i),$	$\boldsymbol{x}_i^\top \boldsymbol{\beta} = \beta_0 + \beta_1 x_i + \beta_2 x_i + \dots$	$g(\mu_i) = \log\left(\frac{p_i}{1-p_i}\right)$

Required R packages

Before we proceed, load all the packages needed for this week:

```
library(dplyr)
library(ggplot2)
library(moderndive)
library(gapminder)
library(sjPlot)
library(stats)
```

```
library(readr)
```

2 Logistic regression with one numerical explanatory variable

Here we shall begin by fitting a logistic regression model with one numerical explanatory variable. Let's return to the `evals` data from the `moderndive` package that we examined in Week 3.

2.1 Teaching evaluation scores

Student feedback in higher education is extremely important when it comes to the evaluation of teaching techniques, materials, and improvements in teaching methods and technologies. However, there have been studies into potential bias factors when feedback is provided, such as the physical appearance of the teacher; see Economics of Education Review for details. Here, we shall look at a study from student evaluations of $n = 463$ professors from The University of Texas at Austin.

Previously, we looked at **teaching score** as our continuous response variable and **beauty score** as our explanatory variable. Now we shall consider **gender** as our response variable, and hence shall have a binary response variable (female/male). We will examine if there is any difference in **gender** by **age** of the teaching instructors within the `evals` data set.

First, let's start by selecting the variables of interest from the `evals` data set:

```
evals.gender <- evals %>%
                  select(gender, age)
```

```
# A tibble: 6 x 2
  gender   age
  <fct>  <int>
1 female    36
2 female    36
3 female    36
4 female    36
5 male      59
6 male      59
```

Now, let's look at a boxplot of `age` by `gender` to get an initial impression of the data:

```
ggplot(data = evals.gender, aes(x = gender, y = age, fill = gender)) +
  geom_boxplot() +
  labs(x = "Gender", y = "Age")
```

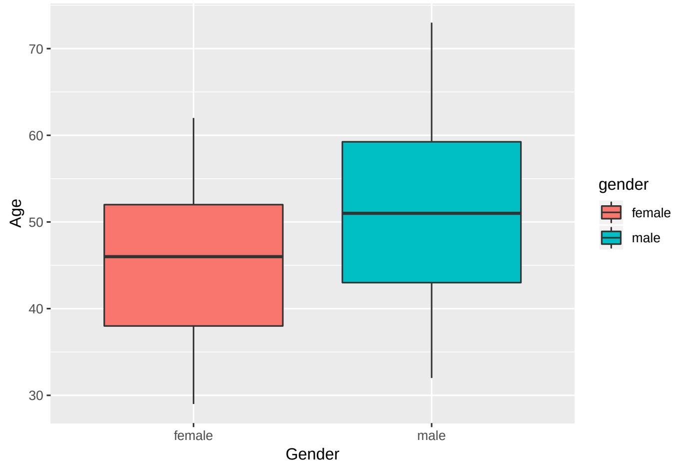


Figure 1: Teaching instructor age by gender.

Here we can see that the age of male teaching instructors tends to be higher than that of their female counterparts. Now, let's fit a logistic regression model to see whether age is a significant predictor of the odds of a teaching instructor being male or female.

2.2 Log-odds

To fit a logistic regression model we will use the generalised linear model function `glm`, which acts in a very similar manner to the `lm` function we have used previously. We only have to deal with an additional argument. The logistic regression model with **gender** as the response and **age** as the explanatory variable is given by:

```
model <- glm(gender ~ age, data = evals.gender, family = binomial(link = "logit"))
```

Here we include the additional `family` argument, which states the distribution and link function we would like to use. Hence `family = binomial(link = "logit")` states we have a binary response variable, and thus have a binomial distribution, with its corresponding **logit link** function. Now, let's take a look at the summary produced from our logistic regression model:

```
model %>%
  summary()
```

```

Call:
glm(formula = gender ~ age, family = binomial(link = "logit"),
    data = evals.gender)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.7134  -1.1815   0.7238   1.0180   1.4778

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.69795    0.51194  -5.270 1.36e-07 ***
age          0.06296    0.01059   5.948 2.71e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 630.30  on 462  degrees of freedom
Residual deviance: 591.41  on 461  degrees of freedom
AIC: 595.41

Number of Fisher Scoring iterations: 4
```

Firstly, the baseline category for our binary response is `female`. This is due to the default baseline in R being taken as the one which comes first alphabetically, which can be seen from the `levels` function:

```r
levels(evals.gender$gender)
```

```
[1] "female" "male"
```

This means that estimates from the logistic regression model are for a change on the **log-odds** scale for `males` in comparison to the response baseline `females`. That is

$$\ln\left(\frac{p}{1-p}\right) = \alpha + \beta \cdot \text{age} = -2.7 + 0.06 \cdot \text{age},$$

where $p = \text{Prob}\left(\text{Male}\right)$ and $1 - p = \text{Prob}\left(\text{Female}\right)$. Hence, the **log-odds** of the instructor being male increase by 0.06 for every one unit increase in `age`. This provides us with a point estimate of how the log-odds changes with age, however, we are also interested in producing a 95% confidence interval for these log-odds. This can be done as follows:

```r
mod.coef.logodds <- model %>%
                      summary() %>%
                      coef()
```

```r
age.logodds.lower <- (mod.coef.logodds["age", "Estimate"]
                        - 1.96 * mod.coef.logodds["age", "Std. Error"])
```

```
[1] 0.04221777
```

```r
age.logodds.upper <- (mod.coef.logodds["age", "Estimate"]
                        + 1.96 * mod.coef.logodds["age", "Std. Error"])
```

```
[1] 0.08371167
```

Hence the point estimate for the log-odds is 0.06, which has a corresponding 95% confidence interval of (0.04, 0.08). This can be displayed graphically using the `plot_model` function from the `sjPlot` package by simply passing our `model` as an argument:

```r
plot_model(model, show.values = TRUE, transform = NULL,
           title = "Log-Odds (Male instructor)", show.p = FALSE)
```

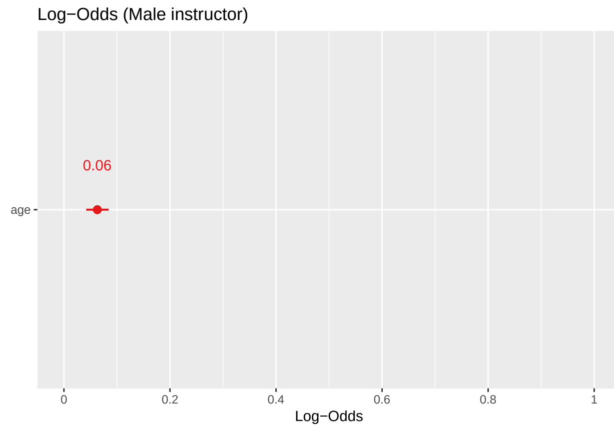


Figure 2: The log-odds of age for male instructors.

Some of the interesting arguments that can be passed to the `plot_model` function are:

- `show.values = TRUE/FALSE`: Whether the log-odds/odds values should be displayed;
- `show.p = TRUE/FALSE`: Adds asterisks that indicate the significance level of estimates to the value labels;
- `transform`: A character vector naming the function that will be applied to the estimates. The default transformation uses `exp` to display the odds ratios, while `transform = NULL` displays the log-odds; and
- `vline.color`: colour of the vertical "zero effect" line.

Further details on using `plot_model` can be found here. Now, let's add the estimates of the log-odds to our data set:

```
evals.gender <- evals.gender %>%
                  mutate(logodds.male = predict(model))
```

```
# A tibble: 6 x 3
  gender   age logodds.male
  <fct>  <int>        <dbl>
1 female    36       -0.431
2 female    36       -0.431
3 female    36       -0.431
4 female    36       -0.431
5 male      59        1.02
6 male      59        1.02
```

2.3 Odds

Typically we would like to work on the **odds** scale as it is easier to interpret an odds-ratio as opposed to the log-odds-ratio. To obtain the odds we simply exponentiate the log-odds, that is

$$\frac{p}{1-p} = \exp\left(\alpha + \beta \cdot \text{age}\right),$$

```
model %>%
 coef() %>%
  exp()
```

```
(Intercept)         age
 0.06734369  1.06498927
```

On the odds scale, the value of the intercept (0.07) gives the odds of a teaching instructor being male given their `age = 0`, which is obviously not a viable age for a teaching instructor, and hence why this value is very close to zero. For `age` we have an odds of 1.06, which indicates that for every 1 unit increase in age, the odds of the teaching instructor being male increase by a factor of 1.06. So how is this calculated? Let's look at the odds-ratio obtained from instructors aged 51 and 52 years old, that is, a one unit difference:

$$\frac{\text{Odds}_{\text{age}=52}}{\text{Odds}_{\text{age}=51}} = \left(\frac{\frac{p_{\text{age}=52}}{1-p_{\text{age}=52}}}{\frac{p_{\text{age}=51}}{1-p_{\text{age}=51}}} \right) = \frac{\exp\left(\alpha + \beta \cdot 52\right)}{\exp\left(\alpha + \beta \cdot 51\right)} = \exp\left(\beta \cdot (52-51)\right) = \exp\left(0.06\right) = 1.06.$$

For example, the odds of a teaching instructor who is 45 years old being male is given by

$$\frac{p}{1-p} = \exp\left(\alpha + \beta \cdot \text{age}\right) = \exp\left(-2.7 + 0.06 \cdot 45\right) = 1.15.$$

This can be interpreted as the chances of an instructor who is 45 being male are 15% greater than them being female. We can obtain a 95% confidence interval for the odds by simply exponentiating the lower and upper bounds of our log-odds interval:

```r
age.odds.lower <- exp(age.logodds.lower)
```

```
[1] 1.043122
```

```r
age.odds.upper <- exp(age.logodds.upper)
```

```
[1] 1.087315
```

Hence the point estimate for the odds is 1.06, which has a corresponding 95% confidence interval of (1.04, 1.09). This can be displayed graphically using the `plot_model` function from the `sjPlot` package by simply passing our `model` as an argument as well as removing `transform = NULL` (the default transformation is exponential):

```r
plot_model(model, show.values = TRUE,
           title = "Odds (Male instructor)", show.p = FALSE)
```

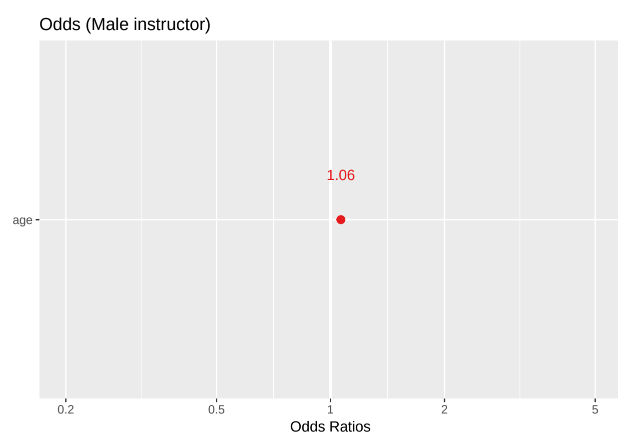


Figure 3: The odds of age for male instructors.

Note: As the 95% confidence interval is so narrow it is hard to see it displayed in the plot, but it is included by default.

Now, let's add the estimates of the odds to our data set:

```
evals.gender <- evals.gender %>%
                  mutate(odds.male = exp(logodds.male))
```

```
# A tibble: 6 x 4
  gender   age logodds.male odds.male
  <fct>  <int>        <dbl>     <dbl>
1 female    36       -0.431     0.650
2 female    36       -0.431     0.650
3 female    36       -0.431     0.650
4 female    36       -0.431     0.650
5 male      59        1.02      2.76
6 male      59        1.02      2.76
```

2.4 Probabilities

We can obtain the probability $p = \text{Prob}(\text{Male})$ using the following transformation:

$$p = \frac{\exp\left(\alpha + \beta \cdot \text{age}\right)}{1 + \exp\left(\alpha + \beta \cdot \text{age}\right)}.$$

For example, the probability of a teaching instructor who is 52 years old being male is

$$p = \frac{\exp\left(\alpha + \beta \cdot \text{age}\right)}{1 + \exp\left(\alpha + \beta \cdot \text{age}\right)} = \frac{\exp\left(-2.697946 + 0.0629647 \cdot 52\right)}{1 + \exp\left(-2.697946 + 0.0629647 \cdot 52\right)} = 0.64,$$

which can be computed in R as follows:

```
p.num <- (exp(mod.coef.logodds["(Intercept)", "Estimate"]
              + mod.coef.logodds["age", "Estimate"] * 52))
p.denom <- 1 + p.num
p.num / p.denom
```

```
[1] 0.6401971
```

The `plogis()` function from the `stats` library can also be used to obtain probabilities from the log-odds:

```
plogis(mod.coef.logodds["(Intercept)", "Estimate"]
        + mod.coef.logodds["age", "Estimate"] * 52)
```

```
[1] 0.6401971
```

Let's add the probabilities to our data, which is done using the `fitted()` function:

```
evals.gender <- evals.gender %>%
                  mutate(probs.male = fitted(model))
```

Note: `predict(model, type = "response")` will also provide the estimated probabilities.

Finally, we can plot the probability of being male using the `geom_smooth()` function by giving `method = "glm"` and `methods.args = list(family = "binomial")` as follows:

```r
ggplot(data = evals.gender, aes(x = age, y = probs.male)) +
  geom_dotplot(dotsize = 0.6, alpha = 0.2, aes(fill = gender),
               binwidth = 1, stackgroups = TRUE) +
  geom_smooth(method = "glm", method.args = list(family = "binomial"), se = FALSE) +
  labs(x = "Age", y = "Probability of instructor being male")
```

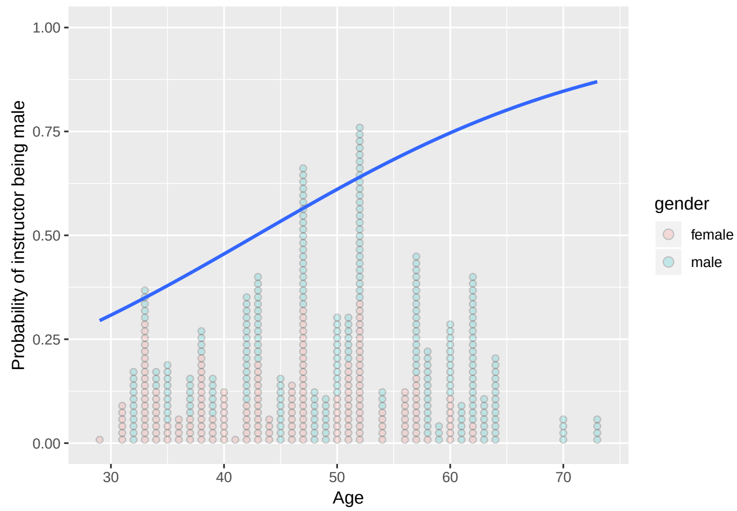


Figure 4: Probability of teaching instructor being male by age.

Note: the ages of all teaching instructors have been superimposed as a dotplot using `geom_dotplot()`.

The `plot_model()` function from the `sjPlot` package can also produce the estimated probabilities by `age` as follows:

```r
plot_model(model, type = "pred", title = "",
           axis.title = c("Age", "Prob. of instructor being male"))
```

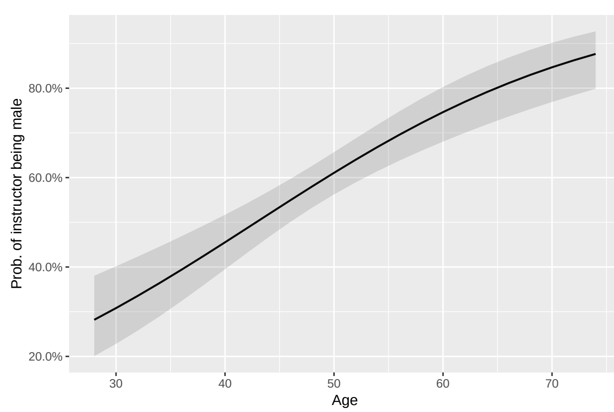


Figure 5: Probability of teaching instructor being male by age.

3 Logistic regression with one categorical explanatory variable

Instead of having a numerical explanatory variable such as `age`, let's now use the binary categorical variable `ethnicity` as our explanatory variable.

```
evals.ethnic <- evals %>%
                  select(gender, ethnicity)
```

```
# A tibble: 6 x 2
  gender ethnicity
  <fct>  <fct>
1 female minority
2 female minority
3 female minority
4 female minority
5 male   not minority
6 male   not minority
```

Now, let's look at a barplot of `gender` and `ethnicity` to get an initial impression of the data. First, we need to create a table providing the proportion of males/females who are in the minority/not minority:

```
gender.ethnic <- table(evals.ethnic$gender, evals.ethnic$ethnicity)
gender.ethnic <- prop.table(gender.ethnic, 2)
gender.ethnic <- as.data.frame(gender.ethnic)
```

```
  gender    ethnicity      prop
1 female     minority 0.5625000
2   male     minority 0.4375000
3 female not minority 0.3984962
4   male not minority 0.6015038
```

```
ggplot(data = gender.ethnic, aes(x = gender, y = prop, fill = ethnicity)) +
  geom_col(position = "dodge") +
  labs(x = "Gender", y = "Proportion")
```

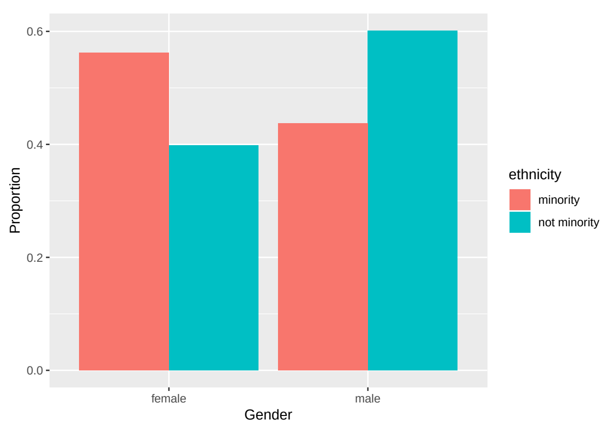


Figure 6: Barplot of teaching instructors' gender by ethnicity.

We can see that a larger proportion of instructors in the `minority` ethnic group are female (56.3% vs 43.8%), while the `not minority` ethnic group is comprised of more male instructors (60.02% vs 39.85%). Now we shall fit a logistic regression model to determine whether the gender of a teaching instructor can be predicted from their ethnicity.

3.1 Log-odds

The logistic regression model is given by:

```
model.ethnic <- glm(gender ~ ethnicity, data = evals.ethnic,
                    family = binomial(link = "logit"))
```

```
model.ethnic %>%
  summary()
```

```

Call:
glm(formula = gender ~ ethnicity, family = binomial(link = "logit"),
    data = evals.ethnic)

Deviance Residuals:
   Min      1Q  Median      3Q     Max
-1.357  -1.357   1.008   1.008   1.286

Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
(Intercept)            -0.2513     0.2520  -0.997   0.3186
ethnicitynot minority   0.6630     0.2719   2.438   0.0148 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 630.30  on 462  degrees of freedom
Residual deviance: 624.29  on 461  degrees of freedom
AIC: 628.29

Number of Fisher Scoring iterations: 4
```

Again, the baseline category for our binary response is `female`. Also, the baseline category for our explanatory variable is `minority`, which, like `gender`, is done alphabetically by default by R:

```
levels(evals.ethnic$ethnicity)
```

```
[1] "minority"     "not minority"
```

This means that estimates from the logistic regression model are for a change on the **log-odds** scale for `males` ($p = \text{Prob}(\text{Males})$) in comparison to the response baseline `females`. That is

$$\ln\left(\frac{p}{1-p}\right) = \alpha + \beta \cdot \text{ethnicity} = -0.25 + 0.66 \cdot \mathbb{I}_{\text{ethnicity}}(\text{not minority}),$$

where $\mathbb{I}_{\text{ethnicity}}(\text{not minority})$ is an indicator function. Hence, the **log-odds** of an instructor being male increase by 0.66 if they are in the ethnicity group `not minority`. This provides us with a point estimate of how the log-odds changes with ethnicity, however, we are also interested in producing a 95% confidence interval for these log-odds. This can be done as follows:

```
mod.ethnic.coef.logodds <- model.ethnic %>%
                            summary() %>%
                            coef()
```

```
ethnic.logodds.lower <- (mod.ethnic.coef.logodds["ethnicitynot minority", "Estimate"]
                         - 1.96 *
                          mod.ethnic.coef.logodds["ethnicitynot minority", "Std. Error"])
```

```
[1] 0.1300587
```

```
ethnic.logodds.upper <- (mod.ethnic.coef.logodds["ethnicitynot minority", "Estimate"]
                         + 1.96 *
                          mod.ethnic.coef.logodds["ethnicitynot minority", "Std. Error"])
```

```
[1] 1.19604
```

Hence the point estimate for the log-odds is 0.66, which has a corresponding 95% confidence interval of (0.13, 1.2). This can be displayed graphically using the `plot_model` function from the `sjPlot` package by simply passing our `model` as an argument:

```
plot_model(model.ethnic, show.values = TRUE, transform = NULL,
           title = "Log-Odds (Male instructor)", show.p = FALSE)
```

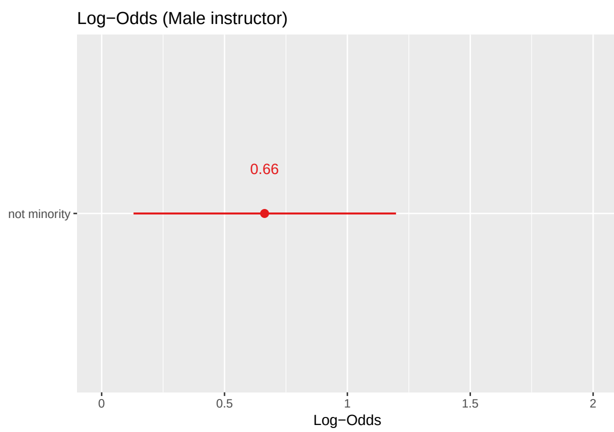


Figure 7: The log-odds for male instructors by ethnicity (not a minority).

Now, let's add the estimates of the log-odds to our data set:

```
evals.ethnic <- evals.ethnic %>%
                  mutate(logodds.male = predict(model.ethnic))
```

```
# A tibble: 6 x 3
  gender ethnicity    logodds.male
  <fct>  <fct>               <dbl>
1 female minority           -0.251
2 female minority           -0.251
3 female minority           -0.251
4 female minority           -0.251
5 male   not minority        0.412
6 male   not minority        0.412
```

3.2 Odds

On the **odds** scale the regression coefficients are given by

```r
model.ethnic %>%
 coef() %>%
  exp()
```

```
          (Intercept) ethnicitynot minority
            0.7777778             1.9407008
```

The `(Intercept)` gives us the odds of the instructor being male given that they are in the `minority` ethnic group, that is, 0.78 (the indicator function is zero in that case). The odds of the instructor being male given they are in the `not minority` ethnic group are 1.94 times greater than the odds if they were in the `minority` ethnic group.

Before moving on, let's take a look at how these values are computed. First, the odds of the instructor being male given that they are in the `minority` ethnic group can be obtained as follows:

$$\frac{p_{\text{minority}}}{1 - p_{\text{minority}}} = \exp\left(\alpha\right) = \exp\left(-0.25\right) = 0.78.$$

```r
# the number of instructors in the minority
pmin <- evals.ethnic %>%
              filter(ethnicity == "minority") %>%
              summarize(n()) %>%
              pull()

# the number of male instructors in the minority
pmin.male <- evals.ethnic %>%
              filter(ethnicity == "minority", gender == "male") %>%
              summarize(n()) %>%
              pull()

# the proportion/probability of males in the minority
prob.min.male <- pmin.male / pmin
odds.min.male <- prob.min.male / (1 - prob.min.male)
odds.min.male
```

```
[1] 0.7777778
```

Now, the odds-ratio of an instructor being male in the `not minority` compared to the `minority` ethnic group is found as follows:

$$\frac{\text{Odds}_{\text{not minority}}}{\text{Odds}_{\text{minority}}} = \frac{\frac{p_{\text{not minority}}}{1-p_{\text{not minority}}}}{\frac{p_{\text{minority}}}{1-p_{\text{minority}}}} = \frac{\exp\left(\alpha+\beta\right)}{\exp\left(\alpha\right)} = \exp\left(\alpha+\beta-\alpha\right) = \exp\left(\beta\right) = \exp\left(0.66\right) = 1.93.$$

```r
# the number of instructors not in the minority
pnotmin <- evals.ethnic %>%
              filter(ethnicity == "not minority") %>%
              summarize(n()) %>%
              pull()

# the number of male instructors not in the minority
pnotmin.male <- evals.ethnic %>%
              filter(ethnicity == "not minority", gender == "male") %>%
              summarize(n()) %>%
              pull()
```

```
# the proportion/probability of males not in the minority
prob.notmin.male <- pnotmin.male / pnotmin
odds.notmin.male <- prob.notmin.male / (1 - prob.notmin.male)
odds.ratio.notmin <- odds.notmin.male / odds.min.male
```

```
[1] 1.940701
```

We can obtain a 95% confidence interval for the odds by simply exponentiating the lower and upper bounds of the log-odds interval:

```
ethnic.odds.lower <- exp(ethnic.logodds.lower)
```

```
[1] 1.138895
```

```
ethnic.odds.upper <- exp(ethnic.logodds.upper)
```

```
[1] 3.306994
```

Hence the point estimate for the odds-ratio is 1.94, which has a corresponding 95% confidence interval of (1.14, 3.31). Again, we can display this graphically using the `plot_model` function from the `sjPlot` package:

```
plot_model(model.ethnic, show.values = TRUE,
           title = "Odds (Male instructor)", show.p = FALSE)
```

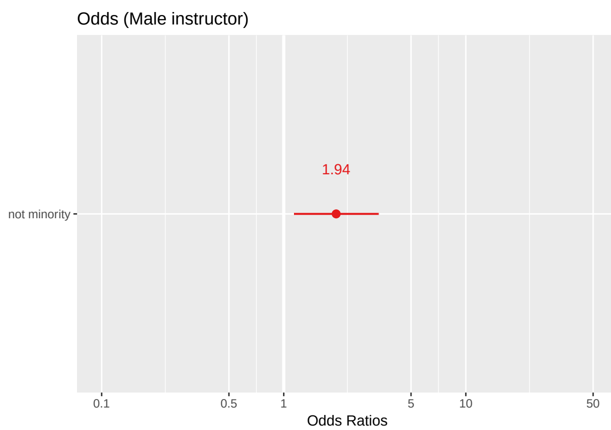


Figure 8: The odds-ratio of a male instructor given they are in the ‘not minority‘ group.

Now, let’s add the estimates of the odds to our data set:

```
evals.ethnic <- evals.ethnic %>%
                  mutate(odds.male = exp(logodds.male))
```

```
# A tibble: 6 x 4
  gender ethnicity    logodds.male odds.male
  <fct>  <fct>               <dbl>     <dbl>
1 female minority           -0.251     0.778
2 female minority           -0.251     0.778
3 female minority           -0.251     0.778
4 female minority           -0.251     0.778
5 male   not minority        0.412     1.51
6 male   not minority        0.412     1.51
```

3.3 Probabilities

The probabilities of an instructor being male given they are in the `minority` and `not minority` groups are

```
plogis(mod.ethnic.coef.logodds[, "Estimate"])
```

```
          (Intercept) ethnicitynot minority
             0.437500              0.659945
```

Hence, the probabilities of an instructor being male given they are in the `minority` and `not minority` ethnic groups are 0.437 and 0.66, respectively. Let's add the probabilities to our data:

```
evals.ethnic <- evals.ethnic %>%
                  mutate(probs.male = fitted(model.ethnic))
```

Finally, we can use the `plot_model()` function from the `sjPlot` package to produce the estimated probabilities by `ethnicity` as follows:

```
plot_model(model.ethnic, type = "pred", title = "",
            axis.title = c("Ethnicity", "Prob. of instructor being male"))
```

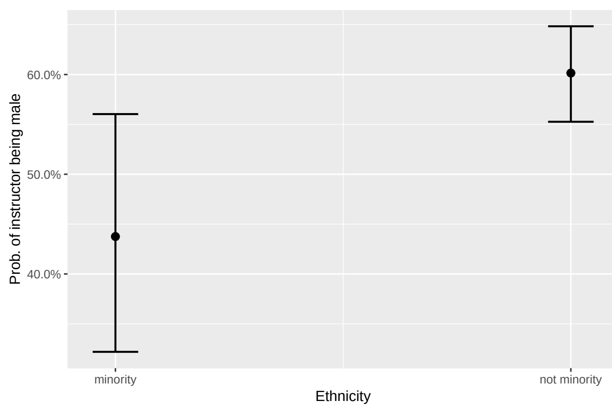


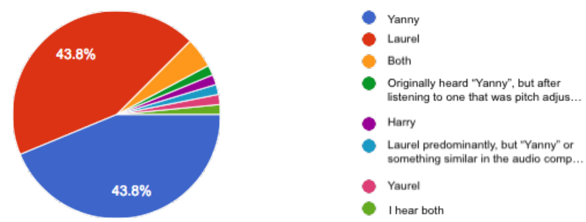
Figure 9: Probability of teaching instructor being male by ethnicity.

4 Further Tasks

4.1 Yanny or Laurel?

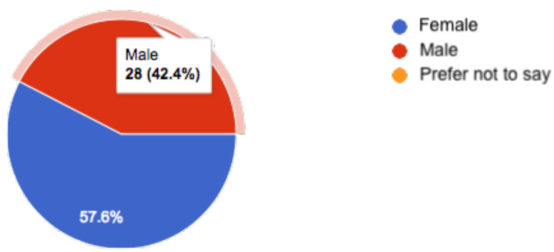
This auditory illusion first appeared on the internet in May 2018. An explanation of why people hear different things can be found in this short video, just one of many internet sources discussing the phenomenon. The main reason behind the difference appears to be that as we age we lose the ability to hear certain sounds. To see if we could find evidence of such an age effect, we asked students and staff at the School of Mathematics and Statistics at the University of Glasgow to fill out a survey on what they hear. Below you can see summaries of the responses.

66 responses



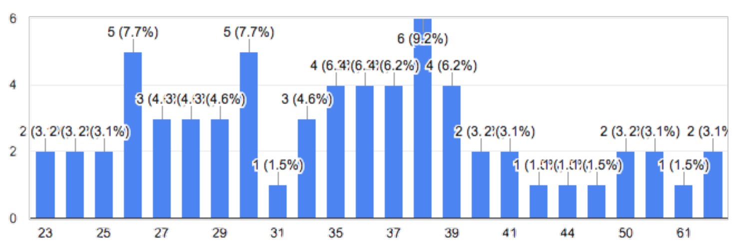
What is your gender?

66 responses



What is your age?

66 responses



The proportions hearing `Yanny` and `Laurel` are very similar to each other, and there are some respondents who hear both or even something completely different. This may be because people do not listen to the audio file using the same device, something we couldn't control for in the survey. Ignoring the responses other than `Yanny` or `Laurel`, we have 53 observations.

Download the data (`yanny.csv`) from Moodle and fit a logistic regression model with `hear` as the binary response variable, and `age` and `gender` as the explanatory variables. What are your findings?

4.2 Titanic

On 15th April 1912, during its maiden voyage, the Titanic sank after colliding with an iceberg, killing 1502 out of 2224 passengers and crew. One of the reasons that the shipwreck led to such loss of life was that there were not enough lifeboats for the passengers and crew. Although there was some element of luck involved in surviving the sinking, some groups of people were more likely to survive than others, such as women, children, and the upper-class.

Download the data (`titanic.csv`) from Moodle for $n = 891$ passengers aboard the Titanic and fit a logistic regression model with `survived` as the binary response variable, and `age`, `gender`, and `passenger.class` as the explanatory variables. What are your findings?