Chapter 1

Introduction to Probability and Counting

Section 1.1

2. $\frac{30}{75}$; Relative Frequency
4. $\frac{2}{50}$; Classical

Section 1.2

6(a)

(b) $S = \{sss, ssf, sfs, sff, fss, fsf, ffs, fff\}$

(c) $A_1 = \{sss, ssf, sfs, sff\}$
$A_2 = \{sss, ssf, fss, fsf\}$
$A_3 = \{sss, ssf, fsf, ffs\}$
no; a successful firing of one shell does not precludes a successful firing of another; $A_I \cap A_j \neq \phi$

(d) A_1' is the event that the first firing is not successful
$A_1' = \{fss, fsf, ffs, fff\}$

(e) $A_1 \cap A_2' \cap A_3'$ is the event that the first firing is not successful and the second and third are not.
$A_1 \cap A_2' \cap A_3' = \{sff\}$

(f) The 8 sample points in S are equally likely.

$\mathrm{P}[A_1 \cap A_2' \cap A_3'] = \frac{1}{8}$

8(a)

8(b) yes, in Example 1.2.3 the sampling continues indefinitely so the tree branches indefinitely. The tree drawn here ends after 5 branches because the missile battery can fire five missiles at a time.

8(c) $S = \{h, mh, mmh, mmmh, mmmmh, mmmmm\}$

8(d) $A_1 = \{mh\}$
$A_2 = \{h, mh\}$
no; $A_1 \cap A_2 = \{mh\} \neq \phi$

Section 1.3

10.(a) $(4)(3) = 12$

(b) $(4)(3)(5) = 60$

(b) $(4)(3)(5)(6) = 360$

12. $(2)(2)(2) = 8$

14.(a) $2^4 = 16$

(b) $2^x = 32 \implies x = 5$

16.(a) $(3)(4)(3) = 36$

(b) $(36)(5) = 180$

18. $\binom{8}{3} = 56; \frac{1}{56}$

20.(a) $\binom{2000}{120}$

(b) $\binom{2000}{119}$

(c) $\frac{1}{\binom{2000}{119}}$

22. $1 - \frac{\binom{17}{5}}{\binom{20}{5}} = 0.601$

24. $\binom{128}{2} = 8,128$

26.(a) $3^6 = 729$

(b) $\frac{1}{729} = 0.00137$

(c) $\binom{6}{2}\binom{4}{2}\binom{2}{2} = 90$
or
$\frac{6!}{2!2!2!} = 90$

Chapter 2

Some Probability Laws

Section 2.1

2.(a) $\frac{5}{35}+\frac{4}{35}+\frac{1}{35}=\frac{10}{35}$

(b) $1-\frac{11}{35}=\frac{24}{35}$

4. P$[B\cap M]$ = .95 + .80 - .99 = .76
P$[M'\cap B]$ = .80 - .76 = .04
P$[B'\cap M]$ = .95 - .76 = .19
P$[(M\cup B)']$ = 1 - .99 = .01

6. P$[O\cap SW]$ = .75 +1.5 - .85 = .05
P$[SW\cap O']$ = .15 - .05 = .10

8. P$[LD\cap SL]$ = .5 - .35 = .15
P$[SL'\cap LD']$ = P$[(SL\cup LD)']$ = 1 - .6 = .4

10. P[H-E] = .8 - .35 = .45

Section 2.2

14.(a) P[B$|M'$] = .8

(b) yes, P[B$|M'$], as we would expect, failure of main engine should not influence the reliability of the backup engine.

16.(a) $10^2=100$

(b) (1)(10) = 10

(c) (10)(1) = 10

(d) (1)(1) = 1

(e) P[number ends with 9 | number begins with a 2]

$$=\frac{P[\text{number ends with 9}|\text{number begins with a 2}]}{P[\text{number begins with a 2}]}$$
$$=\frac{1/100}{10/100}$$
$$=\frac{1}{10}$$

Section 2.3

18. P[$A_1 \cap A_2$] = P[A_1].P[A_2], if A_1 and A_2 are independent events
    = (.5)(.7)
    = .35

20. yes, since P[A_1] = P[$A_1|A_2$]

20. P[Rh negative] = (.39)(.39) = .1521
    P[AB negative] = P[AB].P[Rh negative]
    = (.04)(.1521)
    = .0061

24. P[A ∩ B] = P[A | B].P[A]
    =(.40)(.10)
    =.04

26. P[yes] = (.17)(.5)+(.03)(.5) = .10
    no, P[yes] ≠ P[yes | asked about barn]

28. P[A ∩ ϕ] = P[ϕ] = 0
    =P[A].0
    =P[A].P[ϕ]

30. H : Power line is hit during storm;
    D :Hard derive is damaged;
    P[H∩D] =P[D| H]P[H]
    =(.5)(.001)
    =.0005

32. $A_1 \cap A_2 = \phi$
    $\Longrightarrow$ P[$A_1 \cap A_2$] = P[ϕ]
    = 0 ≠ P[A_1]P[A_2] > 0

Section 2.4

34. P[B | TA] = $\dfrac{(.04)(.09)}{(.88)(.41) + (.04)(.09) + (.10)(.04) + (.o4)(.46)}$

36. D :chip is defective; T : chip is stolen

    P[T|D] = $\dfrac{P[D|T]P[T]}{P[D|T]P[T] + P[D|T']P[T']}$
    = $\frac{(.50)(.01)}{(.50)(.01)+(.05)(.99)}$
    = .0917

Chapter 3

Discrete Distributions

SECTION 3.1

2. discrete

4. not discrete

6. discrete

8. (a) f(8) = 1 -(.02+.03+.05+.2+.4+.2+.07) = .03

(b)

x	1	2	3	4	5	6	7	8
f(x)	.02	.05	.10	.30	.70	.90	.97	1.00

(c) P[$3 \leq X \leq 5$] =F(5) - F(2) = .7 - .05 = .65

(d) P[$X \leq 4$] = F(4) = .3
P[X<4] = F(3) =.1
no

(e) F(-3) = P[X$\leq$ 3] = 0
F(10) = P[X$\leq$ 10] =1

10.(a)

x	f(x)
1	.7
2	.21 =(.3)(.7)
3	.063 = $(.3)^2(.7)$
4	.0189 =$(.3)^3(.7)$

(b) $f(x) = \begin{cases} (0.3)^{x-1}(0.7), & x = 1, 2, 3, ... \\ x, & otherwise \end{cases}$

(c) P[X =6] f(6) = $(.3)^5(.7)$ = .0017

(d) In general, for any real number x_0
F(x_0) = $P[X \leq x_0] = \sum_{x=1}^{x_0} (.3)^{x-1}(.7)$
where $[x_0]$ is the greatest integer less than or equal to x_0.
but for x_0 a positive integer, F(x_0) is sum of the first x_0 term in a geometric series, and is therefore
$F(x_0) = \frac{(.7)(1-.3^{x_0})}{(1-.3)} = 1 - .3^{x_0}$
Thus F(x) =1 -$(.3)^{x_0}$, for = 1, 2, 3.....

(e) P[X$\leq$ 4] = F(4) = 1 - $(.3)^4$ = .9919

(f) P[X$\geq$ 5] = 1 - P[X$\leq$ 4] = $.3^4$ = .0081

12.(a)

x	0	1	2	3	4	5	6
f(x)	.05	.10	.20	.30	.20	.10	.05

Section 3.3

14.(a) E[X] = 0(.7)+1(.2)+2(.05)+3(.03)+4(.01)+5(.01) = .48

(b) μ_X = E[X] = .48

(c) E[X^2] = 0^2(.7)+1^2(.2)+2^2(.05)+3^2(.03)+4^2(.01)+5^2(.01) = 1.08

(d) VarX =E[X^2] - $(E[X])^2$ = .8496

(e) σ_X^2 = VarX =.8496

(f) $\sigma_X = \sqrt{.8496}$ = .9217

(g) grafts that fails

16.(a)

x	0	1	2	3
f(x)	.001	..027	.243	.729

E[X] = 0(.001)+1(.027)+2(.243)+3(.729) = 2.7
VarX = E[X^2] - $(E[X])^2$ = 7.56 - $(2.7)^2$ = .27
E[X] = (n)(p) = (3)(.9) and varX = (n)(p)(1 - p) = (3)(.9)(.1)

18. f(y) = $(.5)^{y-1}$(.5), 1, 2, 3, ...
E[Y] = $\sum_{y=1}^{\infty} y.(.5)^{y-1} = 1(.5) + 2(.5)(.5) + 3(.5)^2(.5) + 4(.5)^3(.5) + ...$
.5E[Y] = (.5)(.5)+2$(.5)^2$(.5) + 3$(.5)^3$(.5) + 4$(.5)^4$(.5)
Thus, E[Y] - .5E[Y] = .5+(.5)(.5)+$(.5)^2$(.5)+$(.5)^3$(.5)+...

$=\sum_{x=1}^{\infty}(.5)(.5)^{x-1}$, which is geometric series
$=\frac{.5}{1-.5}$
= 1
Therefore, .5E[Y] =1 $\implies$ E[Y] = 2

20. VarX = E[c^2] - $(E[c])^2$ = $c^2 - (c)^2 = 0$
VarcX = E[$(cX)^2$] - $(E[cX])^2$ = VarX = E[c^2X^2] - $(cE[X])^2$
$=c^2E[X^2] - c^2(E[X])^2 = c^2(E[X^2] - E[X]^2)$
$= c^2VarX$

22.(a) $\sum_{allx} f(x) = \frac{1}{2}.2^{-|-1|} + \frac{1}{2}.2^{-|1|} + \frac{1}{2}.2^{-|-2|} + \frac{1}{2}.2^{-|2|} + \frac{1}{2}.2^{-|-3|} + \frac{1}{2}.2^{-|3|} + ...$
$= 2(\frac{1}{2}\frac{1}{2}) + 2(\frac{1}{2}\frac{1}{4}) + 2(\frac{1}{2}\frac{1}{8}) + ...$
$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + ..$
$= \sum_{y=1}^{\infty}(\frac{1}{2})(\frac{1}{2})^{y-1}$
$= \frac{\frac{1}{2}}{1-\frac{1}{2}}$
$= 1$
It is obivious that $f(x) > 0$ for all $x = \pm1, \pm2, \pm3, ...$

(b)

x	±1	±2	±3	$\pm4...$
$f(x)$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}...$
$g(x)$	2	$\frac{-4}{3}$	$\frac{8}{7}$	$\frac{-16}{15}...$

$\sum_{allx} g(x)f(x) = 2(2.\frac{1}{4}) + 2(-\frac{4}{3}.\frac{1}{8}) + 2(-\frac{16}{15}\frac{1}{32}) + ...$
$= 1 - \frac{1}{3} + \frac{1}{7} - \frac{1}{15} + ...$
$= \sum_{y=1}^{\infty}(-1)^{y-1}\frac{1}{2^y - 1}$
which is an alternating series whose terms are decreasing and whose n^th term converges to 0.Therefore, the series converges.

(c) $\sum_{allx} |g(x)|f(x) = 1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + ...$
$\sum_{y=1}^{\infty}\frac{1}{2^y - 1} > \frac{1}{3}\sum_{y=1}^{\infty}\frac{1}{y}$
which is divergent.Thus, the series $\sum_{allx} |g(x)|f(x)$ is divergent.

Section 3.4

24.(a)(i) The drilling of a well(trial) results in a strike(success) or not a strike(failure)

(ii) Trials are identical and independent with p = $\frac{1}{13}$ for each well.

(iii) X = the number of trials (well drilled) before the first success(strike)

(b) $f(x) = \begin{cases} (\frac{12}{13})^{x-1}(\frac{1}{13}), & x = 1, 2, 3, ... \\ 0, & otherwise \end{cases}$

(c) $m_x(t) = \frac{\frac{1}{13}e^t}{1 - \frac{12}{13}e^t}, t < -ln\frac{12}{13}$

(d) E[X] = $\frac{1}{p} = \frac{1}{\frac{1}{13}} = 13$
$E[X^2] = \frac{1+q}{p^2} = \frac{1+\frac{12}{13}}{(\frac{1}{13})^2} = 325$

$\sigma^2 = \frac{q}{p^2} = \frac{\frac{12}{13}}{(\frac{1}{13})^2} = 156$
$\sigma = \sqrt{156} = 12.49$

(e) $P[X \geq 2] = 1 - F(1) = 1 - (1 - \left(\frac{12}{13}\right))$,since $F(x) = 1 - q^x$
$= \frac{12}{13}$

26. The density for the geometric random variable X with probability of success p is
$f(x) = q^x p, x = 1, 2, 3, ...$
$F(x_0) = P[X \leq x_0] = \sum_{x=1}^{x_0} q^{x-1}p = \frac{p(1-q^{x_0})}{1-q}$, the sum of the first x_0 terms of a geometric series
$= \frac{p(1-q^{x_0})}{p}$,
=1-q^{x_0}
The density in Example 3.2.4 can be expressed as geometric density f(y) = $(\frac{1}{2})^{y-1}(\frac{1}{2}), y = 1, 2, 3, ...$
Thus
F(y) = 1-q^y = 1-$(\frac{1}{2})^y$

28. no,X is not the number of trials (bits) until the first success (transmission error).

28.(i) Each exposed cell (trial) fuses(success) or doesnt
(ii) The fusion of one cell does not affect the fusion of any other cell;
(iii) Y = the number of trials (exposed cells) before the first success (fusion)
E[Y] $\frac{1}{p} = \frac{1}{1/2} = 2$

32.(a) $\frac{dm_x(t)}{dt} = 2e^t(e^{2(e^t-1)}) = 2$

$$\begin{aligned} E[X] &= \frac{dm_x(t)}{dt}|_{t=0} \\ &= 2e^0(e^{2(e^0-1)}) \\ &= 2 \end{aligned}$$

(b) $\frac{d^2m_x(t)}{dt^2} = 2e^t.2e^t(e^{2(e^t-1)}) + (e^{2(e^t-1)})2e^t$
$= 2e^t(e^{2(e^t-1)})(2e^t + 1)$
$E[X^2] = \frac{d^2m_x(t)}{dt^2}|t = 0$
$= 2e^0(e^{2(e^0-1)})(2e^0 + 1)$
$= 2(1)(2 + 1) = 6$

(c) $\sigma^2 = E[X^2] - (E[X])^2 = 6 - 2^2 = 2 \implies \sigma = \sqrt{2}$

34.(a) $m_x(t) = E[e^{tX}] = \frac{1}{n}\sum_{i=1}^n e^{tx_i}$

(b) $\frac{dm_x(t)}{dt} = \frac{1}{n}(x_1e^{tx_1} + x_2e^{tx_2} + x_3e^{tx_3} + ... + x_ne^{tx_n})$
$E[X] = \frac{dm_x(t)}{dt}|t = 0$
$$\frac{1}{n}(\sum_{i=1}^n x_i)$$
$$\frac{d^2m_x(t)}{dt^2} = \frac{1}{n}(x_1^2e^{tx_1} + x_2^2e^{tx_2} + x_3^2e^{tx_3} + ... + x_n^2e^{tx_n})$$
$$E[X^2] = \frac{d^2m_x(t)}{dt^2}|t = 0$$

$$= \frac{1}{n}(\sum_{i=1}^{n} x_i)$$

$$\sigma^2 = E[X^2] - (E[X])^2 = \frac{1}{n}\left(\sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2\right)$$

(c) $\mu_y = \frac{1}{n}(\sum_{i=1}^{n} y_i) = \frac{1}{10}(\frac{9.10}{6}) = 4.5$

$E[Y^2] = \frac{1}{n}(\sum_{i=1}^{n} y_i^2) = \frac{1}{10}(\frac{9.10.19}{6}) = 28.5$

$\sigma^2 = 28.5 - (4.5)^2 = 8.25$

Section 3.5

36.(a) $f(x) = \begin{cases} \binom{15}{x}(0.2)^x(0.8)^{15-x}, & x = 0, 1, 2, 3, ..., 15 \\ 0, & otherwise \end{cases}$

(b) $m_x(t) = (0.8 + 0.2e^t)^{15}$

(c) $E[X] = np = (15)(0.2) = 3$

$VarX = npq = (15)(0.2)(0.8) = 2.4$

(d) $\frac{dm_x(t)}{dt} = 15(0.8 + 0.2e^t)^{14}(0.2e^t)$

$E[X] = \frac{dm_x(t)}{dt}|t = 0$

$= 15(0.8 + 0.2)^{14}(0.2) = 3$

$\frac{d^2m_x(t)}{dt^2} = 15[(0.8 + 0.2e^t)(0.2e^t) + (0.2e^t)14(0.8 + 0.2e^t)^{13}(0.2e^t)]$

$= 15[(0.2e^t)(.8 + .2e^t)^{13}(.8 + 3e^t)]$

$E[X^2] = \frac{d^2m_x(t)}{dt^2}|t = 0$

$= 15[(0.2)(.8 + .2)^{13}(.8 + 3)] = 11.4$

$VarX = 11.4 - 3^2 = 2.4$

(e) $P[X \le 1] = P[X = 0] + P[X = 1]$

$= \binom{15}{0}(0.2)^0(0.8)^{15} + \binom{15}{1}(0.2)^1(0.8)^{14} = 0.352 + 0.1319 = 0.1671$

(f) $P[X \le 5] = F(5) = 0.9389$

$P[X < 5] = P[X \le 4] = F(4) = 0.8358$

P[$2 \le X \le 7$] = P[$X \le 7$] - P[$X \le 1$] = F(7) - F(1) = 0.9958 - 0.1671 =0.8287

P[$2 \le X < 7$] = P[$2 \le X < 6$] = F(6) - F(1) = 0.9819 - 0.1671 = 0.8148

P[$x \ge 2$] = 1 - F(2) = 1- 0.3980

F(9) = 0.9999

F(20) = 1

$P[X = 10] = F(10) - F(9) = 1 - 0.9999 = 0.0001$

38(a) X is the number of success (operable computer system) out of three trials (computer system) which operate independently of one another, each of which operates successfully with probability 0.9.

$$f(x) = \begin{cases} \binom{3}{x}(0.9)^x(0.1)^{3-x}, & x = 0, 1, 2, 3 \\ 0, & otherwise \end{cases}$$

(b) $E[X] = np = 3(0.9) = 2.7$
$VarX = npq = 3(0.9)(0.1) = 0.27$

40.(a) Let X: the number of the 15 photographs taken by a 35-mm camera that were selected by the judge as better
X is a binomial random variable with parameters n=15 and p=0.5.
E[X] =(15)(.5) = 7.5

(b) yes, the probability of this event occurring is P$[X \geq 12|p = 0.5]$ = 0.0176, which is very small so the event is unlikely to occur.

(c) yes, the probability that the judge selects the photographs at random is so small (0.0176) that there is reason to suspect she is not.

42. Let X: the number of silent paging errors introduced when using the system word processor 20 times
X is binomial with n = 20 and p = 0.1

(a) P[X=0] = F(0) = 0.1216

(b) P$[X \geq 1]$ = 1- F(0) = 0.8784

(c) yes, the probability of more than four errors occurring is
P$[X > 4]$ =1- F(4) = 1- 0.9568 = 0.0432, which is small.

44.(a) Let X: the number of passengers of 15 that are stoped because of change in her or his pocket
X is binomial with n= 15 and p = 0.25

(b) yes, P[X=0] = F(0) = 0.0134

Section 3.6

46. np = 5 and np(1-q) = 4 $\implies$ p = $\frac{1}{5}$ and thus n= 25

48. Let X: the number of pitches (trials) to get four balls outside the strike zone (success)
X is negative binomial with r = 4 and p = 0.10
$\mu = \frac{r}{p} = \frac{4}{0.1} = 40$
P[X=7] = $\frac{6}{3}(0.9)^3(0.1)^4 = 0.00058$

50. $m_x(t) = (pe^t)^r(1 - qe^t)^{-r}$
$\frac{dm_x(t)}{dt} = (pe^t)^r \left(-r(1 - qe^t)^{-(r+1)})(-qe^t)\right) + (1 - qe^t)^{-r}r(pe^t)^{r-1}(pe^t)$
$= rqe^t(pe^t)^r(1 - qe^t)^{-(r+1)} + r(pe^t)^r(1 - qe^t){-}r$
$= r(pe^t)^r(1 - qe^t){-}(r + 1)$

$\frac{d^2m_x(t)}{dt^2} = r\left((pe^t)^r \left(-(r + 1)(1 - qe^t)^{-(r+2)}(-qe^t)\right) + (1 - qe^t)^{-(r+1)}r(pe^t)r - 1(pe^t)\right)$
$= r(pe^t)^r(1 - qe^t){-}(r + 2)(qe^t + r)$
$\frac{d^2m_x(t)}{dt^2}|t = 0 = r(p^r(1 - qe^{-(r+2)}(q + r) = \frac{rq+r^2}{p^2}$
$VarX = \frac{r^2+rq}{p^2} - \frac{r^2}{p^2} = \frac{rq}{p^2}$

52. X is negative binomial with parameter r=3 and p = 0.9
$f(x) = \begin{cases} \binom{x-1}{2}(0.1)^{x-3}(0.9), & x = 3, 4, 5, ... \\ 0, & otherwise \end{cases}$

$\mu = \frac{r}{p} = \frac{3}{0.9} = 3.3333$
$\sigma^2 = \frac{rq}{p^2} = \frac{3(0.1)}{(0.9)^2} = 0.3704$
yes, P$[X \geq 7]$ = 1 - P$[X \leq 6]$

$= 1 - \left(\binom{2}{2}(0.1^0)(0.9)^3 + \binom{3}{2}(0.1)^1(0.9)^3 + \binom{4}{2}(0.1)^2(0.9)^3 + \binom{5}{2}(0.1)^3(0.9)^3\right)$
= 0.00127, which is very small

Section 3.7

54. max[0, 5-(20-17)] $\leq x \leq min(5, 17)$; thus x =2, 3, 4, or 5
E[X] = $5(\frac{17}{20})$ = 4.25
VarX = 5 $(\frac{17}{20})(\frac{3}{20})(\frac{15}{19})$ = 0.5033

56. max[0, 5-(20-10)] $\leq x \leq min(5, 10)$; thus x = 0, 1, 2, 3, 4, or 5
E[X] = $5(\frac{10}{20})$ = 2.5
VarX = 5 $(\frac{10}{20})(\frac{10}{20})(\frac{15}{19})$ = 0.9868

58.(a) X is hypergeometric with parameters N =15, r=4, and n=3.
$f(x) = \frac{\binom{4}{x}\binom{11}{3-x}}{\binom{15}{3}}$, for max [0, 3 -(15-4)]$\leq x \leq min(3, 4) \implies x = 0, 1, 2, 3$

(b) E[X] = $3(\frac{4}{15})$ = 0.80
VarX = 3 $(\frac{4}{15})(\frac{11}{15})(\frac{12}{14})$ = 0.5029

(c) P$[X \leq 1]$ = P[X=0]+P[X=1]
$= \frac{\binom{4}{0}\binom{4}{1}}{\binom{15}{3}} + \frac{\binom{4}{1}\binom{11}{2}}{\binom{15}{3}} = 0.8462$

60.(a) X is hypergeometric with N =150,000, r = 90,000, and n = 15
f(x) = $\frac{\binom{90,000}{x}\binom{60,000}{15-x}}{\binom{150,000}{15}}$

max[0, 15-(150,000-90,000)]$\leq x \leq min(15, 90, 000) \implies x = 0, 1, 2, 3, ..., 15$

(b) E[X] = $15(\frac{90,000}{150,000})$ = 9
VarX = $15(\frac{90,000}{150,000})(\frac{60,000}{150,000})(\frac{149,985}{149,999})$ = 3.599

(c) P$[X \leq 6]$ = 1-P$[X \leq 5]$ = 1- $\sum_{x=0}^{5} \frac{\binom{90,000}{x}\binom{60,000}{15-x}}{\binom{150,000}{15}}$

(d) n= 15, p =$\frac{r}{N}$ = 0.6
p$[X \geq 6]$ =1 - P$[X \leq 5]$ = 1- 0.338 = 0.9662

Section 3.8

62 Let X: the number of emissions in one month
X is Poisson with parameter k = 2 emissions per month
P$[X \leq 4]$ = 0.947 from Table II of Appendix A
Let Y: the number of emissiona in three months
Then E[Y] = λs = 2(3) = 6
yes, P$[Y \geq 12]$ = 1-P$[Y \leq 11]$ = 1-0.98 = 0.02, which is quit small for $\lambda = 2$

64 Let X:the number of destructive earthquakes per year
X is Poisson with k= λ = 1 destructive earthquake per year

Let Y: the number of destructive earthquakes in a six month period
Y is a poisson with parameter λs = 1(0.5) = 0.5
P$[Y \geq 1]$ = 1 - P$[Y \leq 0]$ = 1 - 0.607 = 0.393
Yes, P$[Y \geq 3]$ = 1-P$[Y \leq 2]$ = 1 - 0.986 = 0.014, which indicates a small chance of this event occurring.

66. Let X: the number of burrs on seven metal parts
X is Poisson with parameter λs = 2(7) = 14
P$[X \leq 4]$ = 0.002 from Table II in Appendex A

68. P[X=0] = P[X=1] $\implies \frac{e^{-k}k^0}{0!} = \frac{e^{-k}k^1}{1!} \implies e^{-k} = ke^{-k} \implies k = 1$

70. Let x:the number of times the light will be activated in two weeks
X is Poisson with parameter λ = 2(0.5) = 1
$\mu = \lambda = 1$
yes, P$[X \geq 5]$ = 1 - P$[X \leq]$ = 1- 0.996 = 0.004, which is very small

Chapter 4

Continuous Distributions

2. (a) $P[X \leq 5]$

   (b) $P[X > 5]$

   (c) $P[X = 10]$

   (d) $P[5 \leq X < 10]$

   (e) $P[5 < X < 10]$
   $P[X \leq 5] = 1 - P[X > 5]$
   $P[5 \leq X \leq 10] = P[5 < X < 10]$

4. (a)(i) $\frac{1}{ln2}.\frac{1}{x} > 0$ for all x such that $25 \leq x \leq 50$

   (ii) $\int_{25}^{50} f(x)dx = \frac{1}{ln2}\int_{25}^{50} f(x)dx = \frac{1}{ln2}lnx|_{25}^{50} = \frac{1}{ln2}ln\left(\frac{50}{25}\right). = 1$

   (b) $P[30 \leq X \leq 40] = \frac{1}{ln2}\int_{30}^{40} f(x)dx = \frac{1}{ln2}lnx|_{30}^{40} = \frac{1}{ln2}ln\left(\frac{40}{30}\right) = \frac{0.28768}{0.69315} = 0.415$

6.(a) $f(\theta) = \frac{1}{2\pi - 0} = \frac{1}{2\pi}, 0 < \theta < 2\pi$

   (c) Since the area of ractangle is length x width,
   the shaded area $= 2(\frac{\pi}{4})(\frac{1}{2\pi}) = \frac{1}{4}$

   (d) $P[0 < \theta \leq \frac{\pi}{4} or \frac{7\pi}{4} \leq \theta < 2\pi] = \int_0^{\frac{\pi}{4}} \frac{1}{2\pi}d\theta + \int_{\frac{7\pi}{4}}^{2\pi} \frac{1}{2\pi}d\theta$
   $= \frac{1}{2\pi}(\frac{\pi}{4}) + \frac{1}{2\pi}(\frac{\pi}{4}) = \frac{1}{4}$

   (e) Let X: the number of birds out of that are orienting within $\frac{\pi}{4}$ radian of home.
   X is binomial with n = 10 and p = $\frac{1}{4}$
   yes, $P[X \geq 7] = 1 - P[X \leq 6] = 1 - 0.9965 = -.0035$, which is small enough to consider this a rare event.

8.(a) $P[X \leq 5] = F(5)$

   (b) $P[X > 5] = 1 - F(5)$

   (c) $P[X = 10] = F(10) - F(10)$

   (d) $P[5 \leq X < 10] = F(10) - F(5)$

(e) $P[5 < X < 10] = F(10) - F(5)$

10. $P[X \leq x] = \int_a^x \frac{1}{b-a}dt = \frac{1}{b-a}|_a^x = \frac{x-a}{b-a}$, for $a < x < b$

Thus, $F(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x \geq b \end{cases}$

12. $F(x) = P[X \leq x] = \int_0^x \frac{1}{10}e^{-(\frac{t}{10})} = -e^{-(\frac{t}{10})}|_0^x = -e^{-(\frac{x}{10})} + 1, for\, x > 0$

Thus $F(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-(\frac{x}{10})}, & x > 0 \end{cases}$

$P[1 \leq X \leq 2] = F(2) - F(1) = (1 - e^{-\frac{2}{10}}) - (1 - e^{-\frac{1}{10}})$
$= e^{0.1} - e^{0.2} = 0.9048 - 0.8187 = 0.0861$

14.(a) $f(x) = 1, for -1 \leq x \leq 0$, is a valid continuous density

(b) f(x) $= F'(x) = \begin{cases} 2x, & 0 < x \leq \frac{1}{2} \\ \frac{1}{2}, & \frac{1}{2} < x \leq 1 \end{cases}$

is not a valid density since $\int_0^{\frac{1}{2}} 2xdx + \int_{\frac{1}{2}}^1 \frac{1}{2}dx \neq 1$

Section 4.2

16. $\mu = E[X] = \displaystyle\int_{25}^{50} x\frac{1}{ln2}\frac{1}{x}dx = \frac{1}{ln2}x|_{25}^{50} = \frac{25}{ln2} = 36.23$ pounds

$$E[X^2] = \int_{25}^{50} x^2\frac{1}{ln2}\frac{1}{x}dx = \frac{1}{ln2}\frac{x^2}{2}|_{25}^{50} = \frac{1875}{2ln2} = 1358.6956$$

$\sigma^2 = 1358.6956 - (36.23)^2 = 51.67$
$\sigma = \sqrt{51.67} = 7.188$ pounds

18. $E[X] = \displaystyle\int_a^b \frac{x}{b-a}dx = \frac{x^2}{2(b-a)}|_a^b = \frac{b^2-a^2}{2(b-a)} = \frac{a+b}{2}$

$$E[X^2] = \int_a^b \frac{x^2}{b-a}dx = \frac{x^3}{3(b-a)}|_a^b = \frac{b^3-a^3}{3(b-a)} = \frac{(b-a)(b^2+ba+a^2)}{3(b-a)} = \frac{(b^2+ba+a^2)}{3}$$

Therefore , $VarX = \dfrac{(b^2+ba+a^2)}{3} - \dfrac{(b+a)^2}{4} = \dfrac{(b^2-2ba+a^2)}{12} = \dfrac{(b-a)^2}{12}$

20.(a) 10

(b) 12.5

(c) 12.5

(d) 7 (debatable)

22. $\displaystyle\int_{-\infty}^{\infty} |x|f(x)dx = \int_{-\infty}^{\infty} |x|\frac{1}{\pi}\frac{1}{1+x^2}dx = \frac{1}{\pi}\int_{-\infty}^{0} \frac{-x}{1+x^2}dx + \frac{1}{\pi}\int_0^{\infty} \frac{x}{1+x^2}dx,$

by definition of absolute value
$= -\frac{1}{2\pi}\int_{-\infty}^{0}\frac{2x}{1+x^2}dx + \frac{1}{2\pi}\int_{0}^{\infty}\frac{2x}{1+x^2}dx$
Let $u = 1 + x^2$ and $du = 2x$. Then substitution yields

$\int_{-\infty}^{\infty}|x|f(x)dx = -\frac{1}{2\pi}ln|1+x^2||_{-\infty}^{0} + \frac{1}{2\pi}ln|1+x^2||_{0}^{\infty}$,
which does not exist because $ln\infty \to \infty$

24.(a)(i) $f(x) \geq$ since $x > 0$

(ii) $\int_o^4 f(x)dx = \frac{1}{(64)(4)}x^4|_0^4 = 1$

(b) $F(x) = \int_o^x \frac{1}{64}x^3dx = \frac{1}{(4^4)}x^4|_0^x = \frac{x^4}{256}, 0 < x < 4$

$P[X \leq 2] = F(2) = \frac{16}{256} = 0.0625$

(c) $P[X > 3] = 1 - P[X \leq 3] = 1 - F(3) = 0.6836$

(d) $\int_0^4 \frac{1}{64}x^4dx = \frac{1}{(64)(5)}x^5|_0^4 = \frac{16}{5} = 3.2$

Section 4.3

26. part 1: $\Gamma 1 = \int_0^\infty z^0e^{-z}dz = -e^{-z}|_0^\infty = -(0-1) = 1$

Part 2: $\Gamma\alpha = \int_0^\infty z^{\alpha-1}e^{-z}dz = -z^{\alpha-1}e^{-z}|_0^\infty + \int_0^\infty z^{\alpha-2}\alpha - 1e^{-z}dz$
$= -z^{\alpha-1}e^{-z}|_0^\infty + (\alpha-1)\Gamma(\alpha-1)$
Now by a repeated use of l' Hospital rule,
$lim_{z\to\infty}\frac{-z^{\alpha-1}}{e^z} = lim_{z\to\infty}\frac{-(\alpha-1)-z^{\alpha-1}}{e^z} = lim_{z\to\infty}\frac{-(\alpha-1)(\alpha-2)-z^{\alpha-3}}{e^z}$
$= ...$

$= lim_{z\to\infty}\frac{-(\alpha-1)!}{e^z}$
$lim_{z\to\infty}\frac{0}{e^z} = 0$
Therefore, $\Gamma\alpha = (\alpha-1)\Gamma(\alpha-1)$

28. Let $z = \frac{x}{\beta}$ Then $x = \beta z$ and $dx = \beta dz$
substitution yields
$\int_0^\infty \frac{1}{\Gamma\alpha\beta^\alpha}x^{\alpha-1}e^{-x/\beta}dx$
$= \frac{1}{\Gamma\alpha\beta^\alpha}\int_0^\infty \beta^{(\alpha-1)}z^{\alpha-1}e^{-z}dz$
$= \frac{1}{\Gamma\alpha\beta^\alpha}\beta^\alpha\int_0^\infty z^{\alpha-1}e^{-z}dz = \frac{1}{\Gamma\alpha}.\Gamma\alpha = 1$

30 $\frac{dm_X(t)}{dt} = -\alpha(1-\beta t)^{-(\alpha+1)}(-\beta) = \alpha\beta(1-\beta t)^{-(\alpha+1)}$

$E[X] = \frac{dm_X(t)}{dt}|_t = 0 = \alpha\beta$

$\frac{d^2 m_X(t)}{dt^2} = -(\alpha+1)\alpha\beta(1-\beta t)^{-(\alpha+2)}(-\beta)$

$E[X^2] = \frac{d^2 m_X(t)}{dt^2}|_t = 0 = \alpha(\alpha+1)\beta^2$

$VarX = \alpha(\alpha+1)\beta^2 - \alpha^2\beta^2 = \alpha\beta^2$

32. $\frac{df(x)}{dx} = \frac{1}{\Gamma(\alpha)\beta^2}[x^{\alpha-1}(\frac{-1}{\beta}e^{-x/\beta}) + e^{-x/\beta}(\alpha-1)x^{\alpha-2}]$
Set $\frac{df(x)}{dx} = 0 \implies \frac{-1}{\beta}x^{\alpha-1} = -(\alpha-1)x^{\alpha-2}$
$\implies \frac{x\alpha-1}{x^{\alpha-1}} = \beta(\alpha-1) \implies x = \beta(\alpha-1)$
$\frac{df(x)}{dx} < 0 \implies f(x)$ assumes its maximum at $x = \beta(\alpha-1)$

34 Let X: the time elapsed before the release of the first detectable emission
X is exponential with $\beta = \frac{1}{2}$
$f(x) = 2e^(-2x), x > 0$
$P[X > 3] = 1 - F(3) = 1 - (1 - e^{-2.3})$, since $F(x) = 1 - e^{-x/\beta} = e^{-0.6} = 0.00248$
$\beta = \frac{1}{2}$ month

36. Let X: the time elapsed before any rock noise is recorded
X is exponential with $\beta = \frac{1}{3}$, so $f(x) = 3e^{-3x}$
$P[X \geq] = 1 - F(\frac{1}{2}) = 1 - (1 - e^{\frac{-3}{2}}) = 0.2231$

38.(a) mean $= \gamma = 15$
variance $= 2\gamma = 2(15) = 30$

(b) Since X_r^2 is a gamma random variable with $\beta = 2$ and $\alpha = \frac{\gamma}{2}$,

$$f(x) = \frac{1}{\Gamma(\frac{\gamma}{2})2^((\gamma/2)}x^{(\gamma/2)-1}e^{-x/2}, x > 0$$

So. obviously, when $\gamma = 15$,
$$f(x) = \frac{1}{\Gamma(\frac{15}{2})}2^{15/2)}x^{(15/2)-1}e^{-x/2}, x > 0$$

(c) $m_X(t) = (1-\beta t)^{-\alpha} = (1-2t)^{-15/2}$

(d) $P[X_{15}^2 \leq 5.23] = 0.01$

$P[X_{15}^2 \geq 5.23] = 1 - 0.9 = 0.10$

$P[6.26 \leq X_{15}^2 \leq 27.5] = F(27.5) - F(6.26) = 0.975 - 0.025 = 0.95$

$\chi^2_{0.01} = 30.6$
$\chi^2_{0.05} = 25$
$\chi^2_{0.95} = 7.26$

Section 4.4

40.(a) $f(x) = \dfrac{1}{\sqrt{2\pi}(0.2)} e^{\frac{-1}{2}} (\dfrac{x-15}{2})^2$
$P[1.1 < X < 1.9] = P\left[\dfrac{1.1-1.5}{0.2} < \dfrac{X-1.5}{0.2} < \dfrac{1.9-1.5}{0.2}\right]$
$P[-2 < Z < 2] = F(2) - F(-2) = 0.9772 - 0.0228 = 0.9544$

(b) $P[X < 0.9] = P\left[\frac{X-1.5}{0.2} < \frac{0.9-1.5}{0.2}\right] = P[Z < -3] = 0.0013$

(c) yes, $P[X > 2] = P\left[\dfrac{X-1.5}{0.2} > \dfrac{2-1.5}{0.2}\right] = P[Z > 2.5] = 1 - 0.9938 = 0.0062$
This small probability is indicative of an unlikely event.

(d) Need to find x_0 such that $P[X \geq x_0] = 0.10$
$P[X \geq x_0] = P\left[Z \geq \dfrac{x_0 - 1.5}{0.2}\right] = 0.10 \implies \dfrac{x_0 - 1.5}{0.2} = z_{0.10} = 1.28$
$\implies X_0 = 1.5 + (1.28)(0.2) = 1.756 g/cm^3$

(e) $m_X(t) = e^{1.5t + \frac{(0.2)^2 t^2}{2}} = e^{1.5t + 0.2t^2}$

42.(a) $P[90 < X < 122] = P\left[\frac{90-106}{8} < \frac{X-106}{8} < \frac{122-106}{8}\right]$
$P[-2 < Z < 2] = 0.9772 - 0.0228 = 0.9544$

(b) $P[X \geq 120] = P\left[\dfrac{X-106}{8} > \dfrac{120-106}{8}\right] = P[Z \leq 1.75] = 0.9599$

(c) Need to find x_0 such that $P[X \leq x_0] = 0.25$
$P[X \leq x_0] = P\left[Z \leq \dfrac{x_0 - 106}{8}\right] = 0.25 \implies \dfrac{x_0 - 106}{8} = z_{0.75} = -0.675$
$\implies X_0 = 106 - (0.675)(8) = 100.6 mg/100ml$

(d) yes, $P[X > 130] = P\left[\frac{X-100}{8} > \frac{130-106}{8}\right] = P[Z > 3] = 1 - F(3) = 1 - 0.9987 = 0.0013$,
which indicates that a fasting blood glucose level greater than 130 is quite abnormal.

44.(a) no, $P[X \leq 1875] = P[Z \leq -0.17] = 0.4325$

(b) $P[X \geq 1878] = P[Z > 0.33] = 0.3707$

46.(a) $P[X > 2.7] = P\left[Z > \frac{ln2.7-0.8}{0.1}\right] = 1 - F(1.93) = 0.0268$

(b) Need to find y_1 and y_2 such that$P[y_1 < Y < y_2] = 0.95$. We find y_1 and y_2 that are symmetric with respect to E[Y].
$P[y_1 < Y < y_2] = P\left[\frac{lny_1-0.8}{0.1} < Z < \frac{lny_2-0.8}{0.1}\right] = 0.95$
$\implies \frac{lny_1-0.8}{0.1} = z_{.975} = -1.96 \implies lny_1 = 0.604 \implies y_1 = e^{.604} = 1.829mm$
and $\implies \frac{lny_2-0.8}{0.1} = 1.96 \implies lny_2 = 0.996 \implies y_2 = e^{.996} = 2.707mm$

Section 4.5

48. $P[128 < X < 178] = P[-25 < X - 153 < 25] = P[-\sigma < X-\mu < \sigma] = P[-1 < Z < 1] = .68\ or\ 68\%$
$P[X > 228] = P[(X - 153) > 3(25)] = P[(X - \mu) > 3\sigma] = P[Z > 3] = 0.005\ or\ 0.5\%$

50. Chebyshev's guarantees that $P[|X - \mu| < 3\sigma] \geq 1 - \frac{1}{3^2} = 0.89$
yes, both the normal probability rule and Chebyshev's inequality assign a high probability to a normal random variable being within 3σ of its mean.
The normal probability rule yield a stronger statement.

Section 4.6

50. Let Y be a normal variable with $\mu = (20)(0.3) = 6$ and $\sigma = \sqrt{(20)(.3)(.7)} = 2.049$

(a) $P[X \leq 3] = P[Y \leq 3.5] = P\left[Z \leq \frac{3.5-6}{2.049}\right] = P[Z < -1.22] = 0.1112$
From Table I of Appendix A $P[X < 3] = 0.1071$

(b) $P[3 \leq X \leq 6] = P[2.5 \leq Y \leq 6.5] = P[-1.71 \leq Z \leq 0.24] = 0.5512$
From Table I of Appendix A $P[X \leq 3] = P[X \leq 6] - P[X \leq 2] = 0.5725$

(c) $P[X \geq 4] = P[Y \geq 3.5] = P[Z > -1.22] = 1 - F(-1.22)0.8888$
From Table I of Appendix A $P[X \geq 4] = 1 - P[X \leq 3] = 0.8929$

(d) $P[X = 4]P[3.5 \leq Y \leq 4.5] = p[-1.22 \leq Z \leq -0.73] = 0.1215$
From Table I of Appendix A $P[X = 4] = P[X \leq 4] - P[X \leq 4] = 0.1304$

54.(a yes,$n(1-p) = (60)(0.1) = 6 > 5$

(b) $E[X] = 60(0.9) = 54$ trials

(c) Let Y be a normal random variable with $\mu = 60(0.9) = 54$ and $\sigma = \sqrt{(60)(0.9)(0.1)} = 2.324$
$P[X \geq |p = 0.9] = P[Y \geq 58.5] = P[Z \geq \frac{58.4-58}{3.324}] = P[Z \geq 1.94] = 0.0262$

(d) Let Y be normal with $\mu = 60(0.95) = 57$ and $\sigma = \sqrt{(60)(.95)(.05)} = 1.688$
$P[X \leq 58|p = .95] = P[Y \leq 58.5] = P[Z \leq \frac{58.5-57}{1.688}] = P[Z \leq 0.89] = .8133$

56. Let Y be a normal with $\mu = 15$ and $\sigma = \sqrt{15} = 3.873$
$P[X \leq 12] = 0.268$, from Table II of Appendix A
$P[X \leq 12] = P[Yleq12] = P[Z \leq \frac{12.5-15}{3.873}] = P[Z \leq -0.65] = 0.2578$

Section 4.7

58.(a) $f(x) = 0.02xe^{-.01x^2}, x > 0$

(b) $\mu = (.01)^{-1/2}\Gamma(1 + \frac{1}{2}) = 10\Gamma(1.5) = 10(.5)\Gamma(.5) = 10(.5)(\sqrt{\pi}) = 5\sqrt{\pi}$
$\sigma^2 = (.01)^{-2/2}\Gamma(1 + \frac{2}{2}) - (5\sqrt{\pi})^2 = 100 - 25\pi$

(c) $R(t) = 1 - \int_0^t (0.02)xe^{-0.01x^2}dx = 1 + e^{-0.01x^2}|_0^t = e^{-0.01t^2}$

(d) $R(3) = e^{-0.01(3)^2} = 0.9139$
$R(12) = e^{-0.01(12)^2} = 0.2369$
$R(20) = e^{-0.01(20)^2} = 0.0183$

(e) $\rho(t) = \frac{f(t)}{R(t)} = \frac{0.02te^{-0.01t^2}}{e^{0.01t^2}} = 0.02t$

(f) $\rho(3) = 0.02(3) = 0.06$
$\rho(12) = 0.02(12) = 0.24$
$\rho(20) = 0.02(20) = 0.4$

(g) An increasing hazard rate function seems reasonable because the longer a battery is used, the more likely it is to fail.

60.(a) $f(x) = 0.08xe^{-0.04x^2}, x > 0$

$\mu = (.04)^{-1/2}\Gamma(1 + \frac{1}{2}) = 2.5\sqrt{\pi}$, using the hint in exercise 58(b)
$\sigma^2 = (.04)^{-2/2}\Gamma(1 + \frac{2}{2}) - (2.5\sqrt{\pi})^2 = 25 - 6.25\pi$

(b) $R(t) = 1 - \int_0^t (0.08)xe^{-0.04x^2}dx$, since f(x) is known
$= 1 + e^{-0.04x^2}|_0^t = e^{-0.04t^2}$

(c) $R(5) = e^{-0.04(5)^2} = 0.3679$
$R(10) = e^{-0.04(10)^2} = 0.0183$

(d) $\rho(t) = \frac{0.08te^{-0.04t^2}}{e^{0.04t^2}} = 0.08t$

(e) $\rho(5) = 0.08(5) = 0.4$
$\rho(10) = 0.08(10) = 0.8$

(f) $1 - R(3) = 1 - e^{-0.04(3)^2} = 0.3023$

62. $E[X^2] = \displaystyle\int_0^{\infty} x^2 \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}} dx$
$= \int_0^{\infty} \alpha\beta x^{\beta+1} e^{-\alpha x^{\beta}} dx$

Let $z = \alpha x^{\beta}$.This implies that $x = (\frac{z}{\alpha})^{1/\beta}$ and $dx = \frac{1}{\alpha\beta}(\frac{z}{\alpha})^{1/\beta-1} dz$
substituting, it can be seen that
$E[X^2] = \displaystyle\int_0^{\infty} \alpha\beta \left((\frac{z}{\alpha})^{1/\beta}\right)^{\beta+1} e^{-z} \frac{1}{\alpha\beta}(\frac{z}{\alpha})^{1/\beta-1} dz$
$= \int_o^{\infty} (\frac{z}{\alpha})^{2/\beta} e^{-z} dz$
$= \alpha^{-2/\beta} \int_0^{\inf} z^{2/\beta} e^{-z} dz$
$= \alpha^{-2/\beta} \Gamma(1 + \frac{2}{\beta})$

64.(a) $\beta = 1 \implies \rho(t) = \alpha$

(b) $\rho'(t) = \alpha\beta(\beta-1)t^{\beta-2}$
since $\alpha > 0, \beta > 0$ and $t > 0$, the sign of $\rho'(t)$ is determined by the sign of $\beta - 1$.
$\beta > 1 \implies \beta - 1 > 0 \implies \rho'(t) > 0$
$\beta < 1 \implies \beta - 1 < 0 \implies \rho'(t) < 0$

66.(a) $R_1(t) = 1 - \displaystyle\int_0^t (0.006)(0.5)e^{-0.006x^5} dx = 1 + e{-}0.006x^5|_o^t = e{-}0.006t^5 R_2(t) = 1 - \int_0^t (0.00004)e^{-0.00004x} dx = 1$
Therefore $R_s(t) = e^{-0.006t^5} e^{-0.00004t}$
$R_s(2500) = e^{-0.006(2500)^5} e^{-0.00004(2500)} = e^{-0.3} e^{-0.1} = 0.6703$

(b) $1 - R_(2000) = 1 - \left(e^{-0.006(2000)^5} e^{-0.00004(2000)}\right) = e^{-0.3483} = 0.2941$

(b) $R_s(2500) = 1 - (1 - e^{-0.3})(1 - e^{-0.1}) = 0.9753$

68. $R_1(t) = 0.9$

(a) $R_s(t) =$ P[at least one components is operable]
= 1-P[none are operable]
$= 1 - (1 - 0.9)^3 = 0.999$

(b) $R_s(t)$ =P[at least two components are operable]
= P[two or all three are operable]
$= \binom{3}{2}(0.9)^2(1 - 0.9) + (0.9)^3 = 0.972$
OR
$R_s(t) =$ 1-P[none or one is operable]
$= 1 - \left[0.001 + \binom{3}{1}(0.9)(0.1)^2\right] = 0.972$

(c) $R_s(t) = (0.9)^3$

Section 4.8

70.(a) $E[X] = \int_0^{\sqrt{8}} x.\frac{1}{4}x dx = \frac{x^3}{12}|_0^{\sqrt{8}} = 1.8856$
$E[Y] = E[X] + 3 = 1.8856 + 3 = 4.8856$

(b) $y = x + 3 \implies x = g^{-1}(y) = y - 3$
$and\, 0 \le x \le \sqrt{8} \implies 3 \le y \le 3 + \sqrt{8}$
$f_y(y) = f_x(g^{-1}(y))|\frac{dg^{-1}(y)}{dy}| = \frac{1}{4}(y-3)|1|$
$= \frac{1}{4}(y-3), 3 \le y \le 3 + \sqrt{8}$

(c) $E[Y] = \int_3^{3+\sqrt{8}} y(\frac{1}{4}y - \frac{3}{4})dy = \frac{y^3}{12}|_3^{3+\sqrt{8}} - \frac{3y^2}{8}|_3^{3+\sqrt{8}}$
$= 14.2496 - 9.3640 = 4.8856$

72. $y = e^x \implies x = g^{-1}(y) = lny$ and $x > 0 \implies lny > 0$ or $y > e^0 = 1$
$f_y(y) = e^{-lny}|\frac{1}{y}| = e^{ln\frac{1}{y}}.\frac{1}{y} = \frac{1}{y^2}, y > 1$

74. $y = \frac{1}{2}mx^2 \implies x = (\frac{2}{m}y)^{\frac{1}{2}}$ and $x > 0 \implies y > 0$
$f_y(y) = f_x\left((\frac{2}{m}y)^{\frac{1}{2}}\right)\left(\frac{1}{2}(\frac{2}{m}y)^{\frac{-1}{2}}\right)\frac{2}{m}$
$= c.(\frac{2}{m}y)e^{-\beta(\frac{2}{m}y)}.\frac{1}{(2m)^{1/2}}.\frac{1}{y^{1/2}}$
$= c.\frac{1}{m}(\frac{2}{m}y^{1/2})e^{-\beta(\frac{2}{m}y)}, y > 0$

76.(a) Since $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx$, $\Gamma(\frac{1}{2}) = \int_0^\infty x^{-1/2}e^{-x}dx, x > 0$

(b) Let $x = \frac{t^2}{2}$ such that $dx = tdt$ substituting into $\Gamma(\frac{1}{2})$ in 76(a) yields

$\Gamma(\frac{1}{2}) = \int_0^\infty (\frac{t^2}{2})^{-1/2}e^{-(\frac{t^2}{2})}tdt$
$= \int_0^\infty \sqrt{2}t^{-1}te^{-(\frac{t^2}{2})}dt$
$= 2\sqrt{\pi}\int_0^\infty \frac{1}{\sqrt{2\pi}}e^{-(\frac{t^2}{2})}dt$
$= \sqrt{2\pi}(\frac{1}{2}) = \sqrt{\pi}$
(since $\int_0^\infty \frac{1}{\sqrt{2\pi}}e^{-(\frac{t^2}{2})}dt = 1$ and the standard normal density is symmetric about 0)

(c) $f_y(y) = \frac{1}{2\sqrt{y}}f_x(\sqrt{y}) + f_x(\sqrt{-y})$
$= \frac{1}{2\sqrt{y}}\left[\frac{1}{\sqrt{2\pi}}e^{-1/2}(\sqrt{y})^2 + \frac{1}{\sqrt{2\pi}}e^{-1/2}(\sqrt{-y})^2\right]$
$= \frac{1}{2\sqrt{y}}2.\frac{1}{\sqrt{2\pi}}e^{-y/2} = \frac{1}{2^{1/2}\Gamma(\frac{1}{2})}y^{-1/2}e^{-y/2}$

(d) $f_y(y) =$ is the density of a gamma random variable with $\alpha = \frac{1}{2}$ and $\beta = 2$
A chi- squar random variable with γ degree of freedom is a gamma random variable with $\alpha = \frac{\gamma}{2}$ and $\beta = 2$. Thus, $\gamma = 1$.

78. $y = 2z^2 - 1 \implies |z| = \sqrt{\frac{y+1}{2}}$
$z \ge 0 \implies z = \sqrt{\frac{y+1}{2}}$ and $dz = \frac{1}{4}\sqrt{\frac{2}{y+1}}$

$z < 0 \implies z = -\sqrt{\frac{y+1}{2}}$ and $dz = -\frac{1}{4}\sqrt{\frac{2}{y+1}}$

$h_1(y) = f_x\left(\sqrt{\frac{y+1}{2}}\right)|dz = \frac{1}{4}\sqrt{\frac{2}{y+1}}|$

$= \frac{1}{\sqrt{2\pi}}e^{\frac{-1}{2}(\frac{y+1}{2})}.\frac{\sqrt{2}}{4\sqrt{y+1}}$

$\frac{1}{4\sqrt{\pi(y+1)}}.e^{\frac{-1}{4}(\frac{y+1}{2})}, y > -1$

$f_y(y) = 2.h_1(y) = \frac{1}{2\sqrt{\pi(y+1)}}.e^{\frac{-1}{4}(\frac{y+1}{2})}$

$= \frac{1}{4^{1/2}\Gamma(\frac{1}{2})}(y+1)^{-1/2}e^{\frac{-1}{4}(y+1)}$