Practice Problems
(Partial Order Relation)

1. Let be a relation R is defined as all even number are less than all odd numbers and the usual ordering is applied between the evens and the odds. Is R a total ordering relations. Also, give the order of the elements.

   Ans: This is a well-ordered set : { 0 2 4 6 8 … 1 3 5 7 9 …} with zero being the minimum element

2. xRy iff one of the following condition holds

   x=0, x is positive and y is negative, x and y both are positive and x<=y, x and y both are negative and |x| <= |y|

   Ans: This is a well-ordered set : $\{0\ 1\ 2\ 3\ 4\ \ldots\ -1\ -2\ -3\ \ldots\}$ with zero being the minimum element

3. |x| < |y| or |x| = |y| and x <=y

   Ans: This is a well-ordered set : {0, -1, 1, -2, 2 ….} with zero being the minimum element

4. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} be partially ordered by the division relation (that is, for a, b ∈ A, we say that a R b if a is a divisor of b). How many maximal elements are there for this partial order relation?

   (A) 5 (B) 2 (C) 3 (D) 4

5. Which relation is a total order relation?

   (A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

   Ans: B

6. Let A = {1, 2, 3, 4,5}. Which of the following is a partial order relation on A?

   (A) R = {(a, b) | b mod a = 3} (B) R = {(a, b) | a mod b = 0}

   (C) R = {(a, b) | a + b is even} (D) R = {(a, b) | a mod 3 = b}

7. For which sets A of P(A) with set inclusion (⊆) a total ordering?

   (i) ∅ (ii) {a} (iii) {a, b} (iv) {a, b, c}

   (A) i & ii (B) ii and iii (C) iii and iv (D) i, ii, iii, iv

8. Let (S, ≤) be a partial order with two minimal elements a and b, and a maximum element c. Let P : S →{True, False} be a predicate defined on S. Suppose that P(a) = True, P(b) = False and P (x) ⇒ P(y) for all x, y ∈S satisfying x ≤ y, where ⇒ stands for logical implication. Which of the follwong statements CANNOT be true?

   (A) P(x) = True for all x ∈S such that x ≠ b

   (B) P (x) = False for all x ∈S such that x ≠ a and x ≠ c

   (C) P(x) = False for all x ∈S such that b ≤ x and x ≠ c

   (D) P(x) = False for all x ∈S such that a ≤ x and b ≤ x

9. A relation R is defined on ordered pairs of integers as follows (x, y) R(u, v) if x < u and y> v. Then R is

   Equivalence relation, Total Order relation, Partial Order relation

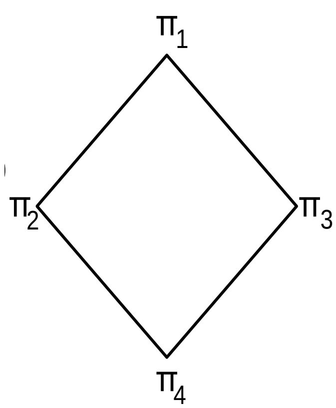
   Ans: Neither a Partial Order not an Equivalence Relation

10. Consider the set S = {a, b, c, d}. Consider the following 4 partitions π_1, π_2, π_3, π_4, on

    S : π_1 = $\overline{\{abcd\}}$, $\pi_2\overline{\{ab,}\overline{cd\}}$, π_3 = $\overline{\{abc,}\ \overline{d\}}$, π_4 = $\{\overline{a},\overline{b},\ \overline{c},\ \overline{d}\}$

    Let $\prec$ be the partial order on the set of partitions S' = (π_1, π_2, π_3, π_4) defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The poset diagram for (S', $\prec$) is

    Ans:



11. Draw the hasse diagram of relation R = {(1,1), (2,2), (3,3), (4,4), (5,5), (3,1), (1,4), (5,1), (3,4), (5,4)}

12. In a partially ordered set, a chain is a totally ordered subset. For example, in the set 1, 2, 3, 4, 5, 6, the divisibility relation is a partial order and 1, 2, 4 and 1, 3, 6 are chains. What is the longest chain on the set {1, 2 . . . n} using the divisibility relation?

    Ans: $\log n + 1$

13. What is the longest chain on the power set of a set A with $|A| = n$ with the $\subseteq$ relation?

    Ans: $n+1$