

Birla Institute of Technology & Science, Pilani

Data Structures & Algorithms (CS F211)

Lab Assignment – 5

KARATSUBA'S DIVIDE AND MULTIPLICATION ALGORITHM

Let N_1 and N_2 be 2 n-digit numbers

$$N_1 = H_1 \times 10^n + L_1,$$

$$N_2 = H_2 \times 10^n + L_2,$$

where,

H_1 is the number representing the high order n digits of N_1 and L_1 is the number representing the low order n digits of N_1 .

Similarly, H_2 is the number representing the high order n digits of N_2 and L_2 is the number representing the low order n digits of N_2 .

We can write the product $N = N_1 \times N_2$ as follows:-

$$N = N_1 \times N_2$$

$$N = (H_1 \times 10^n + L_1) \times (H_2 \times 10^n + L_2)$$

$$N = (H_1 \times H_2 \times 10^n \times 10^n) + (H_1 \times L_2 + H_2 \times L_1) \times 10^n + (L_1 \times L_2)$$

$$N = (A \times 10^n \times 10^n) + (B \times 10^n) + C$$

where,

$$A = (H_1 + L_1) \times (H_2 + L_2),$$

$$B = (H_1 \times H_2)$$

$$C = (L_1 \times L_2)$$

INPUT

You will be given input in single line as $N_1 X N_2$

where

N_1 and N_2 are numbers (**not necessarily of the same length**)

X is the capital English letter X.

Note

Length of $N_1, N_2 < 9$ (If implemented using *long int*)

PROCEDURE

1) Divide Step (Top - Down Approach):

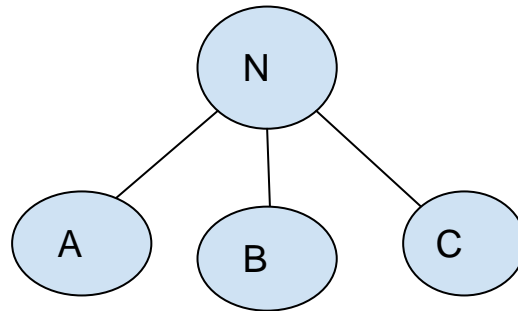
Make a Divide Tree that represents the problems and subproblems .The original problem $N = N_1 \times N_2$ is divided into subproblems

$$A = (H_1 + L_1) \times (H_2 + L_2),$$

$$B = (H_1 \times H_2)$$

$$C = (L_1 \times L_2)$$

We can represent this using tree as follows:



We recursively divide the subproblems A, B and C into smaller subproblems and continue the process until we get a problem of multiplying 2 single digit numbers which will be the leaf nodes of the Divide Tree. For creating the divide Tree you will have to follow the level order traversal similar to Lab Assignment 3 (Game Tree Evaluation). You will have to make use of a queue. You will have to make use of stack also. Whenever you delete a leaf node from the queue you will have to push it on the stack.

2) When the Divide Tree is ready the queue will be empty and stack will have all the leaf nodes. Now pop the leaf nodes and insert them into the queue.

3) Conquer Step (Bottom - Up Approach):-

Now making use of the queue, perform the reverse level order traversal of the Divide tree. Apply the Karatsuba's formula to solve the subproblems and problem together with its solution as follows (one problem and solution per line):

For leaf nodes :

$$N_1 \times N_2 = N$$

For non -leaf nodes :

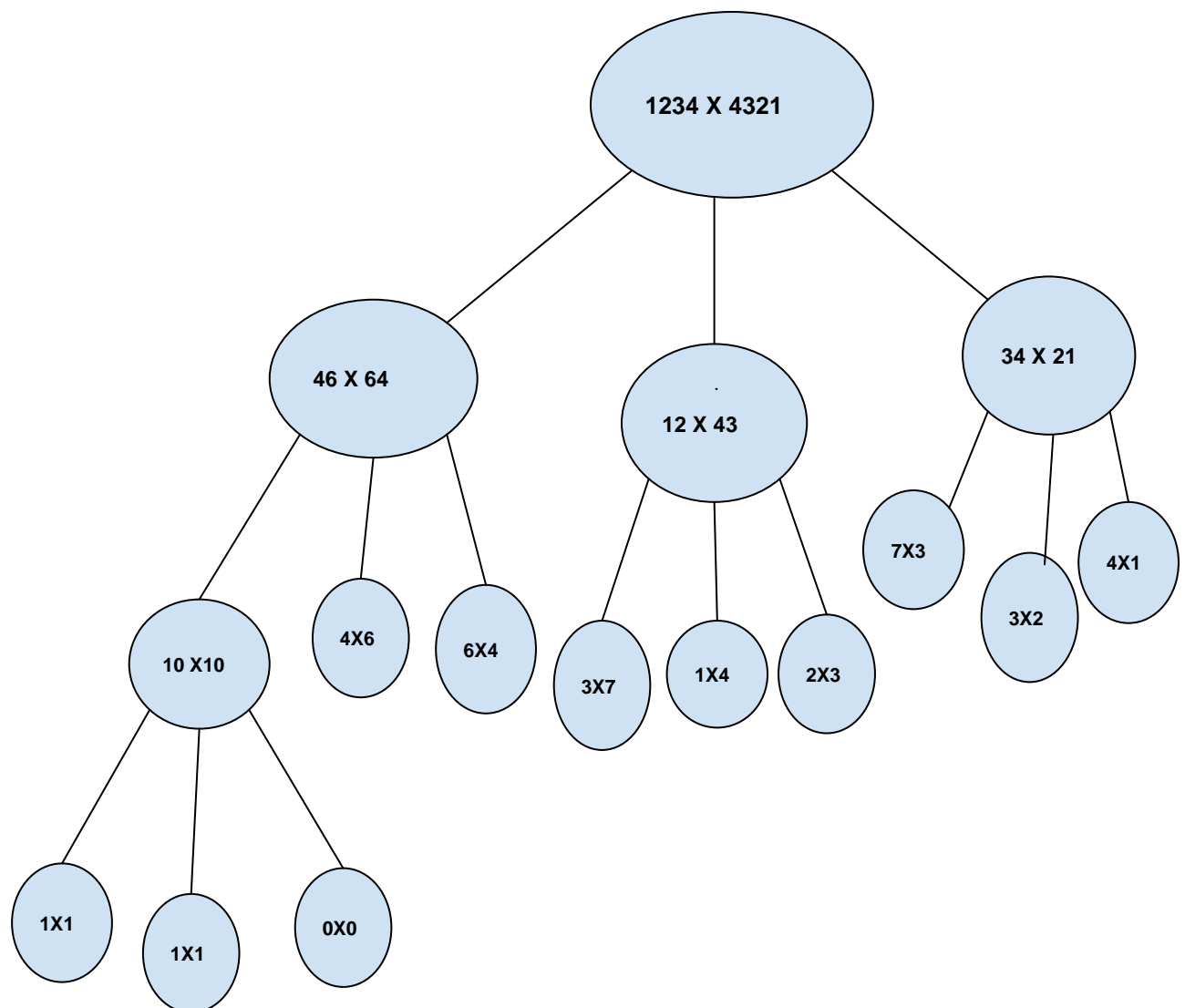
$$N_1 \times N_2 = B \times 10^n \times 10^n + (A - B - C) \times 10^n + C = N$$

(There will not be any space in between and X in the output is English capital letter X)

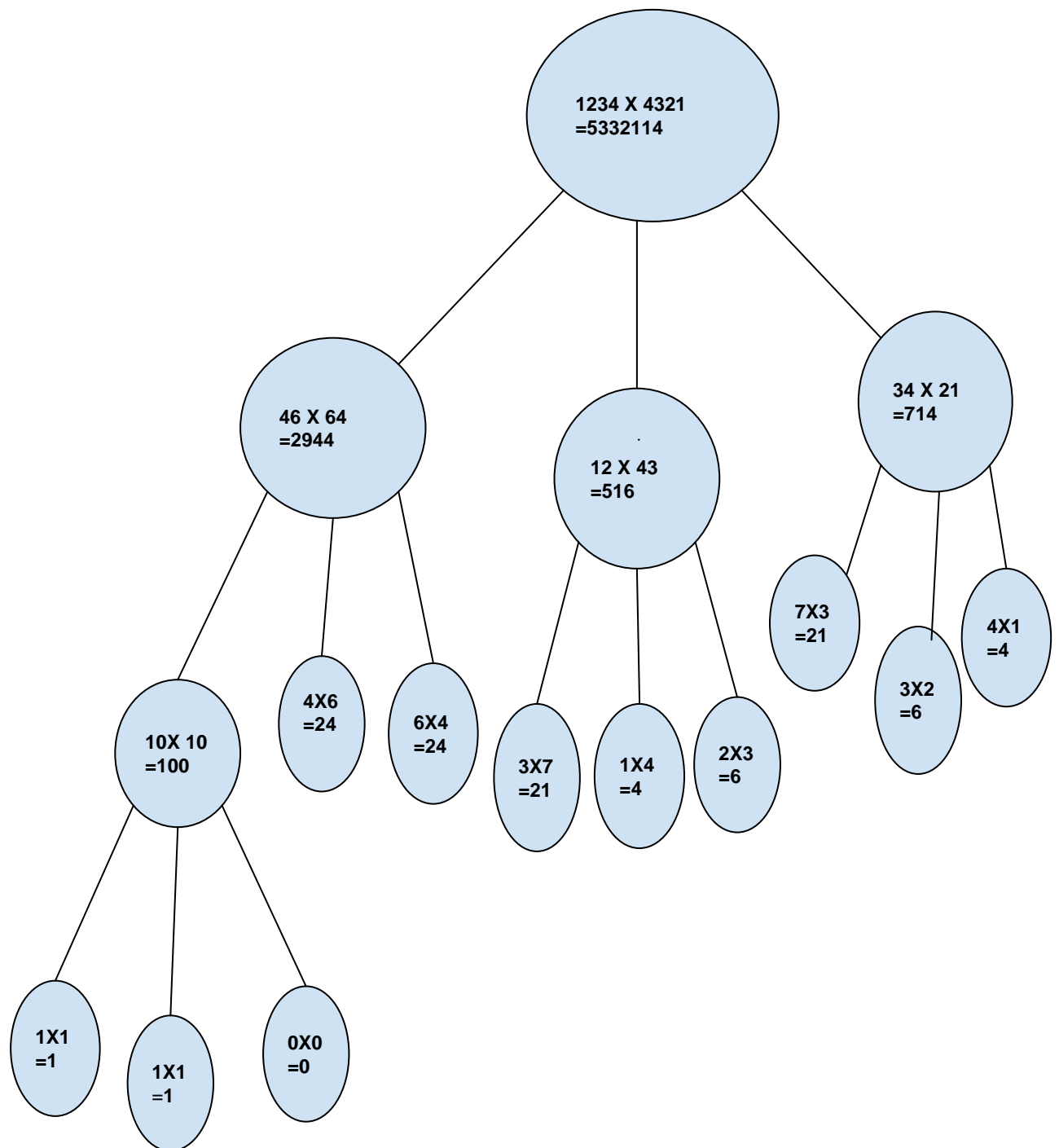
SAMPLE INPUT:

1234X4321

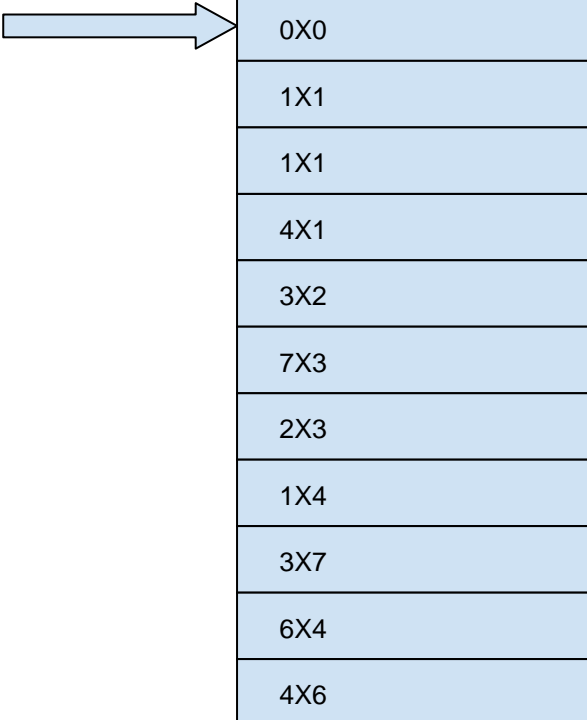
DIVIDE TREE:-



DIVIDE AND CONQUER TREE:-



STACK:-



| |
|-----|
| 0X0 |
| 1X1 |
| 1X1 |
| 4X1 |
| 3X2 |
| 7X3 |
| 2X3 |
| 1X4 |
| 3X7 |
| 6X4 |
| 4X6 |

SAMPLE OUTPUT:

0X0=0

1X1=1

1X1=1

4X1=4

3X2=6

7X3=21

2X3=6

1X4=4

3X7=21

6X4=24

4X6=24

$10 \times 10 = 1 \times 10 \times 10 + (1 - 1 - 0) \times 10 + 0 = 100$

$34 \times 21 = 6 \times 10 \times 10 + (21 - 6 - 4) \times 10 + 4 = 714$

$12 \times 43 = 4 \times 10 \times 10 + (21 - 4 - 6) \times 10 + 6 = 516$

$46 \times 64 = 24 \times 10 \times 10 + (100 - 24 - 24) \times 10 + 24 = 2944$

$1234 \times 4321 = 516 \times 100 \times 100 + (2944 - 516 - 714) \times 100 + 714 = 5332114$

Note:-

1. Continue divide tree process by splitting into subproblems recursively until both the subproblems get single digit.
2. Do not print leading zeros in output.

Example if a number is 01222 then you only need to print 1222