BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
First Semester 2015-2016, MATH F111: MATHEMATICS-I
Assignment - 1

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1. Test the convergence and divergence of the series

$$\frac{4}{1}+\frac{9}{2}+\frac{16}{6}+\frac{25}{24}+\cdots+\frac{(n+1)^2}{n!}+\cdots$$

2. Consider the power series

$$1+\frac{2x}{\sqrt{(5)(5)}}+\frac{4x^2}{\sqrt{(9)(5^2)}}+\frac{8x^3}{\sqrt{(13)(5^3)}}+\cdots+\frac{2^n x^n}{\sqrt{(4n+1)5^n}}+\cdots$$

Find

(i) the radius of convergence,
(ii) the interval of absolute convergence,
(iii) the interval of convergence and
(iv) point/s of conditional convergence of the above series.

3. For the following power series, find the interval of convergence and the radius of convergence:

(i) $\sum_{n=1}^{\infty}(-1)^n n^2 x^n$ (ii) $\sum_{n=1}^{\infty}\frac{2^n}{n^2}(x-3)^n$

4. Determine if each of the following series are absolutely convergent, conditionally convergent or divergent.

(i) $\sum_{n=1}^{\infty}\left[\frac{(-1)^{n+1}\sin nx}{n^3}\right],\ x\in\mathbb{R}$ (ii) $\sum_{n=1}^{\infty}\left[\frac{(-1)^n(n+1)^n}{(2n)^n}\right]$

5. Find the first three non zero terms in the maclaurin series for the function $\sin^2(\tan^{-1}x)$.

6. Find the area of the region that lies inside the circle $r=3a\cos 2\theta$ and outside the cardioid $r=a(1+\cos\theta),\ a>0$.

7. Find the length of the curve $r=8\sin^3\left(\frac{\theta}{3}\right),\quad 0\le\theta\le\frac{\pi}{4}$.

8. Identify the symmetries about x axis, y axis, pole and the line $y=x$ of the following curves

(i) $r=1+2\cos\theta$ and

(ii) $r=2-\ 2\sin\theta$.

9. Show that the function

$$f(x,y)=\begin{cases}(x^2+y^2)\cos\left(\dfrac{1}{\sqrt{x^2+y^2}}\right), & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

is continuous at $(0,0)$ and also find $f_{xy}(0,0)$.

10. Determine the set of points at which the function f is continuous where

$$f(x,y)=\begin{cases}\dfrac{x^2y^3}{2x^2+y^2}, & (x,y)\neq(0,0)\\ 1, & (x,y)=(0,0).\end{cases}$$

11. If the acceleration of an object is given by $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} + 6t\,\boldsymbol{k}$, find the object's velocity and position vectors given that the initial velocity is $\boldsymbol{v}(0) = \boldsymbol{j} - \boldsymbol{k}$, and the initial position vector is $\boldsymbol{r}(0) = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$. Determine the tangential and normal components of the acceleration.

12. Find $\boldsymbol{T}$, $\boldsymbol{N}$ and $\boldsymbol{B}$ for the given space curve $\boldsymbol{r}(t) = (e^{2t}\sin 3t)\boldsymbol{i} + (e^{2t}\cos 3t)\boldsymbol{j} + (3e^{2t})\boldsymbol{k}$ at $t = 0$. Hence find the equation of osculating plane at $t = 0$.

13. For the space curve $\boldsymbol{r}(t) = (6\sin 2t)\boldsymbol{i} + (6\cos 2t)\boldsymbol{j} + 5t\boldsymbol{k}$, find $\boldsymbol{T}$, $\boldsymbol{N}$, $\boldsymbol{B}$, κ and τ.

14. Derive the following expressions of tangential and normal components of acceleration $\boldsymbol{a}(t)$

$$a_T = \frac{\boldsymbol{v}.\boldsymbol{a}}{|\boldsymbol{v}|}$$
$$a_N = \frac{|\boldsymbol{v} \times \boldsymbol{a}|}{|\boldsymbol{v}|}$$

Use these formulae to find a_T and a_N for the particle moving along the curve described by the position vector $\boldsymbol{r}(t) = t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j} + \frac{1}{3}t^3\boldsymbol{k}$.

15. Graph the curve and sketch the velocity and acceleration vectors at the given values of t, then write $\boldsymbol{a}$ in the form of $\boldsymbol{a} = a_T\boldsymbol{T} + a_N\boldsymbol{N}$ at the given value of t without finding $\boldsymbol{T}$ and $\boldsymbol{N}$, and find the value of κ at the given values of t

$$\boldsymbol{r}(t) = (4\cos t)\boldsymbol{i} + (\sqrt{2}\sin t)\boldsymbol{j}, \quad t = 0 \text{ and } t = \pi/4$$

16. Find the domain and the equation for the level surface of the given function passing through the given point:

$$g(x,y,z) = \sum_{n=0}^{\infty} \frac{(x+y)^n}{n!\,z^n}, \qquad (\ln 4, \ln 9, 2).$$

17. Draw a branch diagram and write a chain rule formula for the partial derivatives $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$, where $w = h(x,y,z),\ x = f(u,v),\ y = g(u,v),\ z = k(u,v)$.

18. Find the directions in which the following functions increase and decrease most rapidly at P_0.Then find the derivatives of the functions in these directions.

(i) $f(x,y) = x^2y + e^{xy}\sin y$, $P_0(1,0)$.

(ii) $g(x,y,z) = \ln(xy) + \ln(yz) + \ln(xz)$, $P_0(1,1,1)$

19. Find parametric equations for the line tangent to the curve of intersection of the surfaces $x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0$ and $x^2 + y^2 + z^2 = 11$ at the point $(1,1,3)$.

20. A rectangular box, open at the top, is to have a volume of 32 cubic feet. What must be the dimensions so that the total surface area is a minimum?