Birla Institute of Technology and Science, Pilani

MATH F112, Mathematics-II

Second Semester 2015-16

Assignment -1 (Linear Algebra)

---

1. Suppose A is a 3×3 zero matrix. Describe the solution set of the system $AX = 0$.

2. Solve the homogeneous system of equation $AX = 0$, where A is given by

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 1 & 4 \\ 3 & 2 & -6 & 1 \end{bmatrix}.$$

3. Check whether the inverse of given matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ exists or not. If yes, then find A^{-1} by using Gauss- Jordan method.

4. Let $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$.

(i) If $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ is an eigenvector of A, then find the corresponding eigenvalue.

(ii) If -2 is an eigenvalue of A, then find corresponding eigenvector.

5. Let $V = \mathbb{R}^2$.

(i) Show that V is not a vector space with respect to the vector addition and scalar multiplication defined as

$$[a, b] + [c, d] = [a + c, b + d], \quad k[a, b] = [k^2 a, k^2 b].$$

(ii) Examine whether the set $W = \{[a, b] \in \mathbb{R}^2 \mid a, b \in \mathbb{Q}\}$ is subspace of V.

6. Prove that the set $S = \{[2, 4, 0], [3, 8, 2], [5, 9, -6]\}$ spans $\mathbb{R}^3$.

7. Let $\{\alpha, \beta, \gamma\}$ be a basis of a vector space $\mathbb{R}^3$. Then show that $\{\alpha + \beta, \beta + \gamma, \gamma + \alpha\}$ is also a basis of $\mathbb{R}^3$.

8. Find a basis for the vector space of polynomials $P_3 = \left\{ \sum_{i=0}^{3} a_i x^i \mid a_i \in \mathbb{R} \right\}$. Also find a basis for the subspace $W = \{p(x) \in P_3 \mid p(1) = 0\}$. Justify your answer.

9. Determine whether the following functions are linear transformations or not.

(i) $L : P_2 \to \mathbb{R}$ such that $L(ax^2 + bx + c) = 4abc$.

(ii) $L : M_{22} \to M_{22}$ defined as $L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} -a+2b+c & 2a-d \\ 3a+2b & b+c-3d \end{bmatrix}$.

10. Let B, C and D be a basis of vector spaces V_1, V_2 and V_3, respectively. Suppose that the linear transformations $L_1 : V_1 \to V_2$ and $L_2 : V_2 \to V_3$ are represented by the matrices

$$A_{BC} = \begin{bmatrix} -4 & 3 & -1 \\ 1 & 0 & -2 \end{bmatrix} \text{ and } A_{CD} = \begin{bmatrix} 2 & -2 \\ 1 & 0 \\ -1 & -3 \end{bmatrix}, \text{ respectively.}$$

Find the matrix A_{BD} which represents the composition $L_2 \, o \, L_1 : V_1 \to V_3$.

11. Consider the ordered basis $B = ([-2, 1, 3], [1, 0, 2], [-13, 5, 10])$ for $\mathbb{R}^3$. Suppose that C is another ordered basis for $\mathbb{R}^3$ and the transition matrix from B to C is given by

$$\begin{bmatrix} 1 & 9 & -1 \\ 2 & 13 & -11 \\ -1 & -8 & 3 \end{bmatrix}.$$

Then find C.

12. Prove or disprove the following.

(i) Every linear operator $L : \mathbb{R} \to \mathbb{R}$ is of the form $L(x) = cx$, for some $c \in \mathbb{R}$.

(ii) If $L : V \to W$ is a linear transformation, then the preimage of $\{\mathbf{0}_W\}$ is a subspace of V.

13. Show that the vector space $V = \{c_1 \cos\theta + c_2 \sin\theta : c_1, c_2 \in \mathbb{R}\}$ of functions of θ is isomorphic to the vector space $\mathbb{R}^2$ under the map $\Phi : V \to \mathbb{R}^2$ defined as

$$\Phi(c_1 \cos\theta + c_2 \sin\theta) = [c_1, c_2].$$

14. Determine whether the linear transformation $T : \mathbb{R}^6 \to \mathbb{R}^4$ defined as

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_2 + 2x_3 + x_5 \\ -x_1 - x_2 - x_3 + x_4 + x_6 \\ 4x_2 + 2x_3 + 4x_4 + 3x_5 + 3x_6 \\ x_1 + 3x_2 + 2x_3 - 2x_4 \end{bmatrix}$$

is one-one or not.