

Non-linear models worksheet

Ryan T. Godwin

Non-linear effects

1. What is meant by a “non-linear model” or “non-linear effect”?

One in which the effect of an x variable on the y variable is not constant.

2. What happens if we ignore non-linearities?

If we ignore the non-linearities then the model is “misspecified” and the estimators are biased and inconsistent.

3. Can LS still be used for non-linear models? How?

LS can still be used in some cases. The non-linear effects can be approximated using the polynomials or logarithms. The model can sometimes be “linearized” so that it is linear in the parameters (for example, taking logs of the Cobb-Douglas function).

NLS

4. When *can't* LS be used for non-linear effects/models?

When the model that we want to estimate can't be linearized (for example the CES production function), or when it is inadvisable to linearize the model.

5. How is the NLS estimator *derived*? (Where does the solution to $\hat{\theta}$ come from?)

The Non-Linear Least Squares estimator is the vector, $\hat{\theta}$, that minimizes the quantity:

$$S(X, \theta) = \sum_i \left[y_i - f_i(X, \hat{\theta}) \right]^2$$

6. What are the properties of the NLS estimator?

NLS is consistent, asymptotically efficient, and asymptotically Normal.

7. What is the main *computational* issue with the NLS estimator?

There is typically no closed-form solution for the FOCs from the minimization problem, since the objective function is highly non-linear to begin with.

Newton-Raphson (NR) algorithm

8. Why is the NR algorithm needed?

In order to find the values of θ in which the sum of squared residuals is minimized.

9. Sketch out how the NR algorithm finds the minimum of S .

10. What is the formula for the NR algorithm?

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n - H^{-1}(\boldsymbol{\theta}_n) g(\boldsymbol{\theta}_n)$$

11. When does the NR algorithm *converge*?

The algorithm stops at convergence - when the difference between iterations is within a certain tolerance, that is, if:

$$\left| \frac{(\theta_{n+1}^{(i)} - \theta_n^{(i)})}{\theta_n^{(i)}} \right| < \mathcal{E}^{(i)}; \quad i = 1, 2, \dots, p$$

12. What problems might the NR algorithm encounter?

The algorithm fails if H ever becomes singular at any iteration. The algorithm may locate only a local minimum. The algorithm may oscillate.

Log of Gravity example

13. What is the gravity model of trade?

$$T_{ij} = \alpha_0 Y_i^{\alpha_1} Y_j^{\alpha_2} D_{ij}^{\alpha_3} \eta_{ij}$$

14. Why can't LS be used in this case?

Log-linearizing the model and using LS has been shown to result in inconsistency when there error term is heteroskedastic.

15. Are the LS and NLS estimates for the parameters in the gravity model the same in our example?
No!