Econ 7010 Final Exam Formula Sheet

Standard regression model	$y = X\boldsymbol{\beta} + \boldsymbol{\epsilon}$
OLS estimator	$\boldsymbol{b} = (X'X)^{-1} X' \boldsymbol{y}$
Residual vector	$\boldsymbol{e} = (\boldsymbol{y} - \hat{\boldsymbol{y}}) = \boldsymbol{y} - X\boldsymbol{b}$
Estimator of error variance	$s^2 = \left(e'e\right)/(n-k)$
Covariance matrix for random vector, \boldsymbol{x}	$V(\boldsymbol{x}) = E\left[(\boldsymbol{x} - E(\boldsymbol{x}))(\boldsymbol{x} - E(\boldsymbol{x}))' \right]$

Covariance matrix for errors
$$V(\epsilon) = \sigma^2 I_n$$

"Residual maker" matrix
$$M_X = I_n - X (X'X)^{-1} X'$$

Deviations-from-means matrix
$$M_i = I - \frac{1}{n} \boldsymbol{i} \boldsymbol{i}'$$

Projection matrix
$$P_X = X (X'X)^{-1} X'$$

R-squared
$$R^2 = \frac{\widehat{\mathbf{y}}' M_i \widehat{\mathbf{y}}}{\mathbf{y}' M_i \mathbf{y}} = 1 - \frac{e'e}{\mathbf{y}' M_i \mathbf{y}}$$

t-statistic
$$t_i = (b_i - \beta_i) / (\text{ s.e. } (b_i)) \sim t_{n-k}$$

Confidence interval
$$[b_i - t_c \text{ s.e. } (b_i) \text{ , } b_i + t_c \text{ s.e. } (b_i)]$$

Wald test statistic
$$W = (Rb - q)' \left[R (X'X)^{-1} R' \right]^{-1} (Rb - q)/s^2$$

$$F = \left\{ (Rb - q)' \left[R (X'X)^{-1} R' \right]^{-1} (Rb - q) / J \right\} / s^2$$

$$= [(e^{*\prime}e^* - e'e)/J]/[(e'e)/(n-k)]$$

IV estimator (just-identified)
$$\hat{\beta}_{IV} = (Z'X)^{-1} Z' \boldsymbol{y}$$

F-statistic

IV estimator (over-identified)
$$\hat{\beta}_{IV} = \left[X'Z \left(Z'Z \right)^{-1} Z'X \right]^{-1} X'Z \left(Z'Z \right)^{-1} Z' \boldsymbol{y}$$

Hausman test statistic
$$H = (\boldsymbol{b}_{IV} - \boldsymbol{b})' \left[\hat{V} (\boldsymbol{b}_{IV}) - \hat{V} (\boldsymbol{b}) \right]^{-1} (\boldsymbol{b}_{IV} - \boldsymbol{b})$$

Restricted Least Squares estimator
$$\boldsymbol{b}_* = \boldsymbol{b} - (X'X)^{-1} R' \left[R (X'X)^{-1} R' \right]^{-1} (R\boldsymbol{b} - \boldsymbol{q})$$

Generalized least squares estimator
$$\widehat{\boldsymbol{\beta}}_{\text{GLS}} = \left[X' \Sigma^{-1} X \right]^{-1} X' \Sigma^{-1} \boldsymbol{y} = \left[X' \Omega^{-1} X \right]^{-1} X' \Omega^{-1} \boldsymbol{y}$$

Autoregressive (1) process
$$\epsilon_t = \rho \epsilon_{t-1} + u_t$$
; $u_t \sim \text{i.i.d. } N[0, \sigma_u^2]$; $|\rho| < 1$

Moving average (1) process
$$\epsilon_t = u_t + \phi u_{t-1}$$
; $u_t \sim \text{i.i.d.N } [0, \sigma_u^2]$