Econ 7010 Final Exam Formula Sheet

Standard regression model	$y = X\beta +$
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OLS estimator
$$\boldsymbol{b} = (X'X)^{-1} X' \boldsymbol{y}$$

Residual vector
$$\mathbf{e} = (\mathbf{y} - \hat{\mathbf{y}}) = \mathbf{y} - X\mathbf{b}$$

Estimator of error variance
$$s^2 = (e'e)/(n-k)$$

Covariance matrix for random vector,
$$\mathbf{x} \quad V(\mathbf{x}) = E[(\mathbf{x} - E(\mathbf{x}))(\mathbf{x} - E(\mathbf{x}))']$$

Covariance matrix for errors
$$V(\epsilon) = \sigma^2 I_n$$

"Residual maker" matrix
$$M_X = I_n - X (X'X)^{-1} X'$$

Deviations-from-means matrix
$$M_i = I - \frac{1}{n}ii'$$

Projection matrix
$$P_X = X (X'X)^{-1} X'$$

R-squared
$$R^2 = \frac{\hat{y}' M_i \hat{y}}{y' M_i y} = 1 - \frac{e'e}{y' M_i y}$$

t-statistic
$$t_i = (b_i - \beta_i) / (\text{ s.e. } (b_i)) \sim t_{n-k}$$

Confidence interval
$$[b_i - t_c \text{ s.e. } (b_i) , b_i + t_c \text{ s.e. } (b_i)]$$

Wald test statistic
$$W = (R\mathbf{b} - \mathbf{q})' \left[R(X'X)^{-1} R' \right]^{-1} (R\mathbf{b} - \mathbf{q})/s^2$$

$$F = \left\{ (R\boldsymbol{b} - \boldsymbol{q})' \left[R (X'X)^{-1} R' \right]^{-1} (R\boldsymbol{b} - \boldsymbol{q}) / J \right\} / s^2$$

F-statistic
$$= \left[\left(e^{*\prime} e^{*} - e^{\prime} e \right) / J \right] / \left[\left(e^{\prime} e \right) / (n - k) \right]$$

$$= \left[\left(R^2 - R_*^2 \right) / J \right] / \left[\left(1 - R^2 \right) / (n - k) \right]$$

Restricted Least Squares estimator
$$\boldsymbol{b}_* = \boldsymbol{b} - (X'X)^{-1} R' \left[R (X'X)^{-1} R' \right]^{-1} (R\boldsymbol{b} - \boldsymbol{q})$$

IV estimator (just-identified)
$$\hat{\beta}_{IV} = (Z'X)^{-1} Z' \boldsymbol{y}$$

IV estimator (over-identified)
$$\hat{\beta}_{IV} = \left[X'Z \left(Z'Z \right)^{-1} Z'X \right]^{-1} X'Z \left(Z'Z \right)^{-1} Z' \boldsymbol{y}$$

Hausman test statistic
$$H = (\boldsymbol{b}_{IV} - \boldsymbol{b})' \left[\hat{V} (\boldsymbol{b}_{IV}) - \hat{V} (\boldsymbol{b}) \right]^{-1} (\boldsymbol{b}_{IV} - \boldsymbol{b})$$

Generalized least squares estimator
$$\widehat{\boldsymbol{\beta}}_{\text{GLS}} = \left[X' \Sigma^{-1} X \right]^{-1} X' \Sigma^{-1} \boldsymbol{y} = \left[X' \Omega^{-1} X \right]^{-1} X' \Omega^{-1} \boldsymbol{y}$$

Autoregressive (1) process
$$\epsilon_t = \rho \epsilon_{t-1} + u_t \quad ; \quad u_t \sim \text{ i.i.d. } N\left[0, \sigma_u^2\right] \quad ; \quad |\rho| < 1$$

Moving average (1) process
$$\epsilon_t = u_t + \phi u_{t-1}$$
; $u_t \sim \text{i.i.d. N } [0, \sigma_u^2]$