Utility Theory and Model Formulation

Consumer Behavior Assumptions

- In a multinomial logit, the choices reflect tradeoffs the consumer must face
  - Tide is of high quality but of higher price
  - Cheer is not so good, but the price is lower
- These tradeoffs are captured in the consumer’s utility function for each choice alternative

How the Model Maps Utilities to Choices?

- j indexes the choices (J of them)
  - No need to assume equal choices
- i indexes people (N of them)
- Y_{ij} = 1 if person i selects option j, = 0 otherwise
- U_{ij} is the utility or net benefit of person i if they select option j

How the Model Maps Utilities to Choices?

- j indexes the choices among “Tide,” “Wisk,” “YES,” “Cheer” (J = 4)
  - No need to assume equal choices
- i indexes people (N of them)
- Y_{ij} = 1 if person i selects option j, = 0 otherwise
- U_{ij} is the utility or net benefit of person i if they select option j
- Suppose customer chooses Tide (j = 1)

How the Model Maps Utilities to Choices?

- Then there are a set of 3 (J - 1) inequalities that must be true
- $U_{itide} > U_{iwisk}$
- $U_{itide} > U_{iYES}$
- $U_{itide} > U_{icheer}$
- Choice of Tide dominates the other
- A multinomial logit model will ensure the coefficients reflect these behavioral assumptions about consumers

Compute Choice Scores

Relay C (Binary Logit)
Probability of retaining customer i =

$$P_i = \frac{e^{(U_i)}}{1+e^{(U_i)}}$$

(Multinomial Logit)
Probability of customer i choosing j =

$$P_{ij} = \frac{e^{(U_{ij})}}{\sum_{k=1}^{K} e^{(U_{ik})}}$$

Probability of customer i choosing alternative “Cheer” among choices “Tide,” “Wisk,” “YES,” “Cheer”

$$= P_{icheer} = \frac{e^{(U_{icheer})}}{e^{(U_{itide})} + e^{(U_{iwisk})} + e^{(U_{iYES})} + e^{(U_{iCheer})}}$$

Map to Market Share

$$p_{ij} = \frac{e^{u_{ij}}}{\sum_k e^{u_{ik}}}$$

$$m_j = \frac{\sum_{j=1}^{n} p_{ij}}{n}$$

u_{ij} = Total utility of product bundle j for consumer i

p_{ij} = Proportion of purchases that consumer i makes of product j or

p_{ij} = probability that consumer i will choose product j

Market share for product j (m_j) = average p_{ij} across consumers