

# CKY Parsing

Ling571

Deep Processing Approaches to NLP

January 18, 2017

# Roadmap

- CKY parsing algorithm
  - Dynamic programming structure
  - Algorithm and example
  - Analysis & Discussion
- Probabilistic CFGs
  - Motivation: Ambiguity
  - Augmenting CFGs with probabilities

# CKY

- Given an input string  $S$  of length  $n$ ,
  - Build table  $(n+1) \times (n+1)$
  - Indexes correspond to inter-word positions
    - E.g., 0 Book 1 That 2 Flight 3
- Cells  $[i,j]$  contain SETS of non-terminals of ALL constituents spanning  $i,j$ 
  - $[j-1,j]$  contains pre-terminals
  - If  $[0,n]$  contains Start, the input is recognized

# CKY Algorithm

**function** CKY- PARSE(*words*, *grammar*) **returns** *table*

**for**  $j$  **from** 1 **to** LENGTH(*words*) **do**

$table[j-1, j] \leftarrow \{ A \mid A \rightarrow w_j, w_j \in \text{grammar} \}$

**for**  $i$  **from** 1 **to**  $j-2$  **do**

**for**  $k$  **from**  $i+1$  **to**  $j-1$  **do**

$table[i, j] \leftarrow table[i, j] \cup$

$\{ A \mid A \rightarrow BC, B \in table[i, k],$

$C \in table[k, j] \}$

$C \in table[k, j] \}$

Is this a parser?

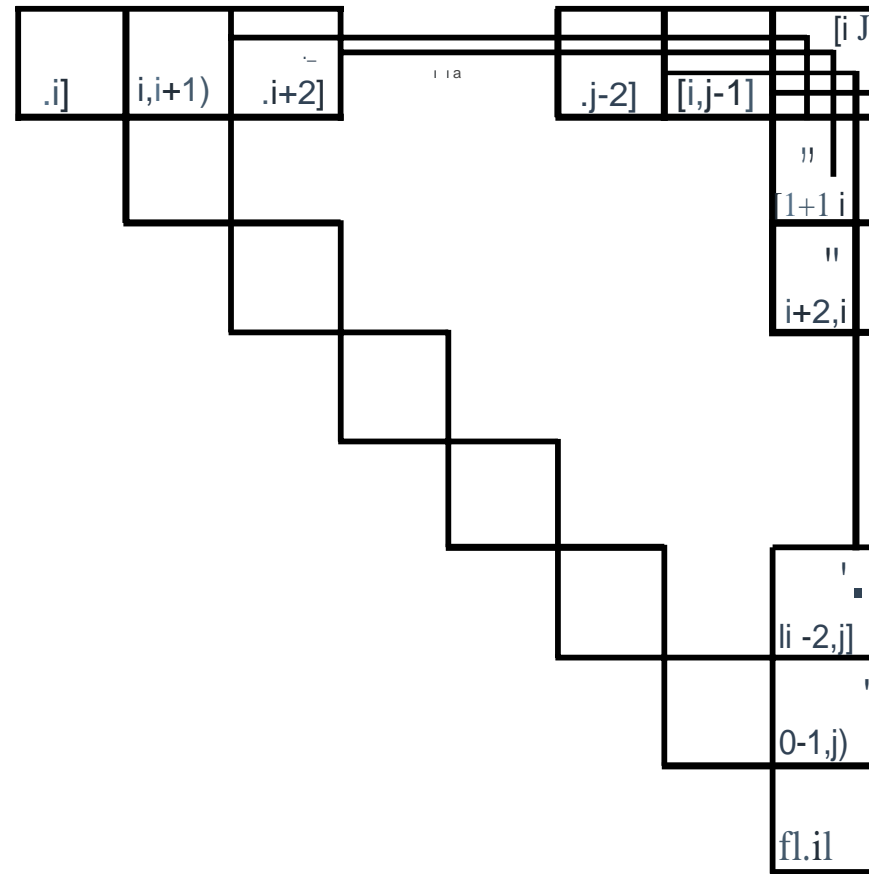


# CKY Algorithm

- Table fills:
  - Column-by-column
  - Left-to-right
  - Bottom-to-top
- Why?
  - Necessary info available (below and left)
  - Allows online sentence analysis
    - Works across input string as it arrives



# Filling CKY cell





0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

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NP, Pronoun  
[0,1]

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

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0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]				
	Verb,VP,S [1,2]				
		Det [2,3]			

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]				
	Verb,VP,S [1,2]	[1,3]			
		Det [2,3]			

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]			
	Verb,VP,S [1,2]	[1,3]			
		Det [2,3]			
			NN, Nom [3,4]		

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]			
	Verb,VP,S [1,2]	[1,3]			
		Det [2,3]	NP [2,4]		
			NN, Nom [3,4]		



0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]			
	Verb,VP,S [1,2]	[1,3]	VP,XP2,S [1,4]		
		Det [2,3]	NP [2,4]		
			NN, Nom [3,4]		

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb,VP,S [1,2]	[1,3]	VP,XP2,S [1,4]		
		Det [2,3]	NP [2,4]		
			NN, Nom [3,4]		

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb,VP,S [1,2]	[1,3]	VP,XP2,S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			NN, Nom [3,4]	[3,5]	
				Prep [4,5]	

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb,VP,S [1,2]	[1,3]	VP,XP2,S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			NN, Nom [3,4]	[3,5]	
				Prep [4,5]	
					NNP, NP [5,6]

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb,VP,S [1,2]	[1,3]	VP,XP2,S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			NN, Nom [3,4]	[3,5]	
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb,VP,S [1,2]	[1,3]	VP,XP2,S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			NN, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb,VP,S [1,2]	[1,3]	VP,XP2,S [1,4]	[1,5]	VP,XP2,S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			NN, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

0 I 1 prefer 2 a 3 flight 4 on 5 TWA 6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb,VP,S [1,2]	[1,3]	VP,XP2,S [1,4]	[1,5]	VP,XP2,S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			NN, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



# From Recognition to Parsing

- Limitations of current recognition algorithm:
  - Only stores non-terminals in cell
    - Not rules or cells corresponding to RHS
  - Stores SETS of non-terminals
    - Can't store multiple rules with same LHS
- Parsing solution:
  - All repeated versions of non-terminals
  - Pair each non-terminal with pointers to cells
    - Backpointers
  - Last step: construct trees from back-pointers in  $[0,n]$

0 I    1 prefer    2 a    3 flight    4 on    5 TWA    6

I	prefer	a	flight	on	TWA
NP,Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S <sub>1</sub> , S <sub>2</sub> , S <sub>3</sub> [0,6]
	Verb,VP,S [1,2]	[1,3]	VP,XP2,S [1,4]	[1,5]	VP <sub>1</sub> ,VP <sub>2</sub> ,VP <sub>3</sub> , XP2,S <sub>1</sub> ,S <sub>2</sub> ,S <sub>3</sub> [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			NN, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

# CKY Discussion

- Running time:

- $O(n^3)$  where  $n$  is the length of the input string
- Inner loop grows as square of # of non-terminals

- Expressiveness:

- As implemented, requires CNF
  - Weakly equivalent to original grammar
  - Doesn't capture full original structure
    - Back-conversion?
      - Can do binarization, terminal conversion
      - Unit non-terminals require change in CKY

# Parsing: PCFGs

Ling 571

Deep Processing Techniques for NLP

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# Roadmap

- Probabilistic Context-free Grammars (PCFGs)
  - Motivation: Ambiguity
  - Approach:
    - Definition
    - Disambiguation
    - Parsing
    - Evaluation
    - Enhancements

— What about ambiguity?

— CKY can represent it

— Can't resolve it

# Probabilistic Parsing

- Provides strategy for solving disambiguation problem
  - Compute the probability of all analyses
  - Select the most probable
- Employed in language modeling for speech recognition
  - N-gram grammars predict words, constrain search
  - Also, constrain generation, translation

# PCFGs

- Probabilistic Context-free Grammars
  - Augmentation of CFGs

$N$  a set of **non-terminal symbols** (or **variables**)

$\Sigma$  a set of **terminal symbols** (disjoint from  $N$ )

$R$  a set of **rules** or productions, each of the form  $A \rightarrow \beta$  [ $p$ ],

where  $A$  is a non-terminal,

$\beta$  is a string of symbols from the infinite set of strings  $(\Sigma \cup N)^*$ ,

and  $p$  is a number between 0 and 1 expressing  $P(\beta|A)$

$S$  a designated **start symbol**



# PCFGs

- Augment each production with probability that LHS will be expanded as RHS
  - $P(A \rightarrow B)$  or  $P(A \rightarrow B | A)$ ,  $P(\text{RHS} | \text{LHS})$
  - Sum over all possible expansions is 1

$$\sum_{\beta} P(A \rightarrow \beta) = 1$$

- A PCFG is consistent if sum of probabilities of all sentences in language is 1.
  - Recursive rules often yield inconsistent grammars

# Example PCFG

## Grammar

## Lexicon

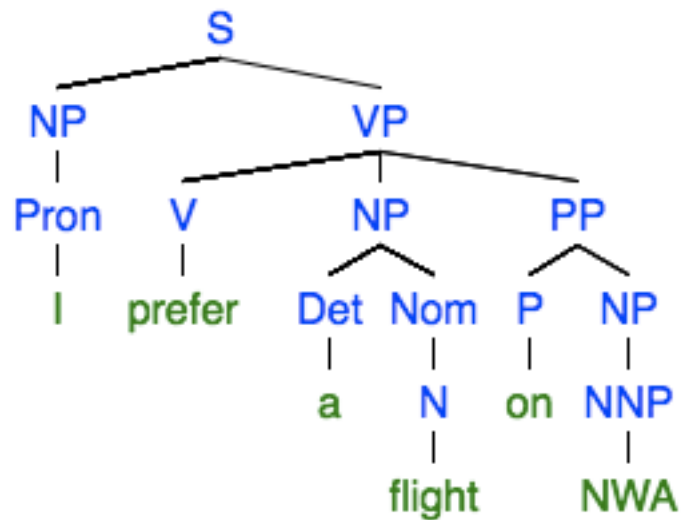
<i>S</i> → <i>NP VP</i>	[.80]	<i>Det</i> → <i>that</i> [.10]   <i>a</i> [.30]   <i>the</i> [.60]
<i>S</i> → <i>Aux NP VP</i>	[.15]	<i>Noun</i> → <i>book</i> [.10]   <i>flight</i> [.30]
<i>S</i> → <i>V P</i>	[.05]	<i>real</i> [.15]   <i>money</i> [.05]
<i>NP</i> → <i>Pronoun</i>	[.35]	<i>flights</i> [.40]   <i>dinner</i> [.10]
<i>NP</i> → <i>Proper-Noun</i>	[.30]	<i>Verb</i> → <i>book</i> [.30]   <i>include</i> [.30]
<i>NP</i> → <i>Det NoIninal</i>	[.20]	<i>prefer</i> [.40]
<i>NP</i> → <i>Nonzinal</i>	[.15]	<i>Pronoun</i> → <i>I</i> [.40]   <i>she</i> [.05]
<i>Norninal</i> → <i>Noun</i>	[.75]	<i>me</i> [.15]   <i>you</i> [.40]
<i>Norninal</i> → <i>Norninal Noun</i>	[.20]	<i>Proper-Noun</i> → <i>Houston</i> [.60]
<i>Norninal</i> → <i>Nonzinal PP</i>	[.05]	<i>NWA</i> [.40]
<i>VP</i> → <i>Verb</i>	[.35]	<i>Aux</i> → <i>does</i> [.60]   <i>can</i> [.40]
<i>VP</i> → <i>Verb TP</i>	[.20]	<i>Preposition</i> → <i>front</i> [.30]   <i>to</i> [.30]
<i>VP</i> → <i>Verb NP PP</i>	[.10]	<i>on</i> [.20]   <i>near</i> [.15]
<i>VP</i> → <i>Verb PP</i>	[.15]	<i>through</i> [.05]
<i>VP</i> → <i>Verb NP NP</i>	[.05]	
<i>VP</i> → <i>VP PP</i>	[.15]	
<i>PP</i> → <i>Preposition NP</i>	[1.0]	

# Disambiguation

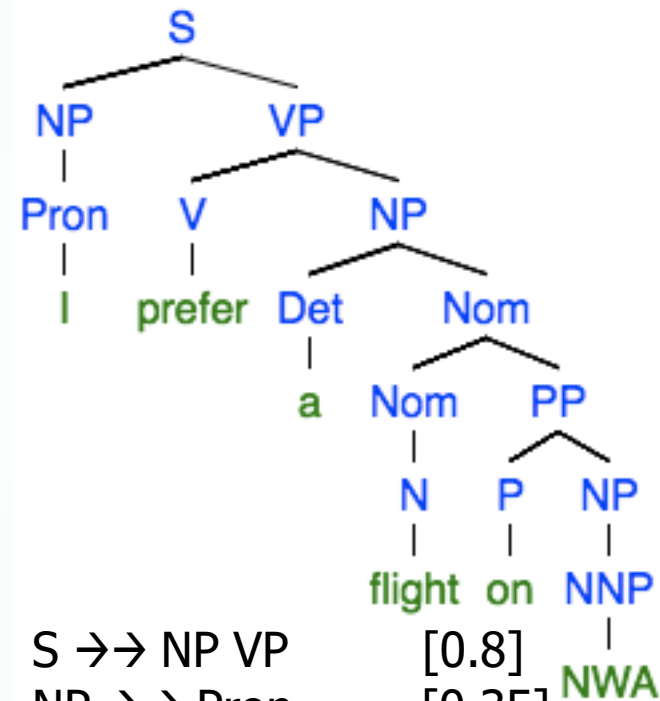
- A PCFG assigns probability to each parse tree  $T$  for input  $S$ .
- Probability of  $T$ : product of all rules to derive  $T$

$$P(T, S) = \prod_{i=1}^n P(RHS_i \mid LHS_i)$$

$$P(T, S) = P(T)P(S \mid T) = P(T)$$



S →→ NP VP	[0.8]
NP →→ Pron	[0.35]
Pron →→ I	[0.4]
<b>VP →→ V NP PP</b>	<b>[0.1]</b>
V →→ prefer	[0.4]
NP →→ Det Nom	[0.2]
Det →→ a	[0.3]
Nom →→ N	[0.75]
N →→ flight	[0.3]
PP →→ P NP	[1.0]
P →→ on	[0.2]
NP →→ NNP	[0.3]
NNP →→ NWA	[0.4]



S →→ NP VP	[0.8]
NP →→ Pron	[0.35]
Pron →→ I	[0.4]
<b>VP →→ V NP</b>	<b>[0.2]</b>
V →→ prefer	[0.4]
NP →→ Det Nom	[0.2]
Det →→ a	[0.3]
<b>Nom →→ Nom PP</b>	<b>[0.05]</b>
Nom →→ N	[0.75]
N →→ flight	[0.3]
PP →→ P NP	[1.0]
P →→ on	[0.2]
NP →→ NNP	[0.3]
NNP →→ NWA	[0.4]

# Parsing Problem for PCFGs

- Select  $T$  such that:

$$\hat{T}(S) = \operatorname{argmax}_{Ts.t, S=\text{yield}(T)} P(T)$$

- String of words  $S$  is *yield* of parse tree over  $S$
- Select tree that maximizes probability of parse