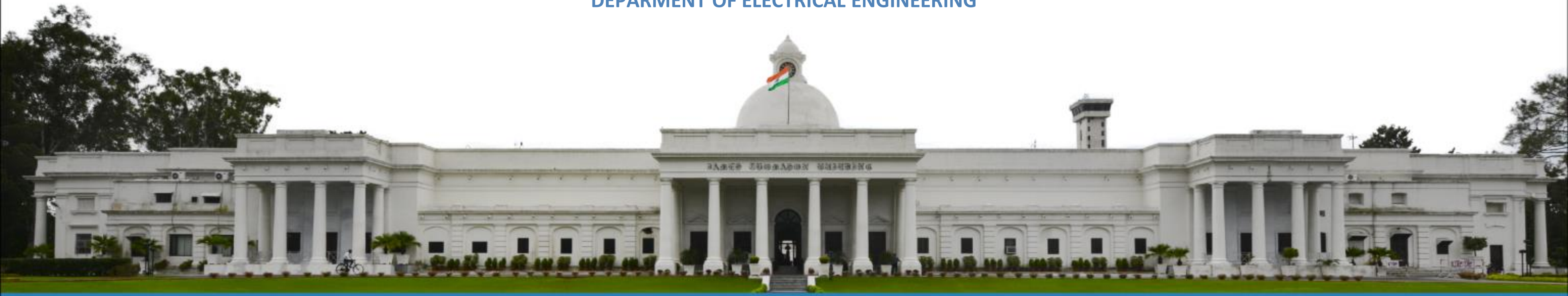
IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Prof. FELIX ORLANDO MARIA JOSEPH

DEPARMENT OF ELECTRICAL ENGINEERING



Fundamentals of Robot Manipulability

Outline

1. Manipulability Ellipsoid and Manipulability Measure

2. Best Configurations of Manipulators based on Manipulability
   - Two Link Planar Manipulator
   - SCARA Type Manipulator
   - PUMA Type Manipulator
   - Four Jointed Robot Finger

3. Various Indices of Manipulability

4. Manipulability Analysis of Human Digits in Object Translation Motion – *Yokogawa and Hara*

Manipulability Ellipsoid and Manipulability Measure

- Consider a manipulator with n-DOF

Manipulability Ellipsoid and Manipulability Measure

- The set of all end-effector velocities v which are realizable by joint velocities such that,

$$\|\dot{\boldsymbol{q}}\| = (\dot{q}_1^2 + \dot{q}_2^2 + \ldots + \dot{q}_n^2)^{1/2}$$

$$\|\dot{\boldsymbol{q}}\| \leq 1$$

- In the direction of the major axis of the ellipsoid, the end effector → moves at high speed.
- In the direction of the minor axis, end effector → moves at low speed.
- If the ellipsoid is almost a sphere, the end effector can move in all directions uniformly.
- Also, the larger the ellipsoid is, the faster the end effector can move.
- The manipulability ellipsoid $\boldsymbol{v}^T (\boldsymbol{J}^+)^T \boldsymbol{J}^+ \boldsymbol{v} \leq 1, \boldsymbol{v} \in \boldsymbol{R}(\boldsymbol{J})$

Manipulability Ellipsoid and Manipulability Measure (cont'd)

•Principle axes of the manipulability ellipsoid by making use of the singular-value decomposition of $\boldsymbol{J}$

$$\boldsymbol{J} = \boldsymbol{SVD}^{\boldsymbol{T}}$$

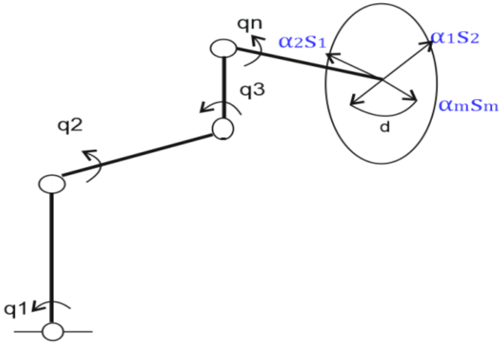
where $\boldsymbol{S}$and **D**are, respectively, m × m and n × n orthogonal matrices, and where **V**is an n × n matrix defined by

$$\boldsymbol{V} = \begin{bmatrix} \alpha_1 & \mathbf{0} & \\ & \ddots & \mathbf{0} \\ \mathbf{0} & \alpha_m & \end{bmatrix}, \alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m \geq 0.$$

• The scalars α_1, α_2,…,α_m are called singular values of $\boldsymbol{J}$, and are equal to the m larger values of the n roots $\{\sqrt{\lambda_i}, i = 1, 2, \ldots, n\}$, where λ_i (i=1,2,…n) are eigenvalues of the matrix $\boldsymbol{J}^{\boldsymbol{T}}\boldsymbol{J}$.

Manipulability Ellipsoid and Manipulability Measure (cont'd)

- Further, we let s_i be the i^{th} column vector of **S**.
- Then the principle axes of the manipulability ellipsoid are $\alpha_1 s_1, \alpha_2 s_2, \ldots, \alpha_{\boldsymbol{m}} s_{\boldsymbol{m}}$



Manipulability Ellipsoid and Manipulability Measure (cont'd)

•From the properties of pseudo inverse,

$$\boldsymbol{J}^{+} = \boldsymbol{D}\boldsymbol{V}^{+}\boldsymbol{S}^{T}$$

where $\boldsymbol{V}^{+}$ is the pseudo-inverse of $\boldsymbol{V}$, given by

$$\boldsymbol{V}^{+} = \begin{bmatrix} \alpha_{1}^{-1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \alpha_{m}^{-1} \\ & \mathbf{0} & \end{bmatrix}$$

We consider the following orthogonal transformation of $\boldsymbol{d}$:

$$\tilde{\boldsymbol{d}} = \mathbf{S}^{T}\mathrm{d} = col[\tilde{d}_i]$$

Then, by the eqn. of manipulability ellipsoid, we have

$$\sum_{\alpha_i \neq 0} \frac{1}{\alpha_i^2}\tilde{d}_i^2 \leq 1$$

Manipulability Ellipsoid and Manipulability Measure (cont'd)

- Thus, direction of the coordinate axis for $\tilde{d}_i$ (i.e., the direction of s_i) is that of a principle axis, and that the radius in that direction is given by α_i.

- Therefore, the principle axes are $\alpha_1 s_1$, $\alpha_2 s_2$, …, $\alpha_m s_m$.

- The manipulability measure is given by

$$w = \alpha_1 \alpha_2 \ldots \alpha_m$$

- The manipulability measure w has the following properties:

  - $w = \sqrt{|\boldsymbol{J}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q})|}$

  - *When $m = n$ (non-redundant manipulators),*

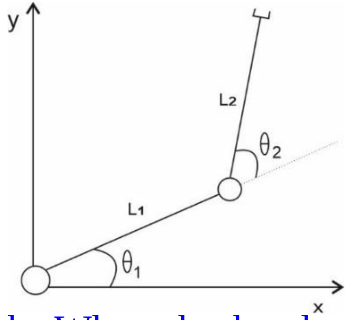
$$w = |\boldsymbol{J}(\boldsymbol{q})|$$

  - *Generally $w \geq 0$ holds, and $w=0$ if and only if*

$$rank\ J(q) < m$$

Best Configurations of Robotic Mechanisms from Manipulability Viewpoint

- Two Link Mechanism



- Let us consider a two-link. When the hand position $[x, y]^T$ is used for **r**, the Jacobian matrix is

$$\boldsymbol{J} = \begin{bmatrix} -L_1 s_1 - L_2 s_{12} & -L_2 s_{12} \\ L_1 c_1 + L_2 c_{12} & L_2 c_{12} \end{bmatrix}$$

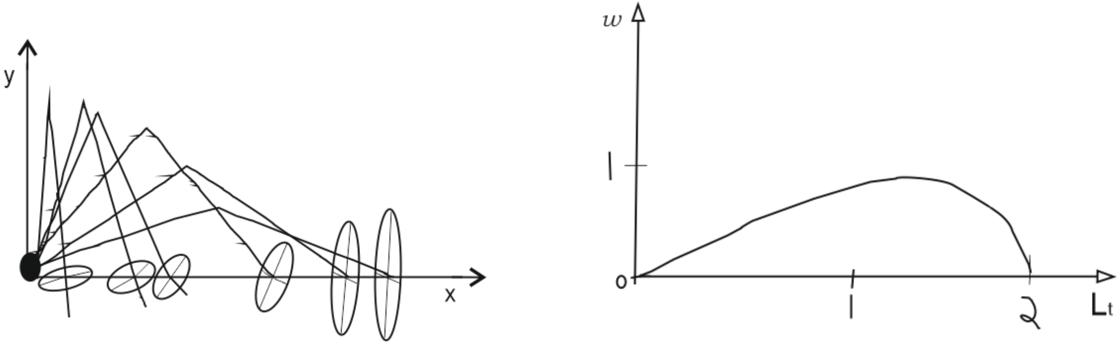
And the manipulability measure w is

$$w = |\boldsymbol{J}| = L_1 L_2 |s_2|$$

- Thus, the manipulator takes its optimal configuration when $\theta_2 = \pm 90^{\circ}$, for any given values of L_1, L_2 and θ_1.

Best Configurations of Robotic Mechanisms from Manipulability Viewpoint

- Thus, schematically:



Best Configurations of Human Finger from Manipulability Viewpoint

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Best Configurations of Robotic Mechanisms from Manipulability Viewpoint

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Best Configurations of Robotic Mechanisms from Manipulability Viewpoint

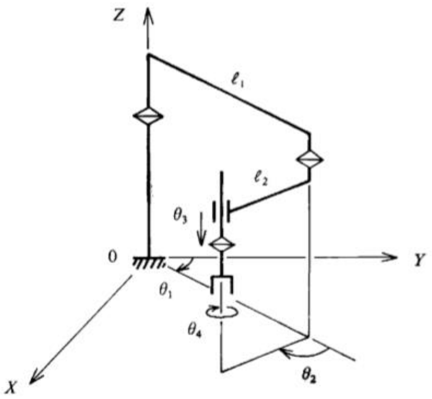
- SCARA Type Mechanism

Scara type mechanism having 4 DOF as $s=[x,y,z,\alpha]^T$ and $[x,y,z]^T$ is the hand position and α is the rotational angle of hand with respect to Z-axis. So the Jacobian matrix for this case is

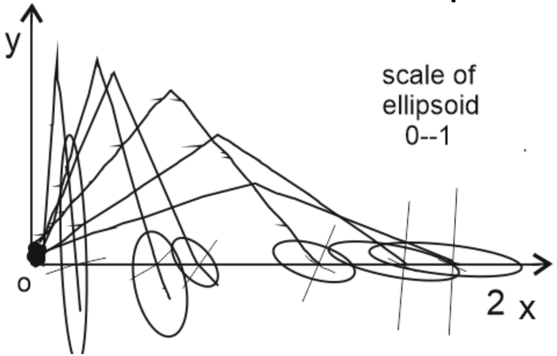
$$J=\begin{bmatrix} L_1 s_1 - L_2 s_{12} & -L_2 s_{12} & 0 & 0 \\ L_1 c_1 + L_2 c_{12} & L_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- The manipulability is $w=L_1L_2|s_2|$
- Like the two link mechanism the best posture is attained when Θ_2 is $\pm 90^o$

Best Configurations of Robotic Mechanisms from Manipulability Viewpoint



Manipulability
Force Ellipsoid



Best Configurations of Robotic Mechanisms from Manipulability Viewpoint

PUMA-Type

here $q=[\Theta_1\Theta_2\Theta_3]^T$ and manipulator vector be $[x\ y\ z]^T$ so

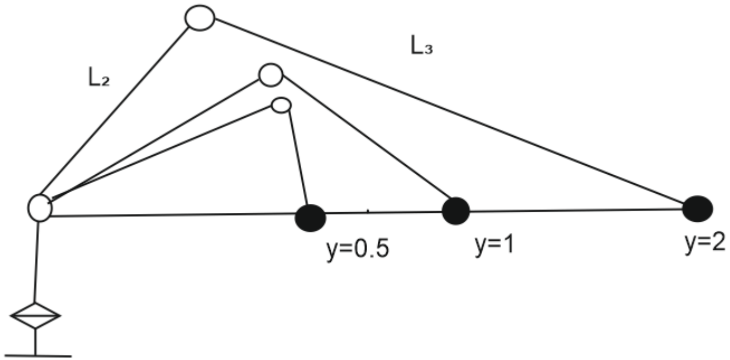
$$J=\begin{bmatrix} -s_1(L_2s_2+L_3s_{23}) & c_1(L_2c_2+L_3c_{23}) & c_1L_3c_{23} \\ C_1(L_2s_2+L_3s_{23}) & S_1(L_2c_2+L_3c_{23}) & S_1L_3c_{23} \\ 0 & -(L_2s_2+L_3s_{23}) & -L_3s_{23} \end{bmatrix}$$

- The manipulability Measure $w_1=L_2L_3|(L_2s_2+L_3s_{23})s_3|$
- For the best posture $\tan\Theta_2=(L_1+L_3c_4)/L_3s_3$

so the manipulability is $w_2=L_2L_3\sqrt{L_{22}+L_{32}+2L_2L_3c_3}|s_3|$

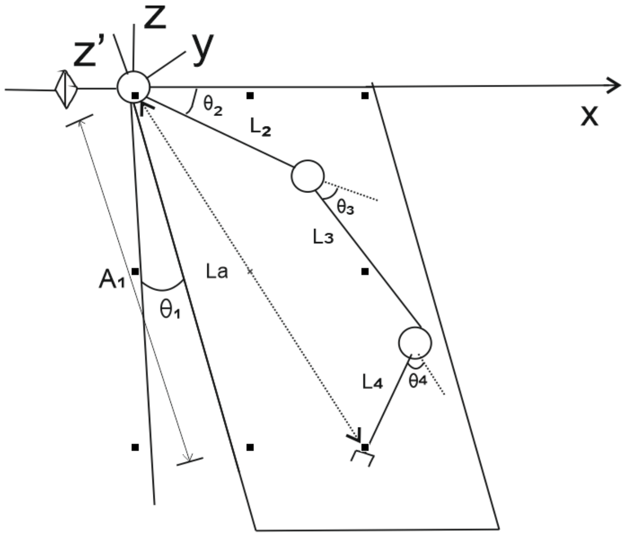
The Θ_3 which maximizes the w_2 is $\cos(\Theta_3)=(\sqrt{(L_{22}+L_{32})^2+12L_{22}L_{32}}-(L_{22}+L_{32}))/6L_2L_3$

Best Configurations of Robotic Mechanisms from Manipulability Viewpoint



Best Configurations of Robotic Mechanisms from Manipulability Viewpoint

- Four-Joint Robotic Finger



Best Configurations of Robotic Mechanisms from Manipulability Viewpoint

Four-Jointed Robot Finger

- Let $q=[\Theta_1\Theta_2\Theta_3\Theta_4]^T$ and manipulator vector be $[x\ y\ z]^T$
- So the Jacobian matrix is

$$\begin{bmatrix} 0 & -A_1 & -A_2 & -A_3 \\ A_1c_1 & B_1s_1 & B_2s_2 & B_3s_1 \\ A_1s_1 & -B_1c_1 & -B_2c_1 & -B_3c_1 \end{bmatrix}$$

Where

$$A_1 = L_2s_2+L_3s_{23}+L_4s_{234}$$

$$A_2 = L_3s_{23}+L_4s_{234}$$

Best Configurations of Robotic Mechanisms from Manipulability Viewpoint

- $A_3 = L_4 s_{234}$
- $B_1 = L_2 c_2 + L_3 c_{23} + L_4 c_{234}$
- $B_2 = L_3 c_{23} + L_4 c_{234}$

and the manipulability measure is

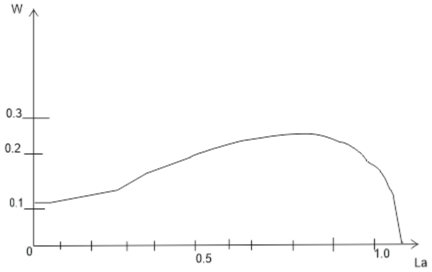
- $w = |A_1| W(\Theta_2 \Theta_3 \Theta_4)$

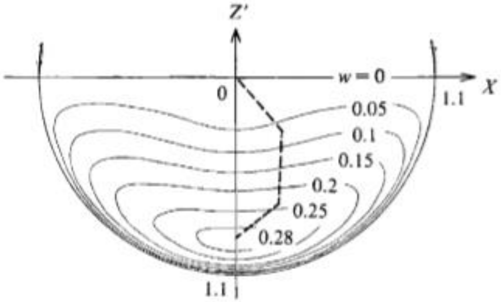
- Where $W(\Theta_2 \Theta_3 \Theta_4) = \sqrt{\det(\boldsymbol{J}\boldsymbol{J}^T)}$
- Where $\boldsymbol{J} = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{bmatrix}$

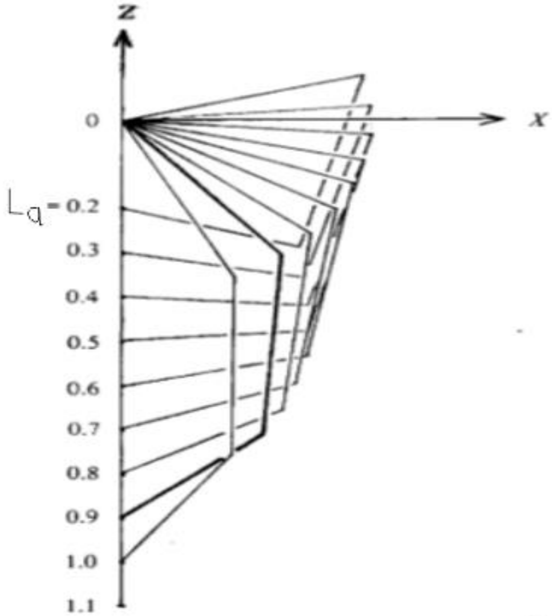
- $W(\Theta_2 \Theta_3 \Theta_4)$ is the manipulability measure of three joint mechanism which consists of joints 2,3,4 and which moves in X and Z' plane.

Best Configurations of Robotic Mechanisms from Manipulability Viewpoint

- Four-Joint Robotic Finger (cont'd)







Various Indices of Manipulability

- w_1: Manipulability measure represents the volume of the manipulability ellipsoid.

•w_2: α_m/α_1, ratio of the minimum and maximum radii of the ellipsoid. The closer to unity this index is, the more spherical the ellipsoid is.

  - An index of the directional uniformity of the ellipsoid and is independent of its size.

- w_3: α_m, is the minimum radius of the ellipsoid. This gives the upper bound of the magnitude of velocity at which the end effector can be moved in any direction.

- w_4: $(\alpha_1\alpha_2 \ldots \alpha_m)^{1/m} = (w_1)^{1/m}$, is the geometric mean of the radii $\alpha_{1,}\alpha_2, \ldots \alpha_m$,and isequal to the radius of the sphere whose volume is the same as that of the ellipsoid.

Research Study on Human Digits

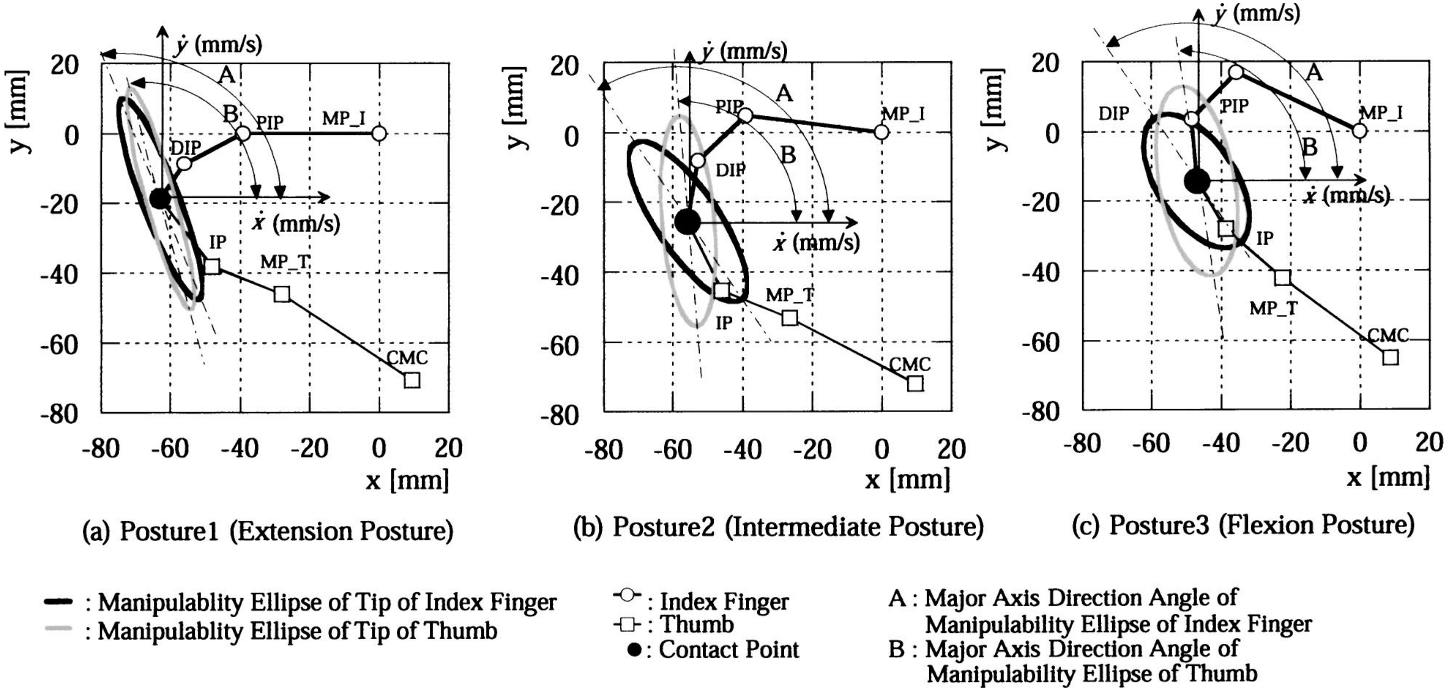
- Tip-pinch involving the human thumb and index finger for translation of a small object is an important function of hand.
  - Yokogawa and Hara [2].
- Objective:
  To investigate how humans affect the manipulabilities of these two digits during the cooperative translation motion of a small object.
- Based on the three criteria of
  (i) Manipulability measure
  (ii) Major axis direction angle of the manipulability ellipsoid
  (iii) Ratio of the minor over major axis length, the collective behavior of the digits was studied.
- It is found that the index finger is active and the thumb is passive.

- Yokogawa and Hara [2].

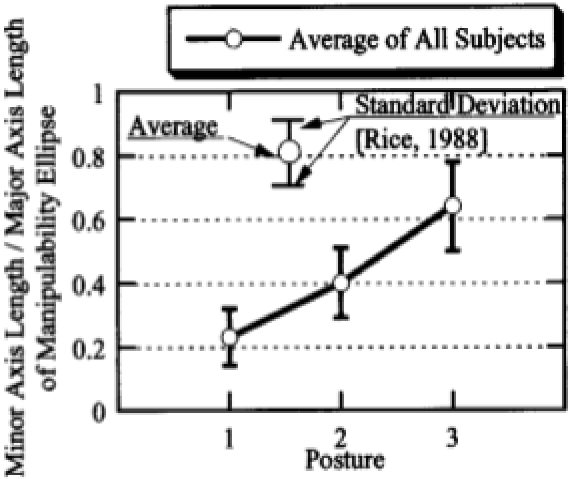
(a) Posture 1 (Extension Posture) (b) Posture 2 (Intermediate Posture) (c) Posture 3 (Flexion Posture)

●: Tip Point of Index Finger and Thumb

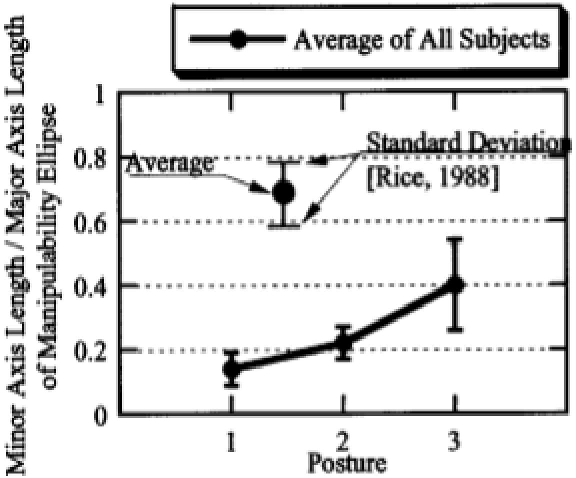
➢ Yokogawa and Hara [2].



- Yokogawa and Hara [2].

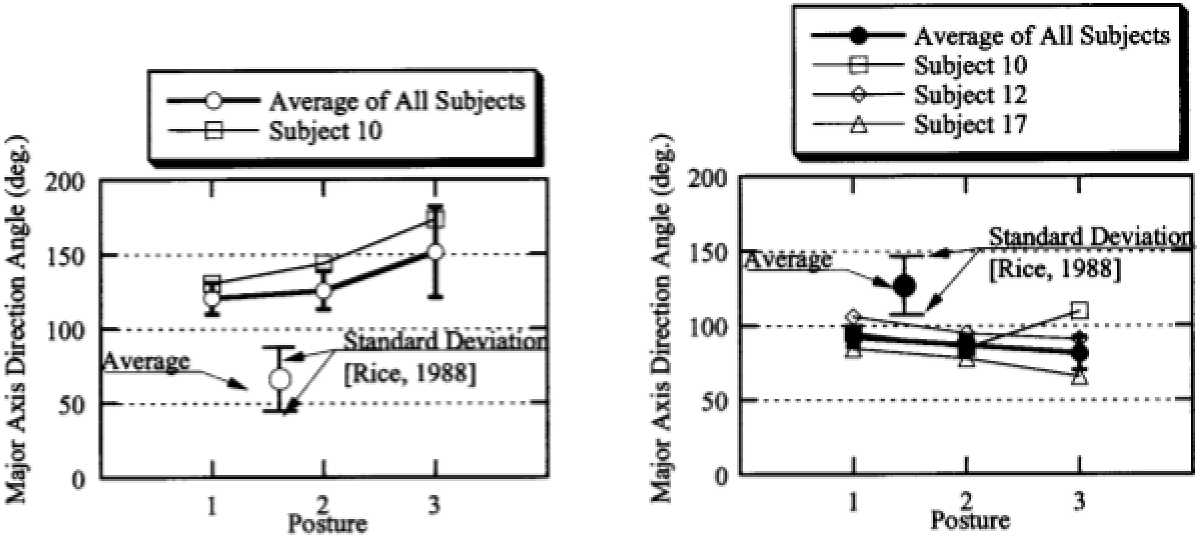


(a) Index Finger



(b) Thumb

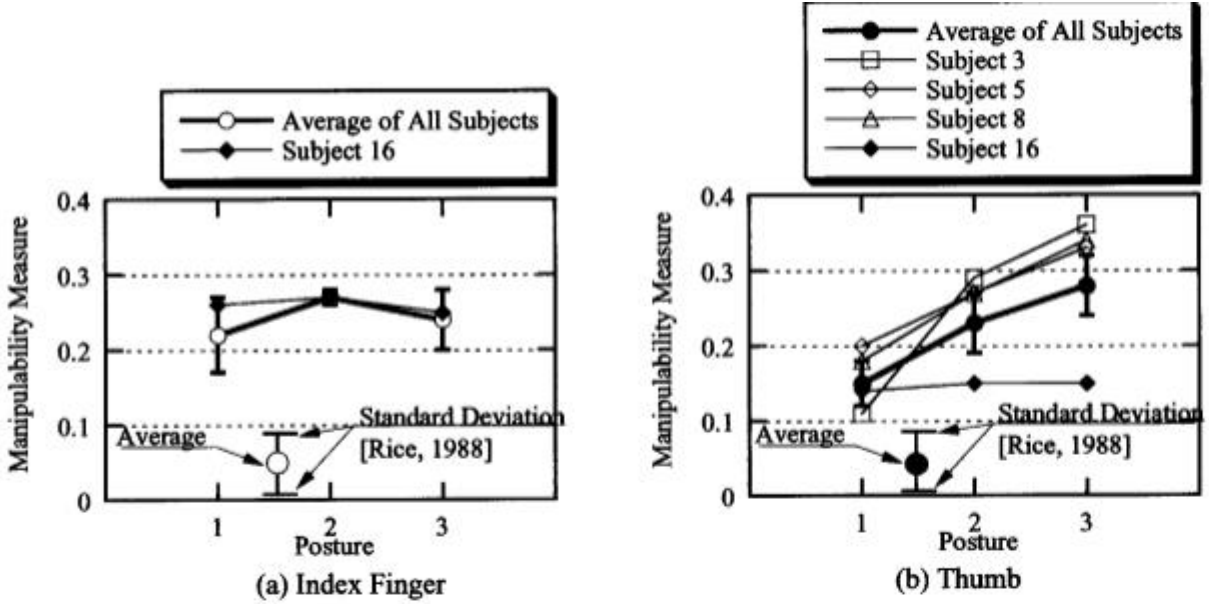
- Yokogawa and Hara [2].



(a) Index Finger

(b) Thumb

- Yokogawa and Hara [2].



References

1. Tsuneo Yoshikawa, “Foundations of Robtics: Analysis and Control”, The MIT Press, Cambridge, Massachusetts, 1990.

2. R. Yokogawa and K. Hara, “Manipulabilities of the index finger and thumb in three tip-pinch postures,” J. Biomech. Eng., vol. 126, no. 2, pp. 212–219, 2004.

Thank You!