IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

# Manipulator Jacobian Example

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DEPARMENT OF MATHEMATICS

# Manipulator Jacobian Example

| | $\boldsymbol{\theta}$ | $\boldsymbol{d}$ | $\boldsymbol{\alpha}$ | $\boldsymbol{a}$ |
|---|---|---|---|---|
| 1 | $\boldsymbol{\theta_1}$ | OA | $\pi/2$ | 0 |
| 2 | 0 | AC | $\pi/2$ | 0 |
| 3 | 0 | CE | 0 | 0 |
| 4 | $\boldsymbol{\theta_4}$ | EF | 0 | 0 |

As we know,

$$^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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So,
$$^{0}T_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & OA \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & AC \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & AC \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & CE \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & CE \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & EF \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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The arm matrix is given by:

$$^{0}T_4 = {}^{0}T_1 . {}^{1}T_2 . {}^{2}T_3 . {}^{3}T_4$$

Let

$$OA = 10,$$
$$BC = 12,$$
$$DE = 6,$$
$$EF = 4,$$
$$AC = d_2,$$
$$CE = d_3$$
$$\theta_1, d_2, d_3 \; and \; \theta_4 \; are \; joint \; variables$$

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Then

$$ {}^{3}T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 4+d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ 0 & 0 & -1 & -4-d_3 \\ \sin\theta_4 & \cos\theta_4 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_4 = \begin{bmatrix} \cos(\theta_1-\theta_4) & -\sin(\theta_1-\theta_4) & 0 & d_2\sin\theta_1 \\ \sin(\theta_1-\theta_4) & -\cos(\theta_1-\theta_4) & 0 & -d_2\cos\theta_1 \\ 0 & 0 & -1 & 6-d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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If the end-effector position and orientation at a time instant t is given by:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 6 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then

$$\theta_1 - \theta_4 = \frac{\pi}{4},$$
$$d_2 \sin\theta_1 = 0,$$
$$-d_2 \cos\theta_1 = 6,$$
$$\theta_1 = \pi,$$
$$d_2 = 6,$$
$$6 - d_3 = 2,$$
$$d_3 = 4$$
$$\theta_4 = \frac{3\pi}{4}$$

Now substituting $\theta_1, d_2, d_3$ *and* $\theta_4$, we get ${}^0T_4$, ${}^1T_4$, ${}^2T_{4,}$, ${}^3T_4$ at that time instant.

# Thanks!