IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Prof. FELIX ORLANDO MARIA JOSEPH

DEPARMENT OF ELECTRICAL ENGINEERING

Sliding Mode Control

Outline

1. Introduction

2. Conventional Sliding Mode Control

Introduction

- Discrepancy between actual plant and mathematical model.
- Unknown external disturbances, parameter variations of the plant and unmodeled dynamics.
- Control law Designing – Challenging task.
- Robust Control – Sliding Mode Control.

Advantages:

- Reduced order compensated dynamics
- Robustness
- Finite time convergence

Introduction

Considering an example…

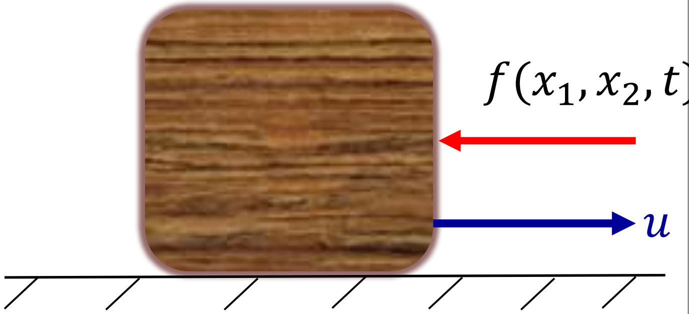
$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u + f(x_1, x_2, t)$$

$u \rightarrow$ control input

$f(x_1, x_2, t) \rightarrow$ bounded disturbance

$$|f(x_1, x_2, t)| \leq L > 0$$



Single dimensional motion of a unit mass

Control Problem

- To design a feedback control law $u = u(x_1, x_2)$

$$\lim_{t \to \infty} x_1, x_2 = 0$$

- $u = -k_1 x_1 - k_2 x_2, \quad k_1, k_2 > 0$ provides asymptotic stability only when disturbance is zero.

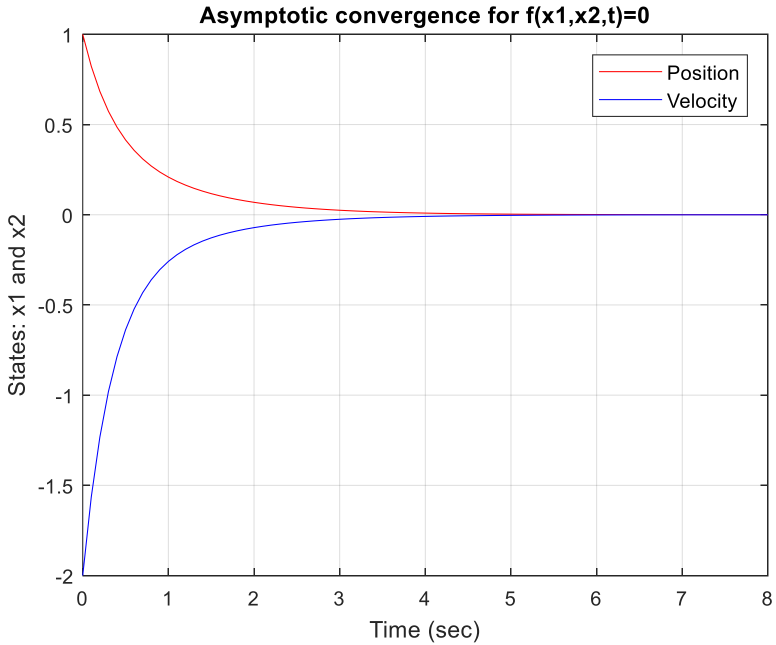
- It drives the system states to a bounded domain δ for the given bounded disturbance.

Introduction

- Asymptotic convergence of states only for $f(x_1, x_2, t) = 0$

- Given initial conditions:

$$x_1(0) = 1$$
$$x_2(0) = -2$$
$$k_1 = 3$$
$$k_2 = 4$$
$$f(x_1, x_2, t) = 0$$

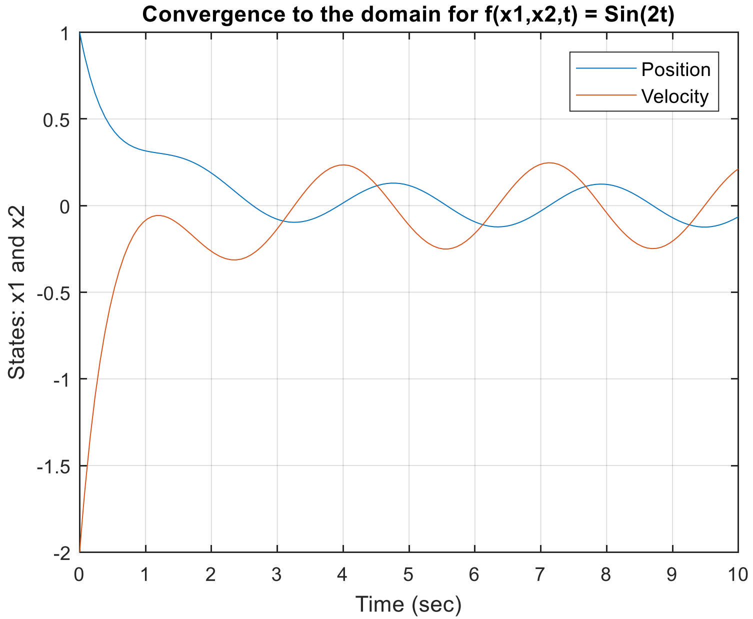


Introduction

- Driving the states only to bounded domain δ for $f(x_1, x_2, t) \neq 0$

- Given initial conditions:

$$x_1(0) = 1$$
$$x_2(0) = -2$$
$$k_1 = 3$$
$$k_2 = 4$$
$$f(x_1, x_2, t) = \sin(2t)$$



Main Concepts of Sliding Mode Control

- Let the desired compensated dynamics for the given system be

$$\dot{x}_1 + cx_1 = 0, \qquad c > 0$$

Since$x_2(t) = \dot{x}_1(t)$,

We have,

$$x_1(t) = x_1(0)\exp(-ct)$$
$$x_2(t) = \dot{x}_1(t) = -cx_1(0)\exp(-ct)$$

- Asymptotic convergence of the states.

- No disturbance effect is observed on the state compensated dynamics.

Main Concepts of Sliding Mode Control (cont'd)

- Introducing a new variable in the state space of the given system:

$$\sigma = \sigma(x_1, x_2) = x_2 + cx_1, \qquad c > 0$$

- To achieve the asymptotic convergence of the state variables in the presence of disturbance, the variable σ must be driven to zero in finite time by u

- Lyapunov function techniques to the σ-dynamics.

Main Concepts of Sliding Mode Control

- Let the Lyapunov function candidate be:

$$V = \frac{1}{2}\sigma^2$$

$$V = \sigma = \sigma(x_1, x_2) = x_2 + cx_1, \qquad c > 0$$

- For the asymptotic convergence, the following conditions must be satisfied:

  i. *Positive Definite*
  ii. $\lim_{\sigma \to \infty} V = \infty$
  iii. $\dot{V} \leq 0$ for asymptotic stability

Main Concepts of Sliding Mode Control

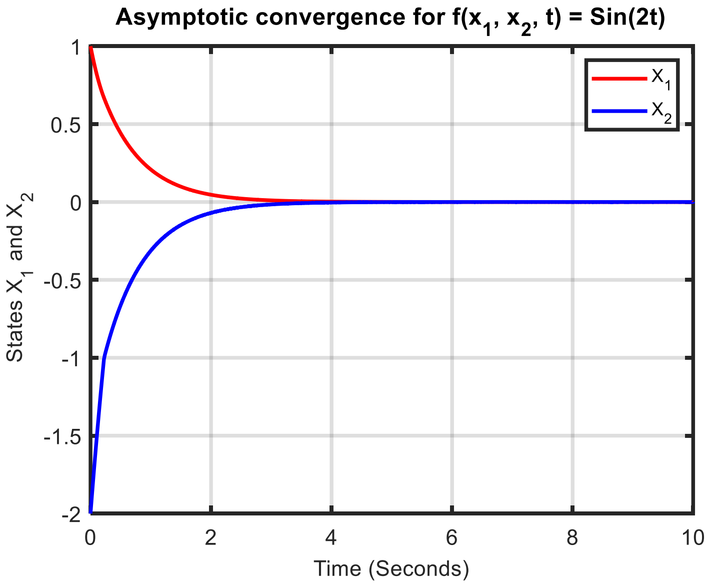
- But, for finite time convergence, we modify the condition iii by,

$$\dot{V} \leq -\alpha V^{1/2}, \qquad \alpha > 0$$

Sliding Mode Control

- Given initial conditions:

$$x_1(0) = 1$$
$$x_2(0) = -2$$
$$f(x_1, x_2, t) = \sin(2t)$$
$$c = 1.5$$
$$\rho = 2$$
$$u = -cx_2 - \rho\, sign(\sigma)$$



Sliding Mode Control

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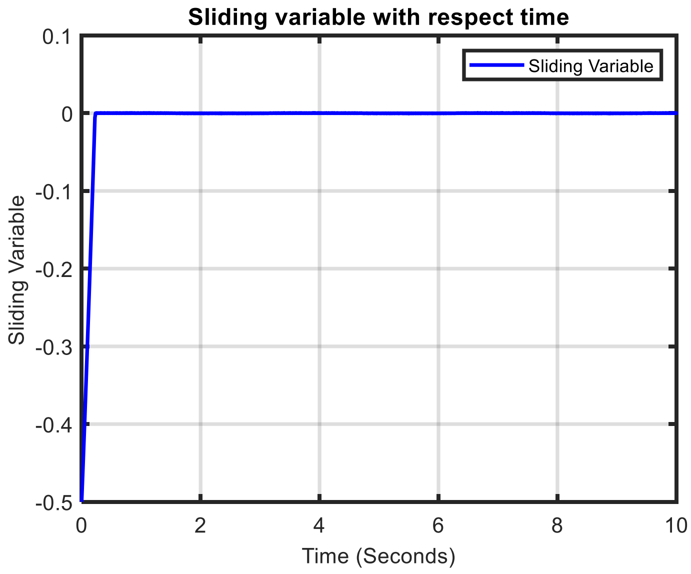

Sliding Mode Control

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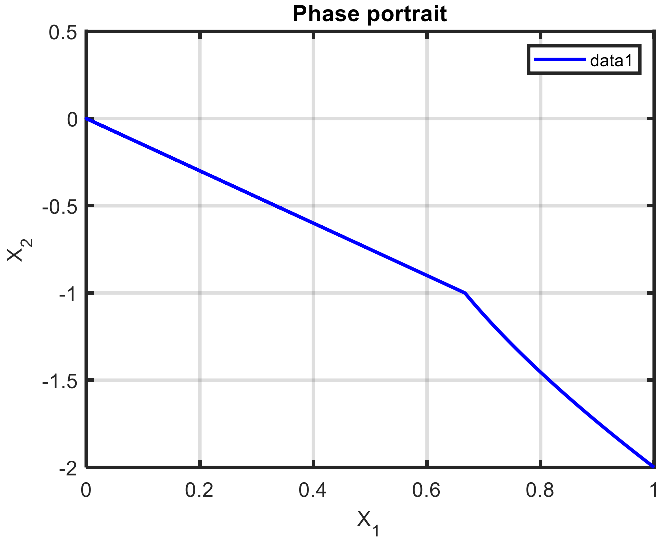

Sliding Mode Control

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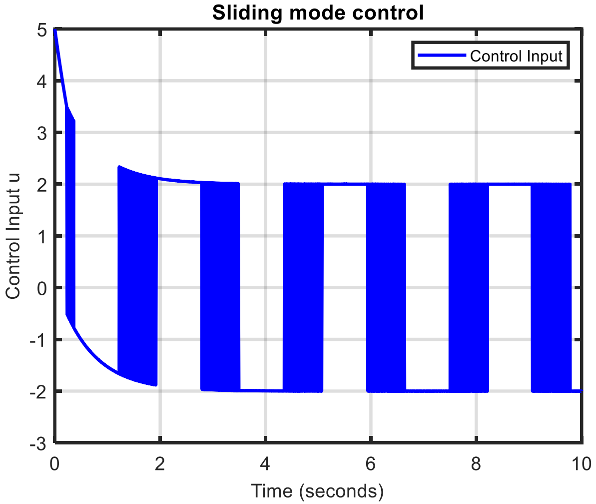

Conclusion

- Two designs are to be considered in SMC:

    1. Design of u
    2. Design of surface

Thank You!