IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Biped Robot Basics and Flat Foot Biped Model

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DEPARMENT OF MATHEMATICS

Biped Robots

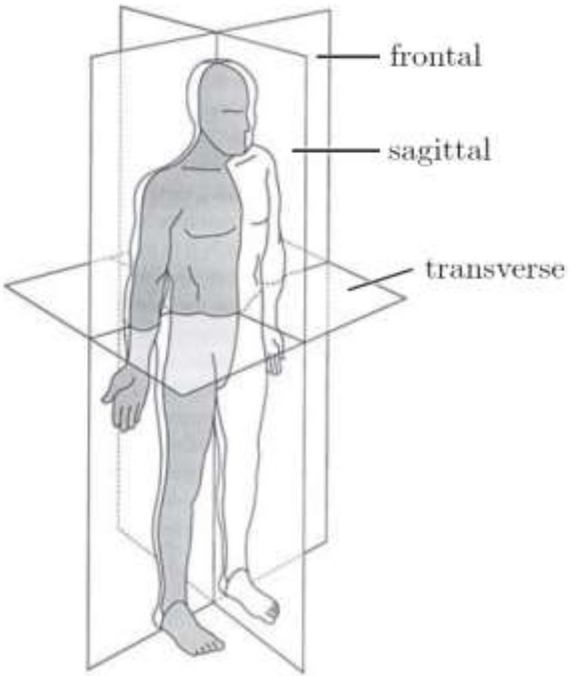
Honda Asimo

Atlas

Introduction

- Locomotion may be defined as the ability of a body to move from one place to another place.
- Naturally locomotion may be flying of birds, swimming of fish, walking of human beings etc.
- In case of complex environments like staircase walking, rough terrain or sloppy surface, most appropriate medium for locomotion are legs.
- Legs can avoid discontinuities in environment either by neglecting them or stepping over such obstacles.
- A subclass of legged robots are **Biped robots**.
- A biped may be defined as an open kinematic chain which consist of two subchains termed as legs and may also have a sub chain called torso, each one is connected at a common point termed as hip.
- To replace humans in hazardous environment like mining, nuclear plant activities, military related work, biped robots can be very beneficial.

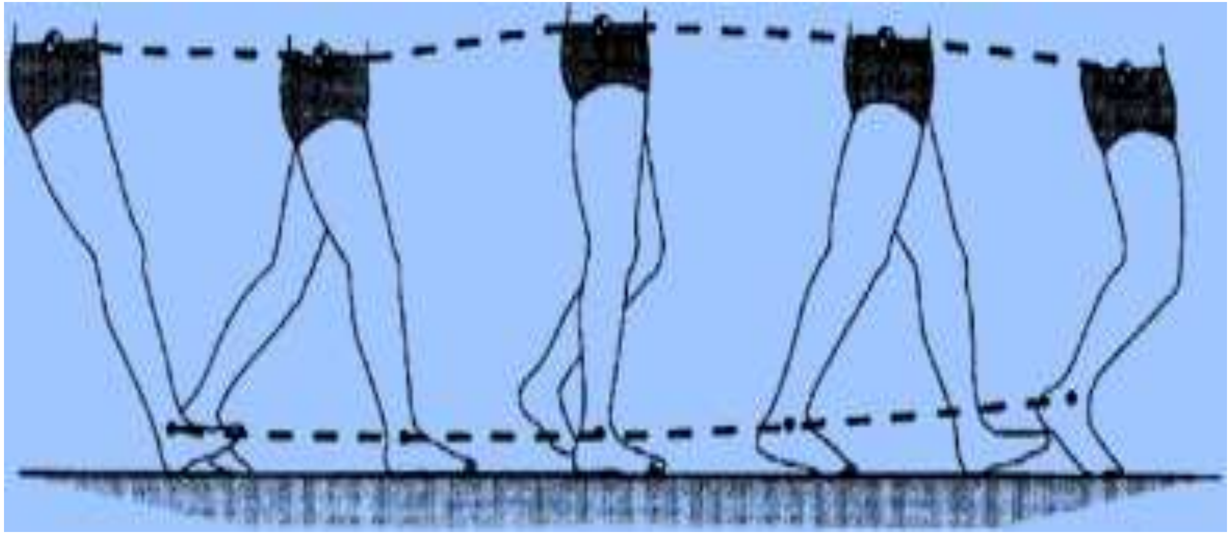
Sagittal, frontal and Transverse Planes



Sagittal, frontal and Transverse Planes

- Sagittal plane is the longitudinal plane which divides the body into left and right subsections.
- Frontal plane is the plane perpendicular to sagittal plane and it divides the body into front and back portions.
- Plane perpendicular to both the sagittal and frontal planes is transverse plane.
- In case of planar biped, motion is only restricted in sagittal plane.
- When a three dimensional motion is taken into consideration, motion is considered both in sagittal and frontal plane.

Human Locomotion
Ankle and Hip Trajectories

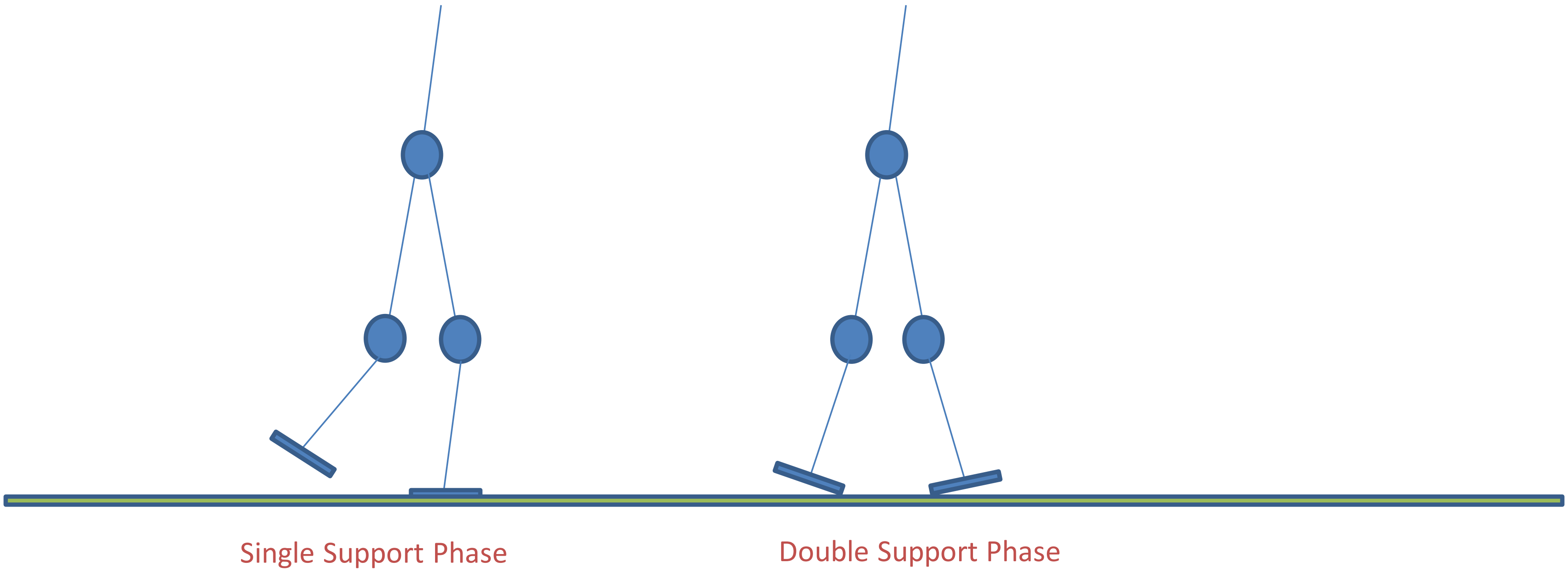


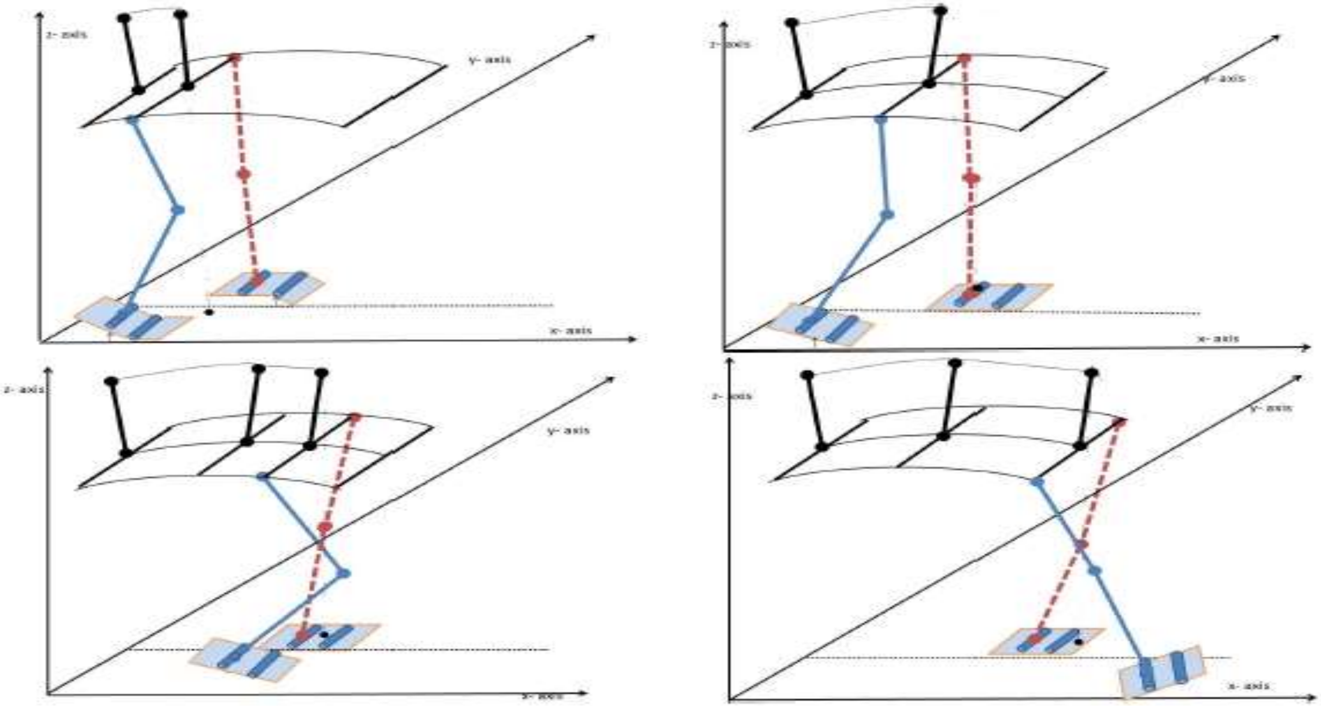
Single Support and Double Support Phase

- At a particular instant one or both the legs can be in contact with ground surface.
- The case when one leg is in contact with the ground surface, the contact leg is known as **stable leg** and other leg is called **swing leg**.
- End of the leg is referred as foot.
- A single cycle of walking can be divided into two phases:

  (i) Single Support Phase(SSP) (ii) Double Support Phase(DSP)
- Single support or swing phase is that phase of biped locomotion when there is only one foot is in contact with the ground.
- On the other hand when both feet are in contact with the ground, that phase is known as double support phase.
- So, alternating single and double support phase results in walking with a condition that the horizontal component of biped robot's center of mass (COM) have strictly monotonic displacement (For example in dancing the horizontal components moves forward and backward).

Single Support and Double Support Phase




z- axis
y- axis
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y- axis
x- axis

Stability

- In order to remain stable, the robot’s Center of Gravity must fall under its polygon of support
- The support polygon is the smallest polygon obtained by connecting all the points of contact of the robot with the walking surface
- In case of periodic locomotion of a biped robot, a statically stable gait is one in which robot’s COM does not leave the support polygon.
- People, and humanoid robots, are not statically stable
- Standing up and walking appear effortless to us, but we are actually using active control of our balance
- We use muscles and tendons. Robots use motors

Dynamic Stability

- In case of dynamically stable periodic gait, the center of pressure (CoP) of the biped robot is on the boundary of support polygon for at least part of cycle and still biped robot does not overturn.
- CoP may be defined as the point on the ground at which resultant of ground reaction force acts.
- In reference to legged robots, CoP is termed as Zero Moment Point (ZMP)
- If this CoP lies strictly in interior of support polygon and not on boundary, that case is quasi-statically stable gait.

Zero Moment Point

- Zero Moment Point (ZMP) may be defined as that point on the surface of the ground about which resultant sum of moments of all forces which are active is zero.
- ZMP can be calculated by following equations:

$$x_{ZMP} = \frac{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)x_i - \sum_{i=1}^{n} m_i\ddot{x}_i z_i - \sum_{i=1}^{n} I_{iy}\ddot{\Omega}_{iy}}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)}$$

$$y_{ZMP} = \frac{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)y_i - \sum_{i=1}^{n} m_i\ddot{y}_i z_i - \sum_{i=1}^{n} I_{ix}\ddot{\Omega}_{ix}}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)}$$

where $m_i$denotes mass of link i, respective inertial components are denoted by I_{ix} and I_{iy}, absolute angular accelerations are denoted by $\ddot{\Omega}_{ix}$ and $\ddot{\Omega}_{iy}$, gdenoted the acceleration due to gravity, $(x_{ZMP}, y_{ZMP}, 0)$ denotes coordinates for zero moment point and (x_i, y_i, z_i)denotes coordinates for center of mass of link i.

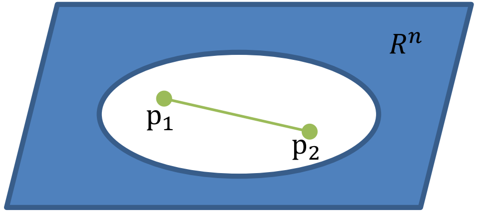
Convex Set and Convex Hull

- A subset $S\ of\ R^n$ will be defined as convex set if

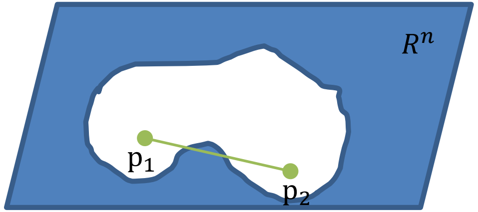
$$\alpha p_1 + (1-\alpha)p_2 \in S$$

is satisfied for any point $\mathrm{p}_1, \mathrm{p}_2 \in S\ and\ \alpha, 0 \leq \alpha \leq 1$. In other words, if a segment formed by connecting any two point in S is also included in S, then the set will be a convex set.

Convex Set



Non- Convex Set



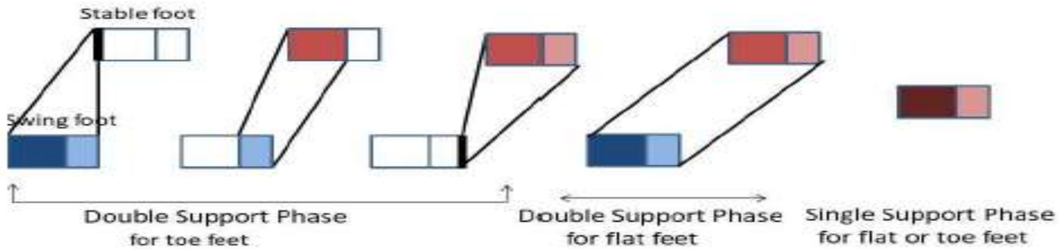
- The minimum convex set including S is defined as Convex Hull.

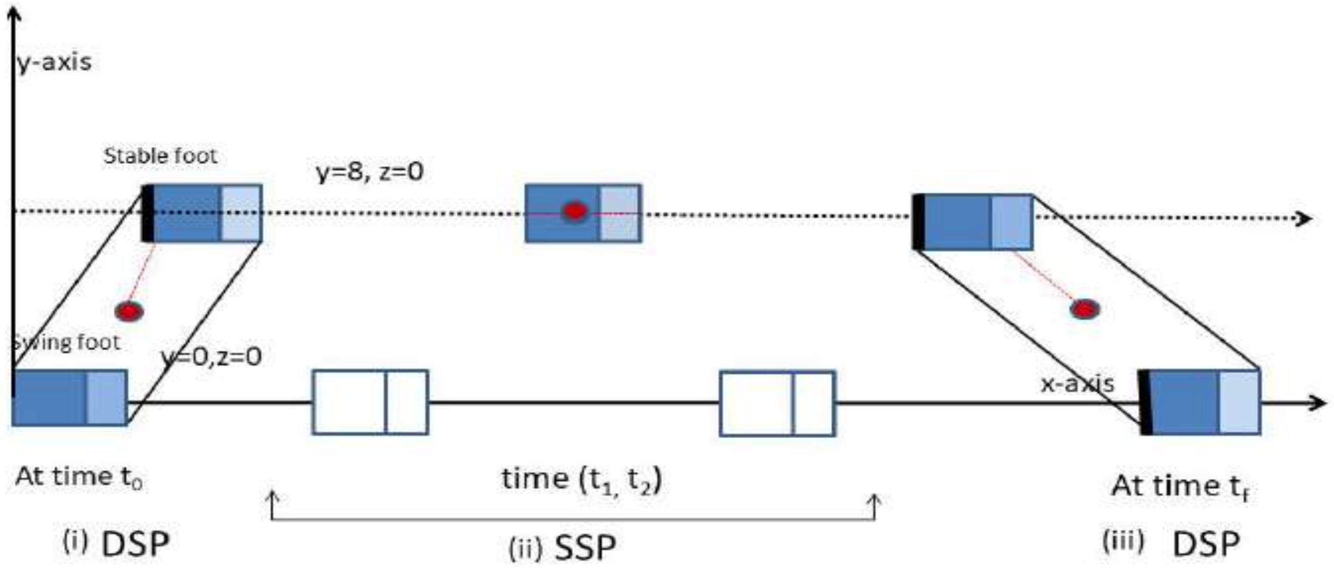
Supported region

The convex hull of all floor contact points is called supported region/polygon.
ZMP should lie within the supported polygon for stable walk.

Stability margin

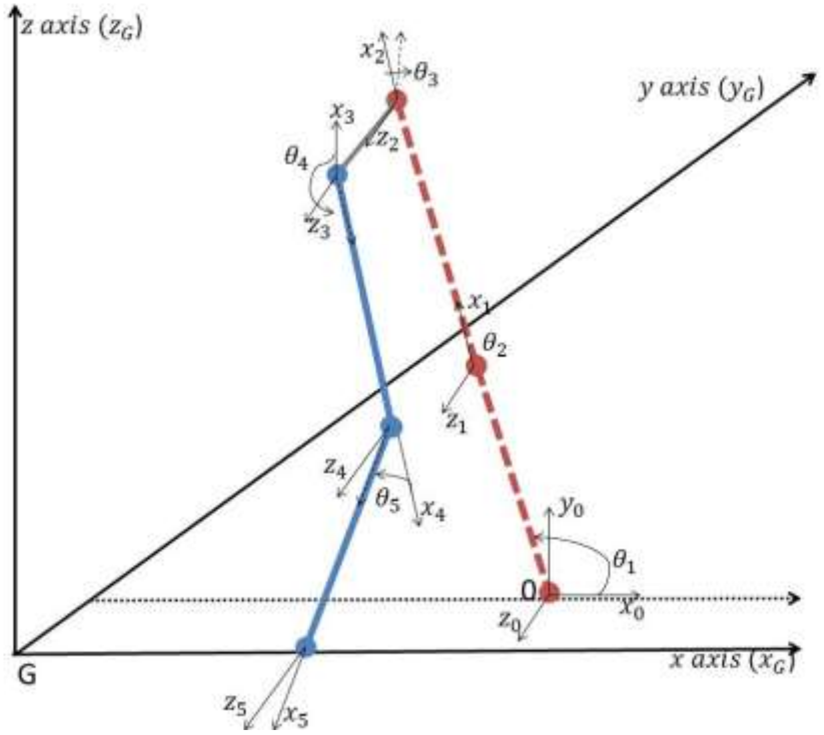
- Stability margin was defined for a given support polygon as the smallest of the distances from the ZMP to the edges of the support polygon.
- The walk is stable if ZMP lies strictly inside supported region with larger stability margin.





- In single support phase, the ZMP should be located inside the foot region.
- In double support phase the ZMP should fall within the supported region.

DH Procedure for Biped Robot



DH Procedure for Biped Robot

- In Figure, coordinate frames (x_i, y_i, z_i), i = 0, 1, 2…5 are assigned at the joints starting from the stable leg's ankle to swing leg's ankle according to Denavit-Hartenberg (DH) procedure and related joint angles i ; i = 0, 1, 2…5 are demonstrated.
- The letter 'G' is assigned to indicate the universal coordinate frame.
- The numeric '0' is assigned to the base coordinate frame at the ankle joint of stable leg with axes $x_0 y_0$ and z_0 where $x_0$is the direction of walking, $z_0$is lying along the axis of rotation
- of joint 1 and $y_0$is the axis according to the right hand thumb rule.
- The DH parameters for the biped are given in Table.

DH Parameter Table

Link	Joint angle(θ_i)	Twist angle(α_i)	Link length(a_i)	Joint length(d_i)
1	θ_1	0	l_1	0
2	θ_2	0	l_2	0
3	θ_3	0	0	l_0
4	θ_4	0	l_2	0
5	θ_5	0	l_1	0

Homogeneous Coordinate Transformation Matrices A_i^{i-1}

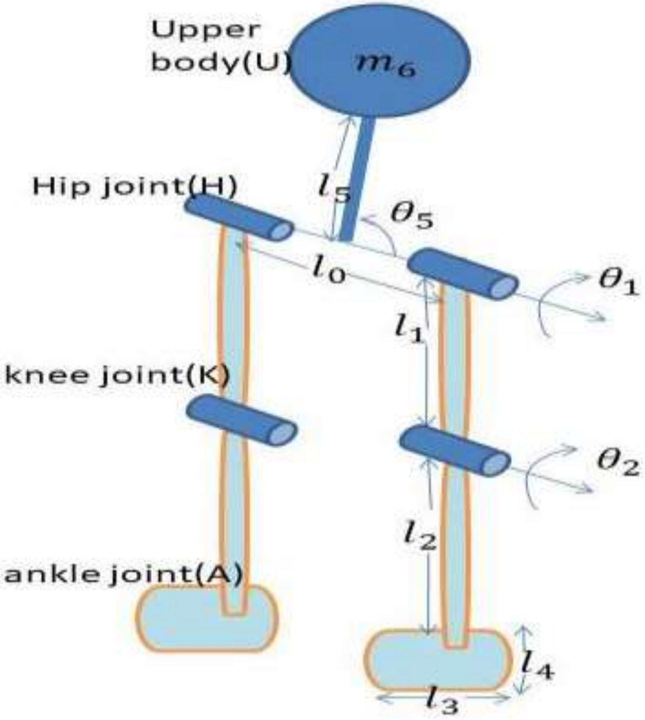
$$A_1^0 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^1 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4^3 = \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & C\theta_4 \\ S\theta_4 & C\theta_4 & 0 & S\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^4 = \begin{bmatrix} C\theta_5 & -S\theta_5 & 0 & C\theta_5 \\ S\theta_5 & C\theta_5 & 0 & S\theta_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Biped Robot
Flat Foot Model

Robot Model



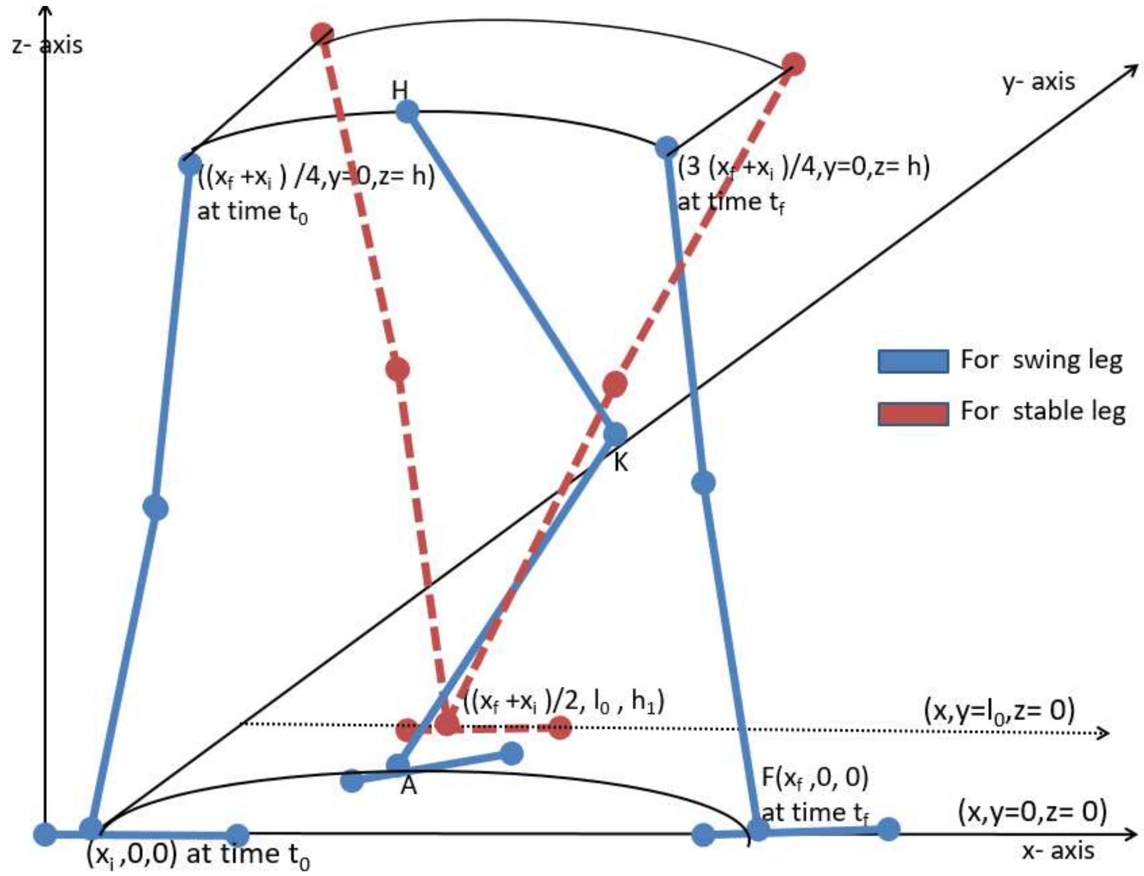
Upper body(U)
m_6
Hip joint(H)
l_5
θ_5
l_0
θ_1
l_1
knee joint(K)
θ_2
l_2
ankle joint(A)
l_4
l_3

Parameters

Link	Length	Value	Mass	Value
HK	l_1	14 inches	m_1	4kg
KA	l_2	14 inches	m_2	4kg
HU	l_5	10 inches	m_6	50kg
HH	l_0	8 inches	m_5	4kg

Table 3.1: Parameters

- In Figure, each leg of biped robot have 2 degrees of freedom (DOF) with flat foot.
- All the joints are revolute which are called hip joint (H), knee joint (K) and ankle joint (A).
- Centre of mass of upper body is denoted by (U).
- Robot's walk can be considered as a repetition of one-step motion.
- The walking sequence can be determined by computing the trajectory of the hip, ankle and upper body joints.
- For hip trajectory, stable ankle joint is considered as a base and hip as the end effector.
- For biped robot walking on a plane, motion of the stable leg is assumed to be like an inverted pendulum considering it's ankle joint as base and hip as end effector.
- While walking, the biped do not fold the stable leg . And the whole body weight is shifted on the stable leg..
- Flat foot is attached at the ankle joint of each leg.
- Let the robot walk in sagittal plane (xz-plane).



z- axis
y- axis
H
((x_f +x_i) /4,y=0,z= h)
at time t_0
(3 (x_f +x_i)/4,y=0,z= h)
at time t_f
For swing leg
For stable leg
K
((x_f +x_i)/2, l_0 , h_1)
(x,y=l_0,z= 0)
A
F(x_f ,0, 0)
at time t_f
(x,y=0,z= 0)
(x_i ,0,0) at time t_0
x- axis

Swing leg's trajectories:

Boundary Conditions of Ankle Trajectory

$x_A(t_0) = x_i$; $x_A(t_f) = x_i + x_f$; $\dot{x}_A(t_0) = 0$; $\dot{x}_A(t_f) = 0$.
$z_A(x_0) = 0$; $z_A(x_f) = 0$; $z_A(x_m) = h_1$; $\dot{z}_A(x_m) = 0$.

Ankle Trajectory

$$
\begin{aligned}
x_A(t) &= x_i + \left(\frac{3x_f}{t_f^2}\right) t^2 - \left(\frac{2x_f}{t_f^3}\right) t^3; \\
z_A(t) &= \frac{h(-(x_f + x_i)^2 x_i)}{(x_m - x_i)(x_m - x_f - x_i)^2} + \frac{h(x_f + x_i)(x_f + 3x_i)x_A(t)}{(x_m - x_i)(x_m - x_f - x_i)^2} \\
&- \frac{h(2x_f + 3x_i)x_A(t)^2 + hx_A(t)^3}{(x_m - x_i)(x_m - x_f - x_i)^2}
\end{aligned}
$$

Stable leg's trajectories:

Boundary Conditions of Hip Trajectory

$x_H(t_0) = x_i + x_f/4$; $x_H(t_f) = x_i + 3x_f/4$; $\dot{x}_H(t_0) = v_s$; $\dot{x}_H(t_f) = v_e$.
$z_H(t_0) = h$; $z_H(t_f) = h$; $\dot{z}_H(t_0) = v_{zs}$; $\dot{z}_H(t_f) = v_{ze}$.

Hip Trajectory

$$x_H(t) = \frac{x_f}{4} + v_s t + \left(\frac{(v_e - v_s)}{2t_f} - r_4\frac{3t_f}{2}\right)t^2 - 2\left(\frac{x_f}{2t_f^3} - \frac{(v_s + v_e)}{2t_f^2}\right)t^3;$$

$$z_H(t) = \sqrt{(l_1 + l_2)^2 - (x_H(t) - (x_i + x_f/2))^2}.$$

where $r_4 = -2\left(\frac{x_f}{2t_f^3} - \frac{(v_s+v_e)}{2t_f^2}\right)$

Forward Kinematics

For Swing leg

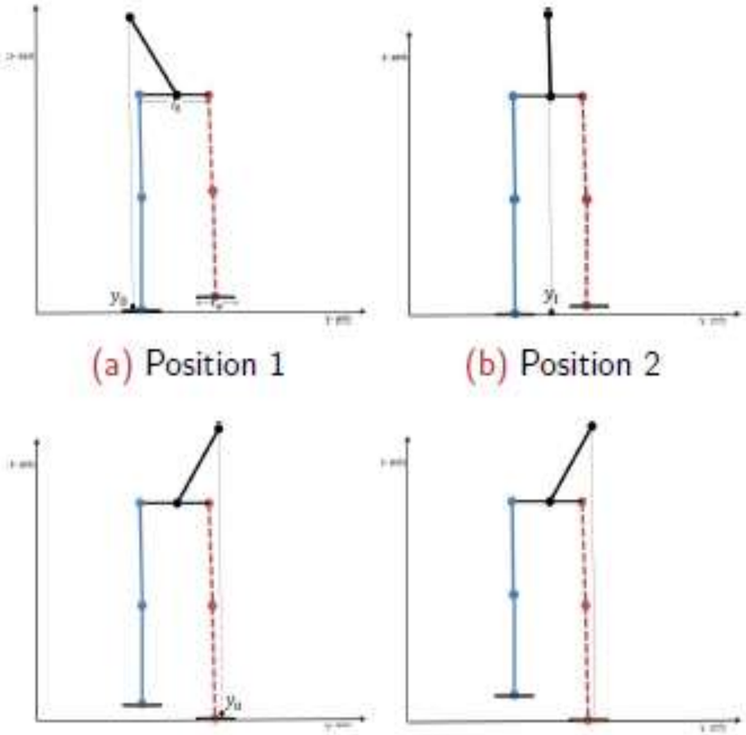
$$x_A(t) - x_H(t) = l_1 cos\theta_1(t) + l_2 cos(\theta_1(t) + \theta_2(t));$$
$$z_A(t) - z_H(t) = l_1 sin\theta_1(t) + l_2 sin(\theta_1(t) + \theta_2(t));$$

where $(x_A(t), z_A(t))$ and $(x_H(t), z_H(t))$ are defined as earlier.

For stable leg

$$x_H(t) - \left(x_i + \frac{x_f}{2}\right) = (l_1 + l_2)\cos\theta_5(t);$$
$$z_H(t) = (l_1 + l_2)\sin\theta_5(t);$$

where $(x_i+x_f/2, l_0, 0)$ is the position of the stable leg's ankle joint which lies on the line $y=l_0$.



(a) Position 1 (b) Position 2

(c) Position 3 (d) Position 4

Upper Body Motion

Case-1

- Upper body start to move from middle to the side of the stable leg's hip during time t_0 to t_1,
- Stay there during time t_1 to t_3.
- Again start moving towards middle of legs between t_3 to t_f time where $t_1 = t_f/4$ and $t_3 = 3t_f/4$.

The moving mass trajectory in y-direction is given below: $y_M(t) =$

$$
\begin{cases}
y_I + y_v t + \left(\frac{3(y_a - y_I)}{t_1{}^2} - \frac{2y_v}{t_1}\right)t^2 + \left(\frac{-2(y_a - y_I)}{t_1{}^3} - \frac{y_v}{t_1{}^2}\right)t^3 & t_0 \le t \le t_1 \\
y_a & t_1 \le t \le t_3 \\
\left(y_a + \frac{(-3t_f t_3^2 + t_3^3)(y_I - y_a)}{(t_3 - t_f)^3} + \frac{t_f t_3^2 y_v}{(t_3 - t_f)^2}\right) & \\
\left(\frac{6t_f t_3(y_I - y_a)}{(t_3 - t_f)^3} - \frac{(t_3^2 + 2t_f t_3)y_v}{(t_3 - t_f)^2}\right)t + \left(\frac{-3((y_I - y_a)(t_3 + t_f)}{(t_3 - t_f)^3}\right. & \\
\left. + \frac{y_v(4t_3 + 2t_f)}{2(t_3 - t_f)^2}\right)t^2 + \left(\frac{2(y_I - y_a)}{(t_3 - t_f)^3} - \frac{y_v}{(t_3 - t_f)^2}\right)t^3 & t_3 \le t \le t_f
\end{cases}
$$

Case-2

- Upper body start to move from middle to the side of the stable leg's hip during time t_0 to $t_f/8$.
- Stay there during time $t_f/8$ to $7t_f/8$.
- Again start moving towards middle of legs between $7t_f/8$ to t_f time.

Then the trajectory can be calculated by case-1 equation.

Case-3

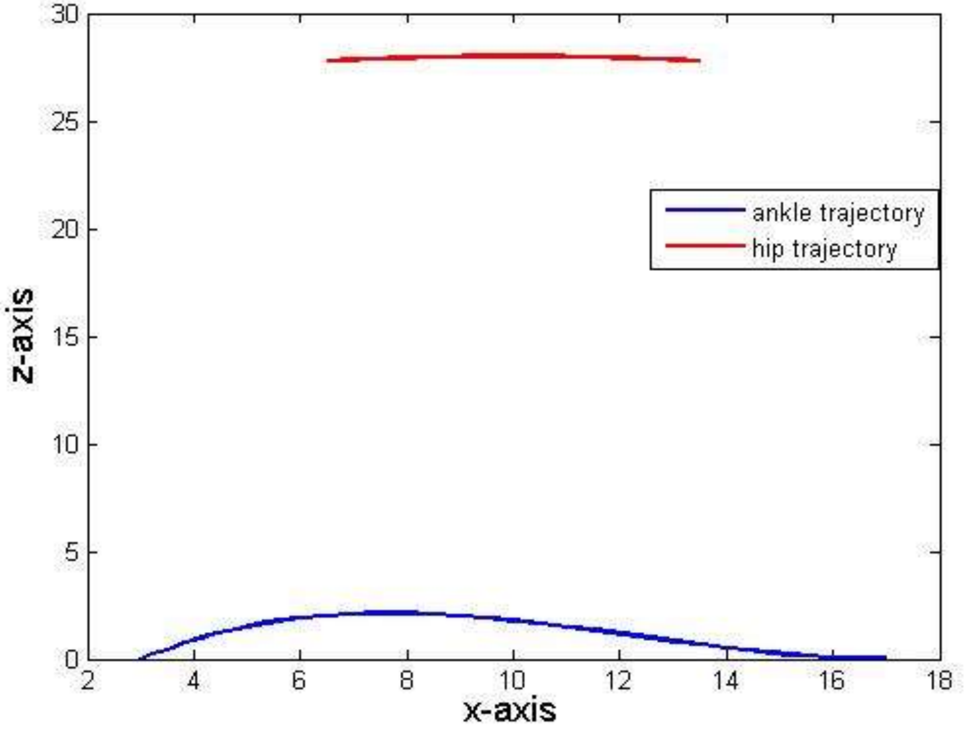
Similarly upper body start to move from middle position to stable foot from time t_0 to t_2, then return back to initial condition. So the moving mass trajectory in y-direction is given below:

$$y_M(t) = \begin{cases} y_l + y_v t + \left(\frac{3(y_a - y_l)}{t_2^2} - \frac{2y_v}{t_2}\right)t^2 + \left(\frac{-2(y_a - y_l)}{t_2^3} - \frac{y_v}{t_2^2}\right)t^3 & t_0 \le t \le t_2 \\ \left(y_a + \frac{(-3t_f t_2^2 + t_2^3)(y_l - y_a)}{(t_2 - t_f)^3} + \frac{t_f t_2^2 y_v}{(t_2 - t_f)^2}\right) & \\ \left(\frac{6t_f t_2 (y_l - y_a)}{(t_2 - t_f)^3} - \frac{(t_2^2 + 2t_f t_2) y_v}{(t_2 - t_f)^2}\right)t + \left(\frac{-3((y_l - y_a)(t_2 + t_f)}{(t_2 - t_f)^3} \right. & \\ \left. + \frac{y_v(4t_2 + 2t_f)}{2(t_2 - t_f)^2}\right)t^2 + \left(\frac{2(y_l - y_a)}{(t_2 - t_f)^3} - \frac{y_v}{(t_2 - t_f)^2}\right)t^3 & t_2 \le t \le t_f \end{cases}$$

Results

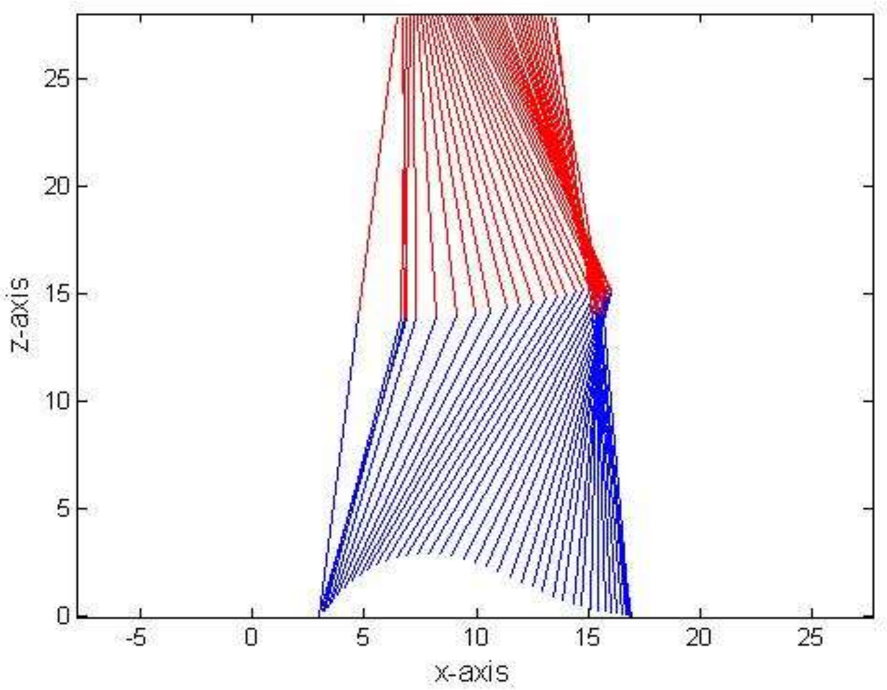
- Let total length of foot is 6 units and width is 4 units, initial and end velocity for ankle is 0 unit/sec.
- Ankle is fixed at the middle point of the foot, so that the initial x coordinate of the ankle is x_i =3 units.
- The ankle joint covers a step length x_f =14 units from initial position $(x_i,0,0)$ to the final position $(x_i + x_f,0,0)$ with step height h=2.5 units.
- Swing foot lies on the xy-plane in the region $0<x<6$ units and $-2<y<2$ units and stable foot lies on the line $y=l_0$ in the region $7<x<13$ units and $6<y<10$ units.

Results

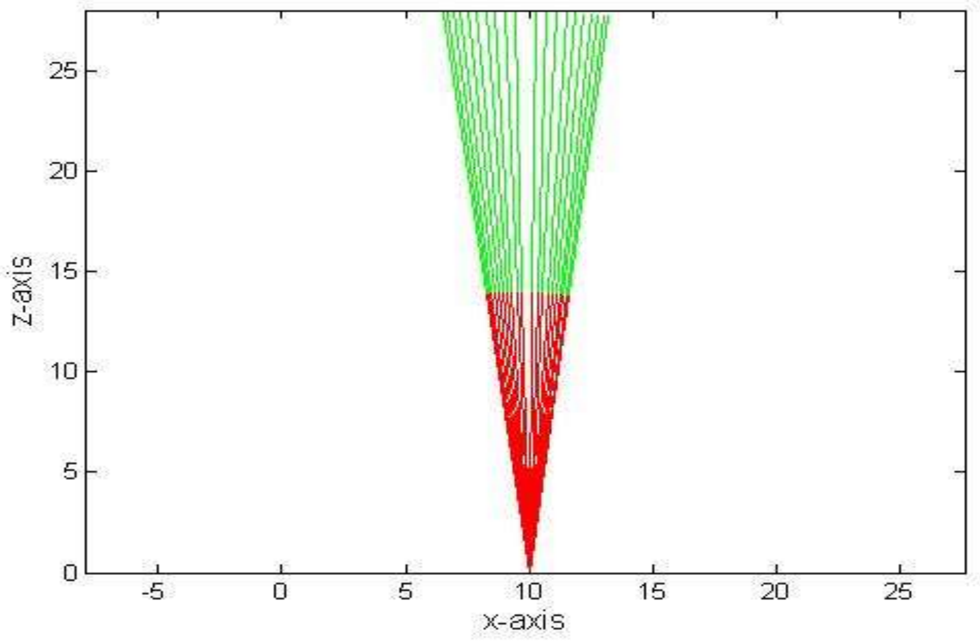


30
25
20
15
10
5
0
2
4
6
8
10
12
14
16
18
z-axis
x-axis
ankle trajectory
hip trajectory

Inverse Kinematics for Swing leg

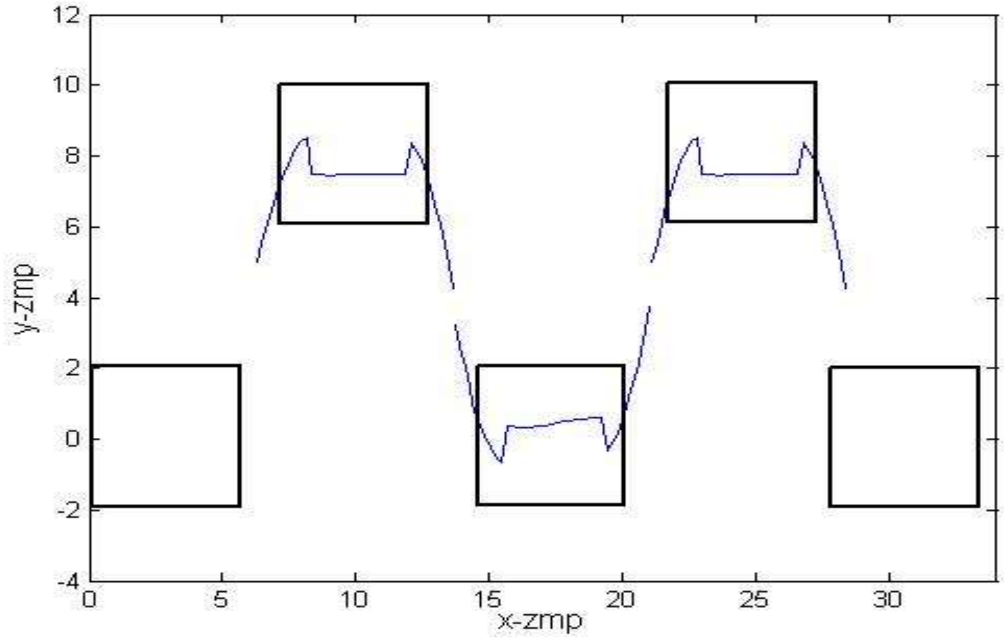


Inverse Kinematics for Stable leg

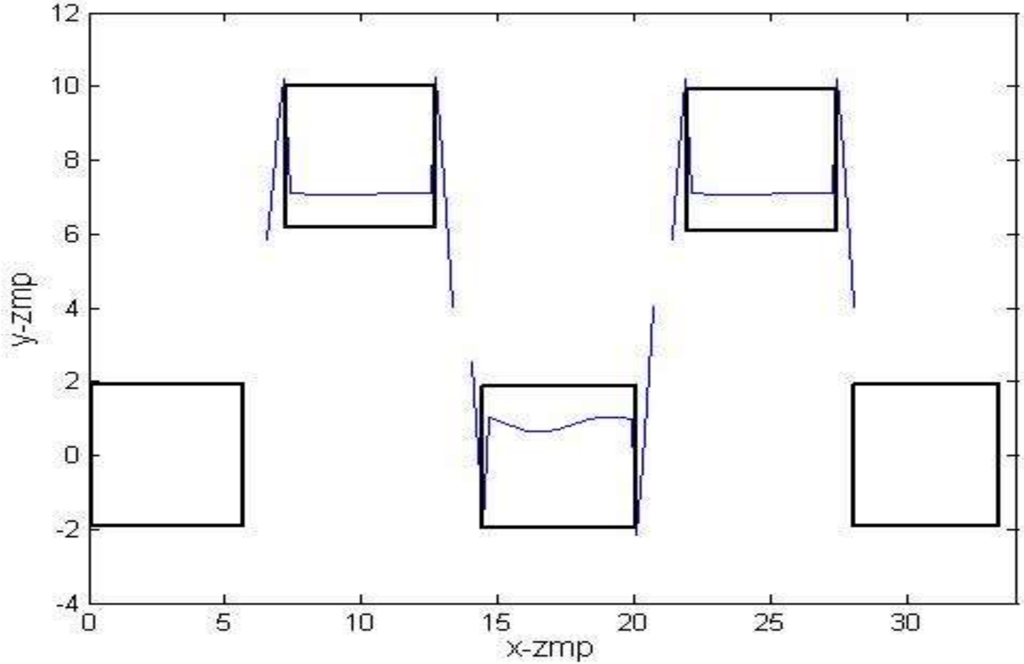


ZMP Trajectory

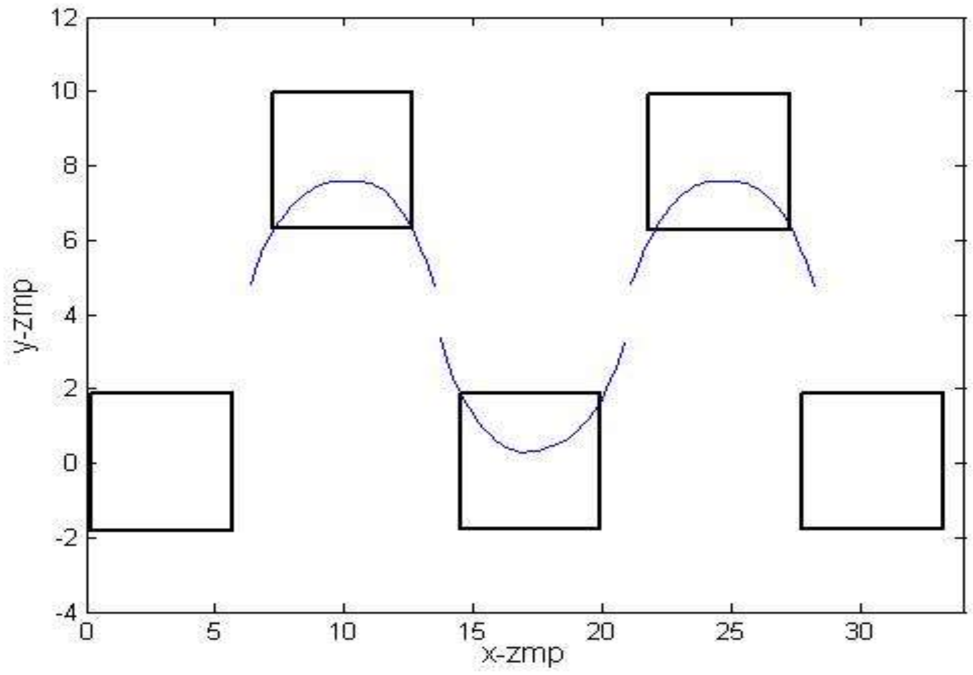
Case-1



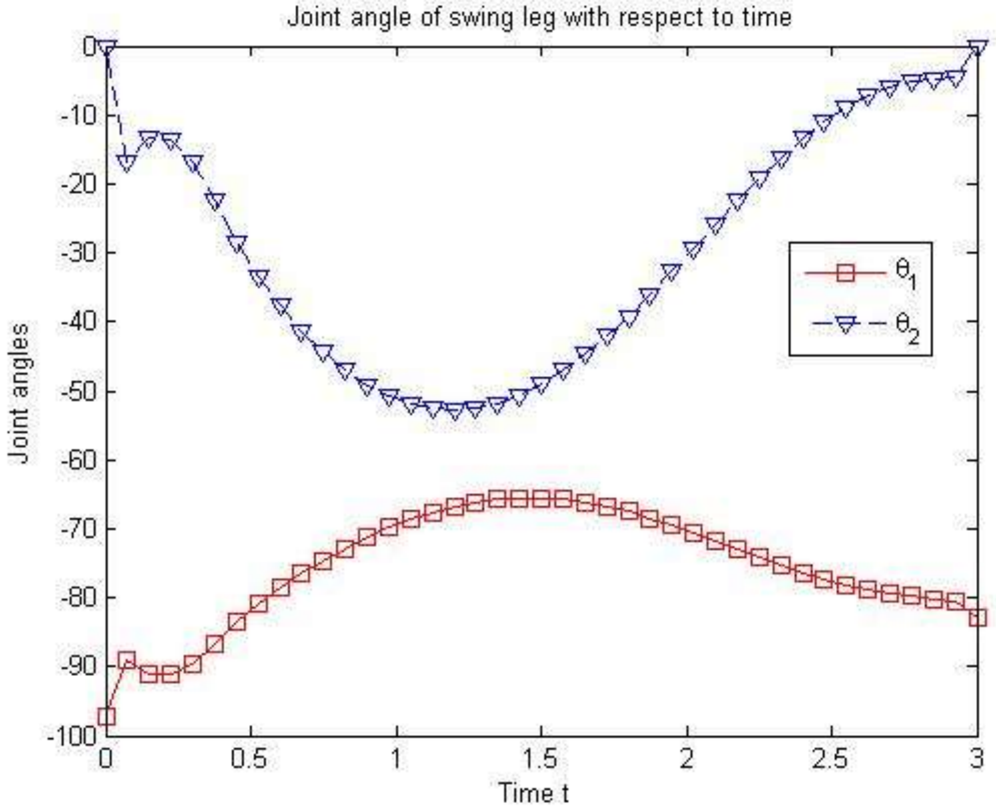
Case-2

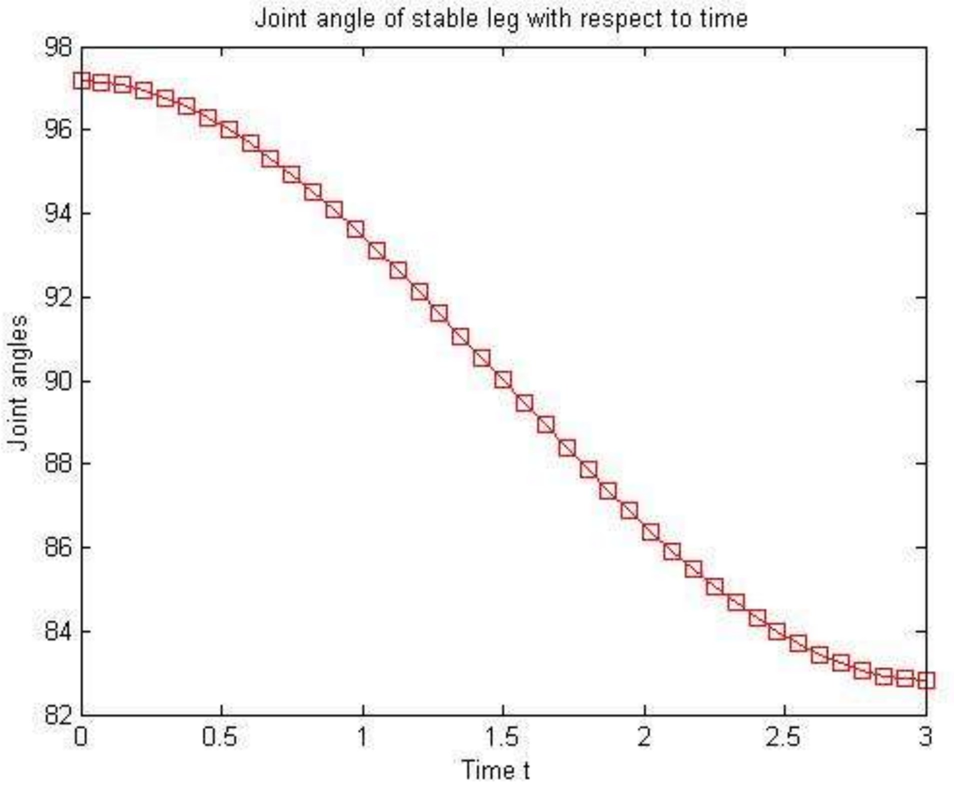


Case-3

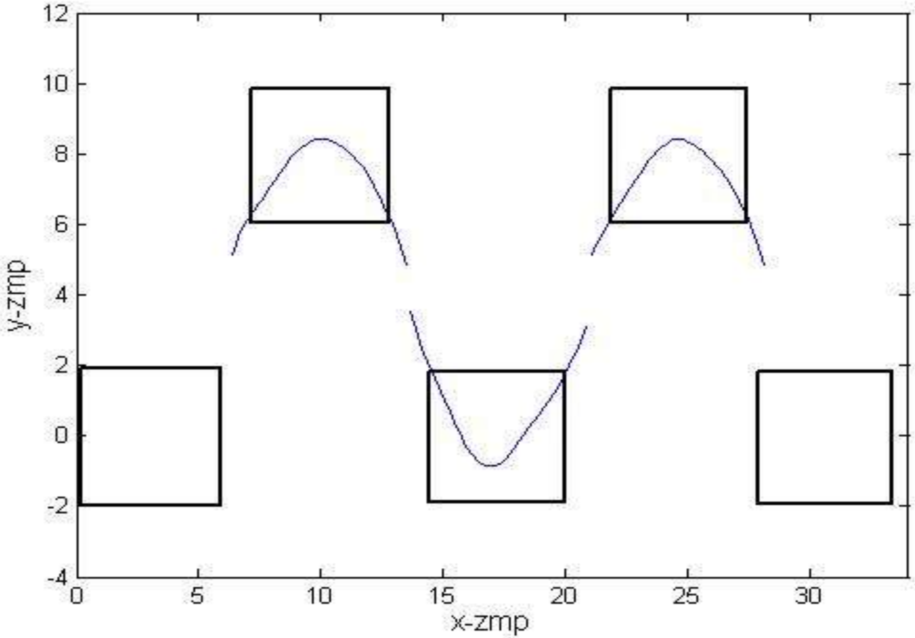


Hip		Upper Body		ZMP
velocity	Time	Trajectory	initial velocity	stability
v_s=2.3in/s	3s	case-1	$y_v = 10 in/s$	stable
v_s=3.5in/s	2s	case-1	$y_v = 15 in/s$	stable but small margin
v_s=4.7in/s	1.5s	case-1	$y_v = 20 in/s$	unstable
v_s=2.4in/s	3s	case-2	$y_v = 16 in/s$	unstable
v_s=3.5in/s	2s	case-2	$y_v = 20 in/s$	unstable
v_s=4.7in/s	1.5s	case-2	$y_v = 22 in/s$	unstable
v_s=2.3in/s	3s	case-3	$y_v = 7.3 in/s$	stable
v_s=3.5in/s	2s	case-3	$y_v = 10.3 in/s$	stable
v_s=4.7in/s	1.5s	case-3	$y_v = 11 in/s$	stable

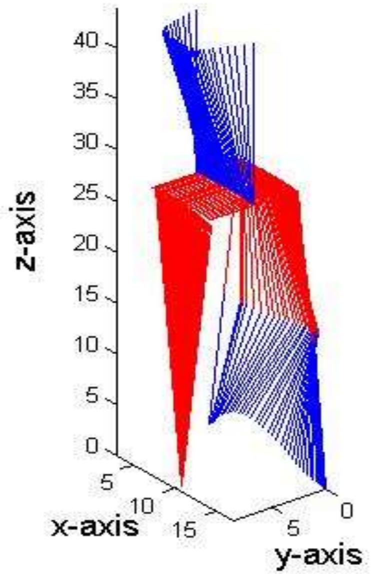

Joint angle of swing leg with respect to time
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
Joint angles
0
0.5
1
1.5
2
2.5
3
Time t
θ1
θ2


Joint angle of stable leg with respect to time
Joint angles
Time t
98
96
94
92
90
88
86
84
82
0
0.5
1
1.5
2
2.5
3

Final ZMP Trajectory



Full Body Motion



Thanks!