IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Direct Kinematics

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DEPARMENT OF MATHEMATICS

3-DoF Manipulator

As we know basic transformation matrix:

$$
{}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

3-DoF Manipulator

$$
{}^{0}T_{1} = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}T_{2} = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

3-DoF Manipulator

$$^{0}T_3 = {}^{0}T_1 . {}^{1}T_2 . {}^{2}T_3$$

$$^{0}T_2 = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & -\sin\theta_1 & \cos\theta_1 \sin\theta_2 & d_3 \cos\theta_1 \sin\theta_2 \\ \sin\theta_1 \cos\theta_2 & \cos\theta_1 & \sin\theta_1 \sin\theta_2 & d_3 \sin\theta_1 \sin\theta_2 \\ -\sin\theta_2 & 0 & \cos\theta_2 & \cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_3 = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & -\sin\theta_1 & \cos\theta_1 \sin\theta_2 & d_3 \cos\theta_1 \sin\theta_2 \\ \sin\theta_1 \cos\theta_2 & \cos\theta_1 & \sin\theta_1 \sin\theta_2 & d_3 \sin\theta_1 \sin\theta_2 \\ -\sin\theta_2 & 0 & \cos\theta_2 & d_3 \cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4-Axis SCARA Robot

4-Axis SCARA Robot

As we know basic transformation matrix:

$$^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_1 = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 & a_1\cos\theta_1 \\ \sin\theta_1 & -\cos\theta_1 & 0 & a_1\sin\theta_1 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4-Axis SCARA Robot

$$^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_{4} = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We can get final transformation matrix by:

$$^{0}T_{4} = {}^{0}T_{1}.\,{}^{1}T_{2}.\,{}^{2}T_{3}.\,{}^{3}T_{4}$$

4-Axis SCARA Robot

$$ {}^{0}T_2 = \begin{bmatrix} \cos(\theta_1 - \theta_2) & \sin(\theta_1 - \theta_2) & 0 & a_1 \cos\theta_1 + a_2 \cos(\theta_1 - \theta_2) \\ \sin(\theta_1 - \theta_2) & -\cos(\theta_1 - \theta_2) & 0 & a_1 \sin\theta_1 + a_2 \sin(\theta_1 - \theta_2) \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_3 = \begin{bmatrix} \cos(\theta_1 - \theta_2) & \sin(\theta_1 - \theta_2) & 0 & a_1 \cos\theta_1 + a_2 \cos(\theta_1 - \theta_2) \\ \sin(\theta_1 - \theta_2) & -\cos(\theta_1 - \theta_2) & 0 & a_1 \sin\theta_1 + a_2 \sin(\theta_1 - \theta_2) \\ 0 & 0 & -1 & d_1 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_4 = \begin{bmatrix} \cos(\theta_1 - \theta_2 - \theta_4) & \sin(\theta_1 - \theta_2 - \theta_4) & 0 & a_1 \cos\theta_1 + a_2 \cos(\theta_1 - \theta_2) \\ \sin(\theta_1 - \theta_2 - \theta_4) & -\cos(\theta_1 - \theta_2 - \theta_4) & 0 & a_1 \sin\theta_1 + a_2 \sin(\theta_1 - \theta_2) \\ 0 & 0 & -1 & d_1 - d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

6-Axis Manipulator Robot

6-Axis Manipulator Robot

$$^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6-Axis Manipulator Robot

$$ {}^{2}T_{3} = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}T_{4} = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & a_4\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & a_4\sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}T_{5} = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

6-Axis Manipulator Robot

$$^{5}T_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So,

$$^{0}T_6 = {}^{0}T_1.{}^{1}T_2.{}^{2}T_3.{}^{3}T_4.{}^{4}T_5.{}^{5}T_6$$

$$^{0}T_6 = \begin{bmatrix} (c_1c_2c_{345}+s_1s_{345})c_6+c_1s_2s_6 & c_1s_2c_6-(c_1c_2c_{345}+s_1s_{345})s_6 & -s_1c_{345}+c_1c_2s_{345} & c_1c_2(a_3c_3+a_4\,c_{34}+d_6s_{345})+s_1(a_3s_3+a_4s_{34}-d_6c_{345}) \\ (s_1c_2c_{345}-c_1s_{345})c_6+s_1s_2s_6 & s_1s_2c_6-(s_1c_2c_{345}-c_1s_{345})s_6 & c_1c_{345}+s_1c_2s_{345} & s_1c_2(a_3c_3+a_4\,c_{34}+d_6s_{345})-c_1(a_3s_3+a_4s_{34}-d_6c_{345}) \\ s_2c_{345}c_6-c_2s_6 & -c_2c_6-s_2c_{345}s_6 & s_2s_{345} & d_1+s_2(a_3c_3+a_4c_{34}+d_6s_{345}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thanks!