IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Robotics and Control: Theory and Practice

Introduction

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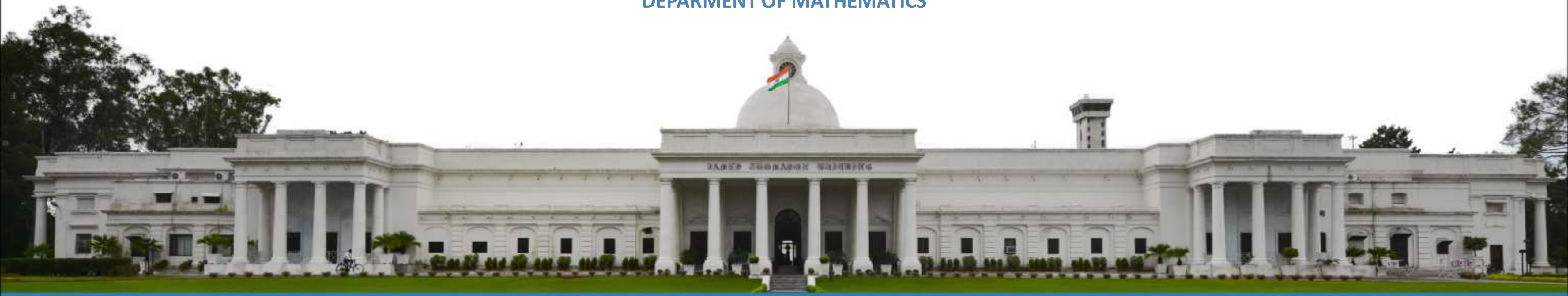
DEPARMENT OF MATHEMATICS



Contents

ROBOTICS

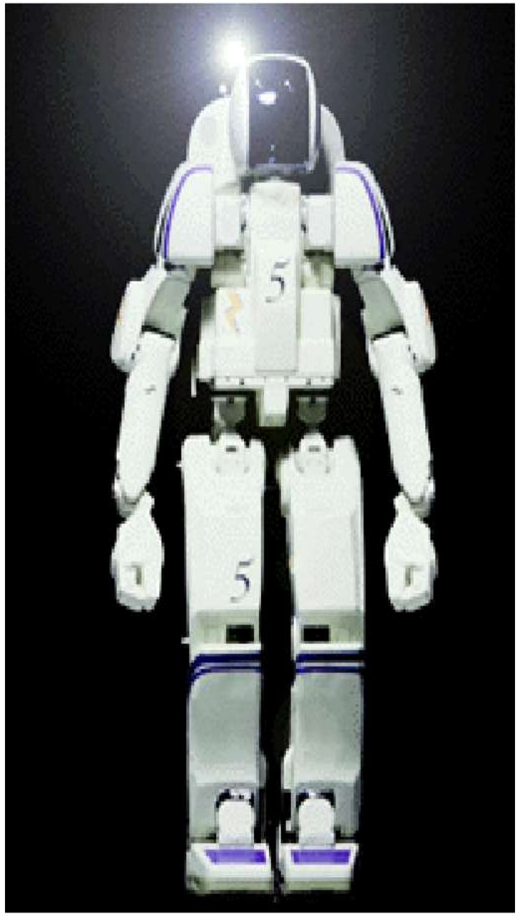
- **Robotics** is an exciting interdisciplinary field of the study of robots.
- Deals with robot design and analysis ,selection of proper sensors and actuators ,suitable control systems to handle a task in a required way.
- **Robot**: A robot is a movable physical structure, with actuators(motors) at joints, various sensor systems, power supply and a computer "brain" that controls all of these elements to perform a task automatically.
- Robot is a machine designed to carry out various tasks in place of humans.

Robot Components

-Body

-Actuators

- Controller



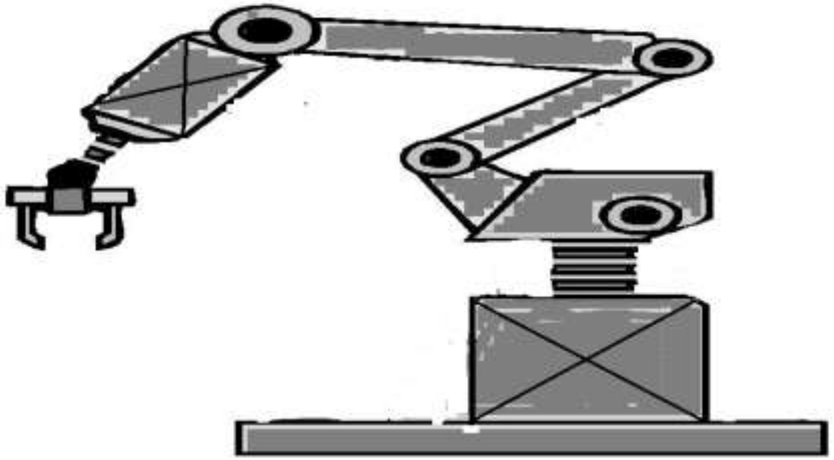
-Effectors

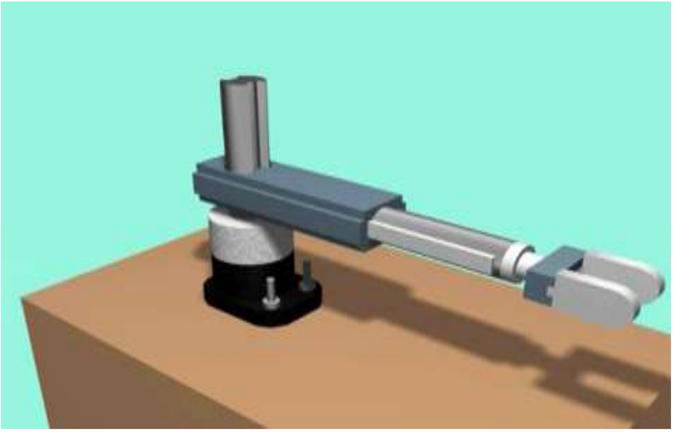
- Sensors

- Software

Manipulator

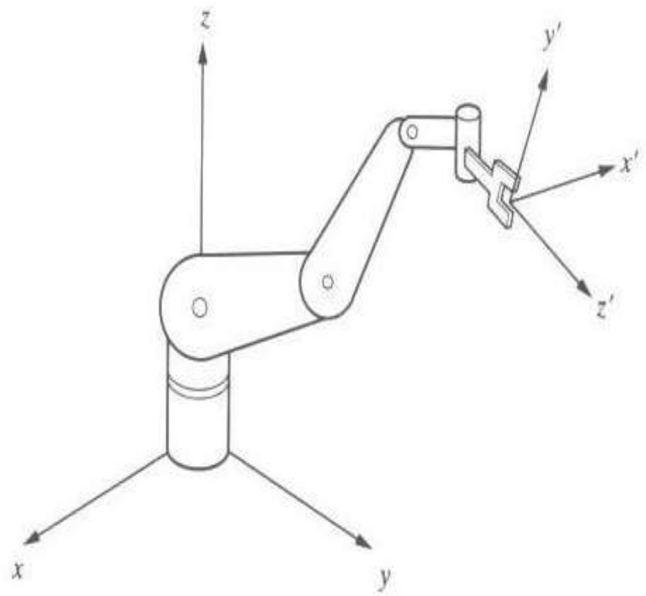
This is the main body of the robot and consists of the links , the joints and other structural element of the robot .





Reference Frames

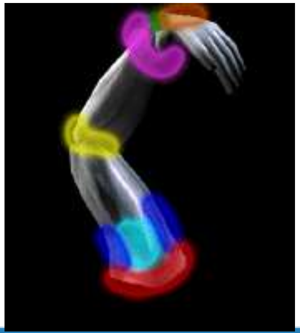
- ❖ World Reference Frame
- ❖ Joint Reference Frame



Tool reference frame

Degrees of Freedom

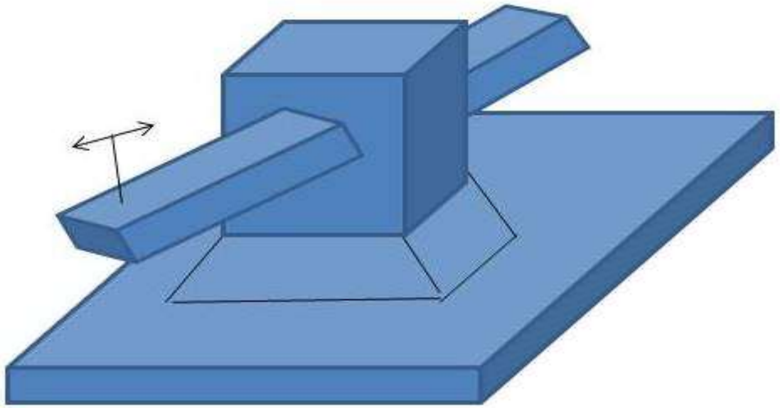
- Degrees of freedom that a manipulator possesses is the number of independent variables that would have to be specified in order to locate all parts of the mechanism.
- A rigid body in space has 6 DOF.
- An object moving on a plane has 3 DOF
- A system with seven degrees of freedom has infinitely many solutions to reach a **position and orientation**. There are infinite number of ways it can position and orientate an object at a desired location. There must be additional decision making routine (for the controller) that allows it to pick the fastest or shortest path to the desired destination.
- As more computing time is needed to control, normally less than 7 DOF robot is used in industry.
- Human arms have seven DOF. (Shoulder – 3 DOF, Elbow – 2 DOF and wrist - 2 DOF).



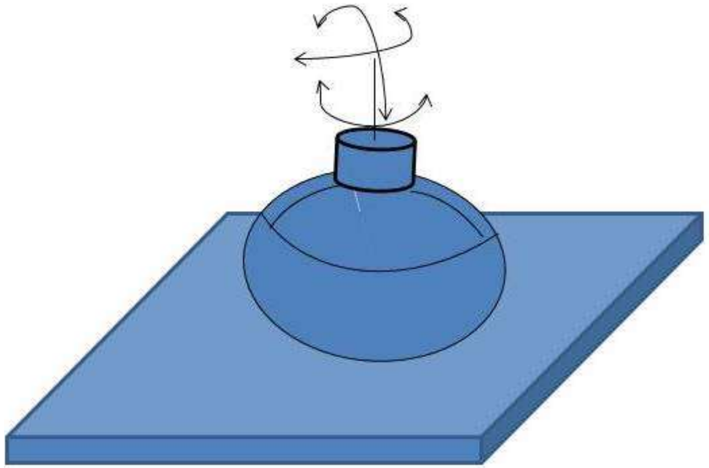
Joints of a Robot

- The Robot Joints are important element in a robot which helps the links to perform different kinds of movements.
- In the case of a rotatory or **revolute joint**, the amount of rotation angle is called **joint angle**.
- Sliding or **prismatic** joints, in which the displacement is a translation and the amount of this linear displacement is called the joint offset or **joint distance.**

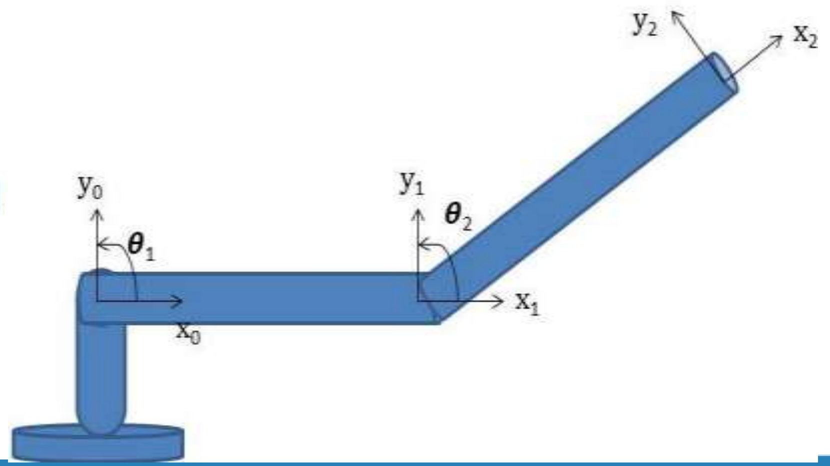
Prismatic Joint
1 DOF (linear)



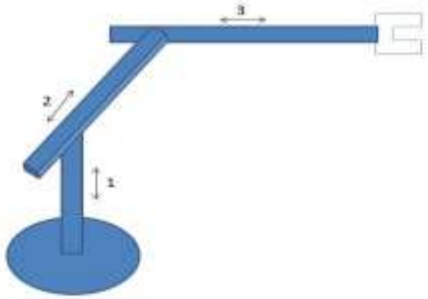
Spherical Joint
3 DOF ,



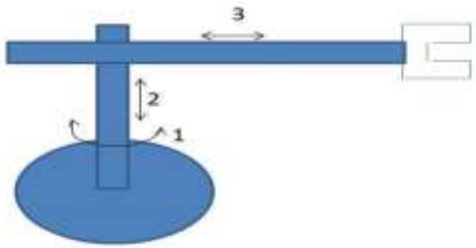
Revolute At each joint
1 DOF



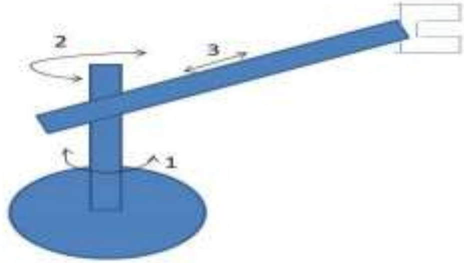
Robot Configurations



Cartesian: PPP

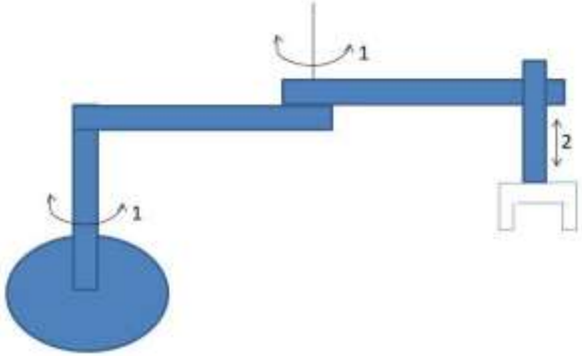


Cylindrical: RPP



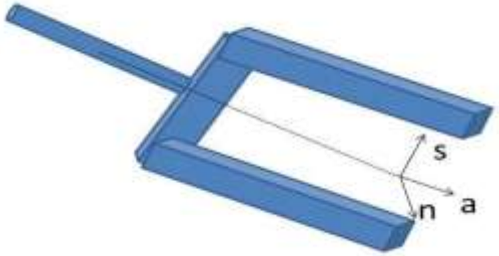
Spherical: RRP

Articulated: RRR



SCARA: RRP

(Selective Compliance Assembly Robot Arm)

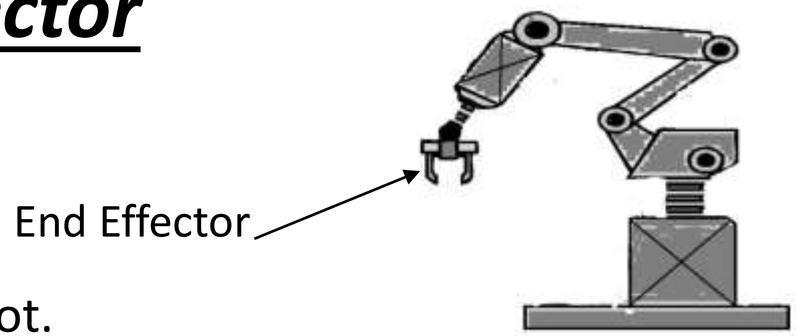


Hand coordinate:

n: normal vector; **s**: sliding vector;

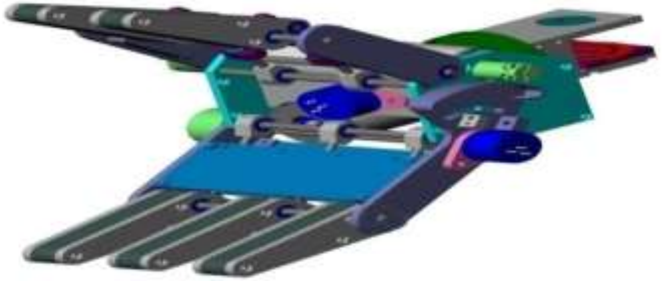
a: approach vector, normal to the tool mounting plate

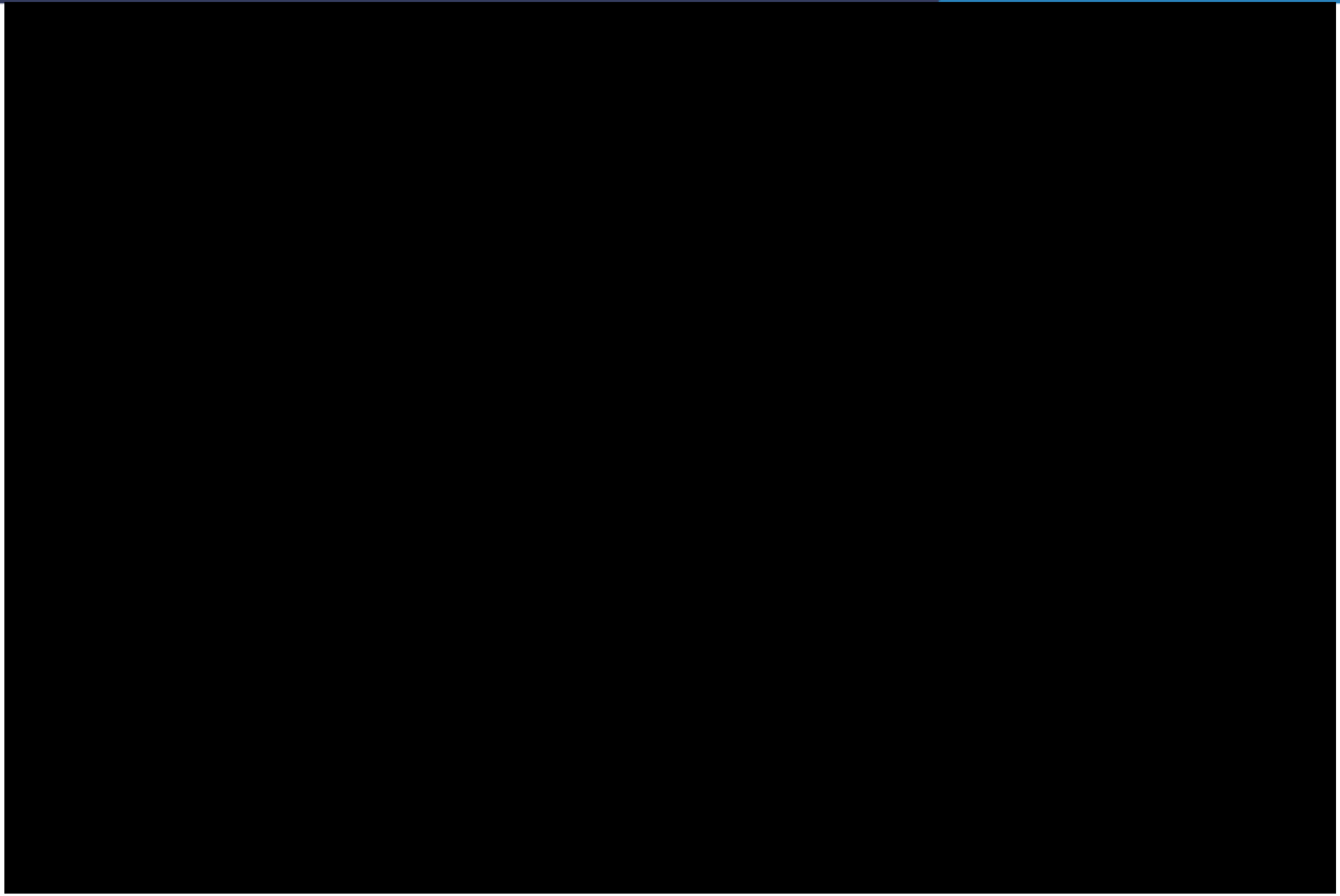
The Hand of a Robot: *End- Effector*



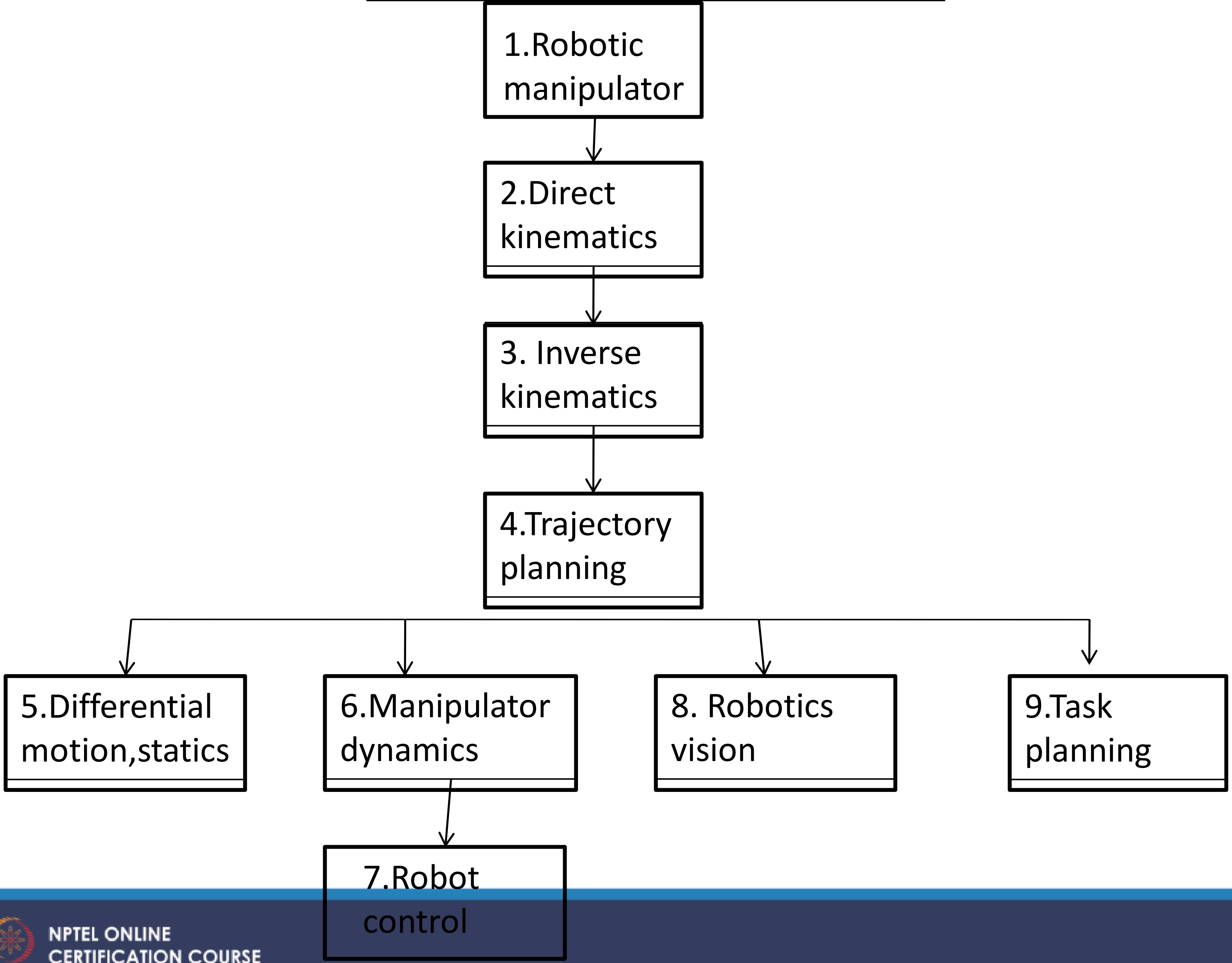
- **End- effector** is the ‘Hand’ that is attached to the end of robot.
- Different end effectors are used in a robot to perform numerous tasks.
- Two different types of ene-effectors are: **Grippers & Tools**

  Used for grasping, drilling, painting, welding, etc. (See Fig. below)
- We generally describe the position of the manipulator by giving a description of the tool frame, which is attached to the end- effector relative to the base frame, which is attached to the nonmoving base of the manipulator.





Robotics Terminology



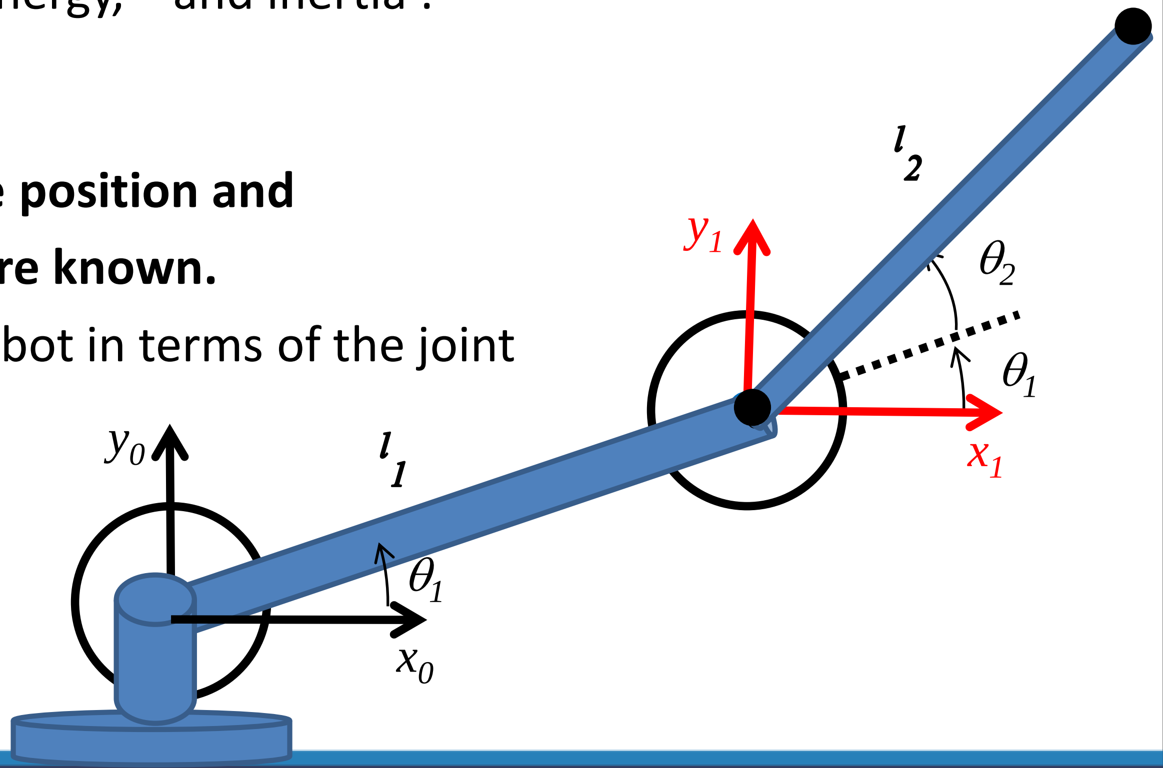
Kinematics

- In kinematics motion of a rigid body is studied without regard to the forces which cause the motion.
- Kinematics deals with position, velocity, acceleration, and all higher order derivatives of the position variables (with respect to time or any other variable(s)).
- It ignores concepts such as torque, force, mass , energy, and inertia .

Forward Kinematics

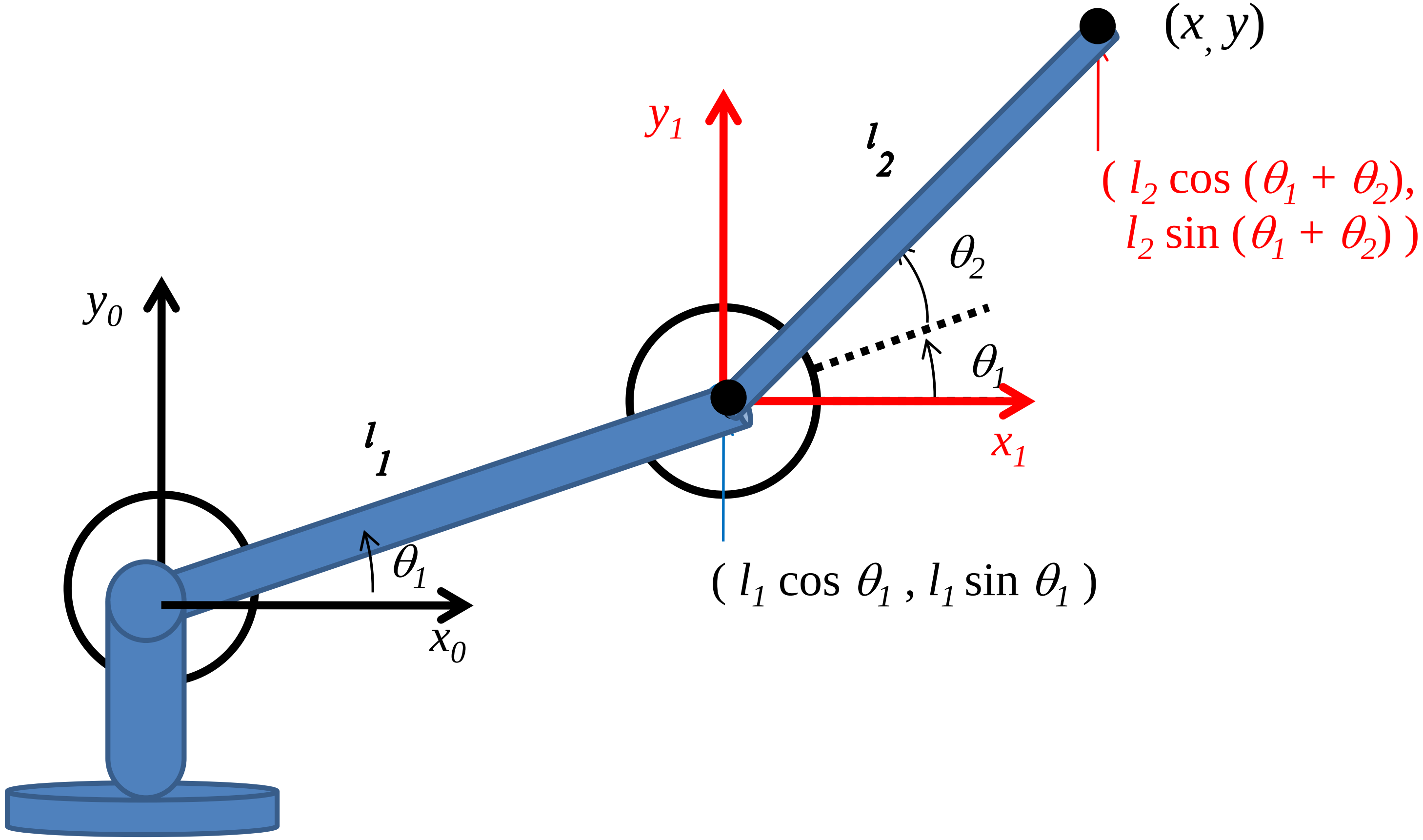
Forward Kinematics is the problem of finding the position and orientation of the end effector If joint variables are known.

Find (x, y) w.r.t the base coordinate frame of the robot in terms of the joint angles.

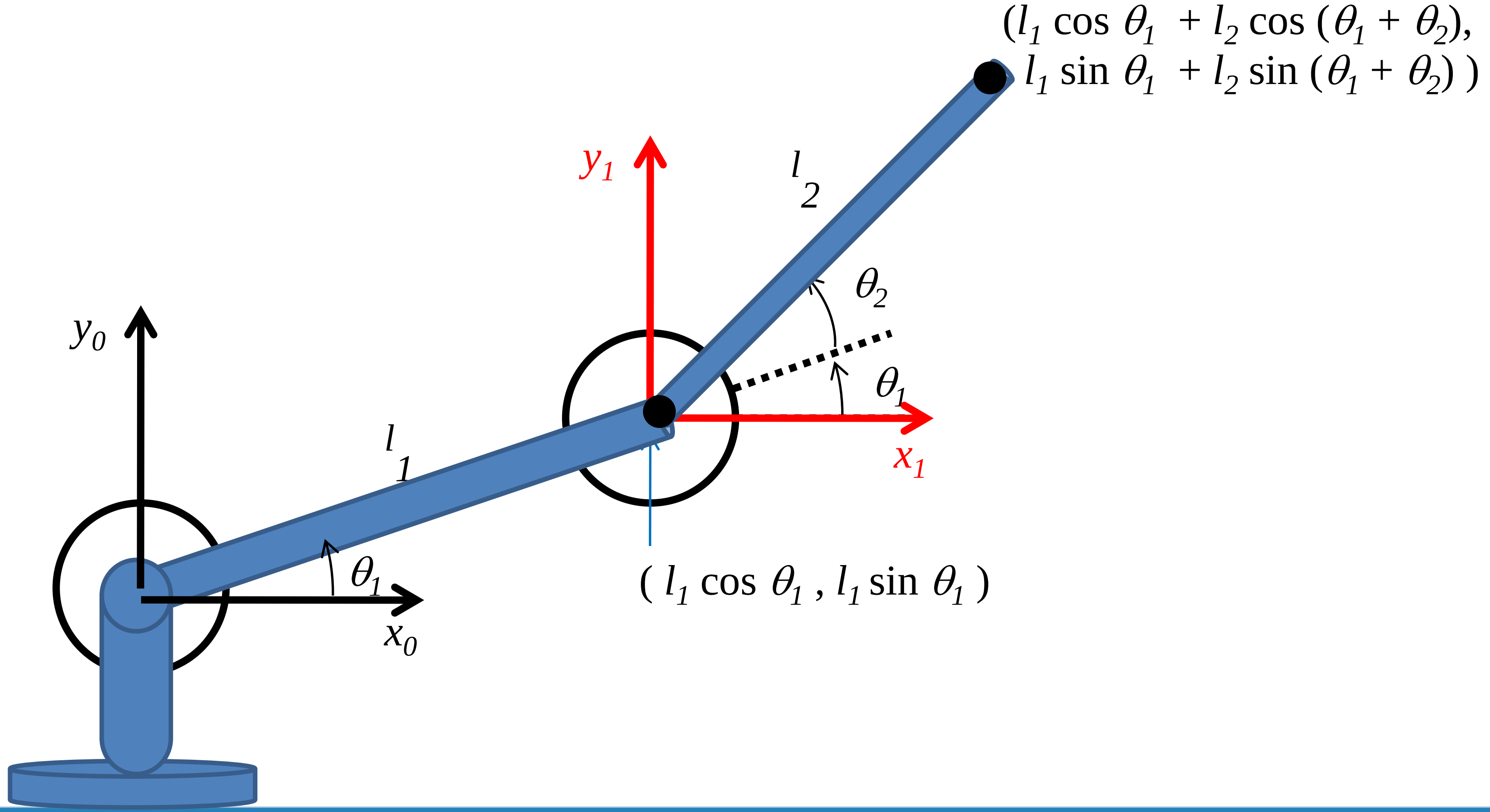


Forward Kinematics

- link *1* moves in a circle centered on the base frame
- link *2* moves in a circle centered on frame 1

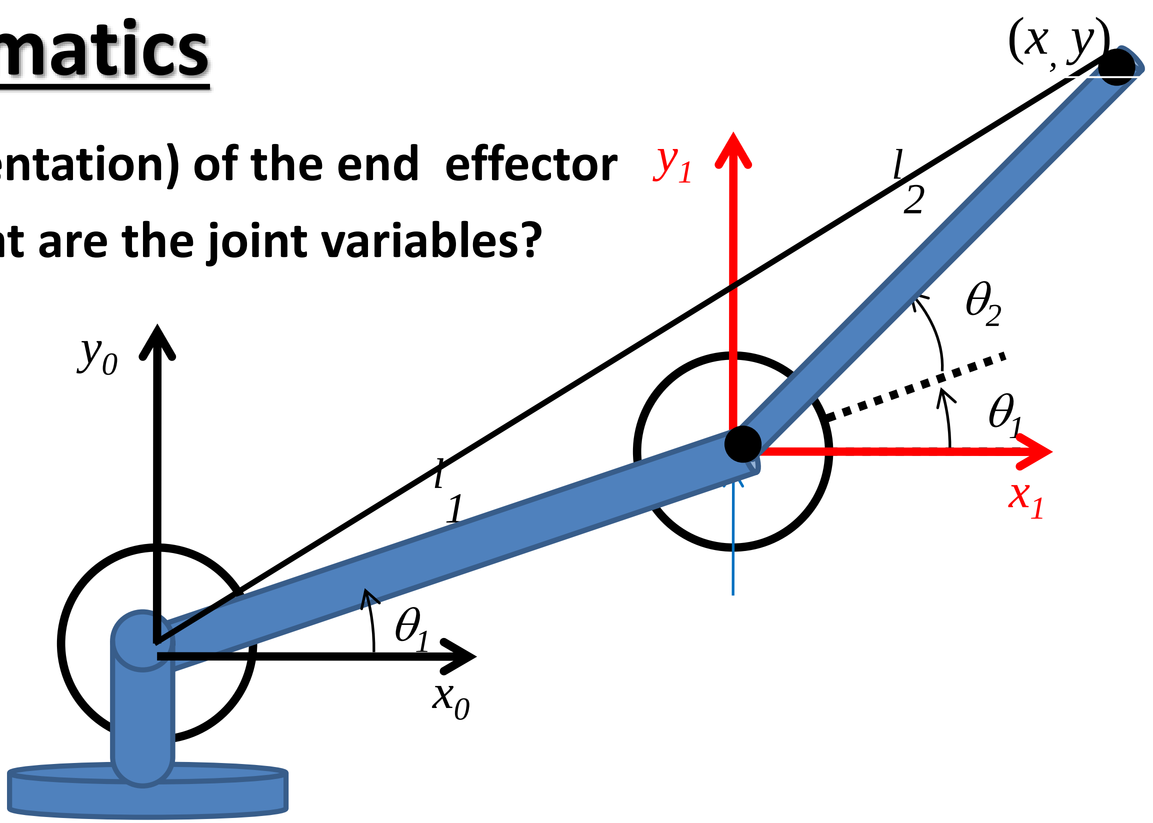


because the base frame and frame 1 have the same orientation, we can sum the coordinates to find the position of the end effector in the base frame



Inverse Kinematics

Given the position (and possibly the orientation) of the end effector and the dimensions of the links, what are the joint variables?



$$c_1 = \cos\theta_1$$
$$c_{1+2} = \cos(\theta_2 + \theta_1)$$
$$(1)\ x = l_1 c_1 + l_2 c_{1+2}$$
$$(2)\ y = l_1 s_1 + l_2 \sin_{1+2}$$
$$(3)\ \theta = \theta_1 + \theta_2$$

from $(1)^2 + (2)^2$

$$x^2 + y^2 =$$
$$= \left(l_1^2 c_1^2 + l_2^2 (c_{1+2})^2 + 2 l_1 l_2 c_1 (c_{1+2})\right) + \left(l_1^2 s_1^2 + l_2^2 (\sin_{1+2})^2 + 2 l_1 l_2 s_1 (\sin_{1+2})\right)$$
$$= l_1^2 + l_2^2 + 2 l_1 l_2 \left(c_1 (c_{1+2}) + s_1 (\sin_{1+2})\right)$$
$$= l_1^2 + l_2^2 + 2 l_1 l_2 c_2 \longleftarrow \text{Only Unknown}$$
$$\therefore \theta_2 = \arccos\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

Note:
$\cos(a \pm b) = (\cos a)(\cos b) \mp (\sin a)(\sin b)$
$\sin(a \pm b) = (\cos a)(\sin b) \pm (\cos b)(\sin a)$

$$
\begin{aligned}
x &= l_1\,c_1 + l_2\,c_{1+2} \\
&= l_1\,c_1 + l_2\,c_1c_2 - l_2 s_1 s_2 \\
&= c_1(l_1 + l_2\,c_2) - s_1(l_2 s_2)
\end{aligned}
$$

$$
\begin{aligned}
y &= l_1\,s_1 + l_2\,\sin_{1+2} \\
&= l_1\,s_1 + l_2\,s_1c_2 + l_2\,s_2c_1 \\
&= c_1(l_2\,s_2) + s_1(l_1 + l_2\,c_2)
\end{aligned}
$$

$$c_1\,x + s_1 y = (l_1 + l_2\,c_2)$$

$$c_1\,y - s_1 x = l_2 s_2$$

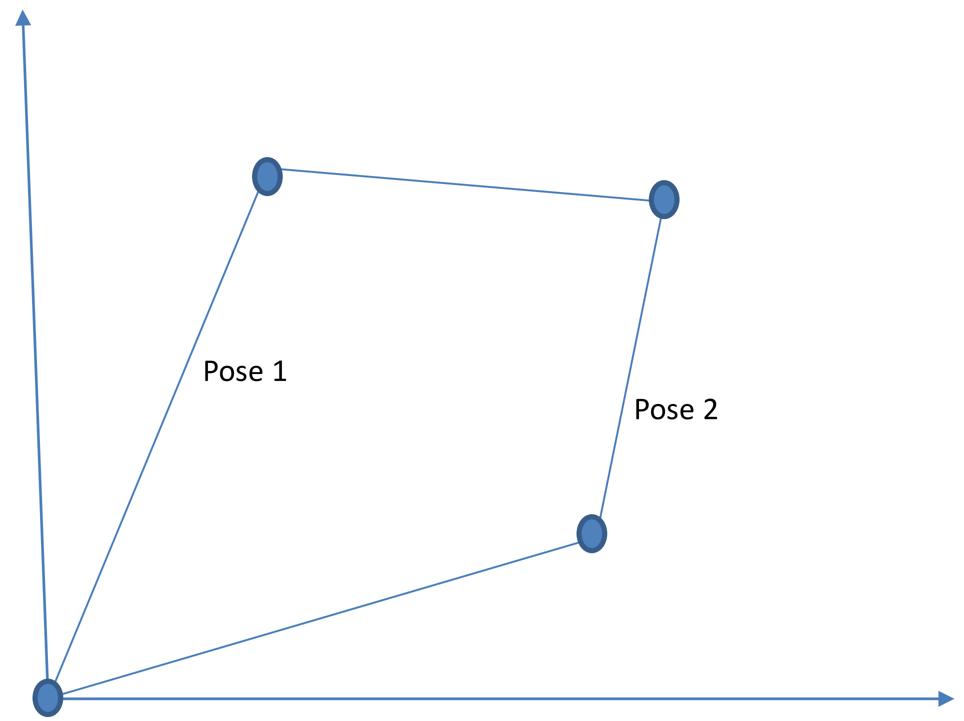
$$\frac{c_1\,x + s_1 y}{c_1\,y - s_1 x} = \frac{(l_1 + l_2\,c_2)}{l_2 s_2} = \frac{a}{b},$$

$$c_1(bx - ay) = -s_1(ax + by),$$

$$\theta_1 = \tan^{-1}\frac{(ay - bx)}{(ax + by)}$$

We know what θ_2 is from the previous slide. We need to solve for θ_1 . Now we have two equations and two unknowns (sin θ_1 and cos θ_1)

Inverse Kinematics has Multiple Solutions



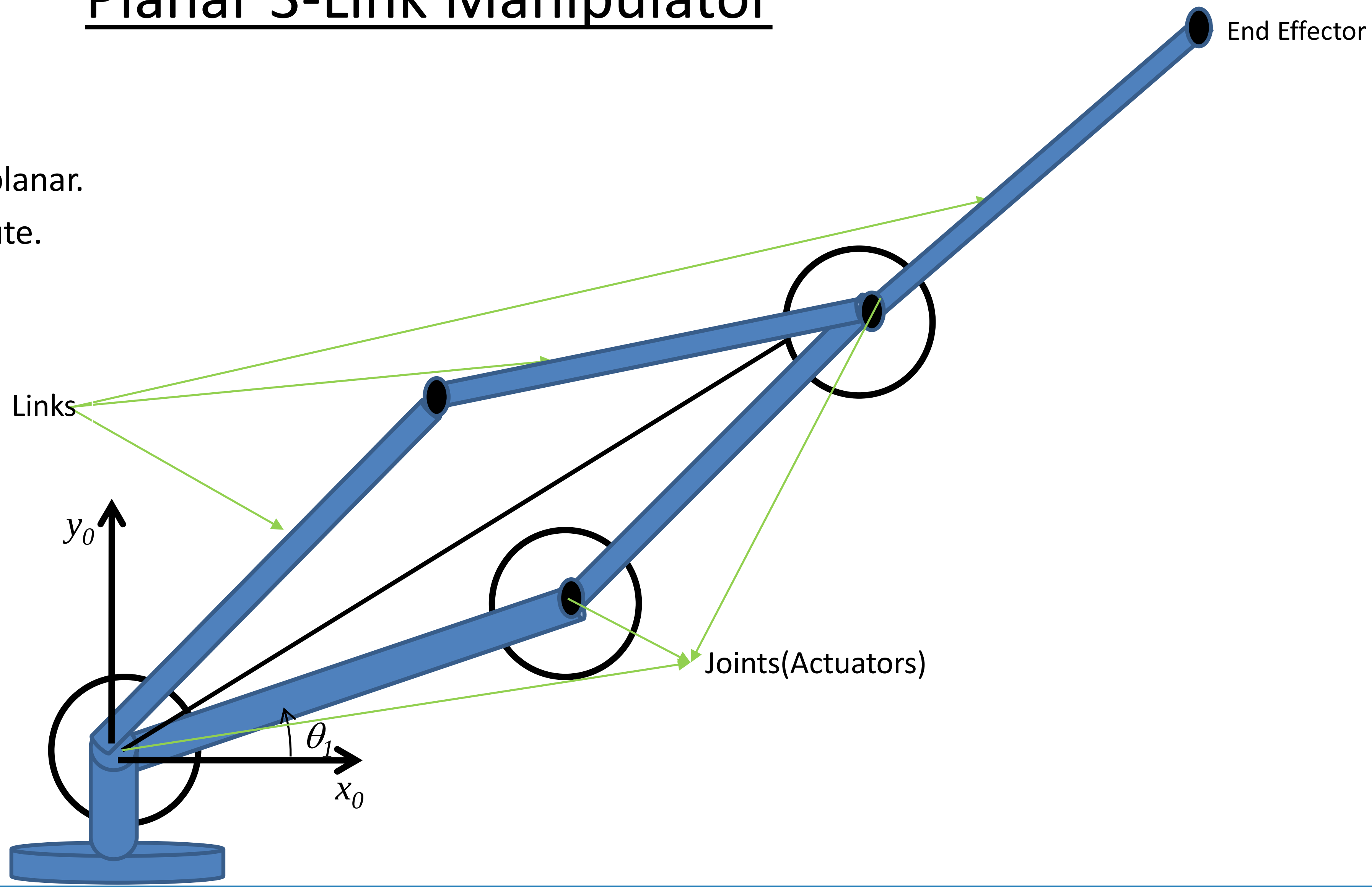
Pose 1 and pose lead to same end effector position, which one is correct leads to multiple solutions

Why Inverse Kinematics is Difficult

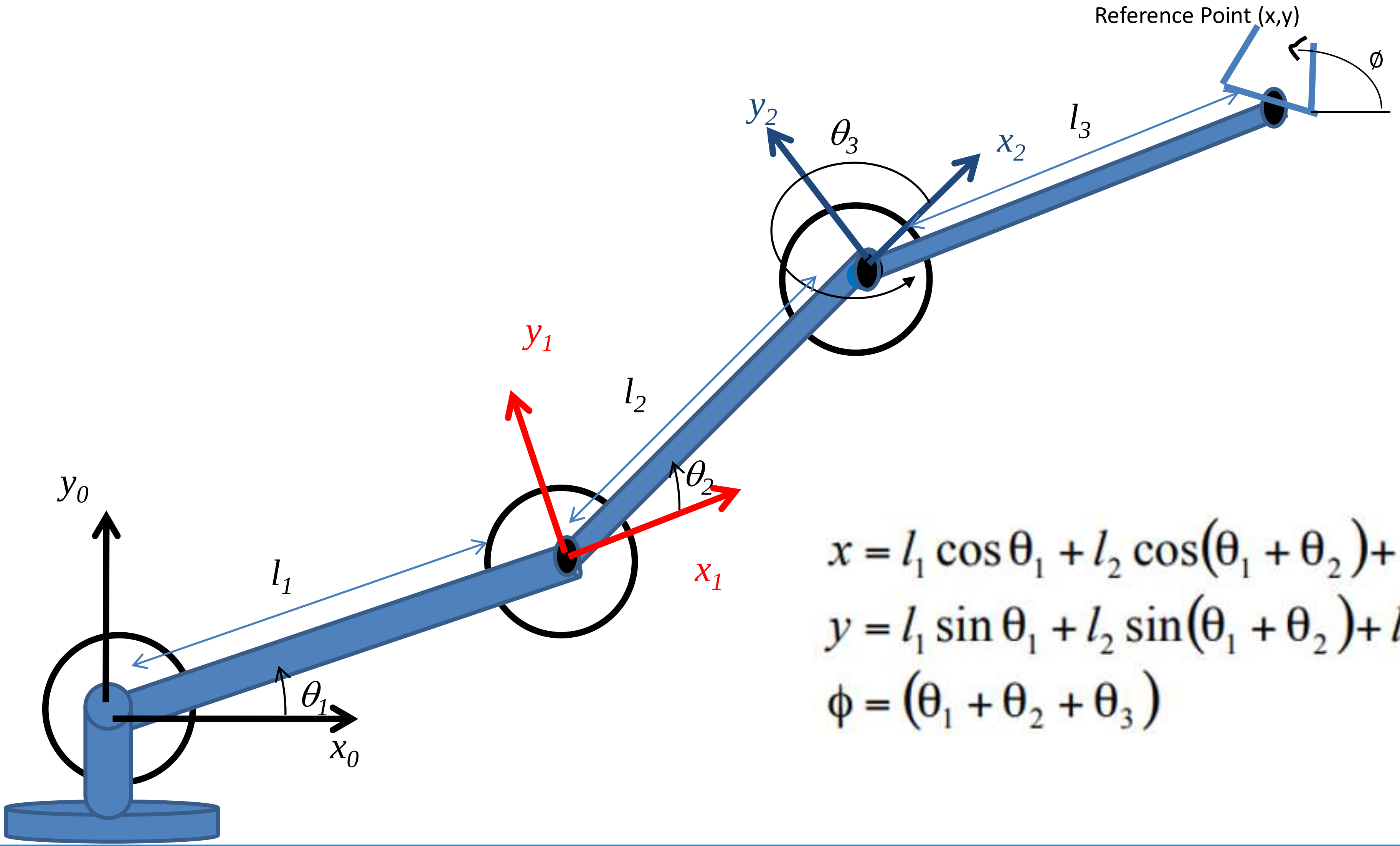
- Non Linear
- Multi-valued
- Discontinuities and Singularities
- Possibly over-constrained (No Exact Solution)

Planar 3-Link Manipulator

- Kinematic chain in planar.
- Every Joint is Revolute.



Direct Kinematics

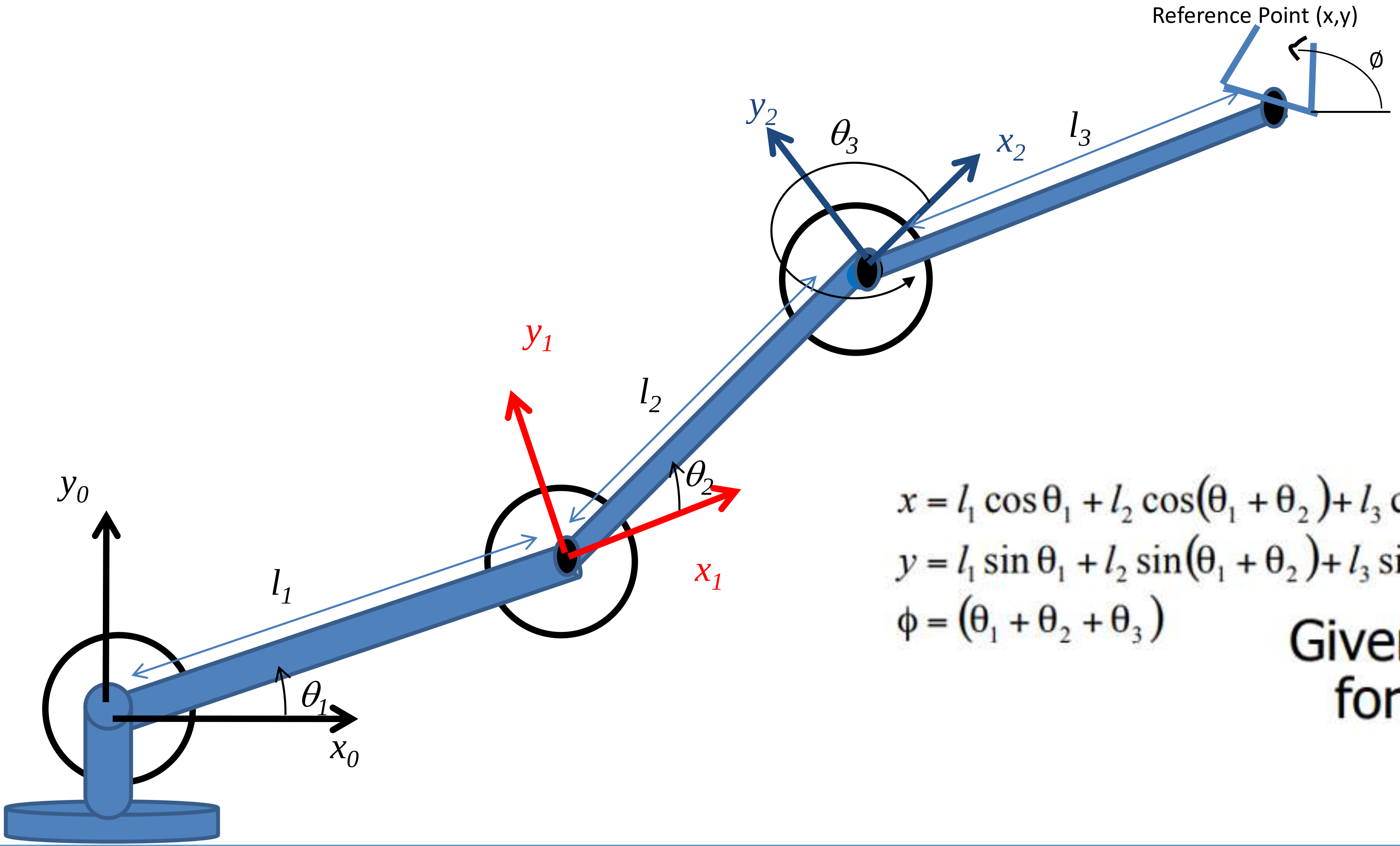


$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\phi = (\theta_1 + \theta_2 + \theta_3)$$

Inverse Kinematics



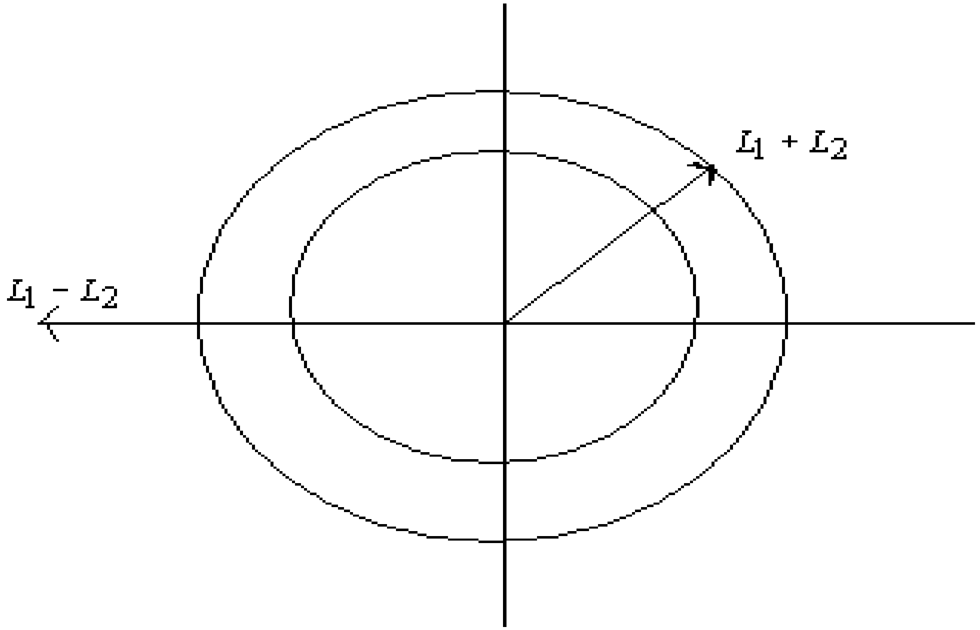
$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\phi = (\theta_1 + \theta_2 + \theta_3)$$

Given x, y, ϕ, solve for θ_1, θ_2, θ_3

Workspace

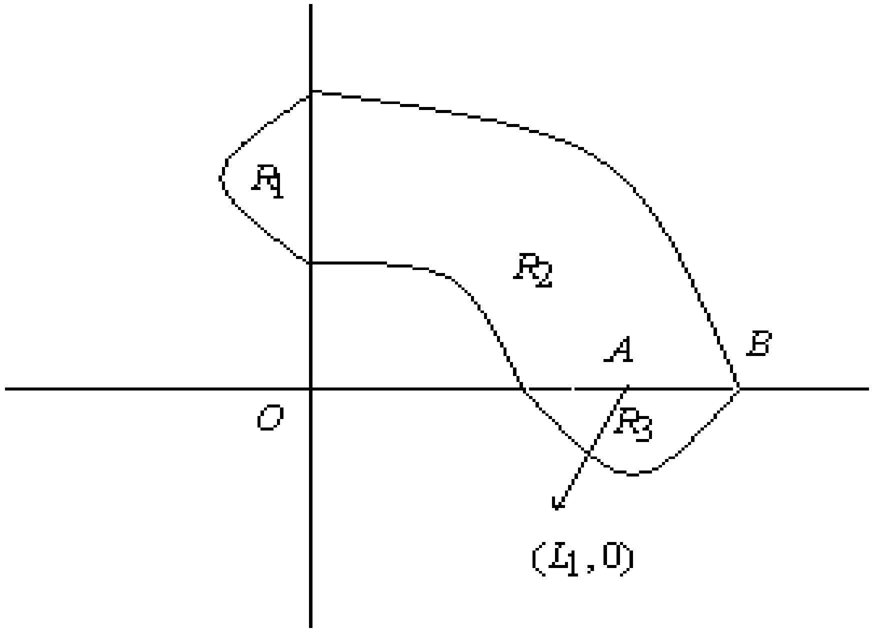


- The collection of all points to which the end – effector can reach is called work space.
- Ex. Find the workspace if $-\pi \leq \theta_1 \leq \pi, -\pi \leq \theta_2 \leq \pi, L_2 \leq L_1$
- Workspace is given by $L_1 - L_2 \leq \sqrt{x^2 + y^2} \leq L_1 + L_2$

Example of Workspace

- Ex. If $0 \le \theta_1 \le \frac{\pi}{2}, -\pi \le \theta_2 \le \pi, L_2 \le L_1$

  Find the Workspace



$$(x - L_1)^2 + y^2 < L_2^2$$

R_1 is $x^2 + (y - L_1)^2 \le L_2^2 \qquad x < 0, y > 0$

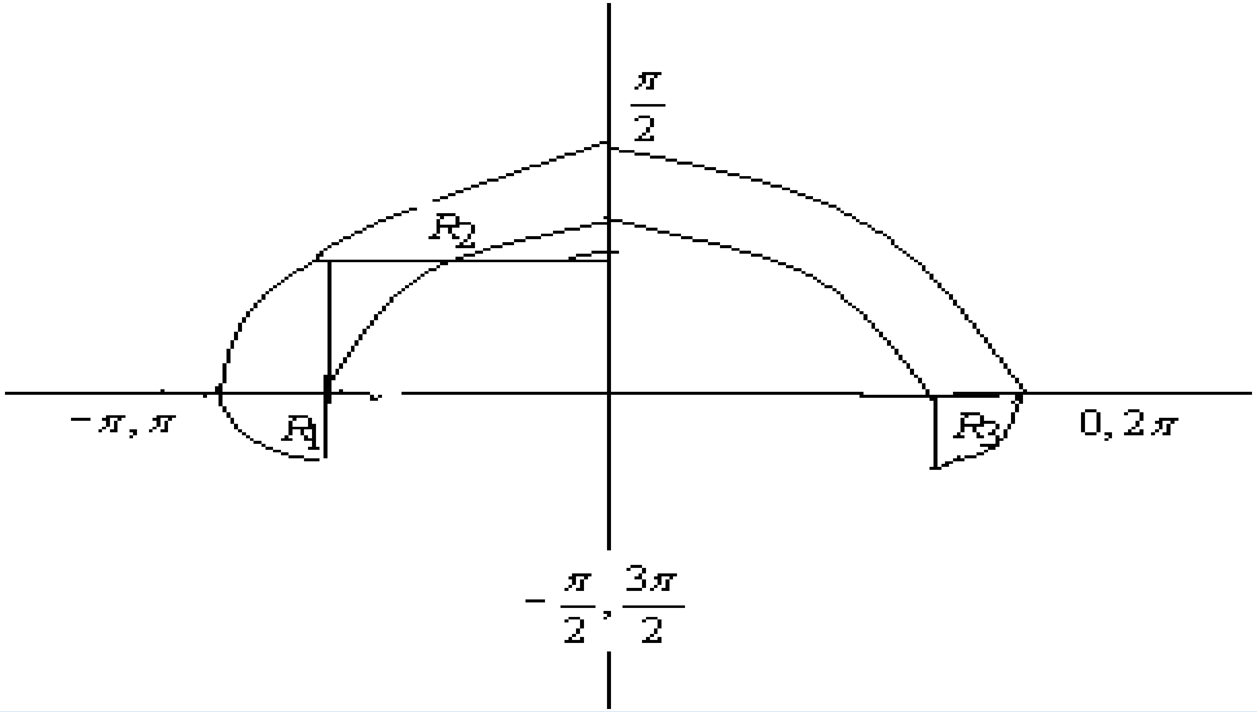
R_2 is $L_1 - L_2 \le \sqrt{x^2 + y^2} \le L_1 + L_2 \qquad x, y \ge 0$

R_3 is $(x - L_1)^2 + y^2 < L_2^2 \qquad x > 0, y < 0$

Example of Workspace

- Ex. If $0 \leq \theta_1 \leq \pi, -\frac{\pi}{2} \leq \theta_2 \leq \frac{\pi}{2}, L_2 \leq L_1$

  Find the Workspace



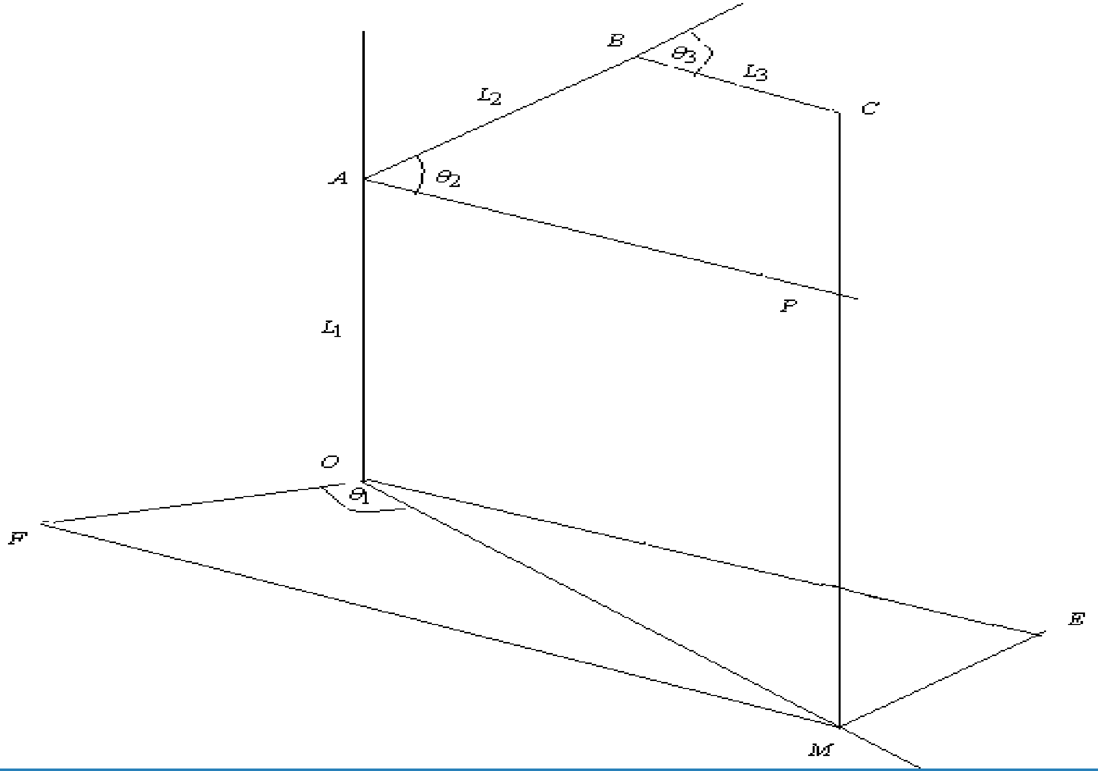
R_1 is $(x+L_1)^2 + y^2 < L_2^2$ $\quad x < -L_1, y < 0$

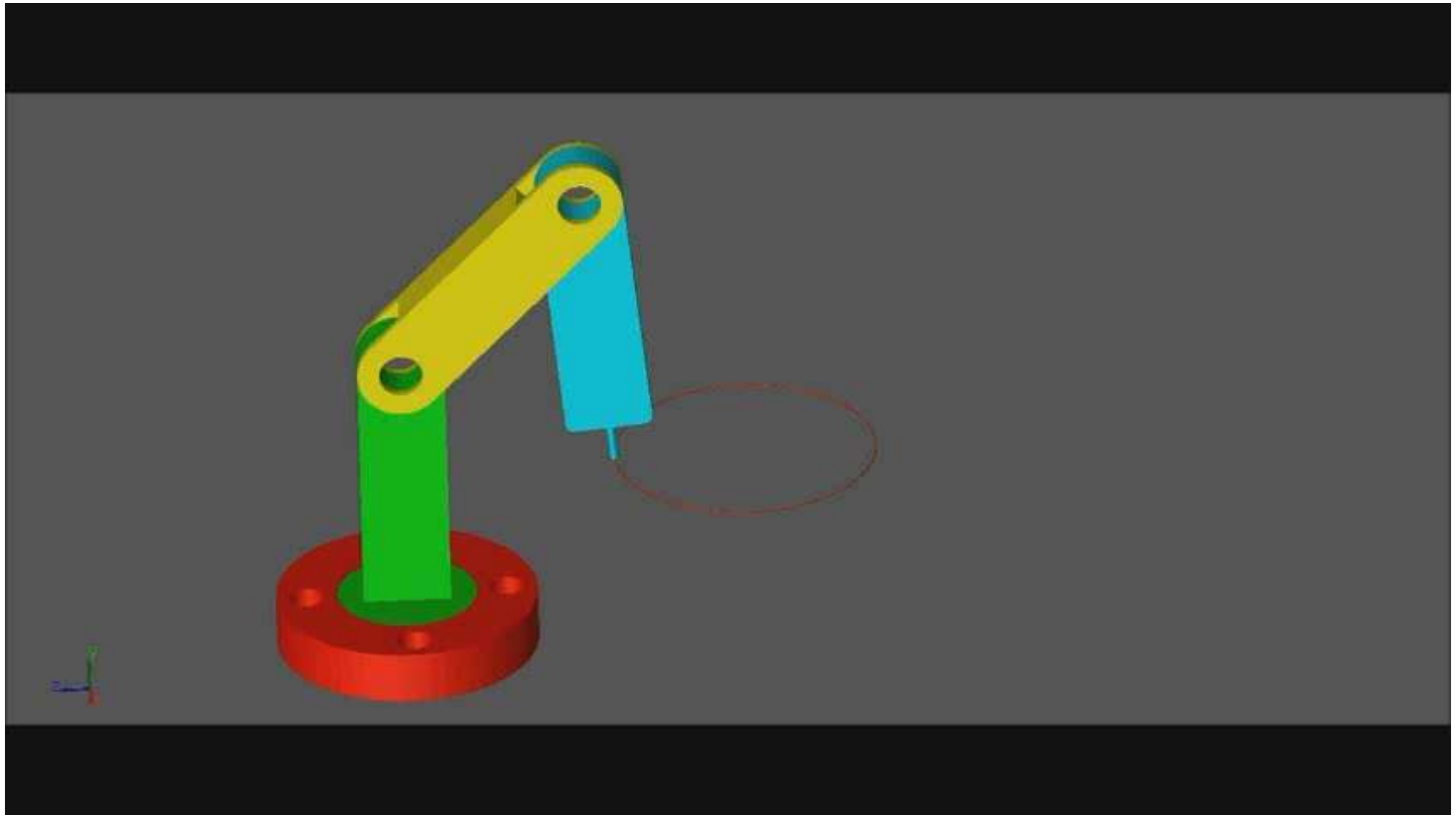
R_2 is $L_1 - L_2 \leq \sqrt{x^2 + y^2} \leq L_1 + L_2$ $\quad y > 0, x > or < 0$

R_3 is $(x-L_1)^2 + y^2 < L_2^2$ $\quad x > L_1, y < 0$

Three Axis Manipulator

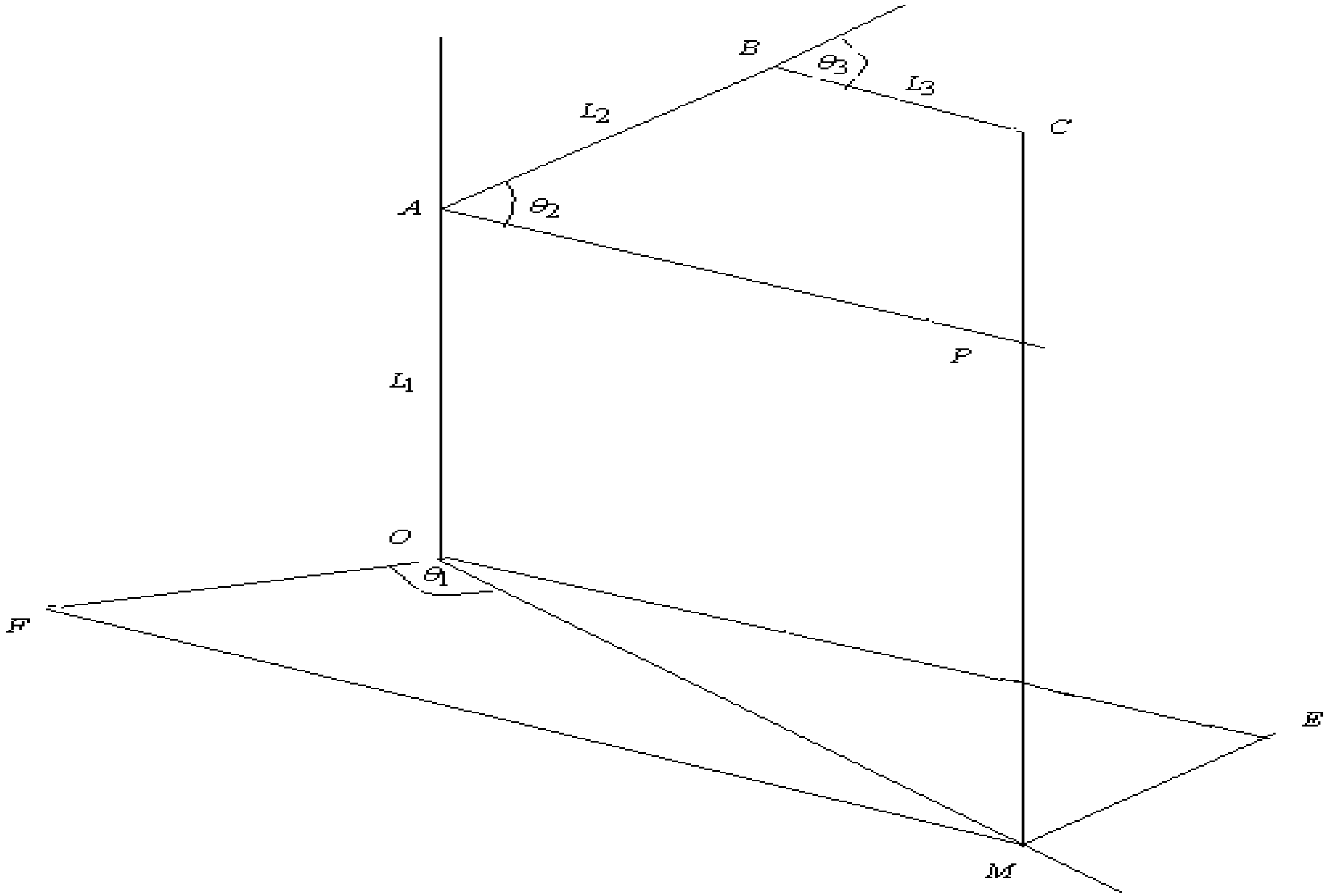
- In the following figure OABC represents a three axis manipulator. O is the base and C is the end effector.
- The links are OA, AB, BC. The joint angles are measured as shown in the figure.
- Let (x,y,z) be the end effector position.





Continue…

- $OF = x, FM = y, MC = z$
- $x = OF = OM\cos\theta_1$
- $y = OE = OM\sin\theta_1$
- $z = MP + PC$
- $OM = AP = L_2C_2 + L_3C_{2+3}$
- $PC = L_2S_2 + L_3S_{2+3}$



Continue…

Direct Kinematic Equations:

$$x = OMcos\theta_1 = [L_2C_2 + L_3C_{2+3}]cos\theta_1 \ldots \ldots .. (1)$$

$$y = OMsin\theta_1 = [L_2C_2 + L_3C_{2+3}]sin\theta_1 \ldots \ldots .. (2)$$

$$z = MP + PC = L_1 + L_2S_2 + L_3S_{2+3} \ldots \ldots \ldots . (3)$$

Continue…

- **Inverse Kinematic Equations:** For given (x, y, z), find $\theta_1, \theta_2\ and \theta_3.$

From (1) and (2)

$$Tan\theta_1 = \frac{y}{x} \Rightarrow \theta_1 = Tan^{-1}\frac{y}{x}$$

$$x^2 + y^2 = [L_2 C_2 + L_3 C_{2+3}]^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

$$(z - L_1)^2 = [L_2 S_2 + L_3 S_{2+3}]^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

Equations (a) and (b) $\Rightarrow$

$$\cos\theta_3 = \frac{x^2 + y^2 + (z - L_1)^2 - L_2^2 - L_3^2}{-2L_2L_3}$$

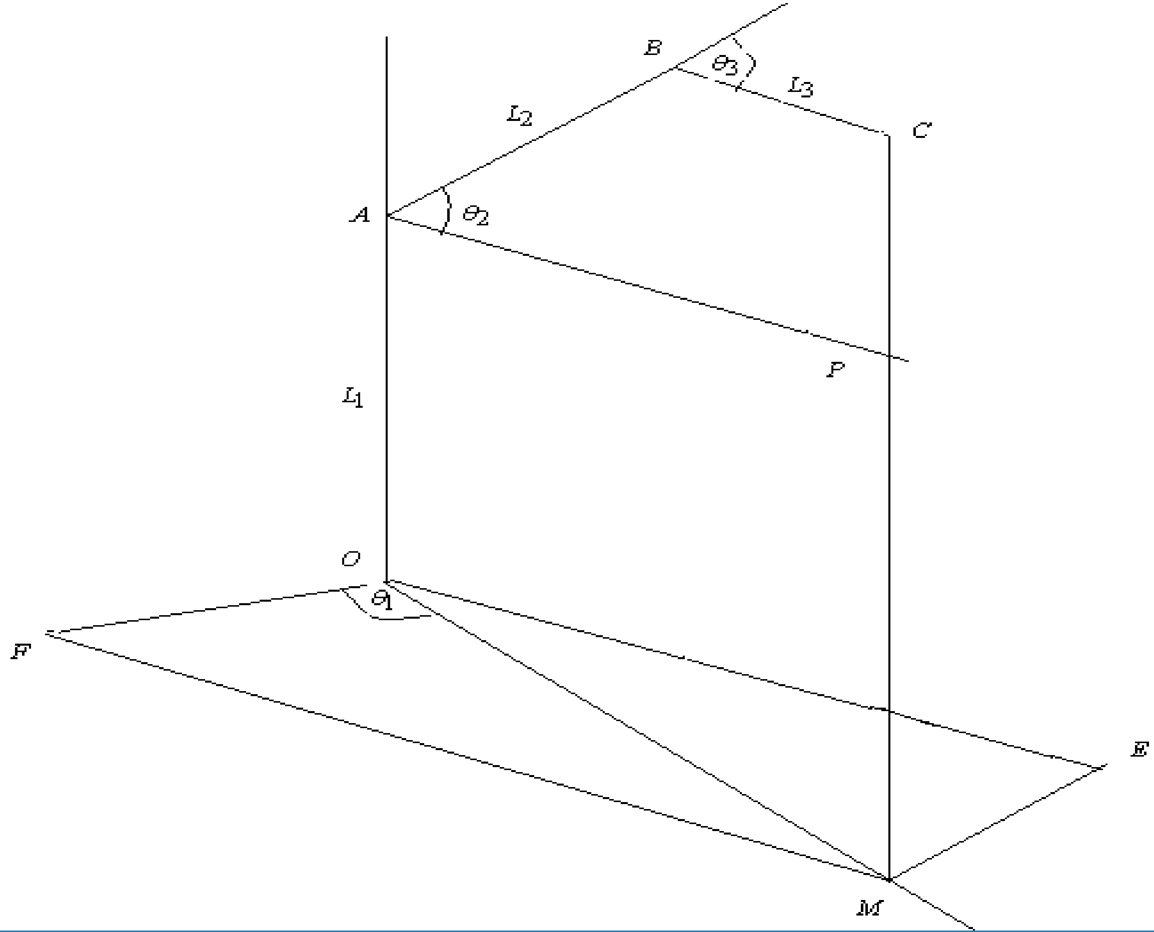
$$\theta_3 = \pm\cos^{-1}\left(\frac{x^2 + y^2 + (z - L_1)^2 - L_2^2 - L_3^2}{-2L_2L_3}\right)$$

$$x^2 + y^2 = [L_2 + L_3C_3]C_2 - L_3S_2S_3$$

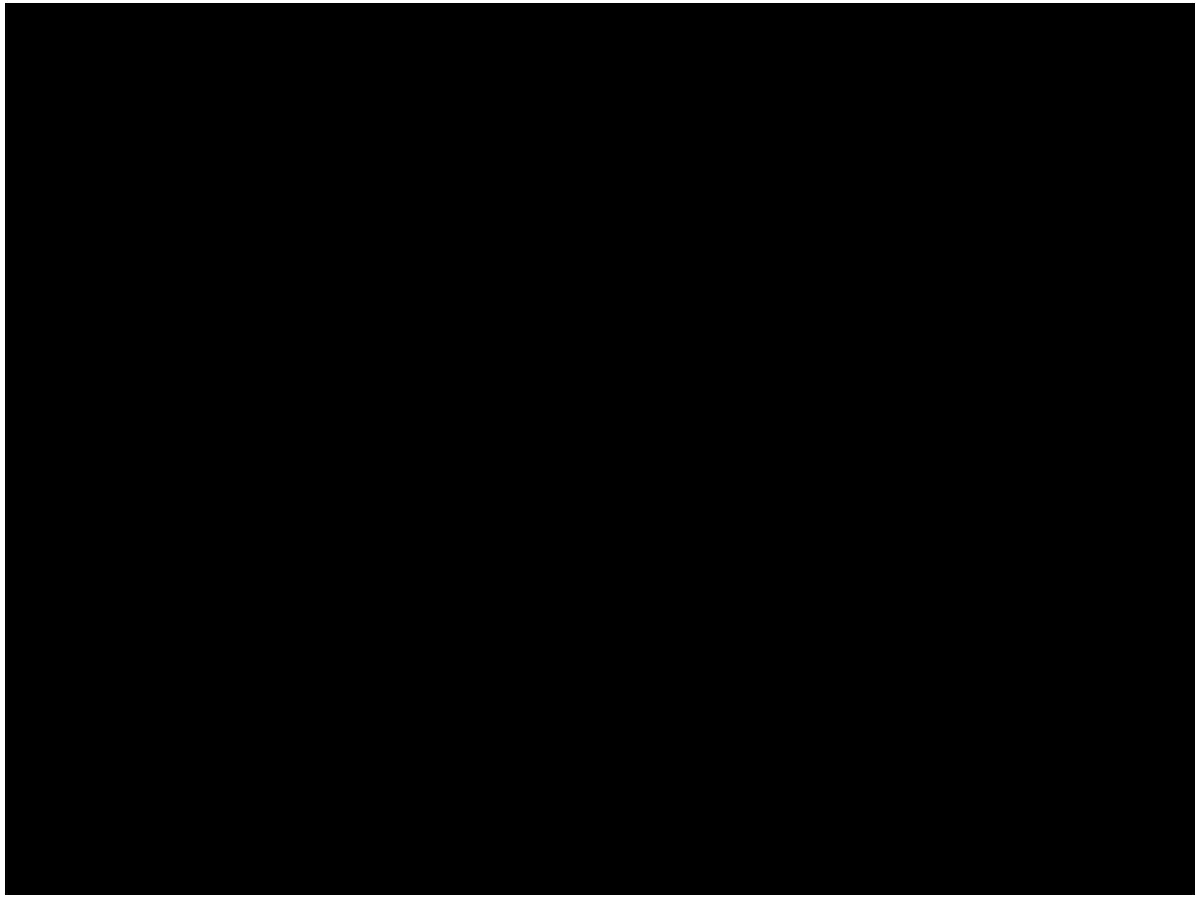
$$(z - L_1)^2 = [L_2 + L_3C_3]S_2 + L_3S_3C_2$$

$$Tan\theta_2 = \frac{(z - L_1)^2[L_2 + L_3C_3] - L_3S_3(x^2 + y^2)}{(x^2 + y^2)[L_2 + L_3C_3] + L_3S_3(z - L_1)^2}$$

$$\theta_2 = Tan^{-1}\left(\frac{(z - L_1)^2}{x^2 + y^2}\right) - Tan^{-1}\left(\frac{L_3S_3}{L_2 + L_3C_3}\right)$$



Example



Thanks!