IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Inverse Kinematics

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DEPARMENT OF MATHEMATICS

6-Axis Manipulator Robot

6-Axis Manipulator Robot

$$ {}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}T_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

6-Axis Manipulator Robot

$$^{2}T_{3} = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_{4} = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & a_4\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & a_4\sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{4}T_{5} = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6-Axis Manipulator Robot

$$
{}^{5}T_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

So,

$$
{}^{0}T_6 = {}^{0}T_1 . {}^{1}T_2 . {}^{2}T_3 . {}^{3}T_4 . {}^{4}T_5 . {}^{5}T_6
$$

$$
{}^{0}T_6 = \begin{bmatrix} (c_1c_2c_{345} + s_1s_{345})c_6 + c_1s_2s_6 & c_1s_2c_6 - (c_1c_2c_{345} + s_1s_{345})s_6 & -s_1c_{345} + c_1c_2s_{345} & c_1c_2(a_3c_3 + a_4\,c_{34} + d_6s_{345}) + s_1(a_3s_3 + a_4s_{34} - d_6c_{345}) \\ (s_1c_2c_{345} - c_1s_{345})c_6 + s_1s_2s_6 & s_1s_2c_6 - (s_1c_2c_{345} - c_1s_{345})s_6 & c_1c_{345} + s_1c_2s_{345} & s_1c_2(a_3c_3 + a_4\,c_{34} + d_6s_{345}) - c_1(a_3s_3 + a_4s_{34} - d_6c_{345}) \\ s_2c_{345}c_6 - c_2s_6 & -c_2c_6 - s_2c_{345}s_6 & s_2s_{345} & d_1 + s_2(a_3c_3 + a_4c_{34} + d_6s_{345}) \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Example 1

i	a	α	d_i	θ
1	a_1	$\frac{\pi}{2}$	d_1	θ_1
2	0	$-\frac{\pi}{2}$	0	θ_2
3	0	0	d_3	0

Joint-Link Parameters

Example 1

As we know basic transformation matrix:

$$^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & a_1\sin\theta_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 1

$$ {}^{2}T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

We can get final transformation matrix by:

$$ {}^{0}T_3 = {}^{0}T_1 . {}^{1}T_2 . {}^{2}T_3 $$

$$ {}^{0}T_3 = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & -\sin\theta_1 & -\cos\theta_1 \sin\theta_2 & a_1 \cos\theta_1 - d_3 \cos\theta_1 \sin\theta_2 \\ \sin\theta_1 \cos\theta_2 & \cos\theta_1 & -\sin\theta_1 \sin\theta_2 & a_1 \sin\theta_1 - d_3 \sin\theta_1 \sin\theta_2 \\ \sin\theta_2 & 0 & \cos\theta_2 & d_1 + d_3 \cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Example 1

We can evaluate inverse kinematics as follows:

$$\Rightarrow \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\sin\theta_1 & -\cos\theta_1\sin\theta_2 & a_1\cos\theta_1 - d_3\cos\theta_1\sin\theta_2 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1 & -\sin\theta_1\sin\theta_2 & a_1\sin\theta_1 - d_3\sin\theta_1\sin\theta_2 \\ \sin\theta_2 & 0 & \cos\theta_2 & d_1 + d_3\cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{n_y}{n_x} = \frac{\sin\theta_1\cos\theta_2}{\cos\theta_1\cos\theta_2} = \tan\theta_1 \Rightarrow \theta_1 = \tan^{-1}\frac{n_y}{n_x}$$

$$\frac{n_z}{a_x} = \frac{\sin\theta_2}{\cos\theta_2}, or\ \theta_2 = \tan^{-1}\frac{n_z}{a_x}$$

$$a_1\cos\theta_1 - d_3\cos\theta_1\sin\theta_2 = p_x \Rightarrow d_3 = \frac{a_1\cos\theta_1 - p_x}{\cos\theta_1\sin\theta_2}$$

Example 2

	θ	d	α	a
1	θ_1	OA	$-\frac{\pi}{2}$	0
2	θ_2	0	0	AB
3	θ_3	0	$\frac{\pi}{2}$	0
4	θ_4	BC	0	0

Example 2

$$
{}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Example 2

$$^{2}T_3 = \begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 & 0 \\ \sin\theta_3 & 0 & -\cos\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So,

$$^{0}T_4 = {}^{0}T_1 . {}^{1}T_2 . {}^{2}T_3 . {}^{3}T_4$$

$$^{0}T_4 = \begin{bmatrix} (c_1c_2c_3 - c_1s_2s_3)c_4 - s_1s_4 & -(c_1c_2c_3 - c_1s_2s_3)s_4 - s_1c_4 & c_1c_2s_3 + c_1s_2c_3 & a_2c_1c_2 + d_4(c_1c_2s_3 + c_1s_2c_3) \\ (s_1c_2c_3 - s_1s_2s_3)c_4 + c_1s_4 & -(s_1c_2c_3 - s_1s_2s_3)s_4 + c_1c_4 & s_1c_2s_3 + s_1s_2c_3 & a_2s_1c_2 + d_4(s_1c_2s_3 + s_1s_2c_3) \\ (-s_2c_3 - c_2s_3)c_4 & -(-s_2c_3 - c_2s_3)s_4 & -s_2s_3 + c_2c_3 & d_4(-s_2s_3 + c_2c_3) - a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 2

If

$$ {}^{0}T_4 = \begin{bmatrix} (c_1c_2c_3 - c_1s_2s_3)c_4 - s_1s_4 & -(c_1c_2c_3 - c_1s_2s_3)s_4 - s_1c_4 & c_1c_2s_3 + c_1s_2c_3 & a_2c_1c_2 + d_4(c_1c_2s_3 + c_1s_2c_3) \\ (s_1c_2c_3 - s_1s_2s_3)c_4 + c_1s_4 & -(s_1c_2c_3 - s_1s_2s_3)s_4 + c_1c_4 & s_1c_2s_3 + s_1s_2c_3 & a_2s_1c_2 + d_4(s_1c_2s_3 + s_1s_2c_3) \\ (-s_2c_3 - c_2s_3)c_4 & -(-s_2c_3 - c_2s_3)s_4 & -s_2s_3 + c_2c_3 & d_4(-s_2s_3 + c_2c_3) - a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_1 \\ r_{21} & r_{22} & r_{23} & p_2 \\ r_{31} & r_{32} & r_{33} & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

We get

$$ c_1(c_2c_3 - s_2s_3)c_4 - s_1s_4 = r_{11} $$
$$ s_1(c_2c_3 - s_2s_3)c_4 + c_1s_4 = r_{21} $$
$$ \Rightarrow s_4 = c_1r_{21} - s_1r_{11} $$

$$ \tan\theta_4 = \frac{r_{32}}{r_{31}} $$

$$ (-c_2s_3 - s_2c_3)c_4 = r_{31} $$
$$ \tan\theta_1 = \frac{r_{23}}{r_{13}} $$

Thanks!