Biped Robot
Flat Foot and Toe Foot Model

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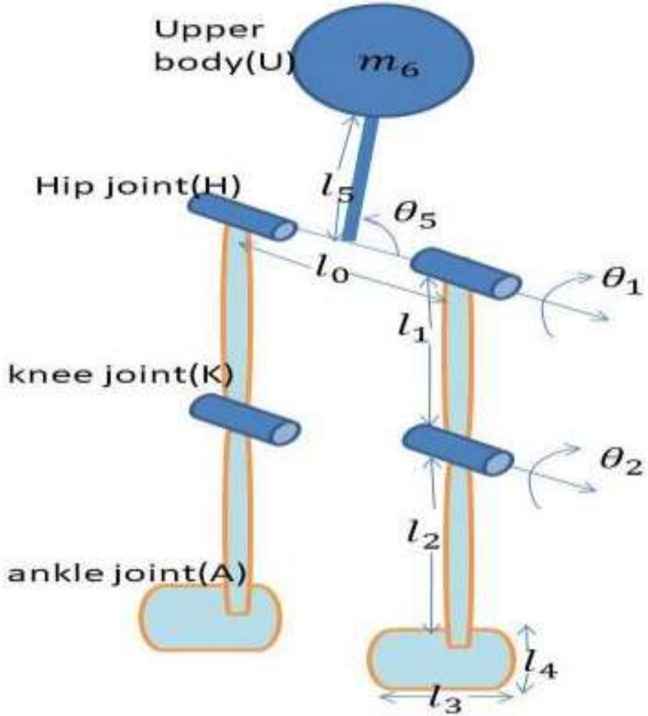
Biped Robot
Flat Foot Model

Robot Model



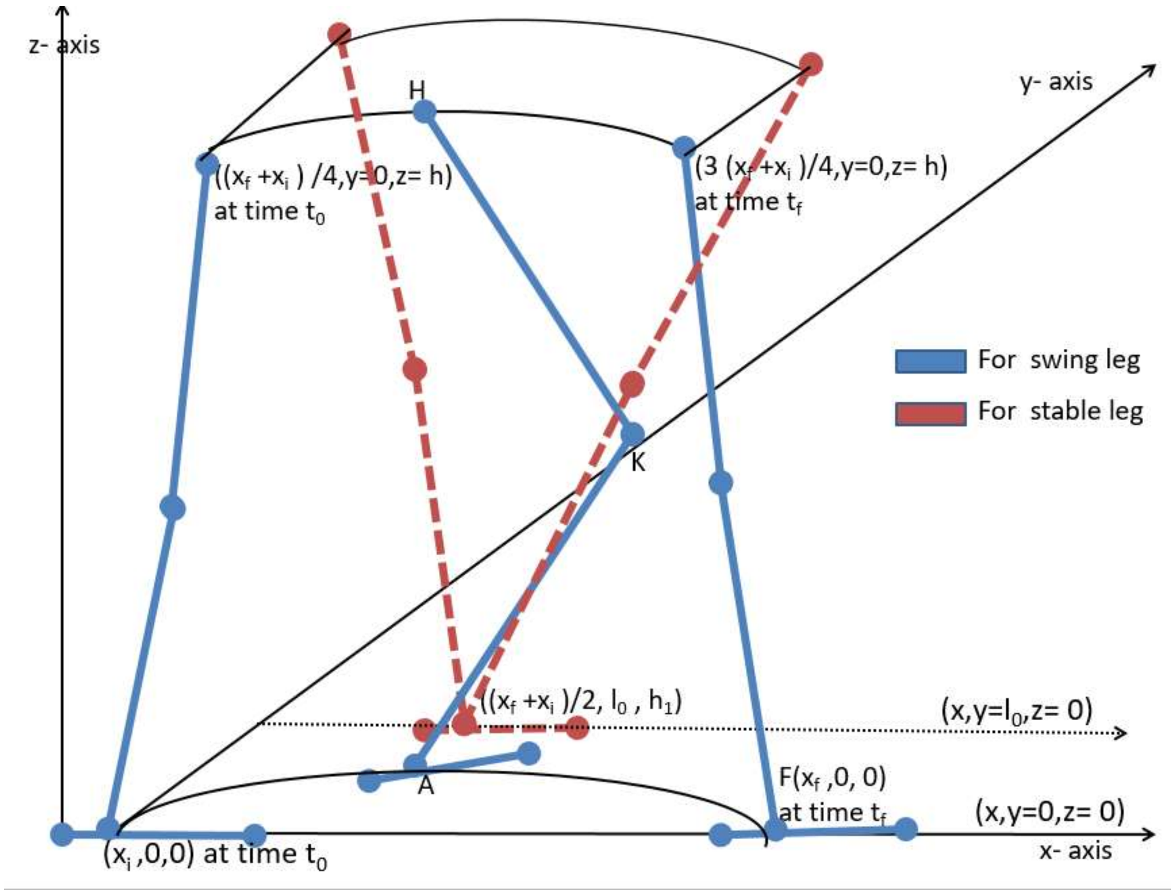
Upper body(U)
m_6
Hip joint(H)
l_5
θ_5
l_0
θ_1
l_1
knee joint(K)
θ_2
l_2
ankle joint(A)
l_4
l_3

Parameters

Link	Length	Value	Mass	Value
HK	l_1	14 inches	m_1	4kg
KA	l_2	14 inches	m_2	4kg
HU	l_5	10 inches	m_6	50kg
HH	l_0	8 inches	m_5	4kg

Table 3.1: Parameters

- In Figure, each leg of biped robot have 2 degrees of freedom (DOF) with flat foot.
- All the joints are revolute which are called hip joint (H), knee joint (K) and ankle joint (A).
- Centre of mass of upper body is denoted by (U).
- Robot's walk can be considered as a repetition of one-step motion.
- The walking sequence can be determined by computing the trajectory of the hip, ankle and upper body joints.
- For hip trajectory, stable ankle joint is considered as a base and hip as the end effector.
- For biped robot walking on a plane, motion of the stable leg is assumed to be like an inverted pendulum considering it's ankle joint as base and hip as end effector.
- While walking, humans do not fold their stable leg as the whole body weight lies on it.
- Flat foot is attached at the ankle joint of each leg.
- Let the robot walk in sagittal plane (xz-plane).


z- axis
y- axis
x- axis
H
K
A
((xf +xi) /4,y=0,z= h)
at time t0
(3 (xf +xi)/4,y=0,z= h)
at time tf
For swing leg
For stable leg
((xf +xi)/2, l0 , h1)
(x,y=l0,z= 0)
F(xf ,0, 0)
at time tf
(x,y=0,z= 0)
(xi ,0,0) at time t0

Swing leg's trajectories:

Boundary Conditions of Ankle Trajectory

$x_A(t_0) = x_i$; $x_A(t_f) = x_i + x_f$; $\dot{x}_A(t_0) = 0$; $\dot{x}_A(t_f) = 0$.
$z_A(x_0) = 0$; $z_A(x_f) = 0$; $z_A(x_m) = h_1$; $\dot{z}_A(x_m) = 0$.

Ankle Trajectory

$$\begin{aligned} x_A(t) &= x_i + \left(\frac{3x_f}{t_f^2}\right) t^2 - \left(\frac{2x_f}{t_f^3}\right) t^3; \\ z_A(t) &= \frac{h(-(x_f + x_i)^2 x_i)}{(x_m - x_i)(x_m - x_f - x_i)^2} + \frac{h(x_f + x_i)(x_f + 3x_i)x_A(t)}{(x_m - x_i)(x_m - x_f - x_i)^2} \\ &- \frac{h(2x_f + 3x_i)x_A(t)^2 + hx_A(t)^3}{(x_m - x_i)(x_m - x_f - x_i)^2} \end{aligned}$$

Stable leg's trajectories:

Boundary Conditions of Hip Trajectory

$x_H(t_0) = x_i + x_f/4$; $x_H(t_f) = x_i + 3x_f/4$; $\dot{x}_H(t_0) = v_s$; $\dot{x}_H(t_f) = v_e$.
$z_H(t_0) = h$; $z_H(t_f) = h$; $\dot{z}_H(t_0) = v_{zs}$; $\dot{z}_H(t_f) = v_{ze}$.

Hip Trajectory

$$x_H(t) = \frac{x_f}{4} + v_s t + (\frac{(v_e - v_s)}{2t_f} - r_4\frac{3t_f}{2})t^2 - 2(\frac{x_f}{2t_f^3} - \frac{(v_s + v_e)}{2t_f^2})t^3;$$
$$z_H(t) = \sqrt{(l_1 + l_2)^2 - (x_H(t) - (x_i + x_f/2))^2}.$$

where $r_4 = -2(\frac{x_f}{2t_f^3} - \frac{(v_s+v_e)}{2t_f^2})$

Forward Kinematics

For Swing leg

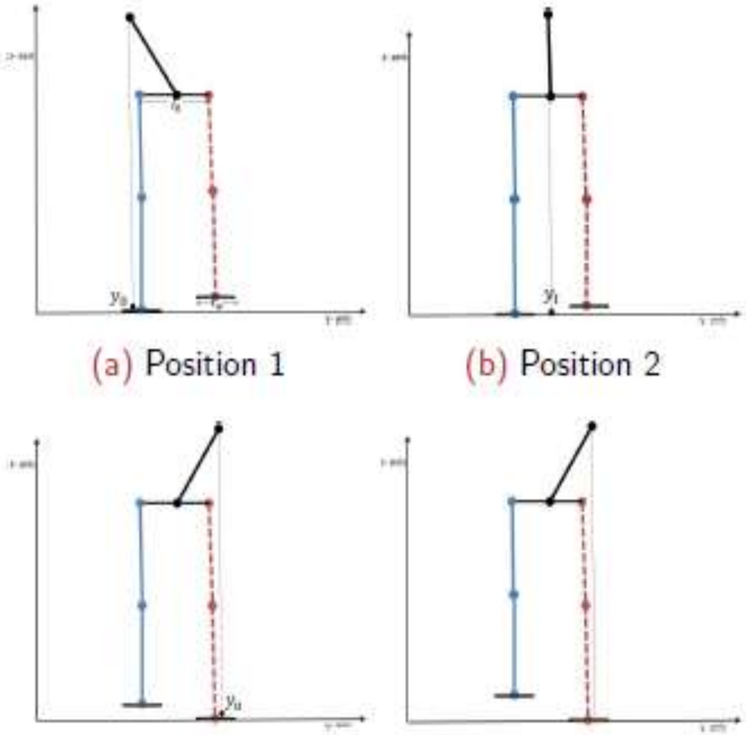
$$x_A(t) - x_H(t) = l_1 cos\theta_1(t) + l_2 cos(\theta_1(t) + \theta_2(t));$$
$$z_A(t) - z_H(t) = l_1 sin\theta_1(t) + l_2 sin(\theta_1(t) + \theta_2(t));$$

where $(x_A(t), z_A(t))$ and $(x_H(t), z_H(t))$ are defined as earlier.

For stable leg

$$x_H(t) - \left(x_i + \frac{x_f}{2}\right) = (l_1 + l_2)\cos\theta_5(t);$$
$$z_H(t) = (l_1 + l_2)\sin\theta_5(t);$$

where $(x_i+x_f/2, l_0, 0)$ is the position of the stable leg's ankle joint which lies on the line $y=l_0$.


(a) Position 1 (b) Position 2
(c) Position 3 (d) Position 4

Trunk Motion

Trunk motion play an important role in ZMP stability.

Case-1

- Upper body start to move from middle to the side of the stable leg's hip during time t_0 to t_1,
- Stay there during time t_1 to t_3.
- Again start moving towards middle of legs between t_3 to t_f time where $t_1 = t_f/4$ and $t_3 = 3t_f/4$.

The moving mass trajectory in y-direction is given below: $y_M(t) =$

$$\begin{cases} y_l + y_v t + \left(\frac{3(y_a - y_l)}{t_1^2} - \frac{2y_v}{t_1}\right)t^2 + \left(\frac{-2(y_a - y_l)}{t_1^3} - \frac{y_v}{t_1^2}\right)t^3 & t_0 \le t \le t_1 \\ y_a & t_1 \le t \le t_3 \\ \left(y_a + \frac{(-3t_f t_3^2 + t_3^3)(y_l - y_a)}{(t_3 - t_f)^3} + \frac{t_f t_3^2 y_v}{(t_3 - t_f)^2}\right) & \\ \left(\frac{6t_f t_3(y_l - y_a)}{(t_3 - t_f)^3} - \frac{(t_3^2 + 2t_f t_3)y_v}{(t_3 - t_f)^2}\right)t + \left(\frac{-3((y_l - y_a)(t_3 + t_f)}{(t_3 - t_f)^3}\right. & \\ \left. + \frac{y_v(4t_3 + 2t_f)}{2(t_3 - t_f)^2}\right)t^2 + \left(\frac{2(y_l - y_a)}{(t_3 - t_f)^3} - \frac{y_v}{(t_3 - t_f)^2}\right)t^3 & t_3 \le t \le t_f \end{cases}$$

Case-2

- Upper body start to move from middle to the side of the stable leg's hip during time t_0 to $t_f/8$.
- Stay there during time $t_f/8$ to $7t_f/8$.
- Again start moving towards middle of legs between $7t_f/8$ to t_f time.

Then the trajectory can be calculated by case-1 equation.

Case-3

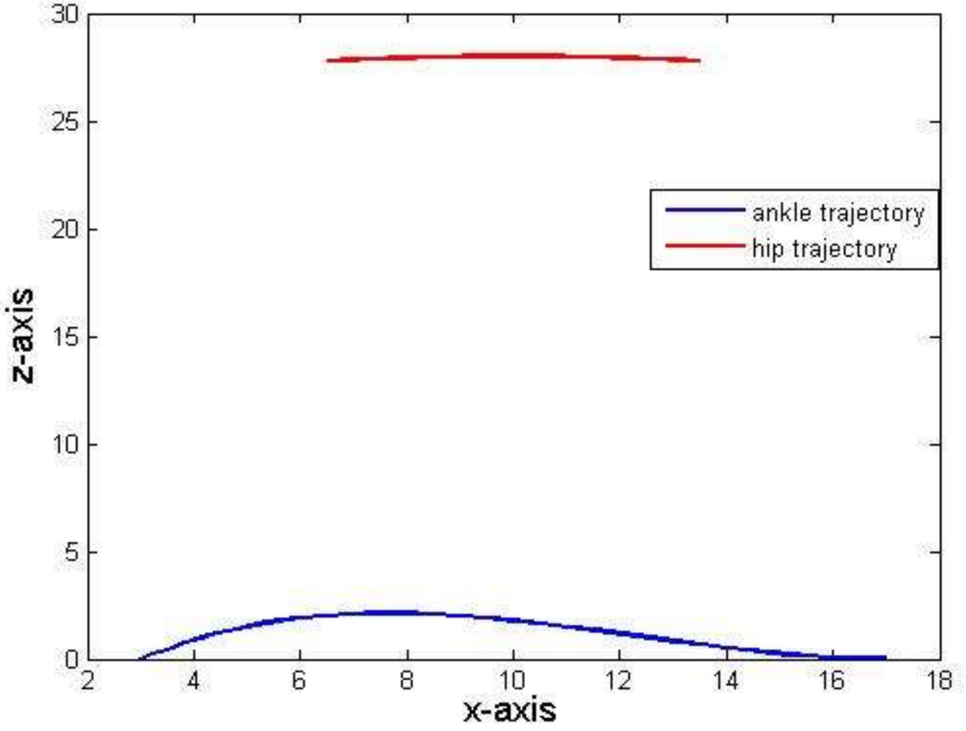
Similarly upper body start to move from middle position to stable foot from time t_0 to t_2, then return back to initial condition. So the moving mass trajectory in y-direction is given below:

$$y_M(t) = \begin{cases} y_l + y_v t + \left(\frac{3(y_a - y_l)}{t_2^2} - \frac{2y_v}{t_2}\right)t^2 + \left(\frac{-2(y_a - y_l)}{t_2^3} - \frac{y_v}{t_2^2}\right)t^3 & t_0 \le t \le t_2 \\ \left(y_a + \frac{(-3t_f t_2^2 + t_2^3)(y_l - y_a)}{(t_2 - t_f)^3} + \frac{t_f t_2^2 y_v}{(t_2 - t_f)^2}\right) & \\ \left(\frac{6t_f t_2 (y_l - y_a)}{(t_2 - t_f)^3} - \frac{(t_2^2 + 2t_f t_2) y_v}{(t_2 - t_f)^2}\right)t + \left(\frac{-3((y_l - y_a)(t_2 + t_f)}{(t_2 - t_f)^3}\right. & \\ \left. + \frac{y_v(4t_2 + 2t_f)}{2(t_2 - t_f)^2}\right)t^2 + \left(\frac{2(y_l - y_a)}{(t_2 - t_f)^3} - \frac{y_v}{(t_2 - t_f)^2}\right)t^3 & t_2 \le t \le t_f \end{cases}$$

Results

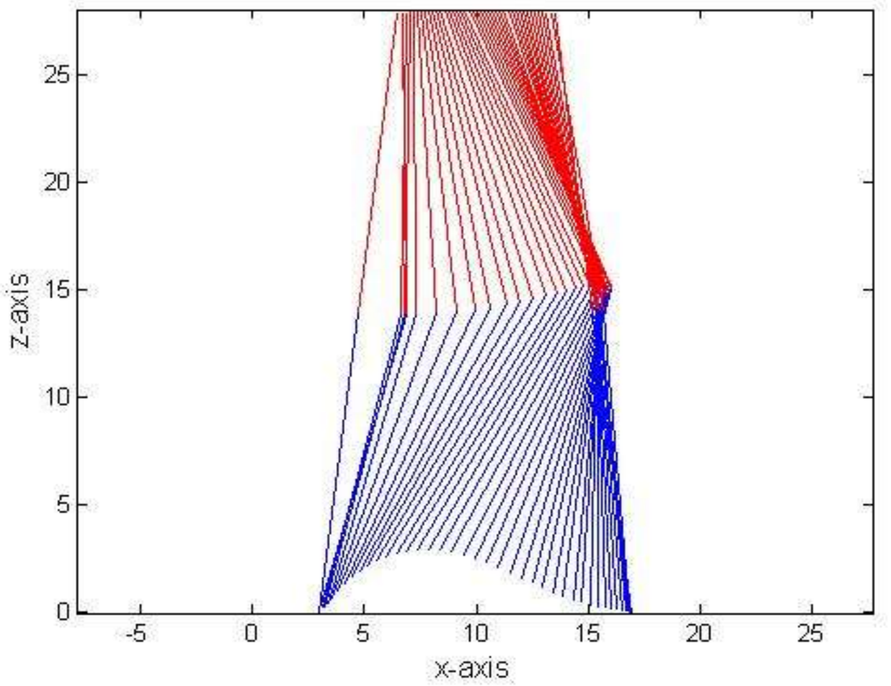
- Let total length of foot is 6 units and width is 4 units, initial and end velocity for ankle is 0 unit/sec.
- Ankle is fixed at the middle point of the foot, so that the initial x coordinate of the ankle is $x_i = 3$ units.
- The ankle joint covers a step length $x_f = 14$ units from initial position $(x_i, 0, 0)$ to the final position $(x_i + x_f, 0, 0)$ with step height h=2.5 units.
- Swing foot lies on the xy-plane in the region $0<x<6$ units and $-2<y<2$ units and stable foot lies on the line $y=l_0$ in the region $7<x<13$ units and $6<y<10$ units.

Results

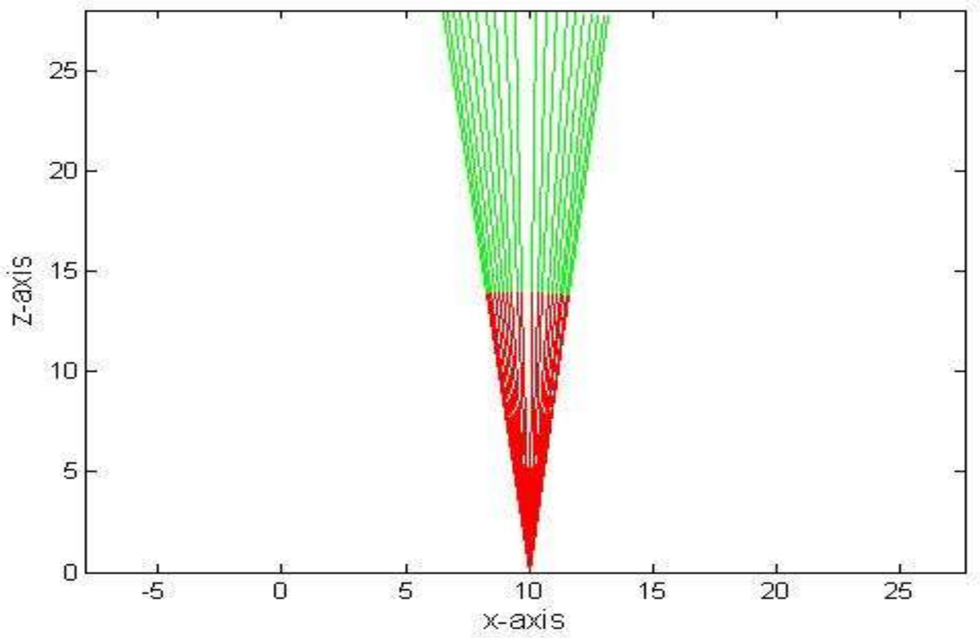


30
25
20
15
10
5
0
2
4
6
8
10
12
14
16
18
z-axis
x-axis
ankle trajectory
hip trajectory

Inverse Kinematics for Swing leg



Inverse Kinematics for Stable leg



Zero Moment Point

- Zero Moment Point (ZMP) may be defined as that point on the surface of the ground about which resultant sum of moments of all forces which are active is zero.
- ZMP can be calculated by following equations:

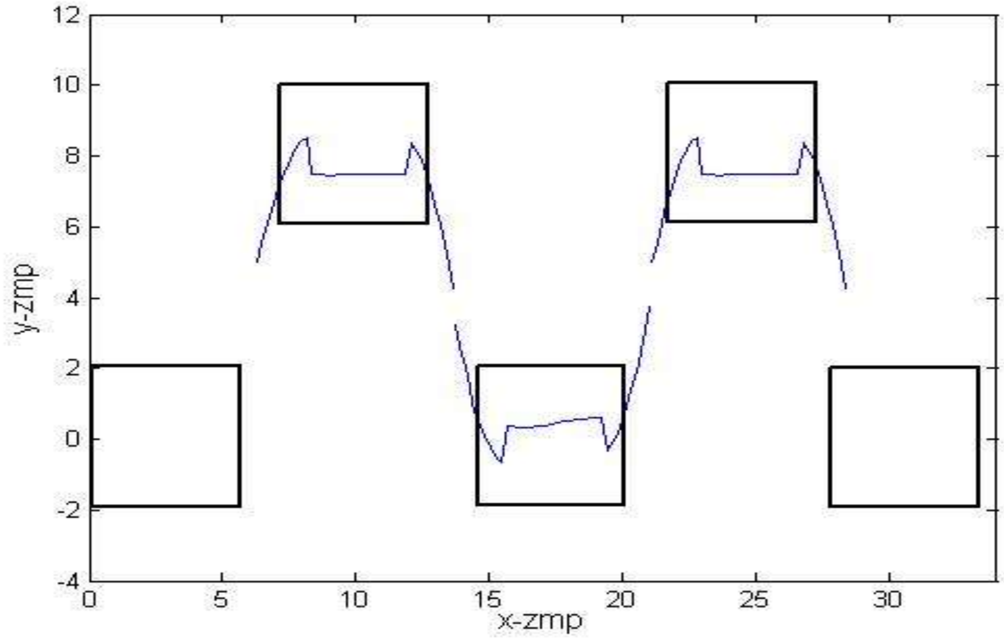
$$x_{ZMP} = \frac{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)x_i - \sum_{i=1}^{n} m_i\ddot{x}_i z_i - \sum_{i=1}^{n} I_{iy}\ddot{\Omega}_{iy}}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)}$$

$$y_{ZMP} = \frac{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)y_i - \sum_{i=1}^{n} m_i\ddot{y}_i z_i - \sum_{i=1}^{n} I_{ix}\ddot{\Omega}_{ix}}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)}$$

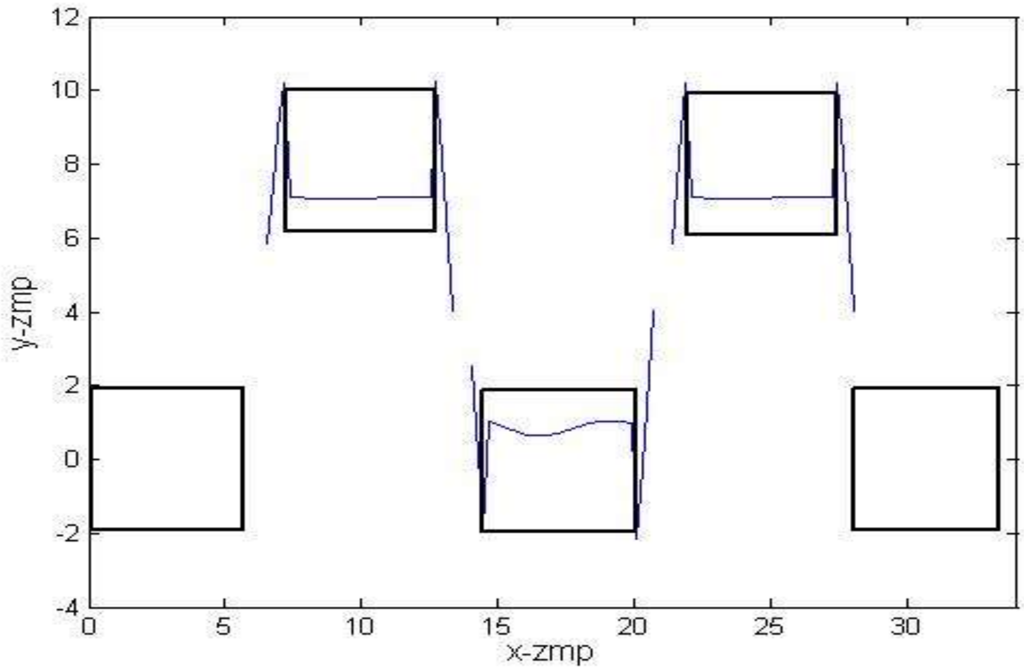
where $m_i$denotes mass of link i, respective inertial components are denoted by $I_{ix}\ and\ I_{iy}$, absolute angular velocities are denoted by $\ddot{\Omega}_{ix}$ and $\ddot{\Omega}_{iy}$, gdenoted the acceleration due to gravity, $(x_{ZMP}, y_{ZMP}, 0)$ denotes coordinates for zero moment point and (x_i, y_i, z_i)denotes coordinates for center of mass of link i.

ZMP Trajectory

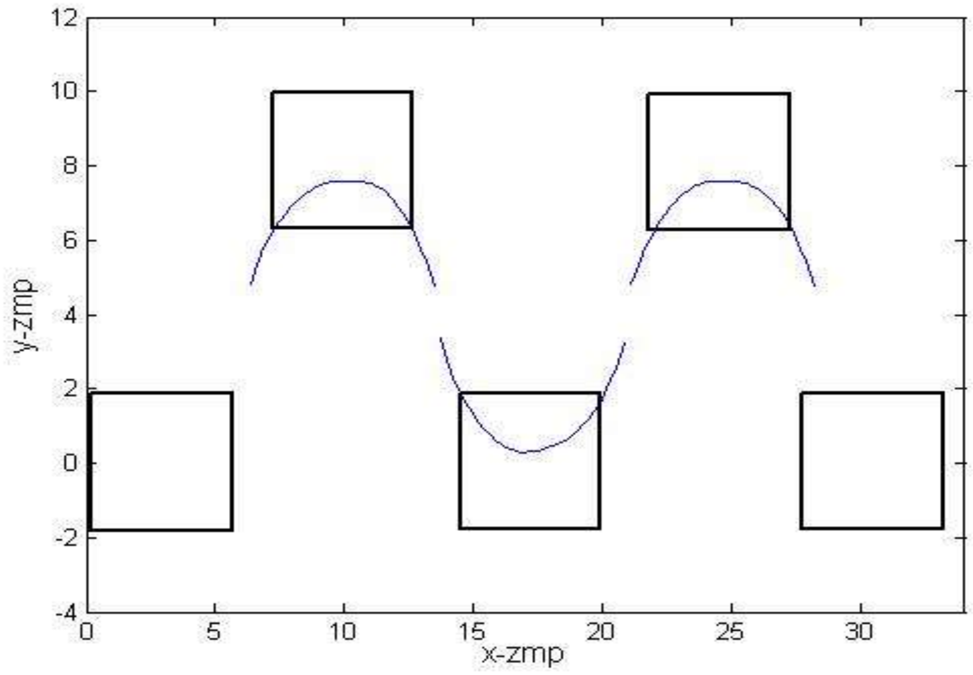
Case-1



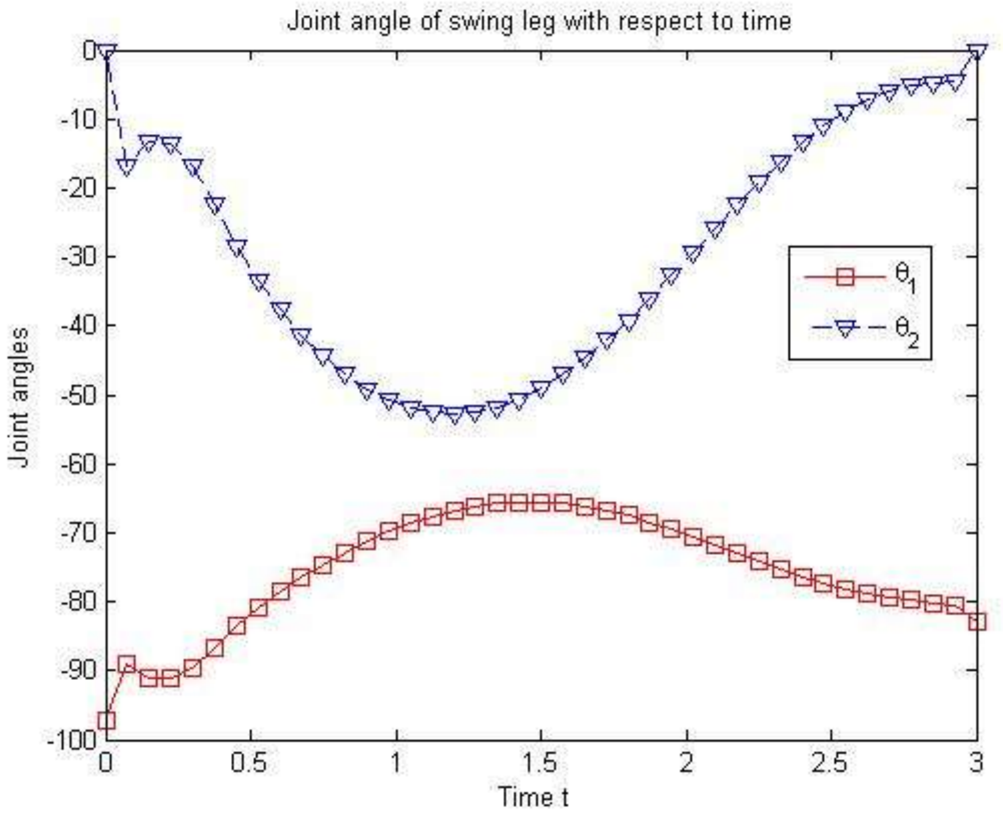
Case-2



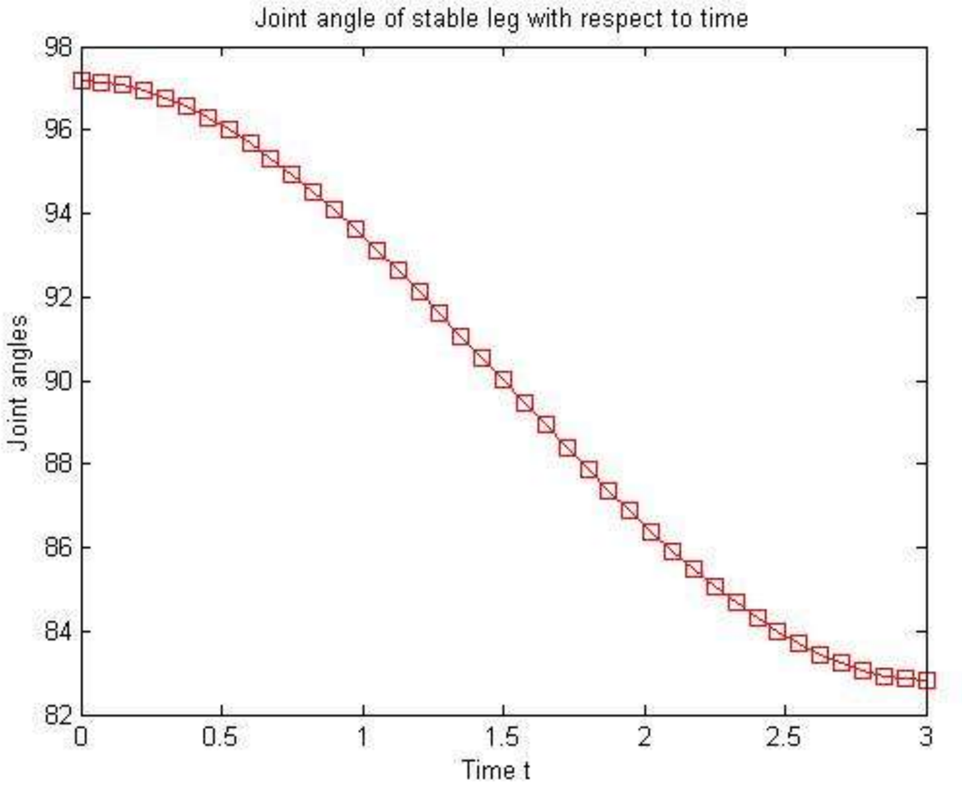
Case-3



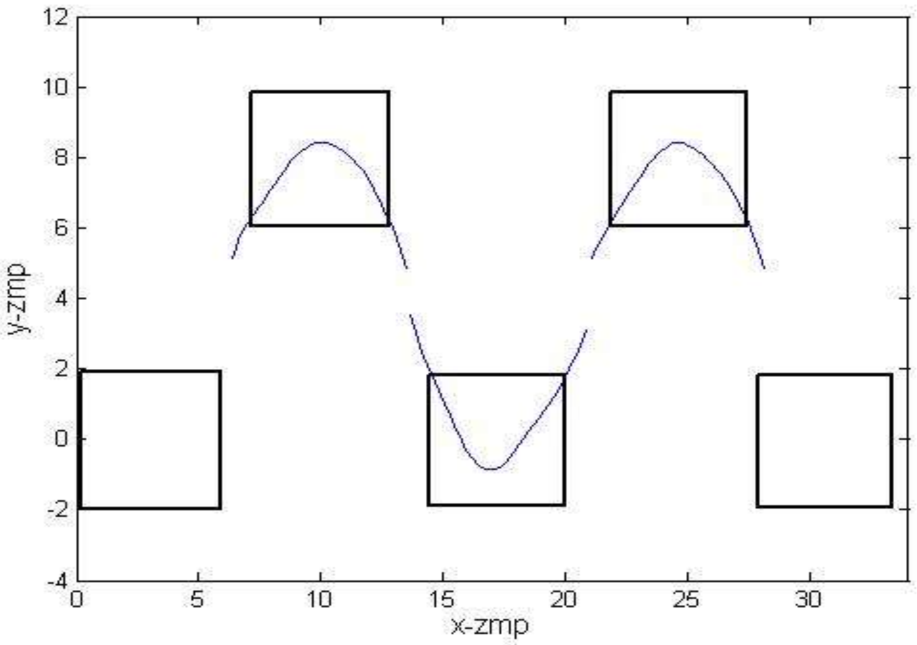
Hip velocity	Time	Upper Body Trajectory	initial velocity	ZMP stability
v_s=2.3in/s	3s	case-1	$y_v = 10 in/s$	stable
v_s=3.5in/s	2s	case-1	$y_v = 15 in/s$	stable but small margin
v_s=4.7in/s	1.5s	case-1	$y_v = 20 in/s$	unstable
v_s=2.4in/s	3s	case-2	$y_v = 16 in/s$	unstable
v_s=3.5in/s	2s	case-2	$y_v = 20 in/s$	unstable
v_s=4.7in/s	1.5s	case-2	$y_v = 22 in/s$	unstable
v_s=2.3in/s	3s	case-3	$y_v = 7.3 in/s$	stable
v_s=3.5in/s	2s	case-3	$y_v = 10.3 in/s$	stable
v_s=4.7in/s	1.5s	case-3	$y_v = 11 in/s$	stable



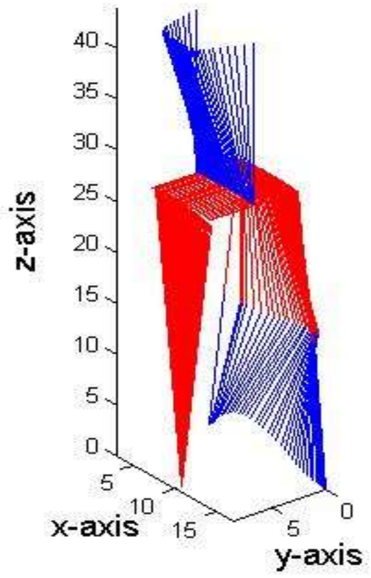
Joint angle of swing leg with respect to time
Joint angles
Time t
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
0
0.5
1
1.5
2
2.5
3
θ1
θ2


Joint angle of stable leg with respect to time
Joint angles
Time t
98
96
94
92
90
88
86
84
82
0
0.5
1
1.5
2
2.5
3

Final ZMP Trajectory

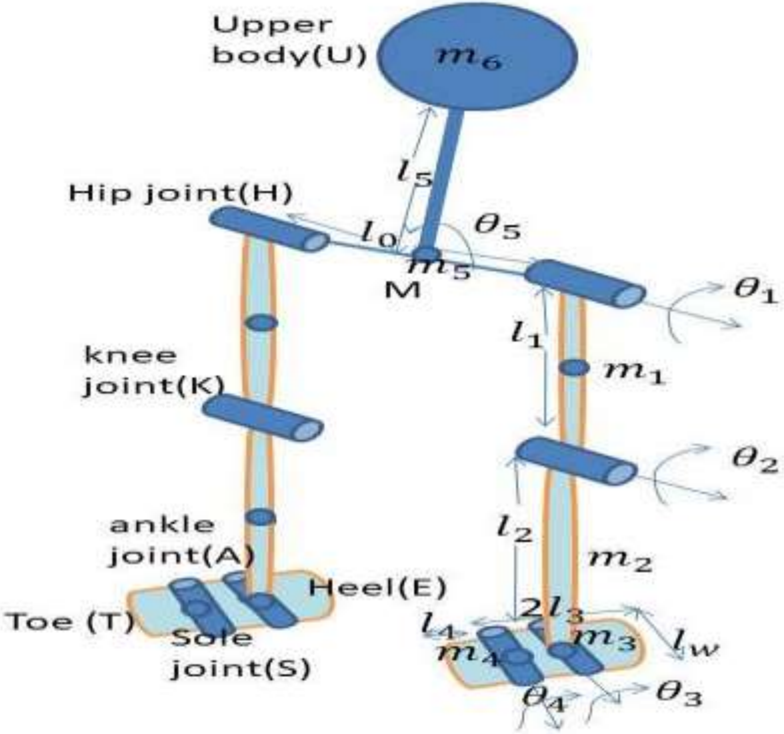


Full Body Motion



Toe-Foot Biped Model

Biped Model

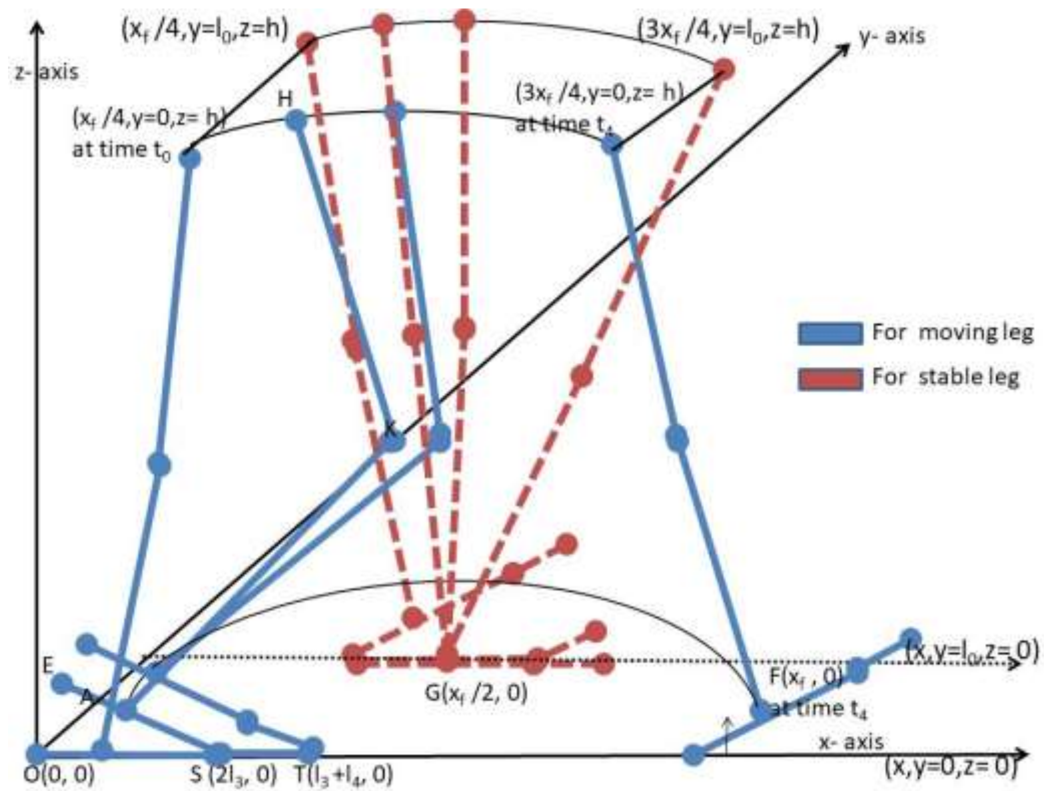


Upper body(U)
m_6
l_5
Hip joint(H)
l_0
θ_5
m_5
M
θ_1
l_1
m_1
knee joint(K)
θ_2
l_2
m_2
ankle joint(A)
Heel(E)
$2l_3$
Toe (T)
l_4
m_3
l_w
Sole joint(S)
m_4
θ_4
θ_3

Parameters

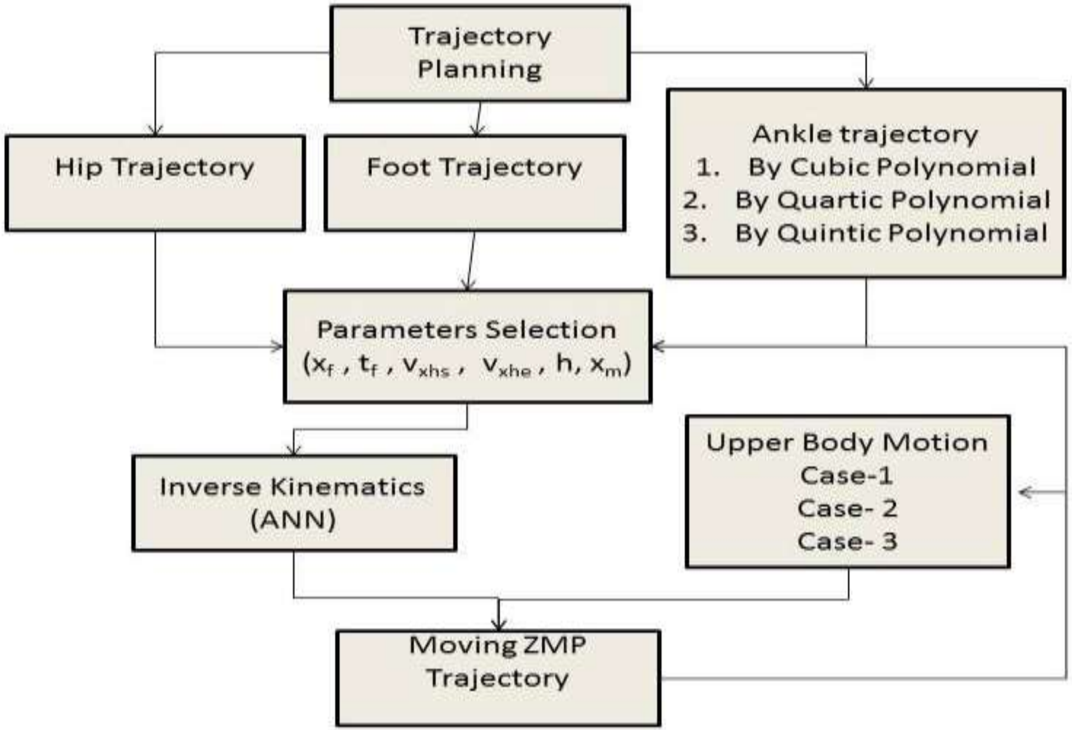
Link	Length	value	mass	value
HK	l_1	14 units	m_1	4 units
KA	l_2	14 units	m_2	4 units
ES	$2l_3$	4 units	m_3	0.8 units
ST	l_4	2 units	m_4	0.2 units
HU	l_5	12 units	m_6	50 units
HH	l_0	8 units	m_5	4 units
foot width	l_w	3 units		

Table 4.1: Description of robot links



(x_f /4,y=l_0,z=h)
(3x_f /4,y=l_0,z=h)
y- axis
z- axis
H
(x_f /4,y=0,z= h)
at time t_0
(3x_f /4,y=0,z= h)
at time t_4
For moving leg
For stable leg
K
E
A
(x,y=l_0,z=0)
F(x_f , 0)
at time t_4
G(x_f /2, 0)
x- axis
(x,y=0,z= 0)
O(0, 0)
S (2l_3, 0)
T(l_3 +l_4, 0)

Trajectory Generation



Swing leg's trajectories:

Ankle Trajectory during DSP

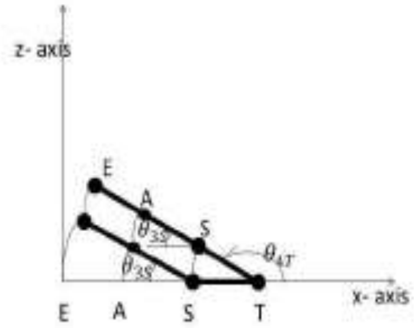
$$x_A(t) = 2l_3 + l_4 + l_4\cos\theta_{4t}(t) + l_3\cos(\theta_{3t}(t) + \theta_{4t}(t));$$
$$z_A(t) = l_4\sin\theta_{4t}(t) + l_3\sin(\theta_{3t}(t) + \theta_{4t}(t))$$

$$\theta_{3t}(t) = \begin{cases} (\theta_a)\left(\frac{3t^2}{T_s^2} - 2\frac{t^3}{T_s^3}\right) & 0 \le t \le t_1 \\ (\theta_a)\left(-4 + 12\frac{t}{T_s} - \frac{9t^2}{T_s^2} + 2\frac{t^3}{T_s^3}\right) & t_1 \le t \le t_2 \end{cases}$$

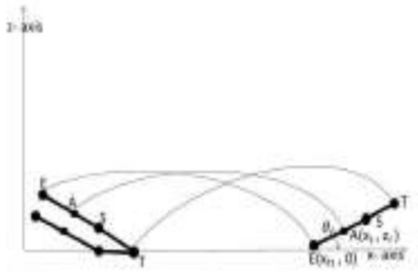
$$\theta_{4t}(t) = \begin{cases} \pi & 0 \le t \le t_1 \\ \pi + (\theta_b)\left(-5 + 12\frac{t}{T_s} - 9\frac{3t^2}{T_s^2} + 2\frac{t^3}{T_s^3}\right) & t_1 \le t \le t_2 \end{cases}$$

Ankle Trajectory during SSP

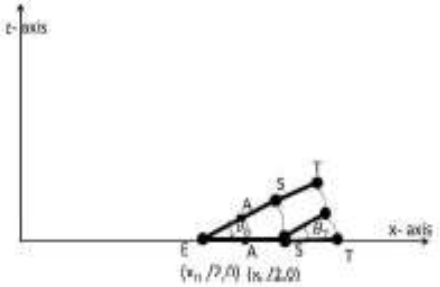
During SSP (t_2, t_f), boundary conditions are
$x_A(t_2) = x_e$; $x_A(t_f) = x_f$; $\dot{x}_A(t_2) = x_v$, $\dot{x}_A(t_f) = 0$
$z_A(x_e) = z_e$; $z_A(x_f) = 0$; $z_A(x_m) = h_1$, $\dot{z}_A(x_f) = 0$



(a) Swing foot during DSP



(b) Swing foot during SSP



(c) Stable foot during DSP

Swing leg's ankle joint trajectory

Ankle Trajectory during SSP

By substituting the boundary conditions, we get

$$n_0 + n_1 t_2 + n_2 t_2^2 + n_3 t_2^3 = x_e; \quad (3)$$

$$n_0 + n_1 t_4 + n_2 t_4^2 + n_3 t_4^3 = x_f; \quad (4)$$

$$n_1 + 2n_2 t_2 + 3n_3 t_2^2 = x_v; \quad (5)$$

$$n_1 + 2n_2 t_4 + 3n_3 t_4^2 = 0. \quad (6)$$

The matrix representation for these equations (3-6) is

$$M_{4\times1} = A_{4\times4}.N_{4\times1}$$

Then, the coefficients of the polynomial can be calculated by

$$N_{4\times1} = A_{4\times4}^{-1}.M_{4\times1}$$

Stable leg's trajectories:

Sole trajectory

$$x_S = \frac{x_f}{2} + l_3 \cos\theta_6(t), \quad z_S = l_3 \sin\theta_6(t)$$

Toe trajectory

$$x_T = \frac{x_f}{2} + l_3 \cos\theta_6(t) + (l_3 + l_4)\cos(\theta_6(t) + \theta_7(t))$$

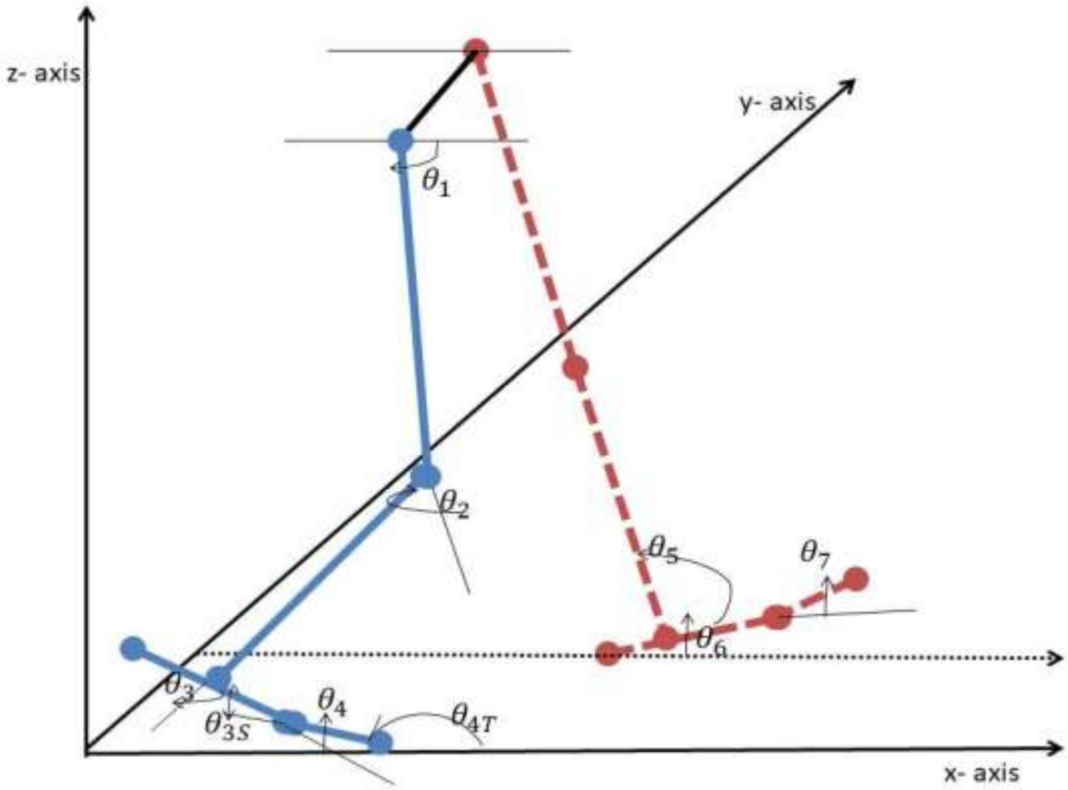
$$z_T = l_3 \sin\theta_6(t) + (l_3 + l_4)\sin(\theta_6(t) + \theta_7(t))$$

Hip Trajectory

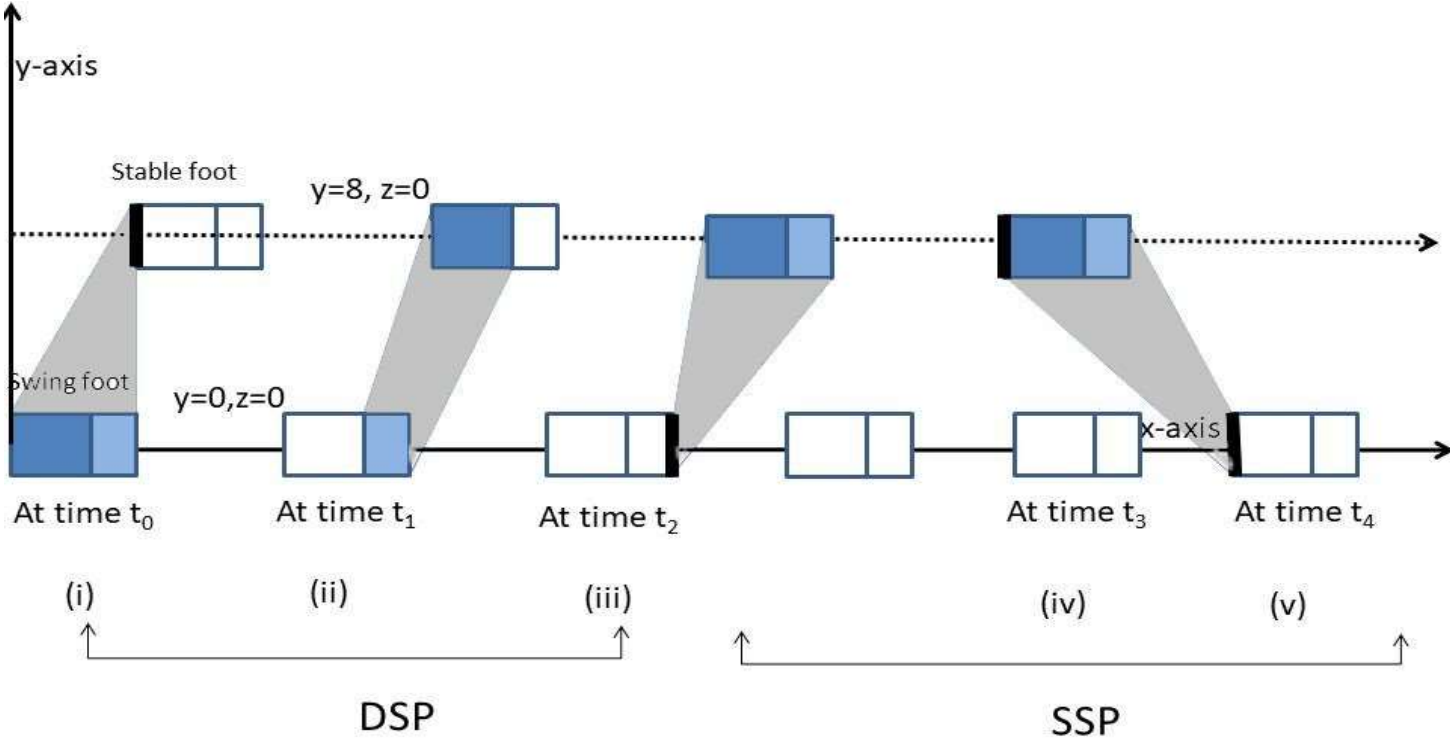
$x_H(t_0) = \frac{x_f}{4}$, $x_H(t_f) = \frac{3x_f}{4}$, $\dot{x}_H(t_0) = v_{xhs}$, $\dot{x}_H(t_f) = v_{xhs}$,

$$x_H(t) = \frac{x_f}{4} + \frac{3(x_f)}{2t_f^2}t^2 - \frac{(x_f)}{t_f^3}t^3$$

$$z_H(t) = \sqrt{(l_1 + l_2)^2 - (x_H(t) - (x_f)/2)^2}$$


z- axis
y- axis
x- axis
θ_1
θ_2
θ_3
θ_{3S}
θ_4
θ_{4T}
θ_5
θ_6
θ_7

Supported region for SSP and DSP during one cycle(step)



ZMP Stability Analysis and Trunk Motion

ZMP

$$x_{ZMP} = \frac{\sum_{i=1}^{n} m_i(x_i(\ddot{z}_i + g) - \ddot{x}_i z_i)}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)}$$

$$y_{ZMP} = \frac{\sum_{i=1}^{n} m_i(y_i(\ddot{z}_i + g) - \ddot{y}_i z_i)}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)}$$

Upper body motion: Case-1

$$y_U(t) = \begin{cases} y_0 + y_v t + \frac{-2y_v t_2 + 3(y_a + y_m/2 - y_0)}{t_2^2} t^2 + \\ \quad \frac{-2(y_a + y_m/2 - y_0) + y_v t_2}{t_2^3} t^3 & t_0 \leq t \leq t_2 \\ y_a & t_2 \leq t \leq t_4 \end{cases}$$

ZMP

$$x_{ZMP} = \frac{\sum_{i=1}^{n} m_i(x_i(\ddot{z}_i + g) - \ddot{x}_i z_i)}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)}$$

$$y_{ZMP} = \frac{\sum_{i=1}^{n} m_i(y_i(\ddot{z}_i + g) - \ddot{y}_i z_i)}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)}$$

Upper body motion: Case-1

U starts to move from swing leg's hip to stable leg's hip during DSP (t_0, t_2) and stays there all the time during SSP (t_2, t_4).

$$y_U(t) = \begin{cases} y_0 + y_v t + \frac{-2y_v t_2 + 3(y_a + y_m/2 - y_0)}{t_2^2} t^2 + \\ \frac{-2(y_a + y_m/2 - y_0) + y_v t_2}{t_2^3} t^3 & t_0 \le t \le t_2 \\ y_a & t_2 \le t \le t_4 \end{cases}$$

Case-2

Upper body start to move from middle to the side of the stable leg's hip during time t_0 to t_1.

Stay there during time t_1 to t_3.

Again start moving towards middle of legs between t_3 to t_f time where $t_1 = t_f/4$ and $t_3 = 3t_f/4$.

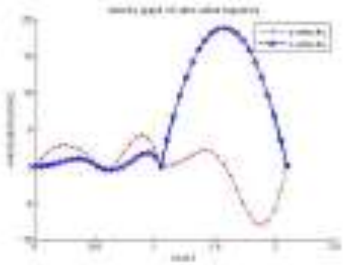
$$y_U(t) = \begin{cases} \frac{y_l}{2} + \frac{3(y_l+y_a)}{(4t_1^2)}t^2 - \frac{y_l+y_a}{2(t_1^3)}t^3 & t_0 \le t \le t_1 \\ y_a & t_1 \le t \le t_3 \\ (\frac{(16(t_3-t_4)^3-(9(y_l+y_a)t_4^2t_3-(y_l+y_a)t_4^3)}{4(t_3-t_4)^3} + (\frac{3(y_l+y_a)}{2(t_3-t_4)^3}t_4t_3)t + \frac{3(y_l+y_a)(t_4+t_3)}{4(t_3-t_4)^3}t^2 - \frac{(y_l+y_a)}{2(t_3-t_4)^3}t^3 & t_3 \le t \le t_4 \end{cases}$$

Case-3

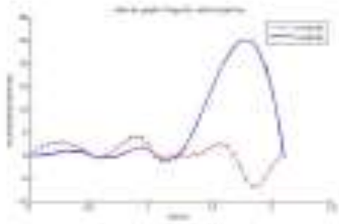
Start to move from middle position to stable leg's hip in half time (t_0, t_2), then return back to initial condition in rest of the time (t_2, t_f).

$$y_U(t) = \begin{cases} \frac{y_l}{2} + \frac{3(y_l+y_a)}{(4t_2^2)}t^2 - \frac{y_l+y_a}{2(t_2^3)}t^3 & t_0 \le t \le t_2 \\ (\frac{(16(t_2-t_4)^3-(9(y_l+y_a)t_4^2t_2-(y_l+y_a)t_4^3)}{4(t_2-t_4)^3} + (\frac{3(y_l+y_a)}{2(t_2-t_4)^3}t_4t_2)t + \frac{3(y_l+y_a)(t_4+t_2)}{4(t_2-t_4)^3}t^2 - \frac{(y_l+y_a)}{2(t_2-t_4)^3}t^3 & t_2 \le t \le t_4 \end{cases}$$

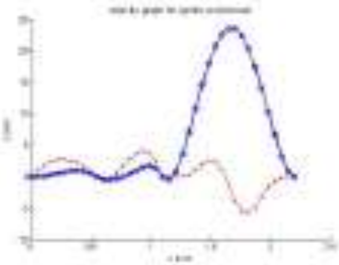
Results



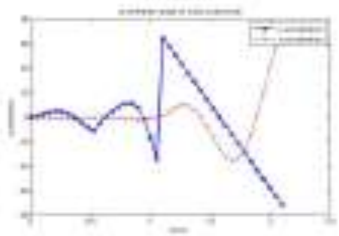
(e) for Cubic



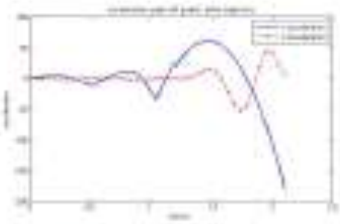
(f) for Quartic



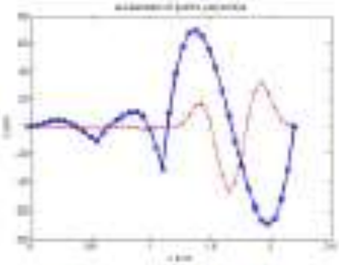
(g) for Quintic



(h) for Cubic



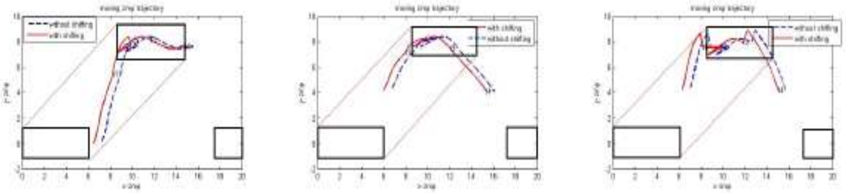
(i) for Quartic



(j) for Quintic

Figure: Ankle velocity and acceleration trajectory graphs

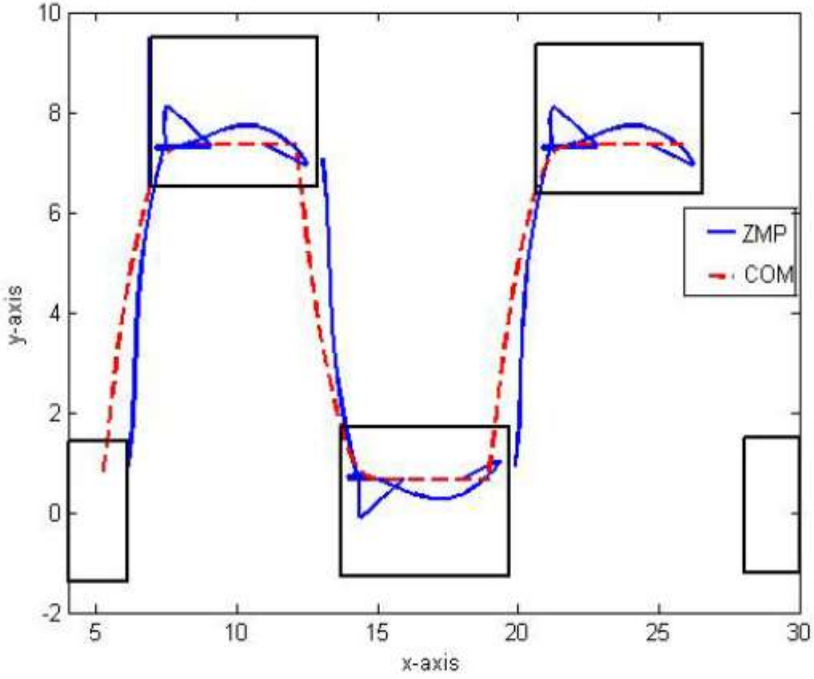
Effect of upper body motion on ZMP stablity for active foot



(a) case 1 (b) case 2 (c) case 3

Figure: ZMP graph for 3 different cases of Upper body motion

Stable ZMP trajectory for case-1 for $t_f = 2.1 sec$.



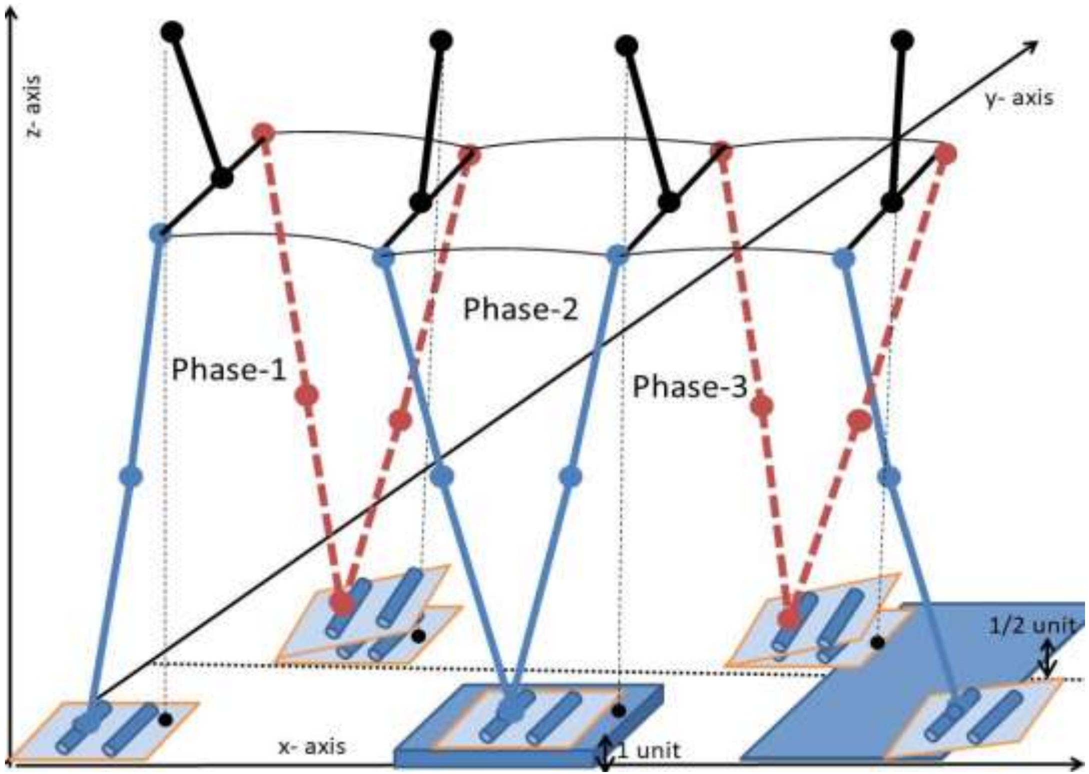
Results

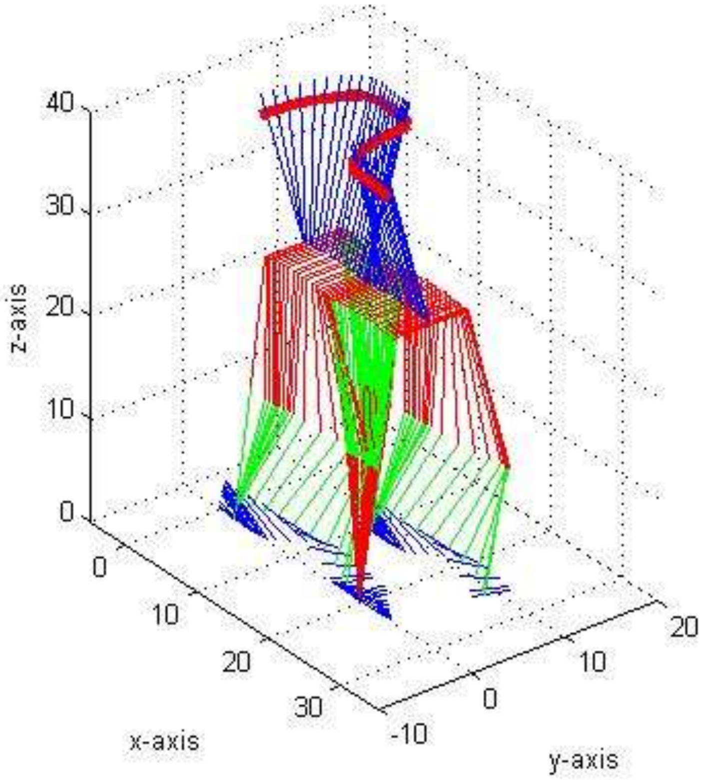
Upper body	Ankle trajectory	Step Time	Initial hip velocity	Conclusion
No upper body	cubic,quartic ,quantic	4.5	v_h=2.95	x-ZMP is in the region for $t > 4.5s$ but y-ZMP at middle of hip
Fixed	cubic,quartic ,quantic	3	v_h=2.55	x-ZMP is in the region for $t > 3s$ but y-ZMP at middle of hip
Moving				

Table: Simulations based observation

Effect of Upper body, ankle trajectory and other Parameters

Upper body	Ankle trajectory	Step Time	Initial hip velocity	Conclusion
case-2	cubic,quartic ,quantic	3	v_h=2 $y_a = 8$	for last 4 time instant ZMP is out of region
case-3	cubic,quartic ,quantic	3	v_h=2	unstable
case-1	cubic	2.5	v_h=2.8	stable
	cubic	2.2	v_h=3.15	unstable
	cubic	2.1	v_h=3.55	unstable
	quartic	2.5	v_h=2.8	stable
	quartic	2.2	v_h=3.15	stable
	quartic	2.1	v_h=3.55	unstable
	quintic	2.5	v_h=2.8	stable
	quintic	2.2	v_h=3.15	stable
	quintic	**2.1**	v_h=3.55	**stable**
	quintic	2	v_h=3.65	stable
	quintic	1.8	v_h=3.65	unstable


z- axis
y- axis
Phase-1
Phase-2
Phase-3
1/2 unit
x- axis
1 unit



40
30
20
10
0
z-axis
0
10
20
30
x-axis
-10
0
10
20
y-axis

Thanks!