IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Kinematic Model for Robotic Manipulator

N.SUKAVANAM

DEPARMENT OF MATHEMATICS

Kinematic Model for Robot Manipulator

Steps to derive kinematics model:

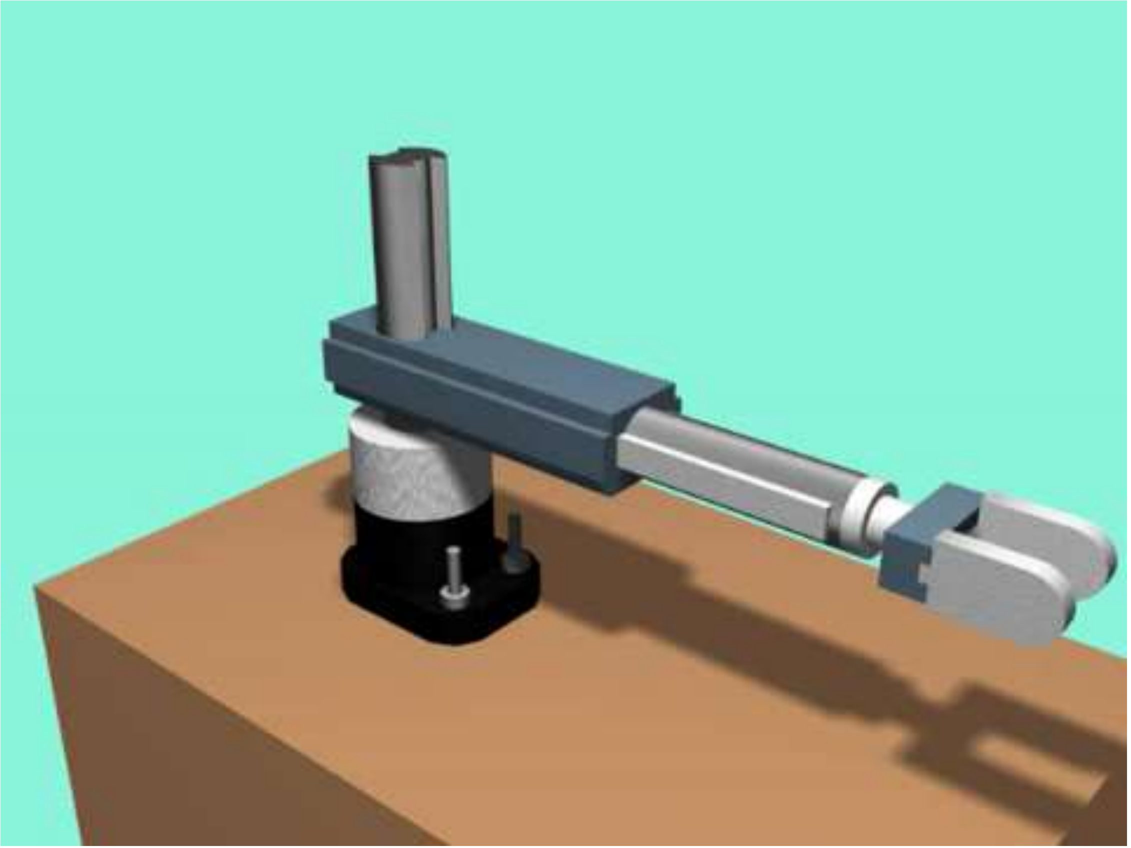
- Assign Denavit-Hartenberg coordinates frames
- Find link parameters
- Find Transformation matrices of adjacent joints
- Find Kinematics Matrix (Arm Matrix)

Denavit-Hartenberg Procedure

- Assign numbers 1 to n for joints starting from the base to the end-effector.
- **Establish the base coordinate system**. Establish a right-handed orthonormal coordinate system (X_0, Y_0, Z_0) at the supporting base with Z_0 axis lying along the axis of motion of joint 1.
- **Establish joint axis**. Align the Z_i with the axis of motion (rotary or sliding) of joint i+1.
- **Establish the origin of the ith coordinate system**. Locate the origin of the i^{th} coordinate system at the intersection of the Z_i & Z_{i-1} or at the intersection of Z_i axis with the common normal between Z_{i-1} & Z_i axes.
- **Establish X_i axis**. Establish $X_i = \pm(Z_{i-1} \times Z_i)/\|Z_{i-1} \times Z_i\|$ or along the common normal between the Z_{i-1} & Z_i axes when they are parallel or non-intersecting.
- **Establish Y_i axis.** Assign $Y_i = +(Z_i \times X_i)/\|Z_i \times X_i\|$ to complete the right-handed coordinate system.
- Find the link and joint parameters

Denavit-Hartenberg Procedure

- Locate point b_i at the intersection of the x_i and z_{i-1} axes. If they do not intersect, use the intersection of x_i with a common normal between x_i and z_{i-1}.
- Define θ_i as the angle of rotation from x_{i-1} to x_i measured about z_{i-1}.
- Define d_i as the distance from the origin o_{i-1} of i-1th frame to point b_i measured along z_{i-1}.
- Define a_i as the distance from point b_i to the origin o_i of ith frame measured along x_i.
- Define α_i as the angle of rotation from z_{i-1} to z_i measured about x_i.
- (θ_i ,d_i,a_i,α_i) *are parameters where* θ_i *is called* joint angle, ,d_i is joint distance,a_i *is link length* ,α_i *is link twist angle*



Transformation between the frames i-1 and i

Four successive elementary transformations are required to relate the i-th coordinate frame to the (i-1)-th coordinate frame:

- Rotate about the Z_{i-1} axis an angle of θ_i to align the X_{i-1} axis with the X_i axis. Here we perform Rot(Z_{i-1}, θ_i)
- Translate along the Z_{i-1} axis a distance of d_i, to bring X_{i-1} and X_i axes into coincidence. We have performed Trans(0, 0, d_i)
- Translate along the X_i axis a distance of a_i to bring the two origins O_{i-1} and O_i as well as the X axis into coincidence. We have performed Trans(a_i, 0, 0)
- Rotate about the X_i axis an angle of α_i (in the right-handed sense), to bring the two coordinates into coincidence. We have performed Rot(X_i, α_i)

D-H transformation matrix for adjacent coordinate frames, *i* and *i-1*.

•The position and orientation of the *i*-th frame coordinate can be expressed in the *(i-1)* th frame by the following homogeneous transformation matrix:

$$T_{i-1}^{i} = T(z_{i-1}, d_i)R(z_{i-1}, \theta_i)T(x_i, a_i)R(x_i, \alpha_i)$$

$$\begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thanks!