



**IIT ROORKEE**



**NPTEL ONLINE  
CERTIFICATION COURSE**

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# Model Based Control of Robot Manipulators



# Outline

## 1. Introduction

- Kinematics
- Dynamics

## 2. Model-Based Control of Robotic Systems

- Kinematic Model Based Scheme
- Dynamic Model Based Scheme

## 3. Conclusion



# Introduction

Kinematics:



# Introduction

Dynamics:



## Kinematic Model Based Control

- Closed Loop Inverse Kinematics (CLIK)

$$e = x_d - x = x_d - k(q)$$

$$\dot{e} = \dot{x}_d - \dot{x}$$

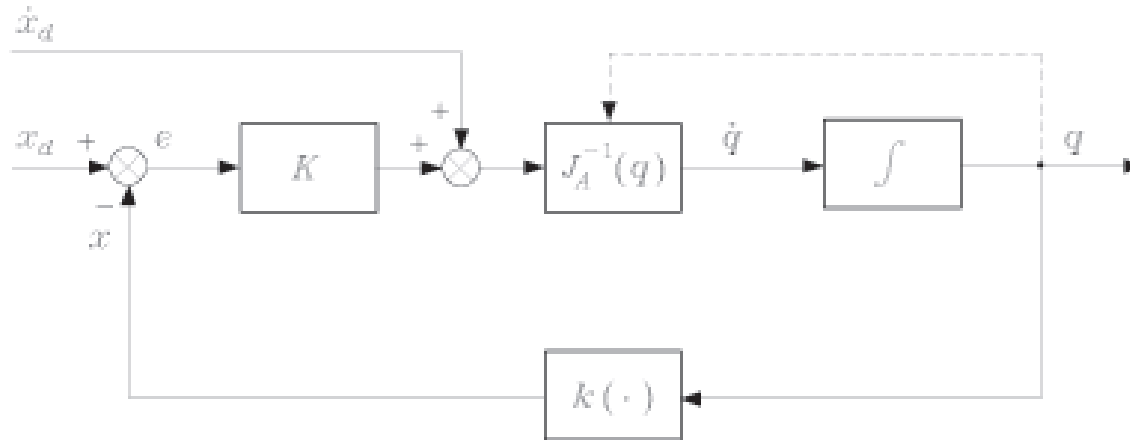
$$\dot{e} = \dot{x}_d - J_A(q)\dot{q}.$$

$$\dot{q} = J_A^{-1}(q)(\dot{x}_d + Ke)$$

$$\dot{e} + Ke = 0.$$

## Kinematic Model Based Control (cont'd)

- Closed Loop Inverse Kinematics (CLIK)



## Dynamic Model Based Control

The rigid-body dynamics have the form

$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta) \quad (1)$$

Assuming that our model of friction is a friction is a function of joint positions and velocities, we add the term  $F(\Theta, \dot{\Theta})$  to eq. (1), to yield the model

$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta) + F(\Theta, \dot{\Theta}) \quad (2)$$



## Dynamic Model Based Control (cont'd)

The problem of controlling a complicated system like (2) can be handled by the partitioned controller scheme. In this case, we have

$$\tau = \alpha \tau' + \beta (3)$$

where  $\tau$  is the  $n \times 1$  vector of joint torques. We choose

$$\alpha = M(\Theta),$$

$$\beta = V(\Theta, \dot{\Theta}) + G(\Theta) + F(\Theta, \dot{\Theta}) \quad (4)$$

## Dynamic Model Based Control (cont'd)

with the servo law

$$\tau' = \ddot{\Theta}_d + K_v \dot{E} + K_p E \quad (5)$$

where

$$E = \Theta_d - \Theta \quad (6)$$

Using (2) through (5), it is quite easy to show that the closed-loop system is characterized by the error equation

$$\ddot{E} + K_v \dot{E} + K_p E = 0 \quad (7)$$

## Dynamic Model Based Control (cont'd)

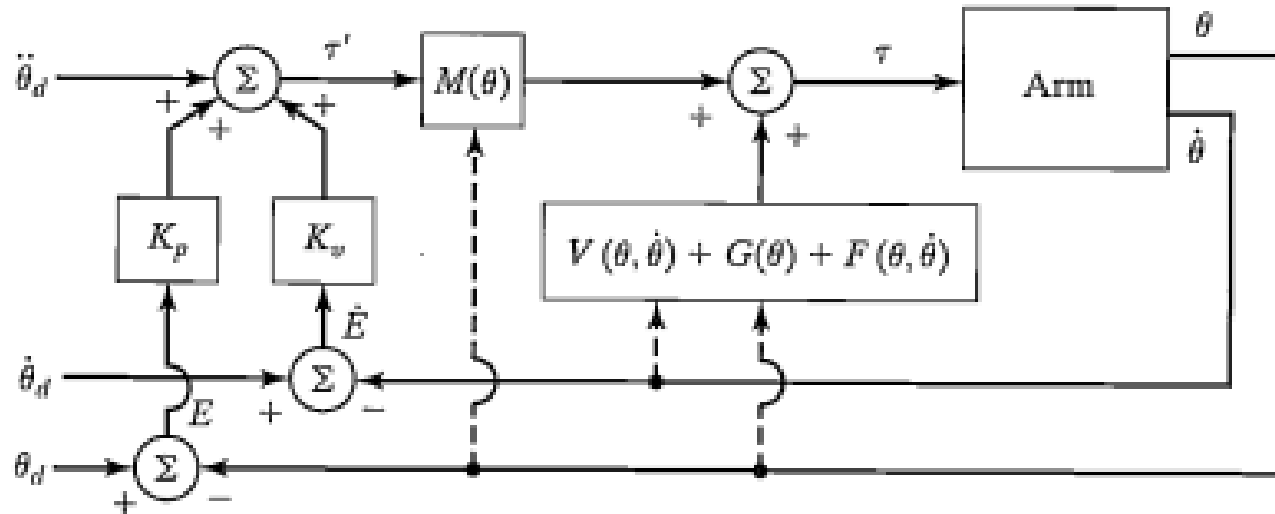
Note that this vector equation is decoupled: The matrices  $K_v$  and  $K_p$  are diagonal, so that (7) could just as well be written on a joint-by-joint basis as

$$\ddot{e}_i + k_{vi}\dot{e} + k_{pi}e = 0 \quad (8)$$



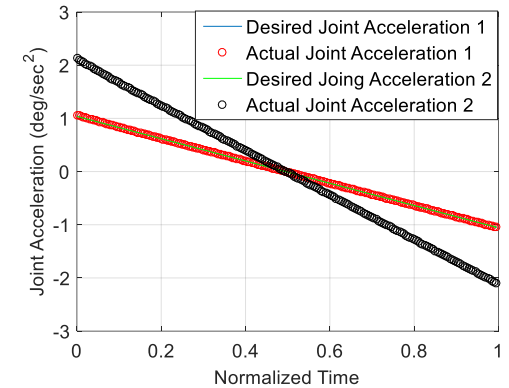
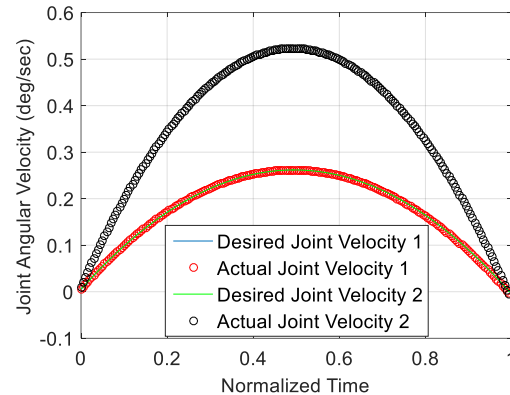
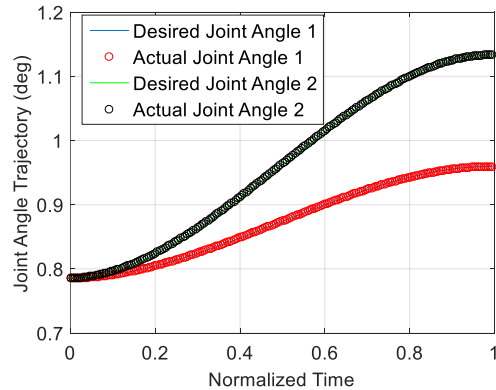
## Dynamic Model Based Control (cont'd)

Block Diagram:



## Dynamic Model Based Control (cont'd)

Results for 2R planar manipulator:



## Dynamic Model Based Control (cont'd)

2R planar manipulator Dynamics:

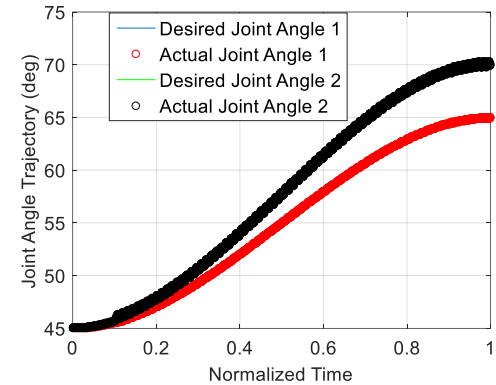
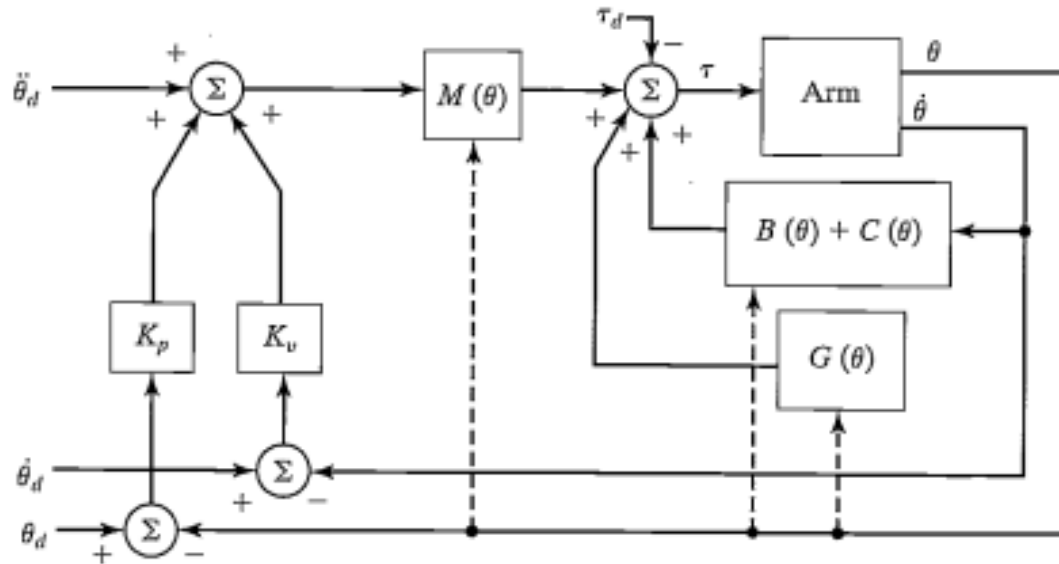


## Dynamic Model Based Control (cont'd)

2R planar manipulator Dynamics:



## With External Disturbance





## Cartesian Model-Based Control Scheme

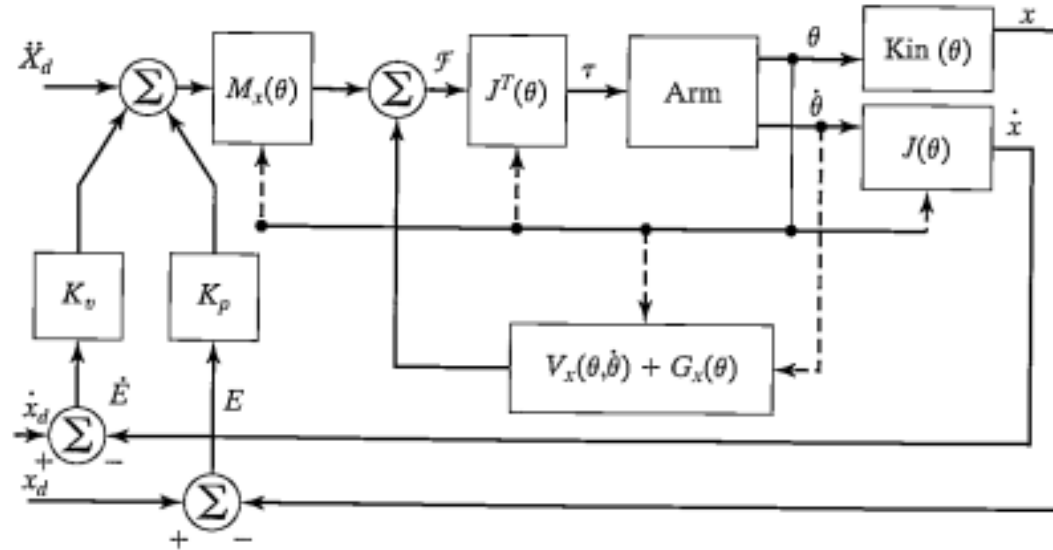
The rigid-body dynamics can be written as

$$\mathcal{F} = M_x(\Theta) \ddot{\chi} + V_x(\Theta, \dot{\Theta}) + G_x(\Theta)$$

$$\tau = J^T(\Theta) \mathcal{F}$$

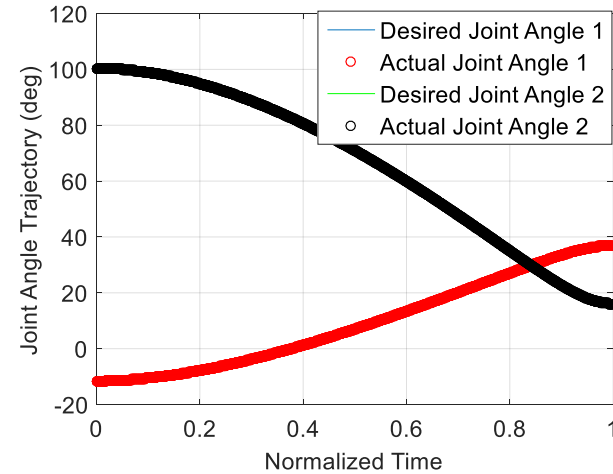
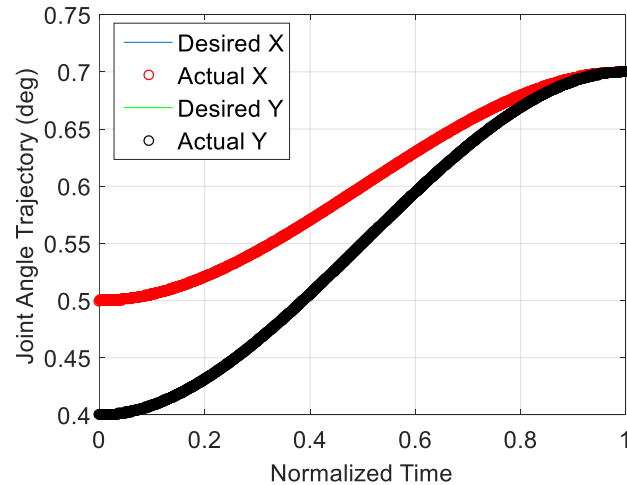
## Cartesian Model-Based Control Scheme (cont'd)

Block Diagram:



## Cartesian Model-Based Control Scheme (cont'd)

Results:



## Conclusions

- **Kinematic Model Based Control**
- **Dynamic Model Based Control**



# Thank You!

