IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Redundancy Resolution of the Human Fingers in Cooperative Object Translation -I

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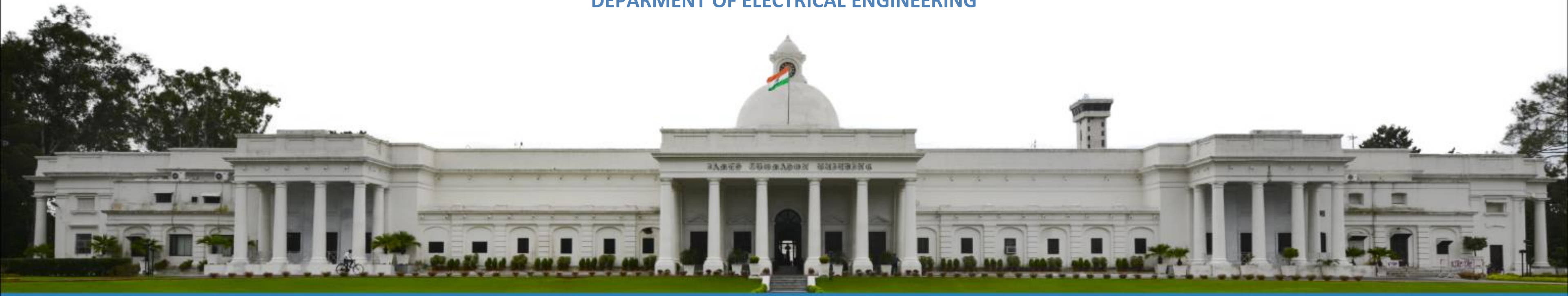
DEPARMENT OF ELECTRICAL ENGINEERING



Outline

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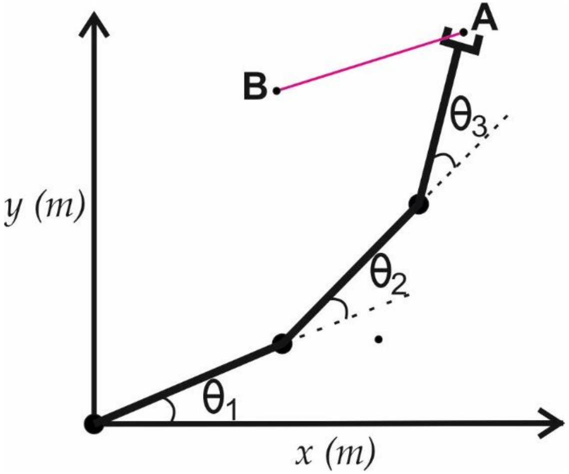
    - What is redundancy?
    - Human Arm
    - Advantages & Disadvantages
2. **Decomposition of Tasks**
    - Fundamental Equations
    - Secondary Subtask → Desired Trajectory
    - Secondary Subtask → Objective Criterion
3. **Applications**
    - Obstacle Avoidance
    - Singularity Avoidance

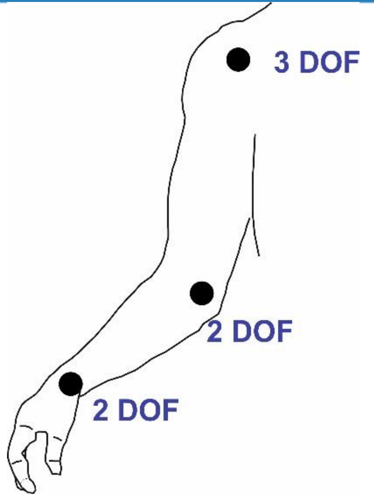
Introduction

❖ A manipulator is redundant, if it has more DOF than are necessary to perform a given primary task.



Introduction (cont'd)

- ❖ Human Arm is redundant → 7 DOF
- ❖ Elbow elevation/down → during object grasping.
- ❖ Highly versatile and broad applicability.



Introduction (cont'd)

Advantages:

- To avoid obstacles
- To avoid singular configurations
- To perform low energy consuming motions
- To perform certain tasks even after the failure of few joints of the robot

Disadvantages:

- More Joints and Actuators → Bulkier in size & Heavier in weight.
- More complex control strategy is required → High increase in necessary computations.

Decomposition of Tasks

Task → Primary Subtask + Secondary Subtask

- Each Secondary Subtask is performed using the remaining DOF after performing the subtasks with higher priority.

- Examples:

Decomposed into **Hand Position control** + **Hand Orientation Control**

Fundamental Equations

Considering n-DOF robotic manipulator with i^{th} input angle as θ_i and configuration given by vector:

$$\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_n]^T$$

Assuming,

first subtask described by m_1-dimensional vector x_1(manipulation vector):

$$\boldsymbol{x_1} = \boldsymbol{f_1}(\boldsymbol{\theta})$$

desired trajectory for x_1 given by: $\boldsymbol{x_{1d}}(\boldsymbol{t})\left(0 \leq t \leq t_f; t_f \; is \; final \; time\right)$

Fundamental Equations(cont'd)

Two cases considered for secondary subtask:

Case 1:

To follow towards given desired configuration denoted by $\boldsymbol{x}_{2d}(t); \left(0 \leq t \leq t_f\right)$, and second subtask represented by $\boldsymbol{x}_2 = \boldsymbol{f}_2(\boldsymbol{\theta})$

Case 2:

Maximizing given performance criterion function:

$$r = \boldsymbol{C}(\boldsymbol{\theta})$$

Fundamental Equations(cont'd)

General solution for differential kinematics $\dot{\boldsymbol{x}}_{\mathbf{1}} = \boldsymbol{J}_{\mathbf{1}}\dot{\boldsymbol{\theta}}$is given by

$$\dot{\boldsymbol{\theta}} = \boldsymbol{J}_{\mathbf{1}}^{+}\dot{\boldsymbol{x}}_{\mathbf{1}\boldsymbol{d}} + (\boldsymbol{I} - \boldsymbol{J}_{\mathbf{1}}^{+}\boldsymbol{J}_{\mathbf{1}})\boldsymbol{O}_{\mathbf{1}} \quad (1)$$

where $\boldsymbol{O}_{\mathbf{1}} \rightarrow n$ dimensional arbitrary constant vector

First term $\boldsymbol{J}_{\mathbf{1}}^{+}\dot{\boldsymbol{x}}_{\mathbf{1}\boldsymbol{d}} \rightarrow$ joint velocity to achieve the first subtask i.e. to track the given trajectory $\boldsymbol{x}_{\mathbf{1}\boldsymbol{d}}(\boldsymbol{t})$

Second term utilized to perform second subtask i.e. it represents redundancy of system

Finding pseudo inverse

The Moore-Penrose pseudo inverse $\rightarrow$ right inverse or left inverse
$\rightarrow$depends on the number of columns and rows of $\boldsymbol{J}$

- $\boldsymbol{J}^{+}=\boldsymbol{J}^{T}(\boldsymbol{J}\boldsymbol{J}^{T})^{-1}$ $\rightarrow$right inverse
  $\rightarrow$ when the number of columns $>$ number of rows

$\rightarrow$here, $\boldsymbol{J}\boldsymbol{J}^{+}=\boldsymbol{I}$

- $\boldsymbol{J}^{+}=(\boldsymbol{J}^{T}\boldsymbol{J})^{-1}\boldsymbol{J}^{T}$ $\rightarrow$left inverse
  $\rightarrow$ when the number of rows $>$ number of columns

$\rightarrow$ here, $\boldsymbol{J}^{+}\boldsymbol{J}=\boldsymbol{I}$

Properties of pseudo inverse

For every finite *m*n* real matrix $\boldsymbol{A}$,there is a unique *n*m* real matrix $\boldsymbol{A}^{+}$satisfying the following properties:

- $(\boldsymbol{A}^{+})^{+} = \boldsymbol{A}$
- $(\boldsymbol{A}^{\boldsymbol{T}})^{+}= (\boldsymbol{A}^{+})^{\mathbf{T}}$, $(\boldsymbol{A}\boldsymbol{A}^{\boldsymbol{T}})^{+}= (\boldsymbol{A}^{+})^{\mathbf{T}}\boldsymbol{A}^{+}$
- $\boldsymbol{A}^{+} = \boldsymbol{A}^{\boldsymbol{T}}(\boldsymbol{A}\boldsymbol{A}^{\boldsymbol{T}})^{-1}$
- $\boldsymbol{A}\boldsymbol{A}^{+}\boldsymbol{A} = \boldsymbol{A}$
- $\boldsymbol{A}^{+}\boldsymbol{A}\boldsymbol{A}^{+} = \boldsymbol{A}^{+}$
- $(\boldsymbol{A}\boldsymbol{A}^{+})^{\mathbf{T}}= \boldsymbol{A}\boldsymbol{A}^{+}$
- $(\boldsymbol{A}^{+}\boldsymbol{A})^{\mathbf{T}}= \boldsymbol{A}^{+}\boldsymbol{A}$

Proof

Proof: $\hat{J}_2^+ = (I - J_1^+ J_1)$

Having, $\hat{J}_2 = J_2(I - J_1^+ J_1)$ and $J_2 = I$

Thus, $\hat{J}_2 = (I - J_1^+ J_1)$

$$\hat{J}_2^+ = (I - J_1^+ J_1)^+$$
$$= (I - J_1^+ J_1)^T [(I - J_1^+ J_1)(I - J_1^+ J_1)^T]^{-1}$$
$$\because A^+ = A^T (AA^T)^{-1}$$

Now, $\quad (I - J_1^+ J_1)^T = \left(I^T - (J_1^+ J_1)^T\right) = (I - J_1^+ J_1)$

$$\because (A^+ A)^T = (A^+ A)$$

Substituting this result in the above equation

$$\hat{J}_2^+ = (I - J_1^+ J_1)[(I - J_1^+ J_1)(I - J_1^+ J_1)]^{-1}$$

Now, $\quad (I - J_1^+ J_1)(I - J_1^+ J_1) = I - J_1^+ J_1 - J_1^+ J_1 + J_1^+ J_1 J_1^+ J_1$

$= I - 2J_1^+ J_1 + J_1^+ J_1 \because A^+ A A^+ = A^+$

$$= I - J_1^+ J_1$$

Proof (cont'd)

Hence, $(\boldsymbol{I} - \boldsymbol{J}_1^{+}\boldsymbol{J}_1)^{+} = (\boldsymbol{I} - \boldsymbol{J}_1^{+}\boldsymbol{J}_1)(\boldsymbol{I} - \boldsymbol{J}_1^{+}\boldsymbol{J}_1)^{-1}$

Letting$(\boldsymbol{I} - \boldsymbol{J}_1^{+}\boldsymbol{J}_1) = \boldsymbol{B}$

Thus the equation reduces to $\boldsymbol{B}^{+} = \boldsymbol{B}\boldsymbol{B}^{-1}$

Post-multiplying both sides by $\boldsymbol{B}\boldsymbol{B}^{+}$

We get: $\boldsymbol{B}^{+}\boldsymbol{B}\boldsymbol{B}^{+} = \boldsymbol{B}\boldsymbol{B}^{-1}\boldsymbol{B}\boldsymbol{B}^{+} = \boldsymbol{B}\boldsymbol{B}^{-1}\boldsymbol{B}\boldsymbol{B}\boldsymbol{B}^{-1}$

Coupling the matrix and their inverse pairs, it reduces to

$$\boldsymbol{B}^{+}\boldsymbol{B}\boldsymbol{B}^{+} = \boldsymbol{B}$$

But we know as a property

$$\boldsymbol{B}^{+}\boldsymbol{B}\boldsymbol{B}^{+} = \boldsymbol{B}^{+}$$

Hence, $\boldsymbol{B}^{+} = \boldsymbol{B}$

i.e. $\hat{\boldsymbol{J}}_2^{+} = (\boldsymbol{I} - \boldsymbol{J}_1^{+}\boldsymbol{J}_1)^{+} = (\boldsymbol{I} - \boldsymbol{J}_1^{+}\boldsymbol{J}_1)$

<u>Case 1</u>: Second Subtask provided by Desired Trajectory

Manipulation vector → $\boldsymbol{x_2} = \boldsymbol{f_2}(\boldsymbol{\theta})$ and desired trajectory $\boldsymbol{x_{2d}}(\boldsymbol{t})$

We select vector $\boldsymbol{O_1}$ to realize desired trajectory $\boldsymbol{x_{2d}}(\boldsymbol{t})$

Time differentiation of $\boldsymbol{x_2}$:

$$\dot{\boldsymbol{x}}_2 = \boldsymbol{J_2}\dot{\boldsymbol{\theta}} \tag{2}$$

Substituting $\dot{\boldsymbol{x}}_2 = \dot{\boldsymbol{x}}_{2d}$ and (1) in (2),

$$\dot{\boldsymbol{x}}_{2d} - \boldsymbol{J_2 J_1^+ \dot{x}_{1d}} = \boldsymbol{J_2}(\boldsymbol{I} - \boldsymbol{J_1^+ J_1})\boldsymbol{O_1} \tag{3}$$

Second Subtask provided by Desired Trajectory(cont'd)

Considering $\hat{\boldsymbol{J}}_2 = \boldsymbol{J}_2(\boldsymbol{I} - \boldsymbol{J}_1^{+}\boldsymbol{J}_1)$ and general solution of system of linear equations $\boldsymbol{Ax} + \boldsymbol{b}$ i.e., $\boldsymbol{x} = \boldsymbol{A}^{+}\boldsymbol{b} + (\boldsymbol{I} - \boldsymbol{A}^{+}\boldsymbol{A})\boldsymbol{O}$

We get,

$$\boldsymbol{O}_1 = \hat{\boldsymbol{J}}_2^{+}(\dot{\boldsymbol{x}}_{2d} - \boldsymbol{J}_2\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_{1d}) + \left(\boldsymbol{I} - \hat{\boldsymbol{J}}_2^{+}\hat{\boldsymbol{J}}_2\right)\boldsymbol{O}_2 \quad (4)$$

where $\boldsymbol{O}_2 \rightarrow n$ dimensional arbitrary constant vector

From [Yoshikawa [1] and [2]], we get, $(\boldsymbol{I} - \boldsymbol{J}_1^{+}\boldsymbol{J}_1)\hat{\boldsymbol{J}}_2^{+} = \hat{\boldsymbol{J}}_2^{+}$

Therefore,

$$\dot{\boldsymbol{\theta}}_d = \boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_{1d} + \hat{\boldsymbol{J}}_2^{+}(\dot{\boldsymbol{x}}_{2d} - \boldsymbol{J}_2\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_{1d}) + (\boldsymbol{I} - \boldsymbol{J}_1^{+}\boldsymbol{J}_1 - \hat{\boldsymbol{J}}_2^{+}\hat{\boldsymbol{J}}_2)\boldsymbol{O}_2 \quad (5)$$

Second Subtask provided by Desired Trajectory(cont'd)

If $\boldsymbol{x_2} = \boldsymbol{\theta}$, $\boldsymbol{J_2} = \boldsymbol{I}$ and second subtask presents the desired trajectory of the whole arm configuration:

$$\hat{J}_2^{+} = (\boldsymbol{I} - \boldsymbol{J_1^{+}J_1})^{+} = (\boldsymbol{I} - \boldsymbol{J_1^{+}J_1})$$

Hence,

$$\dot{\boldsymbol{\theta}}_{\boldsymbol{d}} = \boldsymbol{J_1^{+}}\dot{\boldsymbol{x}}_{\boldsymbol{1d}} + (\boldsymbol{I} - \boldsymbol{J_1^{+}J_1})\dot{\boldsymbol{x}}_{\boldsymbol{2d}} \qquad (6)$$

In general to bring $\boldsymbol{x_2} \rightarrow \boldsymbol{x_{2d}}$,

$$\dot{\boldsymbol{x}}_{\boldsymbol{2d}} = G(\boldsymbol{x_{2d}} - \boldsymbol{x_2})$$

where G → Diagonal Gain Matrix

Case 2: Second Subtask provided by Objective Criterion Function

Choose the value of $\boldsymbol{O_1}$ (1) → To maximize the criterion function as large as possible

One method by Yoshikawa [1] is detailed below:

$$\boldsymbol{O_1} = \boldsymbol{\eta} O_p$$

where $\boldsymbol{\eta} = [\eta_1, \eta_2, \ldots, \eta_n]^T$ and $\eta_1 = \dfrac{\partial C(\theta)}{\partial \theta_1}$

and $O_p \rightarrow$ appropriate positive constant

Desired joint velocity:

$$\dot{\boldsymbol{\theta}}_{\boldsymbol{d}} = \boldsymbol{J_1^+} \dot{\boldsymbol{x}}_{\boldsymbol{1d}} + (\boldsymbol{I} - \boldsymbol{J_1^+ J_1}) \boldsymbol{\eta} O_p$$

where $(\boldsymbol{I} - \boldsymbol{J_1^+ J_1}) \boldsymbol{\eta} O_p$ corresponds to orthogonal projection of $\boldsymbol{O_1}$ on $\boldsymbol{J_1}$

Applications

1. Obstacle Avoidance

Aim: Make the end effector follow desired trajectory **avoidingcollisions with obstacle.**

Given:

Link lengths: $l_1 = l_2 = 1$ and $l_3 = 0.3$

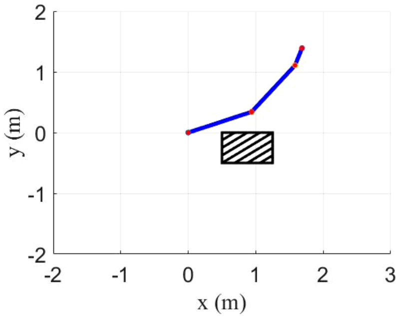
$\theta_0 = [20°, 30°, 20°]^T$

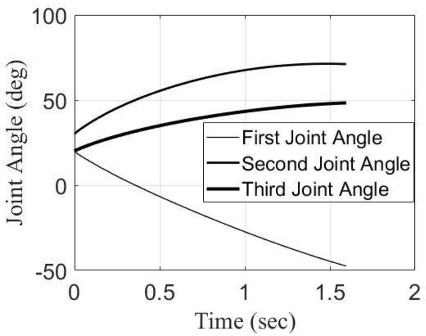
which corresponds to

$r_0 = [x_0, y_0]^T \cong [1.69, 1.39]^T$

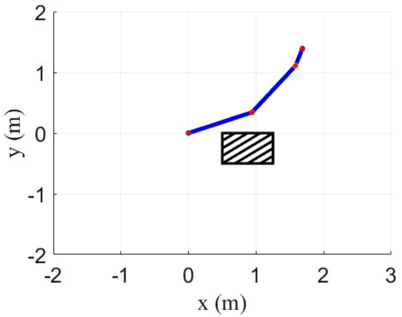
Applications

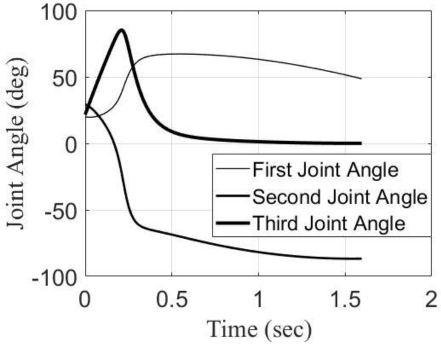
Obstacle Collision





Obstacle Avoidance





Applications

2. Singularity Avoidance

Aim: Make the end effector to avoid **singular configuration**

Given:

Link lengths: $l_1 = l_2 = 1$ and $l_3 = 0.3$

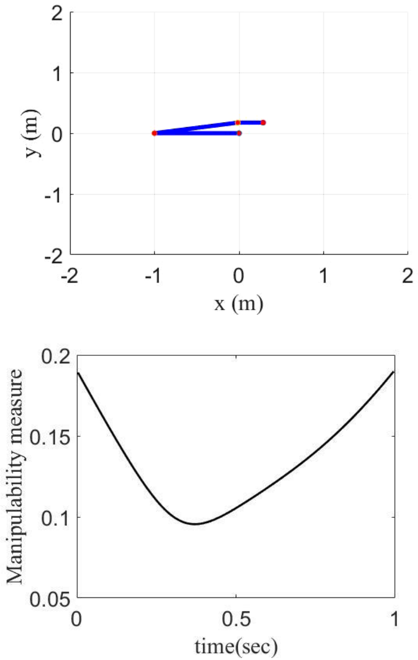
$\theta_0 = [180°, 190°, -10°]^T$

which corresponds to

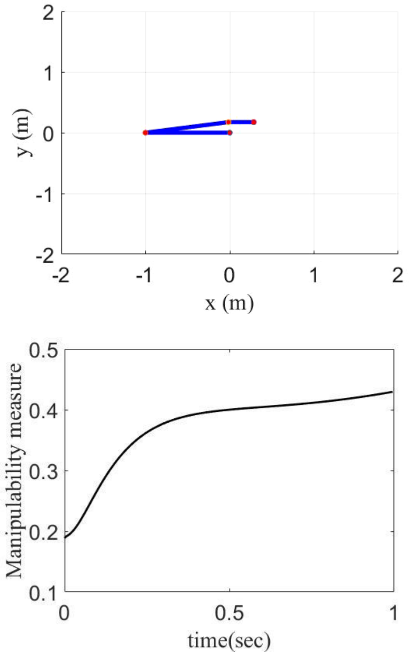
$r_0 = [x_0, y_0]^T \cong [0.28, 0.17]^T$

Applications

Singular Configuration



Singularity Avoidance



Applications

3. Mechanical Joint Limits Avoidance

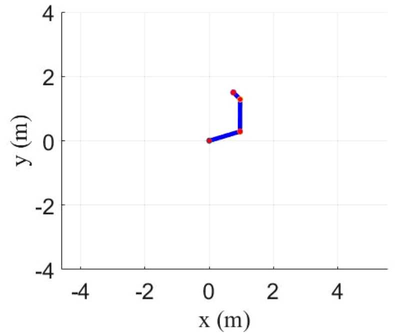
$$M(\theta) = -\frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\theta_i - \bar{\theta}_i}{\theta_{iMax} - \theta_{iMin}}\right)^2$$

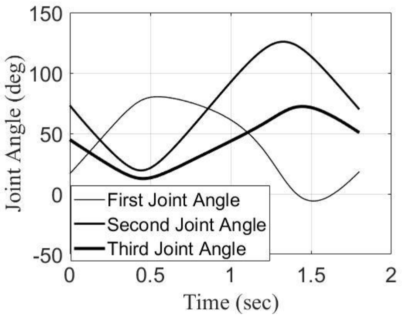
$$0° \leq \theta_1 \leq 180°$$
$$0° \leq \theta_2 \leq 150°$$
$$0° \leq \theta_3 \leq 180°$$

Applications

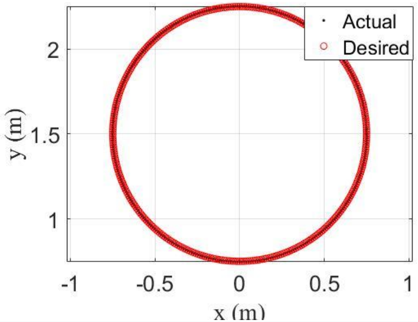
Distance from Mechanical Joint Limits

Joint Limit Violation

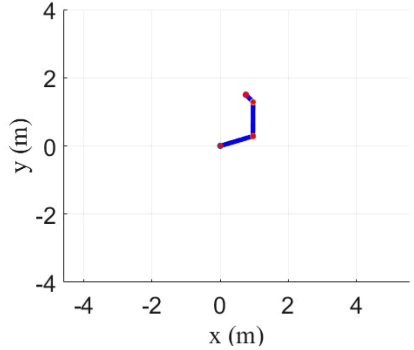


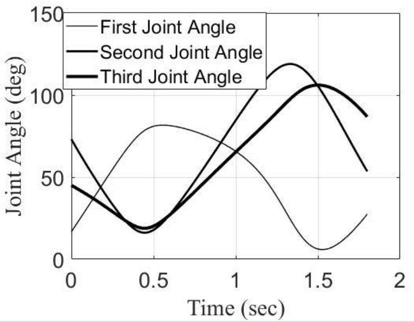


$$0° \leq \theta_1 \leq 180°$$
$$0° \leq \theta_2 \leq 150°$$
$$0° \leq \theta_3 \leq 180°$$



Within Safe Joint Range





References

1. T. Yoshikawa, *Foundations of Robotics: Analysis and Control*, The MIT Press, Cambridge, U.S.A., 1990.

2. A. A. Maciejewski and C. A. Klein, “Obstacle Avoidance for Kinematically Redundant Manipulators,” *International Journal of Robotics Research 4*, no. 3(1985):109-117

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4. Bruno Siciliano, Oussama Khatib, *Handbook of robotics*(Springer)

Thank You!