IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Manipulator Control

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Manipulator Control

Consider the dynamic equation of a n-link manipulator:

$$M(q,\dot{q})\ddot{q} + C(q,\dot{q}) + G(q) = \tau \ldots\ldots\ldots(1)$$

where

$$q(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ . \\ . \\ q_n(t) \end{bmatrix} \quad , \quad \tau(t) = \begin{bmatrix} \tau_1(t) \\ \tau_2(t) \\ . \\ . \\ \tau_n(t) \end{bmatrix}$$

$M_{n\times n}$ is the Inertia Matrix.

$C_{n\times 1}$ is the centrifugal and Coriolis force vector.

$G_{n\times 1}$ is the Gravity term.

$\tau_{n\times 1}$ is the Force/Torque vector.

Manipulator Control

Let $q_d(t)$: $t \geq 0$ be the desired joint trajectory.

If $q(t)$ is the actual joint variable at time t then:

$$e(t) = q(t) - q_d(t)$$

is the error at time t.

To track the desired joint trajectory $q_d(t)$, it is necessary to find the control vector $\tau(t)$ (Force/Torque) such that the error:

$$e(t) \rightarrow 0 \; as \; t \rightarrow \infty$$

The equation (1) is a system of n second order ODE. It can be converted into $2n$ first order ODE.

Let

$$\dot{q} = v$$

Then

$$\dot{v} = M^{-1}(q, v)[\tau - C(q, v) - G(q)]$$

Manipulator Control

Now we can write these equations in terms of e.

$$e = q - q_d,$$
$$w = \dot{e} = \dot{q} - \dot{q}_d = v - \dot{q}_d$$

So,

$$\dot{e} = w$$
$$\dot{w} = \ddot{q} - \ddot{q}_d$$
$$= M^{-1}(e + q_d, w + \dot{q}_d)[\tau - C(e + q_d, w + \dot{q}_d) - G(e + q_d)]$$

$$= \bar{M}^{-1}(e, w)[\tau - \bar{C}(e, w) - \bar{G}(e)]$$

Choose the control $\tau(t)$ to be:

$$\tau(t) = \bar{C}(e, w) + \bar{G}(e) + \bar{M}(e, w)[-Ke - Lw]$$

Manipulator Control

Then the equation becomes:

$$\dot{e} = w$$
$$\dot{w} = -Ke - Lw$$

For this system, the equilibrium point is $(0,0)$ and it is asymptotically stable.

Proof:

Let

$$V(e, w) = Ke^2 + w^2$$

Then

$$\begin{aligned} \dot{V} &= 2Ke\dot{e} + 2w\dot{w} \\ &= 2Kew + 2w(-Ke - Lw) \\ &= -2Lw^2 \end{aligned}$$

Manipulator Control

$$\Rightarrow \dot{V}\ is\ -ve\ definite$$

$$as\ w = 0 \Rightarrow \dot{w} = 0 \Rightarrow e = 0$$

$$So\ \dot{V} = 0\ only\ at\ (0,0)$$

$$and\ \dot{V} < 0\ for\ all\ (e, w)$$

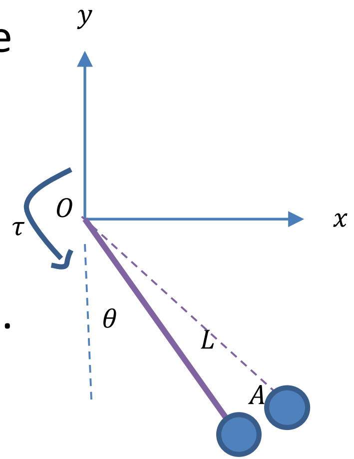
Hence Proved

Manipulator Control Example

- Consider the controlled pendulum OA.
- Let $OA = L \; and \; \tau$ denotes the torque applied.

θ is variable denoted angle with vertical axis.

M denotes the mass of pendulum.



Manipulator Control Example

The dynamic equation is:

$$\frac{1}{3}ML^2\ddot{\theta} + \frac{Mg}{2}L\sin\theta = \tau$$

If we want to find a control τ such that:

$$\theta(0) = \frac{\pi}{6} \quad and \; \theta(5) = \frac{\pi}{3}$$

$$\dot{\theta}(0) = 0 \quad and \; \dot{\theta}(5) = 0$$

Then we can find the desired trajectory as:

$$\theta_d(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

a_0, a_1, a_2, a_3 can be obtained using the four conditions.

If the actual initial position is:

$$\theta(0) = 0 \; and \; \dot{\theta}(0) = 0$$

Manipulator Control Example

$$e = \theta - \theta_d$$
$$w = \dot{e} = \dot{\theta} - \dot{\theta}_d$$

Thus

$$\dot{e} = w$$
$$\dot{w} = \frac{3}{ML^2}\left[\tau - \frac{Mg}{2} L \sin(e + \theta_d)\right]$$

Thus

$$\tau(t) = \frac{M}{2} gL \sin(e + \theta_d) - \frac{ML^2}{3}[-Ke - Lw]$$
$$\Rightarrow \dot{e} = w$$
$$\dot{w} = -Ke - Lw$$

Manipulator Control Example

$$\Rightarrow \ddot{w} = -Kw - L\dot{w}$$
$$\Rightarrow \ddot{w} + L\dot{w} + Kw = 0$$

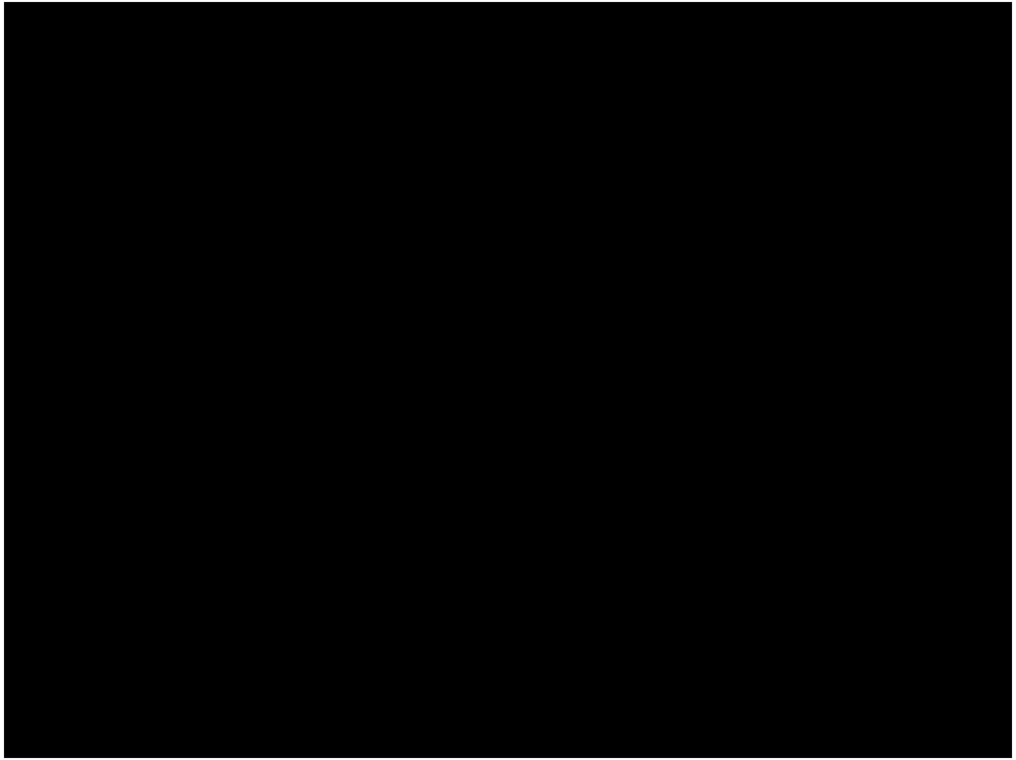
$$w(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$
$$m_1 = -\frac{L}{2} \pm \frac{\sqrt{L^2 - 4K}}{2}$$

If $L^2 - 4K < 0$ then the real parts of $m_1\ and\ m_2$ are $-\frac{L}{2}$

So $e^{m_1 t}\ and\ e^{m_2 t} \to 0\ as\ t \to \infty$

If L is large then $w(t)$ is close to 0 for small value of t.

Manipulator Control Example 2



Manipulator Control Example 2

	θ	d	α	a
1	θ_1	OA	$-\frac{\pi}{2}$	0
2	θ_2	0	0	AB
3	θ_3	0	$\frac{\pi}{2}$	0
4	θ_4	BC	0	0

$$OA = 10$$
$$BC = 6$$
$$AB = 8$$
$$Tool\ Length = 1\ unit$$
$$One\ Rotation = \frac{1}{4}\ units$$

Manipulator Control Example 2

$$^{0}T_4 = \begin{bmatrix} (c_1c_2c_3 - c_1s_2s_3)c_4 - s_1s_4 & -(c_1c_2c_3 - c_1s_2s_3)s_4 - s_1c_4 & c_1c_2s_3 + c_1s_2c_3 & a_2c_1c_2 + d_4(c_1c_2s_3 + c_1s_2c_3) \\ (s_1c_2c_3 - s_1s_2s_3)c_4 + c_1s_4 & -(s_1c_2c_3 - s_1s_2s_3)s_4 + c_1c_4 & s_1c_2s_3 + s_1s_2c_3 & a_2s_1c_2 + d_4(s_1c_2s_3 + s_1s_2c_3) \\ (-s_2c_3 - c_2s_3)c_4 & -(-s_2c_3 - c_2s_3)s_4 & -s_2s_3 + c_2c_3 & d_4(-s_2s_3 + c_2c_3) - a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_1 \\ r_{21} & r_{22} & r_{23} & p_2 \\ r_{31} & r_{32} & r_{33} & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We get

$$c_1(c_2c_3 - s_2s_3)c_4 - s_1s_4 = r_{11}$$
$$s_1(c_2c_3 - s_2s_3)c_4 + c_1s_4 = r_{21}$$
$$\Rightarrow s_4 = c_1r_{21} - s_1r_{11}$$

$$\tan\theta_4 = \frac{r_{32}}{r_{31}}$$

$$(-c_2s_3 - s_2c_3)c_4 = r_{31}$$
$$\tan\theta_1 = \frac{r_{23}}{r_{13}}$$

Manipulator Control Example 2

$$Initial\ position\ of\ end\ effector(t_0) = \{0,14,10\}$$
$$Position\ of\ end\ effector\ at\ t = 1(t_1) = \{4,4,1\}$$

$$t \in [0,1]$$
$$^0T_4\ at\ t = 0$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 14 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_4\ at\ t = 1$$
$$\begin{bmatrix} -1 & 0 & 0 & 4 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Manipulator Control Example 2

$$In\ interval\ t \in [1,2], screw\ makes\ one\ full\ rotation$$

$$End\ effector\ position\ at\ time\ t\ is\ given\ by$$

$$^{0}T_4(t)\begin{bmatrix} \cos\theta(t) & \sin\theta\,(t) & 0 & 4 \\ \sin\theta(t) & -\cos\theta(t) & 1 & 4 \\ 0 & -1 & 0 & z(t) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\theta(1) = \pi \quad z(1) = 1$$

$$\theta(2) = 3\pi \quad z(2) = \frac{3}{4}$$

$$\theta(t) = \pi + (t-1)2\pi$$

Manipulator Control Example 2

$$ {}^{0}T_4(t) = a_0 + a_1 t \quad t \in [0,1] $$
$$ a_0 = {}^{0}T_4(0) $$
$$ a_1 = {}^{0}T_4(1) - {}^{0}T_4(0) $$

$$ t \in [1,2] $$
$$ \theta(1) = \pi \;\; z(1) = 1 $$
$$ \theta(2) = 3\pi \;\; z(2) = \frac{3}{4} $$
$$ z(t) = b_0 + b_1 t $$
$$ 1 = b_0 + b_1 $$
$$ \frac{3}{4} = b_0 + 2b_1 $$

Manipulator Control Example 2

$$\Rightarrow b_1 = -\frac{1}{4} \quad b_0 = \frac{5}{4}$$

$$z(t) = \frac{5}{4} - \frac{1}{4}t$$

$$\theta(t) = \pi(2t - 1)$$

$$\tan\theta_1 = \frac{r_{23}}{r_{13}} \Rightarrow \theta_1 = \tan^{-1}\frac{r_{23}}{r_{13}}$$

$$\tan\theta_4 = \frac{r_{32}}{r_{31}} \Rightarrow \theta_4 = \tan^{-1}\frac{r_{32}}{r_{31}}$$

$Now\ by\ using\ \theta_4\ and\ \theta_1, we\ can\ find\ \theta_2\ and\ \theta_3$

Thanks!