ECE 771: Project Report
Adaptive Noise Cancellation on ECG signals using the Least-Mean-Square Algorithm

Thursday December 16, 2010

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Contents

Chapter 1

Introduction

1.1 Background

One of the easiest and inexpensive methods of monitoring the heart for Cardiovascular Disease (CVD) is the use of an Electrocardiogram (ECG). ECG machines provide a means of quantifying the electrical activity of the heart, which is represented graphically for specialists to analyze. The waveform of an ECG recording is able to provide vital information regarding an individuals heart condition by observing specific features in combination with the known relationship between cardiac contraction and relaxation and electrical activity [4]. The ECG signal provides a great method of monitoring the heart in real-time as well as providing medical specialists with quick and reliable information regarding a patient's condition.

The heart is a four chambered pump that sends deoxygenated blood to the lungs, while simultaneously distributing oxygenated blood to the rest of the body. As illustrated in Figure 1.1, the heart is divided so that two chambers are implicated in the deoxygenation process and the remainder are involved in the oxygenation along with all the corresponding major arteries and veins.

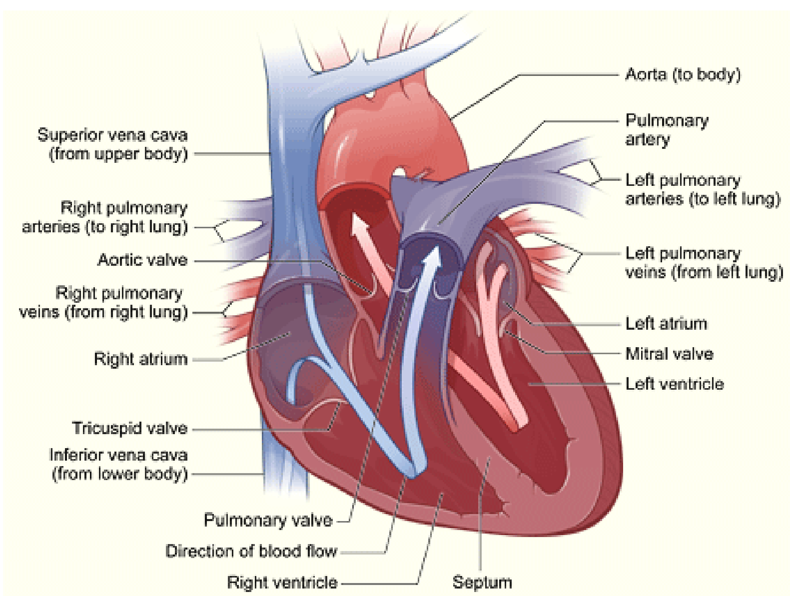

Figure 1.1: The Anatomy of the Heart [5]

The processes involved in pumping blood can be quantified through the ECG signal which measures electrical potentials in respect to time [6]. Figure 1.2 shows the typical ECG waveform which consists of 5 main features: the P, Q, R, S and T waves [6]. Since the timing, shape and amplitude of these waves characterize certain heart diseases, the ECG is essential to classification of CVD.

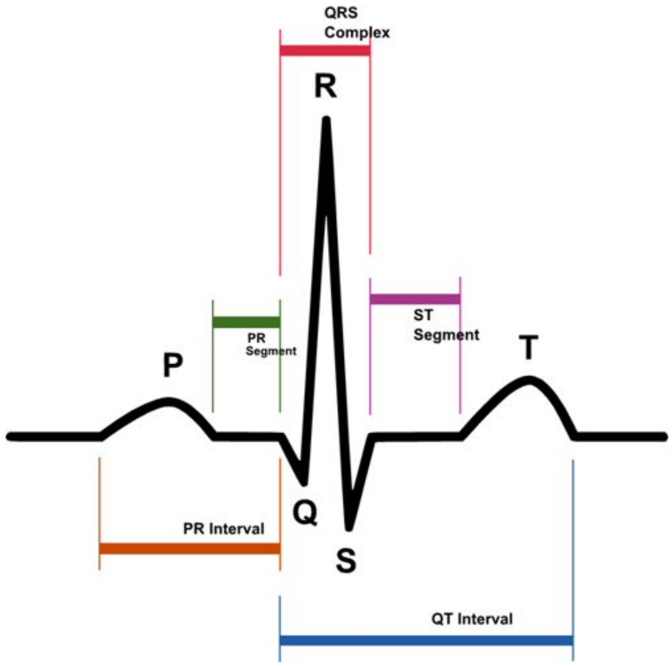

Figure 1.2: Key features of an ECG signal [7]

As well, most classification systems require signals with a high signal-to-noise ratio (SNR). ECG measurements are subject to various noises such as main interference (60 Hz) and gaussian background noise. Since conventional notch filtering is ineffective for mains interference with frequency jitter, a more adaptive system is necessary. An optimal solution to remove noises corrupting ECG signals is adaptive noise cancelation (ANC). The ANC method uses a finite-impulse-response (FIR) filter that has tap-weights that evolve to provide the optimal solution using the least-mean-square (LMS) algorithm [2]. This type of filtering has proven to be effective and has been employed in a wide range of applications.

1.2 Objective

The objective of this project was to apply the topic of adaptive noise cancelation by the implementation of an adaptive filter, to cancel the noises associated with any ECG signal. The ECG signal that was used was synthetically generated using the simulator developed by [1]. Experiments were conducted where gaussian noise and 60 Hz mains noise was added to the clean signal to simulate real-life ECG measurements. The sinusoidal interference was extended by frequency modulating it, which kept the mains noise frequency between 59.5 Hz and 60.5 Hz. Using the noisy signals, the objective is to reproduce the original clean signal with minimal error.

Chapter 2

Methodology

2.1 Adaptive Filter derivation

In order create an adaptive filter, the Wiener filter must be discussed. In this context, the Wiener filter is a linear optimum discrete time filter. The FIR filter output $y(n)$ at the time index ,n, is defined by the linear convolution sum of M delayed terms of the input $u(n)$ as shown in (2.1)[2].

$$y(n) = \sum_{k=0}^{M-1} w_k u(n-k), \quad n = 0, 1, 2, \ldots \tag{2.1}$$

If the error, $e(n)$, is defined as the difference between the desired output, $d(n)$, and the FIR filter output,

$$e(n) = d(n) - y(n) \tag{2.2}$$

then in order to optimize the filter, we must minimize the mean-square value,J, of the estimation error $e(n)$ [2].

$$J(n) = E\left[|e(n)|^2\right] \tag{2.3}$$

If we let $e_o(n)$ denote the value of the estimation error when the filter is optimum, then using the principle of orthogonality, the mean-square-error (MSE), J, is minimized when $e_o(n)$ is orthogonal to each input sample that enters the estimation of the desired response at time n [2].

$$E[u(n-k)e_o(n)] = 0, \quad k = 0, 1, \ldots, M-1 \tag{2.4}$$

Let w_{oi} represent the ith coefficient of the impulse response of the optimum FIR filter [2]. Using the principle of orthogonality, we can substitute (2.1) into (2.2) and then into (2.4) to get

$$E\left[u(n-k)\left(d(n) - \sum_{i=0}^{M-1} w_o i u(n-i)\right)\right] = 0, \quad k = 0, 1, \ldots, M-1 \tag{2.5}$$

which results in

$$\sum_{i=0}^{M-1} w_{oi} E[u(n-k)u(n-i)] = E[u(n-k)d(n)], \quad k = 0, 1, \ldots, M-1 \tag{2.6}$$

If we let,

$$\mathbf{u}(n) = [u(n), u(n-1), \ldots, u(n-(M-1))]^T \tag{2.7}$$

and

$$\mathbf{R} = E\left[\mathbf{u}(n)\mathbf{u}^T(n)\right] \tag{2.8}$$
$$\mathbf{p} = E\left[\mathbf{u}(n)d(n)\right] \tag{2.9}$$

then (2.6)can be represented as

$$\mathbf{R}\mathbf{w_o} = \mathbf{p} \tag{2.10}$$

where $\mathbf{w_o} = \left[w_{o0}, w_{o1}, \ldots, w_{o(M-1)}\right]^T$ represents the optimal tap-weight vector, $\mathbf{R}$ represents the correlation matrix of the tap-input vector $\mathbf{u}(n)$, and $\mathbf{p}$ represents the cross-correlation vector between the tap-input vector $\mathbf{u}(n)$ and the desired response $d(n)$ [2].

Rewriting (2.1) and (2.2), we have

$$y(n) = \mathbf{w}^T(n)\mathbf{u}(n) \tag{2.11}$$
$$e(n) = d(n) - \mathbf{w}^T(n)\mathbf{u}(n) \tag{2.12}$$

and therefore the cost function, J from (2.3), can be rewritten as

$$\begin{aligned} J(n) &= E\left[(d(n) - \mathbf{w}^T(n)\mathbf{u}(n))((d(n) - \mathbf{w}^T(n)\mathbf{u}(n))^T\right] \\ &= E\left[(d(n) - \mathbf{w}^T(n)\mathbf{u}(n))((d(n) - \mathbf{u}^T(n)\mathbf{w}(n))\right] \\ &= E\left[|(d(n)|^2\right] - \mathbf{w}^T(n)E\left[\mathbf{u}(n)d(n)\right] - E\left[d(n)\mathbf{u}^T(n)\right]\mathbf{w}(n) + \mathbf{w}^T(n)E\left[\mathbf{u}(n)\mathbf{u}^T(n)\right]\mathbf{w}(n) \\ &= \sigma_d^2 - \mathbf{w}^T(n)\mathbf{p} - \mathbf{p}^T\mathbf{w}(n) + \mathbf{w}^T(n)\mathbf{R}\mathbf{w}(n) \end{aligned} \tag{2.13}$$

where σ_d^2 is the variance of the desired response $d(n)$ [2].

By taking the gradient of (2.13) with respect to $\mathbf{w}$ we get,

$$\nabla J(n) = -2\mathbf{p} + 2\mathbf{R}\mathbf{w}(n) \tag{2.14}$$

and according to the method of steepest decent, the tap-weight vector at $n+1$ is computed using (2.15) [2].

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{1}{2}\mu\left[-\nabla J(n)\right] \tag{2.15}$$

Therefore, the tap-weight adaptation algorithm is summarized with (2.16), by substituting (2.14) into (2.15) [2].

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\left[\mathbf{p} - \mathbf{R}\mathbf{w}(n)\right], \qquad n = 0, 1, 2, \ldots \tag{2.16}$$

2.2 Application of Adaptive Noise Cancelation

As derived in §2.1, the Weiner filter is made optimal when the last term in (2.16) is zero, hence (2.10). Therefore, the last term in (2.16) represents a correction term to the current tap-weight. Since many signals vary in time, it is essential to have a filter that *adapts* to the input vectors. That is exactly the case for the adaptive filter in §2.1, where the tap-weights are recursively updated to minimize the error function $e(n)$, which clearly demonstrates the LMS algorithm.

As mentioned before, the problem associated with ECG signals are mains interference (which is 60 Hz in Canada). Although a simple solution would be to use a notch filter to attenuate the interference, it does not suffice as the interference frequency is not constant. With the mains interference fluctuating about the 60 Hz mark, an adaptive filter must be introduced whose cut off frequency follows that of the interference frequency. The algorithm from §2.1 is a system that can be implemented recursively to eliminate the mains interference. In addition to the mains interference, background gaussian noise can also be filtered using this system. Since ECG signals contain both of these noise sources, ANC can therefore be used to eliminate them.

The ANC is implemented using (2.11), (2.12), and (2.16), which can be represented in scalar form as,

$$y(n) = \sum_{i=0}^{M-1} w_i u(n-i) \tag{2.17}$$

$$e(n) = d(n) - y(n) \tag{2.18}$$

$$w_i(n+1) = w_i(n) + \mu u(n-i)e(n), \quad i = 0, 1, 2, \ldots, M-1 \tag{2.19}$$

where $w_i(n)$ represents the ith tap-weight at time n and μ is a constant step-size parameter [2]. The general process of ANC is summarized in Figure 2.1

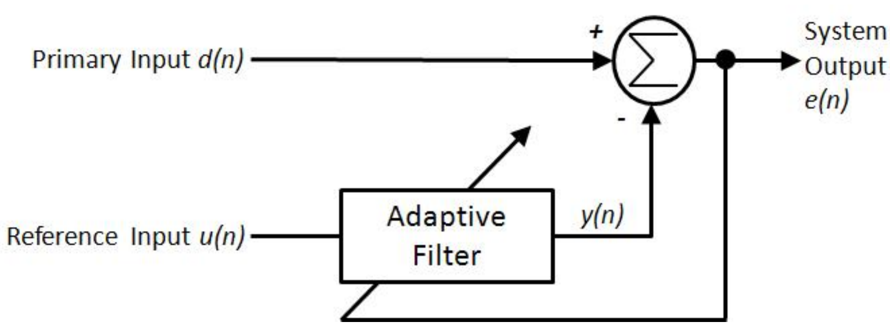

Figure 2.1: A block diagram summarizing the process of ANC [2]

Since the ECG data is real valued, we can represent the primary and reference inputs, respectively, as

$$d(n) = s(n) + A_o \cos(2\pi f_0 n + \phi_0) + \nu(n) \tag{2.20}$$

$$u(n) = A\cos(2\pi f_0 n + \phi) + \nu(n) \tag{2.21}$$

where $s(n)$, represents the ECG signal, A_0 is the amplitude of the sinusoidal interference, f_0 is the interference frequency, ϕ_0 is the phase and $\nu(n)$ is the background noise [2]. In addition, the amplitude A and the phase ϕ in (2.21) are different from those in (2.20), but f_0 and $\nu(n)$ are the same. By referring to Figure 2.1, the output of the system $e(n)$ is the result of the cancelation of the sinusoidal interference term and the noise term which are correlated with the reference input. Since the signal $s(n)$ is uncorrelated with the interference and noise, the ANC is therefore an efficient solution to solving interference and noise problems in ECG signals. Since the interference and noise terms in the primary input and reference input can be measured simultaneously using dual channel measurements (hence they are correlated), the cancelation can be done successfully.

One important factor of designing the filter is stability. Assuming the total number of tap-weights in the filter is large, the filter can be shown to be stable when it's poles are located inside the unit circle. This therefore provides the following condition for adaptive filter stability [2]:

$$\begin{aligned} \frac{\mu M A^2}{4} &\ll 1 \\ \therefore \mu &\ll \frac{4}{M A^2} \end{aligned} \tag{2.22}$$

The expression in (2.22) represents the condition that needs to be met in order for the filter to be stable. It is therefore required that the value of the step-size parameter is small, hence, a slow adaptation rate [2].

To test the effectiveness of the adaptive filter, the reference input is constructed using a sinusoidal interference and gaussian noise. The primary input is constructed by adding the reference input to the clean synthetic ECG signal. The adaptive filter is then tested through different experiments, to justify the effectiveness of the adaptive noise canceler. For more information on the MATLAB code used for experiments, please refer to Appendix A.

Chapter 3

Results

In this chapter, the results of the ANC we be presented. As said before, in order to test the ANC, a number of different experiments must be observed.

3.1 Experiment 1

Figures 3.1 to 3.4 depict the first experiment for when there is mild gaussian noise (SD = 0.09 V), normal amplitude sinusoidal interference (A = 0.1 V) and f_0 = 60 Hz. The MSE of the filter was $\sigma^2_{MSE} = 0.00087944\ V^2$ and $\sigma_{MSE} = 0.0297$ V.
Figure 3.1 plots of the primary input $d(n)$ versus the output of the ANC $e(n)$ for *Experiment 1*. Figure

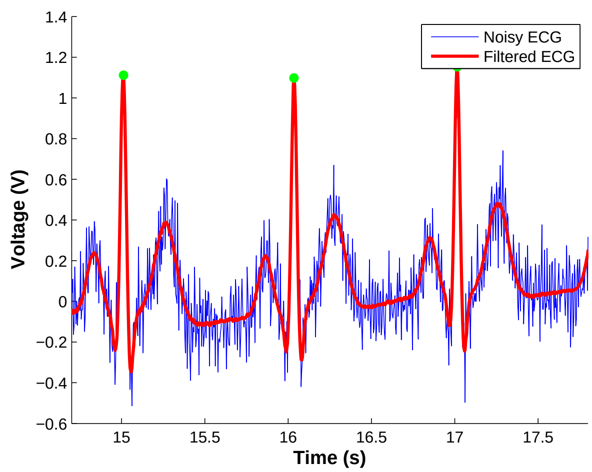

Figure 3.1: Noisy ECG vs Adaptive Filter Output for *Experiment 1*

3.2 is a plot of the original clean synthetic ECG and the filtered ECG corresponding to Figure 3.1 for *Experiment 1*. Figure 3.3 plots the power spectral density (PSD) of the noisy ECG and the filtered ECG for *Experiment 1*. Figure 3.4 represents the PSD of the error obtained by subtracting the filtered signal by the clean signal for *Experiment 1*.

3.2 Experiment 2

Figures 3.5 to 3.8 represents the second experiment for when there is intermediate gaussian noise (SD = 0.3), normal amplitude sinusoidal interference (A = 0.1) and f_0 = 60 Hz. The MSE of the filter was $\sigma^2_{MSE} = 0.0028\ V^2$ and $\sigma_{MSE} = 0.0532$ V.

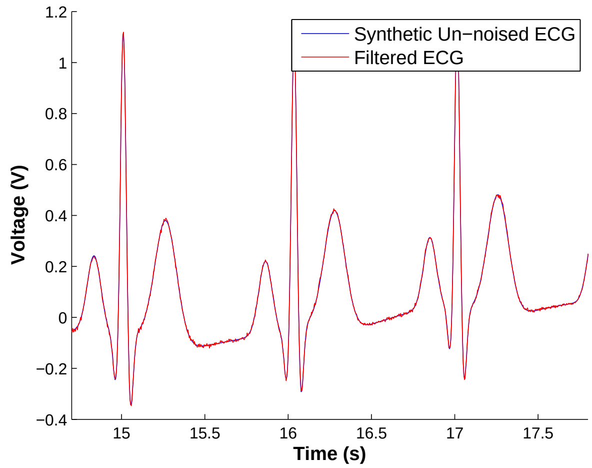

Figure 3.2: Plot of the synthetic clean ECG and filtered ECG for *Experiment 1*

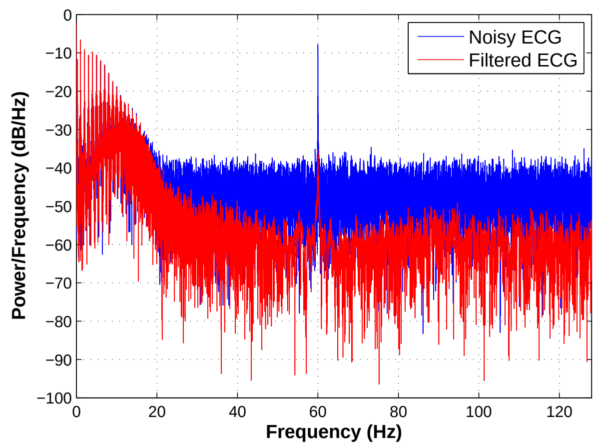

Figure 3.3: PSD of Noisy ECG vs PSD of Filtered ECG for *Experiment 1*

Figure 3.5 plots the primary input $d(n)$ versus the output of the ANC $e(n)$ for *Experiment 2*. Figure 3.6 is a plot of the original clean synthetic ECG and the filtered ECG corresponding to Figure 3.5 for *Experiment 2*. Figure 3.7 plots the PSD of the noisy ECG and the filtered ECG for *Experiment 2*. Figure 3.8 represents the PSD of the error obtained by subtracting the filtered signal by the clean signal for *Experiment 2*.

3.3 Experiment 3

Figures 3.9 to 3.12 represents the third experiment for when there is severe gaussian noise (SD = 0.5), high amplitude sinusoidal interference (A = 0.9) and $f_0 = 60$ Hz. The MSE of the filter was $\sigma^2_{MSE} = 0.0038\ V^2$ and $\sigma_{MSE} = 0.0613$ V.
Figure 3.9 plots the primary input $d(n)$ versus the output of the ANC $e(n)$ for *Experiment 3*. Figure 3.10 is a plot of the original clean synthetic ECG and the filtered ECG corresponding to Figure 3.9 for *Experiment 3*. Figure 3.11 plots the PSD of the noisy ECG and the filtered ECG for *Experiment 3*. Figure 3.12 represents the PSD of the error obtained by subtracting the filtered signal by the clean signal for *Experiment 3*.

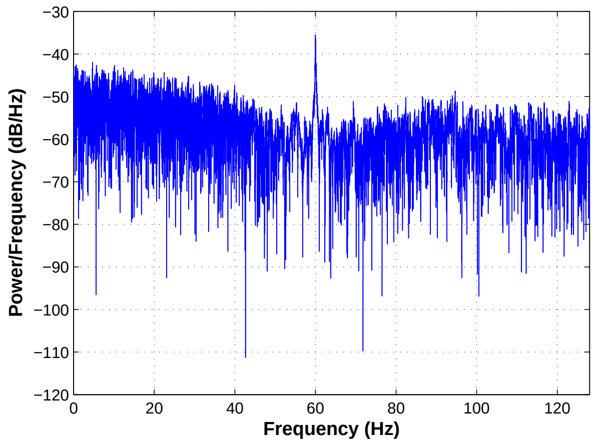

Figure 3.4: PSD of Error for *Experiment 1*

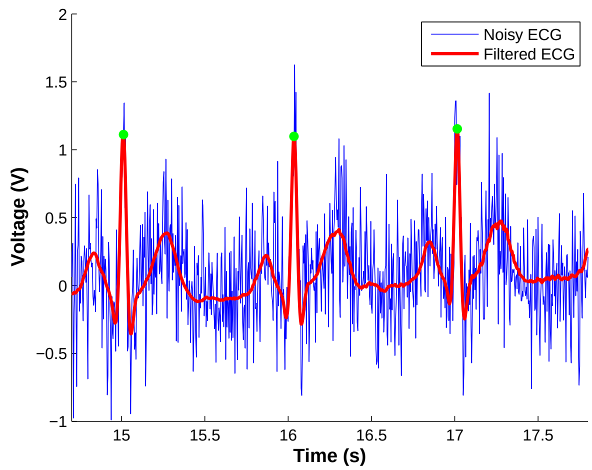

Figure 3.5: Noisy ECG vs Adaptive Filter Output for *Experiment 2*

3.4 Experiment 4

Figures 3.13 and 3.14 represent the fourth experiment for when there is no gaussian noise, abnormal amplitude sinusoidal interference (A = 0.3) and $59.5Hz \leq f_0 \leq 60.5Hz$. The MSE of the filter was $\sigma^2_{MSE} = 0.00040915\ V^2$ and $\sigma_{MSE} = 0.0202$ V.
Figure 3.13 plots the primary input $d(n)$ versus the output of the ANC $e(n)$ for *Experiment 4*. Figure 3.14 is a plot of the original clean synthetic ECG and the filtered ECG corresponding to Figure 3.13 for *Experiment 4*.

3.5 Experiment 5

Figures 3.15 and 3.16 represent the fifth experiment for when there is no gaussian noise, high amplitude sinusoidal interference (A = 1) and $59.5Hz \leq f_0 \leq 60.5Hz$. The MSE of the filter was $\sigma^2_{MSE} = 0.00073661$ V^2 and $\sigma_{MSE} = 0.0271$ V.
Figure 3.15 plots the primary input $d(n)$ versus the output of the ANC $e(n)$ for *Experiment 5*. Figure 3.16 is a plot of the original clean synthetic ECG and the filtered ECG corresponding to Figure 3.15 for *Experiment 5*.

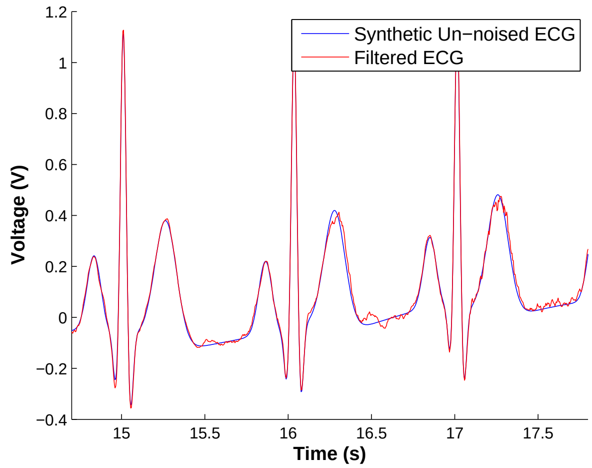

Figure 3.6: Plot of the synthetic un-noised ECG and filtered ECG for *Experiment 2*

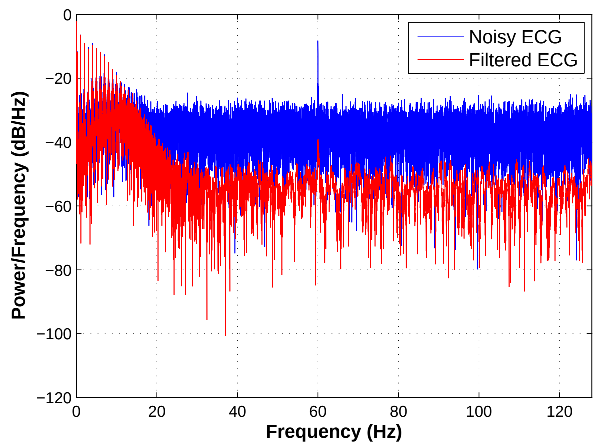

Figure 3.7: PSD of Noisy ECG vs PSD of Filtered ECG for *Experiment 2*

3.6 Experiment 6

Figures 3.17 and 3.18 represent the sixth experiment for when there is normal gaussian noise (SD = 0.09), normal amplitude sinusoidal interference (A = 0.2) and $59.5Hz \leq f_0 \leq 60.5Hz$. The MSE of the filter was $\sigma^2_{MSE} = 0.00064299\ V^2$ and $\sigma_{MSE} = 0.0254$ V.

Figure 3.17 plots the primary input $d(n)$ versus the output of the ANC $e(n)$ for *Experiment 6*. Figure 3.18 is a plot of the original clean synthetic ECG and the filtered ECG corresponding to Figure 3.17 for *Experiment 6*.

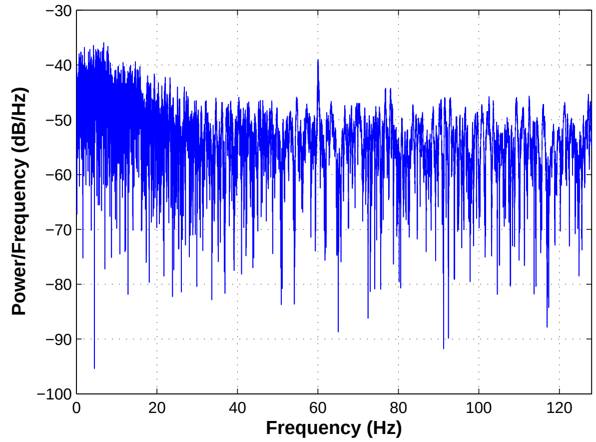

Figure 3.8: PSD of Error for *Experiment 2*

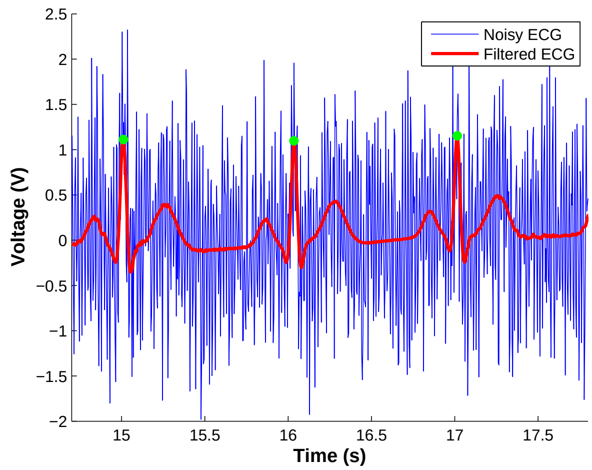

Figure 3.9: Noisy ECG vs Adaptive Filter Output for *Experiment 3*

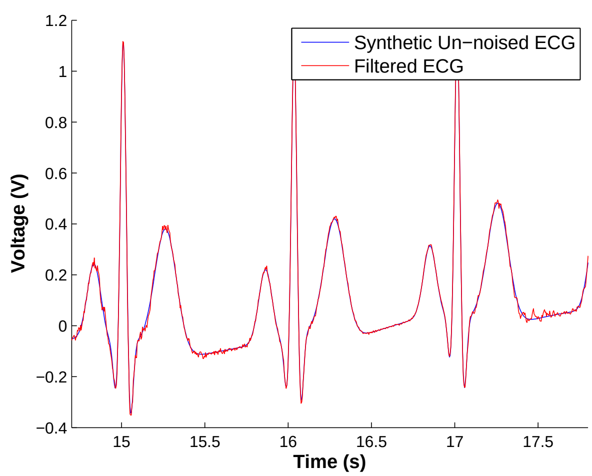

Figure 3.10: Plot of the synthetic un-noised ECG and filtered ECG for *Experiment 3*

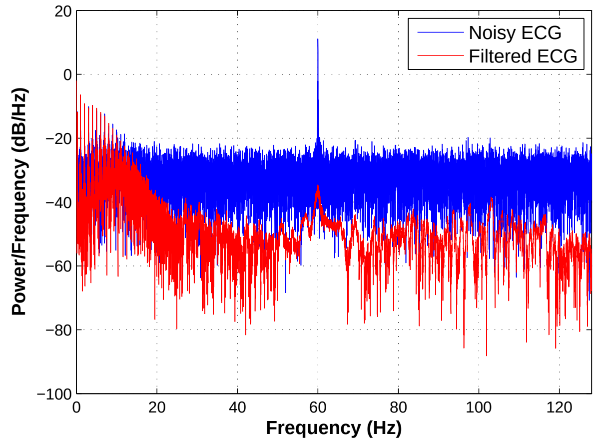

Figure 3.11: PSD of Noisy ECG vs PSD of Filtered ECG for *Experiment 3*

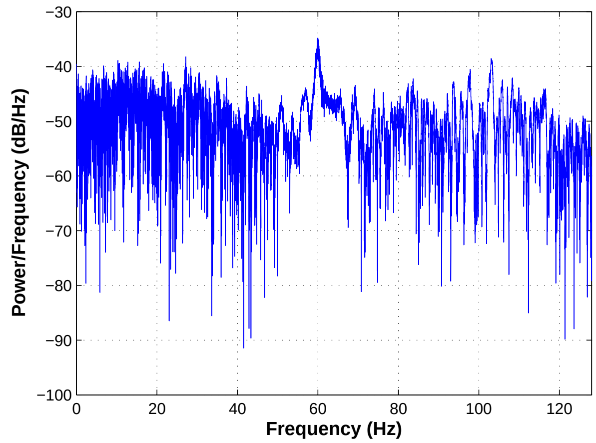

Figure 3.12: PSD of Error for *Experiment 3*

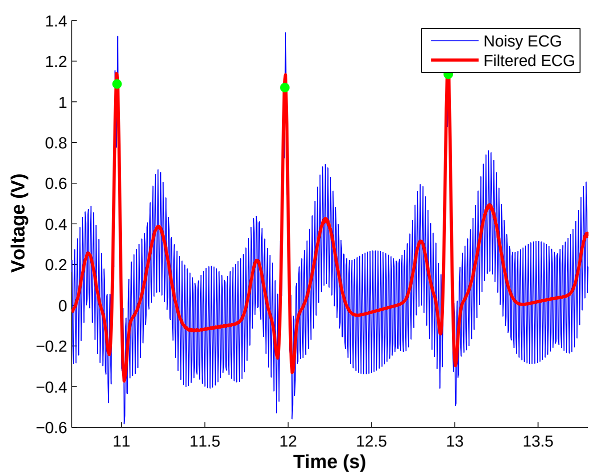

Figure 3.13: Noisy ECG vs Adaptive Filter Output for *Experiment 4*

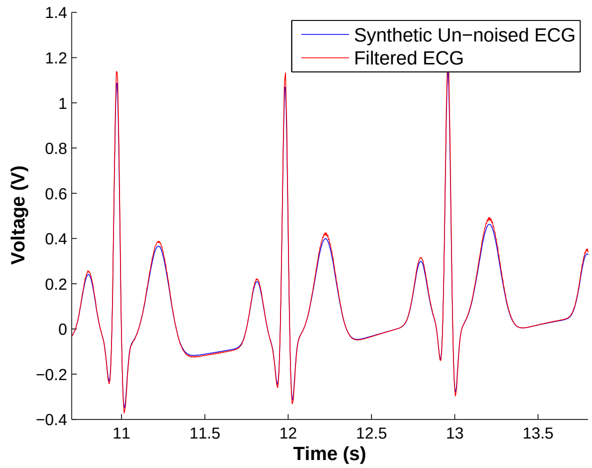

Figure 3.14: Plot of the synthetic un-noised ECG and filtered ECG for *Experiment 4*

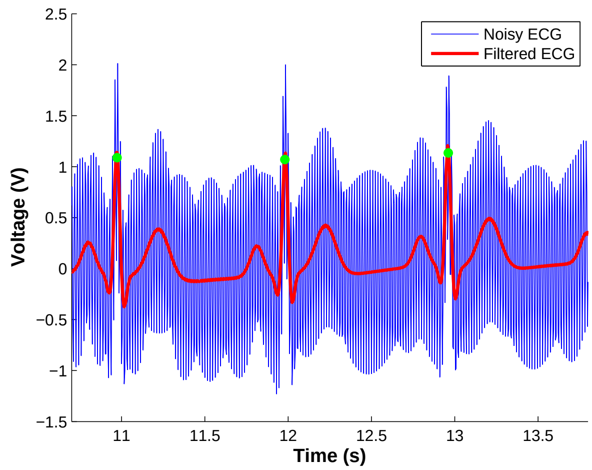

Figure 3.15: Noisy ECG vs Adaptive Filter Output for *Experiment 5*

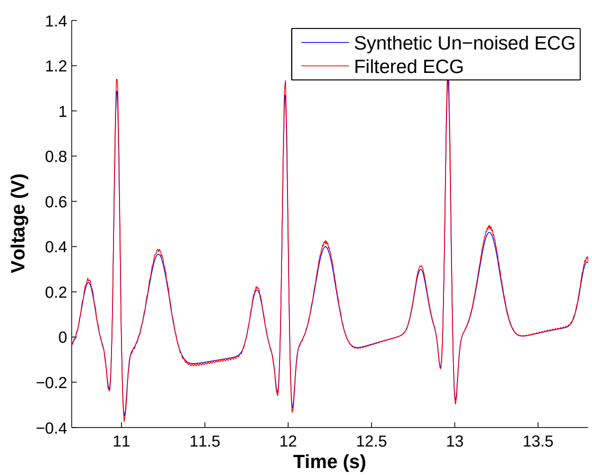

Figure 3.16: Plot of the synthetic un-noised ECG and filtered ECG for *Experiment 5*

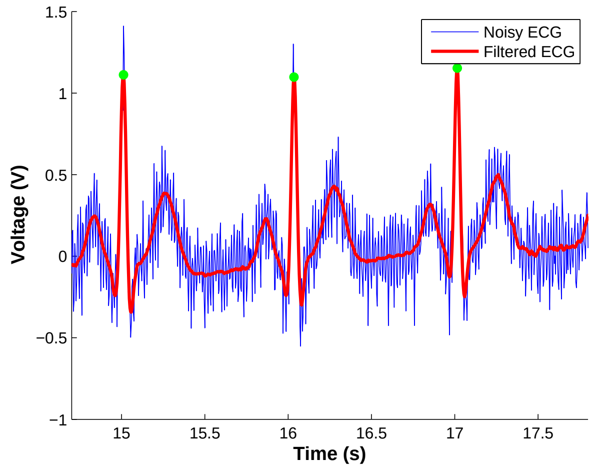

Figure 3.17: Noisy ECG vs Adaptive Filter Output for *Experiment 6*

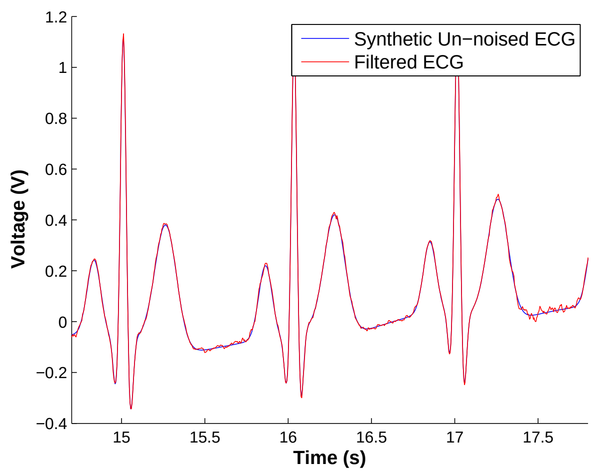

Figure 3.18: Plot of the synthetic un-noised ECG and filtered ECG for *Experiment 6*

Chapter 4

Discussion and Conclusion

As evidenced by the results, the adaptive filter performs very well throughout the experiments. By comparing the figures from *Experiment 1* to *Experiment 2*, the increase in the SD of the noise from 0.09 to 0.3, results in doubling σ_{MSE} associated with the adaptive filter. Although the these errors seem large, they do not affect the ECG morphology by much. *Experiment 3* was an extreme case of the previous experiments as the SD of the noise and the amplitude of the interference were increased. With the amplitude of the interference being on the same order of magnitude as the ECG data (A = 0.9 V), the filter performed very well as σ_{MSE} was 0.0613 V. *Experiment 4* demonstrated the effects of no gaussian noise but with an abnormal (A = 0.3 V) sinusoidal interference whose frequency was modulated with time. With the interference frequency jittering, the adaptive filter was able to modify its cut off frequency by continuously changing its tap-weight vector to optimize the filter performance. *Experiment 5* was a more extreme case of *Experiment 4* since the amplitude of the interference was increased to A = 1 V. The σ_{MSE} of the filters showed little change with an increase in interference amplitude, emphasizing the effectiveness of the adaptive filter. The last experiment, *Experiment 6*, a combination of normal gaussian noise and frequency modulated mains interference, was filtered and resulted in $\sigma_{MSE} = 0.0254\,V$. This error is comparable to the errors of the other experiments and demonstrates the effectiveness of the ANC. Figure 3.17 in *Experiment 6* represents a realistic ECG signal, and therefore is good example of how normal ECG signals can be filtered using ANC.

In the field of signal processing, there are many advantages to using ANC. One great advantage is that a model of the desired signal is not necessary. The model used to generate the ECG signals are extremely complex and therefore reduces computational load for real-time implementation [1]. Since the reference input can be measured in conjunction with the ECG signal, implementation can be real-time. Despite the advantages, some downfalls exist. One such downfall, is that if the step-size parameter μ is too large or too small, a large σ_{MSE} can result. In this project, μ was found using the expression in (2.22) with the addition of minor modifications to decrease σ_{MSE} as much as possible. The design could be extended in filtering motion artifacts using an electromyogram recording (in the same area as the ECG recording) as the reference input, along with modifications to the calculated μ parameter. Overall, the adaptive filtering of gaussian noise and mains interference from the noisy ECG signal was carried out with minor errors, thus making it a suitable implementation for ECG machines.

Appendix A

MATLAB Code

Listing A.1 shows a MATLAB script that was used to generate the figures in Chapter 3.

Listing A.1: MATLAB Script Used to Generate the Figures in Chapter 3

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Adaptive Noise Cancelation using LMS Algorithm
% By Robert Tisma 0658942
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clc
clear
close all

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Initialization of data
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x = open('ECG.mat'); %ECG Data from ECG simulator
ecg = x.ecg;
timesteps = ceil(length(ecg(:,1))/2);
t = ecg(1:timesteps,1);
d = ecg(1:timesteps,2);

Fs = 256; %Sampleing Freq in Hz
k = 1; %counter initialization

%Noise description ( r =0 is constant freq, r =1 is oscilatory)
osc_freq = 0;% 0 = 60Hz and 1 = 60Hz mean with oscillating freq
if (osc_freq==0)
    f_mains = 60; %the mains interference frequency
else
    f_mains = zeros(timesteps,1);
    for i=1:timesteps
        f_mains(i,1) = 60 + 0.5*sin(0.002*i);
    end
end

A = 0.9; %amplitude of the mains interference
SNR = 30; %SNR of the added noise to the clean ecg signal
dc_offset = 0; %dc offset in the noise signal
SD_sig = 0.5; %SD of signal noise
select = 3; %select the method of input noise
```

```
for ii = 1:timesteps
    gnoise(ii,1) = randn;
end
if (select ==1)
    u = A*(sin(2*pi*f_mains.*t) +wgn(length(t),1,0) );
elseif (select == 2)
    u = A*sin(2*pi*f_mains.*t) ;
else
    u = A*sin(2*pi*f_mains.*t) + SD_sig*gnoise;
end
o = d; %original clean synthetic ECG
d = d + u; %noise added to original clean synthetic ECG

%Peak detection algorithm
peak_val = zeros(timesteps,1); %value of the peak (R wave)
peak_time = peak_val; %time value of the R wave
for i = 1:timesteps
    if (ecg(i,3) == 3)
        peak_val(k,1) = ecg(i,2);
        peak_time(k,1) = (i-1)/Fs;
        k = k+1;
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                          ADAPTIVE FILTER                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
M = 35; %number of tap weights
if (osc_freq == 0)
    a = 0.01*(4/(M*A^2));
else
    a = 0.0095*(4/(M*A^2)); %step size parameter
end
w = zeros(M,1); %initial estimate of the weightings

y(M,1) = 0;
y = zeros(timesteps,1);
r = zeros(M,1);
for n = M:timesteps
    r_avg = 0;
    for j=1:M
        r(j,1)=u(n-(j-1),1);
        r_avg = r_avg + r(j,1);
    end

    y(n,1) = w'*r;
    e(n,1) = d(n) - y(n); %estimation error (output)

    w = w + a*r*e(n,1);
end

%to find the variance of the error
error = e -o;
H = ones(1,length(o));
error_avg = (H*error)/length(o);
```

```
error_var = 0;
for i = 1:length(o)
    error_var = error_var + (error(i,1)-error_avg)^2;
end
error_var = error_var/i
error_SD = sqrt(error_var)
```

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