RohanTiwari CFRM521 Homework1

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CFRM 421/521, Spring 2022

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Homework 1

- Due: Monday, April 18, 2022, 11:59 PM
- Total marks: 46
- Late submissions are allowed, but a 20% penalty per day applies. Your last submission is considered for calculating the penalty.
- Use this Jupyter notebook as a template for your solutions. Your solution must be submitted as one Jupyter notebook. The notebook must be already run, that is, make sure that you have run all your code, save the notebook, and then when you reopen the notebook, checked that all output appears as expected. You are allowed to use code from the textbook, textbook website, or lecture notes.

1 1. Preprocessing housing data [7 marks]

Hint: Reading Chapter 2 of the textbook and the accompanying Jupyter notebook on the textbook website may help with this question.

Consider the California housing data from Chapter 2 of the textbook. The following code fetches the data.

```
[]: import os
  import tarfile
  from six.moves import urllib
  import numpy as np
  import pandas as pd
  from sklearn.model_selection import StratifiedShuffleSplit

HOUSING_PATH = os.path.join("datasets", "housing")

def fetch_housing_data(housing_url, housing_path=HOUSING_PATH):
    if not os.path.isdir(housing_path):
        os.makedirs(housing_path)
    tgz_path = os.path.join(housing_path, "housing.tgz")
    urllib.request.urlretrieve(housing_url, tgz_path)
```

Next, we create stratified test and training sets based on different income categories.

```
[]: data["income_cat"] = np.ceil(data["median_income"] / 1.5)
    data["income_cat"].where(data["income_cat"] < 5, 5.0, inplace=True)

split = StratifiedShuffleSplit(n_splits=1, test_size=0.2, random_state=42)
for train_index, test_index in split.split(data, data["income_cat"]):
    strat_train_set = data.loc[train_index]
    strat_test_set = data.loc[test_index]

for set_ in (strat_train_set, strat_test_set):
    set_.drop("income_cat", axis=1, inplace=True)</pre>
```

Let us separate the features (X_raw) and the response variable (y).

```
[]: X_raw = strat_train_set.drop("median_house_value", axis=1)
y = strat_train_set["median_house_value"].copy()
```

1.1 (a) Handling missing values [1 mark]

Let us find out if there are any missing values in our features. As the code below indicates, out of 16512 observations, 158 have missing values. Further inspection shows that total_bedrooms has missing values (only a few missing values are shown below).

10915	-117.87	33.73	45.	.0 22	264.0	NaN
19150	-122.70	38.35	14.	.0 23	313.0	NaN
4186	-118.23	34.13	48.	.0 13	308.0	NaN
16885	-122.40	37.58	26.	.0 32	281.0	NaN
	population	households	median_income	ocean_pro	kimity	
1606	825.0	626.0	2.9330	NE.	AR BAY	
10915	1970.0	499.0	3.4193	<1H	OCEAN	
19150	954.0	397.0	3.7813	<1H	OCEAN	
4186	835.0	294.0	4.2891	<1H	OCEAN	
16885	1145.0	480.0	6.3580	NEAR	OCEAN	

Task: Read the subsection "Data Cleaning" in Chapter 2 of the textbook. Use the sklearn.impute.SimpleImputer class to fill the missing values in the numerical features (everything except ocean_proximity) with the median of the corresponding feature.

- 1. remove the text attributed "ocean_proximity" before fitting SimpleImputer as median calculation would not work for text attribute
- 2. Train a SimpleImputer on the housing training data using fit method.
- 3. Tranform the training data (replace missing values with corresponding median of each feature) using transform method.
- 4. Convert result back to pandas dataframe.

```
from sklearn.impute import SimpleImputer
housing_num = X_raw.drop("ocean_proximity", axis=1)

imputer = SimpleImputer(strategy="median")
imputer.fit(housing_num)
X = imputer.transform(housing_num)
housing_tr = pd.DataFrame(X, columns=housing_num.columns, index=housing_num.
index)
housing_tr.head()
```

	Housin	nousing_tr.nead()							
[]:		longitude	latitude h	nousing_median_age	total_rooms	total_bedrooms	\		
	12655	-121.46	38.52	29.0	3873.0	797.0			
	15502	-117.23	33.09	7.0	5320.0	855.0			
	2908	-119.04	35.37	44.0	1618.0	310.0			
	14053	-117.13	32.75	24.0	1877.0	519.0			
	20496	-118.70	34.28	27.0	3536.0	646.0			
		population	households	s median_income					
	12655	2237.0	706.0	2.1736					
	15502	2015.0	768.0	6.3373					
	2908	667.0	300.0	2.8750					
	14053	898.0	483.0	2.2264					
	20496	1837.0	580.0	4.4964					

1.2 (b) Handling categorical features [2 marks]

ML algorithms generally work with numerical values. So, we need to convert categorical features to numerical values. One way is to simply map each category to an integer. Another alternative is to use **one-hot encoding**.

Task: Read subsection "Handling Text and Categorical Attributes" in Chapter 2 of the textbook. Then use the sklearn.preprocessing.OrdinalEncoder class and the sklearn.preprocessing.OneHotEncoder class to transform the ocean_proximity feature to integers and one-hot vectors. Explain why it is more appropriate to use one-hot encoding.

[Add your solution here]

"ocean_proximity" is the text and categorial attribute to preprocess. We first look at a sample of data.

```
[]: housing_cat = X_raw[["ocean_proximity"]]
housing_cat.head(10)
```

```
[]:
           ocean_proximity
     12655
                     INLAND
                 NEAR OCEAN
     15502
     2908
                     INLAND
                 NEAR OCEAN
     14053
     20496
                  <1H OCEAN
     1481
                   NEAR BAY
     18125
                  <1H OCEAN
     5830
                  <1H OCEAN
     17989
                  <1H OCEAN
     4861
                  <1H OCEAN
```

We can convert the above categories into numbers by using OrdinalEncoder. OrdinalEncoder gives a list of categories in a 1D array (for each categorial attribute). In our case, we can a single 1D array as there is only one categorical attribute.

```
[]: from sklearn.preprocessing import OrdinalEncoder

ordinal_encoder = OrdinalEncoder()
housing_cat_encoded = ordinal_encoder.fit_transform(housing_cat)
housing_cat_encoded[:10]
```

```
[0.]])
```

```
[]: ordinal_encoder.categories_
```

```
[]: [array(['<1H OCEAN', 'INLAND', 'ISLAND', 'NEAR BAY', 'NEAR OCEAN'], dtype=object)]
```

One issue with this representation is that ML algorithms will assume that two nearby values are more similar than two distant values. This may be fine in some cases (e.g., for ordered categories such as "bad," "average," "good," and "excellent"), but it is obviously not the case for the ocean_proximity column (for example, categories 0 and 4 are clearly more similar than categories 0 and 1). To fix this issue, a common solution is to create one binary attribute per category: one attribute equal to 1 when the category is "<1H OCEAN" (and 0 otherwise), another attribute equal to 1 when the category is "INLAND" (and 0 otherwise), and so on. This is called one-hot encoding, because only one attribute will be equal to 1 (hot), while the others will be 0 (cold). The new attributes are sometimes called dummy attributes (From textbook).

```
[]: from sklearn.preprocessing import OneHotEncoder

cat_encoder = OneHotEncoder()
housing_cat_1hot = cat_encoder.fit_transform(housing_cat)
housing_cat_1hot
```

[]: <16512x5 sparse matrix of type '<class 'numpy.float64'>'
with 16512 stored elements in Compressed Sparse Row format>

Note that the above output is a SciPy sparse matrix instead of a NumPy array. This is very useful when you have categorial attributes with thousands of categories. After one-hot encoding we get a matrix with thousands of columns, and the matrix is full of 0s except for a single 1 per row. Using up tons of memory to store zeros would be very wasteful, so instead a sparse matrix only stores the location of the non-zero elements. We can convert it to a dense NumPy array as follows.

Let us check the list of categories.

```
[]: cat_encoder.categories_
```

```
[]: [array(['<1H OCEAN', 'INLAND', 'ISLAND', 'NEAR BAY', 'NEAR OCEAN'], dtype=object)]
```

1.3 (c) Feature scaling [2 marks]

Many ML algorithms do not perform well if the features have different scales (some very small, and others very large). In such scenarios, one should transform the features so that they have similar range of values. There are two common ways to achieve this. In **min-max** scaling, we subtract the feature from its minimum value and then divide by its range (i.e. maximum value minus minimum value) so that the scaled values are between 0 and 1. In **standardization**, we subtract the values from the average and divide by the standard deviation, so that the transformed values has mean 0 and variance 1.

Task: Read the subsection "Feature Scaling" in Chapter 2 of the textbook. Use the sklearn.preprocessing.MinMaxScaler class and sklearn.preprocessing.StandardScaler class to scale the numerical features using min-max scaling and standardization.

[Add your solution here]

Let us try MinMaxScalar first that shifts values so that they end up ranging from 0 to 1

```
from sklearn.preprocessing import MinMaxScaler
minmax_scalar = MinMaxScaler()
minmax_housing = minmax_scalar.fit_transform(housing_tr)
X = pd.DataFrame(minmax_housing, columns=housing_tr.columns,index=housing_tr.
index)
X.head()
```

```
[]:
            longitude
                       latitude
                                  housing_median_age
                                                       total_rooms
                                                                     total_bedrooms
             0.287849
     12655
                       0.635494
                                             0.549020
                                                          0.098362
                                                                           0.128061
     15502
             0.709163
                       0.058448
                                             0.117647
                                                          0.135168
                                                                           0.137403
     2908
             0.528884
                       0.300744
                                             0.843137
                                                          0.041003
                                                                           0.049613
     14053
             0.719124
                       0.022317
                                             0.450980
                                                          0.047591
                                                                           0.083280
             0.562749
                                                          0.089790
                                                                           0.103737
     20496
                       0.184910
                                             0.509804
                                     median_income
            population households
     12655
              0.062614
                           0.131441
                                          0.115426
     15502
              0.056392
                           0.143017
                                          0.402574
     2908
              0.018610
                           0.055639
                                           0.163798
     14053
              0.025085
                           0.089806
                                           0.119067
     20496
              0.051403
                           0.107916
                                           0.275617
```

Let us try StandardScalar which first subtracts the mean value and then divides it by standard deviation so the resulting distribution has unit variance and standardized values always have a zero mean.

```
[]: from sklearn.preprocessing import StandardScaler

standard_scalar = StandardScaler()
standard_housing = standard_scalar.fit_transform(housing_tr)

X = pd.DataFrame(standard_housing, columns=housing_tr.columns,index=housing_tr.

index)
```

```
X.head()
```

```
[]:
                                                                     total bedrooms
            longitude
                       latitude
                                  housing median age
                                                       total rooms
            -0.941350
                       1.347438
                                             0.027564
                                                          0.584777
                                                                           0.640371
     12655
     15502
             1.171782 -1.192440
                                            -1.722018
                                                          1.261467
                                                                           0.781561
     2908
             0.267581 -0.125972
                                                         -0.469773
                                                                          -0.545138
                                             1.220460
     14053
             1.221738 -1.351474
                                            -0.370069
                                                         -0.348652
                                                                          -0.036367
     20496
             0.437431 -0.635818
                                            -0.131489
                                                          0.427179
                                                                           0.272790
            population households
                                     median_income
              0.732602
     12655
                           0.556286
                                          -0.893647
              0.533612
     15502
                           0.721318
                                           1.292168
     2908
             -0.674675
                          -0.524407
                                          -0.525434
             -0.467617
     14053
                          -0.037297
                                          -0.865929
     20496
              0.374060
                           0.220898
                                           0.325752
```

1.4 (d) Transformation pipelines [2 marks]

Usually, we have to perform several steps before the data is ready to be fed to an ML algorithm. Scikit-Learn's Pipeline class provides a systematic way of "packaging" such a sequence of transformations.

Task: Read the subsection "Transformation Pipelines" in Chapter 2 of the textbook. Then, create one pipeline that applies the transformations on part (a), (b) (use one-hot encoding), and (c) (use standardization) to the "raw" features X_raw. Call your transformed data X.

[Add your solution here]

We first create a numerical pipeline and then apply the ColumnTransformer to handle all transformations for columns

```
→BAY', 'NEAR OCEAN'], index=housing_num.index)
     X.head()
[]:
                                                                     total_bedrooms
            longitude
                        latitude
                                  housing_median_age
                                                       total_rooms
            -0.941350
     12655
                        1.347438
                                             0.027564
                                                           0.584777
                                                                            0.640371
     15502
             1.171782 -1.192440
                                            -1.722018
                                                           1.261467
                                                                            0.781561
     2908
             0.267581 -0.125972
                                             1.220460
                                                          -0.469773
                                                                           -0.545138
     14053
             1.221738 -1.351474
                                            -0.370069
                                                                           -0.036367
                                                          -0.348652
     20496
             0.437431 -0.635818
                                            -0.131489
                                                           0.427179
                                                                            0.272790
            population households
                                     median_income
                                                     <1H OCEAN
                                                                 INLAND
                                                                         ISLAND
     12655
              0.732602
                           0.556286
                                          -0.893647
                                                            0.0
                                                                    1.0
                                                                             0.0
     15502
              0.533612
                           0.721318
                                           1.292168
                                                            0.0
                                                                    0.0
                                                                             0.0
     2908
             -0.674675
                          -0.524407
                                          -0.525434
                                                            0.0
                                                                    1.0
                                                                             0.0
     14053
             -0.467617
                          -0.037297
                                          -0.865929
                                                            0.0
                                                                    0.0
                                                                             0.0
```

X = pd.DataFrame(X, columns=num_attribs+['<1H_OCEAN', 'INLAND', 'ISLAND', 'NEAR_

	NEAR	BAY	NEAR	OCEAN
12655		0.0		0.0
15502		0.0		1.0
2908		0.0		0.0
14053		0.0		1.0
20496		0.0		0.0

0.374060

20496

2 2. Fine-tuning models [16 marks]

0.220898

Hint: Reading Chapter 2 of the textbook and the accompanying Jupyter notebook on the textbook website may help with this question.

0.325752

1.0

0.0

0.0

2.1 (a) Linear regression [2 marks]

Task: Use the sklearn.linear_model.LinearRegression class to fit a linear regression to the training set in Question 1. Use the processed features X that you obtained in 1(d) as predictors and y (the median house values) as the response. Return fitted values of the response for the first 10 observations of the training set.

```
[]: from sklearn.linear_model import LinearRegression
lin_reg = LinearRegression()
lin_reg.fit(X, y)

some_data = X.iloc[:10]
some_labels = y.iloc[:10]
print("Predictions:", lin_reg.predict(some_data))
print("Labels:", list(some_labels))
```

```
Predictions: [ 88983.14806384 305351.35385026 153334.71183453 184302.55162102 246840.18988841 208983.6813477 375046.02067267 270259.898592 153462.62515736 12077.73402934]
Labels: [72100.0, 279600.0, 82700.0, 112500.0, 238300.0, 165700.0, 412300.0, 349600.0, 181300.0, 500001.0]
```

2.2 (b) RMSE and MAE [2 marks]

We can measure the out-of-sample performance with root mean square error (RMSE) and mean absolute error (MAE).

Task: Use mean_squared_error and mean_absolute_error functions from sklearn.metrics to calculate the in-sample RMSE and MAE of the linear regression that you fit in part (a).

[Add your solution here]

First let us try RMSE

```
[]: from sklearn.metrics import mean_squared_error
housing_predictions = lin_reg.predict(X)
lin_mse = mean_squared_error(y, housing_predictions)
lin_rmse = np.sqrt(lin_mse)
lin_rmse
```

[]: 69050.56219504567

Now let us try MAE

```
[]: from sklearn.metrics import mean_absolute_error
housing_predictions = lin_reg.predict(X)
lin_mae = mean_absolute_error(y, housing_predictions)
lin_mae
```

[]: 49905.329442715316

2.3 (c) Cross validation [3 marks]

In part (b), we evaluated the in-sample performance of the linear regression model. To obtain out-of-sample performance, we use K-fold cross validation.

Task: Use the cross_val_score function from sklearn.model_selection to perform 10-fold cross validation on the linear regression from part (a) and return the values of **RMSE**. Return the 10 RMSE scores as well as their mean.

```
print("Scores:", scores)
print("Mean:", scores.mean())
print("Standard deviation:", scores.std())
display_scores(lin_rmse_scores)
```

Scores: [72229.03469752 65318.2240289 67706.39604745 69368.53738998

66767.61061621 73003.75273869 70522.24414582 69440.77896541

66930.32945876 70756.31946074]

Mean: 69204.32275494764

Standard deviation: 2372.070791055919

2.4 (d) Alternatives to linear regression [3 marks]

Let us consider fitting two alternative models, namely, **decision trees** and **random forests**. These models will be discussed in future lectures. The following code fits a decision tree and obtains the fitted response for the first 10 observations.

```
[]: from sklearn.tree import DecisionTreeRegressor
dt_reg = DecisionTreeRegressor(random_state=42)
dt_reg.fit(X,y)
```

[]: DecisionTreeRegressor(random_state=42)

```
[]: dt_reg.predict(X.iloc[:10,])
```

```
[]: array([72100., 279600., 82700., 112500., 238300., 165700., 412300., 349600., 181300., 500001.])
```

The following code does the same for the random forest model.

[]: RandomForestRegressor(max_depth=5, random_state=42)

```
[]: rf_reg.predict(X.iloc[:10,])
```

```
[]: array([81652.52309416, 312816.43220775, 103956.26879436, 145669.10673902, 254963.91541844, 223471.15507845, 359454.5338993, 261250.07064825, 178062.03642388, 152509.12871198])
```

Task: Find the in-sample performance of the decision tree and the random forest using RMSE on the training set. Also, find the out-of-sample performance by 10-fold cross validation as you did in part (c). Which of the 3 models has the best in-sample performance? Which has the best out-of-sample performance?

RMSE for decision tree (in-sample)

```
[]: from sklearn.metrics import mean_squared_error
housing_predictions = dt_reg.predict(X)
dt_mse = mean_squared_error(y, housing_predictions)
dt_rmse = np.sqrt(dt_mse)
dt_rmse
```

[]: 0.0

RMSE for random forest (in-sample)

```
[]: from sklearn.metrics import mean_squared_error
housing_predictions = rf_reg.predict(X)
rf_mse = mean_squared_error(y, housing_predictions)
rf_rmse = np.sqrt(rf_mse)
rf_rmse
```

[]: 66750.70697525762

out-of-sample performance of decision tree

Scores: [71177.6601991 69770.07865373 64770.5639395 68536.60203993 67057.08155801 68847.12456973 70977.38255647 69208.86346929 67187.87131535 73280.38732407]

Mean: 69081.361562518

Standard deviation: 2296.288087393378 out-of-sample performance of random forest

```
Scores: [70666.17468232 65975.4104431 63466.98553521 71762.08084765 65644.92773928 70198.63516218 69549.01504688 68795.8612658 65340.03824595 68660.17724972]
```

Mean: 68005.93062180877

Standard deviation: 2585.2080431468353

Decision tree gives best in-sample performance and random forest has best out-of-sample performance.

2.5 (e) Choosing optimal values of hyperparameters using cross validation [4 marks]

Most ML algorithms have **hyperparameters**. These hyperparameters control the learning process and should not be confused with the model parameters (which are the unknown values that are being estimated). For example, the random forest algorithm has 3 hyperparameters, namely bootstrap, n_estimators, and max_depth. To fine-tune a model, we should find good values of the hyperparameters based on out-of-sample performance.

Task: Read the subsections "Grid Search" and "Randomized Search" in Chapter 2 of the textbook. Consider the random forest model with the argument random_state=42. Use the sklearn.model_selection.GridSearchCV class to choose the best hyperparameter values for bootstrap over the values True, False; n_estimators over the values 50, 100, 200; and max_depth over the values 5, 20, 40. Use 4-fold cross-validation with RMSE when searching for the hyperparameters.

Next, use the sklearn.model_selection.RandomizedSearchCV class to perform a randomized search over the same ranges with 18 parameter settings sampled, that is bootstrap is True or False with equal probability, n_estimators is a uniform random integer between 50 and 200, max_depth is a uniform random integer between 5 and 40. RandomizedSearchCV must include the argument random state=42.

In both functions, you may use the argument n_jobs=-1 to use all avaliable processor cores to speed up the calculation. Based on all of your results, what are the best hyperparameter values?

[Add your solution here]

Grid Search

/usr/local/lib/python3.7/dist-

```
packages/joblib/externals/loky/process executor.py:705: UserWarning: A worker
    stopped while some jobs were given to the executor. This can be caused by a too
    short worker timeout or by a memory leak.
      "timeout or by a memory leak.", UserWarning
[]: GridSearchCV(cv=4, estimator=RandomForestRegressor(random_state=42), n_jobs=-1,
                 param_grid=[{'bootstrap': [True, False], 'max_depth': [5, 20, 40],
                               'n_estimators': [50, 100, 200]}],
                 return_train_score=True, scoring='neg_mean_squared_error')
[]: grid_search.best_params_
[]: {'bootstrap': True, 'max_depth': 40, 'n_estimators': 200}
[]: grid_search.best_estimator_
[]: RandomForestRegressor(max_depth=40, n_estimators=200, random_state=42)
[]: np.sqrt(-grid_search.best_score_)
[]: 50271.12629517273
    Randomized Search
[]: from sklearn.model_selection import RandomizedSearchCV
     from scipy.stats import randint
     param grid = [
                   {'n_estimators':randint(low=50, high=200), 'bootstrap':[True,_
     →False], 'max_depth':randint(low=5, high=40)}
     forest_reg = RandomForestRegressor(random_state=42)
     rd_search = RandomizedSearchCV(forest_reg, param_grid, cv=4,_
     ⇒scoring='neg_mean_squared_error', return_train_score=True, n_iter=18,⊔
     →random_state=42,n_jobs=-1)
     rd search.fit(X,y)
[]: RandomizedSearchCV(cv=4, estimator=RandomForestRegressor(random_state=42),
                        n iter=18, n jobs=-1,
                        param_distributions=[{'bootstrap': [True, False],
                                              'max depth':
     <scipy.stats._distn_infrastructure.rv_frozen object at 0x7f1a682791d0>,
                                              'n estimators':
     <scipy.stats._distn_infrastructure.rv_frozen object at 0x7f1a68279110>}],
                        random_state=42, return_train_score=True,
                        scoring='neg_mean_squared_error')
```

[]: rd_search.best_params_

```
[]: {'bootstrap': True, 'max_depth': 28, 'n_estimators': 166}
[]: rd_search.best_estimator_
[]: RandomForestRegressor(max_depth=28, n_estimators=166, random_state=42)
    np.sqrt(-rd_search.best_score_)
```

[]: 50273.28602841223

Grid Search is giving lower RMSE so the best hyperparameter values come from Grid Search and they are: {'bootstrap': True, 'max depth': 40, 'n estimators': 200}

(f) Evaluating your final model using the test set [2 marks]

The last step of a ML project is to evaluate your (fine-tuned) model using the test set. The goal of this step is to estimate the out-of-sample performance of your trained model on new data. Be careful not to train your model using the test set. Do not use .fit() or .fit_tranform() of your estimators or pipelines. Also, do not fine tune your hyperparameters after finding out the performance of a model on the test set as that would be data snooping.

Task: Using the test set, find the RMSE of your best model in part (e).

[Add your solution here]

```
[]: final_model = grid_search.best_estimator_
     X_test = strat_test_set.drop("median_house_value", axis=1)
     y_test = strat_test_set["median_house_value"].copy()
     X_test_prepared = full_pipeline.transform(X_test)
     X_test_prepared = pd.DataFrame(X_test_prepared,
                                    columns=num attribs+['<1H OCEAN', 'INLAND', I
     →'ISLAND', 'NEAR BAY', 'NEAR OCEAN'])
     final_predictions = final_model.predict(X_test_prepared)
     final_mse = mean_squared_error(y_test, final_predictions)
     final_rmse = np.sqrt(final_mse)
     final_rmse
```

[]: 47074.669205115606

3. Regularizing linear regression [6 marks]

Consider the housing data set from Questions 1 and 2.

3.1 (a) Polynomial regression and regularizing [2 marks]

Fit a model that adds degree 3 polynomial terms to the linear regression in 2(a). Next, regularize the model using ridge regression with parameter $\alpha = 1$.

[Add your solution here]

```
[]: from sklearn.preprocessing import PolynomialFeatures
poly_features = PolynomialFeatures(degree=3, include_bias=False)
X_poly = poly_features.fit_transform(X)
poly_reg = LinearRegression()
poly_reg.fit(X_poly, y)
```

[]: LinearRegression()

```
[]: from sklearn.pipeline import Pipeline
     from sklearn.preprocessing import StandardScaler
     from sklearn.compose import ColumnTransformer
     num_pipeline = Pipeline([
                               ('imputer', SimpleImputer(strategy="median")),
                               ('polynomial', ⊔
      →PolynomialFeatures(degree=3,include_bias=False)),
                              ('std scalar', StandardScaler()),
     ])
     num_attribs = list(housing_num)
     cat_attribs = ["ocean_proximity"]
     full_pipeline = ColumnTransformer([
                                      ("num", num_pipeline, num_attribs),
                                      ("cat", OneHotEncoder(), cat attribs),
     1)
     X_ridge = full_pipeline.fit_transform(X_raw)
     X_ridge = pd.DataFrame(X_ridge)
     from sklearn.linear_model import Ridge
     ridge_reg = Ridge(alpha=1, solver="cholesky")
     ridge_reg.fit(X_ridge,y)
```

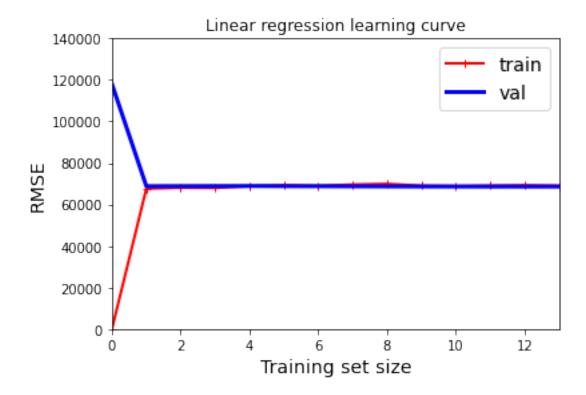
[]: Ridge(alpha=1, solver='cholesky')

3.2 (b) Learning curves [4 marks]

Obtain learning curves for the linear regression model that you fitted in 2(a), the polynomial regression model in 3(a), and the ridge regression model in 3(a). Hence, comment on whether the curves indicate the models are underfitting, overfitting or neither. When plotting the learning curve, use the function train_test_split with the argument random_state=10 to create a validation

set from a random 20% split of the test set. Instead of adding 1 instance at a time in the learning curve, add 1000 instances at a time.

```
[]: from sklearn.metrics import mean_squared_error
    from sklearn.model_selection import train_test_split
    import matplotlib.pyplot as plt
    def plot_learning_curves(model, X, y):
        →random state=10)
        train_errors, val_errors = [], []
        for m in range(1, len(X_train), 1000):
            model.fit(X_train[:m], y_train[:m])
            y_train_predict = model.predict(X_train[:m])
            y_val_predict = model.predict(X_val)
            train_errors.append(mean_squared_error(y_train[:m], y_train_predict))
            val_errors.append(mean_squared_error(y_val, y_val_predict))
        plt.plot(np.sqrt(train_errors), "r-+", linewidth=2, label="train")
        plt.plot(np.sqrt(val_errors), "b-", linewidth=3, label="val")
        plt.legend(loc="upper right", fontsize=14) # not shown in the book
        plt.xlabel("Training set size", fontsize=14) # not shown
        plt.ylabel("RMSE", fontsize=14)
[]: plot_learning_curves(lin_reg, X, y)
    plt.axis([0, 13, 0, 140000]) # not shown in the book
    plt.title("Linear regression learning curve")
    plt.show()
```



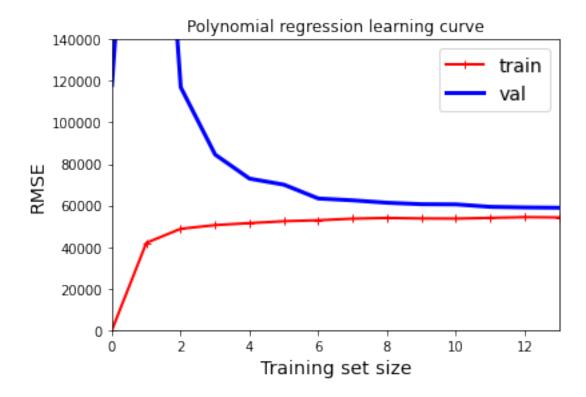
When the training set size is very low, the RMSE on training data is also very low (curve starts at 0). When more and more instances are added to the training set, the RME on training data goes up but then reaches a plateau which means adding more instances does not make the error better or worse.

When the training set size is very low, the validation error is high but then it gradually goes down as more training instances are added. It also plateaus after a while which means adding more and more instances does not make the error better or worse.

There is hardly any gap between the curves (maybe no gap) after we have added a few training instances.

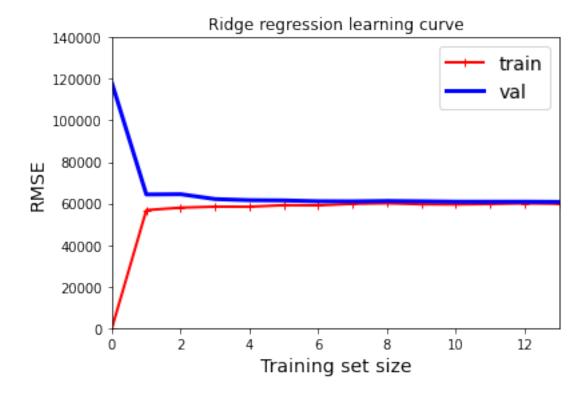
This means that a straight line does not a good job here and **the model is underfitting**. (Referred concepts from textbook).

```
[]: plot_learning_curves(poly_reg, X_poly, y)
  plt.axis([0, 13, 0, 140000]) # not shown in the book
  plt.title("Polynomial regression learning curve")
  plt.show()
```



There is a significant gap between the curves. The model is performing significantly better on the training data than on the validation data. **This indicates an overfitting model**. (Referred concepts from textbook).

```
[]: plot_learning_curves(ridge_reg, X_ridge, y)
  plt.axis([0, 13, 0, 140000]) # not shown in the book
  plt.title("Ridge regression learning curve")
  plt.show()
```



Gap between the curves reduces as more training instances are added. The error is almost as low as what we see for polynomial regression. The curves coincide when more training instances are added indicating that **model** is **neither underfitting nor overfitting**.

4 4. Default dataset [17 marks]

In this question you will work with the Default dataset, which is a simulated dataset of credit card default for 10000 customers. Download the data as a csv file from Canvas. The data was originally obtained from the R library ISLR2, and you can read the documentation for the dataset here.

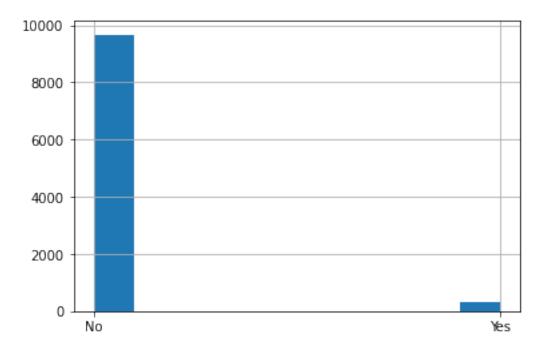
You want to predict credit card defaults. The target for this classification problem is **default** and all other variables are the features.

4.1 (a) [2 marks]

Load the dataset. If it is stored in a file named Default.csv in your working directory, this can be done with the code below. Then split the data into a training and a test set, using 20% of the data for the test set. When doing the splitting, use the random seed 42.

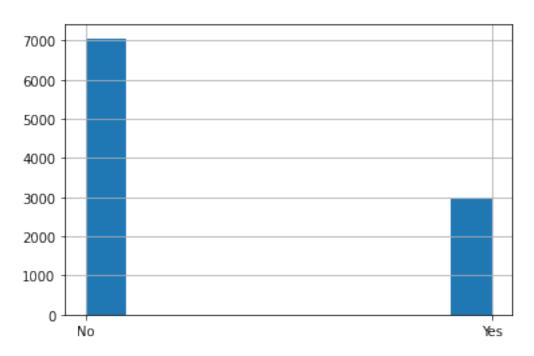
```
[4]: import os
  import numpy as np
  import pandas as pd
  data = pd.read_csv("Default.csv")
  data['default'].hist()
```

[4]: <matplotlib.axes._subplots.AxesSubplot at 0x7fba5549d650>



[5]: data['student'].hist()

[5]: <matplotlib.axes._subplots.AxesSubplot at 0x7fba55419d50>



[Add your solution here]

The below code makes sure that the ratio of student=yes and student=no is same as the original data.

```
[6]:
         student
                      balance
                                     income
    9254
              No
                 1018.568130 34103.879520
    1561
              No
                    62.170050 28660.747508
    1670
              No 1046.743543 40822.447413
    6087
              No
                   763.735280 44125.718725
    6669
             Yes
                   697.248633 25730.917583
```

4.2 (b) [3 marks]

Design a pipeline for preprocessing that applies one hot encoding to the categorical features and a standard scaler to the numerical features.

```
X = full_pipeline.fit_transform(X_raw)
X = pd.DataFrame(X, columns=num_attribs+['NO', 'YES'], index=X_Raw_num.index)
X.head()
```

```
[7]: balance income NO YES
9254 0.381082 0.037308 1.0 0.0
1561 -1.591774 -0.370099 1.0 0.0
1670 0.439202 0.540178 1.0 0.0
6087 -0.144587 0.787420 1.0 0.0
6669 -0.281735 -0.589390 0.0 1.0
```

4.3 (c) [4 marks]

Train a logistic regression classifier and a SGDClassifier, and then using 10-fold cross-validation plot the ROC curve of the classifiers and compute their ROC AUC.

[Add your solution here]

```
[8]: from sklearn.linear_model import LogisticRegression

log_reg = LogisticRegression(random_state=42)
log_reg.fit(X,y_Yes) # maybe remove.
```

[8]: LogisticRegression(random_state=42)

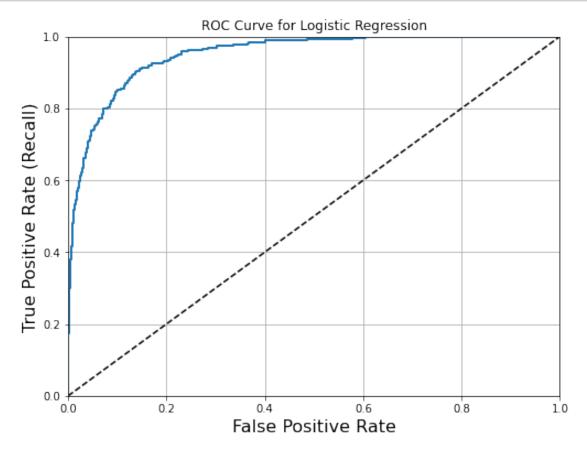
```
[9]: from sklearn.linear_model import SGDClassifier

sgd_clf = SGDClassifier(random_state=42)
sgd_clf.fit(X,y_Yes)
```

[9]: SGDClassifier(random_state=42)

```
import matplotlib.pyplot as plt
%matplotlib inline
def plot_roc_curve(fpr, tpr, label=None):
    plt.plot(fpr, tpr, linewidth=2, label=label)
    plt.plot([0, 1], [0, 1], 'k--')
    plt.axis([0, 1, 0, 1])
    plt.xlabel('False Positive Rate', fontsize=16)
    plt.ylabel('True Positive Rate (Recall)', fontsize=16)
```

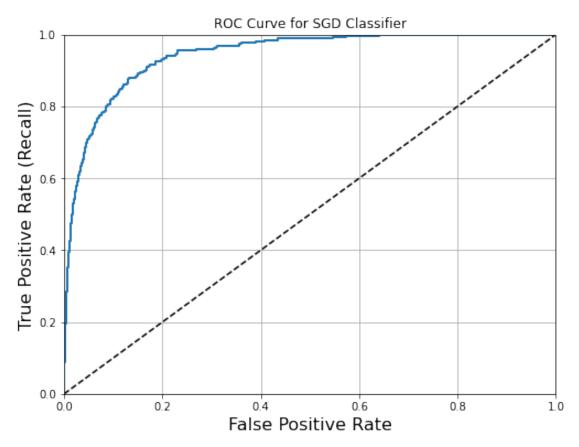
```
plt.figure(figsize=(8, 6))
plot_roc_curve(fpr, tpr)
plt.title('ROC Curve for Logistic Regression')
plt.grid(True)
plt.show()
```



```
[12]: from sklearn.metrics import roc_auc_score roc_auc_score(y_Yes, y_score_logistic_regression)
```

[12]: 0.9501670662154117

```
plt.title('ROC Curve for SGD Classifier')
plt.grid(True)
plt.show()
```



```
[14]: from sklearn.metrics import roc_auc_score roc_auc_score(y_Yes, y_score_sgd_clf)
```

[14]: 0.9433360557488014

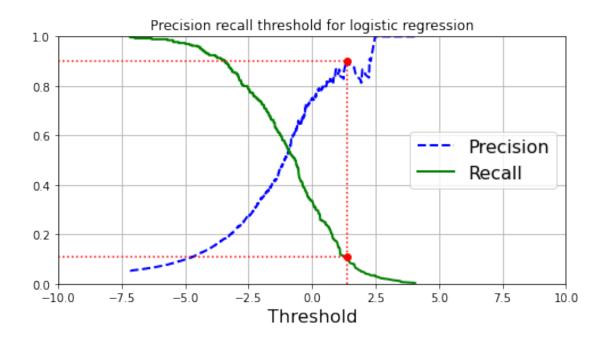
4.4 (d) [5 marks]

For the classifiers in part (c), plot on the same axes the precision as a function of the threshold and the recall as a function of the threshold. Now suppose you want to achieve a recall of 80%. Consider 4 models: 1. The logistic regression classifier in (c), 2. SGDClassifier in (c), 3. A logistic regression with the argument class_weight="balanced", 4. SGDClassifier with the argument class_weight="balanced".

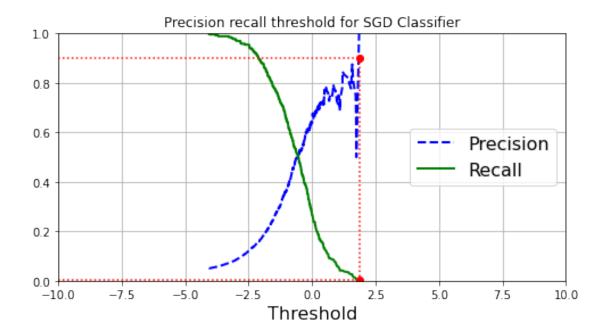
For each of the 4 models, determine the threshold to achieve an 80% recall based on 10-fold cross-validation. Then determine the corresponding precisions. Which model has the best precision?

```
[15]: def plot_precision_recall_vs_threshold(precisions, recalls, thresholds):
    plt.plot(thresholds, precisions[:-1], "b--", label="Precision", linewidth=2)
    plt.plot(thresholds, recalls[:-1], "g-", label="Recall", linewidth=2)
    plt.legend(loc="center right", fontsize=16) # Not shown in the book
    plt.xlabel("Threshold", fontsize=16) # Not shown
    plt.grid(True) # Not shown
    plt.axis([-10, 10, 0, 1]) # Not shown
```

```
[16]: from sklearn.metrics import precision_recall_curve
      precisions, recalls, thresholds = precision_recall_curve(y_Yes,_
       →y_score_logistic_regression)
      recall_90_precision = recalls[np.argmax(precisions >= 0.90)]
      threshold 90_precision = thresholds[np.argmax(precisions >= 0.90)]
      plt.figure(figsize=(8, 4))
                    # Not shown
      plot_precision_recall_vs_threshold(precisions, recalls, thresholds)
      plt.plot([threshold 90_precision, threshold 90_precision], [0., 0.9], "r:")
                    # Not shown
      plt.plot([-10, threshold_90_precision], [0.9, 0.9], "r:")
                                                                                      ш
                 # Not shown
      plt.plot([-10, threshold_90_precision], [recall_90_precision,_
      →recall_90_precision], "r:")# Not shown
      plt.plot([threshold 90 precision], [0.9], "ro")
                    # Not shown
      plt.plot([threshold_90_precision], [recall_90_precision], "ro")
      plt.title("Precision recall threshold for logistic regression")
                    # Not shown
      plt.show()
```



```
[17]: from sklearn.metrics import precision_recall_curve
      precisions, recalls, thresholds = precision_recall_curve(y_Yes, y_score_sgd_clf)
      recall_90_precision = recalls[np.argmax(precisions >= 0.90)]
      threshold_90_precision = thresholds[np.argmax(precisions >= 0.90)]
      plt.figure(figsize=(8, 4))
                    # Not shown
      plot_precision_recall_vs_threshold(precisions, recalls, thresholds)
      plt.plot([threshold_90_precision, threshold_90_precision], [0., 0.9], "r:")
                    # Not shown
      plt.plot([-10, threshold_90_precision], [0.9, 0.9], "r:")
                 # Not shown
      plt.plot([-10, threshold_90_precision], [recall_90_precision,_
      →recall_90_precision], "r:")# Not shown
      plt.plot([threshold_90_precision], [0.9], "ro")
                    # Not shown
      plt.plot([threshold_90_precision], [recall_90_precision], "ro")
                    # Not shown
      plt.title("Precision recall threshold for SGD Classifier")
      plt.show()
```



80% recall with logistic regression

Threshold: -7.168864964582789 Precision: 0.05340886101557758

80% recall with SGD Classifier

```
[19]: from sklearn.metrics import precision_score
    from sklearn.linear_model import SGDClassifier

precisions, recalls, thresholds = precision_recall_curve(y_Yes, y_score_sgd_clf)

threshold_80_recall = thresholds[np.argmax(recalls >= 0.8)]
    print("Threshold:", threshold_80_recall)
```

```
y_train_pred_80 = (y_score_sgd_clf >= threshold_80_recall)
      print("Precision:", precision_score(y_Yes,y_train_pred_80))
     Threshold: -4.065059846392796
     Precision: 0.050555342780543855
     80% recall with Logistic regression and class weight = "balanced"
[20]: from sklearn.linear_model import LogisticRegression
      from sklearn.metrics import precision score
      log_reg_b = LogisticRegression(random_state=42, class_weight="balanced")
      log_reg_b.fit(X,y_Yes)
      y_score_logistic_regression_b = cross_val_predict(log_reg_b, X, y_Yes,_u

cv=10,method="decision_function")
                                               #this is the score
      precisions, recalls, thresholds = precision_recall_curve(y_Yes,__
      →y_score_logistic_regression_b)
      threshold_80_recall = thresholds[np.argmax(recalls >= 0.8)]
      print("Threshold:", threshold_80_recall)
      y_train_pred_80 = (y_score_logistic_regression_b >= threshold_80_recall)
      print("Precision:", precision_score(y_Yes,y_train_pred_80))
     Threshold: -3.7529438988460786
     Precision: 0.052810562112422486
     80% recall with SGD Classifier and class weight = "balanced"
[21]: from sklearn.metrics import precision_score
      from sklearn.linear_model import SGDClassifier
      sgd_clf_b = SGDClassifier(random_state=42, class_weight="balanced")
      sgd_clf_b.fit(X,y_Yes)
      y_score_sgd_clf_b = cross_val_predict(sgd_clf_b, X, y_Yes,_
       ⇒cv=10,method="decision_function")
                                            #this is the score
      precisions, recalls, thresholds = precision_recall_curve(y_Yes,_
       →y_score_sgd_clf_b)
      threshold_80_recall = thresholds[np.argmax(recalls >= 0.8)]
```

print("Threshold:", threshold_80_recall)

```
y_train_pred_80 = (y_score_sgd_clf_b >= threshold_80_recall)
print("Precision:", precision_score(y_Yes,y_train_pred_80))
```

Threshold: -5.4512569350550635 Precision: 0.04080370942812983

Logistic regression without class weight = "balanced" has the best precision.

4.5 (e) [3 marks]

Fit the best classifier in (d) to the whole training data, and use it to predict on the test set. What is its accuracy, precision, recall, ROC curve and ROC AUC over the test set?

[Add your solution here]

Best Classifier: Logistic Regression without class weight="balanced"

```
[22]: X_test = data_strat_test_set.drop("default", axis=1)
y = data_strat_test_set["default"].copy()
y_test_Yes = (y == 'Yes')
X_test_num = X_test.drop("student", axis=1)

X_test = full_pipeline.transform(X_test)
X_test = pd.DataFrame(X_test, columns=num_attribs+['NO', 'YES'])

X_test.head()
```

```
[22]: balance income NO YES

0 1.241465 -0.157056 1.0 0.0

1 -0.127973 0.638724 1.0 0.0

2 -1.720019 -0.882921 1.0 0.0

3 -1.485744 -0.059998 1.0 0.0

4 1.081535 1.219882 1.0 0.0
```

```
[23]: from sklearn.linear_model import LogisticRegression
  log_reg = LogisticRegression(random_state=42)
  log_reg.fit(X,y_Yes)
  y_score = log_reg.decision_function(X_test)
  threshold = -7.168864964582789
  y_pred = (y_score > threshold)

#y_pred = log_reg.predict(X_test)
```

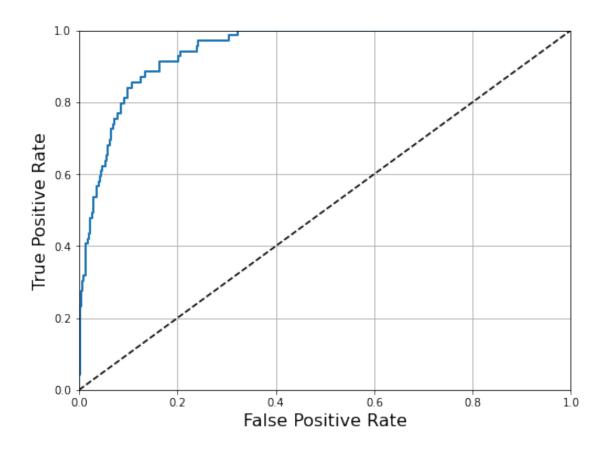
Accuracy

```
[24]: from sklearn.model_selection import cross_val_score from sklearn.metrics import accuracy_score
```

```
#cross_val_score(log_reg, X_test, y_test_Yes, cv=10, scoring = "accuracy")
      accuracy_score(y_test_Yes, y_pred)
[24]: 0.4075
     Precision
[25]: from sklearn.metrics import precision_score
      precision_score(y_test_Yes,y_pred)
[25]: 0.05502392344497608
     Recall
[26]: from sklearn.metrics import recall_score
     recall_score(y_test_Yes,y_pred)
[26]: 1.0
     ROC Curve
[27]: from sklearn.metrics import roc_curve
      y_score = log_reg.decision_function(X_test) #cross_val_predict(log_reg,_
      \rightarrow X_{test}, y_{test} Yes, cv=10, method="decision_function")
      fpr, tpr, thresholds = roc_curve(y_test_Yes, y_score)
      def plot_roc_curve(fpr, tpr, label=None):
          plt.plot(fpr, tpr, linewidth=2, label=label)
          plt.plot([0, 1], [0, 1], 'k--')
          plt.axis([0, 1, 0, 1])
          plt.xlabel('False Positive Rate', fontsize=16)
          plt.ylabel('True Positive Rate', fontsize=16)
```

plt.figure(figsize=(8, 6))
plot_roc_curve(fpr, tpr)

plt.grid(True)
plt.show()



ROC AUC

[28]: from sklearn.metrics import roc_auc_score roc_auc_score(y_test_Yes, y_score)

[28]: 0.9435375528186192