



Customized Retirement Advice

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Problem Statement

- 10 clients, 7 balanced funds or “Mixes”, and 7 asset classes
- Clients have some similarities and dissimilarities.
- Retirement age is the same for all : 67 years
- Minimum target account balance for every client : \$1540.8K
- Shortfall-based utility function based on the target account balance
 - Computed utility of target balance : 6.39
 - Initial utility of each client:

Amy	Bob	Carla	Darrin	Eric	Francine	Giovanni	Heloise	Isaac	Jennifer
2.75	2.59	2.59	1.72	3.11	2.89	2.42	6.10	3.11	6.76

- Forecasting team has provided annual scenarios of market outcomes for different asset classes
- **Goal:** Recommend allocations for each client along with likely consequences to reach the minimum target account balance

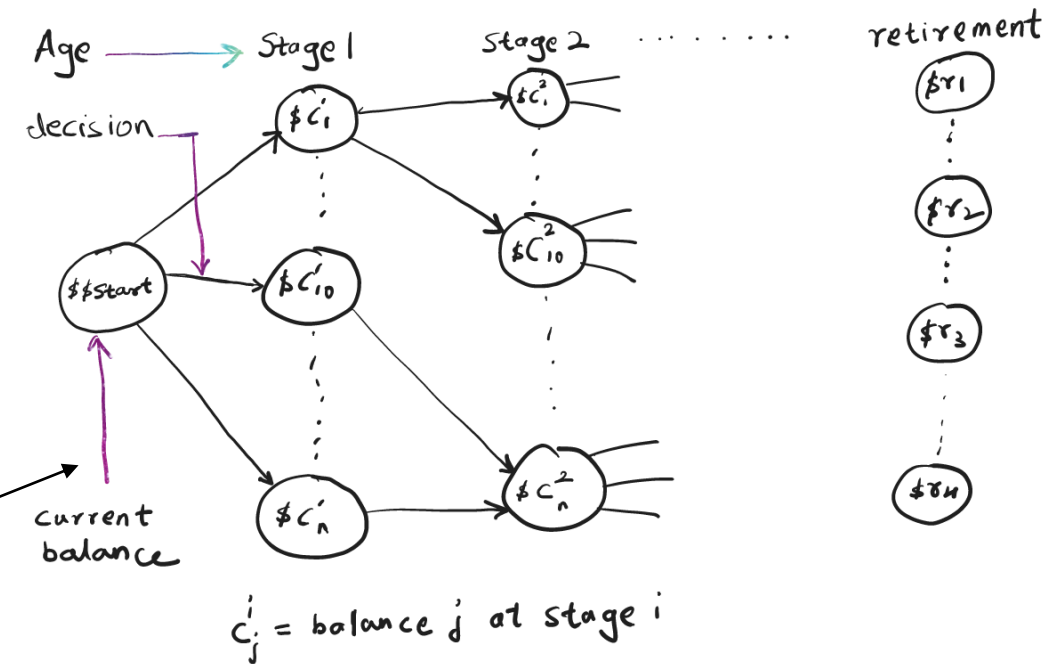
Solution Method

• Big Picture

- Combination of stochastic and Dynamic programming (DP).
- Stage: Age, State: Account Balance, Decision: Best Mix.
- For each DP sub-problem, maximize expected utility.

• Implementation

- Levels of the scenario tree are years from youngest to oldest client. Nodes of tree are the possible account balances – pre-generated 50 balances: from \$100k, with \$100k steps.
- Use backward recursion and “cache” solutions to previous sub problems in a table.
- Using random annual mix returns, calculate potential balances at each level/node, and decide the best mix based on maximizing expected utility.



Scenario Tree

Age	Balance	EVal	Mix
50	100	3.644639	1
50	200	4.692005	1
50	300	5.5723	1
50	400	6.252688	1
50	500	6.765128	1
50	600	7.149113	1
50	700	7.438924	2
50	800	7.658792	2
50	900	7.827376	2
50	1000	7.958343	2
50	1100	8.06167	3

Dynamic Programming Table

Solution Analysis

- **Mix Standard Deviations** (measure of risk):

Mix 1	Mix 2	Mix 3	Mix 4	Mix 5	Mix 6	Mix 7
0.17	0.15	0.12	0.10	0.09	0.07	0.06

- **Starting mix recommendation:**

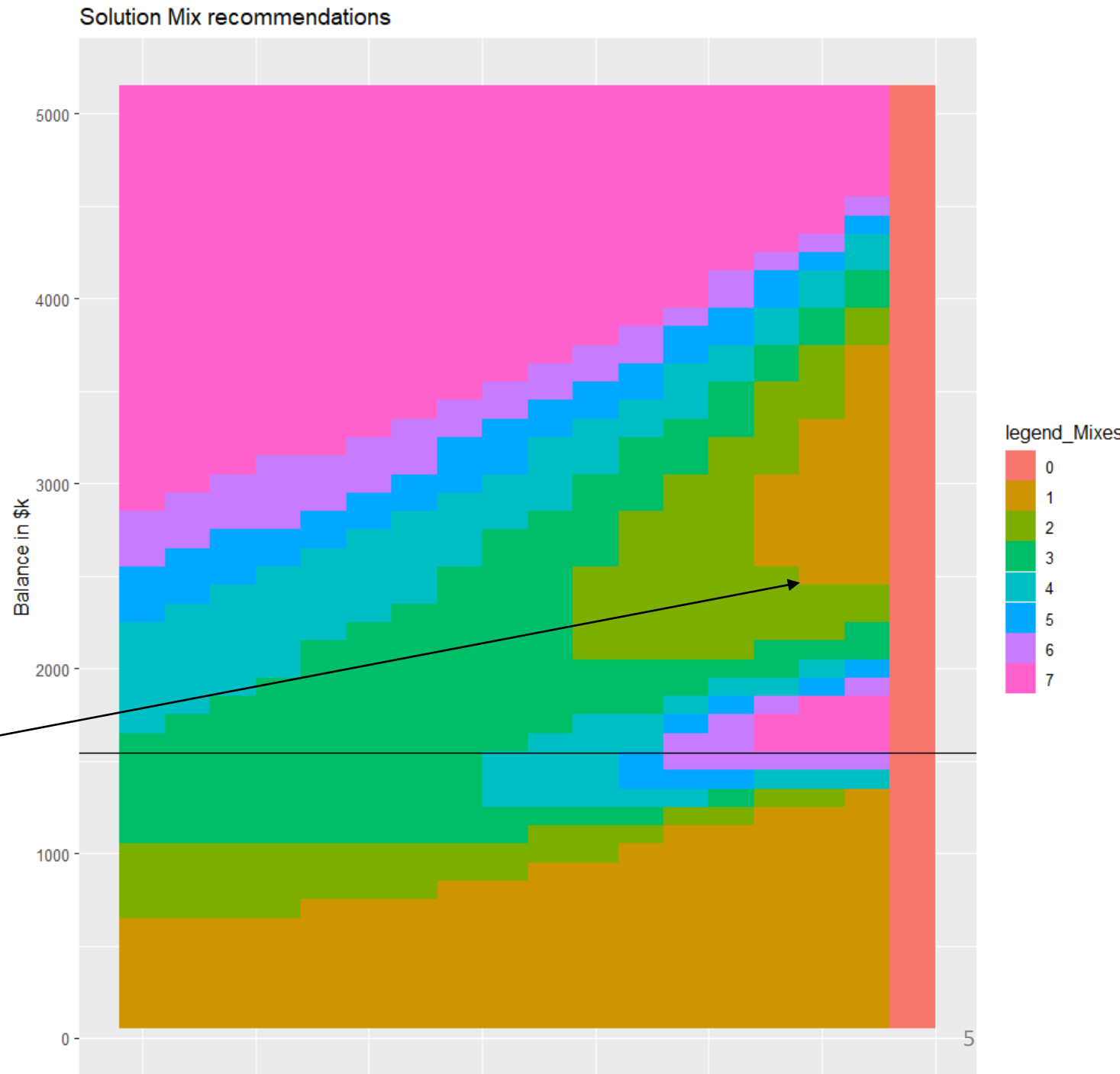
	Amy	Bob	Carla	Darrin	Eric	Francine	Giovanni	Heloise	Isaac	Jennifer
Mix	2	1	1	1	2	2	1	4	1	7
Ex. Util	7.44	6.98	6.79	4.26	7.29	6.70	5.19	7.85	6.13	7.39

- **Key takeaways**

- Jennifer and Heloise are close to retirement, and Jennifer has already met the goal, but Heloise is close to it.
- Bob, Carla, Darrin and Giovanni have low current balance and have several years remaining.
- Isaac has fewer years to retire with a decently high balance.
- Amy, Eric, Francine have a decent current balance and have several years remaining.
- Everybody except Darrin and Giovanni have an expected utility higher than utility of target balance. Implies these two have a low chance of hitting the goal.

Solution Analysis

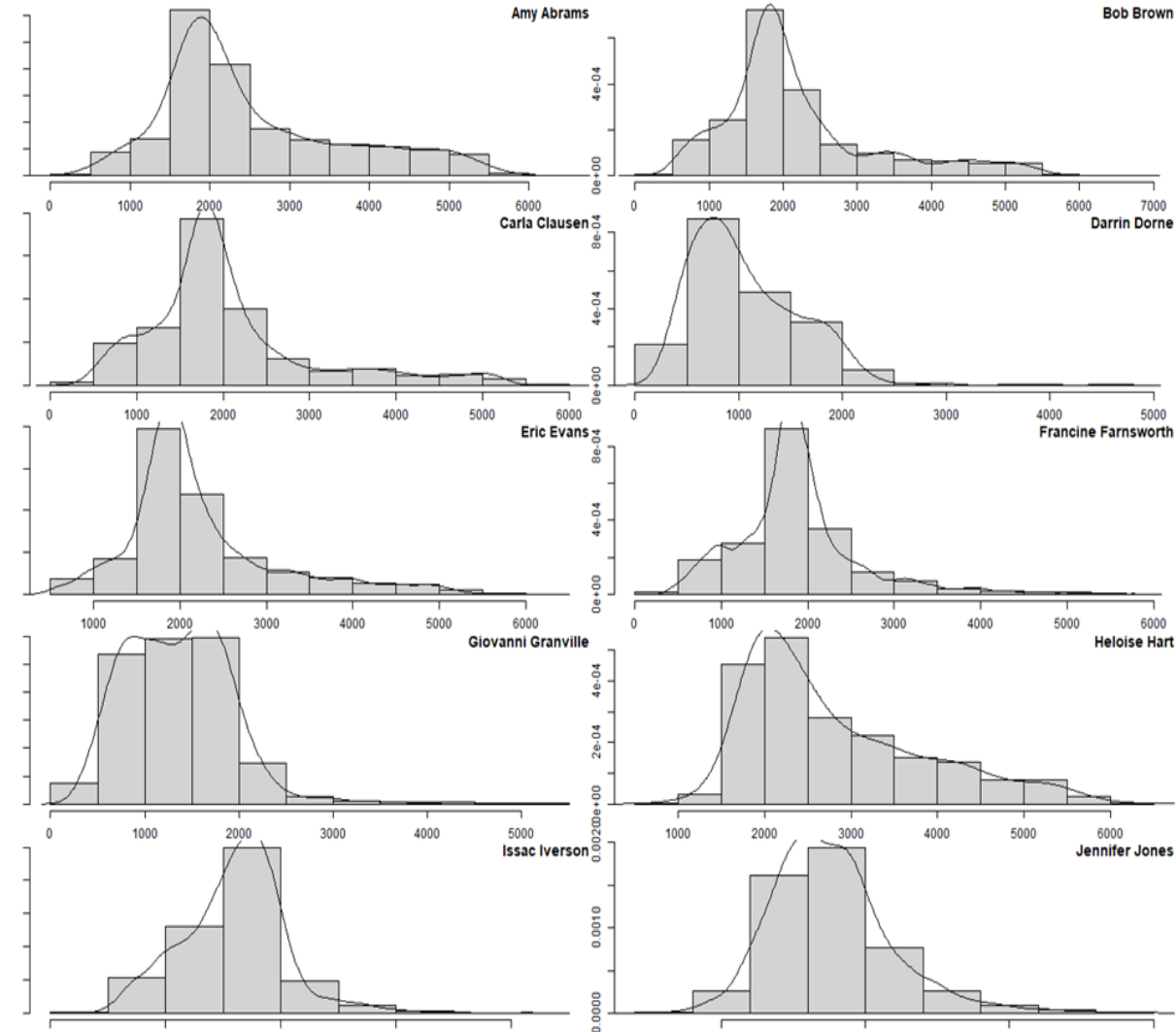
- Dynamic Programming solution given alongside built using 1000 annual return scenarios
- Y-intercept \$1540.8k is the target.
- Invest in riskier mixes when below the target.
- Less time remaining to meet goal, take more risk.
- Roughly around \$2500k, clients tend to invest in mix1 (high risk) with few years remaining – maybe because a “Safety Net” has been achieved?
- At high values of wealth, invest in safer investments. Additional \$\$\$ do not increase happiness (Utility) much - “Diminishing marginal utility”.



Simulation Testing

- Simulated 1000 paths per client to evaluate
- Used out of sample returns
- Testing statistics for future wealth and utility:

Amy	Bob	Carla	Darrin	Eric	Francine	Giovanni	Heloise	Isaac	Jennifer
Probability of hitting the target									
0.41	0.31	0.29	0.04	0.48	0.33	0.11	0.86	0.31	0.96
Expected value of wealth at retirement (\$K)									
2410.6	2166.4	2020.2	1077.5	2206.8	1887.3	1361.3	2758.9	1601.4	1884.1
Expected Utility at retirement									
6.96	6.74	6.57	3.99	7.12	6.58	4.99	7.66	6.05	7.41



Future Wealth Distributions

Simulation Analysis

- **Clients with high probability of hitting the target:**
 - Jennifer's current balance is already \$1600K, could choose to not invest with 3 years to retirement, but expected wealth is \$1884K.
 - Heloise's balance is \$1500K, simply accumulating the savings will be enough but expected wealth is \$1601K.
 - Recall high initial utilities of Jennifer and Heloise. They also have negative constant rate of return.
- **Clients with low probability of hitting the target:**
 - Even though Darrin has several years to go, current balance is very low.
 - Similar situation with Giovanni.
 - Recall low initial utilities of Darrin and Giovanni.

Final Thoughts

- **Performance and testing**

- Model building took 35 mins and out-of-sample testing took 46 mins for longest run. Ran for different combinations of scenarios/simulation paths.
- Test with different scenario sizes, different balance steps and simulation path
- Parallel computing will help with testing. More scenario returns and more simulations.

- **How to convince clients?**

- Using the results of large-scale simulation testing, share % chance of meeting the goal
- Share modeled expected and max wealth (subject to market risk).

- **What else?**

- More fund choices could be useful especially for clients like Darrin and Giovanni.
- Consider client specific risk tolerance parameter in utility function for a more realistic recommendation per client