$D[k] = [x_1[k], \dots, x_D[k]] \in \mathbb{R}^{2\times(D+1)}$ demonstrated states encoded as DMP (one DoF)

$U[k] = [u_1[k], \dots, u_D[k]] \in \mathbb{R}^{1\times D}$ corresponding control inputs (one DoF)

$\bar{A} \in \mathbb{R}^{2\times 2}$ discrete state transition matrix of a DMP (one DoF)

$\bar{B} \in \mathbb{R}^{2\times 1}$ discrete input matrix (one DoF)

$x[k] \in \mathbb{R}^{2\times 1}$ state vector $x[k] = \begin{bmatrix} q[k] \\ \dot{q}[k] \end{bmatrix}$ (one DoF)

$\xi[k] = [x_1[k], \dots, x_L[k]]^T \in \mathbb{R}^{2\cdot L\times 1}$ concatenated state vector of L DoF

$\lambda[k] \in \mathbb{R}^{(D+1)}$ control vector from solving QP (one DoF) $\lambda[k] = [\lambda_1[k], \dots, \lambda_D[k], \tilde{\lambda}[k]]$

auxiliary control input for obstacle avoidance

$\boxed{\text{Combined State Model of } L \text{ DMP}}$

$$\xi[k+1] = \underbrace{\begin{bmatrix} \bar{A} & & 0 \\ & \ddots & \\ 0 & & \bar{A} \end{bmatrix}}_{\phi \in \mathbb{R}^{2L\times 2L}} \xi[k] + \underbrace{\begin{bmatrix} \bar{B} & & 0 \\ & \ddots & \\ 0 & & \bar{B} \end{bmatrix}}_{\mathcal{B} \in ^{2\cdot L\times L}} \underbrace{\begin{bmatrix} [U_1[k],1] & & 0 \\ & \ddots & \\ 0 & & [U_L[k],1] \end{bmatrix}}_{\mathcal{U}[k] \in \mathbb{R}^{L\times(D+1)L}} \underbrace{\begin{bmatrix} \lambda_1[k] \\ \vdots \\ \lambda_L[k] \end{bmatrix}}_{\mu[k] \in \mathbb{R}^{(D+1)L\times 1}}$$

$\boxed{\text{Projection error at time } t_k}$

$$\Delta\xi[k] = \xi[k] - \underbrace{\begin{bmatrix} [D_1[k],0] & & 0 \\ & \ddots & \\ 0 & & [D_L[k],0] \end{bmatrix}}_{\Omega[k] \in \mathbb{R}^{2L\times(D+1)\cdot L}} \mu[k]$$

$$\Delta\xi[k+1] = \xi[k+1] - \Omega[k+1]\mu[k+1] = \phi\xi[k] + \mathcal{B}\mathcal{U}[k]\mu[k] - \Omega[k+1]\mu[k+1]$$

$$\Delta\xi[k+2] = \xi[k+2] - \Omega[k+2]\mu[k+2] = \phi^2\xi[k] + \phi\mathcal{B}\mathcal{U}[k]\mu[k] + \mathcal{B}\mathcal{U}[k+1]\mu[k+1] - \Omega[k+2]\mu[k+2]$$

$$\Delta\xi[k+3] = \xi[k+3] - \Omega[k+3]\mu[k+3] = \phi^3[k] + \phi^2\mathcal{B}\mathcal{U}[k]\mu[k] + \phi\mathcal{B}\mathcal{U}[k+1]\mu[k+1] + \mathcal{B}\mathcal{U}[k+2]\mu[k+2] - $$
$$- \Omega[k+3]\mu[k+3]$$

$\boxed{\text{MPC - scheme}}$ Preview window size P

$$\underbrace{\begin{bmatrix} \Delta\xi[k] \\ \Delta\xi[k+1] \\ \Delta\xi[k+2] \\ \Delta\xi[k+3] \\ \vdots \end{bmatrix}}_{\Delta Z \in \mathbb{R}^{2L(P+1)\times 1}} = \overbrace{\begin{bmatrix} \phi^0 \\ \phi^1 \\ \phi^2 \\ \phi^3 \\ \vdots \end{bmatrix}}^{\varphi \in \mathbb{R}^{2L(P+1)\times 1}} \xi[k] + \underbrace{\begin{bmatrix} -\Omega[k] & 0 & 0 & 0 \\ \mathcal{B}\mathcal{U}[k] & -\Omega[k+1] & 0 & 0 \\ \phi\mathcal{B}\mathcal{U}[k] & \mathcal{B}\mathcal{U}[k+1] & -\Omega[k+2] & 0 \\ \phi^2\mathcal{B}\mathcal{U}[k] & \phi\mathcal{B}\mathcal{U}[k+1] & \mathcal{B}\mathcal{U}[k+2] & -\Omega[k+3] \\ & & \vdots & \end{bmatrix}}_{\Theta \in \mathbb{R}^{2L(P+1)\times(D+1)L(P+1)}} \underbrace{\begin{bmatrix} \mu[k] \\ \mu[k+1] \\ \mu[k+2] \\ \mu[k+3] \\ \vdots \end{bmatrix}}_{\Lambda \in \mathbb{R}^{(D+1)L(P+1)\times 1}}$$

$v = [l_{11}, \ldots, l_{D1}, 0, \ldots, l_{1L} \ldots l_{DL}, 0, 0 \ldots 0] \in \mathbb{R}^{(D+1)L \times 1}$

↳ distance vector from the current state to the demo states

$P_0 \in \mathbb{R}$ weight factor for the auxiliary control inputs

$H \in \mathbb{R}^{2L(P+1) \times 2L(P+1)}$ state weight matrix

$\varepsilon \in \mathbb{R}$ small weight for the distances

$$\min_{\Lambda} \frac{1}{2} \underbrace{\Lambda^T [\cdot]^T H^T H [\cdot] \Lambda + \varphi^T H^T H [\cdot] \Lambda}_{\frac{1}{2} \|_1 Z\|_H^2} + \varepsilon v^T \mu[k] + \underbrace{P_0 \sum_{i=1}^{P+1} \sum_{l=1}^{L} \bar{\lambda}_{il}}_{\text{minimize auxiliary controls}}$$

$$\text{s.t.} \quad \lambda_{dl}[k+i] \geq 0 \qquad d = 1, \ldots, D, \quad i = 0, \ldots, P, \quad l = 1, \ldots, L$$

$$\sum_{d=1}^{D} \lambda_{dl}[k+i] = 1 \qquad i = 0, \ldots, P, \quad l = 1, \ldots, L$$

BASIC FORMULATION

ADDITIONAL STATE CONSTRAINTS FOR OBSTACLE AVOIDANCE

$\xi[k+1] = \phi \xi[k] + B \mathcal{X}[k] \mu[k]$ discrete SS model

$$\Rightarrow \underbrace{\begin{bmatrix} \xi[k+1] \\ \xi[k+2] \\ \vdots \\ \xi[k+P] \end{bmatrix}}_{Z \in \mathbb{R}^{2LP}} = \underbrace{\begin{bmatrix} \phi \\ \phi^2 \\ \vdots \\ \phi^P \end{bmatrix}}_{\delta \in \mathbb{R}^{2LP}} \xi[k] + \underbrace{\begin{bmatrix} B\mathcal{X}[k] & 0 & 0 \\ \phi \mathcal{X}[k] & B\mathcal{X}[k] & 0 \\ & \vdots & \end{bmatrix}}_{Y \in \mathbb{R}^{2LP \times (D+1)L(P+1)}} \underbrace{\begin{bmatrix} \mu[k] \\ \mu[k+1] \\ \vdots \\ \mu[k+P] \end{bmatrix}}_{\Lambda \in \mathbb{R}^{(D+1)L(P+1)}}$$

$H = [H_1, \ldots, H_L] \in \mathbb{R}^L$ L-dimensional normal vector (in position space although it could also be in velocity space)

$$S^q = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ & & \vdots & & \end{bmatrix} \in \mathbb{R}^{L \times 2L}$$

Selection matrices to pick positions/velocities from the state vector ξ

$$S^v = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ & & \vdots & & \end{bmatrix} \in \mathbb{R}^{L \times 2L}$$

$$H[k+1]S^q \xi[k+1] + b[k+1] \leq 0$$
$$\vdots$$
$$H[k+P]S^q \xi[k+P] + b[k+P] \leq 0$$

P constraints over the previous window

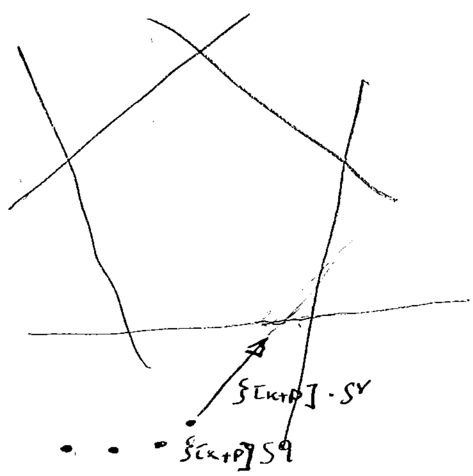
if constraint not active at k+i

$\Rightarrow H[k+i] = 0 \;, \; b[k+i] = 0$

$$\underbrace{\begin{bmatrix} H[k+1]S^q & & \\ & \ddots & \\ 0 & & H[k+P]S^q \end{bmatrix}}_{C \in \mathbb{R}^{P \times 2LP}}$$

$$\boxed{C \cdot \delta + \begin{bmatrix} b[k+1] \\ \vdots \\ b[k+P] \end{bmatrix} + CY\Lambda \leq 0}$$

How to find active constraints



if velocity ray at k+P intersects the obstacle, the closest hyperplane is an active constraint