

$D[k] = [x_1[k], \dots, x_D[k]] \in \mathbb{R}^{2 \times D}$  demonstrated states encoded as DMP (one DoF)

$u[k] = [u_1[k], \dots, u_D[k]] \in \mathbb{R}^{1 \times D}$  corresponding control inputs (one DoF)

$\bar{A} \in \mathbb{R}^{2 \times 2}$  discrete state transition matrix of a DMP (one DoF)

$\bar{B} \in \mathbb{R}^{2 \times 1}$  discrete input matrix (one DoF)

$x[k] \in \mathbb{R}^{2 \times 1}$  state vector  $x[k] = \begin{bmatrix} q[k] \\ \dot{q}[k] \end{bmatrix}$  (one DoF)

$\xi[k] = [x_1[k], \dots, x_L[k]]^T \in \mathbb{R}^{2L \times 1}$  concatenated state vector of  
 $\lambda[k] \in \mathbb{R}^{D \times 1}$  control vector from solving QP  $L$  DoF (one DoF)

Combined state model of  $L$  DMP

$$\xi[k+1] = \underbrace{\begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{A} \end{bmatrix}}_{\phi \in \mathbb{R}^{2L \times 2L}} \xi[k] + \underbrace{\begin{bmatrix} \bar{B} & 0 \\ 0 & \bar{B} \end{bmatrix}}_{B \in \mathbb{R}^{2L \times L}} \underbrace{\begin{bmatrix} u_1[k] & 0 \\ 0 & u_L[k] \end{bmatrix}}_{\mathcal{U}[k] \in \mathbb{R}^{L \times D-L}} \underbrace{\begin{bmatrix} \lambda_1[k] \\ \vdots \\ \lambda_L[k] \end{bmatrix}}_{\mu[k] \in \mathbb{R}^{D-L \times 1}}$$

Projection error at time  $t_k$

$$\Delta \xi[k] = \xi[k] - \underbrace{\begin{bmatrix} D_1[k] & 0 \\ 0 & D_L[k] \end{bmatrix}}_{\Omega[k] \in \mathbb{R}^{2L \times D-L}} \mu[k]$$

$$\Delta \xi[k+1] = \xi[k+1] - \Omega[k+1] \mu[k+1] = \phi \xi[k] + B \mathcal{U}[k] \mu[k] - \Omega[k+1] \mu[k+1]$$

$$\Delta \xi[k+2] = \xi[k+2] - \Omega[k+2] \mu[k+2] = \phi^2 \xi[k] + \phi B \mathcal{U}[k] \mu[k] + B \mathcal{U}[k+1] \mu[k+1] - \Omega[k+2] \mu[k+2]$$

$$\Delta \xi[k+3] = \xi[k+3] - \Omega[k+3] \mu[k+3] = \phi^3 \xi[k] + \phi^2 B \mathcal{U}[k] \mu[k] + \phi B \mathcal{U}[k+1] \mu[k+1] + B \mathcal{U}[k+2] \mu[k+2] - \Omega[k+3] \mu[k+3]$$

MPC - scheme Preview window size  $P$

$$\underbrace{\begin{bmatrix} \Delta \xi[k] \\ \Delta \xi[k+1] \\ \Delta \xi[k+2] \\ \Delta \xi[k+3] \\ \vdots \end{bmatrix}}_{\Delta Z \in \mathbb{R}^{2L(P+1) \times 1}} = \underbrace{\begin{bmatrix} \phi^0 \\ \phi^1 \\ \phi^2 \\ \phi^3 \\ \vdots \end{bmatrix}}_{\phi \in \mathbb{R}^{2L(P+1) \times 1}} \xi[k] + \underbrace{\begin{bmatrix} -\Omega[k] & 0 & 0 & 0 \\ B \mathcal{U}[k] & -\Omega[k+1] & 0 & 0 \\ \phi B \mathcal{U}[k] & B \mathcal{U}[k+1] & -\Omega[k+2] & 0 \\ \phi^2 B \mathcal{U}[k] & \phi B \mathcal{U}[k+1] & B \mathcal{U}[k+2] & -\Omega[k+3] \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}}_{[E] \in \mathbb{R}^{2L(P+1) \times D(L(P+1))}} \underbrace{\begin{bmatrix} \mu[k] \\ \mu[k+1] \\ \mu[k+2] \\ \mu[k+3] \\ \vdots \end{bmatrix}}_{\mu \in \mathbb{R}^{D(L(P+1)) \times 1}}$$

$$v = [l_{11} \dots l_{j1}, \dots, l_{1L} \dots l_{DL}]^T \in \mathbb{R}^{D \times L \times 1}$$

distance vector from current state to demo states

$$\begin{aligned} \min_{\mathcal{L}} \quad & \|z\|_H^2 + v^T \mu_{ck} \\ \text{s.t.} \quad & \mathcal{L} \succeq 0 \\ & \underbrace{\begin{bmatrix} 1^T & & 0 \\ & \ddots & \\ 0 & & 1^T \end{bmatrix}}_{\in \mathbb{R}^{P \times P \cdot D}} \begin{bmatrix} \lambda_{ck} \\ \lambda_{ck+1} \\ \lambda_{ck+2} \\ \lambda_{ck+3} \\ \vdots \end{bmatrix} = 1 \end{aligned}$$