$D[k] = [x_1[k], \dots, x_D[k]] \in \mathbb{R}^{2\times D}$ demonstrated states encoded as DMP (one DoF)

$U[k] = [u_1[k], \dots, u_D[k]] \in \mathbb{R}^{1\times D}$ corresponding control inputs (one DoF)

$\bar{A} \in \mathbb{R}^{2\times 2}$ discrete state transition matrix of a DMP (one DoF)

$\bar{B} \in \mathbb{R}^{2\times 1}$ discrete input matrix (one DoF)

$x[k] \in \mathbb{R}^{2\times 1}$ state vector $x[k] = \begin{bmatrix} q[k] \\ \dot{q}[k] \end{bmatrix}$ (one DoF)

$\xi[k] = [x_1[k], \dots, x_L[k]]^T \in \mathbb{R}^{2\cdot L \times 1}$ concatenated state vector of L DoF

$\lambda[k] \in \mathbb{R}^{D\times 1}$ control vector from solving QP (one DoF)

$\boxed{\text{Combined State Model of } L \text{ DMP}}$

$$\xi[k+1] = \underbrace{\begin{bmatrix} \bar{A} & & 0 \\ & \ddots & \\ 0 & & \bar{A} \end{bmatrix}}_{\phi \in \mathbb{R}^{2L\times 2L}} \xi[k] + \underbrace{\begin{bmatrix} \bar{B} & & 0 \\ & \ddots & \\ 0 & & \bar{B} \end{bmatrix}}_{\mathcal{B} \in ^{2\cdot L\times L}} \underbrace{\begin{bmatrix} U_1[k] & & 0 \\ & \ddots & \\ 0 & & U_L[k] \end{bmatrix}}_{\mathcal{U}[k] \in \mathbb{R}^{L\times D\cdot L}} \underbrace{\begin{bmatrix} \lambda_1[k] \\ \vdots \\ \lambda_L[k] \end{bmatrix}}_{\mu[k] \in \mathbb{R}^{D\cdot L\times 1}}$$

$\boxed{\text{Projection error at time } t_k}$

$$\Delta\xi[k] = \xi[k] - \underbrace{\begin{bmatrix} D_1[k] & & 0 \\ & \ddots & \\ 0 & & D_L[k] \end{bmatrix}}_{\Omega[k] \in \mathbb{R}^{2L\times D\cdot L}} \mu[k]$$

$$\Delta\xi[k+1] = \xi[k+1] - \Omega[k+1]\mu[k+1] = \phi\xi[k] + \mathcal{B}\mathcal{U}[k]\mu[k] - \Omega[k+1]\mu[k+1]$$

$$\Delta\xi[k+2] = \xi[k+2] - \Omega[k+2]\mu[k+2] = \phi^2\xi[k] + \phi\mathcal{B}\mathcal{U}[k]\mu[k] + \mathcal{B}\mathcal{U}[k+1]\mu[k+1] - \Omega[k+2]\mu[k+2]$$

$$\Delta\xi[k+3] = \xi[k+3] - \Omega[k+3]\mu[k+3] = \phi^3[k] + \phi^2\mathcal{B}\mathcal{U}[k]\mu[k] + \phi\mathcal{B}\mathcal{U}[k+1]\mu[k+1] + \mathcal{B}\mathcal{U}[k+2]\mu[k+2] -$$
$$- \Omega[k+3]\mu[k+3]$$

$\boxed{\text{MPC - scheme}}$ Preview window size P

$$\underbrace{\begin{bmatrix} \Delta\xi[k] \\ \Delta\xi[k+1] \\ \Delta\xi[k+2] \\ \Delta\xi[k+3] \\ \vdots \end{bmatrix}}_{\Delta Z \in \mathbb{R}^{2L(P+1)\times 1}} = \overbrace{\begin{bmatrix} \phi^0 \\ \phi^1 \\ \phi^2 \\ \phi^3 \\ \vdots \end{bmatrix}}^{\varphi \in \mathbb{R}^{2L(P+1)\times 1}} \xi[k] + \underbrace{\begin{bmatrix} -\Omega[k] & 0 & 0 & 0 \\ \mathcal{B}\mathcal{U}[k] & -\Omega[k+1] & 0 & 0 \\ \phi\mathcal{B}\mathcal{U}[k] & \mathcal{B}\mathcal{U}[k+1] & -\Omega[k+2] & 0 \\ \phi^2\mathcal{B}\mathcal{U}[k] & \phi\mathcal{B}\mathcal{U}[k+1] & \mathcal{B}\mathcal{U}[k+2] & -\Omega[k+3] \\ & & \vdots & \end{bmatrix}}_{\Theta \in \mathbb{R}^{2L(P+1)\times DL(P+1)}} \underbrace{\begin{bmatrix} \mu[k] \\ \mu[k+1] \\ \mu[k+2] \\ \mu[k+3] \\ \vdots \end{bmatrix}}_{\Lambda \in \mathbb{R}^{DL(P+1)\times 1}}$$

$v = [\ell_{11} \cdots \ell_{D1}, \cdots, \ell_{1L} \cdots \ell_{DL}]^T \in \mathbb{R}^{D \cdot L \times 1}$ distance vector from current state to demo states

$$\min_{\Lambda} \frac{1}{2} \| z \|_H^2 + v^T \mu[k]$$

s.t.

$$\Lambda \geq 0$$

$$\underbrace{\begin{bmatrix} 1^T & & 0 \\ & \ddots & \\ 0 & & 1^T \\ & & & \ddots \end{bmatrix}}_{\in \mathbb{R}^{L\cdot(P+1) \times L(P+1)\cdot D}} \begin{bmatrix} \mu[k] \\ \mu[k+1] \\ \mu[k+2] \\ \mu[k+3] \\ \vdots \end{bmatrix} = 1$$

$$\Rightarrow \min_{\Lambda} \frac{1}{2} \Lambda^T [-]^T H [-] \Lambda + \varphi^T \cdot H \cdot [-] \Lambda + \varepsilon \cdot v^T \mu[k]$$

s.t.

$\vdots$

$$H \in \mathbb{R}^{2L(P+1) \times 2L(P+1)}$$