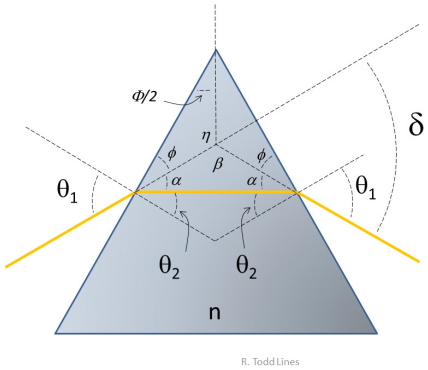
0.1 Calculation of n using a prism

Let's do a problem using the idea of refraction in a prism. Let's find the index of refraction of a the prism material. Suppose we make a prism as shown. We know the angle Φ and can measure the exit angle δ. In terms of these two variables, what is n?



Using the notation indicated in the figure, we choose θ_1 such that the interior ray is horizontal.[1] This is a refraction problem, so we will want to use Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Thus, we need to find θ_1 and θ_2. Knowing Φ and δ, and realizing $n_1 = 1$, we can find θ_2 and θ_1. Then use

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

to find n_2. Let's see one way to do this. In geometry it is fair game to extend lines and even make some new lines of our own. By doing this we can realize that

$$\theta_1 = \theta_2 + \alpha \tag{1}$$

and that

$$\delta = 180 - \beta \tag{2}$$

and it is also true that

$$180 = \beta + 2\alpha \tag{3}$$

Then

$$\delta = 2\alpha \tag{4}$$

[1] WARNING! in the upcoming homework problem you can't make the same assumptions!

and

$$\alpha = \frac{\delta}{2} \tag{5}$$

Now also realize that

$$90 = \alpha + \theta_2 + \phi \tag{6}$$

and

$$180 = \Phi + 2\alpha + 2\phi \tag{7}$$

or

$$90 = \frac{\Phi}{2} + \alpha + \phi \tag{8}$$

Then

$$\alpha + \theta_2 + \phi = \frac{\Phi}{2} + \alpha + \phi \tag{9}$$

$$\theta_2 = \frac{\Phi}{2} \tag{10}$$

We can put these in our equation for θ_1

$$\theta_1 = \theta_2 + \alpha \tag{11}$$

$$= \frac{\Phi}{2} + \frac{\delta}{2} \tag{12}$$

$$= \frac{\Phi + \delta}{2} \tag{13}$$

Now we can use Snell's Law

$$\sin(\theta_1) = n \sin(\theta_2) \tag{14}$$

$$\sin\left(\frac{\Phi + \delta}{2}\right) = n \sin\left(\frac{\Phi}{2}\right) \tag{15}$$

then

$$n = \frac{\sin\left(\frac{\Phi}{2}\right)}{\sin\left(\frac{\Phi+\delta}{2}\right)} \tag{16}$$

and since we know Φ and δ, we can find n.