# Chapter 18

# 18 Cameras and Magnifiers 3.2.6, 3.2.7

Fundamental Concepts

- Cameras and other imaging systems use a strange term called and f-number to tell what the intensity of the image will be

$$I \propto \frac{1}{(f/\#)^2}$$

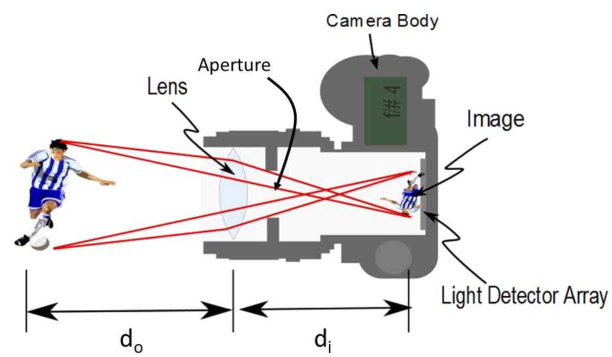
where $f/\#$ is the symbol for f-number and the f-number is given by

$$f/\# \equiv \frac{f}{D}$$

- Angular magnification $M = \frac{\theta}{\theta_o}$ describes how much bigger the image looks with an optical system compared to with just your eye.

## 18.1 The Camera

in 1900 George Eastman introduced the Brownie Camera. This event has changed society dramatically. The idea behind a camera is very simple.




The camera has a lens, often a compound lens made of several individual lenses, and a screen for projecting a real image created by the lens.

Let's take an example camera. Say we are at a family reunion and wish to take a picture of Aunt Sally. Aunt Sally is about 1.5 m tall. She is standing about 5 m away. Then to fit the image of Aunt Sally on our 35 mm detector, we must have

$$h_o = 1.5\,\mathrm{m}$$
$$h_i = 0.035\,\mathrm{m}$$
$$d_o = 5\,\mathrm{m}$$
$$f = 0.058\,\mathrm{m}$$

We wish to find $d_i$ and $m$. Let's do $m$ first.

$$\begin{aligned} m &= \frac{h_i}{h_o} = \frac{-0.035\,\mathrm{m}}{1.5\,\mathrm{m}} \\ &= -2.3333 \times 10^{-2} \end{aligned}$$

so our image is small and inverted. The small size we wanted. But now we know that the image in our cameras is upside down. A digital camera uses it's built-in computer to turn the image right side up for us on the display on the back of the camera.

Now let's find $d_i$.

$$\begin{aligned} d_i &= \frac{f d_o}{d_o - f} \\ &= 5.8681 \times 10^{-2}\,\mathrm{m} \\ &= 58.681\,\mathrm{mm} \end{aligned}$$

so our detector must be 58.681 mm from the lens.

Now suppose we want to photograph a 1000 m tower from 2 km away. Then

$$\begin{aligned} m &= -\frac{0.035\,\mathrm{m}}{1000\,\mathrm{m}} \\ &= -3.5 \times 10^{-5} \end{aligned}$$

and

$$
\begin{aligned}
d_i &= \frac{(0.058\,\mathrm{m})\,(2000\,\mathrm{m})}{2000\,\mathrm{m} - (0.058\,\mathrm{m})} \\
&= 5.800\,2 \times 10^{-2}\,\mathrm{m} \\
&= 58.002\,\mathrm{mm}
\end{aligned}
$$

Notice that the image distance changed, but not by very much. This is why you need a focus adjustment on the lens of a good camera. Objects far away require a different $d_i$ value than objects that are close. Usually you twist the lens housing to make this adjustment. The lens housing has a threaded screw system that increases or decreases $d_i$ as you twist. Cell phones and consumer cameras often have an motor that makes this adjustment for you. In some cameras you may see the lens move back and forth as someone takes a picture.

There are several things that govern whether a picture will be good. When you buy a quality manual lens, it will be marked in $f/\#$. The specification of an automatic lens will be given in terms of $f/\#$. To help us buy such lenses, we should understand what the terminology means.

Most things we want to take a snapshot of are much farther than 58 mm from the camera. For such objects we can revisit the magnification.

$$m = -\frac{d_i}{d_o}$$

but from the thin lens formula

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

If $d_o \gg f$ then we can say that $1/d_o \approx 0$ and so $d_i \approx f$. Then

$$m \approx -\frac{f}{d_o}$$

and we see that the size of the image is directly proportional to the focal distance. If we change the focal distance, we can change the size of the image. This is how a zoom lens works. A zoom lens is a compound lens, and the focal length is changed by increasing the distance between the component lenses (more on compound lenses soon). This is what your camera is doing when it zooms in and out when you push the telephoto button.

Remember we studied intensity

$$I = \frac{P}{A}$$

Photographic film and digital focal plane arrays detect the intensity of light falling on them. We can see that the area of our image depends on our magnification, which depends on $d_i$ and for our distant objects it is proportional to

$f$. The image area is proportional to $d_i^2 \approx f^2$. So we can say that the area is proportional to $f^2$. Then

$$I \propto \frac{P}{f^2}$$

The power entering the camera is proportional to the size of the aperture (hole the light goes through). A bigger aperture lets in more light. A smaller aperture lets in less light. If the camera has a circular opening, this area is proportional to the square of the diameter of the opening, $D^2$ so

$$I \propto \frac{D^2}{f^2}$$

This ratio is useful because it gives us a relative measure of how much intensity we get in terms of things we can easily know. Good cameras have changeable aperture sizes, and good lenses have changeable focal lengths. By using the combination of these two terms, we can ensure we will get enough light (but not too much) when we take the picture.

It would be good to give this ratio a special name. But instead, we named the ratio

$$f/\# \equiv \frac{f}{D} \tag{18.1}$$

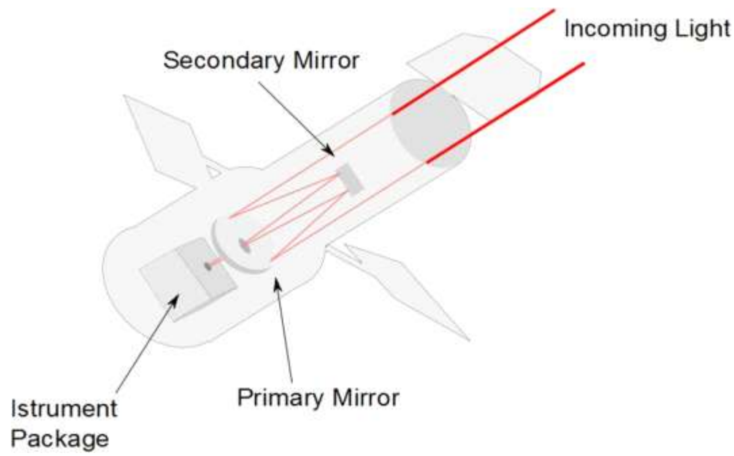
It is called the $f/\#$ (pronounced f-number) so

$$I \propto \frac{1}{(f/\#)^2} \tag{18.2}$$

So good cameras have adjustable lens systems marked in $f/\#'s$. Typical values are $f/2.8$, $f/4$, $f/5.6$, $f/8$, $f/11$, and $f/16$. Notice that since $I$ is proportional to $(1/f/\#)^2$ these common $f/\#$ values give an increase of intensity of a factor of 2 each time you change the $f/\#$ by one marked stop.

| $f/\#$ change | Intensity change factor |
|---|---|
| $f/11$ to $f/8$ | 2 |
| $f/8$ to $f/5.6$ | 2 |
| $f/5.6$ to $f/4$ | 2 |
| $f/4$ to $f/2.8$ | 2 |

This terminology is used for telescope design as well. The Hubble telescope is an $f/24$ Ritchey-Chretien Cassegrainian system with a 2.4 m diameter aperture. The effective focal length is 57.6 m.




It is important to realize that electronic (and biological) sensors don't react instantly to what we see. The intensity is

$$I = \frac{P}{A} = \frac{\Delta E}{\Delta t A}$$

so there is a time involved. The time it takes to collect enough light to form an image on the sensor is called the *exposure time*.

$$\Delta t = \frac{\Delta E}{IA}$$

So changing our $f/\#$ changes the needed exposure time by changing the intensity. How sensitive our camera sensor is also affects the exposure time. Modern sensors have adjustable sensitivity. The photography world gives the three letters ISO for the name for this detector sensitivity. There isn't a standard for exactly what ISO setting gives what exposure. Different manufacturers use slightly different numbers. But a change in ISO settings usually are equivalent to one $f/\#$ change in exposure.

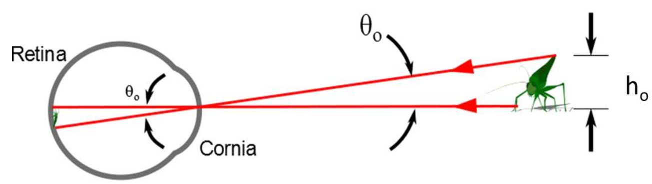
This is part of what a good photographer does in taking a picture. The photographer will adjust the $f/\#$ and the exposure time and ISO to get a photograph that is not too exposed (too light) or underexposed (to dark).

## 18.2 Angular Magnification

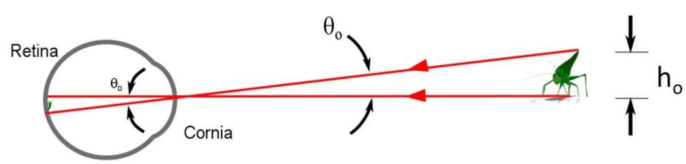
We already encountered the simple magnifier when we studied ray diagrams. But by this point in our study of optics you are probably wondering about our definition of magnification. If you are an Idahoan and are out hunting, when you look through your binoculars or scope you don't want to know how big the image is compared the actual dear, you want to know if you can see the deer better than you could with just your eyes. The magnification on your scope doesn't compare the image size to the object size, but it is comparison between two optical systems, one is just your eyes, the other is your eyes and the scope working together.

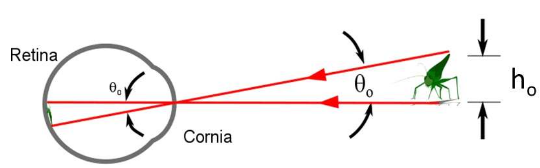
Let's use the simple magnifier that we know to define a new kind of magnification that does this comparison between two optical systems. We can use what we know about easy rays to draw to describe both optical systems. We usually use three principle rays, but for this analysis, let's just use one for each side of an object. Since the image is on the retina, we can see where these singular rays strike the retina and understand the size of the image on our eye light detection system. Let's choose to draw the rays that go straight through the middle of the lens of the eye (because they are the easiest ones!).

If we pick a ray from the top of our object that goes through the center of the lens, that ray won't seem to change direction at all. It will hit the retina to form the top of the image of the object. We can do the same for the bottom of the object. Then we can see from the next figure



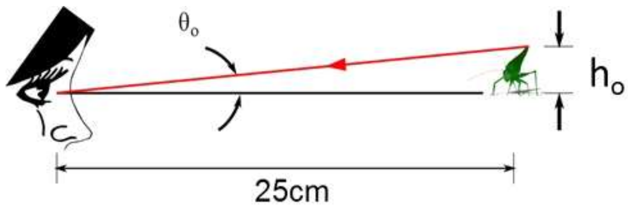

that the angle $\theta_o$ subtends[1] both the object and the image of the object. If you think about the previous figure, you will see that if the angle were to increase, so would the size of the image on the retina. We can increase this angle by, say, moving the object closer to our eyes.



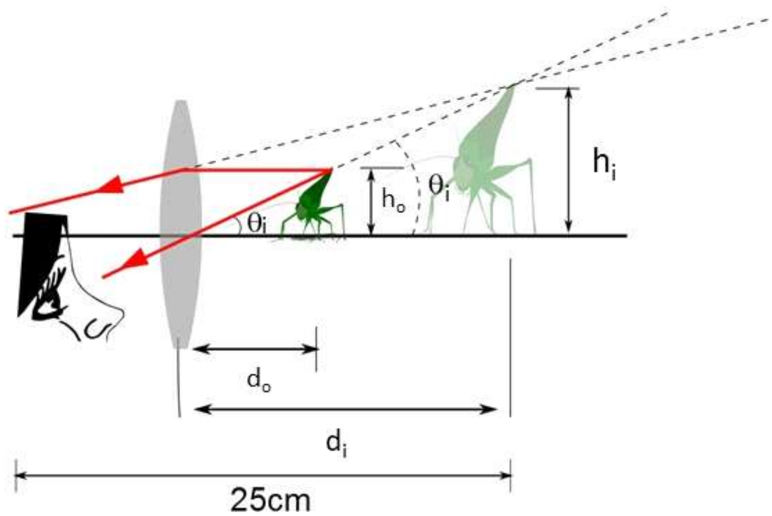

When we get to our 25 cm we reach the limit of the eye for focusing.

[1] The angle that "subtends" and object is the one who's bounding lines just hit either side of the object.




If we move the object any closer, it will appear blurry. We call this position, the closest point where we can place an object and still bring it into focus with our eye, the *near point.* Thus the maximum value of $\theta$ will be at the near point for unaided viewing. We will call this maximum unaided angle $\theta_o$.

But suppose we want to see this object in more detail. That is, we want to spread the image of the object over more of our retina so our retinal pixels do a better job of showing the details. We can use a magnifying glass to spread the light over more of our retina.

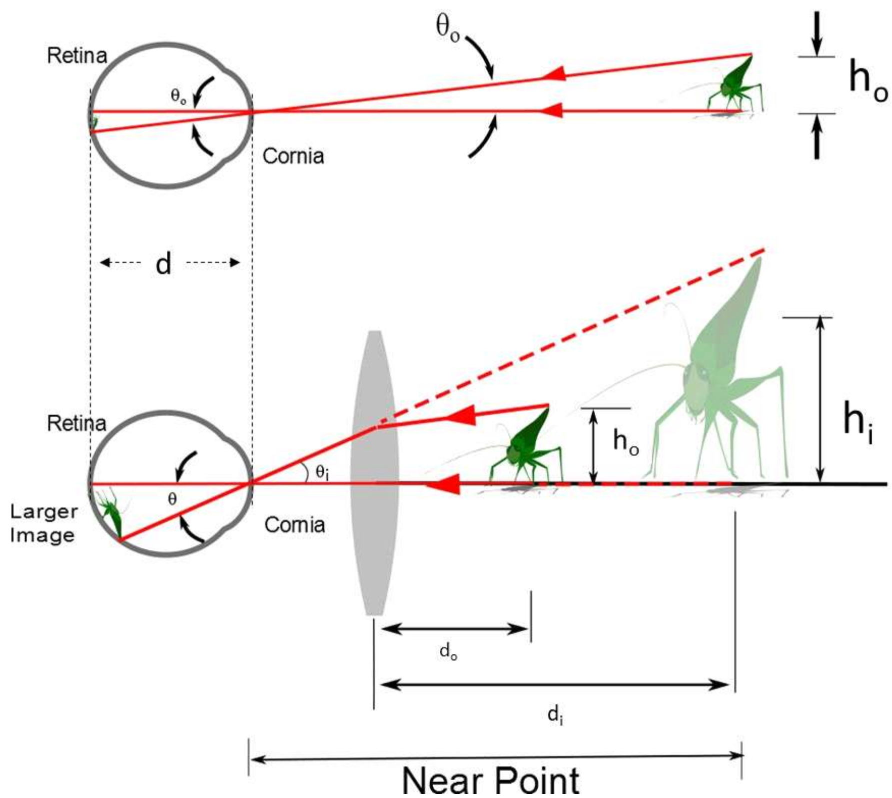



If we place the object closer to the magnifying glass than the focal distance($d_o < f$), then we have a virtual image with magnification

$$m = \frac{-d_i}{d_o} \tag{18.3}$$

which is larger than one and positive (because $d_i$ is negative).

But what we really want to know is how much bigger the image looks with the lens than it did without the lens. By looking at what happens to the rays when they enter our eye, we can see why the image looks bigger.




The magnifying glass has bent the light, and the bent rays make a bigger angle, $\theta_i$, so the image on our retina is bigger. Since the image fills more of our retina, we perceive the image as being bigger.

We need a mathematical formula that will tell us how much bigger the image on our retina will be. Notice from the figure that for just our eye (no lens)

$$\tan\theta_o = \frac{h_{i,eye}}{d}$$

and for the case with the lens

$$\tan\theta_i = \frac{h_{i,lens-eye}}{d}$$

so

$$\begin{aligned} h_{i,eye} &= d\tan\theta_o \\ h_{i,lens-eye} &= d\tan\theta_i \end{aligned}$$

Then if we compare the new, larger image on the retina formed with the lens-eye system to the one formed with just the eye, we get

$$\frac{h_{i,lens-eye}}{h_{i,eye}} = \frac{d\tan\theta_i}{d\tan\theta_o} = \frac{\tan\theta_i}{\tan\theta_o}$$

and if we once again use the small angle approximation

$$\frac{h'_{lens-eye}}{h'_{eye}} \approx \frac{\theta}{\theta_o}$$

This would tell us how much bigger our object looks when viewed with the magnifying glass compared to how it looked without the magnifying glass. This is just what we want! Let's give this a new symbol

$$M = \frac{\theta_i}{\theta_o} \tag{18.4}$$

Remember, this is really different than the magnification we have studied before. The magnification we have been using compared the size of the image with the size of the object. We call $M$ the *angular magnification.*

So, the angular magnification compares how big the object seems to be with and without a lens our lens system. It is really a comparison between the size of the area the diverging set of rays make on the retina formed with just our eye, and the one formed with the magnifier.

If the virtual image formed is farther than the near point of the eye, ($d_i >$ ˜25 cm) the image on our retina will be smaller than it would be at the near point because it is farther away. If the virtual image is closer than the near point, it will be fuzzy because the eye cannot focus closer than the near point. Thus, the value of $M$ will be maximum when $d_i$ for the magnifying glass is at the near point of the eye. We can find where to place the image so that we get maximum magnification. Taking just the magnifier, and placing the image at about $-25\,\text{cm}$,

$$\begin{aligned} \frac{1}{d_o} + \frac{1}{d_i} &= \frac{1}{f} \\ \frac{1}{d_o} + \frac{1}{-25\,\text{cm}} &= \frac{1}{f} \end{aligned}$$

and so

$$\frac{1}{d_o} = \frac{-25\,\text{cm} - f}{-f\,(25\,\text{cm})}$$

or

$$d_o = \frac{(25\,\text{cm})\,f}{25\,\text{cm} + f} \tag{18.5}$$

Using small angle approximations, we can write

$$\tan\theta_o = \frac{h_o}{25\,\text{cm}} \approx \theta_o$$

and

$$\tan\theta_i = \frac{h_o}{d_o} \approx \theta_i$$

then the maximum angular magnification is

$$\begin{aligned}
m_{\max} &= \frac{\theta_i}{\theta_o} = \frac{\frac{h_o}{d_o}}{\frac{h_o}{25\,\mathrm{cm}}} \\
&= \frac{25\,\mathrm{cm}}{\frac{25\,\mathrm{cm}f}{25\,\mathrm{cm}+f}} \\
&= \frac{25\,\mathrm{cm}+f}{f} \\
&= 1+\frac{25\,\mathrm{cm}}{f}
\end{aligned}$$

We can also find the minimum magnification by letting $d_o$ be at $f$. This gives

$$\theta_i = \frac{h}{f}$$

$$\begin{aligned}
m_{\min} &= \frac{\theta_i}{\theta_o} = \frac{\frac{h}{f}}{\frac{h}{25\,\mathrm{cm}}} \\
&= \frac{25\,\mathrm{cm}}{f}
\end{aligned}$$

We use the idea of a simple magnifier in combination with other lenses to make the magnification happen in telescopes, microscopes, and other instruments that magnify. So all of these systems can be described using the idea of an angular magnification.

Angular magnification is important. Many optical systems that we buy are really designed to make things seem bigger (or smaller) than they seem with just our eyes. Things like telescopes and microscopes and binoculars. We will see how to apply this idea of angular magnification to some such systems in our next lecture.