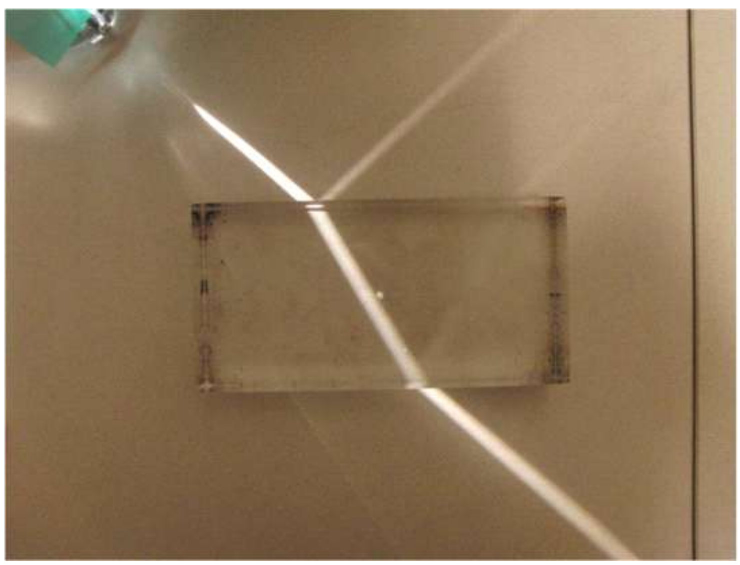
Chapter 13

13 Refraction 3.1.3, 3.1.4



We have studied reflection of light. Now let's consider what happens when light enters a material like glass or water.

Fundamental Concepts

- Refraction
- Total internal reflection

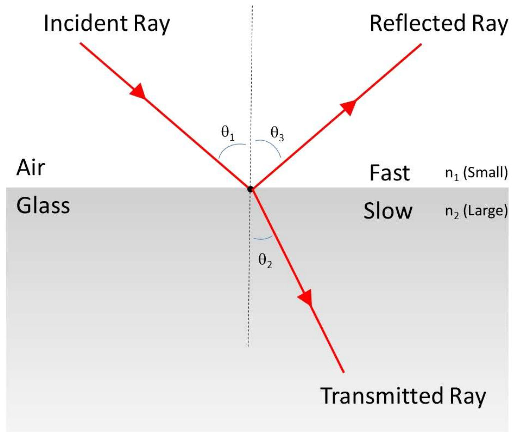
In the last lecture we split up electromagnetic waves into different kinds of light. We categorized the different kinds of light by noting they have different wavelength ranges. We found visible light, of course, but also infrared and ultraviolet and x-rays and radio waves. Not all surfaces reflect all the light. Some, like the lenses shown below, reflect some light at visible wavelengths, but are transparent so most of the light travels through them.

We need a way to deal with transparent materials. This is tricky, because different wavelengths of light penetrate different materials in different ways. As an example, this is also a lens

Infrared lens, but visible mirror (Image courtesy US Navy, image in the public domain)

but it clearly is not transparent at visible wavelengths. It is transparent in the infrared. So what might be transparent at one wavelength might not be at another.

When light travels into a material, we say it is transmitted. This is nothing new. Light is a wave, so the part that goes through is called the transmitted part. The situation is shown schematically below.



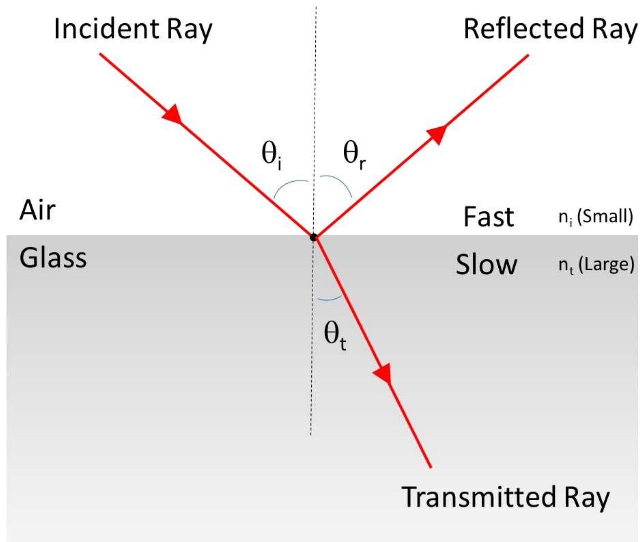
In the figure we see a ray incident on an air-glass boundary. Some of the light is reflected just as we saw before. But some passes into the glass. Notice that the angle between the normal and the new transmitted ray is *not* equal to the incident ray. We say the ray has been bent or *refracted* by the change in material. Many experiments were performed to find a relationship between the incident and the refracted angles. It was found that

$$\frac{\sin(\theta_2)}{\sin(\theta_1)} = \frac{v_2}{v_1} = \text{constant} \tag{13.1}$$

Many optics books write this as

$$\frac{\sin(\theta_t)}{\sin(\theta_i)} = \frac{v_t}{v_i} = \text{constant} \tag{13.2}$$

where the subscript i stands for "incident" and the subscript t stands for "transmitted."

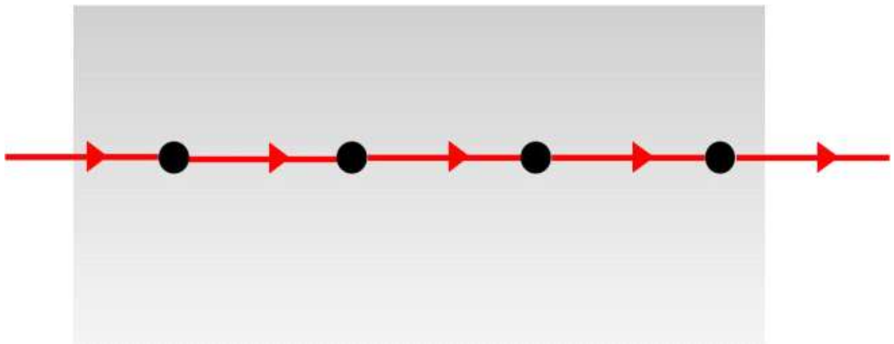


Note that we are using the fact that the speed of light changes in a material. We should probably recall why this should occur

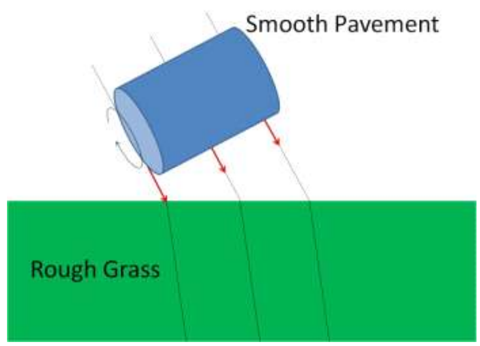
13.1 Speed of light in a material

In a vacuum, light travels as a disturbance in the electromagnetic field with nothing to encounter. In a material (like glass) the light waves continually hit atoms. We have not studied antennas, but I think many of you know that an antenna works because the electrons in the metal act like driven harmonic oscillators. The incoming radio waves drive the electron motion. Here each atom has electrons, and the atoms act like little antennas, their electrons moving and absorbing the light. But the atom cannot keep the extra energy[1], so it is readmitted. It travels to the next atom and the process repeats. Quantum mechanics tells us that there is a time delay in the re-emission of the light. This re-emitted light mixes with what is left of the original wave. And this causes the propagation of the combined light wave to slow down. Thus the speed of light is slower in a material.

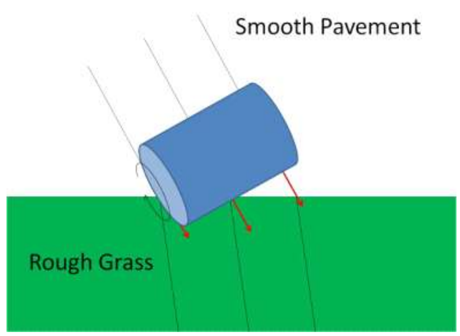
[1] If you get to take Modern Physics (PH279) and Quantum Physics (PH433) you will learn how this works.



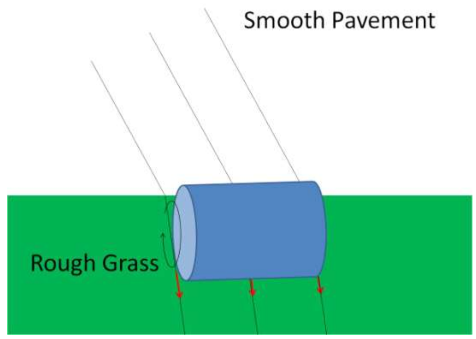
As a mechanical analog, consider a rolling barrel.



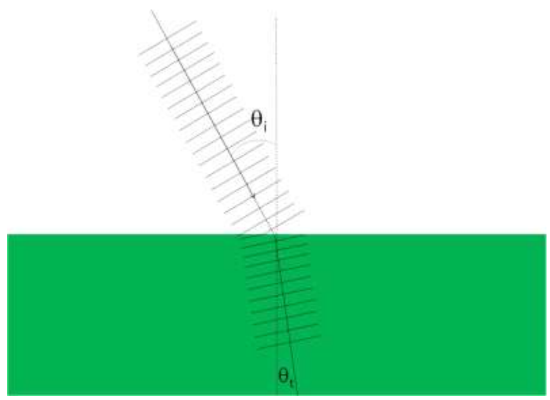
As the barrel rolls from a flat low-friction concrete to a higher-friction grass lawn, the friction slows the barrel. If the barrel hits the lawn parallel to the boundary (so it's velocity vector is perpendicular to the boundary), then the barrel continues in the same direction at the slower speed. But if it hits at an angle, the leading edge is slowed first.



This makes the trailing edge travel faster than the leading edge, and the barrel turns slightly.



We expect the same behavior from light.



We can see that the left hand side of the wave hits the slower (green) material first and slows down. The rest of the wave front moves quicker. The result is the turning of the wave.[2]

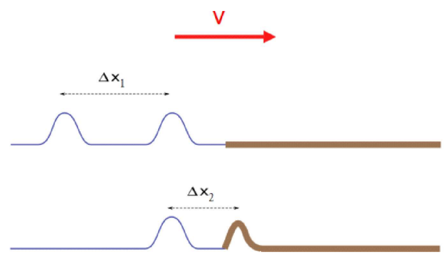
13.2 Change of wavelength

We have found that when a wave enters a material, its speed may change. But we remember from wave theory

$$v = \lambda f \tag{13.3}$$

But it is time to review: does λ change, or does f change? If you will recall, we found that the change in speed at the boundary changes the wavelength. Recall that if we go from a fast material to a slow material, the forward part of the wave slows and the rest of the wave catches up to it.

[2] Once again this is a bit of a simplification, but it will due for now. If you are lucky enough to take a junior level optics class, you will revisit this.



This will compress pulses, and lower the wavelength. Now that we know more about light we can also argue that f cannot change because

$$E = hf$$

If f changed, then we would either require an input of energy or we would store energy at the boundary because

$$\Delta f = \frac{\Delta E}{h}$$

This can't be true. If the wavelength changes, there is no change in energy.

And since

$$v_i = \lambda_i f$$

and

$$v_t = \lambda_t f$$

then the ratio

$$\frac{v_i}{v_t} = \frac{\lambda_i}{\lambda_t}$$

and we have a solution for the wavelength in the material

$$\lambda_t = \lambda_i \frac{v_t}{v_i}$$

which agrees with our previous analysis.

13.3 Index of refraction and Snell's Law

Because the equation

$$\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{v_t}{v_i} = \text{constant} \tag{13.4}$$

has a constant ratio of velocities, it is convenient to define a term that represents that ratio. We already have a concept that can help. The *index of refraction* is just such a term. It assumes that one speed is the speed of light in vacuum, c.

$$n \equiv \frac{c}{v} \tag{13.5}$$

Then for our example

$$\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{1}{n} \tag{13.6}$$

Suppose we don't have a vacuum (or air that is close to a vacuum). We can write our formula as

$$n_i \sin(\theta_i) = n_t \sin(\theta_t) \tag{13.7}$$

where we have

$$n_i \equiv \frac{c}{v_i} \tag{13.8}$$

and

$$n_t \equiv \frac{c}{v_t} \tag{13.9}$$

This is called *Snell's law of refraction*.

Using the index of refraction we can write our equation relating the ratio of velocities and wavelengths as

$$\frac{v_i}{v_t} = \frac{\lambda_i}{\lambda_t} = \frac{\frac{2}{n_i}}{\frac{c}{n_t}} = \frac{n_t}{n_i} \tag{13.10}$$

which gives

$$\lambda_i n_i = \lambda_t n_t \tag{13.11}$$

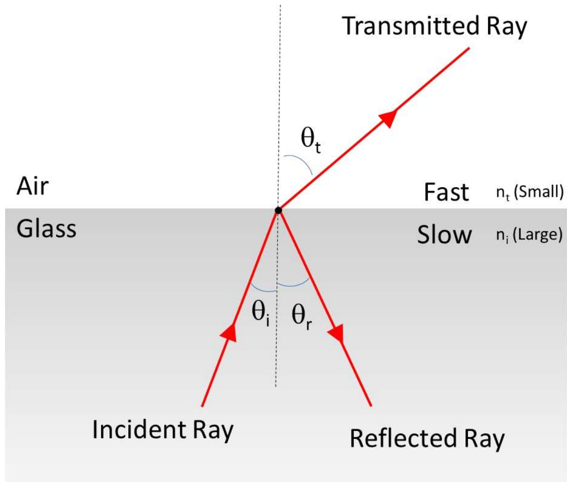
and if we have vacuum and a single material we can find the index of refraction from

$$n = \frac{\lambda}{\lambda_{in}} \tag{13.12}$$

where $\lambda_{in} = \lambda_t$ is the wavelength in the material and $\lambda = \lambda_i$ is the wavelength outside the material.

13.4 Total Internal Reflection

Up to now we have assumed that light was coming from a region of low index of refraction (air) into a region of high index of refraction (water or glass). We should pause to look at what can happen if we go the other way.



We start with Snell's law

$$n_i \sin\theta_i = n_t \sin\theta_t \tag{13.13}$$

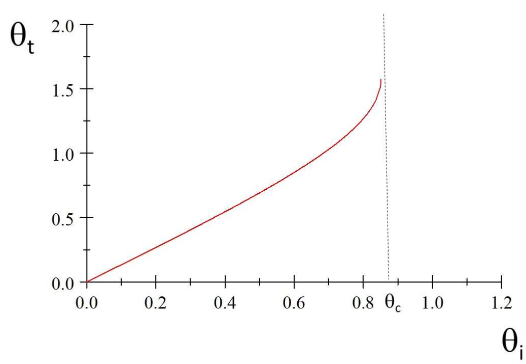
but this time $n = n_i$ and $n_t = 1$ so

$$n \sin\theta_i = \sin\theta_t \tag{13.14}$$

which gives

$$\theta_t = \sin^{-1}(n \sin\theta_i) \tag{13.15}$$

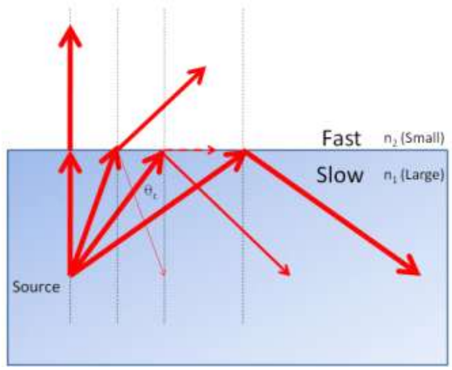
If we take $n = 1.33$ (water) we can plot this expression as a function of θ_i



we see that at $\theta_i = 0.850\,91\,\mathrm{rad}$ $(48.754\,^\circ)$ the curve becomes infinitely steep. If we use this value in our equation this gives

$$\begin{aligned} \theta_t &= \sin^{-1}(n \sin(0.850\,91)) && (13.16)\\ &= 1.570\,8\,\mathrm{rad} && (13.17)\\ &= 90\,^\circ && (13.18) \end{aligned}$$

The light skims along the edge of the water!



We can find the value of θ_i that makes this happen without graphing. Set $\theta_t = 90^\circ$ then

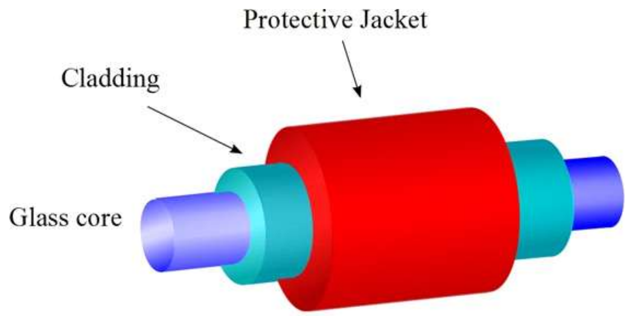
$$\theta_i = \theta_c \equiv \sin^{-1}\left(\frac{1}{n}\right) \tag{13.19}$$

We give this value of θ_i a special name. It is the *critical angle* for internal reflection. But what happens if we go farther than this ($\theta_i > \theta_c$). We will no longer have a transmitted ray. But the wave energy has to go somewhere! The ray will be reflected. This is why when you dive into a pool and look up, you see a region of the roof of the pool area (or sky) but off to the side of the pool the surface looks mirrored. It is also why you sometimes see the sides of a fish tank appear to be mirrored when you look through the front.

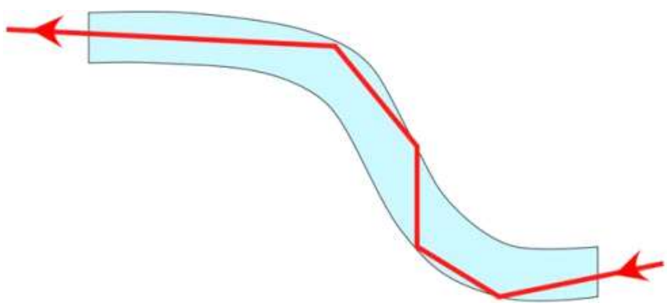
More importantly, it is why cut gems (like diamonds) sparkle. They capture the light with facets that are cut at angles that create total internal reflection. The light that enters the gem comes back out the front.

13.5 Fiber Optics

Beyond pretty pebbles, this effect is very useful! It is the heart and soul of fiber optics.

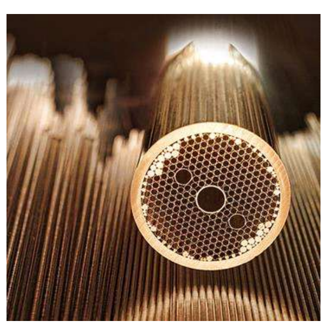


An interior material with a lower index of refraction is inclosed in a cladding with a higher index. This creates a light pipe that traps the light in the fiber.



Modern fibers don't always have a hard boundary. The fibers have a gradual change in index of refraction that changes the direction of the light gradually. This keeps the light in the fiber but tends to direct along the fiber so the beam is not crisscrossing as it goes.

The cutting edge of fiber design today uses hollow fibers or fibers filled with different index material.



Hollow-Core Fiber (Courtesy Defense Advanced Research Projects Agency (DARPA), image in the public domain)