Chapter 25

25 Fluids and Pressure 1.14.1, 1.14.2

We have spent about 2/3 of a semester studying waves. Waves are very regular motion of more than one object. But surely we can have less regular motion of more than one object!

To study such non-regular motion we are changing topics radically. PH121 taught us how individual objects move. We will use some of what we learned to address the question of how many objects move. To get started, let's review some basic properties of matter. Matter is made of many atoms. So matter is an example of many objects that might just move.

Fundamental Concepts

- Compressibility of fluids
- Density of fluids
- Pressure is a force spread over an area
- Pressure increases with depth in a fluid
- Barometers
- Manometers
- Buoyancy

25.1 Fluids

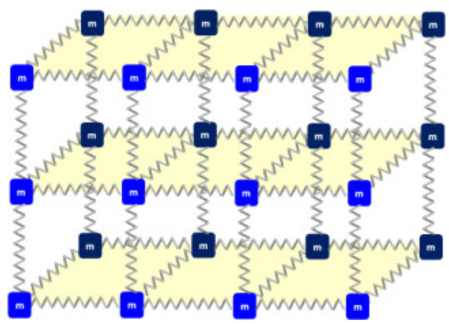
You are probably aware that there are four general states of matter

solid
liquid
gas
plasma

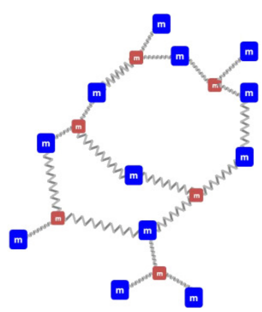
Believe it or not, plasma[1] is the most common, because stars are made of plasma. Planets are sometimes made of solids, liquids, or gases, but the great glowing stars are plasma. (I am ignoring the mysterious form of “dark matter” because so far we don’t know what it is). Plasma is a heated gas that is ionized. We will mostly ignore this state, because unless you are dealing with neon signs, fluorescent lights, or the like, on Earth you don’t encounter plasmas in every day experience.

Solids

We can view solids as having a set of forces that keep the molecules in place much as though they were attached using springs. Solids can have definite organization. If so, they are called crystals. You should observe the crystals around the Romney building if you have not already.



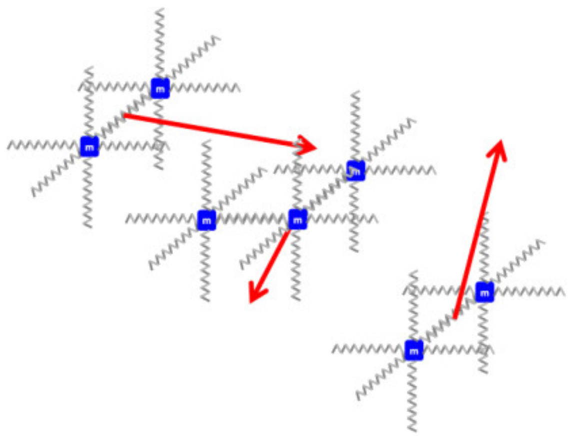
If the solid lacks definite order in its organization, it is called amorphous.



Liquids

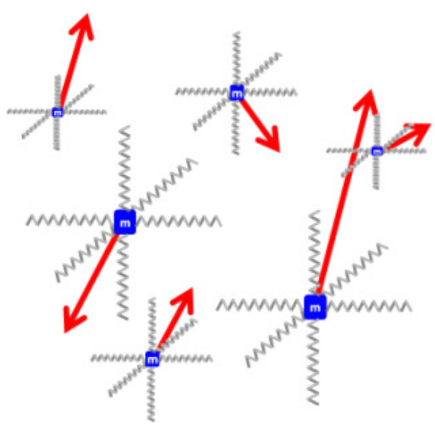
[1] This is not the kind of plasma that you donate!

The molecules in a liquid are less tightly bound than those in a solid. That is why they can flow. In the next figure, the atoms are bound by one spring-like force. But the atoms are not tied together in a tight set of bonds like a solid.



Gasses

The molecules in a gas are not bound to each other–not at all.



We have an intuitive feel for what a fluid is. But let's make a more formal definition.

25.1.1 What is a fluid?

For the next few lectures we will study fluids, but what is a fluid? A **Fluid** is a collection of molecules that are randomly arranged and are at most held together by weak cohesive forces and by forces exerted by the walls of a container.

Thus, liquids and gasses are definitely fluids. Solids are generally considered not fluids. But how about Jell-O^{TM}? or a combination of corn starch and water (sometimes called "ooblick")? These are non-Newtonian fluids. That is, things that are sort of solid and sort of not. But we will stick with things that are definitely fluids (at least at first), and generally fluids with negligible friction. This is a little like in PH121 when we studied frictionless surfaces. How many surfaces are truly frictionless? Very few! you might guess that there are few fluids that have no friction, and you would be right. But just like with PH121, the assumption of frictionless fluids makes the math easier, and that is good when we are starting a new topic.

25.2 Pressure

Vollyball Demo

We have already seen pressure in this course. But it's been a while, so let's review. Consider a situation where we ask six of our class members to come up and press on the ball from all directions. Suppose further that we ask each person to exert a force on the ball. And suppose we ask the person to use the area of their hand to exert the force. The motion of the ball, and even it's shape depended on both the force (magnitude and direction) and the area involved in each push.

Noticing that each person exerts a force but that the force is not acting on one point, but is spread out over an area, we recognize that each person is exerting a pressure on the ball

$$P \equiv \frac{F}{A} \tag{25.1}$$

Hopefully you studied pressure in PH121 or equivalent. It is a force spread over an area. In this case it is the force of a hand spread over the area of the hand.

Now consider a ball sitting in a room surrounded by air. The air is a fluid, so it's molecules are quite free to move around. Because there is some thermal energy in the room[2] the molecules will have some kinetic energy. So the air molecules will hit the ball. This will cause a force on the surface of the ball. And that force will be spread over the entire surface area of the ball. This is a force spread over an area. This is a pressure. We call this *air pressure*. This air force due to the colliding molecules is like having the hands pushing on the pushing on the ball.

This force due to individual molecules is small and only lasts during the collision. But in the room we have many molecules, and many molecules impact the ball. The molecules also impact the walls of the room. Suppose that every time a molecule bounces back from one wall it ends up headed back to the

[2] Even in Rexburg.

opposite wall bounces back again toward the first wall. If the molecules keep coming, there will be a force on the wall quite a bit of the time. At least, on average there is a force, anyway. This is the force that causes air pressure. The molecules impact the walls, and the ball, and us, and everything in the room all over the surface area of each object. The result is air pressure on every object in the room.

Likewise, the water pressure in a swimming pool is caused by moving water molecules. You should convince yourself that the reason the water stays in the pool is partly because the air molecules bounce against the water surface exerting a pressure on the water!

You might notice that there are more molecules bunched together in the water than in the air. We can describe how bunched together the molecules are by giving the mass of the molecules for a unit of volume

$$\frac{m}{V\!\!\!/} = \rho \tag{25.2}$$

We call this measure of bunched-up-ness the *density* of the material and we give it (unfortunately) the symbol ρ (the Greek letter "rho" pronounced like "row" as in "row a boat"). Sometimes it is useful to compare the density of a substance (like air) to the density of water

$$\frac{\rho_{\text{substance}}}{\rho_{water}} = \text{specific gravity}$$

This comparison is (unfortunately) called the *specific gravity*. But we need to be careful. We will find that density can depend on temperature. So let's define specific gravity as the comparison of the substance's density to the density of water at 4 °C at 1 atm of pressure. It turns out at this temperature and pressure

$$\rho_{water} = 1000 \frac{\text{kg}}{\text{m}^3}$$

which makes it convenient for comparison.

25.2.1 Working with the definition of pressure

We can work with equation 25.1 to define the force due to pressure

$$F \equiv PA$$

and can define a force due to the pressure at an element of area dA

$$dF \equiv PdA$$

Where there is a differential, expect that some time in the future we will integrate!

But before we go on, let's see what the units of pressure would be. We have a force divided by an area.

$$1 \frac{\mathrm{N}}{\mathrm{m}^2} = 1\,\mathrm{Pa}$$

The symbol is the Pa, and the unit is called the *pascal*. This is the name of a famous scientist.

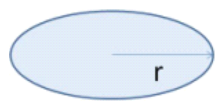
25.2.2 Pressure Example:

Let's do a pressure problem together,

Problem statement:

A 50.0 kg person balances on one heel of a pair of high heeled shoes. If the heel cross section is circular and has a radius of 0.5000 cm, what pressure does she exert on the floor?

Drawing



Variables

	Known	
M	Mass of person	$M = 50\,\mathrm{kg}$
r	Radius of person's heal	$r = 0.500\,\mathrm{cm}$
g	Acceleration do to gravity	$g = 9.8\frac{\mathrm{m}}{\mathrm{s}^2}$
	Unknown	
F	Force	
x	Coordinate Axis	
z	Coordinate Axis	
A	Area of woman's heal	

Basic Equations

$$\begin{aligned} F &= ma \\ A &= \pi r^2 \\ P &= \frac{F}{A} \end{aligned}$$

Symbolic Solution

$$A = \pi r^2$$

$$F = ma = Mg$$

$$P = \frac{F}{A} = \frac{Mg}{\pi r^2}$$

Numerical Solution

$$\begin{aligned}
P &= \frac{Mg}{\pi r^2} \\
&= \frac{50\,\mathrm{kg} * 9.8\frac{\mathrm{m}}{\mathrm{s}^2}}{\pi\,(0.500\,\mathrm{cm})^2} \\
&= \frac{490.0\frac{\mathrm{m}}{\mathrm{s}^2}\,\mathrm{kg}}{0.785\,40\,\mathrm{cm}^2}\frac{(100\,\mathrm{cm})^2}{\mathrm{m}^2} \\
&= \frac{490.0\frac{\mathrm{m}}{\mathrm{s}^2}\,\mathrm{kg}}{0.785\,40}\frac{100000}{\mathrm{m}^2} \\
&= 6.238\,9\times 10^6\,\frac{\mathrm{kg\,m}}{\mathrm{s}^2\,\mathrm{m}^2} \\
&= 6.238\,9\times 10^6\,\mathrm{Pa}
\end{aligned}$$

$$P = 6.24\,\mathrm{MPa}$$

Units Check

$$\frac{\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}}{\mathrm{cm}^2} = \frac{\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}}{\mathrm{cm}^2}\frac{(100\,\mathrm{cm})^2}{\mathrm{m}^2} = 10000\frac{\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}}{\mathrm{m}^2} = 10000\,\mathrm{Pa}$$

Units
Check
Reasonableness

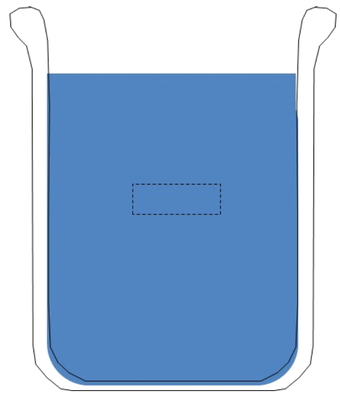
This seems like a large number, but I have had a high heeled person step on my toe, so I believe it! We can compare to average atmospheric pressrue which is about 1.13×10^5 Pa. Compared to this the toe crunch seems pretty reasonable.

25.3 Variation of Pressure with Depth

If you have been in a swimming pool, you probably have noticed that the pressure due to the water feels different at the surface than it does at the bottom of the deep end. Intuitively, we can say that the pressure at the bottom is larger than the pressure at the top. Let's see if we can show that this is true. We will do this in a few different ways.

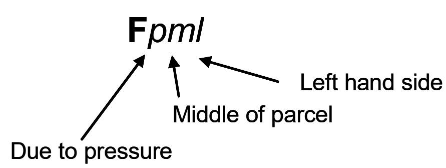
25.3.1 Pressure variation in liquids

Take a glass of water or some other liquid.

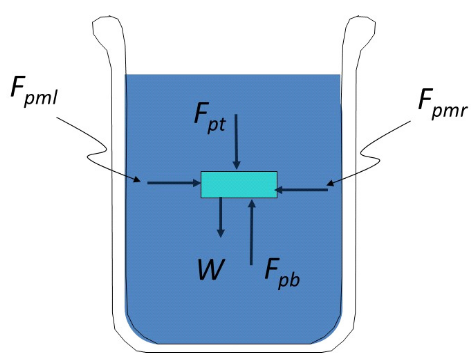


And let's look at just a part of the liquid (the section inside the box-shaped section with the dotted line around it in figure ??). The box shaped part of the liquid looks like, well, a box. And an old word for "box" is " parcel." Let's treat this "parcel of fluid" as a distinct body and look at the forces acting on it.

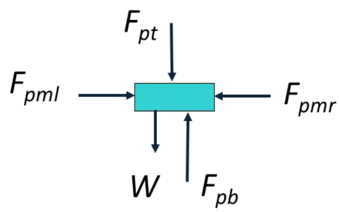
We need some way of labeling our forces. My way is kind of simple, but will work for now.



Consider Newton's second law. The sum of the forces must be equal to zero. Why?–think of the acceleration of the parcel of liquid. It's not accelerating.



And what we have in our drawing looks a lot like a free body diagram for our parcel of liquid. We could draw the free body diagram like this



where you should notice that we are drawing the forces with their points on the part of the parcel where the force acts. That is different than what we did in Principles of Physics I. This is because we are going to turn these forces into pressures using

$$P = \frac{F}{A}$$

so we want to identify the area that the force acts on. Let's write out Newton's second law for our parcel of liquid.

$$\overrightarrow{F}_{net} = m\overrightarrow{a} = 0 = \overrightarrow{F}_{pt} + \overrightarrow{F}_{pb} + \overrightarrow{F}_{pmr} + \overrightarrow{F}_{pml} + \overrightarrow{W}$$

but just like in Principles of Physics I we will break this equation into components.

$$\begin{aligned} F_{net_x} &= 0 = 0 + 0 - F_{pmr} + F_{pml} - 0 \\ F_{net_y} &= 0 = -F_{pt} + F_{pb} - 0 + 0 - W \end{aligned}$$

The x-part tells us that

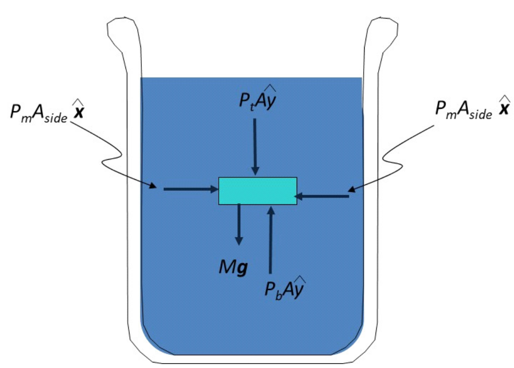
$$F_{pmr} = F_{pml}$$

which is not too much of a surprise. It seems reasonable that the side forces must be equal if the parcel doesn't accelerate. The y-part gives

$$F_{pb} - F_{pt} = W$$

Now let's use our pressure equation. The definition of pressure gives

$$F = PA$$



So, for the top of our parcel of liquid, the force on the top must be

$$F_{pt} = P_t A$$

and for the bottom the force must be

$$F_{pb} = P_b A$$

Recalling that

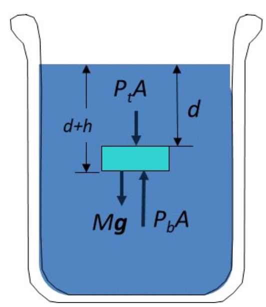
$$m = \rho \cancel{V} \tag{25.3}$$

where ρ is the density and $\cancel{V}$ is the volume, and substituting, we get

$$\begin{aligned}
F_{pb} - F_{pt} &= W \\
P_b A - P_t A &= -mg \\
(P_b - P_t)\, A &= -\rho \cancel{V} g \\
(P_b - P_t)\, A &= -\rho h A g
\end{aligned}$$

where we have used the fact that the volume must be the area of the top or bottom, A, multiplied by the length, h, of our parcel.

$$\cancel{V} = Ah \tag{25.4}$$



We can solve this for the pressure at the bottom

$$P_b = P_t + \rho g h$$

Units Check

$$\text{Pa} \Bumpeq \text{Pa} + \frac{\text{kg}}{\text{m}^3}\frac{\text{m}}{\text{s}^2}\,\text{m}$$

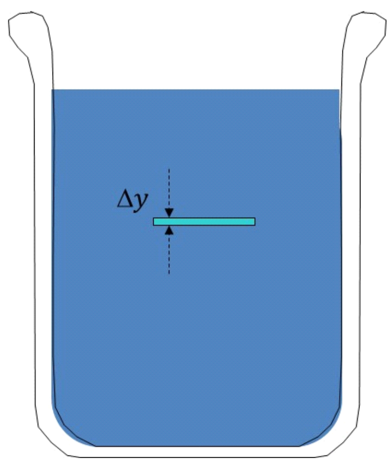
the last term can be written as

$$\left(\frac{\text{kg}\,\text{m}}{\text{s}^2}\right)\frac{1}{\text{m}^2} \Rightarrow \frac{\text{N}}{\text{m}^2} \Rightarrow \text{Pa}$$

so the units check.

This is a profound statement! (and a new basic equation for us). The pressure is larger at the bottom of the pool than the top. And it is larger by the amount ρgh where ρ is the liquid density, g is the acceleration due to gravity, and h is how far down we go to get to the bottom of our parcel.

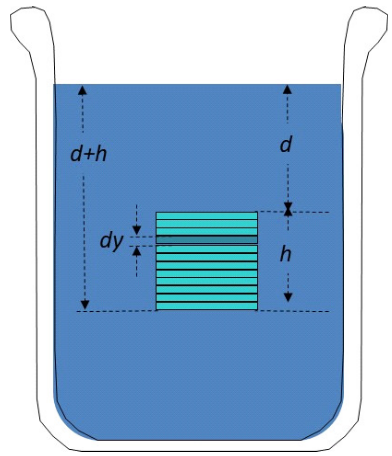
You might have thought of an objection in how we came up with this equation for the pressure at the bottom of a parcel of liquid. We said the sideways forces on the parcel were equal. But now we know the pressure increases with depth. Won't that affect the side forces so that the force on the bottom of the side is larger than the force on the top of the side? Of course the answer is yes. We could solve this problem by making the sides of the box infinitesimally small. Let's call the parcel height Δy



With our parcel of liquid so very thin now the pressure on the side can't have changed much from the top to the bottom of the parcel. So now we know the side forces cancel. We could say that the change in pressure from the top to the bottom

$$\begin{aligned} P_b &= P_t + \rho g \Delta y \\ \Delta P_{small} &= \rho g \Delta y \end{aligned}$$

And we could consider making a larger parcel to be made of many small parcels of thickness Δy.



For each of the little parcels the side forces cancel. We just have top and bottom forces. For the topmost parcel we would have

$$P_{b1} = P_t + \rho g \Delta y_1$$

And note, the bottom pressure from the topmost parcel is the top pressure for the second parcel

$$\begin{aligned} P_{b2} &= P_{t2} + \rho g \Delta y_2 \\ &= P_{b1} + \rho g \Delta y_2 \end{aligned}$$

And the bottom pressure from the second parcel is the top pressure for the third parcel

$$\begin{aligned} P_{b3} &= P_{t3} + \rho g \Delta y_3 \\ &= P_{b2} + \rho g \Delta y_3 \end{aligned}$$

To get to the bottom of the big parcel we would have

$$\begin{aligned} P_{b(total)} &= P_t + \sum_{i=1}^{N} \rho g \Delta y_i \\ &= P_t + \rho g \sum_{i=1}^{N} \Delta y_i \\ &= P_t + \rho g h \end{aligned}$$

just as we thought.

Of course we could take a limit to make $\Delta y \to dy$ and then we would have

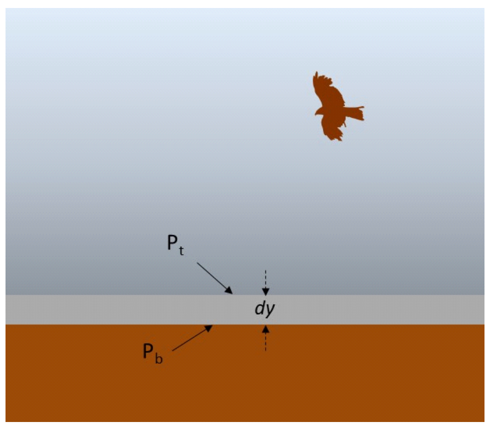
$$\begin{aligned} P_b &= P_t + \int_d^{d+h} \rho g dy \\ &= P_t + \rho g y|_d^{d+h} \\ &= P_t + \rho g \left((d+h) - d\right) \\ &= P_t + \rho g h \end{aligned}$$

just as we said before.

Note that we used a box shaped volume, but the result is general. To see this, view any arbitrary volume as consisting of little boxes. As we make the box size small, we approximate the actual volume better and better until it can be exact. Since it works for each box that makes up the volume individually, it will work for the whole volume (it takes a minute to think about this).

25.3.2 Pressure variation in gasses

You probably noticed that in our last section we didn't let ρ change. Another way to say that the density doesn't change is to call our liquid *incompressible*. Liquids are fluids, but they are difficult to compress. So to a good approximation their density doesn't change. But gasses are also fluids, and it is easy to compress a gas. We can't make the approximation of incompressibility for gasses.



We don't know enough yet to know how ρ changes with y. But if you took high school physics or chemistry we can make an approximation using the Ideal Gas Law. If you have never heard of the Ideal Gas Law, don't despair. We will study it soon. But for now here is the equation

$$P\cancel{V} = Nk_BT$$

where P is the pressure, $\cancel{V}$ is the volume of the air, n is a number density (how many air molecules per unit volume), $k_B = 1.3806568 \times 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$ is a constant, and T is the temperature. We can solve this for pressure

$$P = \frac{Nk_BT}{\cancel{V}}$$

We can change the form of this a bit by realizing that the number of molecus is the total mass of our gas divided by the mass of one gas molecule.

$$N = \frac{m_{total}}{m_{molecule}}$$

so that

$$P = \frac{\frac{m_{total}}{m_{molecule}} k_B T}{\cancel{V}}$$

then

$$P = \frac{\frac{m_{total}}{\cancel{V}} k_B T}{m_{molecule}} = \rho \frac{k_B T}{m_{molecule}}$$

and we can solve this for the density so we can see how density of the gas changes with pressure.

$$\frac{P m_{molecule}}{k_B T} = \rho$$

Now we can use our equation for a small parcel of fluid (gas this time)

$$P_b = P_t + \rho g \Delta y$$

but this time we want to know P_t at the top so let's solve for that.

$$P_t = P_b - \rho g \Delta y$$

and write it as

$$\frac{\Delta P}{\Delta y} = -\rho g = -\frac{P m_{molecule} g}{k_B T}$$

This seems pretty terrible, but we can take the limit as $\Delta y \rightarrow dy$

$$\frac{dP}{dy} = -P \left(\frac{m_{molecule} g}{k_B T} \right)$$

and we note that so long as the temperature doesn't change, all the things in the parentheses are constant. So we could give this group of constants a symbol

$$\alpha = \frac{m_{molecule} g}{k_B T}$$

Then our equation for how pressure varies with height is just

$$\frac{dP}{dy} = -\alpha P$$

and this is a nice differential equation. But rather than use our differential equations class (that we haven't taken) to solve this let's rearrange it

$$\frac{dP}{P} = -\alpha dy$$

and now let's integrate over the height of our parcel of air from $y = 0$ to $y = h$. On the pressure side we need to integrate from P_b to P_t

$$\int_{P_b}^{P_t} \frac{1}{P} dP = -\int_0^h \alpha dy$$

we get

$$\ln P|_{P_b}^{P_t} = -\alpha \; y|_o^h$$
$$\ln(P_t) - \ln(P_b) = -\alpha h$$

Using one of our favorite log identities $\ln a - \ln b = \ln \frac{a}{b}$ we can rewrite this as

$$\ln\left(\frac{P_t}{P_b}\right) = -\alpha h$$

and we can un-log this by exponentiating both sides

$$\frac{P_t}{P_b} = e^{-\alpha h}$$

which gives finally

$$P_t = P_b e^{-ah}$$

We could let our parcel of air be arbitrarily tall by letting the height be $h = y$ above the surface ($y = 0$) and making $P_t = P(y)$ and letting the surface pressure $P_b = P_{surface}$. Then

$$P(y) = P_{surface} e^{-ay}$$

but this can only work for small distances above the Earth's surface because in reality the temperature also changes with y. But in our classrooms this approximation is good enough (because the hvac system is keeping the whole lab at the same temperature).

25.4 Pressure Measurements

Now that we understand pressure and how it changes in fluids, we can study several pressure measurement devices. But there are some quirks in pressure measurement. Humans have measured pressure for a long time. But for most of our existence what we cared about was how much larger a pressure was than the surrounding air pressure. Think of the pressure inside a steam engine. The steam pressure has to be larger than atmospheric pressure to make the steam engine work. So humans invented pressure gages to put on things like steam engines that tell us how much higher than atmospheric pressure our particular pressure is. We call this a *gauge pressure.* But now days we understand that most of the universe is a vacuum. And that vacuum of space has almost no pressure.[3] We need to calculate the *absolute* pressure, and if your gauge measured gauge pressure you need to add atmospheric pressure to your reading.

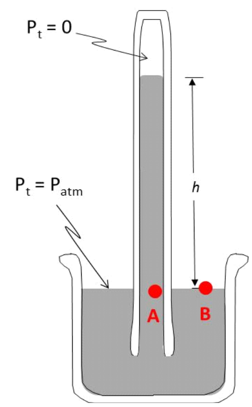
$$P_{abs} = P_{gague} + P_{atmospheric}$$

But with this understanding, let's look at pressure measuring devices.

[3] You might have heard an old saying which says that nature abhors a vacuum. But since most of nature is free space, this old saying only applies to things on or in planets or stars. The universe seems to love vacuum.

25.4.1 The Barometer

The barometer is a common pressure measuring device. It consists of



1. a long tube closed at one end
2. a dish
3. Mercury or another fluid

The dish and the tube are filled with the fluid. The tube is inverted in the dish. The pressure at the top of the tube is essentially zero. This is because the weight of the fluid pulls the mass of fluid downward. If the fluid is mercury, it is massive enough to leave a vacuum behind.

The pressure at point A and point B must be the same (or fluid would flow until $P_A = P_B$). This might be a little hard to see. But remember the air has a pressure, and just because it is a gas it does not mean that pressure is lower than the liquid pressure (think of the gas pressure in a hurricane that blows over trees and buildings).

It would be useful to find out how high up the mercury (or other fluid) will stay. And we have a way to do this! We know

$$P_b = P_t + \rho g h$$

At the top we have zero pressure, so $P_t = 0$. Note, this is not atmospheric pressure, **it is vacuum**. How did we get vacuum at the top? We got it by filling the tube with mercury and then turning the tub upside down. The mercury is heavy. So heavy that it falls down the tube a bit, leaving nothing behind. So there is no air at the top of the tube, just open tube because the heavy mercury moved downward. But the mercury can only fall down so far. The air pressure on the mercury in the dish pushes, keeping the mercury from all falling out. Then for this special case

$$P_b = 0 + \rho g h \tag{25.5}$$

And we can solve for the height of the fluid

$$h = \frac{P_b}{\rho g} \tag{25.6}$$

Let's consider what P_b must be. The air outside the barometer pushes down on the liquid mercury. Remember, when we first construct a barometer, we fill the tall tube with fluid and then turn it upside down in the dish. The fluid will fall until the force due to air pressure matches the weight of the column of fluid. Then the pressure at B must be atmospheric pressure. Since the pressure at point A equals the pressure at point B (think $P_b = P_t + \rho g h$), we can use this as a convenient P_b. So

$$P_b = P_{atm}$$

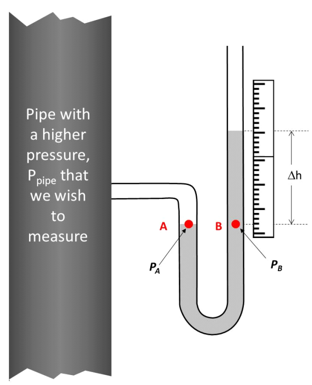
and

$$h = \frac{P_{atm}}{\rho g} \tag{25.7}$$

Thus as the atmospheric air pressure changes, the height of the mercury changes. We often hear of atmospheric pressure given in mm of Hg. This is why.

25.4.2 Manometer

The manometer finds an unknown pressure



The atmospheric pressure P_{atm} is applied on one end of the tube. In our example it is on the right side, where the tube is open at the top. The pressure to be tested is applied to the other side, in our case, the left side. The pressure at point A must equal the pressure at point B. Why? (think of their heights and $\rho g h$). The pressure at point A is the pressure to be measured. Again

$$P_b = P_t + \rho g h$$

where we take P_A to be the test pressure and for this design the tube above B is open to the atmosphere so the top pressure will be atmospheric pressure.

$$P_A = P_B = P_{atm} + \rho g h$$

We can measure the height h, and knowing P_{atm} we can calculate P_A.

These are just two of many types of pressure measuring devices. In the next lecture we will use what we know of pressure to design some useful devices like hydraulic jacks and things that float (like boats).