Chapter 19

19 Optical Systems and back to waves 3.2.8, 3.3 1

So far we have looked at single lenses and how they work. But we hinted that actual camera lenses and even our eyes are have lens systems that are made of more than one lens. We should look at how lens systems work.

We should also return to our study of waves because it turns out that the wave nature of light gives us a fundamental limit on how good our imagery can be from our cameras we design.

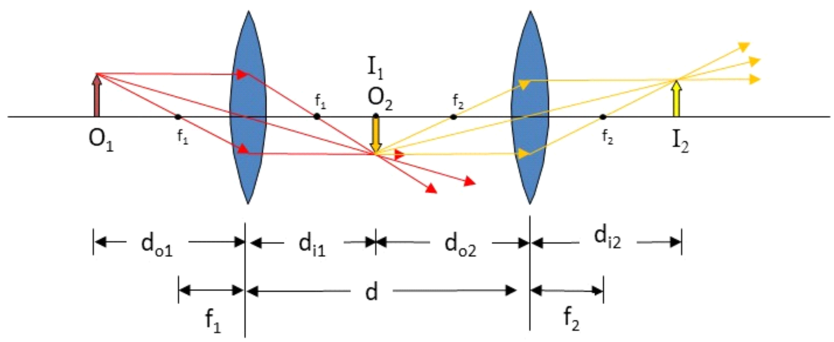
Fundamental Concepts

- The magnification of a two-lens system is just the product of the magnifications of the individual lenses $m_{\text{combined}} = m_1 m_2$
- If the two lenses in a two-lens system are placed so the distance between them is essentially zero, then the focal length of the two-lens system is given by
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$
where f_1 and f_2 are the focal lengths of the individual lenses
- If the two lenses in a two-lens system are not placed so the distance between them is essentially zero, the previous equation is not valid!
- Telescopes and Microscopes are double lens systems
- Light waves experience constructive and destructive interference like other waves
- Young's double slit experiment

19.1 Combinations of Lenses

We found that to correct chromatic aberration we used two lenses. Together they are called an "achromat" and all good cameras use achromats to fix chromatic aberration. We can do something similar to correct for spherical aberration. But in each case, we are combining two lenses (or even more!). And now we have microscopes that have two lenses. How do we predict what the lens system will do? It turns out that we have all we need to know to combine lenses already.

To combine lenses, we do the same thing we did for the two surfaces of a thin lens. We form the image from the first lens as though the second lens is not there. Then we use the image from the first lens as the object for the second lens. Suppose we take two lenses of focal lengths f_1 and f_2 and place them a distance d apart.



Because this system would use a magnified image as the object for lens 2, the final magnification is the product of the two lens magnifications

$$m_{\text{combined}} = m_1 m_2 \tag{19.1}$$

For the first lens we have

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \tag{19.2}$$

where d_{i1} is our first lens image distance. We can solve for d_{i1}

$$d_{i1} = \frac{d_{o1} f_1}{d_{o1} - f_1} \tag{19.3}$$

We then take as the second object distance

$$d_{o2} = d - d_{i1}$$

and we use the lens formula again.

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2}$$

and again find the image distance

$$d_{i2} = \frac{d_{o2} f_2}{d_{o2} - f_2}$$

but we can use our value of d_{o2} to find

$$\begin{aligned} d_{i2} &= \frac{(d - d_{i1}) f_2}{(d - d_{i1}) - f_2} \\ &= \frac{(d - d_{i1}) f_2}{d - d_{i1} - f_2} \end{aligned}$$

We have and expression relating the image distances, d, d_{i1}, d_{i2} and f_2. But we would really like to have an expression that relates d_{o1} and d_{i2}. Lets use

$$d_{i1} = \frac{d_{o1} f_1}{d_{o1} - f_1}$$

and substitute it into our expression for d_{i2}

$$d_{i2} = \frac{\left(d - \frac{d_{o1} f_1}{d_{o1} - f_1}\right) f_2}{d - \frac{d_{o1} f_1}{d_{o1} - f_1} - f_2}$$

This looks messy, but we can do some simplification

$$d_{i2} = \frac{d f_2 - \frac{d_{o1} f_1 f_2}{d_{o1} - f_1}}{d - f_2 - \frac{d_{o1} f_1}{d_{o1} - f_1}} \tag{19.4}$$

Well, it is still a little messy, but we have achieved our goal. We have d_{i2} in therms of the focal lengths, d, and d_{o1}.

Suppose we let $d \rightarrow 0$. That would mean the two lenses are right next to each other, like for an achromat. Then

$$\begin{aligned} d_{i2} &= \frac{-\frac{d_{o1} f_1 f_2}{d_{o1} - f_1}}{-f_2 - \frac{d_{o1} f_1}{d_{o1} - f_1}} \\ &= \frac{\frac{d_{o1} f_1 f_2}{d_{o1} - f_1}}{\frac{f_2 (d_{o1} - f_1)}{d_{o1} - f_1} + \frac{d_{o1} f_1}{d_{o1} - f_1}} \\ &= \frac{d_{o1} f_1 f_2}{f_2 d_{o1} - f_2 f_1 + d_{o1} f_1} \\ &= \frac{d_{o1} f_1 f_2}{d_{o1} (f_2 + f_1) - f_2 f_1} \end{aligned}$$

So

$$d_{i2} = \frac{d_{o1} f_1 f_2}{d_{o1} (f_2 + f_1) - f_2 f_1}$$

Lets undo the math that brought us d_{i2} in the first place

$$\begin{aligned}\frac{1}{d_{i2}} &= \frac{d_{o1}\left(f_2+f_1\right)-f_2f_1}{d_{o1}f_1f_2} \\ &= \frac{d_{o1}\left(f_2+f_1\right)}{d_{o1}f_1f_2}-\frac{f_2f_1}{d_{o1}f_1f_2} \\ &= \frac{\left(f_2+f_1\right)}{f_1f_2}-\frac{1}{d_{o1}}\end{aligned}$$

or

$$\frac{1}{d_{i2}}+\frac{1}{d_{o1}}=\frac{\left(f_2+f_1\right)}{f_1f_2}$$

Which looks very like the lens formula with

$$\frac{1}{f}=\frac{\left(f_2+f_1\right)}{f_1f_2}$$

If we unwind this expression, we find

$$\frac{1}{f}=\frac{f_2}{f_1f_2}+\frac{f_1}{f_1f_2}$$

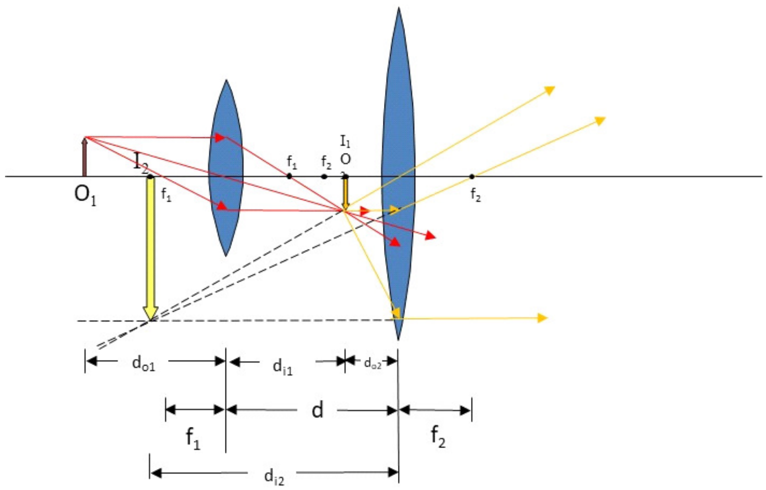
$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \tag{19.5}$$

This is how we combine thin lenses. We see that the two lenses are equivalent to a single lens with focal length f as long as they are close together.

Of course, we had to place our lenses right next to each other for this to work. This is not the case for a telescope or microscope. We should look at such a case. There is no need for more math. We can go back to equation (19.4).

$$d_{i2}=\frac{df_2-\frac{d_{o1}f_1f_2}{d_{o1}-f_1}}{d-f_2-\frac{d_{o1}f_1}{d_{o1}-f_1}}$$

But let's look at a case using ray diagrams. For this case, let's take two lenses, and let's have the first lens make a real image. Once again, let's have that image be the object for the second lens. But this time, let's move the second lens so that the image from the first lens (object for the second lens) is closer to the second lens than f_2. If that is the case, the second lens works like a magnifier. The final image is enlarged.

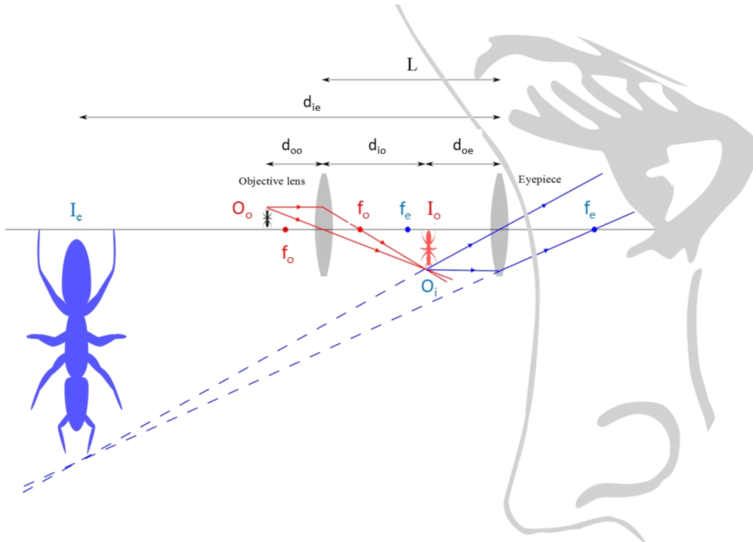


This is the basis of both microscopes and telescopes!

19.2 The Microscope

Microscopes are designed to allow us to see very small things. They go beyond a simple magnifier. But it turns out we don't have to go very far beyond a magnifying glass. To see things that are very small, we add another lens to our simple magnifier. We will place this lens near the object and call this lens the *objective* because it is next to the object. We will keep a simple magnifier and place it near the eye. This lens will be called the *eyepiece* because it is near your eye.

The objective will have a very short focal length. The eyepiece will have a longer focal length (a few centimeters).



We separate the lenses by a distance L where

$$\begin{aligned} L &> f_o \\ L &> f_e \end{aligned}$$

We place the object *just outside* the focal point of the objective lens. The image formed by the objective lens is then real and inverted. Then (and here is the trick we just learned!) we use this real image formed by the objective lens as the new object for the eyepiece. For a real image, there is really light there. So a real image can act like light bouncing off of an object. We can use the real image as an object for the eyepiece.

The image formed by the eyepiece is upright and virtual, but it looks upside down because the object for the eyepiece (the image made by the objective) is upside down.

To say it once more, what we are doing is forming a real image using the objective lens and then using the eyepiece lens as a simple magnifier to make a larger virtual image of the real image made by the objective lens. We could find d_{ie} using equation (19.4). But this is a microscope. We want to know how much bigger our final image is.

Of course, we really want to know the magnification of the system. It's not just the eyepiece that will magnify. We know what the eyepiece will do because it is being used as just a simple magnifier. Recall that magnifications are a factors. Think from our basic equation

$$m = \frac{h_i}{h_o}$$

tells us that

$$h_i = m h_o$$

or in words, m is the factor by which h_i is bigger (or smaller) than h_o. So magnifications are factors. For example, d_i could be 10 time bigger than d_o. And we have two lenses. Each lens will have a magnification of it's object. But how do we combine the magnifications of the two lenses?

To find the overall system magnification we start with the magnification of the eyepiece M_e, say, 20 times bigger than it would look with just our eye. so whatever the object, the eyepiece gives us a virtual image of that object that is M_e times bigger than it would look without the eyepiece. But the object in our case is the real image formed by the objective lens. And this real image may be bigger than the actual object. If you think about it, it makes sense that if the objective makes the object look 10 times bigger, and the eyepiece makes the image look 20 times bigger than if you looked at it with your eye, the system makes it look 200 times bigger. The combined magnification for the two-lens system is

$$m = m_o M_e \tag{19.6}$$

The minimum magnification of the eye piece will be roughly

$$M_{e_{\min}} \approx \frac{25\,\text{cm}}{f_e}$$

and the maximum will be

$$M_{e_{\max}} = 1 + \frac{25\,\text{cm}}{f_e}$$

But remember the object for the eyepiece is the image from the first lens. And that image is larger than the object by an amount

$$m_o = \frac{-d_{io}}{d_{oo}}$$

where d_{io} is the image distance for the objective lens and d_{oo} is the object distance for the objective lens. And let's estimate how big the magnification due to the first lens will be. Because for a microscope $d_{oo} \approx f_o$. and $d_{io} \approx L$ (very roughly)

$$m_o = \frac{-d_{io}}{d_{oo}} \approx -\frac{L}{f_o}$$

The combined magnification for the two-lens system is about

$$m_{system} = m_o M_e \approx -\frac{L}{f_o}\frac{25\,\text{cm}}{f_e} \tag{19.7}$$

this is the minimum magnification (because we used the minimum magnification formula for the eyepiece).

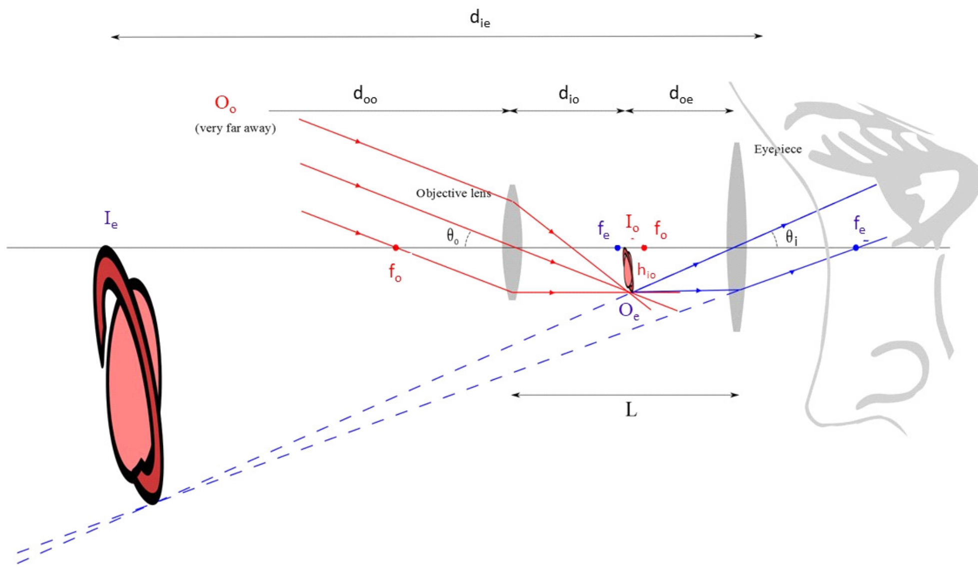
This gives us a rough idea of how a microscope works, let's try telescopes next.

19.3 Telescopes

There are two main types of telescopes *refracting* and *reflecting*. We will study refracting telescopes first.

19.3.1 Refracting Telescopes

Like the microscope, we combine two lenses and call one the objective and the other the eyepiece. The eyepiece again plays the role of a simple magnifier, magnifying the image produced by the objective.



We again form a real, inverted image with the objective. We are now looking at distant objects, so the image distance $d_{io} \approx f_o$. Once again, we use the image from the objective as the object for the eyepiece. The eye piece forms an upright virtual image (that looks inverted because the object for the eyepiece is the image from the objective, and the real image from the objective is inverted). Once again we could find d_{ie} using equation (19.4). But once again we want the magnification.

The largest magnification is when the rays exit the eyepiece parallel to the principal axis. Then the image from the eyepiece is formed at near infinity (but it is very big, so it is easy to see). This gives a lens separation of $f_o + f_e$ which will be roughly the length of the telescope tube.

The angular magnification will be

$$M = \frac{\theta_i}{\theta_o} \tag{19.8}$$

where θ_o is the angle subtended by the object at the objective (see figure above) and θ is subtended by the final image at the viewer's eye. Consider s_o is very large. We see from the figure that

$$\tan\theta_o \approx -\frac{h_{io}}{f_o} \tag{19.9}$$

and with d_{oo} large we can use small angles.

$$\theta_o \approx -\frac{h_{io}}{f_o} \tag{19.10}$$

The angle θ_i will be the angle formed by rays bent by the lens of the eyepiece. This angle will be the same as the angle formed by a ray traveling from the tip of the first image and traveling parallel to the principal axis. This ray is bent by the objective to pass through $f_{e.}$ Then

$$\tan\theta \approx \frac{h_{io}}{f_e} \approx \theta \tag{19.11}$$

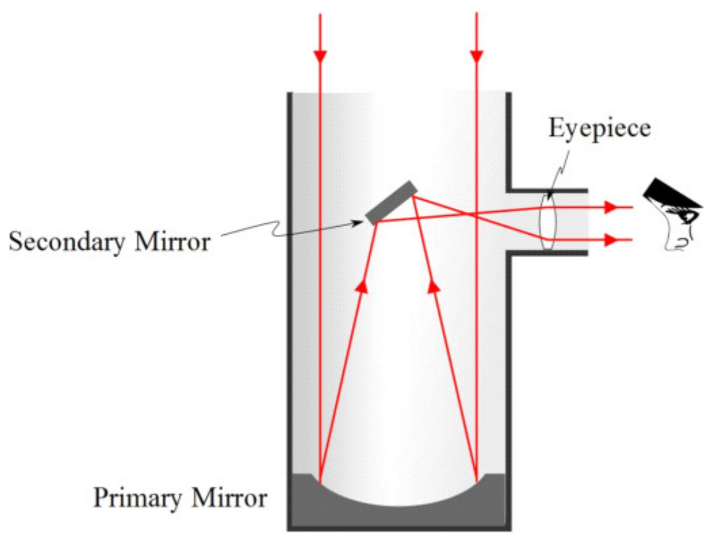
so

The magnification is then

$$M = \frac{\theta_i}{\theta_o} \approx \frac{\frac{h_{io}}{f_e}}{-\frac{h_{io}}{f_o}} \approx -\frac{f_o}{f_e} \tag{19.12}$$

19.3.2 Reflecting Telescopes

Reflecting telescopes use a series of mirrors to replace the objective lens. Usually, there is an eyepiece that is refractive (although there need not be, radio frequency telescopes rarely have refractive pieces).



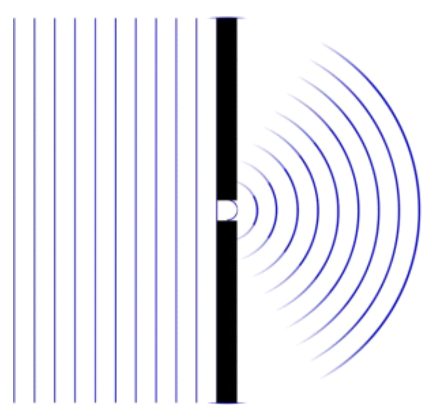
There are two reasons to build reflective telescopes. The first is that reflective optics do not suffer from chromatic aberration. The second is that large mirrors are much easier to make and mount than refractive optics. The Keck Observatory in Hawaii has a 10 m reflective system. The largest refractive system is a 1 m system. The Hubble telescope has a 2.5 m aperture and James Webb has a 6 m. Reflective telescopes are just easier to build.

The telescope pictured in the figure is a Newtonian, named after Newton, who designed this focus mechanism. Many other designs exist. Popular designs for space applications include the cassigrain telescope.

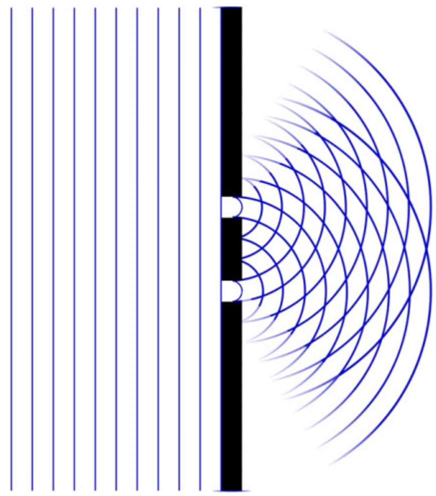
The rough design of a reflective telescope can be worked out using refractive pieces, then the rough details of the reflective optics can be formed.

19.4 Interference and Young's Experiment

Waves do some funny things when they encounter barriers. Think of a water wave. If we pass the wave through a small opening in a barrier, the wave can't all get through the small hole, but it can cause a disturbance right at the opening. We know that a small disturbance will cause a wave. But this wave will be due to a very small–almost point–source. Point sources make spherical waves. So the waves on the other side of the small opening will be nearly spherical. The smaller the opening the more pronounced the curving of the wave, because the source (the hole) is more like a point source.

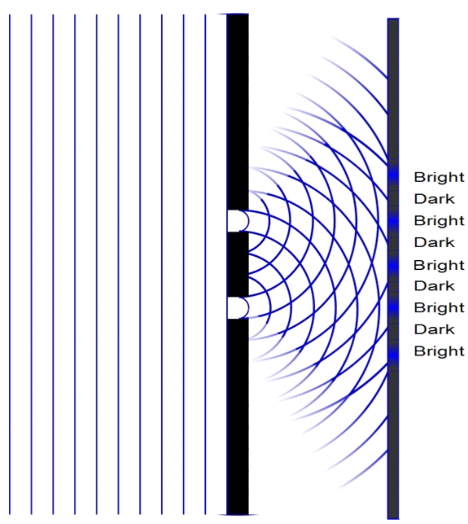


Now suppose we have two of these openings. We expect the two sources to make curved waves and those waves can interfere.



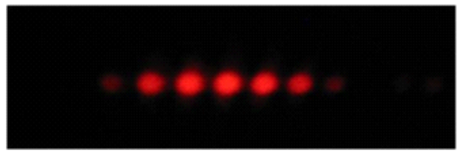
None of this is really new. We know waves generally have a spherical shape, and we have a disturbance in the wave medium where the two holes are that is experiencing simple harmonic motion. The holes are really sources for new spherical waves.

In the figure, we can already see that there will be constructive and destructive interference were the waves from the two holes meet. Thomas young predicted that we should see constructive and destructive interference in light (he drew figures very like the ones we have used to explain his idea).



Young set up a coherent source of light and placed it in front of this source a barrier with two very thin slits cut in it to test his idea.. He set up a screen beyond the barrier and observed the pattern on the screen formed by the light.

This (in part) is what he saw



We see bright spots (constructive interference) and dark spots (destructive interference). Only wave phenomena can interfere, so this is fairly good evidence that light is a wave.

But in our study of mirrors and lenses, beyond the forming of rays we just didn't have constructive and destructive interference. But form Young's experiment, clearly we should sometimes see more of the wave nature of light. It is time to return to waves and see the details of how light waves interact.