Chapter 15

15 Polarization and Image formation 3.1.7, 3.2.1

Fundamental Concepts

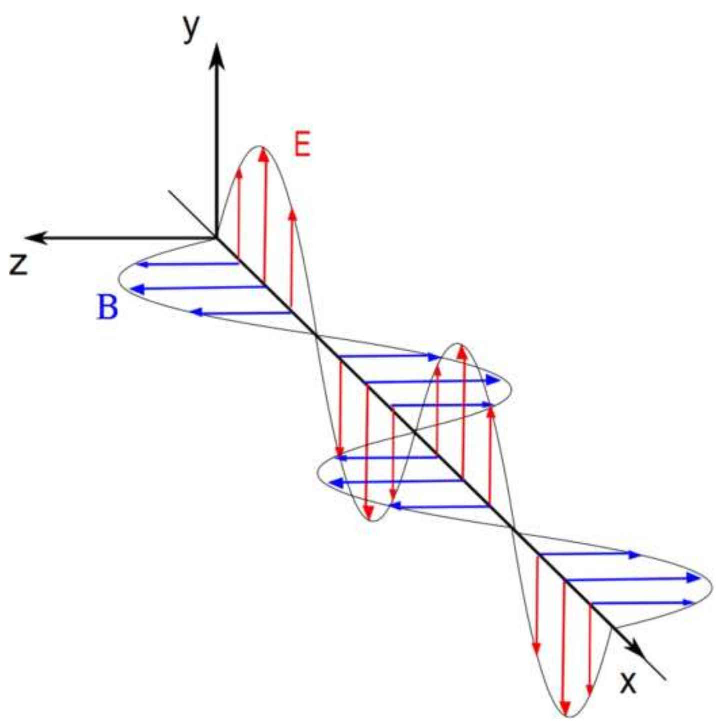
- The direction of the electric field in a plane wave is called the polarization direction.
- Natural light is usually a superposition of many waves with random polarization directions. This light is called unpolarized light.
- Some materials allow light with one polarization to pass through, while stopping other polarizations. The polaroid is one such material polaroids. will have a final intensity that follows the relationship $I = I_{\max} \cos^2(\theta)$
- Light reflecting off a surface may be polarized because of the absorption and re-emission pattern of light interacting with the material atoms.
- Scattered light may be polarized because of anisotropies in the scatterers.
- Birefringent materials have different wave speeds in different directions. This affects the polarization of light entering these materials.

15.1 Polarization of Light Waves

We said much earlier in our study of light that it was a transverse wave and that the medium for this wave is the electromagnetic field. Understanding electric and magnetic fields is a topic for Principles of Physics III (PH220) so we can't go to deep into electric and magnetic fields here. Let's assume that all of space (the whole universe) is filled with this stuff we call electric and magnetic fields.

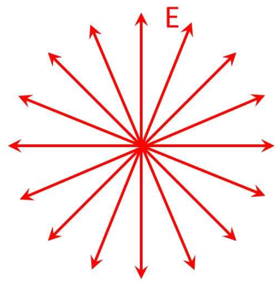
And let's suppose that these fields can be a medium for waves. And let's identify these waves as light.

The light waves are transverse waves. So when we look at a light wave in detail we will see the pieces of the electric and magnetic fields experiencing simple harmonic motion much like the parts of our spring for waves on springs or air molecules for sound waves. It turns out that the electric field medium parts move perpendicular to the direction the wave goes, and also perpendicular to the direction the magnetic field medium parts go. This makes them transverse waves. We will now show some implications of this fact. In a course in electromagnetic theory, we often draw light as in the figure below.



and that looks a bit like our waves on strings or springs only with the electric field part going up and down and the magnetic field part going back and forth.

In what follows we will ignore the magnetic field (marked in the figure as B) because it's amplitude is very much smaller than the electric field amplitude. So, we will look at the E field an notice that it goes up and down in the figure. But we could have light in any orientation. If we look directly at an approaching beam of light we would "see" many different orientations as shown in the next figure.



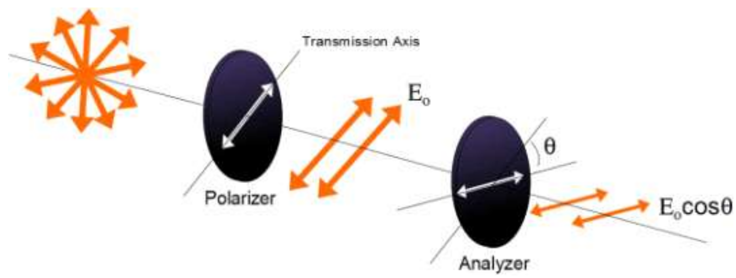
When light beams have waves with many orientations, we say they are *unpolarized*. But suppose we were able to align all the light so that all the waves in the beam were transverse waves in the same orientation. Say, the one in the next figure.

Then we would describe the light as *linearly polarized*. The plane that contains the E-field is known as the *polarization plane*.

15.1.1 Polarization by removing all but one wave orientation

One way to make polarized light is to remove all but one orientation of an unpolarized beam. A material that does this at visible wavelengths is called a *polaroid*. It is made of long-chain hydrocarbons that have been treated with iodine to make them conductive. The molecules are all oriented in one direction by stretching during the manufacturing process. The molecules have electrons that can move when light hits them. They can move farther in the long direction of the molecule, so in this direction the molecules act like little antennas. The molecules' electrons are driven into harmonic motion along the length of the molecule. This takes energy (and therefore, light) out of the beam. Little electron motion is possible in the short direction of the molecule, so light is given a preferential orientation. The light is passed if it is perpendicular to the long direction of the molecules. This direction is called the *transmission axis*.

We can take two pieces of polaroid material to study polarization.



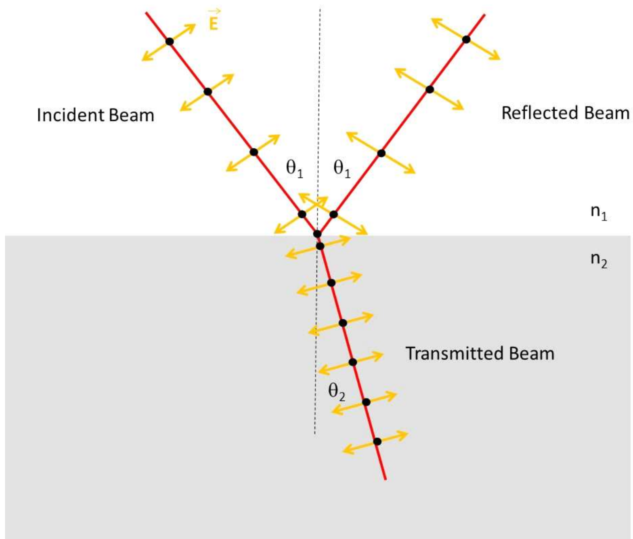
Unpolarized light is initially polarized by the first piece of polaroid called the *polarizer*. The second piece of polaroid then receives the light. This piece is called the *analyzer*. If there is an angular difference in the orientation of the transmission axes of the polarizer and analyzer, there will be a reduction of light through the system. We expect that if the transmission axes are separated by 90° no light will be seen. If they are separated by 0°, then there will be a maximum. It is not hard to believe that the intensity will be given by

$$I = I_{\max} \cos^2(\theta) \tag{15.1}$$

remembering that we must have a squared term because $I \propto E^2$.

15.1.2 Polarization by reflection

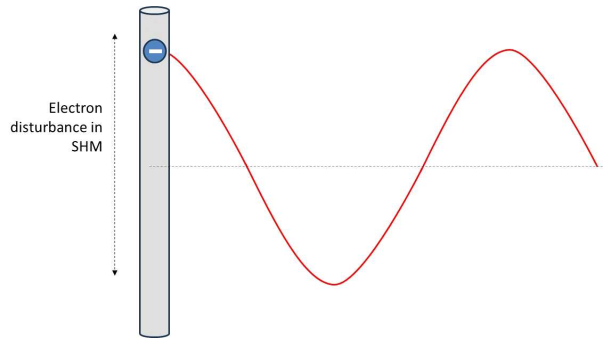
If we look at light reflected off of a desk or table through a piece of polaroid, we can see that at some angles of orientation, the reflection diminishes or even disappears! Light is often polarized on reflection. Let's consider a beam of light made of just two polarizations. We will define a plane of incidence. This plane is the plane of the paper or computer screen. This plane is perpendicular to the reflective or refractive surface in the figure below.



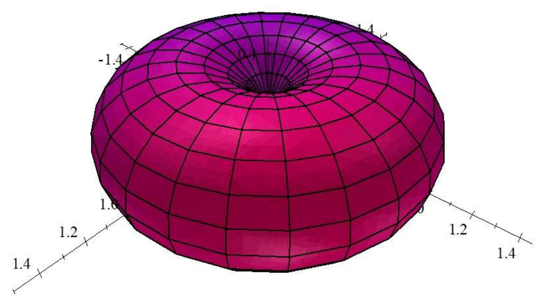
One of our polarizations is defined as parallel to this plane. This direction is represented by orange (lighter grey in black and white) arrows in the figure. The other polarization is perpendicular to the plane of incidence (the plane of the paper). This is represented by the black dots in the figure. These dots are supposed to look like arrows coming out of the paper.

When the light reaches the interface between n_1 and n_2 it drives the electrons in the medium into SHM. The perpendicular polarization finds electrons that are free to move in the perpendicular direction and re-radiate in that direction. The electron orbitals change shape and oscillate with the incoming electromagnetic wave.

The parallel ray is also able to excite SHM, but a electromagnetic analysis tells us that these little "antennas" will not radiate at an angle $90\,^\circ$ from their excitation direction. And this is something new! Before when we used Huygens' principle we thought of the scattered waves as little spheres. But it turns out that the light doesn't go out in all directions equally. The disturbance for our wave is a moving charged particle like an electron. If an electron in a wire experiences simple harmonic motion there will be a wave in the electric field. But think, that wave will go out away from the electron perpendicular to the electron's motion



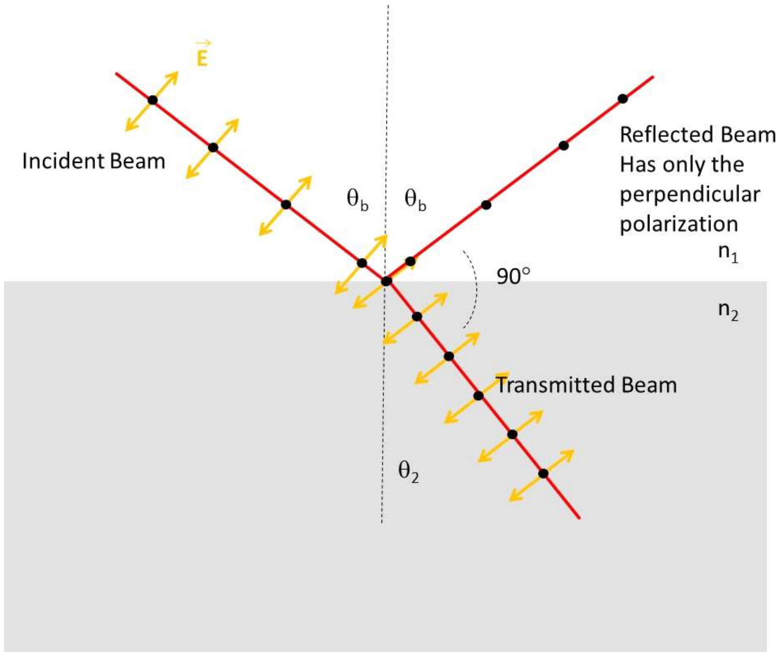
In the figure we can see the light wave won't travel up or down. The actual situation with our light striking a surface is a little more complicated, The pattern of light coming out from the little atomic oscillators looks more like a doughnut.



Angular dependence of S for a dipole scatterer.

But the point is that it's not completely spherical and none of the light wave goes up along the direction of the oscillation.

In the figure you can see that along the antenna axis, the field amplitude is zero. This means that the wave really does not go that direction. So in our case, the amount of polarization in the parallel direction decreases with the angle between the reflected and refracted rays until at 90 ° there is no reflected ray in the parallel direction.



The incidence angle that creates an angular difference between the refracted and reflected rays of 90° is called the *Brewster's angle* after its discoverer. At this angle the reflected beam will be completely linearly polarized.

We can predict this angle. Remember Snell's law.

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Let's re-lable the incidence angle $\theta_i = \theta_b$. We take $n_i = 1$ and $n_t = n$ so

$$n = \frac{\sin\theta_b}{\sin\theta_t}$$

Now notice that for Brewster's angle, we have

$$\theta_b + 90^\circ + \theta_t = 180^\circ$$

so

$$\theta_t = 90^\circ - \theta_b$$

so we have

$$n = \frac{\sin\theta_b}{\sin(90^\circ - \theta_b)}$$

ah, but we remember that $\sin(90^\circ - \theta) = \cos(\theta)$ so

$$n = \frac{\sin\theta_b}{\cos\theta_b}$$

Again we remember that

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

so

$$n = \tan\theta_b \tag{15.2}$$

which we can solve for θ_b.

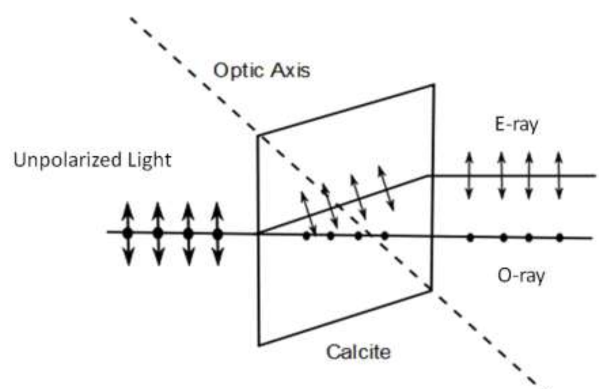
$$\theta_b = \tan^{-1}(n)$$

This phenomena is why we wear polarizing sunglasses to reduce glare.

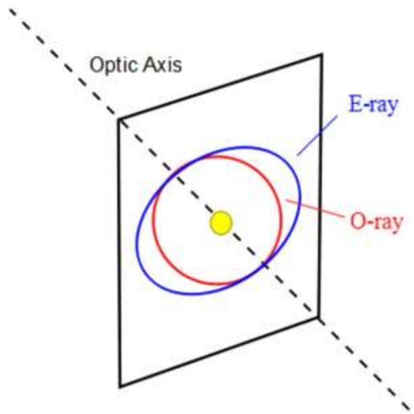
15.1.3 Birefringence

Glass is an amorphic solid–that is–it has no crystal structure to speak of. But some minerals do have definite order. Sometimes the difference in the crystal structure creates a difference in the speed of propagation of light in the crystal. This is not to hard to believe. We said before that the reason light slows down in a substance is because it encounters atoms which absorb and re-emit the light. If there are more atoms in one direction than another in a crystal, it makes sense that there could be a different speed in each direction.

Calcite crystals exhibit this phenomena. We can describe what happens by defining two polarizations. One parallel to the plane of the figure below, and one perpendicular.



With a careful setup, we can arrange things so the perpendicular ray is propagated just as we would expect for glass. We call this the O-ray (for *ordinary*). The second ray is polarized parallel to the incidence plane. It will have a different speed, and therefore a different index of refraction. We call it the *Extraordinary ray* or E-ray.

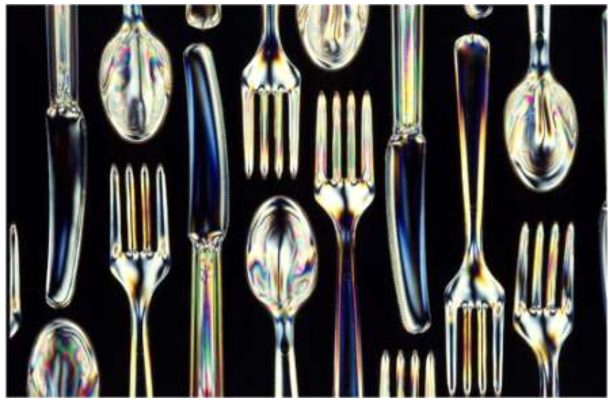


If we were to put a light source in a calcite crystal, we would see the O-ray send out a sphere of light as shown in the figure above. But the E-ray would send out an ellipse. The speed for the E-ray depends on orientation. There is one direction where the speeds are equal. This direction is called the *optic axis* of the crystal.

If our light entering our calcite crystal is unpolarized, then we will have two images leaving the other side that are slightly offset because the O-rays and E-rays both form images.

15.1.4 Optical Stress Analysis

Some materials (notably plastics) become birefringent under stress. A plastic or other stress birefringent material is molded in the form planned for a building or other object (usually made to scale). The model is placed under a stress, and the system is placed between to polaroids. When unstressed, no light is seen, but under stress, the model changes the polarization state of the light, and bands of light are seen.



15.1.5 Polarization due to scattering

It is important to understand that light is also polarized by scattering. It really takes a bit of electromagnetic theory to describe this. So for a moment, lets just comment that blue light is scattered more than red light. In fact, the relative intensity of scattered light goes like $1/\lambda^4$. This has nothing to do with polarization, but it is nice to know.

Now suppose we have long pieces of wire in the air, say, a few microns long. The pieces of wire would have electrons that could be driven into SHM when light hits them. If the wires were all oriented in a common direction, we would expect light to be absorbed if it was polarized in the long direction of the particles and not absorbed in a direction perpendicular to the orientation of the particles. This is exactly what happens when long ice particles in the atmosphere orient in the wind (think of the moment of inertia). We often get impressive halo's around the sun due to scattering from ice particles.

Rain drops also have a preferential scattering direction because they are shaped like oblate spheroids (not "rain drop shape" like we were told in grade school).

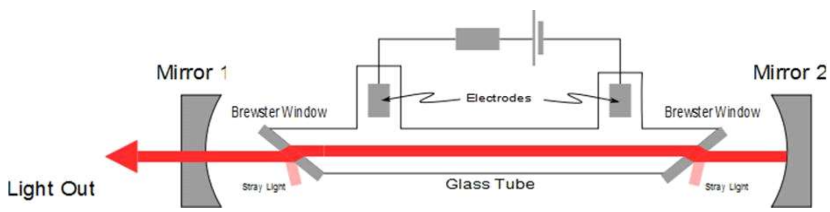
It is also true that small molecules will act like tiny antennas and will scatter light preferentially in some directions and not in others. This is called *Rayleigh scattering* and is very like small dipole antennas.

15.1.6 Optical Activity

Some substances will rotate the polarization of a beam of light. This is called being *optically active*. The polarization state of the light exiting the material depends on the length of the path through the material. Your calculator display works this way. An electric field changes the optical activity of the liquid crystal. There are polarizers over the liquid crystal, so sometimes light passes through the display and sometimes it is black.

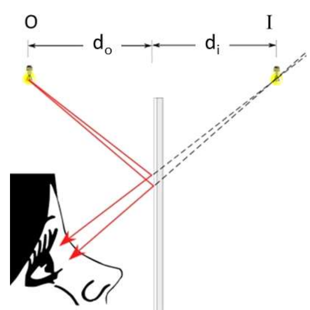
15.1.7 Laser polarization

One last comment. Lasers are usually polarized. This is because the laser light is generated in a *cavity* created by two mirrors. The mirror is tipped so light approaches it at the Brewster angle. Light with the right polarization (parallel to the plane of the drawing) is reflected back nearly completely, but light with the opposite polarization is not reflected at all. This reduces the usual loss in reflection from a mirror, because in one polarization the light must be reflected completely.



15.2 Imaging with Flat Mirrors

All of us have looked in a mirror at some time. We know what to expect. We see an image of ourselves. To study mirrors mathematically, we need to establish a sign convention and some standard notation.

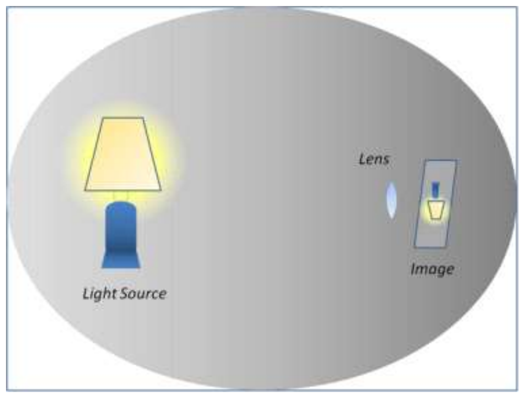


In the figure above, we have a person observing an object O in a mirror. The object is located at a distance s from the mirror. Just like for lenses, we will call this the *object distance*, d_o. The image appears to be located at a point I beyond the mirror a distance d_i.We will call this the *image distance.* Be careful, d_i looks like an incident of initial distance, but it's not. It is the distance from the mirror to the *image.* We know we see an image in the mirror, but to understand this we need to define what we mean by the word "image."

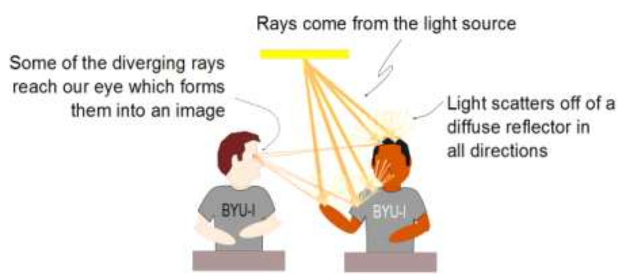
Images

Make Images with Lens Demo

Let's think about what an image is. Take a piece of paper and a lens, and hold up the lens in a darkened room that has some bright object in it. Move the lens or the paper back and forth, and at just the right distance, a miniature picture of the bright object will appear. We should think about what the word "picture" means in this sense.



We have talked about how we see objects. Remember the BYU-I guys from last time.



Our eyes gather rays that are diverging from the object because light has bounced off of the object. Our eyes intersect a diverging set of rays that form a definite pattern. That diverging set of rays forming a pattern is the picture of the object.

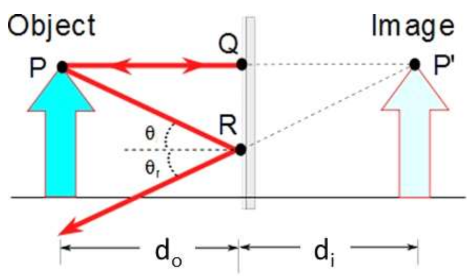
So when we say that the lens has formed a miniature picture of our object, we mean that the lens has somehow formed a diverging set of rays that form a pattern that looks like the pattern formed by the diverging set of rays coming from the object, itself. In other words, the object forms a diverging set of rays. And our lens forms a duplicate set of rays in the same pattern, so we see the same thing. The lens' version is smaller, upside down, and backwards, but it is still essentially the same pattern.

But how can a mirror form a divergensing of rays that form a pattern?

Well, the light that we see in a mirror first bounces off of something, say, a person. TThis makes a recognizable pattern in the light. Then the light reflects off of the mirror. Then the light hits our eyes. So when we see something in a mirror we do see a diverging set of rays that form a pattern. But what we see makes more makes sense if we know how our brains processes the light signals that we see. Our eyes intercept rays of light diverging from an object. Our brain processes those rays as though they traveled only in straight lines. Our brain really believes light travels just in lines, no bouncing or reflecting. This means, if we can create a situation that makes rays diverge in the same way the object did, we will have an image of the object. But our brain will be wrong in where it thinks that image is. We will see an image as though it were behind the mirror.

15.2.1 Virtual Images

We will call We already know that mirrors often create what we call *virtual* images because the image appears to be created from diverging rays from behind the mirror. If we look behind the mirror, no rays exist there (if they did exist, they would not make it through the mirror!).

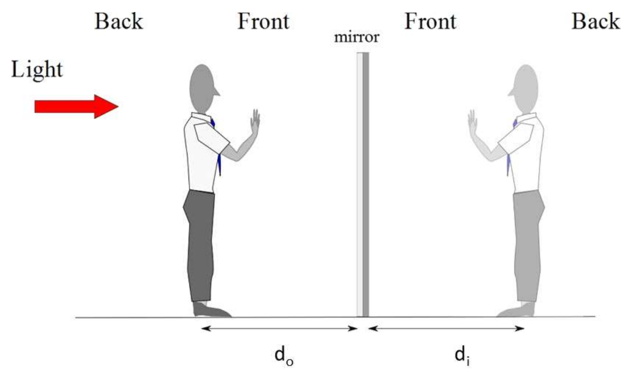


Let's look at a simple image as shown in the figure above. The object is an arrow. We could trace all the rays that diverge from this object and build a very nice representation of the arrow (like ray tracing-based computer graphics–the way movies like *Toy Story* are made) but that would take time and computation power. We only really need to use two rays, and to remember what the object looked like to figure out where the image will be.

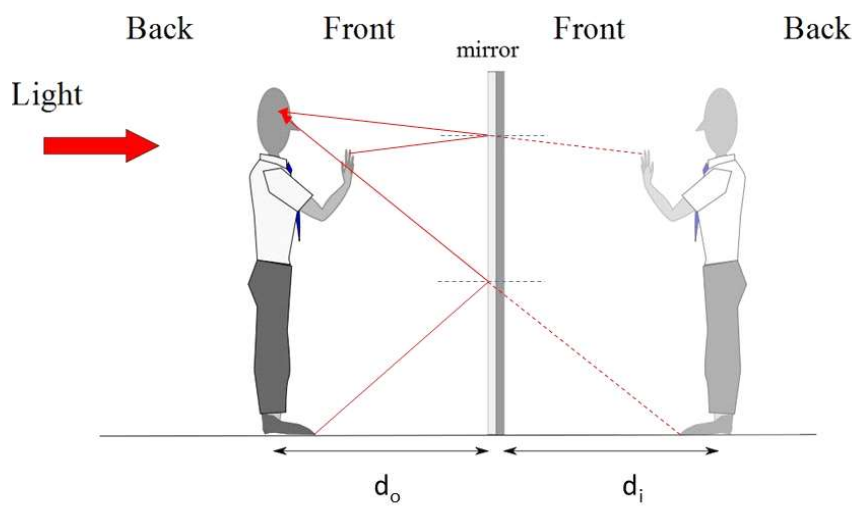
We pick one ray from the top of the arrow that travels straight to the mirror. This ray will travel a distance s and bounce back. Using the law of reflection we know that it will bounce straight back along the path it came in on. We pick a second ray from point P that travels the path PR. This ray bounces off the mirror at an angle $\theta_r = \theta_i$. But our brain doesn't believe the light reflected. It thinks the light traveled straight paths. It traces the rays bacwards along straight paths. And this makes it appear that the tip of the arrow is at position P'. The rays from the tip appear to travel the paths $P'P$ and $P'R$. The light doesn't really come from point P'. But it *appears* to come from point P' prime. We could use similar sets of two rays to find where the light from the middle of the arrow *appears* to come from, and for the bottom of the arrow, and all the arrow parts. The collection of all these apparent locations is image location and the light that appears to come from this location is the image. But, and this is important! light doesn't come from P'. It only appears to come from P'. When an image is formed, but the light does't really come from the image location, we call that a *virtual image.*

15.2.2 Mirror reversal

Look into a mirror. Raise your right hand.



Your image raises what appears to be a left hand. It looks like a mirror switches the left and right sides of the image. But it might be better to note that the image raised hand is on the same side of the room as your real raised hand. Think of lying sideways on the ground in front of the mirror and raise a hand. Note that your image feet are on the same side of the room as your real feet. And note that as you put one hand up, the image hand also goes up. This is not exactly left-right reversal. What is happening?



The light from your raised hand would hit the mirror, reflect, and then be absorbed by your eye. Your brain tells you the light traveled in a straight line. So it looks like you have a raised hand in the mirror. But what has happened is that the light has traveled from your hand, to the mirror and back to your eye. This is not really left-right reversal. We need a name to describe what is really happening. The name is odd. To see why we use this name, notice that from your perspective (moving left to right across the figure) you first have the back part of your hand, then the front of your hand and then the front of the image hand, then the back of the image hand. We call this back-front reversal. It comes from the reversal in the order of parts of the object, like the front and back of your hand. So we say that a flat mirror performs a front-back reversal, not a left-right reversal.