Chapter 26

26 Pascal and Archimedes 1.14.3, 1.14.4

So now we know how pressure changes with depth, and we know that liquids are generally hard to compress, so we can approximate them as incompressible, let's build some things with our new knowledge.

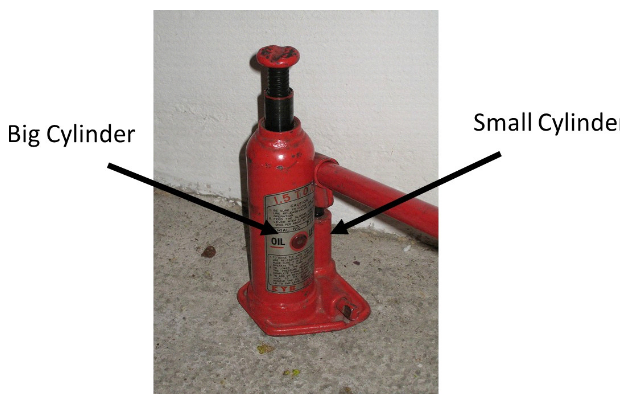
Fundamental Concepts

1. Pascal's Principle
2. Hydraulic Presses
3. Archimedes Principle
4. Buoyant Forces

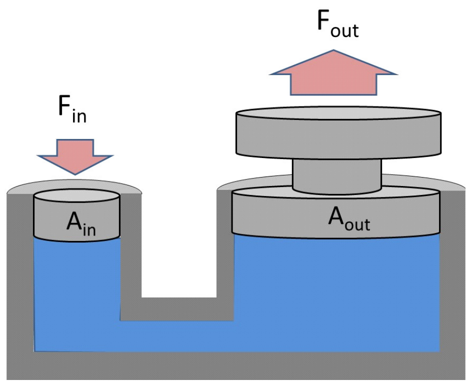
26.1 Hydraulic Press

We use fluids and their incompressibility to convert pressure into forces to smash or lift things. A hydraulic press can smash carbon. into diamonds

A hydraulic jack can lift cars.



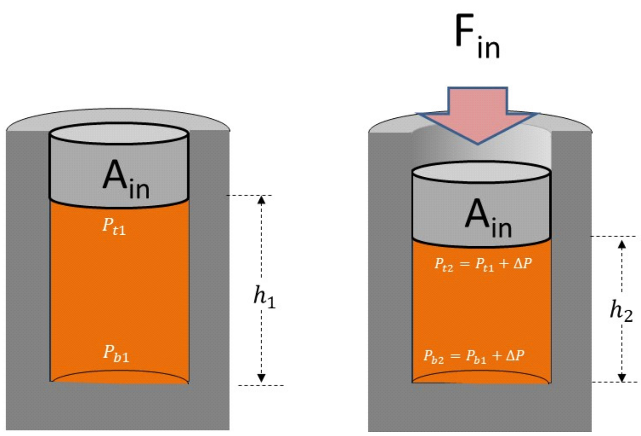
Let's take on the hydraulic jack. In the last figure, the jack is made from two cylinders filled with fluid, usually an oil. The jack in the picture is a little harder to think about. Let's draw a simplified jack:



We usually have a constant force acting on the jack area. And that force will be related to the pressure in the liquid.

$$F = PA \tag{26.1}$$

Let's look at just the left cylinder, the input of our jack. And further suppose we fill the jack with a gas that is compressible



As we press down on the piston, the force spread over the piston area. That causes an increase in pressure, ΔP. A early researcher named Pascal reasoned that at first the gas molecules that are near the piston would go flying. But they would hit other molecules (and the walls) and after a time all the gas molecules would be effected. When the system comes to equilibrium, we find that not only has the top pressure changed by ΔP so that (using the figure)

$$P_{t2} = P_{t1} + \Delta P$$

but also the bottom pressure has changed.

$$P_{b2} = P_{b1} + \Delta P$$

This is called Pascal's principle. We can restate it as the change in pressure in a fluid is transferred to the entire fluid and the magnitude of the change is the same everywhere.

Pascal's Law: a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

Now let's consider the jack again. In the jack we have a liquid, and liquids are not easily compressible. When our jack is just sitting in our garage, we have just atmospheric pressure pushing on the jack input piston,

$$F_{atm} = P_{atm} A_{in} \tag{26.2}$$

so Newton's second law for the input piston is

$$\Sigma F_{in} = F_{fluid} - F_{atm} - W_{in} = F_{fluid} - P_{atm} A_{in} - W_{in} = 0 \tag{26.3}$$

which tells us that

$$F_{fluid} = P_{atm} A_{in} + W_{in}$$

But we wish to add a force F_{in} to this. So on the input side, we have, using Newton's second law

$$\Sigma F_{in} = F_{fluid} - F_{atm} - W_{in} - F_{in} = F_{fluid} - P_{atm} A_{in} - W_{in} - F_{in} \tag{26.4}$$

And once again our F_{in} will cause a change in pressure

$$F_{in} = \Delta P A_{in} \tag{26.5}$$

But the first three forces in our Newton's second law haven't changed, and they sum to zero, so we have

$$\Sigma F_{in} = 0 - \Delta P A_{in}$$

But this time we can't compress the liquid. But we expect our ΔP to be transmitted throughout the entire fluid. By changing the pressure on the input side we have changed the pressure by ΔP on the output side (the lifting side). The only way this can happen is for the output piston to move! So the output will have an amount of force added to it. On this output side, remember before we pushed we had atmospheric pressure pushing down on the output piston.

$$\Sigma F_{out} = F_{fluid} - F_{atm} - W_{out} + F_{out} \tag{26.6}$$

Now think again about the state of our jack before we pushed on the input piston. The output piston wasn't accelerating. So before

$$\Sigma F_{out} = F_{fluid} - F_{atm} - W_{out} = 0 \tag{26.7}$$

and these forces haven't changed, but we do have a new force $F_{out} = \Delta P A_{out}$ that we have added to our net force on the output piston.

$$\Sigma F_{out} = 0 + \Delta P A_{out} \tag{26.8}$$

Let's solve for ΔP in our two net force equations

$$\frac{\Sigma F_{in}}{A_{in}} = \Delta P = \frac{F_{in}}{A_{in}}$$

and

$$\frac{\Sigma F_{out}}{A_{out}} = \Delta P = \frac{F_{out}}{A_{out}} \tag{26.9}$$

Since ΔP in both equations is the same when the two sides are at the same elevation, then

$$\frac{F_{out}}{A_{out}} = \frac{F_{in}}{A_{in}} \tag{26.10}$$

So how does your hydraulic jack work?

$$F_{out} = F_{in}\frac{A_{out}}{A_{in}} \tag{26.11}$$

If $A_{out} > A_{in}$ a much smaller F_{in} can produce a large F_{out} (you already knew that, didn't you!).

It is important to note that we have assumed our jack fluid is not compressible. So the volume of fluid leaving the cylinder at the input side must be the same as the volume of fluid entering the output side. Since $A_{out} > A_{in}$, it is clear that the output piston will travel a much smaller distance than the input piston. This is why you have to pump quite a lot on the input side of your jack to move a car a relatively small distance.

Note that our equation

$$P_b = P_t + \rho g h$$

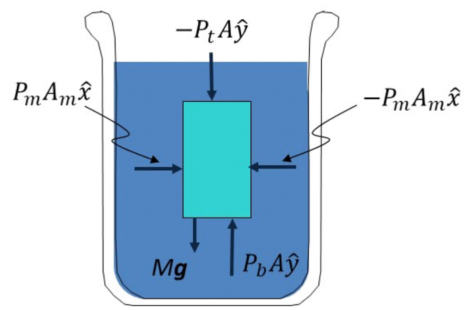
still applies! The pressure is not the same at the top and bottom of the cylinders. Pascal's principle tells us that the change in pressure ΔP was the same in the entire liquid, not that the pressure, itself, is the same. This is an important distinction! We will need to be careful with Pascal's principle.

In many cases we can sum up pressure over an area because the pressure will be the same at every point within the area. Our hydraulic jack is one of these cases.

But let's turn our attention now to pressure variation with depth again and look at things that float.

26.2 Buoyant Forces, Archimedes' Principle

Remember our parcel of liquid from last lecture.



Let's investigate the net force on our parcel and let's say our liquid is water this time. We found the net force must be zero, or it would be accelerating.

$$\begin{aligned}\Sigma F_x &= ma_x = 0 = P_m A_m - P_m A_m \\ \Sigma F_y &= ma_y = 0 = -P_{atm} A + P_{bottom} A - W\end{aligned}$$

But suppose I ignore the weight of the parcel of water, $W = mg$. Then there would be a net force due to all other forces in the $\widehat{\mathbf{y}}$ direction. But our parcel isn't accelerating. That force must be matching the downward force due to gravity.

$$W = P_b A - P_t A$$

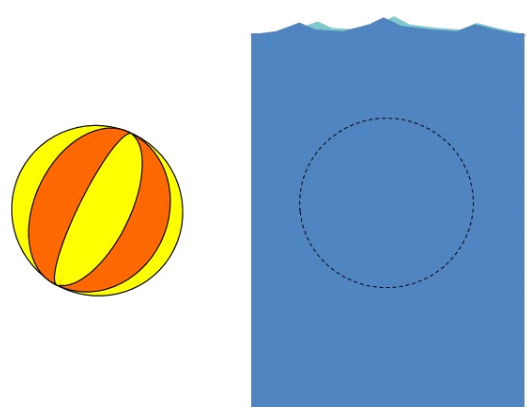
We could give this net force due to all the water pressure forces a symbol, say "B."

$$B = P_b A - P_t A$$

Then B would be equal to the weight of the parcel of water!

$$B = W$$

Now take a beach ball sized parcel of water



The parcel is in equilibrium just as before. We know there is a force on the parcel of water due to gravity. It is downward. Just as before, the ball-shaped parcel of water is not sinking so again there is no net force. Then if we add up all the forces due to the water pressure, only, we must have a net pressure force that is upward to balance the downward force due to gravity. Again this is what we called "B." And again B is equal to the weight of the parcel of water.

The idea of a net force just due to pressure due to a fluid is so useful we give it its own name We will call the net force due to just pressure from the fluid the *Buoyant force*.

Note that this is **not** the net force, it is just the sum of the forces due to the water pressure. It does not include gravity or any tension or spring or any other forces. It just contains the pressure forces due to the fluid.

Note also that the buoyant force is not some new kind of force. It is a name we give to the sum of the pressure forces due acting on an object in the fluid. Like we call a group of soldiers a battalion or a group of cows a heard, the sum of the group of pressure forces due to a fluid is given the name "buoyant force." But herds and battalions are not new things, they are groups of cows and soldiers. A Buoyant force is not a new force, it is the vector sum of a group of pressure forces.

Because the parcel is not sinking, we can determine that in this special case

$$B = W_{\text{fluid parcel}} \qquad \text{Floating} \tag{26.12}$$

for this situation that we call floating (these are magnitudes, are the directions the same?). The term on the right is the weight of the parcel of water or fluid.

Suppose we replace this amount of water with the beach ball. The weight of the beach ball is different than the weight of the water parcel. But will any of the pressure forces be different? The two volumes are exactly the same!

It turns out that the buoyant force will be exactly the same for the beach ball as it would for the beach ball-shaped parcel of water. We can calculate the buoyant force by thinking of replacing the actual object we have with an equal volume parcel of water. The buoyant force will be equal to the weight of that equal volume of the fluid. We could say that to replace the water with the beach ball we have *displaced* a beach ball volume's worth of water. The water that was at the location of the beach ball is now somewhere else, so it is displaced. Then the buoyant force would be equal to the weight of the water that was displaced.

$$B = W_{\text{displaced fluid}} \tag{26.13}$$

Of course, the weight of the beach ball is far less than the weight of the water displaced

$$W_{ball} < W_{\text{displaced fluid}}$$

and this is why the beach ball accelerates upward.

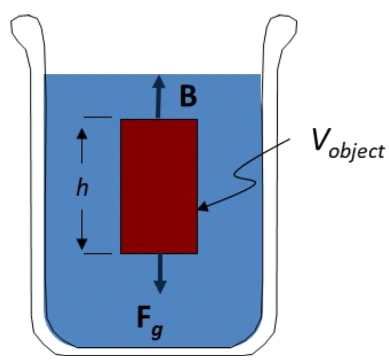
It turns out that this idea of the buoyant force being equal to the weight of an equal volume of fluid is general! This concept of the Buoyant force being equal to the weight of the water that fits inside our volume is called *Archimedes' principle*.

The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

26.3 Buoyant Force: Two Cases

We can gain insight into this concept of a buoyant force by considering more floating objects. Let's consider an object that is floating submerged in the fluid (like a fish or submarine) and one that is floating at the surface and partially extends out of the fluid (like a floating log or boat).

26.3.1 Totally submerged object



Consider a solid object, a block of some material, placed in water. The weight of the solid object is Mg.

The magnitude of the buoyant force is equal to the weight of an equal volume of water. If we imagine replacing the block with a block-sized volume of water, then

$$
\begin{aligned}
B &= m_{fluid}g \\
&= \rho_{fluid}\not{V}_{object}g \\
&= \rho_{fluid}g\not{V}_{block}
\end{aligned}
$$

Think Archimedes: The weight of an equal volume of water was displaced by the block. So we have the volume of the block as part of our equation for the buoyant force.

If the object has mass M then we can write the mass of the block as

$$
\begin{aligned}
M &= \rho_{object}\not{V}_{object} \\
&= \rho_{block}\not{V}_{block}
\end{aligned}
$$

and the weight of the block is then

$$
\begin{aligned}
W &= Mg \\
&= \rho_{object} \cancel{V}_{object} g \\
&= \rho_{block} \cancel{V}_{block} g
\end{aligned}
$$

The net force is then

$$
\begin{aligned}
F_{net} &= B - W = \rho_{fluid} g \cancel{V}_{object} - \rho_{object} \cancel{V}_{object} g \\
&= \rho_{fluid} g \cancel{V}_{block} - \rho_{block} \cancel{V}_{block} g \\
&= g \cancel{V}_{block} (\rho_{fluid} - \rho_{block})
\end{aligned}
$$

Consider this last equation. If the density of the block is large, larger than the density of the fluid, then $(\rho_{fluid} - \rho_{block})$ is negative, and the force will be negative. The block will accelerate downward. If the density of the block is smaller than the density of the fluid, then $(\rho_{fluid} - \rho_{block})$ will be positive and the block will accelerate upward. If the density of the block is just the same as the density of the fluid, the block will float in place (or at least sink or float upward at a constant rate. A submarine adjusts it's density to dive or to surface.

We did this calculation for our block, but it could have been any object. So in general

$$
F_{net} = g \cancel{V}_{object} (\rho_{fluid} - \rho_{object}) \tag{26.14}
$$

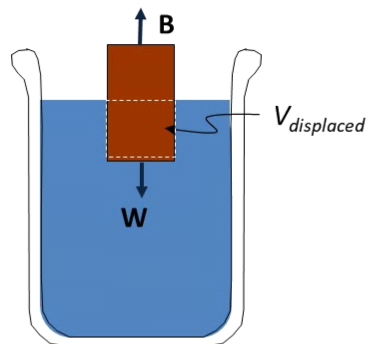
If the object has a density that is less than the fluid, the object will be accelerated upward. And if the density of the object is greater than the density of the fluid the object will be accelerated downward.

bb in a test tube demo

Note that the motion of a totally submerged object is determined only by the relative density! In a way, this could be confusing because we did not allow our object to change volume as it moved. If we had an object that could change size, then it changes density as it changes size. A balloon full of air released from the bottom of a swimming pool does change volume as it rises. So in that case we would need to know $\Delta \cancel{V}_{object}$ and how the density, ρ_{object}, changes as the balloon moves.

26.3.2 Partially submerged object

Now let's consider a partially submerged object



Let's assume $\rho_{object} < \rho_{fluid}$ (why? think of $\rho_{fluid} - \rho_{object}$). We assume static equilibrium, which means we are observing this floating object when it is just sitting still relative to the objects around it, not accelerating.

Since $a = 0$ then

$$\Sigma F = ma = 0$$

and

$$\Sigma F = B - W$$

so for an object floating on the surface

$$B = -W$$

Lets call the volume of the fluid displaced by the object $\not{V}_{fluid}$. Now this is not the same as $\not{V}_{object}$ for this case! Part of the object is sticking out of the fluid, so $\not{V}_{fluid} < \not{V}_{object}$. But, it is still true that the weight of the displaced volume of fluid gives the buoyant force

$$B = \rho_{fluid} g \not{V}_{fluid}$$

Then returning to our force equation

$$\begin{aligned} B &= -W \\ \rho_{fluid} g \not{V}_{fluid} &= -\rho_{object} \not{V}_{object} g \end{aligned}$$

or

$$\frac{\not{V}_{fluid}}{\not{V}_{object}} = \frac{\rho_{object}}{\rho_{fluid}} \tag{26.15}$$

The fraction of the volume of a floating object that is below the fluid surface is equal to the ratio of the density of the object to that of the fluid.

In the case of an iceberg, not all of the ice is visible above the surface. The density of ice is about $920 \frac{\mathrm{kg}}{\mathrm{m}^3}$ and that of sea water is about $1000 \frac{\mathrm{kg}}{\mathrm{m}^3}$. So for ice

$$\frac{\not{V}_{fluid}}{\not{V}_{object}} = \frac{920 \frac{\mathrm{kg}}{\mathrm{m}^3}}{1030 \frac{\mathrm{kg}}{\mathrm{m}^3}} = 0.893\,2$$

$$\forall_{fluid} = 0.89\forall_{object}$$

that is the fluid displaced is 89% of the ice. Just about 10% of the ice sticks up out of the water. I

Iceberg with both beautiful blue-green submerged portion and a reflection of the surface ice and snow. Approximately 90 per cent of the iceberg is submerged. Antarctica, Palmer Peninsula, Northern area. (Dr. Mike Goebel, NOAA NMFS SWFSC, NOAA NMFS SWFSC Antarctic Marine Living Resources (AMLR) Program.)

We have gained some good insight using pressure. But think, pressure is a force spread over an area. And we know forces from Principles of Physics I (PH121). But we didn't just use forces in Principles of Physics I. We also found we could use energy to solve motion problems. In our next lecture, let's try to bring the powerful technique of conservation of energy into our study of fluid flow.