Chapter 28

28 Turbulence and Temperature 1.14.7, 2.1.1

In our last lecture we said we would ignore turbulence and viscosity. But if we are cautious we can at least get some intuition about what will happen if we have friction or non-laminar flow.

Fundamental Concepts

- Number Density
- Atomic Mass
- Atomic Mass Number
- The Mole
- Temperature

28.1 Viscosity

Way back in Principles of Physics I (PH121) we learned about friction. So far we haven't had any friction with our fluids. But we know this can't be true! We know there is drag force. So there must be some friction. And we can think of there being two different chases where friction can matter. There is the friction between parts of the water, itself, and there must be something like a drag force between the water and the solid earth. Let's take on the water/earth friction first.

In the past, we used two different equations for drag for fluids, one for air

$$D = \frac{1}{2}\rho C A v^2$$

where ρ is the air density, C is the drag coefficient, A is the cross sectional area, and v is the speed of an object through air. And we had a different one for a liquid like oil when we had oscillators being damped by the fluid

$$D = bv$$
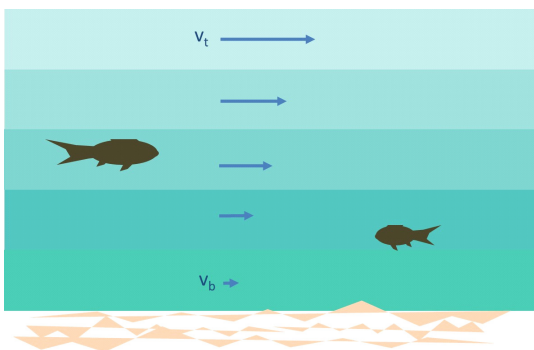
where b is the damping coefficient and v is the speed. These two equations, one for a gas fluid like air and one for a thicker liquid like oil, show that friction for fluids can be quite different. And of course it is much easier to get through air than oil, so that is not a surprise. Recall that we call friction in liquids *viscosity* and we would say the oil is more viscus than the air.

There will be a drag force between the river bed and the lowest water. And that drag force will likely be more of the $D = bv$ type for a liquid. This slows the water that is right next to the river bed down to a v that is quite slow.

But we know fluids don't just have friction with solid surfaces. Because the fluid can move around internally there is internal friction or viscosity. When a river flows down its bed we know the drag force at the bottom causes the water to slow down. But this slower water is also viscus and so it slows the water just above it a little. So the water at the bottom of the river will be slow and the water just above it will be less slow. The water above this second layer will be faster still, and this keeps going until we reach the top of the water where there is some friction with the air, but it is much less. So the fastest water is on top. This works for air too.

You might have tried flying a kite.

Kite flown on the June 19th 2025 BUYU-I High Altitude Research Team flight as part of the high altitude balloon payload retrieval effort. The kite can carry radios that connect to the payload trackers.

When you first start, you have to run with the kite. But once the kite is higher, the winds tend to be higher and the kit can stay up without you running.
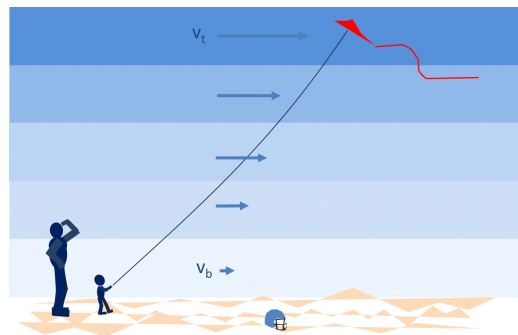
Both cases work well as long as nothing disrupts the fluid flow. This nice well behaved flow we described as being *laminar*. In laminar fluid flow layers may have different speeds, but they flow without mixing.

All this works pretty well if we don't have turbulence and the viscosity isn't too high.

We can measure the viscosity of a fluid by placing it between two sheets
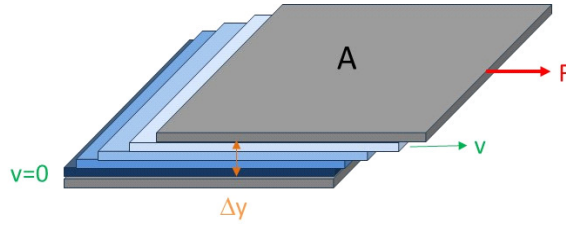
If a force, F is applied to the top sheet we have a sheer force and the sheet moves horizontally. The viscosity makes the fluid move with it. The fluid at the bottom won't move at all, the fluid at the top moves with speed v and the layers in between will have speeds in between v and 0. The viscosity η is given by

$$\eta = \frac{F\Delta y}{vA}$$

In our society we put a lot of effort to make fluids flow in pipes. Our water comes and goes in pipes. Fuel flows in pipes. Air flows in organ pipes during conference, etc. The rate at which the water flows is called a flow rate and unfortunately it is often given the symbol Q. But what makes the fluid flow? Generally is is pressure. We recall that pressure is a stress, a force spread over an area.

$$P = \frac{F}{A}$$
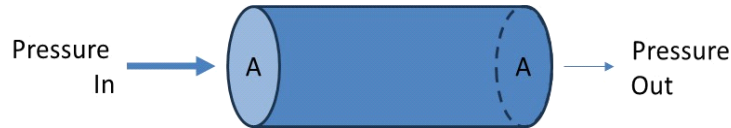
For our fluid in a pipe the area will be the pipe cross sectional area. We would like to know how the pressure affects the flow rate, $\mathbb{Q}$. It turns out that this is fairly hard to solve (there are multivariate differential equations) so for those who are curious or need to be able to do this professionally I recommend taking a fluid dynamics class[1] but for our class let's just borrow a result.

$$\mathbb{Q} = \frac{P_{in} - P_{out}}{R}$$

This is called Poiseuille's Law. We can see the pressure in, P_{in} and the pressure out, P_{out}. The denominator, R is the viscosity resistance. This equation won't work for fluids with no viscosity, because R would be zero and the flow rate infinite. But fluids with no viscosity are rare, so it works for most normal situations. We would not expect the flow to be uniform across the pipe. Fluid

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[1] At BYU-I this class is ME360 and is called Fluid Mechanics.

closer to the pipe walls should move slower. Fluid near the center should move faster. We can model the resistance in the pipe by borrowing from a fluid dynamics class

$$R = \frac{8\eta L}{\pi r^4}$$

where η is the viscosity of the fluid, L is the length of the pipe, and r is the radius of the pipe. We can put this simple model for R into Poiseuille's law

$$
\begin{aligned}
Q &= \frac{P_{in} - P_{out}}{\frac{8\eta L}{\pi r^4}} \\
&= \frac{\pi r^4 (P_{in} - P_{out})}{8\eta L}
\end{aligned}
$$

We can see that this gives us a dependence on the radius. The wider the pipe, the more flow we get. The more viscus the fluid, the less the flow rate. Notice also that longer pipes have lower flow. Friction will have more of an affect if the fluid interacts with the pipe longer, so this makes sense. And of course if we increase the pressure on one end, the flow changes.
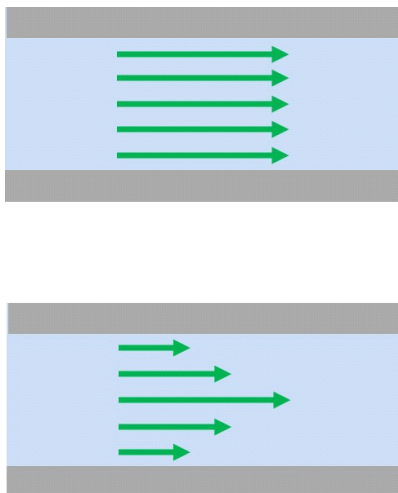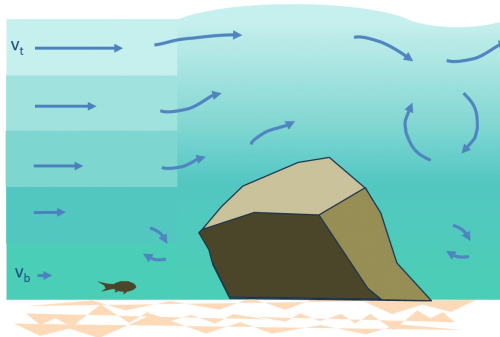
Pipe with no viscosity (top) and with more realistic viscosity (bottom)

The math that lead to this equation assumed laminar flow. But what if we don't have laminar flow?

28.1.1   Turbulence

We can disrupt laminar flow by, say, putting a rock in the way of the fluid. Here is a rock in a river.

Now the laminar flow is disrupted by the rock and the layers will mix. We call a situation like this *turbulent* flow. As you can imagine, the equations for turbulent flow are harder. Finding the flow rate for turbulent flow is really a topic for a fluid dynamics class. But we can give a measure of how turbulent our situation is.

$$R_e = \frac{\rho v D}{\eta}$$

where ρ is the fluid density, v is the flow speed, D is the pipe diameter, and of course η is the viscosity of the fluid. This quantity is called the Reynolds number and if this number is high enough the flow will be turbulent. Engineering books give a table like the following for determining if the flow is still laminar.

laminar	$R_e < 2300$
Transient	$2300 < R_e < 4000$
Turbulent	$R_e > 4000$

although not all books agree on the value for the transition to turbulence. But generally if $R_e < 2300$ our equations for this class work and for $R_e > 2300$ we don't have laminar flow and our equations really don't work. The transient range is a transition to turbulence and for R_2 above 3000 to 4000 most agree that the flow is turbulent. For many engineering designs we really don't want turbulence. So this is enough to do some good work. For cool physics problems turbulence is a form of "complexity" which is a current emphasis in physics research.

Fluid dynamics is the movement of a bulk of fluid. This is all we can do for fluid dynamics in this class. But we can still look at the motion of the molecules and atoms inside of the fluids (and solids). So lets move on to thinking about what we will call internal motion in the fluid. It turns out that this internal motion is related to how hot or cold our fluid is. We will call this the thermodynamics of the fluid!

28.2   Temperature and the Zeroth Law of Thermodynamics

We need a basis for talking about thermal systems. We sort of have an intuition for temperature, so let's use our intuition for now.
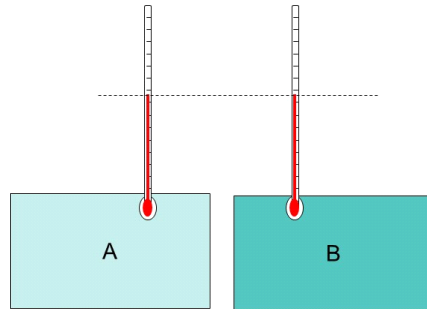
In the figure we have two objects. I measure both objects temperature. I find that they are the same. Suppose I put the two objects in "contact."
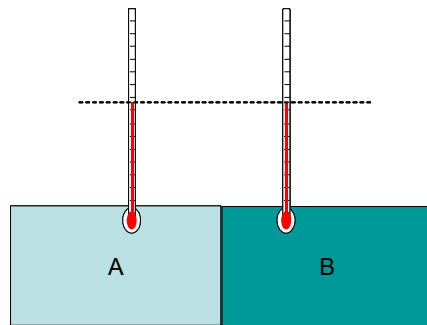
Would you expect there to be energy transferred between the two objects? For most of us our intuition says "no." Given this intuition, lets make some more formal definitions:

- Thermal Contact: Two objects are in thermal contact if they can exchange energy due to a temperature difference

- Thermal Equilibrium : A state where two objects would not exchange energy by heat or electromagnetic radiation if they were placed in thermal contact

- Temperature: The property that determines if an object is in thermal equilibrium with other objects (two objects in thermal equilibrium have the same temperature)

Given all this, we can state a law of thermal dynamics: **If objects A and B are separately in thermal equilibrium with a third object C, then A and B are in thermal equilibrium with each other.**

Again this fits our intuition, if we have two containers of water that are both at the same temperature as a third container of water, we expect they all have the same temperature.

In a sense, this seems obvious, so obvious that it was always assumed without statement. But in retrospect, it is a fundamental concept that deserves it's place as one of the laws of thermodynamics. By the time this was acknowledged, the names "first law" and "second law" were taken, so this became the *Zeroth law of Thermodynamics.*

Before we move on, let's review some things we know from high school chemistry.

28.3   Numbers of Atoms and Number Densities

Atoms are small. They are really incredibly small. A typical atom measures a few Angstroms across. An angstrom is 1×10^{-10} m. That is unbelievably small. Let's try to get an understanding of this scale. Picture a meter stick. Then divide it up into 10000000000 equal units. That is how small an atoms is. Or, consider scaling our meter stick to be the size of the continental US, that is about 4.0877×10^6 m. Then for our US sized meter stick, on this scale an atom would still only be about half a millimeter across! About the size of a cake sprinkle.

So it is no surprise that in a typical sample of a solid we can have something like 10^{25} atoms. We use the capital letter N to mean the number of atoms in a sample of matter.

Because these numbers are so large, it is sometimes convenient to talk about how many atoms there are in a small volume of a substance. This is often less than in the whole sample. We can say, for example, that there are 1000 atoms per cubic micrometer of material. We call such a ratio of number of atoms per volume a *number density.* In the SI system, the units for the number of atoms per unit volume are $1/\mathrm{m}^3$. So number densities can still be quite large! Number densities are more useful for samples where the density is uniform, but we can talk about substances who's number density changes. Unfortunately you will often see a small n used to mean number density. This over use of n is confusing, so sometimes we will write number density as N/V so it is obvious what we mean.

28.3.1   Atomic Mass

You probably ran into atomic mass in High School. The idea of atomic mass is that each nucleon (proton or neutron) has about the same mass, and the electrons don't have much mass at all, so we can sort of count nucleons and say that an atom has a mass in units of nucleon masses. Carbon has 12 nucleons

(six protons and six neutrons) so it would have a mass of 12 u. The unit u is the atomic mas unit, the mass of one nucleon. Because of the electrons (and other effects) this is not an exact approach, and it is not good enough if we attempt to do nuclear physics. But it is close enough for us for now. So we can say that hydrogen $\left(^{1}H\right)$ has an atomic mass of 1 u. If we have a molecule, we can just add the atomic masses of each atom in the molecule together to find the molecule's mass. Note that to be more accurate, we can use a periodic table and find the atomic mass, or, if we need many digits of accuracy, we can go to a table of the nucleotides. We can convert these masses to SI units using

$$1\,\mathrm{u} = 1.66 \times 10^{-27}\,\mathrm{kg}$$

28.3.2 Moles

Chances are that you are familiar with the concept of a mole as well. But experience has shown me that the mole will be new to some people. So let's start with something we know, a dozen.

What is a dozen? Well, it is 12 of something. We could have a dozen doughnuts or a dozen eggs. or a dozen large white pickup trucks. It is just an amount. It is 12 of whatever we are counting.

A mole is like a dozen, but it is a BIG dozen. Is is 6.02×10^{23} of something. I could have 6.02×10^{23} doughnuts, or 6.02×10^{23} eggs. I would need quite a few hungry friends to eat it all. But a mole is just a number of something. The mole is not a very practical amount for everyday objects like doughtnuts or trucks. But think of the large number of atoms in a sample of matter. For this situation, a mole seems like a reasonable number to use.

Like we say I want to buy a dozen eggs, we could say we want to buy a mole of hydrogen atoms. Since atoms are so small, packaging them in big numbers makes sense.

This number, 6.02×10^{23}, is called *Avogadro's number*

$$N_A = 6.02 \times 10^{23}$$

To find the number of dozens of a something, we take the number of items and divide by 12.

$$n_{dozen} = \frac{N}{12}$$

To find the number of moles of a something, we take the number of items and divide by Avogadro's number

$$n_{moles} = \frac{N}{N_A}$$

usually we will use small n to be the number of moles of something. This could be a problem because n can also mean number density. So we will have to be careful to state what we mean with our variables.

28.3.3   Molar mass

We could give a mass for a dozen eggs. If each egg has the same mass (more or less) then the dozenal mass would be the mass of a dozen eggs and it would be twelve times the mass of one egg. The dozenal mass of a dozen trucks would be larger than the dozenal mass of eggs because the mass of a single truck is larger than the mass of an egg, and the mass of a dozen trucks is 12 times the mass of one truck.

We can also define a molar mass, which would be the mass of 6.02×10^{23} of something. Usually, we choose the units for this molar mass to be the mass *in grams* of a mole of something. And when we do this, the molar mass is equal to the numerical value of the atomic or molecular mass. for example, the molar mass of He, with $m = 4\,\mathrm{u}$ is $M = 4\,\mathrm{g/mol}$. The molar mass of diatomic nitrogen would be $28\,\mathrm{g/mol}$ because each nitrogen atom has an atomic mass of $m = 14\,\mathrm{u}$ and we add the two atomic masses together for the molecular mass. With this definition of molar mass we can see that

$$n_{moles} = \frac{M\,(\text{in grams})}{M_{mol}}$$

Now that we have these new terms to use, let's take up measuring temperature in our next lecture.