Chapter 9

9 Sound Standing waves 1.17.4, 1.17.5

Fundamental Concepts

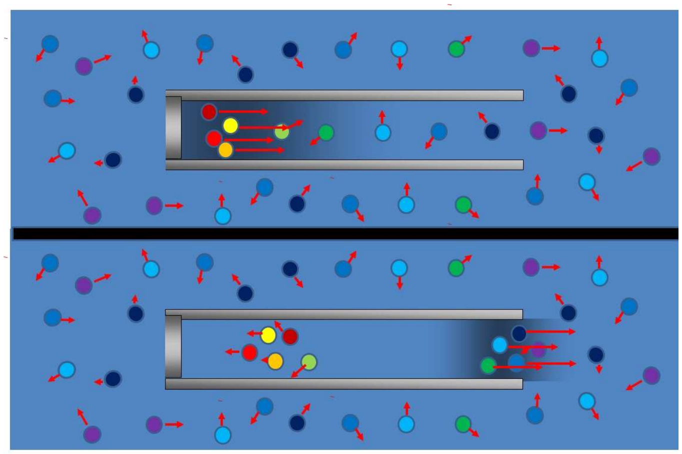
- Sound Standing waves (music)

9.1 Sound Standing waves.

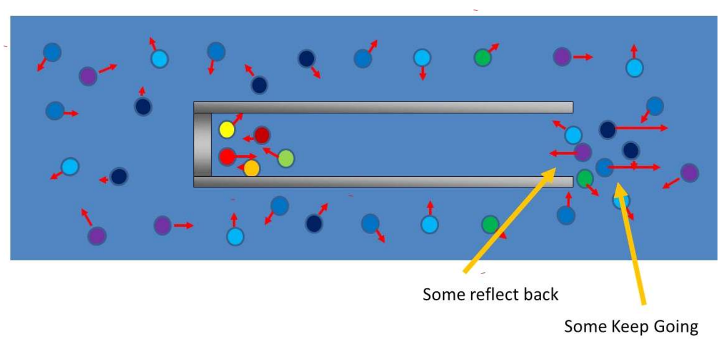
We have spent some time studying standing waves on strings. But we also have studied sound waves, could we make sound standing waves?

For our attempt, let's take a pipe as shown in the next figures. The pipe will control the direction of the sound wave. The sound wave will (mostly) go down the pipe. But will there be a reflection at the end of the pipe like there was for the string?

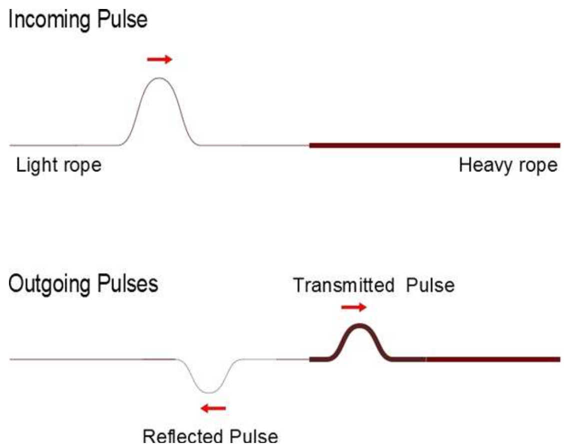
It turns out that there will be a reflection. Let's consider our sound wave as a change in pressure to see why. For the string wave, we had a reflected pulse because the person (or guitar bridge, or whatever is holding the string end) exerted a force on the string. If a sound wave travels down our pipe, the pressure will change as the wave goes down the pipe. Remember that pressure is a force spread over an area. When the sound wave gets to the end of the pipe, the rest of the room's air is waiting. And that air mass pushes back on the air molecules that are trying to leave the pipe. The air mass of the room won't change it's pressure much. That resistance to pressure change (the force due to the rest of the room air molecules colliding with our wave molecules, pushing them back) will send our tube molecules back down the tube. And that starts a reflection. Thin about this for a moment. We need to picture more than we have before. The room is full of air, and it turns out that the air molecules aren't sitting still. So when we make our wave start but hitting molecules with a piston, the molecules at the end of the tube will hit room air molecules that are already moving.



Think of pool balls. If the cue ball and another ball, say, the 6 ball, collide, usually the 6 ball is stationary at the beginning. But if it is not, that will change the outcome of our conservation of momentum problem. if the 6 ball is initially moving toward the cue ball, the cue ball will bounce back the way it came. That will happen with some of the molecules at the end of the pipe. But some will keep going.

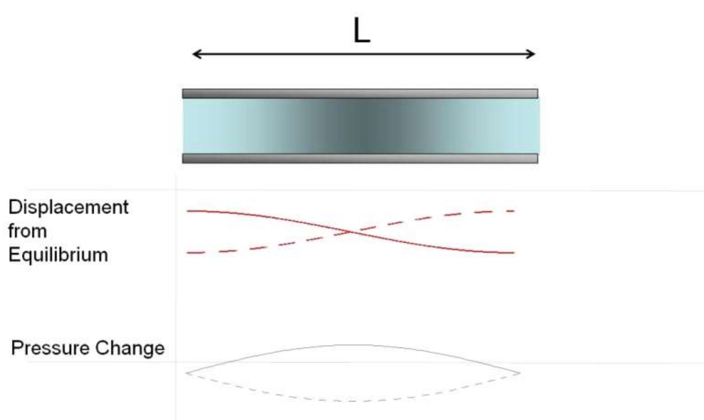


We will split the energy from the wave into to waves, much like we did with waves on strings when we came to an interface.



Now that we have a reflected wave in the tube, we can end up with two waves so long as the piston keeps working at making new waves. And with two waves in the same medium, we have the possibility of having standing waves!

If we have a pipe open at both ends, we can see that air molecules are free to move in and out of the ends of the pipes. If the air molecules can move, the ends must not be nodes. In our string case, the part of the string that experienced destructive interference and did not move was called a node and we always had a node on the end of the string. But the pipe is different than the string case! We can see that air molecules will move out and then bounce back in at the end of the pipe. Still, we expect that there must be a node somewhere. We can reasonably guess that there will be a node in the middle of the pipe. Of course, the pressure on both ends must be atmospheric pressure. So, remembering that pressure and displacement are 90° out of phase for sound waves, we can guess that there are pressure nodes on both ends. But there is a displacement anti-node at the ends of the pipe.



Then for the first harmonic we can draw a displacement node in the middle and we see that

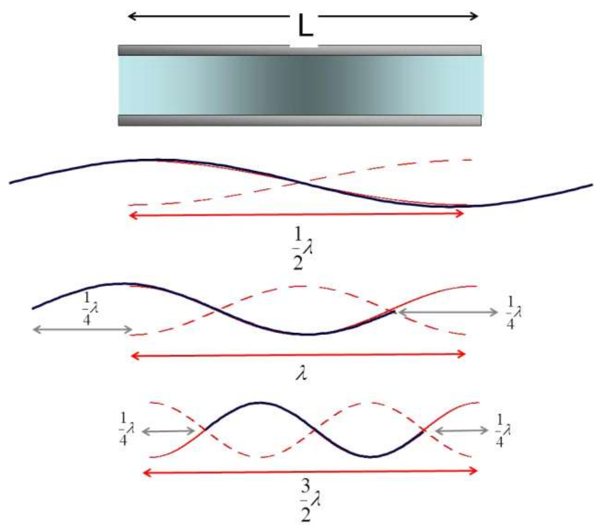
$$\lambda_1 = 2L \tag{9.1}$$

and

$$f_1 = \frac{v}{2L} \tag{9.2}$$

The next mode fits a whole wavelength

$$\lambda_2 = L \tag{9.3}$$
$$f_2 = \frac{v}{L} \tag{9.4}$$



but the next mode fits a wavelength and a half

$$\lambda_3 = \frac{3}{2}L \tag{9.5}$$
$$f_3 = \frac{2v}{3L} \tag{9.6}$$

If we keep going

$$\lambda_n = \frac{2}{n}L \tag{9.7}$$
$$f_n = n\frac{v}{2L} \qquad n = 1, 2, 3, 4 \ldots \tag{9.8}$$

This is the same mathematical form that we achieved for a standing wave on a string!

9.2 Pipes closed on one end

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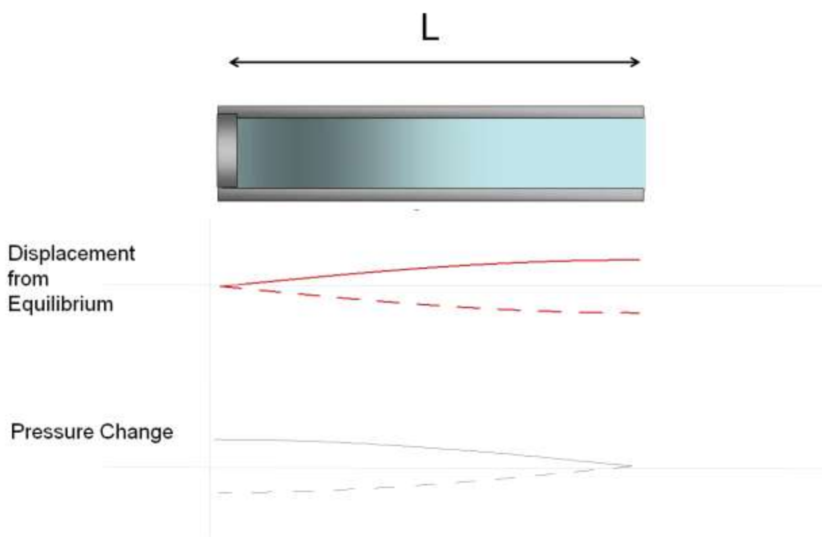
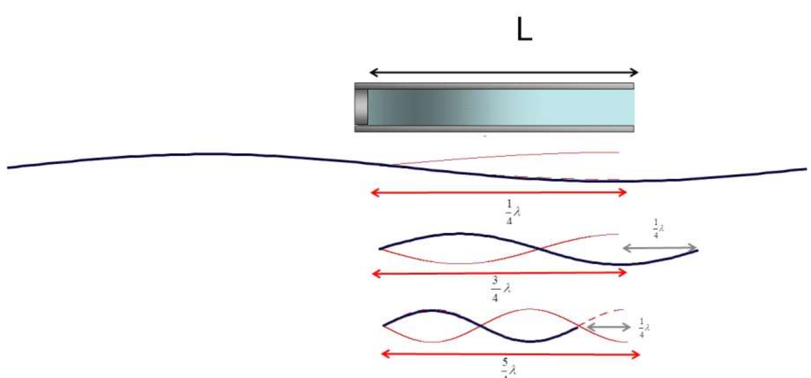
But what happens if we put a cap on one end of the pipe? The air molecules cannot move longitudinally once they hit the cap. And where the molecules don’t move, that is a node. So our capped end must be a displacement node. Question 123.24.2

The open end is still a displacement antinode. Our standing wave will have a node on the capped side, and an anti-node on the open side. It might look like this



In the next figure we draw the first few harmonics for the “closed on one end” case.



The first harmonic for the closed pipe are found by using

$$v = \lambda f \tag{9.9}$$

$$f = \frac{v}{\lambda} \tag{9.10}$$

We know the speed of sound, so we have v. Knowing that the first harmonic has a node at one end and an anti node at the other end gives us the wavelength. If the pipe is L in length, then L must be

$$L = \frac{1}{4}\lambda_1 \tag{9.11}$$

or

$$\lambda_1 = 4L \tag{9.12}$$

then

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \tag{9.13}$$

The next configuration that will have a node on one end and an antinode on the other will have

$$L = \frac{3}{4}\lambda_2 \tag{9.14}$$

which gives

$$\lambda_2 = \frac{4}{3L} \tag{9.15}$$

and

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} \tag{9.16}$$

If we continued, we would find

$$\lambda_n = \frac{4}{n}L \tag{9.17}$$

and

$$f_n = n\frac{v}{4L} \qquad n = 1, 3, 5 \ldots \tag{9.18}$$

Example: organ pipe

Organ Pipe Demo

The organ pipe shown is closed at one end so we expect

$$f_n = n\frac{v}{4L} \qquad n = 1, 3, 5 \ldots \tag{9.19}$$

Measuring the pipe, and assuming about 20 °C for the room temperature we have

$$\begin{aligned} L &= 0.41\,\mathrm{m} \\ R &= 0.06\,\mathrm{m} \\ v &= 343\,\tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned} \tag{9.20}$$

There is a detail we have ignored in our analysis, the width of the pipe matters a little. I will include a fudge factor to account for this. With the fudge factor, the wavelength is

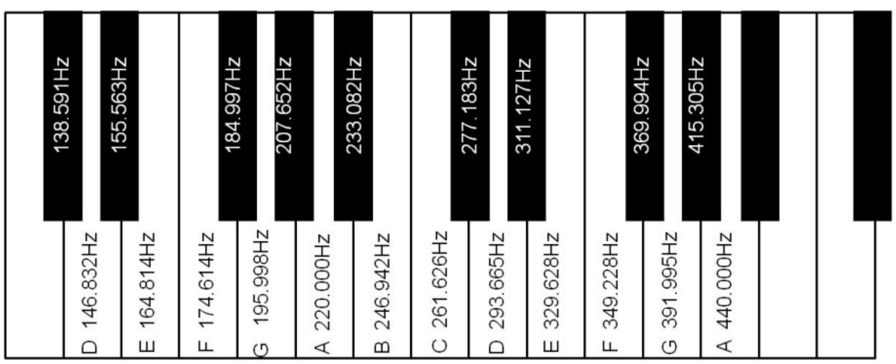
$$\begin{aligned} \lambda_1 &= 4\,(L + 0.6R) \\ &= 178.\,4\,\mathrm{cm} \end{aligned}$$

then our fundamental frequency is

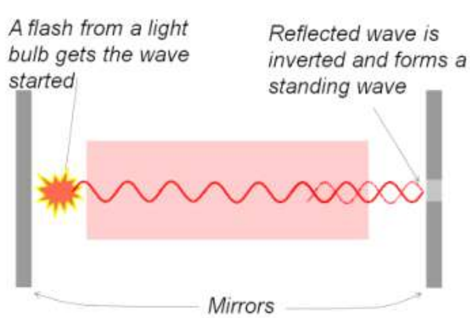
$$f_1 = \frac{v}{\lambda_1} \tag{9.21}$$

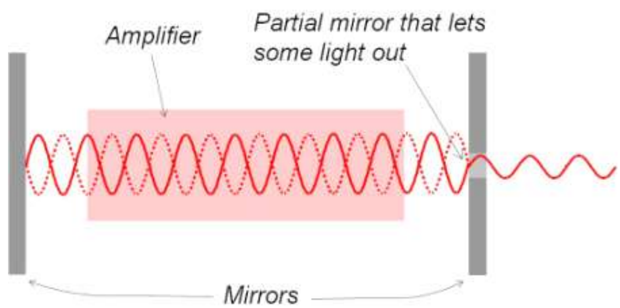
$$= 192.\,26 \tag{9.22}$$

We can identify this note, and compare to a standard, like a tuning fork or a piano to verify our prediction.



Lasers and standing waves

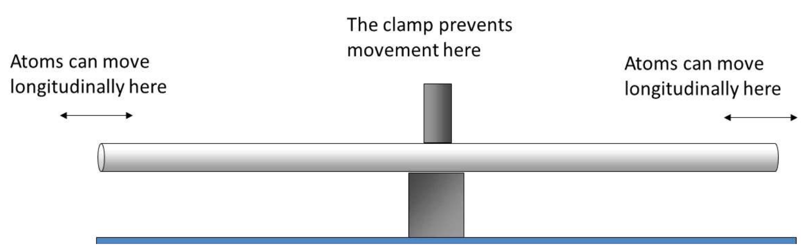




Question 123.24.3
Question 123.24.4

9.3 Standing Waves in Rods and Membranes

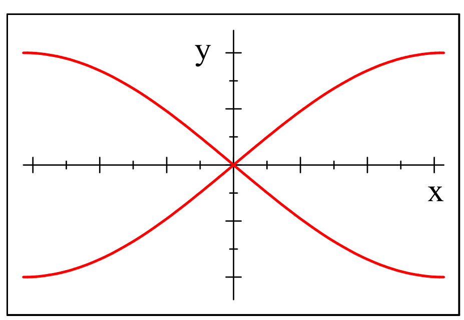
We have hinted all chapter that the analysis techniques we were building apply to structures. We need more math and computational tools to analyze complex structures like bridges and buildings, but we can tackle a simple structure like a rod that is clamped. The atoms in the rod can vibrate longitudinally



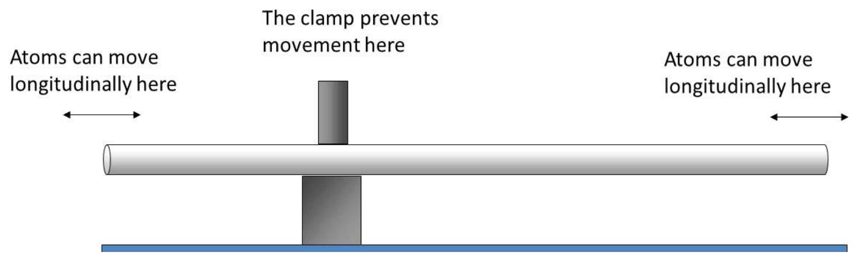
Since we have motion possible on both ends and not in the middle, we surmise that this system will have similar solutions as did the open ended pipe.

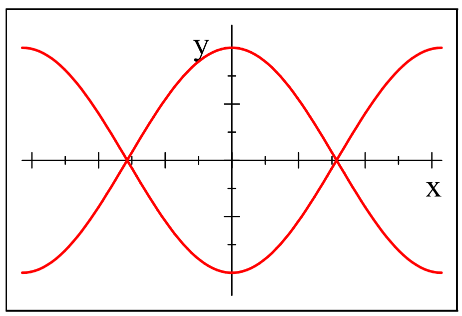
$$f_n = n\frac{v}{2L}$$

The fundamental looks like
$\sin(x)$



But suppose we move the clamp. The clamp forces a node where it is placed. If we place the clap at $L/4$





We can perform a similar analysis for a drum head, but it is much more complicated. The modes are not points, but lines or curves, and the frequencies of oscillation are not integer multiples of each other. See for example http://physics.usask.ca/~hirose/ep225/animation/drum/anim-drum.htm.

Of course structures can also waggle on the ends. the ends can rotate counter to each other, etc. These are more complex modes than the longitudinal modes we have considered.