Chapter 29

29 Thermometers: Measuring Temperature 2.1.2, 2.1.3

Last lecture we reviewed some basics of how materials are made and how chemists and physicists and material scientists determine how many moving parts a sample of a material might have. We also talked about measuring temperature. But didn't actually talk about what temperature is or even how temperature measuring devices work. We will start with measurement techniques in this lecture and build up to a model of what temperature is.

Fundamental Concepts

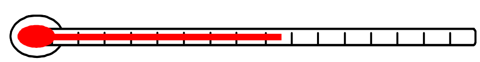
1.
    - Kelvin Temperature Scale
    - volume of a liquid
    - the dimensions of a solid
    - the pressure of a gas at constant volume
    - the volume of a gas at constant pressure
    - The electric resistance of a conductor
    - The color of an object
    - Phase Changes and Phase Diagrams
    - Thermal Expansion

29.1 How to build a thermometer

We will find that several material properties change with temperature

1. Volume of a liquid
2. The dimensions of a solid
3. The pressure of a gas at constant volume
4. The volume of a gas at constant pressure
5. The electric resistance of a conductor
6. The color of an object
7. The size of an object

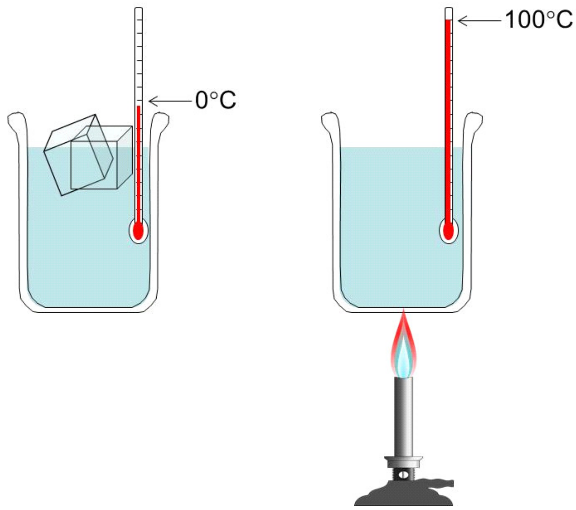
Since these things change with temperature, we can use any of these to make a temperature measuring device. Think of old fashioned thermometers, how do they work?



Old fashioned mercury thermometers (and their glycerine replacements) use the first property, volume of a liquid increases with temperature. The liquid is placed in an evacuated tube, and the amount of the tube that is filled depends on the temperature. The liquid fills more of the tube when the temperature goes up because the liquid volume gets bigger with temperature. By placing a ruled background behind the tube or even by making rule marks on the outside of the tube, we can find out how much the liquid volume changed, and calculate the temperature change.

29.1.1 Celsius temperature scale

To make our old fashioned mercury thermometer work we need to have units of temperature to mark on the tube. Once of the sets of units for temperature is called the *Celsius temperature scale.* The most important thing to say about the Celsius scale is not to use it for doing thermodynamic problems (The same goes double for the Fahrenheit scale). But since it is common to see Celsius temperatures, we will talk about them.



The idea,once again, is to make marks on our thermometer tube that show how the change in liquid volume is related to temperature. To do this for the Celsius scale we put our thermometer in ice water and mark where the top of the liquid is. And we assign the value of $0\,^{\circ}\mathrm{C}$ to this mark. Then we take our thermometer and put it in boiling water. The liquid moves up the tube. Where it stops we make a mark and label this mark $100\,^{\circ}\mathrm{C}$. This scale sets the zero point at the ice point of water and sets $100\,^{\circ}\mathrm{C}$ at the steam point of water. Then we make 100 evenly spaced divisions between these points. We can also mark the tube with the same division spacing below the $0\,^{\circ}\mathrm{C}$ mark and above the $100\,^{\circ}\mathrm{C}$ mark. But this calibration does not work well if high accuracy is required. There are often large discrepancies when temperatures are measured outside of the $0\,^{\circ}\mathrm{C}$ to $100\,^{\circ}\mathrm{C}$ range.

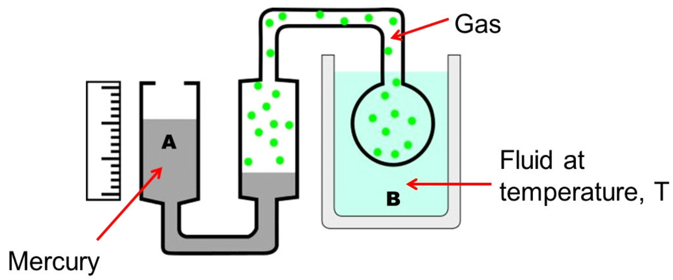
Worse yet, water only boils at $100\,^{\circ}\mathrm{C}$ at sea level, and does not always freeze at $0\,^{\circ}\mathrm{C}$.

Even worse, there is nothing really very 0ish about $0\,^{\circ}\mathrm{C}$. We picked it because it was a nice value for water. But we can certainly have lower temperatures than $0\,^{\circ}\mathrm{C}$.[1] This makes $0\,^{\circ}\mathrm{C}$ more like an origin of a coordinate system. We could say that the intersection of Main St. and Center St. is our origin, but there is noting really zeroish about that intersection. But it turns out there *is* a natural zero point for temperature! A better temperature scale would start at the natural zero point. Let's see if we can make such a scale.

[1] We often do in Rexburg from about December to March.

29.1.2 The Constant-Volume Gas Thermometer and the Absolute Temperature Scale

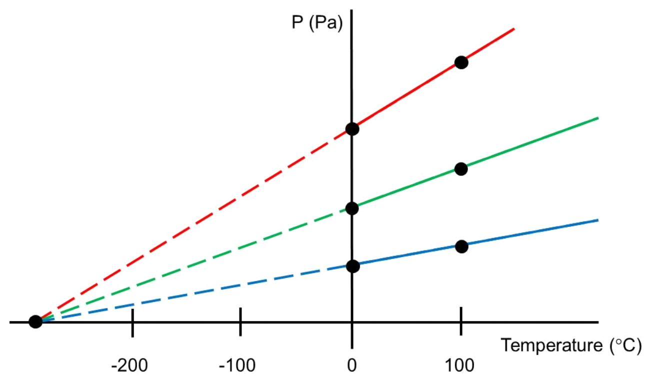
We wish to define a temperature scale that is based on a physical zero point. To do this we need a better thermometer. So here is one in the figure below.



This device uses the rise in pressure at a constant volume (the number 3 property from our list at the beginning of the lecture). It works by placing the gas bulb in thermal contact with the object who's temperature is to be measured (reservoir B in the figure[2]). A scale (like a ruler) is placed in the apparatus such that the mercury in reservoir A on the left is at the zero point. We could take water at its ice point, for example, to find this first part of the scale. Then the bulb is placed in thermal contact with another object by swapping out what is in reservoir B, say we use water at its steam point this time. The mercury will move because the pressure increases in the bulb, and the gas tries to expand. We add mercury into reservoir A (just when you thought you were done with U-tubes!) until the level of reservoir is again at the zero point. The height of the additional mercury we added is proportional to the change in pressure, as we now know. The Pressure for both measurements is calculated and plotted against the temperature and a linear curve is drawn between the two pressure-temperature points. This curve serves as a calibration for other temperatures. We could fill the bulb with different gasses. If we do this, we get different sloped lines. You can see this in the next figure.

But think, it seems like with this new thermometer design we should have zero temperature if we have zero gas in the bulb. We have different lines for each gas, but if we then extend these lines back to where $P = 0$ (so that we have no gas), we find they intersect with $P = 0$ at $T = 273.15\,^{\circ}\mathrm{C}$. This is the basis we need for an absolute temperature scale! We have found a place where the temperature really should be zero (because we have no gas to have a temperature!).

[2] A reservoir is just a container of something.



We will take this common zero pressure point, $T = 273.15\,^{\circ}\mathrm{C}$ as our zero point for our improved temperature scale, but we need another point. We will choose the triple point of water, the one point where water, ice and steam exist in equilibrium; this happens at $0.01\,^{\circ}\mathrm{C}$ and $4.58\,\mathrm{mmHg} = 610.\,61\,\mathrm{Pa}$. We will call this point 273.15 on our new scale which seems kind of random. But think, with this choice each unit of temperature will then be $1/273.15^{th}$ of the distance between our zero point and the water triple point. This way the degrees in this scale are the same size as the degrees in the Celsius scale (which is kind of nice). But this new scale that starts at the physical zero point we will call the Kelvin Temperature scale and the units are Kelvins (K). This is the temperature system we really should use.

29.1.3 Temperature scale conversions

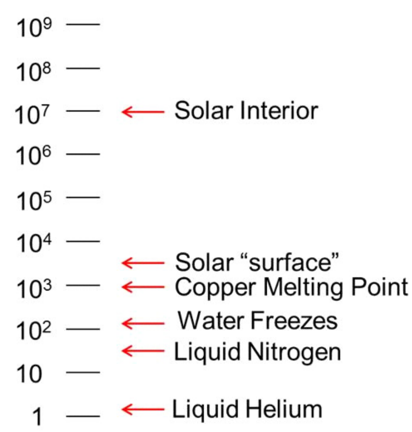
We will need some conversion factors so you can take temperatures from the Celsius and Fahrenheit scales to the Kelvin scale.

$$T_F = \frac{9}{5}T_C + 32\,^{\circ}\mathrm{F} \tag{29.1}$$

$$\Delta T_C = \Delta T_K = \frac{5}{9}\Delta T_F \tag{29.2}$$

$$T_C = T_K - 273.15 \tag{29.3}$$

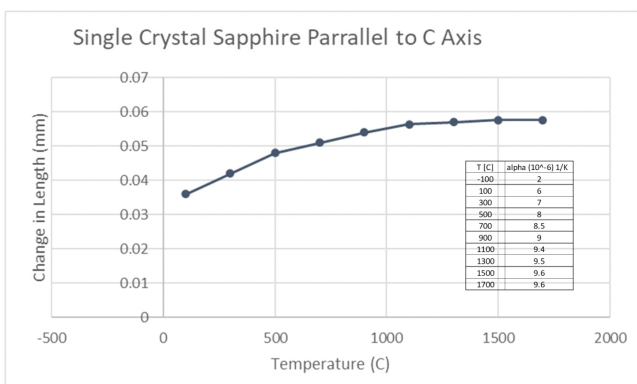
Since the Kelvin scale is not the scale your doctor or your meteorologist uses, we should try to get some feeling for the Kelvin scale. Here are some interesting temperatures placed on the Kelvin scale.



29.2 Thermal Expansion

Ring and Ball Demo

We still only kind of know what temperature is. But we can now measure temperature, so suppose we increase the temperature of a substance. We could take the mercury in a thermometer, for example. When the temperature is higher, the mercury takes up more space. It expands. This isn’t just true for liquids. Here is the change in length for a laser crystal as a function of temperature.



T [C]	alpha (10^-6) 1/K
-100	2
100	6
300	7
500	8
700	8.5
900	9
1100	9.4
1300	9.5
1500	9.6
1700	9.6

Notice that the change in length makes a curve that bends. We would prefer to make our thermometers out of a material that has a linear expansion. That is why we used mercury in thermometers for many years. Mercury has expansion that is linear in temperature for many situations. When the expansion length is much less than the original dimensions of the object (L_i), we can consider the expansion to be linear in the change in temperature ΔT. Of course we could increase the temperature above this nice linear region, and we could get interesting effects like melting or exploding. But for this lecture we will limit our discussion to a linear change in length with temperature for now. Let's see how this linear region of expansion works.

29.2.1 Expansion Coefficient

What does it mean to have a linear change in length with temperature?

$$L_f = L_i + \alpha L_i \Delta T \tag{29.4}$$

It means that the final length of the object is equal to the initial length of the object plus a little bit more that depends on ΔT and not ΔT^2 or ΔT^3 or some other function of ΔT. We will get straight line plots instead of curved plots as T changes. You can see that this is the case for the equation above for thermal expansion. We can write it next to an equation for a straight line to show the linear nature of the equation

$$\begin{aligned} y &= b + mx \\ L_f &= L_i + \alpha L_i \Delta T \end{aligned}$$

If we plot L_f vs. ΔT this would make a straight line. The slope of the line would be αL_i so the amount of expansion depends on how big the object was to begin with. It also depends on what material we have. That is what the α tells us. Different substances expand at different rates even if they have the same initial length. So if you have a metal cavity of length L_i and fill it with a laser crystal of length L_i and heat them both up, you may find the crystal expands at a different rate than the metal fasteners when the temperature rises–and may need to buy a replacement for the shattered crystal.[3]. The coefficient α is different for every substance, because every substance expands a little differently than other substances. We should give α a name. It is called the *average coefficient of linear expansion* where from equation (29.4) we can see that

$$\begin{aligned} \alpha &\equiv \frac{\frac{\Delta L}{L_i}}{\Delta T} \\ &= \frac{\Delta L}{L_i \Delta T} \end{aligned}$$

where ΔL is the change in length due to a temperature change ΔT. And of course we already know L_i is the original length of the sample.

As long as ΔT is "not too big" α is constant. We can write this as

$$L_f - L_i = \alpha L_i \left(T_f - T_i\right)$$

[3] It was a very sad day when this happened to me personally.

where the subscript f means final and i means initial as usual. The units of α are inverse temperature (unfortunately it is often in °C, but we would prefer to use K). Laboratories publish tables of average coefficients of linear expansion. A few are given in the table below

Material	Average $\alpha\left(\mathrm{K}^{-1}\right)$
Aluminum	25×10^{-6}
Copper	17×10^{-6}
Gold	14×10^{-6}
Lead	29×10^{-6}
Steel	11×10^{-6}
Brass	18.7×10^{-6}
quartz	0.4×10^{-6}
Glass	9×10^{-6}
Concrete	12×10^{-6}

Suppose we increase the temperature of something that has a hole in it. Does the hole increase or decrease in size? A cavity in a piece of material expands in the same way as if the cavity were filled with the material. Let's see that this must be true.

29.2.2 Volume expansion

Don't do this in class

suppose we have a cube with sides of length L_i. What happens when we increase the cube's temperature? Each side will become larger. So what happened to the volume?

$$\begin{aligned}
\cancel{V}_i+\Delta\cancel{V} &= \left(L_i+\Delta L\right)\left(L_i+\Delta L\right)\left(L_i+\Delta L\right)\\
&= \left(L_i+\alpha L_i\Delta T\right)\left(L_i+\alpha L_i\Delta T\right)\left(L_i+\alpha L_i\Delta T\right)\\
&= L_i^3\left(1+\alpha\Delta T\right)\left(1+\alpha\Delta T\right)\left(1+\alpha\Delta T\right)\\
&= L_i^3\left(\Delta T^3\alpha^3+3\Delta T^2\alpha^2+3\Delta T\alpha+1\right)\\
&= \cancel{V}_i\left(\Delta T^3\alpha^3+3\Delta T^2\alpha^2+3\Delta T\alpha+1\right)
\end{aligned}$$

Now let's divide both sides by $\cancel{V}_i$

$$\begin{aligned}
1+\frac{\Delta\cancel{V}}{\cancel{V}_i} &= \Delta T^3\alpha^3+3\Delta T^2\alpha^2+3\Delta T\alpha+1\\
\frac{\Delta\cancel{V}}{\cancel{V}_i} &= \Delta T^3\alpha^3+3\Delta T^2\alpha^2+3\Delta T\alpha
\end{aligned}$$

and let's make an approximation. For normal conditions, $\alpha\Delta T\ll1$ (good for $T<373.15\,\mathrm{K}$ $(100\,^\circ\mathrm{C})$). Then $\alpha^2\Delta T^2$ and $\alpha^3\Delta T^3$ are really really small. Let's ignore them.

$$\frac{\Delta\cancel{V}}{\cancel{V}_i}\approx3\alpha\Delta T \tag{29.5}$$

We can write this as

$$\begin{aligned}\Delta \cancel{V} &\approx 3\alpha \cancel{V}_i \Delta T \\ &= \beta \cancel{V}_i \Delta T\end{aligned} \tag{29.6}$$

where β is called the *average coefficient of volume expansion.*

We could do the same for a two dimensional metal plate (in the approximation that a plate is 2D!)

$$\Delta A = 2\alpha A_i \Delta T \tag{29.7}$$

We need to be careful that we don't use these linear expansion equations outside of their valid range. For example, suppose we build a satellite and launch it into a low Earth orbit (LEO). The space environment for a LEO orbit (sun side to shade side) has a very large ΔT.

$$\begin{array}{l} T_{sun} = 120\,^\circ\mathrm{C} = 393.15\,\mathrm{K} \\ T_{shade} = -100\,^\circ\mathrm{C} = 173.15\,\mathrm{K} \end{array} \tag{29.8}$$

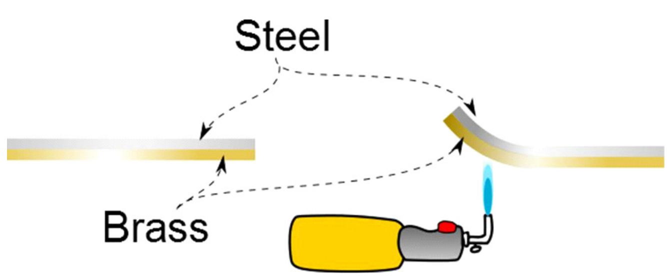
This ΔT range is too large for most materials to still be linear. Special low α materials are used to allow satellite structures to survive

But how about on Earth? Even on Earth we can have extremes

$$\begin{array}{l} T_{\max} = 57.6\,^\circ\mathrm{C} = 330.75\,\mathrm{K} \\ T_{\min} = -89\,^\circ\mathrm{C} = 184.15\,\mathrm{K} \end{array} \tag{29.9}$$

This ΔT range would be a challenge to cover with most materials.[4] So if you are going to build a device or instrument that will travel around the globe, you need to use special low α materials. Often making these materials is very expensive and the process is a tightly held industrial secret. Alternately you can air condition or heat your system. But you do have to do something to stay in the linear range.[5]

Looking at table of average linear expansion coefficients we find see different α values for different materials. Sometimes α values are quite different. Suppose we take two materials with different α's and put them together.



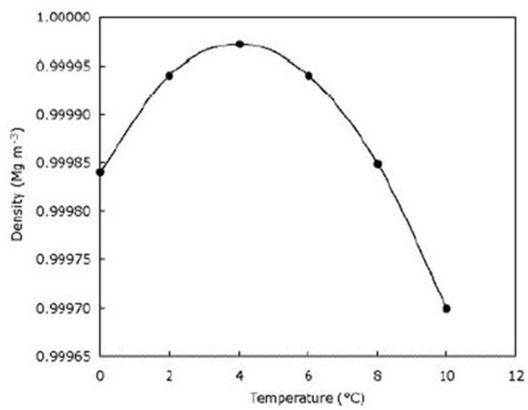
[4] Expecially metal structures and laser crystals. >sigh<

[5] I am told by the BYU-I facilities people that the BYUIC has one huge beam across the entire auditorium that has to stay heated or it shrinks enough to let the balcony fall off the walls!

The figure shows a bimetallic strip. What would happen if we heated this bimetallic strip? The two metals will expand at different rates, forcing the bonded pair of metals to bend. And we could make a temperature measuring device that measures how much this bimetallic strip bends.

29.2.3 Water is weird (and the Lord knows what He is doing!)

Question 123.9.2



Density of water near $0\,°C$. (Public Domain image PD-1923, PD-Australia)

For most substances the volume increases with increasing temperature. Usually liquids increase in volume more than solids. But water is different. Look at the graph of density vs. Temperature.

Just as ice melts, it starts to *increase* in density, but then it becomes less dense with increased temperature after about $277.15\,K$ ($4\,°C$). We expect the behavior we see after $277.15\,K$ ($4\,°C$). The density is decreasing because we have the same number of water molecules, but the temperature change has increased the volume. So the density goes down.

But from $273.15\,K$ ($0\,°C$) to $277.15\,K$ ($4\,°C$) we see the volume decreases with ΔT. This is very odd. But very cool! This is why a lake freezes from the top down. The colder water near $273.15\,K$ ($0\,°C$) is less dense, and so before it freezes it floats to the surface. This is great, because if it became more dense just before it froze, the water would sink to the bottom and freeze, leaving the liquid water exposed to the cold atmosphere to freeze more water. The lake would fill up from the bottom with solid ice, killing fish and water life.

Lake Erie from Space. Notice the ice covering part of the lake. (Public Domain Image courtesy NASA Visible Earth, http://visibleearth.nasa.gov/view_rec.php?id=1286

Because the water becomes less dense before freezing we don't freeze large lakes and oceans solid. And that keeps life going on Earth!

We have made progress. But we still don't know what temperature is. We know some things about what temperature does. And this can give us a clue. Temperature is making things move by expansion. And we know that there must be forces behind making things move. Forces do work. Work is energy. Could it be that temperature has to do with exchanging energy? Question 123.9.2

Question 123.9.3