

Chapter 32

32 Kinetic Theory of Gases

2.2.2, 2.2.3

We now have gases with internal energy. But so far, as we have considered the motion of atoms with thermal energy we have been able to treat whole samples of a gas at a time. But you might guess that we are sort of taking averages of the motions of the atoms or molecules. Let's make this more explicit in this lecture.

Fundamental Concepts

- In a gas, the molecules have different speeds
- The distribution of speeds is given by the Boltzmann distribution law
- the root mean square speed ($v_{rms} = \sqrt{\overline{v^2}}$) is a better measure of the speed of gas molecules than the average velocity
- Pressure for an ideal gas is given by $P = \frac{2}{3} \frac{N}{V} \left(\frac{1}{2} m \overline{v^2} \right)$
- Temperature is related directly to the average kinetic energy of the molecules of a gas
- We call a way the molecule can move a *degree of freedom*.
- For an ideal gas $C_V = \frac{3}{2} R$
- For an ideal gas $E_{int} = \frac{3}{2} k_B T$

32.1 Molecular Model of an Ideal Gas

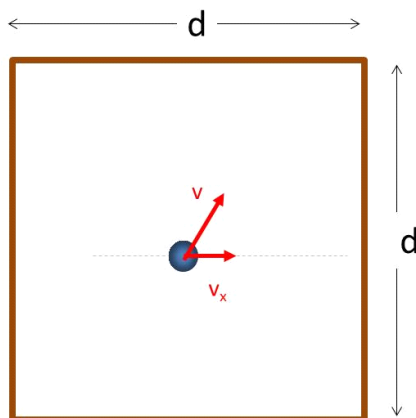
We have hinted that the internal energy of a system must be the energy associated with the atoms and molecules that make the system. By making association in our mental model more concrete, and more mathematical. We will be able to see how the thermodynamic laws we have studied are generated by the basic laws of motion. But we will still limit our study to ideal gases in this class. So let's state some assumptions that follow from the ideal gas approximation.

Ideal Gas Model
The number of molecules in the gas is very large,
The average separation between molecules is large compared the their dimensions
The molecules obey Newton's laws of motion,
On the whole the molecules move randomly
The molecules interact only by short-range forces during elastic collisions
The molecules make elastic collisions with the walls
The gas under consideration is a pure substance; that is; all molecules are identical.

We will often say “molecule” but for an ideal gas atoms and molecules act alike.

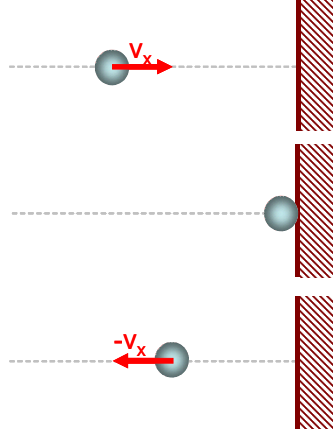
32.2 Microscopic Definition of Pressure

We want to express macroscopic quantities like pressure and volume in terms of microscopic effects. The easiest to start with is pressure



Picture a particle in a cubical box of side length d .

As the particle approaches the wall of the box we may say that it has velocity v_x . The particle impacts the wall and bounces off again. Now it travels with $-v_x$.



We can think of why this would be. We know momentum is conserved. Before the collision we may take the velocity of the wall to be zero, and we may imagine the wall to have a very large mass (which it does, compared to the particle!). If we call the particle mass m and the wall mass M but the wall will not move with any speed after the collision so $v_{wall_f} = 0$

$$\begin{aligned} P_i &= Mv_{wall_i} + mv_{x_i} \\ &= mv_{x_i} \end{aligned}$$

and

$$\begin{aligned} P_f &= Mv_{wall_f} + mv_{x_f} \\ &= -mv_{x_f} \end{aligned}$$

because the wall still isn't going to move. Then for conservation of momentum

$$P_i = P_f$$

We have

$$mv_{x_i} = -mv_{x_f}$$

Then the magnitude of the velocities must be equal and the change in momentum must be

$$\begin{aligned} \Delta p &= mv_{x_i} - (-mv_{x_f}) \\ &= 2mv_{x_i} \end{aligned}$$

Question 123.14.6

Recall that the impulse from the collision of the particle on the wall would be

$$\begin{aligned} \bar{F}_{i,\text{on molecule}} \Delta t_{\text{collision}} &= \Delta p_{x_i} \\ &= -2mv_{x_i} \end{aligned}$$

This is the impulse momentum theorem from Principles of Physics I. The time of the collision is very short. This force is the force of the wall on the particle.

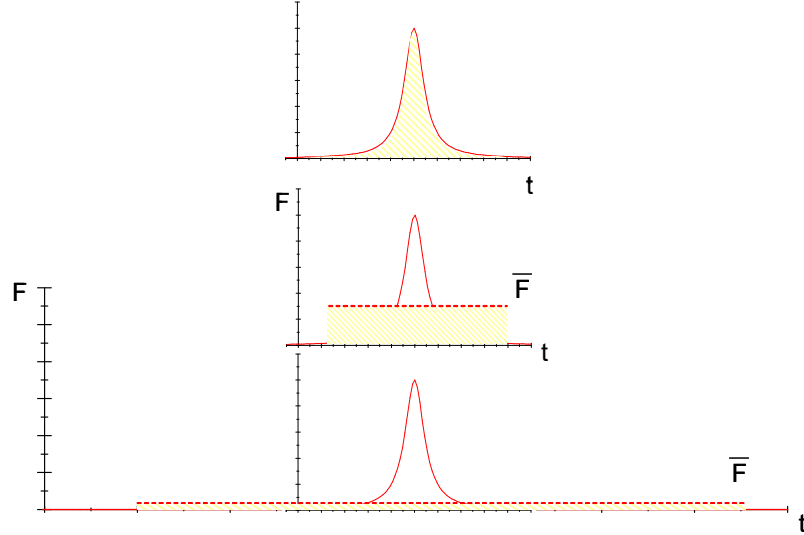
Now consider that if a particle bounces off one wall in the x -direction, it must travel to the opposite wall and back before it can bounce off the same wall again. That is, the time between bounces is roughly

$$\Delta t = \frac{2d}{v_{x_i}} \quad (32.1)$$

Averages are funny things, it is perfectly legal to redefine the time of our average force to be this travel time Δt . What does this mean? In the first graph of the next figure we have the full definition of impulse

$$\Delta p = \int_{t_i}^{t_f} \mathbf{F} dt \quad (32.2)$$

which says the impulse is the area under the F vs. t graph.



The average force \bar{F} is pictured in the second graph. It has the same area as the first graph, but is flat, representing the average force (red dotted line). The final graph has a \bar{F} averaged over a much larger Δt . The area is the same, but the magnitude of the average force is much smaller.

This may seem useless, but at any rate, we can do this. So let's take our average of F over the time

$$\Delta t = \frac{2d}{v_{x_i}}$$

which is the time it takes our particle to travel away from our wall, and bounce back to the wall. We know that sometime within this Δt the collision with our wall actually occurs. The change in momentum is still

$$\bar{F} \Delta t = -2mv_{x_i}$$

so we can write

$$\begin{aligned}\bar{F} \frac{2d}{v_{x_i}} &= -2mv_{x_i} \\ \bar{F} &= -\frac{mv_{x_i}^2}{d}\end{aligned}$$

So far our force has been the force of the wall on the particle, but we know by Newton's third law that the particle force must be equal and opposite.

$$\bar{F}_{i, \text{on molecule}} = -\bar{F}_{i, \text{on the wall}}$$

then if we shorten $\bar{F}_{i, \text{on the wall}}$ to just \bar{F}_{iw}

$$\bar{F}_{iw} = \frac{mv_{x_i}^2}{d}$$

for one particle (molecule or atom).

For many particles

$$\begin{aligned}\bar{F}_w &= \sum_{i=1}^N \frac{mv_{x_i}^2}{d} \\ &= \frac{m}{d} \sum_{i=1}^N v_{x_i}^2\end{aligned}$$

We have found the average force on the container wall!

Question 123.14.7

Now remember how to take an average

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

so if we were to calculate the average velocity, *but square the velocity* before we take the average, it would look like this

$$\overline{v_{x_i}^2} = \frac{1}{N} \sum_{i=1}^N v_{x_i}^2$$

We almost have this in our force equation! Let's put in the missing parts

$$\begin{aligned}\bar{F} &= \frac{m}{d} \frac{N}{N} \sum_{i=1}^N v_{x_i}^2 \\ &= \frac{m}{d} N \overline{v_{x_i}^2}\end{aligned}$$

Going back to a single molecule, we remember that the velocity of that single molecule has components. The velocity squared would be (think of a dot product)

$$v_i^2 = v_{x_i}^2 + v_{y_i}^2 + v_{z_i}^2$$

Then if we take the averages of each piece like we did with $v_{x_i}^2$

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

There is no real reason to prefer the x -direction, we could have chosen the y -direction and had the same value. Then

$$v_x^2 = v_y^2 = v_z^2$$

and

$$\overline{v^2} = 3\overline{v_x^2}$$

So our force on the wall is

$$\begin{aligned}\bar{F} &= \frac{m}{d} N \overline{v_x^2} \\ &= \frac{m}{d} N \frac{1}{3} \overline{v^2} \\ &= \frac{mN}{3d} \overline{v^2} \\ &= \frac{N}{3d} (m\overline{v^2})\end{aligned}$$

Finally we can find the pressure. We know

$$\begin{aligned}P &= \frac{F}{A} \\ &= \frac{F}{d^2} \\ &= \frac{1}{d^2} \left(\frac{N}{3d} (m\overline{v^2}) \right) \\ &= \frac{1}{d^3} \frac{N}{3} (m\overline{v^2}) \\ &= \frac{1}{3} \frac{N}{V} (m\overline{v^2}) \\ &= \frac{2}{3} \frac{N}{V} \left(\frac{1}{2} m\overline{v^2} \right)\end{aligned}$$

That last step might seem unnecessary, but look how cute the $1/2$ looks with a $m\overline{v^2}$ next to it! We can recognize the part in parenthesis as the average kinetic energy of the molecule! So

$$P = \frac{2}{3} \frac{N}{V} \left(\frac{1}{2} m\overline{v^2} \right) \quad (32.3)$$

and we see that the pressure of the gas is proportional to the number of molecules per unit volume and to the average kinetic energy of the molecules! In fact, this equation shows some of the things we have learned about gas laws. For example, P is inversely proportional to V !

32.3 Temperature

Finally, we are ready to define temperature!

We now know that

$$P = \frac{2}{3} \frac{N}{V} \left(\frac{1}{2} m \overline{v^2} \right)$$

Question 123.15.1

Question 123.15.2

Equation on Board

and that this has some of the properties of the ideal gas law. We can rearrange this to look a little more like the ideal gas law. Let's start with our pressure, but put the volume on the other side where it is in the ideal gas law.

$$PV = \frac{2}{3} N \left(\frac{1}{2} m \overline{v^2} \right)$$

But temperature does not appear in this equation. Let's recall that we can write the ideal gas law as

$$PV = Nk_B T$$

If we set these two equations equal to each other

$$Nk_B T = \frac{2}{3} N \left(\frac{1}{2} m \overline{v^2} \right)$$

then

$$T = \frac{2}{3} \frac{1}{k_B} \left(\frac{1}{2} m \overline{v^2} \right) \quad (32.4)$$

This is fantastic! We have finally defined temperature. Temperature is related directly to the average kinetic energy of the molecules! The average kinetic energy is

$$\bar{K}_{mol} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T$$

Note that in each direction we should have

$$\bar{K}_x = \frac{1}{2} m \overline{v_x^2} = \frac{1}{2} k_B T$$

so

$$\bar{K}_y = \frac{1}{2} k_B T$$

and

$$\bar{K}_z = \frac{1}{2} k_B T$$

So in each direction the molecules have

$$\bar{K}_i = \frac{1}{2} k_B T \quad (32.5)$$

We call a way the molecule can move a *degree of freedom*. Our molecules can move in the x , y , and z , direction, so our molecules have three degrees of freedom. Each degree of freedom contributes $\frac{1}{2} k_B T$ worth of energy.

Of course, we only allow translational energy in our formulation for ideal gasses, but for more complex molecules we could have rotational energy (another way to move) and vibrational energy (yet another way to move) etc. And each way the molecule can move is another degree of freedom would contribute $\frac{1}{2}k_B T$. But remember, these more complex molecules would not be ideal gasses.

For the collection of N molecules, we have all together

$$K = N \left(\frac{3}{2} k_B T \right) \quad (32.6)$$

$$\begin{aligned} &= \frac{3}{2} N k_B T \\ &= \frac{3}{2} n R T \end{aligned} \quad (32.7)$$

and for our ideal gas the only form of internal energy we have is the kinetic energy, so we see that the internal energy depends only on the temperature.

This is only true under our assumption that the molecules or atoms are structureless. So this works well for actual monotonic gases, but not as well for diatomic gasses, etc.

Let's do a problem. Suppose a colleague tells you that in her Statistical Mechanics class they defined a root-mean-squared velocity for atoms in a gas. That is, they took the velocity, squared it, and then took the average. They got

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

Let's show that this works from our kinetic theory of gasses.

We know that each degree of freedom contributes $\frac{1}{2}k_B T$ worth of internal energy for the gas molecules. We just found that

$$\bar{K} = \frac{K}{N} = \frac{3}{2} k_B T$$

and we know that

$$\bar{K} = \frac{1}{2} m \overline{v^2}$$

Setting these equal gives

$$\begin{aligned} \frac{3}{2} k_B T &= \frac{1}{2} m \overline{v^2} \\ 3k_B T &= m \overline{v^2} \\ \frac{3k_B T}{m} &= \overline{v^2} \end{aligned}$$

so

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}}$$

which is just what our colleague said. This shows the power of the simple idea of the ideal gas approximation.

The rms speed tells us that if the molecule or atom is tiny (has little mass) it must move very fast. If it is larger, it will move more slowly. Let's look at how molecules travel as they go through a gas.

32.4 Mean Free Path

We started this chapter by saying how molecules interact with the walls of a box. But the molecules really do bounce off each other. The average distance a molecule travels before a molecule-molecule collision is called the *mean free path*. We could give this a symbol, let's re-use the Greek letter λ

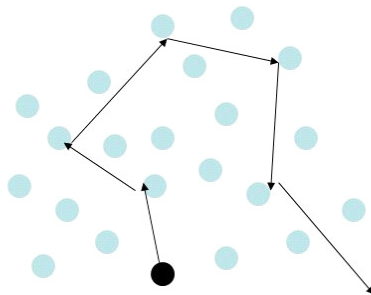


Figure 32.1:

$$\begin{aligned}\lambda &= \frac{\text{Length traveled}}{\text{Number of collisions}} \\ &= \frac{L}{N_{\text{col}}}\end{aligned}$$

This will give the average length before a collision happens.

The average length traveled is easy

$$L = \bar{v}\Delta t$$

But to predict the number of collisions is harder. To do this problem, let's play a geometrical trick. First let's assume the molecules are spheres with diameter d . We see that a collision does not happen unless the distance between the molecules is less than or equal to $2d$. We can model this interaction as a large particle of size $2d$ and many point particles. We let the large particle move in a straight line to create a cylindrical path.

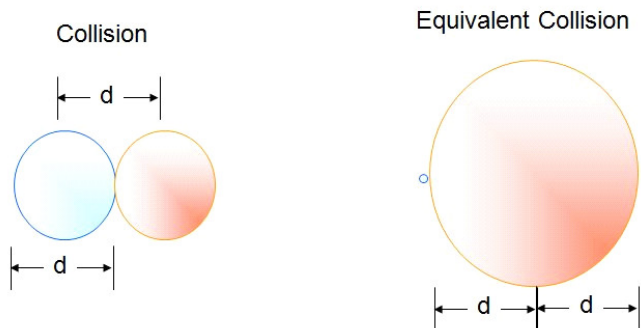
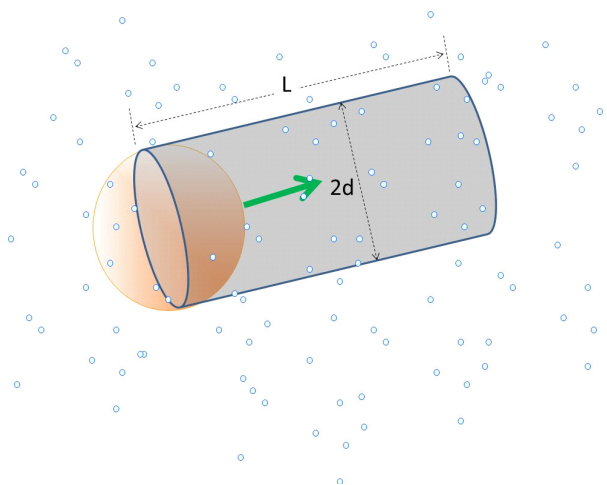


Figure 32.2:



All point particles in the cylinder will collide with our large particle. The volume of such a big cylindrical path is

$$V = (\pi d^2)L$$

but the length L of the cylinder will be

$$L = \bar{v}\Delta t$$

so the volume will be

$$V = \pi d^2 \bar{v} \Delta t \quad (32.8)$$

then, if n_V is the number of molecules per unit volume, our big equivalent particle will hit all of the point particles in the volume.

$$N_{\text{col}} = \pi d^2 \bar{v} \Delta t n_V \quad (32.9)$$

is the number of point molecules in the cylinder. And it is the number of collisions.

The mean free path is the average distance traveled in a time interval Δt divided by the number of collisions that occur in Δt . We now know both of these, so

$$\begin{aligned}\lambda &= \frac{\bar{v}\Delta t}{\pi d^2 \bar{v} \Delta t n_V} \\ &= \frac{1}{\pi d^2 n_V}\end{aligned}\quad (32.10)$$

We can find the the average time between collisions. This is called the mean free time.

$$\begin{aligned}\tau &= \frac{\lambda}{\bar{v}} \\ &= \frac{1}{\pi d^2 n_V \bar{v}}\end{aligned}$$

The frequency of collisions is

$$f = \frac{1}{\tau} = \pi d^2 \bar{v} n_V \quad (32.11)$$

It turns out that our simple model is off by a factor of $\sqrt{2}$ because we assumed the molecules are stationary. We can fix that easily

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n_V} = \frac{V}{\sqrt{2}\pi d^2 N} \quad (32.12)$$

and

$$f = \sqrt{2}\pi d^2 \bar{v} n_V = \frac{\sqrt{2}\pi d^2 \bar{v} N}{V} \quad (32.13)$$

and finally

$$\tau = \frac{1}{\sqrt{2}\pi d^2 n_V \bar{v}} = \frac{V}{\sqrt{2}\pi d^2 N \bar{v}}$$

We can make a further modification if we assume an ideal gas. We know

$$PV = Nk_B T$$

so

$$\frac{V}{N} = \frac{k_B T}{P}$$

then where there is a V/N we can substitute

$$\lambda = \frac{k_B T}{\sqrt{2}\pi d^2 P} \quad (32.14)$$

and

$$f = \frac{\sqrt{2}\pi d^2 \bar{v} P}{k_B T} \quad (32.15)$$

$$\tau = \frac{k_B T}{\sqrt{2}\pi d^2 \bar{v} P} \quad (32.16)$$

I have used \bar{v} as our average speed for our molecules, but we will find it might be good to use the v_{rms} as our average. But we will get to that soon enough (our colleague from the last problem was right to suggest it!).

Now that we know what temperature is, let's go back to thinking about temperature rise as we transfer energy by heat. We studied heat capacities and specific heats a few lectures ago, but we avoided gasses as we did so. We need to find the temperature rise ΔT in a gas after transferring energy by heat Q for gasses. Let's do that in next.

32.5 Molar Specific Heat

We learned that

$$Q = mc\Delta T$$

or that we could use the molar form of this equation

$$Q = nC\Delta T$$

But we need to find the C values for different gasses. To do this, remember for an ideal gas, we only have translational kinetic energy, so it must be true that

$$E_{int} = K_{tran} = \frac{3}{2}nRT$$

But let's take a special case. If we take the system through a process at constant volume,

$$Q = \Delta E_{int}$$

because no work is done (we didn't change the volume) and for this special case we can write C as C_V to remind us that the volume didn't change.

$$Q = nC_V\Delta T$$

so

$$\Delta E_{int} = nC_V\Delta T$$

Notice this is a change in internal energy. We added energy to our gas

$$\begin{aligned} E_{int_f} - E_{int_i} &= nC_V(T_f - T_i) \\ &= nC_VT_f - nC_VT_i \end{aligned}$$

this strongly suggests that if C_V is constant

$$E_{int} = nC_VT$$

we can solve for C_V

$$C_V = \frac{1}{n} \frac{\Delta E_{int}}{\Delta T}$$

and for very small changes in temperature

$$C_V = \frac{1}{n} \frac{dE_{int}}{dT}$$

But for an ideal gas the internal energy is kinetic energy so we know that

$$E_{int} = \frac{3}{2} nRT$$

So we can easily find $\frac{dE_{int}}{dT}$

$$\frac{dE_{int}}{dT} = \frac{3}{2} nR$$

Then our value for C_V is

$$\begin{aligned} C_V &= \frac{1}{n} \frac{dE_{int}}{dT} \\ &= \frac{1}{n} \frac{3}{2} nR \\ &= \frac{3}{2} R \end{aligned}$$

Knowing $R = 8.314 \frac{\text{J}}{\text{mol K}}$, the numerical value is

$$\begin{aligned} C_V &= \frac{3}{2} 8.314 \frac{\text{J}}{\text{mol K}} \\ &= 12.471 \frac{\text{J}}{\text{mol K}} \end{aligned}$$

for all monotonic gasses. It turns out that this is a very good approximation.

You might find the restriction of heating at a constant volume somewhat constrictive. We will lift this restriction in a later lecture.

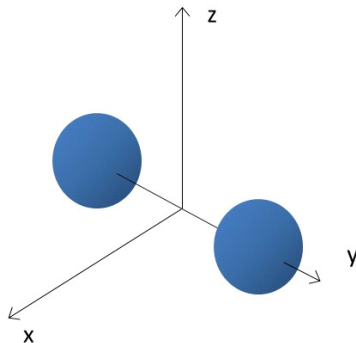
32.6 The Equipartition of Energy

We can extend our ideal gas model a little by using what we know about degrees of freedom. Remember that we found that for each degree of freedom the internal energy was

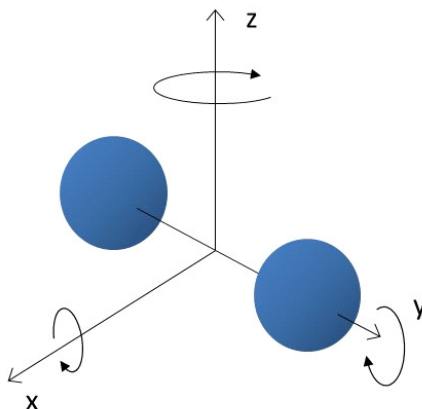
$$E_i = \frac{1}{2} k_B T \quad (32.17)$$

we found for an ideal monotonic gas that the internal energy was

$$E_{int} = 3E_i = \frac{3}{2} k_B T \quad (32.18)$$



where each E_i came from a translational degree of freedom. But a diatomic molecule has several more degrees of freedom. It can rotate about any of the axes.



Here rotation about the y axis does not contribute significantly because the moment of inertia of a sphere (we will take the atom to be roughly spherical) about its axis is

$$\mathbb{I} = \frac{2}{5}mr^2 \quad (32.19)$$

where m is the mass of the atom. Most of the mass is centered in the nucleus (proton mass = 1.67×10^{-27} kg, electron mass = 9.11×10^{-31} kg), which has a radius of about $r = 1.7 \times 10^{-5}$ Å is the radius of the atom. The moment of inertia for rotation about the center of the two mass system is

$$\mathbb{I} = \sum_i m_i R_i^2 \quad (32.20)$$

where R is the distance from the center of mass. For diatomic hydrogen, $R = \frac{1}{2}1.06$ Å so we can see that the rotation about the y -axis is not very important,

so we are left with three translational and two rotational degrees of freedom. This gives

$$E_{int} = \left(\frac{3}{2}k_B T\right)_{trans} + \left(\frac{2}{2}k_B T\right)_{rot} = \frac{5}{2}k_B T \quad (32.21)$$

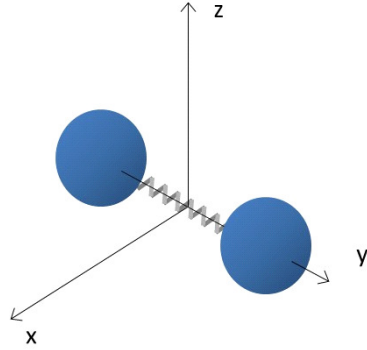
Writing this in molar terms

$$E_{int} = \frac{5}{2}nRT \quad (32.22)$$

then

$$C_V = \frac{1}{n} \frac{dE_{int}}{dT} = \frac{5}{2}R \quad (32.23)$$

BUT WAIT, we did not include vibration! The atoms are bond together with an electrical attraction that acts quite like a spring force. So vibration along the axis is possible and we need to add in one more degree of freedom. We also have potential energy involved for a spring force, so we expect an additional degree of freedom for vibration.



When we add all these up, we get

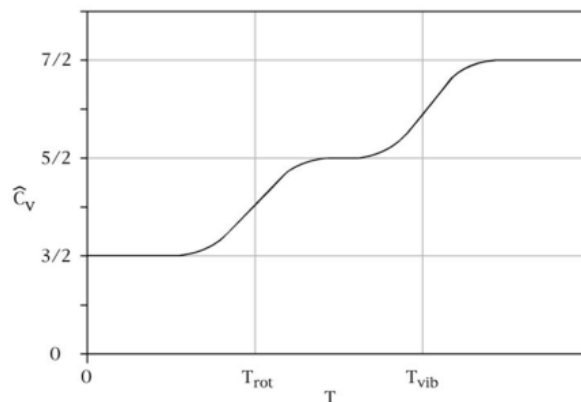
$$E_{int} = \left(\frac{3}{2}k_B T\right)_{trans} + \left(\frac{2}{2}k_B T\right)_{rot} + \left(\frac{2}{2}k_B T\right)_{vib} = \frac{7}{2}k_B T \quad (32.24)$$

which gives

$$C_V = \frac{7}{2}R \quad (32.25)$$

We should pause to ask, what values do we use? for diatomic gasses, is $C_V = \frac{7}{2}R$ all the time?

It turns out that when energy is added to a collection of molecules, it does not pick randomly from the degrees of freedom.

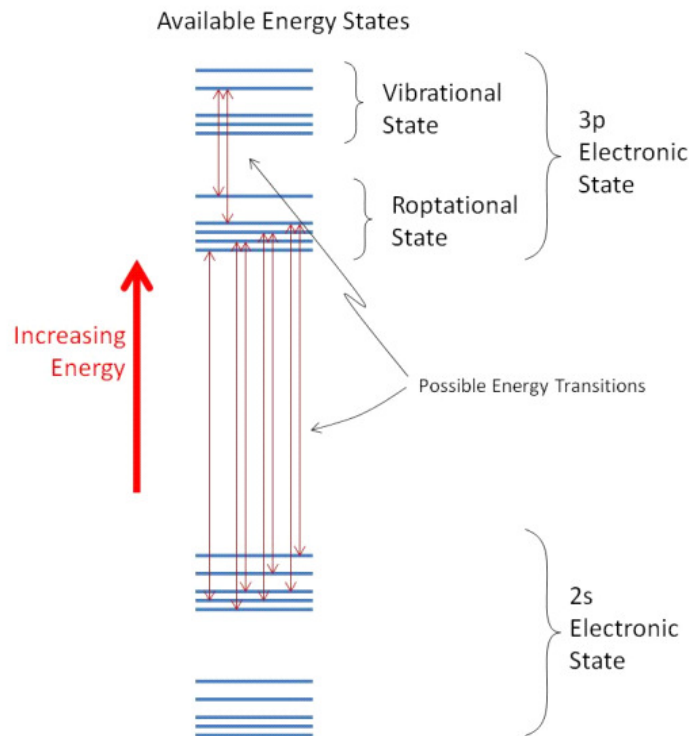


At low temperature, the translational degrees of freedom are preferred. Then, as the temperature rises, the rotational degrees of freedom are filled. Finally the vibrational degrees of freedom are used. The figure shows this relationship for diatomic Hydrogen. Note that there are plateaus at each of our values of C_V that we found ($\frac{3}{2}R$, $\frac{5}{2}R$, $\frac{7}{2}R$).

We have talked about the simplest of molecules. If we had a more complex molecule, there would be more complex rotational and vibrational degrees of freedom. That is why for larger molecules the values of C_V in table in the book don't follow a simple fraction of R .

32.7 Quantization

We have come to the end of what classical theory can explain. We know that different degrees of freedom are filled in groups, but we don't know why. We will only hint at the reason, but it has to do with wave motion that we studied!



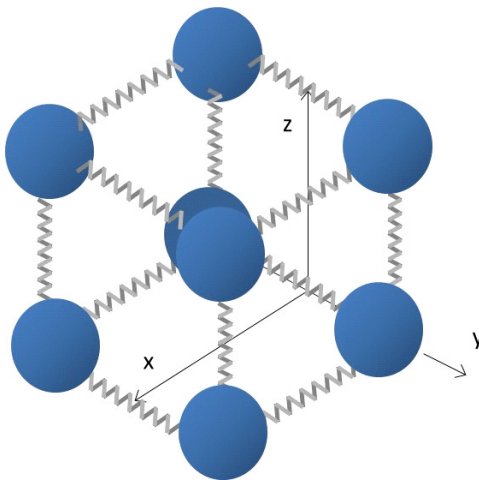
Quantum theory tells us that atoms can be described as waves under boundary conditions. Like a string with fixed ends, these atomic waves have quantized frequencies. The energy of the molecule is proportional to the frequency in quantum theory, so quantum mechanics tells us that the energy of the molecule will be quantized. The figure shows an energy level diagram for a molecule. Higher energy is in the y direction. The vibrational and rotational states are marked. (the maroon arrows show transitions from higher to lower states. These transitions create radiation).

The lowest allowed state is called the *ground state*. It is analogous to the fundamental frequency. At low temperatures the molecule only has enough energy to populate the lower states, but when the temperature rises, there is enough energy transfer due to collisions to push molecules into the rotational states. With higher temperatures, you will see the collisions transfer molecules to the higher vibrational states.

32.8 Molar specific heat of solids

Let's do a problem. Let's find C_V for a solid, and compare it to the specific heat of elemental solids.

We can view the atoms of solids as having a structure of springs and atoms.



We can see that we should have three translational degrees of freedom for each atom. We also have three degrees of freedom associated with the potential energy from the spring-like bonding forces. Recall that

$$E_x = \frac{1}{2}mv_x^2 - \frac{1}{2}kx^2 \quad (32.26)$$

and we have E_y and E_z as well.

From equipartition of energy we have

$$E_i = 6 \left(\frac{1}{2}k_B T \right) \quad (32.27)$$

per atom! So we have

$$E_{int} = 3Nk_B T \quad (32.28)$$

$$= 3nRT \quad (32.29)$$

and then for solids

$$C_V = \frac{1}{n} \frac{dE_{int}}{dT} = 3R \quad (32.30)$$

numerically this is

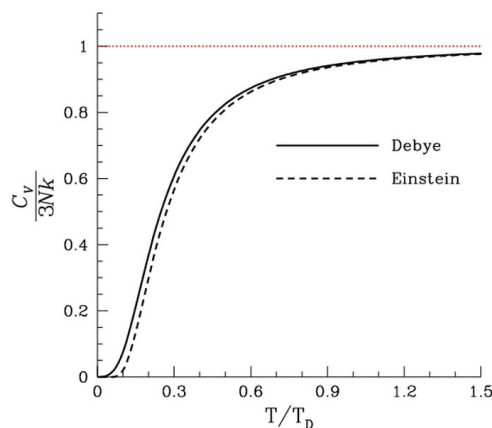
$$\begin{aligned} C_V &= 3 \times 8.314 \frac{\text{J}}{\text{mol K}} \\ &= 24.942 \frac{\text{J}}{\text{mol K}} \end{aligned}$$

This is the *Dulong-Petit law*. Here is our table of specific heat values again.

Substance	$c \left(\frac{\text{J}}{\text{kg K}} \right)$	$C_V \left(\frac{\text{J}}{\text{mol K}} \right)$
Aluminum	900	24.3
Copper	385	24.4
Iron	449	25.1
Gold	129	25.4
Lead	128	26.5
Ice	2090	37.6
Mercury	140	28.1
Water	4190	75.4

we see that our simple analysis did not do too bad! For elemental solids we get about the right number.

This law also breaks down for very cold temperatures where we need even more quantum mechanics to explain what happens. Einstein and Debye both contributed to explaining this departure, but their work is beyond this freshman course. This is a topic for a solid state physics course which is an elective that you might consider if you are interested in seeing a more complete solution.



Debye and Einstein models. (Image in the Public Domain courtesy Frédéric Perez)

32.9 Partial Pressure and Vapor Pressure

So far we have assumed we only have one type of molecule in our gas. But what if we had several kinds of molecules, like in our air that we breathe that is mostly N_2 but also has some very important O_2 that we need to live (and a little bit of CO_2 that we worry about)?

We could define a pressure that the gas would make if occupied the entire volume on its own. We call this a *partial pressure*. But think of our kinetic

theory of gasses that we have just developed. If the gasses are in thermal equilibrium, then they have the same temperature, and therefore the same average kinetic energy. They would each make a pressure on the walls of our container.

$$P = \frac{2}{3} \frac{N}{V} \left(\frac{1}{2} m \overline{v^2} \right)$$

But each gas is doing this at the same time. So the total pressure on the container walls would be the sum of the pressures from the individual gasses in the mix

$$P_{total} = \sum_i P_i$$

This is called Dalton's law. A result of this law is that for any two gasses in equilibrium

$$\frac{P_1}{n_1} = \frac{P_2}{n_2}$$

There is a safety issue hidden in what we have just said. Suppose we have a large container of liquid nitrogen in our lab (because we do). And suppose our container has a leak. The pressure in the room might not change much at all, but the portion of that pressure due to O_2 might change. And we need that O_2 pressure to stay alive. But since the overall pressure didn't change noticeably, we wouldn't have any warning that there was a problem until we felt trouble moving or passed out.

Suppose we have a vapor in a container but also have some of the same material in it's liquid state in the same container. And suppose that the vapor and liquid are in equilibrium. The partial pressure of the vapor under these conditions is called a *vapor pressure*. This is important in many fields including weather prediction. The partial pressure of the water vapor in our atmosphere can't exceed the vapor pressure of water at the same temperature or the water vapor will condense out of the air. This causes dew and the temperature at which the condensation starts to happen is called the *dew point temperature*.

If you listen to a weather report you might hear about the *relative humidity*.

$$R.H. = \frac{\text{Partial pressure of water vapor at } T}{\text{Vapor pressure of water at } T} \times 100$$

Think about what this means. If at a given temperature the partial pressure of water vapor is the same as the vapor pressure of water then the air has as much water as it can hold at that temperature. We say the air is saturated and it feels very muggy.

This almost completes our kinetic theory of ideal gasses. But before we move on we need to look at the speeds of our molecules. We know average speed is related to temperature, but not every molecule is moving at that average speed. What distribution of speeds like?