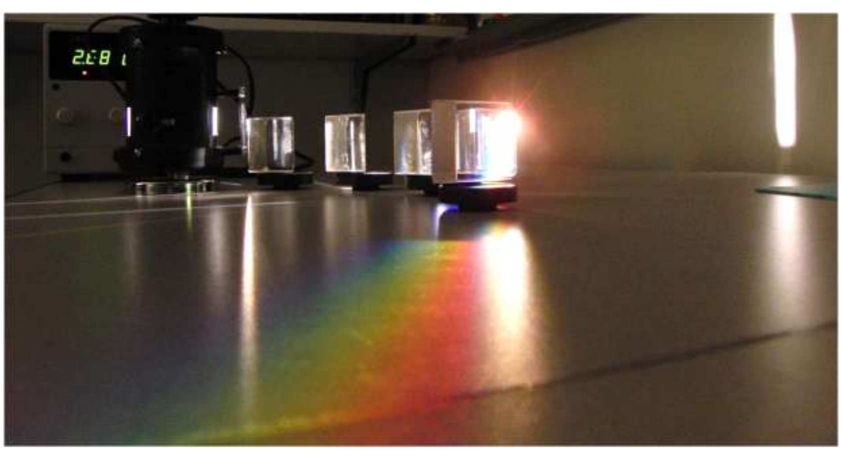
Chapter 14

14 Dispersion and Huygen's Principle 3.1.5 3.1.6

Fundamental Concepts

- The index of refraction is wavelength dependent
- That different wavelengths bend different amounts when refracted is called *dispersion*
- White light is a superposition of many other frequencies

14.1 Dispersion



Question 123.15.1

Who hasn't played with a prism? We immediately recognize a rainbow. But why does the prism make a rainbow? The secret lies in the nature of the refractive index.

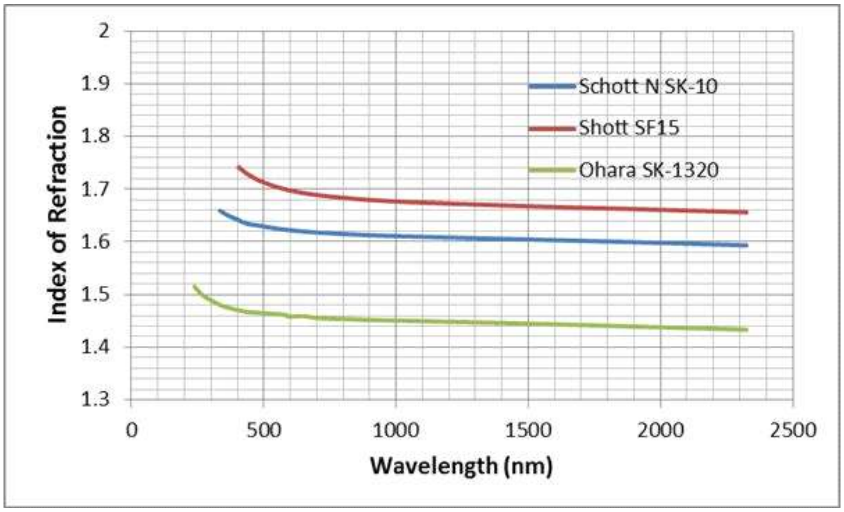
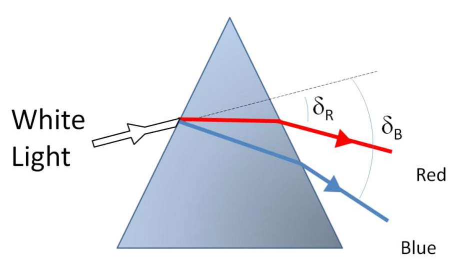


Figure 14.1: Index of refraction as a function of wavelength (Ohara optical glass http://www.oharacorp.com/fused-silica-quartz.html data and Schott optical glass data http://www.uqgoptics.com/materials_glasses_schott.aspx)

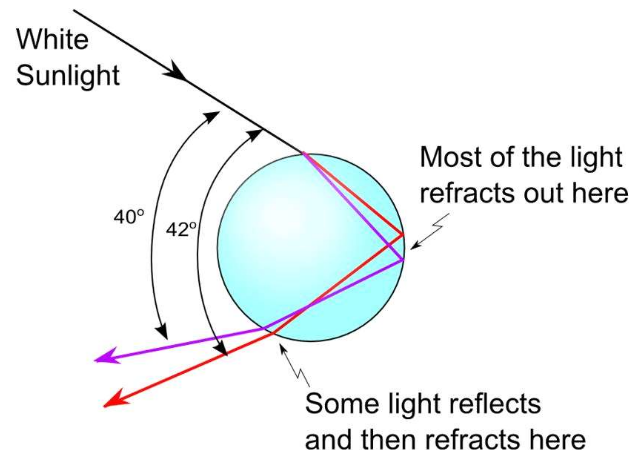
Notice that in the figure, the index of refraction depends on wavelength. This means that as light enters a material, different wavelengths will be refracted at different angles. White light is a superposition of many wavelengths of light. Thus white light is pulled apart by refraction into a rainbow. This process is called *dispersion*.

The graph tells us that blue light bends more than red light.



We call the change in direction measured from the original direction of travel, δ, the *angle of deviation*. The colors we can see are called the visible spectrum. Note that this is a second way to make a visible light spectrum. The first way used a diffraction grating and the wave nature of light. Although this dispersive element (prism) method of making a spectrum uses the ides of geometric optics, it is still the wave nature of light that makes the waves of different wavelengths refract differently. Question 123.15.1

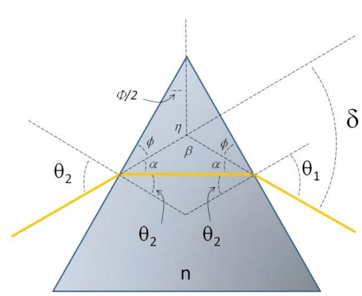
Let's look at a natural rainbow. The dispersion is caused buy small droplets Question 123.15.2
of water. The white sunlight enters the drop and is dispersed. It bounces off the back of the drop and then leaves the drop, again being dispersed. Red light leaves the drop at about 42° from its input direction, and blue light leaves at about 40°.



This effectively separates all the different wavelengths and we see a rainbow at angles between 40° and 42° from the incoming light direction. Question 123.15.3

14.1.1 Calculation of n using a prism

Let's do a problem using the idea of refraction in a prism. Let's find the index of refraction of a the prism material. Suppose we make a prism as shown. We know the angle Φ and can measure the exit angle δ. In terms of these two variables, what is n?



Using the notation indicated in the figure, we choose θ_1 such that the interior ray is horizontal.[1] This is a refraction problem, so we will want to use Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Thus, we need to find θ_1 and θ_2. Knowing Φ and δ, and realizing $n_1 = 1$, we can find θ_2 and θ_1. Then use

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

to find n_2. Let's see one way to do this. In geometry it is fair game to extend lines and even make some new lines of our own. By doing this we can realize that

$$\theta_1 = \theta_2 + \alpha \tag{14.1}$$

and that

$$\delta = 180 - \beta \tag{14.2}$$

and it is also true that

$$180 = \beta + 2\alpha \tag{14.3}$$

Then

$$\delta = 2\alpha \tag{14.4}$$

and

$$\alpha = \frac{\delta}{2} \tag{14.5}$$

Now also realize that

$$90 = \alpha + \theta_2 + \phi \tag{14.6}$$

and

$$180 = \Phi + 2\alpha + 2\phi \tag{14.7}$$

or

$$90 = \frac{\Phi}{2} + \alpha + \phi \tag{14.8}$$

Then

$$\alpha + \theta_2 + \phi = \frac{\Phi}{2} + \alpha + \phi \tag{14.9}$$

$$\theta_2 = \frac{\Phi}{2} \tag{14.10}$$

[1] WARNING! in the upcoming homework problem you can't make the same assumptions!

We can put these in our equation for θ_1

$$\begin{aligned}
\theta_1 &= \theta_2 + \alpha && (14.11)\\
&= \frac{\Phi}{2} + \frac{\delta}{2} && (14.12)\\
&= \frac{\Phi + \delta}{2} && (14.13)
\end{aligned}$$

Now we can use Snell's Law

$$\begin{aligned}
\sin(\theta_1) &= n\sin(\theta_2) && (14.14)\\
\sin\left(\frac{\Phi+\delta}{2}\right) &= n\sin\left(\frac{\Phi}{2}\right) && (14.15)
\end{aligned}$$

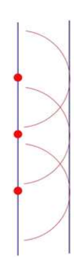
then

$$n = \frac{\sin\left(\frac{\Phi}{2}\right)}{\sin\left(\frac{\Phi+\delta}{2}\right)} \tag{14.16}$$

and since we know Φ and δ, we can find n.

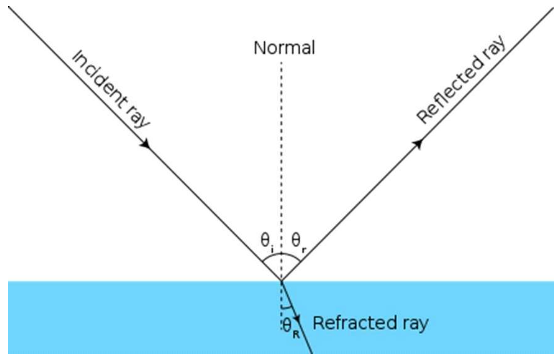
14.2 Huygen's Principle

We have learned the laws of reflection and refraction. But how does the light know to bounce or bend the way it does? To understand what is going on at a fundamental level we need to remember that light is a wave. And waves can hit and splash off of objects. Sound waves could bounce off of things. So can light waves. Only the light waves that we can see are very small. So they bounce off of very small things like atoms. Suppose we send our light into a material. It will hit the atoms and bounce off. We know this, but there is more. The light bounces off the atoms but this makes a new light wave coming from the atoms. And that new wave is spherical (think that most waves start spherical). If we have many atoms, the light will hit many atoms and make many little spherical *scattered* waves. An early researcher had an idea of how to find the direction of the light using this underlying wave nature of light. We could follow the light path by finding the little spherical scattered waves and look for where their crests line up. This would be constructive interference, and this is where the ray of light would end up!

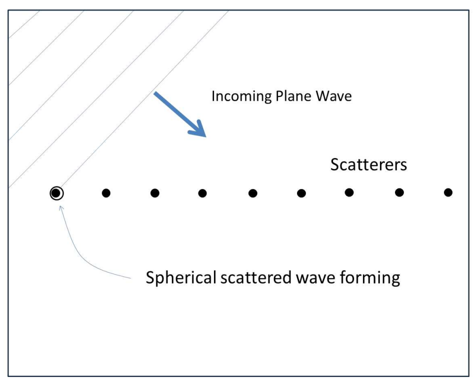


We know that light should reflect off a surface.

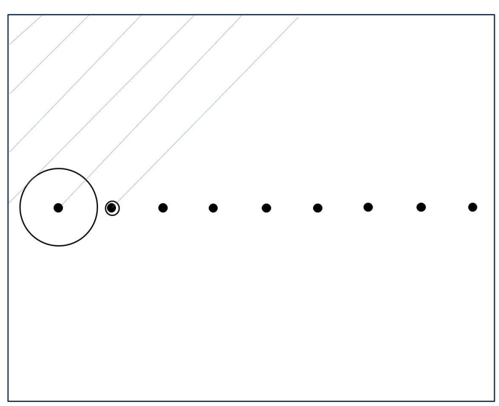
$$\theta_i = \theta_r$$



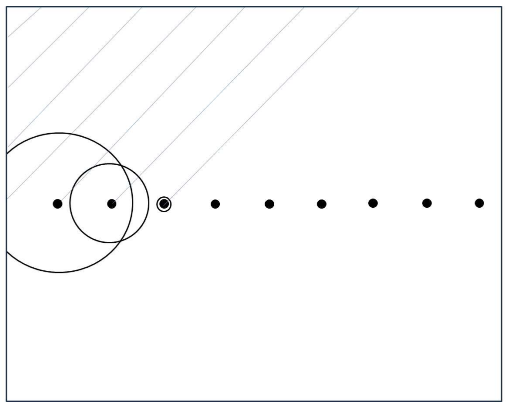
Let's see how Huygen's principle gives this law of reflection. Think about the top row (or about $\lambda/2$ thickness) of scatterers. In the next figure we have such a set of scatters with an incoming plane wave.



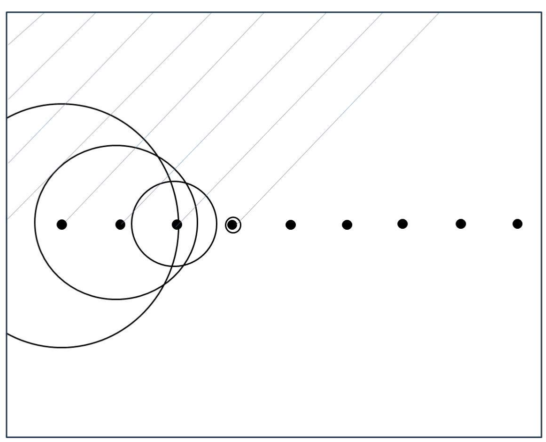
On the left, the plane wave crest is striking a scatter and has started a spherical secondary wave. One period later the situation might look like this



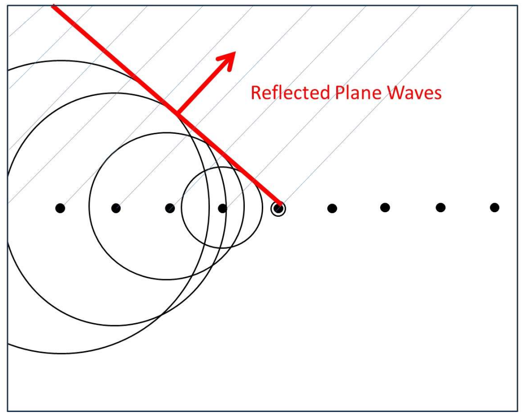
where now the first crest has moved on to the second scatterer and the second crest is at the first scatter. Notice that the first scatter's spherical secondary wave has grown. This process continues as the wave moves onward. In another period, we would have three scattered waves



and in another period, four.

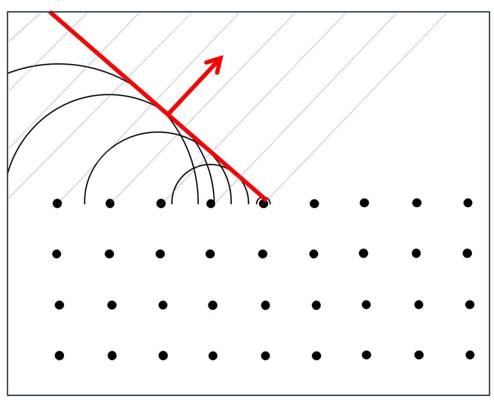


and five.

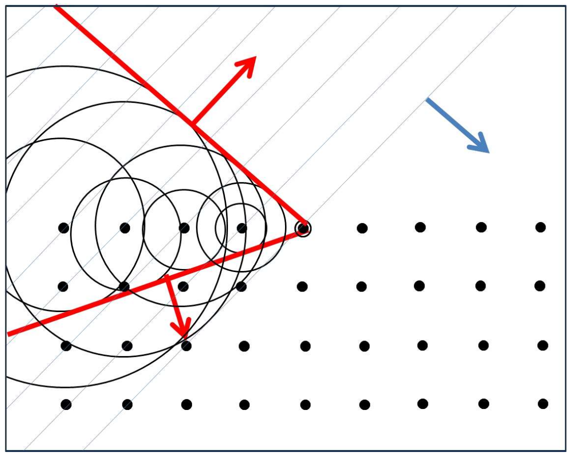


By looking for where the wave crests line up we can find the new crest of the reflected wave. By superimposing these scattered waves we the reflected wave is formed.

We also know *Snel's law of refraction*. But we want to use our scattering model to show this is true. In our discussion on reflection we just used the top layer of scatters, but we know that there are more scatterers in a solid.

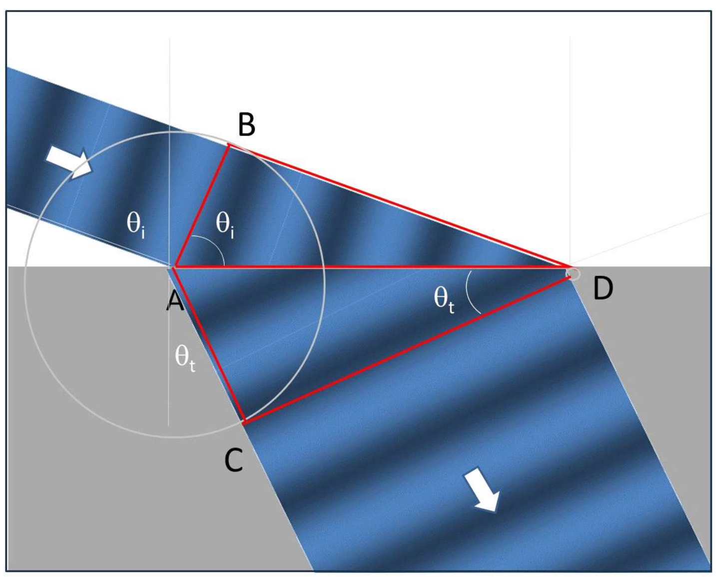


so what happens inside the material with all the other layers of scatterers? Of course our secondary waves are kind of spherical shaped, so the situation is something like this



and we see that we are forming another wave front *inside the material*. Our scattering model predicts the transmitted wave as well as the reflected wave.

We can see that Snel's law follows from basic geometry just as did the law of reflection. Let's go back to our wavefront $\overline{AB}$. This time let's let $\overline{CD}$ be the transmitted wave front shown in the next figure.



The time it takes the wave front to go from B to D would be

$$\Delta t_{BD} = \frac{\overline{BD}}{v_i}$$

but because the new transmitted wave is slower, the transmitted wavefront has only gone the distance $\overline{AC}$ in the same time

$$\Delta t_{AC} = \Delta t_{BD} = \Delta t = \frac{\overline{AC}}{v_t}$$

Once again we have two triangles. And once again they share a side $\overline{AD}$. And they are right triangles. So the two triangles must be similar triangles. Once again

$$\sin\theta_i = \frac{\overline{BD}}{\overline{AD}}$$

and

$$\sin\theta_t = \frac{\overline{AC}}{\overline{AD}}$$

and we can solve both for $1/\overline{AD}$ and set them equal so that

$$\frac{\sin\theta_i}{\overline{BD}} = \frac{\sin\theta_t}{\overline{AC}}$$

but

$$\begin{aligned}\overline{BD} &= v_i\Delta t \\ \overline{AC} &= v_t\Delta t\end{aligned}$$

so

$$\frac{\sin\theta_i}{v_i\Delta t} = \frac{\sin\theta_t}{v_t\Delta t}$$

And the Δt cancels from both sides

$$\frac{\sin\theta_i}{v_i} = \frac{\sin\theta_t}{v_t}$$

We can multiply both sides by c, the speed of light

$$c\frac{\sin\theta_i}{v_i} = c\frac{\sin\theta_t}{v_t}$$

to achieve

$$n_i\sin\theta_i = n_t\sin\theta_t$$

which is Snel's law.

We can (and will) use Huygan's principle to find how light goes through openings. [2] @@@ need new figure of small and large opening and scatterers.

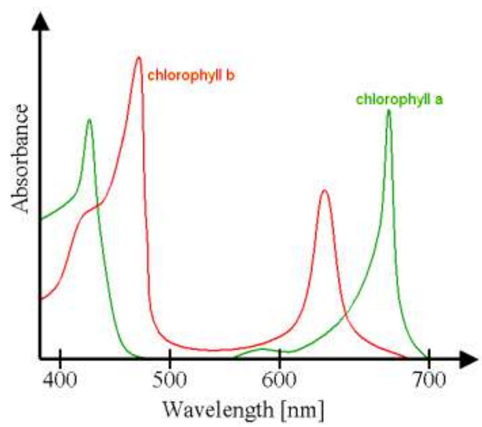
[2] Huygens method is technically not a correct representation of what happens. The actual wave leaving the single opening is a superposition of the original wave, and the wave scattered from the sides of the opening. You can see this scattering by tearing a small hole in a piece of paper and looking through the hole at a light source. You will see the bright ring around the hole where the edges of the paper are scattering the light. But the mathematical result we will get using Huygens method gives a mathematically identical result for the resulting wave leaving the slit with much less high power math. So we will stick with Huygens in this class.

14.3 Filters and other color phenomena

Of course, we have assumed without statement, that white light is made up of all the colors of the rainbow. We should ask why a red shirt is red, or why passing light through a green film makes the light look green as it leaves.

Both of these phenomena are examples of removing wavelengths from white light.

In the case of the red shirt, the red dye in the cloth absorbs all of the visible colors except red. The red is reflected, so the shirt looks red. The filter is much the same. The green pigment in the film causes nearly all visible colors to be absorbed except green. So only green light is transmitted. This is why leaves are green. Chlorophyll absorbs red and blue wavelengths, so the green is reflected or transmitted.



Chlorophyll Spectrum (Public Domain image courtesy Kurzon)