Chapter 24

Induction

Fundamental Concepts

- Conductors moving in magnetic fields separate charge. creating a potential difference that we call "motional emf."
- Motional emfs generate currents, even in solid pieces of conductor. These currents in conductors are called "eddy currents."
- Magnetic flux is found by integrating the dot product of the magnetic field and a differential element of area over the area. $\Phi_B = \int_A \overrightarrow{B} \cdot d\overrightarrow{A}$

24.1 Motional emf

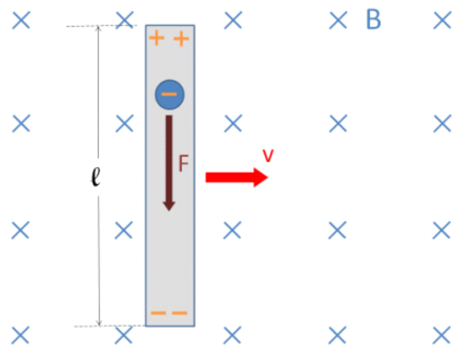
Last lecture, we studied Faraday's experiment. He created a magnetic field, and then used that magnetic field to make a current. But currents are caused by electric fields! Did Faraday's magnetic field create an electric field?

To investigate Faraday's result, let's see if we can find a way to use charge motion and a magnetic field to make an electric field. Let's take a bar of metal and move it in a magnetic field. The bar has free charges in it (electrons). We have given them a velocity. So we expect a magnetic force

$$\overrightarrow{\mathbf{F}}_B = q\overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{B}}$$

The free charges will accelerate together, but the positive stationary charges can't move. We have found another way to separate charge. We know that separated charge creates a potential difference. We often call this induced potential difference the *motional emf* because it is created by moving our apparatus.

Let's take an example to see how it works.



For this example, let's look at a piece of wire moving in a constant field. To make the math easy, let's move the wire with a velocity perpendicular to the B-field.

As the figure shows, the electrons will feel a force. Using our right hand rule, we get an upward force for positive charge carriers, but we know the electrons are negative charge carriers, so the force is downward. We find that the magnitude of the force is

$$F_B = qvB$$

The electrons will bunch up at the bottom of the piece of wire, until their electric force of repulsion forces them to stop. That force is

$$F_E = qE$$

By separating the charges along the wire so that there is excess positive charge on one end and excess negative charge on the other end, we now have and E-field in the wire. We can solve for E when we have reached equilibrium.

$$\Sigma F = 0 = -F_B + F_E$$

or

$$qE = qvB$$

which tells us

$$E = vB \tag{24.1}$$

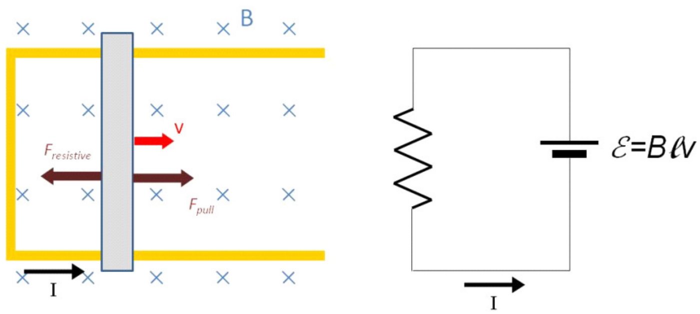
Now, we know that electric fields cause potential differences. The E-field in the wire will be nearly uniform. Then it looks much like a capacitor with separated charges. The potential difference will be

$$\begin{aligned} \Delta V &= \int \overrightarrow{E} \cdot d\overrightarrow{s} \\ &\approx EL \end{aligned}$$

where L is the length of our wire. So

$$\Delta V \approx vBL \tag{24.2}$$

This is like a battery. The magnetic field is "pumping" charge. If we connected the two ends somehow with a wire that is not moving, a current will flow.



Let's take another example. We wish to make a bar of metal move in a B-field. To make the rest of the circuit, we allow the bar to slide along the two long parts of a U-shaped long wire as shown. We will call the two long parts of the wire "rails" since they look a little like railroad rails. Then we have a connection between our moving piece of metal, and the rest of the circuit. What we have is very like the circuit on the right hand side of the last figure.

We will have to apply a force F_{pull} to move the bar. This is because there is another force, marked as $F_{resistive}$ in the figure. This force is one we know, but might not recognize unless we think about it. We now have a current flowing through a wire, and the wire is in a magnetic field. So there will be a force

$$\begin{aligned} F_{resistive} &= I\overrightarrow{L} \times \overrightarrow{B} \\ &= ILB\sin\theta \\ &= ILB \end{aligned}$$

pushing to the left. This force resists our pull.

From Ohm's law, the current in the wire will be

$$\begin{aligned} I &= \frac{\Delta V}{R} \\ &= \frac{vBL}{R} \end{aligned}$$

so the force is

$$\begin{aligned} F_{resistive} &= \left(\frac{vBL}{R}\right) LB \\ &= \frac{vB^2L^2}{R} \end{aligned}$$

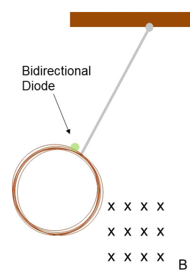
Thus we have to push with an equal force

$$F_{push} = F_{resistive}$$

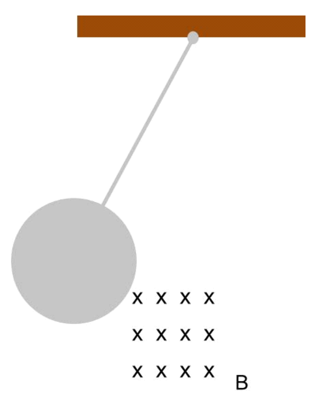
to keep the bar moving along the rails. If $F_{push} < F_{resistive}$ then the bar will have an acceleration, and it will be in the opposite direction from the velocity, so the bar will slow down.

24.2 Eddy Currents

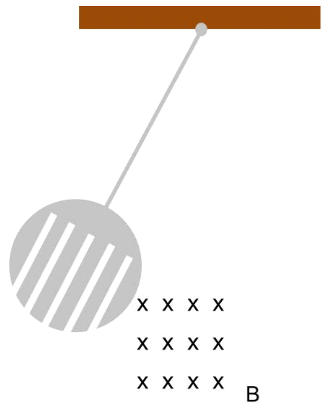
If we have a conductive loop and part of that loop moves in a magnetic field, we get a current. I chose to make our apparatus a pendulum.



So as the pendulum swings, through the magnetic field, we get a current. What if we have a solid sheet of conductor and we move that sheet through the magnetic field, will there be a current?



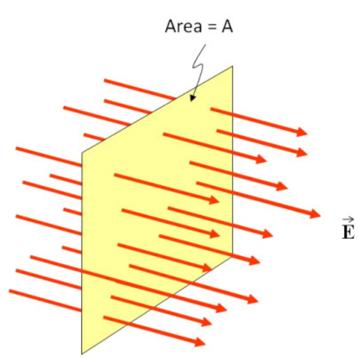
The answer is yes. After all, there are free valence electrons in the metal sheet, and they can flow in the sheet just like they can flow in a wire. We call this current in the sheet an *eddy current*. And if you have a chance to see this demonstrated live, you will notice that it slows the sheet as it moves through the magnetic field. Let's see that this must be true with another experiment. Let's cut groves in the plate.



The current is broken by the grooves, so there is little opposing magnetic field. This effect due to the eddy currents is often used to slow down machines. Rotating blades, and even trains use this effect to provide breaking.

24.3 Magnetic flux

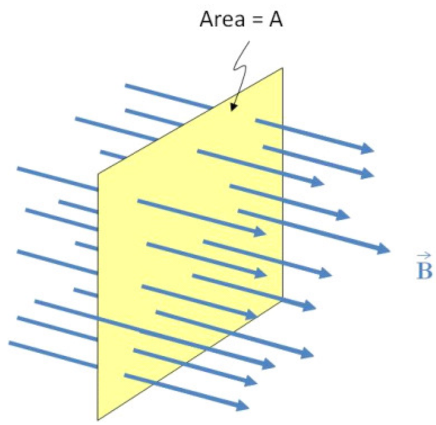
Remember long ago we defined the electric flux.



Recall that the electric flux is given by

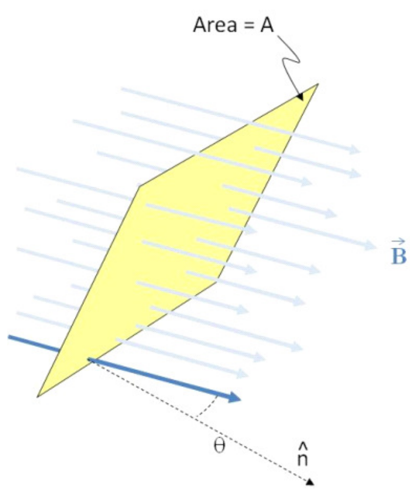
$$\begin{aligned}\Phi_E &= \overrightarrow{E} \cdot \overrightarrow{A} \\ &= EA\cos\theta\end{aligned}$$

But we now have a magnetic field.



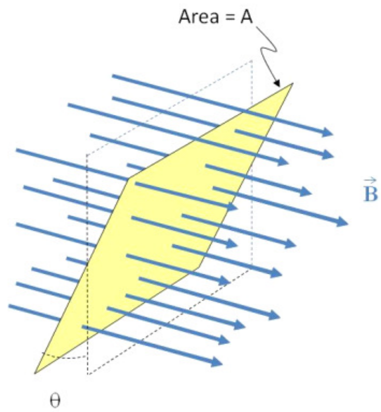
We can define a magnetic flux

$$\Phi_B = \overrightarrow{B} \cdot \overrightarrow{A} \tag{24.3}$$



$$\Phi_B = BA\cos\theta \tag{24.4}$$

where θ is the angle between $\overrightarrow{B}$ and $\overrightarrow{A}$.

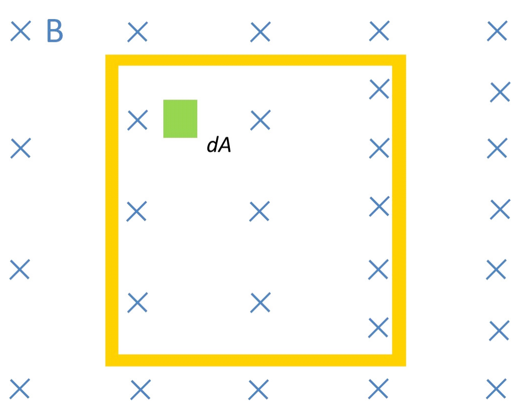


We found that the electric flux was very useful. We used Gauss' law to find fields using the idea of electric flux. It turns out that this magnetic flux is also a very useful idea. There is a difference, though. With electric fluxes, we had imaginary areas that the field penetrated. Often when we measure magnetic flux, we actually have something at the location of our area. We generally want to know the flux through a wire loop.

Just like with electric flux, we expect the flux to be proportional to the number of field lines that pass through the area.

24.3.1 Non uniform magnetic fields

So far in this lecture we have only drawn uniform magnetic fields and considered their flux. But we can easily imagine a non-uniform field. We tackled non-uniform electric field fluxes. We should take on non-uniform magnetic field fluxes as well. Suppose we have the situation shown in the following figure.



We have a loop of wire, and the loop is in a flux that changes from left to right.

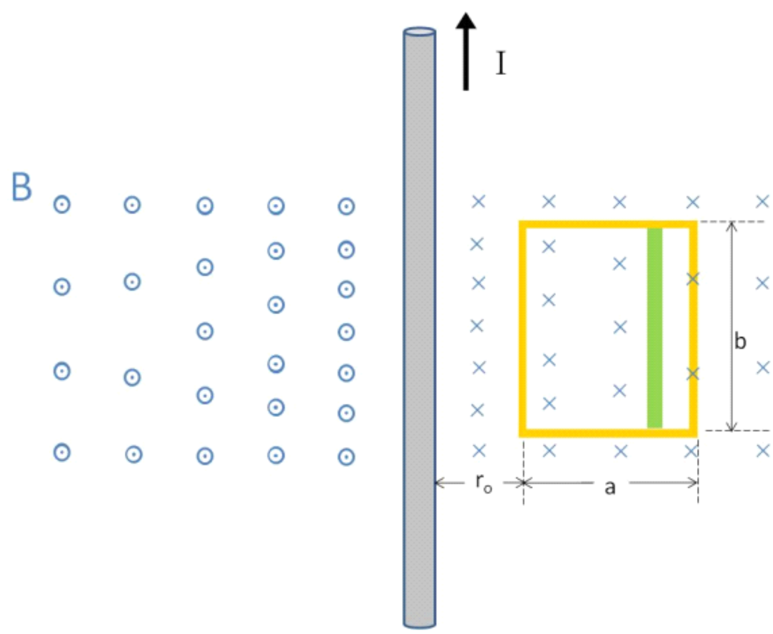
To find the flux through such a loop of wire, we can envision a small element of area, $d\overrightarrow{A}$ as shown. The flux through this area element is

$$d\Phi_B = \overrightarrow{B} \cdot d\overrightarrow{A}$$

We can integrate this to find the total flux

$$\Phi_B = \int_A \overrightarrow{B} \cdot d\overrightarrow{A} \tag{24.5}$$

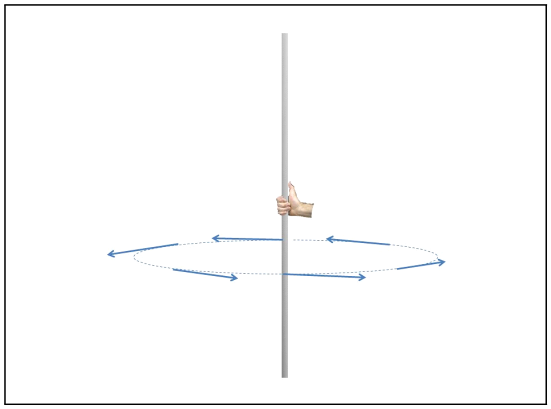
But what could make such a varying B-field? Consider a long straight wire again.



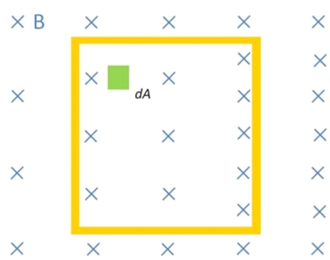
We know that the field due to the current-carrying wire will be

$$B = \frac{\mu_o I}{2\pi r}$$

where r is the distance from the wire and the direction is given by one of our right hand rules.



The flux through the green rectangular area is almost constant. We could take an even smaller piece of our loop area



$$dA = dydr$$

This area is perpendicular to the field, so the angle between B and A is $90\,°$. Then we would have a small amount of flux through dA

$$d\Phi_B = \frac{\mu_o I}{2\pi r} dydr \, (1)$$

and we can integrate this to find the total flux through the whole loop

$$\begin{aligned}
\Phi_B &= \int_{r_o}^{r=a} \int_{o}^{b} \frac{\mu_o I}{2\pi r} dydr \\
&= \frac{\mu_o I}{2\pi} \int_{r_o}^{r=a} \frac{1}{r} dr \int_{o}^{b} dy \\
&= \frac{\mu_o I b}{2\pi} \int_{r_o}^{r_o+a} \frac{1}{r} dr \\
&= \frac{\mu_o I b}{2\pi} \left(\ln\left(r_o + a\right) - \ln\left(r_o\right)\right) \\
&= \frac{\mu_o I b}{2\pi} \ln\left(\frac{r_o + a}{r_o}\right)
\end{aligned}$$

We can even put in some numbers for this case. Suppose our loop has a height of $b = 0.05\,\mathrm{m}$ and a width of $a = 0.01\,\mathrm{m}$ and that it is a distance $r_o = a$ away from the current carrying wire and that the current is $I = 0.5\,\mathrm{A}$. Then

$$\begin{aligned} \Phi_B &= \frac{\left(4\pi \times 10^{-7}\,\frac{\mathrm{T\,m}}{\mathrm{A}}\right)(0.5\,\mathrm{A})\,(0.05\,\mathrm{m})}{2\pi} \ln\left(\frac{0.01\,\mathrm{m} + 0.01\,\mathrm{m}}{0.01\,\mathrm{m}}\right) \\ &= 3.4657 \times 10^{-9}\,\mathrm{Wb} \end{aligned}$$

the unit of magnetic flux is called the weber and it is given by :

$$\mathrm{Wb} = \mathrm{T\,m}^2 = \frac{\mathrm{m}^2}{\mathrm{A}} \frac{\mathrm{kg}}{\mathrm{s}^2}$$

We know now how to calculate magnetic flux, but you should expect that we can do something with this flux to simplify problems. And your expectation would be right. We used electric flux in Gauss' law. We will use magnetic flux to find the induced emf. An induced emf can create a current, and this is the basic idea behind a generator. The law that governs this relationship between induced emf and magnetic flux is called *Faraday's law* after the scientist that discovered it. We will study this law in our next lecture.

Basic Equations

$$\Delta V \approx vBL \tag{24.6}$$

$$F_{resistive} = \frac{vB^2L^2}{R}$$

$$\Phi_B = \overrightarrow{B} \cdot \overrightarrow{A} \tag{24.7}$$

$$\Phi_B = \int_A \overrightarrow{B} \cdot d\overrightarrow{A} \tag{24.8}$$