Chapter 41

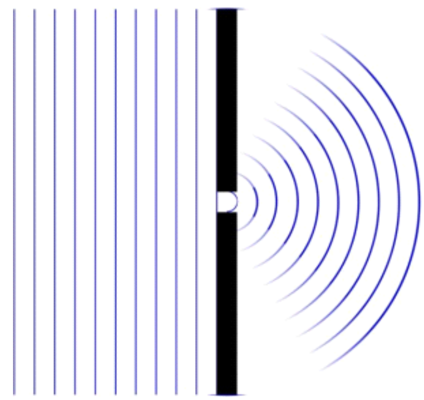
Interference and Young's Experiment

So if light is a wave in the electromagnetic field, does light experience interference?

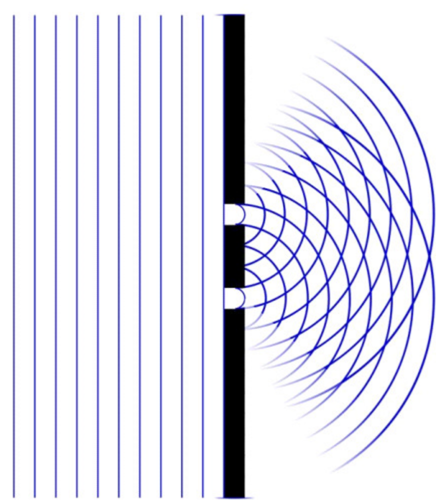
Fundamental Concepts

- Mixing two light waves does create constructive and destructive interference.
- The standard experiment to show this is called "Young's experiment."
- The combination of constructive and destructive interference for light is called an intensity pattern

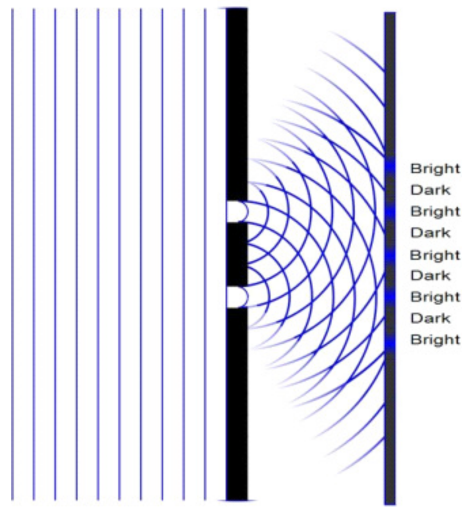
Waves do some funny things when they encounter barriers. Think of a water wave. If we pass the wave through a small opening in a barrier, the wave can't all get through the small hole, but it can cause a disturbance. We know that a small disturbance will cause a wave. But this wave will be due to a very small–almost point–source. So the waves will be spherical leaving the opening. The smaller the opening the more pronounced the curving of the wave, because the source (the hole) is more like a point source.



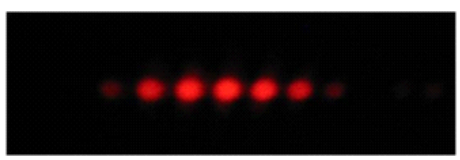
Now suppose we have two of these openings. We expect the two sources to make curved waves and those waves can interfere.



In the figure, we can already see that there will be constructive and destructive interference were the waves from the two holes meet. Thomas young predicted that we should see constructive and destructive interference in light (he drew figures very like the ones we have used to explain his idea).



Young set up a source of light and placed it in front of this source a barrier with two very thin slits cut in it to test his idea.. He set up a screen beyond the barrier and observed the pattern on the screen formed by the light. This (in part) is what he saw.



We see bright spots (constructive interference) and dark spots (destructive interference). Only wave phenomena can interfere, so this is fairly good evidence that light is a wave.

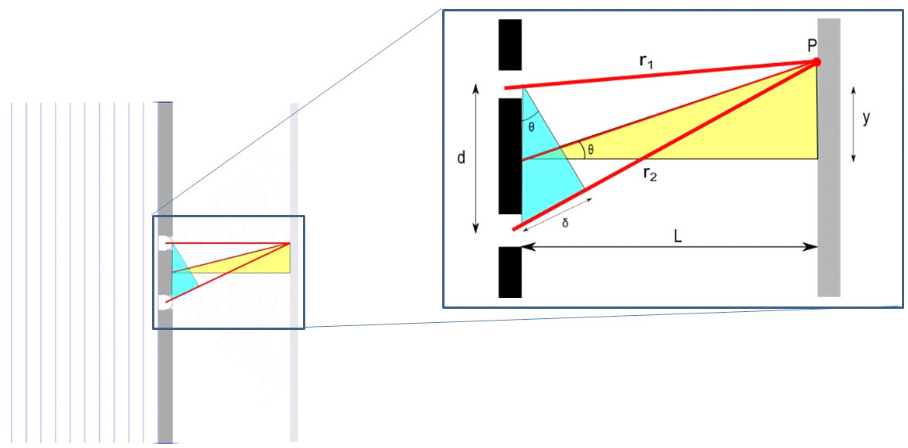
41.0.1 Constructive Interference

We can find the condition for getting a bright or a dark band if we think about it a bit. Here are our equations that we developed for constructive and destructive interference.

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = m2\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Constructive}$$
$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = (2m+1)\,\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Destructive}$$

For constructive interference, the difference in phase, $\Delta\phi$, must be a multiple of 2π. That means the path difference between the two slit-sources must be an even number of wavelengths. We have been calling the path difference in the total phase Δx, or for spherical waves Δr, but in optics it is customary to call this path difference δ. So

$$\delta = \Delta r$$



and our total phase equation becomes

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\delta + \Delta\phi_o\right) = m2\pi$$

Our light going through the slits is all coming from the same light source. So as long as the light hits the slits at a 90° angle, $\Delta\phi_o = 0 - 0 = 0$ so we don't have a change in phase constant. But we do have a change in $\Delta r = \delta$. Let's suppose that the screen is far away so the distance from the slits to the screen, $L \gg d$, the slit distance. Then we can say that the blue triangle is almost a right triangle, and then δ is

$$\delta = r_2 - r_1 \approx d\sin\theta$$

So then for constructive interference

$$\Delta\phi = \left(\frac{2\pi}{\lambda}d\sin\theta + 0\right) = m2\pi$$

We can do a little math to make this simpler.

$$\frac{2\pi}{\lambda}d\sin\theta = m2\pi$$

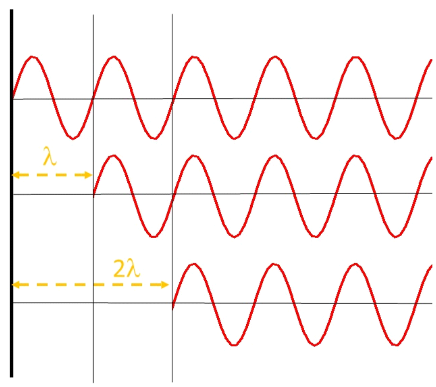
$$\frac{1}{\lambda}d\sin\theta = m$$

$$d\sin\theta = m\lambda$$

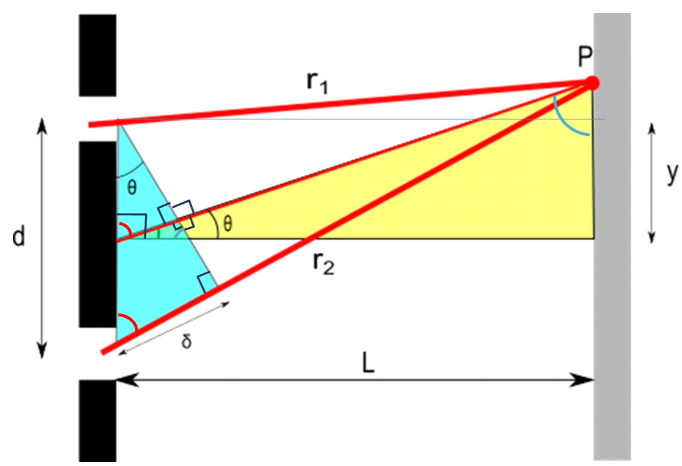
We started by knowing our wave needs to sift by an integer number times 2π radians but now we see that is equivalent to shifting the position of the wave by an integer number times the wavelength, λ. This will make the two waves experience constructive interference (a bright spot).

$$\delta = d\sin\theta = m\lambda \quad (m = 0, \pm1, \pm2 \ldots) \qquad \text{Constructive}$$

where in optics m is called the *order number*. That is, if the two waves are off by any number of whole wavelengths then our total phase due to path difference will be 2π.



In optics, the the bright spots formed by constructive interference are called *fringes*. If we assume that $\lambda \ll d$ we can find the distance from the axis for each fringe more easily. This condition guarantees that θ will be small.



Using the yellow triangle we see

$$\tan\theta = \frac{y}{L}$$

but if θ is small this is just about the same as

$$\sin\theta = \frac{y}{L}$$

because for small angles $\tan\theta \approx \sin\theta \approx \theta$. So if theta is small then

$$\begin{aligned}\delta &= d\sin\theta \\ &\approx d\frac{y}{L}\end{aligned}$$

and for a bright spot or fringe we find

$$d\frac{y}{L} \approx m\lambda$$

Solving for the position of the bright spots gives

$$y_{bright} \approx \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots) \tag{41.1}$$

We can measure up from the central spot and predict where each successive bright spot will be.

41.0.2 Destructive Interference

We can also find a condition for destructive interference. Our destructive interference equation is

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = (2m+1)\,\pi$$

Once again $\Delta\phi_o = 0$ and $\Delta r = \delta$

$$\left(\frac{2\pi}{\lambda}\delta\right) = (2m+1)\,\pi$$

$$\left(\frac{2}{\lambda}\delta\right) = (2m+1)$$

$$\delta = \frac{\lambda}{2}(2m+1)$$

$$\delta = \lambda\left(m + \frac{1}{2}\right)$$

This just shows us again that a path difference of an odd multiple of a half wavelength will give distractive interference.

$$\delta = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2 \ldots)$$

will give a dark fringe. The location of the dark fringes will be

$$y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right) \quad (m = 0, \pm 1, \pm 2 \ldots) \tag{41.2}$$

41.1 Double Slit Intensity Pattern

The fringes we have seen are not just points, but are patterns that fade from a maximum intensity. This is why they are called fringes. We should expect this. Only the center of the bright fringe is total constructive interference and only the center of the dark fringe is total destructive interference. In between we go through partial constructive to partial destructive interference. So we go from the brightest spot to dimmer and dimmer spots until we have the darkest spot in the pattern.

We can calculate the intensity pattern to show this. We need to know a little bit about electric fields to do this.

41.1.1 Superposition of two light waves

Suppose we have two light waves

$$E_1 = E_{\max} \sin(kr_1 - \omega t + \phi_o)$$

$$E_2 = E_{\max} \sin(kr_2 - \omega t + \phi_o)$$

and we combine these two light waves in the electromagnetic field. We would get

$$E_r = E_{\max} \sin(kr_1 - \omega t + \phi_o) + E_{\max} \sin(kr_2 - \omega t + \phi_o)$$

but we know how to make this make ore sense. We can use our favorite trig identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

and we would find the resultant wave is

$$E_r = 2A\cos\left(\frac{1}{2}(\Delta\phi)\right)\sin\left(k\frac{r_2+r_1}{2} - \omega t + \frac{\phi_2+\phi_1}{2}\right)$$

Our light waves are just two waves. They may be the superposition of many individual photons, but the combined wave is just a wave.

At the slits, the waves have the same amplitude $E_{\max}$ and the same phase constant, $\phi_1 = \phi_2 = \phi_o$, but suppose we looking at our screen off center a bit at our point P. And suppose at our chosen y position E_2 has traveled farther than E_1, so $\Delta\phi$ is due to the path difference. We expect to find that the path difference would be

$$\begin{aligned}
\Delta\phi &= k\Delta r + \Delta\phi_o \\
&= k\delta + 0 \\
&= \frac{2\pi}{\lambda} d\sin\theta
\end{aligned}$$

and using our prior result, we have

$$E_P = 2E_{\max}\cos\left(\frac{1}{2}\Delta\phi\right)\sin\left(k\frac{(r_2+r_1)}{2} - \omega t + \phi_o\right)$$

and substituting in our expression for $\Delta\phi$ gives

$$E_P = 2E_{\max}\cos\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\sin\left(k\frac{(r_2+r_1)}{2} - \omega t + \phi_o\right)$$

We have a combined wave at point P that is a traveling wave $\left(\sin\left(k\frac{(r_2+r_1)}{2} - \omega t + \phi_o\right)\right)$ but with amplitude $\left(2E_{\max}\cos\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\right)$ that depends on our total phase $\Delta\phi = \frac{2\pi}{\lambda}d\sin\theta$.

But remember the situation is more complicated because of how we detect light. Our eyes, and most detectors measure the average intensity of the light. We know that

$$I = \frac{\mathcal{P}}{A}$$

and from our last lecture we know that the intensity is

$$I = \frac{E^2}{\mu_o c}$$

so for our combined wave at P

$$\begin{aligned} I &= \frac{1}{\mu_o c}E_P^2 \\ &= \frac{1}{\mu_o c}\left(4E_{\max}^2\cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\sin^2\left(\frac{k(r_2+r_1)}{2} - \omega t + \phi_o\right)\right) \end{aligned}$$

But this is the intensity, not the average intensity. We know that visible light detectors collect energy for a set amount of time. So most visible light detection will be a value averaged over an integration time. We have seen this before. This means that the detector sums up (or integrates) the amount of power received over the detector time. And once again the integration time is much longer than a period, so what is really detected is like a time-average of our intensity.

$$\begin{aligned} \bar{I} &= \int_{\text{many T}} I dt = \frac{1}{\mu_o c}\int_{\text{many T}} 4E_{\max}^2\cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\sin^2\left(\frac{k(r_2+r_1)}{2} - \omega t + \phi_o\right)dt \\ &= \frac{4E_{\max}}{\mu_o c}\cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\int_{\text{many T}}\sin^2\left(\frac{k(rx_2+r_1)}{2} - \omega t + \phi_o\right)dt \end{aligned}$$

but we have seen an integral over $\sin^2(\theta)$ before! (see equation ([**?**])). The term

$$\int_{\text{many T}}\sin^2\left(\frac{k(r_2+r_1)}{2} - \omega t + \phi_o\right)dt = \frac{1}{2} \tag{41.3}$$

So we have

$$\bar{I} = \int_{\text{many periods}} I dt = \frac{2E_{\max}}{\mu_o c}\cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right) \tag{41.4}$$

where $\bar{I}$ is the time average intensity. The important part is that the time varying part has averaged out. So, usually in optics, we ignore the fast fluctuating parts of such calculations because we can't see them. The situation will be different if we use radio waves where we do detect the waves directly (not averaged). But let's continue with visible optics and set

$$I_{\max} = \frac{2E_{\max}}{\mu_o c}$$

so we write our average intensity as

$$I = I_{\max} \cos^2 \left(\frac{1}{2} \left(\frac{2\pi}{\lambda} d \sin\theta \right) \right)$$

where we have dropped the bar from the I, but it is understood that the intensity we report is a time average over many periods.

We should remind ourselves, of our intensity pattern

$$I = I_{\max} \cos^2 \left(\frac{1}{2} \frac{2\pi}{\lambda} d \sin\theta \right)$$

is really

$$I = I_{\max} \cos^2 \left(\frac{\Delta\phi}{2} \right)$$

Which is just our amplitude squared for the mixing of two waves. All we have really done to find the intensity pattern is to find and expression for the phase difference $\Delta\phi$.

Our intensity pattern should give the same location for the center of the bright spots as we got before. Let's check that it works. We used the small angle approximation before, so let's use it again now. For for small angles

$$\begin{aligned} I &= I_{\max} \cos^2 \left(\frac{\pi d}{\lambda} \theta \right) \\ &= I_{\max} \cos^2 \left(\frac{\pi d}{\lambda} \frac{y}{L} \right) \end{aligned}$$

Then we have constructive interference when

$$\frac{\pi d}{\lambda} \frac{y}{L} = m\pi$$

or

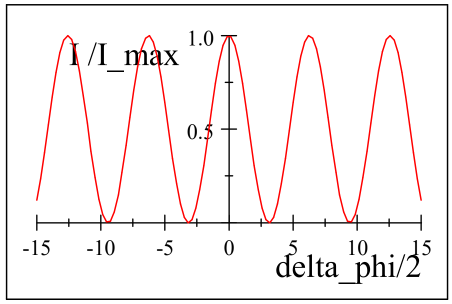
$$y = m \frac{L\lambda}{d}$$

which is what we found before.

The plot of normalized intensity

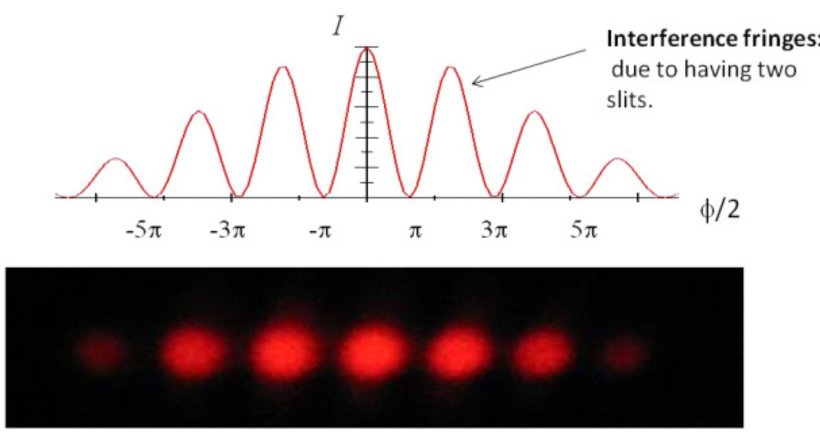
$$\frac{I}{I_{\max}} = \cos^2 \left(\frac{\Delta\phi}{2} \right)$$

verses $\Delta\phi/2$ is given next,



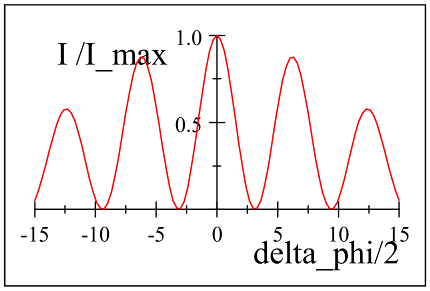
and we get peaks that are the bright fringes and valleys that are the dark fringes and the brightnesses fades from bright to dark just as we expected.

Let's pause to remember what this pattern means. This is the intensity of light due to interference. It is instructive to match our intensity pattern that Young saw with our graph.



The high intensity peaks are the bright fringes and the low intensity troughs are the dark fringes. The pattern moves smoothly and continuously from bright to dark.

We will find that we are not quite through with this analysis. Next time we will find that there is another compounding factor that reduces the intensity as we move away from the midpoint.



We see this in the photograph of our Young's experiment pattern.

Basic Equations

$$y_{bright} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots)$$

$$y_{dark} = \frac{\lambda L}{d} \left(m + \frac{1}{2} \right) \quad (m = 0, \pm 1, \pm 2 \ldots)$$

$$\begin{aligned} I &= I_{\max} \cos^2 \left(\frac{\pi d}{\lambda} \theta \right) \\ &= I_{\max} \cos^2 \left(\frac{\pi d}{\lambda} \frac{y}{L} \right) \end{aligned}$$