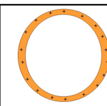
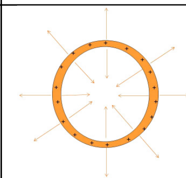
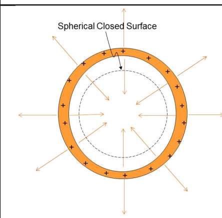
0.0.1 Strategy for Gauss' law problems

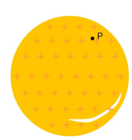
Let's review what we have done before we go on to our last example. For each Gauss' law problem, we

1. Draw the charge distribution	
2. Draw the field lines using symmetry - Don's skip this step, it is essential	
3. Choose (make up, invent) a closed surface that makes $\overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{A}}$ either just EdA or 0. And hopefully on our Gaussian surface E will be constant	
4. Find Q_{in}.	
5. Solve $\oint EdA = \frac{Q_{inside}}{\epsilon_o}$ for the non, zero parts. The integral should be trivial now due to our use of symmetry. Usually, if we picked our surface well, $\oint EdA = EA$	
6. Solve for E i.e. $EA = \frac{Q_{inside}}{\epsilon_o} \to EA = \frac{Q_{inside}}{\epsilon_o}$	

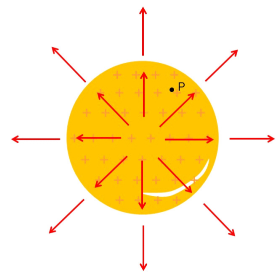
0.0.2 Example

Find the field at a point inside at point P inside a uniformly charged spherical insulator with total charge Q_{total} and radius R. Let's say that P is a distance r from the center of the sphere.

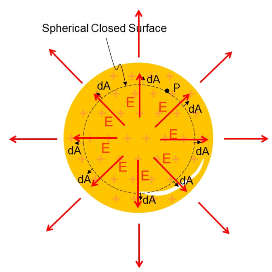
1. draw the charge distribution



2. Draw the field lines using symmetry- Don's skip this step, it is essential



3. Choose (make up, invent) a closed surface that includes point P and makes $\overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{A}}$ either just $\pm EdA$ or 0. And hopefully on our Gaussian surface E will be constant



4. Find Q_{in}. This time it is harder. Since the charge is uniformly distributed

$$\rho = \frac{Q_{total}}{V_{total}} = \frac{Q_{in}}{V_{in}}$$

so

$$Q_{in} = \frac{Q_{total}}{V_{total}} V_{in}$$

We know how to do this for spheres

$$\begin{aligned} V_{total} &= \frac{4}{3}\pi R^3 \\ V_{in} &= \frac{4}{3}\pi r^3 \end{aligned}$$

$$Q_{in} = \frac{Q_{total}}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3 = \frac{Q_{total}}{R^3} r^3$$

5. Solve $\oint EdA = \frac{Q_{inside}}{\epsilon_o}$ for the non, zero parts. The integral should be trivial now due to our use of symmetry. Usually, if we picked our surface well, $\oint EdA = EA = E4\pi r^2$

6. Solve for E

$$\begin{aligned} EA &= \frac{Q_{inside}}{\epsilon_o} \\ E &= \frac{\frac{Q_{total}}{R^3} r^3}{\epsilon_o 4\pi r^2} = \frac{1}{4\pi\varepsilon_o}\frac{Q_{total} r}{R^3} \end{aligned}$$