Chapter 47

Image Formation

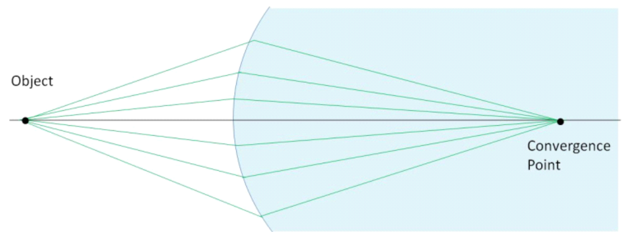
Last lecture we learned how to find an image location graphically, now let's do it algebraically.

Fundamental Concepts

- A curved interface between two media can cause light rays to cross
- The lens-maker's formula is given by $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
- The thin lens formula is given by $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$
- The sign of quantities that go into the lens-maker's equation and the thin lens formula are determined by a sign convention.

47.1 Thin lenses and image equation

In this lecture, we will work toward understanding the equations that allow us to solve for the image location given the object location for a thin lens. Let's start by thinking of a special case for refraction. A circular or spherically curved surface on a very large piece of glass. We will assume that the piece of glass is semi-infinite. Looking at the next figure, this means that on the right hand side of the next figure the glass goes on forever. It doesn't really have to go on forever for our mathematical analysis to work. All it has to be is very large. But let's just assume the glass goes on infinitely to the right. Since we have a spherical bump on the front of our glass, and the glass goes on forever in one direction we can call this a semi-infinite bump of glass.



Take a point object that either glows, or has rays of light reflecting from it. The rays leave the object and reach the surface of the glass. The rays will refract at the surface. Each bends toward the normal, but because of the curvature of the glass, the rays all converge toward the center. We can identify this convergence point as the image of the point object. Since our object is a point, so is our image.[1] Of course we could make an extended object out of many points, and then we would have many image points as well.

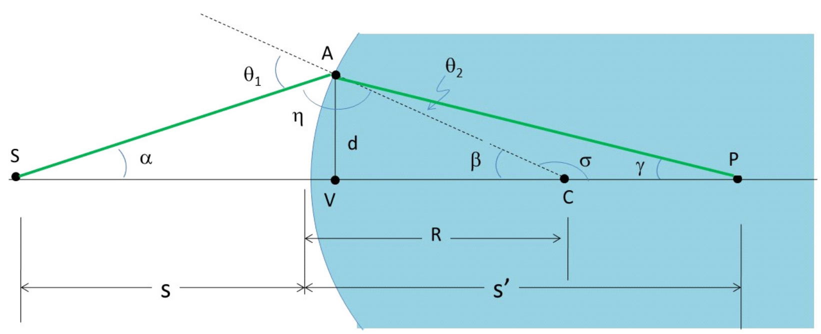
At the surface we can find the refracted angles using Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

We will again use the small angle approximation. Then θ_1 and θ_2 are small and none of the rays are very far away from the axis. This is our paraxial approximation. Snell's law becomes

$$n_1\theta_1 = n_2\theta_2$$

Let's try to see where the image will be using Snell's law.



Using the more detailed figure above, we observe triangles SAC and PAC. We recall that for triangle SAC the top angle labeled η, plus θ_1 must be 180.

$$180\,^{\circ} = \theta_1 + \eta$$

or

$$\eta = 180\,^{\circ} - \theta_1$$

[1] Within the limits of diffraction. So it is really a small circle of light. But it's mostly a point.

We also know that the sum of interior angles must equal 180. So for triangle SAC we know

$$180^\circ = \eta + \alpha + \beta$$

Let's substitute in for η

$$180^\circ = (180^\circ - \theta_1) + \alpha + \beta$$

then

$$\theta_1 = \alpha + \beta$$

Likewise, from triangle PAC,

$$180^\circ = \sigma + \theta_2 + \gamma$$

and

$$180^\circ = \beta + \sigma$$

so

$$\sigma = 180^\circ - \beta$$

and

$$180^\circ = (180^\circ - \beta) + \theta_2 + \gamma$$

which reduced to

$$\beta = \theta_2 + \gamma$$

then,

$$\theta_2 = \beta - \gamma$$

and we can write our paraxial Snell's law as

$$\begin{aligned}
n_1\theta_1 &= n_2\theta_2 \\
n_1(\alpha+\beta) &= n_2(\beta-\gamma) \\
n_1\alpha + n_1\beta &= n_2\beta - n_2\gamma \\
n_1\alpha + n_2\gamma &= n_2\beta - n_1\beta \\
n_1\alpha + n_2\gamma &= \beta(n_2 - n_1)
\end{aligned}$$

Looking at the figure. We see that d is a leg of three different right triangles (SAV, ACV, and PAV). The ray in the figure is clearly not a paraxial ray. If we use an actual paraxial ray, then the point V will approach the air-glass boundary. When this happens, then $SV = s$, $VC = R$, and $VP = s'$. So we can write

$$\begin{aligned}
\tan\alpha &\approx \alpha \approx \frac{d}{s} \\
\tan\beta &\approx \beta \approx \frac{d}{R} \\
\tan\gamma &\approx \gamma \approx \frac{d}{s'}
\end{aligned}$$

so our Snell's law becomes

$$\begin{aligned} n_1\alpha + n_2\gamma &= \beta\left(n_2 - n_1\right) \\ n_1\frac{d}{s} + n_2\frac{d}{s'} &= \frac{d}{R}\left(n_2 - n_1\right) \end{aligned}$$

We can divide out the common factor, d.

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{R} \tag{47.1}$$

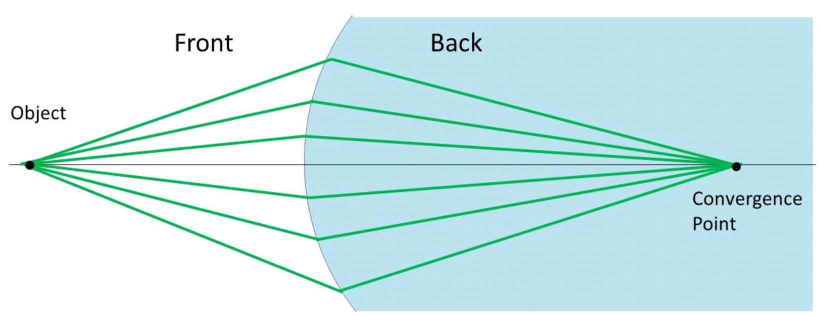
We can use this formula to convince ourselves that no matter what the angle is (providing it is small), the rays will form an image at P. So all the rays in figure (**??**) will converge at P.

Real images will be inside the glass. This may seem a problem. We will fix this with non-infinite lenses soon. But for the case of our eyes, this is exactly what happens. We have fluid (sort of a jelly) in our eyes, and the image is formed in the fluid. The curved surface is our cornea (the spot where your contacts go).

Physicists got together and decided on a mathematical system of signs to make the math easier and consistent. We call such a scheme a sign convention. We started collecting parts of this system last lecture. Let's write it all out in a table so we can use it in today's lecture. Here is the convention for the case of a curved semi-infinte surface.

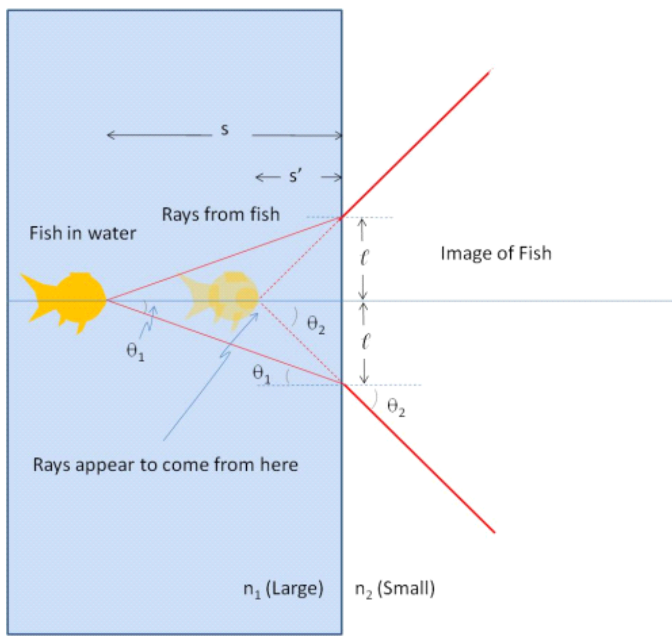
Quantity	**Positive if**	**Negative if**
Object location (s)	Object is in front of surface	Object is in back of surface (virtual object)
Image location (s')	Image is in back of surface (real image)	Image is in front of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R)	Center of curvature is in back of surface	Center of curvature is in front of surface

where the "front" of the lens is the side that gets the light from the object.



We could go through the entire derivation and switch the indices of refraction (in effect, go along the same path, but going backwards). It turns out we get the same equation. The light is bending the other way as it travels the path, but the equation will be the same. So our equation describes light entering a pice of glass, or light leaving a piece of glass.

47.1.1 Flat Refracting surfaces



Let's return to our fist tank. The fish tank has an interface, but it is flat. Still, we have water (that we can pretend goes on forever) on one side and air on the other. This is sort of like a semi-infinate situation. Can we use our equation (47.1) to describe this?

The answer is yes, if we let $R = \infty$. This makes sense for a flat surface. If we have an infinitely large sphere, then our small part of that spherical surface that makes up the fish tank wall will be very flat.

Then

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{\infty}$$

or

$$\frac{n_1}{s} + \frac{n_2}{s'} = 0$$

we see that

$$s' = -s\frac{n_2}{n_1}$$

This is what we got before for this case, except before we just got the distance, and now we have included the effects of our sign convention. The negative sign means that the image is in front of the surface. By "in front" we always mean to follow the light from the source (fish) to the optical boundary. This boundary is the water/air boundary of the tank, so the fact that our image is in the water means that our image is in front of the optical boundary. This means the image is virtual.

47.2 Thin Lenses

Lets' find an equation for a lens made from sections of spherical surfaces once more. But this time, let's let it be more practical and not make the "lens" semi-infinite. We will need to deal with two sides of the lens because (usually) both will be curved.

We found that for refraction

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{R}$$

but we did this for a spherical bump on a semi-infinite piece of glass. For this problem let's make a few assumptions:

- We have two spherical surfaces, with R_1 and R_2 as the radii of curvature
- We have only paraxial rays
- The image formed by one refractive surface serves as the object for the second surface
- The lens is not very thick (the thickness is much smaller than both R_1 and R_2)

The answer we will get is quite simple

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \tag{47.2}$$

where

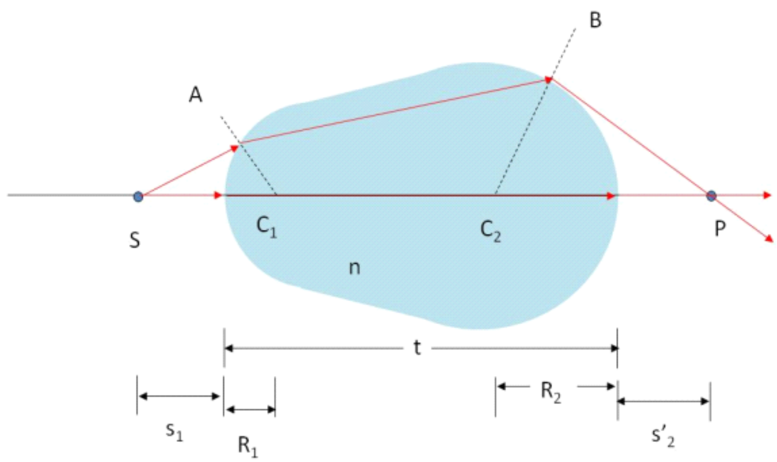
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{47.3}$$

but to appreciate what it means, lets find out where it comes from.

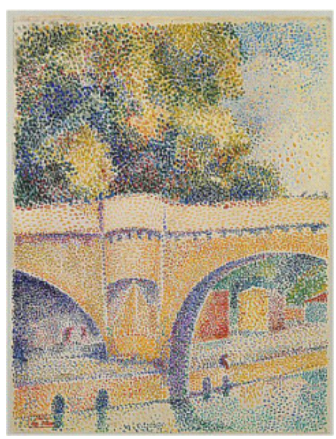
47.2.1 Derivation of the lens equation

Let's find the thin lens formula. Really, we could just assume the formula and be fine, but we are going through the derivation because it will help teach us how to deal with multiple surfaces in an optical system. Telescopes and microscopes all have multiple surfaces. So this will help us understand how they work.

Consider the optical element in the figure below. Notice that our object is a dot, so our image will also be a dot.



By now you have realized that this is not as boring as it sounds if we consider any object can be considered as a collection of dots.



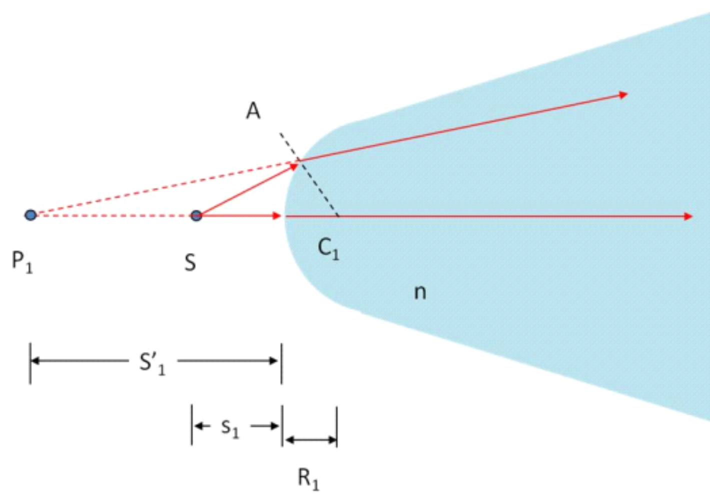
The Pont Neuf by Hippolyte Petitjean. Petitjean was a Pointillist, one who painted with dots of paint instead of continuous application of paint. This illustrates the thought that all objects can be though of collections of small points that reflect or emit light. So we can consider an optical system by considering individual points of light and how the system reacts to those points of light. (Image in the Public Domain)

So we can consider anything as a collection of dots, and work out our formulas for a dot (because it is easier to think about just one dot at a time).

Light enters at a spherical surface on the left hand side. We use a point object located at S on the principal axis, and trace two rays. The ray along the principal axis crosses each spherical surface at right angles, and therefore traves straight through the optic. The second ray hits the first spherical surface at point A. It is refracted and travels to point B. It is again refracted and travels toward the principal axis, crossing at P. The image location is the intersection of these rays, so we have an image at P.

Lets study the surfaces separately

Surface 1:



We treat surface 1 as though surface 2 did not exist. After all, the light does not know about surface 2 as it hits surface 1. So surface 2 won't enter into our calculations (yet).

By considering surface 1 on its own, we have just our semi-infinite bump problem, so we know that

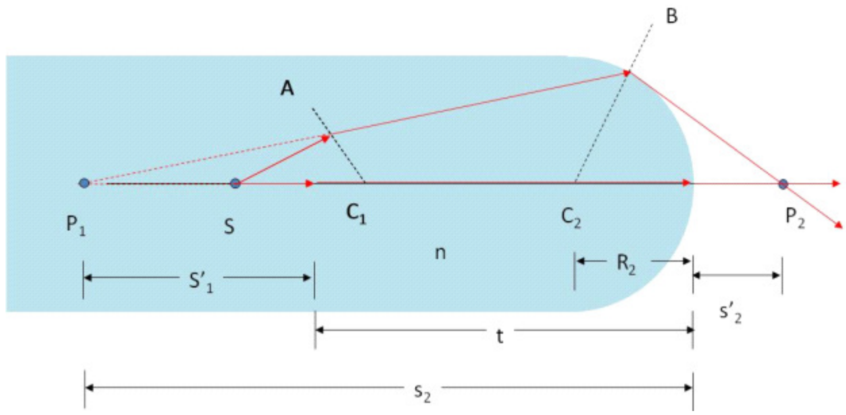
$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{R}$$

We can consider $n_1 = 1$ and $n_2 = n$ for an air-glass interface and noting that s'_1 is negative by our convention. Then

$$\frac{1}{s_1} - \frac{n}{s'_1} = \frac{(n-1)}{R_1} \tag{47.4}$$

Note that our rays are *not* converging in the glass. We have learned that when this happens we can find the virtual image formed by this surface 1 of our lens by tracing the diverging rays backward as we did for the fish tank or magnifying glass. The image formed from the first side of the lens is virtual.

Surface 2: Now consider the second surface.



The second surface sees light diverging as though it came from a semi-infinite piece of glass with the light source at P_1. The virtual image formed by surface 1 serves as the object for surface 2 because the diverging light from A to B is in just the same pattern as if there were a light source at P_1. The distance from P_1 to surface 2 is

$$s_2 = s_1' + t$$

We again use our refractive equation for a semi-infinite bump

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{R}$$

but we identify $n_1 = n$ and $n_2 = 1$. We have for surface 2

$$\frac{n}{s_2} + \frac{1}{s_2'} = \frac{(1-n)}{R_2} \tag{47.5}$$

or

$$\frac{n}{s_1' + t} + \frac{1}{s_2'} = \frac{(1-n)}{R_2} \tag{47.6}$$

Now we take our thin lens approximation. Let $t \to 0$. Then equations 47.4 and 47.6 become

$$\frac{1}{s_1} - \frac{n}{s_1'} = \frac{(n-1)}{R_1}$$

$$\frac{n}{s_1} + \frac{1}{s_2'} = \frac{(1-n)}{R_2}$$

I would like a single equation that gives s_2' in terms of s_1. That is the form of the thin lens equation that we are looking for. Adding these two equations can give me such an equation.

$$\frac{1}{s_1} - \frac{n}{s_1'} + \frac{n}{s_1'} + \frac{1}{s_2'} = \frac{(n-1)}{R_1} + \frac{(1-n)}{R_2}$$

or

$$\frac{1}{s_1}+\frac{1}{s_2'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

This equation is very useful. If we again let $s_1=\infty$ (put the object at ∞ so the rays enter surface 1 parallel) we find

$$\frac{1}{s_2'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

The spot where the rays gather if the object is infinitely far away is the focal point, f, so for the case of $s_1=\infty$ we can identify $s_2'=f$ as the focal length of the lens.

Then we can identify

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

which is known as the *lens makers' equation*. It gives us a way to make a lens that will have a particular focal distance. You grind one side of the lens to have a radius of curvature R_1 and the other side to have radius of curvature R_2. Then with index n, you will have the focal length you desire.

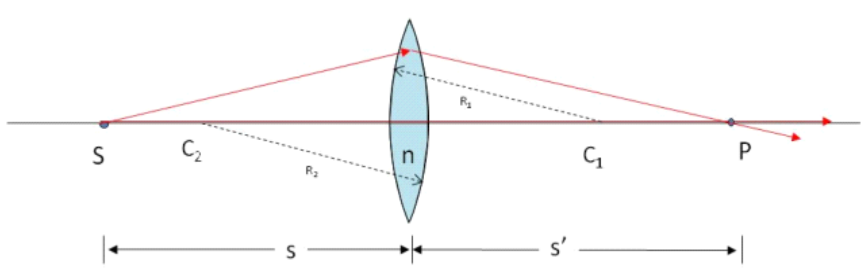
We have a relationship between the object distance in front of the lens, and the final image in back of the lens:

$$\begin{aligned}\frac{1}{s_1}+\frac{1}{s_2'} &= (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\\ &= \frac{1}{f}\end{aligned}$$

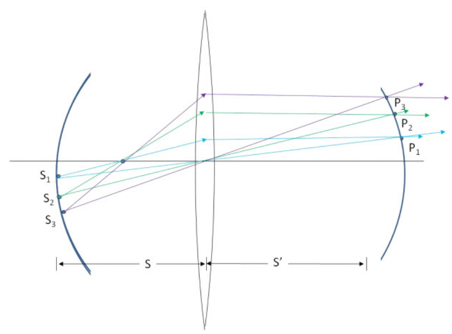
If we drop the subscripts (which we can do now that we let $t=0$ since the internal distances for the inside points are not important) then.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

This is called the *thin lens equation*. The resulting approximate geometry is shown below.



Of course any real object is made of lots of points, but each point is imaged in a corresponding point on the image



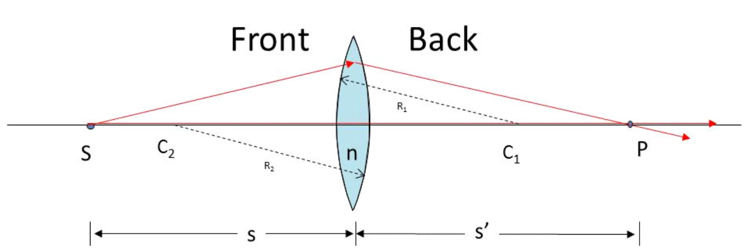
so, as we claimed earlier, our simple analysis explains the formation of actual images and not just point images.

47.2.2 Sign Convention

We need to add to our sign convention table a second radius, and the focal length.

Quantity	**Positive if**	**Negative if**
Object location (s)	Object is in front of surface	Object is in back of surface (virtual object)
Image location (s')	Image is in back of surface (real image)	Image is in front of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius $(R_1$ and $R_2)$	Center of curvature is in back of surface	Center of curvature is in front of surface
Focal length (f)	Converging lens	Diverging lens

Again the front surface is the surface that first gets the light from the object.



Note that each radius has a sign. If the two radii are the same magnitude, it looks like

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

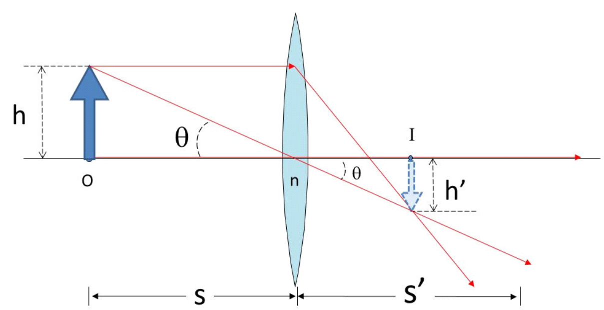
should be undefined (the focal length should be infinite) but usually that is not true because either R_1 or R_2 will be negative.

47.2.3 Magnification

The image is not likely to be the same size as the object. We would like to have a quantity that tells us how big the image is. The measure of choice is the ratio of the two heights.

$$m = \frac{h'}{h} \tag{47.7}$$

where h is the object height and h' is the image height. Note that with our sign convention, if $m > 0$ then the image is upright, and if $m < 0$ the image is inverted (upside down). We call this ratio the *magnification* of the lens.



We can find an expression for the magnification in terms of s and s'. By observing the figure, and using the ray that goes right through the middle of the lens, we can see that

$$\tan\theta = \frac{h}{s}$$

and

$$\tan\theta = \frac{h'}{s'}$$

thus

$$\frac{h}{s} = \frac{h'}{s'}$$

then

$$\frac{s'}{s} = \frac{h'}{h}$$

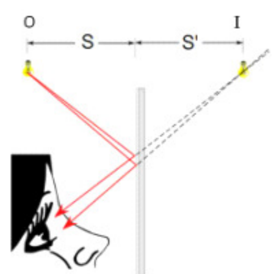
which we can use to form a new equation for the magnification.

$$m = -\frac{s'}{s} \tag{47.8}$$

The minus sign is because h' is negative by our sign convention, but neither s nor s' are negative. The magnification must be negative, so we need the minus sign to satisfy our sign convention.

47.3 Images formed by Mirrors

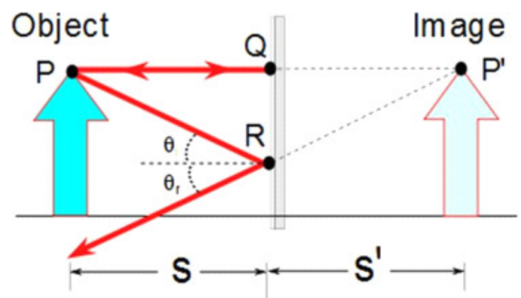
All of us have looked in a mirror at some time. We know what to expect. We see an image of ourselves. To study mirrors we need to establish a sign convention and some standard notation



In the figure above, we have a person observing an object O in a mirror. The object is located at a distance s from the mirror. Just like with lenses, we will call this the *object distance*. The image appears to be located at a point I beyond the mirror a distance s'. This is the *image distance*.

Images are located at a point from which rays of light diverge *or at a point from which rays of light appear to diverge*. This only makes sense. If you remember how we see things, our eyes intercept rays of light diverging from an object. So if we can create a situation that makes rays diverge in the same way the object did, we will have an image of the object.

Mirrors create what we have called *virtual* images because the image appears to be created from diverging rays from behind the mirror, but if we look behind the mirror no rays exist at the image location (if they did exist, they would not make it through the mirror!). Our brain believes the rays traveled in a straight line without reflecting, so brain interprets the diverging reflected waves as a virtual image "behind" the mirror.



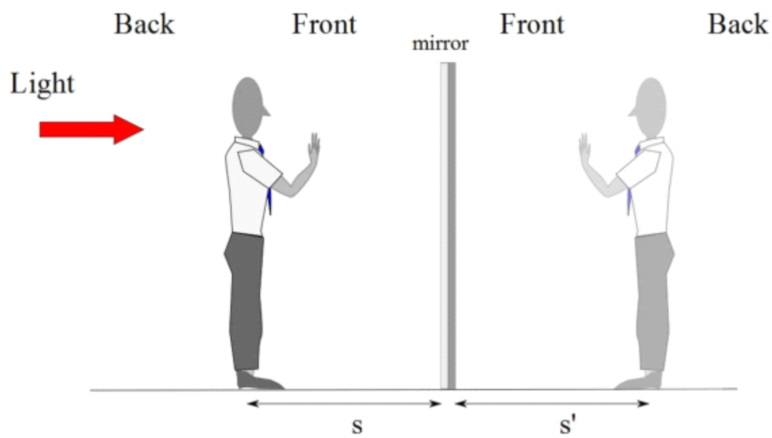
Let's look at a simple image as shown in the figure above. The object (of course) is an arrow. We could trace all the rays that diverge from this object and build a very nice representation of the arrow[2] but that would take time and computation power. We only really need to use two rays, and remember what the object looked like.

We pick one ray from the top of the arrow that travels straight to the mirror. This ray will travel a distance s and bounce back. We pick a second ray from point P that travels the path PR. This ray bounces off the mirror at an angle θ. It appears to our brain that the tip of the arrow is at position P' and the rays from the tip appear to travel the paths $P'P$ and $P'R$. Again, our visual processing center in our brain interprets the rays as traveling in straight lines.

47.4 Mirror reversal

Look into a mirror. Raise your left hand. Your image raises what appears to be a right hand. It looks like a mirror switches the left and right sides of the image. But lie sideways on the ground in front of the mirror and raise a hand. Your hand does not get inverted (and neither do your feet and head). What is happening? A flat mirror performs a front-back reversal. What this means is that, following the light direction, the object is positioned so that the back is encountered first, then the front, but in the image the front of the image is encountered first, then the back.

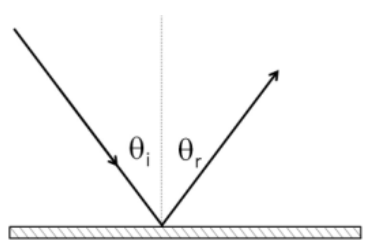
[2] Ray tracing-based computer graphics actually does this–the way movies like *Toy Story* are made.



this has the effect of making it look like the left hand is raised when the object's right hand is raised.

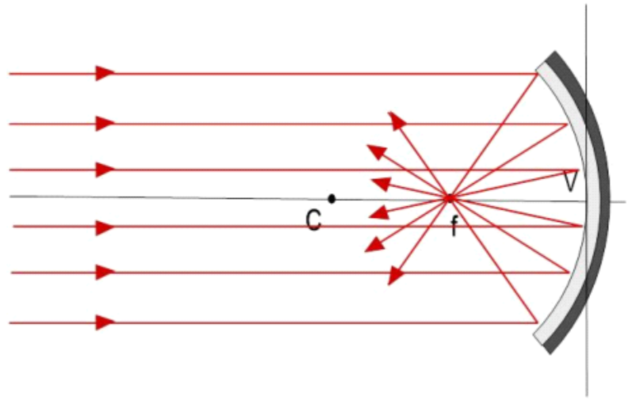
47.4.1 Concave Mirrors

Concave mirrors can form images. I'm sure you know that many telescopes are made with mirrors. We should see how this works. Let's proceed like we did for lenses, but looking at what happens when light strikes the surface of the new material, this time the mirror surface. First, lets look at rays that come from very distant objects so they enter parallel to the optic axis. We recall the law of reflection



$$\theta_i = \theta_r$$

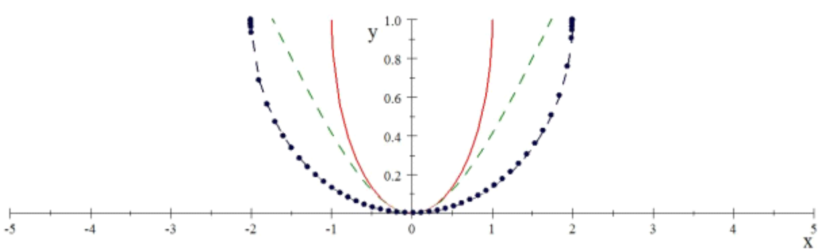
Armed with this, we can see what would happen. Each ray has a different normal due to the curvature of the mirror. The result is that the rays all meet at a spot on the axis.



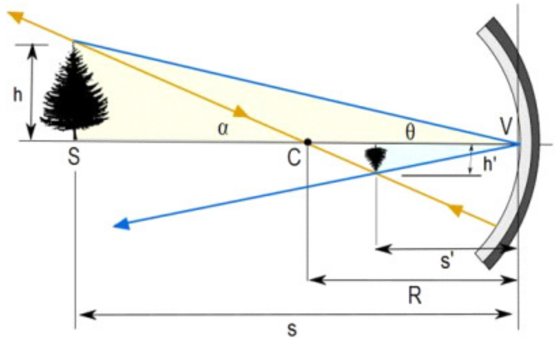
This is a focal point!

47.4.2 Paraxial Approximation for Mirrors

The correct shape of a mirror is more like a parabola, but parabolas are hard to machine or build. Spherical shapes are relatively easy. So we often see spherical mirrors just like we often see spherical lenses. This will work so long as we allow only rays that make small angles with respect to the principal axis. We can see why this works if we plot a sphere and a parabola (and a hyperbola). For small deviations from the center, the shape of the functions all look alike.



We would expect the reflections to be similar under these circumstances, so, if we meet the criteria for the paraxial approximation, our spherical mirrors should work. Note that when you need the entire mirror, say, in a communications antenna, you must do better than a spherical approximation to the correct shape for your mirror.



Like with our flat mirror. we will measure distances from the mirror surface (from point V). We can find the image location by again taking two rays. One convenient ray is the ray that passes through the center of curvature, C. This ray will strike the mirror surface at right angles and bounce back along the same path. Another convenient ray is the ray from the tip of the object to point V. This ray will bounce back with angle θ. Where these two reflected rays cross, we will find the image of the tip of our object (a tree this time, I got tired of imaging arrows). Knowing the shape of the object and that the bottom is on the axis, we can fill in the rest of the image.

We can calculate the magnification for this case. We use the gold triangle to determine that

$$\tan\theta = \frac{h}{s}$$

and the blue triangle to determine that

$$\tan\theta = \frac{h'}{s'}$$

so we have

$$m = \frac{h'}{h} = \frac{-s'\tan\theta}{s\tan\theta} = -\frac{s'}{s}$$

just like for lenses! And like for lenses we indicate that the image is inverted by making the sign of the magnification negative. We recall that h' is negative if the image is inverted. But again neither s nor s' are negative, so we added a negative sign to make this fit with our sign convention.

47.4.3 Mirror Equation

We can further exploit this geometry to get a relationship between s, s', and R. Notice that

$$\tan\alpha = \frac{h}{s-R}$$

and that

$$\tan\alpha = \frac{-h'}{R-s'}$$

Then

$$\frac{h}{s-R}=\frac{-h'}{R-s'}$$

or

$$\frac{R-s'}{s-R}=-\frac{h'}{h}$$

We can use our magnification definition to replace h'/h

$$\frac{R-s'}{s-R}=\frac{s'}{s}$$

we perform some algebra

$$\begin{aligned}
(R-s')\,s &= s'\,(s-R)\\
-s's+Rs &= ss'-Rs'\\
Rs+Rs' &= ss'+s's\\
Rs+Rs' &= 2ss'\\
\frac{Rs'}{Rss'}+\frac{Rs}{Rss'} &= \frac{2ss'}{Rss'}\\
\frac{1}{s}+\frac{1}{s'} &= \frac{2}{R}
\end{aligned}$$

This is called the *mirror equation*. Note how much it looks like the thin lens equation!

Now that we know the mirror equation, let's let s be very large. Then

$$\frac{1}{s'}\approx\frac{2}{R}$$

or

$$s'\approx\frac{R}{2}$$

Using the same logic as with the lens, we can identify this as the *focal point*, F and the distance s' in this case will be called the *focal length*, f. We see that

$$f=\frac{R}{2} \tag{47.9}$$

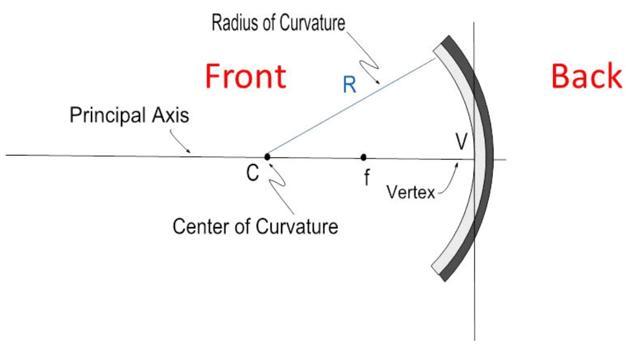
so we can write the mirror equation as

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \tag{47.10}$$

For a mirror, the value of f does not depend on the mirror material. Of course we have a sign convention for mirrors, but it is similar to the convention for lenses. Here is the convention for mirrors.

Quantity	Positive if	Negative if
Object location (s)	Object is in front of surface	Object is in back of surface (virtual object)
Image location (s')	Image is in front of surface (real image)	Image is in back of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R_1 and R_2)	Center of curvature is in front of surface	Center of curvature is in back of surface
Focal length (f)	Concave mirror	Convex mirror

Where the front is, as usual, the part of the mirror that receives the light first from the object.



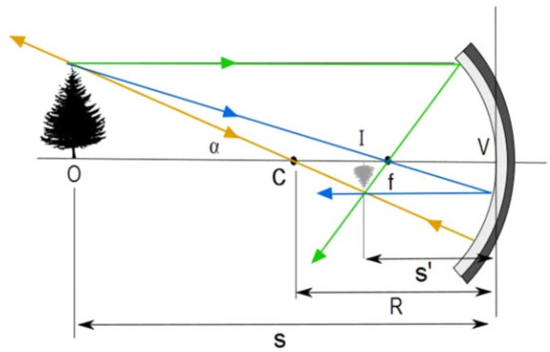
Notice that s' is negative for virtual images as always.

47.4.4 Ray Diagrams for Mirrors

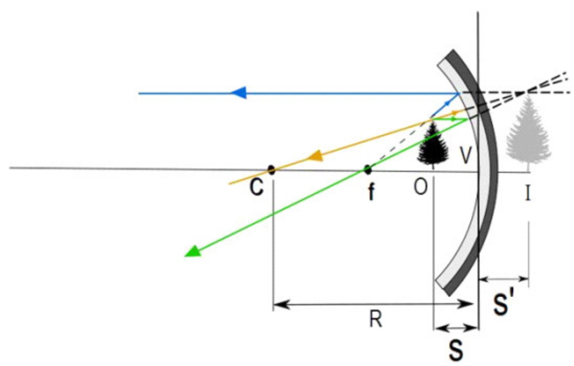
We have been drawing diagrams to find where images are formed for lenses, we should do the same for mirrors. We use a similar set of three rays. These rays are defined as follows:

Principal rays for a concave mirror:

1. Ray 1 is drawn from the top of the object such that its reflected ray must pass through f.
2. Ray 2 is drawn from the top of the object through the focal point to reflect parallel to the principal axis.
3. Ray 3 is drawn from the top of the object through the center of curvature. This ray will be incident on the mirror surface at a right angle and will be reflected back on itself.



We can do the same for an object closer than a focal length

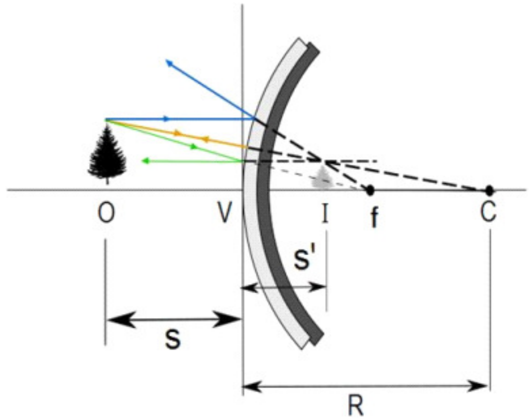


We also may have a mirror that curves, but curves the other way.
Principal rays for a convex mirror:

1. Ray 1 is drawn from the top of the object such that its reflected ray appears to have come from f.

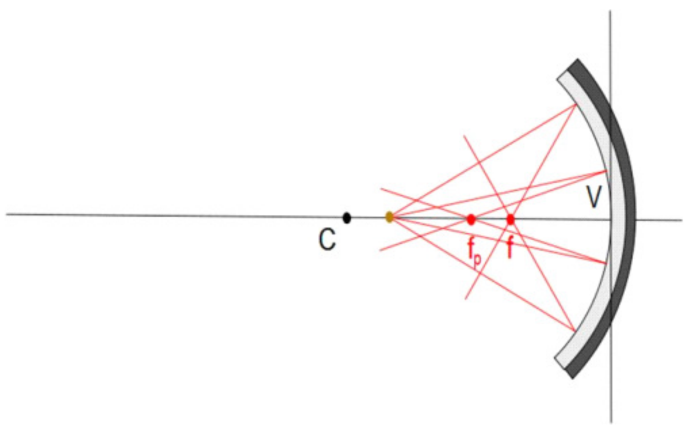
2. Ray 2 is drawn from the top of the object to reflect parallel to the principal axis.

3. Ray 3 is drawn from the top of the object so that it appears to have come from the center of curvature. This ray will be incident on the mirror surface at a right angle and will be reflected back on itself.



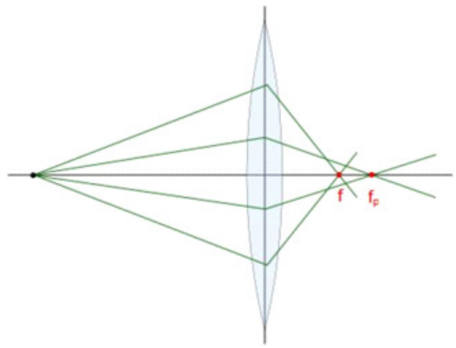
47.4.5 Spherical Aberration

Spherical shapes are easier to make than parabolas or hyperbole, or other shapes. So optics manufacturers have been using spherical optics for centuries.



If we let rays converge from any direction from our spherical mirror we find we have a problem. The rays do not form a single image. Instead, they converge on a volume near where the image should be. Rays from larger angles converge at different distances than rays from small angles. This problem is known as *spherical aberration*. Most of the time, we will point our optics so the object is near the principal axis, so we can make the paraxial approximation that fixes this problem.

The same problem happens with lenses



This problem is called spherical aberration and it was made famous as the main problem with the Hubble Telescope.

There are many aberrations that come from making lenses that are easy to manufacture, but that are not the perfect shape. We won't study these in this class. If you are curious, we cover these in PH375.

Just a note, we have run into another aberration, chromatic aberration, before. Mirrors in optical systems don't experience chromatic aberration. this is because mirrors in optical systems don't include a glass layer in front of the reflective surface like mirrors in your bathroom do. That glass is to protect the reflective surface from damage due to water (or toothpaste, etc.). In an optical system, this glass layer would cause unwanted reflection, refraction, and absorption of the light, so it is not included. So mirrors don't have any refraction associated with them. This means that there will be no dispersion from a mirror.

Basic Equations

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{R}$$

Thin Lenses

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Quantity	**Positive if**	**Negative if**
Object location (s)	Object is in front of surface	Object is in back of surface (virtual object)
Image location (s')	Image is in back of surface (real image)	Image is in front of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R_1 and R_2)	Center of curvature is in back of surface	Center of curvature is in front of surface
Focal length (f)	Converging lens	Diverging lens

Thin Mirrors

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$f = \frac{R}{2} \tag{47.11}$$

Quantity	**Positive if**	**Negative if**
Object location (s)	Object is in front of surface	Object is in back of surface (virtual object)
Image location (s')	Image is in front of surface (real image)	Image is in back of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R_1 and R_2)	Center of curvature is in front of surface	Center of curvature is in back of surface
Focal length (f)	Concave mirror	Convex mirror