Chapter 40

Ampere's law, and Forces on Charges

Fundamental Concepts

- The magnetic field can be found more simply for symmetric currents using Ampere's law $\oint \overrightarrow{B} \cdot d\overrightarrow{s} = \mu_o I_{through}$
- The force due to the magnetic field on a charge, q,is given by $\overrightarrow{\mathbf{F}} = q\overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{B}}$

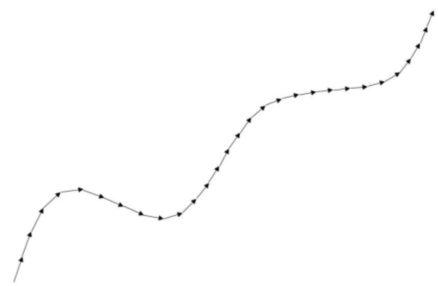
40.1 Ampere's Law

The Biot-Savart law is a powerful technique for finding a magnetic field, but it is more powerful numerically than in closed-form problems. We can only find exact solutions to a few problems with special symmetry. Since problems we can do by hand require special symmetry anyway, we would like to use symmetry as much as possible to remove the need for difficult integration.

We saw this situation before with electrostatics. We did some integration to find fields from charge distributions, but then we learned Gauss' law, and that was easier because it turned hard integration problems into relatively easy ones. This still required special symmetry, but when it worked, it was a fantastic time saver. For non-symmetric problems, there is always the integration method, and a computer.

Likewise, for magnetostatics (constant currents, so constant magnetic fields) there is an easier method. To see how it works, let's review some math.

In the figure there is a line, divided up into many little segments.



We can find the length of the line by adding up all the little segment lengths

$$L = \sum_i \Delta s_i$$

Integration would make this task less tedious

$$L = \int ds$$

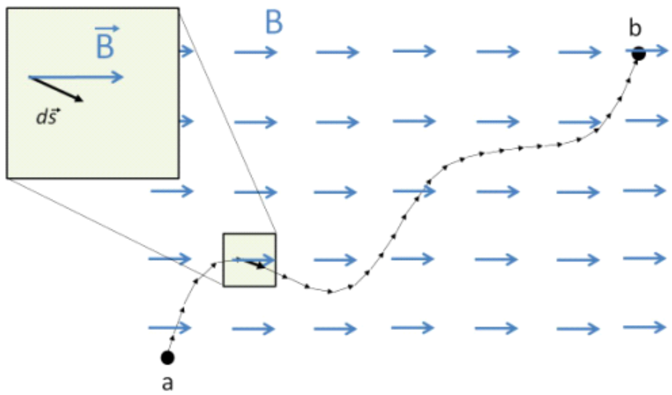
This is called a line integral. Our new method of finding magnetic fields will involve line integrals. The calculation of the length is too simple, however. We will have to integrate some quantity along the line. For example, we could envision integrating the amount of energy lost when pushing a box along a path. The integral would give the total energy loss. The amount of energy lost would depend on the specific path. Thus a line integral

$$w = \int \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{s}}$$

would be useful to find the total amount of work. Each small line segment would give a differential amount of work

$$dw = \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{s}}$$

and we use the integral to add up the contribution to the work for each segment of size ds along the path. Notice the dot product. We need the dot product because only the component of the force in the direction the box is going adds to the total work done.



We wish to do a similar thing for our magnetic field. In order to avoid the kind of integral we got for the Biot-Savart law, we wish to integrate the magnetic field along a path. It's not obvious that this will help. But like with Gauss' law we are going to try to turn a difficult integral into an easy one.

The integral would look like this

$$\int_a^b \overrightarrow{B} \cdot d\overrightarrow{s}$$

Our goal will be to use symmetry to make this integral very easy. The key is in the dot product. We want only the component of the magnetic field that is in the $d\overrightarrow{s_i}$ direction. There are two special cases.

If the field is perpendicular to the $d\overrightarrow{s_i}$ direction, then

$$\int_a^b \overrightarrow{B} \cdot d\overrightarrow{s} = 0$$

because $\overrightarrow{B} \cdot d\overrightarrow{s_i} = 0$ for this case

If the field is in the same direction as $d\overrightarrow{s_i}$, then $\overrightarrow{B} \cdot d\overrightarrow{s_i} = Bds$ and

$$\int_a^b \overrightarrow{B} \cdot d\overrightarrow{s} = \int_a^b Bds$$

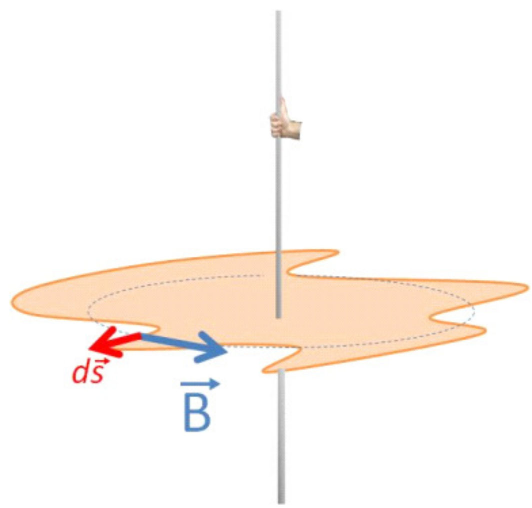
Further if we can make is so that B is constant and everywhere tangent to the path, then

$$\begin{aligned} \int_a^b \overrightarrow{B} \cdot d\overrightarrow{s} &= \int_a^b Bds \\ &= B\int_a^b ds \\ &= BL \end{aligned}$$

This process should look familiar. We used similar arguments to make the integral $\int \overrightarrow{E} \cdot d\overrightarrow{A}$ easy for Gauss' law.

With Gaussian surfaces, we found we could imagine any surface we wanted. In a similar way, for our line integral we can pick any path we want. if we can make B constant and everywhere tangent to the path, then, the integral will be easy. It is important to realize that we get to make up our path. There may be some physical thing along the path, but there is no need for there to be. The paths we will use are imaginary.

Usually we will want our path to be around a closed loop. Let's take the case of a long straight current-carrying wire. We know the field shape for this. We can see that if we take a crazy path around the wire, that $\overrightarrow{B} \cdot \Delta\overrightarrow{s_i}$ will give us the projection of $\overrightarrow{B}$ onto the $\Delta\overrightarrow{s_i}$ direction for each part of the path.



We get

$$\sum_i B_{\|}\Delta s$$

where $B_{\|}$ is the component of B that is parallel to the Δs direction. In integral form this is

$$\int B_{\|}ds$$

The strange shape I drew is not very convenient. This is neither the case where $\overrightarrow{B} \cdot d\overrightarrow{s_i} = 0$ nor where $\overrightarrow{B} \cdot d\overrightarrow{s_i} = Bds$. But if we think for a moment, I do know a shape where $\overrightarrow{B} \cdot d\overrightarrow{s_i} = Bds$. If we choose a circle, then from symmetry B will be constant, and it will be in the same direction as ds so $\overrightarrow{B} \cdot d\overrightarrow{s_i} = Bds$. From our last lecture we even know what the field should be for a long straight wire.

$$B = \frac{\mu_o I}{2\pi r}$$

Let's see if we can use this to form a new general approach. Since B is constant around the loop (because r is constant around the loop), we can write our line integral as

$$\begin{aligned}\int \overrightarrow{B} \cdot d\overrightarrow{s} &= B2\pi r \\ &= \frac{\mu_o I}{2\pi r} 2\pi r \\ &= \mu_o I\end{aligned}$$

This is an amazingly simple result. We integrated the magnetic field around an imaginary loop path, and got that the result is proportional to the current in the wire. This reminds us of Gauss' law where we integrated the electric field around a surface and got that the result is proportional to the amount of charge inside the surface.

$$\int \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{Q_{in}}{\epsilon_o}$$

Let's review. Why did I pick a circle as my imaginary path? Because it made my math easy! I don't want to do hard math to compute the field, so I tried to find a path over which the math was as easy as possible. Since the path is imaginary, I can choose any path I want, so I chose a simple one. I want a path where $\overrightarrow{B} \cdot d\overrightarrow{s_i} = 0$ or where $\overrightarrow{B} \cdot d\overrightarrow{s_i} = Bds$. This is very like picking Gaussian surfaces for Gauss' law. If I chose a harder path I would get the same answer, but it would take more effort. I found the result of my integral $\int \overrightarrow{B} \cdot d\overrightarrow{s}$ to be just $\mu_o I$.

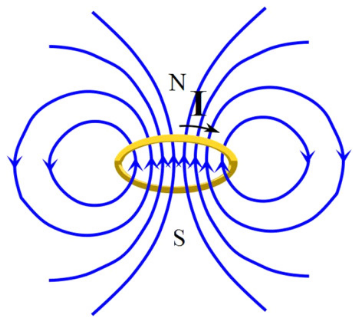
We had to integrate around a closed path, so I will change the integral sign to indicate that we integrated over a closed path.

$$\oint \overrightarrow{B} \cdot d\overrightarrow{s} = \mu_o I_{through} \tag{40.1}$$

and only the current that went through the imaginary surface contributed to the field, so we can mark the current as being the current that goes through our imaginary closed path.

This process was first discovered by Ampere, so it is known as Ampere's law.

Let's use Ampere's law to do another problem. Suppose I have a coil of wire. This coil is effectively a stack of current rings. We know the field from a single ring.



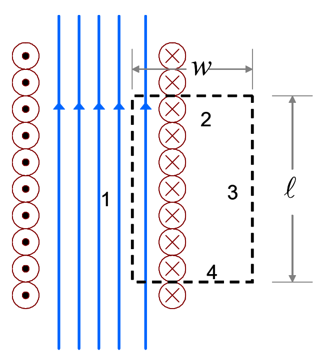
$$B = \frac{\mu_o I}{2R}$$

But what would the field be that is generated by having a current flow through the coil?

Well, looking at the single loop picture, we see that the direction of the field due to a loop is right through the middle of the loop. I think it is reasonable to believe that if I place another loop on top of the one pictured, that the fields would add, making a stronger field down the middle. This is just what happens. So I could write our loop field equation as

$$B = N\frac{\mu_o I}{2r}$$

where N is the number of loops I make. It is customary in electronics to define n as the number of loops per unit length (sort of like the linear mass density we defined in waves on strings, only now it is linear loop density). Suppose I take a lot of loops! In the picture I have drawn the loops like a cross section of a spring. But now the loops are not all at the same location. So we would guess that our field will be different than just N times the field due to one loop. Can we use Ampere's law to find this field?



Consider current is coming out at us on the LHS and is going back into the wires on the RHS. Remember our goal is to use Ampere's law

$$\oint \overrightarrow{B} \cdot d\overrightarrow{s} = \mu_o I_{through}$$

to find the field. Let's imagine a rectangular shaped *Amperian* loop shown as a dotted black line. Note that like Gaussian surfaces, this is an imaginary loop. Nothing is really there along the loop. Let's look at the integral by breaking it into four pieces,

$$\int_1 \overrightarrow{B} \cdot d\overrightarrow{s} + \int_2 \overrightarrow{B} \cdot d\overrightarrow{s} + \int_3 \overrightarrow{B} \cdot d\overrightarrow{s} + \int_4 \overrightarrow{B} \cdot d\overrightarrow{s} = \mu_o I_{through}$$

one for each side of the loop. If I have chosen my loop carefully, then $\overrightarrow{B} \cdot d\overrightarrow{s_i}$ will either be $\overrightarrow{B} \cdot d\overrightarrow{s_i} = 0$ or $\overrightarrow{B} \cdot d\overrightarrow{s_i} = Bds$. Let's start with side 2. We want to consider

$$\overrightarrow{B} \cdot d\overrightarrow{s_2}$$

We see that for our side 2 the field is perpendicular to $d\overrightarrow{s_2}$ So

$$\mathbf{B} \cdot d\ell_2 = 0$$

This is great! I can integrate 0

$$\int 0 = 0$$

The same reasoning applies to

$$\overrightarrow{B} \cdot d\overrightarrow{s_4} = 0$$

From our picture we can see that there is very little field outside of our coil of loops. So B_3 is very small, so $\overrightarrow{B} \cdot d\overrightarrow{s_3} \approx 0$. It is not exactly zero, but it is small enough that I will call it negligible for this problem. For an infinite coil, this would be exactly true (but infinite coils are hard to build).

That leaves path 1. There the B-field is in the same direction as $d\overrightarrow{s_1}$ so

$$\overrightarrow{B} \cdot d\overrightarrow{s_1} = Bds_1$$

Again this is great! B is fairly uniform along the coil. Let's say it is close enough to be considered constant. Then the integral is easy over side 1

$$\int Bds_1 = B\ell$$

We have performed the integral!

$$\begin{aligned}
\oint \overrightarrow{B} \cdot d\overrightarrow{s} &= \int_1 \overrightarrow{B} \cdot d\overrightarrow{s} + \int_2 \overrightarrow{B} \cdot d\overrightarrow{s} + \int_3 \overrightarrow{B} \cdot d\overrightarrow{s} + \int_4 \overrightarrow{B} \cdot d\overrightarrow{s} \\
&= B\ell + 0 + 0 + 0 \\
&= B\ell
\end{aligned}$$

Now we need to find the current in the loop. This is more tricky than it might appear. It is not just I because we have several loops that go through our loop, each on it's own carrying current I and each contributing to the field. We can use a linear loop density[1] n to find the number of loops.

$$N = n\ell$$

and the current inside the loop will be

$$I_{inside} = NI$$

Then, putting the integration all together, we have

$$\oint \overrightarrow{B} \cdot d\overrightarrow{s} = B\ell + 0 + 0 + 0 = \mu_o NI$$

or

$$B\ell = \mu_o NI$$

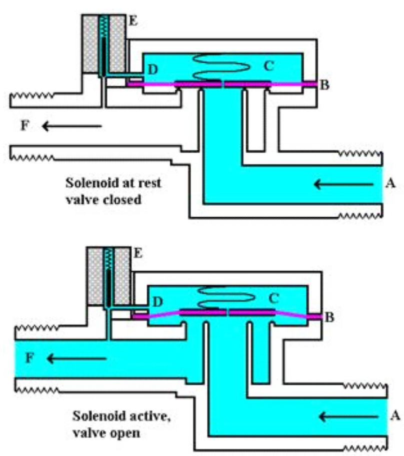
which gives a field of

$$B = \mu_o \frac{N}{\ell} I$$

or

$$B = \mu_o nI$$

This device is so useful it has a name. It is called a *solenoid*. You may have made a coil as a kid and turned it into an electromagnet by hooking it to a battery (a source of potential difference) so that a current ran through it. In engineering solenoids are used as current controlled magnetic switches.

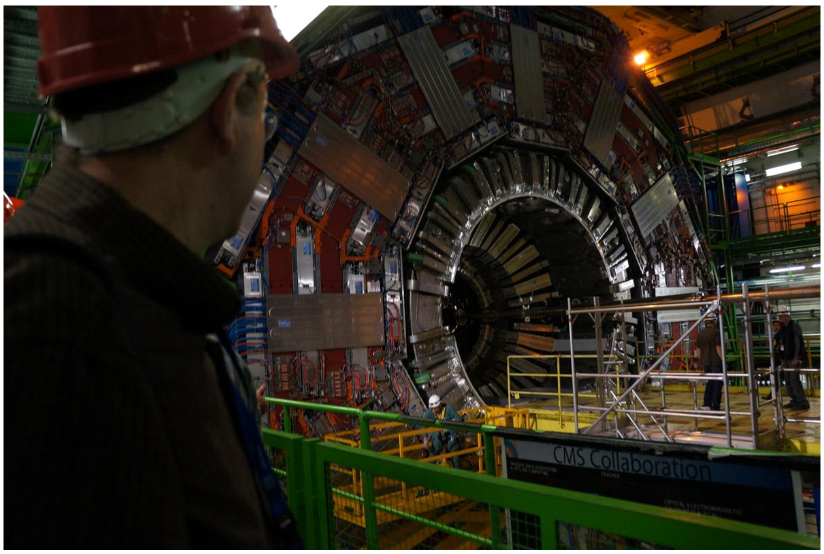


Solenoid operated valve system. (figure courtesy Alfonzo Gonzalez, wilipedia commons)

There is another great thing about a solenoid. In the middle of the solenoid, the field is really nearly constant. Near the ends, there are edge effects, but in

[1] Physicists like densities!

the middle we have a very uniform field. This is analogous to the nearly uniform electric field inside a capacitor. We can therefore see how to generate uniform magnetic fields and consider uniform B-fields in problems. Such a large nearly uniform magnetic field is part of the Compact Muon Solenoid (CMS) experiment at CERN.



CMS Detector at CERN. The detector is constructed of a very large solenoid to bend the path of the charge particles.

40.2 Magnetic Force on a moving charge

Now that we know how to generate a magnetic field, we can return to thinking about magnetic forces on mover charges. Our magnetic field is slightly more complicated than the electric field. We can still use a charge and the force, but now the charge is moving so we expect to have to include the velocity of the charge. We want an expression that relates B and F_{mag} in both magnitude and direction.

Our expression for the relationship between charge, velocity, field and the force comes from experiment (although now we can derive it). The experiments show that when a charged particle moves parallel to the magnetic field, there is no force! This is radically different from our E-field! Worse yet, the force seems to be perpendicular to both v and B when the angle between them is not zero! Here is our expression.

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \tag{40.2}$$

where q is the mover charge and B is the magnetic field environment.

We have a device that can shoot out electrons. The electrons show up because they hit a phosphorescent screen. When we bring a magnet close to our beam of electrons, we find it moves!

But we did this with moving electrons, what happens if they are not moving? We might expect the electrons to accelerate just the same–and we would be wrong! Static charges seem to not notice the presence of the magnet at all!

We expect that, like gravity and electric charge, the force on the moving electrons must be due to a field, but this *magnetic field* does not accelerate stationary electrons. We learned before that the reason we know that there is some force on the electrons came when Oersted, a Dutch scientist experimenting with electric current, found that his compass acted strangely when it was near a wire carrying electric current. This discovery is backwards of our experiment. It implies that moving charges must effect magnets, but given Newton's third law, If moving electrons make a field that makes a force on a magnet, then we would expect a magnet will make a field that makes a force on moving charges as well!

The derivation of the magnitude of the force from the experimental data is tedious. We will just learn the results, but they are exciting enough! The magnitude of the force on a moving charge due to a constant magnetic field is

$$\overrightarrow{\mathbf{F}}_B = q\overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{B}} \tag{40.3}$$

The magnitude is given by

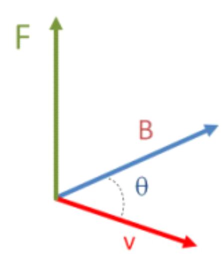
$$F = qvB\sin\theta$$

where q is charge, v is speed, and B is the magnitude of the magnetic field. We need to carefully define θ. Since we have a cross product, θ is the angle between the field direction and the velocity direction.

We can solve the equation for the magnetic field force (equation 40.3) to find the magnitude of the field

$$\frac{F}{qv\sin\theta} = B$$

But the strangeness has not ended. we need a direction of the force. And it turns out that it is perpendicular to both $\overrightarrow{\mathbf{v}}$ and $\overrightarrow{\mathbf{B}}$ as the cross product implies! We use our favorite right hand rule to help us remember.



We start with our hand pointing in the direction of $\overrightarrow{\mathbf{v}}$. Curl your fingers in the direction of $\overrightarrow{\mathbf{B}}$. And your fingers will point in the direction of the force. We saw this type of right hand rule before with torque, but there is one big difference. This really is the direction the charge will accelerate! Note that this works for a positive charge. If the charge is negative, then the q in

$$\overrightarrow{\mathbf{F}}_B = q\overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{B}}$$

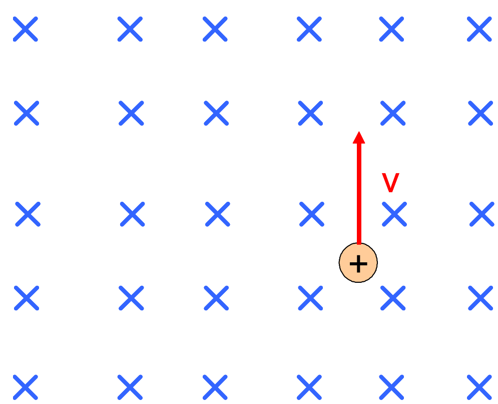
will be negative, and so the force will go in the other way. To keep this straight in my own mind, I still use our right hand rule, and just remember that if F is negative, it goes the opposite way of my thumb.

> Right hand rule #2: We start with our hand pointing in the direction of $\overrightarrow{\mathbf{v}}$. Curl your fingers in the direction of $\overrightarrow{\mathbf{B}}$ And your thumb will point in the direction of the force. The magnitude of the force is given by
>
> $$F = qvB\sin\theta$$

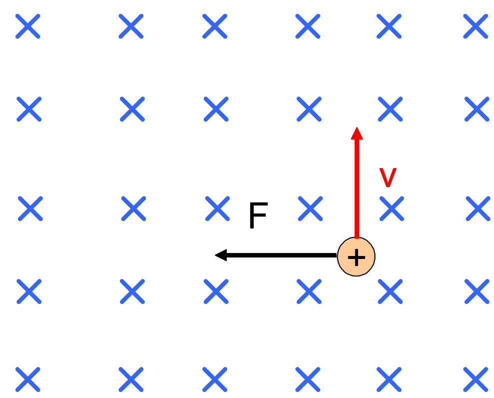
40.3 Motion of a charged particle in a B-Field

We refer to the magnetic field as a B-field for short.

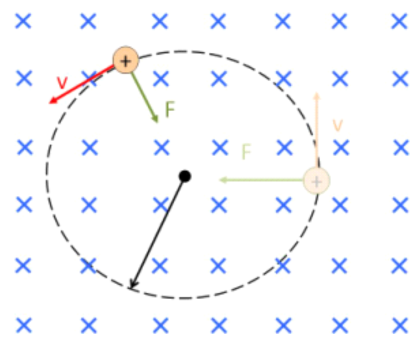
Let's set up a constant B-field as shown in the figure. We draw a B-field as a set of vectors just like we did for electric fields. In the figure, the vectors are all pointing "into the paper" so all we can see is their tails.



If I have a charged particle, with velocity $\tilde{\mathbf{v}}$, what will be the motion of the particle in the field? First off, we should recall that $\tilde{\mathbf{F}}$ is in a direction perpendicular to $\tilde{\mathbf{v}}$ and $\overrightarrow{\mathbf{B}}$.. Using our right hand rule we see that it will go to the left.



Remember that $F = ma$, so the charge will accelerate in the $-x$ direction.



Now, if we allow the charged particle to move, we see that the v direction changes. This makes the direction of F change. Since v and a are always at 90°, the motion reminds us of circular motion! Let's see if we can find the radius of the circular path of the charge.

$$F = qvB\sin\theta$$

will be just

$$F = qvB$$

because θ is always 90°. Then, using Newton's second law

$$F = ma = qvB$$

and noting that the acceleration is center-seeking, and our velocity is always tangential, we can write it as a centripetal acceleration

$$a_C = \frac{v_t^2}{r}$$

Then

$$m\frac{v_t^2}{r} = qv_tB$$

$$m\frac{v_t}{r} = qB$$

We can find the radius of the circle

$$\frac{mv_t}{qB} = r$$

Could we find the angular speed?

$$\omega = \frac{v_t}{r} = \frac{qB}{m}$$

How about the period? We can take the total distance divided by the total time for a revolution

$$v_t = \frac{2\pi r}{T}$$

to find

$$T = \frac{2\pi r}{v_t}$$

and we recognize

$$\frac{1}{\omega} = \frac{r}{v_t}$$

so

$$T = \frac{2\pi}{\omega}$$

so, using our angular speed we can say

$$T = \frac{2\pi m}{qB}$$

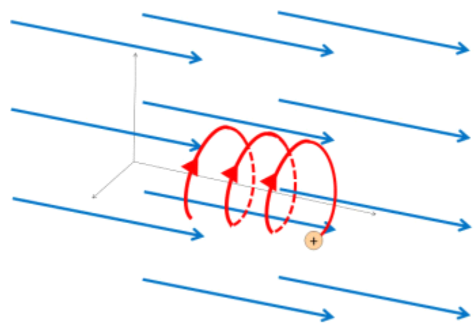
The angular frequency ω that we found is the frequency of a type of particle accelerator called a cyclotron. This type of accelerator is used by places like CERN to start the acceleration of charged particles. The same concept is used to make the charged particles go in a circular path in the large accelerators like the LHC at CERN.

Turning magnets at CERN. This is an actual magnet, but this magnet is at ground level in the testing facility. The tunnel is a mock-up of what the actual beam tunnel looks like.

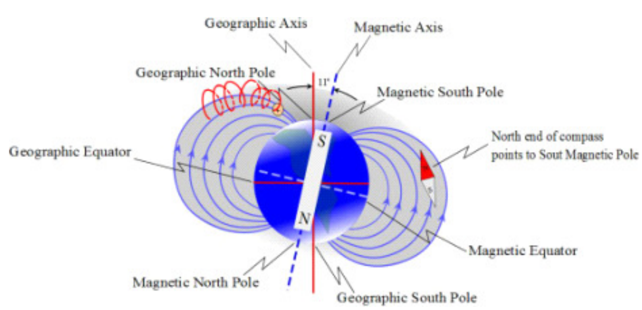
Within the detector systems, like the CMS, charged product particles can be

tracked along curved paths for identification.

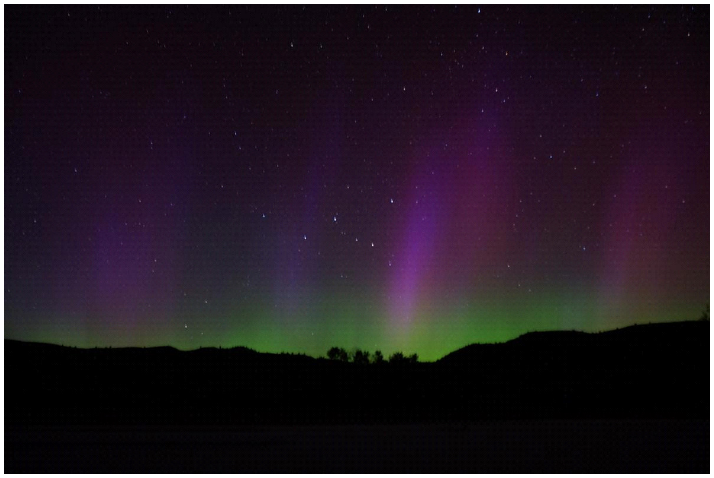
Charged particles that enter a magnetic field with some initial speed will gain a circular motion as well. The combined motion is a helix.



An example is the charged particles from the Sun entering the Earth's magnetic field. the particles will spiral around the magnetic field lines.



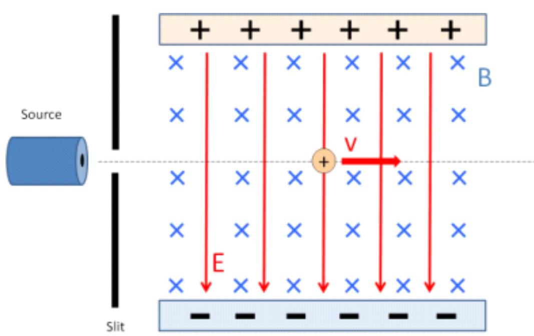
As the helical motion tightens near the poles, the particles will sometimes give off patterns of light as they hit atmospheric atoms.



Aurora Borealis: Sand Creek Ponds Idaho 2013

The light is what we call the aurora borealis. A more high-tech use for this helical motion is the confinement of charged particles in a magnetic field for fusion reaction.

40.3.1 The velocity selector



This device shows up on tests, especially finals, because it has both an electric field and a magnetic field–you test two sets of knowledge at once! But it is also the first stage of a mass spectrometer, which is a device used by chemists. So let's see how it works. Our question should be, what is the velocity of a charged particle that travels through the field without being deflected?

***E*-field**

We remember that the force on a positively charged particle will be

$$F_E = qE$$

directed in the field direction so it is downward.

B-Field

Now we know that

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

and we use our right hand rule to find that the direction will be upward with a magnitude of

$$\begin{aligned} F_B &= qvB\sin\theta \\ &= qvB \end{aligned}$$

So there will be no deflection (no acceleration) when the forces in the y-direction balance.

$$\Sigma F_y = 0 = -F_E + F_B$$

or

$$qE = qvB$$

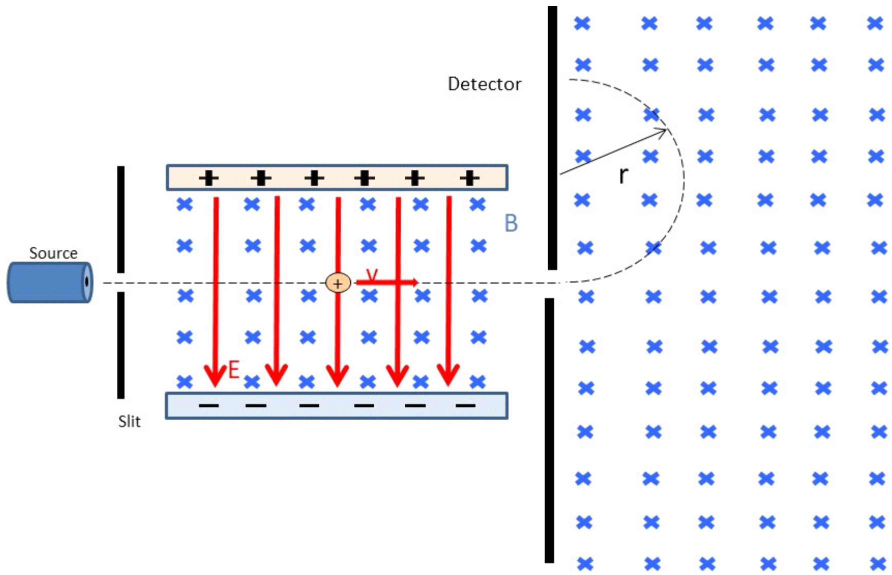
which gives

$$v = \frac{E}{B}$$

as the speed that will be "selected."

40.3.2 Bainbridge Mass Spectrometer

A mass spectrometer is the second most likely place to find a velocity selector (the first was a test). Chemists will likely use a mass-spec often in their careers, but even mechanical engineers may use one some time in your careers. I have had samples identified by mass-spectrometers several times in my industrial career. They are very useful devices–especially when chemical identification is hard or impossible.



The Bainbridge device is one type that we can easily understand. It starts with a velocity selector which sends charged particles at a particular speed into a region of uniform magnetic field. The charged particles then follow curved paths on their way to an array of detectors. When they hit the array, their spatial location is recorded. Where they hit depends on their ratio of charge to mass. From our study of the rotational motion we found

$$r = \frac{mv}{qB_o}$$

so the charge to mass ratio is

$$\frac{q}{m} = \frac{v}{rB_o}$$

Since we know the initial velocity will be

$$v = \frac{E}{B}$$

from the velocity selector, then

$$\frac{q}{m} = \frac{E}{rBB_o}$$

One way this is often used is to separate a sample of substance, say, carbon to find the relative amount of each isotope. The carbon atoms will all ionize to the same charge. Then the position at which they are detected depends on the mass.

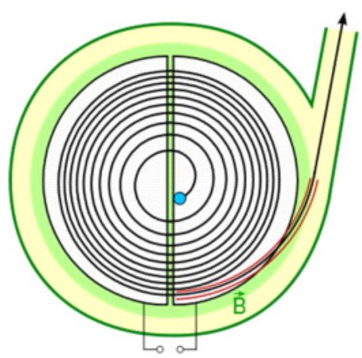
I used a mass-spec in my last industry project to identify large carbon compounds and their relative concentration in complex oil leaks. This data helped us look for possible leak detection targets so pipeline leaks could be detected before the oil was visible to the naked eye.

40.3.3 Classical Cyclotron

We already found the period of rotation of a charged particle in a uniform magnetic field.

$$T = \frac{2\pi m}{qB}$$

Note that this does not depend on the speed of the particle! So it will have the same travel time regardless of how fast it goes. We can use this to accelerate particles. But we add in an electric field to do the acceleration. The device is shown in the figure below



Basic Geometry of the Cyclotron. (Public Domain image courtesy KlausFoehl)

The particle starts in the center circling around in the magnetic field. but the device is divided into halves (called "Ds"). There is a gap between the Ds, and the electric field is created in the gap. One side at high potential and the other at low potential. When the particle is in the gap, it accelerates. It will gain a kinetic energy equal to the potential energy difference across the gap

$$\Delta K = q\Delta V$$

As the particle travels around the D to the other side of the cyclotron, the cyclotron switches the polarity of the potential difference. So as the particle passes the gap on the other side of the cyclotron, it is again accelerated with an additional $\Delta K = q\Delta V$. Since r does depend on the speed,

$$r = \frac{mv}{qB}$$

the radius increases with each "kick." Finally the particle leaves the cyclotron with a velocity of

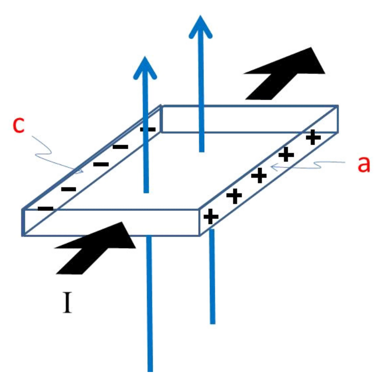
$$\frac{qBr_{\max}}{m} = v$$

Since we often describe the velocity of particles in energy terms, the kinetic energy of the particle

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m\left(\frac{qBr_{\max}}{m}\right)^2 \\ &= \frac{q^2B^2r_{\max}^2}{2m} \end{aligned}$$

40.4 Hall Effect

The Hall probe is a cool little device that measures the magnitude of the magnetic field. It is used in rotation and angle detection in engineering. We should find out how it works.



Let's take a piece of material that has a current going through it. If we place it in a magnetic field, then the charge carriers will feel a force. Suppose it is a metal, and that the charge carriers are electrons. The force is perpendicular to the current direction. So the electrons are accelerated toward the left of the piece of metal as shown in the drawing. This creates a negative charge on the left side of the metal piece. Then the right side will be positively charged relative to the left. With separated charge like this, we think of a capacitor and the electric field created by such a separation of charges. There will be a field in the conductor with a potential difference between the left and right of the conductor. We call this potential difference

$$\Delta V_H$$

the Hall potential after the man who first observed it.

Now if the charge carriers were positive, we would still build up a potential, but it would be in the opposite polarity. We wish to find this hall potential. The electric field of the charges will try to push them back down as more charge builds up. So at some point the upward force due to the magnetic field on the electrons will be balanced by the built up electric field. At that point

$$\Sigma F_y = 0 = F_B - F_E$$

so

$$qv_d B = qE_H$$

where E_H is the field due to the separation of charges.

So

$$E_H = v_d B$$

The potential is nearly equal to

$$\Delta V \approx E_H d$$

where d is the top-to-bottom distance of the conductor , so

$$\Delta V \approx v_d B d$$

Since we know

$$I = nqAv_d$$

then

$$v_d = \frac{I}{nqA}$$

and the area A is

$$A = td$$

where t is the thickness of the conductor, then

$$v_d = \frac{I}{nqtd}$$

and

$$\Delta V \approx \frac{IB}{nqt}$$

You may find this expressed in terms of the Hall coefficient

$$R_H = \frac{1}{nq}$$

so

$$\Delta V \approx R_H \frac{IB}{t}$$

To do a good job of finding R_H for metals and semiconductors, you have to go beyond classical theory. But if we know B, I, t, and ΔV, which can all be measured, then we can find R_H. Once this is done, we can place the Hall probe in different magnetic fields to find their strength. One way to do this is to control I and measure ΔV, so

$$B \approx \frac{t}{R_H I} \Delta V$$

Basic Equations

$$\oint \overrightarrow{B} \cdot d\overrightarrow{s} = \mu_o I_{through}$$

$$\overrightarrow{\mathbf{F}}_B = q\overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{B}}$$

$$\frac{mv_t}{qB} = r$$

$$\omega = \frac{qB}{m}$$

$$v_t = \frac{2\pi r}{T}$$

$$T = \frac{2\pi m}{qB}$$