Chapter 43

Apertures and Interferometers

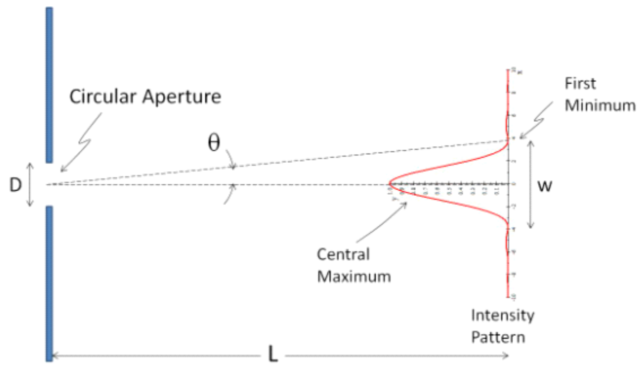
Our analysis of light going through holes has been somewhat limited by squarish holes or slits. But most optical systems, including our eyes don't have rectangular holes. So what happens when the hole is round? The situation is as shown in the next figure.

Fundamental Concepts

- Round apertures act very much like slits, with some added numerical factors
- If $\lambda \ll D$ we see little evidence for the wave nature of light. This limit is called the geometric optics limit or the ray approximation
- Interferometers can measure phenomenally small displacements using the wave nature of light

43.1 Circular Apertures

Before we discuss this situation, let's think about what we know about the width of a single slit diffraction pattern. *Diffraction pattern* is the name we give to an intensity pattern that results from constructive and destructive interference from wave mixing. So let's look again at our single slit diffraction pattern.



We remember that

$$\sin(\theta) = (1)\frac{\lambda}{a}$$

for the first minima, or, because the angles are small,

$$\theta \approx \frac{\lambda}{a}$$

and from the figure we can see that

$$\tan\theta = \frac{y}{L}$$

or

$$\theta \approx \frac{y}{L}$$

so long as θ is small, then we find the position of the first minimum to be

$$y = \frac{\lambda}{a}L$$

This is the distance from the center bright spot to the first dark spot. The width of the bright spot is twice this distance

$$w = 2\frac{\lambda}{a}L$$

We expect something like this for our circular opening. Let's change from the word "slit" to the word *"aperture"* to describe the hole the light goes through no matter the shape. Then we can call our circular hole a circular aperture.

The derivation for the diffraction patter for a circular aperture is not really to hard, but it involves Bessel functions, which are beyond the math requirement for this course. So I will give you the answer

$$\theta = 1.22\frac{\lambda}{D}$$

where D is the diameter of the circular aperture (like a was the width of the slit) and as before. It's still true that

$$\tan\theta = \frac{y}{L}$$

so

$$\theta \approx \frac{y}{L}$$

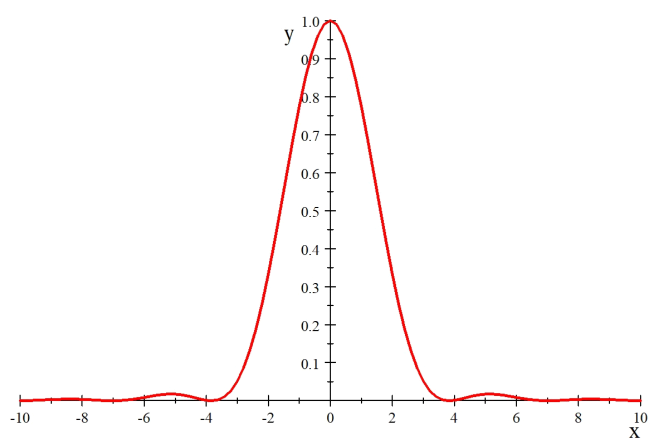
which gives us a first minimum location of

$$y = 1.22\frac{\lambda}{D}L \tag{43.1}$$

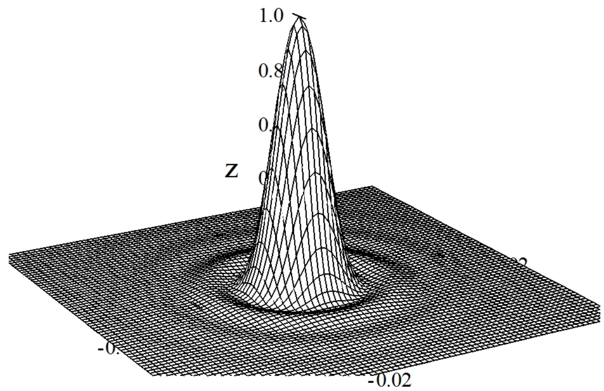
and a width of

$$w = 2.44\frac{\lambda}{D}L \tag{43.2}$$

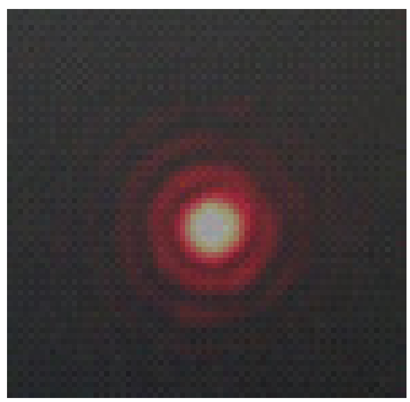
The picture of what this looks like in most books is a little bit deceptive. The pattern looks a little like the slit pattern. But the secondary maxima are actually very small for the circular aperture case. Much smaller than the secondary maxima in the slit case. Here is a larger version of a cross section of the intensity pattern.



Notice how small the secondary and tertiary maxima are. A three dimensional version of the intensity pattern from the circular aperture.



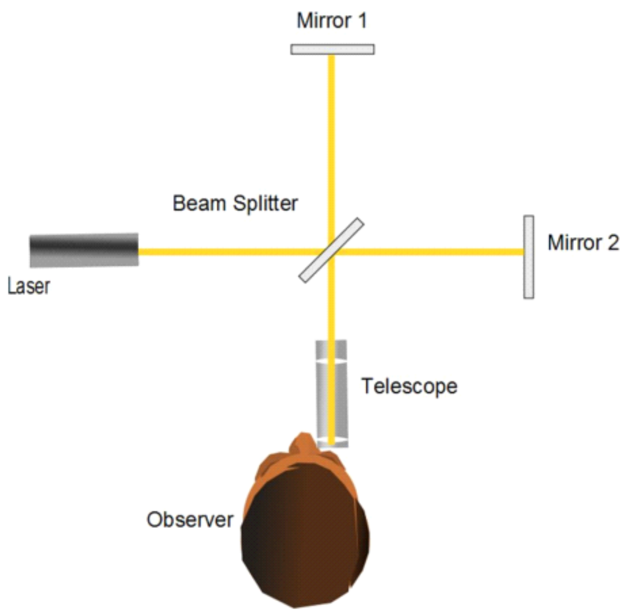
With a bright enough laser, they pattern becomes visible.



but mostly from a circular aperture we see the center bright circle of light. This will become important as we discus imagery and how to make sure your images have enough resolution to do the job you need them to do.

43.2 The Michelson Interferometer

Before we leave wave properties of physics and go to the ray approximation, we should study some devices that use interference. The Michelson interferometer is another device that uses path differences to create interference fringes.



The device is shown in the figure. A coherent light source is used. The light beam is split into two beams that are usually at 90° apart. The beams are reflected off of two mirrors back along the same path and are mixed at the telescope. The result (with perfect alignment) is a target fringe pattern like the first two shown below.

Bright Center Dark Center Small Off-Center Fringes

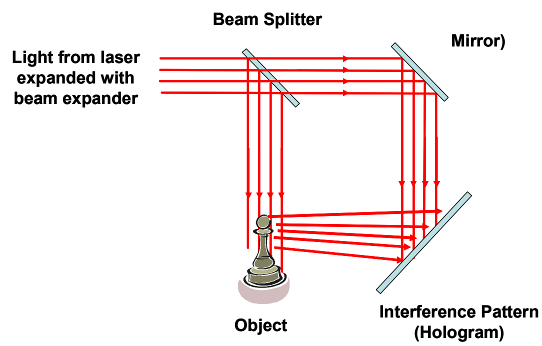
If the alignment is off, you get smaller fringes, but the system can still work. This is shown in the last image in the previous figure.

In the figure, we have constructive interference in the center, but if we move one of the mirrors half a wavelength, we would have destructive interference and would see a dark spot in the center. This device gives us the ability to measure distances on the order of the wavelength of the light. When the distance is continuously changed, the pattern seems to grow from the center (or collapse into the center).

Notice that if the mirror is moved $\frac{\lambda}{2}$, the path distance changes by λ because the light travels the distance to the mirror and then back from the mirror (it travels the path twice!).

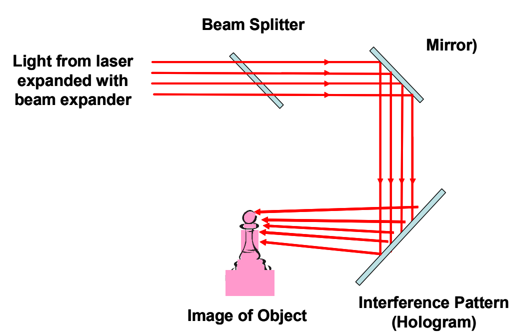
43.2.1 Holography

You may have seen holograms in the past. We have enough understanding of light to understand how they are generated now.



A device for generating a hologram is shown in the figure above. Light from a laser or other coherent source is expanded and split into two beams. One travels to a photographic plate, the other is directed to an object. At the object, light is scattered and the scattered light also reaches the photographic plate. The combination of the direct and scattered beams generates a complicated interference pattern.

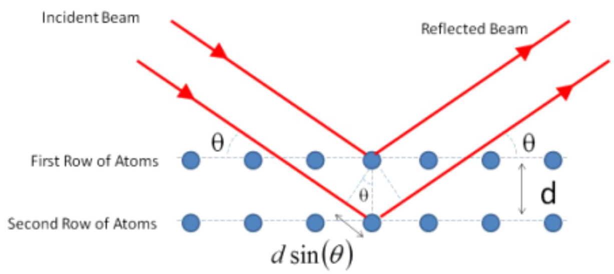
The pattern can be developed (like you develop photographic film). Once developed, it can be re- illuminated with a direct beam. The emulsion on the plate creates complicated patterns of light transmission, which combine to create interference. It is like a very complicated slit pattern or grating pattern. The result is a three-dimensional image generated by the interference. The interference pattern generates an image that looks like the original object.



43.3 Diffraction of X-rays by Crystals

If we make the wavelength of light very small, then we can deal with very small diffraction gratings. This concept is used to investigate the structure of crystals with x-rays. The crystal latus of molecules or atoms creates the regular pattern we need for a grating. The pattern is three dimensional, so the patterns are complex.

Let's start with a simple crystal with a square regular latus. $NaCl$ has such a structure.



If we illuminate the crystal with x-rays, the x-rays can reflect off the top layer of atoms, or off the second layer of atoms (or off any other layer, but for now let's just consider two layers). If the spacing between the layers is d, then the path difference will be

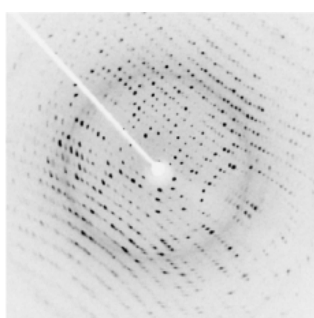
$$\delta = 2\left(d\sin\left(\theta\right)\right) \tag{43.3}$$

then for constructive interference

$$2d\sin\left(\theta\right) = m\lambda \qquad m = 1, 2, 3, \ldots \tag{43.4}$$

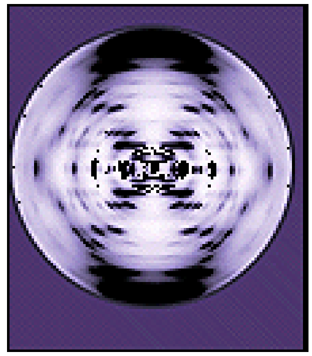
This is known as *Bragg's law.* This relationship can be used to measure the distance between the crystal planes.

A resulting pattern is given in the following figure.



Diffraction image of protein crystal. Hen egg lysozyme, X-ray souce Bruker IμS, $\lambda = 0.154188\,\mathrm{nm}$, $45\,\mathrm{kV}$, Exposure $10\,\mathrm{s}$.

DNA makes in interesting diffraction pattern.

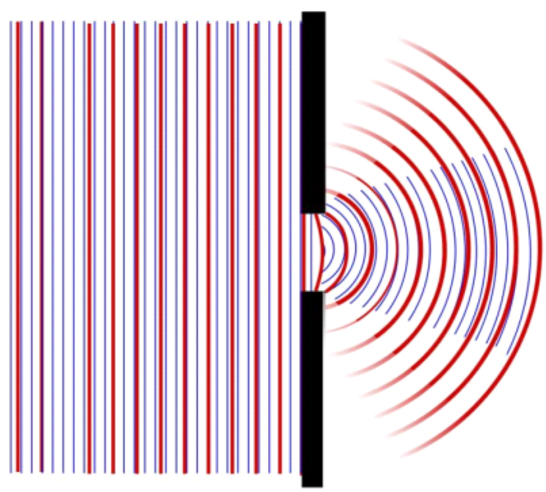


X-ray diffraction pattern of DNA

43.4 Transition to the ray model

But think. On a dark night in Rexburg (say 5:30pm in the winter) you might see two street lights. And where the light from the street lights overlap we don't see constructive and destructive interference. Why not?

In the next figure, two waves of different wavelets go through a single opening. The wave representing the central maximum is shown in each case, but not the secondary maxima.



Notice that the smaller wavelength has a narrower central maxima as we would expect from

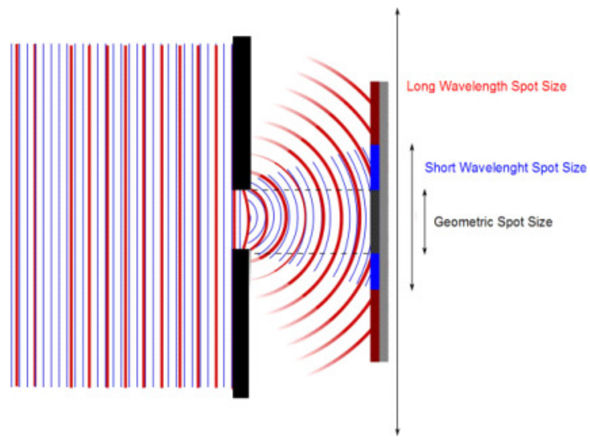
$$\sin(\theta) = 1.22\frac{\lambda}{D}$$

or

$$\theta \approx 1.22\frac{\lambda}{D}$$

we see that the ratio of the wavelength to the hole size determines the angular extent of the central maxima. The smaller the ratio, the smaller the central region. We can use this to explain why we don't see interference for the street lights and why the wave nature of light was so hard to find.

The patch of light on a screen that is created by light passing through the aperture is created by the central maximum.



For the long wavelength (red) the central maximum is larger than the screen. The short wavelength spot will be wholly on the screen as shown. The geometric spot is what we would see if the light traveled straight through the opening. Notice that the short wavelength spot is closer to the size of the geometric spot. In the limit that

$$\lambda \ll a$$

or for circular openings

$$\lambda \ll D$$

then

$$\theta \approx \frac{\lambda}{a} \approx 0$$

or

$$\theta \approx \frac{\lambda}{D} \approx 0$$

and the spot size would be very nearly equal to the geometric spot size.

This is the limit we will call the *ray approximation*.

For most of humankind's time on the Earth, it was very hard to build holes that were comparable to the size of a wavelength of visible light. So it is no wonder that the waviness of light was missed for so many years.

But this ray limit is very useful for apertures the size of camera lenses. So starting next lecture we will begin to use this small λ, large aperture approximation. But this is not the only reason we don't see constructive and destructive interference between street lights or the lights in our classroom. We will continue this discussion in our next lecture.

Basic Equations

Circular Apertures

$$y = 1.22\frac{\lambda}{D}L$$

$$w = 2.44\frac{\lambda}{D}L$$

X-ray diffraction

$$2d\sin(\theta) = m\lambda \qquad m = 1, 2, 3, \ldots \tag{43.5}$$