Chapter 37

Single Frequency Interference, Multiple Dimensions

So far we have had only waves mixed in a one-dimensional medium and we have only allowed for reflections to mix the waves. But surely we can make waves with different sources and mix them. Consider setting up two speakers playing the same frequency. We expect that we will still get regions of constructive and destructive interference. Where these regions will be really depends on the total phase difference between the two waves.

$$\begin{aligned}
\Delta\phi &= (kx_2 - \omega t + \phi_2) - (kx_1 - \omega t + \phi_1) \\
&= k(x_2 - x_1) + (\phi_2 - \phi_1) \\
&= \frac{2\pi}{\lambda}(\Delta x) + \Delta\phi_o
\end{aligned}$$

where we can see that there are at least two sources of phase difference here. One can be from the two waves traveling different paths and then combining (Δx) and the other is from them starting with a different phase to begin with $\Delta\phi$.

If we have two waves

$$\begin{aligned}
y_1 &= A\sin(kx_1 - \omega t + \phi_1) \\
y_2 &= A\sin(kx_2 - \omega t + \phi_2)
\end{aligned}$$

and we look at a particular part of the medium, that part will oscillate with an amplitude that depends on the relative starting points of the two waves, $\Delta\phi_o$ and on how the relative distances the waves have traveled to get to our particular location in the medium, Δx.

Fundamental Concepts

- In two dimensional problems, the total phase difference is given by $\Delta\phi = \left(2\pi\frac{\Delta r}{\lambda} + \Delta\phi_o\right)$

- In the total phase difference $\Delta\phi = \frac{2\pi}{\lambda}(\Delta x) + \Delta\phi_o$, the first term is due to path differences, the second to initial phase differences (whether the two mixed waves start together).

37.1 Mathematical treatment of single frequency interference

It is time to put our treatment of interference on a more general mathematical footing.

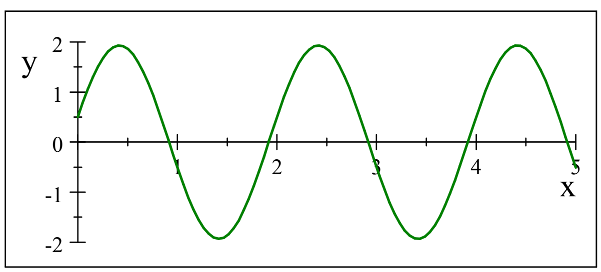
We start with two waves in the same medium

$$\begin{aligned} y_1 &= y_{\max}\sin(kx_1 - \omega t + \phi_1) \\ y_2 &= y_{\max}\sin(kx_2 - \omega t + \phi_2) \end{aligned}$$

Each wave has its own phase constant. Each wave starts from a different position (one at x_1 and the other at x_2), The superposition yields.

$$y_r = y_{\max}\sin(kx_1 - \omega t + \phi_1) + y_{\max}\sin(kx_2 - \omega t + \phi_2)$$

which is graphed in the next figure.



Notice that the wave form is taller (larger amplitude). Noticed it is shifted along the x axis. This graph is not surprising to us now, because we have done a case like this before. We can find the shift in general rewriting y_r. We need a trig identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

then let $a = kx_2 - \omega t + \phi_2$ and $b = kx_1 - \omega t + \phi_1$

$$\begin{aligned}
y_r &= y_{\max}\sin(kx_2 - \omega t + \phi_2) + y_{\max}\sin(kx_1 - \omega t + \phi_1) \\
&= 2y_{\max}\cos\left(\frac{(kx_2 - \omega t + \phi_2) - (kx_1 - \omega t + \phi_1)}{2}\right)\sin\left(\frac{(kx_2 - \omega t + \phi_2) + (kx_1 - \omega t + \phi_1)}{2}\right) \\
&= 2y_{\max}\cos\left(\frac{kx_2 - kx_1}{2} + \frac{\phi_2 - \phi_1}{2}\right)\sin\left(\frac{kx_2 + kx_1 - 2\omega t + \phi_2 + \phi_1}{2}\right) \\
&= 2y_{\max}\cos\left(k\frac{x_2 - x_1}{2} + \frac{\phi_2 - \phi_1}{2}\right)\sin\left(k\frac{x_2 + x_1}{2} - \omega t + \frac{\phi_2 + \phi_1}{2}\right) \\
&= 2y_{\max}\cos\left(k\frac{\Delta x}{2} + \frac{\Delta\phi_o}{2}\right)\sin\left(k\frac{x_2 + x_1}{2} - \omega t + \frac{\phi_2 + \phi_1}{2}\right) \\
&= 2y_{\max}\cos\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}\Delta x + \Delta\phi_o\right)\right)\sin\left(k\frac{x_2 + x_1}{2} - \omega t + \frac{\phi_2 + \phi_1}{2}\right) \\
&= 2y_{\max}\cos\left(\frac{1}{2}(\Delta\phi)\right)\sin\left(k\frac{x_2 + x_1}{2} - \omega t + \frac{\phi_2 + \phi_1}{2}\right)
\end{aligned}$$

where the last line is just a rearrangement to match the form we got last time we did this problem with just one phase constant. As usual, let's look at the sine part first. It is still a wave. We can see that more clearly if we define a variable

$$\mathsf{x} = \frac{x_2 + x_1}{2}$$

and another

$$\overline{\phi} = \frac{\phi_2 + \phi_1}{2}$$

Then the sine part is just

$$\sin\left(k\mathsf{x} - \omega t + \overline{\phi}\right)$$

and we can see it is just our basic wave equation form. The reset of the equation must be the amplitude and we can clearly see that the amplitude depends on what we call the phase difference,

$$\Delta\phi\frac{2\pi}{\lambda}(\Delta x) + \Delta\phi_o$$

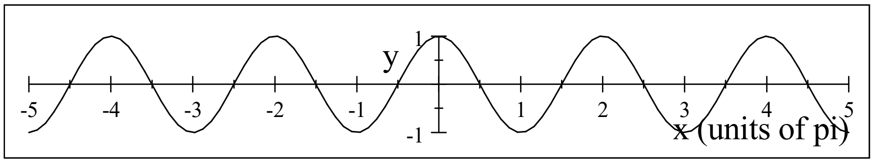
And from this we can see that there can be two sources two sources of phase difference. One can be from the two waves traveling different paths and then combining (Δx) and the other is from the two waves starting with a different phase to begin with, $\Delta\phi_o$. If the total phase difference between the two waves is a multiple of 2π, then the two waves will experience constructive interference

$$\Delta\phi = m2\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots$$

Let's see that this works. Our amplitude is

$$A = 2y_{\max}\cos\left(\frac{1}{2}(\Delta\phi)\right)$$

and if we look at a cosine function we see that $\cos(\theta)$ is either 1 or -1 at $\theta = n\pi$.



So if $\Delta\phi = m2\pi$ then the amplitude is

$$\begin{aligned} A &= 2y_{\max}\left(\frac{1}{2}(m2\pi)\right) \\ &= 2y_{\max}\cos(m\pi) \\ &= \pm 2y_{\max} \end{aligned}$$

We don't really care if the amplitude function is big positively or negatively. So we get constructive interference for $\cos(m\pi)$ being either 1 or -1. Then, this our case for constructive interference.

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta x + \Delta\phi_o\right) = n2\pi \qquad n = 0, \pm1, \pm2, \pm3, \cdots$$

How about for destructive interference? We start again with our amplitude function

$$A = 2y_{\max}\cos\left(\frac{1}{2}(\Delta\phi)\right)$$

but now we want when the cosine part needs to be zero.

$$\cos(\theta) = 0$$

Looking at our cosine graph again, that happens for cosine when $\theta = \frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$, $\cdots$. We could write this as $\theta = \left(m + \frac{1}{2}\right)\pi$ for $m = 0, 1, 2, \cdots$. But remember that in our amplitude function, we already have the $1/2$ in the function, so we want $\Delta\phi$ to have just the odd integer multiple of π. We could write this as

$$\Delta\phi = (2m+1)\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots$$

So our condition for destructive interference is

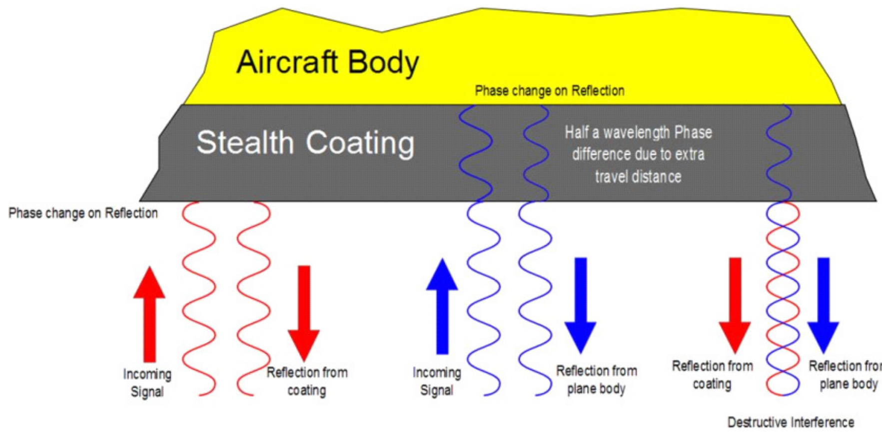
$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta x + \Delta\phi_o\right) = (2m+1)\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots$$

We have developed a useful matched set of equations that will tell us if we mix two waves when we will have constructive and destructive interference:

$$\begin{aligned} \Delta\phi &= \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = m2\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Constructive} \\ \Delta\phi &= \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = (2m+1)\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Destructive} \end{aligned}$$

Let's take an example to see how this can be used.

37.1.1 Example of two wave interference: Stealth Fighter



Suppose the stealth fighter is coated with an anti-reflective polymer.[1] This is part of it's mechanism for making the plane invisible to radar. Suppose we have a radar system with a wavelength of 3.00 cm. Further suppose that the index of refraction of the anti-reflective polymer is $n = 1.50$, and that the aircraft index of refraction is very large, how thick would you make the coating?

We want destructive interference, so let's start with our destructive interference condition

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = (2m+1)\,\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Destructive}$$

The radar waves all hit the plane in phase. From the figure, we see that the radar wave will reflect off of the coating. Because the index of refraction of the coating is large, this is like a fixed end of a rope. There will be an inversion.

But some of the wave will penetrate the polymer. This will reflect off of the plane body. The plane body has a very large index of refraction, so once again the wave will experience an inversion. The outgoing waves would then both be in phase as they leave and create constructive interference (if there were not a path difference) because

$$\Delta\phi_o = \pi - \pi = 0$$

at this point. Thus

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r\right) = (2m+1)\,\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Destructive}$$

[1] I don't really know that this is true. And if I did, I wouldn't be able to use this as an example! So we are reverse engineering the stealth fighter technology. But I think it is a good guess.

But $\Delta\phi_o$ is not the only term in our equation, we also have to remember the path difference, Δr. The part of the wave that entered the polymer travels farther. If that path difference, Δr, is just right we can get destructive interference. For the $m = 0$ case

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r\right) = (2\,(0)+1)\,\pi = \pi$$

then the amplitude function would be

$$\begin{aligned} A &= 2E_{\max}\cos\left(\frac{2\pi}{\lambda}\Delta r\right) \\ &= 2E_{\max}\cos\left(\frac{1}{2}\,(\pi)\right) \\ &= 0 \end{aligned}$$

and we have destructive interference. Note that these are electromagnetic waves, so instead of $y_{\max}$ we have used $E_{\max}$ as the individual wave amplitude. But the important thing is that the plane cannot be seen by the radar! Of course, this works for $m = 1$ and $m = 2$, etc. as well. Any odd multiple of π will work.

$$\frac{2\pi}{\lambda}\Delta r = (2m+1)\,\pi \qquad m = 0, 1, 2, \cdots$$

so that we are guaranteed an odd multiple of π. This is our condition for destructive interference.

But we are interested in the thickness. We realize that Δr is about twice the thickness, since the wave travels though the coating and back out. So let's let $\Delta r \approx 2t$

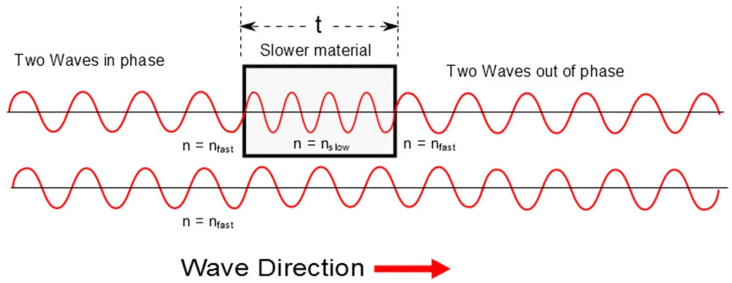
$$\begin{aligned} \frac{2\pi}{\lambda}2t &\approx (2m+1)\,\pi \\ 2t &\approx (2m+1)\,\frac{\lambda}{2} \\ 2t &\approx \left(m+\frac{1}{2}\right)\lambda \\ t &\approx \left(m+\frac{1}{2}\right)\frac{\lambda}{2} \end{aligned}$$

But there is a further complication. We should write our thickness equation as

$$t \approx \left(m+\frac{1}{2}\right)\frac{\lambda_{in}}{2}$$

because Δr has to provide an odd integer times the wavelength *inside the coating* for the phase to be right. After all, the wave is traveling inside the coating. We know that the wavelength will change as we enter the slower material.

To see this, consider two waves traveling to the right in the figure below. One passes through a slower medium. We expect the wavelength to shorten. We can see that, depending on the thickness t, the wave may be in phase or out of phase as it leaves. In the figure, the thickness is just right so that we have destructive interference.



We have such a wavelength shift in the coating. But we don't know the wavelength inside the coating. All we know is the radar wavelength, λ_{out}. We can fix it by writing the wavelength inside in terms of the wavelength outside. Earlier in our studies we found that the new wavelength will be given by equation (33.3)

$$\lambda_f = \frac{v_f}{v_i}\lambda_i$$

Let's rewrite this for our case

$$\lambda_{in} = \frac{v_{in}}{v_{out}}\lambda_{out}$$

We can express this in terms of the index of refraction

$$n = \frac{c}{v}$$

by multiplying the left hand side by c/c then

$$\lambda_{in} = \frac{cv_{in}}{cv_{out}}\lambda_{out}$$

or

$$\begin{aligned}\lambda_{in} &= \frac{\frac{c}{v_{out}}}{\frac{c}{v_{in}}}\lambda_{out}\\ &= \frac{n_{out}}{n_{in}}\lambda_{out}\end{aligned}$$

in the case of our aircraft coating the outside medium is air so $n_{out} \approx 1$

$$\lambda_{in} = \frac{1}{n_{in}}\lambda_{out}$$

This is this wavelength we need to match as the radar signal enters the medium.

Using this expression for λ_{in} in

$$t \approx \left(m + \frac{1}{2}\right)\frac{\lambda_{in}}{2} \qquad m = 0, 1, 2, \cdots$$

will give us the condition for destructive interference. Let's rewrite our λ_{in} equation for our case of a coating and air

$$\lambda_{in} = \lambda_{coating} = \frac{1}{n_{in}}\lambda_{out} = \frac{1}{n_{coating}}\lambda_{air}$$

thus

$$t \approx \left(m + \frac{1}{2}\right)\frac{1}{2}\left(\frac{\lambda_{air}}{n_{coating}}\right) \qquad m = 0, 1, 2, \cdots$$

is our condition for being stealthy.

Let's assume we want the thinnest coating possible, so we set $m = 0$. Then

$$t \approx \left(\frac{1}{4}\right)\left(\frac{\lambda_{air}}{n_{coating}}\right)$$

and our thickness would be

$$t \approx \left(\frac{1}{4}\right)\left(\frac{3.00\,\mathrm{cm}}{1.50}\right) = 0.5\,\mathrm{cm}$$

This seems doable for an aircraft coating!

Of course we could also make a plane that would be more visible to radar by choosing the constructive interference case. Suppose we are building a search and rescue plane. We want to enhance it's ability to be seen by radar in fog. We start with the condition for constructive interference

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = m2\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Constructive}$$

It will still be true that $\Delta\phi_o = 0$.

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r\right) = m2\pi$$

and it is still true that $\Delta r \approx 2t$.

$$\left(\frac{2\pi}{\lambda}2t\right) \approx m2\pi$$

then

$$\left(\frac{1}{\lambda}2t\right) \approx m$$

$$t \approx \frac{1}{2}m\lambda$$

and we still have to adjust for the coating index of refraction

$$t \approx \frac{m}{2}\left(\frac{\lambda_{air}}{n_{coating}}\right)$$

And once again we have several choices for m

$$t \approx \frac{m}{2}\left(\frac{\lambda_{air}}{n_{coating}}\right) \qquad m = 0, 1, 2, \cdots$$

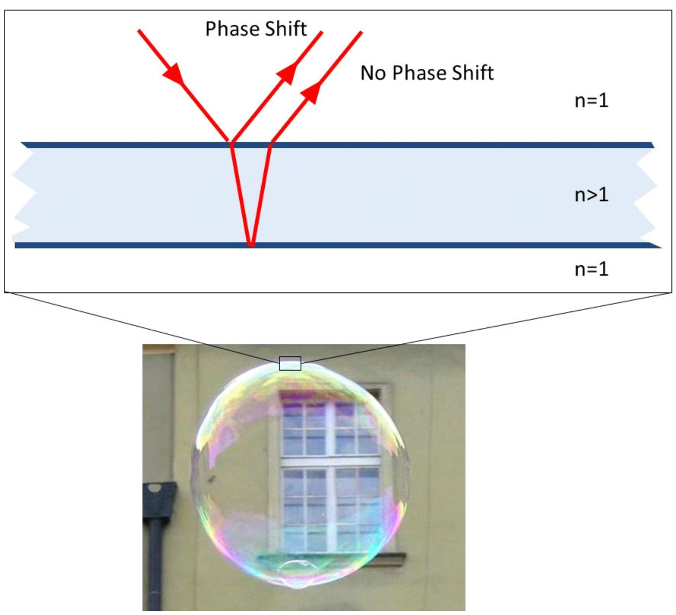
But now the coating will provide constructive interference, making it easier to track on radar from the command center. For the thinnest possibility, set $m = 1$ because the $m = 0$ case doesn't give us any thickness.

$$\begin{aligned} t &\approx m\frac{1}{2}\left(\frac{\lambda_{air}}{n_{coating}}\right) \\ &= \frac{1}{2}\left(\frac{3.00\,\mathrm{cm}}{1.50}\right) \\ &= 1\,\mathrm{cm} \end{aligned}$$

Note that we reasoned out these equations for the boundary conditions that we have in our problem (inversion on reflection from both the coating and the plane body). If the boundary conditions change, so do the equations.

37.1.2 Example of two wave interference: soap bubble

Take a soap bubble for example.



Interference from a soap bubble. (Bubble image in the Public Domain, courtesy Marcin Deręgowski)

Now we have a phase shift on the first reflection, but not one on the reflection from the inside surface of the bubble because the bubble is full of air. The index of refraction of air is less than that for the bubble material. So as we leave the bubble material it is more like having a free end of a rope. As the waves leave the surface, they are half a wavelength out of phase due to $\Delta\phi_o$ because of the single inversion from the bubble outer surface. We would have destructive interference due to just this, but we also have to account for the bubble thickness. If this thickness is a multiple of a wavelength, then we are still have half a wavelength out of phase and we have destructive interference.

Here are our basic equations

$$\begin{aligned}\Delta\phi &= \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = m2\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \text{ Constructive} \\ \Delta\phi &= \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = (2m+1)\,\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \text{ Destructive}\end{aligned}$$

Suppose we want want constructive interference to get our colors, so we take the first

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = m2\pi$$

and this time we have

$$\begin{aligned}\Delta\phi_o &= \phi_{transmitted} - \phi_{reflected} \\ &= 0 - \pi \\ &= -\pi\end{aligned}$$

It is still true that $\Delta r \approx 2t$ so from our constructive interference equation

$$\begin{aligned}\frac{2\pi}{\lambda}2t - \pi &= m2\pi \\ \frac{2}{\lambda}t - \frac{1}{2} &= m \\ \frac{2}{\lambda}t &= m + \frac{1}{2} \\ t &= \frac{\lambda}{2}\left(m + \frac{1}{2}\right)\end{aligned}$$

We again have the problem that this wavelength must be the wavelength inside the bubble material $\lambda = \lambda_{in}$. But we see the outside wavelength λ_{out}. We can reuse our conversion from outside to inside wavelength from our last problem because we are once again in air and $n_{air} \approx 1$.

$$\lambda_{in} = \frac{1}{n_{in}}\lambda_{out}$$

then

$$t = \frac{\lambda_{out}}{2n_{in}}\left(m + \frac{1}{2}\right) \qquad m = 0, 1, 2, \cdots$$

Or writing this with $n_{in} = n_{\text{bubble}}$ to make it clear that the inside material is the bubble solution,

$$t = \left(m + \frac{1}{2}\right)\frac{1}{2}\left(\frac{\lambda_{air}}{n_{\text{bubble}}}\right) \qquad m = 0, 1, 2, \cdots$$

but this was the equation for destructive interference for the plane! We can see that memorizing the thickness equations won't work. We need to start with our conditions on $\Delta\phi$ for constructive and destructive interference to be safe!

How about the dark parts of the bubble with no color (the parts we can see through). These would be destructive interference

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta x + \Delta\phi_o\right) = (2m+1)\,\pi$$

We can fill in the pieces to obtain

$$\begin{aligned}
\left(\frac{2\pi}{\lambda}2t - \pi\right) &= (2m+1)\,\pi \\
\frac{2}{\lambda}2t - 1 &= (2m+1) \\
\frac{2}{\lambda}2t &= (2m+1) + 1 \\
\frac{4}{\lambda}t &= (2m+1) + 1 \\
t &= \frac{\lambda}{4}(2m+2) \\
t &= \frac{\lambda}{2}(m+1) \\
t &= \frac{m+1}{2}\left(\frac{\lambda_{out}}{n_{\text{bubble}}}\right)
\end{aligned}$$

This is our condition for destructive interference for the bubble. We don't have to, but we could write $m + 1 = p$ where p is an integer that starts at 1 instead of zero.

$$t = \frac{p}{2}\left(\frac{\lambda_{out}}{n_{\text{bubble}}}\right) \qquad p = 1, 2, \cdots$$

But this is very like the condition for constructive interference for the plane.

Hopefully, it is apparent that we have to start with our basic equations

$$\begin{aligned}
\Delta\phi &= \left(\frac{2\pi}{\lambda}\Delta x + \Delta\phi_o\right) = m2\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \text{ Constructive} \\
\Delta\phi &= \left(\frac{2\pi}{\lambda}\Delta x + \Delta\phi_o\right) = (2m+1)\,\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \text{ Destructive}
\end{aligned}$$

each time we attempt an interference problem because the outcome depends on both Δx and $\Delta\phi_o$. We have to construct the equation each time for the interference condition we want (constructive or destructive) finding Δx and $\Delta\phi_o$ for the boundary conditions we have.

37.2 Single frequency interference in more than one dimension

Suppose I put two speakers facing each other 3 m apart. And suppose I want four nodes in the middle so we can easily find them. Then I will need

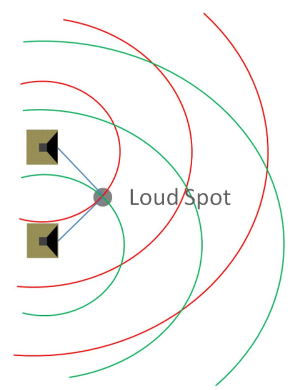
$$2\lambda = 3\,\mathrm{m}$$

or $\lambda = \frac{3}{2}\,\mathrm{m}$. If we are at about $20\,^{\circ}\mathrm{C}$ then $v = 343\,\mathrm{m/\,s}$ and

$$\begin{aligned} f &= \frac{343\,\mathrm{m/\,s}}{\frac{3}{2}\,\mathrm{m.}} \\ &= 228.67\,\mathrm{Hz} \end{aligned}$$

If $\Delta\phi_o = 0$ the nodes should be spaced symmetrically between the two speakers. The rest of the phase comes from the difference in starting positions.

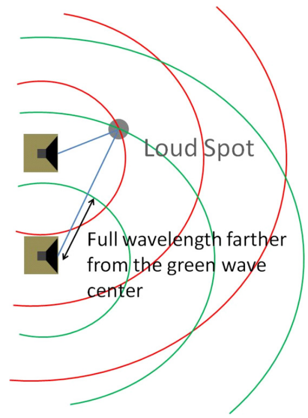
But what happens if our waves don't travel along the same line? Suppose you are at a dance, and there are two speakers. Further suppose that you are testing the system with a constant tone (either that, or you have somewhat boring music with constant tones). Suppose the two speakers make waves in phase. If you are equal distance from the two speakers, you would expect constructive interference because both $\Delta\phi_o = 0$ and $\Delta x = 0$ for this case.



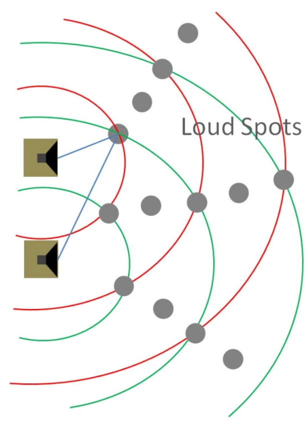
But there are more places where we expect constructive interference, because we know the sound wave is really spherical. Any time the path difference, $\Delta x = n\lambda$, then

$$\Delta\phi = \frac{2\pi}{\lambda}\,(n\lambda) = n2\pi$$

and we will have constructive interference The next figure shows an example where the path difference is one wavelength.



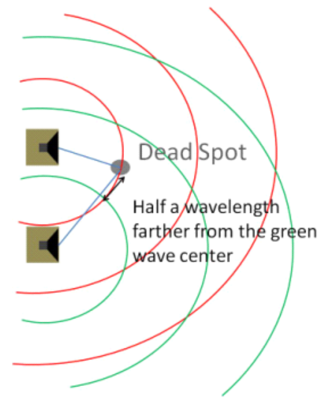
But any of these spots will experience constructive interference. Note the loud spots are where there are two crests or two troughs together.



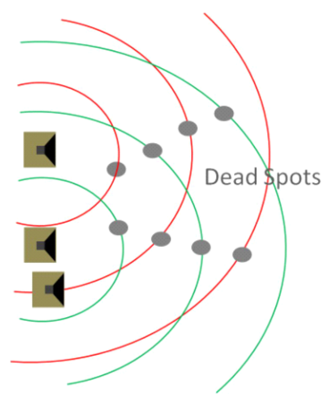
We also expect to see destructive interference. This should occur where path differences are multiples of $\Delta x = \lambda/2$ so that

$$\Delta\phi = \frac{2\pi}{\lambda}\left(n\frac{\lambda}{2}\right) = n\pi$$

The situation of being just half a wavelength off is shown next



but there are many places were we could be a multiple of a wavelength plus and extra half a wavelength off. Each of these will produce destructive interference.



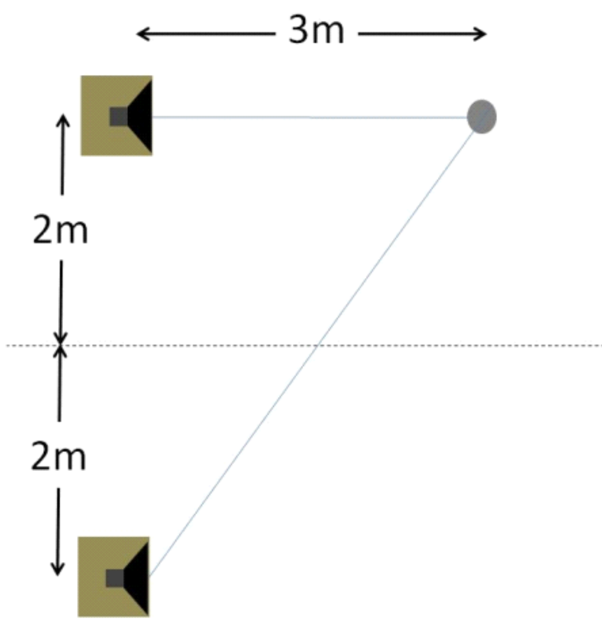
Recall that when you moved from one dimension to two dimensions in PH 121 or Dynamics problems, you changed from the variables x and y to the variable r where

$$r = \sqrt{x^2 + y^2}$$

Thus our phase becomes

$$\Delta\phi = \left(2\pi\frac{\Delta r}{\lambda} + \Delta\phi_o\right)$$

In our dance example, suppose we have speakers that are 4 m apart and we are standing 3 m directly in front of one of the speakers. Further suppose that we play an A just above middle C which has a frequency of 440 Hz. The speed of sound is 343 m/ s. Our speakers are connected to the same stereo with equal length wires. What is the phase difference at this spot?



From the geometry we can tell that the path from the second speaker must be 5 m. So

$$\begin{aligned}\Delta r &= 5\,\mathrm{m} - 3\,\mathrm{m} \\ &= 2\,\mathrm{m}\end{aligned}$$

We can tell that the wavelength is

$$\begin{aligned}\lambda &= \frac{v}{f} \\ &= \frac{343\,\mathrm{m/\,s}}{440\,\mathrm{Hz}} \\ &= 0.779\,55\,\mathrm{m} :\end{aligned}$$

Since the speakers are connected to the same stereo with equal length wires, $\Delta\phi_o = 0$. Then

$$\begin{aligned}\Delta\phi &= \frac{2\pi}{\lambda}\Delta r + \Delta\phi_o \\ &= \frac{2\pi}{0.779\,55\,\mathrm{m}}(2\,\mathrm{m}) + 0 \\ &= 5.\,131\,2\pi \\ &= 2\pi + 3.\,131\,2\pi\end{aligned}$$

We should ask, is this constructive or destructive interference? Well, it is neither purely constructive interference nor total destructive interference. Our amplitude would be

$$2A\cos\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}\Delta r+\Delta\phi_o\right)\right)$$

so in this case we get

$$2A\cos\left(\frac{1}{2}\left(2\pi+3.131\,2\pi\right)\right)=-0.409\,27A$$

which is smaller (in magnitude) than A, so it is partial destructive interference. It would be quieter at this spot than if we had just one speaker operating.

You might guess that this sort of analysis plays a large part in design of concert halls. It also is important in mechanical designs.

But you should have seen a deficit in what we have learned so far. Up to this point, we have only mixed waves that have the same frequency. Can we mix waves that have different frequencies? That will be the subject of our next lecture.

Basic Equations

$$\begin{aligned}
\Delta\phi &= \left(\frac{2\pi}{\lambda}\Delta x+\Delta\phi_o\right)=m2\pi \qquad m=0,\pm1,\pm2,\pm3,\cdots \text{ Constructive}\\
\Delta\phi &= \left(\frac{2\pi}{\lambda}\Delta x+\Delta\phi_o\right)=(2m+1)\,\pi \qquad m=0,\pm1,\pm2,\pm3,\cdots \text{ Destructive}
\end{aligned}$$