Chapter 7

Light and Sound Standing waves

Reading Assignment 21.4, 21.5

Fundamental Concepts

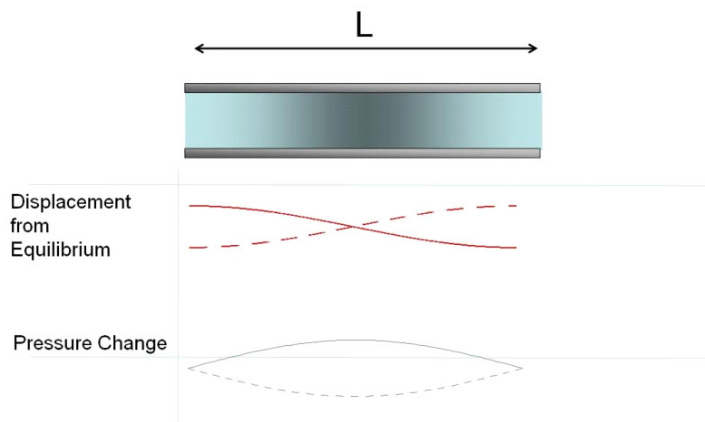
- The harmonic series expressed by a system experiencing standing waves depends on the *boundary conditions.*
- The harmonic series for open pipes is different that the harmonic series for a pipe closed on one end.
- Energy persists in the waves that have the harmonic series frequencies because of resonance.

7.1 Sound Standing waves (music)

Suppose we send a sound wave down a pipe. When the air molecules strike the molecules next to them they end up being reflected back. This happens as the wave goes down the pipe until the wave reaches the end of the pipe. Remember that where the molecules bunch up, the pressure is higher. Question 223.7.1

When we reach the end of the pipe the molecules can't bounce off the walls of the pipe anymore. They travel out into the surrounding air. It is harder to get the room pressure to change because molecules can come from any direction to fill up a vacancy. This is an effective medium change, and there will be some energy reflected back from this pipe-to-room interface. The reflected wave can make a standing wave. This is the basis of wind instruments. Let's repeat the analysis we did last time and find the possible frequencies that can make a standing wave, but this time for a sound wave in a pipe.

Take a pipe as shown in the next figures.



If we have a pipe open at both ends, we can see that air molecules are free to move in and out of the ends of the pipe. If the air molecules can move, the ends must not be nodes. This is different than the string case we studied last time! We expect that there must be a node somewhere. We can reasonably guess that there will be a node in the middle of the pipe due to symmetry. Of course, the pressure on both ends must be atmospheric pressure. So, remembering that pressure and displacement are $90\,^\circ$ out of phase for sound waves, we can guess that there are pressure nodes on both ends.

For the first harmonic we can draw a displacement node in the middle and we see that

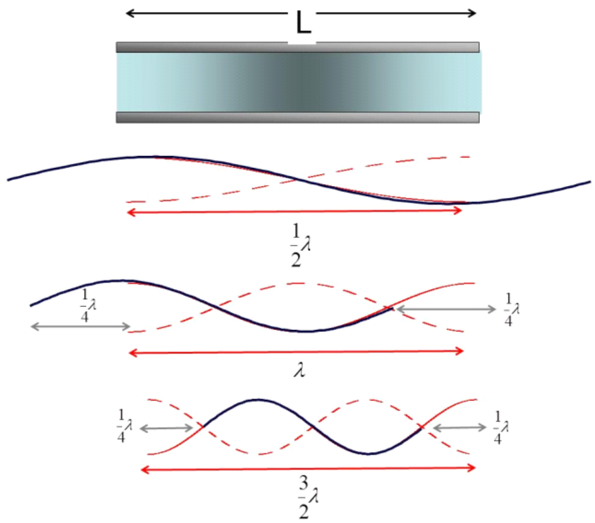
$$\lambda_1 = 2L$$

It takes two lengths of pipe to have the same length as the wavelength that is in our one single pipe. Of course, our wave is hanging out of our pipe. But if we set two additional pipes of length L along side our pipe, these two pipes would be the same length as the wavelength. The frequency would be.

$$f_1 = \frac{v}{2L} \tag{7.1}$$

The next mode fits a whole wavelength

$$\begin{aligned} \lambda_2 &= L \\ f_2 &= \frac{v}{L} \end{aligned}$$



but the next mode fits a wavelength and a half

$$\begin{aligned}\lambda_3 &= \frac{2}{3}L \\ f_3 &= \frac{3v}{2L}\end{aligned}$$

If we keep going

$$\lambda_n = \frac{2}{n}L \tag{7.2}$$

$$f_n = n\frac{v}{2L} \qquad n = 1, 2, 3, 4 \ldots \tag{7.3}$$

This is the same mathematical form that we achieved for a standing wave on a string!

Boom Whacker and Length

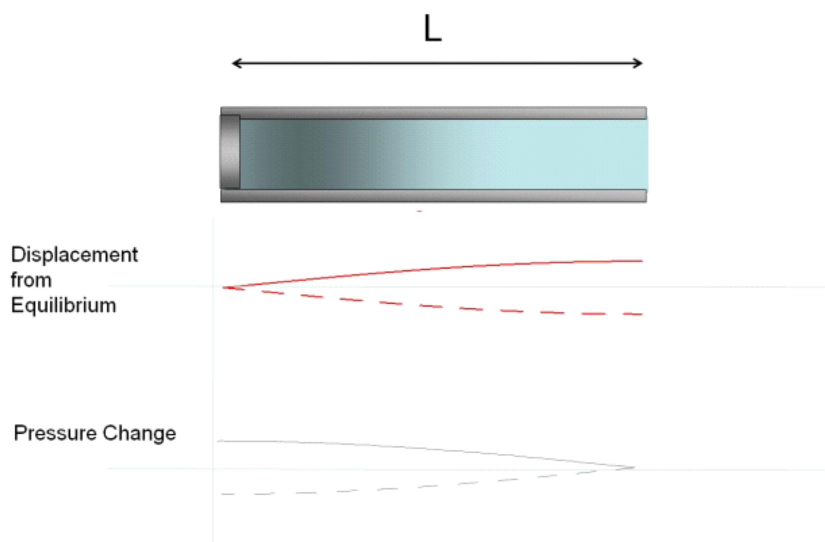
Pipes closed on one end

Question 223.7.2

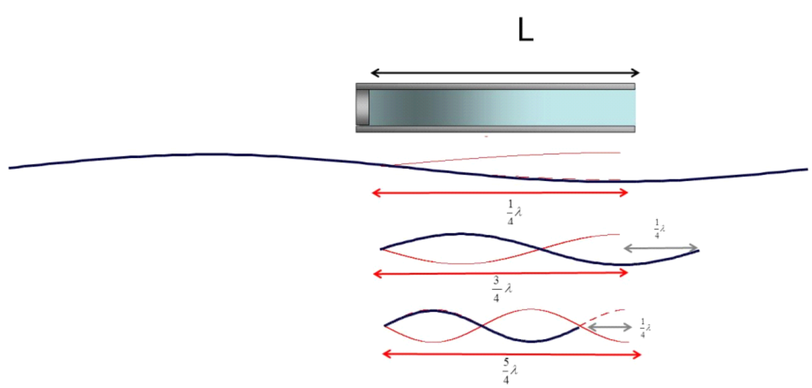
Boom Whacker Demo

But what happens if we put a cap on one end of the pipe? The air molecules cannot move longitudinally once they hit the end. This must be a displacement node. So then it must also be a pressure anti-node.

The open end is a pressure node because it stays at atmospheric pressure. This is just the same as the open ends in the open pipe case we did before. The simplest possible standing wave is shown below.



In the next figure we draw the first few harmonics for this case.



The first harmonic for the closed pipe are found by using

$$v = \lambda f$$

$$f = \frac{v}{\lambda}$$

just as we did for the string and open pipe cases. We know the speed of sound, so we have v. Knowing that the first harmonic has a node at one end and an anti node at the other end gives us the wavelength. If the pipe is L in length, then L must be

$$L = \frac{1}{4}\lambda_1$$

or

$$\lambda_1 = 4L$$

We see it now takes four lengths of of pipe to be the same size as the wave that is in our single pipe! Then the frequency is given by

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

The next configuration that will have a node on one end and an antinode on the other will have

$$L = \frac{3}{4}\lambda_2$$

which gives

$$\lambda_2 = \frac{4}{3L}$$

and

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L}$$

If we continued, we would find

$$\lambda_n = \frac{4}{n}L \tag{7.4}$$

and

$$f_n = n\frac{v}{4L} \qquad n = 1, 3, 5 \ldots \tag{7.5}$$

This is different from the string and open pipe cases. Note that only odd values of n make a standing wave. Changing the end condition changed which frequencies would make standing waves.

7.1.1 Example: organ pipe

The organ pipe shown is closed at one end so we expect

$$f_n = n\frac{v}{4L} \qquad n = 1, 3, 5 \ldots \tag{7.6}$$

Measuring the pipe, and assuming about 20 °C for the room temperature we have

$$\begin{aligned} L &= 0.41\,\mathrm{m} \\ R &= 0.06\,\mathrm{m} \\ v &= 343\,\tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned} \tag{7.7}$$

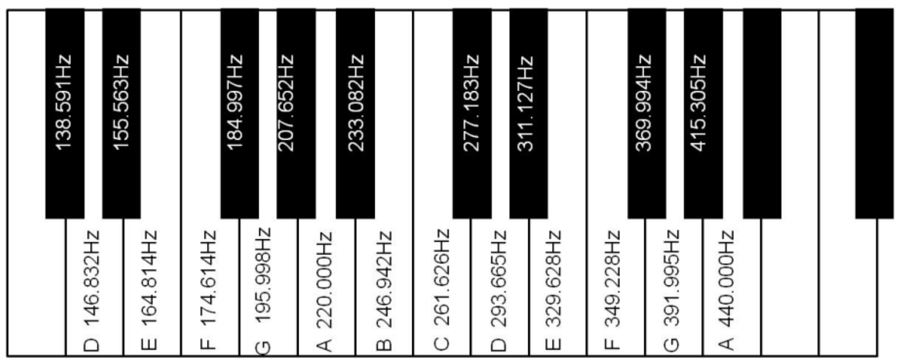
There is a detail we have ignored in our analysis, the width of the pipe matters a little. I will include a fudge factor to account for this. With the fudge factor, the wavelength is

$$\begin{aligned}\lambda_1 &= 4\,(L+0.6R)\\ &= 178.4\,\mathrm{cm}\end{aligned}$$

then our fundamental frequency is

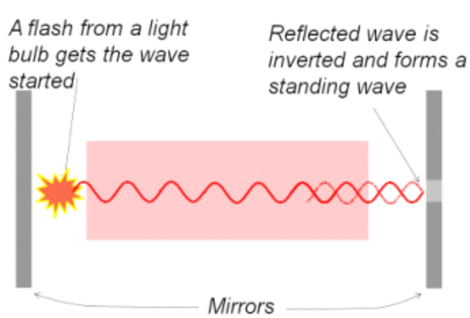
$$f_1 = \frac{v}{\lambda_1} \quad (7.8)$$
$$= 192.26 \quad (7.9)$$

We can identify this note, and compare to a standard, like a tuning fork or a piano to verify our prediction.

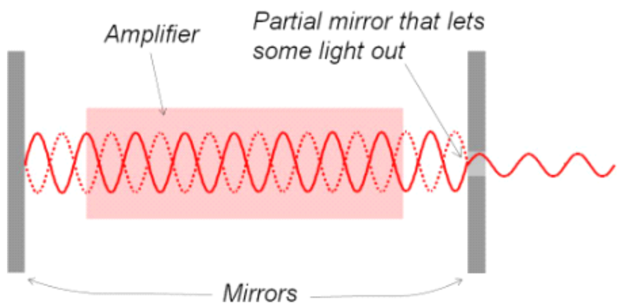


7.2 Lasers and standing waves

Light is a wave. Can we make a standing wave with light? The answer is yes, and surprisingly we do it all the time. A laser creates a standing wave as part of it's amplification system. Here is a laser just getting started.



A flash of light from a flash bulb send light out in all directions. But some of the light goes to the right toward a mirror. This is the light that will eventually become our laser beam. This light is reflected off of the mirror. The wave inverts and travels back along the center of the laser. Because it is inverted, it can cause destructive interference in places along the laser cavity. But this only works if we have just the right frequency of light. The light has to fit an integer number of half wavelengths between the two mirrors for the standing wave to form.



So only certain frequencies will work. That is why lasers usually only have one color, different frequencies of light give us different colors. So a red laser has a frequency of about 4.762×10^{14} Hz. We would expect another frequency to work that is twice this fundamental frequency.

$$\begin{aligned} f_2 &= 2f_1 \\ &= 2 \times 762 \times 10^{14}\,\mathrm{Hz} \\ &= 152\,4 \times 10^{14}\,\mathrm{Hz} \end{aligned}$$

But this frequency is outside the visible range, so we can't see it and chances are it won't go through the glass mirrors. So lasers usually only produce one frequency of light. But gas lasers can be built with special mirrors that allow many harmonics to be produced at once (e.g. CO_2 lasers).

The laser has an additional complication, and that is that it amplifies the light with a laser medium. That medium gives a new photon for every photon that passes through it, doubling the amount of light each time the wave passes through this *gain medium*. How that works is a subject for PH279. But for us, we can see that we can make standing waves in light.

7.3 Standing Waves in Rods and Membranes

We have hinted all chapter that the analysis techniques we were building apply to structures. We need more math and computational tools to analyze complex structures like bridges and buildings, but we can tackle a simple structure like a rod that is clamped. The atoms in the rod can vibrate longitudinally Since

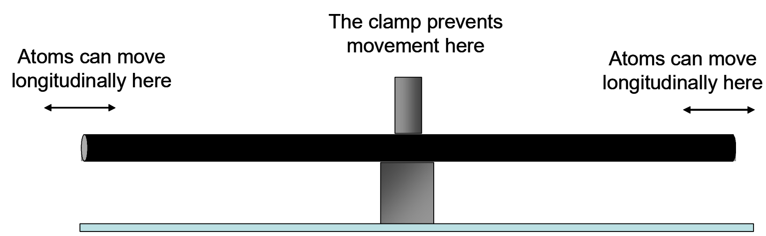


Figure 7.1:

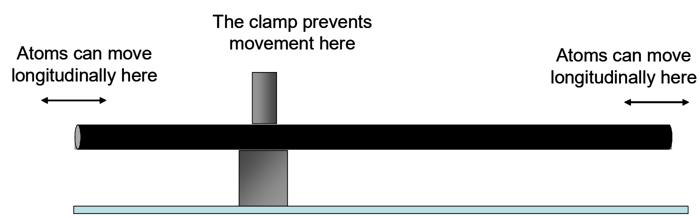
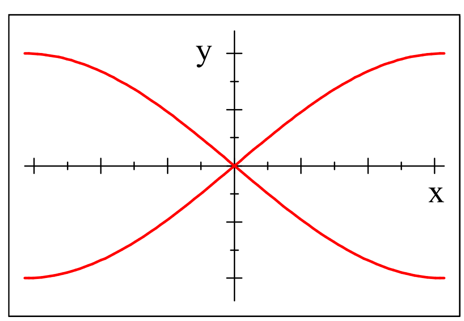


Figure 7.2:

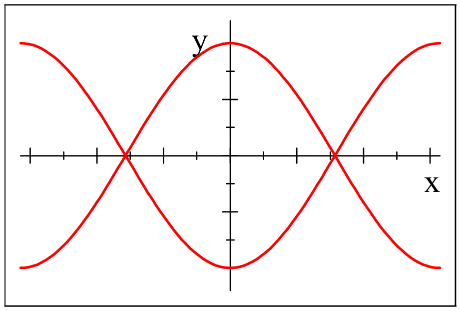
we have motion possible on both ends and not in the middle, we surmise that this system will have similar solutions as did the open ended pipe.

$$f_n = n\frac{v}{2L}$$

The fundamental looks like



But suppose we move the clamp. The clamp forces a node where it is placed. If we place the clap at $L/4$



We can perform a similar analysis for a drum head, but it is much more complicated. The modes are not points, but lines or curves, and the frequencies of oscillation are not integer multiples of each other. See for example http://physics.usask.ca/~hirose/ep225/animation/drum/anim-drum.htm.

Of course structures can also waggle on the ends. the ends can rotate counter to each other, etc. These are more complex modes than the longitudinal modes we have considered.