Chapter 46

Dispersion and Thin Lenses

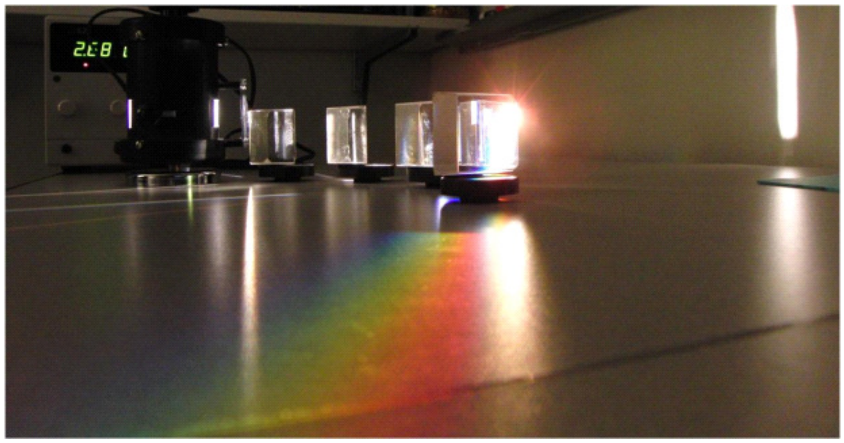
Snell's law tells us that light bends at an interface between two materials. But there is a complication. The separation between atoms in the materials is fixed, but we could send different wavelengths of light into our interface. And compared to the size of a wavelength the distances between atoms will give different effects. We found this was important before. We should worry about the relative size of the atomic spacing now. Once we have refraction understood, we can then make more useful optical devices that use refraction. We will begin this in this lecture, starting with lenses.

Fundamental Concepts

- Index of refraction is wavelength dependent

- We can describe the operation of thin lenses using three easy-to-draw rays.

46.1 Dispersion



Who hasn't played with a prism? We immediately recognize a rainbow. And if you have not done this personally, you have likely seen prisms in the chandeliers in the temple. But why does the prism make a rainbow? The secret lies in the nature of the refractive index.

Notice that in the next figure, the index of refraction depends on wavelength. This means that as light enters a material, different wavelengths will be refracted at different angles. We could have guess this from what we know about openings and wavelength. The relative size of the openings and the wavelength makes a difference. That is in part because of the distribution of scatterers, but more because blue light is nearer the size of the atoms and red light is larger than the atoms. A wave on the beach that is the size of a rock makes a big splash but a larger wave just moves over the rock. The red light is a bigger wave and misses many of the atoms, but the smaller blue waves are likely to hit the atoms. It is this absorption and re-emittion that determines the speed of the wave through the material. So how often the wave is absorbed and re-emitted determines the index of refraction for the material.

White light is not really a color of light. White light is made up of many colors–in fact, all the colors of the rainbow![1] Thus white light is pulled apart by refraction into a rainbow. This process is called *dispersion*. The reason is that for different wavelengths of light it is more likely to for the light waves to be absorbed and re-emitted than for other wavelengths. This has to do with the spacing of the atoms relative to the wavelength, and it has to do with the

[1] Ah, but some light sources fool us. As long as there are the right amounts of red, green, and blue, we can think the light is white. Fluorescent lights and LED lights do this, and the lack of a full spectrum of light explains why plants don't grow well under fluorescent lights and some LED lights.

electron structure of the material. Here is a graph that shows the index of refraction for some materials as a function of wavelength.

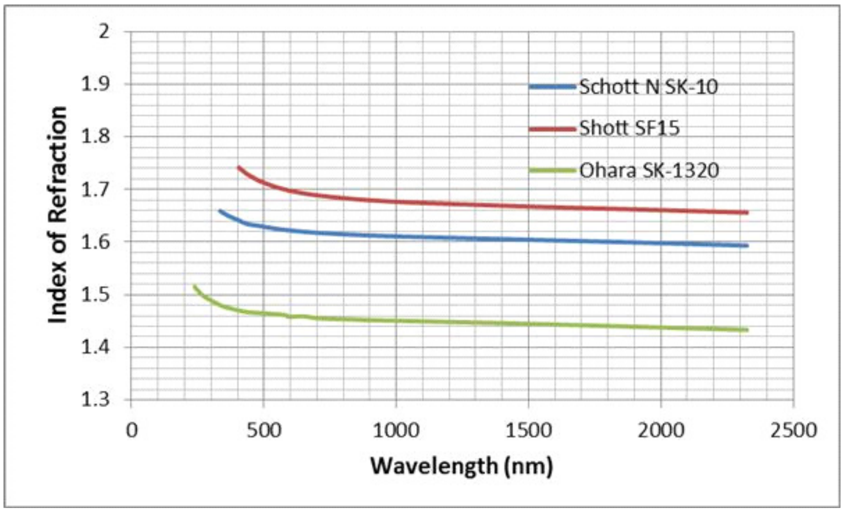
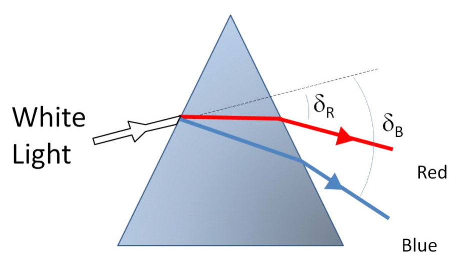


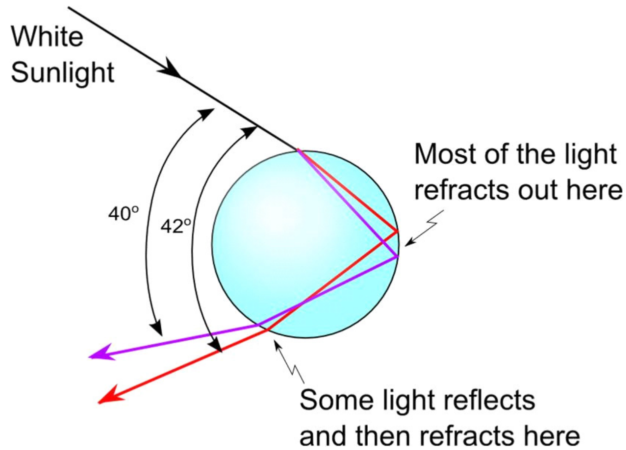
Figure 46.1: Index of refraction as a function of wavelength (Ohara optical glass http://www.oharacorp.com/fused-silica-quartz.html data and Schott optical glass data http://www.uqgoptics.com/materials_glasses_schott.aspx)

The graph tells us that blue light bends more than red light for these materials.



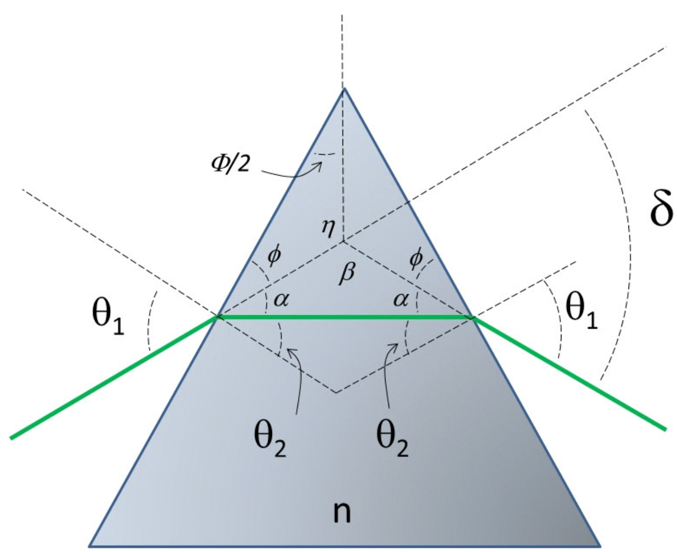
We call the change in direction measured from the original direction of travel, δ, the *angle of deviation*. The colors we can see are called the visible spectrum.

Let's look at a natural rainbow. The dispersion is caused buy small droplets of water. The white sunlight enters the drop and is dispersed. It bounces off the back of the drop and then leaves the drop, again being dispersed. Red light leaves the drop at about 42° from its input direction, and blue light leaves at about 40°.



46.1.1 Calculation of n using a prism

Let's do a problem using the idea of dispersion. Let's find the index of refraction of a the material. Suppose we make a prism as shown[2]. We know the apex angle of the triangle, Φ, and can measure the exit angle δ. In terms of these two variables, what is n?



[2]In this problem, we have carefully arranged the light so it goes horizontally across the prism. This is not always the case–and it is not usually the case in the problems in the homework.

Since the light is refracting our strategy should be to use Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

The figure has a bunch of extra lines and angles marked that might be useful in our problem. Looking at Snell's law, it looks like if we can find θ_1 and θ_2 in terms of Φ and δ then we can solve for the index of refraction of the material (we know $n_1 \approx 1$). Using the notation indicated in the figure, we choose θ_1 such that the interior ray is horizontal.[3] Then note that

$$\theta_1 = \theta_2 + \alpha$$

and that

$$\delta = 180 - \beta$$

It's also true that

$$180 = \beta + 2\alpha$$

Then

$$\delta = \beta + 2\alpha - \beta$$

or

$$\delta = 2\alpha$$

and

$$\alpha = \frac{\delta}{2}$$

Now consider

$$90 = \alpha + \theta_2 + \phi$$

and

$$180 = \Phi + 2\alpha + 2\phi$$

or

$$90 = \frac{\Phi}{2} + \alpha + \phi$$

Then

$$\begin{aligned} \alpha + \theta_2 + \phi &= \frac{\Phi}{2} + \alpha + \phi \\ \theta_2 &= \frac{\Phi}{2} \end{aligned}$$

We can put these in our equation for θ_1

$$\begin{aligned} \theta_1 &= \theta_2 + \alpha \\ &= \frac{\Phi}{2} + \frac{\delta}{2} \\ &= \frac{\Phi + \delta}{2} \end{aligned}$$

[3] WARNING! in the homework problems you can't make the same assumptions!

We now know θ_1 and θ_2 in terms of Φ and δ Now we can use Snell's Law to find n

$$\begin{aligned}
\sin(\theta_1) &= n\sin(\theta_2) \\
\sin\left(\frac{\Phi+\delta}{2}\right) &= n\sin\left(\frac{\Phi}{2}\right)
\end{aligned}$$

then

$$n = \frac{\sin\left(\frac{\Phi+\delta}{2}\right)}{\sin\left(\frac{\Phi}{2}\right)} \tag{46.1}$$

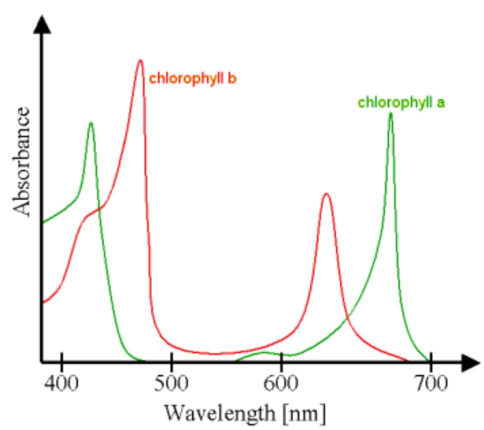
This gives a value for the index of refraction, but it would be better to repeat the analysis for several wavelengths. The resulting values for n can be combined into a n vs. λ curve like the one shown in figure 46.1.

46.1.2 Filters and other color phenomena

We have assumed without proof, that white light is made up of all the colors of the rainbow. But diffraction gratings were pretty good hints that this is true. Now that we know how prisms work, we have additional evidence of this. But knowing that white light is made a superposition of waves of different wavelengths, we should ask why a red shirt is red, or why passing light through a green film makes the light look green as it leaves.

Both of these phenomena are examples of removing wavelengths from white light.

In the case of the red shirt, the red dye in the cloth absorbs all of the visible colors except red. The red is reflected, so the shirt looks red. The filter is much the same. The green pigment in the film causes nearly all visible colors to be absorbed except green. So only green light is transmitted. This is why leaves are green. Chlorophyll absorbs red and blue wavelengths, so the green is reflected or transmitted.

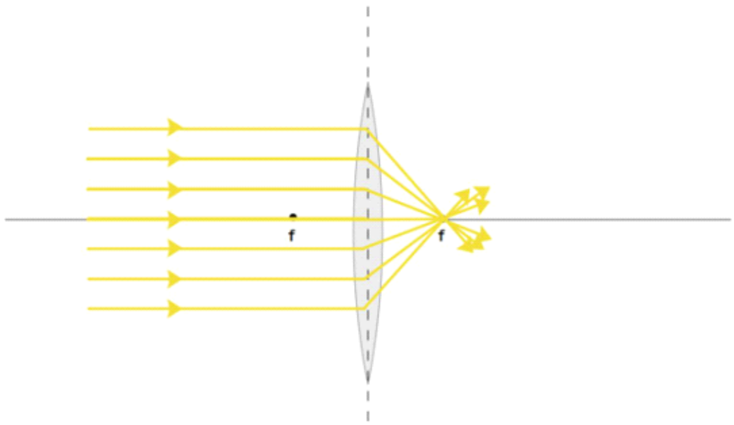


chlorophyll spectrum (Public Domain image courtesy Kurzon)

Knowing the nature of white light, we can start to understand lens systems and their challenges.

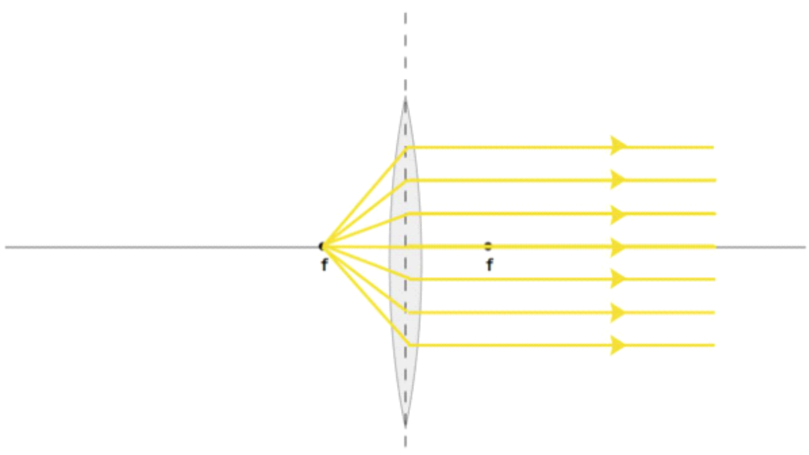
46.2 Ray Diagrams for Lenses

Before we do a lot of math to describe how lenses work, lets think about our early childhood experiences. You may have burned things with a magnifying glass. Using the idea of a ray diagram, here is what happens.

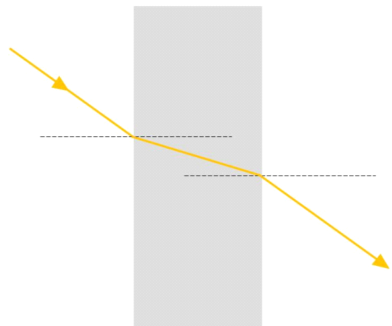


The rays from the Sun come from so far away that they are essentially parallel. We know that these rays come together to a fine point that can start a fire. The point where these rays converge is important to us. We will call this the *focal point*.

Knowing that the light will follow the same paths either direction, we would expect that if we put a light source at the focal distance, the rays should come out parallel.

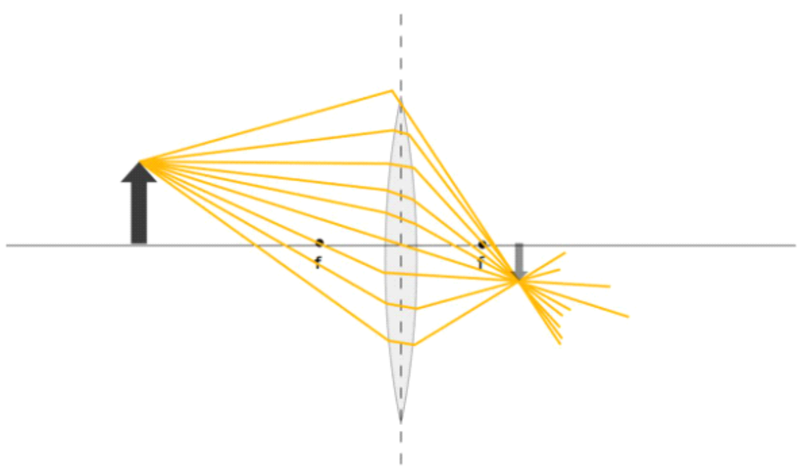


We can use one other bit of information, to understand lenses. We have seen this case before.

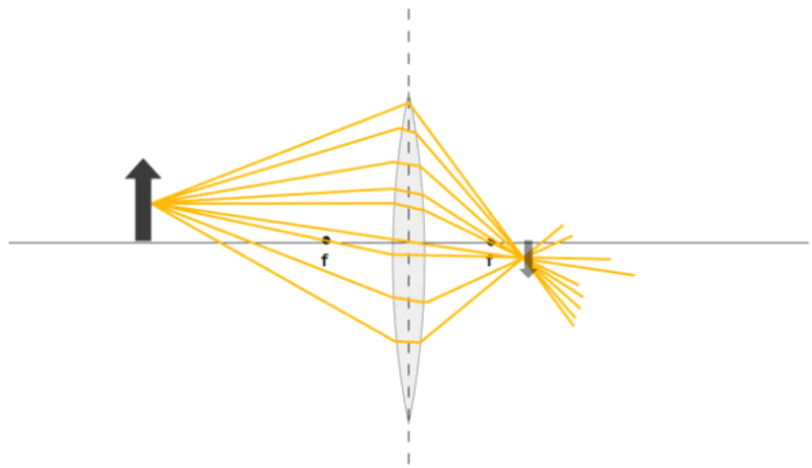


A flat block does refract the light, but when the light leaves the block it is only displaced, it retains the original direction. We will use these three situations to describe what happens when light travels through a lens.

We know that for every point on the object, we get millions of reflected rays that diverge. The lens must collect some of these rays together to form the corresponding point on the image.

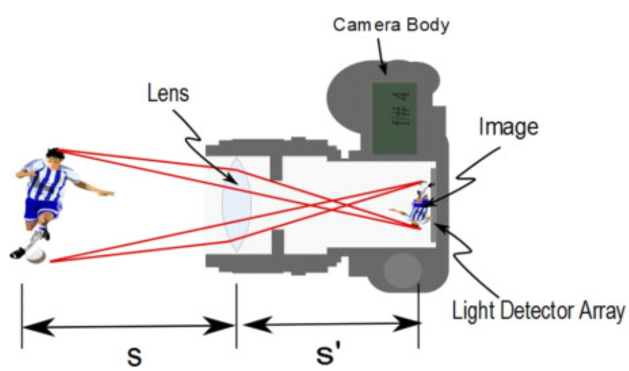


In the figure, the object is an upright arrow. We suppose that the arrow either glows, or that light is reflecting off the arrow. The arrow is a diffuse reflector, so the light bounces off in all directions. In the figure, you see light bouncing off the tip. Of course, this happens for every point on the image. Here is another drawing with light bouncing off the middle of the arrow.



But we usually pick the top of the object. If we place the bottom of the object on the optic axis, the bottom of the image will also be on the optic axis. So knowing where the bottom of the image is, and finding the top of the image gives a pretty good idea of where the rest of the image must be. We will draw diagrams for the top of the object to find the top of the image.

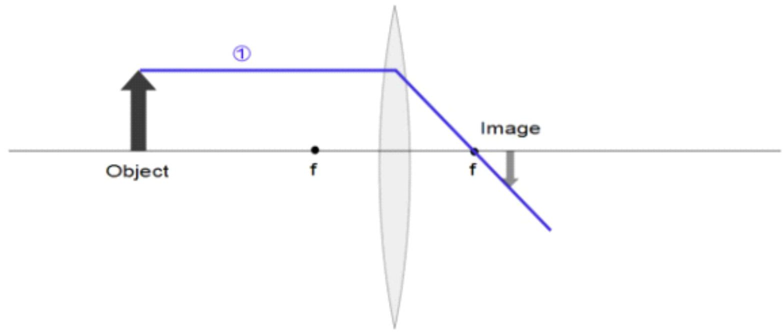
But suppose the bottom of our object isn't on the axis? For example, when we use a camera, we do not align the optical system so the bottom of the subject is in the middle of the lens, on an axis, before we shoot.



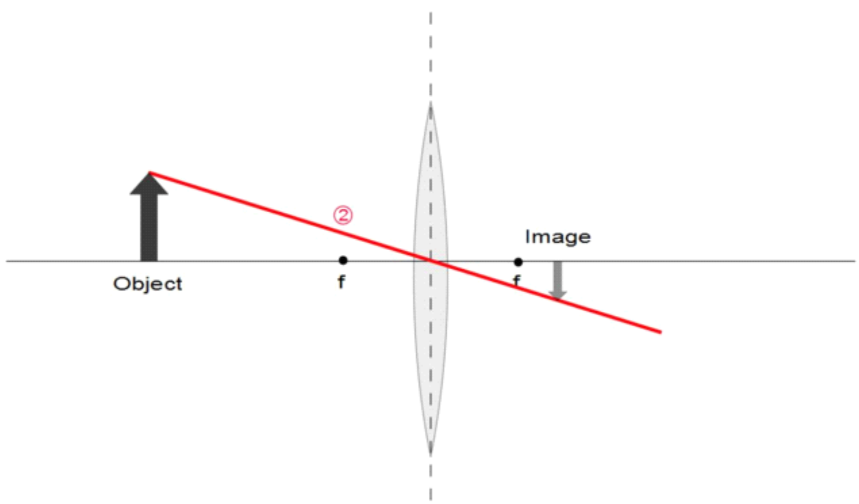
We can, of course, trace some rays for the bottom of the object well as for the top in this case and find the location of the bottom of the image. The middle of the image will still be in between the top and the bottom.

Notice that we said that light bounced off the object (our arrow) in all directions, but we did not draw all the rays going in all directions. Drawing millions of rays is impractical, and fortunately, not needed. We instinctively only drew rays that headed toward the lens. Any ray that does not head toward the lens won't take part in forming the image created by the lens. But could we make due with drawing even fewer rays and still know where the image will be?

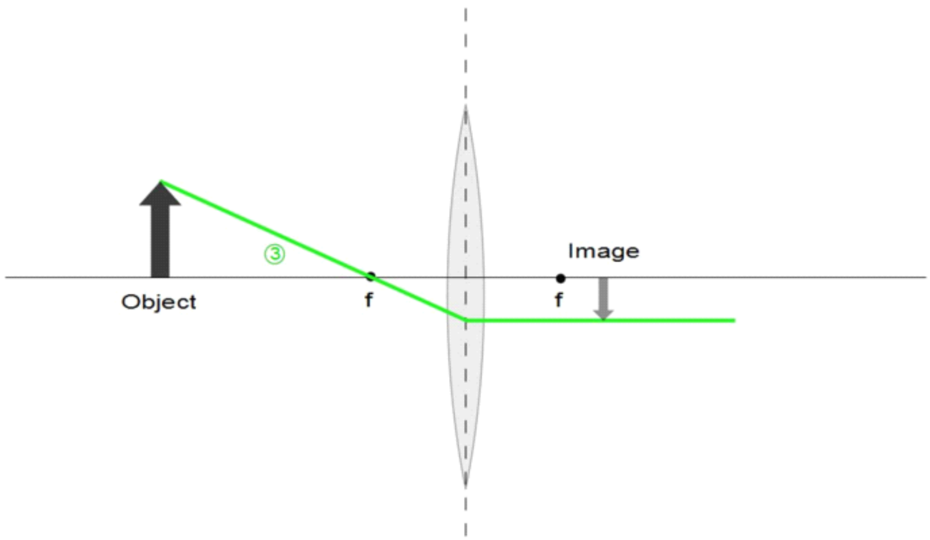
It turns out that we can choose three simple rays that leave the top of the object , and see where these rays converge to form the top of the image. Let's start with a ray that travels from the top of the object and travels parallel to the optic axis. We recognize this ray as being like one of the rays from the Sun. It comes in parallel, so it will leave the lens and travel through the focal point.



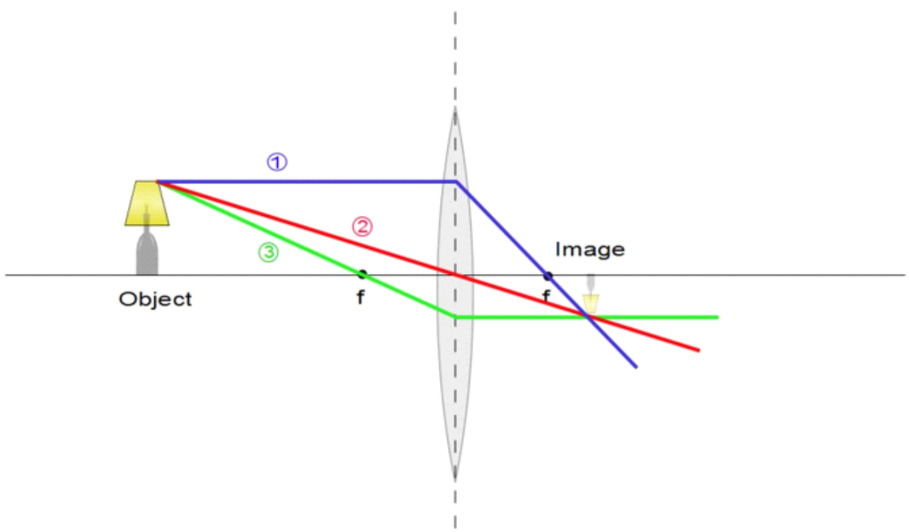
For a second easy ray, lets take the case that is like our flat block. Near the center of the lens, the sides are nearly flat. So we expect that the ray will leave in about the same direction as it was going before it struck the lens.



Two rays are really enough to determine where the top of the image will be, but there is a third ray that is easy to draw, so let's draw it to give us more confidence in our answer. That ray is one that leaves the top of the object and passes through the focal point on the object side of the lens. This situation we also recognize. This ray will leave the lens parallel to the optic axis.



Where all three rays interest, we will have the top of the image.



Because the rays come together or *converge* we call a lens like this a *converging lens.* Notice that in this case, the image is upside down. That is normal. Also notice that it is smaller than the object. We say that the image is magnified, which may seem a little bit strange. But in optics, a magnification of greater than one means that the image is bigger than the object. This is like a movie projector that makes a large image of a small LCD (or DLP) display. The magnification can be equal to one, meaning the object and image are the same size. And finally, the magnification can be less than one. This means that the image is smaller than the object. This is a convenient definition, because then we can use the same equation to describe all three situations.

$$m \equiv \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h}$$

where h is the object height, and h' is the image height. It turns out that we can also write the magnification of a lens in terms of the object distance, s and image distance s' (see more on this below).

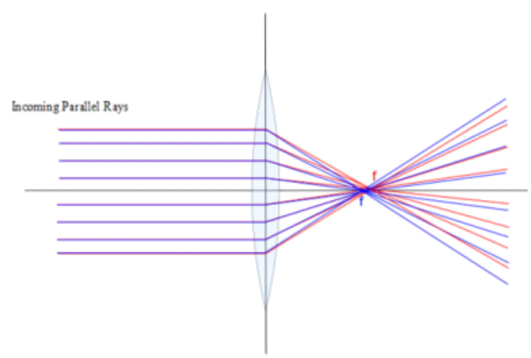
$$m = -\frac{s'}{s}$$

Notice the negative sign. By convention (meaning physicists got together and voted on this) we say that an upside down image has a negative magnification. You just have to memorize this, there is no obvious reason for this except it is mathematically convenient.

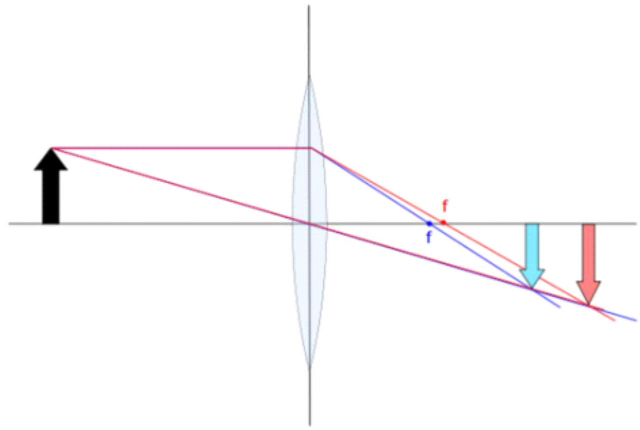
It is time to introduce another approximation Suppose the lens is very thin. Then ray number 2 would travel through the lens with no deviation at all. This is sometimes a good approximation, and will make the math easier, so for this class we will often use it. But there are times when it really does not work, so in practice you have to be careful. PH375 goes beyond the thin lens formulation.

46.2.1 Chromatic Aberration

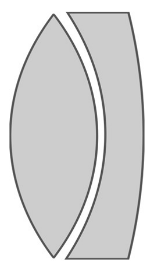
We should pause to realize that our new understanding of Snell's law tells us that we have a problem with our lenses. The index of refraction is wavelength dependent. This means that different wavelengths will focus in different positions. Here is our light from the Sun again, but note that I drew blue light and red light only.



Having removed all the other colors, we can see that the blue light focuses nearer to the lens than the red light. This is because the index of refraction for the blue light is larger. Each visible wavelength will focus somewhere in between these two (except for purple, of course). When we make an image, this means that we get multiple images of our object, one in each color. Usually the images overlap, so we end up with a colored blur.



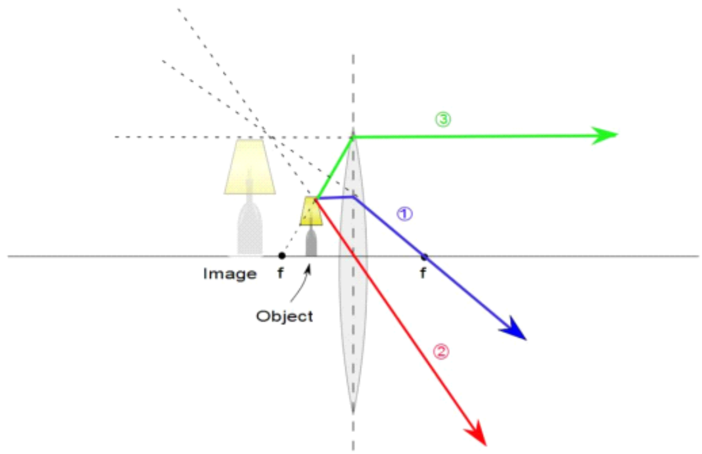
This problem is called Chromatic Aberration. We can fix this by using a combination of lenses.



where each lens has a different index of refraction. The converging lens is designed to form the image, while the diverging lens (a term we explain below) realigns all the colors.

46.2.2 Virtual images

Lets take another case and draw a ray diagram. This time let's place the object closer than the focal distance. This is the case when we use a lens as a magnifying glass. The rays will look like this.



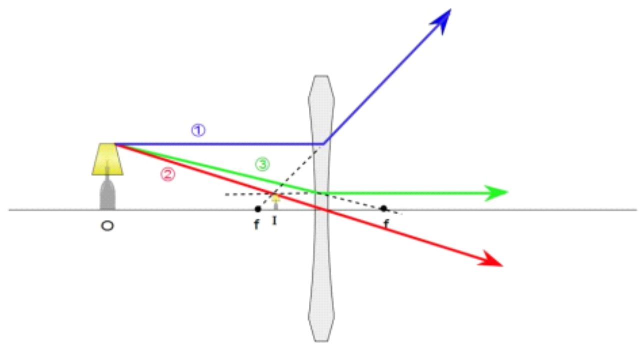
Notice that these rays never converge! We won't get an image that could project on a paper. But we know that there is an image, we can look through the lens and see it! And that is the key. The image does not really exist. We look through the lens, and our mind interprets the diverging rays coming from the lens as though they had only traveled in straight lines. If we extend these rays backwards along straight lines, they appear to come from a common point. This is the point they would have had to have come from if there were no lens. Because our brain believes light travels in straight lines, we believe we see an image at this location. But no light really comes from there! Because this image is not really made from light diverging from this position, we recognize this as a *virtual image*. The image we formed before that could be projected on a screen is called a *real image*.

By convention, we say the distance, s', from the lens to the virtual image has a negative value.

46.2.3 Diverging Lenses

So far our lenses have only been the sort that work as magnifying glasses. We call these *converging lenses*. These lenses are fatter in the middle and thinner on the edges. Because of this they are sometimes called *convex lenses*. By convention, we say the focal distance for this type of lens is positive. For this reason, they are often called *positive lenses*.

But what if we make a lens that is thinner in the middle and thicker on the edges. We can call this sort of lens a *concave lens*, and we will give in a negative focal length by convention, so we can also call it a *negative lens*. But what would this lens do? If we think about our three rays and Snell's law, ray 1 won't be bent toward the optic axis for this type of lens. In fact, if we observe an object through this lens, ray number 1 will appear to come from the focal point. Ray number 2 will still go through the middle of the lens, and if the lens is thin enough, ray 2 will pass through undeviated.



finally ray three will go as if it were aiming for the far focal point, but it will hit the lens and leave parallel to the optic axis. From the figure we see that these three rays will never converge. We expect they will form a virtual image. If we extend the rays backward as shown, we see that the extensions all meet at a point. The rays leaving the lens appear to come from this point. This is the location of the virtual image.

You might wonder what good such a lens could do, but we will find that this type of lens is used to correct vision for nearsighted people.

Basic Equations

$$m \equiv \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h}$$

$$m = -\frac{s'}{s}$$