Chapter 32

Connecting potential and field

Fundamental Concepts

- The potential and the field are manifestations of the same physical thing
- We find the potential from the field using $\Delta V = -\int \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}}$
- Fields and potentials come from separated charge

32.1 Finding the potential knowing the field

It is time to pause and think about the meaning of this electric potential. Let's trace our steps backwards. We defined the electric potential as the potential energy per unit charge:

$$\Delta V = \frac{\Delta U}{q_m}$$

where q_m is our mover and ΔV is a measure of the change in the environment between two points r_1 and r_2 measured from the environmental charge. ΔU is the change in potential energy as q_m moves. But the potential energy change is equal to the negative of the amount of work we have done in moving q_m

$$\Delta V = \frac{-W}{q_m}$$

which is equal to

$$\Delta V = \frac{-1}{q_m} \int \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{s}}$$

where again $d\overrightarrow{\mathbf{s}}$ is a general path length. But this force was a Coulomb force. which we know is related to the electric field

$$\overrightarrow{\mathbf{E}} = \frac{\overrightarrow{\mathbf{F}}}{q_m}$$

so we may rewrite the potential as

$$\begin{aligned}\Delta V &= -\int \frac{\overrightarrow{\mathbf{F}}}{q_m} \cdot d\overrightarrow{\mathbf{s}} \\ &= -\int \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}}\end{aligned}$$

which we found last lecture by analogy with our capacitor potential. Our line of reasoning in this lecture has been more formal, but we arrive at the same conclusion–**and it is an important one!** If we add up the component of field magnitude times the displacement along the path take from r_1 to r_2 we get the electric potential (well, minus the electric potential).

The electric field and the electric potential are not two distinct things. They are really different ways to look at the same thing–and that thing is the environment set up by the environmental charge. It is tradition to say the electric field is the principal quantity. This is because we have good evidence that the electric field *is* something. That evidence we will study at the end of these lectures, but in a nutshell it is that we can make waves in the electric field. If we can make waves in it, it must be something![1]

in our gravitational analogy, the gravitational field is the real thing. Gravitational potential energy is a result of the gravitational field being there. The change in potential energy is an amount of work, and the gravitational force is what does the work. No force, no potential energy. The gravitational field makes that force happen.

It is the same for our electrical force. The electrical potential is due to the Coulomb force, and the Coulomb force exists because the electric field is there.

If the field and the potential are really different manifestations of the same thing, we should be able to find one from the other. We have one way to do this. We can find the potential from the field, but we should be able to find the field from the potential. We will practice the first

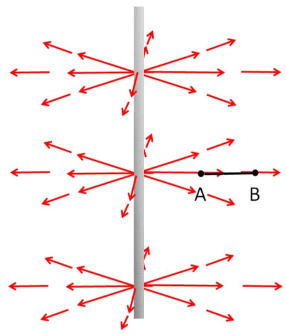
$$\Delta V = -\int \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}}$$

today, and then introduce how to find the field from the potential next lecture.

32.1.1 Finding the potential from the field.

Actually we did an example last lecture. We found the field of a point charge. But let's take on some harder examples in this lecture.

[1] By the end of these lectures, we will try to make this a more convincing (and more mathmatical) statement!



Let's calculate the electric potential do to an infinite line of charge. This is like the potential due to a charged wire. We already found the field due to an infinite line of charge

$$E = \frac{1}{4\pi\epsilon_o}\frac{2\lambda}{r}\hat{\mathbf{r}}$$

so we can use this to find the potential difference.

$$\Delta V = -\int_A^B \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}}$$

We need $d\overrightarrow{\mathbf{s}}$. Of course $d\overrightarrow{\mathbf{s}}$ could be in any direction. We can take components in cylindrical coordinates

$$d\overrightarrow{\mathbf{s}} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}} + dz\hat{\mathbf{z}}$$

Putting in our field gives

$$\begin{aligned}\Delta V &= -\int_A^B \frac{1}{4\pi\epsilon_o}\frac{2\lambda}{r}\hat{\mathbf{r}} \cdot \left(dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}} + dz\hat{\mathbf{z}}\right) \\ &= -\frac{2\lambda}{4\pi\epsilon_o}\int_A^B \frac{dr}{r}\end{aligned}$$

which we can integrate

$$\begin{aligned}\Delta V &= \left(-\frac{1}{2\pi}\frac{\lambda}{\epsilon_o}\ln r_B - \left(-\frac{1}{2\pi}\frac{\lambda}{\epsilon_o}\ln r_A\right)\right) \\ &= -\frac{1}{2\pi}\frac{\lambda}{\epsilon_o}\left(\ln r_B - \ln r_A\right)\end{aligned}$$

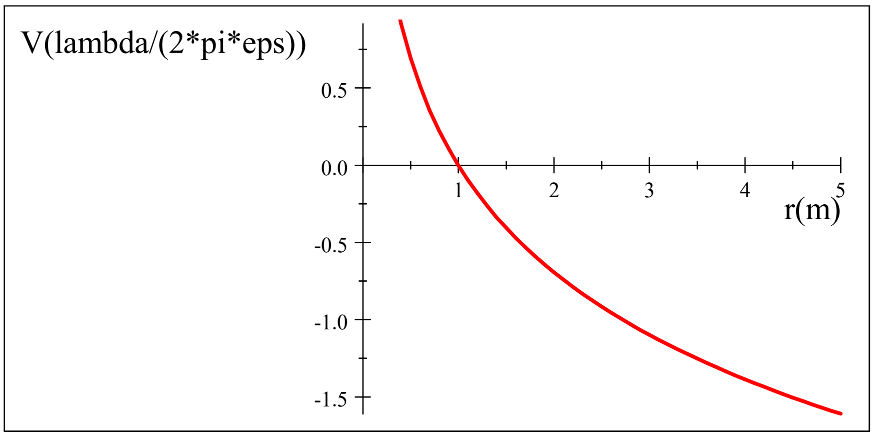
This example gives us a chance to think about our simple geometries and to consider when they are reasonable approximations to real charged objects. So long as neither r_A nor r_B are infinite, this result is reasonable. But remember what it looks like to move away from an infinite line of charge. No matter how far away we go, the line is still infinite. So we never get very far away. The terms

$$V_A = \frac{1}{2\pi}\frac{\lambda}{\epsilon_o}(\ln r_A)$$

or

$$V_B = \frac{1}{2\pi}\frac{\lambda}{\epsilon_o}(\ln r_B)$$

would look something like this



The curve is definitely not approaching zero as r gets large. No matter how far we get from an infinite line of charge, we really never get very far compared with it's infinite length. So the potential is not going to zero!

Our solution is good only when r_A and r_B are much smaller than the length of the line. that is, when our simple geometry is a good representation for something that is real, in this case, a finite length wire. But for $r_A, r_B \ll L$ this works.

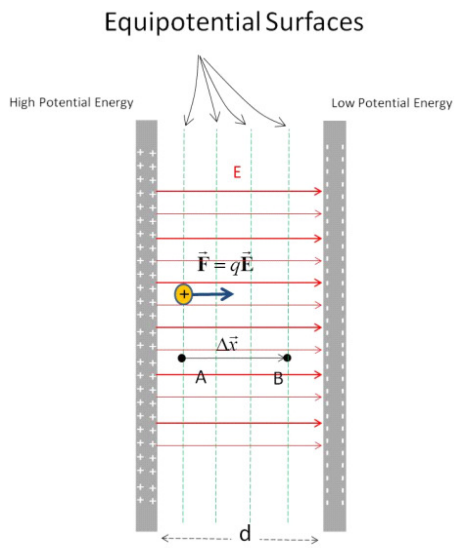
We should also pause to think of the implications of this result for electronic equipment design. Our result means that adjacent wires in a cable or on a circuit board will feel a potential due to their neighbors–something we have to take into consideration in the design to ensure your equipment will work! This is one reason why we use shielded cables for delicate instruments, and for data lines, etc.

As a second example, let's tackle our friendly capacitor problem again. What is the potential difference as we cross the capacitor from point A to point B? We already know the answer

$$\Delta V = E\Delta s = Ed$$

where d is the distance between the plates. But when we found this before, we assumed we knew the potential energy. This time let's practice using

$$\Delta V = -\int_A^B \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}}$$



We know the field is

$$E = \frac{\eta}{\epsilon_o}$$

so

$$\begin{aligned}\Delta V &= -\int_A^B \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}} \\ &= -\int_A^B \frac{\eta}{\epsilon_o} ds \cos\theta_{Es}\end{aligned}$$

where θ_{Es} is the angle between the field direction and our $d\overrightarrow{\mathbf{s}}$ direction. We could write

$$dx = ds\cos\theta_{Es}$$

Then

$$\begin{aligned}\Delta V &= -\frac{\eta}{\epsilon_o}\int_A^B dx \\ &= -\frac{\eta}{\epsilon_o}(x_B - x_A) \\ &= -\frac{\eta}{\epsilon_o}\Delta x\end{aligned}$$

This is just

$$\Delta V = -E\Delta x$$

if we consider the negative side to be the zero potential, and we cross the entire capacitor, then

$$\begin{aligned} \Delta V &= -E\left(x_B - x_A\right) \\ &= -E\left(0 - d\right) \\ &= Ed \end{aligned}$$

as we expect. Note that we can now see how the positive result comes from our choice of the zero voltage point.

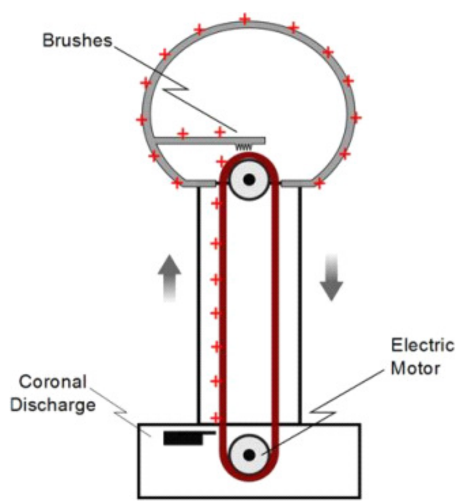
32.2 Sources of electric potential

We know that the electric potential comes from the electric field. And if we think about it, we know where the electric field comes from, charge. But we have found that equal amounts of positive and negative charge produce no net field. So normal matter does not seem to have any net electric field because the protons and electrons create oppositely directed fields, with no net result.

But if we separate the positive and negative charges, we do get a field. This is the source of all electric fields that we see, and therefore all electric potentials are due to separated charge.

We have used charge separation devices already in our lectures. Rubbing a rubber rod with rabbit fur transfers the electrons from the fur to the rod. Some of the charges that were balanced in the fur are now separated. So there is an electric field that creates an electric force. Then there must be an electric potential, since the potential is just a manifestation of the field.

We have also used a van de Graaff generator. It is time to see how this works.



In the base of the van de Graaff, there is a small electrode. It is charged to a large voltage, and charge leaks off through the air to a rubber belt that is very close. The rubber belt is connected to a motor. The motor turns the belt. The extra charge is stuck on the belt, since the belt is not a conductor. The charge is carried up to the top where there is a large round electrode. A conducting brush touches the rubber belt, and the charge is able to escape the belt through the conductor. The charge spreads over the whole spherical electrode surface.

The belt keeps providing charge. Of course the new charge is repelled by the charge all ready accumulated on the spherical electrode, so we must do work to keep the belt turning and the charge ascending to the ball at the top. This is a mechanical charge separation device. It can easily build potential differences between the spherical top and the surrounding environment (including you) of 30000 V.

Much larger versions of this device are used to accelerate sub atomic particles to very high speeds.

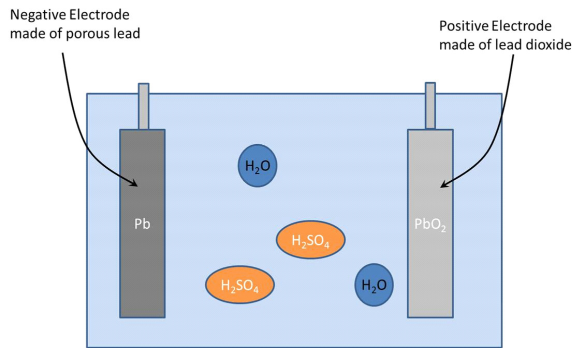
32.3 Electrochemical separation of charge

When you eat table salt, the NaCl ionic bond splits when exposed to polar water molecules, leaving a positively charged Na ion and a negatively charged Cl ion. This is very like the “bleeding” of charge from our charged balloons that we talked about earlier. We already know that the water molecules are polar, and the mostly positive hydrogens are attracted to the negatively charged Cl ions. This causes a sort of tug-o-war for the Cl ions. The positively charged Na ions pull with their coulomb force, and so do the positively charged hydrogens of the water molecules. If we have lots of water molecules, they win and the NaCl is broken apart. Water molecules are polar, but overall neutral. But now, with the Na and Cl ions, we have separated charge. We can make this charge flow, so we can get electric currents in our bodies. Our nervous system uses the positively charged Na ions to form tiny currents into and out of neurons as part of how nerve signaling works. Of course, NaCl is a pretty simple molecule. We could use more complex chemical reactions to separate charge.

32.4 batteries and emf

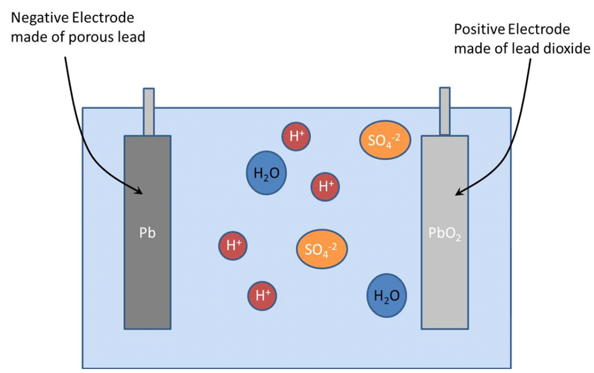
Most of us don’t have a van de Graaff generator in our pockets. But most of us do have a charge separation device that we carry around with us. We call it a battery. But what does this battery do?

Somehow the battery supples positive charge on one side and negative charge on the other side. This is accomplished by doing work on the charges. A lead acid battery is often used in automobiles. The battery is made by suspending two lead plates in a solution of sulfuric acid and water.

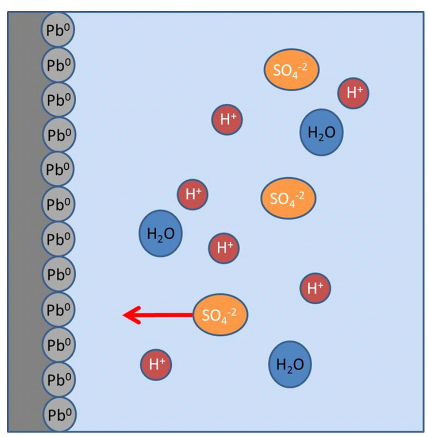


One plate is coated with lead dioxide. There is a chemical reaction at each plate. This ends up separating charge. Let's see the details of how this works.

The sulfuric acid (H_2SO_4) in our battery solution splits into two H^+ ions and an SO_4^{-2} ion.



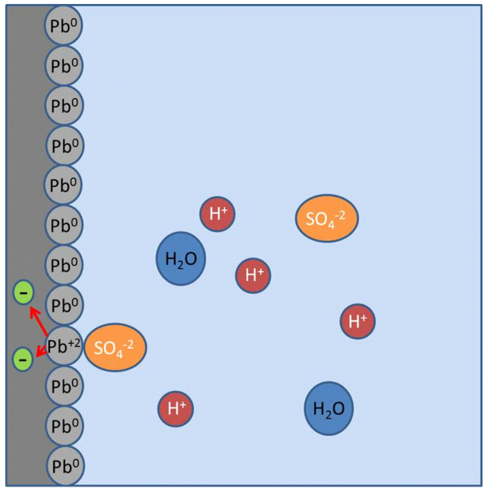
The plain lead plate reacts with the SO_4^{-2} ions.



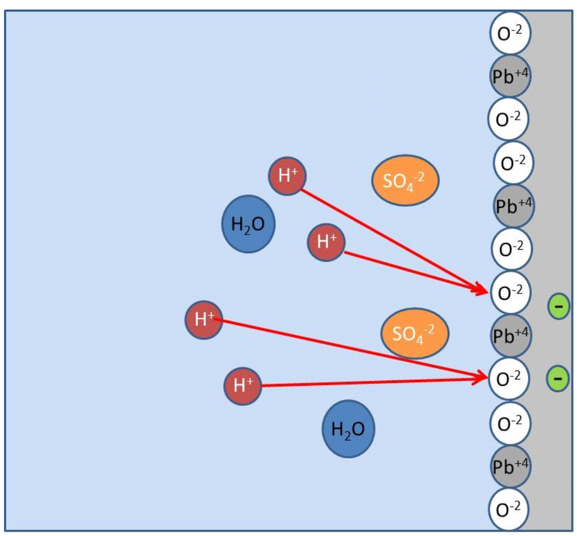
The overall reaction is

$$Pb\,(\text{solid}) + H_2SO_4^-\,(\text{aqueous}) \rightarrow PbSO_4\,(\text{solid}) + 2H^+\,(\text{aqueous}) + 2e^-\,(\text{in conductor})$$

producing lead sulfate on the electrode, some hydrogen ions in solution and some extra electrons that are left in the metal plate. This is the beginning of our separated charge!

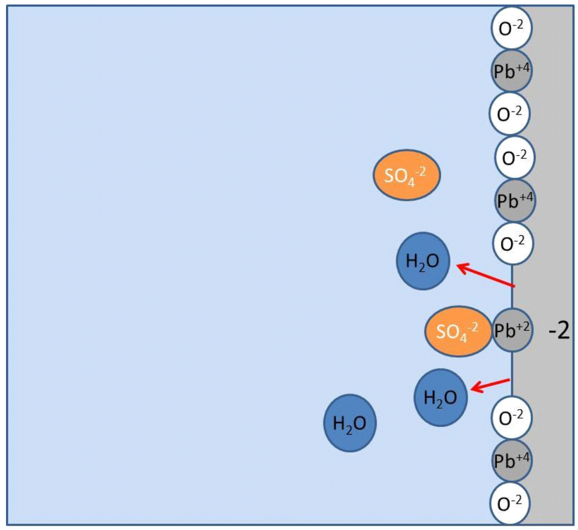


The coated plate's lead dioxide also reacts with the SO_4^{-2} ions and uses the hydrogen ions and the oxygen from the PbO_2 coating.

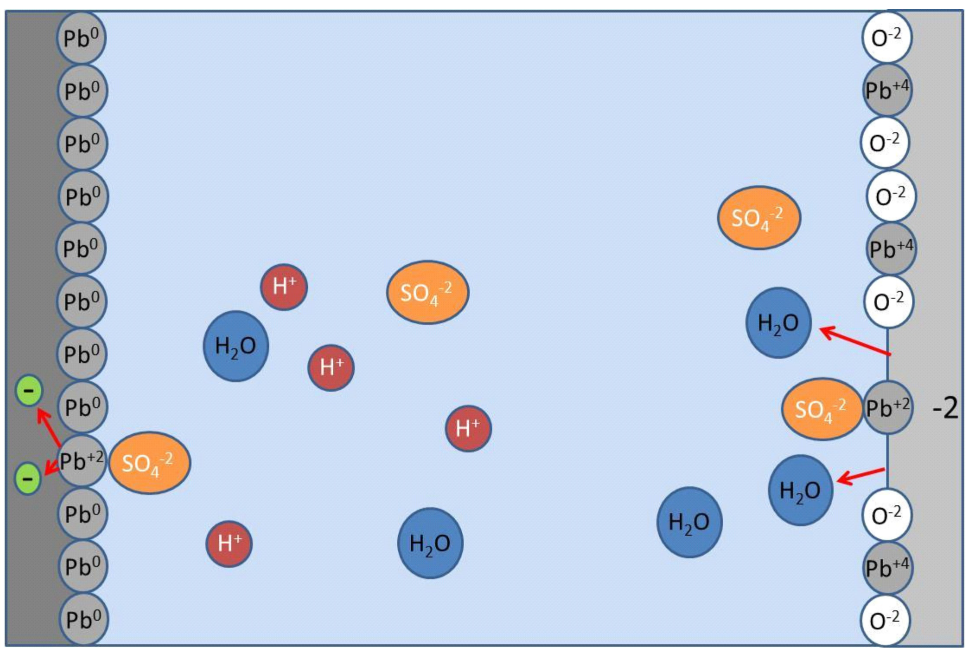


But importantly, It also uses some some electrons from the lead plate. The coated lead plate is neutral, so this reaction is using some of the free electrons from the lead. The reaction goes like this, the PbO_2 splits apart and the Pb^{+4} combines with the SO_4^{-2} and the two electrons. The left over O_2 combines with the hydrogens to form water. The reaction equation is

$$PbO\,(\text{solid}) + H_2SO_4^-\,(\text{aqueous}) + 2H^+\,(\text{solid}) + 2e^-\,(\text{in conductor}) \rightarrow PbSO_4\,(\text{solid}) + 2H_2O\,(\text{liquid})$$



So one lead plate has two extra electrons, and one now lacks two electrons. We have separated charge!



If we connect a wire between the plates, the extra electrons from one plate will move to the other plate, and we have formed a current (something we will discuss in detail later). Lead acid batteries are rechargeable. The recharging process places an electric potential across the two lead plates, and this drives the two chemical reactions backwards.

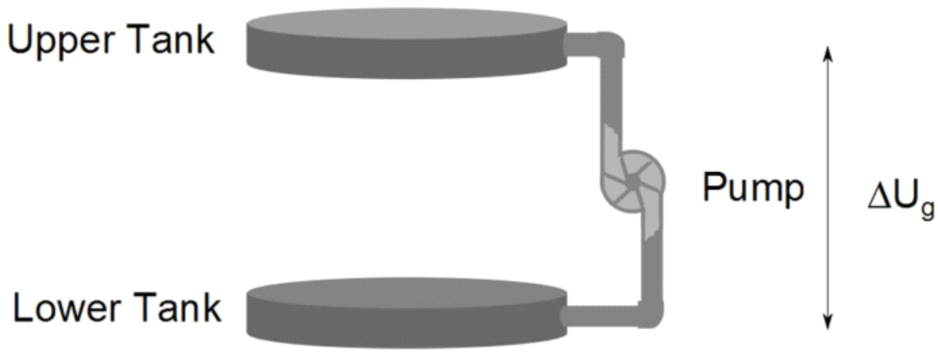
Now that we see that we can use chemistry to separate charge, let's think about what this means for an electric circuit.

$$w_{chem} = \Delta U$$

That work is equivalent to an amount of potential energy, so we have a voltage. That voltage due to the separated charge is

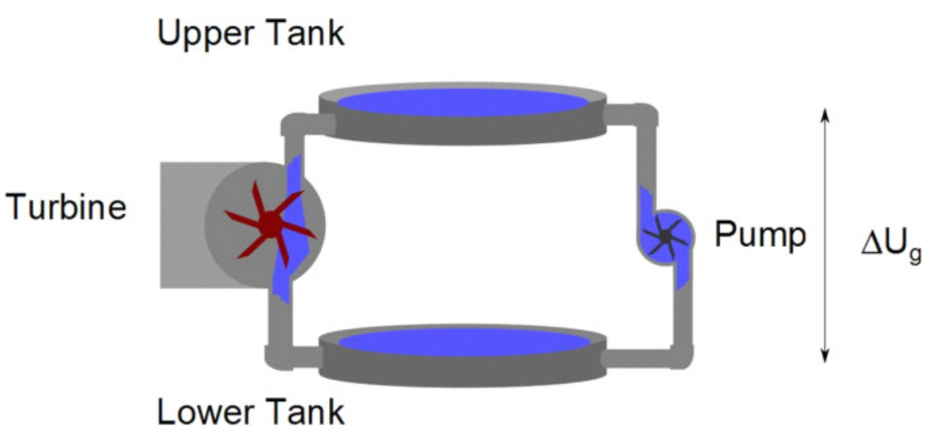
$$\Delta V = \frac{W_{chem}}{q_e}$$

This is not a chemistry class, so we won't memorize the chemical process that does this. Instead, I would like to give a mechanical analogy.

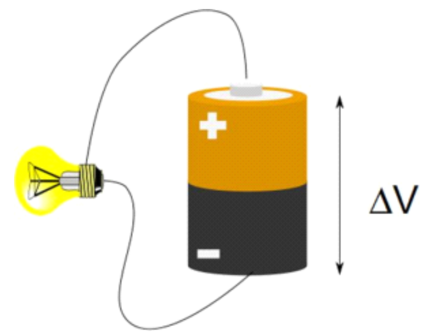


If we have water in a tank and we attach a pump to the tank, we can pump the water to a higher tank. The water would gain potential energy. This is essentially what a battery does for charge. A battery is sort of a "charge pump" that takes charge from a low potential to a high potential.

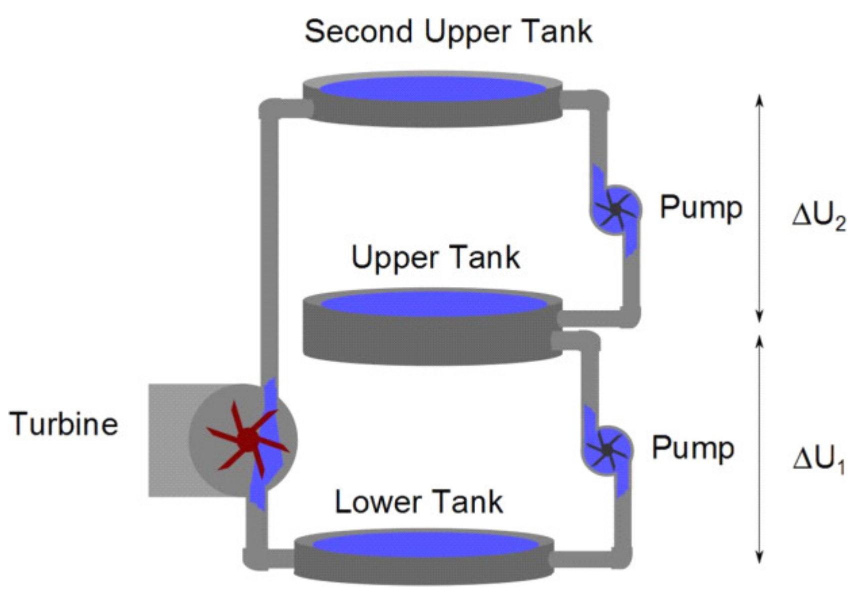
The water in the upper tank can now be put to work. It could, say, run a turbine.



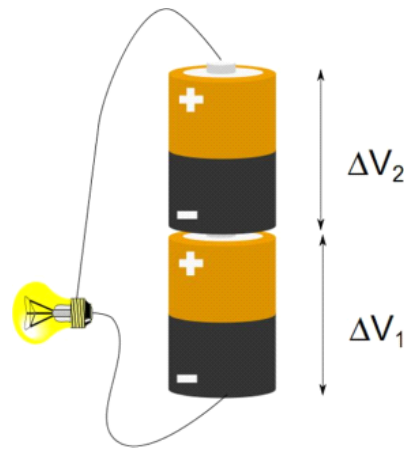
A battery can do the same. The battery "pumps" charge to the higher potential. That charge can be put to work, say, lighting a light bulb.



Of course, we could string plumps together to gain even more potential energy difference.



likewise we can string two batteries to get a larger electrical potential difference.



If we had more batteries, we would have more potential difference. Each battery "pumping" the charge up to a higher potential. Our analogy is not perfect, but it gives some insight into why stringing batteries together increases the voltage. A television remote likely uses two 1.5 V batteries for a total potential difference from the bottom of the first to the top of the last of

$$\Delta V = 2 \times 1.5\,\mathrm{V} = 3\,\mathrm{V}$$

Think for a moment what the potential energy would be if we followed our charge around the circuit. The battery "pumps" us up an amount $\Delta V_{battery}$. Then we lose voltage (think energy) as the charge does work (lights the light bulb). When we get back to the battery again we need to have a net ΔV of zero. For our circuit we can write this as

$$\Delta V_{loop} = \sum_i \Delta V_i = 0$$

That is, if we go around a loop, we should end up at the same potential where we started. It helps to think of our water analogy. If we start at the lower tank, then travel through the pump to the upper tank, then through the turbine to the lower tank we have

$$\Delta U_{total} = \Delta U_{pump} + \Delta U_{turbine} = 0$$

The water gains potential energy by being pumped up higher, then loses that potential energy by falling back down. The water ends up losing all the potential energy the pump gave it. The water is back at the same elevation, it lost all the potential energy it gained by being pumped up when it fell back down through the turbine.

Similarly, the battery pumps the charge up an amount ΔV_{bat} and it "falls" down an amount ΔV_{light} returning to where it started

$$\Delta V_{total} = \Delta V_{bat} + \Delta V_{light}$$

This is just conservation of energy. As we go around the loop we must neither create nor destroy energy. We can convert work into potential through the pump or battery, then we can create movement of water or charge and even useful work by letting the charge or water "fall" back down to the initial state. The change in energy must be zero if there is no loss mechanism. Eventually we must allow some loss to occur, but for now we have ideal batteries and wires and lights, so energy is conserved. This rule for circuits is called *Kirchhoff's loop rule.*

We have a historic name for a charge pump like a battery. We call it an *emf* and give it a new symbol $\mathcal{E}$. This is pronounced "ee em eff," that is, we say the letters. Emf used to stand for something, but that something has turned out to be a poor model for electric current. But the letters describing a charge pump persist. This is a little like Kentucky Fried Chicken changing it's name to KFC because now they bake chicken (and no one wants to think about eating fried foods now days). The letters are the name.

Next lecture we will complete our task. In this lecture we discussed finding the potential if we know the field. Next lecture we will find out how to calculate the field if we know the potential.

Basic Equations

$$\Delta V = -\int_A^B \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}}$$

$$\Delta V = \frac{\Delta U}{q_m}$$

$$\Delta V_{loop} = \sum_i \Delta V_i = 0$$