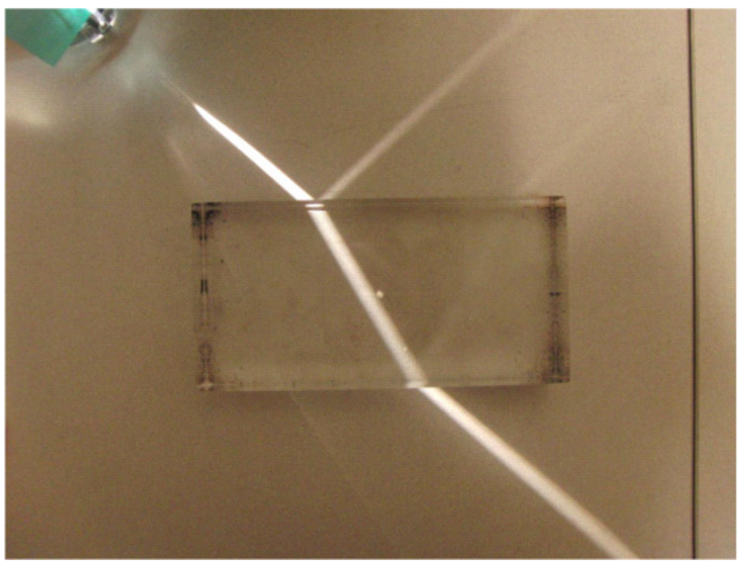
Chapter 45

Refraction and images



We studied light waves reflecting from a surface. We can see the reflection in the image above. But light waves are also transmitted through the piece of glass in the figure. Note the change in direction at the interfaces. This is penetration of a material by light is called refraction, and will be the subject of this lecture.

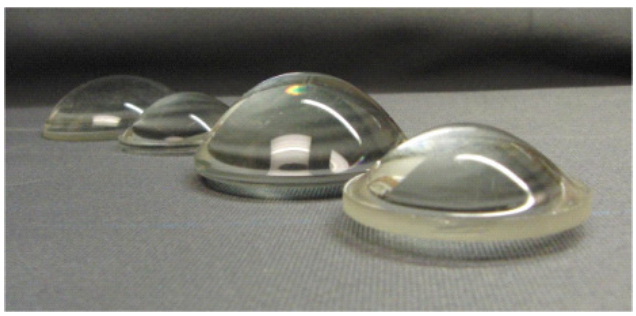
Fundamental Concepts

- Refraction is a change of direction of a light ray as it crosses an interface
- The wavelength of the light changes at an interface

- The angle changes according to Snell's law $n_i \sin\theta_i = n_t \sin\theta_t$
- When going from a high index to a low index material, the light may totally reflect, with no transmission
- Refraction can form images

45.1 Refraction

Not all surfaces reflect all the light. Some, like the lenses shown below, reflect some light at visible wavelengths, but are transparent so most of the light travels through them.

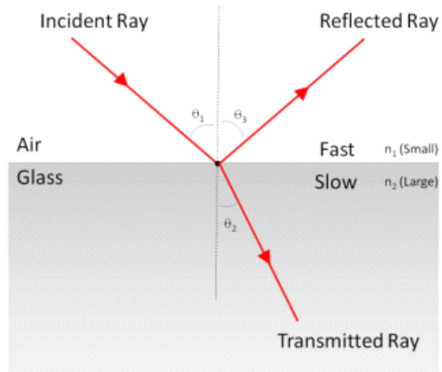


We need a way to deal with transparent materials. This is tricky, because different wavelengths of light penetrate different materials in different ways. As an example, this is also a lens

IR lens. (Image in the Public Domain, courtesy US Navy)

but it clearly is not transparent at visible wavelengths. It is transparent in the infrared part of the electromagnetic spectrum. What might be transparent at one wavelength might not be at another.

When light travels into a material, we say it is transmitted. We saw transmitted waves on strings before, and we saw transmitted radar waves in stealth fighter coatings, and visible light transmitted waves going through soap bubbles. So transmission isn't something new to us. The situation is shown schematically below.



In the figure we see a ray incident on an air-glass boundary. Some of the light is reflected just as we saw before. But some passes into the glass. Notice that the angle between the normal and the new transmitted ray is *not* equal to the incident ray. We say the ray has been bent or *refracted* by the change of media. Many experiments were performed to find a relationship between the incident and the refracted angles. It was found experimentally that

$$\frac{\sin(\theta_2)}{\sin(\theta_1)} = \frac{v_2}{v_1} = \text{constant} \tag{45.1}$$

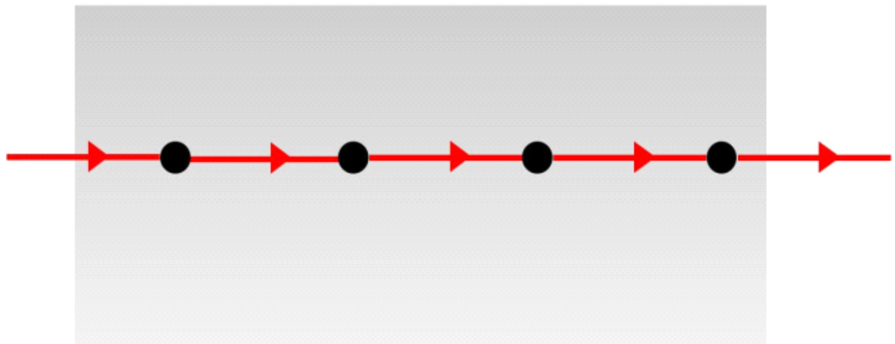
Many optics books write this as

$$\frac{\sin(\theta_t)}{\sin(\theta_i)} = \frac{v_2}{v_1} = \text{constant} \tag{45.2}$$

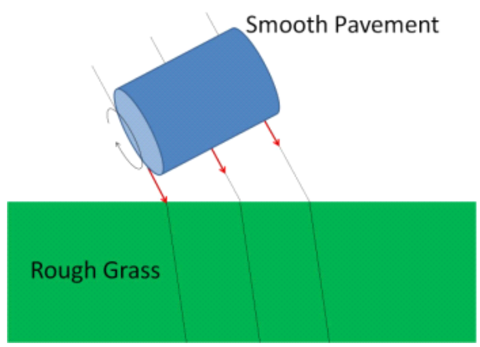
where the subscript i stands for "incident" and the subscript t stands for "transmitted." Note that we are using the fact that the average speed of light changes in a material. We should probably recall why this should occur

45.1.1 Speed of light in a material

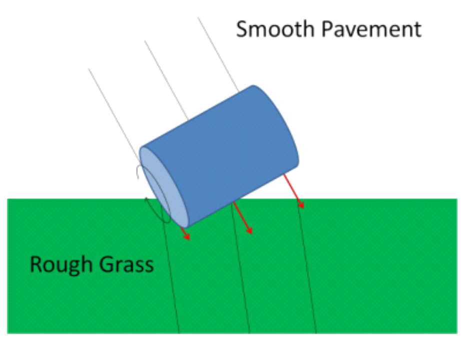
In a vacuum, light travels as a disturbance in the electromagnetic field with nothing to encounter. In a material (like glass) the light waves continually hit atoms. We have not studied antennas, but I think many of you know that an antenna works because the electrons in the metal act like driven harmonic oscillators. The incoming radio waves drive the electron motion. Here each atom has electrons, and the atoms act like little antennas, their electrons moving and absorbing the light. But the atom cannot keep the extra energy, so it is readmitted.



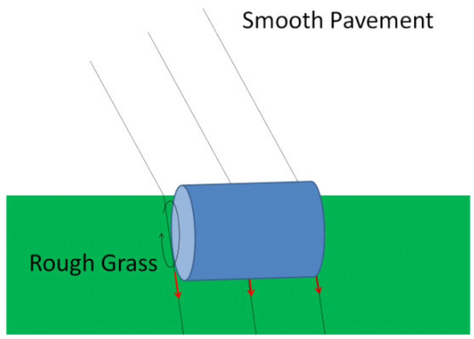
It travels to the next atom and the process repeats. Quantum mechanics tells us that there is a time delay in the re-emission of the light. This causes a secondary wave to mix with the incoming wave. The combined result is that the propagation energy in the light wave slows down. Thus the speed of light is slower in a material. But why does this slowing cause the light ray to bend? As a mechanical analog, consider a rolling barrel.



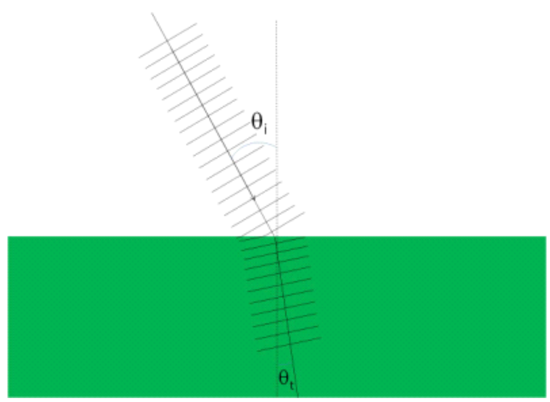
As the barrel rolls from a flat low-friction concrete to a higher-friction grass lawn, the friction slows the barrel. If the barrel hits the lawn parallel to the boundary (so it's velocity vector is perpendicular to the boundary), then the barrel continues in the same direction at the slower speed. But if it hits at an angle, the leading edge is slowed first.



This makes the trailing edge travel faster than the leading edge, and the barrel turns slightly.



We expect the same behavior from light.



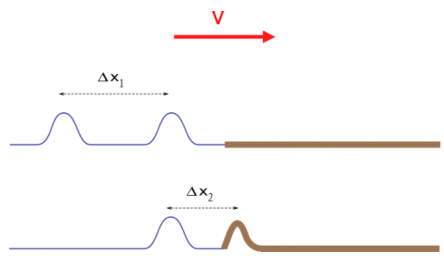
We can see that the left hand side of the wave hits the slower (green) material first and slows down. The rest of the wave front moves quicker. The result is the turning of the wave.

45.1.2 Change of wavelength

We have found that when a wave enters a material, its speed may change. But we remember from wave theory

$$v = \lambda f \tag{45.3}$$

It is time to review: does λ change, or does f change? If you will recall, we found that the change in speed at the boundary changes the wavelength. If we go from a fast material to a slow material, the forward part of the wave slows and the rest of the wave catches up to it.



This will comperes pulses, and lower the wavelength. Now that we know more about light we can also argue that f cannot change because

$$E = hf$$

If f changed, then we would either require an input of energy or we would store energy at the boundary because

$$\Delta f = \frac{\Delta E}{h}$$

This can't be true. If the wavelength changes, there is no such change in energy.

Since

$$v_1 = \lambda_1 f$$

and

$$v_2 = \lambda_2 f$$

then the ratio

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

and we again have our solution for the wavelength in the material

$$\lambda_2 = \lambda_1 \frac{v_2}{v_1}$$

which agrees with our previous analysis.

45.1.3 Index of refraction and Snell's Law

Because the equation

$$\frac{\sin(\theta_2)}{\sin(\theta_1)} = \frac{v_2}{v_1} = \text{constant}$$

has a constant ratio of velocities, it is convenient to define a term that represents that ratio. We already have a concept that can help. The *index of refraction* is just such a term. It assumes that one speed is the speed of light in vacuum, c.

$$n \equiv \frac{c}{v}$$

Then for our example

$$\frac{\sin(\theta_2)}{\sin(\theta_1)} = \frac{cv_2}{cv_1} = \frac{n_1}{n_2}$$

We can write our formula as

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \tag{45.4}$$

where we have

$$n_1 = \frac{c}{v_1}$$

and

$$n_2 = \frac{c}{v_2}$$

This is called *Snell's law of refraction* after the scientist who experimentally determined the relationship.

Again let's consider our wavelength change. Using the index of refraction we can write our equation relating the ratio of velocities and wavelengths as

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\frac{2}{n_1}}{\frac{c}{n_2}} = \frac{n_2}{n_1}$$

which gives

$$\lambda_1 n_1 = \lambda_2 n_2$$

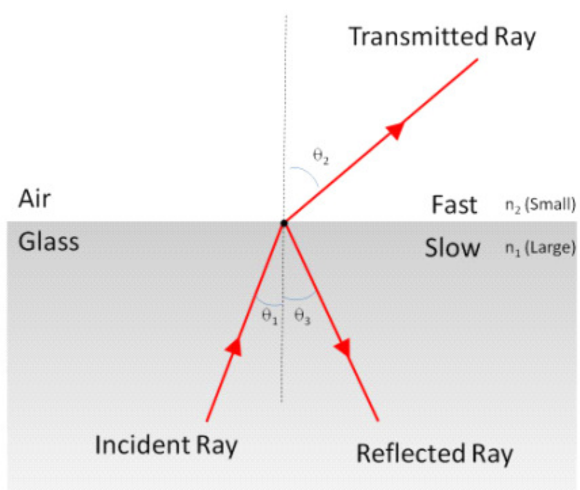
and if we have vacuum and a single material we can find the index of refraction from

$$n = \frac{\lambda}{\lambda_{material}} \tag{45.5}$$

where $\lambda_{material}$ is the wavelength in the material.

45.2 Total Internal Reflection

Up to now we have mostly assumed that light was coming from a region of low index of refraction into a region of high index of refraction. We should pause to look at what can happen if we go the other way.



We start with Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

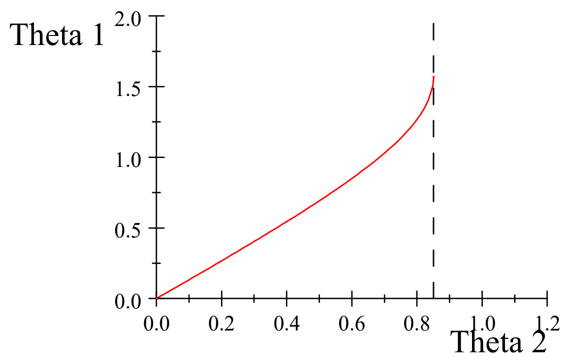
but this time $n = n_1$ and $n_2 \approx 1$ so

$$n \sin\theta_1 = \sin\theta_2$$

which gives

$$\theta_2 = \sin^{-1}(n \sin\theta_1) \tag{45.6}$$

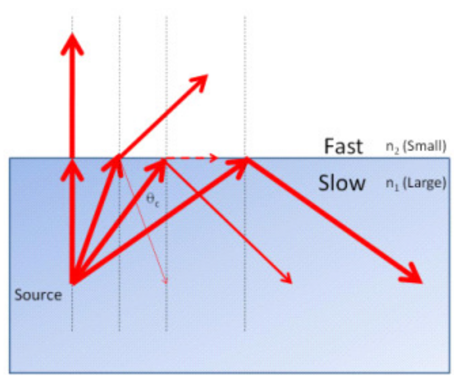
If we take $n = 1.33$ (water) we can plot this expression as a function of θ_1



We see that at $\theta_1 = 0.850\,91\,\mathrm{rad}$ $(48.754\,^\circ)$ the curve becomes infinitely steep. If we use this value in our equation this gives

$$\begin{aligned} \theta_2 &= \sin^{-1}(n \sin(0.850\,91)) \\ &= 1.570\,8\,\mathrm{rad} \\ &= 90\,^\circ \end{aligned} \tag{45.7}$$

The light skims along the edge of the water!



We can find the value of θ_1 that makes this happen without graphing. Set $\theta_2 = 90\,^\circ$ then

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

becomes

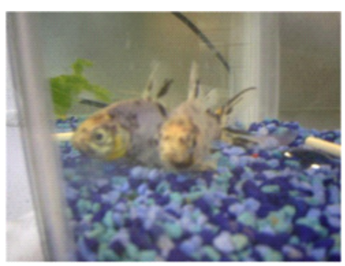
$$n_1 \sin\theta_1 = \sin\left(90\,^\circ\right) = 1$$

$$\sin\theta_1 = \frac{1}{n_1}$$

so then θ_1 is given by

$$\theta_1 = \theta_c \equiv \sin^{-1}\left(\frac{1}{n}\right) \tag{45.8}$$

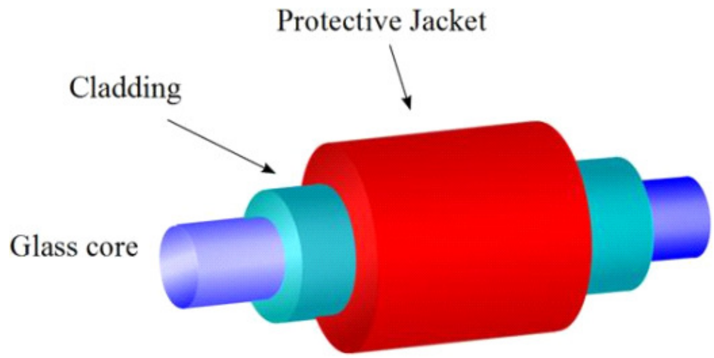
We give this value of θ_1 a special name. It is the *critical angle* for internal reflection. But what happens if we go farther than this ($\theta_1 > \theta_c$)? We will no longer have a transmitted ray. The ray will be reflected. This is why when you dive into a pool and look up, you see a region of the roof of the pool area (or sky) but off to the side of the pool the surface looks mirrored. It is also why you sometimes see the sides of a fish tank appear to be mirrored when you look through the front.



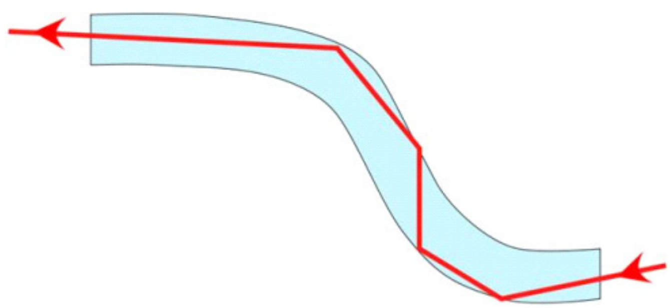
It is also why cut gems (like diamonds) sparkle. They capture the light with facets that are cut at angles that create total internal reflection. The light that enters the gem comes back out the front (We will study how to make the pretty colored sparkles next time).

45.2.1 Fiber Optics

This internal reflection effect is very useful! It is the heart and soul of fiber optics.



An interior material with a lower index of refraction is inclosed in a cladding with a higher index. This creates a light pipe that traps the light in the fiber.

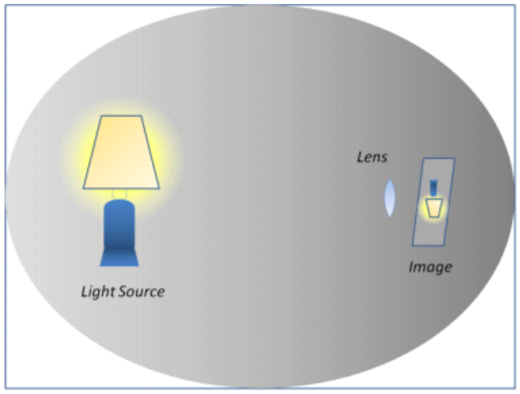


Modern fibers don't always have a hard boundary. The fibers have a gradual change in index of refraction that changes the direction of the light gradually. This keeps the light in the fiber but tends to direct along the fiber so the beam is not crisscrossing as it goes.

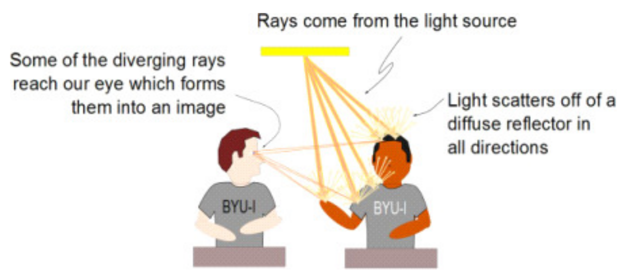
The cutting edge of fiber design today uses hollow fibers or fibers filled with different index material.

45.3 Images Formed by Refraction

Let's think about what an image is. Take a piece of paper and a lens, and hold up the lens is a darkened room that has some bright object in it. Move the lens or the paper back and forth, and at just the right distance, a miniature picture of the bright object will appear. We should think about what the word "picture" means in this sense.



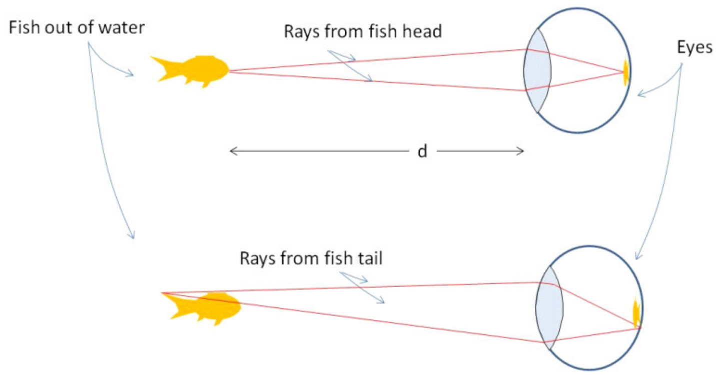
We have talked about how we see something. Remember the BYU-I guys from last time.



Our eyes gather rays that are diverging from the object because light has bounced off of the object. Our eyes intersect a diverging set of rays that form a definite pattern. That diverging set of rays forming a pattern is the picture of the object.

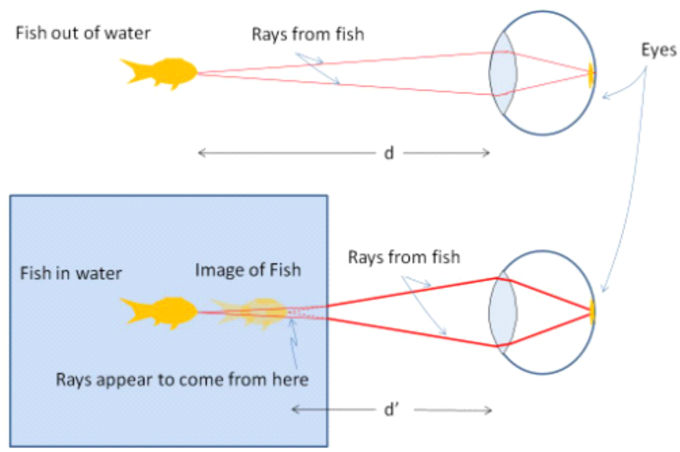
So when we say that the lens has formed a miniature picture of our dark room object, we mean that the lens has somehow formed a diverging set of rays that form a pattern that looks like the pattern formed by the diverging set of rays coming from the object, itself. In other words, the object forms a diverging set of rays, as normal, and our lens forms a duplicate set of rays in the same pattern, so we see the same thing. The lens' version is smaller, and upside down, but it is still essentially the same pattern.

As a first step to see how this works, consider our fish tank again. It would be bad on the fish, but think about looking at a fish in air. The room light would bounce off of the fish, and we would have a diverging set of rays from every point on the fish.



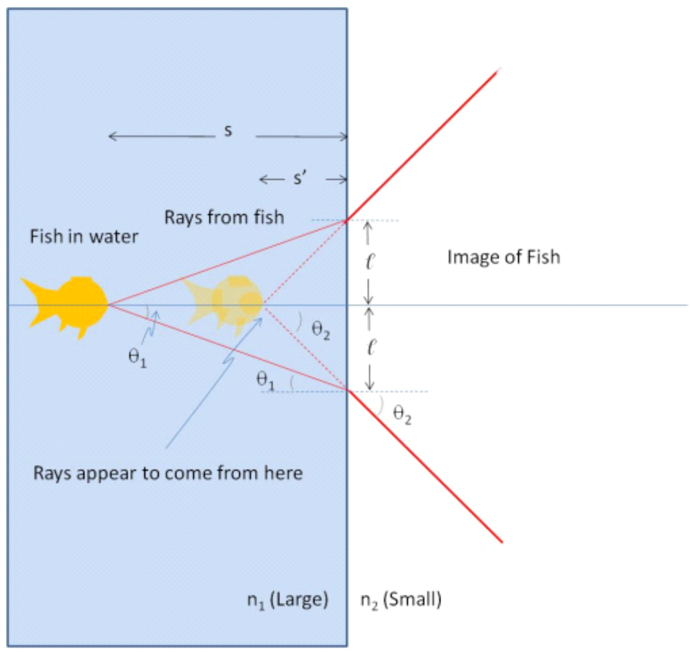
We can see that the picture is made from every point on the fish being "imaged" to a point on the retina. We collect the rays leaving every point on the fish, and bring them to corresponding points on the retina to make the picture.

It will take us a few lectures to see exactly how this is done by the lens system in the eye, but as a first step, let's consider the fish tank, itself. Put the fish back in the tank and look at it.



Rays still come from the fish. But we now know that the change from a slow material to a fast material will bend the light. These bent rays are collected by our eyes, and the picture of the fish is formed on the retina just as before. But our eyes interpret the light as though it went in straight lines with no bends (dotted lines in the last figure). our mind is designed to believe light travels in straight lines, so our mind tells us there is a fish, but that the fish head (and every other part of the fish) is closer than it really is. We call this apparent fish at the closer location an image of the fish, because this is where we think the diverging set of rays come from that form the fish pattern.

The next figure shows the details of the rays leaving a dot on the fish head



The dot on the fish head is our object for this set of rays. The distance from the fish-head dot and the edge of the water/air boundary is called the *object distance* and is given the symbol s.

The distance from the image of the fish-head dot to the edge of the water/air boundary is called the *image distance* and is given the symbol s'. Note that this is not a derivative, it is just a distance like s, because it appears to be where the rays come from, but it is a different distance because of the refraction of the rays. So to make it look different we put a prime mark on it.

We can find where the image is (s') knowing s. We can see from the figure that

$$\begin{aligned}\ell &= s\tan\theta_1 \\ \ell &= s'\tan\theta_2\end{aligned}$$

so

$$s\tan\theta_1 = s'\tan\theta_2$$

or

$$s\frac{\tan\theta_1}{\tan\theta_2} = s'$$

from Snell's law, we know that

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

Usually we can take the small angle approximation. This would limit our analysis to rays that are near the central axes. Let's call this central axis the *optics axis* and the rays that are not too far away from this axis *paraxial rays.* Then for our small angles we can write

$$\tan\theta_i \approx \sin\theta_i$$

so

$$s\frac{\tan\theta_1}{\tan\theta_2} \approx s\frac{\sin\theta_1}{\sin\theta_2} = s\frac{n_2}{n_1} = s'$$

and we have the image distance

$$s' = s\frac{n_2}{n_1}$$

This is not so useful unless you have some burning need to know where your fish are in a tank. But we now have the vocabulary to discuss the larger problem of how a lens works, which we will take up next time.

Basic Equations

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\theta_c \equiv \sin^{-1}\left(\frac{1}{n}\right)$$