Chapter 42

Many slits, and single slits

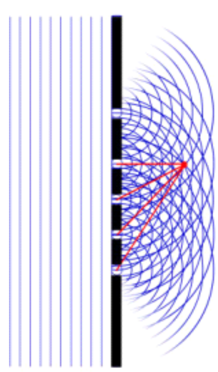
Last lecture we found the pattern that results from sending light through two slits. This lecture takes on many slits, and even the pattern that results from a single slit.

Fundamental Concepts

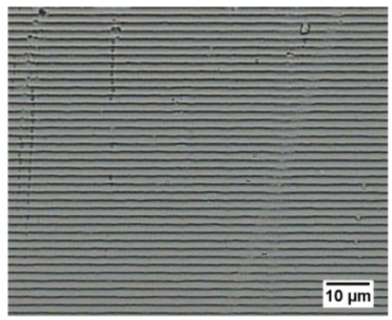
- Many slit devices are called diffraction gratings
- These devices can be use to build spectrometers
- Single slits also produce an interference pattern

42.1 Diffraction Gratings

We have discussed the interference that comes from having two small slits. But what if we have more slits?



A diffraction grating is an optical element with many many parallel slits spaced very close together. Here is a typical diffraction grating created by etching lines in a piece of glass. The etchings scatter the light, but the un-etched part allows the light to pass through. The un-etched parts are essentially a series of slits.



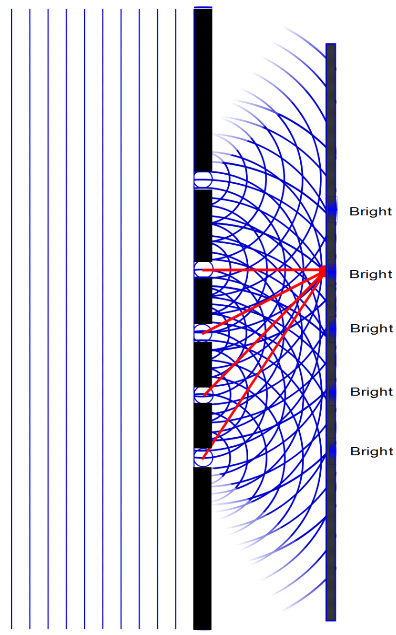
Surface of a diffraction grating (600 lines/mm). Image taken with optical transmission microscope. (Image in the public domain courtesy Scapha)

A typical grating might have 5000 slits per unit centimeter. You have probably used a diffraction grating to see rainbow colors in a beginning science class.

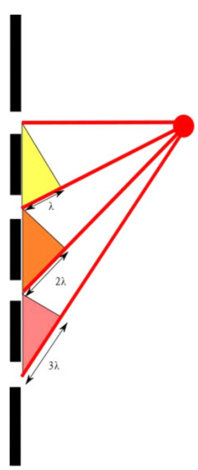
If we use $5000 \frac{\text{slits}}{\text{cm}}$ for an example, we see that the slit spacing is

$$\begin{aligned} d &= \frac{1}{5000}\,\text{cm} && (42.1) \\ &= 2.0 \times 10^{-6}\,\text{m} && (42.2) \end{aligned}$$

Take a section of diffraction grating as shown below



At some point, two of the slits will have a path difference that is a whole wavelength, and we would expect a bright spot. But what about the other slits? If we have a slit spacing such that each of the succeeding slits has a path difference that is just an additional wavelength, then each of the slits will contribute to the constructive interference at our point, and the point will become a very bright spot.

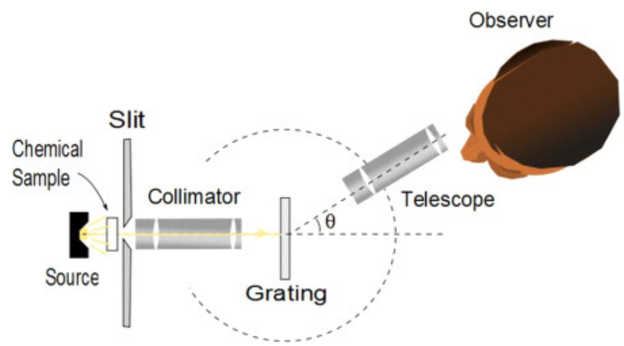


The light leaves each slit in phase with the light from the rest of the slits. At some distance L away and at some angle θ, we will have a path difference

$$\delta = d\sin\left(\theta_{bright}\right) = m\lambda \quad m = 0, \pm 1, \pm 2, \ldots \tag{42.3}$$

This looks a lot like our condition for constructive interference for two slits.

This equation tells us that each wavelength, λ, will experience constructive interference at a slightly different angle θ_{bright}. Knowing d and θ allows an accurate calculation of λ. This may seem a silly thing to do, but suppose we add into our system a sample of a chemical to identify



We could then record the intensity of the transmitted light as a function of angle, which is equivalent to λ. We can again generate a spectrum. This is a traditional way to build a spectrometer and many such devices are available today.

42.1.1 Resolving power of diffraction gratings

For two nearly equal wavelengths λ_1 and λ_2, we say that the diffraction grating can resolve the wavelengths if we can distinguish the two using the grating. The *resolving power* of the grating is defined as

$$R = \frac{(\lambda_1 + \lambda_2)}{2\left(\lambda_1 - \lambda_2\right)} = \frac{\bar{\lambda}}{\Delta\lambda} \tag{42.4}$$

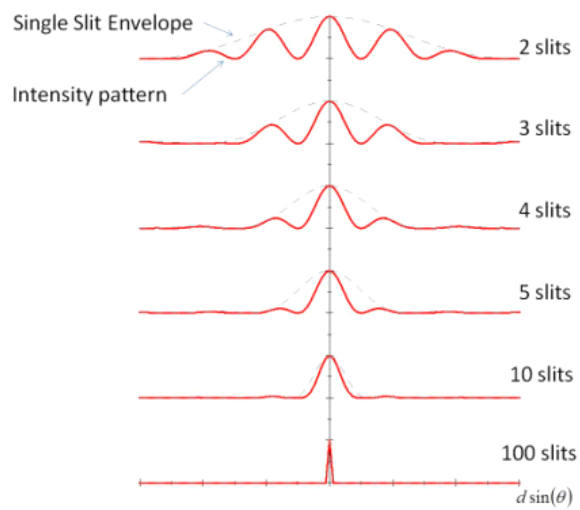
We can show that for the m-th order diffraction, the resolving power is

$$R = Nm \tag{42.5}$$

where N is the number of slits. So our ability to distinguish wavelengths increases with the number of slits and with the order (which is related to how far off-axis we look).

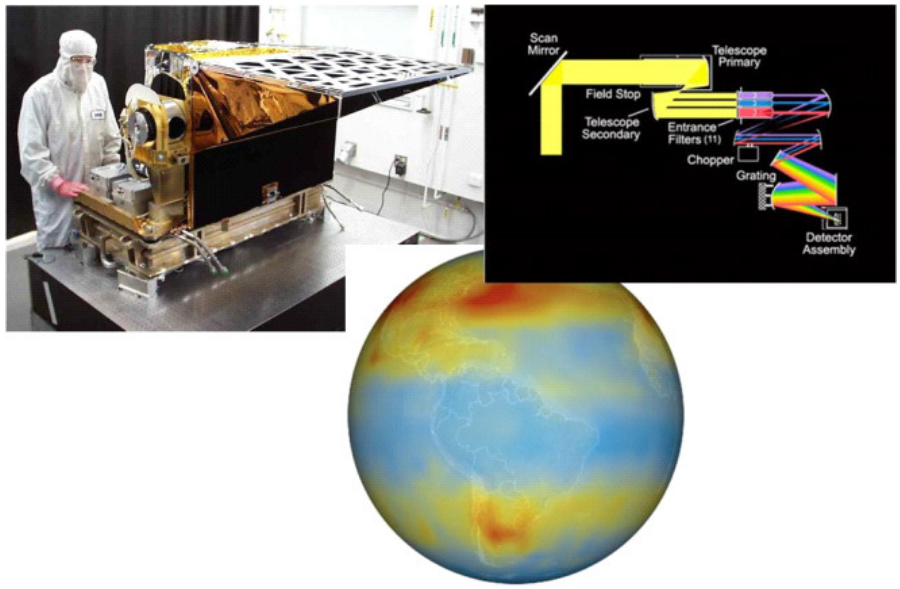
Note that for $m = 0$ we have no ability to resolve wavelengths. The central peak is a mix of all wavelengths and usually looks white for normal illumination.

That the resolution depends on the number of slits, N, means that we can improve our spectrometer by using more lines. Here is a representation of what happens as we increase N



we can see that the peaks get narrower as N increases. These graphs are for a particular λ. If the peaks for a particular λ get narrower, then there will be less overlap with adjacent λ's which means that each wavelength can more easily be resolved.

Spectrometers are used in many places. One that has some public interest today is monitoring the atmosphere. Instruments like the one shown below detect the amount of special gasses in the atmosphere using IR spectrometers.



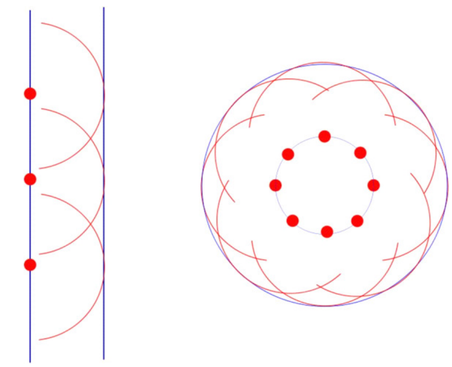
AIRS sensor, spectrometer design, and global CO_2 data. (Images in the Public Domain courtesy NASA)

The instrument shown is the AIRS spectrometer. You can see in the diagram that it uses a grating spectrometer. The picture of the Earth is a composite of AIRS data showing the northern and southern bands of CO_2.

42.2 Single Slits

We have looked at interference from two slits, and for many slits. The two slits acted like two coherent sources. We might expect that a single slit will give only a single bright spot. But let's consider a single slit very closely. To do this, let's return to the work of Huygens.[1] His idea for the nature of light was simple. He suggested that every point on the wave front of a light wave was the source (the disturbance) for a new set of small spherical waves. The next wavefront would be formed by the superposition of the little "wavelets." Here is an example for a plane wave and a spherical wave.

[1] Huygens method is technically not a correct representation of what happens. The actual wave leaving the single opening is a superposition of the original wave, and the wave scattered from the sides of the opening. You can see this scattering by tearing a small hole in a piece of paper and looking through the hole at a light source. You will see the bright ring around the hole where the edges of the paper are scattering the light. But the mathematical result we will get using Huygens method gives a mathematically identical result for the resulting wave leaving the slit with much less high power math. So we will stick with Huygens in this class.



In each case we have drawn spots on the wave front and drawn spherical waves around those spots. where the wavefronts of the little wavelets combine, we have new wave front of our wave. This is sort of what happens in bulk matter. Remember that light is absorbed and re-emitted by the atoms of the material. This is why light slows down in a medium. Because of the time it is absorbed, it effectively goes slower. But the light is not necessarily re-emitted in the same direction. Sometimes it is, but sometimes it is not. This creates a small, spherical wave (called a wavelet) that is emitted by that atom. So Huygens idea is not too bad.

We can use this idea for a single slit and look at what happens as the light goes through. Here is such a slit.

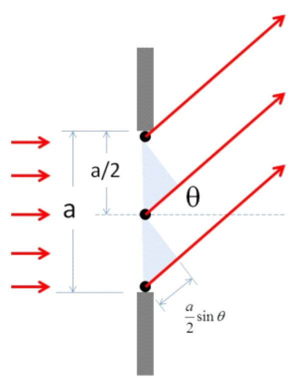
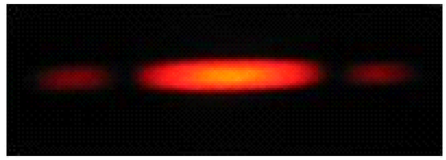


Figure 42.1:

In the figure above, we have divided a single slit of width a into two parts, each of size $a/2$. According to Huygens' principle, each position of the slit acts as a source of light rays. So we can treat half a slit as two coherent sources. These two sources should interfere. So what do we see when we perform such an experiment?



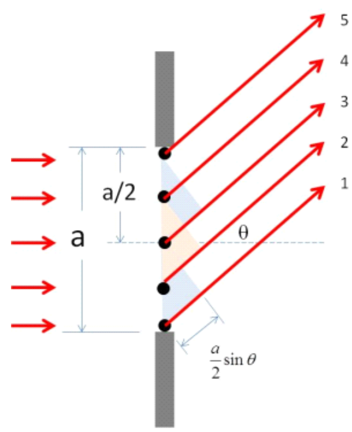
The figure shows a diffraction pattern for a thin slit. There are several terms that are in common use to describe the pattern

1. Central Maximum: The broad intense central band.
2. Secondary Maxima:The fainter bright bands to both sides of the central maxima
3. Minima: The dark bands between the maxima

Let's see how these structures are formed.

42.3 Narrow Slit Intensity Pattern

Let's use figure 42.1 to find the dark minima of the single slit pattern. First we should notice that figure 42.1 could have another set of rays that contribute to the bright spot because they will also have a path difference of $(a/2)\sin\theta$. Let's fill these in. They are rays 2 and 4 of the next figure.



Before we started with what we are now calling rays 1 and 3. Ray 1 travels a distance

$$\delta = \frac{a}{2}\sin(\theta) \tag{42.6}$$

farther than ray 3. As we just argued, rays 2 and 4 also have the same path difference, and so do rays 3 and 5. If this path difference is $\lambda/2$ then we will have destructive interference. The condition for a minima is then

$$\frac{a}{2}\sin(\theta) = \pm\frac{\lambda}{2} \tag{42.7}$$

or

$$\sin(\theta) = \pm\frac{\lambda}{a} \tag{42.8}$$

Now we could also divide the slit into four equal parts. Then we have a path difference of

$$\delta = \frac{a}{4}\sin(\theta) \tag{42.9}$$

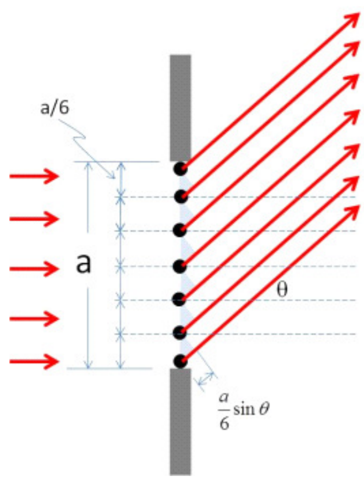
and to have destructive interference we need this path difference to be $\lambda/2$

$$\frac{a}{4}\sin(\theta) = \pm\frac{\lambda}{2} \tag{42.10}$$

or

$$\sin(\theta) = \pm\frac{2\lambda}{a} \tag{42.11}$$

We can keep going to find a minima at



$$\sin(\theta) = \pm\frac{3\lambda}{a} \tag{42.12}$$

and in general at

$$\sin(\theta) = m\frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3 \ldots \tag{42.13}$$

We only found the dark spots in a single slit intensity pattern. The bright spots must be in between the dark spots, but finding them is a little more trouble than finding the dark spots. Do do this, let's look at the intensity function for a single slit interference pattern.

42.3.1 Intensity of the single-slit pattern

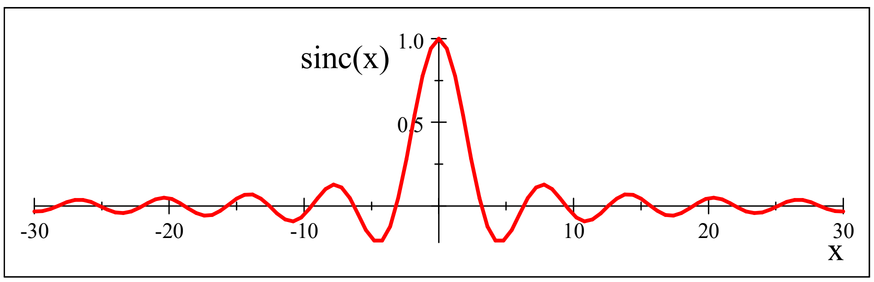
We could derive the single slit intensity pattern, it's not too hard to do. But instead I will just give the result here and we will interpret that result.

$$I = I_{\max}\left(\frac{\sin\left(\frac{\pi}{\lambda}a\sin\theta\right)}{\frac{\pi}{\lambda}a\sin\theta}\right)^2 \tag{42.14}$$

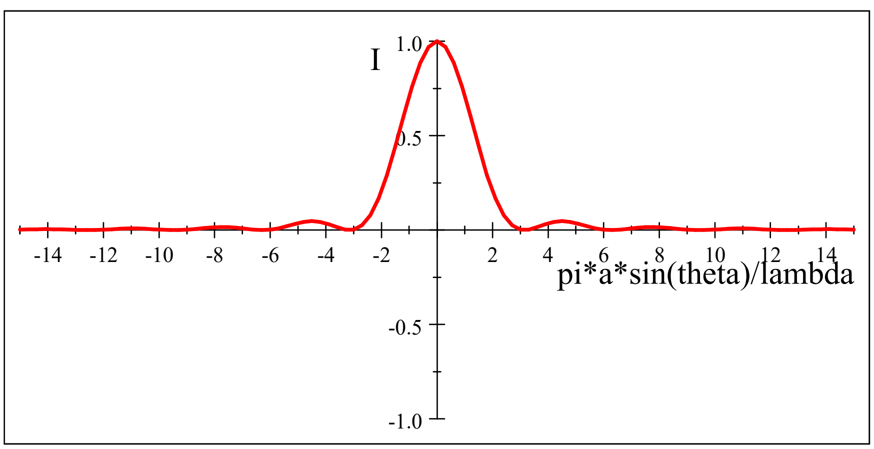
Notice this has the form

$$\frac{\sin x}{x}$$

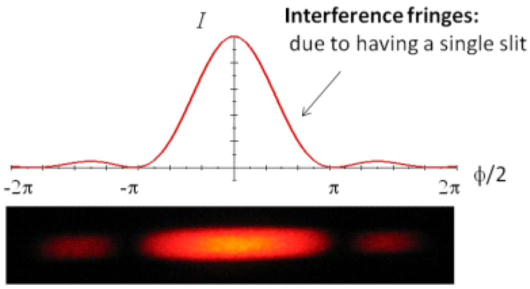
which has a distinctive shape.



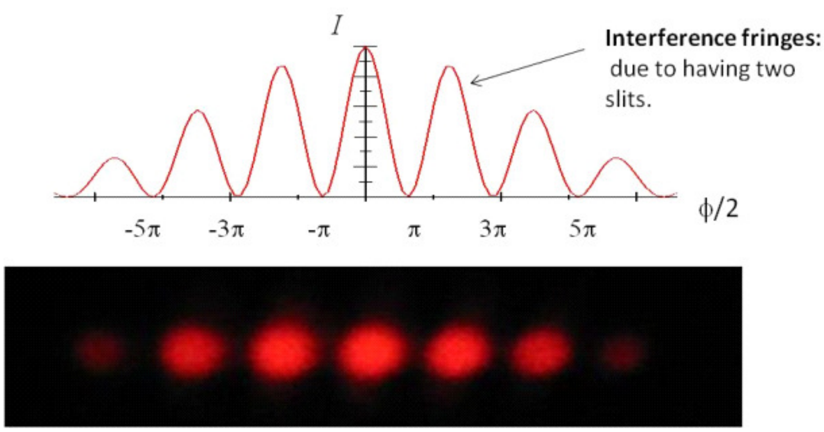
This is known as a sinc function (pronounced like "sink"). It has a central maximum as we would expect. Of course our intensity pattern has a sinc squared



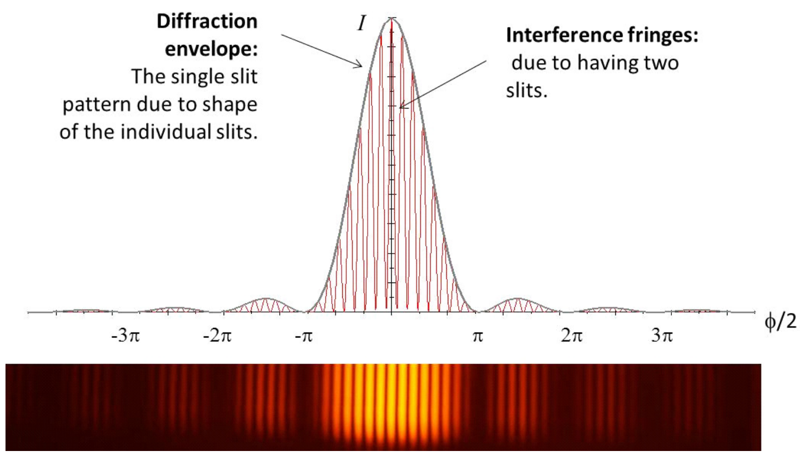
You can see the central maximum and the much weaker minima produced by this function. Indeed, it seems to match what we saw very well. Putting it all together, our pattern looks like this.



This is really an interesting result. You might wonder why, when we found the two slit interference pattern, there was no evidence of the single slit fringing that we discovered in this chapter. After all, a double slit system is made from single slits. Shouldn't there be some effect due to the fact that the slits are individually single slits? The answer is that we did see some hint of the single slit pattern. Remember the figure below.



The intensity of the peaks seems to fall off with distance from the center. We dealt with only the center-most part of the pattern. If we draw the pattern for larger angles, we see the following.



It takes a bright laser or dark room to see the secondary groups of fringes easily, but we can do it. We can also graph the intensity pattern. It is the combination of the two slit and single slit pattern with the single slit pattern acting as an envelope.

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin(\theta)}{\lambda}\right)\left(\frac{\sin\left(\frac{\pi a \sin(\theta)}{\lambda}\right)}{\frac{\pi a \sin(\theta)}{\lambda}}\right)^2 \qquad (42.15)$$

Note that one of the double slit maxima is clobbered by a minimum in the single slit pattern. We can find out the order number of the missing maximum. Recall that

$$d \sin(\theta) = m\lambda$$

describes the maxima from the double slit. But

$$a \sin(\theta) = \lambda$$

describes the minimum from the single slit. Dividing these yields

$$\begin{aligned} \frac{d \sin(\theta)}{a \sin(\theta)} &= \frac{m\lambda}{\lambda} \\ \frac{d}{a} &= m \end{aligned}$$

so the

$$m = \frac{d}{a} \qquad (42.16)$$

double slit maximum will be missing.

Basic Equations

Many slits

$$\delta = d \sin(\theta_{bright}) = m\lambda \quad m = 0, \pm 1, \pm 2, \ldots$$

Single Slit minima

$$\sin(\theta) = m\frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3 \ldots$$