PH223: Physics for Chemists and Mechanical Engineers

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[1] Think of this like the special case of a capacitor made from two flat large plates, the parallel plate capacitor. It was somewhat ideal in the way we treated it. Our treatment of the special case of a coil will likewise be somewhat ideal.

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Chapter 1

Introduction and Electric charge

Fundamental Concepts

- There is a property of matter called "charge."
- There seem to be two types of charges, called "positive" and "negative."
- We have a model for how charge acts. The model tells us there are two types of charge, and that charges of similar type repel and charges of different type attract.
- In metals, the valence electrons are free to move around. We call materials where the charges move "conductors."
- Materials where the valence electrons cannot move are called "insulators."
- In insulators, the atoms can "polarize."

1.1 What is this class?

This class is designed to teach the physics of wave motion, electricity and magnetism, and optics. We have three major goals. One is to teach the physics of electrical attraction that forms the basis for chemical bonding. The same physical principles of electricity and magnetism affect many electrical designs as well, so this is also a goal for our mechanical engineers. The second goal is to teach wave motion well enough that the quantum nature of atoms and molecules make sense as our chemists take physical chemistry. Of course wave motion is also a major design tool for mechanical engineers and is also a major design problem for structural designs. The third goal is to teach optics enough that both chemists and engineers can understand the basics of optical instruments that will be used in chemistry or machine vision. For the chemists we need to understand the fundamental physics. For the engineers we need practical physics that can affect mechanical systems that our engineers will design, build, or test. We will need to spend time both on the fundamentals and on the practical use of those fundamentals.

In engineering, the design parameters are often the goal. In science, the physical relationship is the goal. For design engineers, both views are useful and important. The design is no good if the underlying principles preclude it from working!

As an example, I once worked on an optics project with a strong mechanical component. The system had scanning mechanisms that were fantastic mechanical devices. It was part of an aircraft and integrated into the aircraft system. But the optical system required two lasers that were separated in wavelength by only a few nanometers. The chief engineer knew how to build all the systems, but did not understand the physics that required the close wavelength spacing. He judged that the difficulty in building the device at that wavelength spacing outweighed any benefit, and he changed the specs to give two wavelengths that were fifty nanometers apart. Fifty nanometers is a pretty small tolerance. Surely it would be good enough! The resulting product did not work. For two years he tried to fine tune the scanners, and servos to make it work. After ten million dollars and two years, he finally moved the wavelengths closer. The cost of the change was an extra \$100,000 dollars, about 1/100 of the cost of the mistake. The system worked, but since this was a race to market, the time lost and the reputation lost on the faulty product destroyed the viability of the business. It is a bad day when you and your friends lose your jobs because you made a fundamental physics mistake!

Physics courses stress how we know what we know. They support the discipline called *system engineering*, which deals with the design of new and innovative products. As a more positive example, the National Weather service often releases requests for proposed weather sensing equipment. Their request might say something like the following:

> Measure the moisture of the soil globally from an altitude of 800 km with an accuracy of 5%. The suggested instrument is a passive microwave radiometer.

The job of a system engineer is to determine what type of instrument to build. What is the underlying principle that it will use to do its job? What signal processing will it need? What mechanical and electrical systems will support this? This must all be determined before the bearings and slip-rings, and structures can be designed and built.

The radiometer design that came out of this project is flying today (or one very like it based on the original design) and is a major part of the predictive models that tell us what the weather will be in a few days.

Because this type of reasoning is our goal, we will not only do typical homework problems, but we will also work on our conceptual understanding.

I will also emphasize a problem solving method that I used with my engineering team in industry. It is a structured approach to finding a solution that emphasizes understanding as well as providing a numeric answer for a particular design. When you are part of an innovative design team, you will have to repeat a calculation over and over again each time some other part of the sign changes. If you have produced a symbolic solution, a numerical model, or at least a curve, you are ready for any changes in specifications. But if you have just "found the answer" you will have to find that answer again every time the overall design specs change. This approach is too slow, and, at least in my team, would have you finding a new job because our design efforts were always done against exacting schedules and budgets. By thinking in a structured method, with an eye toward symbolic answers or relationships rather than end numbers, you will learn to be a more valuable engineer. The process we will use is the same approach I used to teach my new engineers in the defense industry. It has been proven useful over and over for decades.

This same problem solving process is useful in chemistry, particularly as you study physical chemistry.

So let's get started. To understand waves, we need to get the waves moving. You studied Oscillation in Dynamics or PH121. Oscillating systems are often the disturbance that starts a wave. We will begin with a review of oscillation.

1.2 Charge model

So far we have claimed that light is a wave in an electromagnetic field. We talked about light waves being made in the electromagnetic field by moving charges. But we have not proved it this to be the case. We will find that it will take the rest of the semester to do so! We need to start by looking at charge and what makes charges move.

But let's think about this conceptually and see if we can motivate our study.

We know that there is an electromagnetic spectrum, and that visible light is just a small part of that spectrum. Radio waves are also part of the spectrum of light.

And we should review, how are radio waves produced? We know electricity is involved.

The answer is that charged particles, like the electrons flowing through the antenna of our radio station, create an electromagnetic field. That field is drug along when the electrons move in the antenna. If we make the electrons oscillate, we can make waves in the field. This is much like having a 3rd grade class all hold the edges of a parachute and having the 3rd graders jump up and down. Waves are made in the parachute.

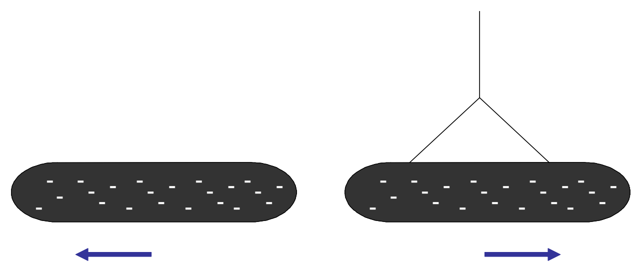
But what is charge? How do we know there are such things as charged particles?

That is the subject we will take up next. Then we will study the motion and actions of these charged particles. Finally we will show that the fields made by charged particles can act as a medium for waves, and that there is good evidence that those waves exist.

1.2.1 Evidence of Charge

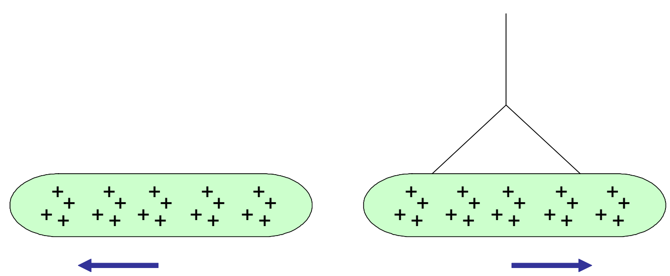
Let's start with something we all know. Let's rub a balloon in someone's hair. If we do this we will find that the balloon sticks to the wall. Why?

We say the balloon and comb have become *charged*. What does this mean? We will have to investigate this more as we learn more about how matter is structured, but for now let's assume charge is some property that provides this phenomena we have observed with the balloon (i.e. it sticks to the wall). Now lets try rubbing other things. We could rub two rubber or plastic rods.



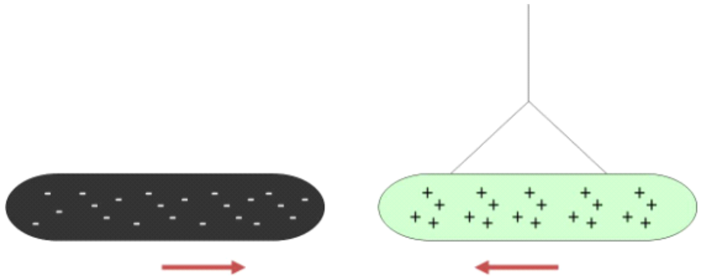
Two charged rubber rods are placed close together. The rods repel each other.

and we could also rub two glass rods



Notice that in each case we have created a force between the two rods. The rods now repel each other.

Now let's try a glass and a rubber rod



Now the two different rods attract each other.

Notice that in our demo, rods that are the same repel and rods that are different attract. We make the intellectual leap that the different rods have different charges. So we are really saying:

1. There are two types of charge.
2. Charges that are the same repel one another and charges that are different attract one another.
3. Friction seems to produce charge, but you have to rub the right materials together.

We will call the rubber or plastic rod charges *negative* and the glass rod charges *positive* but the choice is arbitrary. Ben Franklin is credited with making the choice of names. He really did not know much about charge, so he just picked two names (we will see that in some ways his choice was somewhat unfortunate, but hay, he was an early researcher who helped us understand much about charge , so we will give him a break!).

1.2.2 Types of Charge

We now have reason to believe that there are at least two types of charge, one for rubber and one for glass. But are there more?

Let's start by introducing a new object, only this time we won't rub it with anything.

Now this is strange. The new item is attracted to both rods! What is going on? Have we discovered a new type of charge, one that attracts the other two types we have found?

Maybe, but maybe the explanation of this phenomena is a little different. To understand this, let's consider how charge moves around.

1.2.3 Movement of Charge

One of the strange things about charge is that it is *quantized*. We learned this word in when we found that only certain standing waves could be formed between boundaries. We are using this word in a similar way now. It means that charge has a smallest unit, and that it only comes in whole number multiples of that unit. Charge comes in a basic amount that can't be divided into smaller amounts. So like our standing wave frequencies, only certain amounts are possible As far as we know, the smallest amount of charge possible is the electron charge.[1] This charge we will call negative. We say that the electron is the principle charge carrier for negative charge. This fundamental unit of charge was found to be about

$$e = 1.60219 \times 10^{-19}\,\mathrm{C} \tag{1.1}$$

where the C stands for *Coulomb,* the *SI* unit of charge.

Any larger charge must be a multiple of this fundamental charge

$$Q = n \times e \tag{1.2}$$

The proton is the principle charge carrier for positive charge. From chemistry, you know protons are located in the nucleus of an atom, along with the neutron. In the Bohr model of the atom, the nucleus is surrounded by a cloud of electrons. The proton has the same amount charge as the electron (e), but is opposite in sign.

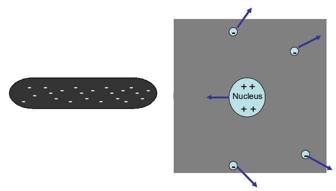
In a gram of mater, there are many, many, units of charge. There are about 5.0125×10^{22} carbon atoms in one gram of carbon. Each carbon atom has twelve protons and about twelve electrons. That is a lot of charge! But notice that the net charge is zero (or very close to it!). It is common for most mater to have zero net charge.

As far as we know, charge is always conserved. We can create charge, but only in plus or minus pairs, so the net charge does not change. We can destroy charge, but we end up destroying both a positive and a negative charge at the same time. The net charge in the universe does not seem to change much. So when something becomes charged, we expect to find that the charge has come from another object.

Lets go back to our rubber rod and glass rod demo. We rubbed the rod that was in our hand, but where did the charge come from? We believe that we are moving charge carriers (usually electrons) from one object to another, stripping them from their atoms. This happens when we use friction (rubbing) to charge the rods.

But what about our object that we did not rub, or our paper (we did not rub the bits of paper). We believe that charge can move, that is why scientists looked for and found charge carriers. Even in an atom, if I bring a charged object near the atom then the negative charge carriers (electrons) will experience a force directed away from the charged object, and the positively charged nucleus will experience a force pulling toward the charge object

[1] I am not counting quarks here, which have a charge of $\frac{1}{3}$ or $\frac{2}{3}$ of the basic electron charge. But still, $\frac{1}{3}$ of the basic electron charge seems to be a real fundamental unit for quark based particles. And quarks aren't stable on their own, so we never see fractional charge in nature.



Notice that the electrons and the nucleus will *attract* each other, so the atom won't split apart. But it will become positively charged on one side because there are more positive charge carriers on that side. It will become more negatively charged on the other side, because there are more negative charge carriers on that side. We could draw the atom like this (figure 1.1)The force due to charge

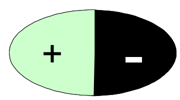
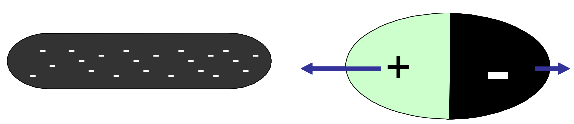
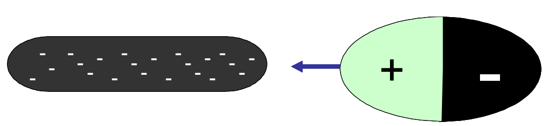


Figure 1.1: Polarized Atom

depends on how far away the charges are from each other. The attractive force between the positively charged side of the atom and the negative rod will have a stronger force than the negatively charged side of the atom and negatively charged rod will experience because the negative side if farther away. We will say that the atom has become *polarized*.

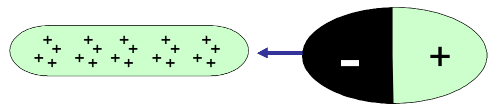


The positive side will experience an attractive force. The negative side will experience a repelling force. The net force due to the charge will be an attractive force. The atom will be accelerated toward the rod! We have seen something like this before. Remember an object in a fluid experiences a downward pressure force on the top, and an upward pressure force on the bottom. The pressure force is larger on the bottom, so there is an upward Buoyant force. The case with our polarized atom is very similar. We have a net electrical attractive force.



Now suppose we have lots of atoms (like our uncharged object or our bits of paper). Will they be attracted to the rod? Yes!

How about if we use a glass rod?



Everything is the same, only we switch the signs. The glass rod is positively charged. It will attract the electrons, and repel the nucleus. The atom becomes charged. The net force is attractive (positive rod and closer negative side of the atom)

We sometimes call the separation of charge in an insulator *polarization*.

1.2.4 Flow of Charge

Let's start by introducing a new object, a salt shaker (my salt shaker is glass with a metal top). We will rub the salt shaker and see if it gets charged by placing it next to our charged rods.

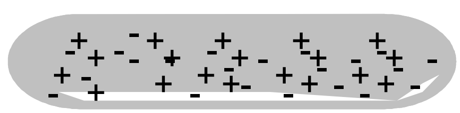
Now this is strange. We rubbed the object, but it was attracted to both rods as if there were no charge. We know glass can be charged. What is the problem?

It turns out that some materials allow charge carriers to flow through them. Our experience with the lighting in our house might suggest that metals will do this. Let's try some other metal objects and see what we find.

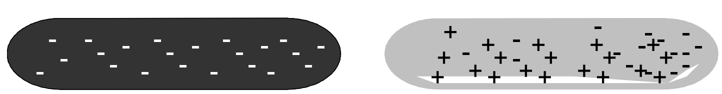
It seems that the atoms are not maintaining a charge separation in these metal atoms! Some materials allow charge carriers to move through them. Usually these materials are metals, but most materials will allow some charge to go through them-even you-which is what is happening in this case. I charge the rod, but the charge leaves through my body. Other materials resist the flow of charge. Materials that allow charge to flow are called *conductors*. Materials that resist the flow of charge are called *insulators*.

1.2.5 Charging by Induction

Knowing that charge carriers can flow though a material, we can think of a way to charge a conductor. Lets suspend a conducting rod.



It is not initially "charged" meaning that it has the same number of positive charges and negative charges, and they are evenly mixed together. I will bring a charged rod next to it.



but let's attach a wire to the other end of the rod to allow the charge to flow away from our conducting rod. We will connect the rod to the ground (in this case, to a water pipe) because the ground seems to be able to accept large amounts of charge carriers. So the charge carriers will flow to the ground.

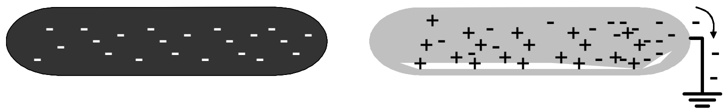
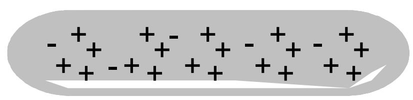


Figure 1.2:

(The strange little triangular striped thing is the electronics sign for a connection to the ground)

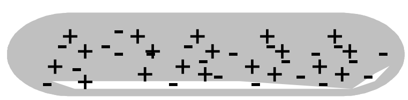
Now let's disconnect the wire from the rod. Is there a net charge on the conducting rod?



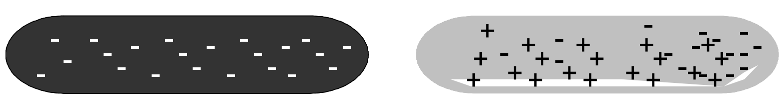
The answer is yes, because we now have more positive charges in the conducting rod than we have negative charges, so the net charge is positive.

1.2.6 Charging by Conduction

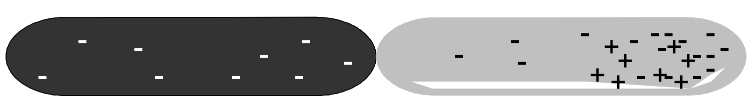
Suppose instead, I perform the same experiment, but I touch the rods. Now charge carriers can flow. Starting with and uncharged conductor,



I again bring in a charged rod. Again the charges separate in our conducting rod.



Then we touch the two rods. The excess charge on our charged rod flows to the conductor. Since in our drawing, the excess charge is negative, then some of the positive charge on the conductor is neutralized.



When we separate the rods, our conducting rod will have an excess of negative charge.

Notice that there is something different in our study of this new force. In the past, it was easy to tell which object was creating the environment and which was the mover. The Earth, being so much larger than normal objects, was the environmental object creating the gravitational acceleration that balls and cars and people move it. Then the balls and cars and people were the movers. Generally the thing causing the force, the environmental object, was much bigger than the mover. That is not true in our charge experiments so far. The rods are about the same size. So which is the environmental object and which is the mover? We will have to pick one to be our environmental object, and the other to be our mover. Sometimes the context of the problem helps. If the problem you are solving asks for the motion or the force on the rod on the right side of the diagram, then it is the mover and the rod on the left is the environmental object. If one charge is much larger than the other, we might be justified in calling this large charge the environmental object and a smaller charge near the big charge would be the mover.

1.3 Our model for Charge

Let's summarize what we tried to learn last time:

Model for Charge
Frictional forces can add or remove charge from an object
There are two, and only two kinds of charge
Two objects with the same kind of charge repel each other
To objects with different kinds of charge attract each other
The force between two charged objects is long ranged
The force between two charged objects decreases with distance
Uncharged objects have an equal mix of both kinds of charge
There are two types of materials, conductors (in which charges can move) and insulators (in which charges are fixed in place)
Charge can be transferred from one object to another by contact between the two objects

A serious shortcoming of this model is that it does not tell us what charge is. This is a shortcoming we will have to live with. We don't know what charge is any more than we can say exactly what mass or energy are. Charge is fundamental, as far as we can tell. We can't find a way to change charge into something else or to change something else into charge. For fundamental particles (like protons and electrons) either a particle has charge, or it does not.

1.3.1 Conservation of charge

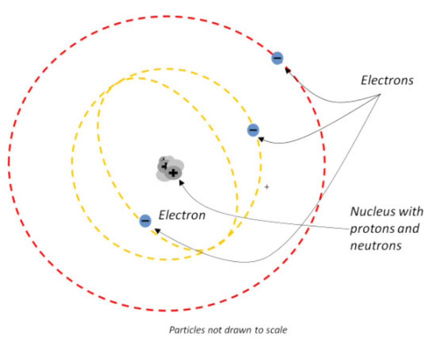
In some ways, this is really great! We have a new quantity that does not ever change. We can say that charge is conserved in the universe. Like energy, we can move charge around, but we don't create or destroy it. When we rubbed the plastic rods with rabbit fur or wool, we were removing charge that was already there in the atoms of the fur. If you take PH279 you might find that there are some caveats to this rule. We can make positron and electron pairs from high energy gamma rays. But when we do this we must always make a pair; one positive, and one negative. So the net charge remains unaffected.

1.4 Insulators and Conductors

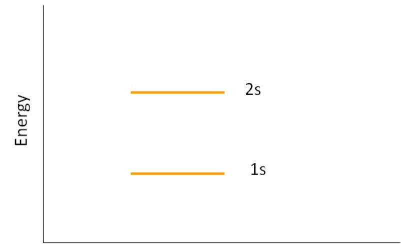
Let's return to charges and atoms. We have an intuitive feeling for what is a conductor and what is an insulator, but let's see why conductors act the way they do.

1.4.1 Potential Diagrams for Molecules

Back in high school or in a collage chemistry class you learned that electrons move around an atom.



In the figure there are two energy states represented. You may even remember the names of these energy states. The orange-yellow lines show one "orbital distance" for the electrons near the nucleus. The red line shows another electron at a larger orbital distance. The inner orbital is a $1s$ state and the outer orbital is a $2s$ state. If these were satellites orbiting the earth, you would recognize that the two orbits have different amounts of potential energy. This is also true for electrons in orbitals. If we plot the potential energy for each state we get something that looks like this



(Schematic)

You can think of this as potential energy "shelves" where we can put electrons. If you were a advanced high school student, you learned that on the first two shelves you can only fit two electrons each. The higher shelves can take six, and so forth. But that won't concern us in this class.

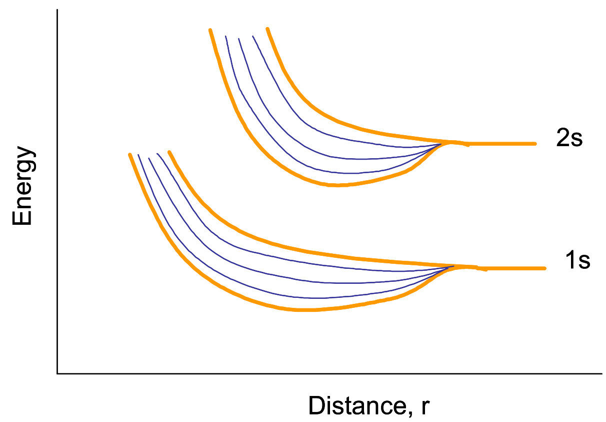
1.4.2 Building a solid

So far I have really only talked about single atoms. What happens when we bind atoms together? Let's take two identical atoms. When they are far apart, they act as independent systems. But when they get closer, they start acting like one quantum mechanical system. What does that mean for the electrons in the atoms?

Electrons are funny things. They won't occupy the exactly the same energy state. I can only have two electrons in a $1s$ state, but as I bring two atoms near each other I will have four! How does the compound solve this problem? The energy "shelves" split into more shelves. As the atoms get closer, we see something like this

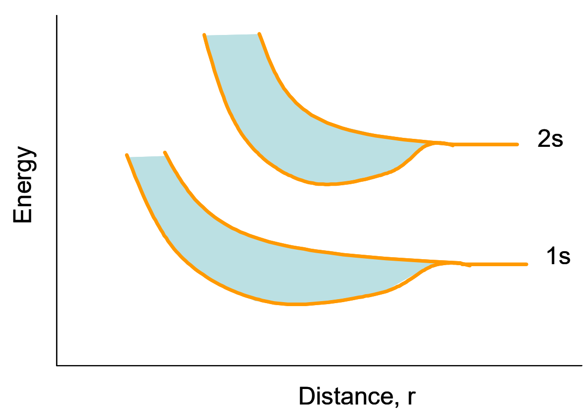
(Schematic)

At some distance, r, the states split. So each electron is now in a different state. Suppose we bring 5 atoms together.



(Schematic)

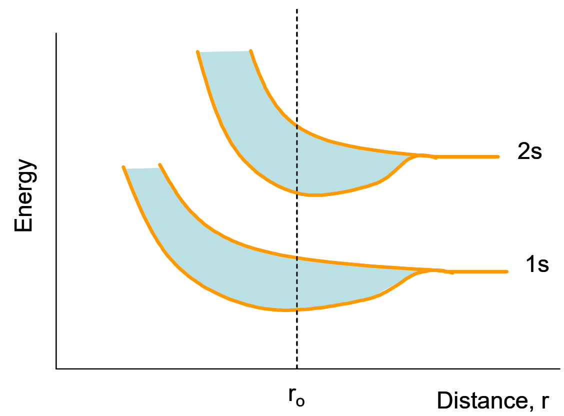
I get additional splitting of states. Now I have five different $1s$ states, enough for 5 atoms worth of $1s$ electrons. But solids have more than five atoms. Let's bring many atoms together.



(Schematic)

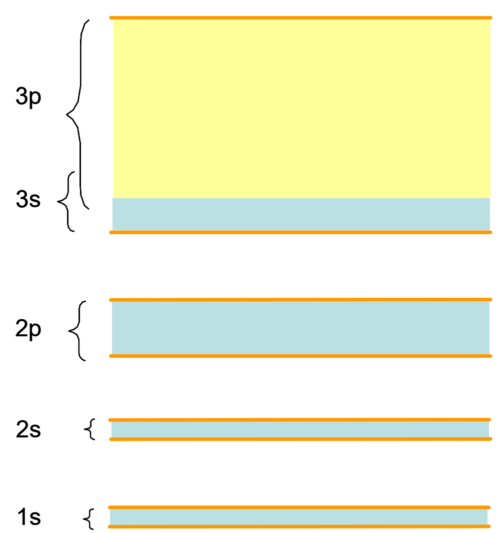
Now there are so many states that we just have a blue blur in between the original two split states. We have created a nearly continuous set of states in two bands. Each electron has a different energy, but those energy differences might be tiny fractions of a Joule. The former two states have almost become continuous bands of allowed energy states.

The atoms won't allow themselves to be too close. They will reach an equilibrium distance, r_o where they will want to stay.



(Schematic)

Since this is where the atoms usually are. We will not draw the whole diagram anymore. We will instead just draw bands at r_o.(along the dotted line). Here is an example.



This means we have *bands* of energies that are allowed, that electrons can use, and *gaps* of energy where no electron can exist.

1.5 Conduction in solids

Notice that in our last picture, the $3s$ and $3p$ bands have grown so much that they overlap. The situation with solids is complicated. Notice also that the lower states are blue. We will let blue mean that they are filled with electrons taking up every available energy state. The upper states are only partially filled. Yellow will mean the energy states are empty. We will call the highest completely filled band the *valance band* and the next higher empty band the *conduction* band.

We have three different conditions possible.

1.5.1 Metals

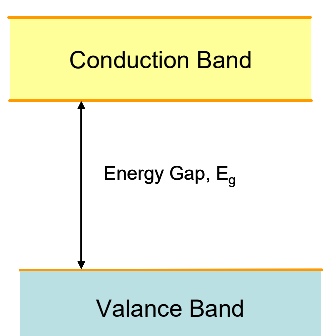
In a metal, the highest occupied band is only partially filled

the electrons in this band require only very little energy to jump to the next state up since they are in the same band and the allowed energies are very closely spaced. Remember that movement requires energy. So if I connect a battery to provide energy, the electrons must be allowed to gain the extra energy, kinetic energy in this case, or they will not move. But in the case of a metal, there are easily accessible energy states, and the electrons flow through the metal.

We can say that the outer electrons are shared by all the atoms of the entire metal, so the electrons are easy to move for metals.

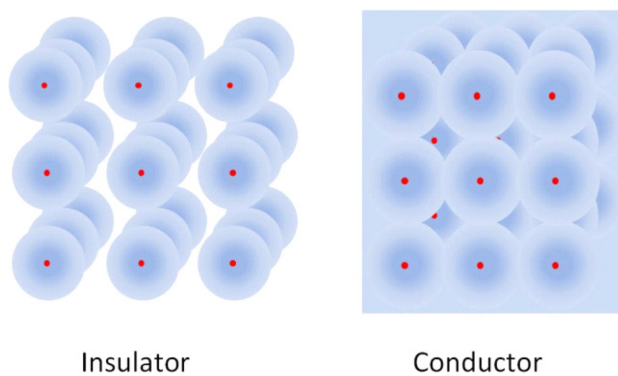
1.5.2 Insulators

A second condition is to have a full valance band and an empty conduction band. The bands are separated by an energy gap of energy E_g.



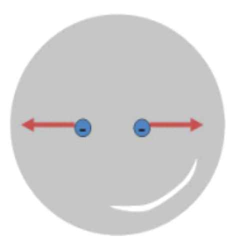
In this case, it would take a whopping big battery to make the electrons move. The battery would have to supply all of the gap energy plus a little more to get the electron to move. You might envision this as if there were an electrical "glue" that keeps the electrons in place. Before they can move, you have to free them from the "glue." It takes an amount of energy, E_g, to free the electrons before they are able to accept kinetic energy. If we do connect a very large battery, say, 33000 V, then we can get electrons to jump the gap to a higher energy "shelf." But high voltages are not normal conditions, so this is not usually the case. A material that has a large energy gap between it's valance band and an empty conduction band is called an insulator.

A mental picture for this might be as shown in the next figure.



The insulator atoms keep their valence electrons bound to the nuclei of the atoms. But for a conductor, the valence electrons are free to travel from atom to atom.

In an isolated conductor, normally the charge is balanced, so the electrons may move but generally they stay near a nucleus. But if a conductor has extra electrons, the electrons that can move will move because they repel each other. So any extra charge will be on the surface of the conductor.



This happens very quickly, generally we do find the extra charge distributed on the outside of a conductor.

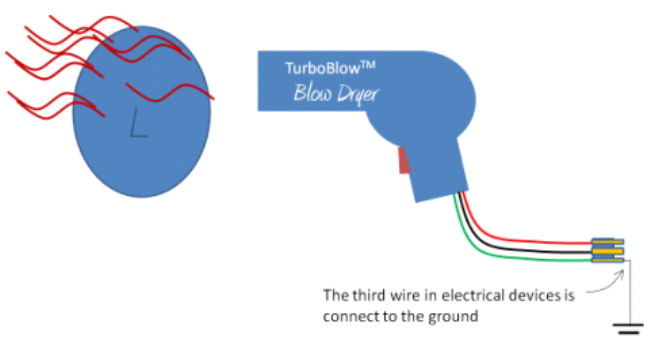
1.5.3 Semiconductors

The third choice is that there is a band gap, but the band gap is small. In this case, some electrons will gain enough thermal energy to cross the gap. Then these electrons will be in the conduction band. Devices that work this way are called semiconductors. We won't deal with semiconductors much in this class, but you probably used many of them in ME210. Diodes, and transistors are made from semiconductors.

1.5.4 Charging and discharging conductors

Conductors can't usually be charged by rubbing. The electrons in the conductor may move when rubbed, but then they are free to move around in the conductor, so they don't leave. But if we rub an insulator, the electrons are not free to travel in the insulator material, so we can break them free. Once this happens, we can take our charged insulator and place it in contact with a conductor. The charge can flow from the insulator to the conductor (and arrange itself on the conductor surface). Once the charge has moved to the exterior, it will reach what we call *electrostatic equilibrium*. All of the repelling electrical forces are in balance, so the charges come to rest with respect to the conductor.

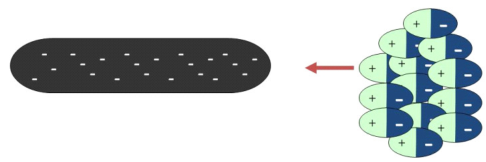
We can remove the extra charge by creating a path for the charge to follow. Consider charging a balloon by rubbing it on your hair. Then you connect a wire to the balloon that is also connected to a metal water pipe. The charge can flow through the metal conducing wire. If there is a large body that can attract extra charge, the charge will flow. The Earth is such a large body that can attract the extra charge. The charge will flow through the wire and pipe and go into the ground.



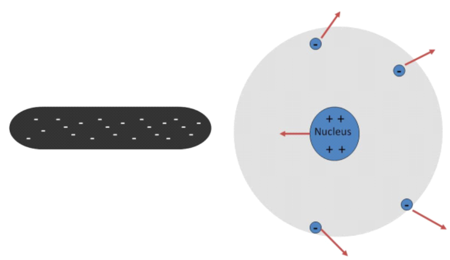
You may have heard of electrical grounds. This literally means tying your device to the Earth through a wire. Since you are made mostly of water that contains positive ions, you are also a conductor. So if we touch a charged object, we will most likely discharge the object. This is also why we must be careful with charge. Large amounts of charge flowing through us leads to death or injury.

If an object is *grounded*, it cannot build up extra charge. This is good for appliances and houses, and people.

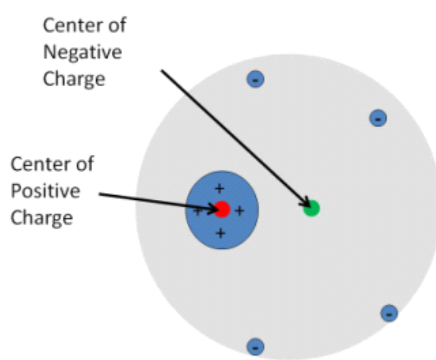
We talked last time about insulator atoms being polarized.



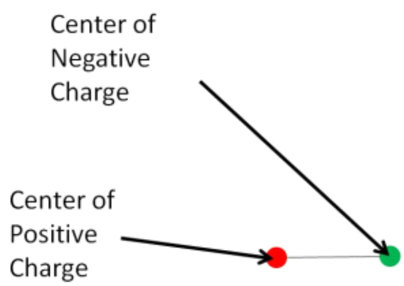
Remember that for each atom the electrons are displaced relative to the nucleus.



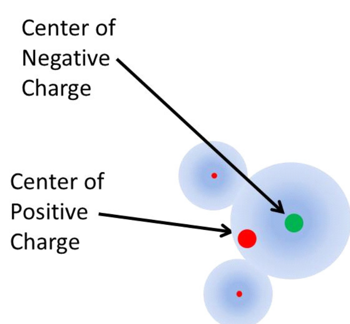
We can define a *center of charge* much like we defined a center of mass. In the case in the figure, we can define a negative center of charge and a positive center of charge.



Notice that the negative and positive center of charge are not in the same place when the atom is polarized. We have a name for a pair of positive and negative charges that are separated by a distance, but that are still bound together. We call it an *electric dipole.* Often we just draw the centers of charge joined by a line.



Using this we can explain why humidity affects our last lecture experiments so much. The water molecule has two hydrogen atoms and one oxygen atom. The covalent bond between the oxygen and hydrogen atoms forms when the oxygen "shares" the hydrogen's electrons. The electrons from the hydrogen atoms spend their time with the oxygen atom making one side of the molecule more positive and the other side more negative.



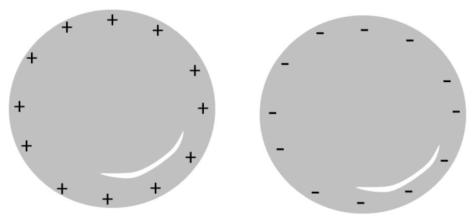
Thus if you have a charged balloon on a humid day, one side of the water molecules in the air will be attracted to the extra charge on the balloon. The extra charge will attach to the water molecules, and float away with them. This will discharge the balloon.

1.6 Note on drawing charge diagrams

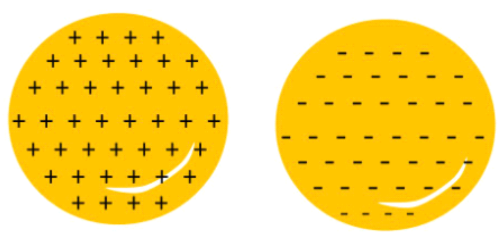
We will have to draw diagrams in our problem solutions. Normally we won't draw atoms, so we will be drawing large objects with or without extra charge. We know that all materials have positive nuclei and negative electrons. When these are balanced, there is an electron for every proton, so if we add up the charges we get zero net charge. These charges don't contribute to net forces because for every attraction there is a repulsion of equal magnitude.

So we won't draw all of these charges, but we should remember they are there. We usually draw a cross section, so here is the cross section of a round, conducting ball.

But if we have extra charge, we should draw it. We will just add plus signs or minus signs. We won't draw little circles to show the electrons (we can't draw them to scale, they are phenomenally small). Here is an example of two round objects, one positive and one negative



If the objects are not conductors, the extra charge may be spread out. We draw the charge throughout the cross section of the object.



Note that if you transfer charge, from one object to another, you should try to keep the same total number of "+"' or "-" signs to show the charge is conserved.

Basic Equations

None so far.