Chapter 34

Doppler Effect and Superposition

Fundamental Concepts

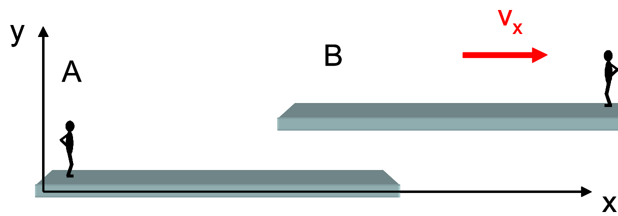
- The frequency of a wave depends on the relative motion between the source and detector.
- Two waves in the same medium add up point for point at every location in the medium. This process is called superposition.

34.1 Doppler Effect

We have learned what happens when a sound wave is generated. But so far, we have assumed that sound emitter was staying still. But we know of many sound emitters that move. What happens if the emitter of the sound is moving? Worse, Back in PH121 or Dynamics we considered the relative motion between two reference frames. What happens to the sound emitter is stationary, but we, the listener, are moving?

Let's start by considering an inertial reference frame like we did when we were studying electric and magnetic fields. Only this time there are no fields so we can go back to calling our reference frames reference frame A and reference frame B.

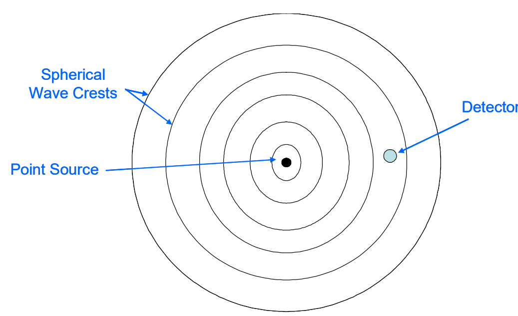
and suppose we position ourselves in reference frame A and we notice reference frame B traveling with a velocity v_x to the right.. Let's also place our reference frames far away from any other object.



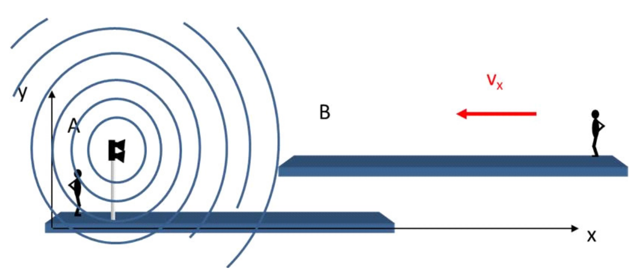
Person A sees himself as stationary and sees person B traveling with velocity $v_{AB} = v_x$. Person B sees himself as stationary, and person A traveling with velocity $v_{BA} = -v_x$. We can't tell which view point is correct. In fact, both are equally valid.

Now suppose one of the reference frames has a sound emitter and one has a sound detector. It seems that whether the emitter moves, or the detector moves, either way if the motion matters, it should matter the same for both cases. From this brief review, it seems that is the *relative* speed v_{AB} that we must consider when thinking about our sound waves.

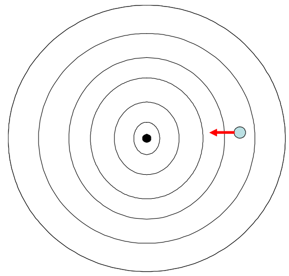
Now suppose we have a wave generator (a point source) creating spherical waves. Let the point source be at rest, say, in frame A.



Let's also assume a detector. If the detector is stationary with respect to the emitter, the detector sees a frequency of the wave, f just as it is created by the emitter. But now lets have the detector be in frame B moving with relative speed $v_{BA} = -v_x$



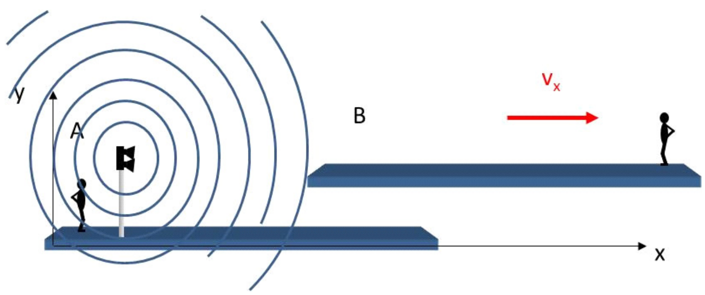
so that it moves relative to the emitter. What will the detector hear? A top view might look like this



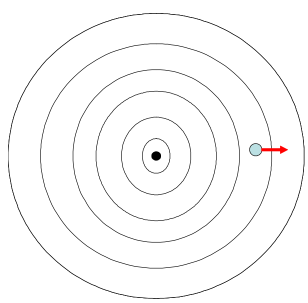
Remember, that the frequency is the number of crests that pass by a given point in a unit time. Does the moving detector see the same number of crests per unit time as when it was stationary?

No, the frequency appears to be higher! That is because every time a wave crest hits the detector, the detector moves toward the next wave crest.

How about if we let the detector move the other way?



The top view might look like this



Again the frequency heard by the detector is different, but this time lower. Each time a wave crest hits the detector, the detector moves away from the next wave crest, giving more time for the wave to catch up to the detector (if the period between wave crests goes up, then the frequency must go down because $f = \frac{1}{T}$).

We can quantify this change. Take our usual variables f_A, λ_A for the stationary emitter, f_B, λ_B for the moving detector, and and the velocity of sound v_{sound}. When the detector moves toward the source, it sees a different velocity.

If you used the subscript system to do relative motion you might identify the speed of the sound wave created by the source in frame A as v_{aA} and the relative speed of the frame A as viewed from frame B as v_{AB}. Then the speed of sound from the source in frame A as viewed in frame B would be.

$$v_{aB} = v_{aA} + v_{AB}$$

This is our normal Galilean transformation. We could abbreviate the subscripts as just.

$$v_B = v_{sound} + v_x$$

Since the detector is riding along with frame B which is moving with speed v_x we could write the detector speed as

$$v_d = v_x$$

$$v_B = v_{sound} + v_d \tag{34.1}$$

In effect, the relative speed adds to the speed of sound making the wave crests come faster from the point of view of the detector. The wavelength will not be changed ($\lambda_A = \lambda_B$), since the distance between wave crests does not change, so

$$v = \lambda f$$

tells us the frequency must change. The new frequency f_B is given by

$$f_B = \frac{v_B}{\lambda} = \frac{v_{sound} + v_d}{\lambda}$$

We can eliminate λ from this expression for the change in f by using $v_A = \lambda f_A$ again, this time solving for λ

$$f_B = \frac{v_{sound} + v_d}{v_{sound}} f_A \qquad \text{observer moving toward the source} \tag{34.2}$$

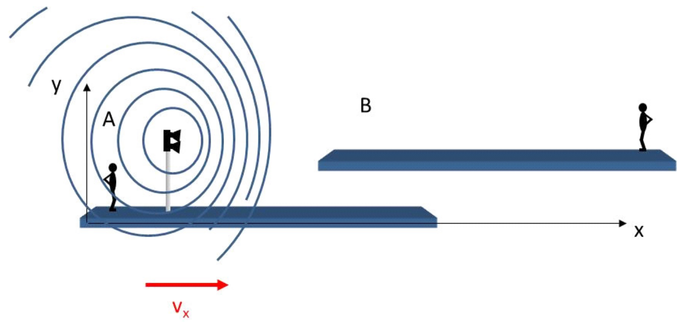
Now if the detector is going the other way v_d is negative.

$$v_B = v_{sound} - v_d$$

We expect that as the wave crest approaches the detector, the detector moves away from it. It takes longer for the crest to reach the detector. The frequency will be smaller.

$$f_B = \frac{v_{sound} - v_d}{v_{sound}} f_A \qquad \text{observer moving away from the source} \tag{34.3}$$

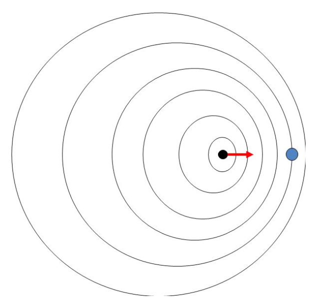
From our thinking about the motion of two inertial reference frames, we expect a similar situation if the detector is stationary and the source moves.



Since the emitter is now riding along with frame A at the relative speed, v_x we could write

$$v_e = v_x$$

where v_e is the speed of the emitter. In this case the detector will see a different wavelength. The top down view might look like this



In fact, if we measure the distance between the crests we must account for the fact that the source moved by an amount

$$\Delta x = v_e T = \frac{v_e}{f_A}$$

during one period. Then the wavelength is seen to be shorter by this amount.

$$\lambda_B = \lambda_A - \frac{v_e}{f_A}$$

Using the basic equation

$$v = \lambda f$$

once more, we can write the frequency as

$$f_B = \frac{v_{sound}}{\lambda_B}$$

and we know λ_B in terms of λ_A

$$f_B = \frac{v_{sound}}{\lambda_A - \frac{v_e}{f_A}}$$

Once more using the basic equation $v = \lambda f$ (rearranged a bit) for reference frame A

$$\lambda_A = \frac{v_{sound}}{f_A}$$

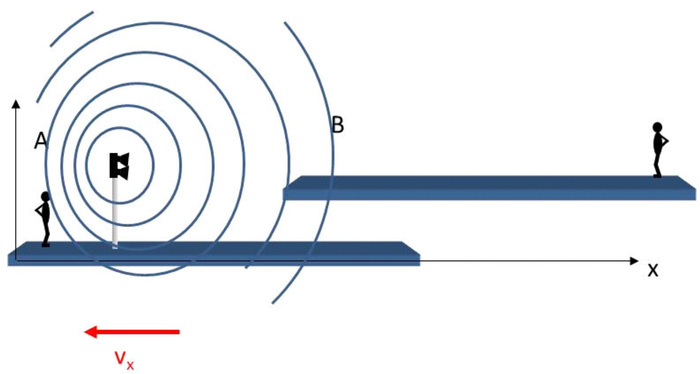
we can write f_B as

$$f_B = \frac{v_{sound}}{\frac{v_{sound}}{f_A} - \frac{v_e}{f_A}}$$

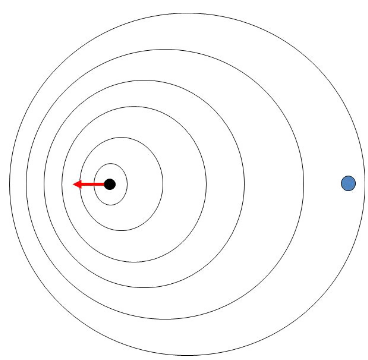
or

$$f_B = \frac{v_{sound}}{v_{sound} - v_e} f_A \qquad \text{Source moving toward observer} \tag{34.4}$$

We should also do the case where the source is moving away from the detector,



The top down view might look like this



we expect the wavelength to be larger. We can go through the math, but we expect it will just change the sine of v_e

$$f_B = \frac{v_{sound}}{v_{sound} + v_e} f_A \qquad \text{Source moving away from observer} \tag{34.5}$$

We can combine these formulae to make one expression

$$f_B = \frac{v_{sound} \pm v_d}{v_{sound} \mp v_e} f_A \tag{34.6}$$

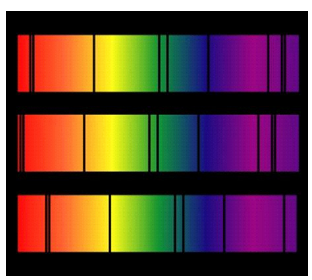
where we use the top sign for the speed when the mover (detector or emitter) is going toward the non-mover.

	Stationary Detector	Detector moves toward Emitter	Detector moves away from Emitter
Stationary Emitter	$f_B = f_A$	$f_B = \frac{v_{sound}+v_d}{v_{sound}} f_A$	$f_B = \frac{v_{sound}-v_d}{v_{sound}} f_A$
Emitter moves toward Detector	$f_B = \frac{v_{sound}}{v_{sound}-v_e} f_A$	$f_B = \frac{v_{sound}+v_d}{v_{sound}-v_e} f_A$	$f_B = \frac{v_{sound}-v_d}{v_{sound}-v_e} f_A$
Emitter moves away from Detector	$f_B = \frac{v_{sound}}{v_{sound}+v_e} f_A$	$f_B = \frac{v_{sound}+v_d}{v_{sound}+v_e} f_A$	$f_B = \frac{v_{sound}-v_d}{v_{sound}+v_e} f_A$

We can see that by moving the emitter or the detector, we get a frequency change. This fact is named after the scientist who studied it. It is called the *Doppler effect* and the change in frequency is called the *Doppler shift*.

34.1.1 Doppler effect in light

Light is also a wave, and so we would expect a Doppler shift in light. Indeed we do see a Doppler shift when we look at moving glowing objects. Here is an optical spectrum of the Sun on the top and a spectrum of a similar star moving away from us in the middle. The final spectrum is for a star moving toward us.

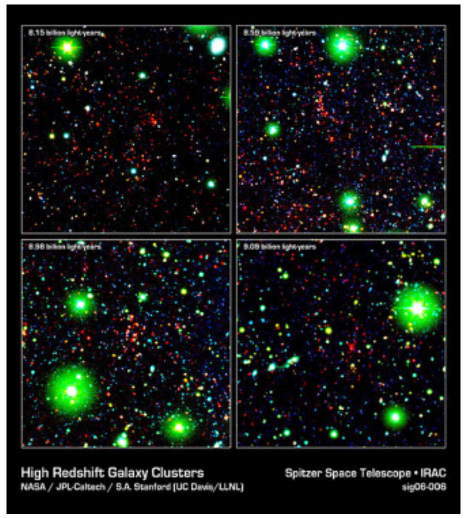


Top: Normal 'dark' spectral line positions at rest. Middle: Source moving away from observer. Bottom: Source moving towards observer. (Public domain image courtesy NASA: http://www.jwst.nasa.gov/education/7Page45.pdf)

Note that the wavelength of the lines is shifted toward the red part of the spectrum when the glowing object moves away from us. This is equivalent to lowering of the frequency of a truck engine noise as it goes away from us. The larger wavelengths indicate a lower frequency of light because

$$f = \frac{c}{\lambda}$$

This gives us a way to determine if distant stars and galaxies are moving toward or away from us. We look for the chemical signature pattern of lines, then see whether they are shifted to the red (moving away from us) or blue (moving toward us) compared to the position in their spectrum of the Sun. This photo is of some of the most distant galaxies that are moving very fast away from us. Their redshift is very large.



High Readshift Galaxy Cluster shown here in false color from the Spitzer Space Telescope. (Public domain image courtacy NASA/JPL-Caltech/S.A. Stanford (UC Davis/LLNL)

Deriving the Doppler equation for light is more tricky because the speed of light is constant in all reference frames. We really tackle this in our Modern Physics (PH279) class. So I will just quote the result here.

$$\lambda_{-} = \lambda_o \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \qquad \text{receding source} \tag{34.7}$$

$$\lambda_{-} = \lambda_o \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \qquad \text{Approaching source} \tag{34.8}$$

For our chemists, think of a group of gas molecules in a sample in a lab. The gas molecules will be moving randomly due to thermal energy. Some will be moving closer to us and some away from us. This means the light emitted from some of the gas molecules will be red shifted and some will be blue shifted. This affects the spectroscopy we use to identify the molecules!

34.2 Superposition Principle

What happens if we have more than one wave propagating in a medium? You probably experienced this as a child. Your parents made you take a bath. You discovered that you could make waves with your arm. But chances are you have two arms, and that you discovered you could make two waves, one with each arm. And when the two waves met in the middle, the water left the bath tub! What happened was that the two wave crests met in the same place and the medium (water) piled up there. We call the combination of two waves in the same medium *superposition.* The word literally means putting one wave on top of another. When we superimpose two waves, their wave functions simply add.

Superposition: If two or more traveling waves are moving through a medium, the resultant wave formed at any point is the algebraic sum of the values of the individual wave forms.

So if we have

$$y_1\left(x,t\right) \tag{34.9}$$

and

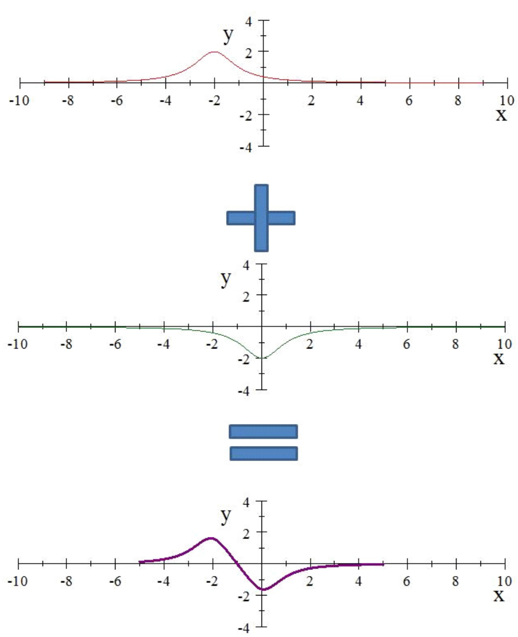
$$y_2\left(x,t\right) \tag{34.10}$$

both propagating on a string, then we would see a resultant wave

$$y_r\left(x,t\right) = y_1\left(x,t\right) + y_2\left(x,t\right) \tag{34.11}$$

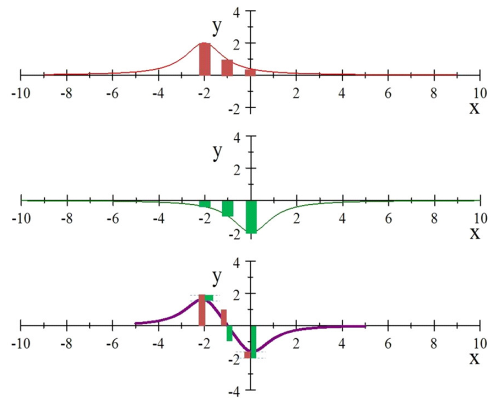
This is a fantastically simple way for the universe to act!

Let's look at an example. let's add the top wave (red) to the middle wave (green). We get the bottom wave (purple)



Of course we are adding these in the snapshot view. So this is all done for just one instant of time.

Let's see how to do this.



As an example, start at $x = -2$. In the figure, I drew a red bar to show the y value at $x = -2$ for the red curve. Likewise, I have a green bar sowing the value of y at $x = -2$ for the green wave. Note that this is negative. On the bottom graph, the bars have been repeated, and we can see that the red bar minus the green bar brings us to the value for the resulting wave at the point $x = -2$. We need to do this at every point along all the waves for this instant of time.

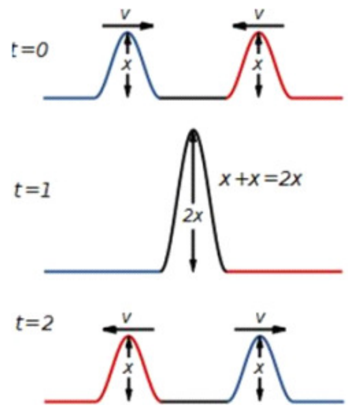
This is tedious by hand, so we won't generally do this calculation by hand. But a computer can do it easily.

Note that this is really only true for *linear* systems. Let's take the example of a Slinky^{TM}. If we form two waves in the Slinky, they behave according to the superposition principle most of the time. But suppose we make the amplitude of the individual waves large. They may travel individually OK, but when the amplitudes add we may overstretch the Slinky. Then it would never return to it's original shape. The wave form would have to change. Such a wave is not linear. There is a good rule of thumb for when waves are linear.

A wave is generally linear when its amplitude is much smaller than its wavelength.

34.2.1 Consequences of superposition

Linear waves traveling in media can pass through each other without being destroyed or altered!



Constructive Interference (Public Domain image by Inductiveload, http://commons.wikimedia.org/wiki/File:Constructive_interference.svg)

Our wave on the string makes the string segments move in the y-direction. Both waves do this. So putting the two waves together just makes the string segments move more! There is a special name for what we observe

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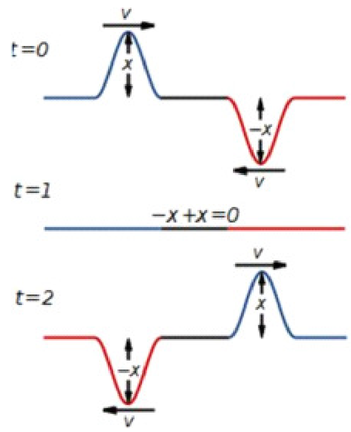
interference: The combination of separate waves in the same region of space to produce a resultant wave.

In our last example the amplitude of the combined wave was larger than the amplitude of the individual waves . We also have a special name for when this happens.

Constructive Interference: interference between waves when the displacement caused by the combined wave is larger that the displacements of the individual waves.

What happens if one of the pulses is inverted?



Destructive Interference (Public Domain image by Inductiveload, http://commons.wikimedia.org/wiki/File:Destructive_interference1.svg)

When the two pulses meet, they “cancel each other out.” But do they go away? No! the energy is still there, the string segment motions have just summed vectorially to zero, the energy carried by each wave is still there in the stretched string. Because we momentarily seem to destroy the wave pulses, we call this type of interference “destructive interference.”

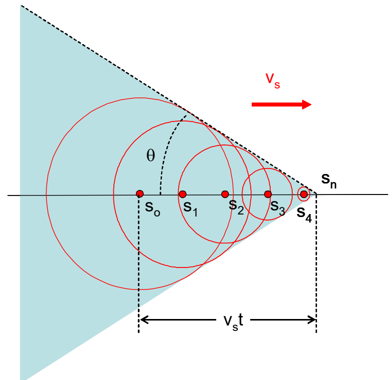
Destructive Interference: *Interference between waves when the displacements caused by the two waves are opposite in direction and the resulting wave displacement is zero.*

34.3 Superposition and Doppler: Shock waves

What happens when the speed of the source is greater than the wave speed?

Remember that the wave speed depends only on the medium. Let’s call the crests of a wave the *wave front*. In the picture below, a point source is generating a wave and the red lines are the wave fronts.

When $v_s = v_{sound}$ the waves superimpose. They begin to pile up. If we allow $v_s > v_{sound}$ then the wave fronts are no longer generated within each other.



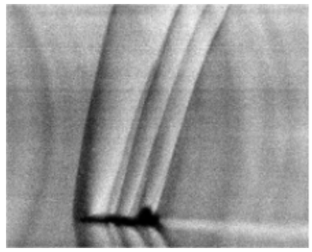
The leading edge of the wave fronts superimpose to form a cone shape. The half angle of this cone is called the *Mach angle*

$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s} \tag{34.12}$$

This ration v/v_s is called the Mach number and the conical wave front is called a shock wave. We see them often in water

Boat wakes as a Doppler cone. Image courtesy US Navy. Image is in the Public Domain.

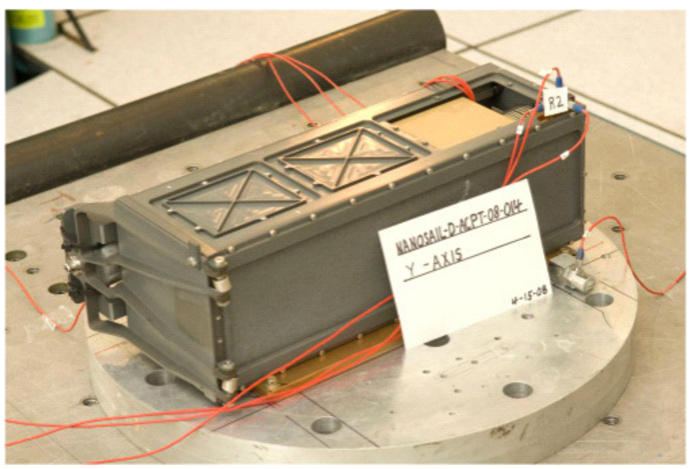
and hear them when jet aircraft go supersonic. In the next figure we can see a picture of a T-38 breaking the sound barrier. You can see the Mach cones, but notice that there are several! Remember that a disturbance creates a wave. There are disturbances created by the nose of the plane, the rudder, and the wings, and perhaps the cockpit in this Schlieren photograph.



Dr. Leonard Weinstein's Schlieren photograph of a T-38 Talon at Mach 1.1, altitude 13,700 feet, taken at NASA Langley Research Center, Wallops in 1993. Image Courtacy NASA, image is in the Public Domain.

34.4 Importance of superposition

The combination of waves is important for both scientists and engineers. In engineering this is the hart of vibrometry.



Marshall and Cal Poly technicians wired the NanoSail-D spacecraft to accelerometers, instruments which measure vibration response during simulated launch conditions. Image couracy NASA, image in the Public Domain.

Mechanical systems have moving parts. These moving parts can be the disturbance that creates a wave. If more than one wave crest arrives at a location in the device, the amplitude at that location could become large. The oscillation of this part of the device could rattle apart welds or bolts, destroying the device. Later, as we study spectroscopy, we will see how to diagnose such a problem and hint at how to correct it.

Basic Equations

Doppler shift

$$f_B = \frac{v_{sound} \pm v_d}{v_{sound} \mp v_e} f_A \tag{34.13}$$

where we use the top sign for the speed when the mover (detector or emitter) is going toward the non-mover.

	Stationary Detector	Detector moves toward Emitter	Detector moves away from Emitter
Stationary Emitter	$f_B = f_A$	$f_B = \frac{v_{sound}+v_d}{v_{sound}} f_A$	$f_B = \frac{v_{sound}-v_d}{v_{sound}} f_A$
Emitter moves toward Detector	$f_B = \frac{v_{sound}}{v_{sound}-v_e} f_A$	$f_B = \frac{v_{sound}+v_d}{v_{sound}-v_e} f_A$	$f_B = \frac{v_{sound}-v_d}{v_{sound}-v_e} f_A$
Emitter moves away from Detector	$f_B = \frac{v_{sound}}{v_{sound}+v_e} f_A$	$f_B = \frac{v_{sound}+v_d}{v_{sound}+v_e} f_A$	$f_B = \frac{v_{sound}-v_d}{v_{sound}+v_e} f_A$

Superposition

$$y_r(x,t) = y_1(x,t) + y_2(x,t)$$