Chapter 19

Resolution and Charge

Fundamental Concepts

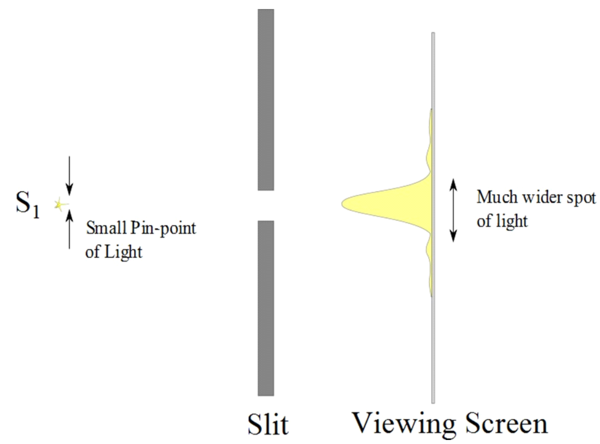
- Two points can be distinguished when imaged if their angular separation is a minimum of $\theta_{\min} = 1.22\frac{\lambda}{D}$
- There is a property of matter called "charge."
- There seem to be two types of charges, called "positive" and "negative."

19.1 Resolution

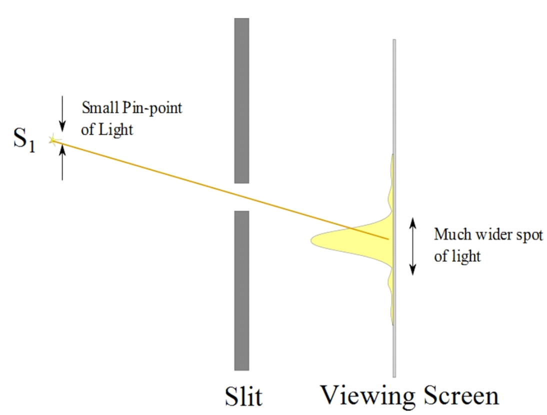
We have emphasized that an extended object can be viewed as a collection of point objects. Then the image is formed from the collection if images of those point objects. It would be great if optical systems could form images with infinite precision–that is, the image of a point object would be a point image. The fact that light acts as a wave prevents this from being true. The quality of our image depends on how poorly a point object is imaged. If each point object makes a large circle of light on the screen or detector array, we get a very confusing image (it will look blurry to us).[1] Let's see why this will happen so we can know how to minimize the effect.

We already know that if we take light and pass it through a single slit, we get an intensity pattern that has a central bright region.

[1] In Fourier Optics, the intensity pattern that comes from imaging a single point is called a *point spread function* because it shows how spread out the light from a single point will be. In mechanical engineering, we might call this an impulse response function. It is the same idea applied to optics.

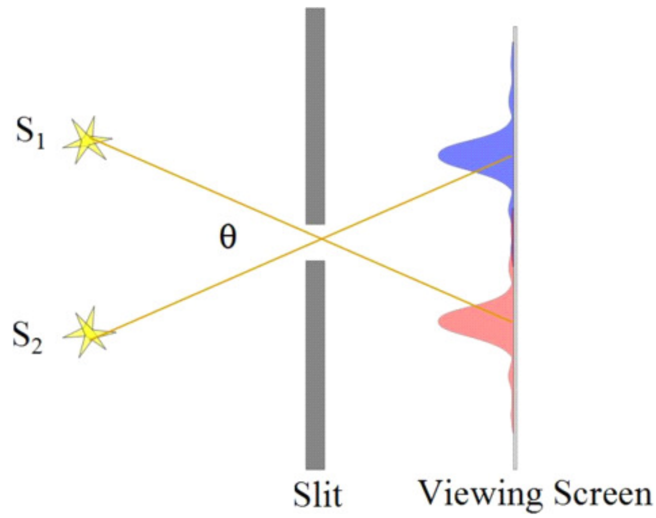


Remember that normal objects will be made up of many small points of light (either due to reflection or glowing) and each of these will form such an intensity pattern on a screen. Here is a bright point source that is not on the axis, and we see that it too makes a bright spot on the screen (and smaller bright spots or rings, depending on the shape of the aperture)

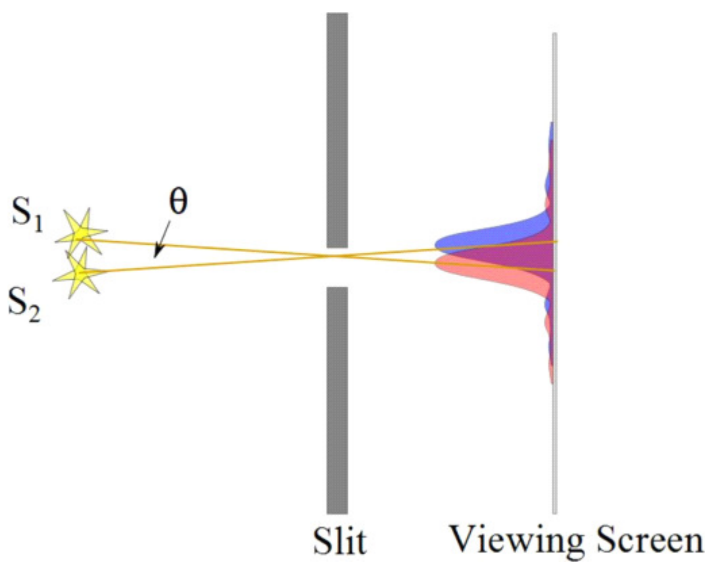


So our images will be made up of many central bright spots, each of which represents a point of light from the object. These central bright spots may overlap, (and their secondary maxima certainly will overlap).

Let's take a simple case of two points of light, S_1 and S_2. If we take a single slit and pass light from two distant point sources through the slit, we do not get two sharp images of the point sources. Instead, we get two diffraction patterns.



If these patterns are formed sufficiently far from each other, it is easy to tell they were formed from two distinct objects. Each point became a small blur, but that is really not so bad. We can still tell that the two blurs came from different sources. If our pixel size is about the same size of the blur, we won't even notice the blurriness in the digital imagery.



But if the patterns are formed close to each other, it gets hard to tell whether they were formed from two objects or one bright object. We now have a problem. Suppose you are trying to look at a star and see if it has a planet. But all you can see is a blur. You can't tell if there is one source of light or two.

Long ago an early researcher titled Lord Rayleigh developed a test to determine if you can distinguish between two diffraction patterns.

> When the central maximum of one point's image falls on the first minimum of anther point's image, the images are said to be just resolved.

This test is known as *Rayleigh's criterion.*

We can find the required separation for a slit. Remember that

$$\sin(\theta) = m\frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3 \ldots \tag{19.1}$$

gives the minima. We want the first minimum, so

$$\sin(\theta) = \frac{\lambda}{a} \tag{19.2}$$

If we place the second image maximum so it is just at this location, the two images will be just barely resolvable. In the small angle approximation, $\sin(\theta) \approx \theta$ so

$$\theta_{\min} = \frac{\lambda}{a} \tag{19.3}$$

Now you may be saying to yourself that you don't often take pictures through single illuminated slits, so this is nice, but not really very interesting.

Suppose, instead, that we image a circular aperture. Again, we won't go through all the math (there are Bessel functions involved) but the criterion becomes

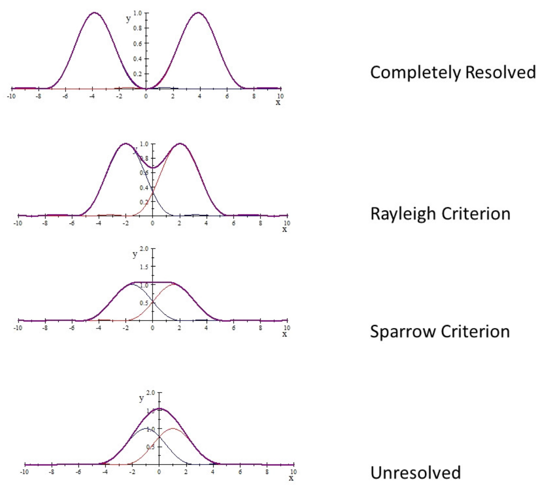
$$\theta_{\min} = 1.22\frac{\lambda}{D} \tag{19.4}$$

where D is the aperture diameter.

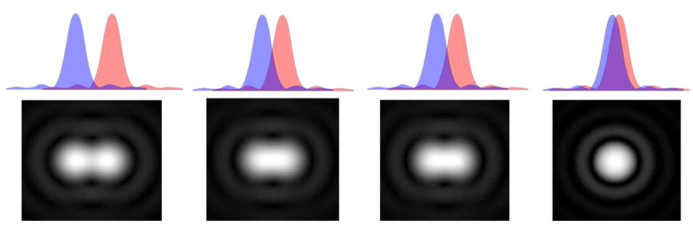
Still, you may say, I don't like pictures taken through small circles any better than through small slits! Yet, in fact, you do. Most cameras have circular apertures. The light that passes into your phone camera must pass though the circular lens. For that matter, the pupil of our eye is a circular aperture. So most images we see are made using circular apertures.

The Rayleigh criteria tells you, based on your camera aperture size, how a point source will be imaged on the film or sensor array. If we consider extended sources (like your favorite car or Aunt Matilda) to be collections of many point sources, then we have a way to tell what features will be clearly resolved on the image and what features will not (like you may not be able to see the lettering on the car to tell what model it is, or you may not be able to distinguish between the gem stones in Aunt Matilda's necklace because the image is too blurry to see these features clearly).

In the next figure you can see two easily resolved intensity patterns, then two that can be resolved using Rayleigh's criterion, and finally a more modern astronomical resolution criterion made by a researcher called Sparrow. Finally there is figure with two intensity patterns that would not be resolvable (would look like just one point of light).



Here are our four cases again but with pictures that show what you would see in an image of the two light sources for each criterion.



From our equation we can see that we have better resolution if D is bigger. This is why professional photographers use large lenses and not their cell phones. The cell phone cameras have apertures that are a few millimeters. Typical professional cameras have 67 mm apertures. We can see that for a cell phone the angle for minimal resolution is about

$$\theta_{\min} = 1.22 \frac{500\,\text{nm}}{3\,\text{mm}} = 2.0333 \times 10^{-4}\,\text{rad}$$

For the professional lens, the minimum resolution is about

$$\theta_{\min} = 1.22 \frac{500\,\text{nm}}{67\,\text{mm}} = 9.1045 \times 10^{-6}\,\text{rad}$$

That is a whopping factor of 22 better resolution. If you need to find a small crack in a structure, or if you want to print a wall sized portrait of your Aunt Miltilda, the extra resolution might be necessary for your application.

19.2 Charge model

So far we have claimed that light is a wave in an electromagnetic field. We talked about light waves being made in the electromagnetic field by moving charges. But we have not proved it this to be the case. We will find that it will take the rest of the semester to do so! We need to start by looking at charge and what makes charges move.

But let's think about this conceptually and see if we can motivate our study.

We know that there is an electromagnetic spectrum, and that visible light is just a small part of that spectrum. Radio waves are also part of the spectrum of light.

And we should review, how are radio waves produced? We know electricity is involved.

The answer is that charged particles, like the electrons flowing through the antenna of our radio station, create an electromagnetic field. That field is drug along when the electrons move in the antenna. If we make the electrons oscillate, we can make waves in the field. This is much like having a 3rd grade class all hold the edges of a parachute and having the 3rd graders jump up and down. Waves are made in the parachute.

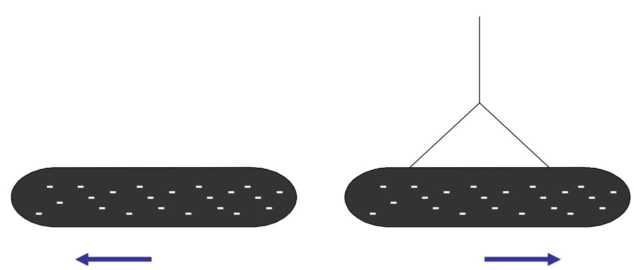
But what is charge? How do we know there are such things as charged particles?

That is the subject we will take up next. Then we will study the motion and actions of these charged particles. Finally we will show that the fields made by charged particles can act as a medium for waves, and that there is good evidence that those waves exist.

19.2.1 Evidence of Charge

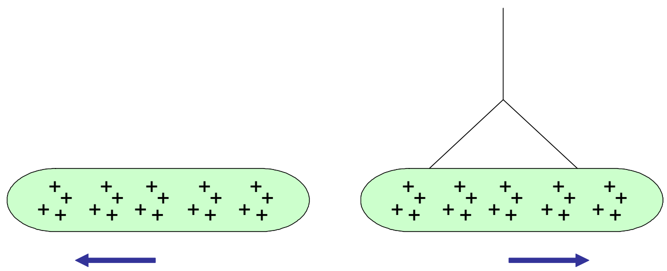
Let's start with something we all know. Let's rub a balloon in someone's hair. If we do this we will find that the balloon sticks to the wall. Why?

We say the balloon and comb have become *charged*. What does this mean? We will have to investigate this more as we learn more about how matter is structured, but for now let's assume charge is some property that provides this phenomena we have observed with the balloon (i.e. it sticks to the wall). Now lets try rubbing other things. We could rub two rubber or plastic rods.



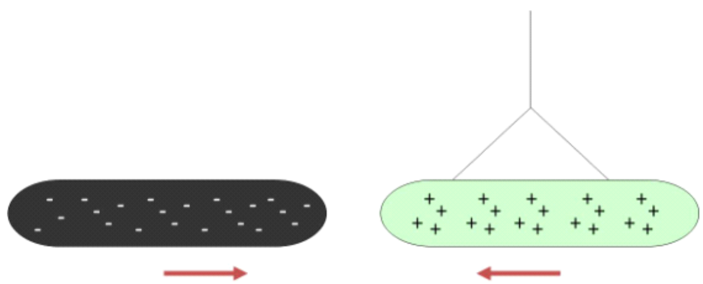
Two charged rubber rods are placed close together. The rods repel each other.

and we could also rub two glass rods



Notice that in each case we have created a force between the two rods. The rods now repel each other.

Now let's try a glass and a rubber rod



Now the two different rods attract each other.

Notice that in our demo, rods that are the same repel and rods that are different attract. We make the intellectual leap that the different rods have different charges. So we are really saying:

1. There are two types of charge.
2. Charges that are the same repel one another and charges that are different attract one another.
3. Friction seems to produce charge, but you have to rub the right materials together.

We will call the rubber or plastic rod charges *negative* and the glass rod charges *positive* but the choice is arbitrary. Ben Franklin is credited with making the choice of names. He really did not know much about charge, so he just picked two names (we will see that in some ways his choice was somewhat unfortunate, but hay, he was an early researcher who helped us understand much about charge , so we will give him a break!).

19.2.2 Types of Charge

We now have reason to believe that there are at least two types of charge, one for rubber and one for glass. But are there more?

Let's start by introducing a new object, only this time we won't rub it with anything.

Now this is strange. The new item is attracted to both rods! What is going on? Have we discovered a new type of charge, one that attracts the other two types we have found?

Maybe, but maybe the explanation of this phenomena is a little different. To understand this, let's consider how charge moves around.

19.2.3 Movement of Charge

One of the strange things about charge is that it is *quantized*. We learned this word in when we found that only certain standing waves could be formed between boundaries. We are using this word in a similar way now. It means that charge has a smallest unit, and that it only comes in whole number multiples of that unit. Charge comes in a basic amount that can't be divided into smaller amounts. So like our standing wave frequencies, only certain amounts are possible As far as we know, the smallest amount of charge possible is the electron charge.[2] This charge we will call negative. We say that the electron is the principle charge carrier for negative charge. This fundamental unit of charge was found to be about

$$e = 1.60219 \times 10^{-19}\,\mathrm{C} \tag{19.5}$$

where the C stands for *Coulomb,* the *SI* unit of charge.

Any larger charge must be a multiple of this fundamental charge

$$Q = n \times e \tag{19.6}$$

The proton is the principle charge carrier for positive charge. From chemistry, you know protons are located in the nucleus of an atom, along with the neutron. In the Bohr model of the atom, the nucleus is surrounded by a cloud of electrons. The proton has the same amount charge as the electron (e), but is opposite in sign.

In a gram of mater, there are many, many, units of charge. There are about 5.0125×10^{22} carbon atoms in one gram of carbon. Each carbon atom has twelve protons and about twelve electrons. That is a lot of charge! But notice that the net charge is zero (or very close to it!). It is common for most mater to have zero net charge.

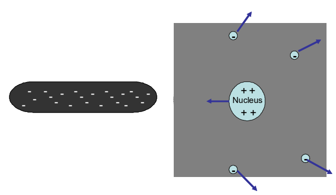
As far as we know, charge is always conserved. We can create charge, but only in plus or minus pairs, so the net charge does not change. We can destroy charge, but we end up destroying both a positive and a negative charge at the

[2] I am not counting quarks here, which have a charge of $\frac{1}{3}$ or $\frac{2}{3}$ of the basic electron charge. But still, $\frac{1}{3}$ of the basic electron charge seems to be a real fundamental unit for quark based particles. And quarks aren't stable on their own, so we never see fractional charge in nature.

same time. The net charge in the universe does not seem to change much. So when something becomes charged, we expect to find that the charge has come from another object.

Lets go back to our rubber rod and glass rod demo. We rubbed the rod that was in our hand, but where did the charge come from? We believe that we are moving charge carriers (usually electrons) from one object to another, stripping them from their atoms. This happens when we use friction (rubbing) to charge the rods.

But what about our object that we did not rub, or our paper (we did not rub the bits of paper). We believe that charge can move, that is why scientists looked for and found charge carriers. Even in an atom, if I bring a charged object near the atom then the negative charge carriers (electrons) will experience a force directed away from the charged object, and the positively charged nucleus will experience a force pulling toward the charge object



Notice that the electrons and the nucleus will *attract* each other, so the atom won't split apart. But it will become positively charged on one side because there are more positive charge carriers on that side. It will become more negatively charged on the other side, because there are more negative charge carriers on that side. We could draw the atom like this (figure 19.1)The force due to charge

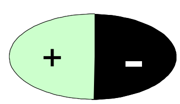
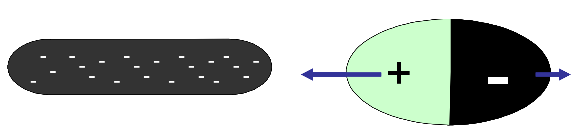
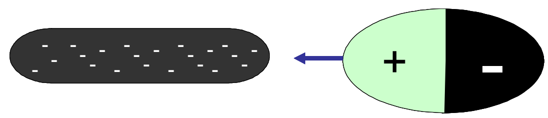


Figure 19.1: Polarized Atom

depends on how far away the charges are from each other. The attractive force between the positively charged side of the atom and the negative rod will have a stronger force than the negatively charged side of the atom and negatively charged rod will experience because the negative side if farther away. We will say that the atom has become *polarized*.

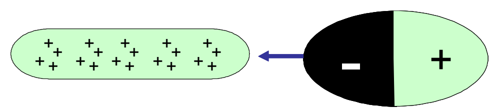


The positive side will experience an attractive force. The negative side will experience a repelling force. The net force due to the charge will be an attractive force. The atom will be accelerated toward the rod! We have seen something like this before. Remember an object in a fluid experiences a downward pressure force on the top, and an upward pressure force on the bottom. The pressure force is larger on the bottom, so there is an upward Buoyant force. The case with our polarized atom is very similar. We have a net electrical attractive force.



Now suppose we have lots of atoms (like our uncharged object or our bits of paper). Will they be attracted to the rod? Yes!

How about if we use a glass rod?



Everything is the same, only we switch the signs. The glass rod is positively charged. It will attract the electrons, and repel the nucleus. The atom becomes charged. The net force is attractive (positive rod and closer negative side of the atom)

We sometimes call the separation of charge in an insulator *polarization*.

19.2.4 Flow of Charge

Let's start by introducing a new object, a salt shaker (my salt shaker is glass with a metal top). We will rub the salt shaker and see if it gets charged by placing it next to our charged rods.

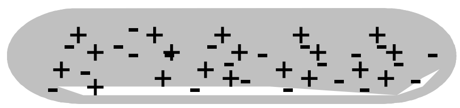
Now this is strange. We rubbed the object, but it was attracted to both rods as if there were no charge. We know glass can be charged. What is the problem?

It turns out that some materials allow charge carriers to flow through them. Our experience with the lighting in our house might suggest that metals will do this. Let's try some other metal objects and see what we find.

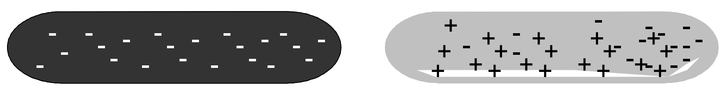
It seems that the atoms are not maintaining a charge separation in these metal atoms! Some materials allow charge carriers to move through them. Usually these materials are metals, but most materials will allow some charge to go through them-even you-which is what is happening in this case. I charge the rod, but the charge leaves through my body. Other materials resist the flow of charge. Materials that allow charge to flow are called *conductors*. Materials that resist the flow of charge are called *insulators*.

19.2.5 Charging by Induction

Knowing that charge carriers can flow though a material, we can think of a way to charge a conductor. Lets suspend a conducting rod.



It is not initially "charged" meaning that it has the same number of positive charges and negative charges, and they are evenly mixed together. I will bring a charged rod next to it.



but let's attach a wire to the other end of the rod to allow the charge to flow away from our conducting rod. We will connect the rod to the ground (in this case, to a water pipe) because the ground seems to be able to accept large amounts of charge carriers. So the charge carriers will flow to the ground.

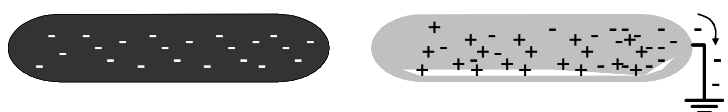
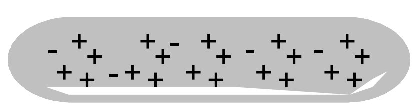


Figure 19.2:

(The strange little triangular striped thing is the electronics sign for a connection to the ground)

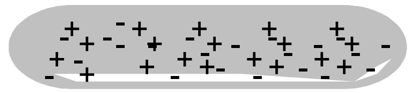
Now let's disconnect the wire from the rod. Is there a net charge on the conducting rod?



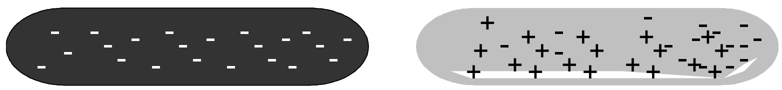
The answer is yes, because we now have more positive charges in the conducting rod than we have negative charges, so the net charge is positive.

19.2.6 Charging by Conduction

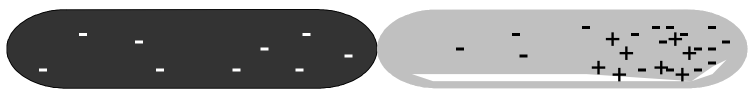
Suppose instead, I perform the same experiment, but I touch the rods. Now charge carriers can flow. Starting with and uncharged conductor,



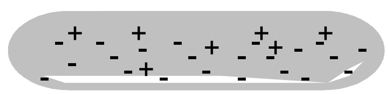
I again bring in a charged rod. Again the charges separate in our conducting rod.



Then we touch the two rods. The excess charge on our charged rod flows to the conductor. Since in our drawing, the excess charge is negative, then some of the positive charge on the conductor is neutralized.



When we separate the rods, our conducting rod will have an excess of negative charge.



Notice that there is something different in our study of this new force. In the past, it was easy to tell which object was creating the environment and which was the mover. The Earth, being so much larger than normal objects, was the environmental object creating the gravitational acceleration that balls and cars and people move it. Then the balls and cars and people were the movers. Generally the thing causing the force, the environmental object, was much bigger than the mover. That is not true in our charge experiments so far. The rods are about the same size. So which is the environmental object and which is the mover? We will have to pick one to be our environmental object, and the other to be our mover. Sometimes the context of the problem helps. If the problem you are solving asks for the motion or the force on the rod on the right side of the diagram, then it is the mover and the rod on the left is the environmental object. If one charge is much larger than the other, we might be justified in calling this large charge the environmental object and a smaller charge near the big charge would be the mover.

Basic Equations

The minimum angle between two objects that can be resolved (according to the Rayleigh criteria) is

$$\theta_{\mathrm{min}} = 1.22\frac{\lambda}{D}$$