# Chapter 44

# Ray Model

We don't see constructive and destructive interference in normal situations, like sitting in class with the room lights on, even though we have more than one light source. But we did see this in our classroom demonstrations. What makes the difference?

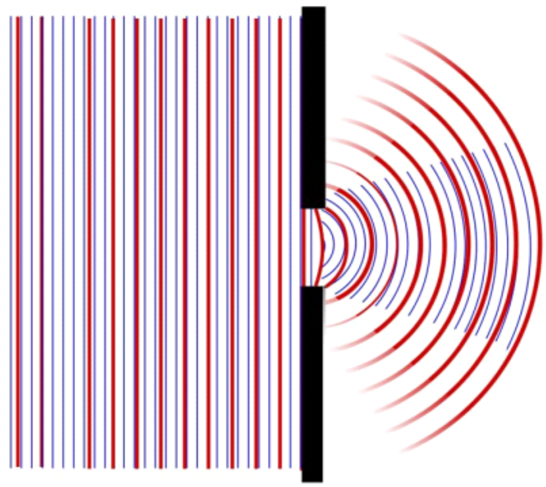
## Fundamental Concepts

- When the aperture size is given by $D_{aperture} = \sqrt{2.44\lambda L}$ we are at a critical size bounding the geometric and wave optics regions
- Coherent light is light that maintains a common phase, direction, and wavelength.
- Light reflects from a specular surface with equal angles

## 44.1 Transition to the ray model

On a dark night in Rexburg (say 5:30pm in the winter) you might see two street lights. And where the light from the street lights overlap we don't see constructive and destructive interference. Why not?

In the next figure, two waves of different wavelets go through a single opening. The wave representing the central maximum is shown in each case, but not the secondary maxima.



Notice that the smaller wavelength has a narrower central maxima as we would expect from

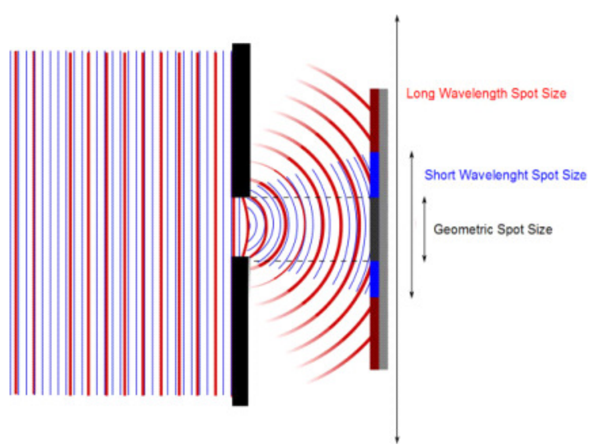
$$\sin(\theta) = 1.22\frac{\lambda}{D}$$

or

$$\theta \approx 1.22\frac{\lambda}{D}$$

we see that the ratio of the wavelength to the hole size determines the angular extent of the central maxima. The smaller the ratio, the smaller the central region. We can use this to explain why we don't see interference for the street lights and why the wave nature of light was so hard to find.

The patch of light on a screen that is created by light passing through the aperture is created by the central maximum.




For the long wavelength (red) the central maximum is larger than the screen.

The short wavelength spot will be wholly on the screen as shown. The geometric spot is what we would see if the light traveled straight through the opening. Notice that the short wavelength spot is closer to the size of the geometric spot. In the limit that

$$\lambda \ll a$$

or for circular openings

$$\lambda \ll D$$

then

$$\theta \approx \frac{\lambda}{a} \approx 0$$

or

$$\theta \approx \frac{\lambda}{D} \approx 0$$

and the spot size would be very nearly equal to the geometric spot size.

This is the limit we will call the *ray approximation*.

For most of humankind's time on the Earth, it was very hard to build holes that were comparable to the size of a wavelength of visible light. So it is no wonder that the waviness of light was missed for so many years.

But this ray limit is very useful for apertures the size of camera lenses. So starting next lecture we will begin to use this small $\lambda$, large aperture approximation. But this is not the only reason we don't see constructive and destructive interference between street lights or the lights in our classroom. We will continue this discussion in our next lecture.

## 44.2 The Ray Approximation in Geometric Optics

Last time we said that when the geometric spot size was about the same size as the spot due to diffraction we could ignore diffraction. This is usually true in our personal experiences. But this may not be true in experiments or devices we design. We should see where the crossover point is.

Intuitively, if the aperture and the geometric spot are the same size, that ought to be some sort of critical point. That is when the aperture size is equal to the spot size

$$D_{aperture} = 2.44 \frac{\lambda}{D_{aperture}} L$$

This gives

$$D_{aperture} = \sqrt{2.44 \lambda L}$$

Of course this is for round apertures, but for square apertures we know we remove the 2.44. This gives about a millimeter for visible wavelengths.

$$\begin{aligned} D_{aperture} &= \sqrt{2.44\,(500\,\mathrm{nm})\,(1\,\mathrm{m})} \\ &= 1.104\,5 \times 10^{-3}\,\mathrm{m} \end{aligned}$$

for apertures much larger than a millimeter, we expect interference effects due to diffraction through the aperture to be much harder to see. We expect them to be easy to see if the aperture is smaller than a millimeter. But what about when the aperture is about a millimeter in size? That is a subject for PH375, and so we will avoid this case in this class. But this is not too restrictive. Most good optical systems have apertures larger than 1 mm. Cell phone cameras may be an exception (but I don't consider cell phone cameras to be good optical systems). Even our eyes have an aperture that varies from about 2 mm to about 7 mm, so most common experiences in visible wavelengths will work fine with what we learn. Note that for microwave or radio wave systems this may really not be true!

How about the other extreme? Suppose $\lambda \gg D$. This is really beyond our class (requires partial differential equations), but in the extreme case, we can use reason to find out what happens. If the opening is much smaller than the wavelength, then the wave does not see the opening, and no wave is produced on the other side. This is the case of a microwave oven door. If the wavelength is much larger than the spacing of the little dots or lines that span the door, then the waves will not leave the interior of the microwave oven. Of course as the wavelength becomes closer to $D$ this is less true, and this case is more challenging to calculate, and we will save it for a 300 level electrodynamics course.

To summarize

| | |
|---|---|
| $\lambda \ll D$ | Wave nature of light is not visible |
| $\lambda \approx D$ | Wave nature of light is apparent |
| $\lambda \gg D$ | Little to no penetration of aperture by the wave |

We can see that early researchers might not have spent a lot of time with sub-millimeter sized holes, so the wave nature of light was not as apparent to Newton and his contemporaries.

### 44.2.1 The ray model and phase

There is a further complication that helps to explain why the wave nature of light was not immediately apparent to early researchers. Let's consider a light source.

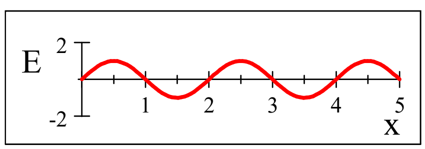
For a typical light source, the filament or light emitting diode (LED) is larger than about a millimeter, which is much larger than the wavelength. So, we should already expect that diffraction might be hard to see. But the filament is made of hot metal (we will leave LED workings for another class). The atoms of the hot metal emit light because of the extra energy they have. The method of producing this light is that the atom's excited electrons are in upper shells because of the extra thermal energy provided by the electricity flowing through the filament. But the electrons eventually fall to their proper shell, and in doing so they give off the extra energy as light. It is not too hard to believe that this process of exciting electrons and having them fall back down is a random process. Each electron that moves starts a wave. The atoms have different positions, so there will be a path difference $\Delta r$ between each atom's waves. There will also be a time difference $\Delta t$ between when the waves start. We can model this with a $\Delta\phi_o$.

It is also true that not all of the electrons fall from the same shell (or even the same shell in different atoms). This gives us different frequencies, so we expect beating between different waves from different atoms. It is also true that we have millions of atoms, so we have millions of waves.

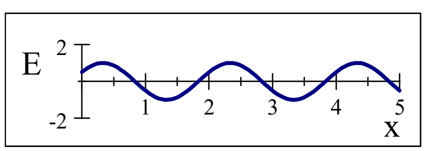
Let's look at just two of these waves

$$\begin{aligned}
&\lambda = 2\\
&k = \tfrac{2\pi}{\lambda}\\
&\omega = 1\\
&\phi_o = \tfrac{\pi}{6}\\
&t = 0\\
&E_o = 1\,\tfrac{\mathrm{N}}{\mathrm{C}}
\end{aligned}$$

$$E_1 = E_{\max}\sin(kx - \omega t)$$




$$E_2 = E_{\max}\sin(kx - \omega t + \phi_o)$$

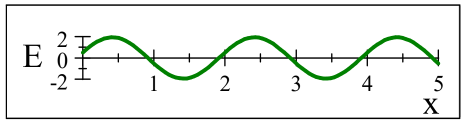



then

$$E_r = E_{\max}\sin(kx - \omega t) + E_o\sin(kx - \omega t + \phi_o)$$

We found a nice meaningful way to write the resultant wave.

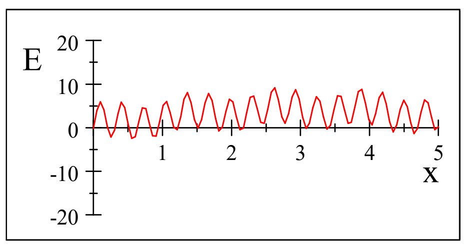
$$E_r = 2E_{\max}\cos\left(\frac{\phi_o}{2}\right)\sin\left(kx - \omega t + \frac{\phi_o}{2}\right)$$




But suppose we complicate the situation by sending out lots of waves at random times, each with different amplitudes and wavelengths. If we look at a single point for a specific time, we might be experiencing interference, but it would be hard to tell. Lets try this mathematically. I will combine many waves with random phases, some coming from the right and some coming from the left.

$$\begin{aligned} E_1 &= E_{\max}\sin\left(5x-\omega t+\frac{\pi}{4}\right)+0.5E_{\max}\sin\left(0.2x-\omega t-\frac{\pi}{6}\right) \\ &\quad +3.6E_{\max}\sin\left(.4x-\omega t+\frac{\pi}{10}\right)+4E_{\max}\sin\left(20x-\omega t-\frac{\pi}{7}\right) \\ &\quad +.2E_{\max}\sin\left(15x-\omega t+1\right)+0.7E_{\max}\sin\left(.7x-\omega t-.25\right) \end{aligned}$$

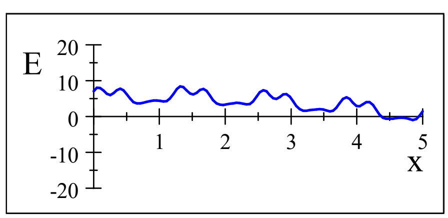
Here is what $E_1$ would look like.



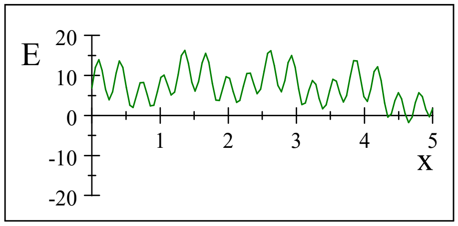

And now let's make another random wave, $E_2$

$$\begin{aligned} E_2 &= E_{\max}\sin\left(0.2x+\omega t+\pi\right)+2E_{\max}\sin\left(5x+\omega t+\frac{\pi}{6}\right) \\ &\quad +6E_{\max}\sin\left(0.4x+\omega t+\frac{\pi}{3.5}\right)+0.4E_{\max}\sin\left(20x+\omega t-0\right) \\ &\quad +E_{\max}\sin\left(15x+\omega t+1\right)+0.7E_{\max}\sin\left(.7x+\omega t-4\right) \end{aligned}$$

Which looks like this



Then $E_1 + E_2$ looks like




In this example, you could think about the superposition of $E_1$ and $E_2$ and predict the outcome, but if there were millions of waves, each with it's own wavelength, phase, and amplitude, the situation would be hopeless. Note that the fluctuations in these waves are much more frequent than our original waves. With all the added waves, we get a rapid change in amplitude.

Now if these waves are light waves, our eyes and most detectors are not able to react fast enough to detect the rapid fluctuations. So if there is constructive or destructive interference that might be simple enough to distinguish, the interference pattern will change so fast that we will miss it due to our detection systems' integration times. To describe this rapidly fluctuating interference pattern that we can't track with our detectors, we just say that light bulbs emit *incoherent light*. The ray approximation assumes incoherent light.

But then light bulbs and hot ovens and most things must emit incoherent light. Does any thing emit coherent light? Sure, today the easiest source of coherent light is a laser. That is why I have used lasers in the class demonstrations so far. Really though, even a laser is not perfectly coherent. One property of the laser is that it produces light with a long *coherence length*, or it produces light that can be treated under most circumstances as begin monochromatic and having a single phase across the wave for much of the beam length. Radar and microwave transmitters emit coherent light (but at frequencies we can't see) and so do radio stations.

In the past, one could carefully create a monochromatic beam with filters. Then split the beam into two beams and remix the two beams. This would generate two mostly coherent sources if the distances traveled were not too large. This is what Young did.

To be coherent,

1. A given part of the wave must maintain a constant phase with respect to the rest of the wave.

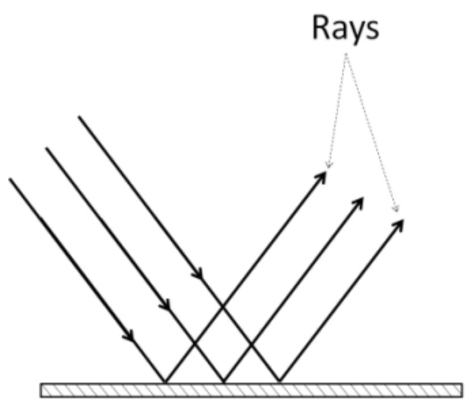
2. The wave must be monochromatic

These are very hard criteria to achieve for visible light. Most visible light, like that from our light bulb, is not coherent.

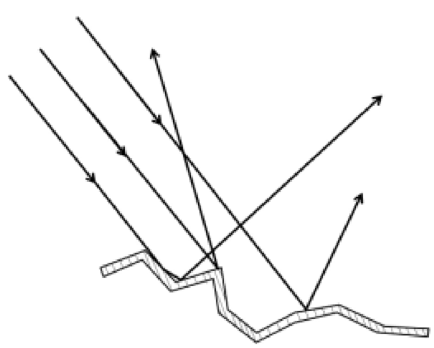
## 44.3 Reflection

In the *Star Wars* movies inter-galactic star ships blast each other with laser cannons. The laser beams streak across the screen. This is dramatic, but not realistic. For us to see the light, some of the light must get to our eyes. The light must either travel directly to our eyes from the source, or it must bounce off of something.

Using the ray approximation we wish to find what happens when a bundle of rays reaches a boundary between media. If the media boundary is very smooth, then the rays are reflected in a uniform way. This is called *specular* reflection




If it is not smooth, then something different happens. The rays are reflected, but they are reflected randomly



This is called *diffuse* reflection.

This difference can be seen in real life

Light reflecting of a specular (left) and diffuse (right) reflector.

We said the surface must be smooth for there to be specular reflection. What does smooth mean? Generally the size of the rough spots must be much smaller than a wavelength to be considered smooth. So suppose we have a red laser. How small do the surface variations have to be for the surface to be considered smooth? The wavelength of a $HeNe$ laser is

$$\lambda_{HeNe} = 633\,\mathrm{nm}$$

This is very small. Modern optics for remote sensing are often manufactured to 1/10 of a wavelength, which would be 63 nm.

How about a microwave beam of light like your cell phone uses?

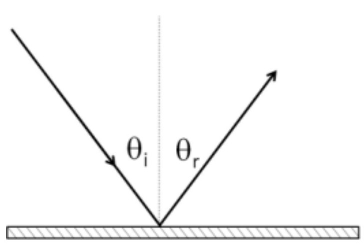
$$\begin{aligned} c &= \lambda f \\ \lambda &= \frac{c}{f} = \frac{3 \times 10^8 \,\frac{\mathrm{m}}{\mathrm{s}}}{1\,\mathrm{GHz}} \\ &= 0.3\,\mathrm{m} \end{aligned}$$

Bumps the size of a third of a meter don't seem very smooth to us humans! We can see that we must be careful in our definition of "smooth."

### 44.3.1 Law of reflection

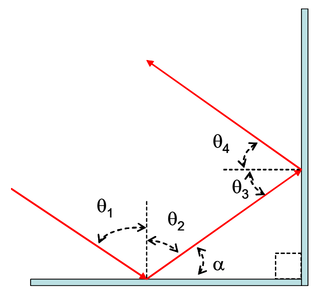
Experience shows that if we do have a smooth surface, that light bounces much like a ball. This is why Newton though light was a particle. Suppose we take a flat surface and we shine a light on it. We have a ray that approaches at an angle $\theta_i$ measured from the normal. Then the reflected ray will leave the surface with an angle $\theta_r$ measured from the normal such that

$$\theta_r = \theta_i$$



This is called the *law of reflection.*

Let's take an example




Let's take our system to be two mirrors set at a right angle. We have a beam of light incident at angle $\theta_1$. By the law of reflection, it must leave the mirror at $\theta_2 = \theta_1$. We can see that $\alpha$ must be $90\,^\circ - \theta_2$ and it is clear that $\theta_3 = \alpha$. By the law of reflection, $\theta_3 = \theta_4$. Then, since

$$\begin{aligned} 90\,^\circ &= \theta_2 + \alpha \\ &= \theta_2 + \theta_3 \end{aligned}$$

and

$$90\,^\circ = \theta_1 + \theta_4$$

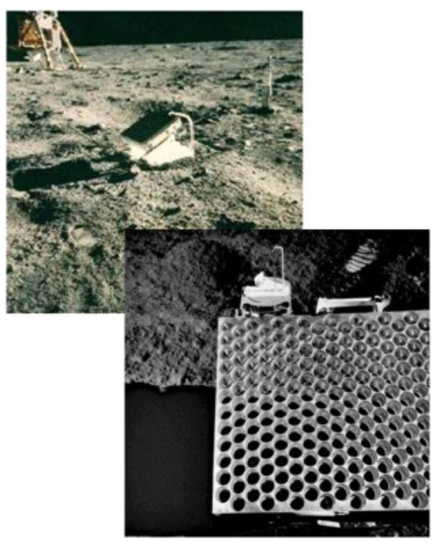
then the total angular change is

$$90\,^\circ + 90\,^\circ = 180\,^\circ$$

or the outgoing ray is sent back toward the source! If we do this in three dimensions we have a corner cube.

Radar retroreflector tower located in the center of Yucca Flat dry lake bed. Used as a radar target by maneuvering aircraft during "inert" contact fusing bomb drops at Yucca Flat. Sandia National Laboratories conducted the tests on the lake bed from 1954 to 1956. (Image in the Public Domain in the United States_

The figure above is a radar corner cube set. The one below is an optical corner cube set on the moon.



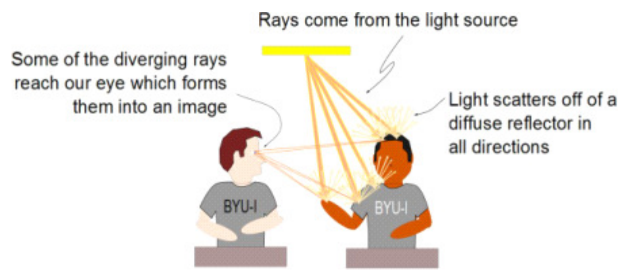
Apollo Retroreflector (Images in the Public Domain courtesy NASA)

We use this optical corner cube array to reflect light off of the moon. The time it takes the light to go to the moon and back can be converted into an Earth-Moon distance for monitoring how close the moon is to the Earth.

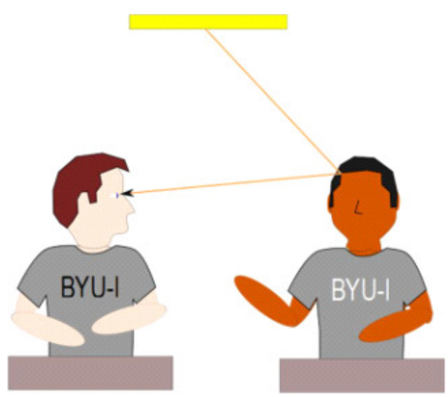
## 44.4 Reflections, Objects, and seeing

Armed with the law of reflection, we can start to understand how we see things. Using the ray concept, we can say that a ray of light must leave the light source. That ray then reflects from something. Suppose you look at the person sitting next to you in class. Light from the ceiling lights has reflected from that person. But is the person a specular or diffuse reflector?

Once again, we can only give an answer relative to the wavelength of light. For visible light, your neighbors do not look like mirrors. The are diffuse reflectors. Light bounces off of them in every direction. Your eye is designed to take this diverging set of rays and condense it into a picture of the person that your brain can interpret.

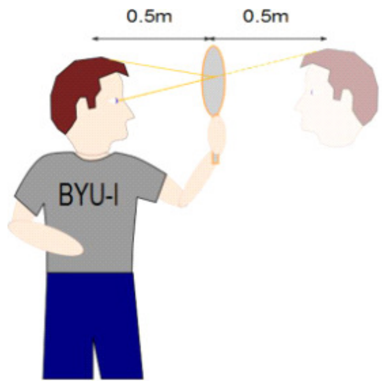



We tend to not draw the rays that bounce off the diffuse reflector but that don't get to our eyes, because we don't see them. So a ray diagram is usually much simpler.

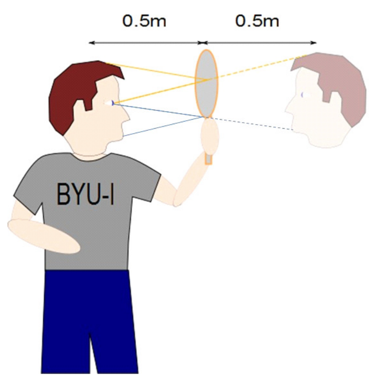



This is easy to understand, but we must keep in mind the wildly fluctuating waviness that is masked by our macroscopic view.

We can use the idea of a ray diagram to solve problems. Suppose you hold a mirror half a meter in front of you and look at your reflection. Where would the reflection appear to be?

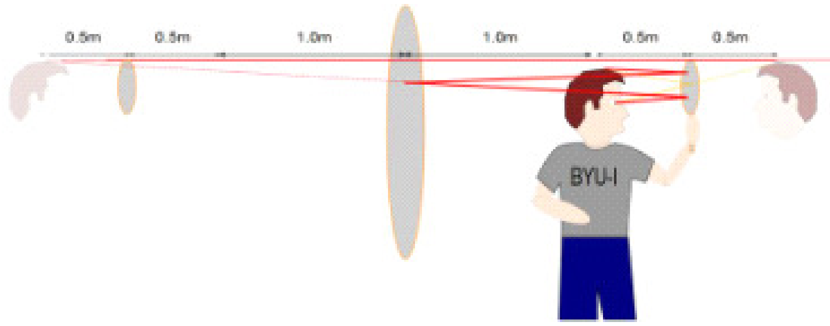



Our visual processing center in our brain[1] knows about light traveling in straight lines. It assumes all rays have traveled straight paths with no bending or even bouncing. Then we can use rays to see where the light appears to be from. We just imagine that the light traveled in a straight line and trace that line backwards through the mirror. Our brain believes the light came from somewhere along that path. The light from the top of your head seems to have come from behind the mirror (yellow path). We can use a second ray (in blue in the next figure)




to get a better location for where our brain believes the light came from. The light from your chin appears to come along a path that comes from behind the mirror as well. We can draw in these imaginary paths that our brain believes are the origin paths for the light. They we can fit a picture of you to along these path so the imaginary path for the light from your hair intersects the picture of your hair in just the right spot. We can make the picture's chin also intersect the chin light path in just the right spot. The location of the picture is where the light appears to come from. We will call the apparent origin of the light an *image*. The image is half a meter behind the mirror. Now suppose we look at an image of that image in a mirror behind us.

[1] In the occipital lobe which is in the back of your brain.




The ray diagram makes it easy to see that the image will appear to be 2 m behind the big mirror.

# Basic Equations

$$D_{aperture} = \sqrt{2.44\lambda L}$$
$$\theta_r = \theta_i$$