Chapter 18

Ohm's law

Fundamental Concepts

- The material property of a conductor that tells us how well the conductor material will allow current to flow through it is called the conductivity
- The inverse of conductivity is the resistivity
- Resistivity may be temperature dependent
- Resistance depends on the resistivity of the material and the geometry of the conductor piece. For a wire it is given by $R = \rho A/L$
- For many conductors, the change in voltage across the conductor is proportional to the current and the resistance. This is called Ohm's law
- The ideal voltage delivered by a battery is called the "emf" and is given the symbol $\mathcal{E}$
- Some materials do not follow Ohm's law. They are called nonohmic
- The Earth has a magnetic field
- Magnets have "magnetic charge centers" called poles and there is a magnetic field.
- Magnetic poles don't seem to exist independently

18.1 Conductivity and resistivity

We defined the current density last lecture

$$J = nq_e v_d$$

but we know that the drift speed is

$$v_d = \left(\frac{q_e \tau}{m_e}\right) E$$

so we can write the current density as

$$\begin{aligned} J &= n q_e \left(\frac{q_e \tau}{m_e}\right) E \\ &= \left(\frac{n q_e^2 \tau}{m_e}\right) E \end{aligned}$$

The factor in parentheses depends only on the properties of the conducting material. For example, if the material is copper, then we would have the $n_{copper} = 8.5 \times 10^{28} \frac{1}{\mathrm{m}^3}$ as the number of valence electrons per meter cubed for copper. The mean time between collisions is something like $\tau_{copper} = 2.5 \times 10^{-14}\,\mathrm{s}$. So our quantity in parentheses is

$$\begin{aligned} \left(\frac{n q_e^2 \tau}{m_e}\right) &= \frac{\left(8.5 \times 10^{28} \frac{1}{\mathrm{m}^3}\right)\left(1.6 \times 10^{-19}\,\mathrm{C}\right)^2 \left(2.5 \times 10^{-14}\,\mathrm{s}\right)}{9.11 \times 10^{-31}\,\mathrm{kg}} \\ &= 5.9715 \times 10^{7} \frac{\mathrm{A}^2}{\mathrm{m}^3} \frac{\mathrm{s}^3}{\mathrm{kg}} \\ &= 5.9715 \times 10^{7} \frac{1}{\Omega\,\mathrm{m}} \end{aligned}$$

The field is due to something outside of the conducting material (e.g. the separated charge pumped up by the battery). Notice that again we have grouped all the properties of the material together. Lets give a name to the quantity in parentheses that contains all the material properties. Since this quantity tells us how easily the charges will go through the conductive material, we can call this the *conductivity* of the material.

$$\sigma = \frac{n q_e^2 \tau}{m_e}$$

Then

$$J = \sigma E$$

The current density depends on two things, how well the material can allow the current to flow (bulk material properties related to conduction, σ) and and the field that motivates the current to flow, E.

The current, then, depends on these two items, as well as the cross sectional area of the wire

$$\begin{aligned} I &= JA \\ &= \sigma E A \end{aligned}$$

Really, the conductivity is more complicated than it appears. The mean time between collisions, τ, depends on the structure of the conductor. Different crystalline structures for the same element will give different values. Think of trying to walk quickly through the Manwering Center crowds during a class break. This takes some maneuvering. But if all the people were placed at equally spaced, regular intervals, it might be easier to make it through quickly. It would also be easier if the crowd stood still. Likewise, the position of the atoms in the conductor make a big difference in the conductivity, and thermal motion of those atoms also makes a large difference. We would expect the conductivity to depend on the temperature of the material.

18.1.1 Resistivity

It is common to speak of the opposite of the concept of conductance. In other words, how hard it is to get the electrons to travel through the conductive material. For example, we might want to build a heating device, like a toaster or space heater. In this case, we want friction in the wires, because that friction will produce thermal energy. So specifying a conductive material by how much friction it has is useful. How much the material impedes the flow of current is the opposite of how much the material allows the current flow, so we expect this new quantity to be the inverse of our conductivity

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq_e^2\tau}$$

Special conductors are often made that use "impurities," that is, trace amounts of other atoms, to increase or decrease the resistivity of those conducive materials by making the electrons collide more often. The thermal dependence can be modeled using the equation

$$\rho = \rho_o \left(1 + \alpha \left(T - T_o\right)\right)$$

where ρ_o is the resistivity at some reference temperature (usually $20\,^{\circ}\mathrm{C}$) and α is a constant that tells us how our particular material changes resistance with temperature. It is kind of like the specific heat in thermodynamics $Q = C\Delta T$. This is an approximation. It is a curve fit that works over normal temperatures. But we would not expect the same resistive properties, say, if we melt the material. The position of the atoms would change if the material goes from solid to liquid. So we will need to be careful in how we use this formula.

Here are some values of the conductivity, resistivity, and temperature coefficients for a few common conductive materials.

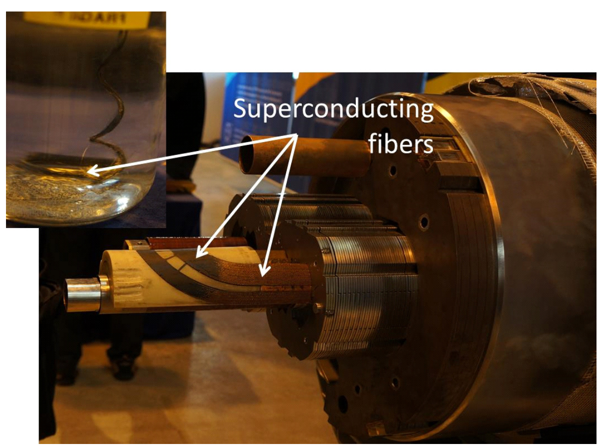
Material	Conductivity $\left(\Omega^{-1}\,\mathrm{m}^{-1}\right)$	Resistivity $(\Omega\,\mathrm{m})$	Temp. Coeff. $\left(\mathrm{K}^{-1}\right)$
Aluminum	3.5×10^7	2.8×10^{-8}	3.9×10^{-3}
Copper	6.0×10^7	1.7×10^{-8}	3.9×10^{-3}
Gold	4.1×10^7	2.4×10^{-8}	3.4×10^{-3}
Iron	1.0×10^7	9.7×10^{-8}	5.0×10^{-3}
Silver	6.2×10^7	1.6×10^{-8}	3.8×10^{-3}
Tungsten	1.8×10^7	5.6×10^{-8}	4.5×10^{-3}
Nichrome	6.7×10^5	1.5×10^{-6}	0.4×10^{-3}
Carbon	2.9×10^4	3.5×10^{-5}	-0.5×10^{-3}

18.1.2 Superconductivity

The relationship

$$\rho = \rho_o \left(1 + \alpha \left(T - T_o\right)\right)$$

also breaks down at low temperatures. The low end is very important these days. For some special materials, the resistivity goes to zero when the material is cold enough. We call these materials superconductors. A superconductor can carry huge currents, because there is no loss of energy, and no heat generated without any friction. Unfortunately most superconducting materials only operate as superconductors at temperatures near absolute zero. But a few "high temperature" superconductors operate at temperatures at high as 125 K. This is still very cold ($-150\,^\circ$C), but these temperatures are achievable, so some superconducting products are possible. As you can guess, there is very active research in making superconductors that operate at even higher temperatures.



Superconducting fiber material and superconducting magnet at CERN. These superconductors operate at 1.9K.

18.1.3 Ohm's law

Let's pause to review, Current density is given by

$$J = \sigma E$$

or now by

$$J = \frac{1}{\rho} E$$

Then the current is given by

$$\begin{aligned} I &= JA \\ &= \frac{A}{\rho} E \end{aligned}$$

If the field is nearly uniform in our conducting wire (similar to our capacitor field), then the potential would be just

$$\Delta V = E \Delta s$$

and then the electric field is approximately given by

$$E = \frac{\Delta V}{\Delta s}$$

For our wire of length L this is

$$E = \frac{\Delta V}{L}$$

Then we can use this field to write our current

$$I = \frac{A}{\rho} \frac{\Delta V}{L}$$

Once again, let's group together all the structural and material properties of the wire. We have

$$I = \left(\frac{A}{L\rho} \right) \Delta V$$

or with a little algebra,

$$\Delta V = I \left(\rho \frac{L}{A} \right)$$

The part in parenthesis contains all the friction terms. It says that the longer the wire, the more friction we will experience. This makes sense if you are familiar with fluid flow. The longer the hose, the more resistance. It also says that the larger the area, the lower the friction. That is also reasonable, since the electrons will have more places to go unrestricted if the area is bigger. In water flow, the larger the pipe, the less the water interacts with the sides of the pipe and therefore the lower the friction. This situation is analogous.

We should give a name to this quantity that describes the frictional properties of the wire. We will call it the *resistance* of the wire.

$$R = \rho \frac{L}{A}$$

so that we can write

$$I = \frac{\Delta V}{R}$$

The resistance has units of

$$\frac{\mathrm{V}}{\mathrm{A}} = \Omega$$

where Ω is given the name of *ohm* after the scientist that did pioneering work on resistance.

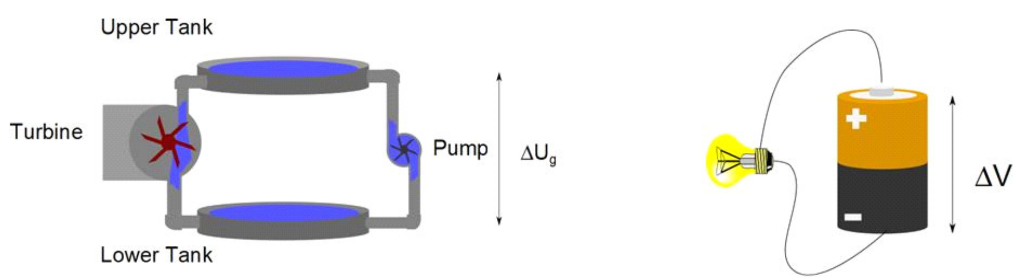
The relationship

$$I = \frac{\Delta V}{R}$$

is called *Ohm's law*.

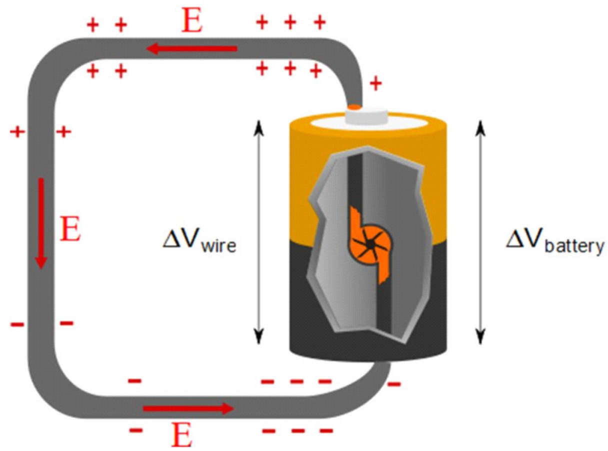
18.1.4 Life History of an electric current

Let's go back and think about our pump model for a battery.



The pump is a source of potential energy *difference*. This is what a battery does as well. The battery is a charge pump. It moves the charges from a low to a high potential. So it is a source of electric potential. The battery's job is to provide the charge separation that creates the electric field that drives the free charges, making the current.

A positive charge in the wire on the negative side of the battery is pumped up to the positive side through a chemical process. We can mentally envision a small charge pump inside of the battery



The battery is the source of the potential. A positive charge near the negative side of the battery would be pumped up to the positive side of the battery, It would gain potential energy

$$\Delta U_{battery} = q\Delta V_{battery}$$

Then it would "fall" down the wire. It must lose all of the potential energy it gained. So it will loose

$$|\Delta U_{wire}| = |\Delta U_{battery}|$$

But if the battery potential energy change is positive, the wire change must be negative. We can see that

$$\Delta V_{wire} = -\Delta V_{battery}$$

so the potential change in the wire is negative. We sometimes call this a potential "drop."

The field forces our charge to move through this wire much like the gravitational field forces rocks to fall. The positive charge ends up at the negative end of the battery again, ready to be pumped up to make another round.

Of course, really this process goes backwards in electrical circuits, since our charge carriers are negative, but we recall that mathematically negative charges going the opposite way is the same. So we will make this picture our mental model of a current.

18.1.5 Emf

We have ignored something in our pump model of a battery. In real water flow, there would be resistance to the flow even inside the pump. This resistance should be small, but not zero. So the actual potential energy gain would be

$$\Delta U = \Delta U_{\text{ideal}} - U_{\text{loss due to friction in the pump}}$$

The same is true for an actual battery. There is some resistance in the battery, itself.

$$\Delta V = \Delta V_{\text{ideal}} - \Delta V_{\text{loss due to resistance in the battery}}$$

Now that we have Ohm's law, we can see what $\Delta V_{\text{loss due to resistance in the battery}}$ would be in terms of the internal resistance of the battery and the current that flows. Referring to the last figure, there is only one way for the current to go. So for this circuit, the current must be the same throughout the entire circuit, even in the battery! If we call the small resistance in the battery r, then

$$\Delta V_{\text{loss due to resistance in the battery}} = Ir$$

then the actual potential energy provided by the battery is

$$\Delta V = \Delta V_{\text{ideal}} - Ir$$

It is traditional to give the ideal voltage a name and a symbol. And we have already encountered this name. It is "emf." Recall that at one time, the letters 'e', 'm', and 'f' stood for something. But not any more. It is just a name. It is pronounced "ē-em-ef," and the symbol is a script capital $\mathcal{E}$. So we can write

$$\Delta V = \mathcal{E} - Ir$$

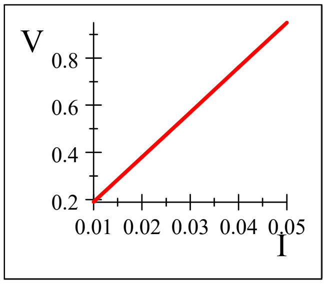
Sometimes you will hear $\mathcal{E}$ referred to as the voltage you would get if the battery is not connected (the "open circuit" voltage). This is the voltage marked on the battery. Notice that the actual voltage provided at the battery terminals depends on how much current is being drawn from the battery. So if you are draining your battery quickly (say, using your electric starter motor to start your car engine) the voltage supplied by your battery might drop (your lights might dim while the starer motor runs). You are not getting 12 V because the current I is large while the starter motor runs. We will change to this new symbol for ideal voltage. But we should keep in mind that actual voltages delivered may be significantly less than this ideal emf unless we plan our designs carefully.

18.1.6 Ohmic or nonohmic

This simple model of resistance is great for understanding simple things. Wires, and resistors do work like this. If we were to take a set of measurements of ΔV and I, we expect a straight line

$$\begin{aligned} y &= mx + b \\ \mathcal{E} &= \Delta V = RI + 0 \end{aligned}$$

where R is the slope.



But there are times when the model fails terribly. An incandescent light bulb is an example that we can quickly understand. The resistance at any one moment fulfils Ohm's law

$$I = \frac{\mathcal{E}}{R}$$

but light bulbs get hot. The resistance will change in time as the bulb heats up. So our relationship is now time dependent. Starting with the resistivity,

$$\rho = \rho_o \left(1 + \alpha \left(T - T_o\right)\right)$$

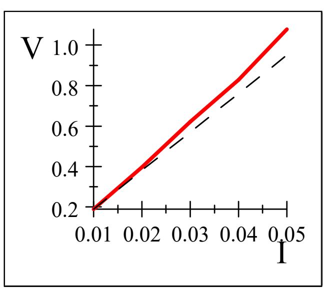
let's multiply both sides by A/L.

$$\frac{A}{L}\rho = \frac{A}{L}\rho_o \left(1 + \alpha \left(T - T_o\right)\right)$$

this gives

$$R = R_o \left(1 + \alpha \left(T - T_o\right)\right)$$

So if the resistance is temperature dependent, the slope of the line will change as we go along making measurements. We might get something like this

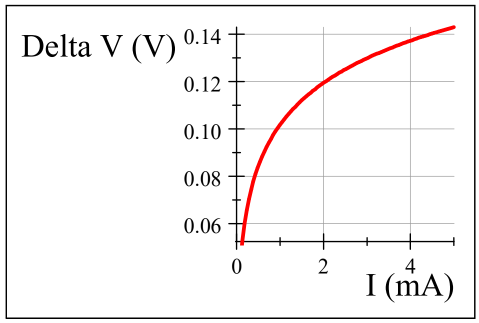


The dashed line is what we expect from Ohm's law. The solid line is what data from a light bulb would actually look like. We could use our temperature dependent resistance, and realize that the temperature is a function of time, to obtain

$$I = \frac{\mathcal{E}}{R_o \left(1 + \alpha \left(T\left(t\right) - T_o\right)\right)}$$

Since unless we acknowledge the temperature dependence this set of measurements is not strictly following Ohm's law, we will say that the light bulb is *nonohmic.*

Many common circuit elements are vary nonohmic. A diode, for example, has a ΔV vs. I relationship that looks like this.



18.2 Power in resisters

We learned that the resistance in a resister depends on the temperature of the resister, and even have an approximate relationship that shows how this works

$$R = R_o(1 + \alpha (T - T_o))$$

so we know that temperature and resistance are related. But most of us have used a toaster, or an electric stove, or an electric space heater, etc. How does an electric circuit produce heat? or even light from a light bulb?

To answer this let's think of the energy expended as an electron travels a circuit. The potential energy expended is

$$\Delta U = q\Delta V$$

where the ΔV comes from the battery, so we could write this as

$$\Delta U = q\mathcal{E}$$

This is the energy lost as the electron travels from one side of the battery to the other. We could describe how fast the energy is lost by dividing by the time it takes the electron to make the trip

$$\frac{\Delta U}{\Delta T} = \frac{q}{\Delta t}\mathcal{E}$$

but of course we want to do this for more than one electron. Let's take a small amount of charge, ΔQ, then

$$\frac{\Delta U}{\Delta T} = \frac{\Delta Q}{\Delta t}\mathcal{E}$$

and if we make the small group of charge very small we have

$$\frac{dU}{dT} = \frac{dQ}{dt}\mathcal{E}$$

and we recognize dU/dt as the power and dQ/dt as current, then

$$\mathcal{P} = I\mathcal{E}$$

This is the power supplied by the battery in moving the group of electrons through the circuit. But from conservation of energy, the charge packet must lose all the energy that the battery provides, so

$$\mathcal{P}_{battery} = \mathcal{P}_R = I\Delta V_R$$

is the energy that leaves the circuit as the packet of charge moves.

This works for any resistance

$$\mathcal{P}_R = I\Delta V_R$$

Then we can use Ohm's law

$$\Delta V_R = IR$$

to find

$$\begin{aligned} \mathcal{P}_R &= I\,(IR) \\ &= I^2 R \end{aligned}$$

But where does this energy go? This is the energy that makes the heat in the space heater, or the light in the light bulb.

We can now understand how an electric current is formed. Hopefully you will take a class like PH250 or Mechatronics where you will learn how to build simple circuits with resistances and capacitances. But for this class, we will now investigate a new force, the magnetic force.

18.3 Magnetism

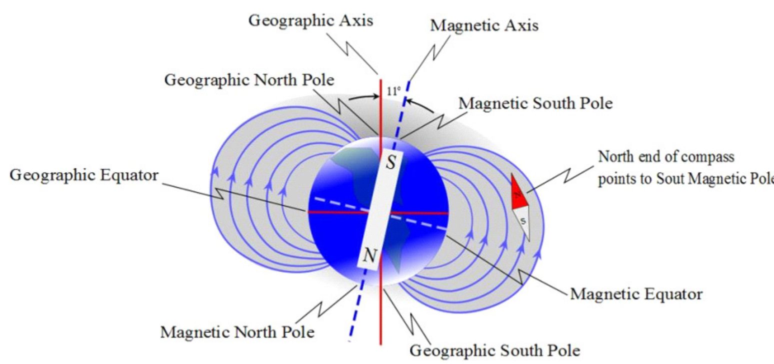
Most people have used a magnet. at some time. They come as ads that stick to a refrigerator. They are the working part of a compass. They hold the pieces of travel games to their boards, etc. So I think we all know that magnets stick to metal things. But do they stick to all metal things?

The answer is no, only a few metals work. Iron and Nickel and Cobalt are some that do. Aluminum and Copper do not. By the time we are done studying magnetism, we should be able to explain this.

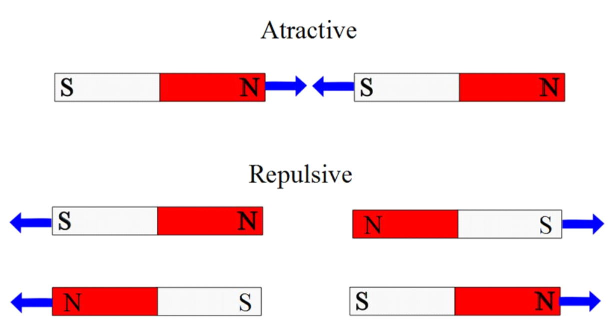
Magnets are very like charged objects in some ways. They can attract or repel each other They attract "unmagnetized" materials like charged things attracted uncharged insulators. But there are some important differences.

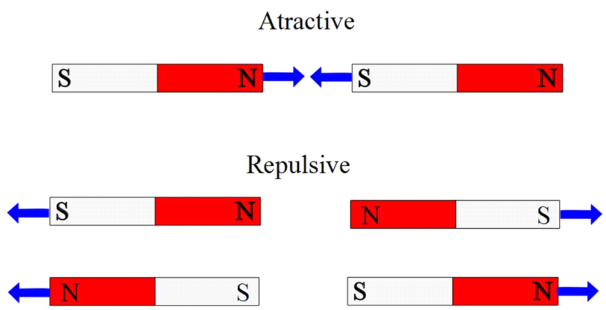
Notice that a "magnetic charge" seems to be induced in some metal objects, but not in other common objects. This is very different than electric charge and electric polarization! And we should state explicitly that for magnets, there seem to be both "charges" in the same object! We call the "charge centers" the *poles* of the magnet. We find that one pole attracts one of the poles of a second magnet and repels the other. If we turn around the first magnet, we find that our pattern of attraction and repulsion reverses. Because magnets were used for centuries in navigational compasses, we call one pole the *north pole* of the compass and the other the *south pole* of the magnet. The north pole is the pole that would orient toward the north. Why does this happen?

I hope your high school science class taught you that the Earth has a magnetic field.



So we constantly live under the influence of a large magnet! Now lets hang both of our magnets from a string, and see which way they like to hang. The north facing end we will label N and the south facing end we will label S. Now we can see that the two N ends repel each other and the two S ends repel each other. But a N end and a S end will attract.



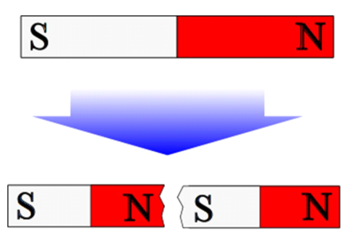


Once again we have a situation where we can define a mover object and an environmental object. We can picture one of the magnets making a magnetic field and the other magnet moving through this field. Once again we will think of this as a mover object and an environmental object. Of course both magnets make magnetic fields, but since a magnet can't make a magnetic field that moves itself, we won't draw this self-field for the mover magnet. We just draw the field for the environmental magnet. We did this in our Earth-compass picture. The Earth was the environmental magnet and the compass was the mover magnet.

One quirk of history is that since a N end of a magnet is attracted to the North part of the Earth. But north end of magnets are attracted to south poles of magnets, the Earth's geographic north pole must be a magnetic south pole!

One common misconception is that there is one specific place that is the magnetic north pole. Really it is a region near Newfoundland where the field strength actually varies quite a bit. You may have heard people discuss how the poles switch every so often. This is true, and we don't fully understand the mechanism for this.

There is a large difference between the magnetic force and the electric force. Electric charges are easy to separate. But magnetic poles are not at all easy to separate. If we break a magnet



we end up with each piece being a magnet complete with both north and south ends. This is very mysterious! something about the source of the magnetic field must be very different than for the source of the electric field. We will investigate the source of a magnetic field as we go.

The Earth's magnetic fields affects many biological systems. One of these is a bacteria that contain small permanent magnets inside of them to help them find the mud they live in.

In the 1990's there was a health fad involving magnets. Many people bought magnets to strap on their bodies. They were supposed to reduce aging and give energy. Mostly they stimulated the economy. But we will find that magnetic fields can alter the flow of blood (but these strap-on magnets did not do so, the FDA would not allow strong enough magnets to be sold as apparel to have this effect). Another common place to find magnetic fields is the MRI devices used in hospitals to make images of the interior of bodies.

Basic Equations

$$\sigma = \frac{n q_e^2 \tau}{m_e}$$

$$J = \sigma E$$

$$\begin{aligned} I &= JA \\ &= \sigma E A \end{aligned}$$

$$\rho = \frac{1}{\sigma} = \frac{m_e}{n q_e^2 \tau}$$

$$\rho = \rho_o \left(1 + \alpha \left(T - T_o\right)\right)$$

$$R = \rho \frac{L}{A}$$

$$I = \frac{\Delta V}{R}$$

$$\Delta V = \mathcal{E} - Ir$$

$$R = R_o \left(1 + \alpha \left(T - T_o\right)\right)$$

$$\mathcal{P} = I\mathcal{E}$$

$$\mathcal{P}_R = I \Delta V_R$$

$$\mathcal{P}_R = I^2 R$$