Chapter 2

Coulomb's Law and Lines of Force

Fundamental Concepts

- Our "charge" force is called the Coulomb force, and is given by $F = k_e \frac{|q_1||q_2|}{r^2}$
- A field is a quantity that has a value (magnitude and direction) at every point in space
- The Coulomb force is caused by an electric field
- We use field lines to give ourselves a mental picture of a field

2.1 Coulomb's Law

Sometime ago in your PH121 or Dynamics class you learned about gravity. Let's review for a moment.

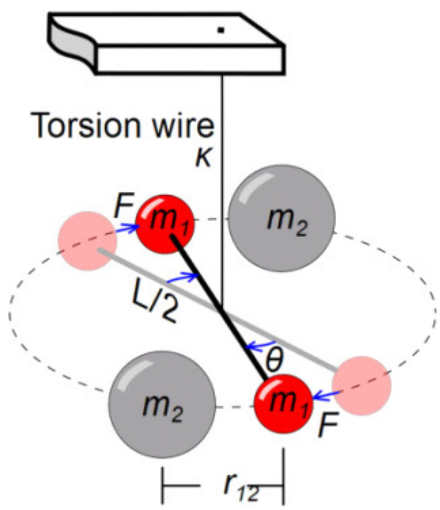
From our experience we know that more massive things exert a stronger gravitational pull than less massive things. We also have some idea that the farther away an object is, the less the gravitational pull. Newton expressed this as

$$F_g = G\frac{m_1 m_2}{r_{12}^2}$$

where the two masses involved (say, the Earth and you) are m_1 and m_2 and the distance between the two masses is r_{12} (e.g. the distance from the center of the Earth to the center of you). The constant G is a constant that puts the force into nice units that are convenient for us to use, like newtons (N). It has a value of

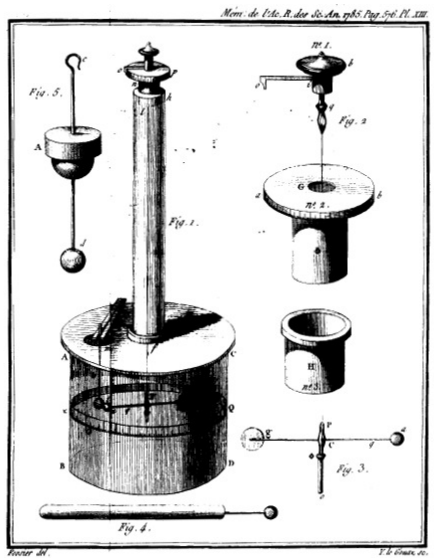
$$G = 6.67428 \times 10^{-11} \frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}$$

You might ask, how do we know this? The answer is that Newton and others performed experiments. Newton's law of gravitation is empirical, meaning that it came from experiment. Lord Cavendish used a clever device to verify this law. He suspended two masses from a wire. Then he placed two other masses near the suspended masses.



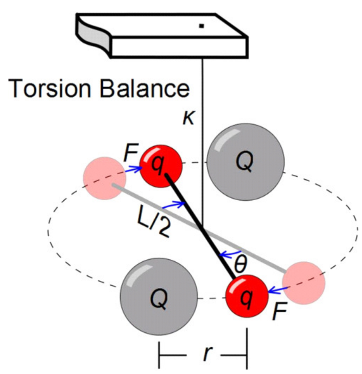
He knew the torsion constant of the wire (how much it resists being twisted). Then by observing how far the suspended masses moved, he could work out the strength of the gravitational force. This is called a torsion balance.

Charles Coulomb thought he could use the same device to measure the strength of the electric force. Here is his experimental design.



Coulomb's Torsion Balance Apparatus

You can see this is really just a torsion balance. This time objects with equal mass *and equal charge* are suspended on either end of a rod. The rod is hung on a wire. Two other *charges* are brought an equal distance, r_{12}, from the other charges. Knowing the torsional properties of the wire, the force due to the charges can be found.



Coulomb determined that the force due to a pair of charges has the following properties:

1. It is directed along a line connecting the two charged particles and is inversely proportional to the distance between their centers

2. It is proportional to the product of the magnitudes of the charges $|q_1|$ and $|q_2|$.

3. It is attractive (the charges accelerate towards each other) if the charges have different signs, and is repulsive (the charges accelerate away from each other) if the charges have the same signs.

We can write this in an equation

$$F = k_e \frac{|q_1|\,|q_2|}{r_{12}^2} \tag{2.1}$$

Note how much this looks like gravitation! In the denominator, we have the distance, r_{12}, between the two charged particles' centers. We have two things in the numerator. But now we have $|q_1|$ and $|q_2|$ instead of m_1 and m_2. We have a constant k_e instead of G, but the equation is very much like Newton's law of gravitation. That should be comforting, because we know how to use Newton's law of gravitation from PH121 or Dynamics. There is a very big difference, though. Gravitation can only attract masses, The Force due to charges can attract *or repel*.

Again there is a constant to fix up the units. Our constant is

$$k_e = 8.9875 \times 10^9 \frac{\mathrm{N\,m^2}}{\mathrm{C^2}} \tag{2.2}$$

which allows us to use more meaningful units (to us humans) in the force equation.

How about strength? Is gravity or is this force due to charge stronger?

Force	Varies with Distance	Attracts	Repels	Acts without contact	Strength
Gravity	Yes	Always	Never	Yes	Weaker
Charge Force	Yes	Sometimes	Sometimes	Yes	Stronger

Lets try an example problem:

Calculate the magnitude of the electric force between the proton and electron in a hydrogen atom. Compare to their gravitational attraction. We expect the electrical force to be larger. We need some facts about Hydrogen

Item	Value
Proton Mass	$1.67 \times 10^{-27}\,\mathrm{kg}$
Electron Mass	$9.11 \times 10^{-31}\,\mathrm{kg}$
Proton Charge	$1.6 \times 10^{-19}\,\mathrm{C}$
Electron Charge	$-1.6 \times 10^{-19}\,\mathrm{C}$
Proton-electron average separation	$5.3 \times 10^{-11}\,\mathrm{m}$

then,

$$\begin{aligned} F_e &= k_e \frac{|q_1|\,|q_2|}{r^2} \\ &= 8.9875 \times 10^9 \frac{\mathrm{N\,m^2}}{\mathrm{C^2}} \frac{\left(-1.6 \times 10^{-19}\,\mathrm{C}\right)\left(1.6 \times 10^{-19}\,\mathrm{C}\right)}{\left(5.3 \times 10^{-11}\,\mathrm{m}\right)^2} \\ &= -8.1908 \times 10^{-8} \frac{\mathrm{m}}{\mathrm{s^2}}\,\mathrm{kg} \end{aligned}$$

and

$$\begin{aligned} F_g &= G\frac{m_1 m_2}{r^2} \\ &= 6.67 \times 10^{-11}\,\mathrm{N}\frac{\mathrm{m}^2}{\mathrm{kg}^2}\frac{\left(1.67 \times 10^{-27}\,\mathrm{kg}\right)\left(9.11 \times 10^{-31}\,\mathrm{kg}\right)}{\left(5.3 \times 10^{-11}\,\mathrm{m}\right)^2} \\ &= 3.6125 \times 10^{-47}\frac{\mathrm{m}}{\mathrm{s}^2}\,\mathrm{kg} \end{aligned}$$

which shows us what we expected, the gravitational force is very small compared to the electric force.

2.1.1 Permittivity of free space

It is customary to define an additional constant

$$\epsilon_o = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12}\frac{\mathrm{C}^2}{\mathrm{N\,m}^2} \tag{2.3}$$

Using this constant

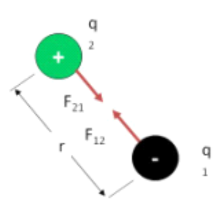
$$F = \frac{1}{4\pi\epsilon_o}\frac{|q_1|\,|q_2|}{r^2} \tag{2.4}$$

which really does not seem to be an improvement. But if you go on to take an advanced class in electrodynamics you will find that this form is more convenient in other unit systems. So we will adopt it even though it is an inconvenience now.

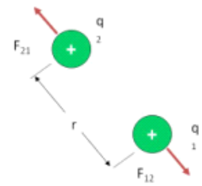
2.2 Direction of the force

What about direction? So far we have only calculated the magnitude of the force. But a force is a vector, so it must have a direction. Notice that our equation has absolute value signs in it. We will only get positive values from Coulomb's law.

To find a strategy for getting the direction, let's observe two charged objects



Experiments show that they seem to be pulled straight toward each other. The force seems to be along the line that passes through the center of charge for each of the two charged objects. We have to find this line from the geometry of our situation and our choice of coordinate systems. To make matters worse, we could have two of the same kind of charge.



The force will still be on the line connecting the centers of charge, but it will be in the opposite direction compared to the last case where the charges were of different sign. This seems complicated, and it is. We must observe the geometry of our situation and note whether the charges are the same or different signs to find the direction. Our equations can't tell us the direction on their own. You can't put the signs of the charges into the formula and expect a direction to come out! You have to draw the picture. Here is the process:

1. Define your coordinate system.

2. Find the line that connects the centers of charge. The force direction will be on that line.

3. Determine the direction by observing the signs of the charges. If the charges have the same sign, the force will be repulsive, if the charges have different signs, it will be attractive.

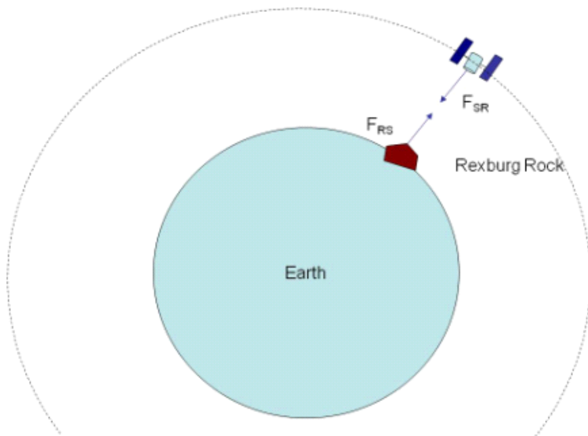
2.3 More than two charges

It is great that we know the force between two charges, but we have learned that there are billions of charges in everything we see or touch. It would be nice to be able to use our simple law of force on more than one or two charges. We did this with gravity. Let's review.

Suppose I have a satellite orbiting the Earth. That satellite feels a force given by

$$\begin{aligned} F_g &= G\frac{M_E m_s}{r^2} \\ &= G\left(\frac{M_E}{r^2}\right) m_s \end{aligned}$$

but consider that on the Earth below the satellite, there is a rock on the surface of the Earth.



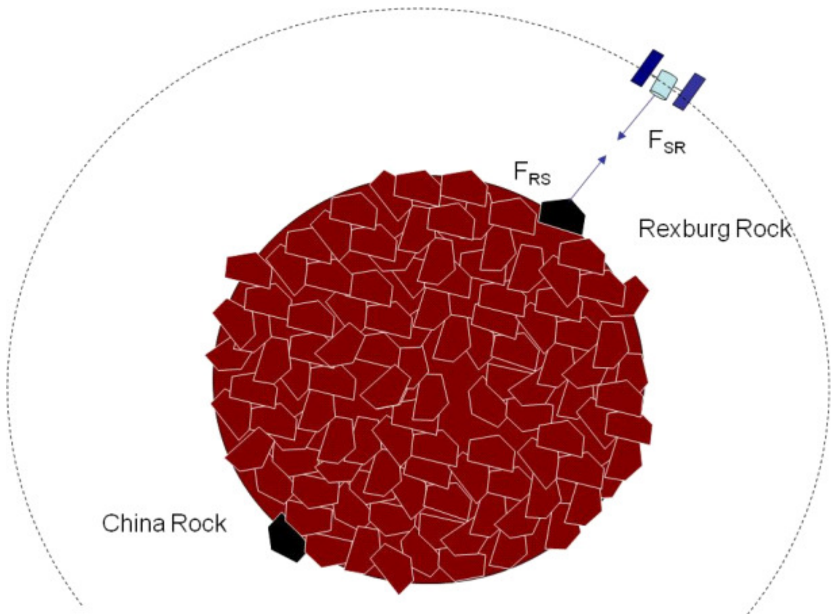
Part of the force due to gravity on the satellite must be due to this rock. We could write our force due to gravity as

$$F_g = G\left(\frac{M_{rest}}{r_{rest}^2}\hat{r}_{rest} + \frac{M_{rock}}{r_s^2}\hat{r}_{rock}\right)m_s$$

where M_{rest} is the mass of all the rest of the Earth, minus the rock. If we take the Earth rock by rock, we would have

$$F_g = G\left(\Sigma_i \frac{M_i}{r_i^2}\hat{\mathbf{r}}_i\right)m_s$$

where M_i is the mass of the i^{th} piece of the Earth and $\hat{\mathbf{r}}_i$ is the direction from M_i to m_s. We would not really want to do this calculation, because it would take a long time. Instead, back in PH121 or Dynamics we found we could add up all the mass and treat the Earth as one big ball of mass and represent it as if the mass was all at it's center of mass (as long as there is no rotation so no torque). But let's think about all this mass. Does the force between a rock in China and our satellite get diminished because our rock in Rexburg is in the way?



No, the force due to gravity is really the sum of all the little forces between all the parts of the Earth and our satellite. One bit of mass does not interfere with the force from another bit of mass.

Now let's look at the electric force. Suppose we have many charges in some configuration (maybe a round ball of charge). We could call the total charge, Q_E. Then our force magnitude on a movable charge q_m, would be

$$F_e = k_e \frac{|Q_E|\,|q_m|}{r^2}$$

The collection of charge Q_E would be the environmental charge. But we can picture this as the individual parts of Q_E all with little forces pairs acting on q_m summing up to get F_e.

$$F_e = k_e \Sigma_i \left(\frac{|Q_i|}{r_i^2} \hat{\mathbf{r}}_i \right) |q_m|$$

where Q_i is a piece of the total charge Q_E.

This is an amazingly simple idea. The force on a mover charge, q_m, due to any number of charges is just the sum of the forces due to each charge acting on q_m. Sometimes the mover charge is called a *test charge*, but we will call it a mover charge and we will call the Q_i environmental charges.

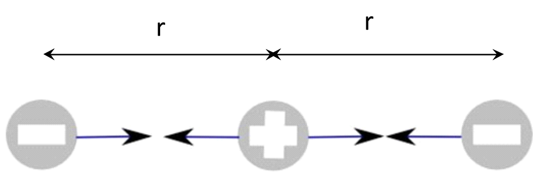
Suppose in our ball of charge, we have an element of charge on the opposite side of the ball and another element of charge close to us. Would the near charge element "screen off" or some how reduce the force due to the far charge element?

Like with gravity, it would not. Note that because one charge is farther away, the force from the far charge is not the same magnitude as that of the near charge. But we calculate both using our formula, and add them up (a vector sum) with all the others.

While we are talking about it, it might seem that the rest of the matter in the ball will screen off the electric force. But matter, itself, does not interfere with our electric force. Only other charges will change the force, and then only following the idea of that their forces add as vectors (remember that for electricity they can cancel, because we have both positive and negative charges).

When we add up the forces on a charge made by several other charges we say the forces are *superimposed.*

Now where there are forces, there will be Newton's second law! Let's consider a problem. Suppose we have three charges, equally spaced apart as shown where each has the charge of one electron (q_e) but the middle charge is positive and the other two are negative



We identify the middle charge as the mover (since we are asked for the force on this charge) and the left and right charges as the environmental charges. We can draw a free body diagram for the mover charge.

and find the net force on the mover charge, then

$$\overrightarrow{\mathbf{F}}_{net} = m\overrightarrow{\mathbf{a}} = \overrightarrow{\mathbf{F}_R} + \overrightarrow{\mathbf{F}}_L$$

We only have x-components so we can write this as

$$F_{net_x} = ma = F_{Rx} - F_{Lx}$$

where the minus sign is used for F_{Lx} because it is pointing to the left and that is usually the minus x direction.

We may ask, is this mover charge accelerating? We may suspect that the answer is no, but here we have something new. We don't know the magnitude of F_R or F_L. We now have to find the magnitudes to know. Back in PH121 you would have been given the magnitude of the forces, but in a charge problem we know how to calculate the magnitudes, so let's do that. We can use the formula for the Coulomb force

$$F = k_e \frac{|q_1|\,|q_2|}{r^2}$$

we can use r as the distance from the middle charge to each of the other charges since in this special case they are both the same distance from the middle charge. Then

$$F_R = k_e \frac{q_e^2}{r^2}$$

$$F_L = k_e \frac{q_e^2}{r^2}$$

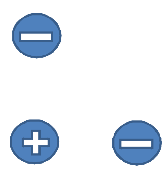
these are the magnitudes. We should notice that F_L points to the left. So we need to include a minus sign in front of it's magnitude.

$$F_{net_x} = ma = F_{Rx} - F_{Lx}$$

$$\begin{aligned} F_{net_x} &= ma = k_e \frac{q_e^2}{r^2} - k_e \frac{q_e^2}{r^2} \\ &= 0 \end{aligned}$$

now we can say that the middle charge is definitely not accelerating.

Of course this is a pretty easy Newton's 2nd law problem. It was all in the x-direction. But suppose that is not true. Then we need to take components of the forces vectors. Let's try one of those.



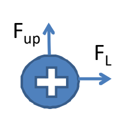
Here is a new configuration of our charges. There will be a Coulomb force between each negative charge the positive charge. What is the net force on the positive charge?

Again we need Newton's second law and the Coulomb force equation. We identify the positive charge as our mover, and the negative charges as the environmental charges. Our basic equations are

$$F = k_e \frac{|q_1|\,|q_2|}{r^2}$$

$$\overrightarrow{\mathbf{F}} = m\overrightarrow{\mathbf{a}}$$

but this time we need an x and a y Newton's second law equation. Let's draw the free body diagram. I have chosen the positive y-direction to be upward and the positive x-direction to be to the right.



The negative charge that is above our positive charge will cause an upward force. The negative charge to the right will cause a force that pulls to the right. This is a two-dimensional problem, so we need to split our Newton's second law into two one-dimensional problems.

$$\begin{aligned} F_{net_x} &= ma_x = F_L \\ F_{net_y} &= ma_y = F_{up} \end{aligned}$$

so

$$\begin{aligned} F_{net_x} &= k_e \frac{q_e^2}{r^2} \\ F_{net_y} &= k_e \frac{q_e^2}{r^2} \end{aligned}$$

We can see that there will be a force in both the x and the y direction. How do we combine these to get the net force? We use our basic equations for combining vectors:

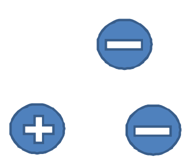
$$\begin{aligned} F_{net} &= \sqrt{F_{net_x}^2 + F_{net_y}^2} \\ &= \sqrt{\left(k_e \frac{q_e^2}{r^2}\right)^2 + \left(k_e \frac{q_e^2}{r^2}\right)^2} \\ &= \sqrt{2} \frac{1}{r^2} k_e q_e^2 \end{aligned}$$

but we are not done. We need a direction. Generally we use the angle with respect to the positive x-axis.

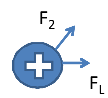
$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{F_{net_y}}{F_{net_x}}\right) \\ &= \tan^{-1}\left(\frac{k_e \frac{q_e^2}{r^2}}{k_e \frac{q_e^2}{r^2}}\right) \\ &= \frac{\pi}{4} \text{ rad} \end{aligned}$$

so we have a net force of $F = \sqrt{2} \frac{1}{r^2} k_e q_e^2$ at a 45° angle with respect to the x-axis.

Of course, this is still fairly simple, we should also review taking components of vectors that are not directed along the x and the y axis. Suppose we move the top charge as shown below



Once again the positive charge is the mover and the negative charges are the environment. Now our free body diagram looks like this:



Once again we have a two-dimensional problem. We need to convert it into two one-dimensional problems.

$$\begin{aligned} F_{net_x} &= ma_x = F_{L_x} + F_{2x} \\ F_{net_y} &= ma_y = F_{L_y} + F_{2y} \end{aligned}$$

but we don't know F_{L_x}, F_{2x}, F_{L_y}, and F_{2y}. But our basic equations should include how to make vector components

$$\begin{aligned} v_x &= v\cos\theta \\ v_y &= v\sin\theta \end{aligned}$$

where θ is measured from the positive x-axis. So

$$\begin{aligned} F_{net_x} &= ma_x = F_L\cos\theta_L + F_2\cos\theta_2 \\ F_{net_y} &= ma_y = F_L\sin\theta_L + F_2\sin\theta_2 \end{aligned}$$

and we realize that

$$\theta_L = 0$$

and that

$$\begin{aligned} \cos(0) &= 1 \\ \sin(0) &= 0 \end{aligned}$$

so

$$\begin{aligned} ma_x &= F_L + F_2\cos\theta_2 \\ ma_y &= 0 + F_2\sin\theta_2 \end{aligned}$$

This gives the x and y components of the net force on the positive charge. Using our Coulomb force for the magnitudes, we have

$$\begin{aligned} F_{net_x} &= k_e\frac{q_e^2}{r^2} + k_e\frac{q_e^2}{r^2}\cos\theta_2 \\ F_{net_y} &= k_e\frac{q_e^2}{r^2}\sin\theta_2 \end{aligned}$$

I will tell you $\theta = \frac{\pi}{4}$ rad (or $45\,^\circ$). So we can find

$$\begin{aligned} F_{net_x} &= k_e\frac{q_e^2}{r^2} + k_e\frac{q_e^2}{r^2}\left(\frac{\sqrt{2}}{2}\right) = k_e\frac{q_e^2}{r^2}\left(1 + \frac{\sqrt{2}}{2}\right) \\ F_{net_y} &= k_e\frac{q_e^2}{r^2}\left(\frac{\sqrt{2}}{2}\right) \end{aligned}$$

and

$$\begin{aligned}
F_{net} &= \sqrt{F_{net_x}^2 + F_{net_y}^2} \\
&= \sqrt{\left(k_e \frac{q_e^2}{r^2}\left(1+\frac{\sqrt{2}}{2}\right)\right)^2 + \left(k_e \frac{q_e^2}{r^2}\left(\frac{\sqrt{2}}{2}\right)\right)^2} \\
&= \frac{k_e q_e^2}{r^2}\sqrt{2+\sqrt{2}}
\end{aligned}$$

This is not so nice and easy. The angle is

$$\begin{aligned}
\theta &= \tan^{-1}\left(\frac{k_e \frac{q_e^2}{r^2}\left(\frac{\sqrt{2}}{2}\right)}{k_e \frac{q_e^2}{r^2}\left(1+\frac{\sqrt{2}}{2}\right)}\right) \\
&= \tan^{-1}\left(\frac{1}{2}\frac{\sqrt{2}}{\frac{1}{2}\sqrt{2}+1}\right) \\
&= 0.392\,70\,\mathrm{rad}
\end{aligned}$$

Note that I am using symbols as long as I can. This will become ever more important in this course. The problems will become very complicated. It is easier to make mistakes if you input numbers early.

Also notice that I carefully placed the charges the same distance, r, from each other. Of course that will not always be true. If the distances are different, we will use subscripts (e.g. r_1, r_2) to distinguish the distances.

2.4 Fields

If you are taking PH223 you should have already taken PH121 or an equivalent class. In PH121, you learned about how things move. You learned about forces and how force relates to acceleration

$$\overrightarrow{\mathbf{F}} = m\overrightarrow{\mathbf{a}}$$

The force, $\overrightarrow{\mathbf{F}}$, is how hard you push or pull. This push or pull changes the motion of the object, represented by it's mass, m. The change in motion is represented by its acceleration, $\overrightarrow{\mathbf{a}}$. Notice that both $\overrightarrow{\mathbf{F}}$ and $\overrightarrow{\mathbf{a}}$ are vectors. We will need all that you learned about vectors in PH121.

Since physics is the study of how things move, we are going to study the motion of objects again in this class. But in this section of our class we will learn about new sources of force, that is, new ways to push or pull something.

Really these new sources of force are not entirely new. You have heard of them and probably experienced them. They are electrical charge and magnetism. You have probably had a sock stick to you after pulling it out of a dryer, and you have probably had a magnet that sticks to your refrigerator. So although these new sources of force are new to our study of physics, they are somewhat familiar in every day lives.

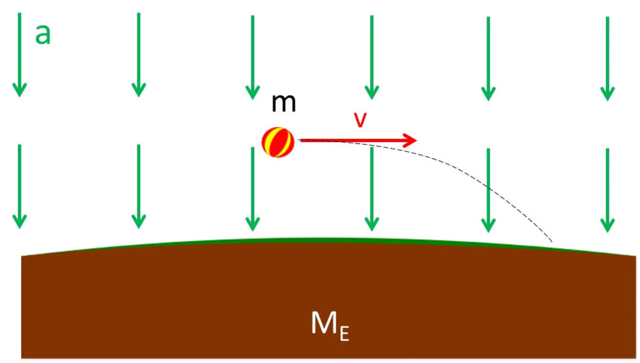
Let's review a particular force, the force due to gravity. This makes sense to do because our equation for electrical force is so very much like Newton's equation for gravity. Think of most of our experience with gravity. We have an object moving near the Earth. There is a force acting on the object, and that force is because of Earth's gravity.

We can think of the Earth as creating an environment in which the object moves, feeling the gravitational force. This is a property of all non-contact forces.

Think of a ball falling, We considered this as an environment of constant acceleration. In this environment, the ball feels a force proportional to its mass

$$\overrightarrow{\mathbf{F}} = m\overrightarrow{\mathbf{g}}$$

where $g = 9.81\frac{\mathrm{m}}{\mathrm{s}}$ is the acceleration due to gravity. This is true anywhere near the Earth's surface. We could draw this situation as follows:



where the environment for constant acceleration is drawn as a series of arrows in the acceleration direction (downward toward the center of the Earth). Anywhere the ball goes the environment is the same. So we draw arrows all around the ball to show that the whole environment around the ball is the same.

Notice that the environment is described by an acceleration, g given by

$$\overrightarrow{\mathbf{g}} = \frac{\overrightarrow{\mathbf{F}}}{m}$$

that is, the environment is described by the force per unit mass.

This environment is caused by the Earth being there. If the Earth suddenly disappeared, then the acceleration would just as suddenly go to zero. So we can say that the Earth creates this constant acceleration environment.

Notice that there are two objects involved, the ball and the Earth. Also notice that one object creates an environment in which the other object moves.

In our case, the Earth created the environment and the ball moved through the environment. This situation will recur many times, so let's give the objects these names, the Earth as the "Environmental object", and the ball as the "mover."

We should ask ourselves, does something like this happen with our electrical force? The electric forces is also a non-contact force. Could we view one charge as creating an environment in which the other charge moves? And if there is an environment, what would that environment be. Would it be an acceleration, or something else?

Michael Faraday came up with answers to this questions. To gain insight into his answers, let's consider our force again.

$$F_e = k_e \frac{|Q_E|\,|q_m|}{r^2}$$

but let's take q_m as a very small test charge that we can place near a larger distribution of charge Q_E. This is like the Earth and our small ball. The large Q_E is the environmental charge and the small q_m is the mover charge. We want q_m to be so small that it can't make any of the parts of Q_E rearrange themselves or any of the atoms forming the body that is charged with Q_E to polarize. Then we define a new quantity

$$\overrightarrow{\mathbf{E}} = \frac{\overrightarrow{\mathbf{F}}}{q_m}$$

This is the force per unit charge. This is very like our gravitational acceleration which is a force per unit mass. Then

$$\overrightarrow{\mathbf{F}} = q_m \overrightarrow{\mathbf{E}} \tag{2.5}$$

This is really like

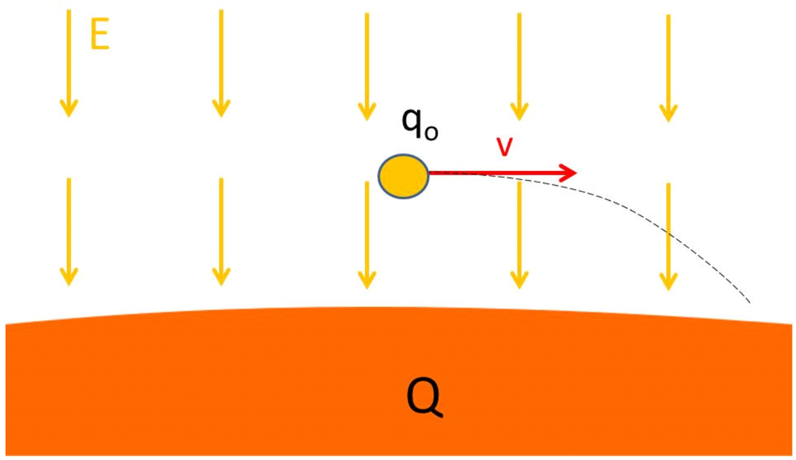
$$\overrightarrow{\mathbf{F}} = m\overrightarrow{\mathbf{g}}$$

but with the mass replaced by q_m and the acceleration replaced by this new force-per-unit-charge thing. For gravity it is the mass that made the gravitational pull. With the electric force it is the charge that creates the pull. So replacing m with q_m makes some sense. But what does it mean that the acceleration has been replaced by $\overrightarrow{\mathbf{E}}$. Well, since $\overrightarrow{\mathbf{g}}$ was the representation of the environment, can see that this new quantity is taking the place of the environment, but it can't be an acceleration. It does not have the right units. Let's investigate what it is.

Let's write the magnitude of E

$$\begin{aligned} E &= \frac{F}{q_m} \\ &= \frac{k_e \frac{|Q_E||q_m|}{r^2}}{q_m} \\ &= k_e \frac{Q_E}{r^2} \end{aligned}$$

But this is really not a quantity that we have seen before It depends on how far away we are from the environmental charge Q_E. It has a value at every point in space–the whole universe! (think of our acceleration environment being all around the moving ball) though it's values for large r are very small. The quantity is only large in the near vicinity of the charge, Q_E.



We can picture this quantity as being like a foot ball field with something (an environmental charge) hidden out there on the grass. If we know where the object is, we can tell a searcher how "warm" or "cold" they are as they wander around looking for the object. For every location, there is a value of "warmness." If we extend this idea to three dimensions, we are close to a picture of $\overrightarrow{\mathbf{E}}$. The environment quantity $\overrightarrow{\mathbf{E}}$ has a value at every point in three dimensional space. Since this is a new quantity, we need to give it a name. We will call it an *electric field.*

A field is a quantity that has a value (magnitude and/or direction) at every point in space.

But we have to add one more complication. It is a vector, so it also has a direction at each point in space as well. This direction is the direction the force would be on q_o, the mover, if we placed it at that location.

But where does this field come from? We say that an environmental charge Q_E creates a field

$$\overrightarrow{\mathbf{E}} = k_e \frac{Q_E}{r^2} \hat{\mathbf{r}} \tag{2.6}$$

centered at the charge location. The field is our environment for our mover.

Now we can understand our classical model about how gravity works! Have you wondered how a satellite knows that the Earth is there and that it should be pulled toward the Earth? We can say the Earth sets up a *gravitational field* because it has mass. The gravitational field shows up as an acceleration field. The satellite (the mover) feels the gravitational field because the field exists at the location of the satellite (it exists at all locations, so it exists at the satellite's location). The satellite does not have to know that the Earth is there, because it feels the field right where it is. The satellite reacts to the field, not directly with the Earth that created the field.[1]

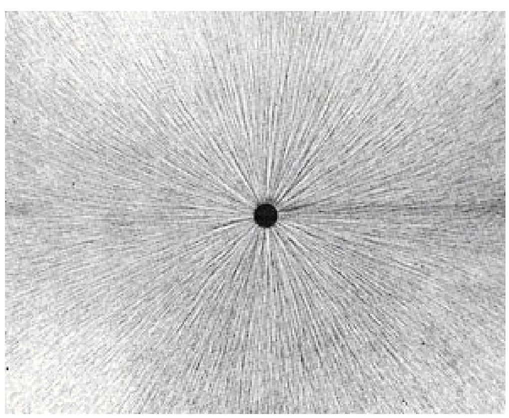
Likewise, our charge Q_E has the property of creating an *electric field* as the environment around it. Other charges (movers) will feel the field at their locations, and therefore will feel a force due to the field created by Q_E.

2.5 Field Lines

We need a way to draw the environment created by the environmental charge Q. We could draw lots of arrows like in the previous pictures. and we will do this sometimes. But there is another way to draw the environment that has become traditional. Have you ever taken iron filings and placed a magnet near them? If you do, you will notice that the filings seem to line up.

If you took PH121 you probably heard that there is a magnetic force. It is a non-contact force, so we expect it has a *magnetic field*. The iron filings are aligning because they are acted upon by the field. It is natural to represent this field as a series of lines like the ones formed by the iron filings. We will do this in a few lectures!

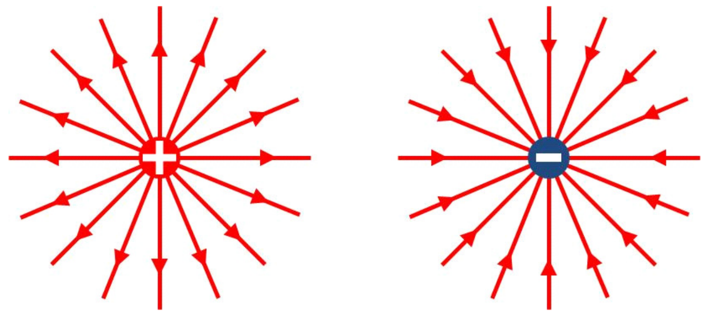
But there is a similar experiment we can do with the electric force. This is harder, but we can use small seeds or pieces of thread suspended in oil. These small things become polarized in an electric field. They line up like the iron filings.



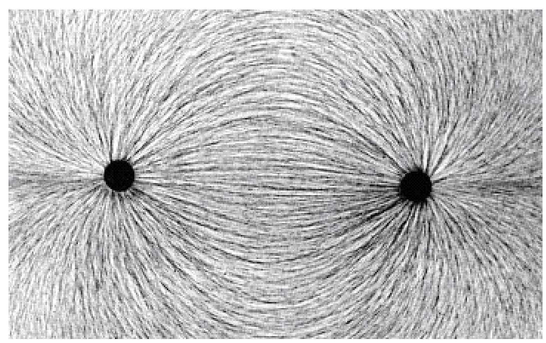
http://stargazers.gsfc.nasa.gov/images/geospace_images/electricity/elec_field_lines.jpg

We can represent the electric field by tracing out these lines. The last figure would look like this

[1] Here I am taking a quantum mechanical view of gravity. In General Relativity, the "field" is space that is warped by the mass of the Earth.

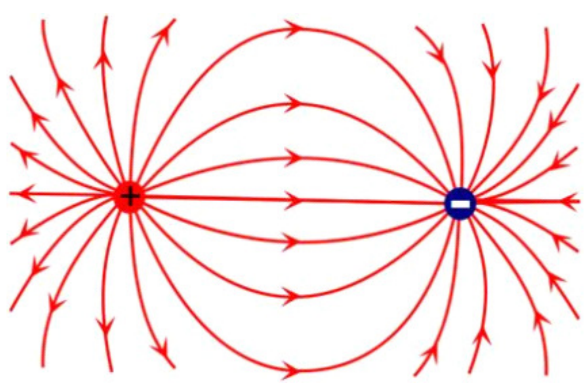


We can't tell if the charge was negative or positive from oil suspension picture, but if it was positive, by convention we draw the field lines as coming out of the charge. If it were negative the field lines would be drawn as going in to the charge. Here is a combination of a negative and a positive charge or dipole.

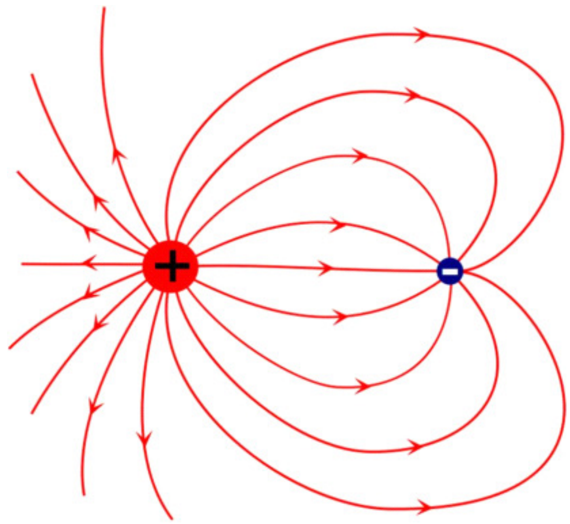


http://stargazers.gsfc.nasa.gov/images/geospace_images/electricity/elec_field_lines2.jpg

In this case both the positive and negative charges are working together to make the environment or field that a third charge could move through. The field line drawing would look like this.



This combination of positive and negative charges had equal charges, the only difference was the sign change. Here is one where the positive charge has more charge than the negative charge.



Notice that the number of field lines is proportional to the field, but there is no set proportionality. If the field from one charge is twice that of the other, we pick a number of field lines for the bigger charge that is twice the number we picked for the smaller charge. You can see this in the last figure. There are more lines attached to the smaller negative charge than there are for the bigger positive charge. So the positive charge has a larger charge amount than the negative charge.

This gives us a way to picture the electric field in our minds!

Some things to notice:

1. The lines begin on positive charges
2. The lines end on negative charges
3. If you don't have matching charges, the lines end infinitely far away (like the single charges in the first picture).
4. Larger charges have more lines coming from them
5. Field lines cannot cross each other
6. The lines are only imaginary, they are a way to form a mental picture of the field.

We only draw the field lines for the environmental charges. Of course the mover charge also makes a field, but this self-field can't cause the mover charge to move. If it could we could have perpetual motion and that violates the second law of thermodynamics. Since the mover's self-field is not participating in making the motion, we won't take the time to draw it![2]

[2] This picture will be a little more complicated when we allow for relativistic motion of charges and other more difficult effects, but that can wait for more advanced physics courses. For most engineering applications, this is a great approximation.

Remember, field lines are not real, but are a nice way to draw the field made by the environmental charge. We will use field lines often in drawing pictures as part of our problem solving process.

2.6 On-Line demonstrations

An applet that demonstrates the electric field of point charges can be found here:

http://phet.colorado.edu/sims/charges-and-fields/charges-and-fields_en.html

If you prefer a video game, try Electric Field Hockey:

http://phet.colorado.edu/en/simulation/electric-hockey

As a wacky example of Coulomb forces, see this video of charged water droplets orbiting charged knitting needles on the Space Shuttle:

http://www.nasa.gov/multimedia/videogallery/index.html?media_id=131554451

Basic Equations

$$F_e = k_e \frac{|Q_E|\,|q_m|}{r^2}$$

$$\overrightarrow{F}_{net} = q_m \overrightarrow{E}$$