Chapter 40

Properties of EM waves

Knowing that the electric and magnetic fields form plane waves, we can investigate these plane wave solutions to see what they imply.

Fundamental Concepts

- Energy is transmitted in EM waves
- The intensity rate of energy transfer in EM waves and it's direction are given by the Poynting vector
- The Poynting vector is proportional to the intensity of the EM wave.
- EM waves can transfer momentum.
- EM waves can cause pressure.

40.0.1 Energy in an EM wave

The electromagnetic (EM) wave is a wave. Waves transfer energy. It is customary find a vector that describes the flow of energy in the electromagnetic wave. We want the vector to be in the direction of every flow and to have its magnitude equal to the energy flow rate.

The rate of at which energy travels with the EM wave is given the symbol $\overrightarrow{\mathbf{S}}$ and is called the Poynting vector after the person who thought of it. It is

$$\overrightarrow{\mathbf{S}} = \frac{1}{\mu_o}\overrightarrow{\mathbf{E}} \times \overrightarrow{\mathbf{B}} \tag{40.1}$$

Let's deal with a dumb name first: The Poynting vector. It is named after a scientist with the last name Poynting. The name is really meaningless. There is nothing particularly "pointy" about this vector more than any other vector.

Instead of a formal derivation, let's just see what we get from Poynting's equation for a plane wave.

For our plane wave case, E and B are at $90\,^{\circ}$ angles[1]. so

$$S = \frac{1}{\mu_o} EB \tag{40.2}$$

and S will be perpendicular to both. Notice from our preceding figures that this is also the direction that the wave travels! That is comforting. That should be true for a EM wave. The energy, indeed, goes the way the Poynting vector points.

Using

$$\frac{E}{B} = c$$

we can write the magnitude of the Poynting vector as

$$S = \frac{E^2}{c\mu_o} \tag{40.3}$$

We could also express this in terms of B only.

Our eyes don't track the oscillations of the electromagnetic waves. Few detectors (if any) can for visible light. That is because for visible light, the frequency is very high. We usually see a time average. This time average of the Poynting vector is called the *intensity* of the wave

$$I = S_{ave}$$

We have seen intensity already for sound waves. Our ears can track signals in the normal frequencies for sound. But our eyes can't track the very large frequencies for visible light. So what we is a time average. Let's look at such an average for many periods of our visible light wave We can call the time over which we integrate an *integration time*. This means that the detector sums up (or integrates) the amount of power received over the detector time. Usually the integration time is much longer than a period, so what is really detected is like a time-average of our intensity. So if our wave is

$$E = E_{\max} \sin\left(kx - \omega t - \phi_o\right)$$

then

$$S = \frac{E_{\max}^2 \sin^2\left(kx - \omega t - \phi_o\right)}{c\mu_o}$$

and

$$\begin{aligned} I &= \int_{\text{many T}} S dt = \int_{\text{many T}} \frac{E_{\max}^2 \sin^2\left(kx - \omega t - \phi_o\right)}{c\mu_o} dt \\ &= \frac{E_{\max}^2}{c\mu_o} \int_{\text{many T}} \sin^2\left(kx - \omega t - \phi_o\right) dt \end{aligned}$$

[1] For other fields this might not be true, but it is generally true for light.

but we have seen an integral over $\sin^2(\theta)$ before! Back when we studied alternating current we found that (see equation ([?]))

$$\int_{\text{many T}} \sin^2(\omega t)\,dt = \frac{1}{2} \tag{40.4}$$

So we have

$$\begin{aligned} \bar{I} &= \frac{E_{\max}^2}{c\mu_o}\int_{\text{many T}} \sin^2(kx-\omega t-\phi_o)\,dt && (40.5)\\ &= \frac{E_{\max}^2}{2c\mu_o} && (40.6) \end{aligned}$$

where $\bar{I}$ is the time average intensity. The important part is that the time varying part has averaged out.

40.0.2 Intensity of the waves

The intensity of electromagnetic waves must relate to the strength of the fields. We can write it as

$$\bar{I} = \frac{E_{\max}^2}{2c\mu_o}$$

with our 2 from the time average, which we can just write as

$$\bar{I} = \frac{E^2}{2c\mu_o}$$

Again using

$$E = cB$$

we can write the intensity as

$$\bar{I} = \frac{(cB)^2}{2\mu_o c} \tag{40.7}$$

And we can write write the average intensity as

$$\bar{I} = \frac{c}{2\mu_o}B^2 \tag{40.8}$$

but this is less traditional. It is also traditional to drop the bar over the I for the average when speaking about light and to just remember I is an average.

We have already found that the intensity, I, is the magnitude of the average Poynting vector S_{ave} and the pointing vector is the amount of energy flow. Recall that we know the energy densities in the fields

$$\begin{aligned} u_E &= \frac{1}{2}\epsilon_o E^2\\ u_B &= \frac{1}{2}\frac{B^2}{\mu_o} \end{aligned}$$

Once again using,

$$E = cB \tag{40.9}$$

for electromagnetic waves we can write the energy density in the magnetic field as

$$\begin{aligned} u_B &= \frac{1}{2}\frac{B^2}{\mu_o} \\ &= \frac{1}{2}\frac{E^2}{c^2\mu_o} \\ &= \frac{1}{2}\epsilon_o E^2 \end{aligned} \tag{40.10}$$

so for a plane electromagnetic wave

$$u_E = u_B \tag{40.11}$$

The total energy in the field is just the sum

$$u = u_E + u_B = \epsilon_o E^2 \tag{40.12}$$

But when we do the time average to find the intensity, we pick up a factor of a half

$$u_{ave} = \frac{1}{2}\epsilon_o E^2 \tag{40.13}$$

Let's go back to our equation for intensity

$$I = S_{ave} = \frac{1}{2\mu_o c}E^2_{\max}$$

we could write this as

$$\begin{aligned} S_{ave} &= \frac{1}{2\mu_o c}\frac{\epsilon_o}{\epsilon_o}E^2_{\max} \\ &= \frac{1}{\mu_o c}\frac{\epsilon_o}{2\epsilon_o}E^2_{\max} \\ &= \frac{1}{\epsilon_o\mu_o c}\left(\frac{1}{2}\epsilon_o E^2_{\max}\right) \\ &= \frac{1}{\epsilon_o\mu_o c}\left(u_{ave}\right) \\ &= \frac{1}{\frac{1}{c^2}c}u_{ave} \\ &= cu_{ave} \end{aligned}$$

If you have already taken your course on thermodynamics you, learned that we could transfer energy by radiation. This is our radiation! And we see that it does indeed transfer energy. One way to use this is to put food in a microwave oven and let the microwave electromangetic waves add energy to your food. But another way to use this is to direct just the right wavelength of electromagnetic waves at a crowd. The Army uses such weapons that apply energy to crowds do disperse crowds (the people aren't actually damaged, but they do hurt when the wave hits them).

US Army Active Denial System (ADS).

40.0.3 Momentum of light

One of the strangest things about light waves is that there is also momentum transferred by the electromagnetic waves. But it's not really so strange if we think about it. Water waves defiantly transfer momentum and so do sound waves. The parts of the medium (water or air) knock into things and transfer momentum. But light doesn't have a material medium. So how can it transfer momentum? We have to go back to our understanding of electric fields. We know

$$\overrightarrow{F} = q\overrightarrow{E}$$

and our electromagnetic wave has an electric field $\overrightarrow{E}$. Matter also is made of charged particles, q. And we know electric fields can polarize atoms, so there would be a force on the polarized atom when an electromagnetic wave hits it. Now recall that

$$\begin{aligned} \overrightarrow{F}_{net} &= m\overrightarrow{a} \\ &= m\frac{d\overrightarrow{v}}{dt} \\ &= \frac{d\overrightarrow{p}}{dt} \end{aligned}$$

so if there is a force, we can have a transfer of momentum.

We won't derive this (it's a good problem for an upper division electricity and magnetism class). But if the waves are absorbed, the momentum is

$$p = \frac{U}{c} \tag{40.14}$$

or if the waves are reflected it is

$$p = \frac{2U}{c} \tag{40.15}$$

(think of balls bouncing off a wall, the change in momentum is always $2mv$ for a bounce).

We can think of the light exerting a pressure on the surface. Once again force is given by

$$\overrightarrow{F}_{net} = \frac{\overrightarrow{dp}}{dt}$$

then using this force, the pressure is

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} \tag{40.16}$$

then

$$P = \frac{F}{A} = \frac{1}{cA}\frac{dU}{dt} \tag{40.17}$$

And $\frac{1}{A}\frac{dU}{dt}$ is the energy rate per unit area, which is the magnitude of the Poynting vector, S. So our pressure due to light is

$$P = \frac{S}{c} \tag{40.18}$$

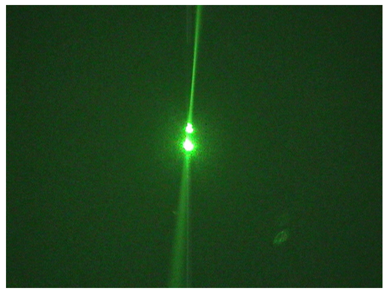
for perfect absorption. If there is perfect reflection

$$P = \frac{2S}{c} \tag{40.19}$$

This may seem a little strange. Water or sound waves would exert a pressure because the water or air particles can strike a surface, exerting a force. But the electromagnetic fields will create forces on the electrons in atoms[2], and most of the electrons are bound to the atoms in materials by the Coulomb force. So there really is a force on the material due to the electromagnetic wave. Quantum mechanics tells us about electrons being knocked out of shells into higher energy shells (absorbing photons of light) and re-emitting the light when the electrons fall back down to lower shells. This is a little like catching a frisbee, and then throwing it. Momentum is transferred both at the catch and at the release.

A cool use of this phenomena is called laser levitation

[2] Protons too, but the protons are more tightly bound due to the nuclear strong force and the nuclei are bound in the material. their resonant frequencies are usually not assessable to visible light, so I will ignore their effect in our treatment. But if you consider x-rays or gamma rays, they would be important.



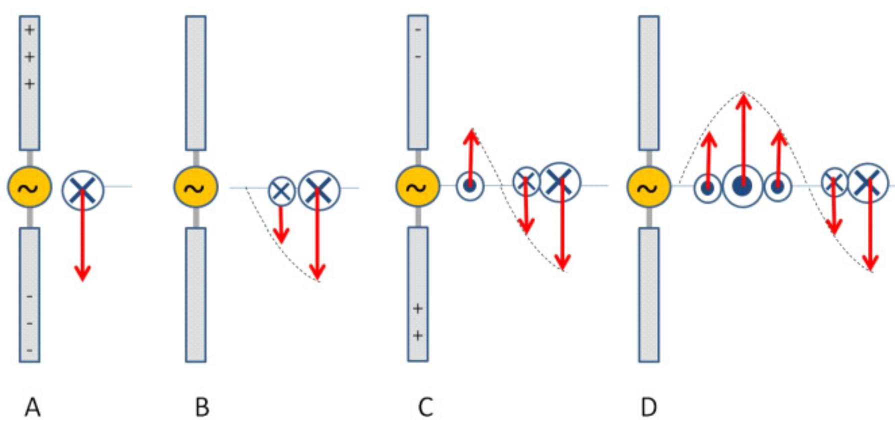
Laser Levation (Skigh Lewis, Larry Baxter, Justin Peatross (BYU), Laser Levitation: Determination of Particle Reactivity, ACERC Conference Presentation, February 17, 2005)

In the picture you are seeing a single small particle that is floating on a laser beam. the laser beam is directed upward. The force due to gravity would make the particle fall, but the laser light keeps it up!

40.0.4 Antennas

We have talked about making electromagnetic waves, but let's consider it again and try to put all we have done together to make a radio wave. First, we know from our analysis that we need changing fields. Neither static charges, nor constant currents will do. If we think about this for a minute, we will realize that the charges will *accelerate* if the current isn't constant. Fundamentally, this is the mechanism for making EM waves.

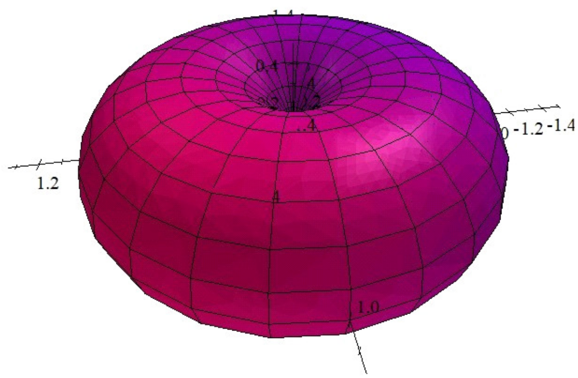
The half wave antenna is simple to understand, so let's take it as our example.



It is made from two long wires connected to an alternating current source (the radio transmitter).The charges are separated in the antenna as shown. But the separation switches as the alternating current changes direction. The charges accelerate back and forth, like a dipole switching direction. Radio people call this antenna a *simple dipole*.

Note the direction of the E and B fields. The Poynting vector is to the right. The antenna field sets up a situation far from the antenna, itself, where the changing electric field continually induces a magnetic field and the changing magnetic field continually induces a changing electric field. The wave becomes self sustaining! And the energy it carries travels outward.

Below you can see a graph of the sort of toroidal angular dependence of the dipole antenna emission pattern.



Angular dependence of S for a dipole scatterer.

From this you can see why we usually stand antennas straight up and down. Then the transmission travels parallel to the Earth's surface, where receivers are more likely to be.

Speaking of receivers, of course the receiver works like a transmitter, only backwards. The EM waves that hit the receiving antenna accelerate the electrons in the wire of the antenna. The induced current passed through an LRC circuit who's resonance frequency allows amplification of just one small band of frequencies (the one your favorite radio station is using) and then the amplified signal is sent to a speaker.

40.1 The Electromagnetic Spectrum

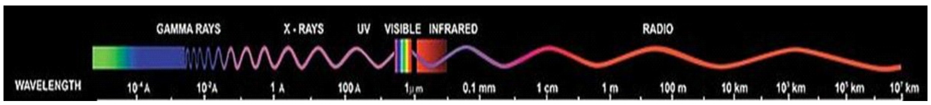
Maxwell predicted how fast his field waves would travel by finding the linear wave equation from the fields and noticing the speed indicated by the result. We have seen how he did this. The answer is

$$v = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \tag{40.20}$$

this speed is so special in physics that it get's its own letter

$$c = 2.99792 \times 10^8 \, \frac{\mathrm{m}}{\mathrm{s}} \tag{40.21}$$

which is of course the speed of light. In fact, that this was the measured speed of light was strong evidence leading us to conclude that light was really a type of these waves. As you know, there are a few more types of electromagnetic waves. In the following chart you can see that visible light is just a small part of what we call the *electromagnetic spectrum.*



Electromagnetic Spectrum (Public Domain image courtesy NASA)

The speed of light is always a constant in vacuum. This is strange. It caused a lot of problems when it was discovered.

$$v = f\lambda \tag{40.22}$$

or

$$c = f\lambda \tag{40.23}$$

where we can see that for light and electromagnetic waves, knowing the wavelength is always enough to know the frequency as well (in a vacuum).

As an example of what problems can come, let's consider a Doppler effect for light. Remember for sound waves, we had a Doppler effect. We will have a Doppler effect for electromagnetic waves too. But light does not change it's speed relative to a reference frame. This is *really weird*. The speed of light in a vacuum is *always c–no matter what frame we measure it in.*

Einstein's theory of Special Relativity is required to deal with this constant speed of light in every reference frame. From special relativity[3], the Doppler equation is

$$f' = f \frac{\sqrt{1 + \frac{v}{c}}}{\sqrt{1 - \frac{v}{c}}} \tag{40.24}$$

or, if we let u be the relative velocity between the source and the detector, and insist that $u \ll c$

$$f' = f \left(\frac{c + u}{c} \right) \tag{40.25}$$

Where of course f' is the observed frequency and f is the frequency emitted by the source. This is usually written as

$$f' = f \left(1 \pm \frac{u}{c} \right) \tag{40.26}$$

[3] At BYU-I we study this in our Modern Physics Class (PH279). So if you would like to know more about how to derive this equation, that would be a good place to start. Really any Modern physics textbook would do for this.

but it is really the same equation[4]. Just like with sound, we use the positive sign when the source and observer are approaching each other.

This means that if things are moving closer to each other the frequency increases. Think of

$$\lambda = \frac{c}{f} \tag{40.27}$$

this means that as a source and emitter approach each other, then the light will have a shorter wavelength. Think of our chart on the electromagnet spectrum. This means the light will get bluer. If they move farther apart, the light will get redder.

This is what gave us the hint that has lead to our cosmological theories like the big bang. Although this theory is now much more complicated, the facts are that as we look at far away objects, we see they are all *red shifted*. That is, they all show absorption spectra for known elements, but at longer wavelengths that we expect from laboratory experiments. We interpret this as meaning they are all going away from us!

40.1.1 Summary

Here is what we have learned so far about the properties of light

1. Electromagnetic waves travel at the speed of light
2. Electromagnetic waves are transverse electric and magnetic waves that are oriented perpendicular to each other.
3. $E = cB$
4. Electromagnetic waves carry energy *and momentum*

Basic Equations

Poynting Vector

$$\mathbf{S} = \frac{1}{\mu_o}\mathbf{E}\times\mathbf{B}$$

$$\frac{E}{B} = c$$

$$S = \frac{E^2}{c\mu_o}$$

[4] This equation is only really true for relative speeds u that are much less than the speed of light. Since is is very hard to make something travel even close to to the speed of light, we will find it is nearly always true.

$$I = \frac{1}{2\mu_o c} E_{\max}^2 = S_{ave}$$

Field energy density

$$\begin{aligned} u_E &= \frac{1}{2}\varepsilon_o E^2 \\ u_B &= \frac{1}{2}\frac{B^2}{\mu_o} \end{aligned}$$

Momentum due to light

$$p = \frac{U}{c}$$

or if the waves are reflected it is

$$p = \frac{2U}{c}$$

Pressure due to light is

$$P = \frac{S}{c}$$

for perfect absorption. If there is perfect reflection

$$P = \frac{2S}{c}$$