17 Optical systems

Combinations of lenses

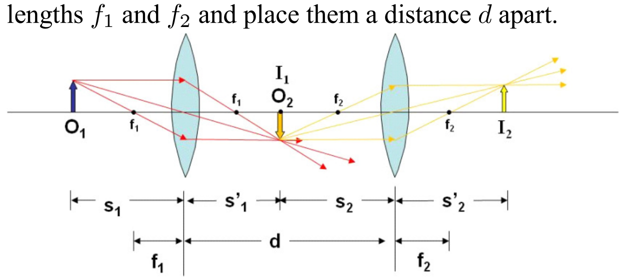
So far we have only used one lens or mirror at a time. But most optical systems are made from several lenses or mirrors (or a combination of both lenses and mirrors). We should think about how lenses work together to form optical systems like telescopes or microscopes or even compound camera lenses.

Question 223.17.1

Question 223.17.2

Question 223.17.3

To combine lenses, we do the same thing we did for the two surfaces of a thin lens. We form the image from the first lens as though the second lens is not there. Then we use the image from the first lens as the object for the second lens. Suppose we take two lenses of focal lengths f_1 and f_2 and place them a distance d apart.



Because this system would use a magnified image as the object for lens 2, the final magnification is the product of the two lens magnifications

$$M_{\text{combined}} = M_1 M_2 \tag{17.1}$$

Let's see that this must be true

$$M_1 = -\frac{s_1'}{s_1} = \frac{h_1'}{h}$$

and

$$M_2 = -\frac{s_2'}{s_2} = \frac{h_2'}{h_1'}$$

then

$$M_1 M_2 = \frac{h'_1}{h}\frac{h'_2}{h'_1} = \frac{h'_2}{h}$$

which is what we mean when we give the magnification of the optical system. It is a little more complicated to show where the final image will be. For the first lens we have

$$\frac{1}{s_1} + \frac{1}{s'_1} = \frac{1}{f_1} \tag{17.2}$$

where s'_1 is our first lens image distance. We can solve for s'_1

$$s'_1 = \frac{s_1 f_1}{s_1 - f_1} \tag{17.3}$$

We then take as the second object distance

$$s_2 = d - s'_1$$

we use the lens formula again.

$$\frac{1}{s_2} + \frac{1}{s'_2} = \frac{1}{f_2}$$

and again find the image distance

$$s'_2 = \frac{s_2 f_2}{s_2 - f_2}$$

but we can use our value of s_2 to find

$$\begin{aligned} s'_2 &= \frac{(d - s'_1) f_2}{(d - s'_1) - f_2} \\ &= \frac{(d - s'_1) f_2}{d - s'_1 - f_2} \end{aligned}$$

We have and expression relating the image distances, d and f_2. But we would really like to have an expression that relates s_1 and s'_2. Lets use

$$s'_1 = \frac{s_1 f_1}{s_1 - f_1}$$

and substitute it into our expression for s'_2

$$s'_2 = \frac{\left(d - \frac{s_1 f_1}{s_1 - f_1}\right) f_2}{d - \frac{s_1 f_1}{s_1 - f_1} - f_2}$$

This looks messy, but we can do some simplification

$$s'_2 = \frac{df_2 - \frac{s_1 f_1 f_2}{s_1 - f_1}}{d - f_2 - \frac{s_1 f_1}{s_1 - f_1}} \tag{17.4}$$

Well, it is still a little messy, but we have achieved our goal. We have s'_2 in therms of the focal lengths, d, and s_1.

Suppose we let $d \rightarrow 0$. Then

$$\begin{aligned} s_2' &= \frac{-\frac{s_1 f_1 f_2}{s_1 - f_1}}{-f_2 - \frac{s_1 f_1}{s_1 - f_1}} \\ &= \frac{\frac{s_1 f_1 f_2}{s_1 - f_1}}{\frac{f_2 (s_1 - f_1)}{s_1 - f_1} + \frac{s_1 f_1}{s_1 - f_1}} \\ &= \frac{s_1 f_1 f_2}{f_2 s_1 - f_2 f_1 + s_1 f_1} \\ &= \frac{s_1 f_1 f_2}{s_1 (f_2 + f_1) - f_2 f_1} \end{aligned}$$

So

$$s_2' = \frac{s_1 f_1 f_2}{s_1 (f_2 + f_1) - f_2 f_1}$$

Lets undo the math that brought us s_2' in the first place

$$\begin{aligned} \frac{1}{s_2'} &= \frac{s_1 (f_2 + f_1) - f_2 f_1}{s_1 f_1 f_2} \\ &= \frac{s_1 (f_2 + f_1)}{s_1 f_1 f_2} - \frac{f_2 f_1}{s_1 f_1 f_2} \\ &= \frac{(f_2 + f_1)}{f_1 f_2} - \frac{1}{s_1} \end{aligned}$$

or

$$\frac{1}{s_2'} + \frac{1}{s_1} = \frac{(f_2 + f_1)}{f_1 f_2}$$

Which looks very like the lens formula with

$$\frac{1}{f} = \frac{(f_2 + f_1)}{f_1 f_2}$$

If we unwind this expression, we find

$$\frac{1}{f} = \frac{f_2}{f_1 f_2} + \frac{f_1}{f_1 f_2}$$

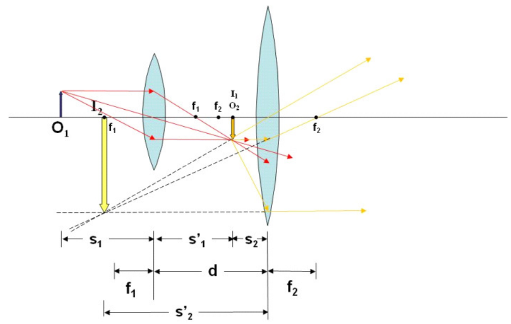
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \tag{17.5}$$

This is how we combine thin lenses. We see that the two lenses are equivalent to a single lens with focal length f as long as they are close together.

Of course, we had to place our lenses right next to each other for this to work. This is not the case for a telescope or microscope. We should look at such a case. There is no need for more math. We can go back to equation (17.4).

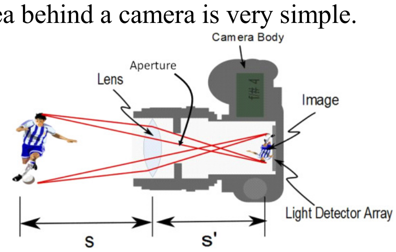
$$s_2' = \frac{d f_2 - \frac{s_1 f_1 f_2}{s_1 - f_1}}{d - f_2 - \frac{s_1 f_1}{s_1 - f_1}}$$

But let's look at a case using ray diagrams. For this case, let's take two lenses, and let's have the first lens make a real image. Once again, let's have that image be the object for the second lens. But this time, let's move the second lens so that the image from the first lens (object for the second lens) is closer to the second lens than f_2. If that is the case, the second lens works like a magnifier. The final image is enlarged.



The Camera

in 1900 George Eastman introduced the Brownie Camera. This event has changed society dramatically. The idea behind a camera is very simple.



The camera has a lens (often a compound lens like the ones we have just discussed) and a screen for projecting a real image created by the lens.

Let's take an example camera. Say we wish to take a picture of Aunt Sally. Aunt Sally is about $1.5\,\mathrm{m}$ tall. She is standing about $5\,\mathrm{m}$ away. Then to fit the image of Aunt Sally on our $35\,\mathrm{mm}$ detector, we must have

$$
\begin{aligned}
h &= 1.5\,\mathrm{m} \\
h' &= 0.035\,\mathrm{m} \\
s &= 5\,\mathrm{m} \\
f &= 0.058\,\mathrm{m}
\end{aligned}
$$

We wish to find s' and m. Let's do m first.

$$\begin{aligned} m &= \frac{h'}{h} = \frac{-0.035\,\mathrm{m}}{1.5\,\mathrm{m}} \\ &= -2.3333 \times 10^{-2} \end{aligned}$$

so our image is small and inverted. The small size we wanted. But now we know that the image in our cameras is upside down. A digital camera uses it's built-in computer to turn the image right side up for us on the display on the back of the camera.

Now let's find s'.

$$\begin{aligned} s' &= \frac{fs}{s-f} \\ &= 5.8681 \times 10^{-2}\,\mathrm{m} \\ &= 58.681\,\mathrm{mm} \end{aligned}$$

so our detector must be $58.681\,\mathrm{mm}$ from the lens.

Now suppose we want to photograph a $1000\,\mathrm{m}$ tower from $2\,\mathrm{km}$ away. Then

$$\begin{aligned} m &= -\frac{0.035\,\mathrm{m}}{1000\,\mathrm{m}} \\ &= -3.5 \times 10^{-5} \end{aligned}$$

and

$$\begin{aligned} s' &= \frac{(0.058\,\mathrm{m})\,(2000\,\mathrm{m})}{2000\,\mathrm{m} - (0.058\,\mathrm{m})} \\ &= 5.8002 \times 10^{-2}\,\mathrm{m} \\ &= 58.002\,\mathrm{mm} \end{aligned}$$

Notice that the image distance changed, but not by very much. This is why you need a focus adjustment on the lens of a good camera. Objects far away require a different s' value than objects that are close. Usually you twist the lens housing to make this adjustment. The lens housing has a threaded screw system that increases or decreases s' as you twist. Consumer cameras often have an motor that makes this adjustment for you. You may see the lens move back and forth as someone takes a picture.

There are several things that govern whether a picture will be good. When you buy a quality manual lens, it will be marked in $f\#s$. The specification of an automatic lens will be given in terms of $f/\#s$. To help us buy such lenses, we should understand what the terminology means.

Question 223.17.4

Most things we want to take a snapshot of are much farther than $58\,\mathrm{mm}$ from the camera. For such objects we can revisit the magnification.

$$m = -\frac{s'}{s}$$

but from the thin lens formula

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Question 223.17.5

If $s \gg f$ then we can say that $1/s \approx 0$ and so $s' \approx f$. Then

$$m=-\frac{f}{s}$$

and we see that the size of the image is directly proportional to the focal distance. If we change the focal distance, we can change the size of the image. This is how a zoom lens works. A zoom lens is a compound lens, and the focal length is changed by increasing the distance between the component lenses. This is what your camera is doing when it zooms in and out when you push the telephoto button.

Remember we studied intensity

$$I=\frac{P}{A}$$

Photographic film and digital focal plane arrays detect the intensity of light falling on them. We can see that the area of our image depends on our magnification, which depends on s' and for our distant objects it is proportional to f. The image area is proportional to $s'^2 \approx f^2$. So we can say that the area is proportional to f^2. Then

$$I \propto \frac{P}{f^2}$$

Question 223.17.6

The power entering the camera is proportional to the size of the aperture (hole the light goes through). A bigger aperture lets in more light. A smaller aperture lets in less light. If the camera has a circular opening, this area is proportional to the square of the diameter of the opening, D^2 so

$$I \propto \frac{D^2}{f^2}$$

This ratio is useful because it tells us how much intensity we get in terms of things we can easily know. Good cameras have changeable aperture sizes, and good lenses have changeable focal lengths. But buy using the combination of these two terms, we can ensure we will get enough light (but not too much) when we take the picture.

It would be good to give this ratio a special name. But instead, we named the ratio

$$f/\# \equiv \frac{f}{D} \tag{17.6}$$

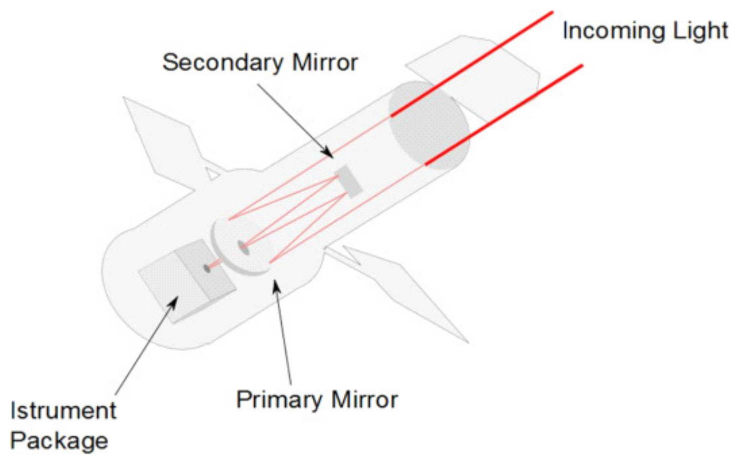
It is called the $f/\#$ (pronounced f-number) so

$$I \propto \frac{1}{(f/\#)^2} \tag{17.7}$$

So good cameras have adjustable lens systems marked in $f/\#'s$. Typical values are $f/2.8$, $f/4$, $f/5.6$, $f/8$, $f/11$, and $f/16$.

This terminology is used for telescope design as well. The Hubble telescope is an $f/24$

Ritchey-Chretien Cassegrainian system with a $2.4\,\mathrm{m}$ diameter aperture. The effective focal length is $57.6\,\mathrm{m}$.



It is important to realize that electronic (and biological) sensor don't react instantly to what we see. The intensity is

$$I = \frac{P}{A} = \frac{\Delta E}{\Delta t A}$$

so there is a time involved. The time it takes to collect enough light to form an image on the sensor is called the *exposure time*.

$$\Delta t = \frac{\Delta E}{IA}$$

So changing our $f/\#$ changes the needed exposure time by changing the intensity. This is part of what a good photographer does in taking a picture. The photographer will adjust the $f/\#$ and the exposure time to get a photograph that is not too exposed (too light) or underexposed (to dark).