Chapter 10

Interference of light waves

Fundamental Concepts

- Light is a wave in the electromagnetic field
- Light is a superposition of many small waves called photons
- The energy in a photon is proportional to the frequency of the photon
- If we mix two coherent light sources, we get interference, with an intensity pattern given by $I = I_{\max} \cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda} d \sin\theta\right)\right)$
- Most detectors cannot follow the fluctuation of light because their integration time is too long.

10.1 The Nature of Light

10.1.1 Physical Ideas of the nature of Light

Before the 19th century (1800's) light was assumed to be a stream of particles. Newton was the chief proponent of this theory. The theory was able to explain reflection of light from mirrors and other objects and therefore explain vision. In 1678 Huygens showed that wave theory could also explain reflection and vision.

In 1801 Thomas Young demonstrated that light had attributes that were best explained by wave theory. We will study Young's experiment later today. The crux of his experiment was to show that light displayed constructive and destructive interference–clearly a wave phenomena! The theory of the nature of light took a dramatic shift

In 1805 Joseph Smith was born in Sharon, Vermont.

In September of 1832 Joseph Smith received a revelation that said in part :

> For the word of the Lord is truth, and whatsoever is truth is light, and whatsoever is light is Spirit, even the Spirit of Jesus Christ. And the Spirit giveth light to every man that cometh into the world; and the Spirit enlighteneth every man through the world, that hearkeneth to the voice of the Spirit. (D&C 84:45-46)

In December of 1832 Joseph Smith received another revelation that says in part:

> This Comforter is the promise which I give unto you of eternal life, even the glory of the celestial kingdom; which glory is that of the church of the Firstborn, even of God, the holiest of all, through Jesus Christ his Son—He that ascended up on high, as also he descended below all things, in that he comprehended all things, that he might be in all and through all things, the light of truth; which truth shineth. This is the light of Christ. As also he is in the sun, and the light of the sun, and the power thereof by which it was made. As also he is in the moon, and is the light of the moon, and the power thereof by which it was made; as also the light of the stars, and the power thereof by which they were made; and the earth also, and the power thereof, even the earth upon which you stand. And the light which shineth, which giveth you light, is through him who enlighteneth your eyes, which is the same light that quickeneth your understandings; which light proceedeth forth from the presence of God to fill the immensity of space—the light which is in all things, which giveth life to all things, which is the law by which all things are governed, even the power of God who sitteth upon his throne, who is in the bosom of eternity, who is in the midst of all things. (D&C 88:5-12)

Light, even real, physical light, seems to be of interest to Latter Day Saints.

In 1847 the saints entered the Salt Lake Valley.

In 1873 Maxwell published his findings that light is an electromagnetic wave (something we will try to show before this course is over!).

Planck's work in quantization theory (1900) was used by Einstein In 1905 to give an explantation of the photoelectric effect that again made light look like a particle.

Current theory allows light to exhibit the characteristics of a wave in some situations and like a particle in others. We will study both before the end of the semester.

The results of Einstein's work give us the concept of a *photon* or a quantized unit of radiant energy. Each "piece of light" or photon has energy

$$E = hf \tag{10.1}$$

where f is the frequency of the light and h is a constant

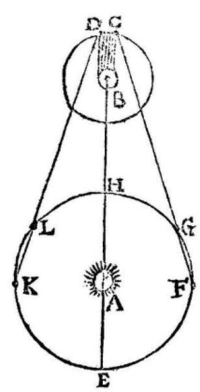
$$h = 6.63 \times 10^{-34}\,\mathrm{J\,s} \tag{10.2}$$

The nature of light is fascinating and useful both in physical and religious areas of thought.

10.2 Measurements of the Speed of Light

One of the great foundations of modern physical theory is that the speed of light is constant in a vacuum. Galileo first tried to measure the speed of light. He used two towers in town and placed a lantern and an assistant on each tower. The lanterns had shades. The plan was for one assistant to remove his shade, and then for the assistant on the other tower to remove his shade as soon as he saw the light from the first lantern. Back at the first tower, the first assistant would use a clock to determine the time difference between when the first lantern was un-shaded, and when they saw the light from the second tower. The light would have traveled twice the inter-tower distance. Dividing that distance by the time would give the speed of light. You can probably guess that this did not work. Light travels very quickly. The clocks of Galileo's day could not measure such a small time difference. Ole Rømer was the first to succeed in measuring the speed of light.

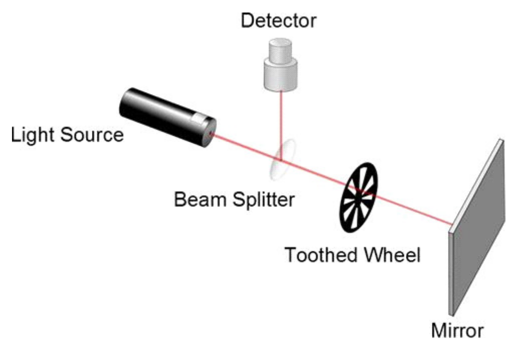
10.2.1 Rømer's Measurement of the speed of light



A diagram illustrating Rømer's determination of the speed of light. Point A is the Sun, piont B is Jupiter. Point C is the immersion of Io into Jupiter's shadow at the start of an eclipse

Rømer performed his measurement in 1675, 269 years before digital devices existed!. He used the period of revolution of Io, a moon of Jupiter, as Jupiter revolved around the sun. He first measured the period of Io's rotation about Jupiter, then he predicted an eclipse of Io three months later. But he found his calculation was off by 600 s. After careful thought, he realized that the Earth had moved in its orbit, and that the light had to travel the extra distance due to the Earth's new position. Given Rømer's best estimate for the orbital radius of the earth and his time difference, Rømer arrived at a estimate of $c = 2.3 \times 10^8 \, \frac{\text{m}}{\text{s}}$. Amazing for 1675!

10.2.2 Fizeau's Measurement of the speed of light



Hippolyte Fizeau measured the speed of light in 1849 using a toothed wheel and a mirror and a beam of light. The light passed through the open space in the wheel's teeth as the wheel rotated. Then was reflected by the mirror. The speed would be

$$v = \frac{\Delta x}{\Delta t}$$

We just need Δx and Δt.

It is easy to see that

$$\Delta x = 2d$$

because the light travels twice the distance to the mirror (d) and back. So the speed is just

$$v = \frac{2d}{\Delta t}$$

If the wheel turned just at the right angular speed, then the reflected light would hit the next tooth and be blocked. Think of angular speed

$$\omega = \frac{\Delta \theta}{\Delta t}$$

so the time difference would be

$$\Delta t = \frac{\Delta \theta}{\omega}$$

We find $\Delta\theta$ by taking the number of teeth on the wheel and dividing by 2π by that number.

$$\Delta \theta = \frac{2\pi}{N_{teeth}}$$

Then the speed of light must be

$$\begin{aligned} c &= v = \frac{2d}{\frac{\Delta\theta}{\omega}} \\ &= \frac{2d\omega}{\Delta\theta} \\ &= \frac{2d\omega N_{teeth}}{2\pi} \\ &= \frac{d\omega N_{teeth}}{\pi} \end{aligned}$$

then if we have 720 teeth and ω is measured to be $d = 7500\,\mathrm{m}$

$$\begin{aligned} c &= \frac{(7500\,\mathrm{m})\,(172.79\,\mathrm{Hz})\,(720)}{\pi} \\ &= 2.97 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

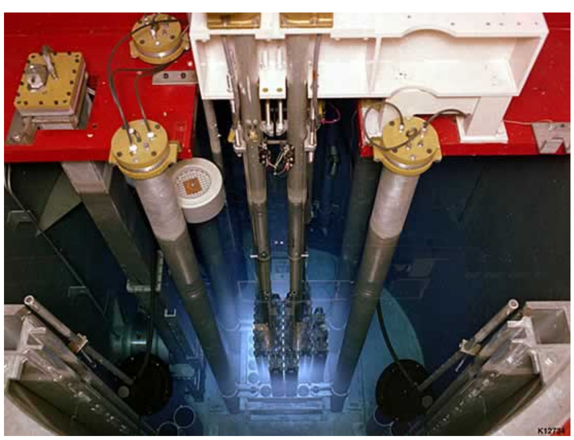
which is Fizeau's number and it is pretty good!

Modern measurements are performed in very much the same way that Fizeau did his calculation. The current value is

$$c = 2.9979 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}} \tag{10.3}$$

10.2.3 Faster than light

The speed of light in a vacuum is constant, but in matter the speed of light changes. We will study this in detail when we look at refraction. But for now, a dramatic example is Cherenkov radiation. It is an eerie blue glow around the core of nuclear reactors. It occurs when electrons are accelerated past the speed of light in the water surrounding the core. The electrons emit light and the light waves form a Doppler cone or a light-sonic boom! The result is the blue glow.

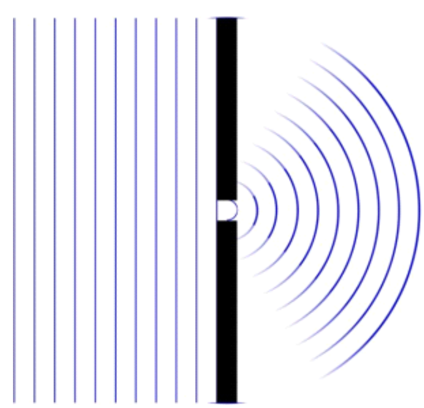


Cherenkov radiation from a 250kW TRIGA reactor. (Image in the Public Domain, courtesy US Department of Energy)

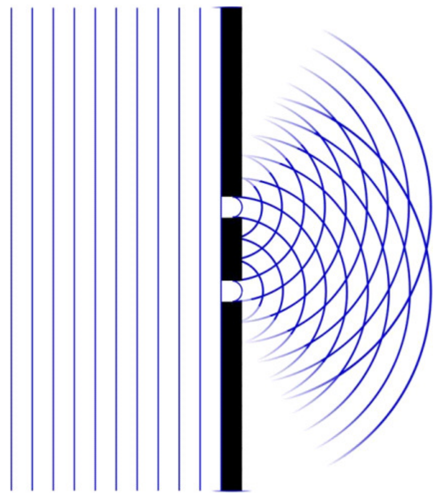
This does bring up a problem in terminology. What does the word "medium" mean? We have used it to mean the substance through which a wave travels. This substance must have the property of transferring energy between it's parts, like the coils of a spring can transfer energy to each other, or like air molecules can transfer energy by collision. For light the wave medium is the electromagnetic field. This field can store and transfer energy (we will see this later in the course). But many books on physics call materials like glass a "medium" through which light travels. The water in our last example is such a medium. Are glass and water wave mediums for light? The answer is no. Light does not need any matter to form it's wave. The wave medium is the electromagnetic field. So we will have to keep this in mind as we allow light to travel through matter. We may call the matter a "medium," but it is not the wave medium.

10.3 Interference and Young's Experiment

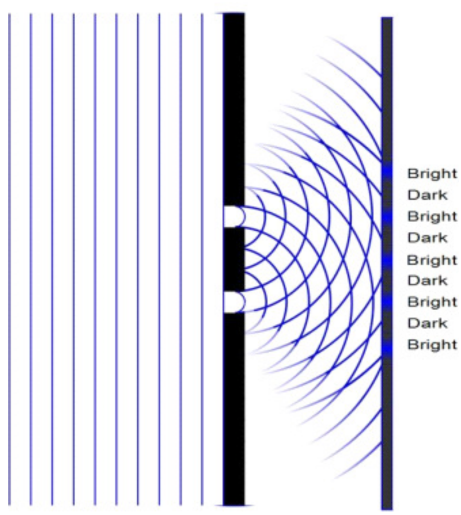
Waves do some funny things when they encounter barriers. Think of a water wave. If we pass the wave through a small opening in a barrier, the wave can't all get through the small hole, but it can cause a disturbance. We know that a small disturbance will cause a wave. But this wave will be due to a very small–almost point–source. So the waves will be spherical leaving the opening. The smaller the opening the more pronounced the curving of the wave, because the source (the hole) is more like a point source.



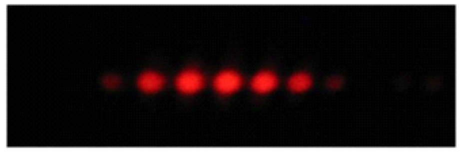
Now suppose we have two of these openings. We expect the two sources to make curved waves and those waves can interfere.



In the figure, we can already see that there will be constructive and destructive interference were the waves from the two holes meet. Thomas young predicted that we should see constructive and destructive interference in light (he drew figures very like the ones we have used to explain his idea).



Young set up a coherent source of light and placed it in front of this source a barrier with two very thin slits cut in it to test his idea.. He set up a screen beyond the barrier and observed the pattern on the screen formed by the light. This (in part) is what he saw.



We see bright spots (constructive interference) and dark spots (destructive interference). Only wave phenomena can interfere, so this is fairly good evidence that light is a wave.

10.3.1 Constructive Interference

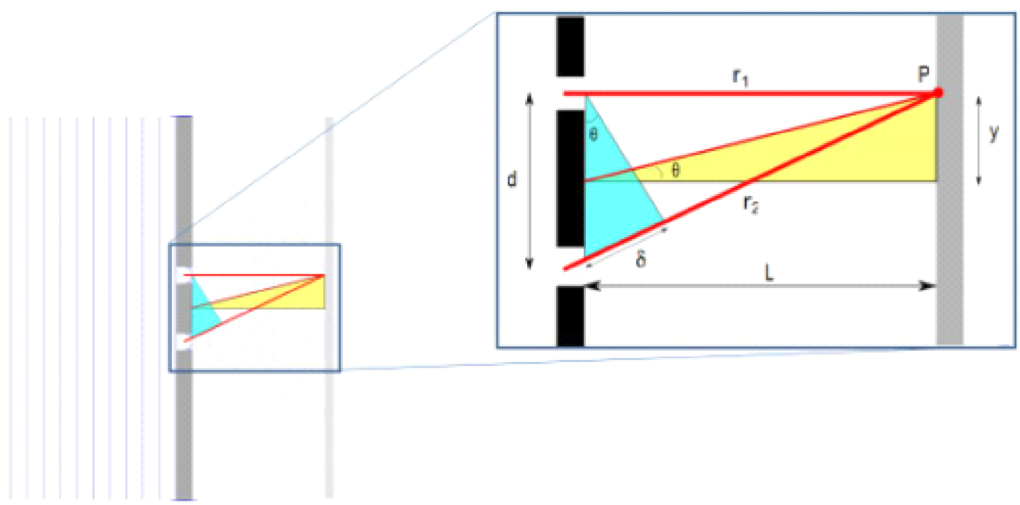
We can find the condition for getting a bright or a dark band if we think about it a bit. Here are our equations that we developed for constructive and destructive interference.

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = m2\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Constructive}$$

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = (2m+1)\,\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Destructive}$$

For constructive interference, the difference in phase, $\Delta\phi$, must be a multiple of 2π. That means the path difference between the two slit-sources must be an even number of wavelengths. We have been calling the path difference in the total phase Δx, or for spherical waves Δr, but in optics it is customary to call this path difference δ. So

$$\delta = \Delta r$$



and our total phase equation becomes

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\delta + \Delta\phi_o\right) = m2\pi$$

Our light going through the slits is all coming from the same light source. So as long as the light hits the slits at a 90° angle, $\Delta\phi_o = 0 - 0 = 0$ so we don't have a change in phase constant. But we do have a change in $\Delta r = \delta$. Let's suppose that the screen is far away so the distance from the slits to the screen, $L \gg d$, the slit distance. Then we can say that the blue triangle is almost a right triangle, and then δ is

$$\delta = r_2 - r_1 \approx d \sin\theta$$

so then

$$\Delta\phi = \left(\frac{2\pi}{\lambda} d \sin\theta + 0\right) = m 2\pi$$

We can do a little math to make this simpler.

$$\frac{2\pi}{\lambda} d \sin\theta = m 2\pi$$

$$\frac{1}{\lambda} d \sin\theta = m$$

$$d \sin\theta = m\lambda$$

We started by knowing our wave needs to sift by an integer number times 2π radians but now we see that is equivalent to shifting an integer number times the wavelength, λ. This will make the two waves experience constructive interference (a bright spot).

$$\delta = d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2 \ldots) \qquad \text{Constructive}$$

where in optics m is called the *order number*. That is, if the two waves are off by any number of whole wavelengths then our total phase due to path difference will be 2π. In optics, the the bright spots formed by constructive interference are called *fringes.*

If we assume that $\lambda \ll d$ we can find the distance from the axis for each fringe more easily. This condition guarantees that θ will be small. Using the yellow triangle we see

$$\tan\theta = \frac{y}{L}$$

but if θ is small this is just about the same as

$$\sin\theta = \frac{y}{L}$$

because for small angles $\tan\theta \approx \sin\theta \approx \theta$. So if theta is small then

$$\begin{aligned} \delta &= d \sin\theta \\ &= d\frac{y}{L} \end{aligned}$$

and for a bright spot or fringe we find

$$d\frac{y}{L} = m\lambda$$

Solving for the position of the bright spots gives

$$y_{bright} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots) \tag{10.4}$$

We can measure up from the central spot and predict where each successive bright spot will be.

10.3.2 Destructive Interference

We can also find a condition for destructive interference. Our destructive interference equation is

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = (2m+1)\,\pi$$

Once again $\Delta\phi_o = 0$ and $\Delta r = \delta$

$$\left(\frac{2\pi}{\lambda}\delta\right) = (2m+1)\,\pi$$

$$\left(\frac{2}{\lambda}\delta\right) = (2m+1)$$

$$\delta = \frac{\lambda}{2}(2m+1)$$

$$\delta = \lambda\left(m + \frac{1}{2}\right)$$

This just shows us again that a path difference of an odd multiple of a half wavelength will give distractive interference.

$$\delta = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2 \ldots)$$

will give a dark fringe. The location of the dark fringes will be

$$y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right) \quad (m = 0, \pm 1, \pm 2 \ldots) \tag{10.5}$$

10.4 Double Slit Intensity Pattern

The fringes we have seen are not just points, but are patterns that fade from a maximum intensity. This is why they are called fringes. We can calculate the intensity pattern to show this. We need to know a little bit about electric fields to do this.

10.4.1 Electric field preview

We can represent an electromagnetic wave using just the electric field (the magnetic field pattern is very similar and can be derived from the electric field pattern).

We represent the field by an equation like

$$y = y_o \sin(kr - \omega t)$$

but since the medium for light waves is the electric field, let's use the symbol E instead of y so we can see that we have a change in the field strength and not a displacement of some material thing.

$$E = E_{\max} \sin(kr - \omega t) \tag{10.6}$$

where the amplitude of the wave is $E_{\max}$ and ω is the angular frequency. This is just our traveling wave equation, but with electric field strength, labeled E, for the amplitude.

Then to find the intensity pattern, we take two waves in the electric field, one from slit one

$$E_1 = E_{\max} \sin(kr_1 - \omega t + \phi_o)$$

and the other from slit two.

$$E_2 = E_{\max} \sin(kr_2 - \omega t + \phi_o)$$

This is mathematically just like superposition of sound waves.

10.4.2 Superposition of two light waves

Remember when we superimposed waves before, we mixed the waves

$$\begin{aligned} y_1 &= A \sin(kr_1 - \omega t + \phi_1) \\ y_2 &= A \sin(kr_2 - \omega t + \phi_2) \end{aligned}$$

and using

$$\sin a + \sin b = 2 \cos\left(\frac{a-b}{2}\right) \sin\left(\frac{a+b}{2}\right)$$

we found the resultant wave

$$y_r = 2A \cos\left(\frac{1}{2}(\Delta\phi)\right) \sin\left(k\frac{r_2 + r_1}{2} - \omega t + \frac{\phi_2 + \phi_1}{2}\right)$$

Our light waves are just two waves. They may be the superposition of many individual photons, but the combined wave is just a wave.

At the slits, the waves have the same amplitude $E_{\max}$ and the same phase constant, $\phi_1 = \phi_2 = \phi_o$, but E_2 travels farther than E_1, so $\Delta\phi$ is due to the path difference. We expect to find that the path difference would be

$$\begin{aligned}\Delta\phi &= k\Delta r + \Delta\phi_o \\ &= k\delta + 0 \\ &= \frac{2\pi}{\lambda} d\sin\theta\end{aligned}$$

Now superimposing E_1 and E_2 at point P on the screen gives

$$\begin{aligned}E_P &= E_2 + E_1 \\ &= E_{\max}\sin(kr_2 - \omega t) + E_o \sin(kr_1 - \omega t)\end{aligned}$$

and using our prior result, we have

$$E_P = 2E_{\max}\cos\left(\frac{1}{2}\Delta\phi\right)\sin\left(k\frac{(r_2+r_1)}{2} - \omega t + \phi_o\right)$$

and using our equation for $\Delta\phi$ above we get

$$E_P = 2E_{\max}\cos\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\sin\left(k\frac{(r_2+r_1)}{2} - \omega t + \phi_o\right)$$

We have a combined wave at point P that is a traveling wave $\left(\sin\left(k\frac{(r_2+r_1)}{2} - \omega t + \phi_o\right)\right)$ but with amplitude $\left(2E_{\max}\cos\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\right)$ that depends on our total phase $\Delta\phi = \frac{2\pi}{\lambda}d\sin\theta$.

But the situation is more complicated because of how we detect light. Our eyes, and most detectors measure the intensity of the light. We know that

$$I = \frac{\mathcal{P}}{A}$$

later in the course we will show that the power in an electromagnetic field wave is proportional to the square of the electric field displacement.

$$\mathcal{P} \propto E_P^2 \tag{10.7}$$

For now, let's just assume this is true. Then the intensity must be proportional to the amplitude of the electric field squared.

$$\begin{aligned}I &\propto E_P^2 \\ &= 4E_{\max}^2\cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\sin^2\left(\frac{k(r_2+r_1)}{2} - \omega t + \phi_o\right)\end{aligned}$$

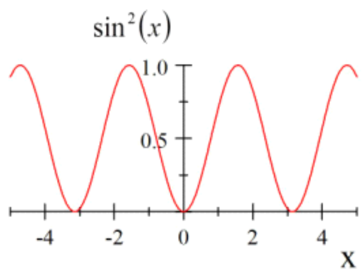
Light detectors collect energy for a set amount of time. So most light detection will be a value averaged over a set *integration time*. This means that the detector sums up (or integrates) the amount of power received over the detector time. Usually the integration time is much longer than a period, so what is really detected is like a time-average of our intensity.

$$\begin{aligned}\int_{\text{many T}} I dt \quad &\propto \quad = \int_{\text{many T}} 4E_{\max}^2 \cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda} d\sin\theta\right)\right) \sin^2\left(\frac{k\left(r_2+r_1\right)}{2} - \omega t + \phi_o\right) dt \\ &= \quad 4E_{\max}\cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda} d\sin\theta\right)\right)\int_{\text{many T}} \sin^2\left(\frac{k\left(rx_2+r_1\right)}{2} - \omega t + \phi_o\right) dt\end{aligned}$$

but the term

$$\int_{\text{many T}} \sin^2\left(\frac{k\left(r_2+r_1\right)}{2} - \omega t + \phi_o\right) dt = \frac{1}{2} \tag{10.8}$$

To convince yourself of this, think that $\sin^2(\omega t)$ has a maximum value of 1 and a minimum of 0. Looking at the graph



should be convincing that the average value over a period is 1/2. The average over many periods will still be 1/2.

So we have

$$\bar{I} = \int_{\text{many periods}} I dt \propto 2E_{\max}\cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda} d\sin\theta\right)\right) \tag{10.9}$$

where $\bar{I}$ is the time average intensity. The important part is that the time varying part has averaged out.

So, usually in optics, we ignore the fast fluctuating parts of such calculations because we can't see them and so we write

$$I = I_{\max}\cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda} d\sin\theta\right)\right)$$

where we have dropped the bar from the I, but it is understood that the intensity we report is a time average over many periods.

We should remind ourselves, of our intensity pattern

$$I = I_{\max}\cos^2\left(\frac{1}{2}\frac{2\pi}{\lambda} d\sin\theta\right)$$

is really

$$I = I_{\max}\cos^2\left(\frac{\Delta\phi}{2}\right)$$

Which is just our amplitude squared for the mixing of two waves. All we have done to find the intensity pattern is to find and expression for the phase difference $\Delta\phi$.

Our intensity pattern should give the same location for the center of the bright spots as we got before. Let's check that it works. We used the small angle approximation before, so let's use it again now. For for small angles

$$\begin{aligned} I &= I_{\max}\cos^2\left(\frac{\pi d}{\lambda}\theta\right) \\ &= I_{\max}\cos^2\left(\frac{\pi d}{\lambda}\frac{y}{L}\right) \end{aligned}$$

Then we have constructive interference when

$$\frac{\pi d}{\lambda}\frac{y}{L} = m\pi$$

or

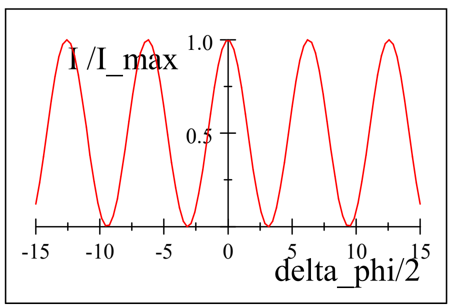
$$y = m\frac{L\lambda}{d}$$

which is what we found before.

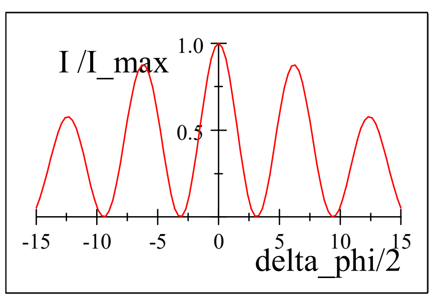
The plot of normalized intensity

$$\frac{I}{I_{\max}} = \cos^2\left(\frac{\Delta\phi}{2}\right)$$

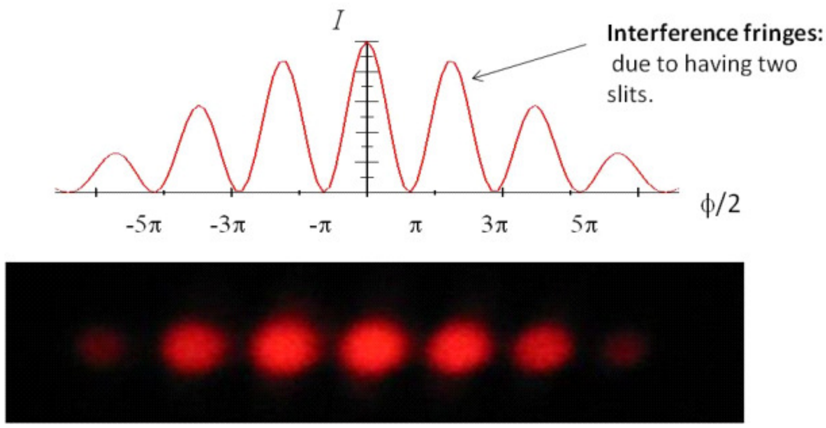
verses $\Delta\phi/2$ is given next,



but we will find that we are not quite through with this analysis. Next time we will find that there is another compounding factor that reduces the intensity as we move away from the midpoint.



Let's pause to remember what this pattern means. This is the intensity of light due to interference. It is instructive to match our intensity pattern that Young saw with our graph.



The high intensity peaks are the bright fringes and the low intensity troughs are the dark fringes. The pattern moves smoothly and continuously from bright to dark.