Chapter 17

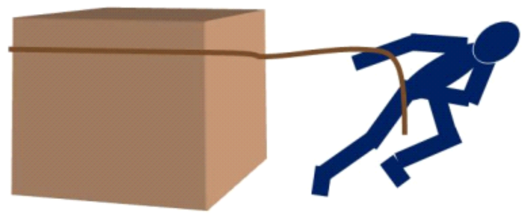
Current, Resistance, and Electric Fields

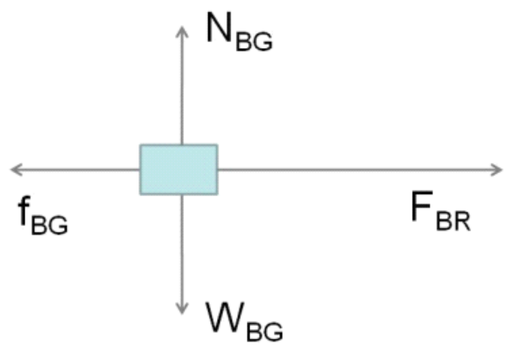
Fundamental Concepts

- There is a nonconservative (friction-like) force involved in current flow called *resistance*.
- A nonuniform charge distribution creates an electric field, which provides the force that makes current flow
- Current flow direction is defined to be the direction positive charge carriers would go
- The current density is defined as $J = nq_e v_d$
- Charge is conserved, so in a circuit, current is conserved.

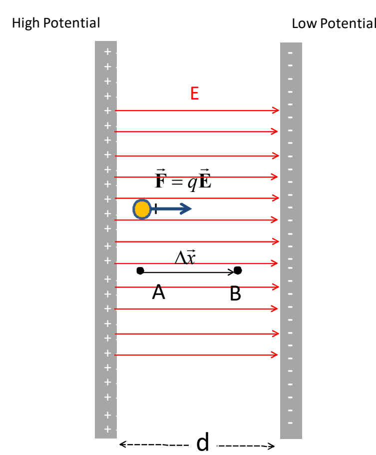
17.1 Current and resistance

We now have flowing charges, but our PH121 or Dynamics experience tells us that there is more. If we push or pull an object, we expect that most of the time there will be dissipative forces. There will be friction.





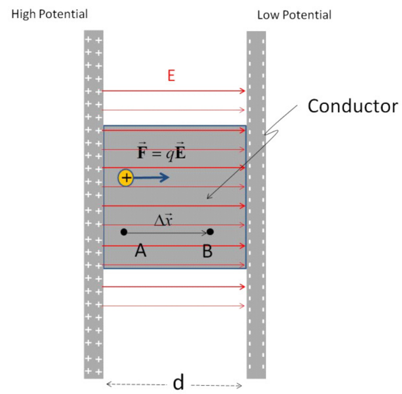
We should ask, is there a friction involved in charge movement? We already know how to push a charge, we use an electric field



The force is

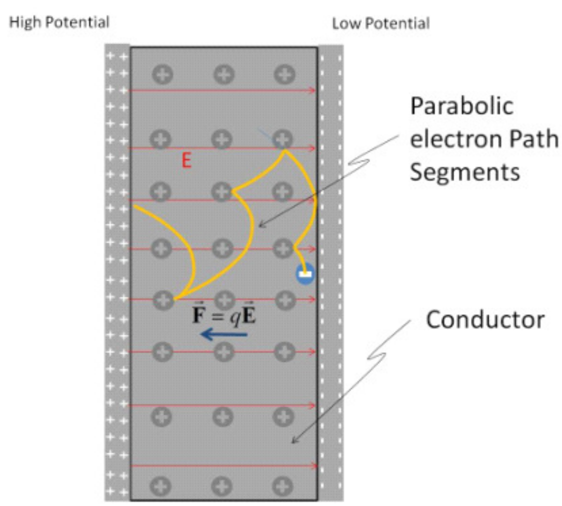
$$F = qE$$

If we push or pull a box, it will eventually come to rest. In our capacitor there are no resistive forces for our charge to encounter. But suppose we place a conductor inside our capacitor, hooked to both plates



Of course, in conductors we now know the charge carrier is an electron and it is negative, so let's try to redraw this picture to show the actual charge motion.

Now the charge is free to move inside of the conductor, but it is not totally unencumbered. The free charges will run into the nuclei of the atoms. The charges will bounce off. So as they travel through the material we will expect to see some randomness to their motion. This is compounded by the fact that the electrons already have random thermal motion. So the path the charge takes looks somewhat like this



We can recognize that each path segment after a collision must be parabolic because the acceleration will be constant

$$
\begin{aligned}
F_{netx} &= ma_x = qE \\
F_{nety} &= ma_y = 0
\end{aligned}
$$

so

$$\overrightarrow{a} = \frac{qE}{m}\hat{\imath}$$

we can describe the electron motion using the kinematic equations.

$$\begin{array}{ll}
x_f = x_i + v_{ix}\Delta t + \frac{1}{2}\left(\frac{qE}{m}\right)\Delta t^2 & y_f = y_i + v_{iy}\Delta t + \frac{1}{2}(0)\Delta t^2 \\
v_{fx} = v_{ix} + \left(\frac{qE}{m}\right)\Delta t & v_{fy} = v_{iy} + (0)\Delta t \\
v_{fx}^2 = v_{ix}^2 + 2\left(\frac{qE}{m}\right)(x_f - x_i) & v_{fy}^2 = v_{iy}^2 + 2(0)(y_f - y_i) \\
x_f = x_i + \frac{v_{fx}+v_{ix}}{2}\Delta t & y_f = y_i + \frac{v_{fy}+v_{iy}}{2}\Delta t
\end{array}$$

or

$$\begin{array}{ll}
x_f = x_i + v_{ix}\Delta t + \frac{qE}{2m}\Delta t^2 & y_f = y_i + v_{iy}\Delta t \\
v_{fx} = v_{ix} + \frac{qE}{m}\Delta t & v_{fy} = v_{iy} \\
v_{fx}^2 = v_{ix}^2 + \frac{2qE}{m}(x_f - x_i) & v_{fy}^2 = v_{iy}^2 \\
x_f = x_i + \frac{v_{fx}+v_{ix}}{2}\Delta t & y_f = y_i + \frac{v_{fy}+v_{iy}}{2}\Delta t
\end{array}$$

and the path will be

$$\begin{aligned}
x_f &= x_i + v_{ix}\Delta t + \frac{qE}{2m}\Delta t^2 \\
y_f &= y_i + v_{iy}\Delta t
\end{aligned}$$

which is parabolic.

Of course, this is just for one electron, and only for a segment between collisions. We will have millions of electrons, and therefore, many millions of bounces. But for each electron, between bounces we expect a parabolic path.[1] For considering current flow, we don't care about motion perpendicular to the current direction. So we can look only at the component of the motion in the flow direction. The net flow in the current direction is toward the positive plate. Let's see how this works.

If we average the velocities of all the electrons we find

$$\begin{aligned}
v_d &= \bar{v}_x \\
&= \bar{v}_{ix} + a_x\Delta\bar{t}
\end{aligned}$$

the first term $\bar{v}_{ix} = 0$ because the initial velocities are random from the thermal and scattering processes. That is, on average, the electrons have no preferred direction after a bounce. This leaves

$$v_d = \left(\frac{qE}{m}\right)\Delta\bar{t}$$

The average time between collisions, $\Delta\bar{t}$, is sometimes given the symbol τ. Let's use this. Then

$$v_d = \left(\frac{q\tau}{m}\right)E$$

[1] Chemists, this is a classical model for current flow. You might get an updated quantum model for current flow in your physical chemistry class.

Recall that current is

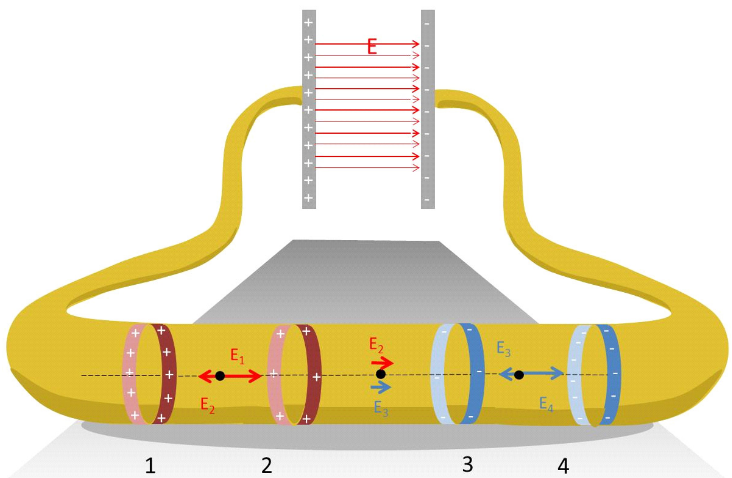
$$I = nAv_dq_c$$

And we can write our current equation using our new equation for v_d

$$I = nqA\left(\frac{q\tau}{m}\right)E$$

We have shown that the current is directly proportional to the field inside the conductor. It is this field that causes the charges to flow. In equilibrium we would have no field inside a conductor, so to have a current we need to not have the conductor in equilibrium. Let's see how that works.

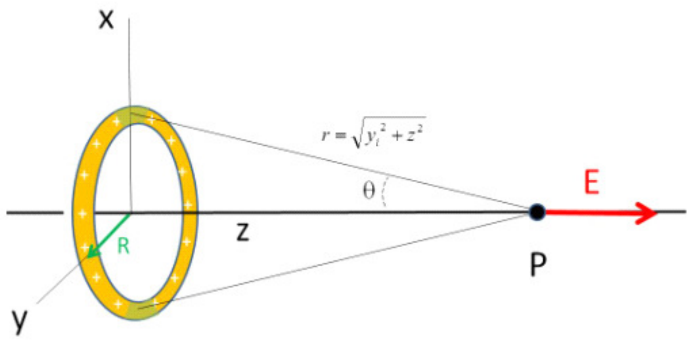
Suppose we connect our two plates with a wire instead of filling their gap with a conductor. If current flows through the wire, we can't have an equilibrium, so there must be a field in the wire. That field is started by our capacitor charge.



This figure is supposed to show our copper wire connected to the capacitor. The capacitor is in the background, and the wire loops close to us. The end of the wire that is connected to the positive side of the capacitor will become positively charged, and the end connected to the negative side of the capacitor will become negatively charged. If we look at the wire an infinitesimal time after the connection has happened, the wire will not be uniformly charged. It won't be in equilibrium! It will take some time for the charges to reach equilibrium. In the mean time, the charge is stronger near the plates, and diminishes toward the middle.

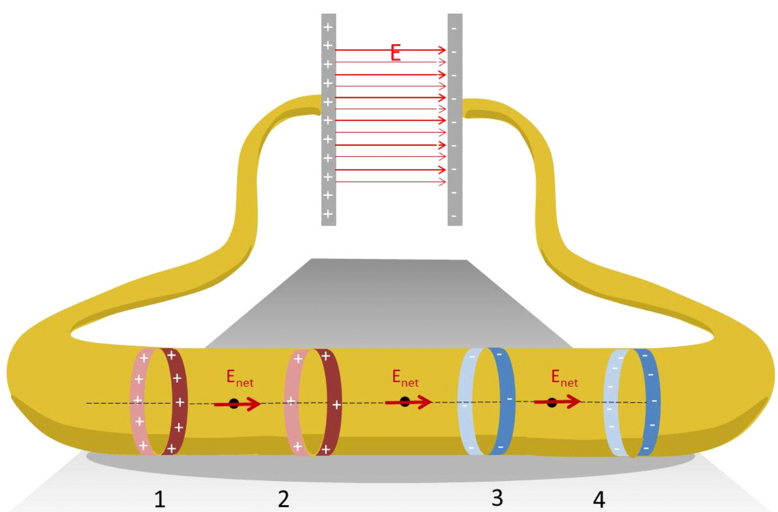
We can't find the exact field in the conductor without resorting to a computational solution, but we can mentally model the situation by viewing the wire as consisting of rings of charge that vary in linear charge density. The extra charge from the capacitor will want to go to the outside of the wire, so this isn't a crazy model.

And we know the field along the axis due to a ring of charge because we have done this problem in the past!



$$\overrightarrow{\mathbf{E}} = \frac{1}{4\pi\epsilon_o} \frac{zQ}{(R^2 + z^2)^{\frac{3}{2}}} \hat{\mathbf{k}}$$

We know the field is along the axis and that it diminishes with distance from the ring. Now consider the field due to ring 1. As we move to the right, away from ring 1 that field will diminish with distance. Also consider the field due to ring 2. As we move to the right toward ring 2 the field due to ring two will grow. The field due to ring two grows at the same rate that the field from ring 1 diminishes. The fields 1, and 2 add up to a constant value along the axis for every point in between the two rings. Now consider the field on the right side of ring 2 and the field on the left side of ring 3. A little thought shows that the situation is the same as that for rings 1 and 2. We will have a constant net field between the two rings.



Likewise for the region between rings 3 and 4. There is a constant net electric field at all points along the wire. This field points from positive to negative. It will exert a force

$$F = qE_{net}$$

on the free charges *inside* the wire. These free charges are not the extra charge from the capacitor. They are the free electrons that are loosely attached to the metal atoms that make up the wire. So these free charges are distributed throughout the volume of the wire. These free charges will accelerate, forming a current inside the wire.

Note that these free charges are not just on the surface, they are inside the wire, even on the axis of the wire in the center. We no longer have a static equilibrium, so our existing free charge in the wire begins to move. The extra charge from the capacitor has made a field that moves the existing free charge in the wire.

All this usually happens very fast, so when we switch on a light, we don't notice the time it takes for the current to start. But this uneven distribution of charge is the reason we get a current.

17.2 Current density

We now realize that when there is an electric field inside a wire, there will be current flow inside the wire. The flow goes through the volume of the wire. The rate of flow is given by

$$I = \frac{\Delta Q}{\Delta t} = nq_e A \left(\frac{q_e \tau}{m_e}\right) E$$

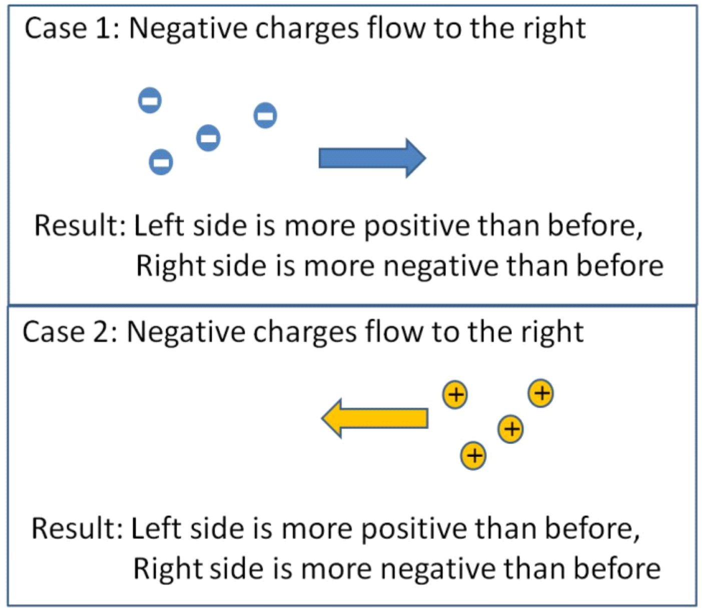
for steady current flow. Here we are writing $q = q_e$ for the electron charge and $m = m_e$ for the electron mass, since our charge carrier is an electron (in chemistry, you will need to think about what the charge carrier is).

The unit for current flow is

$$\frac{\mathrm{C}}{\mathrm{s}} = \mathrm{A}$$

where A is the symbol for an *Ampere* or, for short, an *amp*.

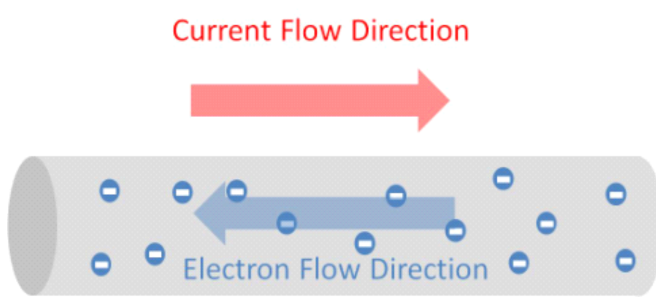
Historically there was no way to tell whether negative charges were flowing or whether positive charges were flowing. It really did not matter so much in the early days, since a flow of positive charges one way is equivalent to a flow of negative charges the other way.



Worse, we know that for some systems there are positive charge carriers and for others negative charge carriers.

By convention, we assign the direction of current flow as though the charge carrier were positive.[2]

This is great for biologists, where the charge carriers are positive ions. But for electronics this gives us the uncomfortable situation that the actual charge carriers, electrons, move in the direction opposite to that of the current.



Let's look again at our definition of current

$$I = \frac{\Delta Q}{\Delta t} = nq_e A \left(\frac{q_e \tau}{m_e}\right) E$$

If we, once again, write this in terms of v_d

$$v_d = \left(\frac{q\tau}{m}\right) E$$

[2] >sigh< Thank you Mr. Franklin.

then after rearranging, we have

$$I = (nq_e v_d)\, A$$

The part in parentheses contains only bulk properties of the conductor material, the number of free charges, the charge of the charge carrier, and the drift speed which depends on the material structure of the conductor. The final factor is just the cross sectional area of the wire. It gives the geometry of the wire we have made out of the bulk material (say, copper). It is convenient to group all the factors that are due to bulk material properties

$$J = nq_e v_d$$

then the current would be

$$I = JA$$

Note how similar this is to a surface charge density

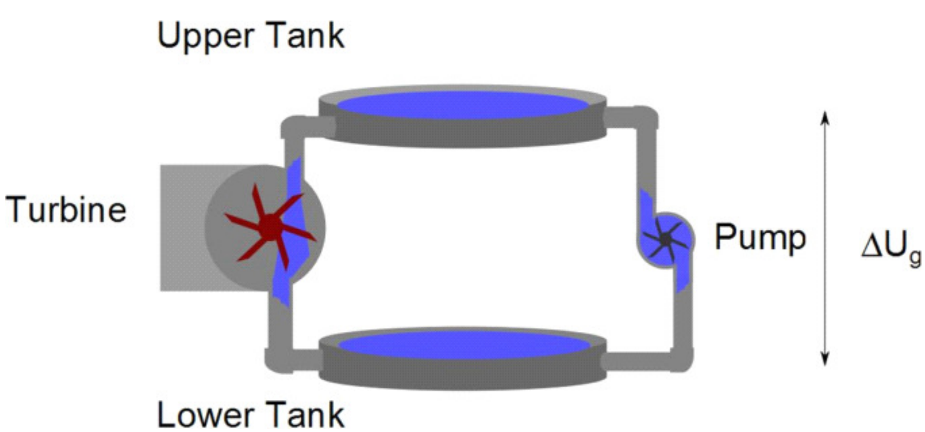
$$Q = \eta A$$

For a static charged surface, Q is the surface charge density multiplied by the particular area. For our case we have a total current, I that is the material properties multiplied by an area. By analogy we could call this new quantity, J, a kind of density, but now our charges are moving. So let's call it the *current density.*

Notice that it is the cross sectional area of the wire that shows up in our current equation. This is another indication that the charge is not flowing along the surface, but that it is deep within the wire as it flows.

17.3 Conservation of current

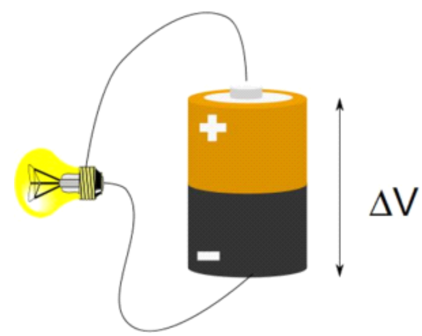
Let's go back to our pumps and turbines.



How much of the water is "used up" in turning the turbine? Another way to say this is to ask if there are 20 l of water entering the turbine, how much water leaves the turbine through the lower pipe?

If the turbine leaks, then we might lose some water, but if all is going well, then you can guess that 20 l of water must also leave the turbine We can't lose or gain water as the turbine is turned. But we must be losing something! We must be giving up something to get useful work out of the system. That something that we lose is potential energy.

Now consider a battery. How much of the current is "used up" in making the light bulb light up?



This case is really the same as the water case. The electric current is a flow of electrons. The flow loses potential energy, but we don't create or destroy electrons as we convert the potential energy of the battery to useful work (like making light) just like we did not create or destroy water in making the turbine turn.

But surely the water slowed down as it traveled through the turbine–didn't it? Well, no, if the water slows down as it goes through the turbine, then the pipe below the turbine would run dry. This does not happen. The flow rate through a pipe does not change under normal conditions, and under abnormal conditions, we would destroy the pump or the turbine! If we throw rocks off a hill, they actually gain speed when the water loses potential energy. Now the flow rate is slower with a turbine in the pipe than it would be with no turbine in the pipe! But with the turbine in the pipe, the flow rate is the same throughout the whole pipe system.

Like the water case, the flow rate of charge does not change from point to point in the wire. The same amount of charge per unit time leaves the wire as went in.

This explains the reasoning behind one of the great laws of electronics

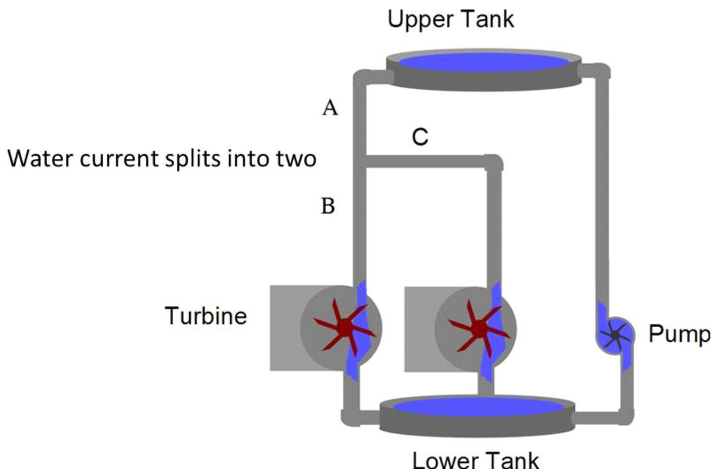
The current is the same at all points in a current-carrying wire.

Like in the water case, the electrons would flow faster if there were no light bulb and just a continuous wire. We can have different flow rates in our wire depending on how much resistance there is to the flow. But the flow rate will be the same in all parts of the wire system.

This leads to the second of the pair of rules called Kirchhoff's laws:

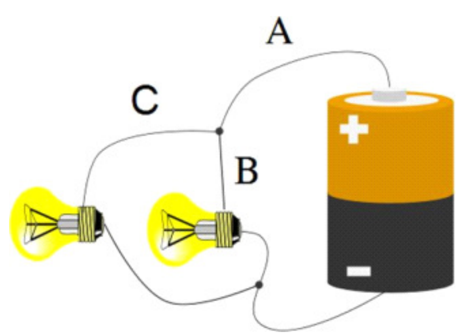
$$\sum I_{in} = \sum I_{out}$$

If the wire branches into two or more pieces, the current will divide. This is not too surprising. The same is true for water in a pipe



In the figure the flow through pipe segment A is split into two smaller currents that flow through pipe segments B and C. We would expect that the flow through B and C combined must be equal to the flow through A.

The same must be true for electrical current. The situation is shown in the next figure.



The current that flows through wires B and C combined must be equal to the current that came through wire A.

Basic Equations

$$J = nq_e v_d$$

$$I = JA$$