Chapter 16

Dielectrics and Current

Fundamental Concepts

- Dielectrics modify the capacitance
- Microscopic nature of electric current
- Current direction is defined as the direction positive charges would go, regardless of the actual sign of the charge.
- In a capacitor, the stored energy is $w = \frac{1}{2}C\Delta V^2$
- The energy density in the electric field is $u = \frac{1}{2}\epsilon_o E^2$

16.1 Energy stored in a capacitor

We have convinced ourselves that ΔV is the change in potential energy per unit charge, so when a capacitor is charged, and the wires connecting it to the battery are removed, is there potential energy "stored" in the capacitor? The answer is yes, and we can see it by considering what would happen if we connected a wire (no battery) between the two plates. Charge would rush from one plate to the other. This is like storing a tank of water on a hill. If we connect a pipe from the tank at the top of the hill to a tank at the bottom of the hill, the water will rush through the pipe to the lower tank.

BE CAREFUL, you are enough of a conductor that by touching different ends of a capacitor you could create a serious current through your body. The capacitors in old computer monitors or old TV sets can store enough charge to kill you!

But how much energy would there be stored in the capacitor? Clearly it must be related to the amount of energy it takes to move the charge onto the plates.

By analogy, the energy stored in the water was the minimum amount of energy it took to pump the water to the upper tank (mgh). It is the minimum, because our pipes might have some resistance, and then we would have to include more work to overcome the resistance.

But for a capacitor it is a little bit more tricky. When the capacitor is not charged, it takes no work (or very little) to move charge from one plate to the other. But once there is a charge there is an electric field between the plates (think of my poorly designed water storage system from the beginning of last lecture). This creates a potential difference. And we must fight against this potential difference to add more charge. This is sort of like transferring rocks up a hill. The more rocks that we carry, the higher the hill gets, and the more work it takes to bring up more rocks.

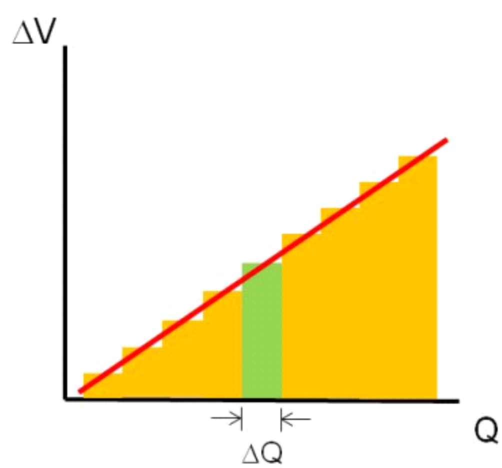
From our formula, the amount of work we get will be

$$w = q\left(V_B - V_A\right)$$

And we can see that if we have just a small amount of charge, ΔQ, we will have a small amount of work

$$\Delta w = \Delta Q \Delta V$$

to move it onto the capacitor.[1] If we start with no charge, then go in small ΔQ steps, we would see a potential rise as shown in the graph below.



The quantity $\Delta Q \Delta V$ is the area of the shaded (green) rectangle. So ΔW is given by the area of a rectangle under a stair-step on our graph. The shaded rectangle is just one of many rectangles in the graph. For our capacitor we can write

$$C = \frac{\Delta Q}{\Delta V} \tag{16.1}$$

[1] Agh! here ΔQ is a small amount of charge, and ΔV is $V_f - V_i$. We have used Δ in two different ways in the same equation.

or

$$\Delta V = \frac{\Delta Q}{C}$$

As ΔQ gets small we can go to a continuous charge model. Our amount of work is

$$\Delta w = \Delta Q \Delta V$$

but for a very small amount of charge we can replace the small unit of charge ΔQ with a continuous variable q to obtain

$$dw = dq\,(\Delta V)$$

Recall that

$$\Delta V = \frac{q}{C}$$

so we can write dW as

$$\begin{aligned} dw &= dq\left(\frac{q}{C}\right) \\ dw &= \frac{1}{C}qdq \end{aligned}$$

Of course, we will integrate this

$$\begin{aligned} w &= \int_0^Q \frac{1}{C}qdq \\ w &= \frac{1}{C}\int_0^Q qdq \\ &= \frac{Q^2}{2C} \end{aligned} \tag{16.2}$$

or sometimes using

$$Q = C\Delta V$$

this is written as

$$w = \frac{1}{2}C\Delta V^2 \tag{16.3}$$

There is a limit to how much energy we can store. That is because even air can conduct charge if the potential difference is high enough. We called this air conduction a spark or coronal discharge. At some point charge jumps from one plate to another through the air in between. If the potential difference is very high, the Coulomb force between the charges on opposite plates will force charge to leave one plate and jump to the other even if there is no air!

16.1.1 Field storage

We consider the energy stored in the capacitor to be stored in the electric field. The field is proportional to the amount of charge and related to the potential energy, so this seems reasonable. Let's find the potential energy stored in the field in the capacitor. Recall for an ideal parallel plate capacitor

$$\Delta V = Ed$$

and

$$C = \epsilon_o \frac{A}{d}$$

We assume that energy provided by the work to move the charges on the capacitor is all stored as potential energy. We just found that work to be

$$w = \frac{1}{2} C \Delta V^2 \tag{16.4}$$

and we know that

$$w = -\Delta U$$

tells us how much energy we can store. In the previous example we had to provide a battery or something to force the charge onto the capacitor. This outside force did the work to move the charge, ω. This means that work done *by the electric field* would be the negative of the work we calculated $(-\omega)$, so the potential energy stored would be positive.

$$\Delta U_{stored} = \frac{1}{2} C \Delta V^2 \tag{16.5}$$

and since we started with $U_i = 0$ then

$$\begin{aligned} U_{stored} &= \frac{1}{2} \left(\epsilon_o \frac{A}{d} \right) (Ed)^2 \\ &= \frac{1}{2} \epsilon_o A d E^2 \end{aligned}$$

It might seem strange but is can be useful to think of how much energy we have in the volume of space inside the capacitor.

$$u = \frac{U_{stored}}{\mathbb{V}}$$

In this case the volume $\mathbb{V}$ is just Ad so

$$u = \frac{1}{2} \epsilon_o E^2 \tag{16.6}$$

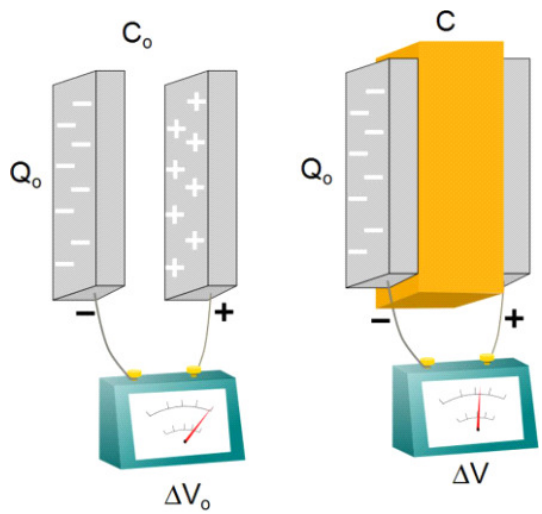
This is the density of energy in the electric field. It turns out that this is a general formula (not just true for ideal parallel plate capacitors).

This is a step toward our goal of showing that electric fields are a physically real thing. They can store energy, so they must be a real thing.

16.2 Dielectrics and capacitors

We should ask ourselves a question about our capacitors, does it matter that there is air in between the plates? For making capacitors, it might be convenient to coat two sides of a plastic block with metal and solder wires to the coated sides. Does the plastic have an effect?

Plastic is an insulator, and another name for insulator is *dielectric*. If we perform the experiment, we will find that when an insulator or dielectric material is placed in the plates, the potential difference decreases!



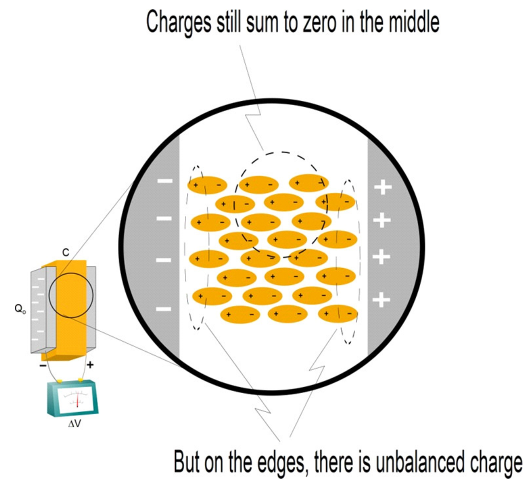
We are lucky, though, from experimentation we have found that it seems to decrease in a nice, linear way. We can write this as

$$\Delta V = \frac{\Delta V_{wo}}{\kappa} \tag{16.7}$$

where κ is a constant that depends on what material we choose as our dielectric[2] and ΔV_{wo} is the potential difference without the dielectric (the subscripts wo will stand for "(w)ith(o)ut the dielectric." But what is happening?

The plates of the capacitor are becoming charged. These charges will polarize the material in the middle.

[2] This symbol κ, is the greek letter "kappa."



Notice how the polarized molecules or atoms sill have a net zero charge in the middle, but on the ends, there is a net charge. It is like we have oppositely charged plates next to our capacitor plates. That reduces the net charge seen by the capacitor, and so the potential difference is less. There is effectively less separated charge.

But since our capacitor is not connected to a battery (or any other electrical device that changes the amount of charge), the amount of actual charge on the capacitor plates can't have changed. So if ΔV changed, but Q did not, we can reason that since

$$Q = C\Delta V$$

the material properties part–the capacitance–must have changed.

$$C = \frac{Q_{wo}}{\Delta V} = \frac{Q_{wo}}{\frac{\Delta V_{wo}}{\kappa}} = \frac{\kappa Q_{wo}}{\Delta V_{wo}}$$

but this is just

$$C = \kappa C_{wo} \tag{16.8}$$

For a parallel plate capacitor, we can account for all this by inserting a κ in our parallel plate capacitor equation

$$C = \kappa\varepsilon_o \frac{A}{d} \tag{16.9}$$

and let $\kappa = 1$ if there is no dielectric.

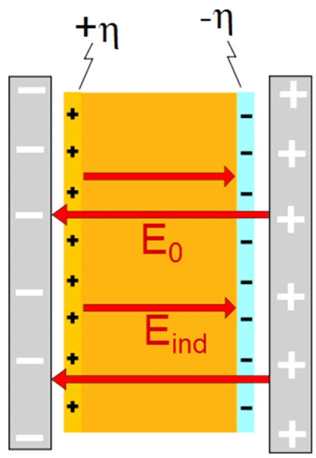
But if we do have a dielectric we need its κ value. So where do you find values for κ? For this class, we will look them up in the tables in books or on the internet. Here are a few values for our use.

Material	κ		Material	κ
Vacuum	1.00000		Paper	3.7
Dry Air	1.0006		Waxed Paper	3.5
Fused quartz	3.78		Polystyrene	2.56
Pyrex glass	$4.7 - 5.6$		PVC	3.4
Mylar	3.15		Teflon	2.1
Nylon	3.4		Water	80

16.3 Induced Charge

In the last discussion we discovered that if we put a dielectric inside a capacitor, we end up with polarized charges with the net result that there will be excess negative charge near the positive plate of the capacitor, and excess negative charge near the positive plates of the capacitor. In the middle of the dielectric, the charges are polarized in each atom, but still for any volume inside, the net charge is zero. The excess charge near each plate we will call the *induced charge*.

Since we have an induced positive charge on one side and an induced negative charge on the other side, we expect there will be an electric field directed from the positive to negative charge inside the dielectric.



The total field inside the dielectric is

$$E = E_{wo} - E_{ind} \tag{16.10}$$

where E_{wo} is the field due to the capacitor plates without the dielectric. From our previous discussion, we recall that

$$\Delta V = \frac{\Delta V_{wo}}{\kappa}$$

and we recall that we found magnitude of the potential difference for an empty capacitor as

$$\Delta V_{wo} = Ed$$

Then our new net field can be found

$$Ed = \frac{E_{wo}d}{\kappa}$$

or

$$E = \frac{E_{wo}}{\kappa}$$

and, recalling for a parallel plate capacitor (near the center) the field is approximately

$$E = \frac{\eta}{\varepsilon_o}$$

then

$$E = E_o - E_{ind}$$

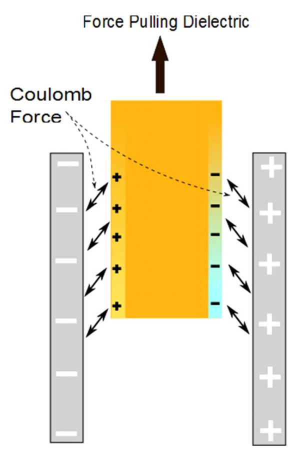
gives

$$\frac{\eta}{\kappa\epsilon_o} = \frac{\eta}{\epsilon_o} - \frac{\eta_{ind}}{\epsilon_o}$$

and we can find the induced surface charge density as

$$\eta_{ind} = \eta\left(1 - \frac{1}{\kappa}\right)$$

You might guess that the induced charge is attracted to the charge on the plates, so a force is required (and work is required) to remove the dielectric once it is in place. If we draw out the dielectric, we can see that the weaker field outside the capacitor causes little induced charge, but the stronger field inside the capacitor causes a large induced charge. A net inward force will result.

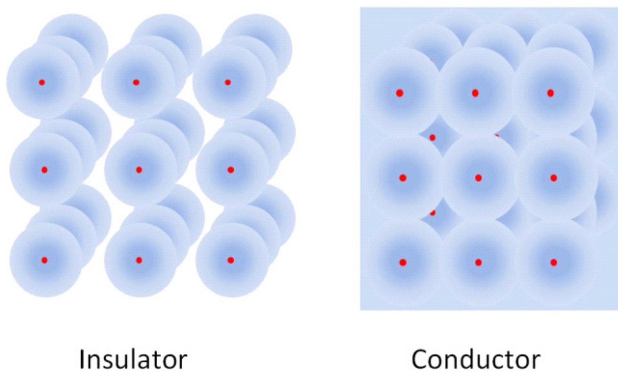


Engineers use this force on a dielectric between capacitor plates to make clever actuators.

16.4 Electric current

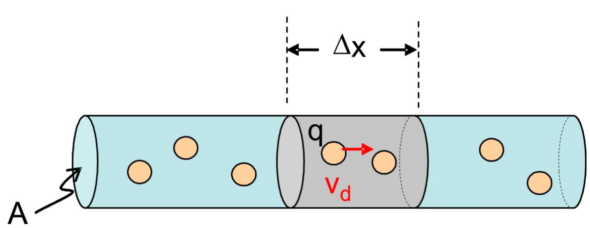
For some time now, we have been talking about charge moving. We have had charge move from a battery to the plates of a conductor. We have had charge flow from one side to another of a conductor, etc. It is time to become more exact in describing the flow of charge. We should take some time to figure out why charge will move.

Let's consider a conductor again.



We remember that in the conductor, the valance electrons are free to move. In fact, they do move all the time. The electrons will have some thermal energy just because the conductor is not at absolute zero temperature. This thermal energy causes them to move in random directions. (think of air molecules in a room).

Let's take a piece of a wire Δx long. The speed of the electrons along the wire (in the x-direction in this case) is called the *drift speed*, v_d, because the electrons just drift from place to place with a fairly small speed. This drift speed could be due mostly to thermal energy, so it can be very small or even zero (if no electric potential is applied). Of course, v_d, must be an average, each electron will be moving random directions with slightly different speeds, so the x-component of the velocity will be different for each electron, but on average they will move at a speed v_d.



So we will suppose that there are charge carriers of charge q_c that are moving through the wire with velocity v_d. Then we can write some length of wire, Δx, as

$$\Delta x = v_d \Delta t$$

The volume of the shaded piece of wire is

$$= A\Delta x$$

if there are

$$n = \frac{\#}{\mathbb{V}}$$

charge carriers per unit volume, a *volume charge carrier number density*, then the total charge in our volume is

$$\Delta Q = nA\Delta x q_c$$

If we have electrons as our charge carrier, then q_c is just q_e.

We can substitute for Δx

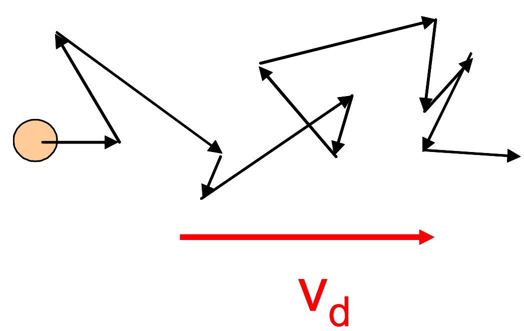
$$\Delta Q = nAv_d \Delta t q_e$$

This gives the charge within our small volume. But it would be nice to know how much charge is going by, because we want moving charge. We can divide by Δt

$$\frac{\Delta Q}{\Delta t} = nAv_d q_e \tag{16.11}$$

to get a charge flow rate. This is very like a volume or mass flow rate in fluid flow. We have an amount of charge going by in a time Δt.

I gave the flow velocity a special name, v_d. But I did not give all the reasons for using an average x-component of the velocity. If we think about it, we will realize that the electrons don't really flow in a straight line. They continually bump into atoms[3]. So the actual path the electrons take looks more like this.



We only care about the forward part of this motion. It is that forward component that we call the *drift speed* of the electrons. It is much slower than the

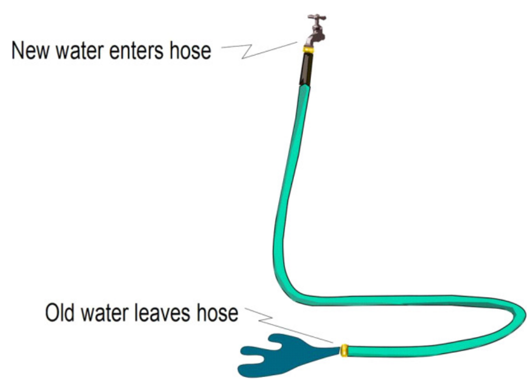
[3] We will refine this picture in the next lecture.

actual speed the electrons travel, and it depends on the type of conductor we are using.

We already know the name for the flow rate of charge, it is the electric current.

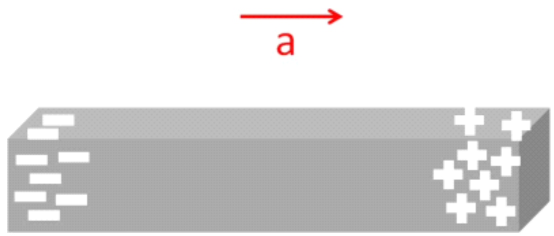
$$\frac{\Delta Q}{\Delta t} = I \tag{16.12}$$

We should take a minute to think about what to expect when we allow charge to flow. Think of a garden hose. If the hose is full of water, then when we open the faucet, water immediately comes out. The water that leaves the faucet is far from the open end of the hose, though. We have to wait for it to travel the entire length of the hose. But we get water out of the hose immediately! Why? Well, from Pascal's principle we know that a change in pressure will be transmitted uniformly throughout the fluid. This is like your hydraulic breaks. The new water coming in causes a pressure change that is transmitted through the hose. The water at the open end is pushed out.



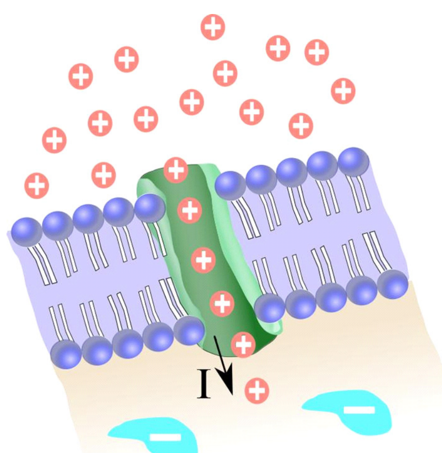
Current is a little bit like this. When we flip a light switch, the electrons near the near the switch start to flow at v_d. But there are already free electrons in all the wire. These experience a Pascal's-principle-like-push that makes the light turn on almost instantly.

There is a historical oddity with current flow. It is that the current direction is the direction positive charges would flow. This may seem strange, since in good conductors, we have said that electrons are doing the flowing! The truth is that it is very hard to tell the difference between positive charge flow and negative charge flow. In fact, only one experiment that I know of shows that the charge carriers in metals are electrons.



That experiment accelerates a conductor. The experiment is easier to perform using a centrifuge, but it is easier to visualize with linear motion. If we accelerate a bar of metal as shown in the preceding figure, the electrons are free to move about in the metal but the nuclei are all bound together. If the nuclei are accelerated they must go as a group. But the electrons will tend to stay with their initial motion (Newton's first law) until the end of the bar reaches them. At this point they must move because the electrical force of the mass of nuclei will keep them bound to the whole mass of metal. But the electrons will pile up at the tail end of the bar–that is–if it is the electrons that are free. When this experiment is performed, it is indeed the electrons that pile up at the tail end, and the forward end is left positive. This can be measured with a voltmeter.

Ben Franklin chose the direction we now use. He had a 50% chance if getting the charge carrier right. All this shows just how hard it is to deal with all these things we can't see or touch. And even more importantly, in semiconductors and in biological systems, it *is* positive charge that flows. In many electro-chemical reactions *both* positive and negative charges flow. We will stick with the convention that the current direction is the direction that positive charges would flow regardless of the actual charge carrier sign.



Flow of positive charge through a gate into a neural cell.

Basic Equations

Voltage if a dielectric is placed between the plates of the capacitor(equation 16.7)

$$\Delta V = \frac{\Delta V_o}{\kappa}$$

Capacitance increases (equation 16.7)

$$C = \kappa C_o$$

For parallel plate capacitors we get

$$C = \kappa \varepsilon_o \frac{A}{d}$$

The induced field in a dielectric is (equation 16.10)

$$E = E_o - E_{ind}$$

Current is the rate of charge flow (equation 16.12)

$$\frac{\Delta Q}{\Delta t} = I$$

Definition of current (equation 16.11)

$$I = nAv_d q_c$$