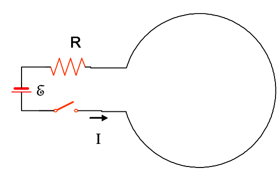
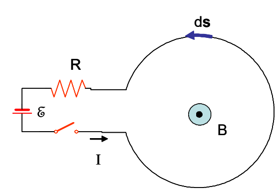
Appendix A

More insight into inductance and non-conservative fields

To try to make this idea of inductance make more sense, let's take another strange circuit.



There is a battery, and resister, and a single loop inductor. When the switch is thrown, the current will flow as shown. The current will create a magnetic field that is out of the page in the center of the loop. Since the loop, itself, is creating this field, let's call this field a *self field*.



Consider this self-field for a moment. When we studied charge, we found that charge created an electric field. That electric field could make *another* charge accelerate. But the electric field crated by a charge does not make the charge that created it accelerate. This is an instance of a self-field, an electric self-field. Now with this background, let's return to our magnetic self-field.

Let's take Faraday's law and apply it to this circuit. Let me choose an area vector **A** that is the area of the big loop and positive out of the page. Again, let's use conservation of energy (Kirchhoff's loop law). Let's find $\oint \mathbf{E} \cdot d\mathbf{s}$ for the entire circuit. We can start with the battery. Since there is an electric field inside the battery we will have a component of $\oint_{bat} \mathbf{E} \cdot d\mathbf{s}$ as we cross it. The battery field goes from positive to negative. If we go counter-clockwise, our $d\mathbf{s}$ direction traverses this from negative to positive, so the electric field is up and the $d\mathbf{s}$ direction is down, we have

$$\oint_{bat} \mathbf{E} \cdot d\mathbf{s} = -\mathcal{E}_{bat}$$

for this section of the circuit. Suppose we have ideal wires. If the wire has no resistance, then it takes no work to move the charges through the wire. In this case, an electron launched by the electric field in the battery just coasts from the battery to the resister. There is no need to have an acceleration in the ideal wire. The electric potential won't change from the battery to the resister. So there doesn't need to be a field in this ideal wire part to keep the charges going. But let's consider the resister. There is a potential change as we go across it. And if there is a change in potential, there must be an electric field. So the resister also has an electric field inside of it. We have a component of $\oint_R \mathbf{E} \cdot d\mathbf{s}$ that is equal to $\mathcal{E}_R = IR$ from this field.

$$\oint_R \mathbf{E} \cdot d\mathbf{s} = IR$$

Now we come to the big loop part. Since we have ideal wire, there is no resistance in this part so there is no voltage drop for this part of the circuit. All the energy that was given to the electrons by the battery was lost in the resister. They just coast back to the other terminal of the battery. Since there is no voltage drop in the big loop,

$$\mathcal{E}_{\text{big loop}} = 0$$

there is no electric field in the big loop either. Along the big loop, $d\mathbf{s}$ is certainly not zero. so

$$\mathcal{E}_{\text{big loop}} = \oint_{\text{big loop}} \mathbf{E} \cdot d\mathbf{s} = 0$$

For the total loop we would have

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\mathcal{E}_{batt} + IR + 0 \tag{A.1}$$

Normally, conservation of energy would tell us that all this must be zero, since the sum of the energy changes around the loop must be zero if no energy is lost. But now we know energy *is* lost in making a magnetic field.

Consider the magnetic flux through the circuit. The magnetic field is made by the current in the circuit. Note that we arranged the circuit so the battery and resister are in a part that has very little area, so we can ignore the flux through that part of the circuit. Most of the flux will go through the big loop part. The magnetic field is out of the paper inside of the loop. The flux is

$$\Phi_B = \oint \mathbf{B} \cdot d\mathbf{A} \tag{A.2}$$

and $\mathbf{B}$ and $\mathbf{A}$ are in the same direction. Φ_B is positive.

Then from Biot-Savart

$$\mathbf{B} = \frac{\mu_o I}{4\pi} \oint \frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \tag{A.3}$$

Let's write this as

$$\begin{aligned} \mathbf{B} &= I \left(\frac{\mu_o}{4\pi} \oint \frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \right) \\ &= I \,(\text{geometry factor}) \end{aligned} \tag{A.4}$$

If the geometry of the situation does not change, then B and I are proportional. Since $B \propto I$, then $\Phi_B \propto I$ since the integral in Biot-Savart is the surface integral of $\mathbf{B}$, and $\mathbf{B}$ is everywhere proportional to I. Instead of using Biot-Savart, let's just define a constant of proportionality that will contain all the geometric factors. We could give it the symbol, L. Then

$$\Phi_B = LI \tag{A.5}$$

where L is my geometry factor. But we recognize this geometry factor. It is just our inductance! This is what inductance is. It is all the geometry factors that make up our loop that will make the magnetic field if we put a current through it.

Assuming I don't change the geometry, then the inductance won't change and we have

$$\frac{d\Phi_B}{dt} = L\frac{dI}{dt} \tag{A.6}$$

and Faraday's law gives us

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \tag{A.7}$$

Which says that we should not have expected $\oint \mathbf{E} \cdot ds = 0$ for our case as we traverse the entire circuit. Integrating $\oint \mathbf{E} \cdot ds$ around the whole circuit including the big loop should not bring us back to zero voltage. We have lost energy in making the field. Instead it gives

$$\oint \mathbf{E} \cdot ds = -L\frac{dI}{dt}$$

We are dealing with non-conservative fields. So we have some energy loss like we would with a frictional force. It took some energy to make the magnetic field!

With this insight, we can now make a new statement of conservation of energy for such a situation. Integrating around the whole circuit gives

$$\oint \mathbf{E} \cdot ds = -\mathcal{E}_{bat} + \mathcal{E}_R$$

Which we now realize should give $-L\frac{dI}{dt}$ so

$$\oint \mathbf{E} \cdot ds = -\mathcal{E}_{bat} + \mathcal{E}_R = -L\frac{dI}{dt}$$

or more succinctly

$$-\mathcal{E}_{batt} + IR = -L\frac{dI}{dt}$$

Now I can take the RHS to the left and find

$$\mathcal{E}_{batt} - IR - L\frac{dI}{dt} = 0 \tag{A.8}$$

which accounts for all of the energy in the situation, so now we see that energy is conserved. For those of you who go on in your study of electronics. this looks like a Kirchhoff's rule with $-L\frac{dI}{dt}$ being a voltage drop across the single loop inductor. Under most conditions we can just treat $-L\frac{dI}{dt}$ as a voltage drop and it works fine. Most of the time thinking this way does not cause much of a problem. But technically it is not right!

We should consider where our magnetic flux came from. The magnetic flux was created by the current. It is a self-field. The current can't make a magnetic flux that would then modify that current. This self-flux won't make an electric field in the wire. So there is no electric field in the big loop, so there is no potential drop in that part of the circuit. It is just that $\oint \mathbf{E} \cdot ds \neq 0$ because our field is not conservative. We had to take some energy to create the magnetic field.

Now, if you are doing simple circuit design, you can pretend you don't know about Faraday's law and this complication and just treat $-L\frac{dI}{dt}$ as though it were a voltage drop. But really it is just that going around the loop we should expect

$$\oint \mathbf{E} \cdot ds = L\frac{dI}{dt}$$

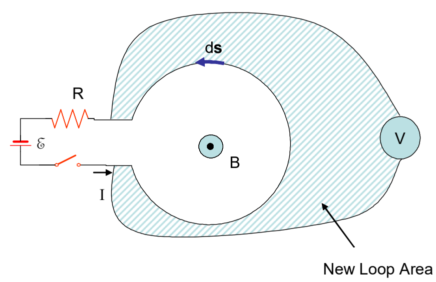
not

$$\oint \mathbf{E} \cdot ds = 0$$

The danger is that if you are designing a complicated device that depends on there being an electric field in the inductor, your device will not work. We have *no external magnetic field,* our only magnetic field is the *self-field* which will not produce an electric field (or at least will form a very small electric field compared to the electric fields in the resistor and the battery, due to the small resistance in the real wire we use to make the big loop). And perhaps just as important, there *is* a magnetic field that would not be predicted by just treating $L\frac{dI}{dt}$ as a voltage drop. This magnetic field could interfere with other parts of your circuits!

This is very subtile, and I struggle to remember this! Fortunately in most circuit design it does not matter. We just treat the inductor as though it were a true voltage drop.

I can make it even more exasperating by asking what you will see if you place a voltmeter across the inductor. What I measure is a "voltage drop" of LdI/dt, so maybe the there is a voltage drop after all! But no, that is not right. The problem is that in introducing the voltmeter, we have created a new loop. For this loop, the field from our big loop *is* an external field. .



So the changing magnetic field through this voltmeter loop will produce an emf that will just match LdI/dt. And there will be an electric field–but it will be in the internal resistor in the voltmeter. And that is what you will measure!

This may all seem very far fetched. But if you are designing radio communications you *want* to have a loss into the magnetic field, because that energy transferred to the magnetic field becomes your radio signal. This could be important!

The bottom line is that for non-conservative fields you need to be careful. If you are just designing simple circuits, you can just treat LdI/dt *as though it were a voltage drop*, but you may be badly burned by this if your system is more complicated, depending on the existence of a real electric field. You can see that if you are designing complicated sensing devices, you may need to deeply understand the underlying physics to get them to work. When in doubt, consult with a really good electrical engineer!

Appendix B

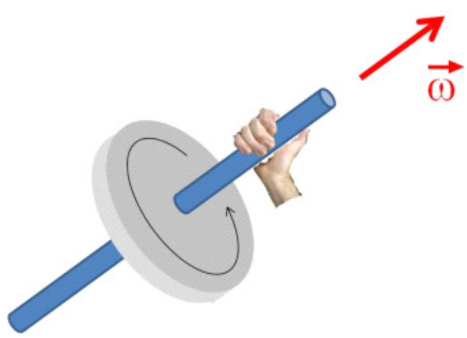
Summary of Right Hand Rules

B.1 PH121 or Dynamics Right Hand Rules

We had two right hand rules on PH121 We didn't give them numbers back then, so we will do that now.

B.1.1 Right hand rule #0:

We found that angular velocity had a direction that was given by imagining you grab the axis of rotation with your right hand so that your fingers seem to curl the same way the object is rotating. Then your thumb gives the direction of $\overrightarrow{\omega}$

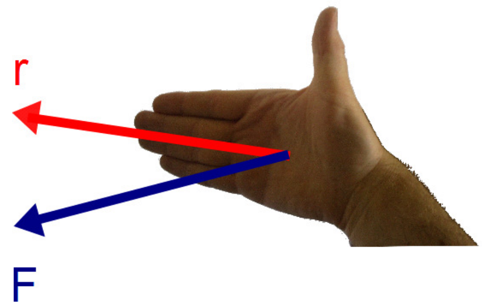


You curl the fingers of your right hand (sorry left handed people, you have to use your right hand for this) in the direction of rotation. Then your thumb points in the direction of the vector.

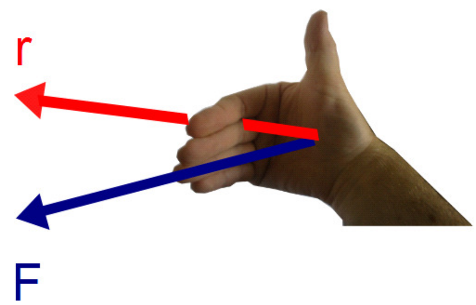
B.1.2 Right hand rule #0.5:

To find the direction of torque, we used the following procedure

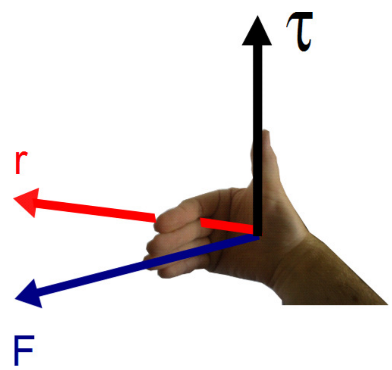
1. Put your fingers of your right hand in the direction of $\tilde{\mathbf{r}}$



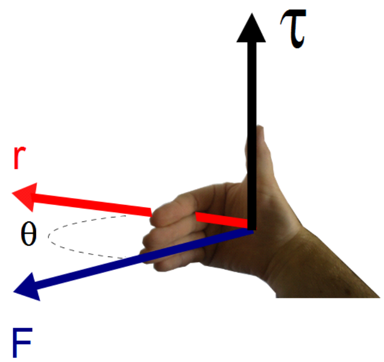
2. Curl them toward $\tilde{\mathbf{F}}$



3. The direction of your thumb is the torque direction



4. The angle θ is the angle *between* $\tilde{\mathbf{r}}$ and $\tilde{\mathbf{F}}$



The magnitude of the torque is

$$\tau = rF\sin\theta$$

B.2 PH223 Right Hand Rules

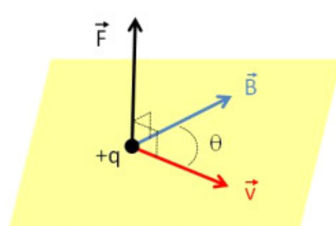
We have four more right hand rules this semester having to do with charges and fields.

B.2.1 Right hand rule #1:

From this rule we get the **direction of the force on a moving charged particle** as it travels thorough a **magnetic field**.

This rule is very like torque. We start with our hand pointing in the direction of $\tilde{\mathbf{v}}$. Curl your fingers in the direction of $\tilde{\mathbf{B}}$. And your thumb will point in the direction of the force. The magnitude of the force is given by

$$F = qvB\sin\theta \tag{B.1}$$



B.2.2 Right hand rule #2:

From this rule we get the direction of the force on current carrying wire that is in a magnetic field.

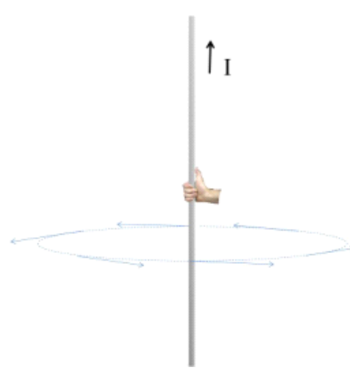
This rule is very like right hand rule #1 above. We start with our hand pointing in the direction of **I**. Curl your fingers in the direction of $\tilde{\mathbf{B}}$. And your thumb will point in the direction of the force. The magnitude of the force is given by

$$F = ILB\sin\theta \tag{B.2}$$

B.2.3 Right hand rule #3:

From this rule we get the **direction of the magnetic field that surrounds a long current carrying wire**.

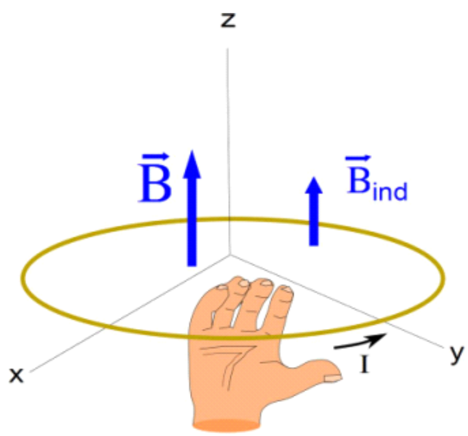
This rule is quite different. It is reminiscent of the rule for angular velocity, but there are some major differences as well. The field is a magnitude and a direction at every point in space. We can envision drawing surfaces of constant field strength. They will form concentric circles (really cylinders) centered on the wire. At any one point on the circle the field direction will be along a tangent to the circle. The direction of the vector is given by imaging you grab the wire with your right hand (don't really do it). Grab such that your right thumb is in the direction of the current. Your fingers will naturally curl in the direction of the field.



B.2.4 Right Hand Rule #4:

From this rule we get the **direction of the induced current when a loop is in a changing magnetic field**.

This rule is only used when we have a loop with a changing external magnetic field. The rule gives the direction of the induced current. The induced magnetic field will oppose the change in the external field, trying to prevent a change in the flux. The current direction is found by imagining we stick our right hand into the loop in the direction of the induced field. Keeping our hand inside the loop we grab a side of the loop. The current goes in the direction indicated by our thumb.



In the figure above, the external field is upward but decreasing. So the induced field is upward. The current flows because there is an induced *emf* given by

$$
\begin{aligned}
\mathcal{E} &= -N\frac{\Delta\Phi}{\Delta t} \\
&= -N\frac{(B_2A_2\cos\theta_2 - B_1A_1\cos\theta_1)}{\Delta t}
\end{aligned}
$$

Appendix C

Some Helpful Integrals

$$\int \frac{rdr}{\sqrt{r^2+x^2}} = \sqrt{r^2+x^2}$$

$$\int \frac{dx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

$$\int \frac{xdx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx}{x^2} = -\frac{1}{x}$$

$$\int_0^{2\pi} \int_0^{\pi} \sin\theta d\theta d\phi = 4\pi$$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^{R} r^2 dr \sin\theta d\theta d\phi = \frac{4}{3}\pi R^3$$

$$\int_0^{2\pi} \int_0^{R} rdrd\phi = \pi R^2$$

Appendix D

Some Physical Constants

Charge and mass of elementary particles

Proton Mass	$m_p = 1.6726231 \times 10^{-27}\,\mathrm{kg}$
Neutron Mass	$m_n = 1.6749286 \times 10^{-27}\,\mathrm{kg}$
Electron Mass	$m_e = 9.1093897 \times 10^{-31}\,\mathrm{kg}$
Electron Charge	$q_e = -1.60217733 \times 10^{-19}\,\mathrm{C}$
Proton Charge	$q_p = 1.60217733 \times 10^{-19}\,\mathrm{C}$

α-particle mass[1]	$m_\alpha = 6.64465675(29) \times 10^{-27}\,\mathrm{kg}$
α-particle charge	$q_\alpha = 2q_e$

Fundamental constants

Permittivity of free space	$\epsilon_o = 8.854187817 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N\,m}^2}$
Permeability of free space	$\mu_o = 4\pi \times 10^{-7} \frac{\mathrm{T\,m}}{\mathrm{A}}$
Coulomb Constant	$K = \frac{1}{4\pi\epsilon_o} = 8.98755 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$
Gravitational Constant	$G = 6.67259 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Speed of light	$c = 2.99792458 \times 10^8\,\mathrm{m\,s^{-1}}$
Avogadro's Number	$6.0221367 \times 10^{23}\,\mathrm{mol^{-1}}$
Fundamental unit of charge	$q_f = 1.60217733 \times 10^{-19}\,\mathrm{C}$

Astronomical numbers

Mass of the Earth[2]	$5.9726 \times 10^{24}\,\mathrm{kg}$
Mass of the Moon[3]	$0.07342 \times 10^{24}\,\mathrm{kg}$
Earth-Moon distance (mean)[4]	$384400\,\mathrm{km}$
Mass of the Sun[5]	$1,988,500 \times 10^{24}\,\mathrm{kg}$
Earth-Sun distance[6]	$149.6 \times 10^6\,\mathrm{kg}$

[1] http://physics.nist.gov/cgi-bin/cuu/Value?mal
[2] http://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html
[3] http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html
[4] http://solarsystem.nasa.gov/planets/profile.cfm?Display=Facts&Object=Moon
[5] http://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html
[6] http://nssdc.gsfc.nasa.gov/planetary/factsheet/index.html

Conductivity and resistivity of various metals

Material	Conductivity $(\Omega^{-1}\,\mathrm{m}^{-1})$	Resistivity $(\Omega\,\mathrm{m})$	Temp. Coeff. (K^{-1})
Aluminum	3.5×10^{7}	2.8×10^{-8}	3.9×10^{-3}
Copper	6.0×10^{7}	1.7×10^{-8}	3.9×10^{-3}
Gold	4.1×10^{7}	2.4×10^{-8}	3.4×10^{-3}
Iron	1.0×10^{7}	9.7×10^{-8}	5.0×10^{-3}
Silver	6.2×10^{7}	1.6×10^{-8}	3.8×10^{-3}
Tungsten	1.8×10^{7}	5.6×10^{-8}	4.5×10^{-3}
Nichrome	6.7×10^{5}	1.5×10^{-6}	0.4×10^{-3}
Carbon	2.9×10^{4}	3.5×10^{-5}	-0.5×10^{-3}