Chapter 33

Light, Sound, Power

Fundamental Concepts

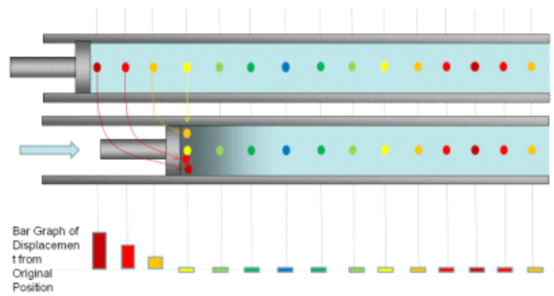
- Sound waves are formed when a disturbance causes a chain-reaction of collisions in the molecules of the air or other substance.
- Power is an amount of energy expended in an amount of time
- Intensity is an amount of power spread over an area
- The human auditory system is not a linear , but rather a logarithmic detector with perceived sound level given by $\beta = 10 \log_{10}\left(\frac{I}{I_o}\right)$

33.1 Waves in matter-Sound

We have said that sound is a longitudinal wave with a medium of air. Really any solid, liquid, or gas will work as a medium for sound. For our study, we will take sound to be a longitudinal wave and treat liquids and gasses. Solids have additional forces involved due to the tight bonding of the atoms, and therefore are more complicated. Technically in a solid sound can be a transverse wave as well a longitudinal wave, but we usually call transverse waves of this nature *shear waves.*

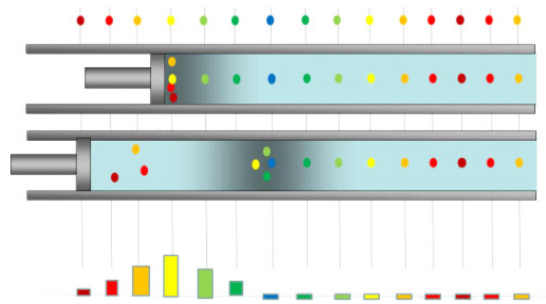
33.1.1 Periodic Sound Waves

Let's go back to making sounds. Suppose we push our piston as we did before.



When we push in the piston, it creates a region of higher pressure next to it.

When we pull back the piston the fluid expands to fill the void.



We create a rarefaction next to the piston.

Suppose we drive the piston sinusoidally. Can we describe the motion of the particles and of the wave? Notice that we end up with small bunches of particles with open spaces in between. Where the particles are bunched up the pressure will be higher. Where there are fewer particles the pressure will be lower. The higher pressure area is called a *compression* and the lower pressure area is called a *rarefaction.*

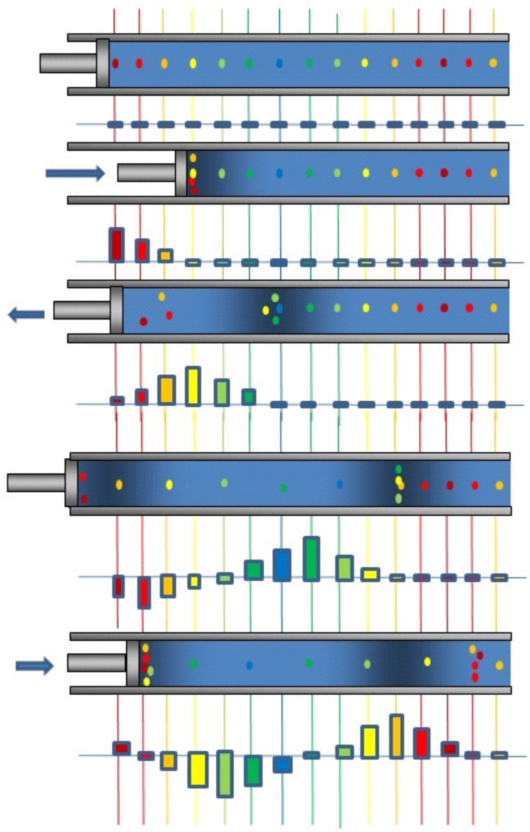
Compression: A local region of higher pressure in a fluid
Rarefaction: A local region of lower pressure in a fluid

We can identify the distance between two compressions as λ. We define $s\left(x,t\right)$ like we defined a wave function, $y\left(x,t\right))$ as the displacement of a particle of fluid relative to its equilibrium position.

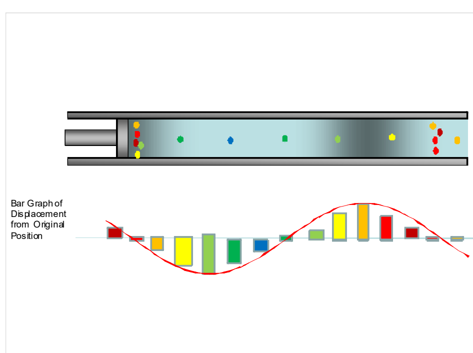
$$s\left(x,t\right)=s_{\max}\cos\left(kx-\omega t\right) \tag{33.1}$$

but what is $s_{\max}$?

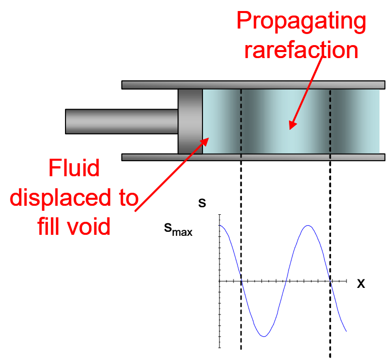
We remember that $s_{\max}$ is the maximum displacement of a particle of fluid from its equilibrium position. We plotted this using a bar graph to show displacement from the equilibrium position for our molecules. As we push the piston in and out we will get something like this.



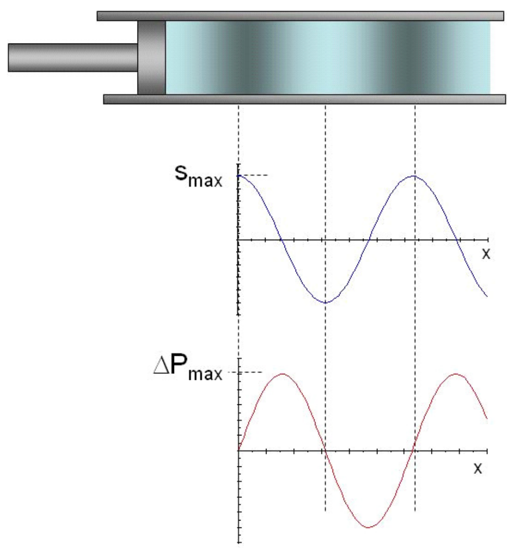
We found before that we get something that looks like a sine wave, but remember what the bars represent. They represent the displacement from original position.



We don't usually draw bar graphs to represent sound waves, we usually just draw the sine wave.



When the air molecules bunch up to form a compression, the pressure will be higher. And, as we know, when the air molecules spread out to form a rarefaction, the pressure will be lower than normal. The variation of the gas pressure ΔP measured from its equilibrium is also periodic



which is why we often refer to a sound wave as a pressure wave. Think of when the wave gets to your ear. The wave consists of a group of particles all headed for your ear drum. When they hit, they exert a force. Pressure is a force spread over an area,

$$P = \frac{F}{A}$$

so in a sense, we hear changes in air pressure!

33.2 Speed of Sound Waves

The speed of sound in air is around $340\,\mathrm{m/s}$. The speed changes when we change media, and even when we are in the same media but the temperature changes. For sound in air, a good approximation near standard pressure and temperature is

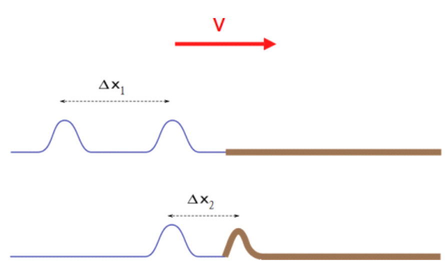
$$v = v_o\sqrt{1+\frac{T_c}{T_o}} \tag{33.2}$$

where $v_o = 331\,\frac{\mathrm{m}}{\mathrm{s}}$ and $T_o = 273\,\mathrm{K}$ $(0\,°\mathrm{C})$.[1]

Why temperature? The density and pressure of air change with temperature. The air molecules gain kinetic energy and tend to move farther apart from each other when they are warm. This changes the time it takes to transfer energy.

33.2.1 Boundaries

Suppose two pulses travel in the same medium, say, on a rope, and they approach a different rope with a different linear mass density. If the new rope is heavier, we expect the wave speed to slow down. So as one pulse reaches the boundary, it will go slower. This allows the second pulse to catch-up before it, too, slows down at the boundary.



Now suppose a sinusoidal wave approaches the boundary. We can envision the crests like pulses, and we expect the first crest to slow down when it reaches the boundary, letting the other crests catch up. Once the wave passes the boundary, the crests will be closer together. The wavelength changes as we move to the slower medium.

But does the frequency change? We know that

$$v = \lambda f$$

so

$$f = \frac{v}{\lambda}$$

both the speed and the wavelength have changed, but did they change proportionately so f is constant? This must be so. Think that the change in

[1] $v = v_o\sqrt{1+\frac{T_C}{T_o}} = v_o\sqrt{\frac{T_o}{T_o}+\frac{T_C}{T_o}} = v_o\sqrt{\frac{T_o+T_C}{T_o}} = v_o\sqrt{\frac{T_K}{T_o}}$

wavelength is due to the relative speed of the wave in the two media. If Δv is small, the change in λ will be small because the crests are not delayed too long. If Δv is large, the crests are delayed by a large amount and so the change in λ is large. We won't derive the fact that f is constant, but we can see that it is very believable that it is true.

This is true for all waves, even light. When a wave crosses a boundary from a fast to a slow or a slow to a fast medium, λ will change and f will remain constant.

Let's find an expression for the new wavelength. The frequency of the light must be the same.

$$f_i = f_f$$

and we know that in general

$$f = \frac{v}{\lambda}$$

so

$$\frac{v_i}{\lambda_i} = \frac{v_f}{\lambda_f}$$

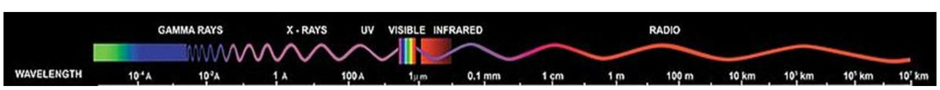
so

$$\lambda_f = \frac{v_f}{v_i}\lambda_i \tag{33.3}$$

and we can see that if v_f is slower than v_i our wavelength does get shorter.

33.3 Waves in fields-Light

We already know light is a wave in the electromagnetic field from our study of electrodynamics. The light we see is just one small part of a whole class of waves that are possible in this electromagnetic field medium. Radio waves, and microwaves, and x-rays are all just different types of electromagnetic waves. The next figure shows where all of these electromagnetic waves fit ordered by wavelength (and frequency).



Electromagnetic Spectrum (Public Domain image courtesy NASA)

There is something very unique about this electromagnetic field medium. The waves in this medium travel at a constant speed-no matter what frame of reference we are in. This fact leads to the formation of the Special Theory of Relativity and the famous equation

$$E = mc^2$$

where c is this speed of light

$$c = 299792458 \frac{\mathrm{m}}{\mathrm{s}}$$

Light does slow down when it enters a material medium, like glass, or even air. The actual speed that light travels does not change. What happens is that light is absorbed by the electrons in the atoms of the material substance. The electron temporarily takes up all the energy from a bit of the light wave–but only temporarily. It eventually has to give up the energy and the light wave reforms and mixes with the incoming wave. This process is complicated and we need more wave theory to understand how the waves will mix. But for now, let's just note that the wave has lost some time in the process, so it's average speed is less. How much less depends on how long the electrons in the atoms can hang-on to the light (and how the incoming and new wave mix). Each substance is different.

We can devise a way to express how much slower light will appear to go in a substance using the ratio

$$\frac{c}{v}$$

the ratio of the speed of light, c, to the average speed in the substance, v. This ratio is so useful that we give it a name, the *index of refraction.*

$$n = \frac{c}{v}$$

33.4 Power and Intensity

We know that energy is being transferred by the wave, whether it is a light or sound wave. We should wonder, how fast is energy transferring? This can mean the difference between sunlight on a warm summer day and being burned by a laser beam. We will start by considering the rate of energy transfer, *power.*

Power

The concept of power should be familiar to us from Principles of Physics I (PH121). We can find the power by

$$\mathcal{P} = \frac{\Delta E}{\Delta t}$$

where ΔE is the energy transferred and Δt is the time it takes to make the transfer.

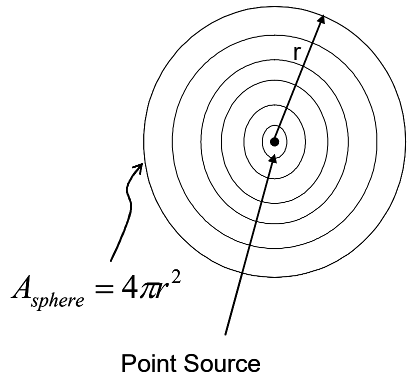
33.4.1 Intensity

We now define something new.

$$\mathcal{I} \equiv \frac{\mathcal{P}}{A} \tag{33.4}$$

that is, the power divided by the area. But what does it mean?

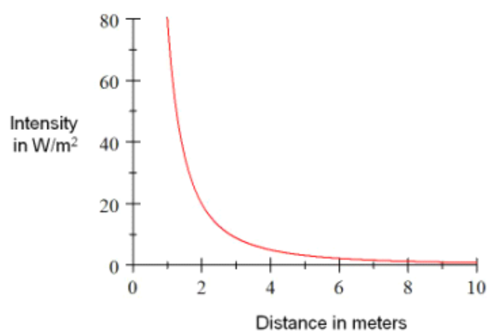
Consider a point source.



it sends out waves in all directions. The wave crests will define a sphere around the points source (the figure shows a cross section but remember it is a wave from a point source, so we are really drawing concentric spheres like balloons inside of balloons.). Then form our point source

$$\mathcal{I} = \frac{\mathcal{P}}{4\pi r^2} \tag{33.5}$$

As the wave travels, its power per unit area decreases with the square of the distance because the area is getting larger. The energy gets spread our over a lager area the farther we get away from the source.



This quantity that tells us how spread our our power has become is called the *intensity* of the wave.

Suppose we cup our hand to our ear. We can now hear fainter sounds. But what are we doing that makes the difference?

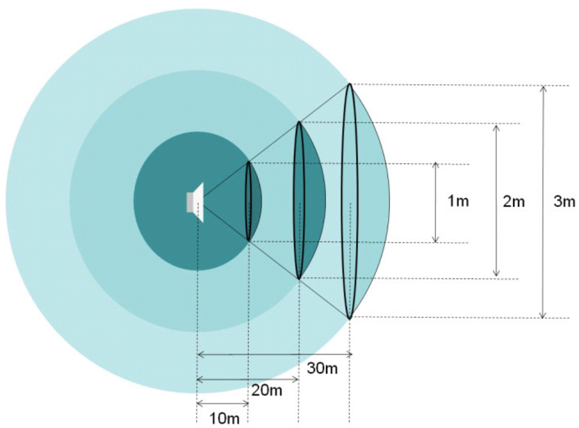
We are increasing the area of our ear. Our ears work by transferring the energy of the sound wave to a electro-chemical-mechanical device that creates a nerve signal. The more energy, the stronger the signal. If we are a distance r away from the source of the sound then the intensity is

$$\mathcal{I} = \frac{\mathcal{P}_{source}}{A_{wave}}$$

But we are collecting the sound wave with another area, the area of our hand. The power received is

$$\begin{aligned}\mathcal{P}_{received} &= \mathcal{I}A_{hand} \\ &= \frac{A_{hand}}{A_{wave}}\mathcal{P}_{source}\end{aligned}$$

and we can see that, indeed, the larger the hand, the more power, and therefore more energy we collect. This is the idea behind a dish antenna for communications and the idea behind the acoustic dish microphones we see at sporting events. In next figure, we can see that it would take an increasingly larger dish to maintain the same power gathering capability as we get farther from the source.



33.4.2 Sound Levels in Decibels

Our Design Engineer made an interesting choice in building us. We need to hear very faint sounds, and very loud sounds too. In order to make us able to hear the soft sounds without causing extreme discomfort when we hear the loud, He gave up linearity. That is, we don't hear twice the sound intensity as twice as loud. The mathematical expression that matches our perception of loudness to the intensity is

$$\beta = 10\log_{10}\left(\frac{I}{I_o}\right) \tag{33.6}$$

where the quantity I_o is a reference intensity. We are comparing the intensity of our sound with some reference intensity, I_o, to see how much louder our sound seems to be.

We call β the *sound level*. I_o we choose to be the *threshold of hearing*, the intensity that is just barely audible. Measured this way, we say that intensity is in units of decibels (dB). The decibel, is an engineer's friend (and useful for physicists too!) because it can describe a non-linear response in a linear way that is easy to match to our human experience.

Suppose we double the intensity by a factor of 2.

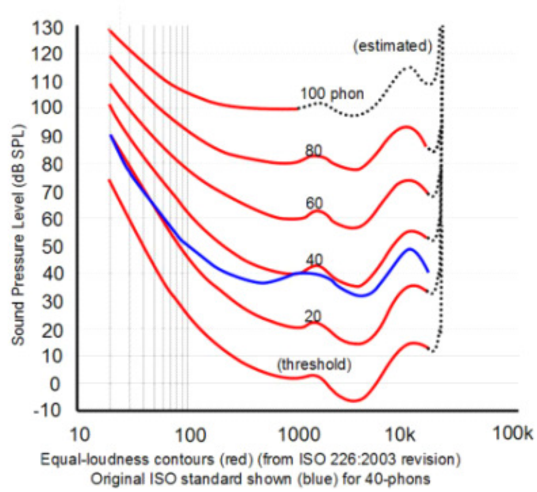
$$\begin{aligned}
\beta &= 10\log_{10}\left(\frac{2I_o}{I_o}\right) \\
&= 10\log_{10} 2 \\
&= 3.010\,3dB
\end{aligned}$$

The sound intensity level is not twice as large, but only $3dB$ larger. It is a tiny increase. This is what we hear. A good rule to remember is that $3dB$ corresponds to a doubling of the intensity.

The tables that follow give some common sounds in units of dB and $\mathrm{W/m^2}$. Just for reference, I have measured a Guns n Roses concert at 120 dB outside the stadium.

Sound	Sound Level (dB)
Jet Airplane at 30m	140
Rock Concert	120
Siren at 30m	100
Car interior when Traveling 60 mi/h	90
Street Traffic	70
Talk at 30 cm	65
Whisper	20
Rustle of Leaves	10
Quietest thing we can hear (I_o)	0

Loudness and frequency



Robinson-Dadson equal loudness curves (Image in the Public Domain courtesy Lindosland)

Our ears are truly amazing in their range and ability. But, sounds with the same intensity at different frequencies do not appear to us to have the same loudness. The frequency response graph above show how this relationship works for test subjects. We don't hear high or low frequencies as well. We have a peak response around 4000 Hz.

Basic Equations

$$P = \frac{F}{A}$$

$$v = v_o\sqrt{1 + \frac{T_c}{T_o}}$$

$$\lambda_f = \frac{v_f}{v_i}\lambda_i$$

$$E = mc^2$$

$$c = 299792458\frac{\text{m}}{\text{s}}$$

$$n = \frac{c}{v}$$

$$\mathcal{I} \equiv \frac{\mathcal{P}}{A}$$

$$\mathcal{I} = \frac{\mathcal{P}}{4\pi r^2}$$

$$.\beta = 10 \log_{10} \left(\frac{I}{I_o} \right)$$