Chapter 9

Conductors in Equilibrium, Electric Potentials

Fundamental Concepts

- Conductors in Equilibrium
- Electric Potential Energy

9.1 Conductors in Equilibrium

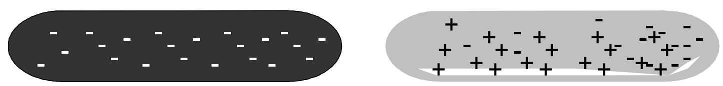
Conductors have some special properties because they have movable charge. Here they are

1. Any excess static charge (charge added to an uncharged conductor) will stay on the surface of the conductor.
2. The electric field is zero everywhere *inside* a conductor.
3. The electric field just outside a charged conductor is perpendicular to the conductor surface.
4. Charge tends to accumulate at sharp points where the radius of curvature of the surface is smallest.

It is our job to convince ourselves that these are true. Lets take these one at a time.

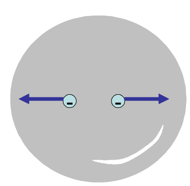
9.1.1 In Equilibrium, excess charge is on the Surface

Let's think about what we know about conductors. Most good conductors are metals. The reason they are good conductors is that the outer electrons in metals are in open valence bands where there are many energy states available to the electrons. These electrons are free to travel around. This means that if we place a charge near a metal object, the free charges will experience an acceleration. Of course, the charge does not fly out of the conductor. It will have to stop when it reaches the end of the metal object. Suppose we go back to our experiment from the first lecture. We took a charged rod, and placed it near an uncharged conductor.



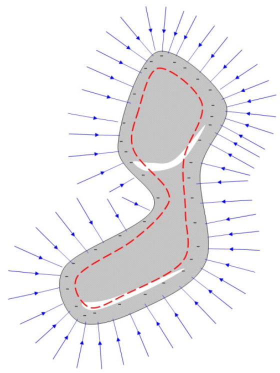
The free electrons moved. We ended up with a bunch of electrons all on the right hand side. They all repel each other. So at some point the force between a free electron and the charged rod, and the force between a free electrons and the rest of the free electrons will balance. At that point, there is zero net force (think of Newton's second law). The free electrons stop moving. We have a word from Principles of Physics I (PH121) for when all the forces balance. We say the charges are in *equilibrium*.

Now suppose we have a conductor just on it's own and suppose we add charge to it. Where would the extra charge go? We have considered this before. In the picture below, I have a spherical conductor with two extra negative charges shown. The pair of charges will repel each other. Now because of the r^2 in our electric force equation, the closer the extra charges are, the stronger the repulsive force. The result is that they will try to go as far from each other as possible. So the extra charge on a spherical conductor will all end up on the surface.



9.1.2 The Electric Field is Zero *Inside* a Conductor

We can use Gauss' law to find the field in a conductor. We know that the extra charge will all be on the surface if there is no electric current.

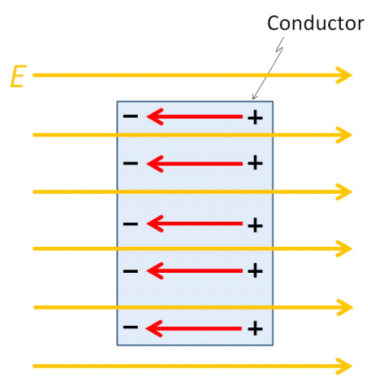


We can then draw a Gaussian surface, to match the symmetry of the conductor. What is the charge inside the Gaussian surface? It is net zero, since the reaming charge is all bound up in atoms and balances out. Since there is no net charge, there is no net flux. If there is no flux, there is no net field inside a conductor that is in static equilibrium.

Note that if we connected this conductor to both ends of a battery, we would have a field in the conductor generated by the battery and the charge flow it creates, so we must remember that static equilibrium is a special case.

If we don't connect the conductor to the ground or a battery, we can say: *The net electric field is zero everywhere inside the conducting material.*

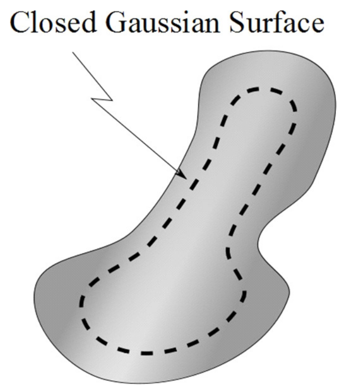
Consider if this were not true! If there were an electric field inside the conductor, the free charge there would accelerate and there would be a flow of charge. If there were a movement of charge, the conductor would not be in equilibrium. Suppose we place a brick of conductor in a field. We expect that the charges will be accelerated. Negative charges will move opposite the field direction. We end up with the situation shown in the next figure.



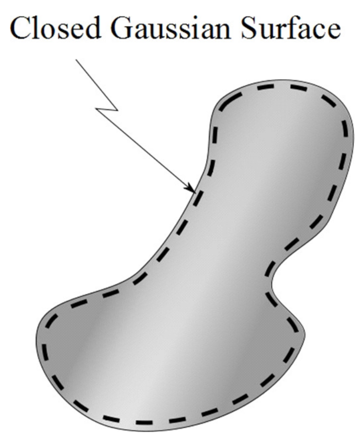
Since the negative charges moved, the other side has a net positive charge. This separation of the charges creates a new field in the opposite direction of the original field. In equilibrium, just enough charge is moved to create a field that cancels the original field.

Suppose we have a conductor in equilibrium. We can now ask, what does it mean that the charge is "on the surface?" Is there a small distance within the metal where we would find extra charge? or is it all right at the edge of the metal?

Let's look at this again now that we know Gauss' law. Let's envision a conducting object with a matching Gaussian surface.



We know the field inside the conductor is zero. So no field lines can leave or enter the Gaussian surface. So no charge can be inside or we would have a net flux, and, therefor, a field. We can move the Gaussian surface from the center of the conductor and grow it until it is just barely smaller than the surface of the conductor, and there still must be no field, so no charge inside.

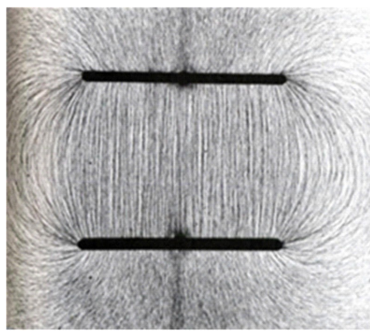


We can make this Gaussian surface as close to the actual surface as we like, and still there must be no field inside. Thus all the excess charge must be on the surface. It is not distributed at any depth in the material.[1]

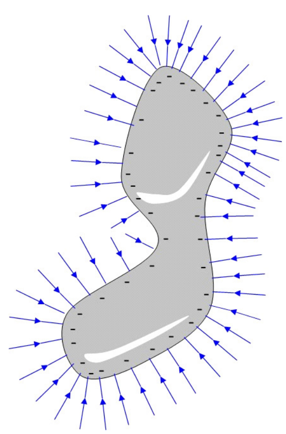
[1] For our chemists, our quantum picture will modify this reasoning a little, since we will view electrons as waves that extend out into space a bit.

9.1.3 Field lines leave normal to the surface

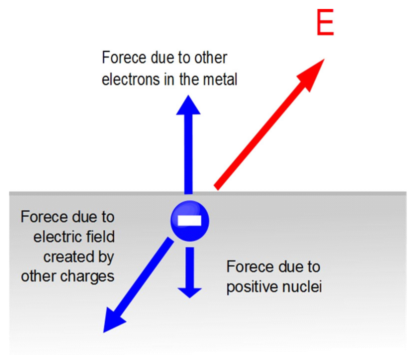
In the following picture, we can see that the field lines seem to leave the surface of these charged conductors at right angles (remember that sometimes we call this *normal* to the surface).



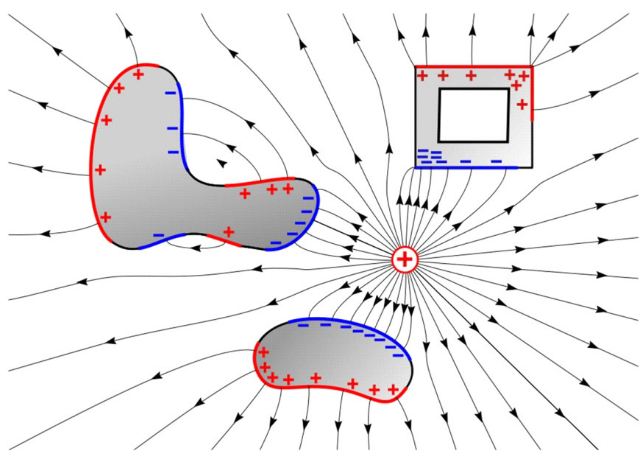
We have charges all along the surface, and neighboring charges cancel all but the normal components of the field, so the field lines go straight out. Notice that farther from the conductor the field lines may bend, but they start out leaving the surface perpendicular to the surface. Let's draw a conducting object.



Consider what would happen if it were not true that the field lines left perpendicular to a conductor surface when the conductor was in equilibrium.

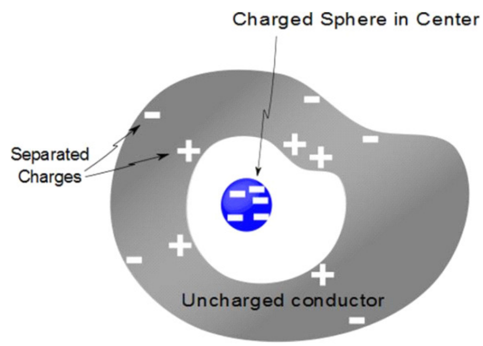


There would be a horizontal component of the field in such a case. The component of the field along the surface would cause the charge to move. In the figure there would be a net force to the left. This force would rearrange the charge until there was no force. But since $F_x = qE_x$, then when F_x is zero, so is E_x. Suppose we place a conductor in an external field. We would see that the charges within the conductor will rearrange themselves until the field lines will leave perpendicular to the surface of the conductors.



Notice the square box in the last figure. There is an opening inside the conductor, but there is no net field inside. The conductor charges rearrange themselves so that the external field is canceled out. This is part of what is known as a *Faraday cage* which allows us to cancel out an external electric field. This is used to protect electronic devices that must operate in strong electric fields. To complete the effect, we will also need to show that magnetic fields are canceled by such a conducting box.

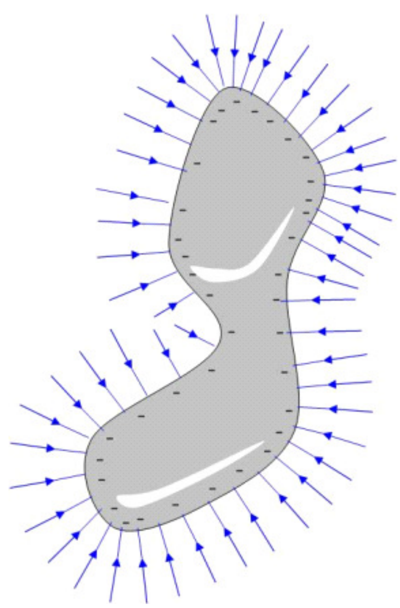
We should also consider what happens when we place a charge in a conductive container. Does this charge get screened off? That is, would the conductive container prevent us from telling if there was a charge inside?



In this case, the answer is no. The charges in the conductor will move because of the charge contained inside the conducting container. The negative charge will move as shown, and it will move to the outside of the container surface. This leaves positive charges behind on the inner surface. We know that there will be no field inside the conductor material, itself. But think of placing a Gaussian surface around all of the container and charge. There will be a net charge inside the Gaussian surface, so there will be a field. The inner surface charge does cancel the charge from the charged sphere. But the negative charge on the conductor surface creates a new field.

9.1.4 Charge tends to accumulate at sharp points

Let's go back to our charged conductor. Notice that the field lines bunch up at the corners! Where the field lines are closer together, there must be more charge and the field strength must be higher.



Now that we have an idea of how charge and conductors act in equilibrium, we would like to motivate charge to move. To see how this happens, let's review energy.

9.2 Electrical Work and Energy

We remember studying energy back in PH121 or Statics and Dynamics. Remember the Work-Energy theorem?

$$W_{nc} = \Delta K + \Delta U \tag{9.1}$$

We started with gravitational force and found the gravitational potential energy, and, as we found conservative forces, we defined new potential energies to describe the work done by those forces. For example, we learned about spring forces and added spring potential energy

$$W_{nc} = \Delta K + \Delta U_g + \Delta U_s \tag{9.2}$$

I bet you can guess what we will do with our electrical or Coulomb force!

$$W_{nc} = \Delta K + \Delta U_g + \Delta U_s + \Delta U_C \tag{9.3}$$

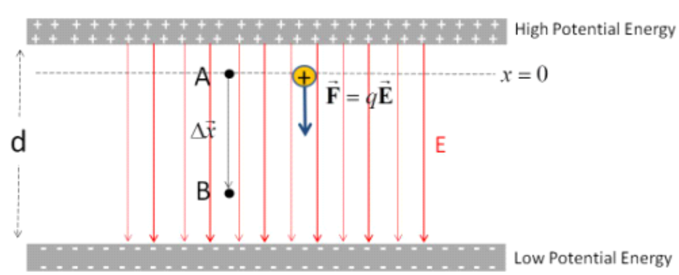
When we do this, we mean that the work done by the Coulomb force (W_C) is the negative of the electrical potential energy change

$$W_C = -\Delta U_C \tag{9.4}$$

and we are saying that the Coulomb force is conservative. But is the Coulomb force conservative? Remember that the equation for the force due to gravity and the equation for the Coulomb force are very alike. So we might guess that the Coulomb force is conservative like gravity–and we would be right!

9.2.1 Energy of a Charge in a uniform field

Let's use our Coulomb force to calculate work. We always found potential energy as a stored energy from work. So starting again with work seems like a good plan. I would like a simple example, so let's assume we have a uniform electric field. We know that we can almost really make a uniform electric field by building a large capacitor.



We draw some field lines (from the + charges to the - charges). The field lines will be mostly straight lines in between the plates. Of course, outside the plates, they will not be at all straight, but we will ignore this because we want to calculate work just in the uniform part of the field.

I want to place a charge, q_m, in this uniform field. The charge will accelerate. Work will be done. I want to find out how much work is done on the charge. From our Principles of Physics I (PH121) experience, we know that

$$\begin{aligned} W &= \int \overrightarrow{\mathbf{F}} \cdot \mathbf{d}\overrightarrow{\mathbf{x}} \\ &= F \Delta x \cos\theta \end{aligned} \tag{9.5}$$

for constant forces. Because we have a constant field, we will have a constant force.

I will choose the x direction to be vertical and $x = 0$ to be near the positive plate. Then we can write the force due to the electric field as

$$\begin{aligned} W &= F \Delta x \cos\theta \\ &= (q_m E)\, \Delta x \cos(0\,^\circ) \\ &= q_m E \Delta x \end{aligned}$$

If there are no non-conservative forces, and we ignore gravity, then we can say

$$\begin{aligned} W_{nc} &= \Delta K + \Delta U_g + \Delta U_s + \Delta U_C \\ 0 &= \Delta K + 0 + \Delta PE_C \\ 0 &= \Delta K + 0 - q_m E \Delta x \end{aligned}$$

so

$$\Delta K = q_m E \Delta x \tag{9.6}$$

This is very interesting! This means that for this simple geometry I could ask you questions like, "after the charge travels Δx, how fast is it going?"

9.2.2 Electric and Gravitational potential energy compared

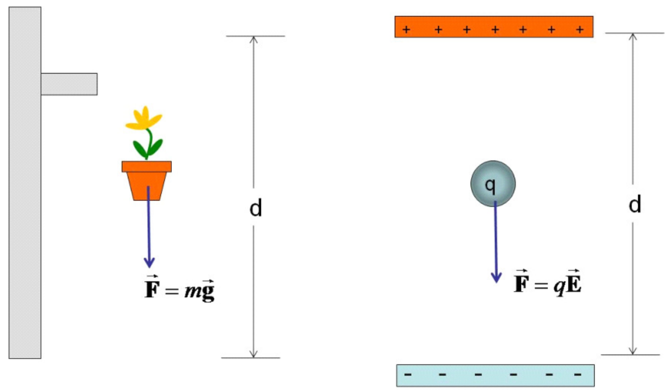
We have found that the potential energy for the Coulomb force is given by

$$\Delta U_C = -q_m E \Delta x$$

for a *uniform* electric field (it will change for non-uniform fields). Let's compare this to the gravitational potential energy

$$\Delta U_g = -mgh$$

Let's set up a situation where the electric field and gravitational field are almost uniform and we have a positively charged particle with charge q and mass m. The height, h, we will call d to match our gravitational and electrical cases.



The gravitational potential difference is

$$\Delta U_g = -mgd \tag{9.7}$$

and the electrical potential difference is

$$\Delta U_C = -q_m E d \tag{9.8}$$

These equations look a lot alike. We should expect that if we push the charge q_m "up," we will increase both potential energies. We will have to do positive work to do that $(W = -\Delta U)$. This is just like doing work in a gravitational field, so we are familiar with this behavior.

There is a difference, however. We have assumed that our charge q_m was positive. Suppose it is negative? There is only one kind of mass, but we have two kinds of charge. We will have to get used to negative charges "falling up" to make the analogy continue.

This analogy helps us to understand how the electric potential energy will act, and we will continue to use it. There is a difficulty, however, in that most engineering classes only study gravitation in nearly uniform gravitational fields. But if we look at large objects (like whole planets) that are separated from other objects by some distance, then we have very non-uniform gravitational fields. Unless you are an aerospace engineer, these cases are less common. So to help us understand electric potential energy, we will study gravitational potential energy of large things first, then study the energy associated with individual charges and their very non-uniform fields. We will take this on next time.

Basic Equations

$$W_{nc} = \Delta K + \Delta U_g + \Delta U_s + \Delta U_C \tag{9.9}$$
$$\Delta U_C = -q_m E d$$