# Chapter 22

# Magnetic forces on wires

## Fundamental Concepts

- The magnetic force on moving charges extends to wires with currents
- The force on a wire with current is given by $\mathbf{F}_I = I\mathbf{L} \times \mathbf{B}$
- The torque on a current loop is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ where $\boldsymbol{\mu} = I\mathbf{A}$

## 22.1 Magnetic forces on Current-Carrying wires

If there is a force on a single moving charge due to a magnetic field, then there must be a force on lots of moving charges! We call lots of moving charges an electric current

$$I = \frac{\Delta Q}{\Delta t}$$

For charges in a wire, we know that the charges move along the wire with a velocity $v_d$. We would expect the total force on all the charges to be the sum of all the forces on the individual charges.

$$F_I = \sum_i F_{q_i} = \sum_i q_i v B \sin\theta$$

but, since in our wire all the charge carriers are the same, this is just

$$F_I = N q_i v_d B \sin\theta$$

where here $N$ is the number of charge carriers in the part of the wire that is experiencing the field. We used a charge density $n$ before. Let's use it again to make an expression for $N$

$$N = nV = nAL$$

where $A$ is the cross sectional area of the wire and $L$ is the length of the wire. So

$$F_I = nALq_iv_dB\sin\theta$$

Now let's think back to our definition of current. We know that

$$I = nq_iv_dA$$

so our force on the current carrying wire is

$$\begin{aligned} F_I &= (nq_iv_dA)\, LB\sin\theta \\ &= ILB\sin\theta \end{aligned}$$

Remember that $\theta$ is the angle between the field direction and the velocity. In this case $I$ is in the direction of the velocity (we still assume positive charge carriers, even though we know they are electrons going the other way). So $\theta$ is the angle between the field direction and the direction of the current. We can write this as a cross product

$$\overrightarrow{\mathbf{F}}_I = I\overrightarrow{\mathbf{L}}\times\overrightarrow{\mathbf{B}} \tag{22.1}$$

where $\overrightarrow{\mathbf{L}}$ is in the current direction.

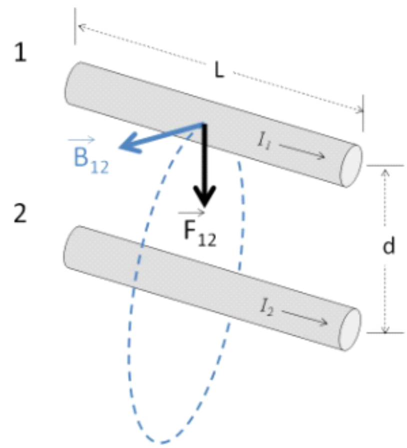
You might wonder how $\overrightarrow{\mathbf{L}}$ became a vector. We started with $\overrightarrow{\mathbf{F}}_B = q\overrightarrow{\mathbf{v}}\times\overrightarrow{\mathbf{B}}$ and ended up with $\overrightarrow{\mathbf{F}}_I = I\overrightarrow{\mathbf{L}}\times\overrightarrow{\mathbf{B}}$. The velocity $\overrightarrow{\mathbf{v}}$ has a direction and so does $I$. So why did we define a vector $\overrightarrow{\mathbf{L}}$ that is in the direction of the current flow instead of just using $\overrightarrow{\mathbf{I}}$? But if we remember the Biot-Savart law that started all of our magnetic study

$$\overrightarrow{\mathbf{B}} = \frac{\mu_o I}{4\pi}\int\frac{d\overrightarrow{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

we see the same pattern. We have $Id\overrightarrow{\mathbf{s}}$ where $d\overrightarrow{\mathbf{s}}$ is in the direction of the current. And $\overrightarrow{\mathbf{L}} = \int d\overrightarrow{\mathbf{s}}$ so this is not so crazy as it seems. Let's use our new force equation to get experience with how it works!

### 22.1.1 Force between two wires

If I have two wires with current, I will have a field created by each wire. Let's suppose that $I_1$ and $I_2$ are in the same direction




and let's calculate the force on wire 1 due to the field of wire 2. The field due to wire 2 at the location of wire 1 will be like

$$B_{12} = \frac{\mu_o I_2}{2\pi d}$$

if the wire is sufficiently long, where $d$ is how far away wire 1 is from wire 2. This is the field at the location of wire 1 due to the current in wire 2. We can use our new force on a wire equation to find the force on wire 1 due to the field from wire 2.

$$F_{12} = I_1 L B_2 \sin\theta$$

We can see that $\sin\theta = 1$ since $I_1$ will be perpendicular to $B_{12.}$

$$F_{12} = I_1 L B_{12}$$

and using our expression for $B_{12}$

$$\begin{aligned} F_{12} &= I_1 L \frac{\mu_o I_2}{2\pi d} \\ &= L \frac{\mu_o I_2 I_1}{2\pi d} \end{aligned} \qquad (22.2)$$

Would you expect $F_{21}$ to be very different?

## 22.2 Torque on a Current Loop

Remember that in PH121 or Statics and Dynamics we defined angular displacement

$$\Delta\theta = \theta_f - \theta_i \qquad (22.3)$$

and this told us how far in angle we had traveled from a starting point $\theta_i$.

We also defined the angular velocity

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad (22.4)$$

which told us how fast an object was spinning in radians per second. The direction of this angular velocity we found using a right hand rule.

We also defined an angular acceleration

$$\alpha = \frac{\Delta\omega}{\Delta t} \tag{22.5}$$

and we used angular acceleration ($\alpha$) in combination with a moment of inertia ($\mathbb{I}$) to express a rotational form of Newton's second law

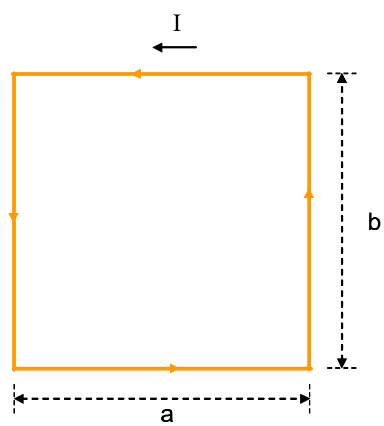
$$\sum \tau = \mathbb{I}\alpha \tag{22.6}$$

where $\tau$ is a torque. We found torque with the expression

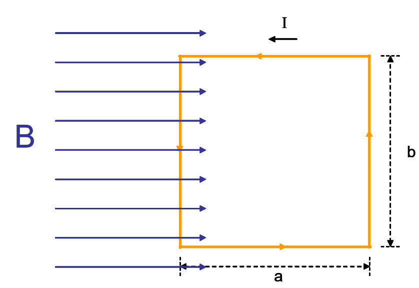
$$\overrightarrow{\boldsymbol{\tau}} = \overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}} \tag{22.7}$$

We wish to apply these ideas to our new force on wires due to magnetism.

Let's take a specific example. I want to use a current loop. This is just the simple loop of current we have seen before.




I want to place this into a magnetic field.



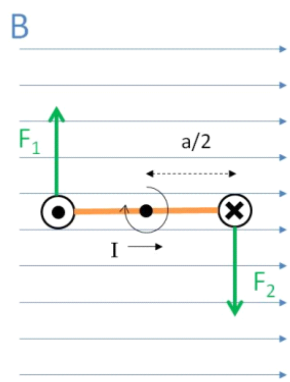
I drew the current loop as a rectangle on purpose, I want to look at the force on the current for each part of the loop. Each side of our loop is a straight wire segment. Remember that the magnitude of the force on a wire is given by

$$F_I = ILB\sin\theta$$

where $\theta$ is the angle between $I$ and $B$ so if $\theta = 0$ or if $\theta = \pi\,\text{rad}$, then $\sin\theta$ will be zero. The magnitude of the force will then be zero. So the top and bottom parts of the loop will not experience a force. The sides will, though, and since for $\theta = \frac{\pi}{2}$ or $\theta = -\frac{\pi}{2}$ then $\sin\theta = 1$ and the force will be a maximum

$$F_I = IbB$$

on each side wire segment. But we need to consider direction. The force will be perpendicular to both $I$ and $B$. We use our right hand rule. Fingers in the direction of $I$, curl to the direction of $B$. We see the force is out of the figure for the left hand side and into the figure for the right hand side. The next figure is a bottom-up view.




Clearly the loop will want to turn! This looks like a nice problem for us to describe with a torque. We have a force acting at a distance from a pivot. We have a torque

$$\tau = rF\sin\psi$$

We have already used $\theta$, and our torque angle is the angle between $r$ and $F$, so we needed a new greek letter. I have used $\psi$[1]. Then $\psi$ is the angle between $r$ and $F$.

Let's fill in the details of our total torque. Remember we have two torques, one for the left hand side, and one for the right and side. Their magnitudes are the same, and the directions we need to get from yet another right hand rule. Both are in the same direction so

$$\begin{aligned}\tau &= \frac{a}{2}F_I\sin(\psi) + \frac{a}{2}(F_I)\sin(\psi)\\ &= aF_I\sin(\psi)\end{aligned}$$

[1] which is a *psi*

Putting in the force magnitude gives

$$\tau = a\,(IbB)\sin\psi$$

and rearranging lets us see

$$\begin{aligned}\tau &= (ab)\,IB\sin\psi \\ &= (A)\,IB\sin\psi\end{aligned}$$

where $A = ab$ is the area of our loop. Of course we can write this as

$$\overrightarrow{\boldsymbol{\tau}} = I\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}} \tag{22.8}$$

The torque is the cross product of the area vector and the magnetic field multiplied by the current.

We did this for a square loop. It turns out that it works for any loop shape.

When things rotate, we expect to use moments. We defined a magnetic dipole moment for a current loop. Now we can see why it is useful. The magnetic moment tells us about how much torque we will get for a particular current loop.

$$\overrightarrow{\boldsymbol{\mu}}_d = I\overrightarrow{\mathbf{A}}$$

using this we have

$$\overrightarrow{\boldsymbol{\tau}} = \overrightarrow{\boldsymbol{\mu}}_d \times \overrightarrow{\mathbf{B}}$$

We could envision our loop as a single circle of wire connected to a battery. But we could just as easily double up the wire. If we do this, what is our torque? Well we would have twice the force, because we now have twice the current (the current goes trough both turns of the wire). So now we have

$$\tau = 2\,(A)\,IB\sin\psi$$

But why stop there? We could make three loops all together.

$$\tau = 3\,(A)\,IB\sin\psi$$

or many more, say $N$ loops,

$$\tau = NAIB\sin\psi$$

Thinking of our magnetic dipole moment, we see that

$$\tau = N\mu_d B\sin\psi$$

for a coil. We could combine the effects of all the loops into one magnetic moment that represents the coil.

$$\overrightarrow{\boldsymbol{\mu}} = N\overrightarrow{\mathbf{A}}I \tag{22.9}$$

then

$$\tau = \mu B \sin\psi$$

or in cross product form

$$\overrightarrow{\boldsymbol{\tau}} = \overrightarrow{\boldsymbol{\mu}} \times \overrightarrow{\mathbf{B}} \tag{22.10}$$

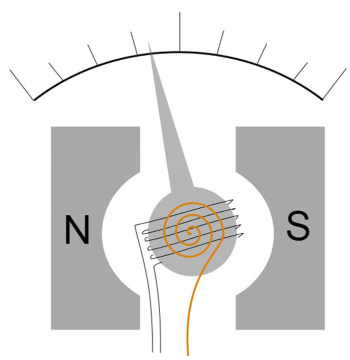
Using this total magnetic moment, we can more easily do problems with coils in magnetic fields.

For example, we found that there was a potential energy associated with spinning dipoles, for a spinning current loop we also expect a potential energy. We have a simple formula for this potential energy in terms of the magnetic moment.

$$U = -\overrightarrow{\boldsymbol{\mu}} \cdot \overrightarrow{\mathbf{B}} \tag{22.11}$$

### 22.2.1 Galvanometer

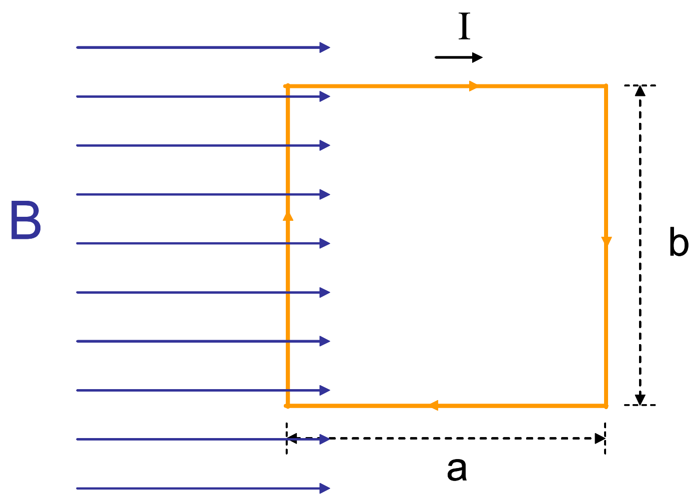
We finally know enough to understand how to measure a current. The device is called a *galvanometer*.



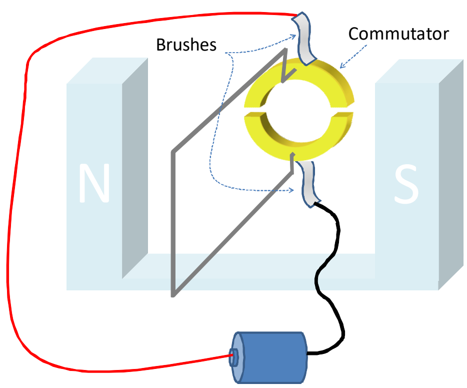

In the picture, we see the typical design of a galvanometer. It has a coil of wire (shown looking at the side of the coil) and a spring. The coil is placed between the ends of a magnet. When there is a current in the wire, there will be a torque on the coil that will compress the spring. The amount of torque depends on the current. As the current increases, the spring is more compressed. A marker (large needle) is attached to the apparatus. As the spring is compressed, the indicator moves across the scale. Since this movement is proportional to the current, a galvanometer can easily measure current.

### 22.2.2 Electric Motors

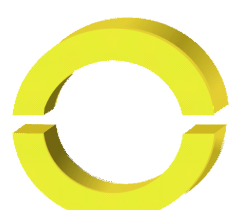
With our new understanding of torque on a current loop, we should be able to see how an electric motor works. A current loop is placed in between two magnets to form a magnetic field. The loop will turn because of the torque due to the $B$-field. But we have to get clever. What happens when the loop turns half way around so the current is now going the opposite way?



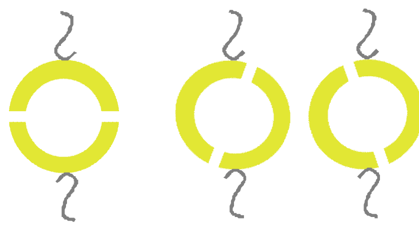

Now the torque switches direction and the loop will come to rest. We don't want that if we are building a motor, so we have to switch the current direction every time the loop turns half way.




The way we do this is to have electrical contacts that are flexible, called brushes. The brushes contact a metal ring. The metal ring is connected to the loop. But the ring has two slits cut out of it.



The ring with slits is called a commutator As the loop turns, the commutator turns, and when it has turned a half turn, the brushes switch sides. This changes the current direction, which puts us back at maximum torque.



This keeps the motor going the same direction.

# Basic Equations

$$\overrightarrow{\mathbf{F}}_I = I\overrightarrow{\mathbf{L}}\times\overrightarrow{\mathbf{B}} \tag{22.12}$$

$$F_{12} = L\frac{\mu_o I_2 I_1}{2\pi d} \tag{22.13}$$

$$\overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\boldsymbol{\mu}} \times \overrightarrow{\mathbf{B}} \tag{22.14}$$

$$U = -\overrightarrow{\boldsymbol{\mu}} \cdot \overrightarrow{\mathbf{B}} \tag{22.15}$$