Strategy for Gauss' law problems

Let's review what we have done before we go on to our last example. For each Gauss' law problem, we

| 1. Draw the charge distribution | |
|--|--------------------------|
| 2. Draw the field lines using symmetryDon's skip this step, it is essential | |
| 3. Choose (make up, invent) a closed surface that makes $\overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{A}}$ either just EdA or 0. And hopefully on our Gaussian surface E will be constant | Spherical Closed Surface |
| 4. Find Q_{in} . | |
| 5. Solve \$\int EdA = \frac{Q_{inside}}{\epsilon_0}\$ for the non, zero parts. The integral should be trivial now due to our use of symmetry. Usually, if we picked our surface well, \$\int EdA = EA\$ 6. Solve for \$E\$ i.e. \$EA = \frac{Q_{inside}}{\epsilon_0} \rightarrow EA = \frac{Q_{inside}}{\epsilon_0}\$ | |

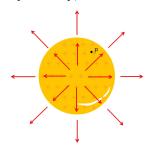
Example

Find the field at a point inside at point P inside a uniformly charged spherical insulator with total charge Q_{total} and radius R. Let's say that P is a distance r from the center of the sphere.

1. draw the charge distribution



2. Draw the field lines using symmetry- Don's skip this step, it is essential



3. Choose (make up, invent) a closed surface that includes point P and makes $\overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{A}}$ either just $\pm EdA$ or 0. And hopefully on our Gaussian surface E will be constant



4. Find Q_{in} . This time it is harder. Since the charge is uniformly distributed

$$\rho = \frac{Q_{total}}{V_{total}} = \frac{Q_{in}}{V_{in}}$$

so

$$Q_{in} = \frac{Q_{total}}{V_{total}} V_{in}$$

We know how to do this for spheres

$$V_{total} = \frac{4}{3}\pi R^3$$
$$V_{in} = \frac{4}{3}\pi r^3$$

$$Q_{in} = \frac{Q_{total}}{\frac{4}{3}\pi R^3} \frac{4}{3}\pi r^3 = \frac{Q_{total}}{R^3} r^3$$

- 5. Solve $\oint EdA = \frac{Q_{inside}}{\epsilon_o}$ for the non, zero parts. The integral should be trivial now due to our use of symmetry. Usually, if we picked our surface well, $\oint EdA = EA = E4\pi r^2$
- 6. Solve for E

$$EA = \frac{Q_{inside}}{\epsilon_o}$$

$$E = \frac{Q_{total}}{\frac{R^3}{R^3}} r^3 = \frac{1}{4\pi\epsilon_o} \frac{Q_{total}r}{R^3}$$