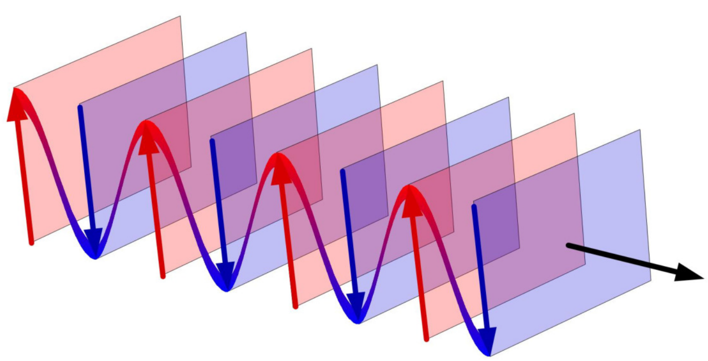
Chapter 49

Waves in the Field

We started this class with a study of waves. We learned about optics, and finally electromagnetic field theory. In this lecture we will take on a case study that involves all three. We will have come full circle and in the process, hopefully understand all three topics a little better.

Fundamental Concepts

- Maxwell's equation lead directly to the liner wave equation for both the electric and the magnetic field with the speed of light being the speed of the waves.
- The magnitude of the E and B fields are related in an electromagnetic wave by $E_{\max} = cB_{\max}$



A representation of a plane wave. Remember that the planes are really of infinite extent. Image is public domain.

Let's picture our wave front far from the source. No matter what the total shape, if we look at a small patch of the fields far away, they will look like the plane wave in the last figure. Since this is a useful and common situation (except if you use lasers), we will perform some calculations assuming plane wave geometry.

We will assume we are in empty space, so the charge q and current I will both be zero. Then our Maxwell Equations become

$$\begin{array}{ll} \oint \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{A}} = 0 & \text{Gauss's law for electric fields} \\ \oint \overrightarrow{\mathbf{B}} \cdot d\overrightarrow{\mathbf{A}} = 0 & \text{Gauss's law for magnetic fields} \\ \oint \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}} = -\frac{d\Phi_B}{dt} & \text{Faraday's law} \\ \oint \overrightarrow{\mathbf{B}} \cdot d\overrightarrow{\mathbf{s}} = \varepsilon_o \mu_o \frac{d\Phi_E}{dt} & \text{Ampere-Maxwell Law} \end{array} \tag{49.1}$$

Our goal is to show that these equations tell us that we can have waves in the field. To do this, we will show that Maxwell's equations really contain the linear wave equation within them. As a reminder, here is the linear wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

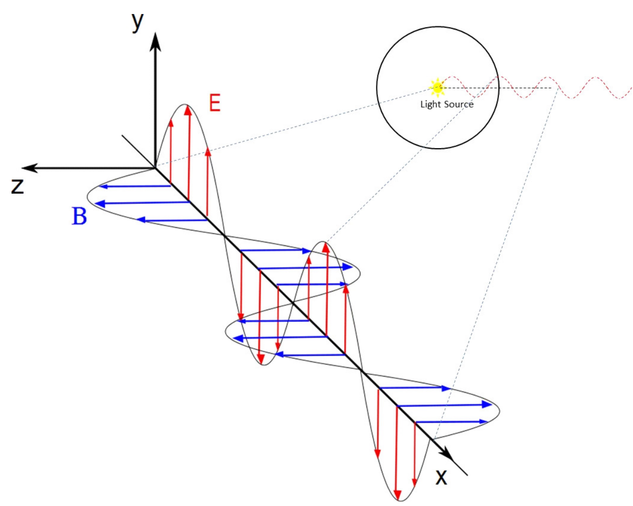
it is a second order differential equation where the left side derivatives are take with respect to position, and the right side derivatives are taken with respect to time. The quantity, v, is the wave speed. In this form of the equation y is the displacement of a medium. Our medium will be the electromagnetic field.

49.0.1 Rewriting of Faraday's law

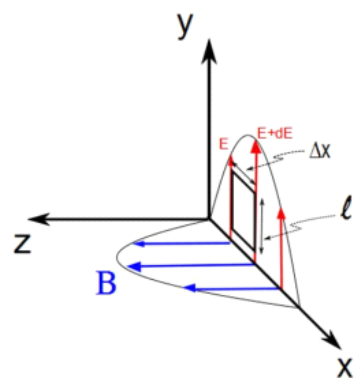
Let's start with Faraday's law

$$\oint \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}} = -\frac{d\Phi_B}{dt} \tag{49.2}$$

Given our geometry, we can say the wave is traveling in the x direction with the $\overrightarrow{\mathbf{E}}$ field positive in the y direction. From our discussion of displacement currents we have a strong hint that the $\overrightarrow{\mathbf{E}}$ and $\overrightarrow{\mathbf{B}}$ fields will be perpendicular. So let's take the magnetic field as positive in the z direction. So as the light wave moves from the source along a line we could draw the $\overrightarrow{\mathbf{E}}$ and $\overrightarrow{\mathbf{B}}$ fields something like this.



Let's take a small rectangle of area to find $\oint \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}}$



The top and bottom of the rectangle don't contribute because $\overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}} = \mathbf{0}$ along these paths. On the sides, the field is either in the $d\overrightarrow{\mathbf{s}}$ or it is in the opposite direction. So

$$\oint \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}} = \oint Eds$$

or

$$\oint \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}} = -\oint Eds$$

along the sides. Let's say we travel counter-clockwise along the loop. Then the left side will be negative and the right side will be positive.

$$\oint \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{s}} = \int_{right} Eds - \int_{left} Eds$$

On the left side, we are at a position x away from the axis, and on the right side we are a position $x + \Delta x$ away from the axis. Then the field of the left side is $E(x,t)$ and the field on the right hand side is approximately

$$E(x+\Delta x,t)\approx E(x,t)+\frac{\partial E}{\partial x}\Delta x \tag{49.3}$$

so if our loop is small, then ℓ is small and E won't change much so we can write approximately

$$\begin{aligned}
\oint \overrightarrow{\mathbf{E}}\cdot d\overrightarrow{\mathbf{s}} &= \int_{right} Eds-\int_{left} Eds && (49.4)\\
&\approx E(x+\Delta x,t)\,\ell-E(x,t)\,\ell && (49.5)\\
&= \left(E(x,t)+\frac{\partial E}{\partial x}\Delta x\right)\ell-E(x,t)\,\ell\\
&= \left(E(x,t)+\frac{\partial E}{\partial x}\Delta x\right)\ell-E(x,t)\,\ell\\
&= \ell\frac{\partial E}{\partial x}\Delta x && (49.6)
\end{aligned}$$

So far then, Faraday's law [1]

$$\oint \overrightarrow{\mathbf{E}}\cdot d\overrightarrow{\mathbf{s}}=-\frac{d\Phi_B}{dt}$$

becomes

$$\ell\frac{\partial E}{\partial x}\Delta x=-\frac{d\Phi_B}{dt}$$

Let's move on to the right hand side of Faraday's law. We need to find Φ_B so that we can find the time rate of change of the flux. We can say that B is nearly constant over such a small area, so

$$\begin{aligned}
\Phi_B &= \mathbf{B}\cdot\mathbf{A}\\
&= BA\cos\theta\\
&= BA\\
&= B\ell\Delta x
\end{aligned}$$

where here Δx means "a small distance" as it did above. Then

$$\begin{aligned}
\frac{d\Phi_B}{dt} &= \frac{d}{dt}(B\ell\Delta x)\\
&= \ell\Delta x\left.\frac{dB}{dt}\right|_{x\text{ constant}}\\
&= \ell\Delta x\frac{\partial B}{\partial t}
\end{aligned}$$

where we have held x constant because we are not changing our small area, so Faraday's law

$$\oint \overrightarrow{\mathbf{E}}\cdot d\overrightarrow{\mathbf{s}}=-\frac{d\Phi_B}{dt}$$

[1] We need ds to be very small, much smaller than the wavelength of the wave.

becomes

$$\ell\frac{\partial E}{\partial x}\Delta x = -\ell\Delta x\frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \qquad (49.7)$$

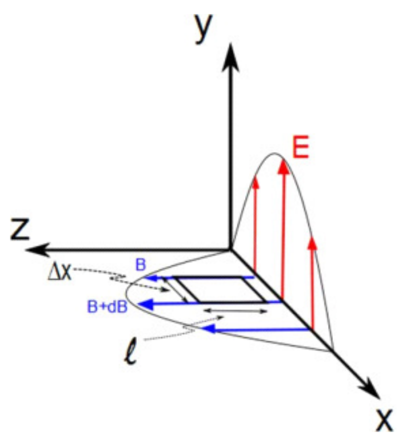
We have made some progress, we have a differential equation relating the fields, but it is a mixed equation containing both the electric and magnetic fields. We are only half way there.

49.0.2 Rewriting of the Maxwell-Ampere Law

We have used one field equation so far and that took us part of the way. We have the Maxwell-Ampere law as well. We can use this to modify our result from Faraday's law to find the linear wave equation that we expect. The Maxwell-Ampere law with no sources (charges or currents) states

$$\oint \overrightarrow{\mathbf{B}} \cdot d\overrightarrow{\mathbf{s}} = \varepsilon_o\mu_o\frac{d\Phi_E}{dt}$$

This time we must consider the magnetic field path integral



We can do the same thing we did with Faraday's law with an area, but this time we will use the area within the magnetic field (shown in the figure above). Again, let's start with the left hand side of the equation. We see that the sides of our area that are parallel to the x-axis do not matter because $\overrightarrow{\mathbf{B}} \cdot d\overrightarrow{\mathbf{s}} = 0$ along these sides, but the other two are in the direction (or opposite direction) of the field. They do contribute to the line integral.

$$\oint \overrightarrow{\mathbf{B}} \cdot d\overrightarrow{\mathbf{s}} = B(x,t)\,\ell - B(x+\Delta x,t)\,\ell \qquad (49.8)$$

$$\approx -\ell\frac{\partial B}{\partial x}\Delta x$$

Now for the left hand side, we need the electric flux. For such a small area, the field is nearly constant so

$$\begin{aligned}\Phi_E &\approx EA\cos\theta \\ &= EA \\ &= E\ell\Delta x\end{aligned}$$

so

$$\frac{\partial \Phi_E}{\partial t} = \ell\Delta x\frac{\partial E}{\partial t} \tag{49.9}$$

Combining both sides

$$\begin{aligned}\oint \overrightarrow{\mathbf{B}}\cdot d\overrightarrow{\mathbf{s}} &= \varepsilon_o\mu_o\frac{d\Phi_E}{dt} \\ -\ell\frac{\partial B}{\partial x}\Delta x &= \varepsilon_o\mu_o\ell\Delta dx\frac{\partial E}{\partial t} \\ \frac{\partial B}{\partial x} &= -\varepsilon_o\mu_o\frac{\partial E}{\partial t} \qquad (49.10)\end{aligned}$$

We now have a second differential equation relating B and E. But it is also a mixed differential equation.

49.1 Wave equation for plane waves

This leaves us with two equations to work with

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \tag{49.11}$$

$$\frac{\partial B}{\partial x} = -\varepsilon_o\mu_o\frac{\partial E}{\partial t} \tag{49.12}$$

Remember that these are all partial derivatives. Taking the derivative of the first equation with respect to x gives

$$\begin{aligned}\frac{\partial}{\partial x}\frac{\partial E}{\partial x} &= \frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right) \\ \frac{\partial^2 E}{\partial x^2} &= -\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t}B\right) \\ \frac{\partial^2 E}{\partial x^2} &= -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right)\end{aligned}$$

In the last equation we swapped the order of differentiation for the right hand side. In parenthesis, we have $\partial B/\partial x$ on the right hand side. But we know what $\partial B/\partial x$ is from our second equation. We substitute from our second equation to obtain

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial t}\left(-\varepsilon_o\mu_o\frac{\partial E}{\partial t}\right)$$

$$\frac{\partial^2 E}{\partial x^2}=\varepsilon_o\mu_o\frac{\partial^2 E}{\partial t^2} \tag{49.13}$$

We can do the same thing, but taking derivatives with respect to time to give

$$\frac{\partial^2 B}{\partial x^2}=\varepsilon_o\mu_o\frac{\partial^2 B}{\partial t^2} \tag{49.14}$$

You will recognize both of these last equations as being in the form of the linear wave equation.

$$\frac{\partial^2 y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

This means that both the E field and the B field are governed by the linear wave equation with the speed of the waves given by

$$v=\frac{1}{\sqrt{\varepsilon_o\mu_o}} \tag{49.15}$$

We have studied waves, so we know the solution to this equation is a sine or cosine function

$$E = E_{\max}\cos\left(kx-\omega t\right) \tag{49.16}$$
$$B = B_{\max}\cos\left(kx-\omega t\right) \tag{49.17}$$

with

$$k=\frac{2\pi}{\lambda}$$

and

$$\omega=2\pi f$$

then

$$\frac{\omega}{k}=\frac{2\pi f}{\frac{2\pi}{\lambda}}=\lambda f$$

which is the wave speed.

We can show that the magnitude of E is related to B.

Lets take derivatives of E and B with respect to x and t.

$$\begin{aligned}\frac{\partial E}{\partial x} &= -kE_{\max}\sin\left(kx-\omega t\right)\\ \frac{\partial B}{\partial t} &= \omega B_{\max}\sin\left(kx-\omega t\right)\end{aligned}$$

then we can use one of our half-way-point equations from above

$$\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$$

and by substitution obtain

$$\begin{aligned}-kE_{\max}\sin\left(kx-\omega t\right) &= -\omega B_{\max}\sin\left(kx-\omega t\right)\\ -kE_{\max} &= -\omega B_{\max}\end{aligned}$$

or

$$\frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = v$$

The speed is the speed of light, c, so

$$\frac{E_{\max}}{B_{\max}} = c \tag{49.18}$$

It is one of the odd things about the universe that speed of electromagnetic waves is a constant. It does not vary in vacuum, and the in-vacuum value, c is the maximum speed. It was a combination of Maxwell's work in predicting c and the observations confirming the predictions that launched Einstein to form the Special Theory of Relativity!

Note that the last equation shows why we often only deal with the electric field wave when we do optics. Since the magnetic field is proportional to the electric field, we can always find it from the electric field.

49.2 Properties of EM waves

Knowing that the electric and magnetic fields form plane waves, we can investigate these plane wave solutions to see what they imply.

49.2.1 Energy in an EM wave

The electromagnetic (EM) wave is a wave. Waves transfer energy. It is customary find a vector that describes the flow of energy in the electromagnetic wave. This is like the ray vectors we have been drawing for some time, but with the magnitude of the vector giving the energy flow rate.

The rate of at which energy travels with the EM wave is given the symbol **S** and is called the Poynting vector after the person who thought of it. It is

$$\mathbf{S} = \frac{1}{\mu_o}\mathbf{E} \times \mathbf{B} \tag{49.19}$$

Let's deal with a dumb name first: The Poynting vector. It is named after a scientist with the last name Poynting. The name is really meaningless. There is nothing particularly "pointy" about this vector more than any other vector.

Instead of a formal derivation, let's just see what we get from Poynting's equation for a plane wave.

For our plane wave case, E and B are at $90\,^{\circ}$ angles[2]. so

$$S = \frac{1}{\mu_o}EB \tag{49.20}$$

and S will be perpendicular to both. Notice from our preceding figures that this is also the direction that the wave travels! That is comforting. That should

[2] For other fields this might not be true, but it is generally true for light.

be true for a EM wave. The energy, indeed, goes the way the Poynting vector points.

Using

$$\frac{E}{B} = c$$

we can write the magnitude of the Poynting vector as

$$S = \frac{E^2}{c\mu_o} \tag{49.21}$$

We could also express this in terms of B only.

You will remember that our eyes don't track the oscillations of the electromagnetic waves. Few detectors (if any) can. For visible light, the frequency is very high. We usually see a time average. This time average of the Poynting vector is called the *intensity* of the wave

$$I = S_{ave}$$

49.2.2 Intensity of the waves

When we studied waves, we learned that waves have an intensity. The intensity of electromagnetic waves must relate to the strength of the fields. We can write it as

$$I = \frac{EB}{2\mu_o}$$

(can you remember where the "1/2" came from?)[3]. Again using

$$E = cB$$

we can write the intensity as

$$I = \frac{1}{2\mu_o c} E^2 \tag{49.22}$$

We remember that I is proportional to the square of the maximum electric field strength from our previous consideration of light intensity. But before we only said that it was proportional. Now we know the constant of proportionality. Of course we could also write the intensity as

$$I = \frac{c}{2\mu_o} B^2 \tag{49.23}$$

but this is less traditional. We have said already that the intensity, I, is the magnitude of the average Poynting vector S_{ave}.

[3] This is because the average value of $\sin^2(\omega t)$ over a period is given by $\frac{1}{T}\int_0^T \sin^2(\omega t)\, dt = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \sin^2(\omega t)\, dt = \frac{1}{2}$

Recall that we know the energy densities in the fields

$$\begin{aligned} u_E &= \frac{1}{2}\varepsilon_o E^2 \\ u_B &= \frac{1}{2}\frac{B^2}{\mu_o} \end{aligned}$$

again, since

$$E = cB \tag{49.24}$$

we can write

$$\begin{aligned} u_B &= \frac{1}{2}\frac{B^2}{\mu_o} \\ &= \frac{1}{2}\frac{E^2}{c^2\mu_o} \\ &= \frac{1}{2}\varepsilon_o E^2 \end{aligned} \tag{49.25}$$

so for a plane electromagnetic wave

$$u_E = u_B \tag{49.26}$$

The total energy in the field is just the sum

$$u = u_E + u_B = \varepsilon_o E^2 \tag{49.27}$$

But when we do the time average to find the intensity, we pick up a factor of a half

$$u_{ave} = \frac{1}{2}\varepsilon_o E^2 \tag{49.28}$$

Comparing this to our equation for intensity gives

$$I = \frac{1}{2\mu_o c}E_{\max}^2 = S_{ave}$$

and then

$$\begin{aligned} S_{ave} &= \frac{1}{\epsilon_o \mu_o c}\frac{1}{2}\epsilon_o E^2 \\ &= \frac{1}{\epsilon_o \mu_o c}u_{ave} \\ &= \frac{1}{\frac{1}{c^2}c}u_{ave} \\ &= c u_{ave} \end{aligned} \tag{49.29}$$

If you have already taken your course on thermodynamics you, learned that we could transfer energy by radiation. This is our radiation! And we see that it does indeed transfer energy. We learned about this by discussing solar heating and by talking about Army weapons that apply energy to crowds.

US Army Active Denial System (ADS).

but we really use this every day when we microwave something. Microwaves are electromagnetic waves!

49.2.3 Momentum of light

One of the strangest things is that there is also momentum in the electromagnetic waves. If the waves are absorbed, the momentum is

$$p = \frac{U}{c} \tag{49.30}$$

or if the waves are reflected it is

$$p = \frac{2U}{c} \tag{49.31}$$

(think of balls bouncing off a wall, the change in momentum is always $2mv$ for a bounce).

We can think of the light exerting a pressure on the surface. Force is given by

$$\begin{aligned} F &= ma \\ &= m\frac{dv}{dt} \\ &= \frac{dp}{dt} \end{aligned}$$

then using this force, the pressure is

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} \tag{49.32}$$

then

$$P = \frac{F}{A} = \frac{1}{cA}\frac{dU}{dt} \tag{49.33}$$

We found $\frac{1}{A}\frac{dU}{dt}$ to be the energy rate per unit area, which is the magnitude of the Poynting vector, S. So our pressure due to light is

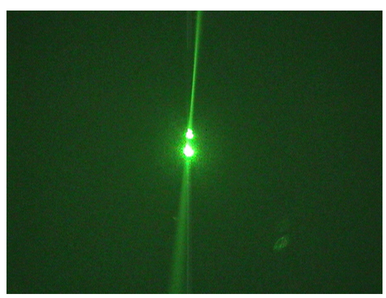
$$P = \frac{S}{c} \tag{49.34}$$

for perfect absorption. If there is perfect reflection

$$P = \frac{2S}{c} \tag{49.35}$$

This may seem a little strange. Water or sound waves would exert a pressure because the water or air particles can strike a surface, exerting a force. But remember the electromagnetic fields will create forces on the electrons in atoms[4], and most of the electrons are bound to the atoms in materials by the Coulomb force. So there really is a force on the material due to the electromagnetic wave. Quantum mechanics tells us about electrons being knocked out of shells into higher energy shells (absorbing photons of light) and re-emitting the light when the electrons fall back down to lower shells. This is a little like catching a frisbee, and then throwing it. Momentum is transferred both at the catch and at the release.

A cool use of this phenomena is called laser levitation



Laser Levation (Skigh Lewis, Larry Baxter, Justin Peatross (BYU), Laser Levitation: Determination of Particle Reactivity, ACERC Conference Presentation, February 17, 2005)

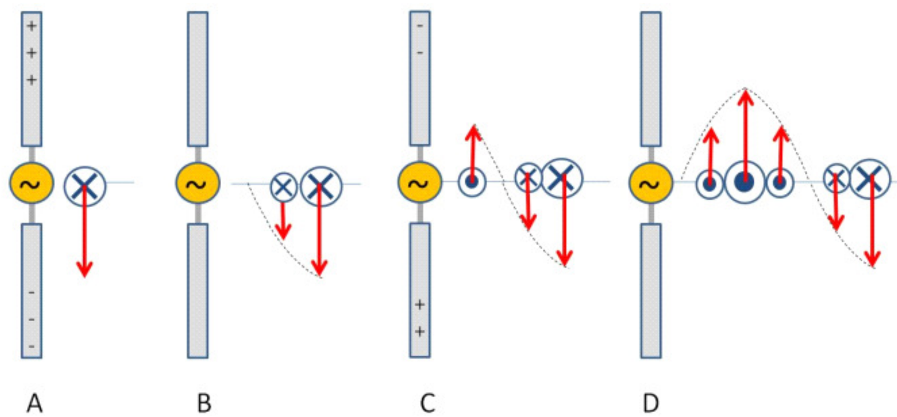
In the picture you are seeing a single small particle that is floating on a laser beam. the laser beam is directed upward. The force due to gravity would make the particle fall, but the laser light keeps it up!

[4]Protons too, but the protons are more tightly bound due to the nuclear strong force and the nuclei are bound in the material. their resonant frequencies are usually not assessable to visible light, so I will ignore their effect in our treatment. But if you consider x-rays or gamma rays, they would be important.

49.2.4 Antennas Revisited

We talked about antennas before. Let's try to put all we have done together to make a radio wave. First, we know from our analysis that we need changing fields. Neither static charges, nor constant currents will do. If we think about this for a minute, we will realize that the charges will *accelerate*. Fundamentally, this is the mechanism for making EM waves.

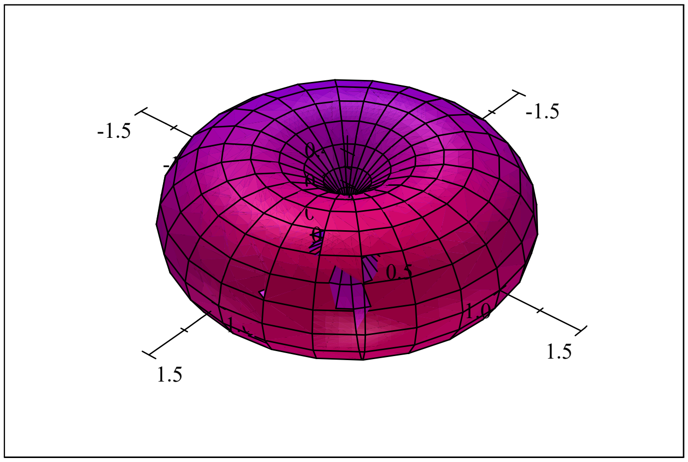
The half wave antenna is simple to understand, so let's take it as our example.



It is made from two long wires connected to an alternating current source (the radio transmitter).The charges are separated in the antenna as shown. But the separation switches as the alternating current changes direction. The charges accelerate back and forth, like a dipole switching direction. Radio people call this antenna a *simple dipole*.

Note the direction of the E and B fields. The Poynting vector is to the right. The antenna field sets up a situation far from the antenna, itself, where the changing electric field continually induces a magnetic field and the changing magnetic field continually induces a changing electric field. The wave becomes self sustaining! And the energy it carries travels outward.

Below you can see a graph of the sort of toroidal angular dependence of the dipole antenna emission pattern.



Angular dependence of S for a dipole scatterer.

From this you can see why we usually stand antennas straight up and down. Then the transmission travels parallel to the Earth's surface, where receivers are more likely to be.

Speaking of receivers, of course the receiver works like a transmitter, only backwards. The EM waves that hit the receiving antenna accelerate the electrons in the wire of the antenna. The induced current passed through an LRC circuit who's resonance frequency allows amplification of just one small band of frequencies (the one your favorite radio station is using) and then the amplified signal is sent to a speaker.

49.3 The Electromagnetic Spectrum

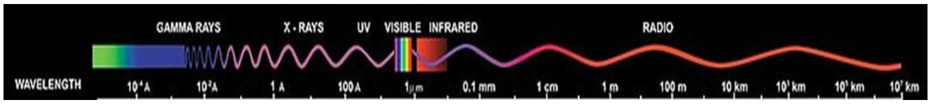
Maxwell predicted how fast his field waves would travel by finding the linear wave equation from the fields and noticing the speed indicated by the result. We have seen how he did this. The answer is

$$v = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \tag{49.36}$$

this speed is so special in physics that it get's its own letter

$$c = 2.99792 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}} \tag{49.37}$$

which is of course the speed of light. In fact, that this was the measured speed of light was strong evidence leading us to conclude that light was really a type of these waves. There are a few more types of electromagnetic waves. In the following chart you can see that visible light is just a small part of what we call the *electromagnetic spectrum.*



Electromagnetic Spectrum (Public Domain image courtesy NASA)

The speed of light is always a constant in vacuum. This is strange. It caused a lot of problems when it was discovered.

$$v = f\lambda \tag{49.38}$$

or

$$c = f\lambda \tag{49.39}$$

where we can see that for light and electromagnetic waves, knowing the wavelength is always enough to know the frequency as well (in a vacuum).

As an example of what problems can come, let's consider a Doppler effect for light. Remember for sound waves, we had a Doppler effect. We will have a Doppler effect for electromagnetic waves too. But light does not change it's speed relative to a reference frame. This is *really weird*. The speed of light in a vacuum is *always c–no matter what frame we measure it in*.

Einstein's theory of Special Relativity is required to deal with this constant speed of light in every reference frame. From Relativity, the Doppler equation is

$$f' = f\frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}} \tag{49.40}$$

or, if we let u be the relative velocity between the source and the detector, and insist that $u \ll c$

$$f' = f\left(\frac{c+u}{c}\right) \tag{49.41}$$

Where of course f' is the observed frequency and f is the frequency emitted by the source. This is usually written as

$$f' = f\left(1 \pm \frac{u}{c}\right) \tag{49.42}$$

but it is really the same equation[5]. Just like with sound, we use the positive sign when the source and observer are approaching each other.

This means that if things are moving closer to each other the frequency increases. Think of

$$\lambda = \frac{c}{f} \tag{49.43}$$

this means that as a source and emitter approach each other, then the light will have a shorter wavelength. Think of our chart on the electromagnet spectrum.

[5] This equation is only really true for relative speeds u that are much less than the speed of light. Since is is very hard to make something travel even close to to the speed of light, we will find it is nearly always true.

This means the light will get bluer. If they move farther apart, the light will get redder.

This is what gave us the hint that has lead to our cosmological theories like the big bang. Although this theory is now much more complicated, the facts are that as we look at far away objects, we see they are all *red shifted*. That is, they all show absorption spectra for known elements, but at longer wavelengths that we expect from laboratory experiments. We interpret this as meaning they are all going away from us!

49.3.1 Summary

Here is what we have learned so far about the properties of light

1. Electromagnetic waves travel at the speed of light
2. Electromagnetic waves are transverse electric and magnetic waves that are oriented perpendicular to each other.
3. $E = cB$
4. Electromagnetic waves carry energy *and momentum*

49.3.2 Photons

Our understanding of light is not complete yet. If you went on to take PH279 you would find that light still operates much like a particle at times. This should not be a surprise, since Newton and others explained much of optics (the study of light) assuming light was a particle.

Einstein and others noticed that for some metals, light would strike the surface and electrons would leave the surface. The energy of a wave is proportional to the amplitude of the wave. It was expected that if the amplitude of the electromagnetic wave was increased, the number of electrons leaving the surface would increase. This proved to be true most of the time. But Hertz and others decided to try different frequencies of light. It turns out that as you lower the frequency, all of a sudden no electrons leave no matter how big the amplitude of the wave. Something was wrong with our wave theory of light. The answer came from Einstein who used the idea of a "packet" of light to explain this *photoelectric effect*. For now, we should know just that the waves of light exist in *quantized* packets called *photons*. The energy of a photon is

$$E = hf \tag{49.44}$$

where E is the energy, f is the frequency of the light wave, and h is a constant

$$h = 6.63 \times 10^{-34}\,\mathrm{J\,s} \tag{49.45}$$

A beam of light is many, many photons all superimposing. We know how waves combine using superposition, so it is easy to see that we can get a big wave from many little waves.

Knowing that light is made from electric and magnetic fields, and that these fields are vector fields, we should expect some directional quality in light. And there is such a directional quality that we will study next lecture.