

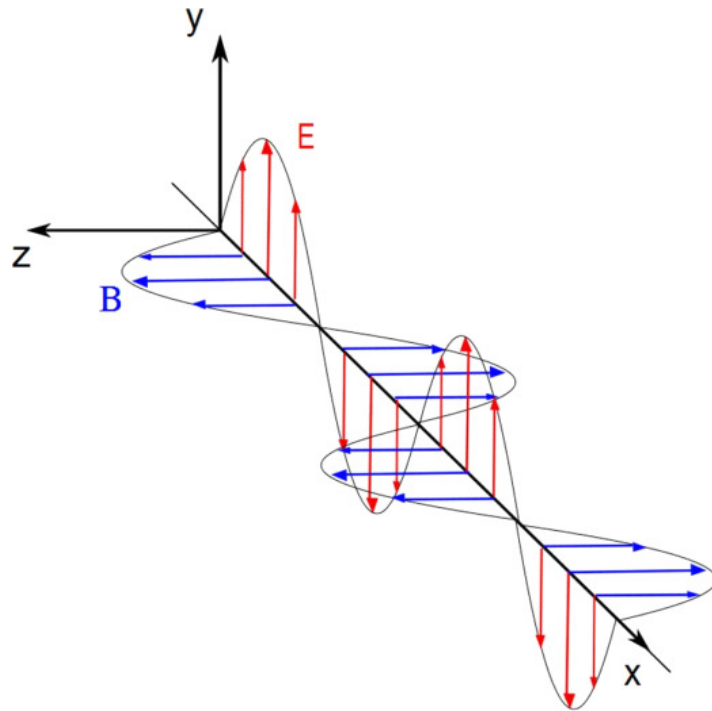
# 50 Polarization

## Fundamental Concepts

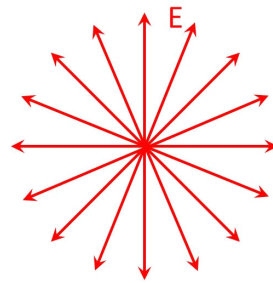
- The direction of the electric field in a plane wave is called the polarization direction.
- Natural light is usually a superposition of many waves with random polarization directions. This light is called unpolarized light.
- Some materials allow light with one polarization to pass through, while stopping other polarizations. The polaroid is one such material. Polaroids will have a final intensity that follows the relationship  $I = I_{\max} \cos^2(\theta)$
- Light reflecting off a surface may be polarized because of the absorption and re-emission pattern of light interacting with the material atoms.
- Scattered light may be polarized because of anisotropies in the scatterers.
- Birefringent materials have different wave speeds in different directions. This affects the polarization of light entering these materials.

## Polarization of Light Waves

We said much earlier in our study of light that it was a transverse wave. Last lecture we saw that we have an electric and magnetic field direction, and that these directions are perpendicular to each other and the direction of energy flow. We will now show some implications of this fact. In a course in electromagnetic theory, we often draw light as in the figure below.



We will continue to ignore the magnetic field (marked in the figure as  $B$ ). We will look at the  $E$  field and notice that it goes up and down in the figure. But we could have light in any orientation. If we look directly at an approaching beam of light we would “see” many different orientations as shown in the next figure.



When light beams have waves with many orientations, we say they are *unpolarized*. But suppose we were able to align all the light so that all the waves in the beam were transverse waves in the same orientation. Say, the one in the next figure.

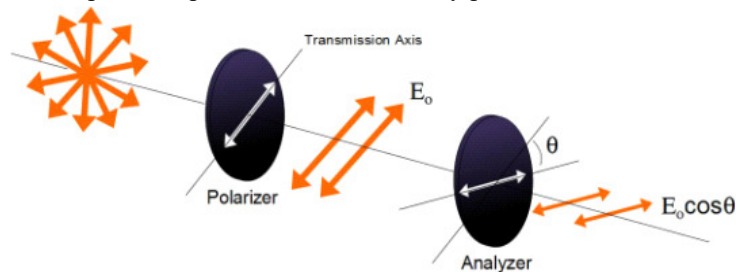


Then we would describe the light as *linearly polarized*. The plane that contains the  $E$ -field is known as the *polarization plane*.

### Polarization by removing all but one wave orientation

One way to make polarized light is to remove all but one orientation of an unpolarized beam. A material that does this at visible wavelengths is called a *polaroid*. It is made of long-chain hydrocarbons that have been treated with iodine to make them conductive. The molecules are all oriented in one direction by stretching during the manufacturing process. The molecules have electrons that can move when light hits them. They can move farther in the long direction of the molecule, so in this direction the molecules act like little antennas. The molecules' electrons are driven into harmonic motion along the length of the molecule. This takes energy (and therefore, light) out of the beam. Little electron motion is possible in the short direction of the molecule, so light is given a preferential orientation. The light is passed if it is perpendicular to the long direction of the molecules. This direction is called the *transmission axis*.

We can take two pieces of polaroid material to study polarization.



Unpolarized light is initially polarized by the first piece of polaroid called the *polarizer*. The second piece of polaroid then receives the light. This piece is called the *analyzer*. If

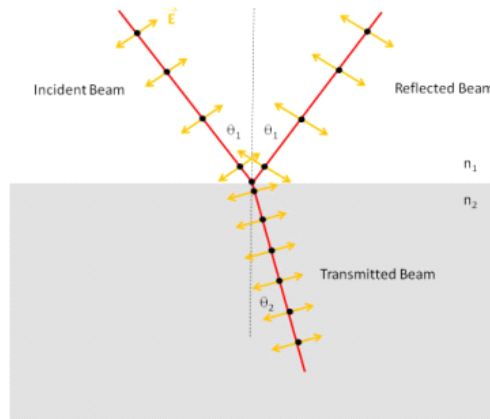
there is an angular difference in the orientation of the transmission axes of the polarizer and analyzer, there will be a reduction of light through the system. We expect that if the transmission axes are separated by  $90^\circ$  no light will be seen. If they are separated by  $0^\circ$ , then there will be a maximum. It is not hard to believe that the intensity will be given by

$$I = I_{\max} \cos^2(\theta) \quad (50.1)$$

remembering that we must have a squared term because  $I \propto E^2$ .

### Polarization by reflection

If we look at light reflected off of a desk or table through a piece of polaroid, we can see that at some angles of orientation, the reflection diminishes or even disappears! Light is often polarized on reflection. Let's consider a beam of light made of just two polarizations. We will define a plane of incidence. This plane is the plane of the paper or computer screen. This plane is perpendicular to the reflective or refractive surface in the figure below.

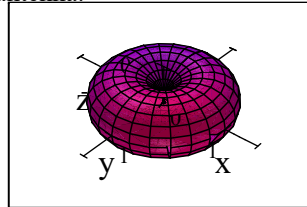


One of our polarizations is defined as parallel to this plane. This direction is represented by orange (lighter grey in black and white) arrows in the figure. The other polarization is perpendicular to the plane of incidence (the plane of the paper). This is represented by the black dots in the figure. These dots are supposed to look like arrows coming out of the paper.

When the light reaches the interface between  $n_1$  and  $n_2$  it drives the electrons in the medium into SHM. The perpendicular polarization finds electrons that are free to move in the perpendicular direction and re-radiate in that direction. Even for a dielectric, the

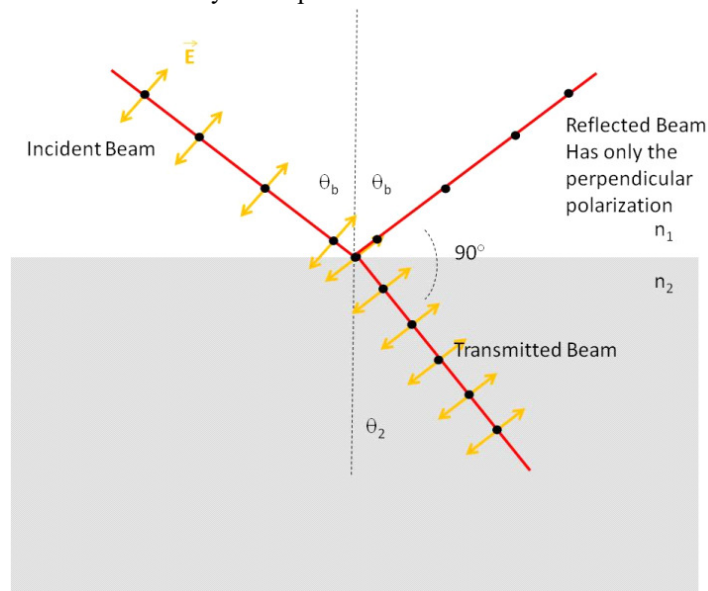
electron orbitals change shape and oscillate with the incoming electromagnetic wave.

The parallel ray is also able to excite SHM, but a electromagnetic analysis tells us that these little “antennas” will not radiate at an angle  $90^\circ$  from their excitation direction. Think of little dipole radiators. We can plot the amplitude of the electric field as a function of direction around the antenna.



Angular dependence of  $S$  for a dipole scatterer.

We see that along the antenna axis, the field amplitude is zero. This means that the wave really does not go that direction. So in our case, the amount of polarization in the parallel direction decreases with the angle between the reflected and refracted rays until at  $90^\circ$  there is no reflected ray in the parallel direction.



The incidence angle that creates an angular difference between the refracted and reflected rays of  $90^\circ$  is called the Brewster's *angle* after its discoverer. At this angle the reflected beam will be completely linearly polarized.

We can predict this angle. Remember Snell's law.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Let's re-label the incidence angle  $\theta_1 = \theta_b$ . We take  $n_1 = 1$  and  $n_2 = n$  so

$$n = \frac{\sin \theta_b}{\sin \theta_2}$$

Now notice that for Brewster's angle, we have

$$\theta_b + 90^\circ + \theta_2 = 180^\circ$$

so

$$\theta_2 = 90^\circ - \theta_b$$

so we have

$$n = \frac{\sin \theta_b}{\sin (90^\circ - \theta_b)}$$

ah, but we remember that  $\sin (90^\circ - \theta) = \cos (\theta)$  so

$$n = \frac{\sin \theta_b}{\cos \theta_b}$$

but again we remember that

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

so

$$n = \tan \theta_b \tag{50.2}$$

which we can solve for  $\theta_b$ .

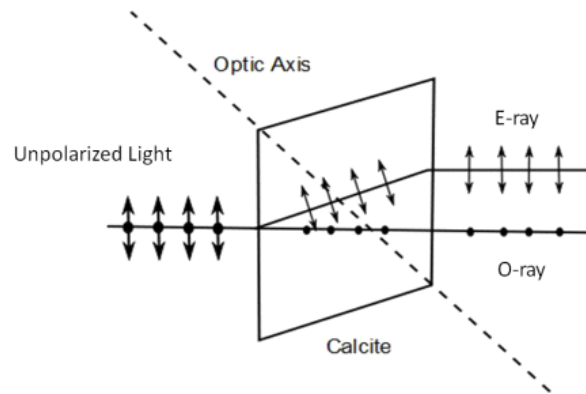
$$\theta_b = \tan^{-1}(n)$$

This phenomena is why we wear polarizing sunglasses to reduce glare.

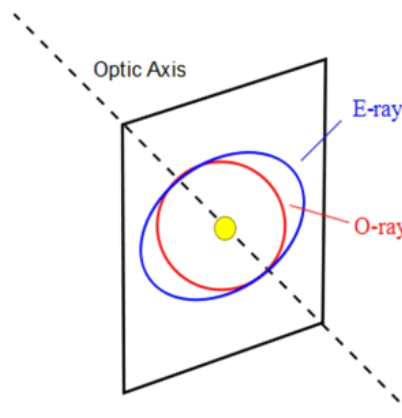
## Birefringence

Glass is an amorphous solid—that is—it has no crystal structure to speak of. But some minerals do have definite order. Sometimes the difference in the crystal structure creates a difference in the speed of propagation of light in the crystal. This is not too hard to believe. We said before that the reason light slows down in a substance is because it encounters atoms which absorb and re-emit the light. If there are more atoms in one direction than another in a crystal, it makes sense that there could be a different speed in each direction.

Calcite crystals exhibit this phenomena. We can describe what happens by defining two polarizations. One parallel to the plane of the figure below, and one perpendicular.



With a careful setup, we can arrange things so the perpendicular ray is propagated just as we would expect for glass. We call this the *O-ray* (for *ordinary*). The second ray is polarized parallel to the incidence plane. It will have a different speed, and therefore a different index of refraction. We call it the *Extraordinary ray* or *E-ray*.



If we were to put a light source in a calcite crystal, we would see the *O-ray* send out a sphere of light as shown in the figure above. But the *E-ray* would send out an ellipse. The speed for the *E-ray* depends on orientation. There is one direction where the speeds are equal. This direction is called the *optic axis* of the crystal.



If our light entering our calcite crystal is unpolarized, then we will have two images leaving the other side that are slightly offset because the *O*-rays and *E*-rays both form images.

### Optical Stress Analysis

Some materials (notably plastics) become birefringent under stress. A plastic or other stress birefringent material is molded in the form planned for a building or other object (usually made to scale). The model is placed under a stress, and the system is placed between two polaroids. When unstressed, no light is seen, but under stress, the model changes the polarization state of the light, and bands of light are seen.



### Polarization due to scattering

It is important to understand that light is also polarized by scattering. It really takes a bit of electromagnetic theory to describe this. So for a moment, let's just comment that blue light is scattered more than red light. In fact, the relative intensity of scattered light goes like  $1/\lambda^4$ . This has nothing to do with polarization, but it is nice to know.

Now suppose we have long pieces of wire in the air, say, a few microns long. The pieces of wire would have electrons that could be driven into SHM when light hits them. If the wires were all oriented in a common direction, we would expect light to be absorbed if it was polarized in the long direction of the particles and not absorbed in a direction perpendicular to the orientation of the particles. This is exactly what happens when long ice particles in the atmosphere orient in the wind (think of the moment of inertia).



We often get impressive halo's around the sun due to scattering from ice particles.

Rain drops also have a preferential scattering direction because they are shaped like oblate spheroids (not “rain drop shape” like we were told in grade school).

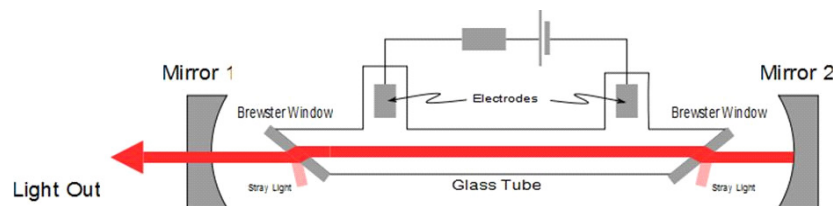
It is also true that small molecules will act like tiny antennas and will scatter light preferentially in some directions and not in others. This is called *Rayleigh scattering* and is very like small dipole antennas.

## Optical Activity

Some substances will rotate the polarization of a beam of light. This is called being *optically active*. The polarization state of the light exiting the material depends on the length of the path through the material. Your calculator display works this way. An electric field changes the optical activity of the liquid crystal. There are polarizers over the liquid crystal, so sometimes light passes through the display and sometimes it is black.

## Laser polarization

One last comment. Lasers are usually polarized. This is because the laser light is generated in a *cavity* created by two mirrors. The mirror is tipped so light approaches it at the Brewster angle. Light with the right polarization (parallel to the plane of the drawing) is reflected back nearly completely, but light with the opposite polarization is not reflected at all. This reduces the usual loss in reflection from a mirror, because in one polarization the light must be reflected completely.



## Retrospective

We have thought about many things in this class. It has been a class *about* science. It has not been a class where we have tried to discover new science, or practiced the scientific method. This is on purpose, this being an engineering class designed to teach the principles of physics for use in designing machines.

But we should pause to think, just for a moment, about the philosophy of science. Is everything in these lectures true? We did not perform experiments to show every principle we learned. So does it all work?

The answer is—maybe. Experiments have been done to show that the equations we have learned work at least sometimes. But science is an inductive process. We can't prove anything true with science. We can only prove things false. So what we have studied is what has not been proven false, yet. Of course, even then, we have taken approximations from time to time, but we pointed these out along the way. You will know when the approximations will fail, because we talked about their valid ranges.

It is important to remember that we are not done discovering new things, and proving old things false. The laws of Newton are approximations that work at low speeds. Relativity provides mechanical equations for very high speeds (e.g. the satellite motion involved in the GPS system). But is Relativity correct? We think it works pretty well, but really we don't know. We may never know for sure. But we know it works within the range of things we have tried.

There are physicists today that are working on a fundamentally new model of the universe. It is called "String Theory" and it would replace most of our thoughts about how matter is made and how it interacts. The equations would reduce to the ones we used in class for the conditions we considered. That is because the new equations have to match the results of the experiments that we have already done or they can't be correct. But the explanations might be very different.

Often, it is in using physics to build something that we learn about the limitations of physical theory. You may be part of that process. It is a happy process because extending our understanding allows us to build new things. But don't be surprised if some of the things we learned in this class are different by the time your children take their engineering physics course. That is what we should expect of an inductive process.

It is also important to note that revealed truth is not an inductive process. It is still not

static (see article of faith 9), but it *can* prove something true as well as prove things false. I hope your FDSCI 101 experience gave you some insight into doing science as well as learning about science.

Some members view science and revelation as in opposition. But I think they are complementary. The scientific process allows us to eliminate things that are not true, allowing us to follow D&C 9:8 in preparation for seeking revelation. During a recent convocation speech, Elder Scott described using this process as a nuclear engineer during his engineering career . We can use this combination in our personal lives as well. I hope you will consider this in your careers and lives.

I have tried to give at least equal time to conceptual understanding and mathematical solving. I hope you review and refresh the conceptual understanding of the physics of what you build. Most of my industrial career, we built what we designed very well. We always did our calculations well. But we did, at times, build the wrong thing because the conceptual basis of the design was wrong. Such mistakes are difficult to fix. Conceptual understanding is a guiding principle for a successful design career. I hope this class has contributed to that conceptual understanding.



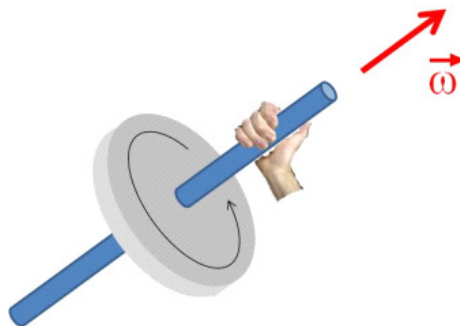
# 51 Summary of Right Hand Rules

## PH121 or Dynamics Right Hand Rules

We had two right hand rules on PH121. We didn't give them numbers back then, so we will do that now.

### Right hand rule #0:

We found that angular velocity had a direction that was given by imagining you grab the axis of rotation with your right hand so that your fingers seem to curl the same way the object is rotating. Then your thumb gives the direction of  $\vec{\omega}$ .

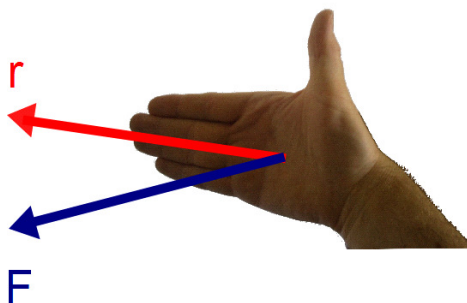


You curl the fingers of your right hand (sorry left handed people, you have to use your right hand for this) in the direction of rotation. Then your thumb points in the direction of the vector.

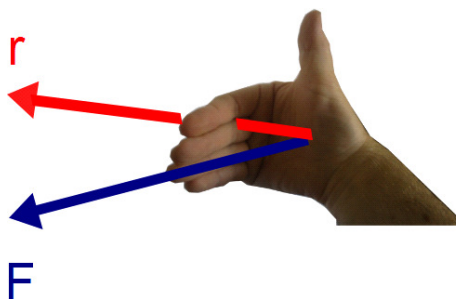
### Right hand rule #0.5:

To find the direction of torque, we used the following procedure

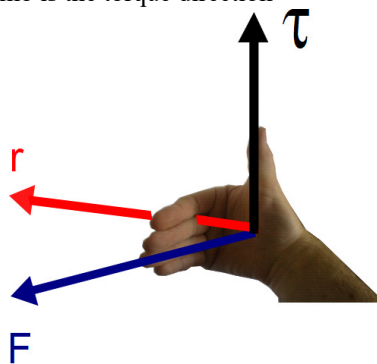
1. Put your fingers of your right hand in the direction of  $\tilde{\mathbf{r}}$



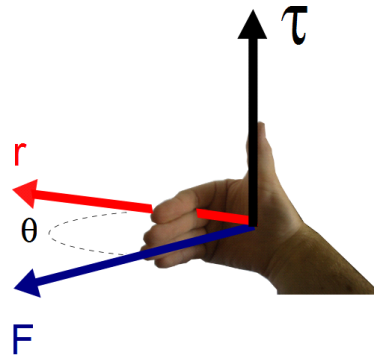
2. Curl them toward  $\tilde{\mathbf{F}}$



3. The direction of your thumb is the torque direction



4. The angle  $\theta$  is the angle *between*  $\tilde{\mathbf{r}}$  and  $\tilde{\mathbf{F}}$



The magnitude of the torque is

$$\tau = rF \sin \theta$$

## PH223 Right Hand Rules

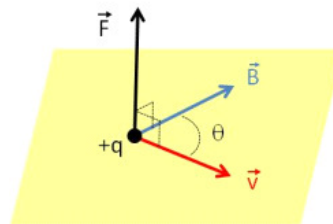
We have four more right hand rules this semester having to do with charges and fields.

### Right hand rule #1:

From this rule we get the **direction of the force on a moving charged particle** as it travels through a **magnetic field**.

This rule is very like torque. We start with our hand pointing in the direction of  $\vec{v}$ . Curl your fingers in the direction of  $\vec{B}$ . And your thumb will point in the direction of the force. The magnitude of the force is given by

$$F = qvB \sin \theta \quad (51.1)$$



**Right hand rule #2:**

From this rule we get the direction of the force on current carrying wire that is in a magnetic field.

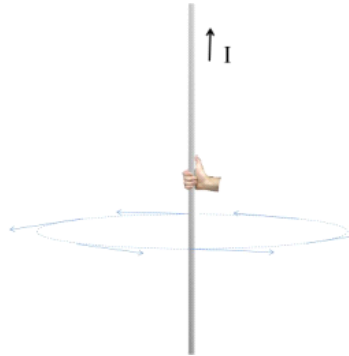
This rule is very like right hand rule #1 above. We start with our hand pointing in the direction of  $\mathbf{I}$ . Curl your fingers in the direction of  $\tilde{\mathbf{B}}$ . And your thumb will point in the direction of the force. The magnitude of the force is given by

$$F = ILB \sin \theta \quad (51.2)$$

**Right hand rule #3:**

From this rule we get the **direction of the magnetic field that surrounds a long current carrying wire**.

This rule is quite different. It is reminiscent of the rule for angular velocity, but there are some major differences as well. The field is a magnitude and a direction at every point in space. We can envision drawing surfaces of constant field strength. They will form concentric circles (really cylinders) centered on the wire. At any one point on the circle the field direction will be along a tangent to the circle. The direction of the vector is given by imaging you grab the wire with your right hand (don't really do it). Grab such that your right thumb is in the direction of the current. Your fingers will naturally curl in the direction of the field.

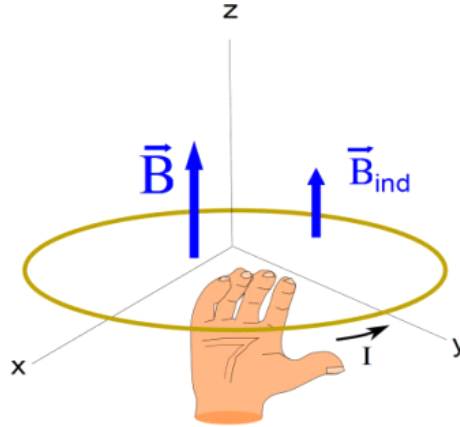
**Right Hand Rule #4:**

From this rule we get the **direction of the induced current when a loop is in a**



**changing magnetic field.**

This rule is only used when we have a loop with a changing external magnetic field. The rule gives the direction of the induced current. The induced magnetic field will oppose the change in the external field, trying to prevent a change in the flux. The current direction is found by imagining we stick our right hand into the loop in the direction of the induced field. Keeping our hand inside the loop we grab a side of the loop. The current goes in the direction indicated by our thumb.



In the figure above, the external field is upward but decreasing. So the induced field is upward. The current flows because there is an induced *emf* given by

$$\begin{aligned}\mathcal{E} &= -N \frac{\Delta\Phi}{\Delta t} \\ &= -N \frac{(B_2 A_2 \cos \theta_2 - B_1 A_1 \cos \theta_1)}{\Delta t}\end{aligned}$$



## 52 Some Helpful Integrals

$$\int \frac{r dr}{\sqrt{r^2 + x^2}} = \sqrt{r^2 + x^2}$$

$$\int \frac{dx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}}$$

$$\int \frac{xdx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx}{x^2} = -\frac{1}{x}$$

$$\int_0^{2\pi} \int_0^\pi \int_0^R \sin \theta d\theta d\phi = 4\pi$$

$$\int_0^{2\pi} \int_0^\pi \int_0^R r^2 dr \sin \theta d\theta d\phi = \frac{4}{3} \pi R^3$$

$$\int_0^{2\pi} \int_0^R r dr d\phi = \pi R^2$$



# 53 Table of Physical Constants

## Charge and mass of elementary particles

Proton Mass	$m_p = 1.6726231 \times 10^{-27} \text{ kg}$
Neutron Mass	$m_n = 1.6749286 \times 10^{-27} \text{ kg}$
Electron Mass	$m_e = 9.1093897 \times 10^{-31} \text{ kg}$
Electron Charge	$q_e = -1.60217733 \times 10^{-19} \text{ C}$
Proton Charge	$q_p = 1.60217733 \times 10^{-19} \text{ C}$

$\alpha$ -particle mass <sup>41</sup>	$m_\alpha = 6.64465675(29) \times 10^{-27} \text{ kg}$
$\alpha$ -particle charge	$q_\alpha = 2q_e$

## Fundamental constants

Permittivity of free space	$\epsilon_o = 8.854187817 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$
Permeability of free space	$\mu_o = 4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}$
Coulomb Constant	$K = \frac{1}{4\pi\epsilon_o} = 8.98755 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$
Gravitational Constant	$G = 6.67259 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Speed of light	$c = 2.99792458 \times 10^8 \text{ m s}^{-1}$
Avogadro's Number	$6.0221367 \times 10^{23} \text{ mol}^{-1}$
Fundamental unit of charge	$q_f = 1.60217733 \times 10^{-19} \text{ C}$

## Astronomical numbers

Mass of the Earth <sup>42</sup>	$5.9726 \times 10^{24} \text{ kg}$
Mass of the Moon <sup>43</sup>	$0.07342 \times 10^{24} \text{ kg}$
Earth-Moon distance (mean) <sup>44</sup>	384400 km
Mass of the Sun <sup>45</sup>	$1,988,500 \times 10^{24} \text{ kg}$
Earth-Sun distance <sup>46</sup>	$149.6 \times 10^6 \text{ km}$

## Conductivity and resistivity of various metals

<sup>41</sup> <http://physics.nist.gov/cgi-bin/cuu/Value?mal>

<sup>42</sup> <http://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html>

<sup>43</sup> <http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html>

<sup>44</sup> <http://solarsystem.nasa.gov/planets/profile.cfm?Display=Facts&Object=Moon>

<sup>45</sup> <http://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html>

<sup>46</sup> <http://nssdc.gsfc.nasa.gov/planetary/factsheet/index.html>

Material	Conductivity ( $\Omega^{-1} \text{ m}^{-1}$ )	Resistivity ( $\Omega \text{ m}$ )	Temp. Coeff. ( $\text{K}^{-1}$ )
Aluminum	$3.5 \times 10^7$	$2.8 \times 10^{-8}$	$3.9 \times 10^{-3}$
Copper	$6.0 \times 10^7$	$1.7 \times 10^{-8}$	$3.9 \times 10^{-3}$
Gold	$4.1 \times 10^7$	$2.4 \times 10^{-8}$	$3.4 \times 10^{-3}$
Iron	$1.0 \times 10^7$	$9.7 \times 10^{-8}$	$5.0 \times 10^{-3}$
Silver	$6.2 \times 10^7$	$1.6 \times 10^{-8}$	$3.8 \times 10^{-3}$
Tungsten	$1.8 \times 10^7$	$5.6 \times 10^{-8}$	$4.5 \times 10^{-3}$
Nichrome	$6.7 \times 10^5$	$1.5 \times 10^{-6}$	$0.4 \times 10^{-3}$
Carbon	$2.9 \times 10^4$	$3.5 \times 10^{-5}$	$-0.5 \times 10^{-3}$