Chapter 11

Electric Potentials

Fundamental Concepts

We defined electrical potential energy last time. We used an analogy with gravitational fields and gravitational potential energy. But there is a missing piece. The gravitational environment property

$$g = \left(G\frac{M_E}{R_E^2}\right)$$

(where here the subscript E is for the environmental object and m is for our mover object) showed up in our equation for the gravitational potential energy

$$U = -\left(G\frac{M_E}{R_E}\right)m_m$$

We found the same form for the electrical potential energy.

$$U_{12} = \frac{1}{4\pi\epsilon_o}\frac{Q_E q_m}{r_{Eq}}$$

(where the subscript E is still for the environmental object and m is still our mover object, and r_{Em} is the distance between the mover and environmental object), We could write this as

$$U_{12} = \left(\frac{1}{4\pi\epsilon_o}\frac{Q_E}{r_{Eq}}\right)q_m$$

where charge q_2 would be our mover charge. By analogy, then

$$\frac{1}{4\pi\epsilon_o}\frac{Q_E}{r_{Eq}}$$

must represent the environment set up by Q_E And sure enough, it has a Q_E in it. But this does not have the units of electric field. So it must be a new quantity. We will need a name for this new representation of the environment created by Q_E.

Fundamental Concepts

- Electric potential is a scaler representation of the electric field environment.
- Electric potential is defined as the potential energy per unit charge.
- Equipotential lines are drawn to show constant electric potential surfaces
- The volt as a unit of electric potential
- The electron-volt as a measure of energy (and speed).

11.0.1 Electric Potential Difference

Let's give a symbol and a name to our new environment quantity.

$$V_{12} = \frac{1}{4\pi\epsilon_o}\frac{Q_E}{r_{Eq}}$$

where we understand that Q_E is making the environment and we are measuring that environment a distance r_{Eq} from Q_E. Thus, Q_E is the environmental charge.

Then

$$\begin{aligned} U_{12} &= \left(\frac{1}{4\pi\epsilon_o}\frac{Q_E}{r_{Eq}}\right) q_m \\ &= \left(V_{Em}\right) q_{m2} \end{aligned}$$

It's traditional to drop the subscripts on the V

$$V = \frac{1}{4\pi\epsilon_o}\frac{Q}{r}$$

where we understand that an environmental charge labeled just Q is making the environment and q_m is a distance r from Q. In that case we can write

$$U_{Em} = (V)\, q_m$$

or

$$V = \frac{U_{Em}}{q_m}$$

This new environment representation appears to be an amount of potential energy per unit charge. In general, any electrical potential energy (U) per unit charge (q) is is called an *electric potential*.

$$V = \frac{U}{q}$$

This is a somewhat unfortunate name, because it sounds like electric potential energy. But it is not, it is a representation of the environment set up by the electric field. We don't get electric potential energy without multiplying by a charge. $U = Vq_m$.

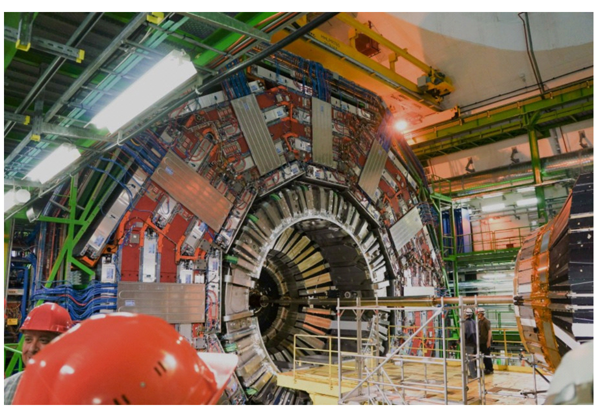
We will give electric potential the symbol V but usually the important quantity is a change in potential energy, then

$$\Delta V = \frac{\Delta U}{q_m} \tag{11.1}$$

If I know ΔV for a configuration of charge (like our capacitor plates) then I can find the ΔU of different charges by multiplying by the amount of charge in each case

$$\begin{aligned} \Delta U_1 &= q_1 \Delta V \\ \Delta U_2 &= q_2 \Delta V \\ &\vdots \end{aligned}$$

which is convenient if I am accelerating many different charges. We do this in linear accelerators or at the Large Hadron Collider at CERN so this is important to physicists!



CMS detector under repair. This is part of the Large Hadron Collider at CERN.

We can see that the units of ΔV must be

$$\frac{\mathrm{J}}{\mathrm{C}} = \mathrm{V} \tag{11.2}$$

which has been named the *Volt* and is given the symbol, V.

Now this may seem familiar. Can you think of anything that carries units of volts? Let's consider a battery. In our cell phones we have something like a 3.8 V lithium-ion battery. Inside the battery we would expect that a charge would experience a potential energy difference. We use the battery so we can convert that potential energy into some other form of energy (e.g. radio wave energy for our phone's wifi). The potential energy achieved depends on the charge carrier. We would have electrons in metals but we would have ions in a solution. This is so convenient to express the potential energy per unit charge, that it is the common form or expressing the energy given by most electrical sources.

11.0.2 Using Electric Potential

Let's write out the electric potential difference between points A and B. It is the change in potential energy per unit charge as the charge travels from point A to point B

$$\Delta V = V_B - V_A = \frac{\Delta U}{q_m} \tag{11.3}$$

This is clearly a measure of how the environment changes along our path from A to B.

Let's reconsider gravitational potential energy. We remember that if the field is uniform (that is, if we are near the Earth's surface so the gravitational field seems uniform) we can set the zero point of the potential energy anywhere we find convenient for our problem, with the provision that once it is set for the problem, we have to stick with our choice.

One logical choice for many electrical appliances is to set the Earth's potential equal to zero. Note! this is not true for point mass problems where we have already set the potential energy $U = 0$ at $r = \infty$. But if our moving mass is not too bit, and we can approximate the Earth as infinitely big so we have a uniform gravitational field, setting the Earth's potential to zero works.

In our gravitational analogy, this is a little bit like mean sea level. Think of river flow. The lowest point on the planet is not mean sea level. But any water above mean sea level will tend to flow downward to this point. Of course, if we have land below mean sea level, the water would tend to continue downward (like water flows to the Dead Sea). The direction of water flow is given by the potential energy difference, not that actual value of the potential energy. It is the same way with electric potential. If we have charge at a potential that is higher than the Earth's potential, then charge will flow toward the Earth.

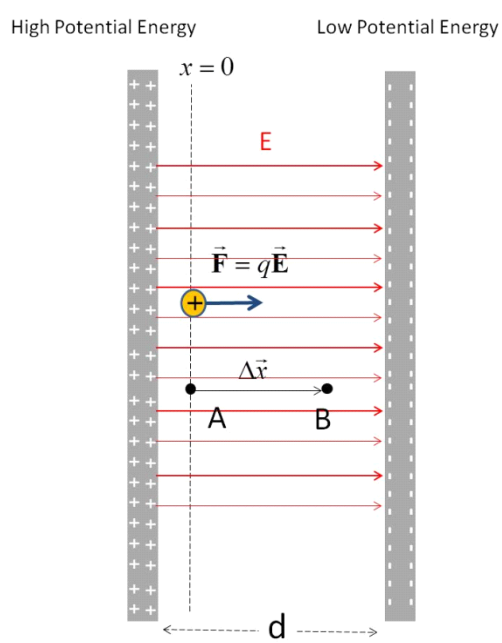
Consider a 9 V battery. If the negative terminal is connected to a grounding rod or metal water pipe, it will be at the electric potential of the Earth while it's positive terminal will be at $\Delta V = 9\,\mathrm{V}$ above the Earth's potential. Likewise, in your home, you probably have a 110 V outlet. One wire is likely set to the potential of the Earth by connecting it to a ground rod. The others are at

$\Delta V = 110\,\mathrm{V}$ above it[1].

In our phones, we don't have a ground wire, so we cannot guarantee that the negative terminal of the battery is at the same potential as the Earth. If our appliances in our house are not all grounded to the same potential, there is a danger that there will be a large enough difference in their potentials (think potential energy per unit charge) to cause the charges to accelerate from one appliance to another. It is the difference in potential that counts! This can produce a spark or shock that could hurt someone or damage equipment. That is why we now use grounded outlets. These outlets have a third wire that is tied to all the other outlet's third wire and also tied physically to the ground near your house or apartment. This way, all appliances are ensured to have the same low electric potential point.

11.1 Example, potential of a capacitor

Let's calculate the potential difference of our favorite device, the capacitor.



The nice uniform field makes this a useful device for thinking about electric potentials. We have found that field to be

$$E = \frac{\eta}{\epsilon_o}$$

[1] House voltages are alternating voltages. We will deal with them later in this course.

with a direction from positive to negative. The work to push a mover charge from one side to the other is given by

$$w = \int F_e \cdot dx$$

The force is uniform since the field is uniform (near the middle at least)

$$F_e = q_m E$$

then our work becomes

$$\begin{aligned} w &= \int q_m E \cdot dx \\ &= \int q_m E \cos(\theta_{Ex})\, dx \\ &= -q_m E \Delta x \end{aligned}$$

because the field and the displacement are in opposite directions. And recall that the amount of potential energy is minus this amount because $w = -\Delta U$ as we learned in PH121.

$$\Delta U = +q_m E \Delta x$$

We can set the zero potential energy point any where we want, but it is tradition to set $U = 0$ at the negative plate. If we do this we end up with the potential energy difference going from the negative plate to the positive plate being

$$\Delta U = q_m E d$$

Then if we go from the negative plate to the positive plate we have a positive ΔU.

We have seen all this before when we compared the electric potential energy of a uniform gravitation field and a uniform electrical field. Now let's calculate the electric potential difference

$$\Delta V = \frac{\Delta U}{q_m} = \frac{q_m E d}{q_m} = Ed$$

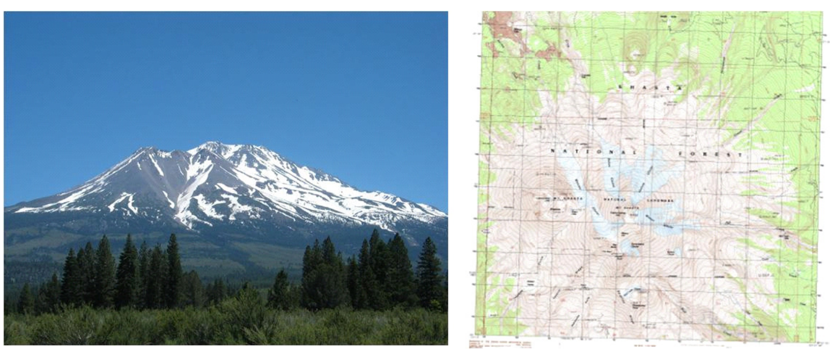
Remember that the field is created by the charges on the capacitor plates, so it exists whether we put any q_m inside of the capacitor or not. Then the potential difference must exist whether or not there is a charge q_m inside the capacitor.

You probably already know that a voltmeter can measure the electric potential difference between two points, say, the plates of a capacitor. If we use such a meter we could find the field inside the capacitor (well, almost, remember our approximation is good for the center of the plates).

$$E = \frac{\Delta V}{d}$$

11.1.1 Equipotential Lines

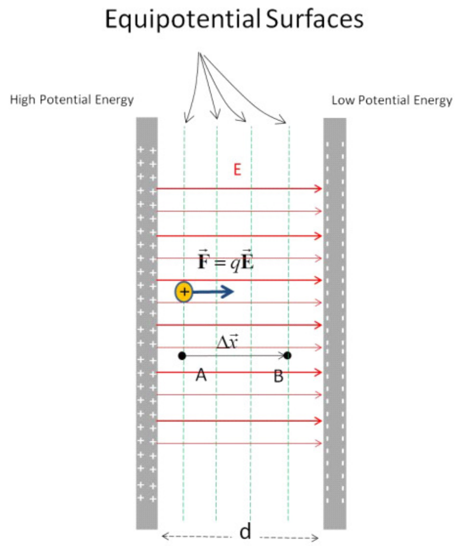
We need a way to envision this new environmental quantity that, like a field, has a value throughout all space. Our analogy with gravity gives us an idea. Suppose we envision the height potential energy as the top of a hill. Then the low potential energy would be the bottom of the hill. We know from our Young Men and Young Women's Camp experiences how to show a change in gravitational potential energy. We plot on a map lines of constant potential energy. We call it constant elevation, but since near the Earth's surface $U_g = mgh$ the potential energy is proportional to the height, so we can say these lines are lines of constant potential energy. Here is an example for Mt. Shasta.



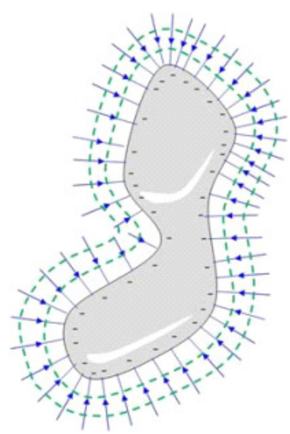
Map courtesy USGS, Picture is in the Public Domain.

We can think of these lines of constant potential energy as paths over which the gravitational field does no work. If we walked along one of these lines we would get neither higher nor lower and though we might do work to move us to overcome some friction, the gravitational field would do no work. And we would do no work in changing elevation.

Likewise we can draw lines of equal potential for our capacitor. When moving along these lines the electric field would do no work.



Of course we could draw these lines for a crazier device. Say, for our charged conductor



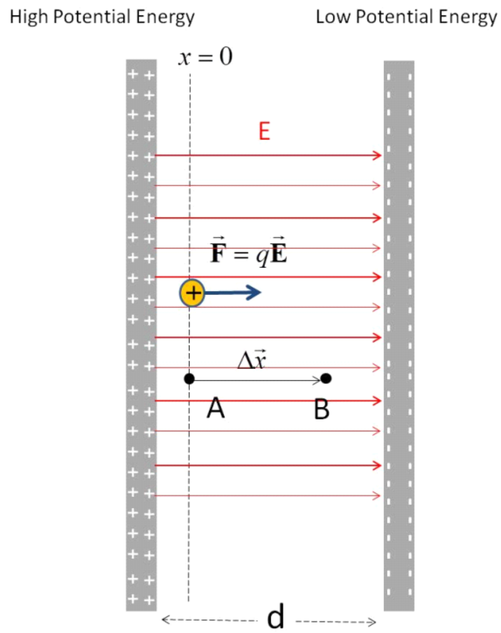
Notice that our equal potential lines are always perpendicular to the field. From

$$w = \int q_o \overrightarrow{\mathbf{E}} \cdot d\overrightarrow{\mathbf{x}}$$

we can see that if the path we travel is perpendicular to the field, no work is done. This is like us marching along around the mountain neither going up nor down.

11.2 Electron Volt

Suppose I set up our uniform electric field device again



We are not including any gravitational field, so the directions involved are all relative to the placement of the capacitor plate orientation.

This time, suppose I make the potential difference $\Delta V = 1\,\text{V}$. I release a proton near the high potential side. What is the kinetic energy of the proton as it hits the low potential side? From the work energy theorem

$$W_{nc} = \Delta K + \Delta U$$

and if we do this in a vacuum so there is no non-conservative work,

$$\begin{aligned} \Delta K &= -\Delta U \\ K_f - K_i &= -\Delta U \\ K_f &= -\Delta U \end{aligned}$$

We can find the potential energy loss from what we just studied

$$\Delta V = \frac{\Delta U}{q_m}$$

so we can find the potential energy as

$$\Delta U = q_m \Delta V$$

but remember we are going from a high to a low potential

$$\Delta V = V_f - V_i$$

this will be negative, so the potential energy change will be negative too.

$$\begin{aligned} K_f &= -\Delta U \\ &= -q_m \Delta V \end{aligned}$$

which will be a positive value (which is good, because I don't know what negative kinetic energy would mean).

$$K_f = -q_m \Delta V$$

We can find the amount of energy in Jules

$$\begin{aligned} K_f &= -\left(1.6 \times 10^{-19}\,\mathrm{C}\right)(-1\,\mathrm{V}) \\ &= 1.6 \times 10^{-19}\,\mathrm{J} \end{aligned}$$

since we defined a volt as $\mathrm{V} = \frac{\mathrm{J}}{\mathrm{C}}$.

You might think this is not very useful, but remember that $K = \frac{1}{2}mv^2$. The kinetic energy is related to how fast the proton is going. In a way, the kinetic energy tells us how fast the particle is going (we know it's mass). If you read about the Large Hadron Collider at CERN, in Switzerland the "speeds"of the particles will be given in energy units that are multiples of 1.6×10^{-19} J. We call this unit an electron-volt (eV).

Beam magnet and Section of the Beam Pipe of the LHC. This section is actually no longer used and is in a service area 100 m above the operating LHC. The people you see are part of a BYU-I Physics Department Tour of the facility.

We can finish this problem by finding the speed of the particle

$$K = \frac{1}{2}mv^2$$

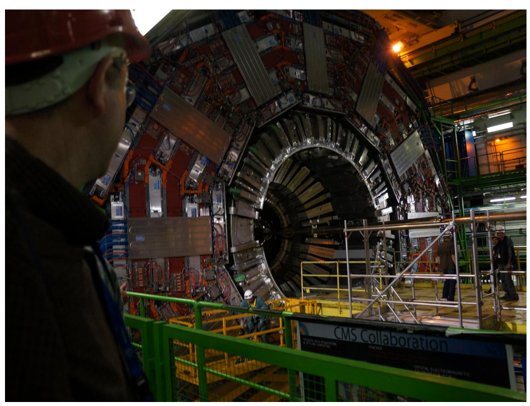
so

$$\frac{2K}{m} = v^2$$

or

$$\begin{aligned} v &= \sqrt{\frac{2K}{m}} \\ &= \sqrt{\frac{2\,(1.6 \times 10^{-19}\,\mathrm{J})}{1.00728\,\mathrm{u}\frac{1.6605\times 10^{-27}\,\mathrm{kg}}{1\,\mathrm{u}}}} \\ &= 13832.\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

Which is pretty fast, but the Large Hadron Collider at CERN can provide energies up to 7×10^{14} eV which would give our proton a speed of 99.9999991% of the speed of light.



CERN CMS detector during a maintenance event. The bright metal pipe seen in the middle of the detector is the beam pipe through which the accelerated protons travel. Note the workers near the scaffolding for scale.

Note that this energy would seem to provide a faster speed – faster than light!

But with energies this high we have to use Einstein's theory of Special Relativity to calculate the particle speed. And, sadly, that is not part of this class. If you are planning to work on the GPS system, or future space craft, you might need to take yet another physics class so you can do this sort of calculation.

You might guess that we will want to know the electric potential of more complex configurations of charge. We will take on this job in the next lecture.

Basic Equations

The electric potential is the electrical potential per unit charge

$$\Delta V = V_B - V_A = \frac{\Delta U}{q}$$

$$V_{12} = \frac{1}{4\pi\epsilon_o}\frac{q_1}{r_{12}}$$

For the special case of a constant electric field in a capacitor the electrical potential is just

$$\Delta V = E\Delta s$$

where Δs is the distance traveled from one side of the capacitor to the other.

The unit

$$1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$$