Chapter 38

Magnetic Field

Fundamental Concepts

We have now experience with two non-contact forces, the gravitational force and the electric or Coulomb force. In both cases, we have found that there is a field involved with the production of this force. We can guess that this is true for the magnetic force as well.

The discovery of this field involved an accidental experiment, and understanding this experiment gives us great insight into the nature of this field and where it comes from. So we will spend a little time describing it.

38.1 Fundamental Concepts in the Lecture

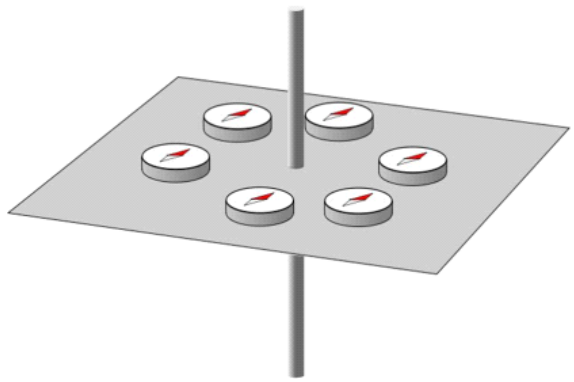
- A long wire that carries a current produces a magnetic field
- The magnetic field due to a long wire with current becomes weaker with distance and forms concentric cylinders of constant magnetic field strength
- The direction of the long-wire-with-current field is given by a right-hand-rule.
- The field due to a moving charge is given by the *Biot-Savart law*

$$B = \frac{\mu_o}{4\pi}\frac{qv\sin\theta}{r^2}$$

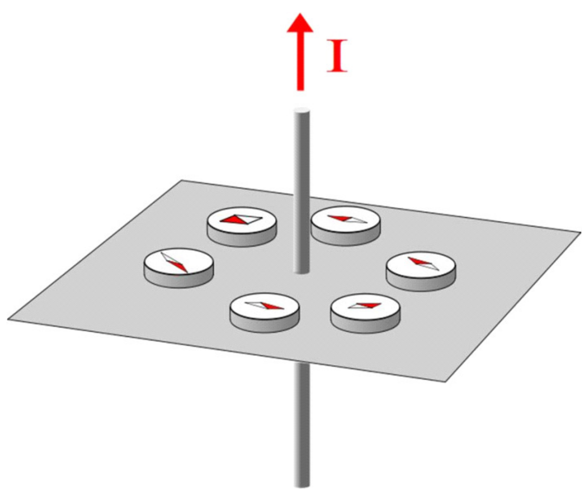
38.2 Discovery of Magnetic Field

In 1819 a Dutch scientist named Oersted was lecturing on electricity. He was actually making the point that there was no connection between electricity and magnetism. He had a large battery connected to a wire. A large current flowed through the wire. By chance, Oersted placed a compass near the wire. He had done this before, but this time the wire was in a different orientation than in previous demonstrations. To his great surprise, the compass needle changed direction when it was placed near the wire!

A similar experiment, but this time with several compasses, is shown in the next figure.



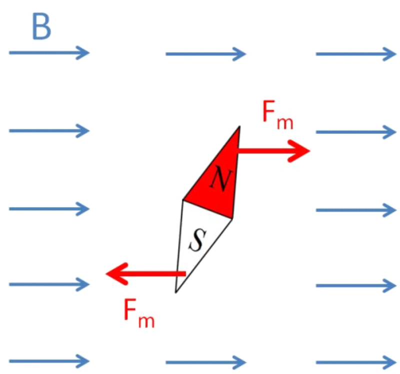
When the current is turned on, the compasses change direction.



This is a very good clue that there really *is* a connection between electricity and magnetism.

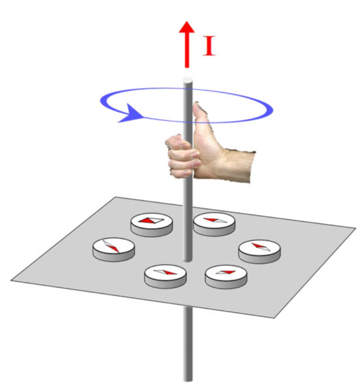
Oersted's Experiment Demo: Use the 106 boards and compasses

We know that a compass orients itself in the Earth's magnetic field. We can infer that the compass needle will orient in any magnetic field. In the next figure you can see that there is a force on each end of the needle due to the magnetic field.

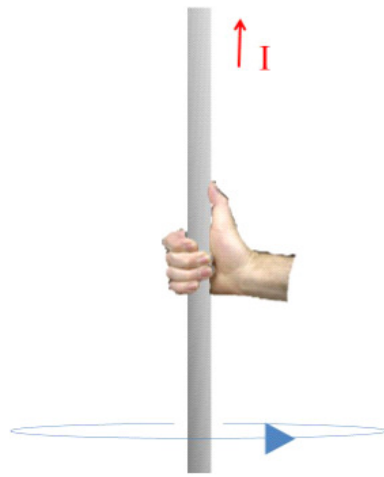


Notice that we have marked the environmental magnetic field with the letter B. This is traditional. Magnetic fields are often called B-fields for this reason. But more importantly, this looks very like an electric dipole in a constant electric field. We know enough about the dipole situation to predict that there will be a torque, and that there will be a stable equilibrium when the compass needle is aligned with the magnetic field.

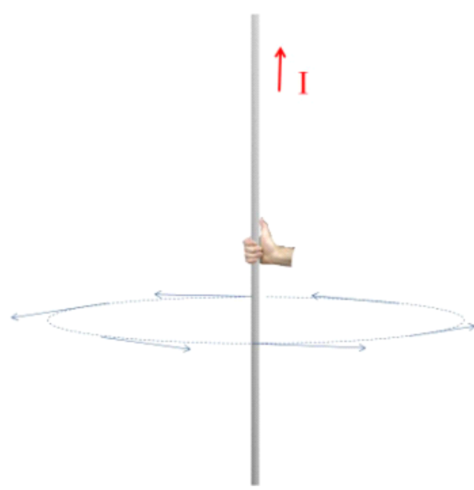
Since our compasses oriented themselves near the current carrying wire, there must be a magnetic field caused by the current in the wire. The field shown in the last figure is uniform, but the field of our wire cannot be uniform. The compasses pointed different directions. A common way to describe this field is with a right-hand-rule. We imagine grabbing the wire with our right hand with our thumb pointing in the current direction. The field direction is given by our fingers.



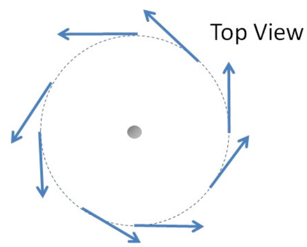
Although this is true, it takes some interpretation Let's take some time to see what it means. Let's redraw the figure.



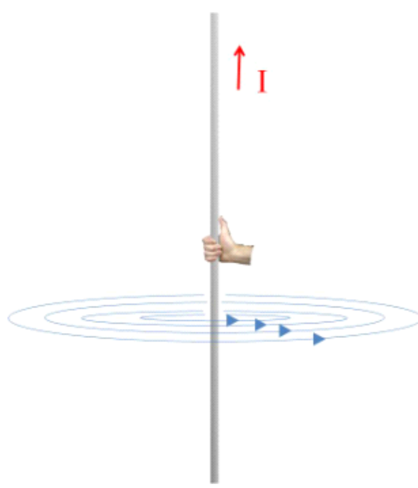
Now that we have a new figure, let's reconsider what our right hand rule means. What we mean is that the magnetic field is constant in magnitude around a circle, and that the direction of the field is tangent to the circle, with the arrow pointing in the direction your fingers go with the right-hand-rule.



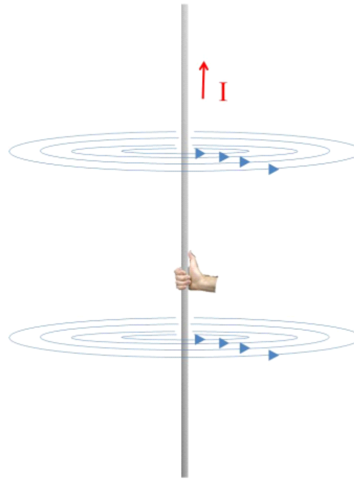
This is easier to see in a top-down view.



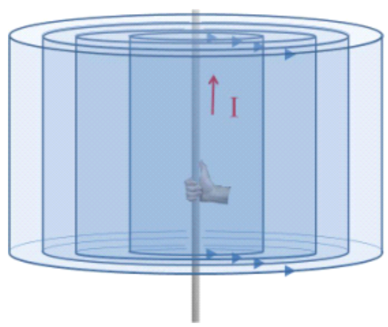
But in the first figure we only drew the field around one circle. By using symmetry, we can guess that the field magnitude must be constant around any circle. It must depend only on r,if the current is constant. So we could draw constant field lines at any distance, r, away from the wire.



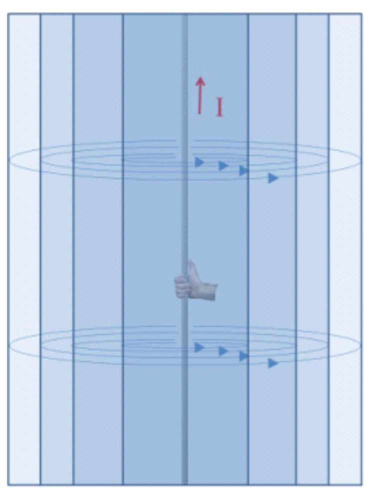
But again, this figure is not so good, because the entire wire makes a field that has a constant value for B at a distance r away. So we could also draw the field above our hand.



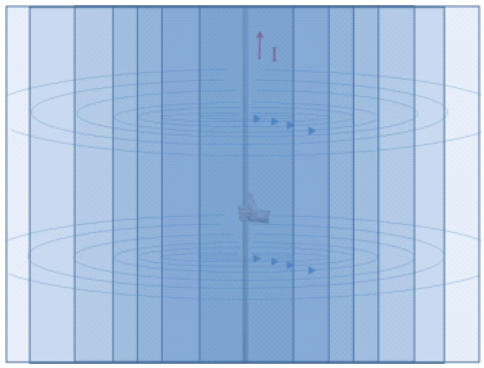
Maybe a better way to draw this field would be a set of concentric cylinders. Along the surface of the cylinder (but not the end caps) the field will be constant.



Of course, if our wire is infinitely long, the cylinders will be infinitely long too...



And the field does not stop after a few cylinders, it reaches $B = 0$ only when $r = \infty$. So the field fills all of space.



This is a more accurate way to draw the magnetic field due to a long straight wire, but it takes a long time to draw such a diagram, so usually we will just draw one circle, and you will have to mentally fill in the other circles and the concentric cylinders that they represent.

To use the right hand rule, remember to place your thumb in the current direction. Then the field direction is given tangent to the circle and pointing in our finger direction.

38.2.1 Making the field–moving charges

But how does a current in a wire make a magnetic field?

The secret is to look at the individual charges that are moving. When early scientists caused individual charges to move, they found they created magnetic fields. The experimental results gave a relationship for the strength of this field

$$B = \frac{\mu_o}{4\pi}\frac{qv\sin\theta}{r^2}$$

and the direction is given by the right hand rule by pointing the thumb in the direction the charges are going and using the figures to indicate the field direction as we have described above. In a sense, this is a very small current (one moving charge!). So the field should look very similar.

This relationship was found by two scientists, Biot and Savart, and it carries their name, the *Biot-Savart law.* The factor μ_o is a constant very like ϵ_o. It has a value

$$\mu_o = 4\pi \times 10^{-7}\frac{\mathrm{T\,m}}{\mathrm{A}}$$

and is called the *permeability of free space,* The unit T is called a *tesla* and is

$$\mathrm{T} = \frac{\mathrm{N}}{\mathrm{A\,m}}$$

The charges already had an electric field before they were accelerated, but now they have two fields, an electric and a magnetic field. We used unit vectors to write our E-field.

$$\overrightarrow{\mathbf{E}} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{\mathbf{r}}$$

It is convenient to do the same for the magnetic case. We can remember that a vector cross product is given by

$$\overrightarrow{\mathbf{a}} \times \overrightarrow{\mathbf{b}} = ab\sin\theta \qquad \perp \overrightarrow{\mathbf{a}}, \perp \overrightarrow{\mathbf{b}}$$

where the resulting vector is perpendicular to both $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$. Thinking about this for a while allows us to realize this is just what we want for the magnetic field. If the velocity of the charges is up (say, in the $\hat{z}$ direction) then we can use our right hand rule to realize we need a vector perpendicular to both $\hat{z}$ and $\hat{r}$. This is given by

$$\hat{\mathbf{z}} \times \hat{\mathbf{r}}$$

which is always tangent to the circle indicated by our fingers. Since v is in the z direction we can use

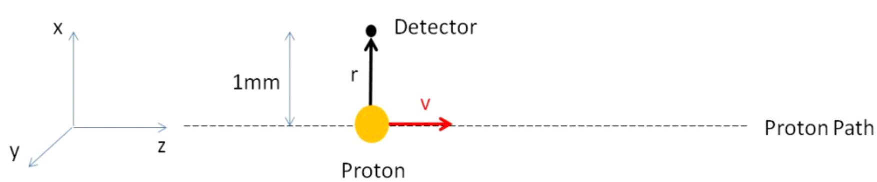
$$\overrightarrow{\mathbf{v}} \times \hat{\mathbf{r}} = v\sin\theta \qquad \perp \overrightarrow{\mathbf{v}}, \perp \hat{\mathbf{r}}$$

to write the Biot-Savart law as

$$\overrightarrow{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\overrightarrow{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

We should do a problem to see how this works.

Suppose we accelerate a proton and send it in the z-direction to a speed of $1.0 \times 10^7\,\mathrm{m/s}$. Let's further suppose we have a magnetic field detector placed $1\,\mathrm{mm}$ from the path of the proton. What field would it measure?



We know

$$\overrightarrow{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\overrightarrow{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

and by symmetry we know that v is perpendicular to $\hat{r}$ just as the proton passes the detector. So, using the right hand rule for cross products, we put our hand in the v-direction and bend our fingers into the r-direction. Then our thumb shows the resulting direction. In this case it is in the positive y-direction, or out of the page. The magnitude would be

$$\begin{aligned}
\overrightarrow{\mathbf{B}} &= \frac{4\pi \times 10^{-7}\,\frac{\mathrm{T\,m}}{\mathrm{A}}}{4\pi} \frac{\left(1.6 \times 10^{-19}\,\mathrm{C}\right)\left(1.0 \times 10^{7}\,\mathrm{m/\,s}\right)}{\left(0.001\,\mathrm{m}\right)^{2}} \hat{\mathbf{y}} \\
&= 1.6 \times 10^{-13}\,\mathrm{T}\hat{\mathbf{y}}
\end{aligned}$$