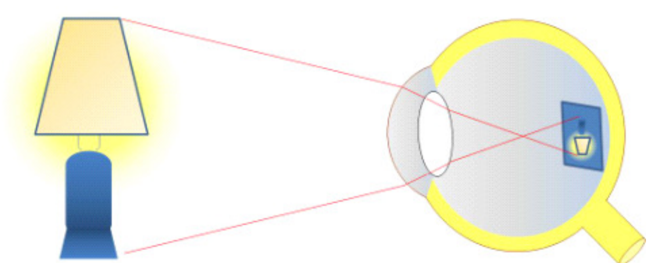
Chapter 49

Eyes and magnifiers

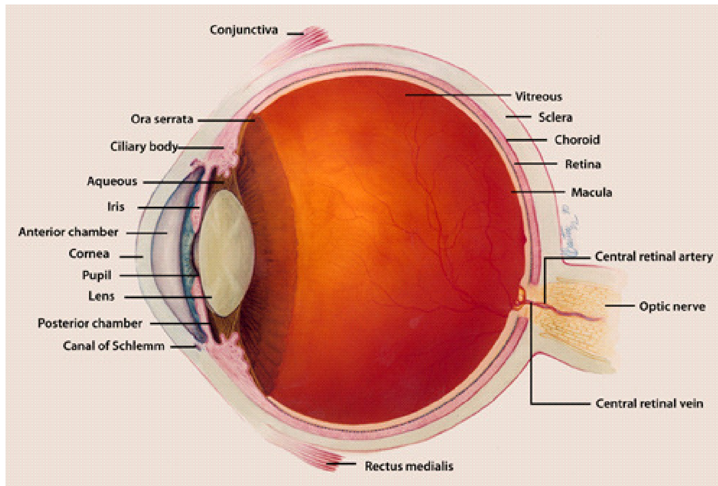
Fundamental Concepts

- Angular magnification compares the apparent size of an image with and without an optical system.
- The power of a lens is measured in Dipopters which are defined to be $1/f\,(\mathrm{m})$
- Compound magnifiers use an objective lens to form an image, and an eyepiece to magnify the image.

49.1 The Eye

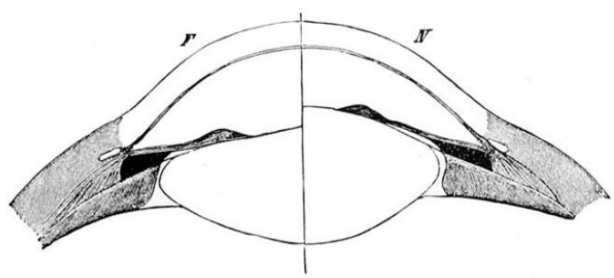


The figure above shows the parts of the eye. The eye is like a camera in its operation, but is much more complex. It is truly a marvel. The parts that concern us in this class are the cornea, crystalline lens, pupil, and the retina.



Details of the eye (more than we need in our class). Image courtesy of the US National Institutes of Health. (Image in the Public Domain)

The Cornea-lens system refracts the light onto the retina, which detects the light. The lens is focused by a set of mussels that flatten the lens to change it's focal length. The focusing process is different from a standard camera. The camera moves the lens to achieve a different image distance. Our eye can't change the distance between the lens system and the retina. So our eye changes the shape of the lens, changing it's focal length.



The crystaline lens becomes thicker, and therefore more curved when the cilliary musscle flexes. Austin Flint, "The Eye as an Optical Instrument," Popular Science Monthly, Vol. 45, p203, 1894 (Image in the public domain)

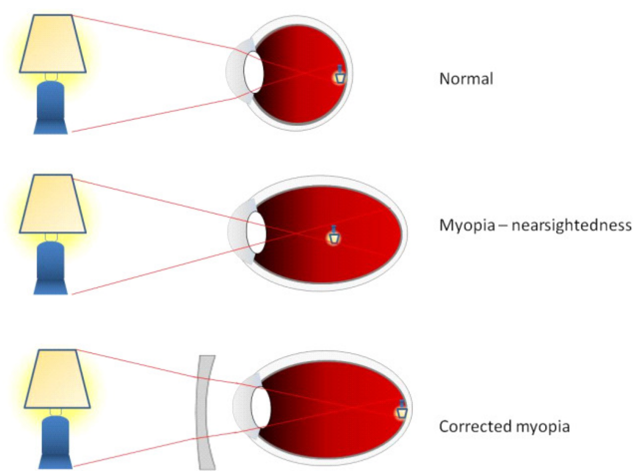
The focusing system is called accommodation. This system becomes less effective at about 40 because the lens becomes less flexible. The closest point that can be focused by accommodation is called the near point. It is about 25 cm on average. There is, of course, no such thing as an average person. All of us are a little bit different. You young students probably have a much shorter near point than 25 cm. For those of us that are a little older, 25 cm or more is more likely.

The farthest point that can be focused is ideally a long way away. It is called the far point. Both the near and far points degrade with years leading to bifocals.

The iris changes the area of the pupil (the aperture of the eye). The pupil is, on average, about 7 mm in diameter.

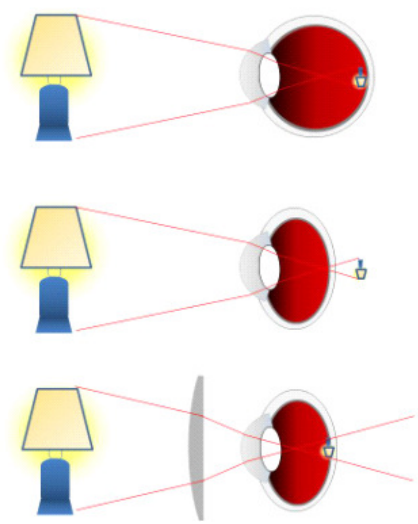
49.1.1 Nearsightedness

In some people the cornea-lens system focuses in front of the retina. This is called nearsightedness or myopia. This is usually because the eye, itself, is elongated into the eye socket, so the retina is too far away from the lens system.



49.1.2 Farsightedness

Sometimes the cornea-lens system does not focus quickly enough so the focus point is in back of the retina. This is often caused by the eye socket being too shallow so the retina is not far enough away from the lens system. This is called farsightedness or hyperopia. It is corrected with a converging lens



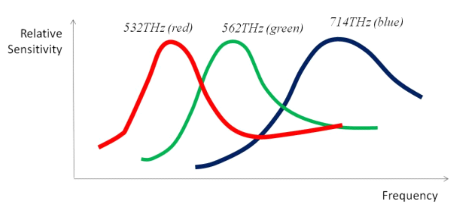
49.1.3 Diopters

Eye glass prescriptions use a different unit of measure to describe how they bend light. The unit is the *diopter* and it is equal one over the focal length

$$\text{diopter} = \frac{1}{f\,(\mathrm{m})} \tag{49.1}$$

49.1.4 Color Perception

The eye detects different colors. The respecters called cones can detect red, green, and blue light.



The eye combines the red, green, and blue response to allow us to perceive many different colors.

Most digital cameras also have red, green, and, blue pixels to provide color to images. The detectors in digital cameras are often have much narrower frequency bands than the eye. Likewise, television displays and monitors have red, green, and blue pixels. By targeting the eye receptors, power need not be wasted in creating light that is not detected well by the eye. The difference in band width can cause problems in color mixing. Yellow school busses (perceived as different amounts of green and red light) may be reddish or green if the bandwidths are chosen poorly.

The science of human visual perception of imagery is called *image science.* There are many applications for this field, from forensics to intelligence gathering.

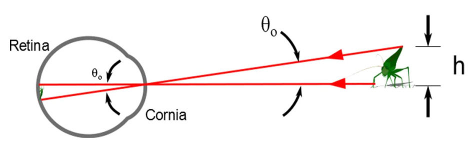
49.2 Optical Systems that Magnify

Magnification is so useful to scientists and engineers that we have designed many optical systems to do this job. We will look at three basic types, the simple magnifier, the microscope, and the telescope.

49.2.1 Simple Magnifier

You may have noticed that, so far, when we say "magnification" we are defining it in a way that is different than every-day usage of the word. We defined magnification to be how big the image is compared to how big the object is. But in every-day speech, magnification means how big the image is using a lens or optical system compared to how big it looks without the lens or optical system. We will call this kind of magnification the *angular magnification.*

We already encountered the simple magnifier when we studied ray diagrams. Let's use the simple magnifier to define angular magnification. To understand angular magnification, we can use what we know about easy rays to draw the rays that go straight through the lens of the eye. If we pick a ray from the top of our object that goes through the center of the lens, that ray won't seem to change direction at all. It will hit the retina to form the top of the image of the object. We can do the same for the bottom of the object. Then we can see from the next figure



that the angle θ_o subtends both the object and the image of the object. If the angle increases, so does the size of the image on the retina.

If we move the object closer, θ_o increases, and so does the size of the image. When we get to about 25 cm, we reach the limit of the eye for focusing. If we move the object any closer, it will appear fuzzy. We called this position, the closest point where we can place an object and still bring it into focus with our eye, the *near point.* Thus the maximum value of θ_o will be at the near point for unaided viewing.

But suppose we want to see this object in more detail. We can use a magnifying glass. If we place the object closer to the magnifying glass than the focal

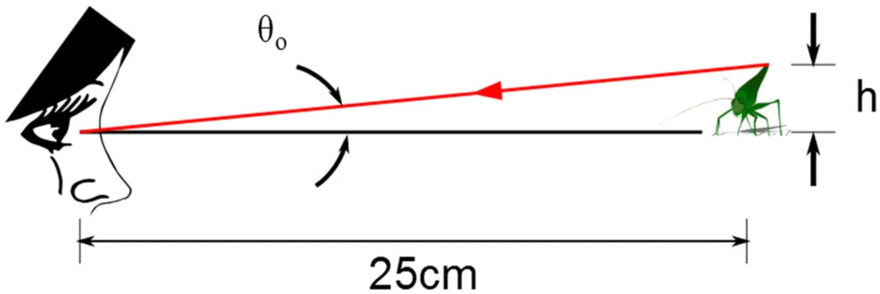


Figure 49.1:

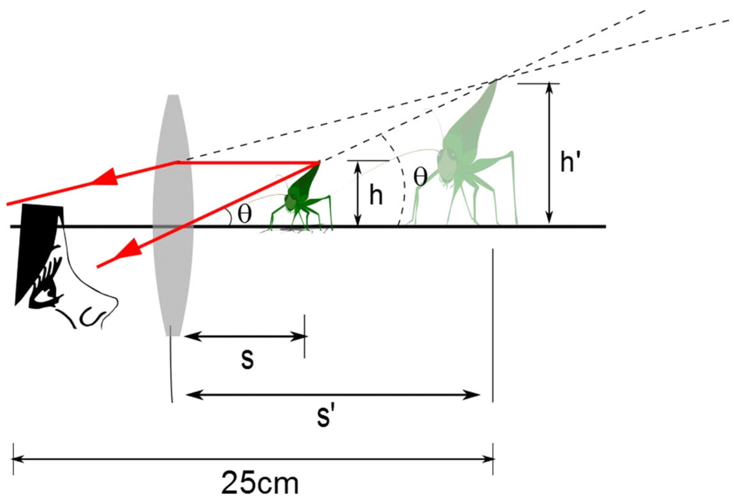


Figure 49.2:

distance ($s < f$), then (lower part of the figure) we have a virtual image with magnification

$$m = \frac{-s'}{s} \tag{49.2}$$

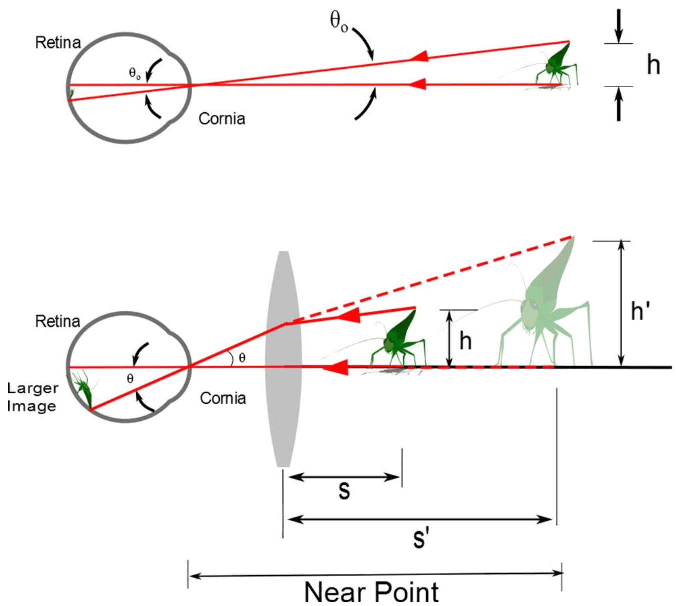
which is larger than one and positive (because s' is negative).

But what we really want to know is how much bigger the image looks with the lens than it did without the lens. We define the angular magnification to do this job.

$$M = \frac{\theta}{\theta_o} \tag{49.3}$$

This is the ratio of the image sizes *with and without the magnifier lens*.

This is really different than the magnification we have studied before. The magnification we have been using compared the size of the image with the size of the object. So, the angular magnification compares how big the object seems to be with and without a lens or lens system. We can think of this as a comparison between the size of the real image on the retina formed with just our eye, and the one formed with the magnifier.



If the virtual image formed is farther than the near point of the eye, ($s' >$ ˜25 cm) it will seem smaller than it would be at the near point because it is farther away. If the virtual image is closer than the near point, it will be fuzzy because the eye cannot focus closer than the near point. Thus, the value of M will be maximum when s' is at the near point of the eye. We can find where to place the image so that we get maximum magnification. Taking just the magnifier,

$$\begin{aligned} \frac{1}{s}+\frac{1}{s'} &= \frac{1}{f} \\ \frac{1}{s}+\frac{1}{-25\,\mathrm{cm}} &= \frac{1}{f} \end{aligned}$$

and

$$\frac{1}{s}=\frac{-25\,\mathrm{cm}-f}{-f\,(25\,\mathrm{cm})}$$

or

$$s=\frac{(25\,\mathrm{cm})\,f}{25\,\mathrm{cm}+f} \tag{49.4}$$

Returning to figure (49.2). Note that the person has adjusted her viewpoint so the ray that passes through the middle of the lens also passes through the middle of her eye lens (cornea). So the angle θ in the figure is also the angle that subtends the image on her retina. This was nice of her because it makes our math easier. Using small angle approximations, we can write

$$\tan\theta_o = \frac{h}{25\,\text{cm}} \approx \theta_o$$

and

$$\tan\theta \approx \frac{h}{s} \approx \theta$$

then the maximum angular magnification is

$$\begin{aligned}
m_{\max} &= \frac{\theta}{\theta_o} = \frac{\frac{h}{s}}{\frac{h}{25\,\text{cm}}} \\
&= \frac{25\,\text{cm}}{\frac{25\,\text{cm}f}{25\,\text{cm}+f}} \\
&= \frac{25\,\text{cm}+f}{f} \\
&= 1+\frac{25\,\text{cm}}{f}
\end{aligned}$$

We can also find the minimum magnification by letting s be at f. This gives

$$\theta = \frac{h}{f}$$

$$\begin{aligned}
m_{\min} &= \frac{\theta}{\theta_o} = \frac{\frac{h}{f}}{\frac{h}{25\,\text{cm}}} \\
&= \frac{25\,\text{cm}}{f}
\end{aligned}$$

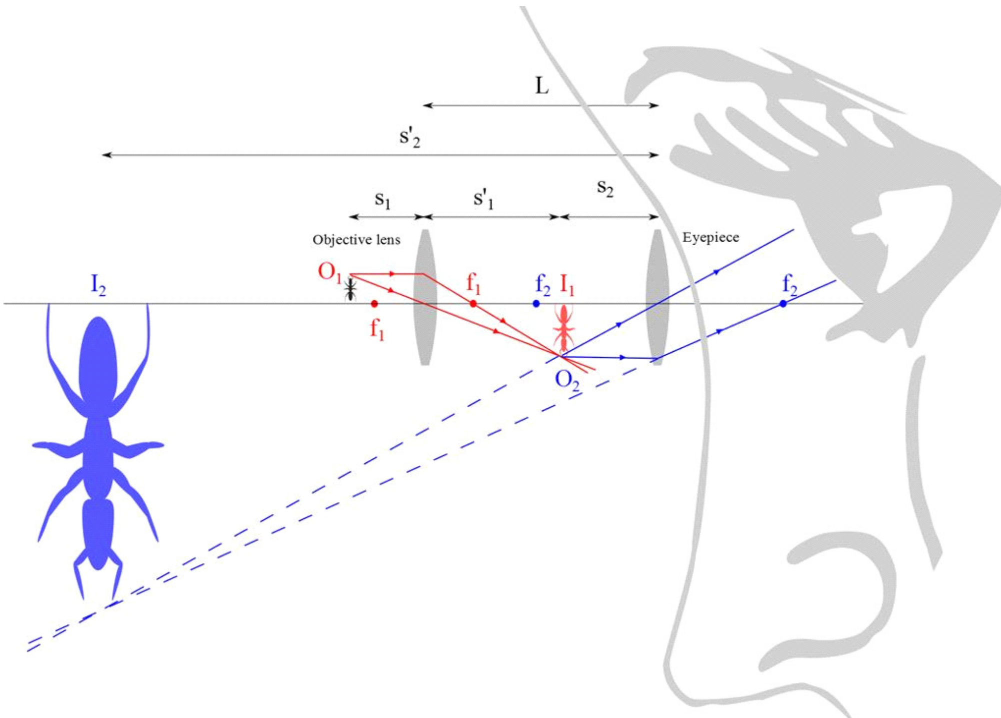
When you use a magnifying glass, notice that you move the lens back and forth between these extremes until you can see the level of detail you want.

But the idea of a magnifier is more than just seeing the details small objects. We use the idea of a simple magnifier in combination with other lenses to make the magnification happen in telescopes, microscopes, and other instruments that magnify.

49.2.2 The Microscope

To see things that are very small, we add another lens to our simple magnifier. We will place this lens near the object. Since this new lens is near the object, let's give it the name *objective lens* or just *objective*. We will keep a simple magnifier and place it near the eye. Since our simple magnifier is near our eye, let's call it the *eyepiece*.

The objective will have a very short focal length. The eyepiece will have a longer focal length (a few centimeters).



We separate the lenses by a distance L where

$$
\begin{aligned}
L &> f_o \\
L &> f_e
\end{aligned}
$$

We place the object just outside the focal point of the objective. The image formed by the objective lens is then real and inverted. We use this image as the object for the eyepiece. The image formed is upright and virtual, but it looks upside down because the object for the eyepiece (first image for the objective) is upside down.

The magnification of the first lens is

$$m_o = \frac{-s_1'}{s_1} \approx -\frac{L}{f_1}$$

because $s_1 \approx f_1$. and $s_1' \approx L$ The magnification of the eyepiece is just that of a simple magnifier when the object is placed at the focal point f_1

$$m_e = \frac{-s_2'}{s_2} \approx \frac{25\,\mathrm{cm}}{f_2}$$

The combined magnification is

$$m = m_o m_e = -\frac{L}{f_1}\frac{25\,\mathrm{cm}}{f_2} \tag{49.5}$$

this is the minimum magnification.

49.3 Telescopes

There are two types of telescopes *refracting* and *reflecting*. We will study refracting telescopes first.

49.3.1 Refracting Telescopes

Like the microscope, we combine two lenses and call one the objective and the other the eyepiece. The eyepiece again plays the role of a simple magnifier, magnifying the image produced by the objective.We again form a real, inverted image with the objective. We are now looking at distant objects, so the image distance $s_o' \approx f_o$. We use the image from the objective as the object for the eyepiece. The eye piece forms an upright virtual image (that looks inverted because the object for the eyepiece is the image from the objective, and the real image from the objective is inverted). The largest magnification is when the rays exit the eyepiece parallel to the principal axis. Then the image from the eyepiece is formed at infinity (but it is very big, so it is easy to see). This gives a lens separation of $f_o + f_e$ which will be roughly the length of the telescope tube.

The angular magnification will be

$$M = \frac{\theta}{\theta_o} \tag{49.6}$$

where θ_o is the angle subtended by the object at the objective. That is the angle we would have with no lenses and just our eye, because it is the angle subtended by the object without the optical system. The angle θ is subtended by the final image at the viewer's eye using the optical system. Consider s_o is very large. We see from figure (49.3) that

$$\tan\theta_o = -\frac{h'}{f_o} \tag{49.7}$$

and with s_o large we can use small angles.

$$\theta_o = -\frac{h'}{f_o} \tag{49.8}$$

The angle θ will be the angle formed by rays bent by the lens of the eye. This angle will be the same as the angle formed by a ray traveling from the tip

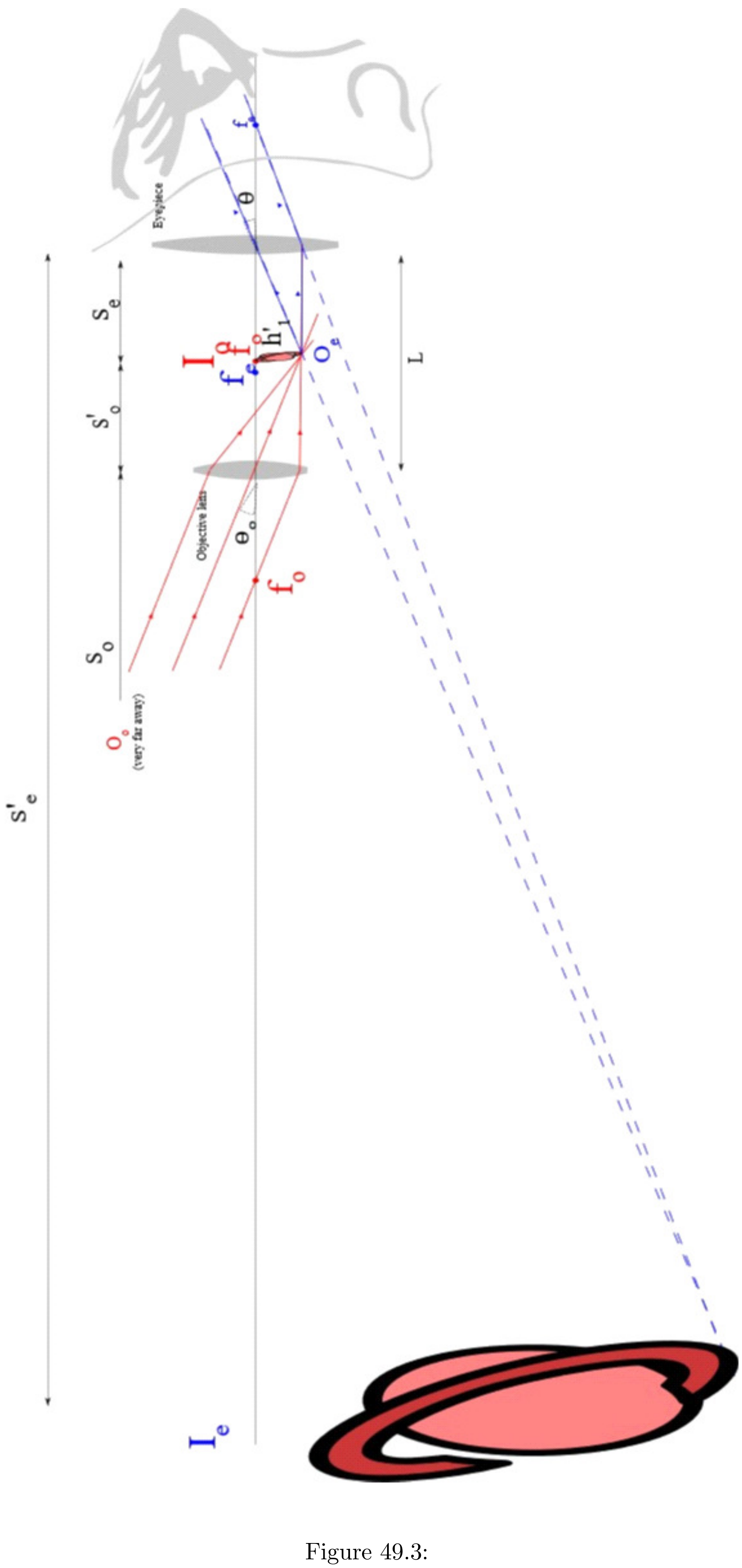


Figure 49.3:

of the first image and traveling parallel to the principal axis. This ray is bent by the objective to pass through $f_{e.}$ Then

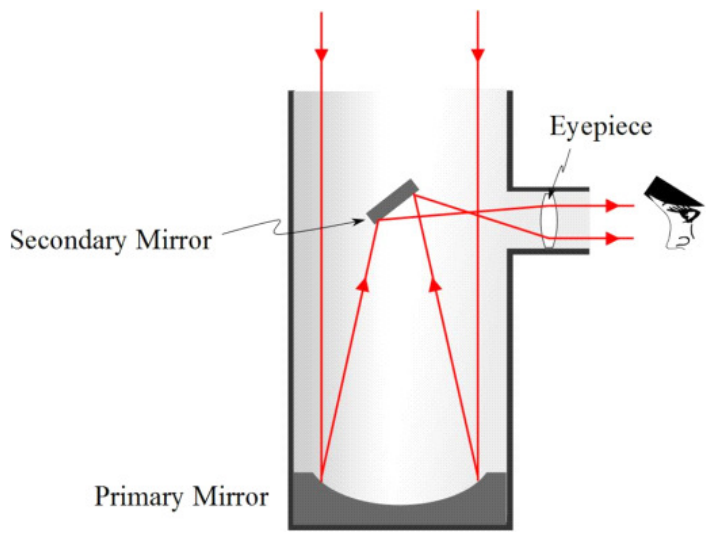
$$\tan\theta = \frac{h'}{f_e} \approx \theta \tag{49.9}$$

A good estimate for the magnification is then

$$m = \frac{\theta}{\theta_o} = \frac{\frac{h'}{f_e}}{-\frac{h'}{f_o}} = -\frac{f_o}{f_e} \tag{49.10}$$

49.3.2 Reflecting Telescopes

Reflecting telescopes use a series of mirrors to replace the objective lens. Usually, there is an eyepiece that is refractive (although there need not be, radio frequency telescopes rarely have refractive pieces).



There are two reasons to build reflective telescopes. The first is that reflective optics do not suffer from chromatic aberration. The second is that large mirrors are much easier to make and mount than refractive optics. The Keck Observatory in Hawaii has a 10 m reflective system. The largest refractive system is a 1 m system. The Hubble telescope has a 2.5 m aperture.

The telescope pictured in the figure is a Newtonian, named after Newton, who designed this focus mechanism. Many other designs exist. Popular designs for space applications include the Cassegrain telescope.

The rough design of a reflective telescope can be worked out using refractive pieces, then the rough details of the reflective optics can be formed.

Basic Equations

$$\text{diopter} = \frac{1}{f\,(\text{m})}$$

$$M = \frac{\theta}{\theta_o}$$

Microscopes

$$m_{\text{max}} = \frac{\theta}{\theta_o} = 1 + \frac{25\,\text{cm}}{f}$$

$$m_{\text{min}} = \frac{25\,\text{cm}}{f}$$

Telescopes

$$m = \frac{\theta}{\theta_o} = \frac{\frac{h'}{f_e}}{-\frac{h'}{f_o}} = -\frac{f_o}{f_e}$$