# Chapter 41

# Interference and Young's Experiment

So if light is a wave in the electromagnetic field, does light experience interference?

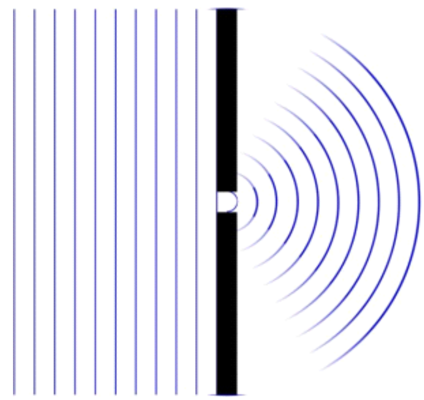
## Fundamental Concepts

- Mixing two light waves does create constructive and destructive interference.

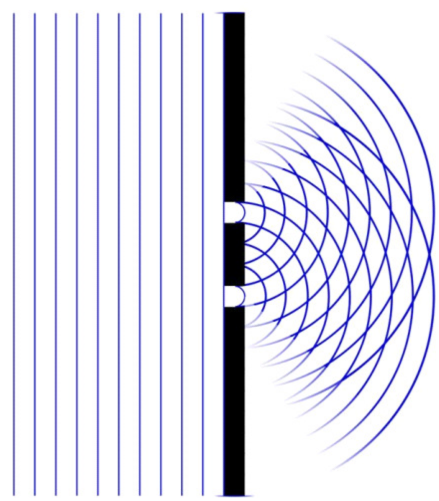
- The standard experiment to show this is called "Young's experiment."

- The combination of constructive and destructive interference for light is called an intensity pattern

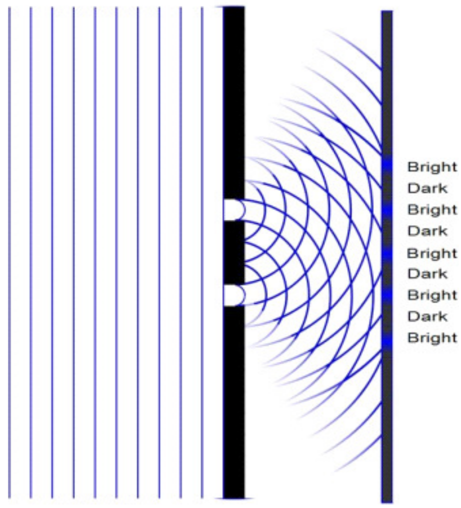
Waves do some funny things when they encounter barriers. Think of a water wave. If we pass the wave through a small opening in a barrier, the wave can't all get through the small hole, but it can cause a disturbance. We know that a small disturbance will cause a wave. But this wave will be due to a very small–almost point–source. So the waves will be spherical leaving the opening. The smaller the opening the more pronounced the curving of the wave, because the source (the hole) is more like a point source.



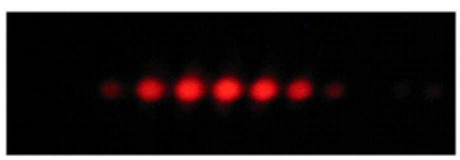
Now suppose we have two of these openings. We expect the two sources to make curved waves and those waves can interfere.



In the figure, we can already see that there will be constructive and destructive interference were the waves from the two holes meet. Thomas young predicted that we should see constructive and destructive interference in light (he drew figures very like the ones we have used to explain his idea).




Young set up a coherent source of light and placed it in front of this source a barrier with two very thin slits cut in it to test his idea.. He set up a screen beyond the barrier and observed the pattern on the screen formed by the light. This (in part) is what he saw.



We see bright spots (constructive interference) and dark spots (destructive interference). Only wave phenomena can interfere, so this is fairly good evidence that light is a wave.

### 41.0.2 Constructive Interference

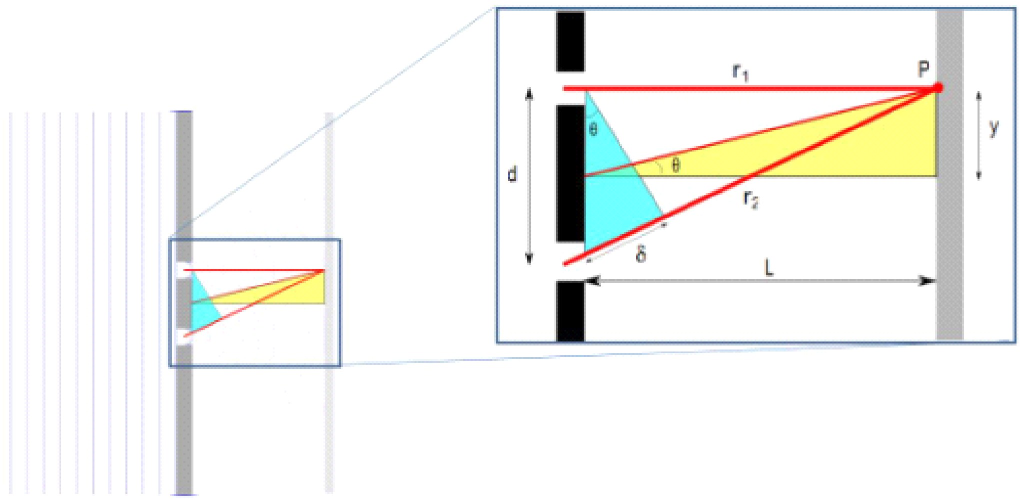
We can find the condition for getting a bright or a dark band if we think about it a bit. Here are our equations that we developed for constructive and destructive interference.

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = m2\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Constructive}$$

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = (2m+1)\,\pi \qquad m = 0, \pm1, \pm2, \pm3, \cdots \qquad \text{Destructive}$$

For constructive interference, the difference in phase, $\Delta\phi$, must be a multiple of $2\pi$. That means the path difference between the two slit-sources must be an even number of wavelengths. We have been calling the path difference in the total phase $\Delta x$, or for spherical waves $\Delta r$, but in optics it is customary to call this path difference $\delta$. So

$$\delta = \Delta r$$




and our total phase equation becomes

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\delta + \Delta\phi_o\right) = m2\pi$$

Our light going through the slits is all coming from the same light source. So as long as the light hits the slits at a $90°$ angle, $\Delta\phi_o = 0 - 0 = 0$ so we don't have a change in phase constant. But we do have a change in $\Delta r = \delta$. Let's suppose that the screen is far away so the distance from the slits to the screen, $L \gg d$, the slit distance. Then we can say that the blue triangle is almost a right triangle, and then $\delta$ is

$$\delta = r_2 - r_1 \approx d \sin\theta$$

so then

$$\Delta\phi = \left(\frac{2\pi}{\lambda}d\sin\theta + 0\right) = m2\pi$$

We can do a little math to make this simpler.

$$\frac{2\pi}{\lambda}d\sin\theta = m2\pi$$

$$\frac{1}{\lambda}d\sin\theta = m$$

$$d\sin\theta = m\lambda$$

We started by knowing our wave needs to sift by an integer number times $2\pi$ radians but now we see that is equivalent to shifting an integer number times the wavelength, $\lambda$. This will make the two waves experience constructive interference (a bright spot).

$$\delta = d\sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2 \ldots) \qquad \text{Constructive}$$

where in optics $m$ is called the *order number*. That is, if the two waves are off by any number of whole wavelengths then our total phase due to path difference will be $2\pi$. In optics, the the bright spots formed by constructive interference are called *fringes.*

If we assume that $\lambda \ll d$ we can find the distance from the axis for each fringe more easily. This condition guarantees that $\theta$ will be small. Using the yellow triangle we see

$$\tan\theta = \frac{y}{L}$$

but if $\theta$ is small this is just about the same as

$$\sin\theta = \frac{y}{L}$$

because for small angles $\tan\theta \approx \sin\theta \approx \theta$. So if theta is small then

$$\begin{aligned} \delta &= d\sin\theta \\ &= d\frac{y}{L} \end{aligned}$$

and for a bright spot or fringe we find

$$d\frac{y}{L} = m\lambda$$

Solving for the position of the bright spots gives

$$y_{bright} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots) \tag{41.1}$$

We can measure up from the central spot and predict where each successive bright spot will be.

### 41.0.3 Destructive Interference

We can also find a condition for destructive interference. Our destructive interference equation is

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\Delta r + \Delta\phi_o\right) = (2m+1)\,\pi$$

Once again $\Delta\phi_o = 0$ and $\Delta r = \delta$

$$\left(\frac{2\pi}{\lambda}\delta\right) = (2m+1)\,\pi$$

$$\left(\frac{2}{\lambda}\delta\right) = (2m+1)$$
$$\delta = \frac{\lambda}{2}(2m+1)$$
$$\delta = \lambda\left(m+\frac{1}{2}\right)$$

This just shows us again that a path difference of an odd multiple of a half wavelength will give distractive interference.

$$\delta = d\sin\theta = \left(m+\frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2 \ldots)$$

will give a dark fringe. The location of the dark fringes will be

$$y_{dark} = \frac{\lambda L}{d}\left(m+\frac{1}{2}\right) \quad (m = 0, \pm 1, \pm 2 \ldots) \tag{41.2}$$

## 41.1 Double Slit Intensity Pattern

The fringes we have seen are not just points, but are patterns that fade from a maximum intensity. This is why they are called fringes. We can calculate the intensity pattern to show this. We need to know a little bit about electric fields to do this.

### 41.1.1 Electric field preview

We can represent an electromagnetic wave using just the electric field (the magnetic field pattern is very similar and can be derived from the electric field pattern).

We represent the field by an equation like

$$y = y_o \sin(kr - \omega t)$$

but since the medium for light waves is the electric field, let's use the symbol $E$ instead of $y$ so we can see that we have a change in the field strength and not a displacement of some material thing.

$$E = E_{\max}\sin(kr - \omega t) \tag{41.3}$$

where the amplitude of the wave is $E_{\max}$ and $\omega$ is the angular frequency. This is just our traveling wave equation, but with electric field strength, labeled $E$, for the amplitude.

Then to find the intensity pattern, we take two waves in the electric field, one from slit one

$$E_1 = E_{\max}\sin(kr_1 - \omega t + \phi_o)$$

and the other from slit two.

$$E_2 = E_{\max}\sin(kr_2 - \omega t + \phi_o)$$

This is mathematically just like superposition of sound waves.

### 41.1.2 Superposition of two light waves

Remember when we superimposed waves before, we mixed the waves

$$\begin{aligned} y_1 &= A\sin(kr_1 - \omega t + \phi_1) \\ y_2 &= A\sin(kr_2 - \omega t + \phi_2) \end{aligned}$$

and using

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

we found the resultant wave

$$y_r = 2A\cos\left(\frac{1}{2}(\Delta\phi)\right)\sin\left(k\frac{r_2+r_1}{2} - \omega t + \frac{\phi_2+\phi_1}{2}\right)$$

Our light waves are just two waves. They may be the superposition of many individual photons, but the combined wave is just a wave.

At the slits, the waves have the same amplitude $E_{\max}$ and the same phase constant, $\phi_1 = \phi_2 = \phi_o$, but $E_2$ travels farther than $E_1$, so $\Delta\phi$ is due to the path difference. We expect to find that the path difference would be

$$\begin{aligned} \Delta\phi &= k\Delta r + \Delta\phi_o \\ &= k\delta + 0 \\ &= \frac{2\pi}{\lambda}d\sin\theta \end{aligned}$$

Now superimposing $E_1$ and $E_2$ at point $P$ on the screen gives

$$\begin{aligned} E_P &= E_2 + E_1 \\ &= E_{\max}\sin(kr_2 - \omega t) + E_o\sin(kr_1 - \omega t) \end{aligned}$$

and using our prior result, we have

$$E_P = 2E_{\max}\cos\left(\frac{1}{2}\Delta\phi\right)\sin\left(k\frac{(r_2+r_1)}{2} - \omega t + \phi_o\right)$$

and using our equation for $\Delta\phi$ above we get

$$E_P = 2E_{\max}\cos\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\sin\left(k\frac{(r_2+r_1)}{2} - \omega t + \phi_o\right)$$

We have a combined wave at point $P$ that is a traveling wave $\left(\sin\left(k\frac{(r_2+r_1)}{2} - \omega t + \phi_o\right)\right)$ but with amplitude $\left(2E_{\max}\cos\left(\frac{1}{2}\left(\frac{2\pi}{\lambda}d\sin\theta\right)\right)\right)$ that depends on our total phase $\Delta\phi = \frac{2\pi}{\lambda}d\sin\theta$.

But the situation is more complicated because of how we detect light. Our eyes, and most detectors measure the intensity of the light. We know that

$$I = \frac{\mathcal{P}}{A}$$

later in the course we will show that the power in an electromagnetic field wave is proportional to the square of the electric field displacement.

$$\mathcal{P} \propto E_P^2 \tag{41.4}$$

For now, let's just assume this is true. Then the intensity must be proportional to the amplitude of the electric field squared.

$$\begin{aligned} I &\propto E_P^2 \\ &= 4E_{\max}^2 \cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda} d\sin\theta\right)\right)\sin^2\left(\frac{k\left(r_2+r_1\right)}{2} - \omega t + \phi_o\right) \end{aligned}$$

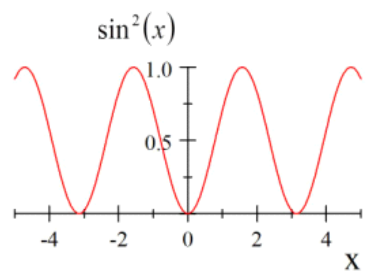
Light detectors collect energy for a set amount of time. So most light detection will be a value averaged over a set *integration time*. This means that the detector sums up (or integrates) the amount of power received over the detector time. Usually the integration time is much longer than a period, so what is really detected is like a time-average of our intensity.

$$\begin{aligned} \int_{\text{many T}} I dt &\propto = \int_{\text{many T}} 4E_{\max}^2 \cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda} d\sin\theta\right)\right)\sin^2\left(\frac{k\left(r_2+r_1\right)}{2} - \omega t + \phi_o\right) dt \\ &= 4E_{\max} \cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda} d\sin\theta\right)\right)\int_{\text{many T}} \sin^2\left(\frac{k\left(rx_2+r_1\right)}{2} - \omega t + \phi_o\right) dt \end{aligned}$$

but the term

$$\int_{\text{many T}} \sin^2\left(\frac{k\left(r_2+r_1\right)}{2} - \omega t + \phi_o\right) dt = \frac{1}{2} \tag{41.5}$$

To convince yourself of this, think that $\sin^2(\omega t)$ has a maximum value of 1 and a minimum of 0. Looking at the graph




should be convincing that the average value over a period is 1/2. The average over many periods will still be 1/2.

So we have

$$\bar{I} = \int_{\text{many periods}} I dt \propto 2E_{\max} \cos^2\left(\frac{1}{2}\left(\frac{2\pi}{\lambda} d\sin\theta\right)\right) \tag{41.6}$$

where $\bar{I}$ is the time average intensity. The important part is that the time varying part has averaged out.

So, usually in optics, we ignore the fast fluctuating parts of such calculations because we can't see them and so we write

$$I = I_{\max} \cos^2 \left( \frac{1}{2} \left( \frac{2\pi}{\lambda} d \sin \theta \right) \right)$$

where we have dropped the bar from the $I$, but it is understood that the intensity we report is a time average over many periods.

We should remind ourselves, of our intensity pattern

$$I = I_{\max} \cos^2 \left( \frac{1}{2} \frac{2\pi}{\lambda} d \sin \theta \right)$$

is really

$$I = I_{\max} \cos^2 \left( \frac{\Delta\phi}{2} \right)$$

Which is just our amplitude squared for the mixing of two waves. All we have done to find the intensity pattern is to find and expression for the phase difference $\Delta\phi$.

Our intensity pattern should give the same location for the center of the bright spots as we got before. Let's check that it works. We used the small angle approximation before, so let's use it again now. For for small angles

$$\begin{aligned} I &= I_{\max} \cos^2 \left( \frac{\pi d}{\lambda} \theta \right) \\ &= I_{\max} \cos^2 \left( \frac{\pi d}{\lambda} \frac{y}{L} \right) \end{aligned}$$

Then we have constructive interference when

$$\frac{\pi d}{\lambda} \frac{y}{L} = m\pi$$

or

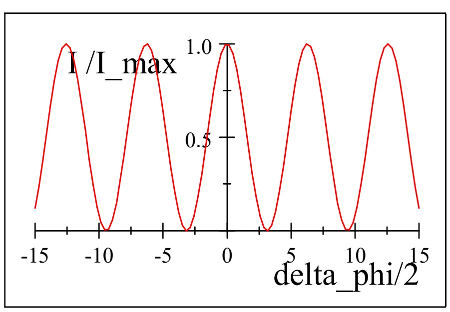
$$y = m \frac{L\lambda}{d}$$

which is what we found before.

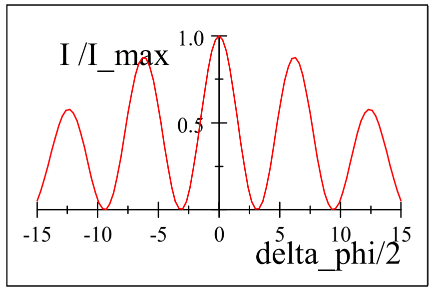
The plot of normalized intensity

$$\frac{I}{I_{\max}} = \cos^2 \left( \frac{\Delta\phi}{2} \right)$$

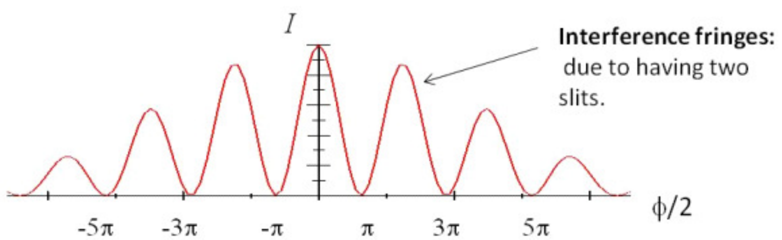
verses $\Delta\phi/2$ is given next,

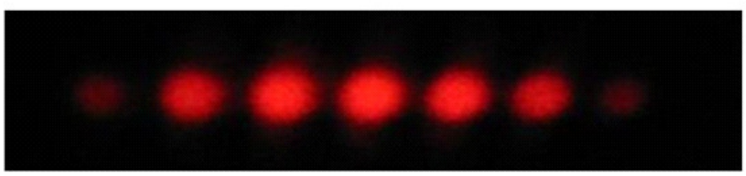


but we will find that we are not quite through with this analysis. Next time we will find that there is another compounding factor that reduces the intensity as we move away from the midpoint.




Let's pause to remember what this pattern means. This is the intensity of light due to interference. It is instructive to match our intensity pattern that Young saw with our graph.



The high intensity peaks are the bright fringes and the low intensity troughs are the dark fringes. The pattern moves smoothly and continuously from bright to dark.

# Basic Equations

$$y_{bright} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots)$$

$$y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right) \quad (m = 0, \pm 1, \pm 2 \ldots)$$

$$\begin{aligned} I &= I_{\max} \cos^2\left(\frac{\pi d}{\lambda}\theta\right) \\ &= I_{\max} \cos^2\left(\frac{\pi d}{\lambda}\frac{y}{L}\right) \end{aligned}$$