Chapter 50

Resolution and Polarization

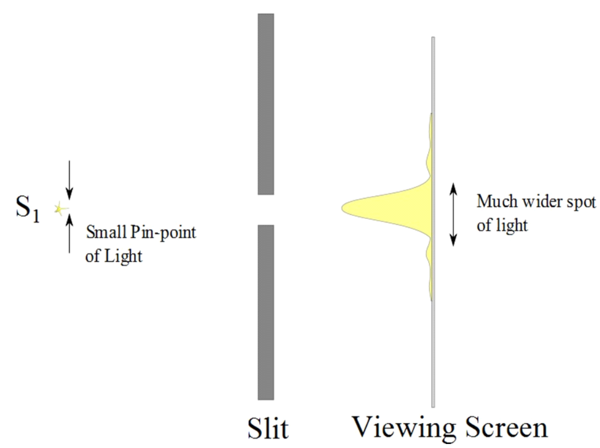
Fundamental Concepts

- Two points can be distinguished when imaged if their angular separation is a minimum of $\theta_{\min} = 1.22\frac{\lambda}{D}$
- light waves are really superpositions of many small waves that we call photons.
- The direction of the electric field in a plane wave is called the polarization direction.
- Natural light is usually a superposition of many waves with random polarization directions. This light is called unpolarized light.
- Some materials allow light with one polarization to pass through, while stopping other polarizations. The polaroid is one such material polaroids. will have a final intensity that follows the relationship $I = I_{\max} \cos^2(\theta)$
- Light reflecting off a surface may be polarized because of the absorption and re-emission pattern of light interacting with the material atoms.
- Scattered light may be polarized because of anisotropies in the scatterers.
- Birefringent materials have different wave speeds in different directions. This affects the polarization of light entering these materials.

50.1 Resolution

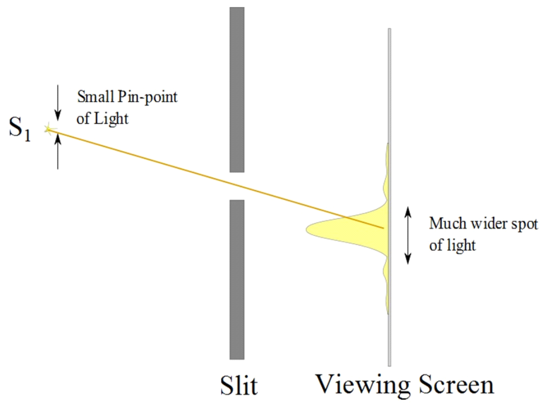
We have emphasized that an extended object can be viewed as a collection of point objects. Then the image is formed from the collection if images of those point objects. It would be great if optical systems could form images with infinite precision–that is, the image of a point object would be a point image. The fact that light acts as a wave prevents this from being true. The quality of our image depends on how poorly a point object is imaged. If each point object makes a large circle of light on the screen or detector array, we get a very confusing image (it will look blurry to us).[1] Let's see why this will happen so we can know how to minimize the effect.

We already know that if we take light and pass it through a single slit, we get an intensity pattern that has a central bright region.



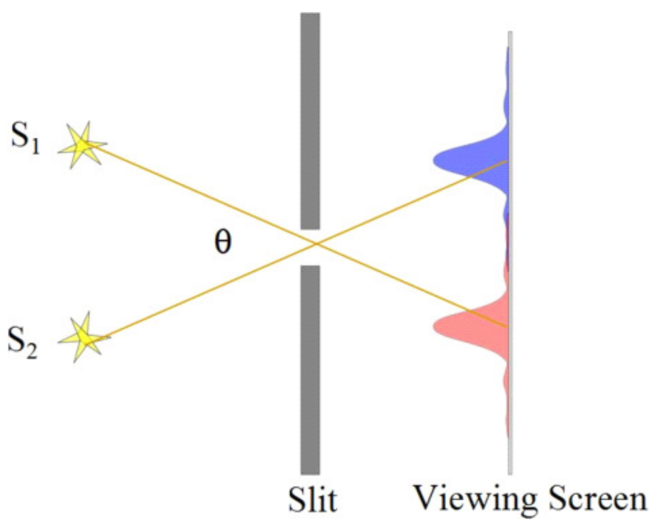
Remember that normal objects will be made up of many small points of light (either due to reflection or glowing) and each of these will form such an intensity pattern on a screen. Here is a bright point source that is not on the axis, and we see that it too makes a bright spot on the screen (and smaller bright spots or rings, depending on the shape of the aperture)

[1] In Fourier Optics, the intensity pattern that comes from imaging a single point is called a *point spread function* because it shows how spread out the light from a single point will be. In mechanical engineering, we might call this an impulse response function. It is the same idea applied to optics.

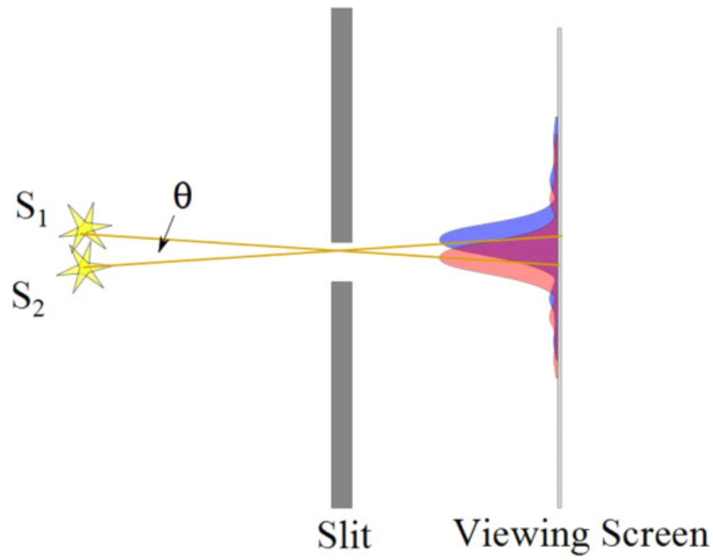


So our images will be made up of many central bright spots, each of which represents a point of light from the object. These central bright spots may overlap, (and their secondary maxima certainly will overlap).

Let's take a simple case of two points of light, S_1 and S_2. If we take a single slit and pass light from two distant point sources through the slit, we do not get two sharp images of the point sources. Instead, we get two diffraction patterns.



If these patterns are formed sufficiently far from each other, it is easy to tell they were formed from two distinct objects. Each point became a small blur, but that is really not so bad. We can still tell that the two blurs came from different sources. If our pixel size is about the same size of the blur, we won't even notice the blurriness in the digital imagery.



But if the patterns are formed close to each other, it gets hard to tell whether they were formed from two objects or one bright object. We now have a problem. Suppose you are trying to look at a star and see if it has a planet. But all you can see is a blur. You can't tell if there is one source of light or two.

Long ago an early researcher titled Lord Rayleigh developed a test to determine if you can distinguish between two diffraction patterns.

> When the central maximum of one point's image falls on the first minimum of anther point's image, the images are said to be just resolved.

This test is known as *Rayleigh's criterion*.

We can find the required separation for a slit. Remember that

$$\sin(\theta) = m\frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3 \ldots \tag{50.1}$$

gives the minima. We want the first minimum, so

$$\sin(\theta) = \frac{\lambda}{a} \tag{50.2}$$

If we place the second image maximum so it is just at this location, the two images will be just barely resolvable. In the small angle approximation, $\sin(\theta) \approx \theta$ so

$$\theta_{\min} = \frac{\lambda}{a} \tag{50.3}$$

Now you may be saying to yourself that you don't often take pictures through single illuminated slits, so this is nice, but not really very interesting.

Suppose, instead, that we image a circular aperture. Again, we won't go through all the math (there are Bessel functions involved) but the criterion becomes

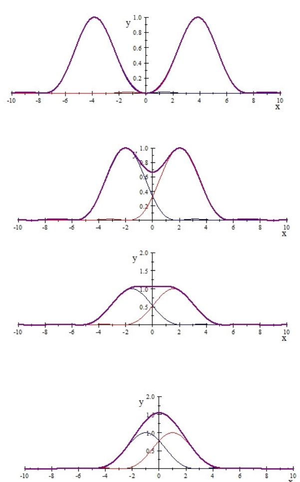
$$\theta_{\min} = 1.22\frac{\lambda}{D} \tag{50.4}$$

where D is the aperture diameter.

Still, you may say, I don't like pictures taken through small circles any better than through small slits! Yet, in fact, you do. Most cameras have circular apertures. The light that passes into your phone camera must pass though the circular lens. For that matter, the pupil of our eye is a circular aperture. So most images we see are made using circular apertures.

The Rayleigh criteria tells you, based on your camera aperture size, how a point source will be imaged on the film or sensor array. If we consider extended sources (like your favorite car or Aunt Matilda) to be collections of many point sources, then we have a way to tell what features will be clearly resolved on the image and what features will not (like you may not be able to see the lettering on the car to tell what model it is, or you may not be able to distinguish between the gem stones in Aunt Matilda's necklace because the image is too blurry to see these features clearly).

In the next figure you can see two easily resolved intensity patterns, then two that can be resolved using Rayleigh's criterion, and finally a more modern astronomical resolution criterion made by a researcher called Sparrow. Finally there is figure with two intensity patterns that would not be resolvable (would look like just one point of light).



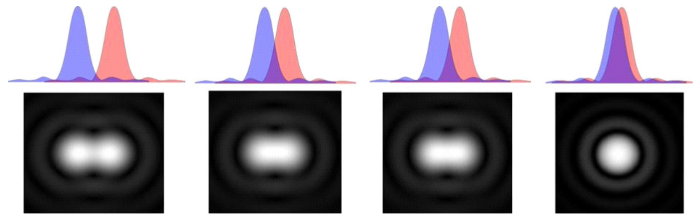
Completely Resolved

Rayleigh Criterion

Sparrow Criterion

Unresolved

Here are our four cases again but with pictures that show what you would see in an image of the two light sources for each criterion.



From our equation we can see that we have better resolution if D is bigger. This is why professional photographers use large lenses and not their cell phones. The cell phone cameras have apertures that are a few millimeters. Typical professional cameras have 67 mm apertures. We can see that for a cell phone the angle for minimal resolution is about

$$\theta_{\min} = 1.22 \frac{500\,\mathrm{nm}}{3\,\mathrm{mm}} = 2.0333 \times 10^{-4}\,\mathrm{rad}$$

For the professional lens, the minimum resolution is about

$$\theta_{\min} = 1.22 \frac{500\,\mathrm{nm}}{67\,\mathrm{mm}} = 9.1045 \times 10^{-6}\,\mathrm{rad}$$

That is a whopping factor of 22 better resolution. If you need to find a small crack in a structure, or if you want to print a wall sized portrait of your Aunt Miltilda, the extra resolution might be necessary for your application.

50.1.1 Photons

Our understanding of light is not complete yet. If you went on to take PH279 you would find that light still operates much like a particle at times. This should not be a surprise, since Newton and others explained much of optics (the study of light) assuming light was a particle.

Einstein and others noticed that for some metals, light would strike the surface and electrons would leave the surface. The energy of a wave is proportional to the amplitude of the wave. It was expected that if the amplitude of the electromagnetic wave was increased, the number of electrons leaving the surface would increase. This proved to be true most of the time. But Hertz and others decided to try different frequencies of light. It turns out that as you lower the frequency, all of a sudden no electrons leave no matter how big the amplitude of the wave. Something was wrong with our wave theory of light. The answer came from Einstein who used the idea of a "packet" of light to explain this *photoelectric effect*. For now, we should know just that the waves of light exist in *quantized* packets called *photons*. The energy of a photon is

$$E = hf \tag{50.5}$$

where E is the energy, f is the frequency of the light wave, and h is a constant

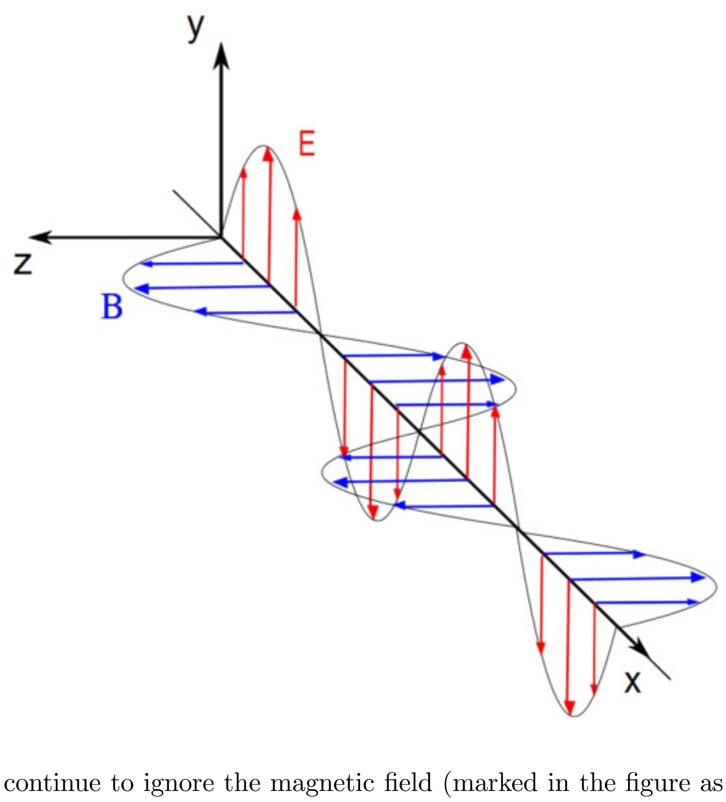
$$h = 6.63 \times 10^{-34}\,\mathrm{J\,s} \tag{50.6}$$

A beam of light is many, many photons all superimposing. We know how waves combine using superposition, so it is easy to see that we can get a big wave from many little waves.

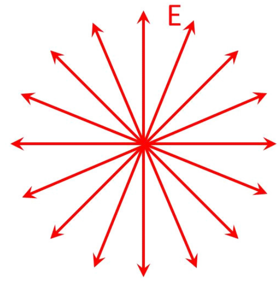
Knowing that light is made from electric and magnetic fields, and that these fields are vector fields, we should expect some directional quality in light. And there is such a directional quality that we will study next lecture.

50.2 Polarization of Light Waves

We said much earlier in our study of light that it was a transverse wave. Last lecture we saw that we have an electric and magnetic field direction, and that these directions are perpendicular to each other and the direction of energy flow. We will now show some implications of this fact. In a course in electromagnetic theory, we often draw light as in the figure below.



We will continue to ignore the magnetic field (marked in the figure as B). We will look at the E field an notice that it goes up and down in the figure. But we could have light in any orientation. If we look directly at an approaching beam of light we would "see" many different orientations as shown in the next figure.



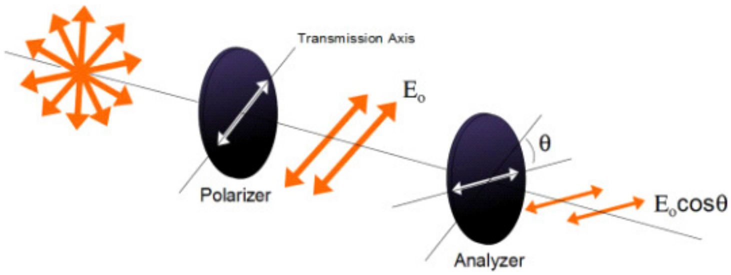
When light beams have waves with many orientations, we say they are *unpolarized*. But suppose we were able to align all the light so that all the waves in the beam were transverse waves in the same orientation. Say, the one in the next figure.

Then we would describe the light as *linearly polarized*. The plane that contains the E-field is known as the *polarization plane*.

50.2.1 Polarization by removing all but one wave orientation

One way to make polarized light is to remove all but one orientation of an unpolarized beam. A material that does this at visible wavelengths is called a *polaroid*. It is made of long-chain hydrocarbons that have been treated with iodine to make them conductive. The molecules are all oriented in one direction by stretching during the manufacturing process. The molecules have electrons that can move when light hits them. They can move farther in the long direction of the molecule, so in this direction the molecules act like little antennas. The molecules' electrons are driven into harmonic motion along the length of the molecule. This takes energy (and therefore, light) out of the beam. Little electron motion is possible in the short direction of the molecule, so light is given a preferential orientation. The light is passed if it is perpendicular to the long direction of the molecules. This direction is called the *transmission axis*.

We can take two pieces of polaroid material to study polarization.



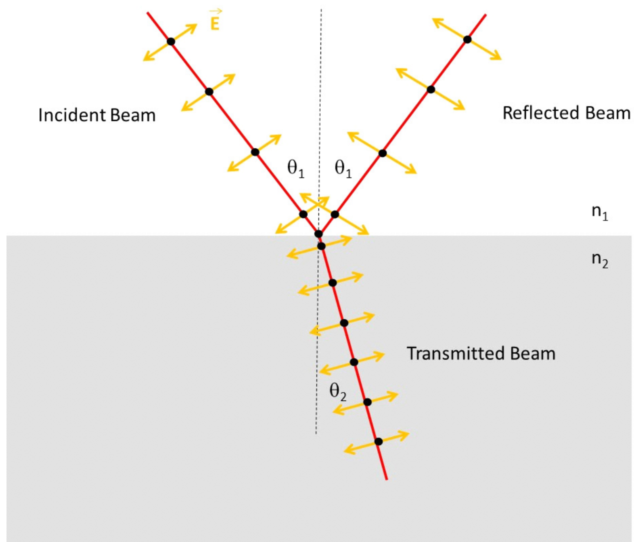
Unpolarized light is initially polarized by the first piece of polaroid called the *polarizer*. The second piece of polaroid then receives the light. This piece is called the *analyzer*. If there is an angular difference in the orientation of the transmission axes of the polarizer and analyzer, there will be a reduction of light through the system. We expect that if the transmission axes are separated by 90 ° no light will be seen. If they are separated by 0 °, then there will be a maximum. It is not hard to believe that the intensity will be given by

$$I = I_{\max} \cos^2(\theta) \tag{50.7}$$

remembering that we must have a squared term because $I \propto E^2$.

50.2.2 Polarization by reflection

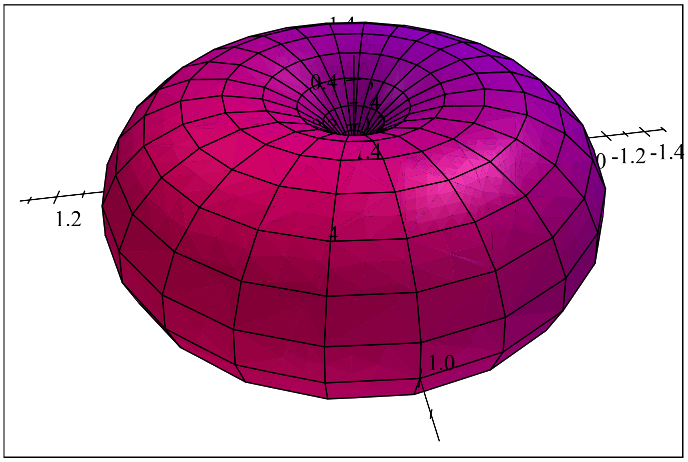
If we look at light reflected off of a desk or table through a piece of polaroid, we can see that at some angles of orientation, the reflection diminishes or even disappears! Light is often polarized on reflection. Let's consider a beam of light made of just two polarizations. We will define a plane of incidence. This plane is the plane of the paper or computer screen. This plane is perpendicular to the reflective or refractive surface in the figure below.



One of our polarizations is defined as parallel to this plane. This direction is represented by orange (lighter grey in black and white) arrows in the figure. The other polarization is perpendicular to the plane of incidence (the plane of the paper). This is represented by the black dots in the figure. These dots are supposed to look like arrows coming out of the paper.

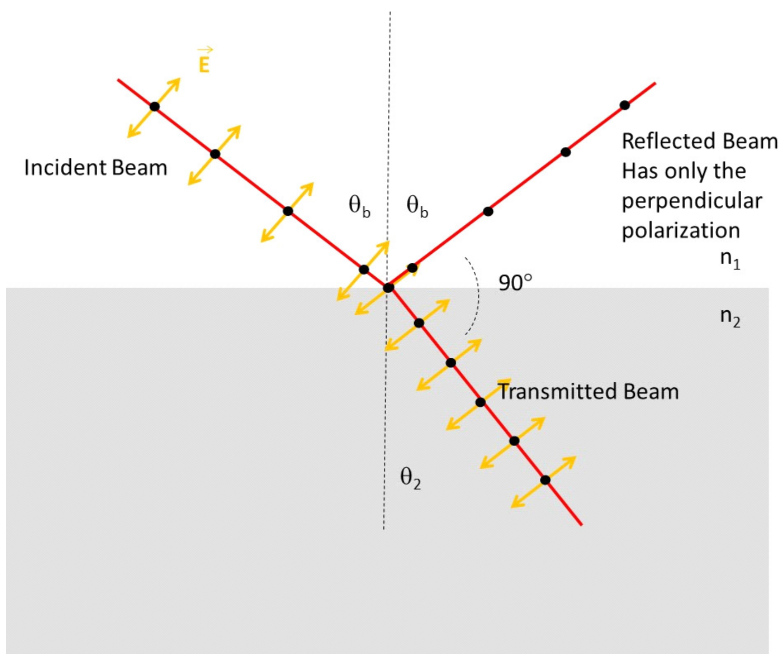
When the light reaches the interface between n_1 and n_2 it drives the electrons in the medium into SHM. The perpendicular polarization finds electrons that are free to move in the perpendicular direction and re-radiate in that direction. Even for a dielectric, the electron orbitals change shape and oscillate with the incoming electromagnetic wave.

The parallel ray is also able to excite SHM, but a electromagnetic analysis tells us that these little "antennas" will not radiate at an angle 90° from their excitation direction. Think of little dipole radiators. We can plot the amplitude of the electric field as a function of direction around the antenna.



Angular dependence of S for a dipole scatterer.

We see that along the antenna axis, the field amplitude is zero. This means that the wave really does not go that direction. So in our case, the amount of polarization in the parallel direction decreases with the angle between the reflected and refracted rays until at $90\,^\circ$ there is no reflected ray in the parallel direction.



The incidence angle that creates an angular difference between the refracted and reflected rays of $90\,^\circ$ is called the Brewster's *angle* after its discoverer. At this angle the reflected beam will be completely linearly polarized.

We can predict this angle. Remember Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Let's re-lable the incidence angle $\theta_1 = \theta_b$. We take $n_1 = 1$ and $n_2 = n$ so

$$n = \frac{\sin\theta_b}{\sin\theta_2}$$

Now notice that for Brewster's angle, we have

$$\theta_b + 90\,^\circ + \theta_2 = 180\,^\circ$$

so

$$\theta_2 = 90\,^\circ - \theta_b$$

so we have

$$n = \frac{\sin\theta_b}{\sin\left(90\,^\circ - \theta_b\right)}$$

ah, but we remember that $\sin\left(90\,^\circ - \theta\right) = \cos\left(\theta\right)$ so

$$n = \frac{\sin\theta_b}{\cos\theta_b}$$

but again we remember that

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

so

$$n = \tan\theta_b \tag{50.8}$$

which we can solve for θ_b.

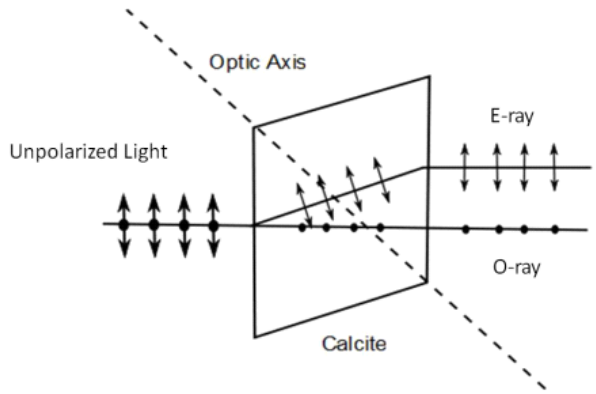
$$\theta_b = \tan^{-1}\left(n\right)$$

This phenomena is why we wear polarizing sunglasses to reduce glare.

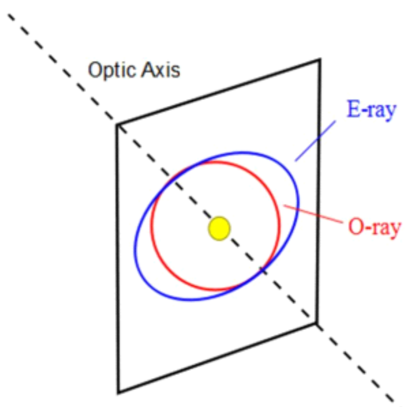
50.2.3 Birefringence

Glass is an amorphic solid–that is–it has no crystal structure to speak of. But some minerals do have definite order. Sometimes the difference in the crystal structure creates a difference in the speed of propagation of light in the crystal. This is not to hard to believe. We said before that the reason light slows down in a substance is because it encounters atoms which absorb and re-emit the light. If there are more atoms in one direction than another in a crystal, it makes sense that there could be a different speed in each direction.

Calcite crystals exhibit this phenomena. We can describe what happens by defining two polarizations. One parallel to the plane of the figure below, and one perpendicular.



With a careful setup, we can arrange things so the perpendicular ray is propagated just as we would expect for glass. We call this the O-ray (for *ordinary*). The second ray is polarized parallel to the incidence plane. It will have a different speed, and therefore a different index of refraction. We call it the *Extraordinary ray* or E-ray.



If we were to put a light source in a calcite crystal, we would see the O-ray send out a sphere of light as shown in the figure above. But the E-ray would send out an ellipse. The speed for the E-ray depends on orientation. There is one direction where the speeds are equal. This direction is called the *optic axis* of the crystal.

If our light entering our calcite crystal is unpolarized, then we will have two images leaving the other side that are slightly offset because the O-rays and E-rays both form images.

50.2.4 Optical Stress Analysis

Some materials (notably plastics) become birefringent under stress. A plastic or other stress birefringent material is molded in the form planned for a building or other object (usually made to scale). The model is placed under a stress, and the system is placed between to polaroids. When unstressed, no light is seen, but under stress, the model changes the polarization state of the light, and bands of light are seen.

50.2.5 Polarization due to scattering

It is important to understand that light is also polarized by scattering. It really takes a bit of electromagnetic theory to describe this. So for a moment, lets just comment that blue light is scattered more than red light. In fact, the relative intensity of scattered light goes like $1/\lambda^4$. This has nothing to do with polarization, but it is nice to know.

Now suppose we have long pieces of wire in the air, say, a few microns long. The pieces of wire would have electrons that could be driven into SHM when light hits them. If the wires were all oriented in a common direction, we would expect light to be absorbed if it was polarized in the long direction of the particles and not absorbed in a direction perpendicular to the orientation of the particles. This is exactly what happens when long ice particles in the atmosphere orient in the wind (think of the moment of inertia). We often get impressive halo's around the sun due to scattering from ice particles.

Rain drops also have a preferential scattering direction because they are shaped like oblate spheroids (not "rain drop shape" like we were told in grade school).

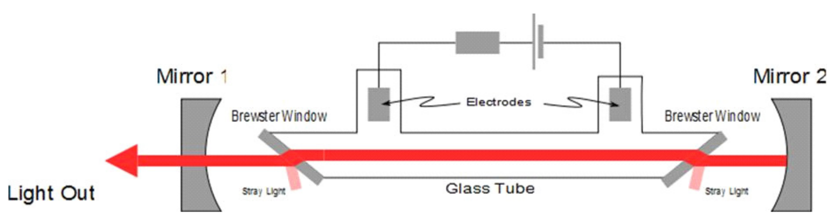
It is also true that small molecules will act like tiny antennas and will scatter light preferentially in some directions and not in others. This is called *Rayleigh scattering* and is very like small dipole antennas.

50.2.6 Optical Activity

Some substances will rotate the polarization of a beam of light. This is called being *optically active*. The polarization state of the light exiting the material depends on the length of the path through the material. Your calculator display works this way. An electric field changes the optical activity of the liquid crystal. There are polarizers over the liquid crystal, so sometimes light passes through the display and sometimes it is black.

50.2.7 Laser polarization

One last comment. Lasers are usually polarized. This is because the laser light is generated in a *cavity* created by two mirrors. The mirror is tipped so light approaches it at the Brewster angle. Light with the right polarization (parallel to the plane of the drawing) is reflected back nearly completely, but light with the opposite polarization is not reflected at all. This reduces the usual loss in reflection from a mirror, because in one polarization the light must be reflected completely.



50.3 Retrospective

We have thought about many things in this class. It has been a class *about* science. It has not been a class where we have tried to discover new science, or practiced the scientific method. This is on purpose, this being an engineering class designed to teach the principles of physics for use in designing machines.

But we should pause to think, just for a moment, about the philosophy of science. Is everything in these lectures true? We did not perform experiments to show every principle we learned. So does it all work?

The answer is–maybe. Experiments have been done to show that the equations we have learned work at least sometimes. But science is an inductive process. We can't prove anything true with science. We can only prove things false. So what we have studied is what has not been proven false, yet. Of course, even then, we have taken approximations from time to time, but we pointed these out along the way. You will know when the approximations will fail, because we talked about their valid ranges.

It is important to remember that we are not done discovering new things, and proving old things false. The laws of Newton are approximations that work at low speeds. Relativity provides mechanical equations for very high speeds or in gravitational acceleration fields (e.g. transmitting signals from the GPS satellites to the ground). But is Relativity correct? We think it works pretty well, but really we don't know. We may never know for sure in our Earth life. But we know it works within the range of things we have tried.

There are physicists today that are working on a fundamentally new model of the universe. It is called "String Theory" and it would replace most of our thoughts about how matter is made and how it interacts. The equations would reduce to the ones we used in class for the conditions we considered. That is because the new equations have to match the results of the experiments that we have already done or they can't be correct. But the explanations might be very different.

Often, it is in using physics to build something that we learn about the limitations of physical theory. You may be part of that process. It is a happy process because extending our understanding allows us to build new things. But don't be surprised if some of the things we learned in this class are different by the time your children take their engineering physics course. That is what we should expect of an inductive process.

It is also important to note that revealed truth is not an inductive process. It is still not static (see article of faith 9), but it *can* prove something true as well as prove things false. I hope your BYU-Idaho experience gave you some insight into doing science as well as learning about science.

Some members view science and revelation as in opposition. But I think they are complementary. The scientific process allows us to eliminate things that are not true, allowing us to follow D&C 9:8 in preparation for seeking revelation. During a BYU-I convocation speech, Elder Richard G. Scott described using this process as a nuclear engineer during his engineering career . We can use this combination in our personal lives as well. I hope you will consider this in your careers and lives.

I have tried to give time to both conceptual understanding and mathematical solving. I hope you review and refresh the conceptual understanding of the physics of what you build. Most of my industrial career, we built what we designed very well. We always did our calculations well. But we did, at times, build the wrong thing because the conceptual basis of the design was wrong.

Such mistakes are difficult to fix. Conceptual understanding is a guiding principle for a successful design career. I hope this class has contributed to that conceptual understanding for your careers.

Basic Equations

$$\theta_{\min} = 1.22\frac{\lambda}{D}$$

$$I = I_{\max}\cos^2(\theta)$$