# Chapter 42

Permanent Magnets, Induction

Fundamental Concepts

- Using classical physics, we can't quite explain a permanent magnet.

- Using a semiclassical model, the permanent magnet's field is due to spinning electrons.

- Alignment of the spinning electrons creates what we call magnetism.

- Temporary alignment results in paramagnetism and diamagnetism.

- More permanent alignment yields ferromagnetism.

- A changing magnetic field can create an emf.

42.1   Finally, why bar magnets work

We started our study of magnetism by looking at bar magnets and considering that if we break one, we end up with two magnets. We don't end up with a north end and a south end as separate pieces. This is different than charge and electric fields. Then we studied how moving charge makes a magnetic field. But we didn't say how bar magnets work yet. Can a permanent magnet have something to do with current loops?
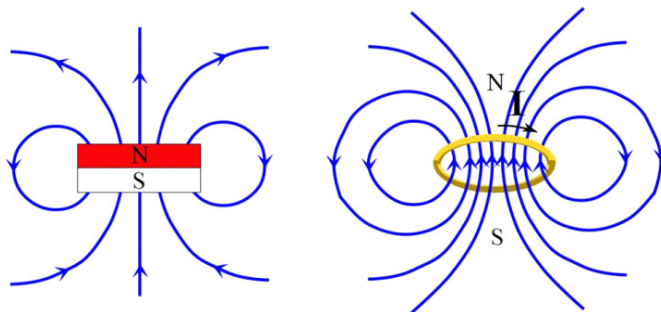
Well, lets look at the field due to a current loop. It looks a lot like the field due to a magnet. Could there be current loops inside a bar magnet? The answer is well, sort of... We have electrons that sort of travel around the atom. Suppose the electrons orbit like planets. (Chemists, just restrain your dismay at this idea for a moment). Then there would be a current as they travel. For one electron the current would be

$$I = \frac{q_e}{T}$$

where T is the period of rotation. And we recall from PH121 or Statics and Dynamics that the period of rotation can be found from

$$\omega = \frac{2\pi}{T}$$

so that

$$T = \frac{2\pi}{\omega}$$

Then the current is

$$I = \frac{q_e \omega}{2\pi}$$

It is an amount of charge per unit time. We can write this as

$$I = q_e \frac{\omega}{2\pi}$$

and recalling

$$v_t = \omega r$$

then

$$I = q_e \frac{v_t}{2\pi r}$$

We can find a magnetic moment (a good review of what we have learned!)

$$
\begin{aligned}
\mu &= NIA = (1)\, IA \\
&= q_e \frac{v_t}{2\pi r} \left(\pi r^2 \right) \\
&= \frac{q_e v_t r}{2}
\end{aligned}
$$

Physicists often write this in terms of angular momentum. Just to review, angular momentum is given by

$$L = \mathbb{I}_m \omega$$

where $\mathbb{I}_m$ is the moment of inertia. Then

$$
\begin{aligned}
L &= \mathbb{I}_m \omega \\
&= \left(mr^2\right)\left(\frac{v_t}{r}\right) \\
&= mrv_t
\end{aligned}
$$

so the magnetic moment of the orbiting electron would be

$$\mu = \frac{q_e L}{2m} \tag{42.1}$$

which gives us a magnetic moment related to the angular momentum of the electron. And if we have a magnetic moment this not only means the atoms would orient in an external field but it also means that the atoms work as little magnets. We will have a magnetic field.

42.1.1 Quantum effects

All of this kind of works for Hydrogen. We find that individual hydrogen atoms do act like small magnets. But if the hydrogen is in a compound, it is more complicated because we then have many electrons and they are "orbiting" in different directions. It is even true that most atoms have many electrons, and within the atom these electrons fly around in all different directions. The magnetic field due to one electron in the atom cancels out the magnetic field due to another, so there is no net magnetic field do to the "motion" of the electrons in their orbits. So in general there is no net magnetic field from orbital motion. Even for Hydrogen in a compound the overall magnetic moment of the compound tends to cancel out.

Worse yet, the electrons don't really move like little planets around the nucleus. So it is not clear that orbital "motion" makes much sense. The electrons form standing waves around the nucleus that oscillate in time. You have probably seen these standing waves drawn in chemistry books.
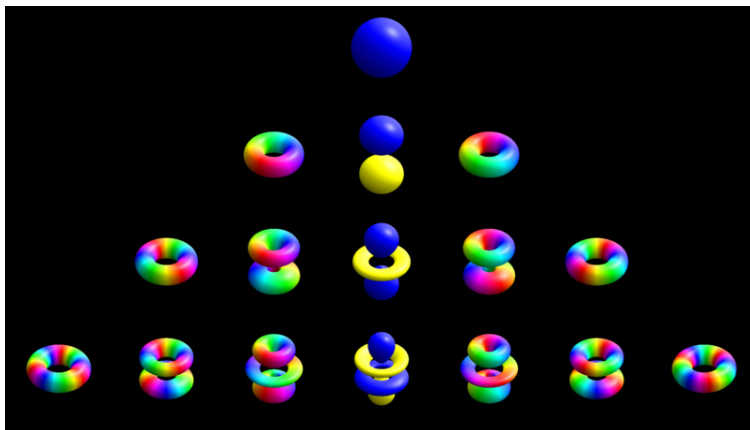
Collection of 16 atomic single-electron orbitals for the lowest four n quantum numbers. Image courtesy Geek3 (https://commons.wikimedia.org/wiki/File:Atomic_orbitals_spdf_m-eigenstates.png)

Because they are three dimensional standing waves their shapes are strange. But these odd standing waves do have angular momentum, and that orbital angular momentum cancels out for most atoms. For more on this, take PH279, Modern Physics.

So our model for magnetism is not really correct yet. To understand the current model of electron orbitals takes some quantum mechanics (and a few more years of physics or one wonderful semester of physical chemistry). But we can understand a little, because quantum mechanics does tell us that the electrons have angular momentum. The big difference is that the angular momentum is *quantized* meaning it can have only certain values (think of the quantized standing wave modes for waves on strings). This means that the magnetic moment has a smallest value and that the other values will be (complicated) multiples of this value. The smallest magnetic moment for an electron turns out to be

$$\mu = \sqrt{2}\frac{q_e}{2m_e}\hbar \tag{42.2}$$

where

$$\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}\,\mathrm{J\,s}$$

is pronounced "h-bar" and is a constant.

It would seem that with only certain values being available the magnetic moments might be more likely to line up. But it turns out that even in quantum mechanics, the magnetic moments of the electrons due to their orbitals cancel each other out most of the time.

But there is another contribution to the magnetic moment, this time from the electron, itself. The electron has an amount of angular momentum. It is as though it spins on an axis. This spin angular momentum is also quantized. It

can take values of

$$S = \pm \frac{\sqrt{3}}{2}\hbar \tag{42.3}$$

My mental picture of this is a charged ball spinning on an axis.[1]
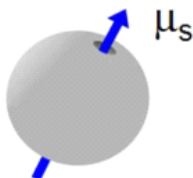
The magnetic moment due to spin is

$$\mu_s = \frac{q_e \hbar}{2m_e} \tag{42.4}$$

This means that electrons, themselves are little magnets. Where does this magnetic moment come from? Well it is *as though* the electron is constantly spinning. It is not really so far as we can tell, but this is a semi-classical mental model that we can use to envision the source of the electron's magnetic field. The "spinning" electron is charged, so the electron acts like a miniscule current loop. The electron, itself is a source of the magnetic field for permanent magnets.

The spin magnetic moment was given the strange name *Bohr magneton* in honor of Niels Bohr. If there are many electrons in the atom, there will be many contributions to the total atomic magnetic moment. The nucleus also has a magnetic moment (a detail we will not discuss at any length in our class) and there are other details like electron spin states pairing up. But those are topics for PH279 and our senior quantum mechanics class (and probably our physical chemistry class). It turns out that this spin magnetic moment is the major cause that produces permanent magnetism in some metals because this spin contribution does not all cancel out for those metals. We don't want to wade though a senior level physics class now (well, you probably don't anyway) so we need a more macroscopic description of magnetism. But fundamentally, if we can get the electrons spins in a material to line up, we will have a magnet.
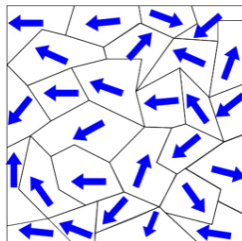
42.1.2 Ferromagnetism

Because of the spin magnetic moment, we can see some hope for how a permanent might work. But these spin magnetic moments are also mostly randomly arranged. So again, most atoms won't have an overall magnetic moment. But some atoms do have a slight net field. They have an odd number of electrons. So the last electron can have an unbalanced magnetic moment. That atom would act as a magnet

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[1] But this is just a mental model. Electrons don't spin. They do have angular momentum. But how electrons generate their angular momentum is still hard to tell. Physicists are working on this.

Still, this does not produce much of an effect, because neighboring atoms all are oriented differently. So neighboring atoms cancel each other out. In a few materials, though, the atoms within small volumes will align their magnetic moments. These little domains form small magnets. But still the overall effect is very small because the domains are all oriented in different directions.
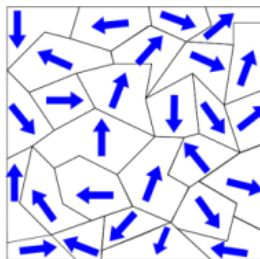
If we place these materials in a magnetic field, we can make the domains align like little compass needles, and then we have something!

Few materials can do this. The ones that can are called ferromagnetic. Iron is one such material. While the piece of iron is next to the magnet that piece of iron is also a magnet. We can make the domains align, but the alignment decays quickly. That is why iron objects stick to a magnet, but don't stick to each other when they are taken away from the magnet. But if we can force the domains to stay in one direction, say, by heating the ferromagnetic metal while it is in a magnetic field and letting it cool and form crystals, then we can make a magnet that will last longer. The magnetic moments will get stuck all pointing about the same direction as the ferromagnetic metal cools. Some materials like Cobalt form very long lasting permanent magnets.

42.1.3   Magnetization vector

We now know that each atom of a substance may have a magnetic moment. For a block of the material, it is useful to think of the magnetic moment per unit volume. We will call this $\overrightarrow{\mathbf{M}}$. It must be a vector, so that if there is an overall magnetic moment, we have a magnet! Let's see how to use this new quantity.

Suppose I have a current carrying wire that produces a field $\overrightarrow{\mathbf{B}}_o$. But I also have a material where $\overrightarrow{\mathbf{M}}$ is not zero. Then there must be a a field due to the

magnetic material $\overrightarrow{\mathbf{B}}_m$. So the total field will be

$$\overrightarrow{\mathbf{B}} = \overrightarrow{\mathbf{B}}_o + \overrightarrow{\mathbf{B}}_m \tag{42.5}$$

and all we have to do is determine the relationship between $\overrightarrow{\mathbf{B}}_m$ and $\overrightarrow{\mathbf{M}}$.

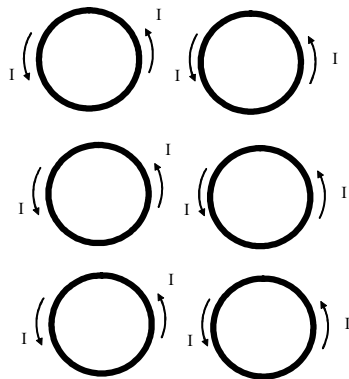### 42.1.4 Solenoid approximation

Lets look at two atoms, We will model them as little current loops, since they have magnetic moments.
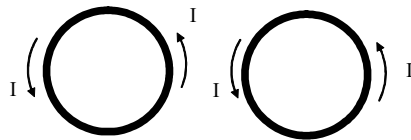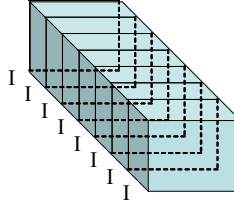
notice that in between the loops, the currents go opposite directions. We could think of them as canceling in the middle. We get a net current that is to the outside of the loops
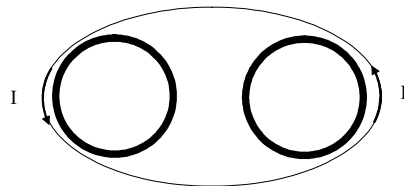
Now let's take many current loops.

again, the inside currents cancel, leaving an overall current along the outside. Now if we view a material as a stack of such current loops

we can model a magnetic material like a solenoid! That is great, because we know how to find the field of a solenoid.

$$\begin{aligned} B_m &= \mu_o n I \\ &= \mu_o \frac{NIA}{\ell A} \end{aligned}$$

I didn't cancel the A's because I want to recognize the numerator as the magnetic moment

$$\mu = NIA$$

so

$$B_m = \mu_o \frac{\mu}{\ell A}$$

But note that ℓA is just the volume of the piece of magnetic material, so

$$B_m = \mu_o \frac{\mu}{\mathbb{V}}$$

which is gives us our new quantity, the magnetization vector

$$M = \frac{\mu}{\mathbb{V}} \tag{42.6}$$

Well, this is the magnitude, anyway, so

$$B_m = \mu_o M \tag{42.7}$$

and of course the directions must be the same, since μ_o is just a scalar constant

$$\overrightarrow{\mathbf{B}}_m = \mu_o \overrightarrow{\mathbf{M}} \tag{42.8}$$

So the total field is given by

$$\overrightarrow{\mathbf{B}} = \overrightarrow{\mathbf{B}}_o + \mu_o \overrightarrow{\mathbf{M}} \tag{42.9}$$

42.1.5   Magnetic Field Strength (another confusing name)

Sometimes we physicists just can't let things alone. So when we arrived at the equation

$$\overrightarrow{\mathbf{B}} = \overrightarrow{\mathbf{B}}_o + \mu_o \overrightarrow{\mathbf{M}} \tag{42.10}$$

someone wanted to define a new term

$$\frac{\overrightarrow{\mathbf{B}}_o}{\mu_o} \tag{42.11}$$

so we could write the equation

$$\overrightarrow{\mathbf{B}} = \mu_o \left(\frac{\overrightarrow{\mathbf{B}}_o}{\mu_o} + \overrightarrow{\mathbf{M}} \right) \tag{42.12}$$

This new term is given an unfortunate name. The *magnetic field strength*. **It is not the magnitude of the magnetic field**, but is the magnitude divided by the constant μ_o. It has it's own symbol, $\overrightarrow{\mathbf{H}}$. So you may write our total field equation as

$$\overrightarrow{\mathbf{B}} = \mu_o \left(\overrightarrow{\mathbf{H}} + \overrightarrow{\mathbf{M}} \right) \tag{42.13}$$

You might find this change unnecessary and confusing (I do) but it is tradition to use this notation, and is not bad once you get used to it.

42.1.6 Macroscopic properties of magnetic materials

We want a way to describe how "magnetic" different substances are without doing quantum mechanics. This will allow us to classify materials, and choose the proper material for whatever experiment or device we are designing.

For many substances we find that the magnetization vector is proportional to the field strength (which is why field strength hangs around in usage)

$$\overrightarrow{\mathbf{M}} = \chi \overrightarrow{\mathbf{H}} \tag{42.14}$$

For many materials, this nice linear relationship applies, and we can look up the constant of proportionality in a table. The name of the constant χ is the *magnetic susceptibility*.

If χ is positive (M is in the same direction as H), we call the material *paramagnetic*.

If χ is negative (M is in the opposite direction as H), we call the material *diamagnetic*.

Using this new notation, our total field becomes

$$\overrightarrow{\mathbf{B}} = \mu_o \left(\overrightarrow{\mathbf{H}} + \overrightarrow{\mathbf{M}} \right)$$

$$\overrightarrow{\mathbf{B}} = \mu_o \left(\overrightarrow{\mathbf{H}} + \chi \overrightarrow{\mathbf{H}} \right)$$

$$\overrightarrow{\mathbf{B}} = \mu_o \left(1 + \chi \right) \overrightarrow{\mathbf{H}} \tag{42.15}$$

The quantity $\mu_o \left(1 + \chi \right)$ is also given a name,

$$\mu_m = \mu_o \left(1 + \chi \right) \tag{42.16}$$

it is called the magnetic permeability. Now you see why μ_o is called the permeability of free space! (the name was not so random after all!). If $\chi = 0$ then

$$\mu_m = \mu_o \qquad (42.17)$$

and this is the case for free space. We can write definitions of paramagnetism and diamagnetism in terms of the permeability.

$$
\begin{array}{ll}
\text{Paramagnetic} & \mu_m > \mu_o \\
\text{Diamagnetic} & \mu_m < \mu_o \\
\text{Free Space} & \mu_m = \mu_o
\end{array}
$$

For paramagnetic and diamagnetic materials, μ_m is usually not too different from μ_o but for ferromagnetic materials μ_m is much larger than μ_o. Note that we have not included ferromagnetic substances in this discussion. That is because

$$\vec{\mathbf{M}} = \chi \vec{\mathbf{H}}$$

is not true for ferromagnetic materials.

42.1.7   Ferromagnetism revisited

But why is ferromagnetism different? To try to understand, let's take a iron toroid (doughnut shape) and wrap it with a coil as shown.
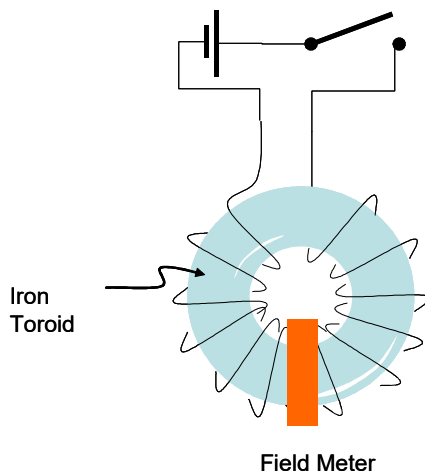
We have a magnetic field meter that measures the field inside the windings of the coil. When we throw the switch, the coil produces a magnetic field. The field will produce a magnetization vector in the iron toroid and, therefore, a field strength. We can plot the applied magnetic field vs. the field strength to see how much effect the applied field has on the magnetic properties of the iron toroid. We won't do this mathematically, but the result is shown in the figure.
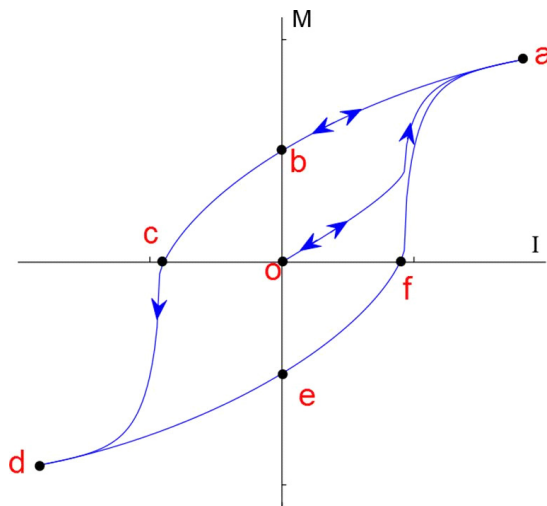
As we throw the switch, we go from no alignment of the domains so zero M and therefore zero induced field in the iron toroid to a value that represents the almost complete alignment of the magnetic moments of each atom of the iron. This is point a. It may take a bit of current, but in theory we can always do this. All the domains are aligned and M is maximum.

Now we reduce the current from our battery, and we find that the field due to the aligned domains drops as expected, but not along the same path that we started on! We go from a to b as the current decreases. At point b there is no current, but we still have a magnetic field in the toroid!

We can even keep going and reverse the field by changing the polarity of our power supply contacts. Since we still have some field in the toroid, it actually takes some reverse current to overcome the residual field. But if we apply enough reverse current, then we get alignment in the other direction. Almost complete alignment is at point d. If we again reduce the current and find that–once again– it does not retrace the same path!

Each time we align the domains with our applied external field from the coil, the domains in the iron toroid seem to want to stay aligned. Most do lose alignment, but some stay put. We have created a weak permanent magnet by placing our ferromagnetic material in a strong external magnetic field.

This strangely shaped curve is the *magnetization curve* for the material. The fact that the path is a strange loop instead of always following the same path is called *magnetic hysteresis*. We can see now that the external field (represented by the current I, since $B_{extrnal} \propto I$) and magnetization don't behave in a simple relationship like they did for diamagnetic or permanganic materials.

The thickness of the area traced by the hysteresis curve depends on the material. It also represents the energy required to take the material through the hysteresis cycle.

If we add enough thermal energy, it is hard to keep the atomic dipole moments aligned. The next figure shows this effect.
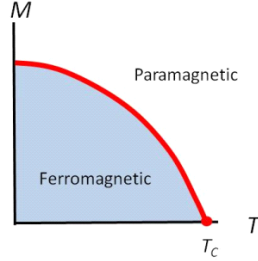
At a temperature called the Curie temperature, the material no longer acts ferromagnetic. It becomes simply paramagnetic. So if we heat up a permanent magnet, we expect it to lose it's alignment and therefore to stop being a magnet. This is what happens to ferromagnetic materials when they are heated due to volcanism. The domains are destroyed and all the atoms lose alignment. Whey the material cools, the Earth's magnetic field acts as an external field and some of the domains will be aligned with this field. This is how we know that the Earth's magnetic field switches polarity. We can see which way the magnetization vector points in the cooled lava deposits from places like the Mid-Atlantic Trench.

This is also how old fashioned magnetic tapes and disks work.

42.1.8   Paramagnetism

We said that if $\mu_m > \mu_o$ we get paramagnetism. But what is paramagnetism? It comes from the material having a small natural magnetic susceptibility.

$$0 < \chi \ll 1 \tag{42.18}$$

So in the presence of an external magnetic field, you can force the little magnetic moments to line up. You are competing with thermal motion as we saw in ferromagnetism, so the effect is usually weak. A rule of thumb for paramagnetism is that

$$M = C\frac{B_o}{T} \tag{42.19}$$

where C contains the particular material properties of the substance you are investigating (another thing to look up in tables in data sets), B_o is the applied field, and T is the temperature. In other words, if it is cool enough, a paramagnetic material becomes a magnet in the presence of an external magnetic field. This is a little like polarization of neutral insulators in the presence of an electric field. For paramagnetic materials, the induced magnetic field is in the same direction as the external field.

Some examples of paramagnetic materials and their susceptibilities are given

below

Material	Susceptibility
Tungsten	6.8×10^{-5}
Aluminium	2.2×10^{-5}
Sodium	0.72×10^{-5}

42.1.9 Diamagnetism.

If $\mu_m < \mu_o$ we said we would have diamagnetism. This is fundamentally quite different from paramagnetism. It comes from the material having paired electrons that orbit the atom (classical model). The magnetic moments of the electrons will have equal magnitudes, but opposite directions (a little bit of quantum mechanics to go with our classical model). When the external field is applied, one electron's orbit is enhanced by the field, and the other is diminished (think $q\mathbf{v} \times B$). So there will be a net magnetic moment. If you think about this for a while, you will realize that the new net magnetic moment is in the opposite direction of the applied external field! So diamagnetism will always repel.

There is always some diamagnetism in all mater. We can enhance the effect using a superconductor. The diamagnetism of the superconductor repeals the external field entirely! Why does this happen only for superconductors? Well, that will take more theory to discover (a great topic for our junior level electrodynamics class). But the phenomena is called the Meissner effect.

42.2 Back to the Earth

So now we can see that the Earth is a magnet and we know how magnets are formed. But wait, why is the Earth a magnet? The real answer is that we don't know. But we believe that again it is because of current loops. We believe there is a current of ionized Nickel and Iron near the center of the Earth. So the flow of these charged liquid metals will create a magnetic field. This is a vary large current loop! The evidence for this is that magnetic field seems proportional to the spin rate of the planet. But this is an area of active research.

It is curious that the magnetic pole and the geographic pole are not in the same place. The magnetic pole also moves around like a precession. Then, every couple of hundred thousand years, the polarity of the Earth's field switches altogether!

There is still plenty of good research to do in this area.

The location of the magnetic pole explains the declination adjustment you have to use when using a compass. What you are really doing is accounting for the difference in pole location.

42.3   Induced currents

We spend most of the last two lectures building a relationship between moving charge (current) and magnetic fields. But suppose we have moving magnetic fields. Could a moving magnetic field make a current?

If we think of relative motion, it seems like it should. After all, how do we know that it is the charge that is moving and not a moving *B*-field. In fact, moving *B*-fields *do* cause a current. We say that a moving or changing magnetic field *induces* a current.

Faraday discovered this effect. He described it as an *induced emf.* An emf is something that "pumps" the charges in the wire. It takes them from a lower to a higher potential so they can form a current. The changing magnetic field must be "pumping" the charges as it changes!

What is really going on here? Think for a minute what must be happening.
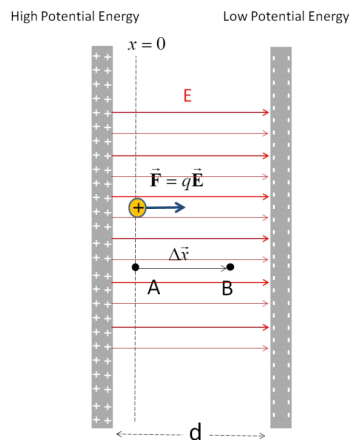
When we defined the electric potential, we use a capacitor. We found that there was a field directed from the + charges to the − charges. And in this field, charges had an amount of potential energy. When a current flows from the + end of the battery to the − end. there must be an electric field acting on the charge in the wire! That is what creates the electric potential. So, then, does a moving magnetic field create an electric field?

The answer is yes! We say that an electric field is *induced* by a moving magnetic field. This is really the same as saying that there is an induced emf for our current loop.
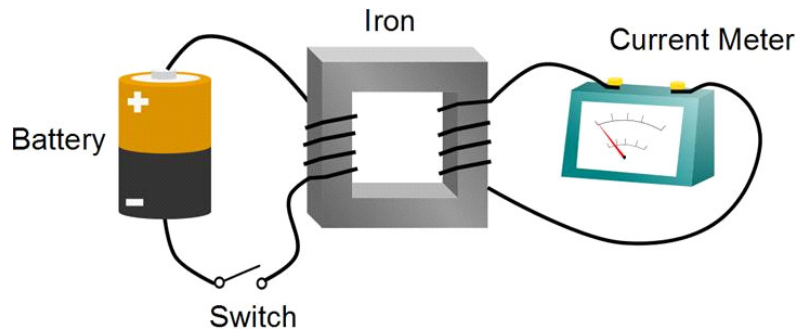
Faraday actually set up his experiment with two coils of wire. One coil was connected to a battery. We now know this coil will make a magnetic field. As the current starts flowing the field will form. While it is forming, it will induce an emf in the second coil. But this is just using an electromagnet instead of a permanent magnet.

To be able to calculate how much current flows, we will need to investigate changing magnetic fields. We will do this next lecture with our concept of flux.

Basic Equations

$$\mu = \frac{q_e v_t r}{2}$$

$$\mu = \frac{q_e L}{2m} \tag{42.20}$$

$$M = \frac{\mu}{\mathbb{V}} \tag{42.21}$$

$$\overrightarrow{\mathbf{H}} = \frac{\overrightarrow{\mathbf{B}}_o}{\mu_o}$$

$$\overrightarrow{\mathbf{B}} = \overrightarrow{\mathbf{B}}_o + \mu_o \overrightarrow{\mathbf{M}} \tag{42.22}$$

$$\overrightarrow{\mathbf{B}} = \mu_o \left(\overrightarrow{\mathbf{H}} + \overrightarrow{\mathbf{M}} \right) \tag{42.23}$$

$$\overrightarrow{\mathbf{B}} = \mu_o \left(1 + \chi \right) \overrightarrow{\mathbf{H}} \tag{42.24}$$

$$\mu_m = \mu_o \left(1 + \chi \right) \tag{42.25}$$

Paramagnetic $\quad \mu_m > \mu_o$
Diamagnetic $\quad \mu_m < \mu_o$
Free Space $\quad \mu_m = \mu_o$

$$\overrightarrow{\mathbf{M}} = \chi \overrightarrow{\mathbf{H}}$$

$$M = C \frac{B_o}{T} \tag{42.26}$$