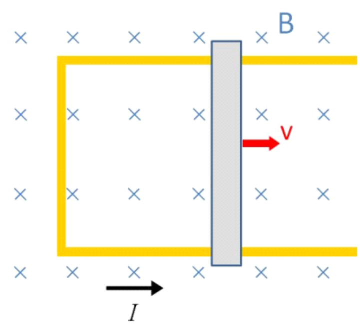
Chapter 25

Faraday and Lenz

We talked about an induced electric field created by a magnetic field last lecture. We want to formalize that relationship in this lecture. Let's go back to our motional emf problem.



We have a sliding bar, and a u-shaped conductor and a magnetic field. The moving bar makes the current flow because of the normal magnetic force on the free charges in the bar. But now we know another way to think about this situation. We can see that there is a magnetic flux through the loop consisting of the u-shaped conductor and the sliding bar. This flux going through the loop is changing. The area is getting larger, so the amount of field going through the loop is increasing. And the motion of the bar was what caused the current to flow and what changed the area of the loop. So the change in the area of the loop is tied to the creation of the current. We can say the induced current is due to the changing loop area in the presence of the magnetic field, or a changing magnetic flux.

An important thing we learned is that the moving bar feels a resistive force due to the current and magnetic field. It seems like the magnetic field and current are resisting any change in our set up. We will see in this lecture that this is true in general.

It turns out that there is more than one way to cause an induced current.

Any change in the magnetic flux is found to make a current flow. Remember in class we found that putting a magnet into or pulling the magnet out of a coil makes a current. In this case, the strength of the magnetic field changes, so the flux changes. Really any change in magnetic flux makes a current flow.

Fundamental Concepts

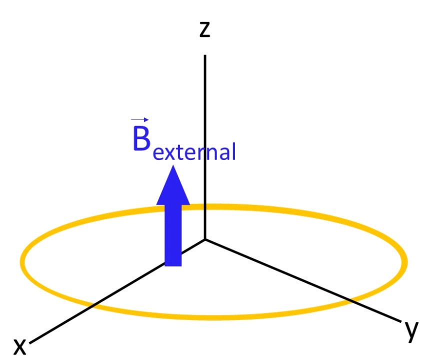
- Changing magnetic flux makes an electric field–which has an associated potential difference or emf.
- The current caused by the induced emf travels in the direction that creates a magnetic field with flux opposing the change in the original flux through the circuit.
- The emf (potential difference) generated by a changing magnetic field is given by $\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$

25.1 Lenz

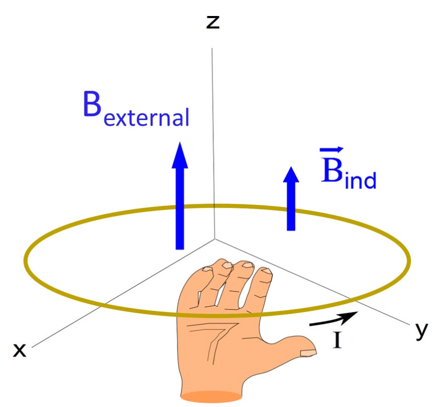
What we are saying is that if we change the magnetic flux through a loop, we will get a current. The direction of current flow is not obvious. We found it last lecture in our motional emf example. But we couldn't just look at the situation to tell which way the current would go. Lenz experimentally found a rule to tell which way it will go. Here is his rule

> The current caused by the induced emf travels in the direction that creates a magnetic field with flux opposing the change in the original flux through the circuit.

This takes a moment to digest. Let's take an example



Consider the case shown in the picture. We have an environmental B-field. One made by, say, some electromagnet that isn't shown in the picture. We will call this an *external* B-field. Suppose the external B-field gets smaller in time (e.g. someone turns down the current in the electromagnet). If that is the case, then the induced current direction will be as if the current wants to keep the same number of magnetic field lines going through the loop. To do this, the induced current makes it's own magnetic field with the field in a direction that will try to keep the flux through the loop the same. So in this case, the induced field $\tilde{\mathbf{B}}_{ind}$ created by the current in the wire will be in the same direction as $\tilde{\mathbf{B}}_{external}$ to try to keep the number of field lines the same.



We find the current direction using our current-carrying wire right hand rule for magnetism. We imagine grabbing the wire such that our fingers curl into the loop the way $\tilde{\mathbf{B}}_{ind}$ goes through the loop. Then our thumb is in the direction of the current.

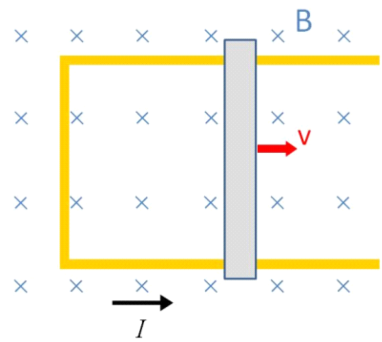
We can think of this as a two step process. The changing external field makes a current in the wire. The current in the wire makes an induced magnetic field. That induced field will be in the direction that tries to keep the flux through the loop from changing.

Of course, current loops don't really "want" anything. The current direction is really determined by looking at the electrical and magnetic forces. But it is *as if* the current wanted to oppose the change in the external field. And we can mentally think of it that way.

25.2 Faraday and Magnetic Induction

In our motional emf problem, the sliding bar in the magnetic field creates a potential difference, ΔV. It becomes an emf. We can use the symbol $\mathcal{E}$ for our emf.

But then in considering Lenz's law, it was experimentally found that any change in flux causes a current. Then any change in flux must create an emf.



In this case the area is getting larger, and so the flux is getting larger. The induced current will oppose the change. So the induced magnetic field should go up through the center of the loop towards us. Imagine sticking your fingers through the loop out of the page, then grabbing the loop (fingers still out of the page in the inside of the loop). Anywhere you grab the wire, your thumb is in the induced current direction.

25.2.1 Faraday's law

Faraday wrote an equation to describe the emf that was given by changing a B-field. It combines what we know about magnetic flux and current from Lenz's law. Faraday did not know the source of the emf, it was a purely empirical equation. Here it is

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

The N is the number of turns in the coil (he used a coil, not just one loop). $\Delta\Phi_B$ is the change in the magnetic flux. Our definition of magnetic flux is

$$\Phi_B = \int \overrightarrow{\mathbf{B}} \cdot d\overrightarrow{\mathbf{A}}$$

but for simple open surfaces and uniform (but changing) fields we can gain some insight by writing the flux as

$$\Phi_B = BA\cos\theta$$

Then the induced emf would be given by

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} \tag{25.1}$$

$$= -N\frac{(B_2A_2\cos\theta_2 - B_1A_1\cos\theta_1)}{\Delta t} \tag{25.2}$$

and we see that we get an emf if B, A, or θ change. We can write this as a differential if we let Δt get very small.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \tag{25.3}$$

Suppose we have a simple flux $\Phi_B = \overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{A}}$, then for this simple case

$$\begin{aligned}\mathcal{E} &= -N\frac{d}{dt}\left(\overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{A}}\right) \\ &= -N\left(\overrightarrow{\mathbf{B}} \cdot \frac{d}{dt}\overrightarrow{\mathbf{A}} + \overrightarrow{\mathbf{A}} \cdot \frac{d}{dt}\overrightarrow{\mathbf{B}}\right)\end{aligned}$$

The first term shows our motional emf case. The area is changing in time. But the second term shows that if the field changes, we get an emf. This is the moving magnet in the coil case.

There are some great applications of induced emfs, from another design for circuit breakers to electric guitar pickups!

25.2.2 Return to Lenz's law

Remember that Lenz's law says the current caused by the induced emf travels in the direction that creates a magnetic field with flux opposing the change in the original flux through the circuit. What if the current went the other way?

If that happened, then we could set up our bar on the rails, and give it a push to the right. With the current going down instead of up (for positive charge carriers) then we would have a force on our bar-like segment of wire

$$F_I = BIL\sin\phi$$

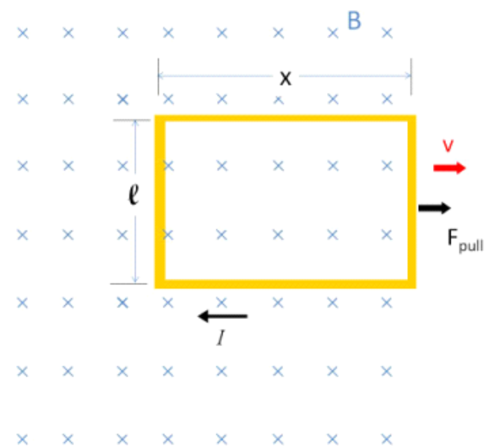
here $\sin\phi = 1$ so

$$F_I = BIL$$

It will be directed to the right. So the bar would accelerate to the right. That would increase the size of the loop, increasing the current. That would increase the force to the right, and our bar would soon zip off at amazing speed. But that does not happen. It would take ever more energy to make the bar go faster, with no input energy. So this would violate conservation of energy. Really Lenz's law just gives us conservation of energy again.

25.3 Pulling a loop from a magnetic field.

Let's try a problem. Suppose we have a wire loop. The loop is rectangular, with side lengths ℓ and x. Further suppose that the loop is in a region with magnetic field, but that it is on the edge of that field, so that if we pull it to the right, it will leave the field.



Let's see if we can find the induced emf and current.

The Magnetic flux through the loop is changing. We can find an expression for the flux. In this case the field is constant and the area isn't changing or tilting.

$$\Phi_B = \overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{A}}$$

We get just where x is the length of the loop and ℓ is the width.

$$\Phi_B = B\ell x$$

We know the emf from Faraday's law

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

then

$$\mathcal{E} = -(1)\frac{d}{dt}(B\ell x)$$

The field is not changing strength, and the length ℓ is not changing. But along the x side, we are losing field. Remember that A in our flux equation is the area that actually has field and we have less area that has field all the time. We can see that

$$\mathcal{E} = -(1)\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt} = -B\ell v$$

where v is the speed at which we are pulling the wire loop. That is the speed at which our flux changes.

We can use Ohm's law to find the current,

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R}$$

or

$$I = \frac{B\ell v}{R}$$

We could ask, how much work does it take to pull the wire out of the field? This is like our capacitor problem where we pulled a dielectric out of the middle of the capacitor.

The net force on the loop is not zero, because the field is no longer uniform. The right hand side of the loop is outside the field, and the left hand side is not. Of course, the top and bottom of the loop have opposite forces that balance each other. So the net force is due to the left hand side of the loop. Recall that

$$\overrightarrow{\mathbf{F}} = I\overrightarrow{\mathbf{L}} \times \overrightarrow{\mathbf{B}}$$

We can see that in this case I is upward, and B is into the page. So there is a force to the left resisting our change flux. We must pull to overcome this force. The magnitude of this force is

$$F = I\ell B$$

and we know I so

$$F = \frac{B\ell v}{R}\ell B = \frac{B^2\ell^2 v}{R}$$

Now we need to find the work done.

$$W = \int F dx$$

or, since our force will be constant until the loop leaves the magnetic field entirely,

$$W = F\int dx$$

which is not a hard integral to do. But instead of performing the integral, let's look at the integrand.

$$dW = F dx$$

if we divide both sides of our equation by dt we have

$$\frac{dW}{dt} = F\frac{dx}{dt}$$

we know that $P = dW/dt$ and $\frac{dx}{dt} = v$ and so we can write our equation as

$$\begin{aligned} P &= Fv \\ &= \frac{B^2\ell^2 v^2}{R} \end{aligned}$$

which is how much power the magnetic field force provides in resisting. We must provide at least an equal power to move the loop. It will take time

$$\Delta t = \frac{\Delta x}{v}$$

to pull the loop a distance Δx. If we define our coordinates such that $x_i = 0$ then to pull out the loop, we will write this time as

$$\Delta t = \frac{x}{v}$$

so the work is

$$\begin{aligned} W &= P\Delta t \\ &= \frac{B^2\ell^2 v^2}{R}\frac{x}{v} \\ &= \frac{B^2\ell^2 xv}{R} \end{aligned}$$

Now think, induced currents must take energy out of a system (it takes energy to make the current go) and because there will be some resistance in our wire this energy will resurface as heat energy. From Ohm's law the power lost due to resistive heating would be

$$\begin{aligned} P &= I^2 R \\ &= \left(\frac{B\ell v}{R}\right)^2 R \\ &= \frac{B2\ell^2 v^2}{R} \end{aligned}$$

which is just the power we had to provide to make our loop move. So our work has moved the loop and heated up the wire.

So, what have we learned? We have created a current in a wire by changing the magnetic field (or the area, or the angle). This is the first step in building a generator. It cost us work to do this. In the next lecture, we will tackle more practical design and build generators and transformers. Then we will pause to think philosophically about what it means that a changing magnetic flux creates an electric field.

Basic Equations

$$\begin{aligned} \mathcal{E} &= -N\frac{\Delta\Phi}{\Delta t} \\ &= -N\frac{(B_2 A_2\cos\theta_2 - B_1 A_1\cos\theta_1)}{\Delta t} \end{aligned}$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$$I = \frac{B\ell v}{R}$$

$$W = \frac{B^2\ell^2 xv}{R}$$

$$P = \frac{B^2\ell^2 v^2}{R}$$