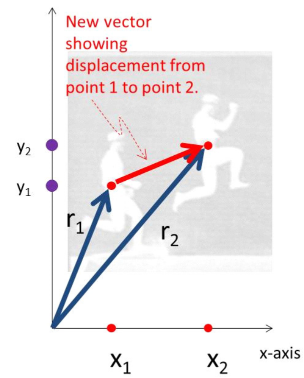
2.1 Vector Addition (and Subtraction)

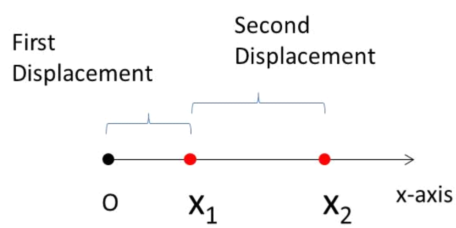
In our example of the jumping man, the displacement from point 1 to point 2 would be given by another arrow. After all, a displacement tells us how far we got and in what direction. The man jumping went a distance and in a particular direction. We use arrows to show how far and in what direction.



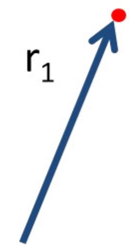
So this new arrow is also a vector, it has a length and a direction. it is the displacement that we need to add to $\overrightarrow{r}_1$ in order to get to $\overrightarrow{r}_2$ or in equation form

$$\overrightarrow{r}_2 = \overrightarrow{r}_1 + \overrightarrow{\Delta r}_{21}$$

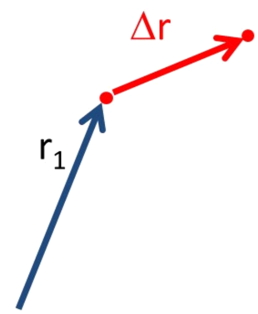
But what does it mean to add vectors? We thought about this before in one dimension. In just the x-direction to add displacements we would go the first displacement, then go the second displacement starting at the end of the first displacement.



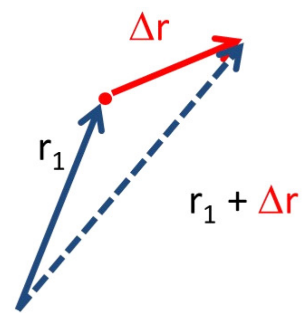
This works the same way for a set of displacements in the y-direction. And this was easy in one dimension, but now we are in two dimensions! But it's not different. We start by thinking of r_1 as the distance from the origin and imagine going that distance so we are at the locatoin of r_1



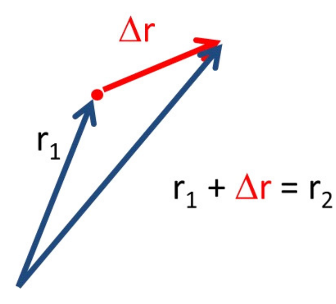
Then from the locatoni of r_1 we start going a distance of Δr in the direction of $\overrightarrow{\Delta r}_{21}$



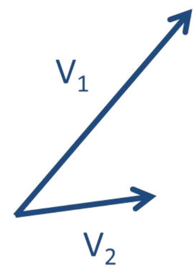
The sum $\overrightarrow{r}_1 + \overrightarrow{\Delta r}_{21}$ is found by drawing a line from the tail of $\overrightarrow{r}_1$ to the tip of $\overrightarrow{\Delta r}_{21}$.



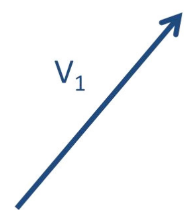
But notice! This dotted line is just $\overrightarrow{r}_2$.



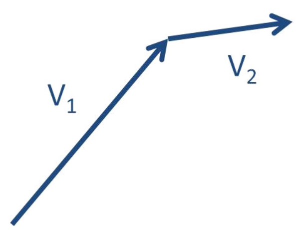
So by adding $\overrightarrow{r}_1 + \overrightarrow{\Delta r}_{21}$ we did get $\overrightarrow{r}_2$. This works with any two vectors. For a sum of any two vectors, $\overrightarrow{\mathbf{V}}_1 + \overrightarrow{\mathbf{V}}_2$ We



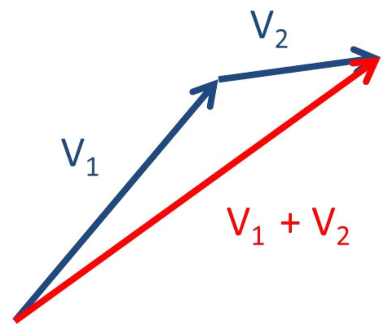
1. Draw $\vec{\mathbf{V}}_1$



2. Draw $\vec{\mathbf{V}}_2$ but start $\vec{\mathbf{V}}_2$ at the place $\vec{\mathbf{V}}_1$ stopped. We sometimes call this drawing the vectors “tip-to-tail.” It’s like two directions in a compass course for those of you who were Boy Scouts.



3. Finally draw a new vector from the tail of $\vec{\mathbf{V}}_1$ to the tip of $\vec{\mathbf{V}}_2$.



this new vector is $\overrightarrow{\mathbf{V}}_1 + \overrightarrow{\mathbf{V}}_2$.

Let's think about if this makes sense. If I tell you to walk in the $\overrightarrow{\mathbf{V}}_1$, a distance V_1 and then to stop and turn into the $\overrightarrow{\mathbf{V}}_2$ direction and walk a distance V_2. You would get to the same place as if you walked in the direction of the red arrow marked $\overrightarrow{\mathbf{V}}_1 + \overrightarrow{\mathbf{V}}_2$. So this does seem to be the sum of two vector displacements.

But in the last section we said that

$$\Delta\overrightarrow{\mathbf{r}} = \overrightarrow{\mathbf{r}}_2 - \overrightarrow{\mathbf{r}}_1$$

That make some sense. If we know that

$$\overrightarrow{r}_2 = \overrightarrow{r}_1 + \overrightarrow{\Delta r}_{21}$$

then is seems we should be able to solve for $\overrightarrow{\Delta r}_{21}$

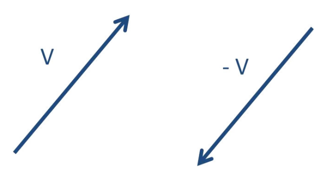
$$\overrightarrow{r}_2 - \overrightarrow{r}_1 = \overrightarrow{\Delta r}_{21}$$

so then

$$\overrightarrow{\Delta r}_{21} = \overrightarrow{r}_2 - \overrightarrow{r}_1$$

But then somehow we need to take $\overrightarrow{\mathbf{r}}_1$ and $\overrightarrow{\mathbf{r}}_2$, subtract them, and end up with $\overrightarrow{\Delta r}_{21}$. How do we subtract vectors?

Let's make a helpful definition: The negative of a vector is a vector of the same size going the opposite direction. So if I have a vector, $\overrightarrow{V}$, as shown in the next figure



then $-\overrightarrow{V}$ will be the vector shown. Think, a negative sign gives us the opposite of a number (at least additively). The opposite of North is South, so with directions the negative of a direction should be "the other way."

The negative of $\overrightarrow{V}$ is just as long as $\overrightarrow{V}$, and goes the opposite direction. Then in our jumping man case we can see that

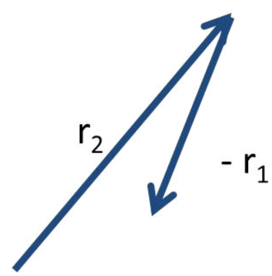
$$\begin{aligned}\Delta\overrightarrow{\mathbf{r}} &= \overrightarrow{\mathbf{r}}_2 - \overrightarrow{\mathbf{r}}_1 \\ &= \overrightarrow{\mathbf{r}}_2 + \left(-\overrightarrow{\mathbf{r}}_1\right)\end{aligned}$$

must mean to take $\overrightarrow{\mathbf{r}}_2$ and add to it a vector that has the length r_1 but goes the opposite direction of $\overrightarrow{\mathbf{r}}_1$. Let's draw $-\overrightarrow{\mathbf{r}}_1$ first

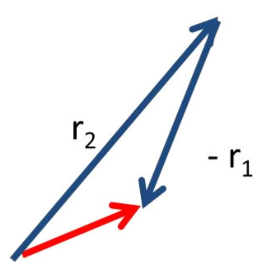
Our displacement is

$$\Delta\overrightarrow{\mathbf{r}} = \overrightarrow{\mathbf{r}}_2 + \left(-\overrightarrow{\mathbf{r}}_1\right)$$

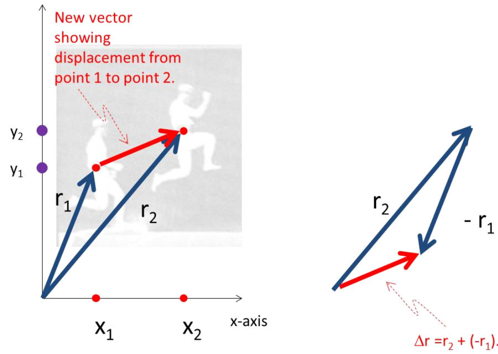
We will travel $\overrightarrow{\mathbf{r}}_2$ and then travel $-\overrightarrow{\mathbf{r}}_1$.



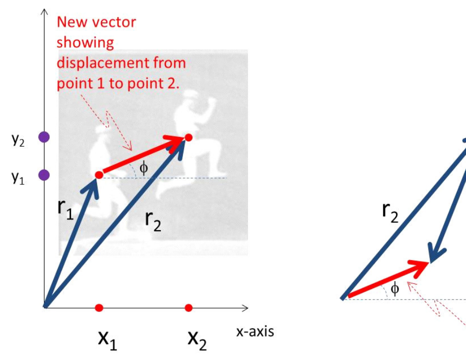
So after going the distance r_2 in the direction of r_2 we then turn into the direction of $-r_1$ and travel the distance r_1. The result is our red vector shown in the next figure



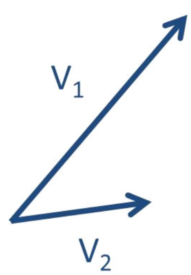
Notice that if we compare this to the displacement of our jumping man



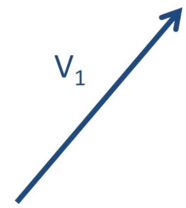
that the two red vectors are exactly the same length and that they point at exactly the same angle! This process seems to have worked!



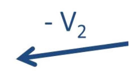
For subtracting two vectors, we just add one additional step. We have to reverse one of the vectors because we are adding a negative displacement For a difference of two vectors, $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2$



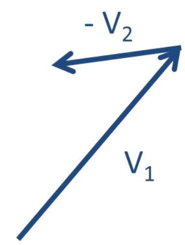
1. Draw $\vec{\mathbf{V}}_1$



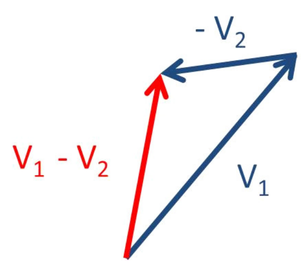
2. Draw $-\vec{\mathbf{V}}_2$ the inverse of $\vec{\mathbf{V}}_2$



3. Now move $-\vec{\mathbf{V}}_2$ so that it starts at the place $\vec{\mathbf{V}}_1$ stopped. We are adding $-\vec{\mathbf{V}}_2$ to $\vec{\mathbf{V}}_1$



4. Finally draw a new vector from the tail of $\vec{\mathbf{V}}_1$ to the tip of $-\vec{\mathbf{V}}_2$.



this new vector is $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2$.

We will use vectors for the rest of this class, for much of PH 123 and all of PH220. If you are a physics major or a mechanical engineering major, you will use vectors for the rest of your career. So it is worth getting used vectors and how to use them.

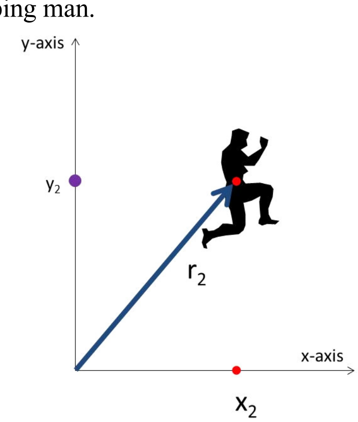
Now that we can describe how the time in an experiment changes, and we can describe how the position in an experiment changes, we can mathematically describe motion. In our next lecture, we will find that the velocity of an object is a combination of position (using displacement) and time

$$\overrightarrow{\mathbf{v}} = \frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t}$$

but this is not too much of a surprise. After all displacement is measured in miles sometimes, and time in hours, and we have been measuring velocities in miles per hour for years now. So we will be on familiar ground!

Position vs. time graphs

We have already used graphs in this class. But so far our graphs have given the position of an object, say, our jumping man.



There is another kind of graph that is useful in describing motion. Since motion requires position and time, a graph of the position and time of the moving object helps us to know how the object is moving. We choose one of the axis of our graph to be time, usually the horizontal axis. Then the vertical axis will be position.