Chapter 1

Problem Solving in Physics

So far we have learned to draw motion diagrams, and we have learned that we can know a lot about the motion of an object by drawing a motion diagram. But we want to apply the power of mathematics to finding the motion of an object. It is time to get ready to do that.

We already know a few basic equations, like our equations for displacement and velocity, and we will learn many more. In what follows for this lecture, I will outline a proven problem solving process, then explain the parts of the process, then give an example of following that process.

1.1 Parts of a problem:

Here are the steps of our problem solving process. You received a copy of this process as part of the Syllabus package on the first day and it is posted on I-Learn under *Course Description*. Here is another copy in figure **??**

1.2 Restate the Problem

The first thing to do when working a homework problem (or a problem given to you in your job, or a problem to test with an experiment, or whatever problem you are assigned to solve), is to restate the problem. You don't get as many points if you solve a different problem than was asked! It is common, especially on tests, to misread the problem. So take some time and make sure you are answering what was asked. In industry, I used to email my boss a restatement of each assignment to make sure I understood what I had been assigned!

1.3 Identify the type of Problem

If you can look at the problem and see it as part of a class of problems, then you know which equations to try, and what techniques to attempt. So far all

Problem Solving Process

Process Step	Purpose	Value if Present	Value if Absent
Label the problem with chapter and problem number	This is essential if I am to figure out what problem to grade	0	0 to -25
Restate the problem in your own words. One line may do! List any assumptions you are making. You may wish to classify the problem in your problem statement	Most major mistakes come from misinterpreting the problem. This step asks you to slow down and determine what the problem really is asking	1	-1
Identify the type of problem. Pick a strategy, is it a Newton's Second Law problem? Is it a rotational problem with constant rotational acceleration?	By identifying the type of problem it is, you are more likely to be able to find the right equations to get started and to successfully complete the problem. The idea is to pause and see what approach will likely be involved in forming the solution. This is not as hard as it sounds. If the word "force" appears, you can reasonably assume Newton's Second Law will be involved, for example.	2	-2
Draw a picture, label items, define coordinate systems, etc. *This picture should be a visual restatement of the problem. View this as a graphical restatement of the problem.*	Many mistakes happen because we do not have a clear picture of the problem. This step may save hours of grief. Also, many physics problems will have different symbolic answers because of the freedom to choose coordinate systems, etc. Drawing a diagram gives the reader the ability to understand your vision of the problem.	5	-5
Define variables used, Identify known and unknown quantities	Choose reasonable names for physical quantities, and let me know what they are. Don't forget to include units.	2	-2
List basic equations that apply to the problem	This step gives you a firm starting place.	2	-2
Solve the problem algebraically starting from the basic equations,	This is the heart of the solution. The symbolic answer tells you the relationships between physical quantities.	10	-10
Determine numerical answer	The specific numerical answer is not the point of doing the problem in this class, but is a great indicator that you have succeeded in understanding the physics.	1	-1
Check units. If you have not done the algebra on the units earlier, do it here.	Many mistakes are evident in a units analysis. It is a good habit to always check units.	1	-1
Determine if the numerical answer is reasonable. Indicate if you are comfortable with the result, if you have little experience with the result and can't tell if it is reasonable, or if it is not reasonable, but you don't know why (or else you would fix it).	From your understanding of the physics, state whether the answer is reasonable. For example, if you are calculating the mass of a ping pong ball, and get an answer that is many times the mass of the earth, you should note that there may be a problem even if you do not know where you went wrong.	1	-1 to -25
Total Possible		25	

Figure 1.1 Problem Solving Process

we have are displacement, velocity, and acceleration problems. But soon we will have many other types.

Suppose you are asked to find the displacement of an object that starts at position $x_i = 5\,\mathrm{m}$ and ends at position $x_f = 12\,\mathrm{m}$. The equation

$$\vec{\mathbf{a}} = \frac{\overrightarrow{\Delta \mathbf{v}}}{\Delta t}$$

is a great resource if you are looking for acceleration, but not so great if you trying to find the displacement. Identifying that the problem asks for displacement helps you realize that the equation

$$\Delta x = x_f - x_i$$

might help.

1.4 Drawing the question

We have spent three lectures learning how to draw diagrams describing motion. We will spend more lectures learning how to draw diagrams for force problems, and equilibrium problems, and rotational problems. You would think it was an art class!

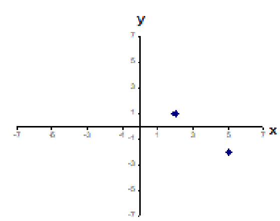
But seriously, learning to express the problem as a visualization is a large part of "doing physics," and the diagram is often the key to seeing how to solve the problem. It is tempting to skip this step. But you must convince yourself to learn how to make the diagrams and to use the diagrams in solving problems.

1.4.1 Coordinate Systems

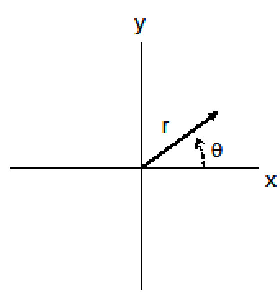
We need a context in which to describe motion graphically. Like a city map, there needs to be a way to tell where we are going and where we have come from. In a western city, we often find a city grid with addresses marked with a number of streets from a central location. Here in Rexburg we have such a system. We have addresses like 300N and 200E. We need such a system for our use.

Often we will use a Cartesian coordinate system (much like the Rexburg system, which is patterned after a Cartesian system).

We have used this system already in our diagrams (even in the context of city blocks).



We could also make a coordinate system by using the distance from the origin and an angle from the x-axis.



Polar

I'm sure you will recognize this as the polar coordinate system. And of course you recognize our position vectors as just position expressed in polar coordinates!

We could extend these to three dimensions (and we will later). Einstein used a very complicated curved three dimensional coordinate system to describe General Relativity. So what seems like a simple idea can become very complicated. Fortunately, we can usually use Cartesian coordinates for most of what we will do.

We will need to think about coordinates a bit. Is there really a zero point in the universe? If not, then are we always free to choose one?

For distances, we do not believe there is a ultimate zero point. When we get to other quantities (like temperature) we will see that sometimes there is a physical zero point.

We have already noticed that our position vector is really just a use of polar coordinates. And in remembering polar coordinates, you probably recall that there is trigonometry involved.

Given the triangle below, we recall that

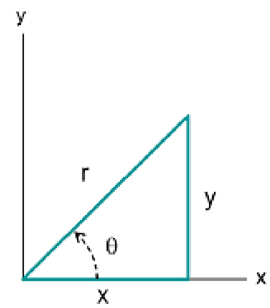


Figure 1.2 Triangle

$$\sin(\theta) = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{y}{r} \tag{1.1}$$

$$\cos(\theta) = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{x}{r} \tag{1.2}$$

$$\tan(\theta) = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{y}{x} \tag{1.3}$$

Suppose I know r and θ but not y, I can find y using

$$y = r\sin(\theta) \tag{1.4}$$

We can use cosine and tangent in a similar way.

Suppose we know r and y, but not θ. We can find this using

$$\theta = \arcsin\left(\frac{y}{r}\right) \tag{1.5}$$

which is often written as

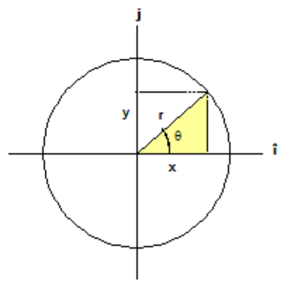
$$\theta = \sin^{-1}\left(\frac{y}{r}\right) \tag{1.6}$$

Also recall the Pythagorean theorem

$$r^2 = x^2 + y^2 \tag{1.7}$$

The combination of these ideas will be used over and over in our study of vectors. If you are a little rusty with trig, it is a good idea to look at the review in the back of our text book.

A nice way to remember the trig functions is to think of our triangle inscribed in a unit circle. The cosine of the angle gives the projection of r onto the x axis. Likewise, the sine functions the projection of r onto the y axis.



As an example of the use of trig functions, we can use them to convert from Cartesian coordinates (x, y) to polar coordinates (r, θ). We start with

$$r = \sqrt{x^2 + y^2} \tag{1.8}$$

from the Pythagorean theorem,

Then we note that

$$\tan(\theta) = \frac{x}{y} \tag{1.9}$$

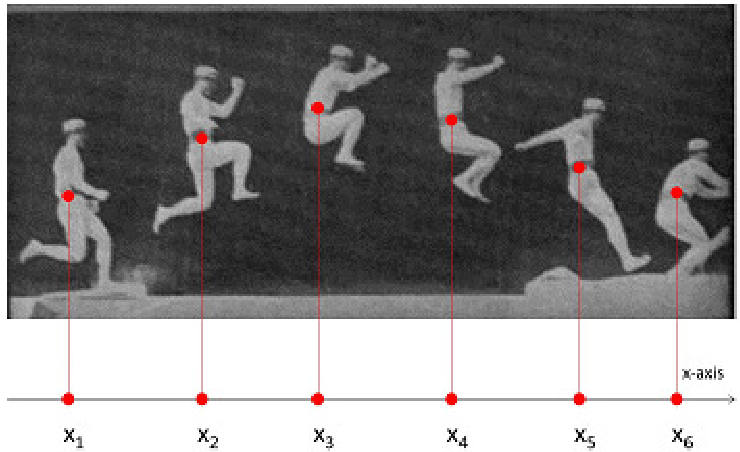
to yield

$$\theta = \tan^{-1}\left(\frac{x}{y}\right) \tag{1.10}$$

You probably remember that we divide circles into angles as shown in the figure above. We often divide the circle into $360\,^\circ$, like 360 pieces of pizza. By adding up 360^{th}'s if a circle we can describe larger angles. This is one way to describe angles. If you took trigonometry, you remember that there are other ways to divide the circle. One that will be very important to us is the *radian*. It is just a slightly bigger pizza piece as a base unit (think of pizza pieces for large foot ball players, you want to start with larger basic units for them!). Your calculator will do trigonometry in degrees or radians, but you have to change the settings to tell the calculator which you want. We will deal with radians a great deal later, but for now you should find out how to set your calculator in degrees mode.

1.5 Defining Variables

Let's look at our jumping man again. It is a great help to realize what you know. Suppose that we know $x_i = 5\,\mathrm{m}$ and $x_f = 12\,\mathrm{m}$ for our man. And suppose we



want to find the man's total displacement. It is much easier to see the known values if we call them out

$$\begin{aligned} x_i &= 5\,\mathrm{m} \\ x_f &= 12\,\mathrm{m} \end{aligned}$$

By placing these on your paper so you can see them, it is easier to find an answer. Seeing the positions like this it is obvious that all you need to do is subtract to get the answer. As the problems become more complicated, this will be an even more important step. It also lets the grader (or in your job, the reader of your report) know what the variable symbols mean. Not every field uses the same letters for the same quantities. And we reuse some letters! The letter T could be "tension" or "period of oscillation." By writing down what the letters mean, you avoid confusing yourself and others.

But what kinds of things are variables? Let's take some time to see what quantities we might define.

1.5.1 Objects

We have been talking about objects, like our jumping man, the bird, or a ball. But what is an object? What is the universe made of?

The startling answer is, we're not entirely sure! Oh, we know that the universe is made of stars and planets and galaxies and dust and many other things, but what are the things made of?

Answering that question is the job of particle physics, and the answer is still in the making. For our study of motion, we will assume there are fundamental things in the universe, and more complicated things are made of these fundamental things. Lets look at three quantities as our initial building blocks. *Mass, Length*, and *Time*.

Mass

When we think of an object, we usually think of something that has mass. Mass is an amount of matter. Exactly what matter is, still somewhat of a question (Job security for Physicists). Einstein equated mass and energy. The great experiments at the Conseil Européen pour la Recherche Nucléaire (CERN) are trying to understand exactly what mass is. You may have heard of the *Higgs boson*, a particle discovered at CERN that gives us a hint that our theory of what mass is might be right. But that is current on-going research. So for this class we shall take mass as just the amount of matter and trust our intuition on what that means. The standard unit of mass is the kilogram, abbreviated " kg." It is the mass of a standard piece of platinum alloy, again kept in France.

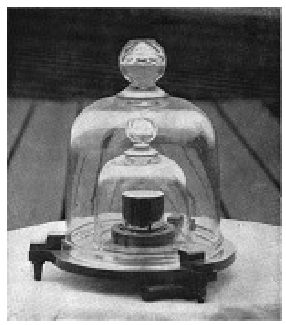


Figure 1.3 US National Institute of Standards standard kilogram.

Note that mass and weight are very different quantities. you can see this if we use a bathroom scale. On earth the scale gives a reading that is proportional to the amount of matter in our bodies, but if we put it on a space craft in orbit, it would not measure any weight at all. Yet the amount of matter in our bodies has not changed!

Length

Perhaps we should really say "space" here. We need to have some idea of how far away things are or how long or tall things are. In this class our view of space will be that it is a container in which things happen. When you study Einstein's Relativity we will change that a little, but for now space is a container, and length is a measure of how far away in this container something is.

In ancient Egypt, the standard of measuring length was when Pharaoh took his ceremonial reed and measured the length of the foundation for the temple (a little like our standard kilogram for mass). This might sound strange, but in essence this is what we all did until 1960. Prior to this, a meter, our unit of length, was defined as one ten millionth of the distance from the North Pole to the equator. Since this was not a very practical day-to-day measuring device, a standard "reed" (this time made of platinum) was kept in France, and meter sticks were made to match this standard. There are terrible problems with this! Each stick of a different materials changes length with temperature! So in 1983 the meter was defined as the distance light traves in vacuum during a time interval of

$$\frac{1}{299792458}\ \mathrm{s}$$

The abbreviation for meter is "m."

Time

You might object that time is not a thing. But we have already used time in our study of motion. It must be something! We should define it before we get to familiar with using time. But what is time? It turns out that time is hard to define. We usually use the idea that time is how long we wait.[1] That can be tricky to measure. Let's start with something simple. How much time will you spend in this class today? That is a time we can wait, about an hour. But it is harder to answer questions like "how long does it take for light to travel a foot." The answer is about a nanosecond. We cannot perceive of times this small. Likewise, we cannot wait for a million years (well, we could, but our vantage point might change after the first 70 years or so).

To measure time we use events that are *periodic*, that is, they occur at regular intervals. An early example is the pendulum of a clock. From a fundamental periodic phenomena, we can build up larger or smaller units of time.

The current unit is the *second*, abbreviated "s," which is given as 91926317000 times the period of oscillation of radiation from the cesium atom. Fortunately we can just use a clock or watch to measure seconds.

The standard for time is the atomic clock.

Figure 1.4 Atomic Clock

[1]Feynman, Richard, Robert Leighton, Matthew Sands, *The Feynman Lectures on Physics*, Vol. I, Addison-Sesley, Reading Massachusetts, 1963

1.5.2 Derived quantities

So we have objects made of mass and space (length) and time to use in describing their motion.

Question 1.1.1

When we combine quantities we derive new quantities that are useful from the basic length, time, mass set. For example, speed is a combination of length and time.

$$v = \frac{\Delta x}{\Delta t} \tag{1.11}$$

quantities like acceleration, force, momentum, etc. are derived quantities.

1.5.3 Dimensional Analysis

Analysis of the units in a measurement can be very useful. For example, if we take our first equation

$$v = \frac{\Delta x}{\Delta t} \tag{1.12}$$

and look at the units, we find that x is a length in, say, meters. We find that t is a time in, say, seconds. Then when we calculate v we should have units of $\mathrm{m/s}$. If, instead, we have m^3 at the end of our calculation, something must have gone wrong! Sometimes it is useful to use generic units for our analysis. That is, any length is given a unit L and any time is given a unit T. So our equation gives

$$\frac{\Delta x}{\Delta t} \Rightarrow \frac{L}{T}$$

To see this lets take and acceleration . It is given by

$$a = \frac{\Delta v}{\Delta t} \Rightarrow \frac{\frac{L}{T}}{T} = \frac{L}{T^2}$$

so from dimensional analysis, we expect that acceleration would be something like

$$a = c\frac{x}{t^2}$$

and we could deduce that

$$x = \frac{1}{c}at^2$$

note that I included a constant, c. Dimensional analysis cannot tell you the constants in an equation. We will see later that in this case $c = 2$ so

$$x = \frac{1}{2}at^2$$

We will find out that our dimensional analysis did not hit too far off the mark. This is part of the equation for position for an accelerating object.

1.5.4 Units

No value in physics is useful without a unit. For example, if I tell you to jump from a height of 100, it makes a difference whether it is 100 cm or 100 m! Units tell us what standard was used to make the measurement so all who see the result can correctly interpret what it means.

1.5.5 System of Units

You will notice that we have only given metric units. We will use the *Système International* or *SI* units. There are, of course, other systems of units. We will try to ignore them in this class. Occasionally we may use feet for length and slugs (yes, slugs) for mass. we will usually use the following SI units for our basic quantities.

Quantity	Unit	Symbol
Mass	Kilogram	kg
Length	meter	m
Time	second	s

The SI system makes use of prefixes to modify the basic unit, like *centi*meter to mean 1/100 of a meter. You should be familiar with the following prefixes.

Prefix	Symbol	Power		Prefix	Symbol	Power
nano-	n	10^{-9}		giga-	G	10^{9}
micro-	μ	10^{-6}		mega-	m	10^{6}
mili-	m	10^{-3}		kilo-	k	10^{3}
centi-	c	10^{-2}		deka-	da	10^{1}
deci-	d	10^{-1}				

We can already see that our unit for mass, the kilogram, must be 1000 grams. A centimeter must be $1/100^{th}$ of a meter. We obviously will need to be able to convert from centimeters to meters from time to time. We should be able to convert from any prefixed unit to any other prefixed unit. We nee a strategy to do this.

1.5.6 Unit Conversions

Let's do a unit conversion that most of you do in your head. Let's convert hours to seconds. We know that

$$1\,\mathrm{h} = 60\,\mathrm{min}$$

and we know that

$$1\,\mathrm{min} = 60\,\mathrm{s}$$

Suppose we have 5 hours. How many seconds is this?

Most of us would say multiply by 3600, and that is right, but let's do it one step at a time so you can see the process. I want to multiply 5 by something, but I can't change the duration. At the end of our calculation, it still has to be a wait of 5 h even though we now give the value in seconds. For a person waiting 5 hours and a second one waiting 18 000 s they must feel the same amount of time. So we need to adjust our 5 hours by something that does not change the wait.

I think you will agree that if I multiply by 1 nothing changes

$$5 \times 1 = 5$$

We can do this with units

$$5\,\mathrm{h} \times 1 = 5\,\mathrm{h}$$

now let's take our equation relating hours to minutes.

$$1\,\mathrm{h} = 60\,\mathrm{min}$$

and let me divide by 1 h

$$\begin{aligned} \frac{1\,\mathrm{h}}{1\,\mathrm{h}} &= \frac{60\,\mathrm{min}}{1\,\mathrm{h}} \\ 1 &= \frac{60\,\mathrm{min}}{1\,\mathrm{h}} \end{aligned}$$

I can multiply by the right hand side of this equation and all I am doing is multiplying by 1!

$$5\,\mathrm{h} \times \frac{60\,\mathrm{min}}{1\,\mathrm{h}} = 5\,\mathrm{h}$$

This still must be true, but let's do the math

$$5\,\mathrm{h} \times \frac{60\,\mathrm{min}}{1\,\mathrm{h}} = 300\,\mathrm{min}$$

this is how many minutes are in 5 hours. We can play the same trick with minutes to seconds

$$\begin{aligned} \frac{1\,\mathrm{min}}{1\,\mathrm{min}} &= \frac{60}{1\,\mathrm{min}}\,\mathrm{s} \\ 1 &= \frac{60\,\mathrm{s}}{1\,\mathrm{min}} \end{aligned}$$

so we can take our 300 min and find out how many seconds we have!

$$300\,\mathrm{min} \times \frac{60\,\mathrm{s}}{1\,\mathrm{min}} = 18000.0\,\mathrm{s}$$

Now we could do this all in one equation, using our strange way of writing 1 that converts from hours to minutes and our strange way of writing 1 that converts from minutes to seconds.

$$\begin{aligned} 5\,\mathrm{h} \times 1 \times 1 &= 5\,\mathrm{h} \\ 5\,\mathrm{h} \times \frac{60\,\mathrm{min}}{1\,\mathrm{h}}\frac{60\,\mathrm{s}}{1\,\mathrm{min}} &= 18000.0\,\mathrm{s} \end{aligned}$$

Notice that the units cancel like variables in algebra!

We will treat units like algebraic quantities that can be canceled. Let's do another example. We want to convert 10 mi (ten miles) to kilometers. We know that

$$1\,\mathrm{mi} = 1609\,\mathrm{m} \tag{1.13}$$

and we know that

$$1\,\mathrm{km} = 1000\,\mathrm{m} \tag{1.14}$$

Start with conversion from miles to meters. We recognize that with a small use of algebra

$$1 = \frac{1609\,\mathrm{m}}{1\,\mathrm{mi}} \tag{1.15}$$

then we can write

$$10\,\mathrm{mi} = 10\,\mathrm{mi}\frac{1609\,\mathrm{m}}{1\,\mathrm{mi}} = 16\,090\,\mathrm{m} \tag{1.16}$$

Then we also recognize that

$$1 = \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} \tag{1.17}$$

or

$$1 = \frac{1\,\mathrm{km}}{1000\,\mathrm{m}} \tag{1.18}$$

then

$$16\,090\,\mathrm{m} = 16\,090\,\mathrm{m}\frac{1\,\mathrm{km}}{1000\,\mathrm{m}} = 16090.0\,\mathrm{m} \tag{1.19}$$

so

$$10\,\mathrm{mi} = 16\,\mathrm{km} \tag{1.20}$$

We could do this all in one large, chained, conversion

$$10\,\mathrm{mi} = 10\,\mathrm{mi}\frac{1609\,\mathrm{m}}{1\,\mathrm{mi}}\frac{1\,\mathrm{km}}{1000\,\mathrm{m}} = 16\,\mathrm{km} \tag{1.21}$$

If you think about it, to convert units, we have multiplied by 1 several times. So as you multiply to convert units, make sure your factors multiply are equal to 1.

1.5.7 Uncertainty in measurements

In science, we must face the fact that no measurement is completely accurate. The reasons for uncertainty are limitations in our human sensory system or sensing apparatus. For example, if I measure a square of metal with a ruler. I am likely not able to tell the length to better than a tenth of a centimeter (1 mm). This is because of inaccuracies in the ruler and in my own ability to see the ruler clearly and consistently. So suppose I have a measurement of 16.3 cm. I can really only tell you that the measurement is between 16.4 cm and 16.2 cm. we could write this as 16.3 ± 0.1 cm. We will study uncertainty in measurements in some detail in PH150. But for PH121 we will need some provisional rules that let us make a guess on now good our answers are.

Significant figures

Scientists have devised a clever way to include the level of uncertainty in the statement of the measurement result. This is referred to as *significant figures* and it basically means to keep only the digits in a number that contain well known information. In the above example, we would say that in 16.3 cm that the 3 is the least significant digit. Now suppose we use the same ruler to measure the same object, but I tell you that the measurement is 16.3259357 cm. If we know the measurement is only good to ± 0.1 cm, what can we say about the digits 259357? We can say they are worthless! They are nonsense, so we cleverly leave them off! There are a series of rules to tell us which digits are significant. It is important to realize that zeros that just mark where the decimal place goes are not significant (e.g. in 0.00163 cm the three 0's are not significant, but in 1.400 cm the digits mean that the measurement is known to ± 0.001 cm).

We usually express numbers in scientific notation.

Propagation of uncertainty

Suppose we take two measurements, like measuring the sides of a rectangle.

$$\begin{aligned} l &= (2.3 \pm 0.1)\ \text{cm} \\ w &= (4.5 \pm 0.1)\ \text{cm} \end{aligned}$$

and we wish to find the area

$$A = l \times w$$

$$\begin{aligned} A &= 2.3\,\text{cm} \times 4.5\,\text{cm} \\ &= 10.35\,\text{cm}^2 \end{aligned}$$

But we were a little uncertain about the length and width, wouldn't we also be uncertain about the area that we made from the uncertain length and the width? Of course there is some uncertainty in the area. Let's see how we could deal with this.

The length could have been as much as

$$l = 2.3\,\text{cm} + 0.1\,\text{cm} = 2.4\,\text{cm}$$

and the width could be as much as

$$w = (4.5 + 0.1)\ \text{cm} = 4.6\,\text{cm}$$

So the area could be as much as

$$\begin{aligned} A_+ &= 2.4\,\text{cm} \times 4.6\,\text{cm} \\ &= 11.04\,\text{cm}^2 \end{aligned}$$

But the length could be as little as

$$l = 2.3\,\text{cm} - 0.1\,\text{cm} = 2.2\,\text{cm}$$

and the width could be as little as

$$w = (4.5 - 0.1)\ \text{cm} = 4.4\,\text{cm}$$

So the area could be as little as

$$\begin{aligned} A_- &= 2.2\,\text{cm} \times 4.4\,\text{cm} \\ &= 9.68\,\text{cm}^2 \end{aligned}$$

We can see that these differ by about $\pm 1\,\text{cm}^2$ total.

$$A_+ - A_- = 11.04\,\text{cm}^2 - 9.68\,\text{cm}^2 = 1.36\,\text{cm}^2$$

Thus the tenths and hundredths places in our calculated area cannot be very certain. We drop these and write

$$A = 10 \pm 1\,\text{cm}$$

Notice that our length and width had two digits,

$$\begin{aligned} l &= (2.3 \pm 0.1)\ \text{cm} \\ w &= (4.5 \pm 0.1)\ \text{cm} \end{aligned}$$

with the uncertainty in the second digit, and notice that our answer for the area has two digits, and the uncertainty is in the second digit!

In general:

> *In multiplying or dividing two quantities, the number of significant figures in the product or quotient is the same as the number of significant figures in the least accurate of the factors being combined.*

In our example, l and w both have two significant figures, so the result should be limited to two significant figures

For addition and subtraction the rules is:

> *The number of decimal places in the result should equal the smallest number of decimal places in any term in the sum or difference.*
>
> These two rules will help us determine how many digits to keep for most problems. You may know that there are several other rules, and thought we wont derive them like we did for multiplication, we

will use them in our problems. Here they are in a table

Significant Figure Rules
1. Non-zero digits are always significant
2. Embedded (i.e. captive) zero-digits are always significant
3. For the *number* zero only the zero-digits after the decimal are significant
4. Leading zero-digits are not significant
5. Trailing zero digits:
If the number has a decimal, the trailing zero digits are significant
If the number does not have a decimal, the trailing zero digits are not significant
6. The final result of multiplication or division operation should have the same number of significant digits as the measured quantity with the least number of significant figures used in the calculation.
7. The final result of an addition or subtraction operation should have the same number digits to the right of the decimal as the measured quantity with the least number of decimal used in the calculation.
8. For a mixture of operations, work from left to right, do mathematical hierarchy of operations ($\times$ or $\div$, then $+$ or $-$).

1.6 Basic Equations

Equations are relationships in physics. The equation

$$\vec{\mathbf{v}} = \frac{\overrightarrow{\Delta \mathbf{x}}}{\Delta t}$$

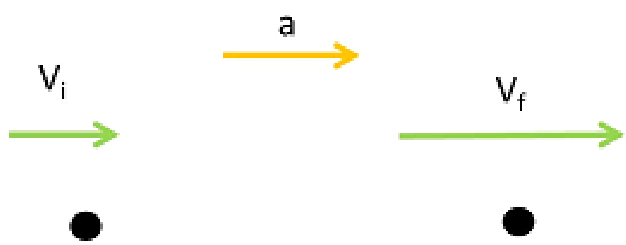
tells us how the displacement and duration combine to form velocity. These equations are our way of expressing motion. They are the tools in our toolbox for solving problems. Once you have identified the type of problem you have, you can quickly write down a list of equations (tools) that you could use to solve that problem. You might write down more equations than you end up using for a particular problem. That's OK. You don't empty your tool box of all tools but the ones you think you might use when you start a fix-up job in your house! You should not do so when starting a problem. List your equations, then choose the ones that seem to work given your known values in your list of variables.

1.7 Solving with symbols

For years now, you have worked with numbers and answers that are numbers. And that is what the teacher was looking for. But in physics the equation is the important thing. It tells you how things relate to each other. Let's try an example:

`The initial speed of an object is` $3\,\mathrm{m/s}$ `and it's acceleration is` $2\,\mathrm{m/s^2}$ `in the same direction as the velocity. Find the speed half a second after the experiment start.`

We want to start by restating the problem:
`Find the final speed knowing` a `and` v_i
Next identify the type of problem. I think it is an acceleration problem:
`PT acceleration`
Next we want to draw the picture



Our variables list is next:
`VAR`

$$\begin{aligned} v_i &= 3\,\frac{\mathrm{m}}{\mathrm{s}} \\ a &= 2\,\frac{\mathrm{m}}{\mathrm{s}^2} \\ \Delta t &= 0.5\,\mathrm{s} \end{aligned}$$

and now basic equations:
`BE:`

$$\begin{gathered} \underline{\vec{\mathbf{a}}} = \frac{\overrightarrow{\Delta \mathbf{v}}}{\underline{\Delta t}} \\ \overrightarrow{\Delta \mathbf{v}} = \vec{\mathbf{v}}_f - \underline{\vec{\mathbf{v}}_i} \\ \vec{\mathbf{v}} = \frac{\overrightarrow{\Delta \mathbf{x}}}{\Delta t} \\ \overrightarrow{\Delta \mathbf{x}} = \vec{\mathbf{x}}_f - \vec{\mathbf{x}}_i \end{gathered}$$

Note that I underlined the known values from my list variables. The first two equations have v_f in them and they contain my known values, so it looks like they are the ones to use in my solution.

1.7.1 Solve Algebraically

Now we try to solve for the speed, but we do so symbolically. We already believe that the first two equations in our basic equation list will be helpful, so let's start with

$$\underline{\vec{\mathbf{a}}} = \frac{\overrightarrow{\Delta \mathbf{v}}}{\underline{\Delta t}}$$

and put in

$$\overrightarrow{\Delta \mathbf{v}} = \overrightarrow{\mathbf{v}}_f - \underline{\overrightarrow{\mathbf{v}}_i}$$

$$\underline{\overrightarrow{\mathbf{a}}} = \frac{\overrightarrow{\mathbf{v}}_f - \underline{\overrightarrow{\mathbf{v}}_i}}{\underline{\Delta t}}$$

At this point I recognize that I can solve for v_f and everything else is known. I could plug things in my calculator and have it solve for v_f using numbers, but we won't do that! We will continue with algebra

$$\overrightarrow{\mathbf{a}}\Delta t = \overrightarrow{\mathbf{v}}_f - \overrightarrow{\mathbf{v}}_i$$

and

$$\overrightarrow{\mathbf{a}}\Delta t + \overrightarrow{\mathbf{v}}_i = \overrightarrow{\mathbf{v}}_f$$

or

$$\overrightarrow{\mathbf{v}}_f = \overrightarrow{\mathbf{v}}_i + \overrightarrow{\mathbf{a}}\Delta t$$

Since $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{v}}_i$ are in the same direction, their magnitudes just add so

$$v_f = v_i + a\Delta t$$

This is the symbolic answer. It has the thing I want, v_f, an equals sign, and then symbols for what v_f is equal to.

1.8 Numeric answer

The numeric answer is easy. Just plug in numbers to your symbolic answer

$$v_f = v_i + a\Delta t$$

$$\begin{aligned} v_f &= 3\frac{\mathrm{m}}{\mathrm{s}} + \left(2\frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.5\,\mathrm{s}) \\ &= 4.0\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

1.9 Reasonableness check

If we had gotten 400000000000 m/ s in our example, we would know something went wrong. Nothing can go this fast! It is good to check your answer and see if it seems to make sense. But how do we do that? One way is to do a quick estimate.

1.9.1 Estimates

There are times when we simply do not have all the information we need, but we need a number for something anyway. There are also times when we wish to make a quick calculation (like checking on the reasonableness of a calculation). In such cases, we estimate. Many people in science are somewhat uncomfortable with estimates, because they are not "correct" (In business and politics, people may be a little too comfortable with estimates).

Let's do a few examples together.

Example 1: How many sheets of paper fit between the Earth and Moon?

To do this calculation, we need to know how far away the Moon is and how thick a piece of paper is.

$$D_{EM} = 4000000\,\mathrm{km} \tag{1.22}$$

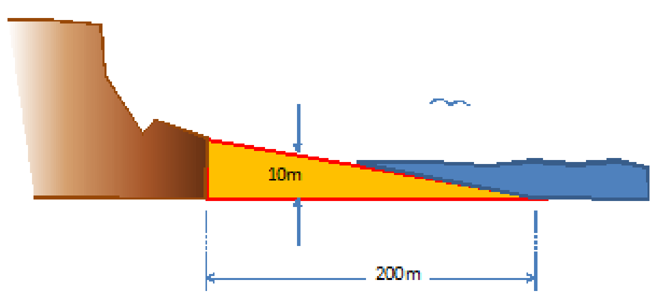
$$\begin{aligned} t &= \frac{3\,\mathrm{mm}}{28} = 1.071\,4 \times 10^{-4}\,\mathrm{m} \qquad (1.23) \\ &\approx 1 \times 10^{-4}\,\mathrm{m} \end{aligned}$$

so the number of pieces of paper would be

$$\begin{aligned} N &= \frac{D_{EM}}{t} = \frac{4000000\,\mathrm{km}}{1 \times 10^{-4}\,\mathrm{m}} \frac{1000\,\mathrm{m}}{\mathrm{km}} \qquad (1.24) \\ &= 40\,000\,000\,000\,000 \\ &= 4.0 \times 10^{13} \\ &\approx 1 \times 10^{13} \qquad (1.25) \end{aligned}$$

Is this reasonable? Well look at Example 1.7 in the book and see what you think (and explain why our answers are different).

Example 2 How much sand is in the world's beaches?



We start by looking for a fundamental element of a beach, say, a grain of sand. We can calculate the total volume of the beaches, and divide by the volume of a grain of sand. This will tell us how many grains of sand there are in the world's beaches. If we can get an estimate of the mass of a grain of sand, then we can answer how much sand is in the world's beaches.

Lets estimate the grain of sand to have a mass of

$$m = 0.005\,\mathrm{kg} \tag{1.26}$$

Let's guess a volume of

$$V_s = 1.0 \times 10^{-15}\,\mathrm{m}^3 \tag{1.27}$$

We can estimate the length of the world's coastline to be

$$L = 40000000000\,\mathrm{m} \tag{1.28}$$

we need the volume of the coast line, lets say the beach is

$$w = 200\,\mathrm{m} \tag{1.29}$$

wide and

$$d = 10\,\mathrm{m} \tag{1.30}$$

deep.

Then the volume of the world's beaches would be

$$\begin{aligned} V &= Lwd = (40000000000\,\mathrm{m})\,(200\,\mathrm{m})\,(10\,\mathrm{m}) && (1.31)\\ &= 8.0 \times 10^{13}\,\mathrm{m}^3 && (1.32) \end{aligned}$$

and the number of grains of sand would be

$$N = \frac{V}{V_s} = \frac{Lwd}{V_s} = 8.0 \times 10^{28} \tag{1.33}$$

which gives us

$$\begin{aligned} M_{beach} &= Nm = \left(8.0 \times 10^{28}\right)(0.005\,\mathrm{kg}) && (1.34)\\ &= 4.0 \times 10^{26}\,\mathrm{kg} && (1.35) \end{aligned}$$

Mass of the Earth

$$M_E = 5.98 \times 10^{24}\,\mathrm{kg} \tag{1.36}$$

where did we go wrong?

Consider silicon oxide. It has a density of

$$\rho = 2200\frac{\mathrm{kg}}{\mathrm{m}^3} \tag{1.37}$$

If our estimate of

$$m = 0.005\,\mathrm{kg} \tag{1.38}$$

is good, then we should have used a volume of

$$V = \frac{m}{\rho} = \frac{0.005\,\mathrm{kg}}{2200\,\frac{\mathrm{kg}}{\mathrm{m}^3}} \tag{1.39}$$

$$= 2.2727 \times 10^{-6}\,\mathrm{m}^3 \tag{1.40}$$

So one problem is that our estimate of the volume of a grain of sand is very bad.

In general, you can be creative in making estimates, but you do have to be careful.

1.10 Units Check

We already have discussed units. But it is important to check your units in your final answer. In our case, the final units must be a length unit divided by a time unit. For speed this must be the case. If we had gotten a length unit divided by a time squared, then we would know something went wrong in our algebra. If the units don't work the answer is wrong. So especially on a test (or in your real job) checking units is important!

1.11 Problem Solving Process

I have assembled all of the problem solving pieces that we have studied into a process that leads us to our solution. Here is the process we will use in a table:

Notice that I have assigned point values to each part. So if you leave out a part you will know how many points you will miss. Also notice that there are a lot of points for the symbolic answer and only a one for the numeric answer. Physics is about relationships that describe motion, the numbers just aren't that important. So it is not a winning strategy to only give me the numeric answer for a problem. you only get one out of twenty five points!

Also notice that if I ask for the mass of a ping pong ball, and you give me an answer that is three times the mass of Jupiter, I can take off more than one point for the very wrong numeric answer! Since you will be doing a reasonableness check, you won't have this problem. But what if you don't know if an answer is reasonable? Then say you don't know! You will act differently as a physicist, engineer, doctor, etc. if you admit in your calculations that you are not certain of the result. And this is very valuable! It can save your job! So if you are not sure, say so.

We will use this process for the rest of the semester, and this or a similar process for PH123, and PH220 (or PH223) and if you are a physicist for the rest of your career. This is also the process I used in engineering in industry. So it is worth practicing in our problems. It is also how I will grade the tests!

Problem Solving Process

Process Step	Purpose	Value if Present	Value if Absent
Label the problem with chapter and problem number	This is essential if I am to figure out what problem to grade	0	0 to -25
Restate the problem in your own words. One line may do! List any assumptions you are making. You may wish to classify the problem in your problem statement	Most major mistakes come from misinterpreting the problem. This step asks you to slow down and determine what the problem really is asking	1	-1
Identify the type of problem. Pick a strategy, is it a Newton's Second Law problem? Is it a rotational problem with constant rotational acceleration?	By identifying the type of problem it is, you are more likely to be able to find the right equations to get started and to successfully complete the problem. The idea is to pause and see what approach will likely be involved in forming the solution. This is not as hard as it sounds. If the word "force" appears, you can reasonably assume Newton's Second Law will be involved, for example.	2	-2
Draw a picture, label items, define coordinate systems, etc. *This picture should be a visual restatement of the problem. View this as a graphical restatement of the problem.*	Many mistakes happen because we do not have a clear picture of the problem. This step may save hours of grief. Also, many physics problems will have different symbolic answers because of the freedom to choose coordinate systems, etc. Drawing a diagram gives the reader the ability to understand your vision of the problem.	5	-5
Define variables used, Identify known and unknown quantities	Choose reasonable names for physical quantities, and let me know what they are. Don't forget to include units.	2	-2
List basic equations that apply to the problem	This step gives you a firm starting place.	2	-2
Solve the problem algebraically starting from the basic equations,	This is the heart of the solution. The symbolic answer tells you the relationships between physical quantities.	10	-10
Determine numerical answer	The specific numerical answer is not the point of doing the problem in this class, but is a great indicator that you have succeeded in understanding the physics.	1	-1
Check units. If you have not done the algebra on the units earlier, do it here.	Many mistakes are evident in a units analysis. It is a good habit to always check units.	1	-1
Determine if the numerical answer is reasonable. Indicate if you are comfortable with the result, if you have little experience with the result and can't tell if it is reasonable, or if it is not reasonable, but you don't know why (or else you would fix it).	From your understanding of the physics, state whether the answer is reasonable. For example, if you are calculating the mass of a ping pong ball, and get an answer that is many times the mass of the earth, you should note that there may be a problem even if you do not know where you went wrong.	1	-1 to -25
Total Possible		25	