`PHSX815_Project4`: Determining Binary Separation

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1 Introduction

One of the important problems in modern astronomy is the detection of a binary system. If we detect a binary system using direct imaging, the most basic information we can ask is about the position of the stars. Given multiple images of a binary system taken at similar times, we would like to determine the separation distance between the two stars. By seeing how the separation distance changes in time, we can attempt to determine further orbital parameters, such as the total mass of the system, orbital period, and orbital inclination. However, atmospheric seeing and detector limits can impede our measurement [1]. Therefore, determining the position of two sources is the first step towards determining these more interesting physical parameters.

2 Problem Statement

Suppose a sequence of two-dimensional images with two sources on it. We assume that the data has been cleaned such that the detector noise has been removed and that the two sources are sufficiently bright so that they are the only two sources in the frame. If the images are taken in a small period of time, then the source positions are fixed. However, because counting photons on the detector is a stochastic process, as well as atmospheric seeing, the apparent positions of the objects may vary over time. If we wish to determine the separation distance between two stars, this problem reduces to finding the most likely position of each individual star given the data, then determining the distance.

We must have a model of how point sources appear on our detector. For point sources passing through a circular aperture, the point spread function is the Airy disk (Equation 1).

$$I\left(x\right) = I_0 \left(\frac{2J_1\left(x\right)}{2x}\right)^2 \tag{1}$$

In Equation 1, x is proportional to the radial distance from the center of the disk and J_1 is the Bessel function of the first kind. We will simulate each observation by taking samples from this distribution using fixed positions for the two sources. Using this simulated data, we will determine the positions of the sources and then calculate the separation distance.

3 Algorithm Analysis

3.a Data Generation

To generate the data, we generate a 100×100 grid of pixels. The two stars have fixed true positions. We approximate seeing as a circularly symmetric, normally-distributed process so that the x and y coordinates of the stars are shifted according to a normal distribution with a single standard deviation σ. Knowing the central x and y values of the stars, we can translate the Airy disk pattern to be centered on the stars. We can then find the intensity of the Airy disk pattern at each position on the detector and use those values as Poisson rate parameters. The number of counts from the Poisson distribution gives us our data. An example of the generated data is presented in Figure 1.

3.b Finding Components

If we are handed an image with two sources on it, we must algorithmically find which pixels on the detector correspond to each source. In the present case, where the sources on the detector are sufficiently bright, we can use a version of k-means clustering to assign each pixel with a star. The procedure goes as follows:

1. Guess two points to be the cluster centers.
2. With fixed centers, calculate the square distance between each point and each center.
3. Correspond each point with a center by minimizing the square distance.
4. With fixed correspondence, calculate new center positions by taking the flux-weighted average of the points (Equation 2).
5. Go to 2. Repeat until the correspondences converge.

In Equation 2, I_j is the flux (or number of photon counts) at point x_j. An example of the result is presented in Figure 2, while the separation of the data is presented in Figure 3.

$$\bar{x}_i = \frac{\sum_j I_j x_j}{\sum_j I_j} \tag{2}$$

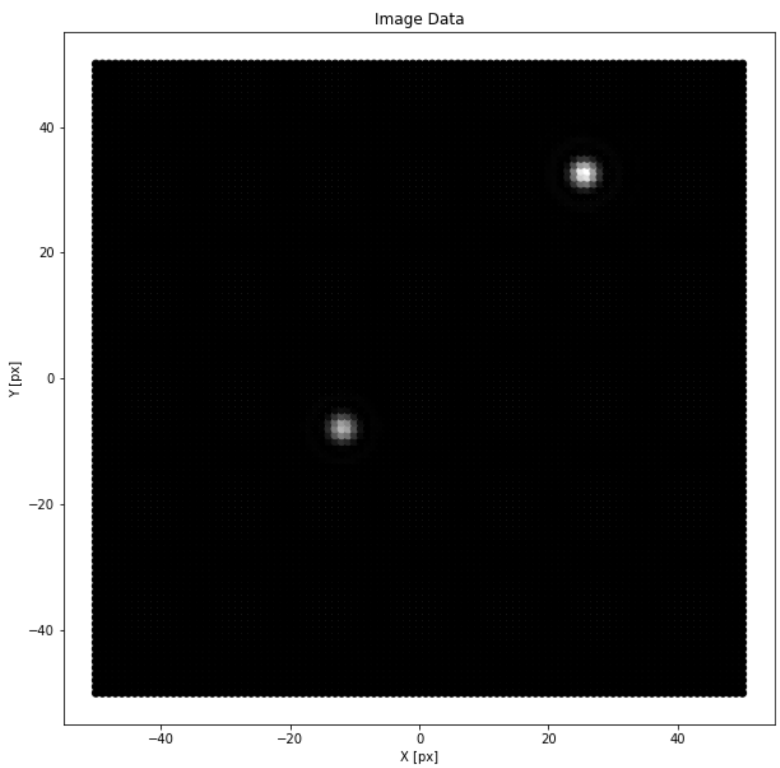

Figure 1: Simulated image data of two point sources.

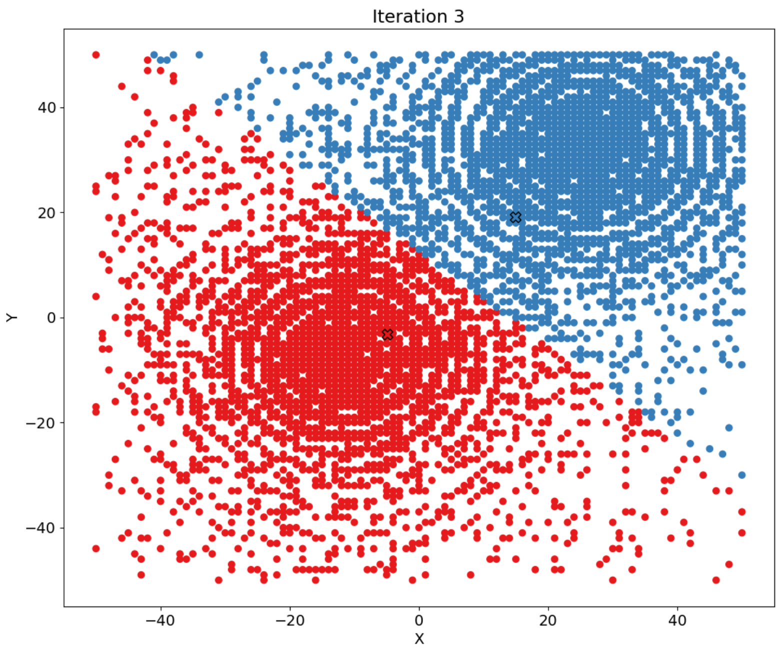

Figure 2: Applying k-means clustering to the image data.

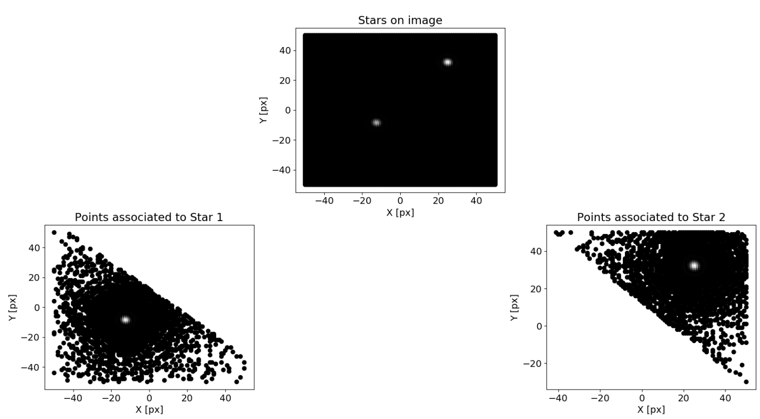

Figure 3: Separating the image data by star.

3.c Calculating the Optimal Position

Once the data is separated by star, we can calculate the position of the star on the image. A common strategy in astronomy is to fit a Gaussian profile to the point spread function. Since we have a set of data, $I(x, y)$, defined on a set of given points, we will follow this strategy. We use the `numpy` nonlinear least-squares algorithm to fit a general two-dimensional Gaussian (Equation 3) to our data.

$$f(x, y, \mu_x, \mu_y, A, a, b, c) = A \exp\left(a(x - \mu_x)^2 + b(x - \mu_x)(y - \mu_y) + c(y - \mu_y)^2\right) \tag{3}$$

This fit gives us $\bar{x}$ and $\bar{y}$, the optimal coordinates of the star's center. It also gives us the covariance matrix of the fit parameters, which allows us to calculate the errors on the position.

3.d Calculating the Mean Position

We obtain values of $\bar{x}$, $\bar{y}$, and their errors from each image. We now want to combine these values from a set of images to produce one optimal position. If the positions are normally distributed, we do this using the variance-weighted average. Let the weights be

$$w_i = \frac{1}{\sigma_i^2}$$

The weighted average is then

$$\langle \bar{x} \rangle = \frac{\sum_i w_i \bar{x}_i}{\sum_i w_i}$$

and its error is

$$\sigma = \sqrt{\frac{1}{\sum_i w_i}}$$

Therefore, for each set of images, we obtain a single values for the star positions.

3.e Calculating the Separation Distance

With the optimal positions of the stars, we can finally calculate the separation distance. It will simply be the Euclidean distance

$$s = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We can calculate the error on this distance using formal error propagation. The formula is

$$\delta s = \sqrt{\frac{(\delta x_1^2 + \delta x_2^2)(x_1 - x_2)^2 + (\delta y_1^2 + \delta y_2^2)(y_1 - y_2)^2}{(x_1 - x_2)^2 + (y_1 - y_2)^2}}$$

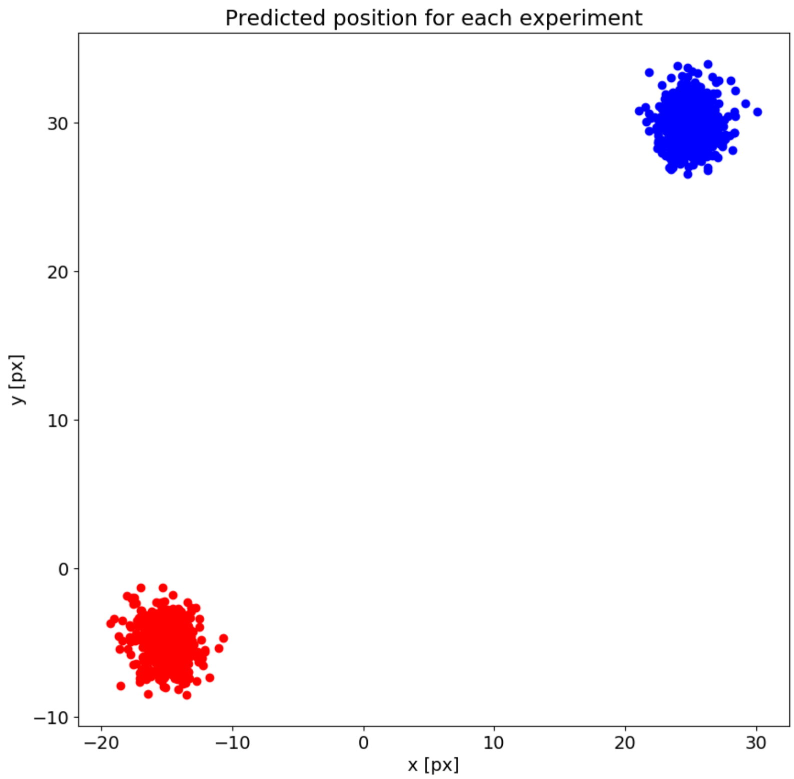

Figure 4: The predicted star positions using 5 images per experiment.

4 Results

With all of these ingredients, we can calculate the separation distance given a set of images. We simulated 500 experiments with 5 images per experiment. We used a mean seeing of 3 pixels, and let the two stars have intensities of 10000 counts per second and 7000 counts per second. The predicted positions over all of the experiments is presented in Figure 4, and the distribution of separation distances is presented in Figure 5. Unfortunately, it appears that only using 5 images does not allow us to sufficiently localize the star positions. As seen in Figure 4, each cluster of points has a large radius. In addition, the distribution of separation distances over the experiments is quite wide. To defeat our 3 pixel seeing, we would have to take more measurements. Using 500 experiments with 20 images per experiment, we obtain Figures 6 and 7. Here, we see that the point clusters are tighter, and and the distribution of separation distances is

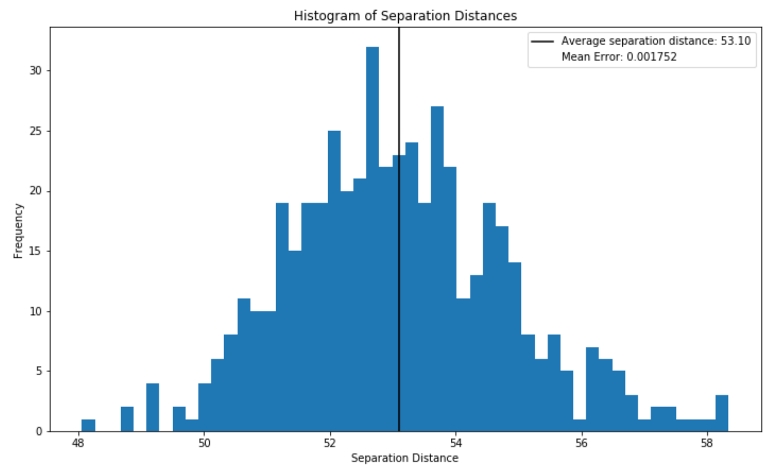

Figure 5: Distribution of separation distances using 5 images per experiment and 500 experiments.

much more narrowly peaked.

5 Conclusions

We saw that even if the seeing is relatively small compared to the size of the image, we need to take lots of images to sufficiently localize the stellar positions. Taking many images may be necessary to produce good astrometry from these images. This reflects the fact that atmospheric seeing is one of the major contributions to error in direct imaging. Hence, the best solution is to remove the atmosphere and use space-based telescopes for this sort of study.

References

[1] P. Martinez, J. Kolb, M. Sarazin, and A. Tokovinin. On the Difference between Seeing and Image Quality: When the Turbulence Outer Scale Enters the Game. *The Messenger*, 141:5–8, September 2010.

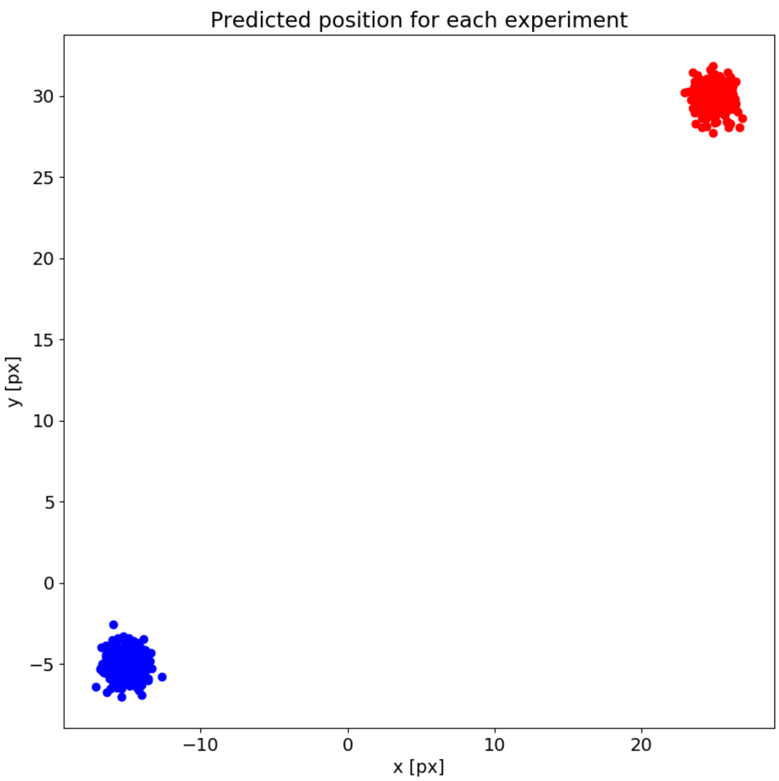

Figure 6: The predicted star positions using 20 images per experiment.

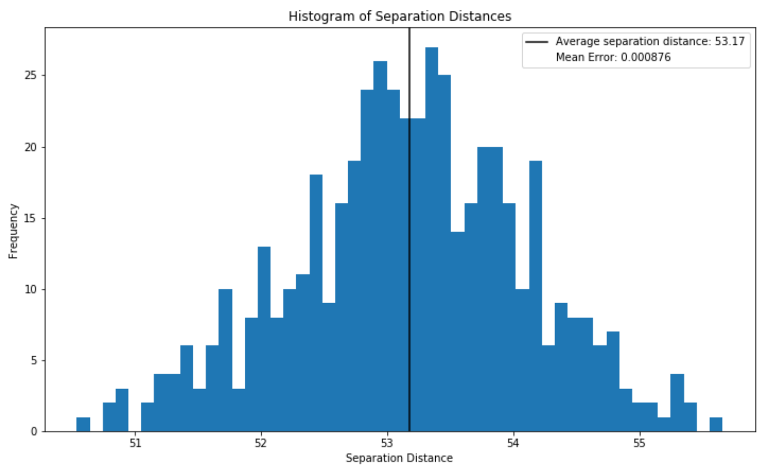

Figure 7: Distribution of separation distances using 20 images per experiment and 500 experiments.