Exercise 2.1

Question 1. Express each of the following as a rational number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:.

(i) 2^{-3}

(ii) $(-4)^{-2}$

(iii) $\frac{1}{3^{-2}}$

(iv) $\left(\frac{1}{2}\right)^{-5}$

(v) $\left(\frac{2}{3}\right)^{-2}$

Answer:

(i) $2^{-3}=\frac{1}{2^{3}}=\frac{1}{8}$

(ii) $(-4)^{-2}=\frac{1}{(-4)^{2}}=\frac{1}{16}$

(iii) $\frac{1}{3^{-2}} = 3^2 = 9$

(iv) $\left(\frac{1}{2}\right)^{-5} = 2^5 = 32$

(v) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Question 2. Find the values of the following:

(i) $3^{-1} + 4^{-1}$

(ii) $\left(3^0 + 4^{-1}\right) \times 2^2$

(iii) $\left(3^{-1} + 4^{-1} + 5^{-1}\right)^0$

(iv) $\left(\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right)^{-1}$

Answer:

(i) We know from the property of powers that for every natural number a, $a^{-1} = \frac{1}{a}$, Then:

$3^{-1} + 4^{-1} = \frac{1}{3} + \frac{1}{4}$

= $\frac{4+3}{12}$

= $\frac{7}{12}$

(ii) We know from the property of powers that for every natural number a, $a^{-1} = \frac{1}{a}$.

Moreover, a^0 is 1 for every natural number a not equal to 0. Then,

$(3^0 + 4^{-1}) \times 2^2$

= $\left(1 + \frac{1}{4}\right) \times 4$

= $\frac{5}{4} \times 4$

= 5

(iii) We know from the property of powers that for every natural number a, $a^{-1} = \frac{1}{a}$.

Moreover, a^0 is 1 for every natural number a not equal to 0. Then,

$(3^{-1} + 4^{-1} + 5^{-1})^0$ = 1 —> (Ignore the expression inside the bracket and use $a^0 = 1$)

(iv) We know from the property of powers that for every natural number a, $a^{-1} = \frac{1}{a}$.

Then:

$\left(\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right)^{-1} = (3 - 4)^{-1}$

= $(-1)^{-1}$

= -1

Question 3.Find the value of each of the following:

(i) $\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-1}+\left(\frac{1}{4}\right)^{-1}$

(ii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

(iii) $\left(2^{-1}\times 4^{-4}\right)\div 2^{-2}$

(iv) $\left(5^{-1}\times 2^{-1}\right)\div 6^{-1}$

Answer:

(i) $\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-1}+\left(\frac{1}{4}\right)^{-1}$

$=\frac{1}{\frac{1}{2}}+\frac{1}{\frac{1}{3}}+\frac{1}{\frac{1}{4}}$

= 2 + 3 + 4 = 12

(ii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

$=\frac{1}{\left(\frac{1}{2}\right)^2}+\frac{1}{\left(\frac{1}{3}\right)^2}+\frac{1}{\left(\frac{1}{4}\right)^2}$

$=\frac{1}{\frac{1}{4}}+\frac{1}{\frac{1}{9}}+\frac{1}{\frac{1}{16}}$

= 4 + 9 + 16 = 29

(iii) $\left(2^{-1} \times 4^{-4}\right) \div 2^{-2}$

= $\frac{1}{2} \times \frac{1}{4} \div \frac{1}{2^2}$

= $\frac{1}{8} \times 4$ = $\frac{1}{2}$

(iv) $(5^{-1} \times 2^{-1}) \div 6^{-1}$

= $\left(\frac{1}{5} \times \frac{1}{2}\right) \div \frac{1}{6}$

= $\frac{1}{10} \times 6$ = $\frac{3}{5}$

Question 4. Simplify:

(i) $(4^{-1} \times 3^{-1})^2$

(ii) $(5^{-1} \div 6^{-1})^3$

(iii) $(2^{-1} + 3^{-1})^{-1}$

(iv) $(3^{-1} + 4^{-1})^{-1} \times 5^{-1}$

Answer:

(i) $\left(4^{-1} \times 3^{-1}\right)^{2}$

= $\left(\frac{1}{4} \times \frac{1}{3}\right)^{2}$

= $\left(\frac{1}{12}\right)^{2}$

= $\left(\frac{1^{2}}{12^{2}}\right)$ = $\left(\frac{1}{24}\right)$

(ii) $\left(5^{-1} \div 6^{-1}\right)^{3}$

= $\left(\frac{1}{5} \div \frac{1}{6}\right)^{3}$

= $\left(\frac{1}{5} \times 6\right)^{3}$

= $\left(\frac{6}{5}\right)^{3}$ = $\frac{216}{125}$

(iii) $\left(2^{-1} + 3^{-1}\right)^{-1}$

= $\left(\frac{1}{2} + \frac{1}{3}\right)^{-1}$

= $\left(\frac{5}{6}\right)^{-1}$

= $\left(\frac{1}{\frac{5}{6}}\right)$ = $\frac{6}{5}$

(iv) $\left(3^{-1} + 4^{-1}\right)^{-1} \times 5^{-1}$

= $\left(\frac{1}{3} \times \frac{1}{4}\right)^{-1} \times \frac{1}{5}$

= $\left(\frac{1}{12}\right)^{-1} \times \frac{1}{5}$ = $\frac{12}{5}$

Question 5. Simplify:

(i) $\left(3^2+2^2\right)\times\left(\frac{1}{2}\right)^3$

(ii) $\left(3^2-2^2\right)\times\left(\frac{2}{3}\right)^{-3}$

(iii) $\left(\left(\frac{1}{3}^{-3}\right)-\left(\frac{1}{2}\right)^{-3}\right)\div\left(\frac{1}{4}\right)^{-3}$

(iv) $\left(2^2+3^2-4^2\right)\div\left(\frac{3}{2}\right)^2$

Answer:

(i) $\left(3^2+2^2\right)\times\left(\frac{1}{2}\right)^3$

= $(9+4)\times\frac{1}{8}$ = $\frac{13}{8}$

(ii) $\left(3^2-2^2\right)\times\left(\frac{2}{3}\right)^{-3}$

= $(9-4)\times\frac{1}{(2/3)^3}$

= $5\times\frac{1}{(8/27)}$ = $\frac{135}{8}$

(iii) $\left(\left(\frac{1}{3}^{-3}\right)-\left(\frac{1}{2}\right)^{-3}\right) \div \left(\frac{1}{4}\right)^{-3}$

= $\left(3^3-2^3\right) \div 4^3$

= $(27-8) \div 64$

= $19 \times \frac{1}{64}$ = $\frac{19}{64}$

(iv) $\left(2^2+3^2-4^2\right) \div \left(\frac{3}{2}\right)^2$

= $(4+9-16) \div \left(\frac{9}{4}\right)$

= $-3 \times \frac{4}{9}$ = $-\frac{4}{3}$

Question 6. By what number should 5^{-1} be multiplies so that the product may be equal to -7^{-1}?

Answer:

Using the property $a^{-1}=\frac{1}{a}$ for every natural number a, we have $5^{-1}=\frac{1}{5}$ and $(-7)^{-1}=-\frac{1}{7}$. We have to find a number x such that

$\frac{1}{5} \times x=\frac{-1}{7}$

Multiply bith sides by 5, we get

$x=\frac{-5}{7}$

Hence, the required number is $\frac{-5}{7}$

Question 7. By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplies so that the product may be equal to $\left(-\frac{4}{7}\right)^{-1}$?

Answer:

Using the property $a^{-1}=\frac{1}{a}$ for every natural number a, we have $\left(\frac{1}{2}\right)^{-1}=2\ and\ \left(\frac{-4}{7}\right)^{-1}=\frac{-7}{4}$.
We have to find the number x such that

$2x=\frac{-7}{4}$
Dividing both sides by 2, we get

$x=\frac{-7}{8}$
Hence, the required number is $\frac{-7}{8}$

Question 8. By what number should $(-15)^{-1}$ be multiplies so that the product may be equal to $(-5)^{-1}$

Answer:

Using the property $a^{-1}=\frac{1}{a}$ for every natural number a, we have $(-15)^{-1}=-\frac{1}{15}$ and $(-5)^{-1}=-\frac{1}{5}$. We have to find a number x such that

$\frac{-\frac{1}{15}}{\frac{x}{1}}=\frac{-1}{5}$
Or $\frac{1}{15}\times\frac{1}{x}=-\frac{-1}{5}$

Or $x=\frac{1}{3}$

Hence, $(-15)^{-1}$ should be divided by $\frac{1}{3}$ to obtain $(-5)^{-1}$.

Exercise 2.2

Q1. Write each of the following in exponential form:

(i) $\left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1}$

(ii) $\left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2}$

Solution:

(i) $\left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} = \left(\frac{3}{2}\right)^{-1+(-1)+(-1)+(-1)}$

$a^m \times a^n = a^{m+n} = \left(\frac{3}{2}\right)^{-4}$

(ii) $\left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} = \left(\frac{2}{5}\right)^{-1+(-2)+(-2)}$

$a^m \times a^n = a^{m+n} = \left(\frac{2}{5}\right)^{-6}$

Q2. Evaluate:

(i) 5^{-2}

(ii) $(-3)^{-2}$

(iii) $\left(\frac{1}{3}\right)^{-4}$

(iv) $\left(\frac{-1}{2}\right)^{-1}$

Solution:

(i) $5^{-2}=\frac{1}{5^2}$ = $\frac{1}{25}$

(ii) $(-3)^{-2}=\frac{1}{(-3)^2}$ = $\frac{1}{9}$

(iii) $\left(\frac{1}{3}\right)^{-4}=\frac{1}{\left(\frac{1}{3}\right)^4}$ = $\frac{1}{\frac{1}{81}}$ = 81

(iv) $\left(\frac{-1}{2}\right)^{-1}=\left(\frac{1}{\frac{-1}{2}}\right)$ = -2

Q3. Express each of the following as a rational number in the form $\frac{p}{q}$:

(i) 6^{-1}

(ii) -7^{-1}

(iii) $\left(\frac{1}{4}\right)^{-1}$

(iv) $(-4)^{-1}\times\left(\frac{-3}{2}\right)^{-1}$

(v) $\left(\frac{3}{5}\right)^{-1}\times\left(\frac{5}{2}\right)^{-1}$

Solution:

(i) $6^{-1} = \frac{1}{6}$

(ii) $-7^{-1} = \frac{1}{-7}$ = $\frac{-1}{7}$

(iii) $\left(\frac{1}{4}\right)^{-1} = \frac{1}{\frac{1}{4}} = 4$

(iv) $(-4)^{-1} \times \left(\frac{-3}{2}\right)^{-1} = \frac{1}{-4} \times \frac{1}{\frac{-3}{2}}$

= $\frac{1}{-4} \times = \frac{2}{-3}$ = $\frac{1}{6}$

(v) $\left(\frac{3}{5}\right)^{-1} \times \left(\frac{5}{2}\right)^{-1} = \frac{1}{\frac{3}{5}} \times \frac{1}{\frac{5}{2}}$

= $\frac{5}{3} \times \frac{2}{5}$ = $\frac{2}{3}$

Q4. Simplify:

(i) $\left\{4^{-1} \times 3^{-1}\right\}^2$

(ii) $\left\{5^{-1} \div 6^{-1}\right\}^3$

(iii) $\left\{2^{-1} + 3^{-1}\right\}^{-1}$

(iv) $\left\{3^{-1}+4^{-1}\right\}^{-1} \times 5^{-1}$

(v) $\left\{4^{-1}+5^{-1}\right\}^{-1}+3^{-1}$

Solution:

(i) $\left\{4^{-1} \times 3^{-1}\right\}^{2}=\left(\frac{1}{4} \times \frac{1}{3}\right)^{2}$

= $\left(\frac{1}{12}\right)^{2}$ = $\left(\frac{1}{144}\right)$

(ii) $\left(5^{-1} \div 6^{-1}\right)^{3}=\left(\frac{1}{5} \div \frac{1}{6}\right)^{3}$

= $\left(\frac{6}{5}\right)^{3}$ = $\left(\frac{216}{125}\right)$

(iii) $\left\{2^{-1}+3^{-1}\right\}^{-1}=\left(\frac{1}{2}+\frac{1}{3}\right)^{-1}$

= $\left(\frac{5}{6}\right)^{-1}$ = $\left(\frac{6}{5}\right)$

(iv) $\left\{3^{-1}+4^{-1}\right\}^{-1} \times 5^{-1}=\left(\frac{1}{3} \times \frac{1}{4}\right)^{-1} \times \frac{1}{5}$

= $\left(\frac{1}{12}\right)^{-1} \times \frac{1}{5}$

= $12 \times \frac{1}{5}$ = $\frac{12}{5}$

(v) $\left\{4^{-1}+5^{-1}\right\}^{-1}+3^{-1}=\left(\frac{1}{4}-\frac{1}{5}\right) \div \frac{1}{3}$

= $\left(\frac{5-4}{20}\right) \times 3$

= $\frac{1}{20} \times 3$ = $\frac{3}{20}$

Q5. Express each of the following rational numbers with a negative exponent:

(i) $\left(\frac{1}{4}\right)^{3}$

(ii) $(3)^{5}$

(iii) $\left(\frac{3}{5}\right)^{4}$

(iv) $\left\{\left(\frac{3}{2}\right)^{4}\right\}^{-3}$

(v) $\left\{\left(\frac{7}{4}\right)^{4}\right\}^{-3}$

Solution:

(i) $\left(\frac{1}{4}\right)^{3}$

$= \left(\frac{4}{1}\right)^{-3}$

(ii) $(3)^{5}$

$= \left(\frac{1}{3}\right)^{-5}$

(iii) $\left(\frac{3}{5}\right)^{4}$

$= \left(\frac{5}{3}\right)^{-4}$

(iv) $\left\{\left(\frac{3}{2}\right)^{4}\right\}^{-3}$

$= \left(\frac{3}{2}\right)^{-12}$

(v) $\left\{\left(\frac{7}{4}\right)^{4}\right\}^{-3}$

$= \left(\frac{7}{3}\right)^{-12}$

Q6. Express each of the following rational numbers with a positive exponent.

(i) $\left(\frac{3}{4}\right)^{-2}$

(ii) $\left(\frac{5}{4}\right)^{-3}$

(iii) $4^{3} \times 4^{-9}$

(iv) $\left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4}$

(v) $\left\{\left(\frac{3}{2}\right)^{4}\right\}^{-2}$

Solution:

(i) $\left(\frac{3}{4}\right)^{-2}$

= $\left(\frac{4}{3}\right)^{2}$

(ii) $\left(\frac{5}{4}\right)^{-3}$

= $\left(\frac{4}{5}\right)^{3}$

(iii) $4^{3} \times 4^{-9}$

= $4^{3-9} = 4^{-6}$

= $\left(\frac{1}{4}\right)^{6}$

(iv) $\left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4}$

= $\left(\frac{4}{3}\right)^{-4\times-3}$

= $\left(\frac{4}{3}\right)^{12}$

(v) $\left\{\left(\frac{3}{2}\right)^{4}\right\}^{-2}$

= $\left(\frac{3}{2}\right)^{4\times-2}$

= $\left(\frac{3}{2}\right)^{-8}$

= $\left(\frac{2}{3}\right)^{8}$

Q7. Simplify:

(i) $\left\{\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right\} \div\left(\frac{1}{4}\right)^{-3}$

(ii) $\left(3^2-2^2\right) \times\left(\frac{2}{3}\right)^{-3}$

(iii) $\left\{\left(\frac{1}{2}\right)^{-1} \times(-4)^{-1}\right\}^{-1}$

(iv) $\left[\left\{\left(\frac{-1}{4}\right)^2\right\}^{-2}\right]^{-1}$

(v) $\left\{\left(\frac{2}{3}\right)^2\right\}^3 \times\left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1}$

Solution:

(i)

$$\left\{\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right\} \div\left(\frac{1}{4}\right)^{-3}=\left(\frac{1}{(1/3)^3}-\frac{1}{(1/2)^3}\right) \div \frac{1}{(1/4)^3}$$

$$=\left(\frac{1}{(1/27)}-\frac{1}{(1/8)}\right) \div \frac{1}{(1/64)}$$

$$=\left(\frac{27}{1}-\frac{8}{1}\right) \div 64$$

$$=(19) \times \frac{1}{64}$$

$$=\frac{19}{64}$$

(ii)

$$(3^2-2^2) \times \left(\frac{2}{3}\right)^{-3} = (9-4) \times \frac{1}{(2/3)^2}$$

$$= 5 \times \frac{27}{8}$$

$$= \frac{135}{8}$$

(iii)

$$\left(\left(\frac{1}{2}\right)^{-1} \times (-4)^{-1}\right)^{-1} = \left(\left(\frac{1}{1/2}\right) \times \left(\frac{1}{-4}\right)\right)^{-1}$$

$$= \left(2 \times \left(\frac{1}{-4}\right)\right)^{-1}$$

$$= \left(\frac{1}{-2}\right)$$

$$= \frac{1}{1/(-2)}$$

$$= -2$$

(iv) $\left(\left(\left(\frac{-1}{4}\right)^2\right)^{-2}\right)^{-1} = \left(\left(\frac{(-1)^2}{4^2}\right)^{-2}\right)^{-1}$

= $\left(\left(\frac{1}{16}\right)^{-2}\right)^{-1}$

= $\left(\left(\frac{1}{(1/16)^2}\right)\right)^{-1}$

= $\left(\frac{1}{(1/256)}\right)^{-1}$

= $256^{-1} = \frac{1}{256}$

(v)

$$\left\{\left(\frac{2}{3}\right)^2\right\}^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1}$$

$$= \left(\frac{2^2}{3^2}\right)^3 \times \frac{1}{(1/3)^4} \times \frac{1}{3} \times \frac{1}{6}$$

$$= \frac{4^3}{9^3} \times 81 \times \frac{1}{18}$$

$$= \frac{64}{729} \times 81 \times \frac{1}{18}$$

$$= \frac{64}{9} \times \frac{1}{18}$$

$$= 64 \times \frac{1}{162}$$

$$= \frac{64}{162}$$

$$= \frac{32}{81}$$

Q8. By what number should 5^{-1} be multiplies so that the product may be equal to $(-7)^{-1}$?

Solution:

Expressing in fraction form, we get:

$5^{-1} = \frac{1}{5}$

And $(-7)^{-1} = \frac{1}{-7}$

We have to find a number x such that

$\frac{1}{5}x = \frac{-1}{7}$

Multiplying both side by 5, we get:

x = $-\frac{5}{7}$

Hence, 5^{-1} be multiplied by $-\frac{5}{7}$ to obtain $(-7)^{-1}$.

Q9. By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplies so that the product may be equal to $\left(\frac{-4}{7}\right)^{-1}$?

Solution:

Expressing in fraction form, we get

$\left(\frac{1}{2}\right)^{-1} = 2,$

And $\left(\frac{-4}{7}\right)^{-1} = \frac{7}{}$

And $\left(\frac{-4}{7}\right)^{-1} = -\frac{7}{4}$

We have to find a number x such that:

$2x = -\frac{7}{4}$

Dividing both side by 2, we get

$x = -\frac{7}{8}$

Hence, $\left(\frac{1}{2}\right)^{-1}$ should be multiplies by $-\frac{7}{8}$ to obtain $\left(\frac{-4}{7}\right)^{-1}$.

Q10. By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$

Solution:

Expressing in fraction form, we get:

$(-15)^{-1} =- \frac{1}{15}$ (using $a^{-1} = \frac{1}{a}$)

And

$(-5)^{-1} =- \frac{1}{5}$ (using $a^{-1} = \frac{1}{a}$)

We have to find a number xx such that

$-\frac{1}{15} \div x = -\frac{1}{5}$

Solving this equation, we get:

$-\frac{1}{15} \times \frac{1}{x} =- \frac{1}{5} -\frac{1}{15} =- \frac{x}{5} \frac{-5}{-15} = x x = \frac{1}{3}$

Hence, $(-15)^{-1}$ should be divided by $\frac{1}{3}$ to obtain $(-5)^{-1}$

Q11. By what number should $\left(\frac{5}{3}\right)^{-2}$ be multiplies so that the product may be $\left(\frac{7}{3}\right)^{-1}$?

Solution:

Expressing as a positive exponent, we have:

$$\left(\frac{5}{3}\right)^{-2}=\frac{1}{(5/3)^2}$$
$$=\frac{1}{25/9}$$
$$=\frac{9}{25}$$
$$and$$
$$=\left(\frac{7}{3}\right)^{-1}=\frac{3}{7}$$
$$We\ have\ to\ find\ a\ number\ x\ such\ that$$
$$\frac{9}{25}\times x=\frac{3}{7}$$

Multiplying both sides by 25/9, we get:

$$x=\frac{3}{7}\times\frac{25}{9}=\frac{1}{7}\times\frac{25}{3}=\frac{25}{21}$$

Hence, $\left(\frac{5}{3}\right)^{-2}$ should be multiplies by $\frac{25}{21}$ top obtain $\left(\frac{7}{3}\right)^{-1}$.

Q12. Find x, if:

(i) $\left(\frac{1}{4}\right)^{-4}\times\left(\frac{1}{4}\right)^{-8}=\left(\frac{1}{4}\right)^{-4x}$

(ii) $\left(\frac{-1}{2}\right)^{-19}\times\left(\frac{-1}{2}\right)^{8}=\left(\frac{-1}{2}\right)^{-2x+1}$

(iii) $\left(\frac{3}{2}\right)^{-3}\times\left(\frac{3}{2}\right)^{5}=\left(\frac{3}{2}\right)^{2x+1}$

(iv) $\left(\frac{2}{5}\right)^{-3}\times\left(\frac{2}{5}\right)^{15}=\left(\frac{2}{5}\right)^{2+3x}$

(v) $\left(\frac{5}{4}\right)^{-x}\div\left(\frac{5}{4}\right)^{-4}=\left(\frac{5}{4}\right)^{5}$

(vi) $\left(\frac{8}{3}\right)^{2x+1}\times\left(\frac{8}{3}\right)^{5}=\left(\frac{8}{3}\right)^{x+2}$

Answer:

(i) We have:

$$\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$$

$$\left(\frac{1}{4}\right)^{-12} = \left(\frac{1}{4}\right)^{-4x}$$

$$-12 = -4x$$

$$3 = x$$

Therefore, x = 3

(ii) We have:

$$\left(\frac{-1}{2}\right)^{-19} \times \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{-2x+1}$$

$$\left(\frac{-1}{2}\right)^{-11} = \left(\frac{-1}{2}\right)^{-2x+1}$$

$$-11 = -2x + 1$$

$$-12 = -2x$$

$$6 = x$$

Therefore, x = 6

(iii) We have:

$$\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{5} = \left(\frac{3}{2}\right)^{2x+1}$$

$$\left(\frac{3}{2}\right)^{2} = \left(\frac{3}{2}\right)^{2x+1}$$

$$2 = 2x + 1$$

$$1 = 2x$$

$$\frac{1}{2} = x$$

Therefore, $x = \frac{1}{2}$

(iv) We have:

$$\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$$

$$\left(\frac{2}{5}\right)^{12} = \left(\frac{2}{5}\right)^{2x+1}$$

$$12 = 2 + 3x$$

$$10 = 3x$$

$$\frac{10}{3} = x$$

Therefore, $x = \frac{10}{3}$

(v) We have:

$$\left(\frac{5}{4}\right)^{-x} \div \left(\frac{5}{4}\right)^{-4} = \left(\frac{5}{4}\right)^{5}$$

$$\left(\frac{5}{4}\right)^{-x+4} = \left(\frac{5}{4}\right)^{5}$$

$$-x + 4 = 5$$

$$-x = 1$$

$$x = -1$$

Therefore, x = -1

(vi) We have:

$$\left(\frac{8}{3}\right)^{2x+1} \times \left(\frac{8}{3}\right)^{5} = \left(\frac{8}{3}\right)^{x+2}$$

$$\left(\frac{8}{3}\right)^{2x+6} = \left(\frac{8}{3}\right)^{x+2}$$

$$2x + 6 = x + 2$$

$$x = -4$$

Therefore, x = -4

Q13.

(i) if $x = \left(\frac{3}{2}\right)^{2} \times \left(\frac{2}{3}\right)^{-4}$, find the value of x^{-2}.

(ii) If $x = \left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$, find the value of x^{-1}.

Answer:

(i) First, we have to find x.

$$x=\left(\frac{3}{2}\right)^{2}\times\left(\frac{2}{3}\right)^{-4}$$
$$=\left(\frac{3}{2}\right)^{2}\times\left(\frac{3}{2}\right)^{4}$$
$$=\left(\frac{3}{2}\right)^{6}$$

Hence, x^{-2} is:

$$x^{-2}=\left(\left(\frac{3}{2}\right)^{6}\right)^{-2}$$
$$=\left(\frac{3}{2}\right)^{-12}$$
$$=\left(\frac{2}{3}\right)^{12}$$

(ii) First we will have to find x.

$$x=\left(\frac{4}{5}\right)^{-2}\div\left(\frac{1}{4}\right)^{2}$$
$$=\left(\frac{4^{-2}}{5^{-2}}\right)\times 4^{2}$$
$$=\frac{4^{0}}{5^{-2}}$$
$$\left(5^{2}\right)^{-1}$$
$$=\frac{1}{5^{2}}$$

Q14. Find the value of x for which $5^{2x}\div 5^{-3}=5^{5}$.

Answer: We have:

$5^{2x}\div 5^{-3}=5^{5}$

$5^{2x+3}=5^{5}$

$2x+3=5$

$2x=2$

$x=1$

Hence, x is 1.

Exercise 2.3

1. Express the following numbers in standard form:

(i) 602000000000000

(ii) 0.000000000943

(iii) 0.00000000085

(iv) 846×10^7

(v) 3759×10^{-4}

(vi) 0.00072984

(vii) 0.000437×10^4

(viii) $4 \div 100000$

Answers:

To express a number in the standard for, move the decimal point such that there is only one digit to the left of the decimal point.

(i) 602000000000000 = 6.02×10^{15} (The decimal point is moved 15 places to the left.)

(ii) 0.000000000943 = 9.43×10^{-12} (The decimal point is moved 12 places to the right.)

(iii) 0.00000000085 = 8.5×10^{-10} (The decimal point is moved 10 places to the right.)

(iv) $846 \times 10^{7} = 8.46 \times 10^{2} \times 10^{7} = 8.46 \times 10^{9}$ (The decimal point is moved two places to the left.)

(v) $3759 \times 10^{-4} = 3.759 \times 10^{3} \times 10^{-4} = 3.759 \times 10^{-1}$ (The decimal point is moved three places to the left.)

(vi) $0.00072984 = 7.984 \times 10^{-4}$ (The decimal point is moved four places to the right.)

(vii) $0.000437 \times 10^{4} = 4.37 \times 10^{-4} \times 10^{4} = 4.37 \times 10^{0} = 4.37$ (The decimal point is moved four places to the right.)

(viii) $4 \div 100000 = 4 \times 100000^{-1} = 4 \times 10^{-5}$ (Just count the number of zeros in 1,00,000 to determine the exponent of 10.)

2. Write the following numbers in the usual form:

(i) 4.83×10^{7}

(ii) 3.02×10^{-6}

(iii) 4.5×10^{4}

(iv) 3×10^{-8}

(v) 1.0001×10^{9}

(vi) 5.8×10^{2}

(vii) 3.61492×10^{6}

(viii) 3.25×10^{-7}

Answers:

(i) $4.83 \times 10^7 = 4.83 \times 1,00,00,000 = 4,83,00,000$

(ii) $3.02 \times 10^{-6} = \frac{3.02}{10^6} = \frac{3.02}{10,00,000} = 0.00000302$

(iii) $4.5 \times 10^4 = 4.5 \times 10,000 = 45,000$

(iv) $3 \times 10^{-8} = \frac{3}{8} = \frac{3}{10,00,00,000} = 0.00000003$

(v) $1.0001 \times 10^9 = 1.0001 \times 1,00,00,00,000 = 1,00,01,00,000$

(vi) $5.8 \times 10^2 = 5.8 \times 100 = 580$

(vii) $3.61492x10^6 = 3.61492x10,00,000 = 3614920$

(viii) $3.25x10^{-7} = \frac{3.25}{10^7} = \frac{3.25}{1,00,00,000} = 0.000000325$