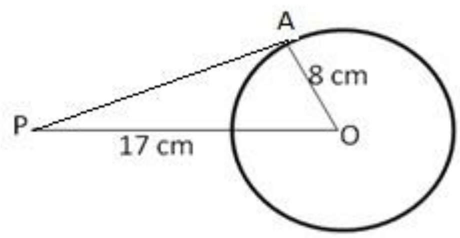
Chapter 8 - Circles

Exercise 8A

Question 1
Find the length of tangent drawn to a circle with radius 8 cm from a point 17 cm away from the centre of the circle.

Solution 1
PA is the tangent to the circle with center O and radius AO = 8 cm. The point P is at a distance of 17 cm from O.
In Δ PAO, $\angle$ A = 90



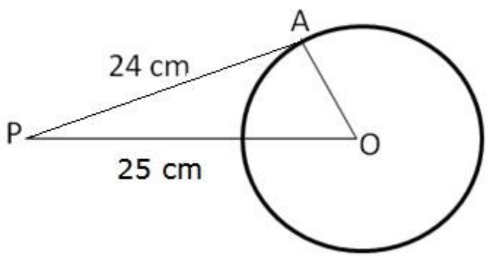
By Pythagoras theorem:

$$\begin{aligned} PO^2 &= PA^2 + AO^2 \\ \text{or } PA^2 &= PO^2 - AO^2 \\ \Rightarrow PA &= \sqrt{(17)^2 - (8)^2}\ \text{cm} \\ &= \sqrt{289 - 64}\ \text{cm} \\ &= \sqrt{225}\ \text{cm} = 15\text{cm} \end{aligned}$$

Hence, the length of the tangent = 15 cm.

Question 2
A point P is 25 cm away from the centre of a circle and the length of tangent drawn from P to the circle is 24 cm. Findthe radius of the circle.

Solution 2
PA is the tangent to the circle with centre O and radius, such that PO = 25 cm, PA = 24 cm
In Δ PAO, $\angle$ A = 90,



By Pythagoras theorem:

$PO^2 = PA^2 + AO^2$

or $OA^2 = PO^2 - PA^2$

$\Rightarrow OA^2 = (25)^2 - (24)^2$ cm

$= (25+24)(25-24)$ cm

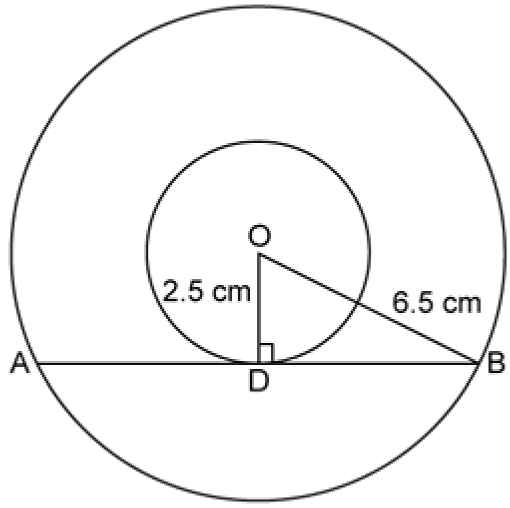
$= 49$ cm

$\therefore$ OA = 7cm

Hence, the radius of the circle is 7 cm.

Question 3

Two concentric circles are of radii 6.5 cm and 2.5 cm. Find the length of the chord of the larger circle which touches the smaller circle.



Solution 3

Since AB is a tangent to the inner circle.

$\angle ODB = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)

AB is a chord of the outer circle.

We know that, the perpendicular drawn from the centre to a chord of a circle, bisects the chord.

So, AB = 2DB.

In ΔODB,

By Pythagoras theorem,

$OB^2 = OD^2 + DB^2$

$\Rightarrow 6.5^2 = 2.5^2 + DB^2$

$\Rightarrow DB^2 = 6.5^2 - 2.5^2$

$\Rightarrow DB^2 = 42.25 - 6.25$

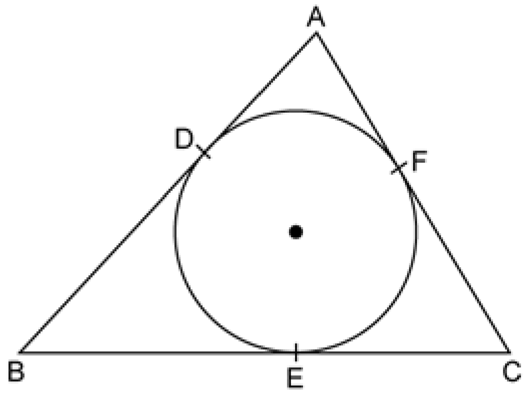
$\Rightarrow DB^2 = 36$ cm

$\Rightarrow DB = 6$ cm

$AB = 2DB = 2(6) = 12$ cm

Question 4

In the given figure, a circle inscribed in a triangle ABC, touches the sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, find the lengths of AD, BE and CF.



Solution 4

We know that tangents from an external point to the circle are equal.

AD = AF = x

BD = BE = y

CE = CF = z

Given that AB = 12 cm, BC = 8 cm and AC = 10 cm

$\Rightarrow x + y = 12,\ \ y + z = 8,\ \ z + x = 10$

Adding the three equation, we get

$2(x + y + z) = 30$

$\Rightarrow x + y + z = 15 \quad ....(i)$

Using (i) we get,

$\Rightarrow x + 8 = 15$

$\Rightarrow x = 7 = AD$

So, AD = 7 cm

Using (i) we get,

$x + y + z = 15$

$\Rightarrow 12 + z = 15$

$\Rightarrow z = 3$

So, CF = 3 cm

Using (i) we get,

$x + y + z = 15$

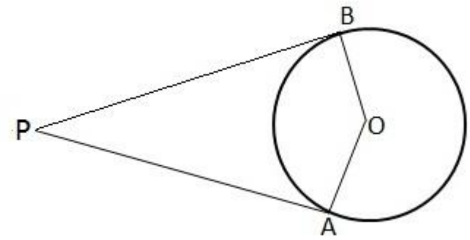
$\Rightarrow 10 + y = 15$

$\Rightarrow y = 5$

So, BE = 5 cm

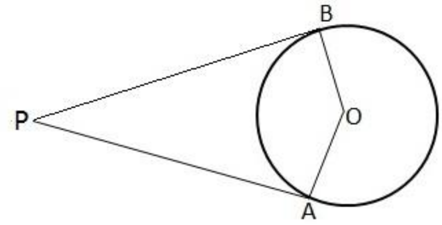
Question 5

In the given figure, PA and PB are the tangent segments to a circle with centre O. Show that the points A, O, B and P are concyclic.



Solution 5

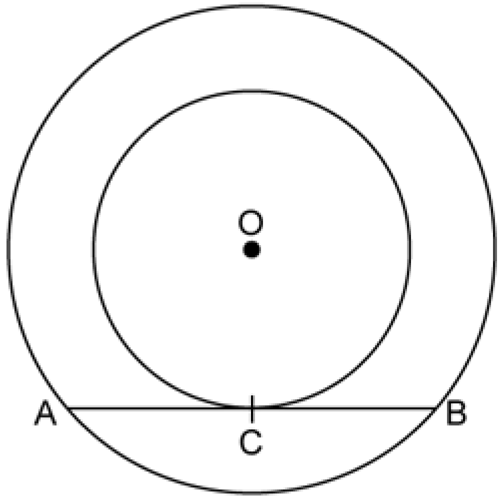
Given AP is a tangent at A and OA is radius through A and PA and PB are the tangent segments to circle with centre O.
Therefore, OA is perpendicular to AP, similarly, OB is perpendicular to BP.



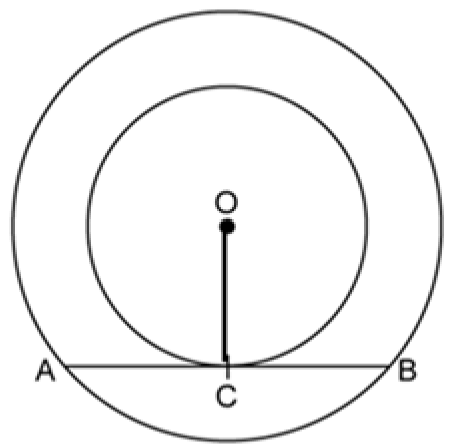
$\therefore \angle OAP = 90$
And $\angle OBP = 90$
So, $\angle OAP = \angle OBP = 90$
$\therefore \angle OBP + \angle OAP = (90 + 90) = 180$
Thus, the sum of opposite angles of quad. AOBP is 180
$\therefore$ AOBP is a cyclic quadrilateral

Question 6

In the given figure, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.



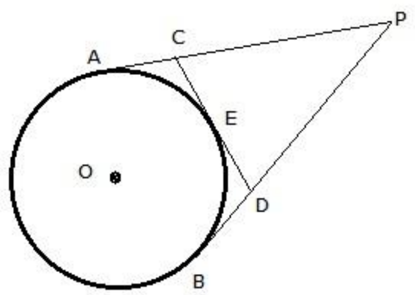
Solution 6



Since AB is the tangent for the smaller circle,
OC ⊥ AB
We know that perpendicular drawn from the centre to the chord bisects the chord.
Since AB is the chord of the larger circle,
AC = CB.
Hence proved.

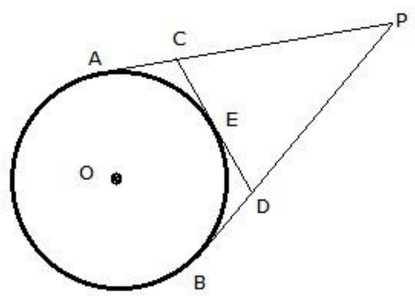
Question 7

From an external point P, tangents PA and PB are drawn to a circle with centre O. if CD is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of Δ PCD.



Solution 7

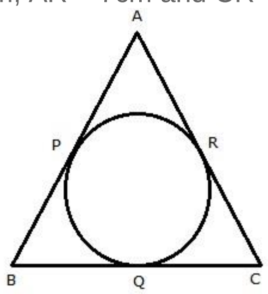
Given: From an external point P, tangent PA and PB are drawn to a circle with centre O. CD is the tangent to the circle at a point E and PA = 14cm.



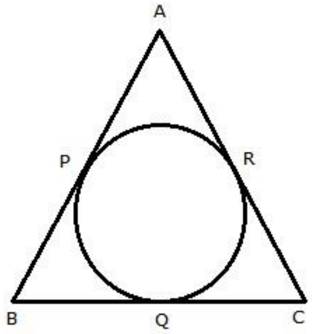
Since the tangents from an external point are equal, we have
PA = PB,
Also, CA = CE and DB = DE
Perimeter of Δ PCD = PC + CD + PD
=(PA - CA) + (CE + DE) +(PB - DB)

= (PA - CE) + (CE + DE) + (PB - DE)
= (PA + PB) = 2PA = (2 14) cm
= 28 cm
Hence, Perimeter of ΔPCD = 28 cm

Question 8
A circle is inscribed in a ΔABC, touching AB, BC and AC at P, Q and R respectively. If AB = 10 cm, AR = 7cm and CR = 5 cm, find the length of BC.

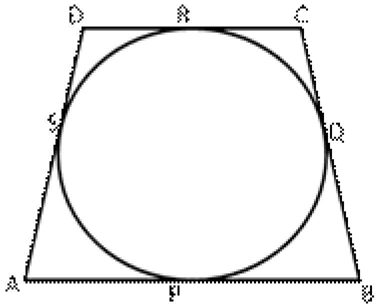


Solution 8
A circle is inscribed in a triangle ABC touching AB, BC and CA at P, Q and R respectively.



Also, AB = 10 cm, AR = 7cm, CR = 5cm
AR, AP are the tangents to the circle
$\therefore$ AP = AR = 7cm
AB = 10 cm
$\therefore$ BP = AB - AP = (10 - 7)= 3 cm
Also, BP and BQ are tangents to the circle
$\therefore$ BP = BQ = 3 cm
Further, CQ and CR are tangents to the circle
$\therefore$ CQ = CR = 5cm
BC = BQ + CQ = (3 + 5) cm = 8 cm
Hence, BC = 8 cm

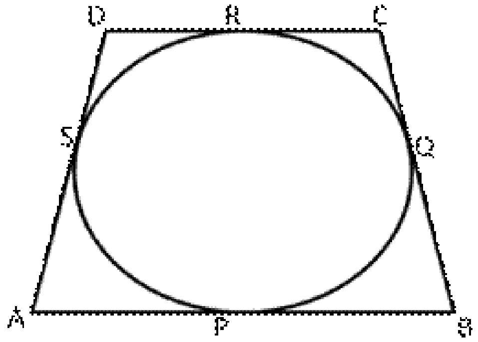
Question 9
In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6cm, BC = 7cm and CD = 4 cm. Find AD.



Solution 9

Let the circle touches the sides AB, BC, CD and DA at P, Q, R, S respectively

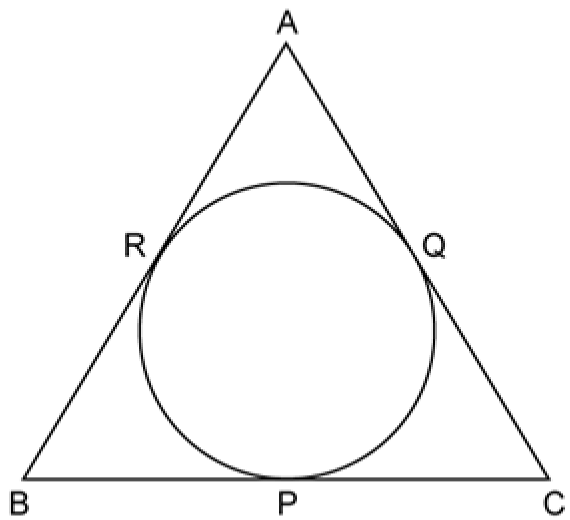
We know that the length of tangents drawn from an exterior point to a circle are equal



AP = AS ----(1)   {tangents from A}
BP = BQ ---(2)   {tangents from B}
CR = CQ ---(3)   {tangents from C}
DR = DS----(4)   {tangents from D}
Adding (1), (2) and (3) we get
∴ AP + BP + CR + DR = AS + BQ + CQ + DS
⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
⇒ AB + CD = AD + BC
⇒ AD = (AB + CD) - BC = {(6 + 4) - 7} cm = 3 cm
Hence, AD = 3 cm

Question 10

In the given figure, an isosceles triangle ABC with AB = AC, circumscribes a circle. Prove that the point of contact P bisects the base BC.



Solution 10

We know that tangents drawn from an external point to the circle are equal.

So,

AR = AQ

BR = BP .....(i)

CP = CQ .....(ii)

Given that AB = AC

$\Rightarrow AR + BR = AQ + CQ \; ....(\because AR = AQ)$

$\Rightarrow BR = CQ$

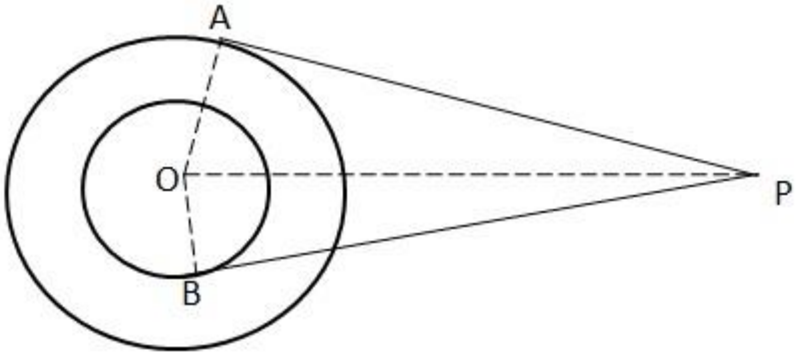
From (i) and (ii) we get

BP = CP

Hence, P bisects base BC.

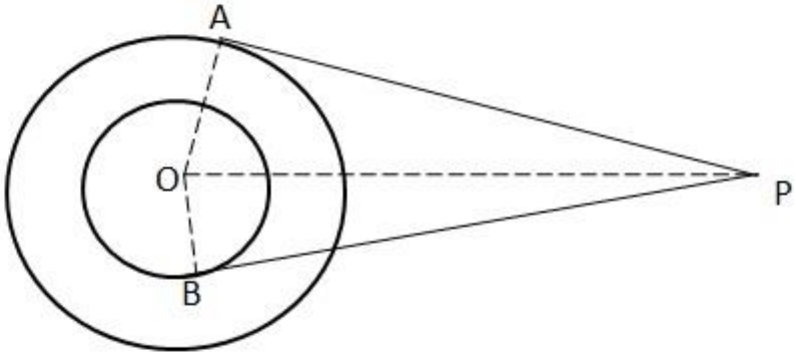
Question 11

In the given figure, O is the centre of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to the outer and inner circle respectively. If PA = 10 cm, find the length of PB up to one place of decimal



Solution 11

Given O is the centre of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to the outer and inner circle respectively. PA = 10cm. Join OA, OB and OP.
Then, OB = 4 cm, OA= 6 cm and PA = 10 cm

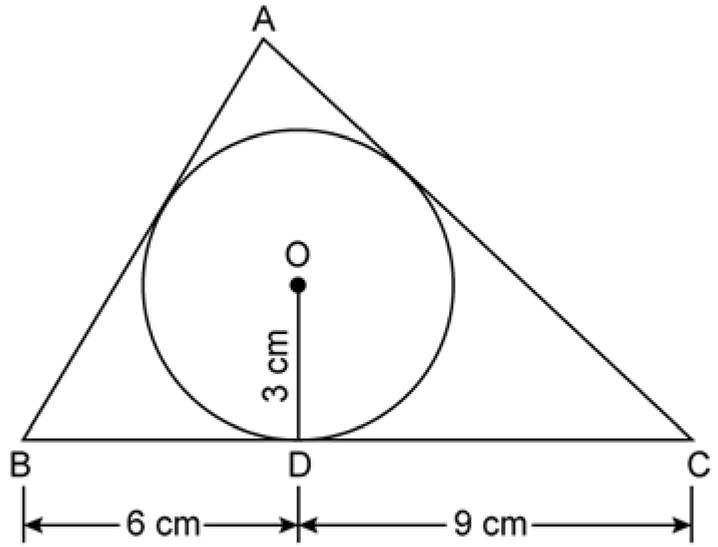


In triangle OAP,

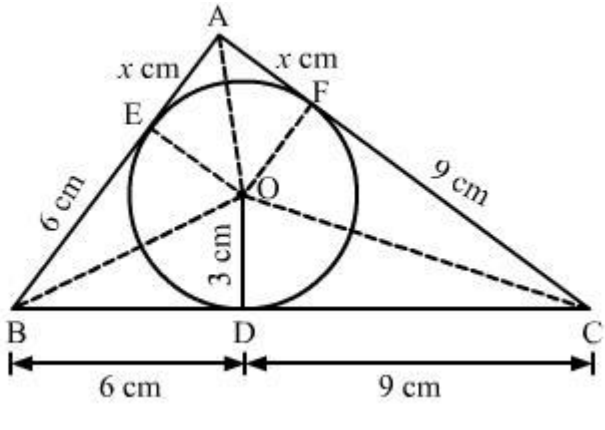
$$OP^2 = OA^2 + PA^2$$
$$= (6)^2 + (10)^2 = 136 \text{ cm}^2$$
In ΔOBP,
$$BP = \sqrt{OP^2 - OB^2} = \sqrt{136 - 16} \text{ cm}$$
$$= \sqrt{120} \text{ cm} = 10.9 \text{ cm}$$
Hence, BP = 10.9 cm

Question 12

In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm such that the segments BD and DC into which BC is divided by the point of contact D, are of lengths 6 cm and 9 cm respectively. If the area of $\Delta ABC = 54 \text{ cm}^2$ then find the lengths of sides AB and AC.

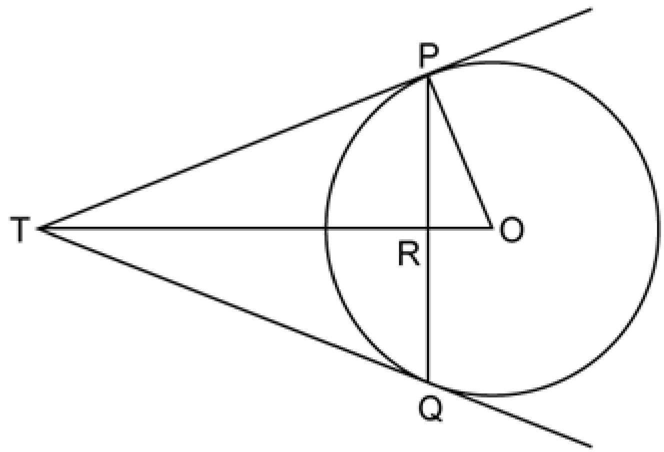


Solution 12

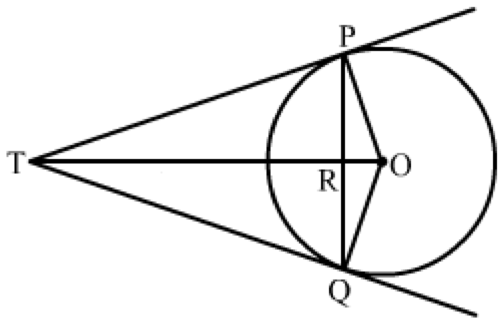


Construction : Join OA, OB, OC.
Draw $OD \perp BC$, $OF \perp AC$ and $OE \perp AB$.
We know that tangents drawn from an external point to the circle are equal.
So,
AE = AF = x (say)
BD = BE = 6 cm
CD = CF = 9 cm
Now,
$ar(\Delta ABC) = ar(\Delta BOC) + ar(\Delta AOC) + ar(\Delta AOB)$
$\Rightarrow 54 = \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OF + \frac{1}{2} \times OE \times AB$
$\Rightarrow 54 = \frac{1}{2}\left[BC \times OD + AC \times OF + OE \times AB\right]$
$\Rightarrow 54 = \frac{1}{2}\left[15 \times 3 + 3(9 + x) + 3(6 + x)\right]$
$\Rightarrow 108 = 15 \times 3 + 3(9 + x) + 3(6 + x)$
$\Rightarrow 108 = 45 + 27 + 3x + 18 + 3x$
$\Rightarrow 6x = 18$
$\Rightarrow x = 3$
So, AB = 6 + 3 = 9 cm and AC = 9 + 3 = 12 cm.

Question 13
PQ is a chord of length 4.8 cm of a circle of radius 3 cm. The tangents at P and Q intersect at a point T as shown in the figure. Find the length of TP.



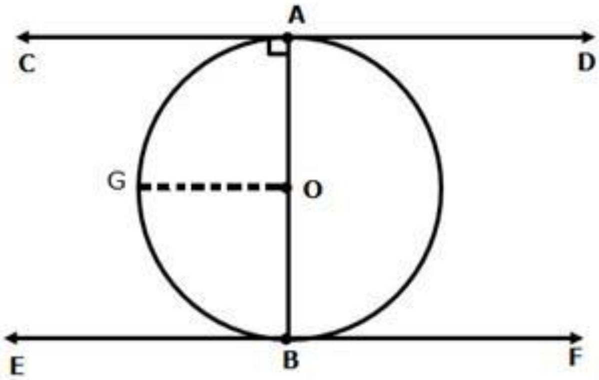
Solution 13


P
T
R
O
Q

Construction : Join OQ.
In ΔTPO and ΔTQO,
$TP = TQ$ ....(tangents from an external point to the circle are equal)
$OT = OT$ ....(common side)
$OP = OQ$ .....(radii of the same circle)
$\Rightarrow \Delta TPO \cong \Delta TQO$ ....(SSS congruence criterion)
$\Rightarrow \angle PTO = \angle QTR$ ....(cpct)
....(i)
In ΔTRP and ΔTRQ,
$TP = TQ$ ....(tangents from an external point to the circle are equal)
$TR = TR$ ....(common side)
$\angle PTR = \angle QTR$ ....(from (i))
$\Rightarrow \Delta TRP \cong \Delta TRQ$ ....(SAS congruence criterion)
$\Rightarrow \angle TRP = \angle TRQ$
Since PRQ is a straight line segment,
$\angle TRP + \angle TRQ = 180°$
$\Rightarrow \angle TRP = \angle TRQ = 90°$
So, $OR \perp PQ$
We know that the perpendicular from the centre to the chord of a circle bisects the chord.
So, $PR = 2.4$ cm
In ΔORP,
$OR^2 = OP^2 - RP^2$ ....(By Pythagoras theorem)
$\Rightarrow OR^2 = 3^2 - 2.4^2$
$\Rightarrow OR^2 = 3.24$
$\Rightarrow OR = 1.8$ cm
In right ΔPRT,
$PT^2 = TR^2 + PR^2$
$\Rightarrow PT^2 = TR^2 + 2.4^2$ .....(i)
In right ΔPOT,
$OT^2 = PT^2 + OP^2$
$\Rightarrow (TR + 1.8)^2 = PT^2 + OP^2$
$\Rightarrow TR^2 + 3.6TR + 3.24 = PT^2 + 3^2$ ....(ii)
Solving (i) and (ii), we get
$TR = 3.2$ cm and $TP = 4$ cm

Question 14
Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

Solution 14



Given : CD and EF are the tangents are two parallel tangents with centre O.

To prove: AOB passes through the centre, that is, to prove that AB is a diameter.

Construction: Join OA and OB. Draw OG||CD.

Proof:

OG||CD and AO cuts them.

$\therefore \angle CAO + \angle GOA = 180°$ ....(interior angles)

$\Rightarrow 90° + \angle GOA = 180°$ ....$[\because OA \perp CD]$

$\Rightarrow \angle GOA = 90°$

Similarly, $\angle GOB = 90°$

$\therefore \angle GOA + \angle GOB = 180°$

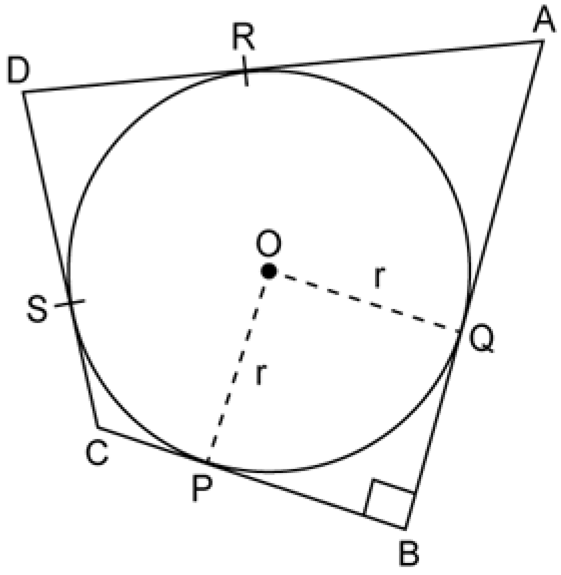
$\Rightarrow$ AOB is a straight line.

$\Rightarrow$ AOB is a diameter of the circle with centre O.

Hence, AOB passes through the centre.

Question 15

In the given figure, a circle with centre O, is inscribed in a quadrilateral ABCD such that it touches the side BC, AB, AD and CD at points P, Q, R and S respectively. If AB = 29 cm, AD = 23 cm, ∠B = 90° and DS = 5 cm then find the radius of the circle.



Solution 15

PB and BQ are tangents to the circle.

So, $\angle PBQ = \angle BPO = \angle BQO = 90^\circ$

and OP = OR = r

$\Rightarrow$ POQB is a square.

$\Rightarrow r = OQ = QB$

We know that tangents drawn from an external point to a circle are equal.

$\therefore DS = DR,\ AR = AQ$

Now, $AR = AD - DR$

$\Rightarrow AR = AD - DS$

$\Rightarrow AR = 23 - 5$

$\Rightarrow AR = 18$ cm

$r = QB$

$\Rightarrow r = AB - AQ$

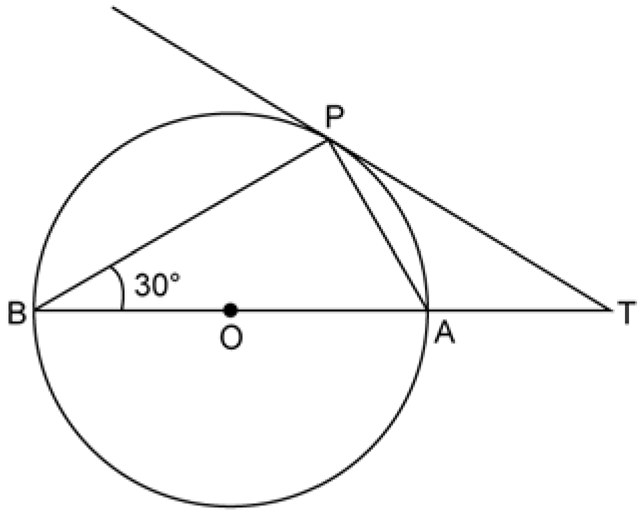
$\Rightarrow r = AB - AR$

$\Rightarrow r = 29 - 18$

$\Rightarrow r = 11$ cm

Question 16

In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT = 30^\circ$, prove that BA: AT = 2:1.



Solution 16

Since AB is the diameter of the circle, $\angle APB = 90^\circ$.

$\therefore \angle PAB = 90^\circ - \angle PBA$

$= 90^\circ - 30^\circ$

$= 60^\circ \quad \ldots.[\because \angle PBA = \angle PBT = 30^\circ]$

$\angle PAT + \angle PAB = 180^\circ \quad \ldots.$(linear pair)

$\Rightarrow \angle PAT = 180^\circ - \angle PAB$

$\Rightarrow \angle PAT = 180^\circ - 60^\circ$

$\Rightarrow \angle PAT = 120^\circ$

$\angle APT = \angle PBA = 30^\circ \quad \ldots.$(angles in alternate segments)

In ΔPAT, we have

$\angle APT + \angle PAT + \angle PTA = 180^\circ$

$\Rightarrow \angle PTA = 180^\circ - (\angle APT + \angle PAT)$

$\Rightarrow \angle PTA = 180^\circ - (30^\circ + 120^\circ)$

$\Rightarrow \angle PTA = 30^\circ$

Now, $\angle APT = \angle PTA = 30^\circ$

$\Rightarrow AT = AP \quad \ldots.(i) \quad \ldots$(Since sides opposite equal angles are equal)

In right ΔAPB, we have

$\cos(\angle PAB) = \frac{AP}{BA}$

$\Rightarrow \cos 60^\circ = \frac{AP}{BA}$

$\Rightarrow \frac{1}{2} = \frac{AP}{BA}$

$\Rightarrow BA = 2AP \quad \ldots..(ii)$

From (i) and (ii), we get

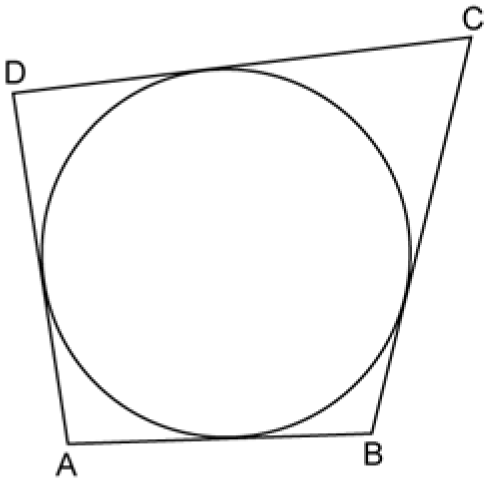
$\frac{BA}{AT} = \frac{2AP}{AP} = \frac{2}{1}$

Hence proved.

Excercise 8B

Question 1

In the adjoining figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.



Solution 1

Using the property, tangents from an external point to the circle are equal.

We can say, AB + CD = AD + BC

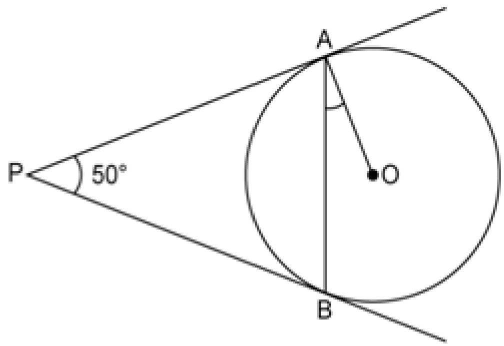
$\Rightarrow$ AD = AB + CD − BC

$\Rightarrow$ AD = 6 + 8 − 9

$\Rightarrow$ AD = 5 cm

Question 2

In the given figure, PA and PB are two tangents to the circle with centre O. If ∠APB = 50° then what is the measure of ∠OAB.



Solution 2

We know that tangents from an external point to a circle are equal.

So,

$PA = PB$

$\Rightarrow \angle PAB = \angle PBA$ ....(angles opposite equal sides are equal)

Now in ΔPAB,

$\angle PAB + \angle PBA + \angle APB = 180^\circ$ ....(Angle Sum Property)

$\Rightarrow \angle PAB + \angle PAB + 50^\circ = 180^\circ$

$\Rightarrow 2\angle PAB + 50^\circ = 180^\circ$

$\Rightarrow 2\angle PAB = 130^\circ$

$\Rightarrow \angle PAB = 65^\circ$

Since AP is a tangent to the circle,

$\angle OAP = 90^\circ$

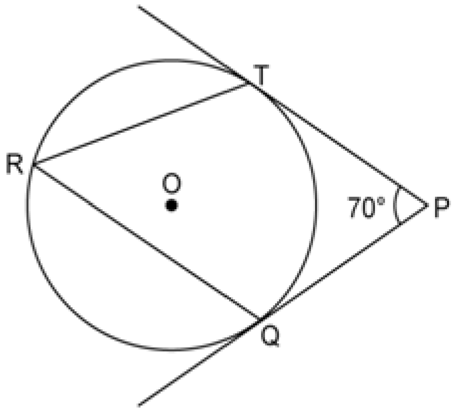
$\Rightarrow \angle OAB + \angle PAB = 90^\circ$

$\Rightarrow \angle OAB + 65^\circ = 90^\circ$

$\Rightarrow \angle OAB = 25^\circ$

Question 3

In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, find ∠TRQ.



Solution 3

Construction : Join OT and OQ

OT and OQ are perpendicular to PT and PQ since radii are perpendicular to the tangents.

In quad. OTPQ,

$\angle OTP + \angle TPQ + \angle OQP + \angle TOQ = 360^\circ$

$\Rightarrow 90^\circ + 70^\circ + 90^\circ + \angle TOQ = 360^\circ$

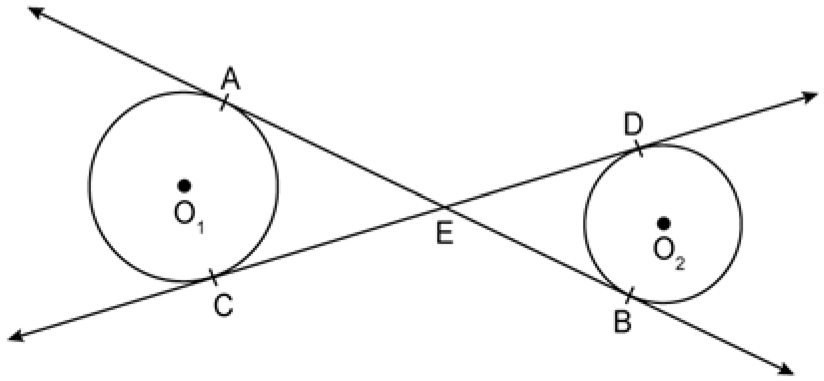
$\Rightarrow \angle TOQ = 110^\circ$

We know that,

$\angle TRQ = \frac{1}{2}\angle TOQ = 55^\circ$ .....(Inscribed angle theorem)

Question 4

In the given figure, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD.



Solution 4

We know that tangents drawn from an external point to a circle are equal.

EA = EC and ED = EB

Adding the two equations, we get

EA + ED = EC + EB

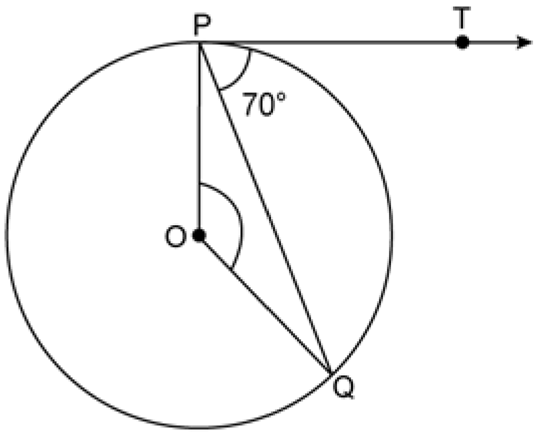
$\Rightarrow$ EA + EB = EC + ED

$\Rightarrow$ AB = CD

Hence proved.

Question 5

If PT is a tangent to a circle with centre O and PQ is a chord of the circle such that ∠QPT = 70°, then find the measure of ∠POQ.



Solution 5

We know that the radius is perpendicular to the tangent of a circle.

$\Rightarrow \angle OPT = 90°$

Now, $\angle OPQ = \angle OPT - \angle TPQ = 90° - 70° = 20°$

Since $OP = OQ$ ....(radii of the same circle)

$\Rightarrow \angle OPQ = \angle OQP = 20°$ ....(angles opposite equal sides are equal)

In ΔPOQ,

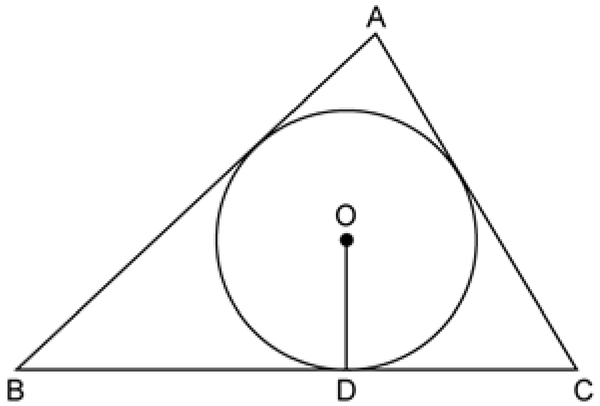
$\angle OPQ + \angle OQP + \angle POQ = 180°$ ...(Angle Sum Property)

$\Rightarrow 20° + 20° + \angle POQ = 180°$

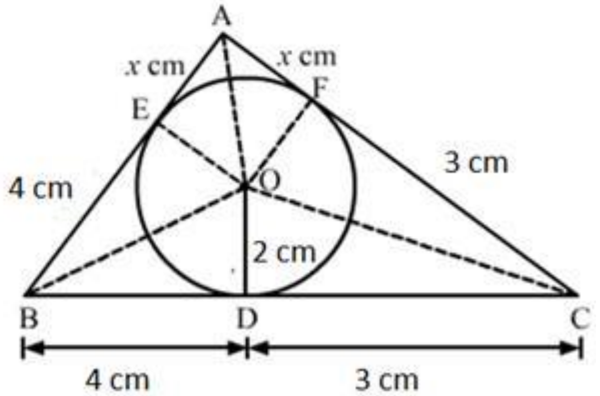
$\Rightarrow \angle POQ = 140°$

Question 6

In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D, are of lengths 4 cm and 3 cm respectively. If the area of $\Delta ABC = 21\ cm^2$ then find the lengths of sides AB and AC.



Solution 6



Construction : Join OA, OB, OC.
Draw $OD \perp BC$, $OF \perp AC$ and $OE \perp AB$.
We know that tangents drawn from an external point
to the circle are equal.
So,
$AE = AF = x$ (say)
$BD = BE = 4$ cm
$CD = CF = 3$ cm
Now,
$ar(\Delta ABC) = ar(\Delta BOC) + ar(\Delta AOC) + ar(\Delta AOB)$

$$\Rightarrow 21 = \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OF + \frac{1}{2} \times OE \times AB$$

$$\Rightarrow 21 = \frac{1}{2}\left[BC \times OD + AC \times OF + OE \times AB\right]$$

$$\Rightarrow 21 = \frac{1}{2}\left[7 \times 2 + 2(3 + x) + 2(4 + x)\right]$$

$$\Rightarrow 42 = 7 \times 2 + 2(3 + x) + 2(4 + x)$$

$$\Rightarrow 42 = 14 + 6 + 2x + 8 + 2x$$

$$\Rightarrow 4x = 14$$

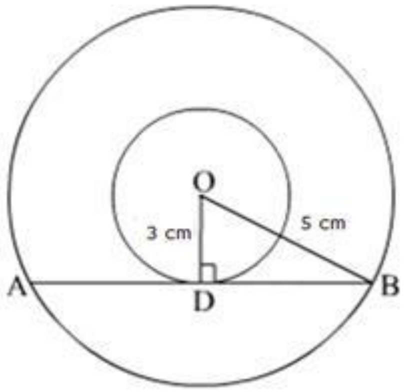
$$\Rightarrow x = 3.5$$

So, $AB = 4 + 3.5 = 7.5$ cm and $AC = 3 + 3.5 = 6.5$ cm.

Question 7

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle (in cm) which touches the smaller circle.

Solution 7



Since AB is a tangent to the inner circle.

$\angle ODB = 90°$ ....(tangent is perpendicular to the radius of a circle)

AB is a chord of the outer circle.

We know that, the perpendicular drawn from the centre to a chord of a circle, bisects the chord.

So, AB = 2DB.

In ΔODB,

By Pythagoras theorem,

$OB^2 = OD^2 + DB^2$

$\Rightarrow 5^2 = 3^2 + DB^2$

$\Rightarrow DB^2 = 5^2 - 3^2$

$\Rightarrow DB^2 = 25 - 9$

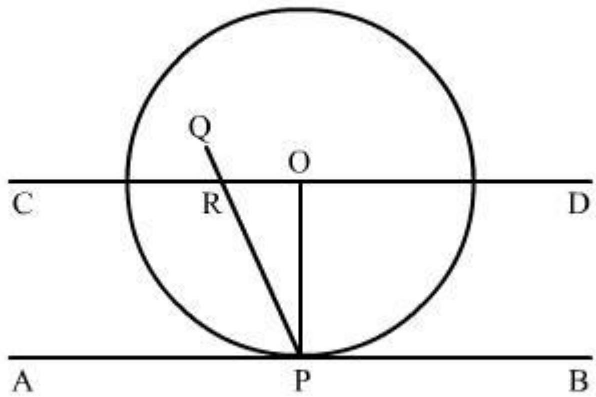
$\Rightarrow DB = 4$ cm

$AB = 2DB = 2(4) = 8$ cm

Question 8

Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre.

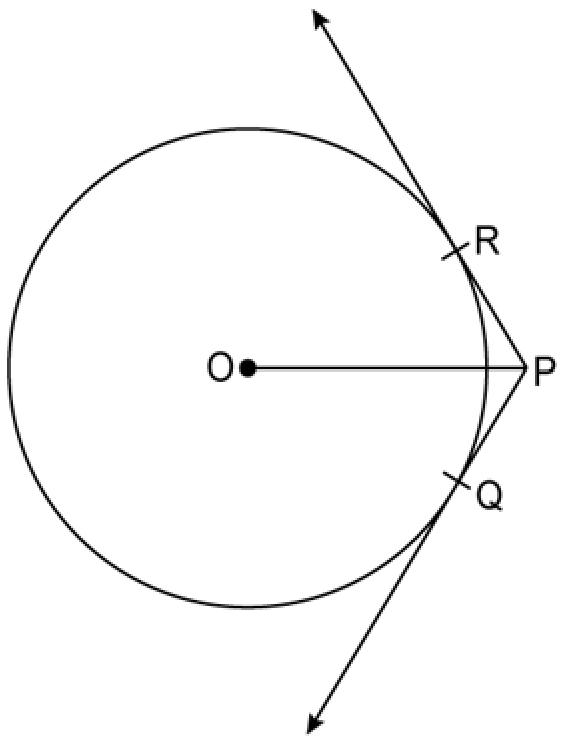
Solution 8



Given: AB is a tangent to the circle at point P with centre O.
To prove: PQ passes through the point O.
Construction: Join OP. Through O, draw a straight line CD parallel to the tangent AB.
Proof:
Suppose PQ does not pass through the point O.
PQ intersects CD at R and also intersects AB at P.
Since CD||AB, PQ is the line of intersection.
$\angle ORP = \angle RPA$ .....(Alternate interior angles)
But, $\angle RPA = 90°$ ....($\because OP \perp AB$)
$\Rightarrow \angle ORP = 90°$
$\angle ROP + \angle OPA = 180°$ ....(Interior angles)
$\Rightarrow \angle ROP + 90° = 180°$
$\Rightarrow \angle ROP = 90°$
$\Rightarrow \Delta ORP$ has two right angles,
that is, $\angle ROP$ and $\angle ORP$. This is not possible.
Thus, our assumption is wrong.
Hence, PQ passes through the point O.

Question 9

In the given figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120°$, then prove that OR = PR + RQ.



Solution 9

Construction : Join PO and OQ.

In ΔPOR and ΔQOR,

$RP = RQ$

....(Since tangents from an external point to the circle are equal)

$OR = OR$ ...(common side)

$OP = OQ$ ...(radii of the same circle)

$\Rightarrow \Delta POR \cong \Delta QOR$ .....(SSS congruence criterion)

$\angle PRO = \angle QRO$ ....(cpct)

Now, $\angle PRO + \angle QRO = \angle PRQ$

$\Rightarrow 2\angle PRO = 120°$

$\Rightarrow \angle PRO = 60°$

In ΔPRO,

$\cos 60° = \dfrac{PR}{OR}$

$\Rightarrow \dfrac{1}{2} = \dfrac{PR}{OR}$

$\Rightarrow OR = 2PR$

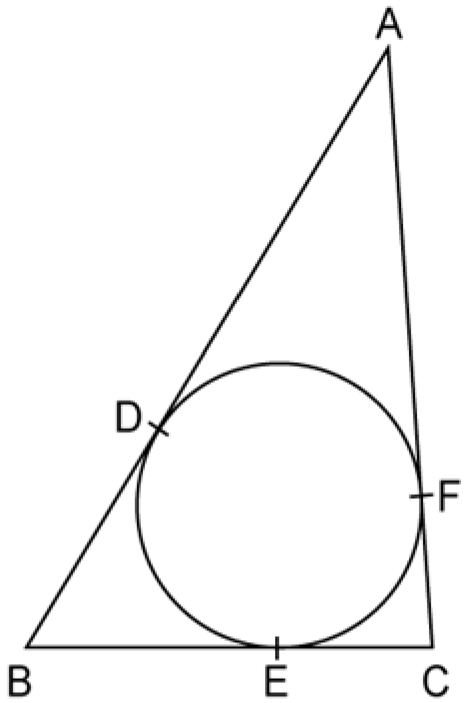
$\Rightarrow OR = PR + PR$

$\Rightarrow OR = PR + RQ$ ....(Since $RP = RQ$)

Hence proved.

Question 10

In the given figure, a circle inscribed in a triangle ABC touches the sides AB, BC and CA at points D, E and F respectively. If AB = 14 cm, BC = 8 cm and CA = 12 cm. Find the lengths AD, BE and CF.



Solution 10

We know that tangents from an external point to the circle are equal.

$AD = AF = x$

$BD = BE = y$

$CE = CF = z$

Given that $AB = 14$ cm, $BC = 8$ cm and $AC = 12$ cm

$\Rightarrow x + y = 14,\ y + z = 8,\ z + x = 12$

Adding the three equation, we get

$2(x + y + z) = 34$

$\Rightarrow x + y + z = 17 \quad \ldots.(i)$

Using (i) we get,

$\Rightarrow x + 8 = 17$

$\Rightarrow x = 9 = AD$

So, $AD = 9$ cm

Using (i) we get,

$x + y + z = 17$

$\Rightarrow 14 + z = 17$

$\Rightarrow z = 3$

So, $CF = 3$ cm

Using (i) we get,

$x + y + z = 17$

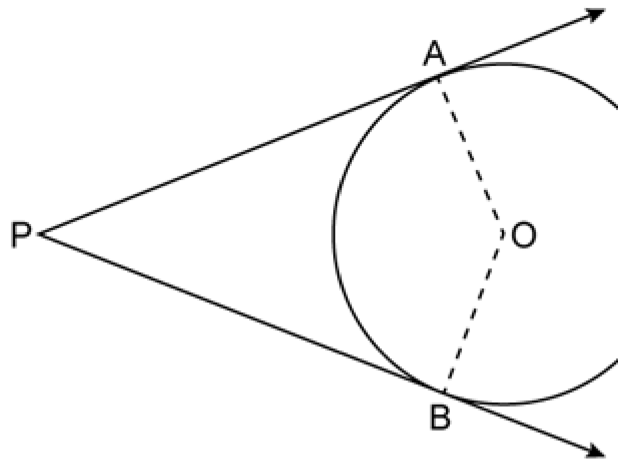
$\Rightarrow 12 + y = 17$

$\Rightarrow y = 5$

So, $BE = 5$ cm

Question 11

In the given figure, O is the centre of the circle. PA and PB are tangents. Show that AOBP is a cyclic quadrilateral.



Solution 11

OA = OB .....(radii of the same circle)

Since PA and PB are tangents to the circle,

$\angle OAP = \angle OBP = 90^\circ$

Consider,

$\angle OAP + \angle OBP$

$= 90^\circ + 90^\circ$

$= 180^\circ$

In quad. AOBP,

$\angle PAO + \angle PBO + \angle AOB + \angle APB = 360^\circ$

$\Rightarrow 180^\circ + \angle AOB + \angle APB = 360^\circ$

$\Rightarrow \angle AOB + \angle APB = 180^\circ$

Since the sum of the opposite angles of quad. AOBP are supplementary, AOBP are concyclic.

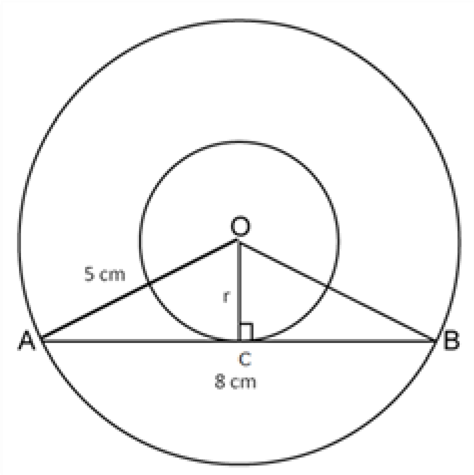
That is, a circle passes through A, O, B and P.

Hence proved.

Question 12

In two concentric circles, a chord of length 8 cm of the larger circle touches the smaller circle. If the radius of the larger circle is 5 cm then find the radius of the smaller circle.

Solution 12



Since AB is a tangent to the inner circle.

$\angle OCB = 90°$ ....(tangent is perpendicular to the radius of a circle)

AB is a chord of the outer circle.

We know that, the perpendicular drawn from the centre to a chord of a circle, bisects the chord.

So, $AB = 2AC \Rightarrow AC = 4$ cm

In ΔOCB,

By Pythagoras theorem,

$OA^2 = OC^2 + AC^2$

$\Rightarrow 5^2 = r^2 + 4^2$

$\Rightarrow r^2 = 5^2 - 4^2$

$\Rightarrow r^2 = 25 - 16$

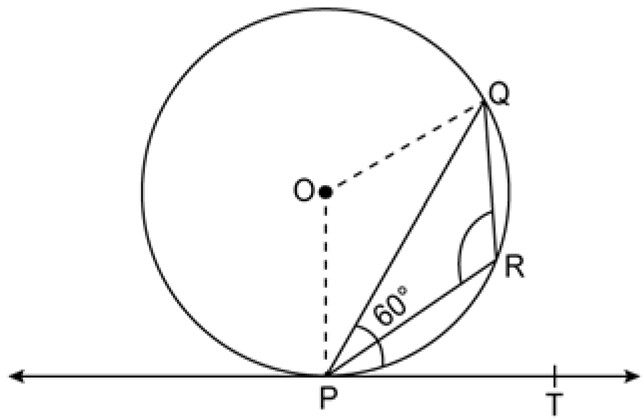
$\Rightarrow r^2 = 9$ cm

$\Rightarrow r = 3$ cm

Hence, the radius of the smaller circle is 3 cm.

Question 13

In the given figure, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$.



Solution 13

Since PT is a tangent to the circle,

$\angle OPT = 90°$ ....(tangent is perpendicular to the radius of a circle)

So,

$\angle OPT = \angle QPT + \angle OPQ$

$\Rightarrow 90° = 60° + \angle OPQ$

$\Rightarrow \angle OPQ = 30°$

Since OP = OQ, $\angle OPQ = \angle OQP = 30°$

In ΔOQP,

$\angle QPQ + \angle OQP + \angle POQ = 180°$

$\Rightarrow 30° + 30° + \angle POQ = 180°$

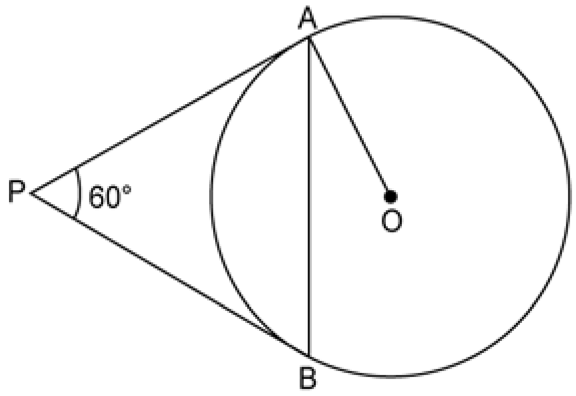
$\Rightarrow \angle POQ = 120°$

So, $\angle POQ$ on the major arc $= 360° - 120° = 240°$

So, $\angle PRQ = \frac{1}{2}$ central angle $= \frac{1}{2} \times 240° = 120°$

Question 14

In the given figure, PA and PB are two tangents to the circle with centre O. If ∠APB = 60° then find the measure of ∠OAB.



Solution 14

We know that tangents from an external point
to a circle are equal.
So,
$PA = PB$
$\Rightarrow \angle PAB = \angle PBA$ ....(angles opposite equal sides are equal)
Now in ΔPAB,
$\angle PAB + \angle PBA + \angle APB = 180°$ ....(Angle Sum Property)
$\Rightarrow \angle PAB + \angle PAB + 60° = 180°$
$\Rightarrow 2\angle PAB + 60° = 180°$
$\Rightarrow 2\angle PAB = 120°$
$\Rightarrow \angle PAB = 60°$
Since AP is a tangent to the circle,
$\angle OAP = 90°$
$\Rightarrow \angle OAB + \angle PAB = 90°$
$\Rightarrow \angle OAB + 60° = 90°$
$\Rightarrow \angle OAB = 30°$

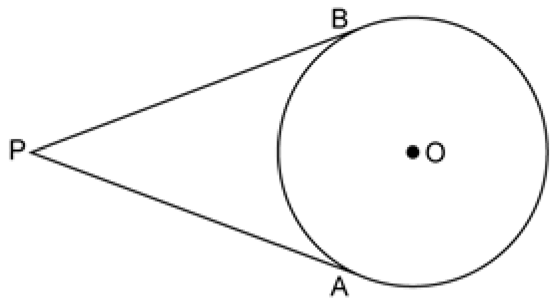
Chapter 12 - Circles Excercise MCQ

Question 1

The number of tangents that can be drawn from an external circle is
(a) 1
(b) 2
(c) 3
(d) 4

Solution 1

Correct option : (b)



We can draw only 2 tangents from an external point to a circle.

Question 2

In the given figure, RQ is a tangent to the circle with centre O. If SQ = 6 cm and QR = 4 cm, then OR is equal to

(a) 2.5 cm
(b) 3 cm
(c) 5 cm
(d) 8 cm

Solution 2

Correct option: (c)

SQ = 6 cm ⇒ OQ = 3 cm

QR = 4 cm

Since RQ is a tangent to the circle at Q.

$\angle RQO = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)

In ΔRQO,

By Pythagoras theorem,

$OR^2 = QR^2 + OQ^2$

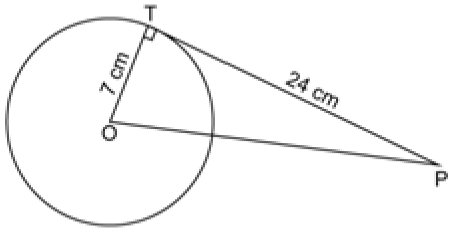
$\Rightarrow OR^2 = 4^2 + 3^2$

$\Rightarrow OR^2 = 25$

$\Rightarrow OR = 5$ cm

Question 3

In a circle of radius 7 cm, tangent PT is drawn from a point P such that PT = 24 cm. If 0 is the centre of the circle, then length OP = ?



(a) 30 cm
(b) 28 cm
(c) 25 cm
(d) 18 cm

Solution 3

Correct option: (c)

PT = 24 cm

OT = 7 cm

Since PT is a tangent to the circle at T.

$\angle PTO = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)

In ΔPTO,

By Pythagoras theorem,

$OP^2 = PT^2 + OT^2$

$\Rightarrow OP^2 = 24^2 + 7^2$

$\Rightarrow OP^2 = 576 + 49$

$\Rightarrow OP^2 = 625$

$\Rightarrow OP = 25$ cm

Question 4

Which of the following pairs of lines in a circle cannot be parallel?
(a) two chords
(b) a chord and a tangent
(c) two tangents
(d) two diameters

Solution 4

Correct option: (d)

The diameter of the circle always passes through the centre. This means all the diameters of a given circle will intersect at the centre, and hence they cannot be parallel.

Question 5

The chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord (in cm) is

(a) $\frac{5}{\sqrt{2}}$

(b) $5\sqrt{2}$

(c) $10\sqrt{2}$

(d) $10\sqrt{3}$

Solution 5

Correct option: (c)

In ΔPOQ,

By Pythagoras theorem,

$PQ^2 = PO^2 + OQ^2$

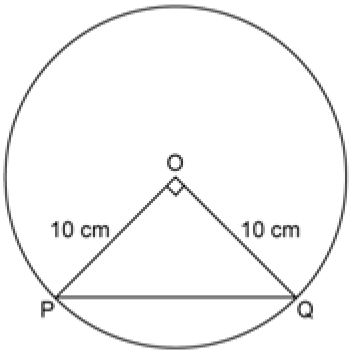
$\Rightarrow PQ^2 = 10^2 + 10^2$

$\Rightarrow PQ^2 = 100 + 100$

$\Rightarrow PQ^2 = 200$

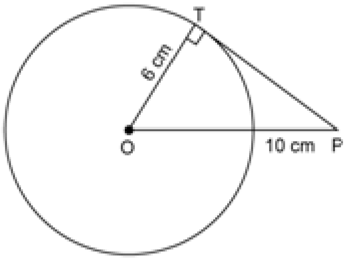
$\Rightarrow PQ = 10\sqrt{2}$ cm

So, the length of the chord is $10\sqrt{2}$ cm.



Question 6

In the given figure, PT is a tangent to the circle with centre O. If OT = 6 cm and OP =10 cm, then the length of tangent PT is



(a) 8 cm
(b) 10 cm
(c) 12 cm
(d) 16 cm

Solution 6

Correct option: (a)

OT = 6 cm

OP = 10 cm

Since PT is a tangent to the circle at T.

$\angle PTO = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)

In ΔPTO,

By Pythagoras theorem,

$OP^2 = PT^2 + OT^2$

$\Rightarrow PT^2 = OP^2 - OT^2$

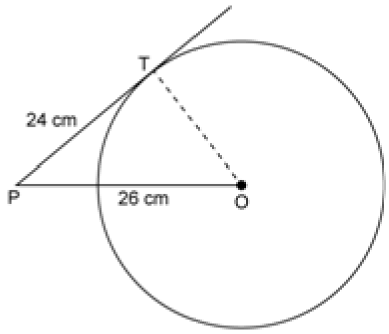
$\Rightarrow PT^2 = 10^2 - 6^2$

$\Rightarrow PT^2 = 100 - 36$

$\Rightarrow PT = 8$ cm

Question 7

In the given figure, point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then, the radius of the circle is



(a) 10 cm
(b) 12 cm
(c) 13 cm
(d) 15 cm

Solution 7

Correct option: (a)
Construction: Join OT.
PT = 24 cm
OP = 26 cm
Since PT is a tangent to the circle at T.
$\angle PTO = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)
In ΔPTO,
By Pythagoras theorem,
$OP^2 = PT^2 + OT^2$
$\Rightarrow OT^2 = OP^2 - PT^2$
$\Rightarrow OT^2 = 26^2 - 24^2$
$\Rightarrow OT^2 = 676 - 576$
$\Rightarrow OT^2 = 100$
$\Rightarrow OT = 10$ cm

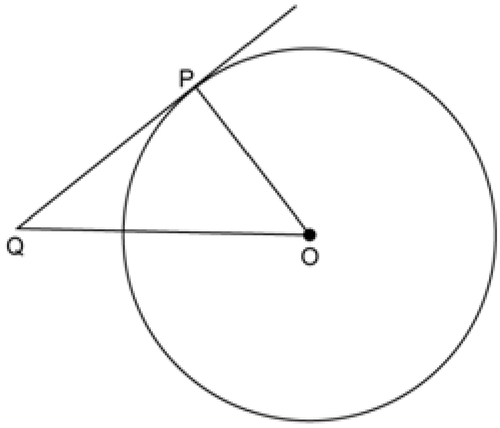
Question 8

PQ is a tangent to a circle with centre O at the point P. If ΔOPQ is an isosceles triangle, then $\angle OQP$ is equal to

(a) 30°
(b) 45°
(c) 60°
(d) 90°

Solution 8

Correct option: (b)
Given that ΔPQO is an isosceles triangle.
Since PQ is a tangent to the circle at P.
$\angle OPQ = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)
In ΔOPQ,
OP = OQ
$\Rightarrow \angle OQP = \angle POQ$
Using Angle Sum Property,
$\angle OQP + \angle POQ + \angle OPQ = 180^\circ$
$\Rightarrow \angle OQP + \angle OQP + 90^\circ = 180^\circ$
$\Rightarrow 2\angle OQP = 90^\circ$
$\Rightarrow \angle OQP = 45^\circ$



Question 9

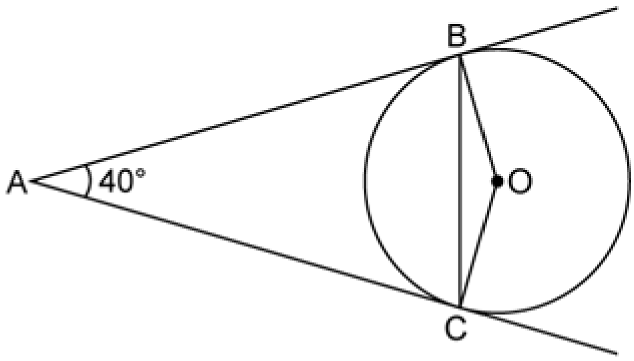
In the given figure, AB and AC are tangents to the circle with centre O such that ∠BAC = 40°. Then, ∠BOC is equal to

(a) 80°

(b) 100°

(c) 120°

(d) 140°



Solution 9

Correct option: (d)

Since AB and AC are the tangents to the circle.

$\angle OBA = \angle OCA = 90°$ ....(tangent is perpendicular to the radius of a circle)

In ABOC,

$\angle OBA + \angle BAC + \angle OCA + \angle BOC = 360°$

$\Rightarrow 90° + 40° + 90° + \angle BOC = 360°$

$\Rightarrow \angle BOC = 140°$

Question 10

If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents to the circle drawn from A and B is

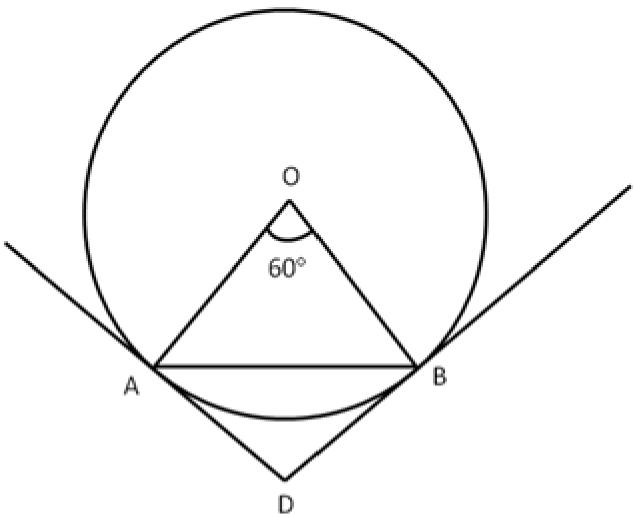
(a) 30°

(b) 60°

(c) 90°

(d) 120°

Solution 10



Correct option: (d)

Since AD and DB are the tangents to the circle.

$\angle OAD = \angle OBD = 90°$ ....(tangent is perpendicular to the radius of a circle)

In ABOD,

$\angle OAD + \angle ADB + \angle OBD + \angle AOB = 360°$

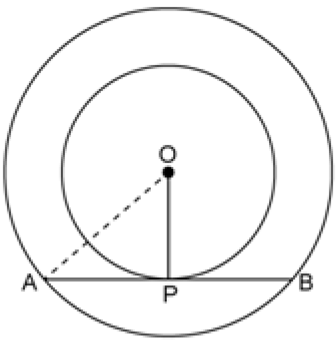
$\Rightarrow 90° + \angle ADB + 90° + 60° = 360°$

$\Rightarrow \angle ADB = 120°$

Question 11

In the given figure, O is the centre of two concentric circles of radii 6 cm and 10 cm. AB is a chord of outer circle which touches the inner circle. The length of chord AB is

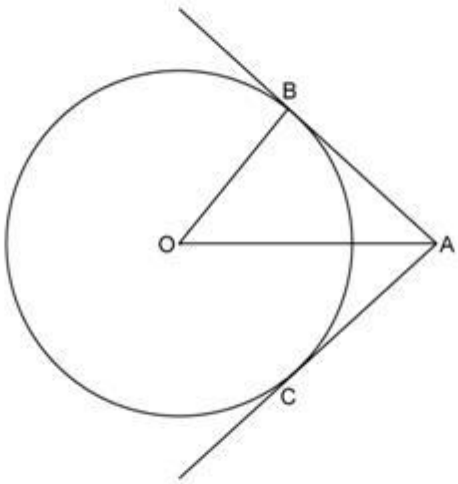
(a) 8 cm
(b) 14 cm
(c) 16 cm
(d) 136 cm



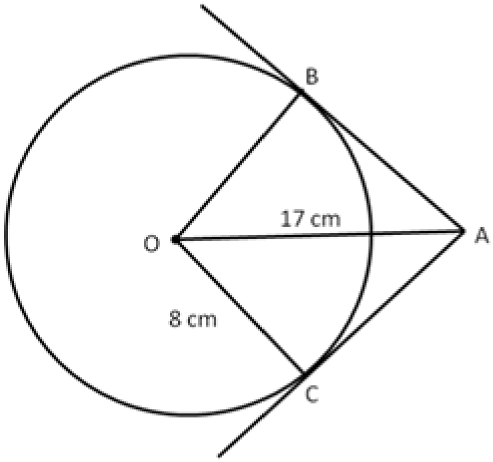
Solution 11

Correct option: (c)
Since AB is a tangent to the inner circle.
$\angle OPA = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)
AB is a chord of the outer circle.
We know that, the perpendicular drawn from the
centre to a chord of a circle, bisects the chord.
In ΔOPA,
By Pythagoras theorem,
$OA^2 = OP^2 + AP^2$
$\Rightarrow 10^2 = 6^2 + AP^2$
$\Rightarrow AP^2 = 10^2 - 6^2$
$\Rightarrow AP^2 = 64$
$\Rightarrow AP = 8$ cm
$AB = 2AP = 2(8) = 16$ cm

Question 12
In the given figure, AB and AC are tangents to a circle with centre O and radius 8 cm. If OA =17 cm, then the length of AC (in cm) is
(a) 9
(b) 15
(c) $\sqrt{353}$
(d) 25



Solution 12



Correct option: (b)

Construction: Join OC.

Since AC is a tangent to the inner circle.

$\angle OCA = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)

In ΔOCA,

By Pythagoras theorem,

$OA^2 = OC^2 + AC^2$

$\Rightarrow 17^2 = 8^2 + AC^2$

$\Rightarrow AC^2 = 289 - 64$

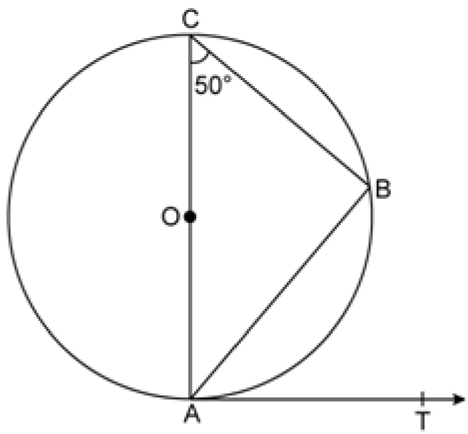
$\Rightarrow AC^2 = 225$

$\Rightarrow AC = 15$ cm

Question 13

In the given figure, O is the centre of a circle, AOC is its diameter such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A then $\angle BAT$=?

(a) 40°
(b) 50°
(c) 60°
(d) 65°



Solution 13

Correct option: (b)

Since AC is a diameter of the circle.

$\angle ABC = 90^\circ$ ...(angle in a semicircle is 90°)

In ΔABC,

$\angle ABC + \angle BCA + \angle BAC = 180^\circ$ ....(Angle Sum Property)

$\Rightarrow 90^\circ + 50^\circ + \angle BAC = 180^\circ$

$\Rightarrow \angle BAC = 40^\circ$

Since AC is a tangent to the inner circle.

$\angle OAT = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)

$\Rightarrow \angle BAO + \angle BAT = 90^\circ$

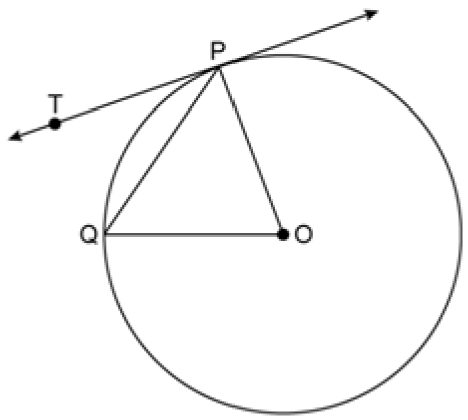
$\Rightarrow 40^\circ + \angle BAT = 90^\circ$

$\Rightarrow \angle BAT = 50^\circ$

Question 14

In the given figure, O is the centre of a circle, PQ is a chord and PT is the tangent at P. If $\angle POQ = 70^\circ$, then $\angle TPQ$ is equal to

(a) 35°
(b) 45°
(c) 55°
(d) 70°



Solution 14

Correct option: (a)

In ΔOPQ,

$OP = OQ$ ....(radii of the same circle)

$\Rightarrow \angle OQP = \angle OPQ$ ....(angles opposite equal sides are equal)

In ΔOPQ,

$\angle OQP + \angle OPQ + \angle POQ = 180°$ ....(Angle Sum Property)

$\Rightarrow \angle OPQ + \angle OPQ + 70° = 180°$

$\Rightarrow 2\angle OPQ = 110°$

$\Rightarrow \angle OPQ = 55°$

Since PT is a tangent to the inner circle.

$\angle OPT = 90°$ ....(tangent is perpendicular to the radius of a circle)

$\Rightarrow \angle OPQ + \angle TPQ = 90°$

$\Rightarrow 55° + \angle TPQ = 90°$

$\Rightarrow \angle TPQ = 35°$

Question 15

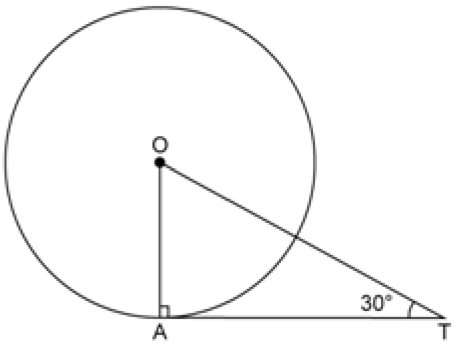
In the given figure, AT is a tangent to the circle with centre O such that OT = 4 cm and $\angle OTA$ = 30°. Then, AT= ?

(a) 4 cm

(b) 2 cm

(c) $2\sqrt{3}$ cm

(d) $4\sqrt{3}$ cm



Solution 15

Correct option: (c)

Since $\angle OAT = 90°$ and $\angle OTQ = 30°$

Clearly, $\angle AOT = 60°$

So, ΔAOT is a 30°-60°-90° triangle.

Side opposite $60° = \frac{\sqrt{3}}{2}$ hypotenuse

$\Rightarrow AT = \frac{\sqrt{3}}{2} OT$

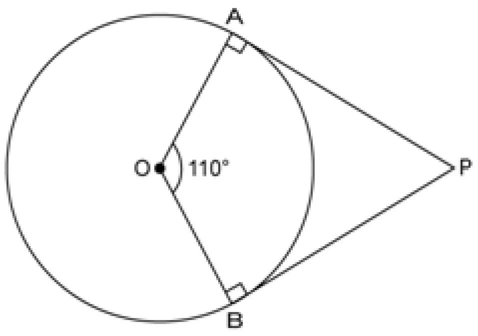
$\Rightarrow AT = \frac{\sqrt{3}}{2}(4)$

$\Rightarrow AT = 2\sqrt{3}$ cm

Question 16

If PA and PB are two tangents to a circle with centre O such that ∠AOB =110° then ∠APB is equal to

(a) 55°
(b) 60°
(c) 70°
(d) 90°



Solution 16

Correct option: (c)

Since PA and PB are the tangents to the circle.

$\angle OAP = \angle OBP = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)

In AOBP,

$\angle OAP + \angle APB + \angle OBP + \angle AOB = 360^\circ$

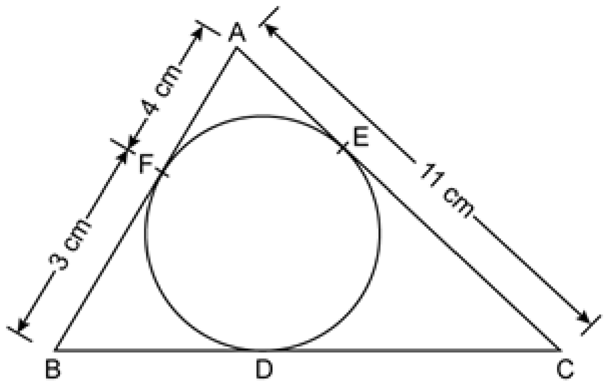
$\Rightarrow 90^\circ + \angle APB + 90^\circ + 110^\circ = 360^\circ$

$\Rightarrow \angle APB = 70^\circ$

Question 17

In the given figure, the length of BC is

(a) 7 cm
(b) 10 cm
(c) 14 cm
(d) 15 cm



Solution 17

Correct option: (b)

We know that tangents from an external point to a circle are equal.

So,

$AF = AE = 4$ cm

$\Rightarrow EC = AC - AE = 11 - 4 = 7$ cm

Now,

$CD = EC = 7$ cm and

$BF = BD = 3$ cm

$BD = BD + CD$

$\Rightarrow BD = 3 + 7$

$\Rightarrow BD = 10$ cm

Question 18

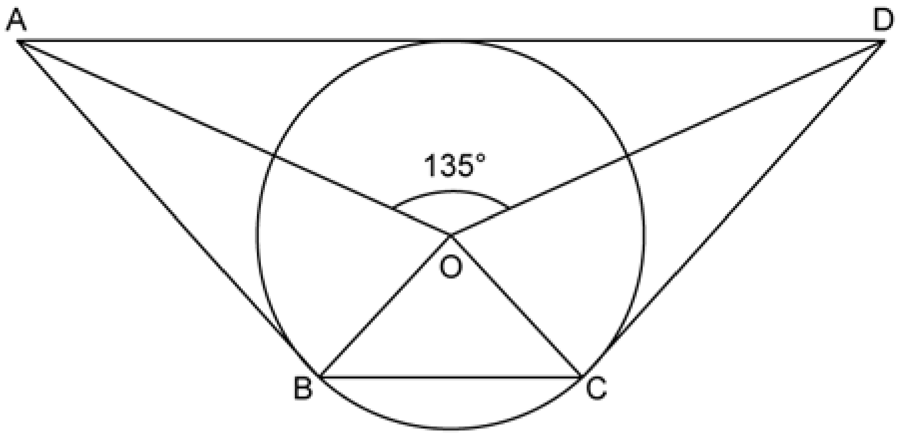
In the given figure, if $\angle AOD = 135°$ then $\angle BOC$ is equal to

(a) 25°

(b) 45°

(c) 52.5°

(d) 62.5°



Solution 18

Correct option: (b)

We know that sum of the angles subtended by opposite sides of a quadrilateral having a circumscribed circle is 180°.

$\Rightarrow \angle AOD + \angle BOC = 180°$

$\Rightarrow 135° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 45°$

Question 19

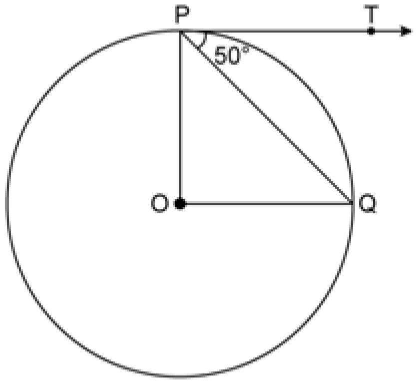
In the given figure, O is the centre of a circle and PT is the tangent to the circle. If PQ is a chord such that $\angle QPT = 50°$ then $\angle POQ = ?$

(a) 100°

(b) 90°

(c) 80°

(d) 75°



Solution 19

Correct option: (a)

Since PT is the tangent to the circle,

$\angle OPT = 90^\circ$

$\Rightarrow \angle TPQ + \angle OPQ = 90^\circ$

$\Rightarrow 50^\circ + \angle OPQ = 90^\circ$

$\Rightarrow \angle OPQ = 40^\circ$

In ΔOPQ,

$OP = OQ$ ...(radii of the same circle)

$\Rightarrow \angle OPQ = \angle OQP = 40^\circ$ ....(angles opposite equal sides are equal)

Now,

$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$ ....(Angle Sum Property)

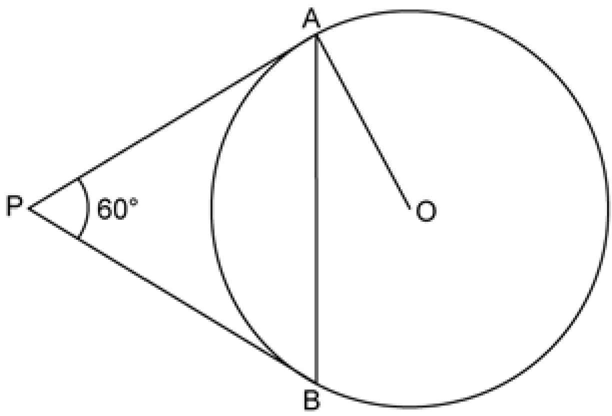
$\Rightarrow 40^\circ + 40^\circ + \angle POQ = 180^\circ$

$\Rightarrow \angle POQ = 100^\circ$

Question 20

In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 60^\circ$ then $\angle OAB$ is

(a) 15°
(b) 30°
(c) 60°
(d) 90°



Solution 20

Correct option: (b)

We know that tangents from an external point to a circle are equal.

So,

PA = PB

$\Rightarrow \angle PAB = \angle PBA$ ....(angles opposite equal sides are equal)

Now in ΔPAB,

$\angle PAB + \angle PBA + \angle APB = 180°$ ....(Angle Sum Property)

$\Rightarrow \angle PAB + \angle PAB + 60° = 180°$

$\Rightarrow 2\angle PAB + 60° = 180°$

$\Rightarrow 2\angle PAB = 120°$

$\Rightarrow \angle PAB = 60°$

Since AP is a tangent to the circle,

$\angle OAP = 90°$

$\Rightarrow \angle OAB + \angle PAB = 90°$

$\Rightarrow \angle OAB + 60° = 90°$

$\Rightarrow \angle OAB = 30°$

Question 21

If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm then the length of each tangent is

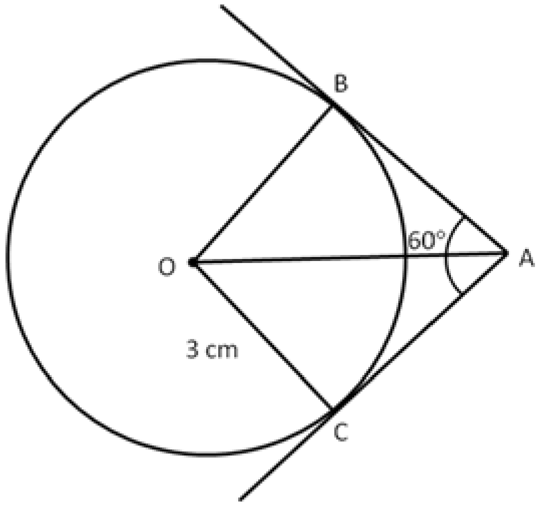
(a) 3 cm

(b) $\frac{3\sqrt{3}}{2}$ cm

(c) $3\sqrt{3}$

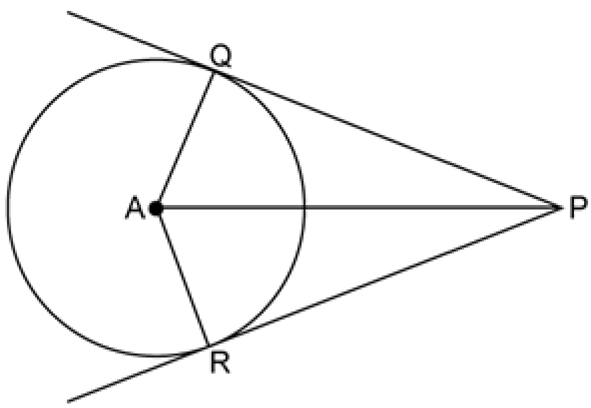
(d) 6 cm

Solution 21



Correct option: (c)
In ΔBAO and ΔCAO,
$\angle OBA = \angle OCA = 90°$ ....(Since AB and AC are tangent to the circle)
$OA = OA$ ...(common side)
$OB = OC$ ...(radii of the same circle)
$\Rightarrow \Delta BAO \cong \Delta CAO$ .....(RHS congruence criterion)
$\angle OAB = \angle OAC$ ....(cpct)
$\Rightarrow \angle OAB = \frac{1}{2}\angle BAC = 30°$
So, ΔBAO is a 30-60-90 triangle.
side opposite $30° = \frac{1}{2}$hypotenuse
$\Rightarrow OB = \frac{1}{2}$hypotenuse
$\Rightarrow$ hypotenuse $= 2OB = 2(3) = 6$ cm
side opposite $60° = \frac{\sqrt{3}}{2}$hypotenuse
$\Rightarrow AB = \frac{\sqrt{3}}{2}(6) = 3\sqrt{3}$ cm
$AB = AC = 3\sqrt{3}$ cm
....(Since tangents from an external point to the circle are equal)
Hence, the length of each tangent is $3\sqrt{3}$ cm.

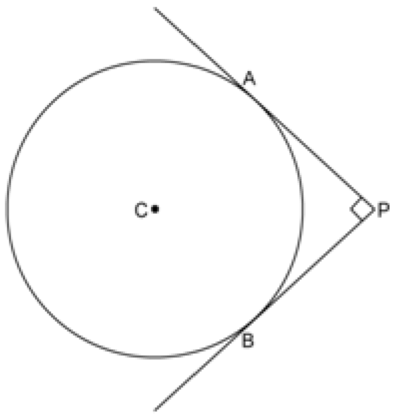
Question 22
In the given figure, PQ and PR are tangents to a circle with centre A. If ∠QPA = 27° then ∠QAR equals
(a) 63°
(b) 117°
(c) 126°
(d) 153°



Solution 22

Correct option: (c)
In ΔPAQ and ΔPAR,
$\angle PQA = \angle PRA$ ....(Since PQ and PR are tangent to the circle)
$AP = AP$ ...(common side)
$AQ = AR$ ...(radii of the same circle)
$\Rightarrow \Delta PAQ \cong \Delta PAR$ .....(RHS congruence criterion)
$\angle QAP = \angle RAP$ ....(cpct)
In ΔPAQ,
$\angle QAP + \angle PQA + \angle APQ = 180^\circ$ ...(Angle Sum Property)
$\Rightarrow \angle QAP + 90^\circ + 27^\circ = 180^\circ$ ...($\angle PQA = 90^\circ$, since radius is perpendicular to the tangent)
$\Rightarrow \angle QAP = 63^\circ$
So, $\angle QAR = \angle QAP + \angle RAP$
$\Rightarrow \angle QAR = 63^\circ + 63^\circ$
$\Rightarrow \angle QAR = 126^\circ$

Question 23
In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$PB, then the length of each tangent is
(a) 3 cm
(b) 4 cm
(c) 5 cm
(d) 6 cm

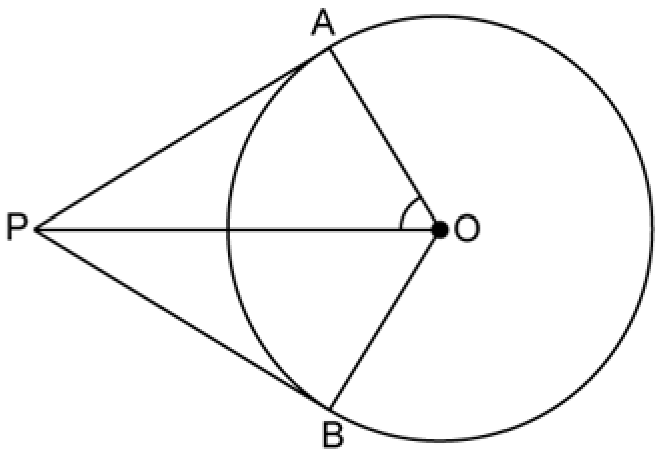


Solution 23
Correct option: (b)
Construction : Join CA and CB.
Since AP and PB are tangent to the circle,
$\angle CAP = \angle CBP = 90^\circ$
Given that $\angle APB = 90^\circ$
We know that tangents drawn from an external point to the circle
are equal.
$\Rightarrow AP = PB$
Also, $CA = CB$ ....(radii of the same circle)
So, quadrilateral APBC is a square.
Thus, $AP = PB = CA = CB = 4$ cm
Hence, the length of each tangent is 4 cm.

Question 24

If PA and PB are two tangents to a circle with centre O such that ∠APB = 80°. Then, ∠AOP=?

(a) 40°
(b) 50°
(c) 60°
(d) 70°



Solution 24

Correct option: (b)

Construction : Join CA and CB.

In ΔPAO and ΔPBO,

$\angle OAP = \angle OBP = 90°$ ....(Since AP and PB are tangent to the circle)

$OP = OP$ ...(common side)

$AO = BO$ ...(radii of the same circle)

$\Rightarrow \Delta PAO \cong \Delta PBO$ .....(RHS congruence criterion)

$\angle OPA = \angle OPB$ ....(cpct)

$\Rightarrow \angle OPA = \frac{1}{2}\angle APB = 40°$

In ΔPAO,

$\angle OPA + \angle PAO + \angle AOP = 180°$ ....(Angle Sum Property)

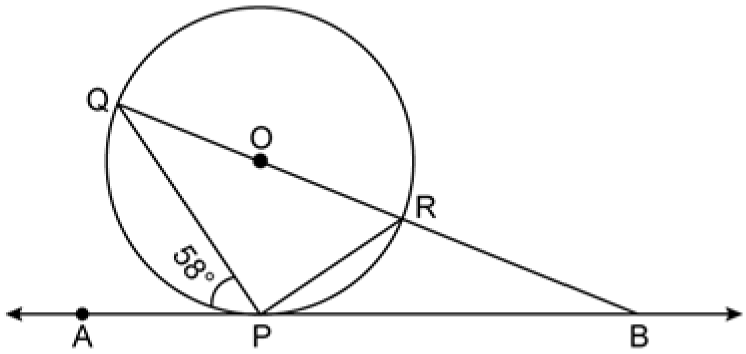
$\Rightarrow 40° + 90° + \angle AOP = 180°$

$\Rightarrow \angle AOP = 50°$

Question 25

In the given figure, Q is the centre of the circle. AB is the tangent to the circle at the point P. If ∠APQ = 58° then the measure of ∠PQB is

(a) 32°
(b) 58°
(c) 122°
(d) 132°



Solution 25

Correct option: (a)

$\angle APQ = 58°$

$\angle QPR = 90°$ ....(angle inscribed in a semicirde)

Since APB is a straight line,

$\angle APQ + \angle QPR + \angle RPB = 180°$

$\Rightarrow 58° + 90° + \angle RPB = 180°$

$\Rightarrow \angle RPB = 32°$

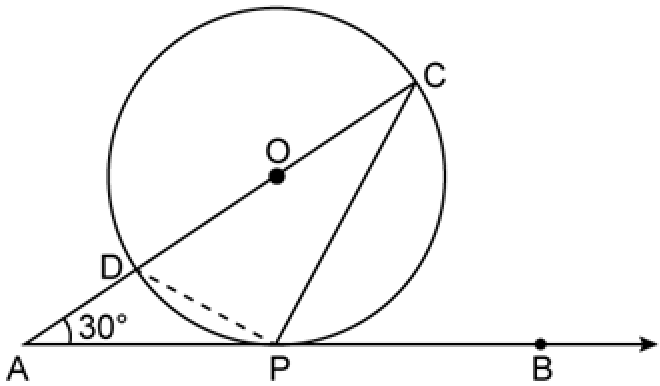
We know that angles that subtend the same arc are equal.

So, $\angle PQB = \angle RPB = 32°$

Question 26

In the given figure, O is the centre of the circle. AB is the tangent to the circle at the point P. If ∠PAO = 30° then ∠CPB + ∠ACP is equal to

(a) 60°
(b) 90°
(c) 120°
(d) 150°



Solution 26

Correct option: (b)

Since APB is a straight line,

$\angle APD + \angle DPC + \angle CPB = 180°$

We know that angles that subtend the same arc are equal.

$\Rightarrow \angle APD = \angle ACP$

$\Rightarrow \angle ACP + 90° + \angle CPB = 180°$ ....(Since $\angle DPC$ is inscribed in a semicircle)

$\Rightarrow \angle CPB + \angle ACP = 90°$

Question 27

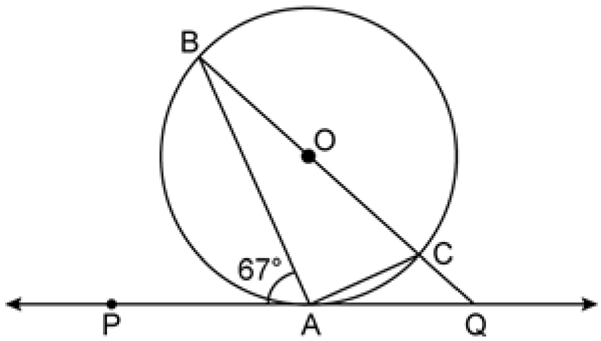
In the given figure, PQ is a tangent to a circle with centre O. A is the point of contact. If ∠PAB = 67°, then the measure of ∠AQB is

(a) 73°

(b) 64°

(c) 53°

(d) 44°



Solution 27

Correct option: (d)

Since $\angle BAC$ is inscribed in a semicircle, $\angle BAC = 90°$.

Since PAQ is a straight line,

$\angle PAB + \angle BAC + \angle CAQ = 180°$

$\Rightarrow 67° + 90° + \angle CAQ = 180°$

$\Rightarrow \angle CAQ = 23°$

We know that angles that subtend the same arc are equal.

$\Rightarrow \angle CBA = \angle CAQ = 23°$

In ΔBAQ,

$\angle BAQ + \angle QBA + \angle AQB = 180°$ ....(Angle Sum Property)

$\Rightarrow (90° + 23°) + 23° + \angle AQB = 180°$

$\Rightarrow (90° + 23°) + 23° + \angle AQB = 180°$

$\Rightarrow \angle AQB = 44°$

Question 28

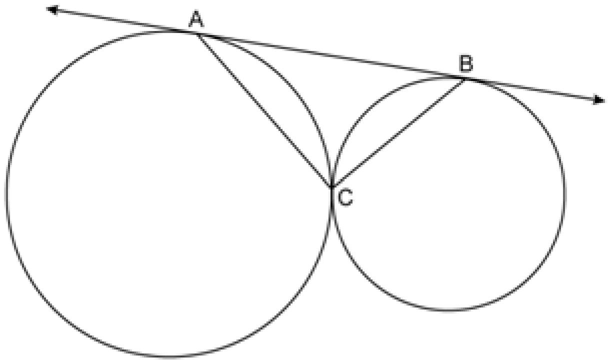
In the given figure, two circles touch each other at C and AB is a tangent to both the circles. The measure of ∠ACB is

(a) 45°

(b) 60°

(c) 90°

(d) 120°



Solution 28

Correct option: (c)

Draw a tangent to the circles at point C which meet AB at P.

Then,

PA = PC

$\Rightarrow \angle PAC = \angle PCA$

And PB = PC

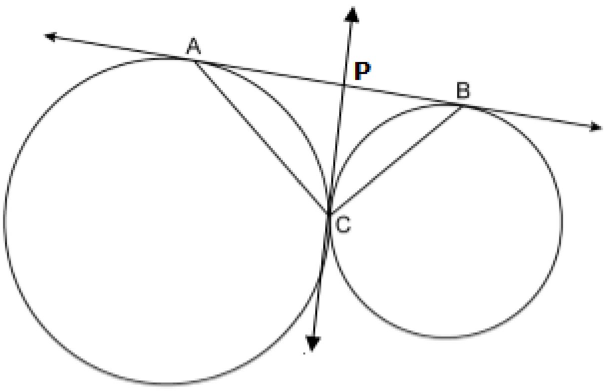
$\Rightarrow \angle PBC = \angle PCB$

$\therefore \angle PAC + \angle PBC = \angle PCA + \angle PCB = \angle ACB$

$\Rightarrow \angle PAC + \angle PBC + \angle ACB = 2\angle ACB$

$\Rightarrow 180° = 2\angle ACB$

$\Rightarrow \angle ACB = 90°$



Question 29

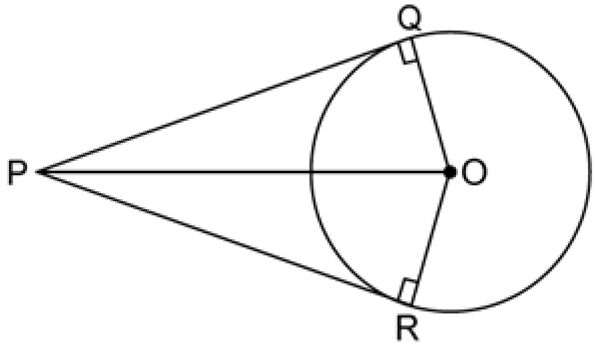
O is the centre of a circle of radius 5 cm. At a distance of 13 cm from O, a point P is taken. From this point, two tangents PQ and PR are drawn to the circle. Then, the area of quad. PQOR is

(a) 60 cm^2

(b) 32.5 cm^2

(c) 65 cm^2

(d) 30 cm^2



Solution 29

Correct option: (a)

In ΔOPQ and ΔORP,

$\angle OQP = \angle ORP = 90^\circ$ ....(Since OP and RP are tangent to the circle)

$OP = OP$ ...(common side)

$OQ = OR$ ...(radii of the same circle)

$\Rightarrow \Delta OPQ \cong \Delta ORP$ .....(RHS congruence criterion)

So, the areas of both the triangle will be the same.

In ΔOPQ,

By Pythagoras theorem,

$OP^2 = OQ^2 + PQ^2$

$\Rightarrow PQ^2 = OP^2 - OQ^2$

$\Rightarrow PQ^2 = 13^2 - 5^2$

$\Rightarrow PQ^2 = 169 - 25$

$\Rightarrow PQ^2 = 144$

$\Rightarrow PQ = 12$ cm

$$\begin{aligned} ar(\Delta OPQ) &= \frac{1}{2} \times PQ \times OQ \\ &= \frac{1}{2} \times 12 \times 5 \\ &= 30 \text{ cm}^2 \end{aligned}$$

$ar(\text{quad } PQOR) = ar(\Delta OPQ) + ar(\Delta ORP)$

$\Rightarrow ar(\text{quad } PQOR) = 30 \text{ cm}^2 + 30 \text{ cm}^2$

$\Rightarrow ar(\text{quad } PQOR) = 60 \text{ cm}^2$

Question 30

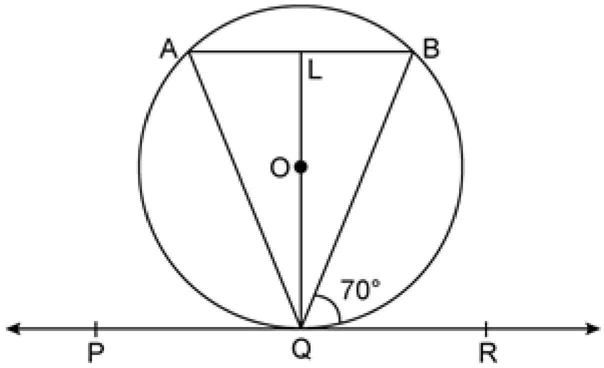
In the given figure, PQR is a tangent to the circle at Q, whose centre is O and AB is a chord parallel to PR such that $\angle BQR = 70°$. Then, $\angle AQB$ = ?

(a) 20°

(b) 35°

(c) 40°

(d) 45°



Solution 30

Correct option: (c)

Since $AB || PQ$

$\angle BQR = \angle ABQ = 70^\circ$ ....(alternate angles)

and $\angle PQA = \angle BAQ = 70^\circ$ ....(alternate angles)

In ΔABQ,

$\angle ABQ + \angle BAQ + \angle AQB = 180^\circ$ ....(Angle Sum Property)

$\Rightarrow 70^\circ + 70^\circ + \angle AQB = 180^\circ$

$\Rightarrow \angle AQB = 40^\circ$

Question 31

The length of the tangent from an external point P to a circle of radius 5 cm is 10 cm. The distance of the point from the centre of the circle is

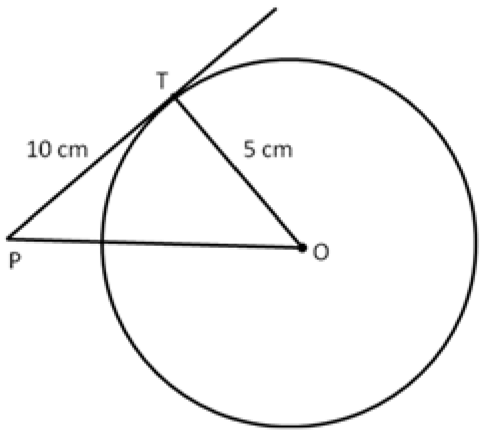
(a) 8 cm

(b) $\sqrt{104}$ cm

(c) 12 cm

(d) $\sqrt{125}$ cm

Solution 31



Correct option: (d)

In ΔPTO,

By Pythagoras theorem,

$OP^2 = PT^2 + OT^2$

$\Rightarrow OP^2 = 10^2 + 5^2$

$\Rightarrow OP^2 = 100 + 25$

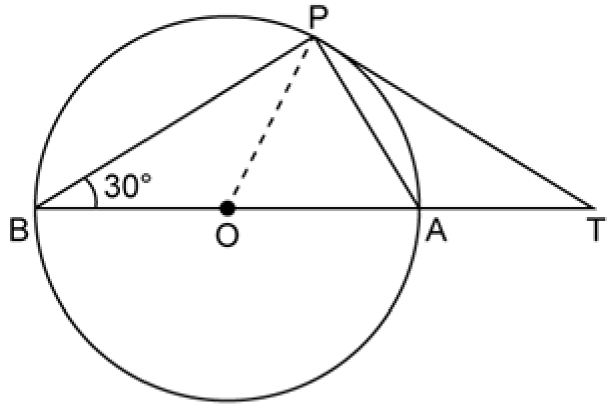
$\Rightarrow OP = \sqrt{125}$ cm

Hence, the distance of the point from the centre of the circle is $\sqrt{125}$ cm.

Question 32

In the given figure, O is the centre of a circle, BOA is its diameter and the tangent at the point P meets BA extended at T. If $\angle PBO = 30°$ then $\angle PTA$ =?

(a) 60°
(b) 30°
(c) 15°
(d) 45°



Solution 32

Correct option: (b)

In ΔOBP,

$OB = OP$ ....(radii of the same circle)

$\Rightarrow \angle OBP = \angle OPB = 30°$ ....(angles opposite equal sides are equal)

Since PT is a tangent,

$\angle OPT = 90°$

In ΔBPT,

$\angle BPT + \angle PBT + \angle PTB = 180°$ ....(Angle Sum Property)

$\Rightarrow (30° + 90°) + 30° + \angle PTB = 180°$

$\Rightarrow \angle PTB = 30°$

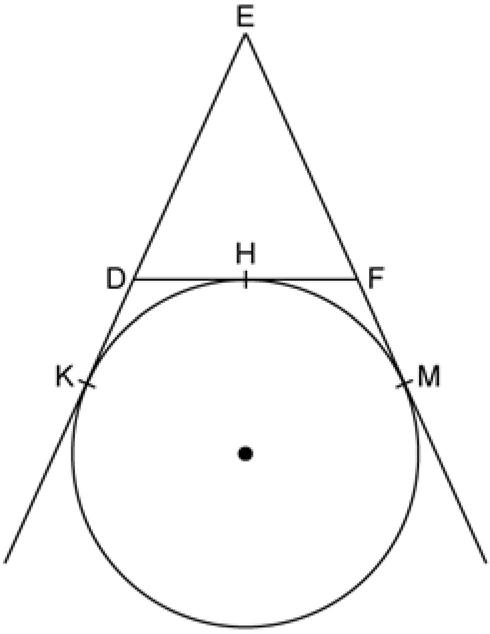
that is, $\angle PTA = 30°$

Question 33

In the given figure, a circle touches the side DF of ΔEDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm then the perimeter of ΔEDF is

(a) 9 cm
(b) 12 cm

(c) 13.5 cm
(d) 18 cm



Solution 33

Correct option: (d)

We know that tangents from an external point to the circle are equal.

So,

EK = EM = 9 cm

DK = DH

FH = FM

Perimeter of ΔEDF

$= ED + EF + DF$

$= ED + EF + DH + HF$

$= (ED + DH) + (EF + HF)$

$= (ED + DK) + (EF + FM)$

$= EK + EM$

$= 9 + 9$

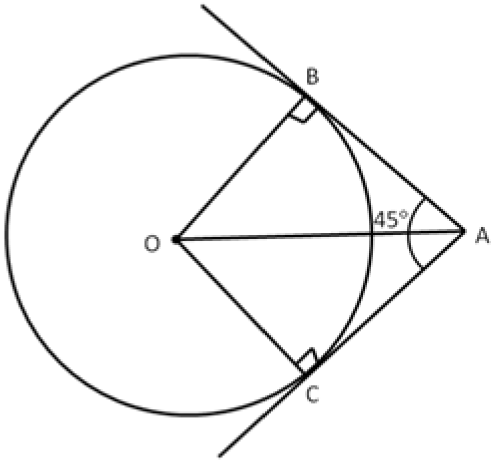
$= 18$ cm

Question 34

To draw a pair of tangents to a circle, which are inclined to each other at an angle of 45°, we have to draw tangents at the end points of those two radii, the angle between which is

(a) 105°
(b) 135°
(c) 140°
(d) 145°

Solution 34



Correct option: (b)

Since AB and AC are the tangents to the circle.

$\angle OBA = \angle OCA = 90°$ ....(tangent is perpendicular to the radius of a circle)

In ACOB,

$\angle OBA + \angle BAC + \angle OCA + \angle BOC = 360°$

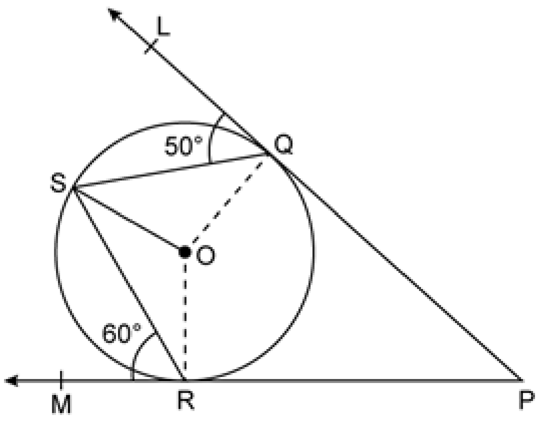
$\Rightarrow 90° + 45° + 90° + \angle BOC = 360°$

$\Rightarrow \angle BOC = 135°$

Question 35

In the given figure, O is the centre of a circle; PQL and PRM are the tangents at the points Q and R respectively and S is a point on the circle such that $\angle SQL = 50°$ and $\angle SRM = 60°$.
Then, $\angle QSR$ =?

(a) 40°
(b) 50°
(c) 60°
(d) 70°



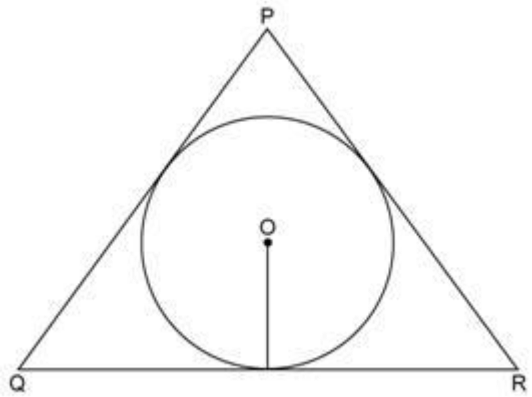
Solution 35

Correct option: (d)
Since PL and PM are the tangents to the circle.
$\angle OQL = \angle ORM = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)
So,
$\angle OQL = \angle SQL + \angle OQS$
$\Rightarrow 90^\circ = 50^\circ + \angle OQS$
$\Rightarrow \angle OQS = 40^\circ$
Similarly, we can find $\angle ORS = 30^\circ$.
In ΔOQS,
$OQ = OS$
$\angle OQS = \angle OSQ = 40^\circ$ ...(angles opposite equal sides are equal)
In ΔORS,
$OR = OS$
$\angle ORS = \angle OSR = 30^\circ$ ...(angles opposite equal sides are equal)
So, $\angle QSR = \angle OSQ + OSR = 40^\circ + 30^\circ = 70^\circ$.

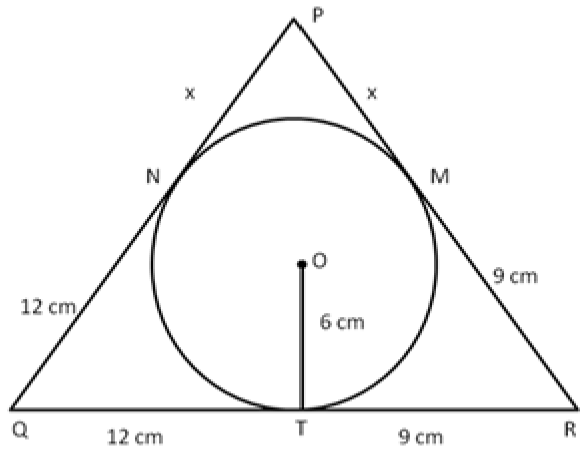
Question 36

In the given figure, a triangle PQR is drawn to circumscribe a circle of radius 6 cm such that the segments QT and TR into which QR is divided by the point of contact T, are of lengths 12 cm and 9 cm respectively. If the area of $\Delta PQR = 189\ cm^2$ then the length of side PQ is

(a) 17.5 cm
(b) 20 cm
(c) 22.5 cm
(d) 25 cm



Solution 36



Correct option: (c)

We know that tangents from an external point to the circle are equal.

So,

QT = QN = 12 cm

TR = RM = 9 cm

Now,

$$ar(\Delta PQR) = \frac{1}{2}(\text{Perimeter of } \Delta PQR) \times r$$

$$\Rightarrow ar(\Delta PQR) = \frac{1}{2}(12 + 12 + 9 + 9 + x + x) \times r$$

$$\Rightarrow ar(\Delta PQR) = \frac{1}{2}(42 + 2x) \times 6$$

$$\Rightarrow 189 = 3(42 + 2x)$$

$$\Rightarrow 63 = 42 + 2x$$

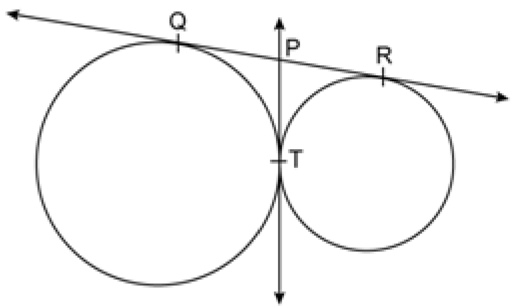
$$\Rightarrow 2x = 21$$

$$\Rightarrow x = 10.5$$

So, $PQ = 12 + 10.5 = 22.5$ cm

Question 37

In the given figure, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm then the length of QR is (a) 1.9 cm
(b) 3.8 cm
(c) 5.7 cm
(d) 7.6 cm



Solution 37

Correct option: (d)

We know that tangents from an external point to the circle are equal.

PQ = PT = 3.8 cm

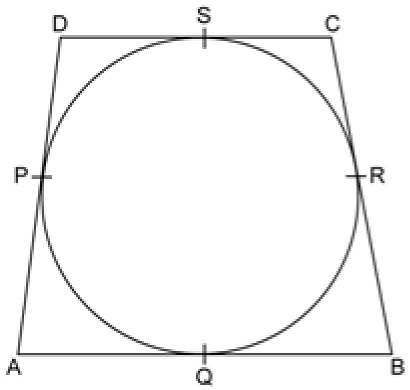
PR = PT = 3.8 cm

$$QR = PQ + PR$$
$$= 3.8 + 3.8$$
$$= 7.6 \text{ cm}$$

Question 38

In the given figure, quad. ABCD is circumscribed, touching the circle at P, Q, R and S. If AP = 5 cm, BC =7 cm and CS = 3 cm. Then, the length AB =?

(a) 9 cm
(b) 10 cm
(c) 12 cm
(d) 8 cm



Solution 38

Correct option: (a)

We know that tangents from an external point to the circle are equal.

AP = AQ = 5 cm

CS = CR = 3 cm

$$RB = BC - CR$$
$$= 7 - 3$$
$$= 4 \text{ cm}$$

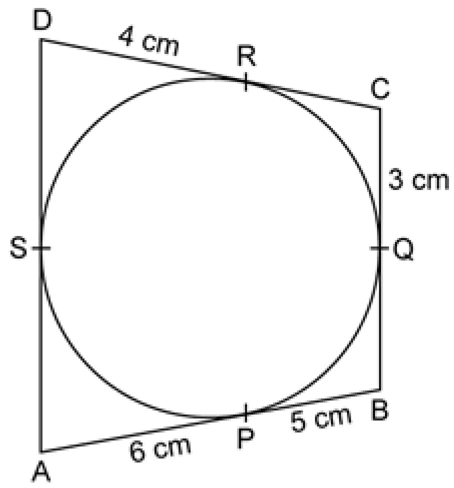
So, BQ = RB = 4 cm

Thus, AB = AQ + RB = 5 + 4 = 9 cm

Question 39

In the given figure, quad. ABCD is circumscribed, touching the circle at P, Q, R and S. If AP = 6cm, BP = 5 cm, CQ = 3 cm and DR = 4 cm then perimeter of quad. ABCD is

(a) 18 cm
(b) 27 cm
(c) 36 cm
(d) 32 cm



Solution 39

Correct option: (c)

We know that tangents from an external point to the circle are equal.

RC = CQ = 3 cm

PB = BQ = 5 cm

AP = AS = 6 cm

SD = DR = 4 cm

Perimeter of quad. ABCD

= AB + BC + CD + AD

= (AP + PB) + (BQ + CQ) + (CR + DR) + (AS + SD)

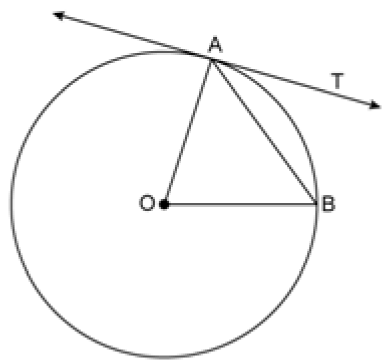
= (6 + 5) + (5 + 3) + (3 + 4) + (6 + 4)

= 36 cm

Question 40

In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If ∠AOB =100° then ∠BAT is equal to

(a) 40°
(b) 50°
(c) 90°
(d) 100°



Solution 40

Correct option: (b)

In ΔOAB,

$OA = OB$ ....(radii of the same circle)

$\Rightarrow \angle OAB = \angle OAB$ ....(angles opposite equal sides are equal)

$\angle AOB + \angle OAB + \angle OBA = 180°$ ....(Angle Sum Property)

$\Rightarrow \angle AOB + \angle OAB + \angle OAB = 180°$

$\Rightarrow 100° + 2\angle OAB = 180°$

$\Rightarrow 2\angle OAB = 80°$

$\Rightarrow \angle OAB = 40°$

Since AT is the tangent,

$\angle OAT = 90°$

$\Rightarrow \angle OAB + \angle BAT = 90°$

$\Rightarrow 40° + \angle BAT = 90°$

$\Rightarrow \angle BAT = 50°$

Question 41

In a right triangle ABC, right-angled at B, BC = 12 cm and AB =5 cm. The radius of the circle inscribed in the triangle is

(a) 1 cm
(b) 2 cm
(c) 3 cm
(d) 4 cm

Solution 41

Correct option: (b)

In right ΔABC,

$AC^2 = AB^2 + BC^2$ ....(By Pythagoras theorem)

$\Rightarrow AC^2 = 5^2 + 12^2$

$\Rightarrow AC^2 = 169$

$\Rightarrow AC = 13$ cm

We know that,

$$ar(\Delta ABC) = \frac{1}{2} \times (\text{perimeter of } \Delta ABC) \times r$$

$$\Rightarrow \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (\text{perimeter of } \Delta ABC) \times r$$

$$\Rightarrow \frac{1}{2} \times 12 \times 5 = \frac{1}{2} \times (5 + 12 + 13) \times r$$

$$\Rightarrow 12 \times 5 = 30 \times r$$

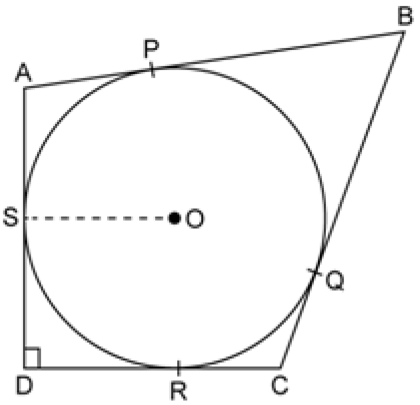
$$\Rightarrow r = 2 \text{ cm}$$

Question 42

In the given figure, a circle is inscribed in a quadrilateral ABCD touching its sides AB, BC, CD and AD at P, Q, R and S respectively. If the radius of the circle is 10 cm, BC = 38 cm, PB = 27 cm and AD $\perp$ CD then the length of CD is

(a) 11 cm
(b) 15 cm

(c) 20 cm
(d) 21 cm



Solution 42

Correct option: (d)

We know that tangents from an external point to the circle are equal.

$BQ = PB = 27$ cm

So, $CQ = BC - BQ = 38 - 27 = 11$ cm

$\Rightarrow CR = CQ = 11$ cm

In quad. SORD,

$\angle SDR = 90^\circ$ ....($\because AD \perp CD$)

$\angle OSD = \angle ORD = 90^\circ$

Also, $OS = OR$ and $SD = SR$

So, quad. SORD is a square.

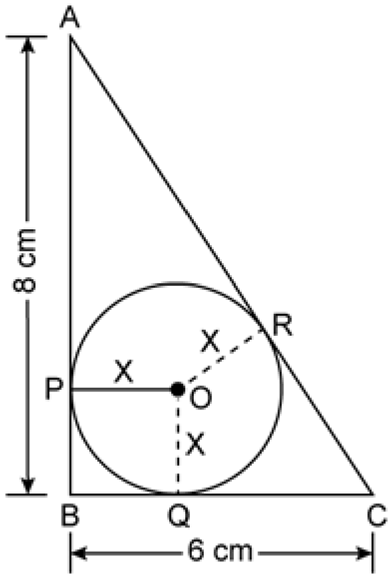
Thus, $DR = SO = 10$ cm

Hence, $CD = DR + CR = 10 + 11 = 21$ cm.

Question 43

In the given figure, ΔABC is right-angled at B such that BC= 6 cm and AB = 8 cm. A circle with centre O has been inscribed inside the triangle. $OP \perp AB$, $OQ \perp BC$ and $OR \perp AC$. If OP = OQ =OR= x cm then x = ?

(a) 2 cm
(b) 2.5 cm
(c) 3 cm
(d) 3.5 cm



Solution 43

Correct option: (a)

In right ΔABC,

$AC^2 = AB^2 + BC^2$ ....(By Pythagoras theorem)

$\Rightarrow AC^2 = 8^2 + 6^2$

$\Rightarrow AC^2 = 100$

$\Rightarrow AC = 10$ cm

We know that tangents from an external point to the circle are equal.

$\Rightarrow CR = CQ = BC - BQ = (6 - x)$ cm

$\Rightarrow AR = AP = AB - BP = (8 - x)$ cm

$AC = (AR + CR) = (8 - x) + (6 - x) = (14 - 2x)$ cm

$\Rightarrow 14 - 2x = 10$

$\Rightarrow 2x = 4$

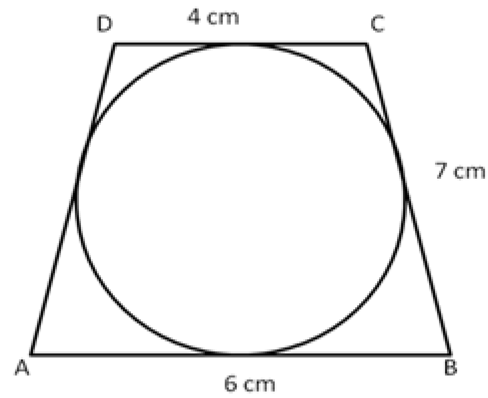
$\Rightarrow x = 2$

Question 44

Quadrilateral ABCD is circumscribed to a circle. If AB = 6 cm, BC =7 cm, and CD = 4 cm then the length of AD is

(a) 3 cm
(b) 4 cm
(c) 6 cm
(d) 7 cm

Solution 44



Correct option: (a)

Using the property, tangents from an external point to the circle are equal.

We can say, AB + CD = AD + BC

$\Rightarrow$ AD = AB + CD − BC

$\Rightarrow$ AD = 6 + 4 − 7

$\Rightarrow$ AD = 3 cm

Question 45

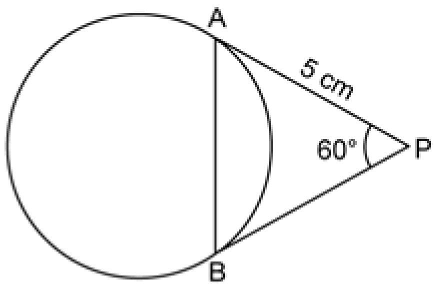
In the given figure, PA and PB are tangents to the given circle such that PA = 5 cm and ∠APB = 60°. The length of chord AB is

(a) $5\sqrt{2}$ cm

(b) 5 cm

(c) $5\sqrt{3}$ cm

(d) 7.5 cm



Solution 45

Correct option: (b)

We know that tangents from an external point to the circle are equal.

PA = PB

$\Rightarrow \angle PBA = \angle PAB = x^\circ$

In ΔPAB,

$\angle PBA + \angle PAB + \angle APB = 180^\circ$ ....(Angle Sum Property)

$\Rightarrow \angle PAB + \angle PAB + 60^\circ = 180^\circ$

$\Rightarrow 2\angle PAB = 120^\circ$

$\Rightarrow \angle PAB = 60^\circ = \angle PBA$

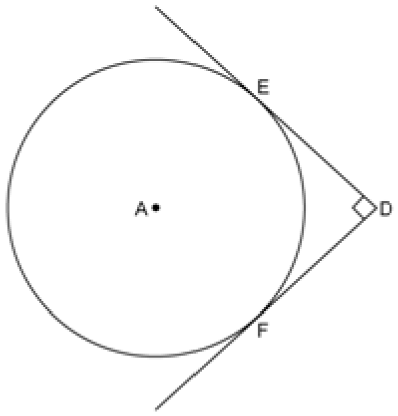
So, ΔPAB is an equilateral triangle.

Thus, AB = PA = 5 cm.

Question 46

In the given figure, DE and DF are tangents from an external point D to a circle with centre A. If DE = 5 cm and DE ⊥ DF then the radius of the circle is

(a) 3 cm
(b) 4 cm
(c) 5 cm
(d) 6 cm



Solution 46

Correct option: (c)

Construction: Join AE and AF.

Since DE and DF are tangent to the circle,

$\angle AED = \angle AFD = \angle EDF = 90^\circ$

Also, AE = AF ....(radii of the same circle)

and ED = EF

....(Since tangents drawn from an external point to the circle are equal.)

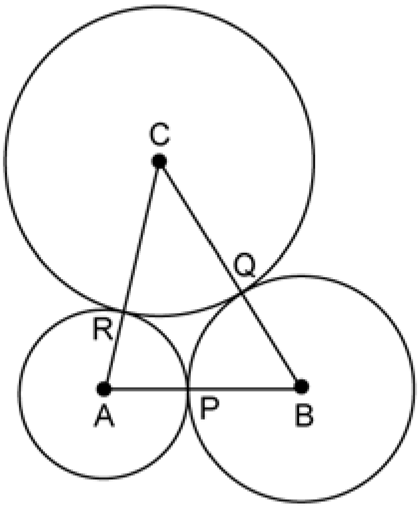
So, quadrilateral AEDF is a square.

Thus, AE = DF = 5 cm

Hence, the length of the radius is 5 cm.

Question 47

In the given figure, three circles with centres A, B, C respectively touch each other externally. If AB = 5 cm, BC = 7 cm and CA = 6 cm then the radius of the circle with centre A is
(a) 1.5 cm
(b) 2 cm
(c) 2.5 cm
(d) 3 cm



Solution 47

Correct option: (b)

Let the radii of the circle with centres A, B and C be x, y and z respectively.

We know that radii of the same circle are equal.

$x + y = 5$

$y + z = 7$

$z + x = 6$

Adding the three equations, we get

$2(x + y + z) = 18$

$\Rightarrow x + y + z = 9$

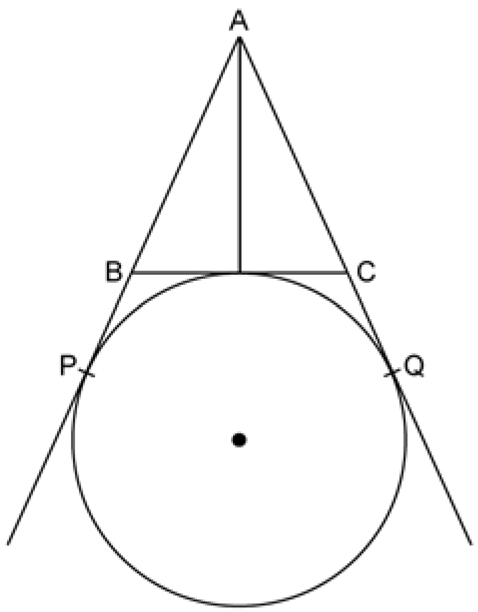
$\Rightarrow x + 7 = 9$

$\Rightarrow x = 2$

So, the radius of the circle with centre A is 2 cm.

Question 48

In the given figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm then the length of AP is
(a) 15 cm
(b) 10 cm
(c) 9 cm
(d) 7.5 cm



Solution 48

Correct option: (d)

Let BC intersect the circle at D.

We know that tangents from an external point to the circle are equal.

BP = BD

CD = CQ

AP = AQ

Perimeter of ΔABC

$= AB + BC + AC$

$= AB + (BD + CD) + AC$

$= AB + (BP + CQ) + AC$

$= (AB + BP) + (AC + CQ)$

$= AP + AQ$

Since perimeter of $\Delta ABC = AB + BC + AC = 5 + 6 + 4 = 15$ cm

$\Rightarrow AP + AQ = 15$

$\Rightarrow 2AP = 15$

$\Rightarrow AP = 7.5$ cm

Question 49

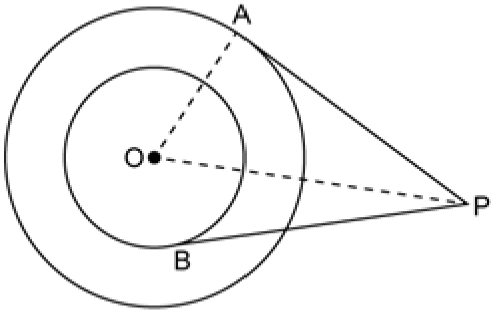
In the given figure, O is the centre of two concentric circles of radii 5 cm and 3 cm. From an external point P tangents PA and PB are drawn to these circles. If PA =12 cm then PB is equal to

(a) $5\sqrt{2}$ cm

(b) $3\sqrt{5}$ cm

(c) $4\sqrt{10}$ cm

(d) $5\sqrt{10}$ cm



Solution 49

Correct option: (c)

Construction : Join OB.

We know that the tangent is perpendicular to the radius of a circle.

In ΔOPA,

By Pythagoras theorem,

$OP^2 = OA^2 + AP^2$

$\Rightarrow OP^2 = 5^2 + 12^2$

$\Rightarrow OP^2 = 169$

$\Rightarrow OP = 13$ cm

In ΔOPB,

By Pythagoras theorem,

$OP^2 = OB^2 + PB^2$

$\Rightarrow PB^2 = OP^2 - OB^2$

$\Rightarrow PB^2 = 13^2 - 3^2$

$\Rightarrow PB^2 = 160$

$\Rightarrow PB = 4\sqrt{10}$ cm

Question 50

Which of the following statements is not true?
a. If a point P lies inside a circle, no tangent can be drawn to the circle, passing through P.
b. If a point P lies on the circle, then one and only one tangent can be drawn to the circle at P.
c. If a point P lies outside the circle, then only two tangents can be drawn to the circle from P.
d. A circle can have more than two parallel tangents, parallel to a given line.

Solution 50

Correct option: (d)
Options (a), (b) and (c) are all true.
However, option (d) is false since we can draw only parallel tangents on either side of the diameter, which would be parallel to a given line.

Question 51

Which of the following statements is not true?

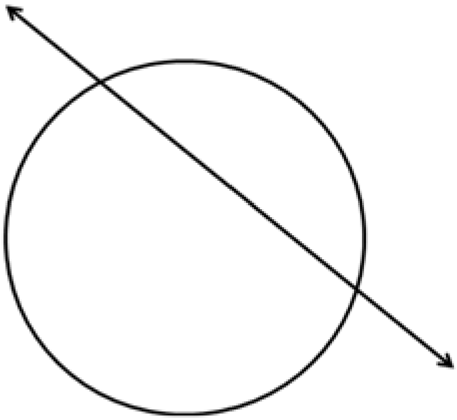
a. A tangent to a circle intersects the circle exactly at one point.
b. The point common to the circle and its tangent is called the point of contact.
c. The tangent at any point of a circle is perpendicular to the radius of the circle through the point of contact.
d. A straight line can meet a circle at one point only.

Solution 51

Correct option: (d)

Options (a), (b) and (c) are all true.

However, option (d) is false since a straight line can meet a circle at two points even as shown below.



Question 52

Which of the following statements is not true?

a. A line which intersects a circle in two points, is called a secant of the circle.
b. A line intersecting a circle at one point only, is called a tangent to the circle.
c. The point at which a line touches the circle, is called the point of contact.
d. A tangent to the circle can be drawn from a point inside the circle.

Solution 52

Correct option: (d)

Options (a), (b) and (c) are true.

However, option (d) is false since it is not possible to draw a tangent from a point inside a circle.

Question 53

Assertion-and-Reason

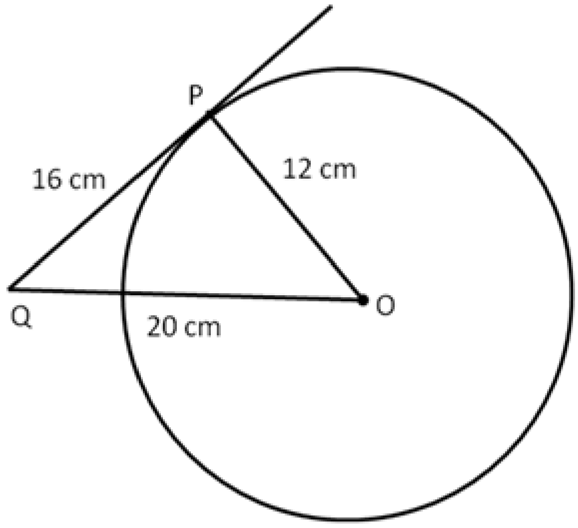
Type Each question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer, use the following code:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
c. Assertion (A) is true and Reason (R) is false.
d. Assertion (A) is false and Reason (R) is true.

Assertion (A)	Reason (R)
At a point P of a circle with centre O and radius 12 cm, a tangent PQ of length 16 cm is drawn. Then, OQ = 20 cm.	The tangent at any point of a circle is perpendicular to the radius through the point of contact.

The correct answer is (a)/(b)/(c)/(d).

Solution 53



Correct option: (a)

We know that the tangent is perpendicular to the radius of a circle.

In ΔOPQ,

By Pythagoras theorem,

$OQ^2 = QP^2 + OP^2$

$\Rightarrow OP^2 = 16^2 + 12^2$

$\Rightarrow OP^2 = 256 + 144$

$\Rightarrow OP^2 = 400$

$\Rightarrow OP = 20$ cm

So, the Assertion (A) is true.

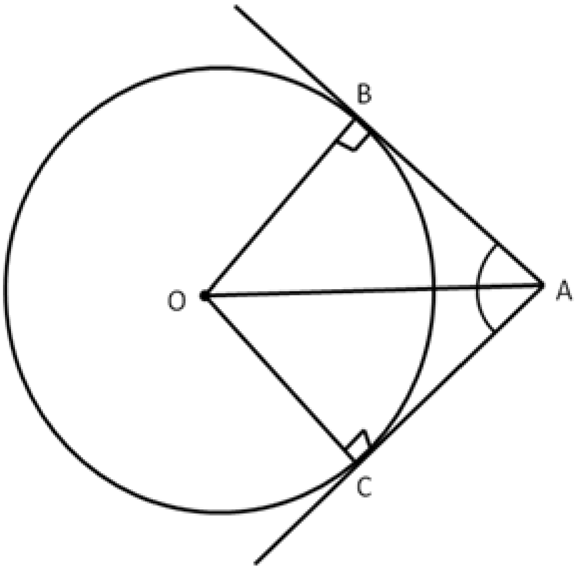
The Reason (R) is also true and is the correct explanation for the Assertion (A).

Question 54

Assertion (A)	Reason (R)
If two tangents are drawn to a circle from an external point then they subtend equal angles at the centre.	A parallelogram circumscribing a circle is a rhombus.

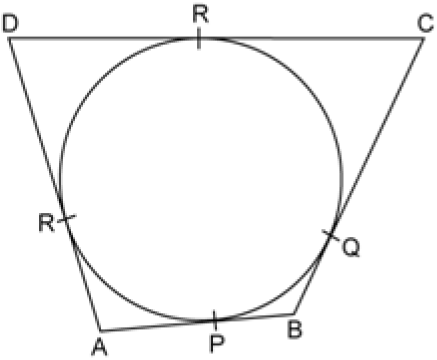
The correct answer is (a)/(b)/(c)/(d).

Solution 54



Correct option: (b)
Consider tangents AB and AC drawn to the circle with centre O.
In ΔOBA and ΔOCA,
$AO = AO$ ...(common side)
$OB = OC$ ....(radii of the same circle)
$\angle B = \angle C = 90^{\circ}$
$\Rightarrow \Delta OBA \cong \Delta OCA$ ....(RHS congruence criterionnnnnnnn)
So, $\angle OBA = \angle COA$ ....(cpct)
Thus, the Assertion (A) is true.
The Reason (R) is also true and can be proved using the property, 'tangents from an external point to a circle are equal'
But, the Reason (R) is not the correct explanation for the Assertion (A).

Question 55

Assertion (A)	Reason (R)
In the given figure, a quad. ABCD is drawn to circumscribe a given circle, as shown. Then, AB + BC = AD + DC 	In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

The correct answer is (a) / (b) / (c) / (d).

Solution 55

Correct option: (d)

The Assertion (A) is false since sum of the opposite sides of a quadrilateral circumscribing a circle are equal, and not the adjacent sides.

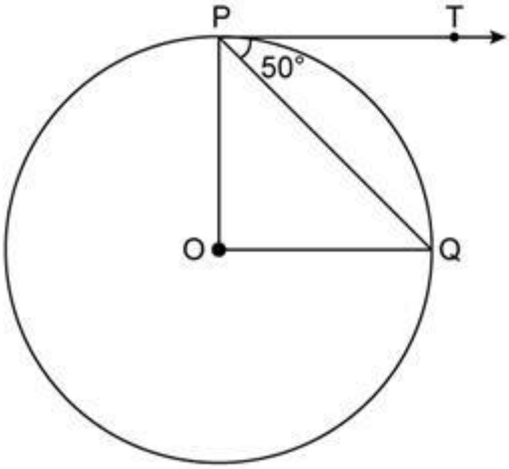
The chord of the larger circle is the tangent to the smaller circle.
We know that the perpendicular drawn from the centre to the chord of a circle, bisects the chord.
So, the Reason (R) is true.
But is not the correct explanation for the Assertion (A).

Chapter 12 - Circles Excercise FA

Question 1

In the given figure, O is the centre of a circle, PQ is a chord and the tangent PT at P makes an angle of 50° with PQ. Then, ∠POQ = ?

(a) 130°
(b) 100°
(c) 90°
(d) 75°



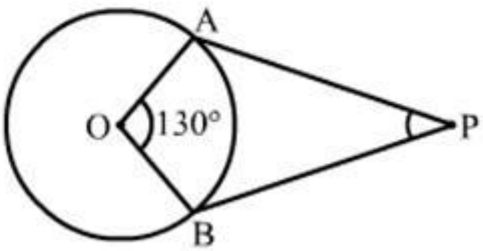
Solution 1

Correct option: (b)
Since PT is the tangent to the circle,
$\angle OPT = 90°$
$\Rightarrow \angle TPQ + \angle OPQ = 90°$
$\Rightarrow 50° + \angle OPQ = 90°$
$\Rightarrow \angle OPQ = 40°$
In ΔOPQ,
$OP = OQ$ ...(radii of the same circle)
$\Rightarrow \angle OPQ = \angle OQP = 40°$ ....(angles opposite equal sides are equal)
Now,
$\angle OPQ + \angle OQP + \angle POQ = 180°$ ....(Angle Sum Property)
$\Rightarrow 40° + 40° + \angle POQ = 180°$
$\Rightarrow \angle POQ = 100°$

Question 2

If the angle between two radii of a circle is 130° then the angle between the tangents at the ends of the radii is
(a) 65°
(b) 40°
(c) 50°
(d) 90°

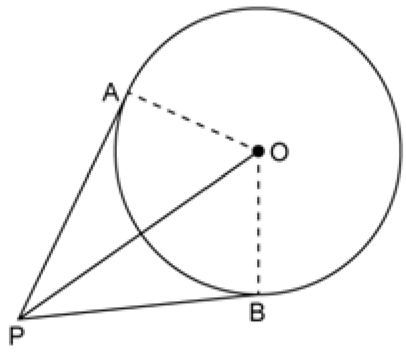
Solution 2



Correct option: (c)
In quad. AOBP
$\angle PAO + \angle PBO + \angle AOB + \angle APB = 360°$
$\Rightarrow 90° + 90° + 130° + \angle APB = 360°$
....(Since radius of a circle is perpendicular to the tangent)
$\Rightarrow \angle APB = 50°$

Question 3

If tangents PA and PB from a point P to a circle with centre O are drawn so that ∠APS = 80° then ∠POA = ?
(a) 40°
(b) 50°
(c) 80°
(d) 60°



Solution 3

Correct option: (b)

In ΔPAO and ΔPBO,

$\angle PAO = \angle PBO = 90^\circ$ ....(Since PQ and PR are tangent to the circle)

$OP = OP$ ...(common side)

$AO = OB$ ...(radii of the same circle)

$\Rightarrow \Delta PAO \cong \Delta PBO$ .....(RHS congruence criterion)

$\angle POA = \angle BOP$ ....(cpct)

In quad. AOBP

$\angle PAO + \angle PBO + \angle AOB + \angle APB = 360^\circ$

$\Rightarrow 90^\circ + 90^\circ + \angle AOB + 80^\circ = 360^\circ$

$\Rightarrow \angle AOB = 100^\circ$

So, $\angle POA = 50^\circ$

Question 4

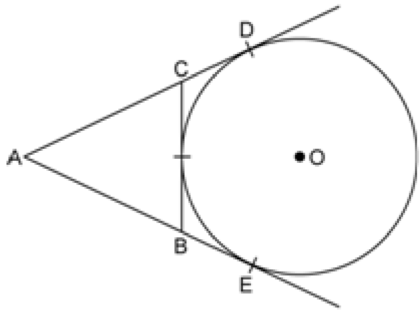
In the given figure, AD and AE are the tangents to a circle with centre O and BC touches the circle at F. If AE = 5 cm then perimeter of ΔABC is

(a) 15 cm

(b) 10 cm

(c) 22.5 cm

(d) 20 cm



Solution 4

Correct option: (b)

We know that tangents from an external point to the circle are equal.

So,

AE = AD = 5 cm

BF = BE

CF = CD

Perimeter of ΔABC

$= AB + BC + AC$

$= AB + (BF + FC) + AC$

$= AB + (BE + DC) + AC$

$= (AB + BE) + (AC + DC)$

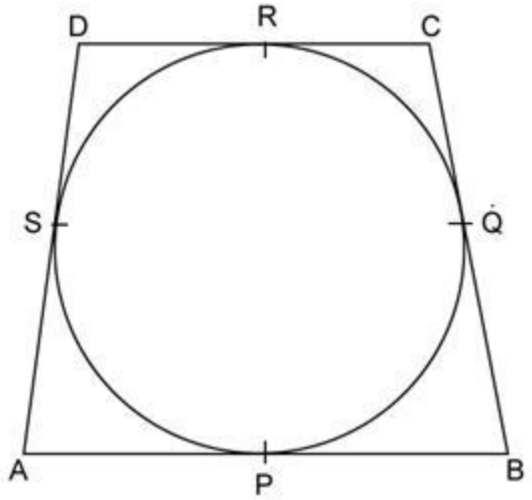
$= AE + AD$

$= 5 + 5$

$= 10$ cm

Question 5

In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If AB = x cm, BC = 7 cm, CR = 3 cm and AS = 5 cm, find x.



Solution 5

We know that tangents from an external point
to the circle are equal.

$AP = AS = 5$ cm

$CQ = CR = 3$ cm

$BQ = BP$

Now,

$BC = CQ + BQ$

$BQ = BC - CQ$

$= 7 - 3$

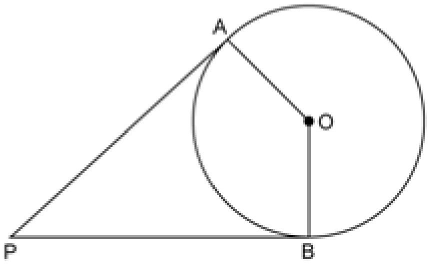
$= 4$ cm

So, $BQ = PB = 4$ cm

Thus, $AB = AP + PB = 5 + 4 = 9$ cm

Question 6

In the given figure, PA and PB are the tangents to a circle with centre O. Show that the points A, O, B, P are concyclic.



Solution 6

$OA = OB$ .....(radii of the same circle)

Since PA and PB are tangents to the circle,

$\angle OAP = \angle OBP = 90°$

Consider,

$\angle OAP + \angle OBP$

$= 90° + 90°$

$= 180°$

In quad. AOBP,

$\angle PAO + \angle PBO + \angle AOB + \angle APB = 360°$

$\Rightarrow 180° + \angle AOB + \angle APB = 360°$

$\Rightarrow \angle AOB + \angle APB = 180°$

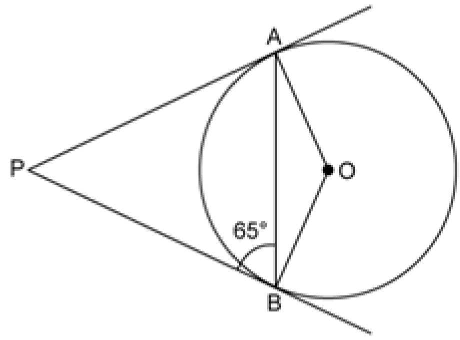
Since the sum of the opposite angles of quad. AOBP
are supplementary, AOBP are concyclic.

That is, a circle passes through A, O, B and P.

Hence proved.

Question 7

In the given figure, PA and PB are two tangents from an external point P to a circle with centre O. If $\angle PBA = 65°$, find $\angle OAB$ and $\angle APB$.



Solution 7

Since PB is a tangent to the circle, $\angle OBP = 90°$.

Now,

$\angle OBA = \angle OBP - \angle ABP$

$= 90° - 65°$

$= 25°$

Since $OB = OA$, $\angle OAB = \angle OBA = 25°$.

In ΔAOB,

$\angle AOB + \angle ABO + \angle OAB = 180°$

$\Rightarrow \angle AOB + 25° + 25° = 180°$

$\Rightarrow \angle AOB = 130°$

In quad. AOBP

$\angle PAO + \angle PBO + \angle AOB + \angle APB = 360°$

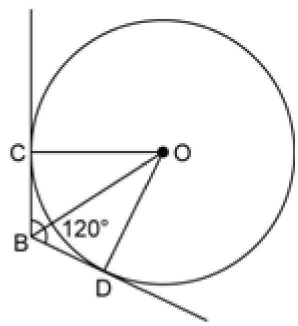
$\Rightarrow 90° + 90° + 130° + \angle APB = 360°$

$\Rightarrow \angle APB = 50°$

Thus, $\angle OAB = 25°$ and $\angle APB = 50°$.

Question 8

Two tangent segments BC and BD are drawn to a circle with centre O such that $\angle CBD = 120°$. Prove that $OB = 2BC$.



Solution 8

In ΔBCO and ΔBDO,

$\angle BCO = \angle BDO = 90^\circ$ ....(Since BC and BD are tangent to the circle)

$OB = OB$ ...(common side)

$OC = OD$ ...(radii of the same circle)

$\Rightarrow \Delta BCO \cong \Delta BDO$ .....(RHS congruence criterion)

$\angle OBC = \angle OBD = 60^\circ$ ....(cpct)

So, $\angle COB = 30^\circ$

So, ΔBCO is a 30-60-90 triangle.

side opposite $30^\circ = \frac{1}{2}$hypotenuse

$\Rightarrow BC = \frac{1}{2}OB$

$\Rightarrow OB = 2BC$

Hence proved.

Question 9

Fill in the blanks.

i. A line intersecting a circle in two distinct points is called a _______ .

ii. A circle can have _________ parallel tangents at the most.

iii. The common point of a tangent to a circle and the circle is called the _________ .

iv. A circle can have __________ tangents.

Solution 9

i. A line intersecting a circle in two distinct points is called a **secant**.

ii. A circle can have **two** parallel tangents at the most.

iii. This is since we can draw only parallel tangents on either side of a diameter.

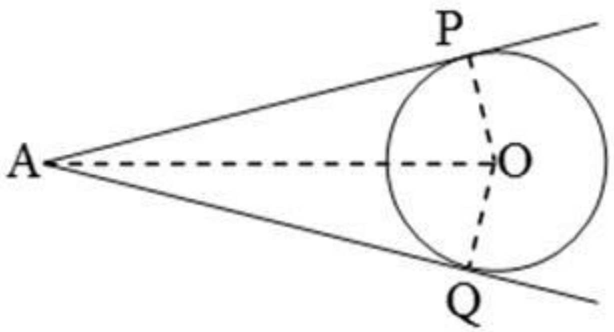
iv. The common point of a tangent to a circle and the circle is called the **point of contact**.

v. A circle can have **infinitely many** tangents.

Question 10

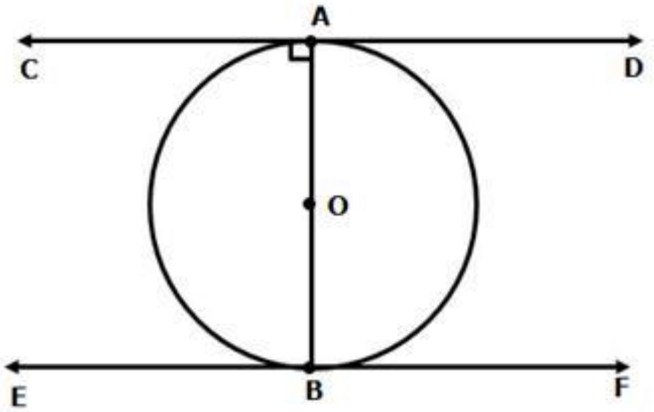
Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Solution 10



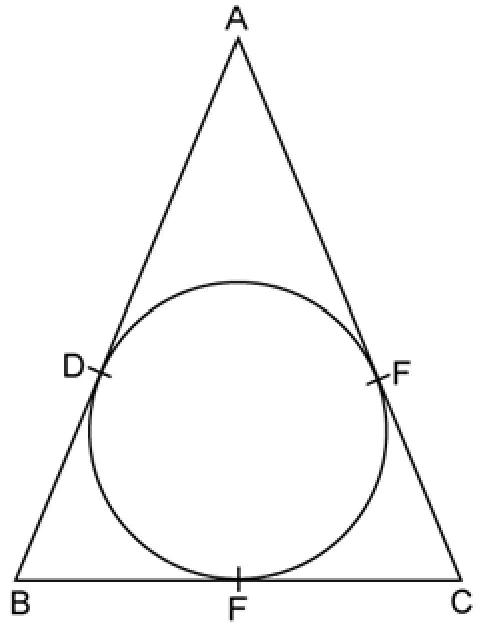
Given : Two tangents AP and AQ are drawn from a point A to a circle with centre O.
To prove: AP = AQ
Construction : Join OP, OQ and OA.
Proof:
Since AP and AQ are the tangents to the circle,
$OP \perp AP$ and $OQ \perp AQ$
In ΔOPA and ΔOQA,
$\angle OPA = \angle OQA = 90°$
OA = OA ...(common side)
OP = OP ...(radii of the same circle)
$\Rightarrow \Delta OPA \cong \Delta OQA$ .....(RHS congruence criterion)
So, AP = AQ

Question 11
Prove that the tangents drawn at the ends of the diameter of a circle are parallel.

Solution 11


Given : CD and EF are the tangents at endpoints A and B of the diameter AB of a circle with centre O.
To prove: CD || EF
Proof:
Since CD is the tangent to the circle at the point A,
$\angle BAD = 90°$
Since EF is the tangent to the circle at the point B,
$\angle ABE = 90°$
Thus, $\angle BAD = \angle ABE = 90°$
But these are alternate interior angles.
$\Rightarrow$ CD || EF
Hence proved.

Question 12
In the given figure, if AB = AC, prove that BE = CE.



Solution 12

We know that tangents drawn from an external point to the circle are equal.

BE = BD

CE = CF

AD = AF

Given AB = AC

$\Rightarrow$ AD + BD = AF + CF

$\Rightarrow$ AF + BE = AF + CE

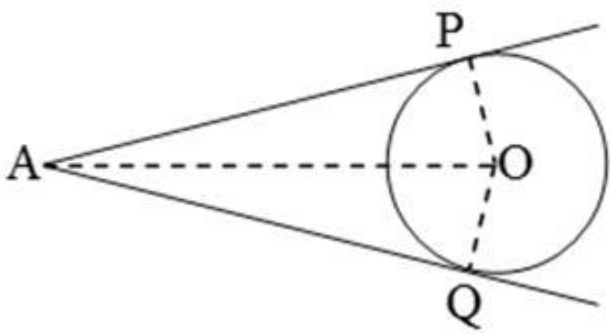
$\Rightarrow$ BE = CE

Hence proved.

Question 13

If two tangents are drawn to a circle from an external point, show that they subtend equal angles at the centre.

Solution 13



Given : Two tangents AP and AQ are drawn from a point A to a circle with centre O.

To prove: AP = AQ

Construction : Join OP, OQ and OA.

Proof:

Since AP and AQ are the tangents to the circle,

OP ⊥ AP and OQ ⊥ AQ

In ΔOPA and ΔOQA,

∠OPA = ∠OQA = 90°

OA = OA ...(common side)

OP = OP ...(radii of the same circle)

⇒ ΔOPA ≅ ΔOQA .....(RHS congruence criterion)

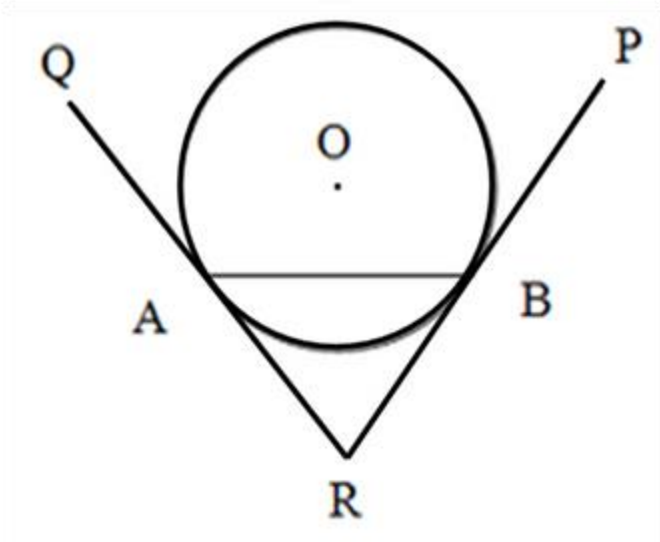
∠AOP = ∠AOQ ....(cpct)

Hence proved.

Question 14

Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

Solution 14



Given : Two tangents RA and RA are drawn from a point A to a circle with centre O.

To prove: ∠RAB = ∠RBA

Proof:

We know that tangents drawn from an external point to the circle are equal.

So, in ΔRAB,

RA = RB

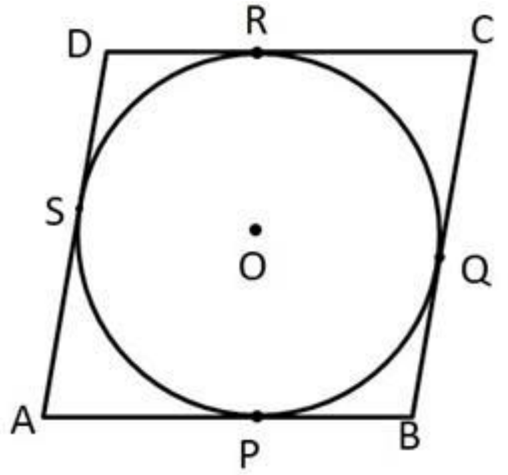
⇒ ∠RAB = ∠RBA .....(sides opposite equal angles are equal)

Hence proved.

Question 15

Prove that the parallelogram circumscribing a circle, is a rhombus.

Solution 15



Given: A parallelogram ABCD circumscribes a circle with centre O.

To prove: AB = BC = CD = AD

Proof:

We know that tangents drawn from an external point to the circle are equal.

$\therefore$ AP = AS ...(tangent from A)

BP = BQ ...(tangent from B)

CR = CQ ...(tangent from C)

DR = DS ...(tangent from D)

$$\Rightarrow AB + CD = AP + BP + CR + DR$$
$$= AS + BQ + CQ + DS$$
$$= (AS + DS) + (BQ + CQ)$$
$$= AD + BC$$

Since opposite sides of a parallelogram are equal,

2AB = 2AD

$\Rightarrow$ AB = AD

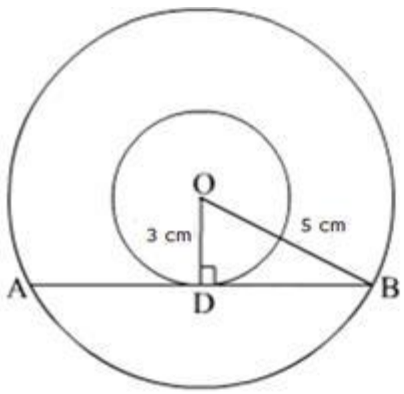
So, AB = BC = CD = AD

Hence proved.

Question 16

Two concentric circles are of radii 5 cm and 3 cm respectively. Find the length of the chord of the larger circle which touches the smaller circle.

Solution 16



Since AB is a tangent to the inner circle.

$\angle ODB = 90°$ ....(tangent is perpendicular to the radius of a circle)

AB is a chord of the outer circle.

We know that, the perpendicular drawn from the
centre to a chord of a circle, bisects the chord.

So, AB = 2DB.

In ΔODB,

By Pythagoras theorem,

$OB^2 = OD^2 + DB^2$

$\Rightarrow 5^2 = 3^2 + DB^2$

$\Rightarrow DB^2 = 5^2 - 3^2$

$\Rightarrow DB^2 = 25 - 9$

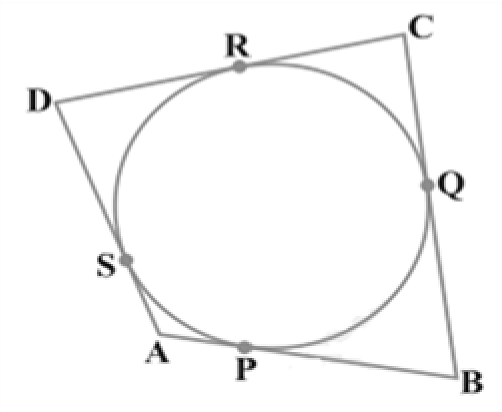
$\Rightarrow DB = 4$ cm

$AB = 2DB = 2(4) = 8$ cm

Question 17

A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal.

Solution 17



Given : ABCD is a quadrilateral in which a circle is inscribed.
To prove: AB + CD = AD + BC
Proof:
We know that the lengths of tangents drawn from an external point to a circle are equal.
$\therefore$ AP = AS ...(i)
BP = BQ ...(ii)
CR = CQ ...(iii)
DR = DQ ...(iv)

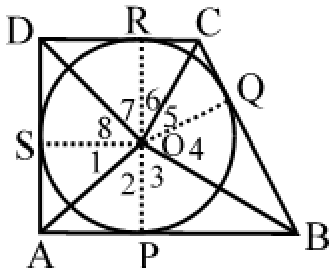
$$\Rightarrow AB + CD = (AP + BP) + (CR + DR)$$
$$= (AS + BQ) + (CQ + DS) \quad ....(\text{from (i), (ii), (iii), (iv)})$$
$$= (AS + DS) + (CQ + BQ)$$
$$= AD + BC$$

Hence proved.

Question 18

Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution 18



Given: A quad. ABCD circumscribes a circle with centre O.
To prove: $\angle AOB + \angle COD = 180^\circ$
and $\angle AOD + \angle BOC = 180^\circ$
Construction : Join OP, OQ, OR and OS.
Proof:
We know that tangents drawn from an external point to a circle subtend equal angles at the centre.
$\therefore \angle 1 = \angle 2, \ \angle 3 = \angle 4, \angle 5 = \angle 6$ and $\angle 7 = \angle 8$
and $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

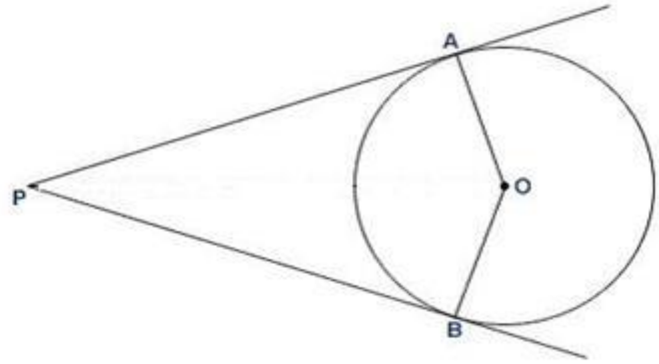
$$\Rightarrow 2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360^\circ \text{ and } 2(\angle 1 + \angle 8) + 2(\angle 4 + \angle 5) = 360^\circ$$
$$\Rightarrow \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180^\circ \text{ and } \angle 1 + \angle 8 + \angle 4 + \angle 5 = 180^\circ$$
$$\Rightarrow \angle AOB + \angle COD = 180^\circ \text{ and } \angle AOD + \angle BOC = 180^\circ$$

Hence proved.

Question 19

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre.

Solution 19



Given: PA and PB are the tangents drawn from a point P to a circle with centre O. Also, the line segments OA and OB are shown.

To prove: $\angle APB + \angle AOB = 180°$

Proof:

We know that the tangent is perpendicular to the radius through the point of contact.

$\therefore PA \perp OA \Rightarrow \angle OAP = 90°$

$\therefore PB \perp OB \Rightarrow \angle OBP = 90°$

$\therefore \angle OAP + \angle OBP = 90° + 90° = 180° \quad ....(i)$

But, we know that the sum of all the angles of a quadrilateral is 360°.

$\therefore \angle OAP + \angle OBP + \angle APB + \angle AOB = 360° \quad ....(ii)$

From (i) and (ii), we get

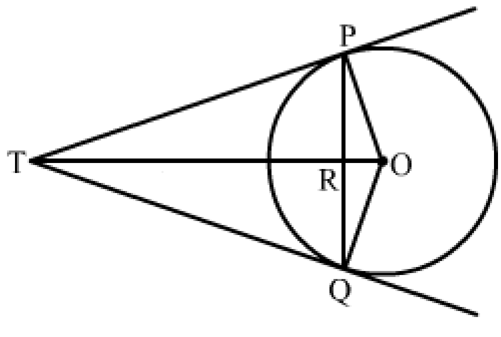
$\angle APB + \angle AOB = 180°$

Hence proved.

Question 20

PQ is a chord of length 16 cm of a circle of radius 10 cm. The tangents at P and Q intersect at a point T as shown in the figure. Find the length of TP.

Solution 20



Construction : Join OQ.

In ΔTPO and ΔTQO,

TP = TQ ....(tangents from an external point to the circle are equal)

OT = OT ....(common side)

OP = OQ .....(radii of the same circle)

$\Rightarrow \Delta TPO \cong \Delta TQO$ ....(SSS congruence criterion)

$\Rightarrow \angle PTO = \angle QTR$ ....(cpct)

....(i)

In ΔTRP and ΔTRQ,

TP = TQ ....(tangents from an external point to the circle are equal)

TR = TR ....(common side)

$\angle PTR = \angle QTR$ ....(from (i))

$\Rightarrow \Delta TRP \cong \Delta TRQ$ ....(SAS congruence criterion)

$\Rightarrow \angle TRP = \angle TRQ$

Since PRQ is a straight line segment,

$\angle TRP + \angle TRQ = 180^\circ$

$\Rightarrow \angle TRP = \angle TRQ = 90^\circ$

So, $OR \perp PQ$

We know that the perpendicular from the centre to the chord of a circle bisects the chord.

So, PR = 8 cm

In ΔORP,

$OR^2 = OP^2 - RP^2$ ....(By Pythagoras theorem)

$\Rightarrow OR^2 = 10^2 - 8^2$

$\Rightarrow OR^2 = 36$

$\Rightarrow OR = 6$ cm

In right ΔPRT,

$PT^2 = TR^2 + PR^2$

$\Rightarrow PT^2 = TR^2 + 8^2$ .....(i)

In right ΔPOT,

$OT^2 = PT^2 + OP^2$

$\Rightarrow (TR + 6)^2 = PT^2 + OP^2$

$\Rightarrow TR^2 + 12TR + 36 = PT^2 + 10^2$ ....(ii)

Solving (i) and (ii), we get

TP = 10.7 cm