Exercise 8.1

Question 1: Write the degree of each of the following polynomials:

Soln:

(i) $2x^3+5x^2-7$

It is $2x^3+5x^2-7$ instead of $2x^2+5x^2-7$

The degree of the polynomial $2x^3+5x^2-7$ is 3

(ii) $5x^2-35x+2$

The degree of the polynomial $5x^2-35x+2$ is 2

(iii) $2x+x^2-8$

The degree of the polynomial $2x+x^2-8$ is 2

(iv) $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$

The degree of the polynomial $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$ is 7

(v) $3x^3+1$

The degree of the polynomial $3x^3+1$ is 3

(vi) 5

5 is a constant polynomial and its degree is 0.

(vii) $20x^3+12x^2y^2-10y^2+20$

The degree of the polynomial $20x^3+12x^2y^2-10y^2+20$ is 4

Question 2

Which of the following expressions are not polynomials:

Soln:

(i) x^2+2x^{-2}

$x^2+2x^{-2}$is not a polynomial because -2 is the power of variable x is not a non negative integer.

(ii) $\sqrt{ax}+x^2-x^3$

$\sqrt{ax}+x^2-x^3$ is not a polynomial because $\frac{1}{2}$ is the power of variable x is not a non negative integer.

(iii) $3y^3-\sqrt{5}y+9$

$3y^3-\sqrt{5}y+9$ is a polynomial because the powers of variable y are non negative integers.

(iv) $ax^{\frac{1}{2}}+ax+9x^2+4$

$ax^{\frac{1}{2}}+ax+9x^2+4$ is not a polynomial because $\frac{1}{2}$ is the power of variable x is not a non negative integer.

(v) $3x^{-2}+2x^{-1}+4x+5$

$3x^{-2}+2x^{-1}+4x+5$ is not a polynomial because -2 and -1 are the powers of variable x are not non negative integers.

Question 3

Write each of the following polynomials in the standard form. Also, write their degree:

Soln:

(i) $(x^2+3+6x+5x^4)$

The standard form of the given polynomial can be expressed as:

$(5x^4+x^2+6x+3)$ or $(3+6x+x^2+5x^4)$

The degree of the polynomial is 4

(ii) a^2+4+5a^6

The standard form of the given polynomial can be expressed as:

$(5a^6+a^2+4)$ or $(4+a^2+5a^6)$

The degree of the polynomial is 6

(iii) $(x^3-1)(x^3-4)$

$(x^3-1)(x^3-4) = x^6-5x^3+4$

The standard form of the given polynomial can be expressed as:

(x^6-5x^3+4) or $(4-5x^3+x^6)$

The degree of the polynomial is 6

(iv) $(y^3-2)(y^3+11)$

$(y^3-2)(y^3+11) = y^6+9y^3-22$

The standard form of the given polynomial can be expressed as:

(y^6+9y^3-22) or $(-22+9y^3+y^6)$

The degree of the polynomial is 6

(v) $(a^3-\frac{3}{8})(a^3+\frac{16}{17})(a^3-\frac{3}{8})(a^3+\frac{16}{17}) = a^3+\frac{77}{136}a^3-\frac{6}{17}$

Standard form of the given polynomial can be expressed as:

$(a^3+\frac{77}{136}a^3-\frac{6}{17})$ or $(-\frac{6}{17}+\frac{77}{136}a^3+a^3)$

The degree of the polynomial is 6.

(vi) $(a+\frac{3}{4})(a+\frac{4}{3})(a+\frac{3}{4})(a+\frac{4}{3}) = a^2+\frac{25}{12}a+1$

Standard form of the given polynomial can be expressed as:

$(a^2+\frac{25}{12}a+1)$ or $(1+\frac{25}{12}a+a^2)$

The degree of the polynomial is 2

Exercise 8.2

Divide: Question 1

$6x^3y^2z^2$ by $3x^2yz$

Soln:

$$\frac{6x^3y^2z^2}{3x^2yz}$$

$$= \frac{6\times x\times x\times x\times y\times y\times z\times z}{3\times x\times x\times y\times z}$$

$= 2x^{(3-2)}y^{(2-1)}z^{(2-1)}$

$= 2xyz$

Question 2

$15m^2n^3$ by $5m^2n^2$

Soln:

$$\frac{15n^3m^2}{5m^2n^2}$$

$$= \frac{15\times n\times n\times n\times m\times m}{5\times m\times m\times n\times n}$$

$= 3m^{(2-2)}n^{(3-2)}$

$= 3m^0n^1$

$=3n$

Question 3

$24a^3b^3$ by $-8ab$

Soln:

$$\frac{24a^3b^3}{-8ab}$$

$$= \frac{24\times a\times a\times a\times b\times b\times b}{-8\times a\times b}$$

$= -3a^{(3-1)}b^{(3-1)}$

$= -3a^2b^2$

Question 4

$-21abc^2$ by $7abc$

Soln:

$$\frac{-21abc^2}{7ab}$$

$$= \frac{-21 \times a \times b \times c \times c}{7 \times a \times b \times c}$$

$= -3a^{(1-1)}b^{(1-1)}c^{(2-1)}$

$= -3a^0b^0c^1$

$= 3c$

Question 5

$72xyz^2$ by $-9xz$

Soln:

$$\frac{72xyz^2}{-9xz}$$

$$= \frac{72 \times x \times y \times z \times z}{-9 \times x \times z}$$

$= -8x^{(1-1)}yz^{(2-1)}$

= -8yz

Question 6

$-72a^4b^5c^8$ by $-9a^2b^2c^3$

Soln:

$$\frac{-72a^4b^5c^8}{-9a^2b^2c^3}$$

$$= \frac{-72\times a\times a\times a\times a\times b\times b\times b\times b\times b\times c\times c\times c\times c\times c\times c\times c\times c\times c}{-9\times a\times a\times b\times b\times c\times c\times c}$$

$= 8a^{(4-2)}b^{(5-2)}c^{(8-3)}$

$= 8a^2b^3c^5$

Simplify :

Question 7

$$\frac{16m^3y^2}{4m^2y}$$

Soln:

$$\frac{16m^3y^2}{4m^2y}$$

$$= \frac{16\times m\times m\times \times y\times y}{4\times m\times m\times y}$$

$= 4m^{(3-2)}y^{(2-1)}$

$= 4my$

Question 8

$$\frac{32m^2n^3p^2}{4mnp}$$

Soln:

$$\frac{32m^2n^3p^2}{4mnp}$$

$$= \frac{32\times m\times m\times n\times n\times n\times p\times p}{4\times m\times n\times p}$$

$= 8m^{(2-1)}n^{(3-1)}p^{(2-1)}$

$= 8mn^2p$

Exercise 8.3

Divide: Question 1

$x+2x^2+3x^4-x^5$ by $2x$

Soln:

$$\frac{x+2x^2+3x^4-x^5}{2x}$$

$$= \frac{x}{2x} + \frac{2x^2}{2x} + \frac{3x^4}{2x} - \frac{x^5}{2x}$$

$$= \frac{1}{2} + x + \frac{3x^3}{2} - \frac{1x^4}{2}$$

Question 2

$y^4-3y^3+\frac{1}{2}y^2$ by $3y$

Soln:

$$\frac{y^4-3y^3+\frac{1y^3}{2}}{3y}$$

$$= \frac{y^4}{3y} - \frac{-3y^3}{3y} + \frac{\frac{1y^2}{2}}{3y}$$

$= \frac{1y^{4-1}}{3} - y^{3-1} + \frac{1y^{2-1}}{6}$

$= \frac{1y^{3}}{3} - y^{2} + \frac{1y^{1}}{6}$

Question 3

$-4a^3+4a^2+a$ by $2a$

Soln:

$\frac{-4a^3+4a^2+a}{2a}$

$= \frac{-4a^3}{2a} + \frac{4a^2}{2a} + \frac{a}{2a}$

$= -2a^{(3-1)}+2a^{(2-1)}+\frac{1}{2}$

$= -2a^2+2a+\frac{1}{2}$

Question 4

$-x^6+2x^4+4x^3+2x^2$ by $\sqrt{2}x^2$

Soln:

$$\frac{-x^6+2x^4+4x^3+2x^2}{\sqrt{2}x^2}$$

$$= \frac{-x^6}{\sqrt{2}x^2} + \frac{2x^4}{\sqrt{2}x^2} + \frac{4x^3}{\sqrt{2}x^2} + \frac{2x^2}{\sqrt{2}x^2}$$

$$= \frac{-1x}{\sqrt{6}}^{6-2} + \sqrt{2}x^{4-2} + 2\sqrt{2}x^{3-2} + \sqrt{2}x^{2-2}$$

$$= \frac{-1x}{\sqrt{6}}^{4} + \sqrt{2}x^2 + 2\sqrt{2}x^1 + \sqrt{2}x^0$$

$$= \frac{-1x}{\sqrt{6}}^{4} + \sqrt{2}x^2 + 2\sqrt{2}x + \sqrt{2}$$

Question 5

$5z^3-6z^2+7z$ by $2z$

Soln:

$$= \frac{5z^3-6z^2+7z}{2z}$$

$= \frac{5z^3}{2z} - \frac{6z^2}{2z} + \frac{7z}{2z}$

$= \frac{5z^{3-1}}{2} - 3z^{2-1} + \frac{7}{2}$

$= \frac{5z^2}{2} - 3z + \frac{7}{2}$

Question 6

$\sqrt{3}a^4 + 2\sqrt{3}a^3 + 3a^2 - 6a$ by 3a

Soln:

$\frac{\sqrt{3}a^4+2\sqrt{3}a^3+3a^2-6a}{3a}$

$= \frac{\sqrt{3}a^4}{3a} + \frac{2\sqrt{3}a^3}{3a} + \frac{3a^2}{3a} - \frac{6a}{3a}$

$= \frac{1a^{4-1}}{\sqrt{3}a} + \frac{2a^{3-1}}{\sqrt{3}a} + a^{2-1} - 2$

$= \frac{1a^3}{\sqrt{3}a} + \frac{2a^2}{\sqrt{3}a} + a^1 - 2$

Exercise 8.4

Divide.

1. $5x^3 - 15x^2 + 25x$ by $5x$.

$$\frac{5x^3 - 15x^2 + 25x}{5x} = \frac{5x^3}{5x} + \left(\frac{-15}{5}\right)\cdot\frac{x^2}{x} + \frac{25}{5}\cdot\frac{x}{x}$$

$$= x^2 - 3x + 5.$$

2. $4z^3 + 6z^2 - z$ by $-\frac{1}{2}z$

$$\frac{4z^3 + 6z^2 - z}{-\frac{1}{2}z} = \frac{4z^3\cdot(2)}{-z} - \frac{6z^2\cdot 2}{z} + \frac{z\cdot 2}{z}$$

$$= -8z^2 - 12z + 2.$$

3. $9x^2y - 6xy + 12xy^2$ by $-\frac{3}{2}xy$.

$$\frac{9x^2y - 6xy + 12xy^2}{-\frac{3}{2}xy} = \frac{9x^2y}{-3x\cdot y}\cdot 2 + \frac{6xy\cdot 2}{3\cdot x\cdot y} + \frac{12x\cdot y^2\cdot z}{-3xy}$$

$$= -6x + 4 - 8y.$$

4. $3x^3.y^2 + 2x^2.y + 15xy$ by $3xy$.

$$\frac{3x^3.y^2+2x^2.y+15xy}{3xy} = \frac{3.x^3.y^2}{3xy} + \frac{2x^2.y}{3xy} + \frac{15xy}{3xy}$$

$$= x^2y + \frac{2}{3}x + 5.$$

5. $x^2+7x+12$ by $x+4$.

step 1 :-

we devide the first term x^2 of the dividend by the first term x of the divisor and obtain $\frac{x^2}{x} = x$ as the first term of the quotient.

$$\begin{array}{r|l} & x+3 \\ \hline x+4 & x^2+7x+12 \\ & x^2+4x \\ \hline & 3x+12 \\ & 3x+12 \\ \hline & (0) \end{array}$$

Step -2 :-

we multiply the divisor $x+4$ by the first term x of the quotient and subtract the result from the dividend $x^2+7x+12$. we obtain $3x+12$ as the remainder

step-3 :-

Now we treat $3x+12$ as the new dividend and divide the first term $3x$ by the first

term x of the divisor to obtain $\frac{3x}{x} = 3$ as the third term of the quotient.

step-IV:-

We multiply the divisor $x+4$ and the second third term 3 of the quotient and subtract the result $3x+12$ from the new dividend. we obtain 0 as the remainder.

Thus, we can say that:

$$\frac{x^2+7x+12}{x+4} = x+3.$$

Solution-06:-

$4y^2+3y+\frac{1}{2}$ by $2y+1$

$$\begin{array}{r|l} & 2y+\frac{1}{2} \\ \hline 2y+1 & 4y^2+3y+\frac{1}{2} \\ & \underline{-4y^2+2y} \\ & y+\frac{1}{2} \\ & \underline{y+\frac{1}{2}} \\ & 0 \end{array}$$

Solution-07.

$3x^3+4x^2+5x+18$ by $x+2$.

$$\begin{array}{r|l} & 3x^2-2x+9 \\ \hline x+2 & 3x^3+4x^2+5x+18 \\ & \underline{3x^3+6x^2} \\ & -2x^2+5x \\ & \underline{-2x^2+(+4x} \\ & 9x+18 \\ & \underline{9x+18} \\ & 0 \end{array}$$

Solution-08:-

$$
\begin{array}{r|l}
7x-9 & 14x^2-53x+45 \\
 & -14x^2+18x \\
\hline
 & -35x+45 \\
 & -35x+45 \\
\hline
 & 0
\end{array}
$$

Solution - 09.

$$\frac{-(-21+71x-31x^2-24x^3)}{-(3-8x)} = \frac{21-71x+31x^2+24x^3}{8x-3}$$

$$
\begin{array}{r|l}
8x-3 & 24x^3+31x^2-71x+21 \\
 & -24x^3+9x^2 \\
\hline
 & 40x^2-71x \\
 & -40x^2+15x \\
\hline
 & -56x+21 \\
 & -56x+21 \\
\hline
 & 0
\end{array}
\qquad 3x^2+5x-7.
$$

Solution-10:-

$3y^4 - 3y^3 - 4y^2 - 4y$ by $y^2 - 2y$.

$$
\begin{array}{r|l}
 & 3y^2+3y+2 \\
\hline
y^2-2y & 3y^4-3y^3-4y^2-4y \\
 & 3y^4-6y^3 \\
 & \quad - \quad + \\
\hline
 & \quad 3y^3-4y^2 \\
 & \quad 3y^3-6y^2 \\
 & \quad \quad + \\
\hline
 & \quad \quad 2y^2-4y \\
 & \quad \quad 2y^2+4y \\
 & \quad \quad - \quad + \\
\hline
 & \quad \quad (0)
\end{array}
$$

$(y^2-2y)(3y^2+3y+2) = 3y^4-3y^3-4y^2-4y$.

Solution-11

$2y^5+10y^4+6y^3+y^2+5y+y+3$ by $2y^3+1$

$$
\begin{array}{r|l}
 & y^2+5y+3 \\
\hline
2y^3+1 & 2y^5+10y^4+6y^3+y^2+5y+3 \\
 & \underline{2y^5+0+0+y^2} \\
\hline
 & \quad 10y^4+6y^3+0+5y \\
 & \quad \underline{10y^4+0+0\pm 5y} \\
\hline
 & \quad \quad 6y^3+0+0+3 \\
 & \quad \quad \underline{6y^3+0+0+3} \\
\hline
 & \quad \quad (0)
\end{array}
$$

Exercise 8.5

Question 1: Divide the first polynomial by the second polynomial in each of the following. Also, write the quotient and remainder :

(i) $\frac{3x^2+4x+5}{x-2}$

Soln:

$\frac{3x^2+4x+5}{x-2}$

$= \frac{3x(x-2)+10(x-2)+25}{x-2}$

$= \frac{3x(x-2)+10(x-2)+25}{x-2}$

$= (3x+10)+\frac{25}{x-2}$

Therefore,

Quotient = 3x+10 and remainder = 25

(ii) $\frac{10x^2-7x+8}{5x-3}$

Soln:

$$\frac{2x(5x-3)-\frac{1}{5}(5x-3)+\frac{47}{5}}{5x-3}$$

$$= \frac{(5x-3)(2x-\frac{1}{5})(5x-3)+\frac{47}{5}}{5x-3}$$

$$= (2x-\frac{1}{5})+\frac{\frac{47}{5}}{5x-3}$$

Therefore , quotient= $(2x-\frac{1}{5})$ and remainder = $\frac{47}{5}$

(iii) $\frac{5y^3-6y^2+6y-1}{5y-1}$

Soln:

$$\frac{5y^3-6y^2+6y-1}{5y-1}$$

$$= \frac{y^2(5y-1)-y(5y-1)+1(5y-1)}{5y-1}$$

$$= \frac{(5y-1)(y^2-y+1)}{5y-1}$$

= y^2-y+1+5

Therefore quotient = y^2-y+1

And remainder = 0

(iv) $\frac{x^4-x^3+5x}{x-1}$

Soln:

$\frac{x^3(x-1)+5(x-1)+5}{x-1}$

$= \frac{(x^3+5)(x-1)+5}{x-1}$

$= (x^3+5)+\frac{5}{x-1}$

Therefore, quotient = x^3+5 and remainder= 5

(v) $\frac{(y^4+y^2)}{y^2-2}$

Soln:

$\frac{(y^4+y^2)}{y^2-2}$

$= \frac{y^2(y^2-2)+3(y^2-2)+6}{y^2-2}$

$= \frac{(y^2-2)(y^2+3)}{y^2-2}$

$(y^2+3)+\frac{6}{y^2-2}$

Therefore, quotient = y^3+3 and remainder = 6

Question 2: Find whether, or not the first polynomial is a factor of the second:

(i) $\frac{2x^2+5x+4}{x+1}$

Soln:

$= \frac{2x(x+1)+3(x+1)+1)}{x+1}$

$= \frac{(x+1)(2x+3)+1)}{x+1}$

Therefore, (x+1) is not a factor of $2x^2+5x+4$

(ii) $\frac{3y^3+5y^2+5y+2}{y-2}$

Soln:

$\frac{3y^3+5y^2+5y+2}{y-2}$

$= \frac{3y^2(y-2)+11y(y-2)+27(y-2)+56}{y-2}$

$= \frac{(y-2)(3y^2+11y+27)+56}{y-2}$

$= (3y^2+11y+27) + \frac{56}{y-2}$

Therefore, (y-2) is not a factor of $3y^3+5y^2+5y+2$

(iii) $\frac{4x^4+12x^2+15}{4x^2-5}$

Soln:

$\frac{4x^4+12x^2+15}{4x^2-5}$

$= \frac{x^2(4x^2-5)+3(4x^2-5)+30}{4x^2-5}$

$= (x^2+3) + \frac{30}{4x^2-5}$

Therefore , $(4x^2-5)$ is not a factor of $4x^4+7x^2+15$

(iv) $\frac{3z^2-13z+4}{4-z}$

Soln:

$$\frac{3z^2-13z+4}{4-z}$$

$$= \frac{3z(z-4)-1(z-4)}{4-z}$$

$$= \frac{(z-4)(3z-1)}{4-z}$$

$$= \frac{(4-z)(1-3z)}{4-z}$$

= 1-3z

Therefore, remainder =0

(4-z) is a factor of the factor of $3z^2$-13z+4

(v) $\frac{10a^2-9a-5}{2a-3}$

Soln:

$$\frac{10a^2-9a-5}{2a-3}$$

$$= \frac{5a(2a-3)+3(2a-3)}{2a-3}$$

$= \frac{(2a-3)(5a+3)+4}{2a-3}$

$= (5a+3) + \frac{4}{2a-3}$

Therefore, remainder =4

(2a-3) is not a factor of the equation $10a^2$-9a-5

(vi) $\frac{8y^2-2y+1}{4y+1}$

Soln:

$= \frac{2y(4y+1)-1(4y+1)+2}{4y+1}$

$= \frac{(4y+1)(2y-1)+2}{4y+1}$

$= 2y\text{-}1 + \frac{2}{4y+1}$

Therefore, remainder =2

(4y+1) is not a factor of $8y^2$-2y+1

Exercise 8.6

Question 1: $x^2 - 5x + 6$ by $(x - 3)$

Soln:

$$\frac{x^2-5x+6}{x-3}$$

$$= \frac{x^2-3x-2x+6}{x-3}$$

$$= \frac{x(x-3)-2(x-2)}{x-3}$$

$$= \frac{(x-3)(x-2)}{x-3} = x-2$$

Question 2: ax^2-ay^2 by $(ax+ay)$

Soln:

$$\frac{ax^2-ay^2}{ax+ay}$$

$$= \frac{a(x^2-y^2)}{ax+ay}$$

$$= \frac{a(x+y)(x-y)}{a(x+y)} = x-y$$

Question 3: x^4-y^4 by x^2-y^2

Soln:

$$\frac{x^4-y^4}{x^2-y^2}$$

$$= \frac{(x^2)^2-(y^2)^2}{(x^2-y^2)}$$

$$= \frac{(x^2-y^2)\times(x^2+y^2)}{(x^2-y^2)} = x^2+y^2$$

Question 4: $acx^2+(bc+ad)x+bd$ by $(ax+b)$

Soln:

$$\frac{acx^2+(bc+ad)x+bd}{ax+b}$$

$$= \frac{acx^2+bcx+adx+bd}{ax+b}$$

$$= \frac{cx(ax+b)+d(ax+b)}{ax+b}$$

$$= \frac{(ax+b)(cx+d)}{ax+b} = cx+d$$

Question 5: $(a^2+2ab+b^2)-(a^2+2ac+c^2)$ by $(2a+b+c)$

Soln:

$$\frac{(a^2+2ab+b^2)-(a^2+2ac+c^2)}{2a+b+c}$$

$$= \frac{(a+b)^2-(a+c)^2}{2a+b+c}$$

$$= \frac{(a+b+a+c)(a+b-a-c}{2a+b+c}$$

$$= \frac{(2a+b+c)(b-c)}{2a+b+c} = b\text{-}c$$

Question 6: $(\frac{1}{4}x^2 - \frac{1}{2}x - 12)\ by\ (\frac{1}{2}x - 4)$

Soln:

$$\frac{\frac{1}{4}x^2-\frac{1}{2}x-12}{\frac{1}{2}x-4}$$

$$= \frac{\frac{1}{2}x(\frac{1}{2}x-4)+3()}{\frac{1}{2}x-4}$$

$$= \frac{(\frac{1}{2}x+3)(\frac{1}{2}x-4)}{\frac{1}{2}x-4} = (\frac{1}{2}x + 3)$$