Exercise 7.1

Find the greatest common factor of the following polynomials (Q.1 – Q.14)

Q.1) $2x^2$ and $12x^2$

Soln.:

The numerical coefficients of the given monomials are 2 and 12.

So, the greatest common factor of 2 and 12 is 2.

The common literal appearing in the given monomials is x.

The smallest power of x in the two monomials is 2.

The monomial of the common literals with the smallest powers is x^2.

Hence, the greatest common factor is $2x^2$.

Q.2) $6x^3y$ and $18x^2y^3$

Soln.:

The numerical coefficients of the given monomials are 6 and 18.

The greatest common factor of 6 and 18 is 6.

The common literals appearing in the two monomials are x and y.

The smallest power of x in the two monomials is 2.

The smallest power of y in the two monomials is 1.

The monomial of the common literals with the smallest powers is x^2y.

Hence, the greatest common factor is $6x^2y$.

Q.3) $7x$, $21x^2$ and $14xy^2$

Soln.:

The numerical coefficients of the given monomials are 7, 21 and 14.

The greatest common factor of 7, 21 and 14 is 7.

The common literal appearing in the three monomials is x.

The smallest power of x in the three monomials is 1.

The monomial of the common literals with the smallest powers is x.

Hence, the greatest common factor is 7x.

Q.4) $42x^2yz$ and $63x^3y^2z^3$

Soln.:

The numerical coefficients of the given monomials are 42 and 63.

The greatest common factor of 42 and 63 is 21.

The common literals appearing in the two monomials are x, y and z.

The smallest power of x in the two monomials is 2.

The smallest power of y in the two monomials is 1.

The smallest power of z in the two monomials is 1.

The monomial of the common literals with the smallest powers is x^2yz.

Hence, the greatest common factor is $21x^2yz$.

Q.5) $12ax^2$, $6a^2x^3$ and $2a^3x^5$

Soln.:

The numerical coefficients of the given monomials are 12, 6 and 2.

The greatest common factor of 12, 6 and 2 is 2.

The common literals appearing in the three monomials are a and x.

The smallest power of a in the three monomials is 1.

The smallest power of x in the three monomials is 2.

The monomial of common literals with the smallest powers is ax^2.

Hence, the greatest common factor is $2ax^2$.

Q.6) $9x^2$, $15x^2y^3$, $6xy^2$ and $21x^2y^2$

Soln.:

The numerical coefficients of the given monomials are 9, 15, 6 and 21.

The greatest common factor of 9, 15, 6 and 21 is 3.

The common literal appearing in the three monomials is x.

The smallest power of x in the four monomials is 1.

The monomial of common literals with the smallest powers is x.

Hence, the greatest common factor is 3x.

Q.7) $4a^2b^3$, $-12a^3b$, $18a^4b^3$

Soln.:

The numerical coefficients of the given monomials are 4, -12 and 18.

The greatest common factor of 4. -12 and 18 is 2.

The common literals appearing in the three monomials are a and b.

The smallest power of a in the three monomials is 2.

The smallest power of b in the three monomials is 1.

The monomial of the common literals with the smallest powers is a^2b.

Hence. the greatest common factor is $2a^2b$.

Q.8) $6x^2y^2$, $9xy^3$, $3x^3y^2$

Soln.:

The numerical coefficients of the given monomials are 6, 9 and 3.

The greatest common factor of 6, 9 and 3 is 3.

The common literals appearing in the three monomials are x and y.

The smallest power of x in the three monomials is 1.

The smallest power of y in the three monomials is 2.

The monomial of common literals with the smallest powers is xy^2.

Hence, the greatest common factor is $3xy^2$.

Q.9) a^2b^3 , a^3b^2

Soln.:

The numerical literals in the three monomials are a and b.

The smallest power of x in the three monomials is 2.

The smallest power of y in the three monomials is 2.

The monomial of common literals with the smallest powers is a^2b^2.

Hence, the greatest common factor is a^2b^2.

Q.10) $36a^2b^2c^4$, $54a^5c^2$, $90a^4b^2c^2$

Soln.:

The numerical coeff. of the given monomials are 36, 54, and 90.

The greatest common factors of 36, 54, and 90 is 18.

The common literals appearing in the three monomials are a and c.

The smallest power of a in the three monomials is 2.

The smallest power of c in the three monomials is 2.

The monomial of common literals with the smallest powers is a^2c^2.

Hence, the greatest common factor is $18a^2c^{2.}$

Q.11) x^3 , $-yx^2$

Soln.:
The common literal appearing in the two monomials is X.
The smallest power of X in both the monomials is 2.

Hence, the greatest common factor is x^2.

Q.12) $15a^3$, $-45a^2$, $-150a$

Soln.:

The numerical coeff. of the given monomials are -15, -45 and -150.

The greatest common factor of 15, -45 and -150 is 15.

The common literal appearing in the three monomials is a.

The smallest power of a in the three monomials is 1.

Hence, the greatest common factor is 15a.

Q.13) $2x^3y^2$, $10x^2y^3$, $14xy$

Soln.:

The numerical coeff. of the given monomials are 2, 10 and 14.

The greatest common factor of 2, 10 and 14 is 2.

The common literals appearing in the three monomials are x and y.

The smallest power of X in the three monomials is 1.

The smallest power of y in the three monomials is 1.

The monomials of common literals with the smallest power is xy.

Hence, the greatest common factor is 2xy.

Q.14) $14x^3y^5$, $10x^5y^3$, $2x^2y^2$

Soln.:

The numerical coeff. of the given monomials are 14, 10 and 2.

The greatest common factor of 14, 10 and 2 is 2.

The common literals appearing in the three monomials are x and y.

The smallest power of X in the three monomials is 2.

The smallest power of Y in the three monomials is 2.

The monomials of common literals with the smallest powers is x^2y^2.

Hence, the greatest common factor is $2x^2y^2$.

Find the greatest common factor of the terms in each of the following expressions :

Q.15) $5a^4 + 10a^3 – 15a^2$

Soln.:

The numerical coeff. of the given monomials are $5a^4$, $10a^3$, and $15a^2$.

The greatest common factor of $5a^4$, $10a^3$, and $15a^2$ is 5.

The common literal appearing in the three monomials is a.

The smallest power of a in the three monomials is 2.

The monomials of common literals with the smallest powers is a^2.

Hence, the greatest common factor is $5a^2$.

Q.16) $2xyz + 3x^2y + 4y^2$

Soln.:

The numerical coeff. of the given monomials are 2xyz, $3x^2y$ and $4y^2$.

The greatest factor of 2xyz, $3x^2y$ and $4y^2$ is 1.

The common literal appearing in the three monomials is y.

The smallest power of y in the three monomials is 1.

The monomials of common literals with the smallest power is y.

Hence, the greatest common factor is y.

Q.17) $3a^2b^2 + 4b^2c^2 + 12a^2b^2c^2$.

Soln.:

The numerical coeff. of the given monomials are $3a^2b^2$, $4b^2c^2$ and $12a^2b^2c^2$.

The greatest common factor of $3a^2b^2$, $4b^2c^2$ and $12a^2b^2c^2$ is 1.

The common literal appearing in the three monomials is b.

The smallest power of b in the three monomials is 2.

The monomials of common literals with the smallest powers is b^2.

Hence, the greatest common factor is b^2.

Exercise 7.2

Factorize the following :

Q.1) $3x - 9$

Soln.:

The greatest common factor of the terms 3x and -9 of the expression $3x - 9$ is 3.

Now,

$3x = 3x$

and

$-9 = 3(-3)$

Hence, the expression $3x - 9$ can be factorised as $3(x - 3)$.

Q.2) $5x - 15x^2$

Soln.:

The greatest common factor of the terms 5x and $15x^2$ of the expression $5x - 15x^2$ is 5x.

Now,

$5x = 5x.(1)$

and

$-15x^2 = 5x.(-3x)$

Hence, the expression $5x - 15x^2$ can be factorised as $5x(1 - 3x)$

Q.3) $20a^{12}b^2 - 15a^8b^4$

Soln.:

The greatest common factor of the terms

$20a^{12}b^2$ and $-15a^8b^4$ of the expression $20a^{12}b^2 - 15a^8b^4$ is $5a^8b^2$.

$20a^{12}b^2 = 5x4xa^8xa^4xb^2 = 5a^8xb^2x4a^4$ and $-15a^8xb^4 = 5x(-3)xa^8xb^2xb^2 = 5a^8b^2\ x(-3)b^2$

Hence, the expression $20a^{12}b^2 - 15a^8b^4$ can be factorised as $5a^8b^2(4a^4 - 3b^2)$

Q.4) $\mathbf{72x^6y^7 – 96x^7y^6}$

Soln.:

The greatest common factor of the terms $72x^6y^7$ and $-96x^7y^6$ of the expression $72x^6y^7 – 96x^7y^{64}$ is $24x^6y^6$

Now,

$72x^6y^7 = 24x^6y^6 . 3y$

And, $– 96x^7y^{64}$ is $24x^6y^6 . – 4x$

Hence, the expression $72x^6y^7 – 96x^7y^6$ can be factorised as $24x^6y^6 . (3y – 4x)$.

Q.5) $\mathbf{20x^3 – 40x^2 + 80x}$

Soln.:

The greatest common factor of the terms $20x^3$, $-40x^2$ and $80x$ of the expression $20x^3 – 40x^2 +80x$ is $20x$.

Now, $20x^3 = 20x . x^2$

$-40x^2 = 20x . -2x$

And, $80x = 20x . 4$

Hence, the expression $20x^3 – 40x^2 +80x$ can be factorised as $20x(x^2 – 2x + 4)$

Q.6) $\mathbf{2x^3y^2 – 4x^2y^3 + 8xy^4}$

Soln.:

The greatest common factor of the terms $2x^3y^2$, $-4x^2y^3$ and $8xy^4$ of the expression

$2x^3y^2 – 4x^2y^3 + 8xy^4$ is $2xy^2$.

Now,

$2x^3y^2 = 2xy^2 . x^2$

$-4x^2y^3 = 2xy^2 . (-2xy)$

$8xy^4 = 2xy^2 . 4y^2$

Hence, the expression $2x^3y^2 – 4x^2y^3 + 8xy^4$ can be factorised as $2xy^2(x^2 – 2xy + 4y^2)$

Q.7) $\mathbf{10m^3n^2 + 15m^4n – 20m^2n^3}$

Soln.:

The greatest common factor of the terms 10^3n^2, $15m^4n$ and $-20m^2n^3$ of the expression

$10m^3n^2 + 15m^4n – 20m^2n^3$ is $5m^2n$.

Now,

$10m^3n^2 = 5m^2n . 2mn$

$15m^4n = 5m^2n . 3m^2$

$-20m^2n^3 = 5m^2n . -4n^2$

Hence, $10m^3n^2 + 15m^2n – 20m^2n^3$ can be factorised as $5m^2n(2mn + 3m^2 – 4n^2)$

Q.8) $2a^4b^4 – 3a^3b^5 + 4a^2b^5$

Soln.:

The greatest common factor of the terms $2a^4b^4$, $-3a^3b^5$ and $4a^2b^5$ of the expression

$2a^4b^4 – 3a^3b^5 + 4a^2b^5$ is a^2b^5.

Now,

$2a^4b^4 = a^2b^5 . 2a^2$

$-3a^3b^5 = a^2b^4 . (-3ab)$

$4a^2b^5 = a^2b^4 . 4b$

Hence, $2a^4b^4 – 3a^3b^5 + 4a^2b^5$ can be factorised as $a^2b^4(2a^2 – 3ab + 4b)$

Q.9) $28a^2 + 14a^2b^2 – 21a^4$

Soln.:

The greatest common factor of the terms$28a^2$, $14a^2b^2$ and $21a^4$ of the expression

$28a^2 + 14a^2b^2 – 21a^4$ is $7a^2$.

Also, we can write $28a^2 = 7a^2. 4$, $14a^2b^2 = 7a^2 . 2b^2$ and $21a^4 = 7a^2 . 3a^2$.

Therefore, $28a^2 + 14a^2b^2 – 21a^4 = 7a^2. 4 + 7a^2 . 2b^2 – 7a^2 . 3a^2$

$= 7a^2 (4 + 2b^2 – 3a^2)$

Q.10) $a^4b – 3a^2b^2 – 6ab^3$

Soln.:

The greatest common factor of the terms a^4b, $3a^2b^2$ and $6ab^3$ of the expression

$a^4b – 3a^2b^2 – 6ab^3$ is ab.

Also, we can write $a^4b = ab . a^3$, $3a^2b^2 = ab . 3ab$ and $6ab^3 = ab . 6b^2$.

Therefore, $a^4b - 3a^2b^2 - 6ab^3 = ab . a^3 - ab . 3ab - ab . 6b^2$.

$= ab (a^3 - 3ab - 6b^2)$

Q.11) $2L^2mn - 3Lm^2n + 4Lmn^2$

Soln.:

The greatest common factor of the terms $2L^2mn$, $3Lm^2n$ and $4Lmn^2$ of the expression $2L^2mn - 3Lm^2n + 4Lmn^2$ is Lmn.

Also, we can write $2L^2mn = Lmn . 2L$, $3Lm^2n = Lmn . 3m$ and $4Lmn^2 = Lmn . 4n$

Therefore, $2L^2mn - 3Lm^2n + 4Lmn^2 = (Lmn . 2L) - (Lmn . 3m) + (Lmn . 4n)$

$= Lmn(2L - 3m + 4n)$

Q.12) $x^4y^2 - x^2y^4 - x^4y^4$

Soln.:

The greatest common factor of the terms x^4y^2, x^2y^4 and x^4y^4 of the expressinon $x^4y^2 - x^2y^4 - x^4y^4$ is x^2y^2

Also, we can write $x^4y^2 = (x^2y^2 . x^2)$, $x^2y^4 = (x^2y^2 . y^2)$ and $x^4y^4 = (x^2y^2 . x^2y^2)$

Therefore, $x^4y^2 - x^2y^4 - x^4y^4 = (x^2y^2 . x^2) - (x^2y^2 . y^2) - (x^2y^2 . x^2y^2)$

$= x^2y^2 (x^2 - y^2 - x^2y^2)$

Q.13) $9x^2y + 3axy$

Soln.:

The greatest common factor of the terms $9x^2y$ and $3axy$ of the expression $9x^2y + 3axy$ is $3xy$.

Also, we can write $9x^2y = 3xy . 3x$ and $3axy = 3xy . a$

Therefore, $9x^2y + 3axy = (3xy . 3x) + (3xy . a)$

$= 3xy (3x + a)$

Q.14) $16m - 4m^2$

Soln.:

The greatest common factor of the terms $16m$ and $4m^2$ of the expression $16m - 4m^2$ is $4m$.

Also, we can write $16m = 4m . 4$ and $4m^2 = 4m . m$

Therefore, $16m - 4m^2 = (4m . 4) - (4m . m)$

$= 4m(4 - m)$

Q.15) $-4a^2 + 4ab - 4ca$

Soln.:

The greatest common factor of the terms $-4a^2$, $4ab$ and $-4ca$ of the expression

$-4a^2 + 4ab - 4ca$ is $-4a$.

Also, we can write $-4a^2 = (-4a . a)$, $4ab = -4a . (-b)$,and $4ca = (-4a . c)$

Therefore, $-4a^2 + 4ab - 4ca = (-4a . a) + (-4a . (-b)) - (4a . c)$

$= -4a (a - b + c)$

Q.16) $x^2yz + xy^2z + xyz^2$

Soln.:

The greatest common factor of the terms x^2yz , xy^2z and xyz^2 of the expression

$x^2yz + xy^2z + xyz^2$ is xyz.

Also, we can write $x^2yz = (xyz . x)$, $(xy^2z = xyz . y)$, $xyz^2 = (xyz . z)$

Therefore, $x^2yz + xy^2z + xyz^2 = (xyz . x) + (xyz . y) + (xyz . z)$

$= xyz(x + y + z)$

Q.17) $ax^2y + bxy^2 + cxyz$

Soln.:

The greatest common factor of the terms ax^2y , bxy^2 and $cxyz$ of the expression

$ax^2y + bxy^2 + cxyz$ is xy.

Also, we can write $ax^2y = (xy . ax)$, $bxy^2 = (xy . by)$, $cxyz = (xy . cz)$

Therefore, $ax^2y + bxy^2 + cxyz = (xy . ax) + (xy . by) + (xy . cz)$

$= xy (ax + by + cz)$

Exercise 7.3

Factorize each of the following algebraic expressions :

Q.1) $6x(2x - y) + 7y(2x - y)$

Soln.:

$6x(2x - y) + 7y(2x - y)$

$= (6x + 7y)(2x - y)$ (taking $(2x - y)$ as common factor)

Q.2) $2r(y - x) + s(x - y)$

Soln.:

$2r(y - x) + s(x - y)$

$= 2r(y - x) - s(y - x)$ [since, $(x - y) = -(y - x)$]

$= (2r - s)(y - x)$ [taking $(y - x)$ as the common factor]

Q.3) $7a(2x - 3) + 3b(2x - 3)$

Soln.:

$7a(2x - 3) + 3b(2x - 3)$

$= (7a + 3b)(2x - 3)$ [taking $(2x - 3)$ as the common factor]

Q.4) $9a(6a - 5b) - 12a^2(6a - 5b)$

Soln.:

$9a(6a - 5b) - 12a^2(6a - 5b)$

$= (9a - 12qa^2)(6a - 5b)$ [taking $(6a - 5b)$ as the common factor]

$= 3a(3 - 4a)(6a - 5b)$ [taking $3a$ as the common factor of the quadratic eqn. $(9a - 12a^2)$]

Q.5) $5(x - 2y)^2 + 3(x - 2y)$

Soln.:

$5(x - 2y)^2 + 3(x - 2y)$

$= [(x - 2y) + 3](x - 2y)$ [taking $(x - 2y)$ as the common factor]

$= (5x - 10y + 3)(x - 2y)$

Q.6) $16(2L - 3m)^2 - 12(3m - 2L)$

Soln.:

$16(2L - 3m)^2 - 12(3m - 2L)$

$= 16(2L - 3m)^2 + 12(2L - 3m)$ [(3m – 2L) = -(2L – 3m)]

$= [16(2L - 3m) + 12](2L - 3m)$ [taking (2L – 3m) as the common factor]

$= 4[4(2L - 3m) + 3](2L - 3m)$ [taking 4 as the common factor (16(2L – 3m) + 12)]

$= 4(8L - 12m + 3)(2L - 3m)$

Q.7) $3a(x - 2y) - b(x - 2y)$

Soln.:

$3a(x - 2y) - b(x - 2y)$

$= (3a - b)(x - 2y)$ [taking (x – 2y) as the common factor]

Q.8) $a^2(x + y) + b^2(x + y) + c^2(x + y)$

Soln.:

$a^2(x + y) + b^2(x + y) + c^2(x + y)$

$= (a^2 + b^2 + c^2)(x + y)$ [taking (x +y) as the common the factor]

Q.9) $(x - y)^2 + (x - y)$

Soln.:

$(x - y)^2 + (x - y)$

$= (x - y)(x - y) + (x - y)$ [taking (x – y) as the common factor]

$= (x - y + 1)(x - y)$

Q.10) $6(a + 2b) - 4(a + 2b)^2$

Soln.:

$6(a + 2b) - 4(a + 2b)^2$

$= [6 - 4(a + 2b)](a + 2b)$ [taking (a + 2b as the common factor)]

$= 2[3 - 2(a + 2b)](a + 2b)$ [taking 2 as the common factor of [6 – 4(a + 2b)]]

$= 2(3 - 2a - 4b)(a + 2b)$

Q.11) $a(x - y) + 2b(y - x) + c(x - y)^2$

Soln.:

$a(x - y) + 2b(y - x) + c(x - y)^2$

$= a(x - y) - 2b(x - y) + c(x - y)^2$ [$(y - x) = -(x - y)$]

$= [a - 2b + c(x - y)](x - y)$

$= (a - 2b + cx - cy)(x - y)$

Q.12) $-4(x - 2y)^2 + 8(x - 2y)$

Soln.:

$-4(x - 2y)^2 + 8(x - 2y)$

$= [-4(x - 2y) + 8](x - 2y)$ [taking $(x - 2y)$ as the common factor]

$= 4[-(x - 2y) + 2](x - 2y)$ [taking 4 as the common factor of $[-4(x - 2y) + 8]$]

$= 4(2y - x + 2)(x - 2y)$

Q.13) $x^3(a - 2b) + x^2(a - 2b)$

Soln.:

$x^3(a - 2b) + x^2(a - 2b)$

$= (x^3 + x^2)(a - 2b)$ [taking $(a - 2b)$ as the common factor]

$= x^2(x + 1)(a - 2b)$ [taking x^2 as the common factor of $(x^3 + x^2)$]

Q.14) $(2x - 3y)(a + b) + (3x - 2y)(a + b)$

Soln.:

$(2x - 3y)(a + b) + (3x - 2y)(a + b)$

$= (2x - 3y + 3x - 2y)(a + b)$ [taking $(a + b)$ as the common factor]

$= (5x - 5y)(a + b)$

$= 5(x - y)(a + b)$ [taking 5 as the common factor of $(5x - 5y)$]

Q.15) $4(x + y)(3a - b) + 6(x + y)(2b - 3a)$

Soln.:

$4(x + y)(3a - b) + 6(x + y)(2b - 3a)$

$= 2(x + y)[2(3a - b) + 3(2b - 3a)]$ [taking $(2(x + y))$ as the common factor]

$= 2(x + y)(6a - 2b + 6b - 9a)$

$= 2(x + y)(4b - 3a)$

Exercise 7.4

Factorize each of the following expressions :

Q.1) qr – pr + qs – ps

Soln.:

qr – pr + qs – ps

= (qr – pr) + (qs – ps)

= r(q – p) + s(q – p)

= (r + s) (q – p)   [taking (q – p) as the common factor]

Q.2) $p^2q – pr^2 – pq + r^2$

Soln.:

$p^2q – pr^2 – pq + r^2$

= $(p^2q – pq) + (r^2 – pr^2)$

$pq(p – 1) + r^2(1 – p)$

$pq(p – 1) – r^2(p – 1)$ [since, (1 – p) = -(p – 1)]

= $(pq – r^2)(p – 1)$ [taking (p – 1) as the common factor]

Q.3) $1 + x + xy + x^2y$

Soln.:

$1 + x + xy + x^2y$

= $(1 + x) + (xy + x^2y)$

=(1 + x) + xy(1 + x)

= (1 + xy)(1 +x)   [taking (1 +x) as the common factor]

Q.4) ax + ay – bx – by

Soln.:

ax + ay – bx – by

$= (ax + ay) – (bx + by)$

$= a(x + y) – b(x + y)$

$= (a – b)(x + y)$ [taking $(x + y)$ as the common factor]

Q.5) $xa^2 + xb^2 – ya^2 – yb^2$

Soln.:

$xa^2 + xb^2 – ya^2 – yb^2$

$= (xa^2 + xb^2) – (ya^2 + yb^2)$

$= x(a^2 + b^2) – y(a^2 + b^2)$

$= (x – y)(a^2 + b^2)$ [taking $(a^2 + b^2)$ as the common factor]

Q.6) $x^2 + xy +xz + yz$

Soln.:

$x^2 + xy +xz + yz$

$= (x^2 + xy) + (xz + yz)$

$= x(x + y) + z(x + y)$

$= (x + z)(x + y)$ [taking $(x + y)$ s the common factor]

$= (x +y)(x +z)$

Q.7) $2ax + bx + 2ay + by$

Soln.:

$2ax + bx + 2ay + by$

$= (2ax + bx) + (2ay + by)$

$= x(2a + b) + y(2a + b)$

$= (x +y)(2a + b)$ [taking $(2a +b)$ as the common factor]

Q.8) $ab – by – ay + y^2$

Soln.:

$ab – by – ay + y^2$

$= (ab – ay) + (y^2 – by)$

$= a(b – y) + y(y – b)$ [since, $(y – b) = – (b – y)$]

$= a(b – y) – y(b – y)$ [taking $(b – y)$ as the common factor]

$= (a – y)(b – y)$

Q.9) axy + bcxy – az – bcz

Soln.:

$axy + bcxy – az – bcz$

$= (axy + bcxy) – (az – bcz)$

$= xy(a + bc) – z(a + bc)$

$= (xy – z)(a + bc)$ [taking $(a + bc)$ as the common factor]

Q.10) $Lm^2 – mn^2 – Lm + n^2$

Soln.:

$Lm^2 – mn^2 – Lm + n^2 = (Lm^2 – Lm) + (n^2 – mn^2)$

$= Lm(m – 1) + n^2(1 – m)$

$= Lm(m – 1) – n^2(m – 1)$ [since, $(1 – m) = -(m – 1)$]

$= (Lm – n^2)(m – 1)$ [taking $(m – 1)$ a sthe common factor]

Q.11) $x^3 – y^2 + x – x^2y^2$

Soln.:

$x^3 – y^2 + x – x^2y^2$

$= (x^3 + x) – (x^2y^2 + y^2)$

$= x(x^2 + 1) – y^2(x^2 + 1)$

$= (x – y^2)(x^2 + 1)$ [taking $(x^2 + 1)$ as the common factor]

Q.12) 6xy + 6 – 9y – 4x

Soln.:

$6xy + 6 – 9y – 4x = (6xy – 4x) + (6 – 9y)$

$= 2x(3y - 2) + 3(2 - 3y)$

$= 2x(3y - 2) - 3(3y - 2)$ [since, $(2 - 3y) = -(3y - 2)$]

$= (2x - 3)(3y - 2)$ [taking $(3y - 2)$ as the common factor]

Q.13) $x^2 - 2ax - 2ab + bx$

Soln.:

$x^2 - 2ax - 2ab + bx$

$= (x^2 - 2ax) + (bx - 2ab)$

$= x(x - 2a) + b(x - 2a)$

$= (x + b)(x - 2a)$ [taking $(x - 2a)$ as the common factor]

$= (x - 2a)(x + b)$

Q.14) $x^3 - 2x^2y + 3xy^2 - 6y^3$

Soln.:

$x^3 - 2x^2y + 3xy^2 - 6y^3$

$= (x^3 - 2x^2y) + (3xy^2 - 6y^3)$

$= x^2(x - 2y) + 3y^2(x - 2y)$

$= (x^2 + 3y^2)(x - 2y)$ [taking $(x - 2y)$ as the common factor]

Q.15) $abx^2 + (ay - b)x - y$

Soln.:

$abx^2 + (ay - b)x - y = abx^2 + axy - bx - y$

$= (abx^2 - bx) + (axy - y)$

$= bx(ax - 1) + y(ax - 1)$

$= (bx + y)(ax - 1)$ [taking $(ax - 1)$ as the common factor]

Q.16) $(ax + by)^2 + (bx - ay)^2$

Soln.:

$(ax + by)^2 + (bx - ay)^2 = a^2x^2 + 2abxy + b^2y^2 + b^2x^2 - 2abxy + a^2y^2$

$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$

$= (a^2x^2 + a^2y^2) + (b^2x^2 + b^2y^2)$

$= a^2(x^2 + y^2) + b^2(x^2 + y^2)$

$= (a^2 + b^2)(x^2 + y^2)$ [taking $(x^2 + y^2)$ as the common factor]

Q.17) $16(a – b)^3 – 24(a – b)^2$

Soln.:

$16(a – b)^3 – 24(a – b)^2$

$= 8(a – b)^2 [2(a - b) - 3]$ [taking $8(a - b)^2$ as the common factor]

$= 8(a – b)^2(2a – 2b – 3)$

Q.18) $ab(x^2 + 1) + x(a^2 + b^2)$

Soln.:

$ab(x^2 + 1) + x(a^2 + b^2) = abx^2 + ab + a^2x + b^2x$

$= (abx^2 + a^2x) + (b^2x + ab)$

$= ax(bx + a) + b(bx + a)$

$= (ax + b)(bx + a)$ [taking $(bx + a)$ as the common factor]

Q.19) $a^2x^2 + (ax^2 + 1)x + 1 + a$

Soln.:

$a^2x^2 + (ax^2 + 1)x + 1 + a = a^2x^2 + ax^3 + x + a$

$= (ax^3 + a^2x^2) + (x + a)$

$= ax^2(x + a) + (x + a)$

$= (ax^2 + 1)(x + a)$ [taking $(x + a)$ as the common factor]

Q.20) $a(a – 2b – c) + 2bc$

Soln.:

$a(a – 2b – c) + 2bc = a^2 – 2ab – ac + 2bc$

$= (a^2 – ac) + (2bc – 2ab)$

$= a(a – c) + 2b(c – a)$ [since, $(c - a) = -(a – c)$]

$= a(a – c) – 2b(a - c)$

$= (a - 2b)(a - c)$ [taking $(a - c)$ as the common factor]

Q.21) $a(a + b - c) - bc$

Soln.:

$a(a + b - c) - bc = a^2 + ab - ac - bc$

$= (a^2 - ac) + (ab - bc)$

$= a(a - c) + b(a - c)$

$= (a + b)(a - c)$ [taking $(a - c)$ as the common factor]

Q.22) $x^2 - 11xy - x + 11y$

Soln.:

$x^2 - 11xy - x + 11y = (x^2 - x) + (11y - 11xy)$

$= x(x - 1) + 11y(1 - x)$

$= x(x - 1) - 11y(x - 1)$ [since, $(1 - x) = -(x - 1)$]

$= (x - 11y)(x - 1)$ [taking out the common factor]

Q.23) $ab - a - b + 1$

Soln.:

$ab - a - b + 1 = (ab - b) + (1 - a)$

$= b(a - 1) + (1 - a)$

$= b(a - 1) - (a - 1)$ [since, $(1-a) = -(a - 1)$]

$= (a - 1)(b - 1)$ [taking out the common factor $(a - 1)$]

Q.24) $x^2 + y - xy - x$

Soln.:

$x^2 + y - xy - x = (x^2 - xy) + (y - x)$

$= x(x - y) + (y - x)$

$= x(x - y) - (x - y)$ [$(y - x) = -(x - y)$]

$= (x - 1)(x - y)$ [taking $(x - y)$ as the common factor]

Exercise 7.5

Q.1) $16x^2 - 25y^2$

Soln.:

$16x^2 - 25y^2$

$= (4x)^2 - (5y)^2$

$= (4x - 5y)(4x + 5y)$

Q.2) $27x^2 - 12y^2$

Soln.:

$27x^2 - 12y^2$

$= 3(9x^2 - 4y^2)$

$= 3[(3x)^2 - (2y)^2]$

$= 3(3x - 2y)(3x + 2y)$

Q.3) $144a^2 - 289b^2$

Soln.:

$144a^2 - 289b^2$

$= (12a)^2 - (17b)^2$

$= (12a - 17b)(12a + 17b)$

Q.4) $12m^2 - 27$

Soln.:

$12m^2 - 27$

$= 3(4m^2 - 9)$

$= 3[(2m)^2 - 3^2]$

$= 3(2m - 3)(2m + 3)$

Q.5) $125x^2 - 45y^2$

Soln.:

$125x^2 - 45y^2$

$= 5(25x^2 - 9y^2)$

$= 5[(5x)^2 - (3y)^2]$

$= 5(5x - 3y)(5x + 3y)$

Q.6) $144a^2 - 169b^2$

Soln.:

$144a^2 - 169b^2$

$= (12a)^2 - (13b^2)$

$= (12a - 13b)(12a + 13b)$

Q.7) $(2a - b)^2 - 16c^2$

Soln.:

$(2a - b)^2 - 16c^2$

$= (2a - b)^2 - (4c)^2$

$= [(2a - b) - 4c][(2a - b) + 4c]$

$= (2a - b - 4c)(2a - b + 4c)$

Q.8) $(x + 2y)^2 - 4(2x - y)^2$

Soln.:

$(x + 2y)^2 - 4(2x - y)^2 = (x + 2y)^2 - [2(2x - y)]^2$

$= [(x + 2y) - 2(2x - y)][(x + 2y) + 2(2x - y)]$

$= (x + 2y - 4x + 2y)(x + 2y + 4x - 2y)$

$= 5x(4y - 3x)$

Q.9) $3a^5 - 48a^3$

Soln.:

$3a^5 – 48a^3$

$= 3a^3(a^2 – 16)$

$= 3a^3(a^2 – 4^2)$

$= 3a^3(a -4)(a + 4)$

Q.10) $a^4 – 16b^4$

Soln.:

$a^4 – 16b^4$

$= a^4 – 2^4b^4$

$= (a^2)^2 – (2^2b^2)^2$

$= (a^2 – 2^2b^2)(a^2 + 2^2b^2)$

$= [a^2 – (2b)^2](a^2 + 4b^2)$

$(a – 2b)(a + 2b)(a^2 + 4b^2)$

Q.11) $x^8 – 1$

Soln.:

$x^8 – 1 = (x^4)^2 – 1^2$

$= (x^4 – 1)(x^4 + 1)$

$= [(x^2)^2 – 1^2](x^4 + 1)$

$= (x^2 – 1)(x^2 + 1)(x^4 + 1)$

$= (x^2 – 1^2)(x^2 + 1)(x^4 + 1)$

$=(x – 1)(x +1)(x^2 + 1)(x^4 + 1)$

Q.12) $64 – (a + 1)^2$

Soln.:

$64 – (a + 1)^2$

$= (8)^2 – (a +1)^2$

$= [8 – (a + 1)][8 + (a + 1)]$

$= (8 – a – 1)(8 + a + 1)$

$= (7 –a)(9 + a)$

Q.13) $36L^2 – (m + n)^2$

Soln.:

$36L^2 – (m + n)^2$

$= (6L)^2 – (m + n)^2$

$= [6L – (m +n)][6L + (m +n)]$

$= (6L – m – n)(6L + m + n)$

Q.14) $25x^4y^4 – 1$

Soln.:

$25x^4y^4 – 1$

$= (5x^2y^2)^2 – 1$

$= (5x^2y^2 – 1)(5x^2y^2 +1)$

Q.15) $a^4 – 1/b^4$

Soln.:

$a^4 – 1/b^4$

$= (a^2)^2 – 1/(b^2)^2$

$= a^2 – 1/b^2a^2 + 1/b^2$

$= a – 1/ba + 1/ba^2 + 1/b^2$

Q.16) $x^3 – 144x$

Soln.:

$x^3 – 144x$

$= x(x^2 – 144)$

$= x(x^2 – 12^2)$

$= x(x – 12)(x + 12)$

Q.17) $(x - 4y)^2 - 625$

Soln.:

$(x - 4y)^2 - 625$

$= (x - 4y)^2 - 25^2$

$= [(x - 4y) - 25][(x - 4y) + 25]$

$= (x - 4y - 25)(x - 4y + 25)$

Q.18) $9(a - b)^2 - 100(x - y)^2$

Soln.:

$9(a - b)^2 - 100(x - y)^2$

$= [3(a - b)]^2 - [10(x - y)]^2$

$= [3(a - b) - 10(x - y)][3(a - b) + 10(x - y)]$

$= (3a - 3b - 10x + 10y)(3a - 3b + 10x - 10y)$

Q.19) $(3 + 2a)^2 - 25a^2$

Soln.:

$(3 + 2a)^2 - 25a^2$

$= (3 + 2a)^2 - (5a)^2$

$= [(3 + 2a) - 5a][(3 + 2a) + 5a]$

$= (3 + 2a - 5a)(3 + 2a + 5a)$

$= (3 - 3a)(3 + 7a)$

$= 3(1 - a)(3 + 7a)$

Q.20) $(x + y)^2 - (a - b)^2$

Soln.:

$(x + y)^2 - (a - b)^2$

$= [(x + y) - (a - b)][(x + y) + (a - b)]$

$= (x + y - a + b)(x + y + a - b)$

Q.21) 116 **x^2y^2 – 449 y^2z^2**
Soln.:

116 x^2y^2 – 449 y^2z^2
= y^2(116 x^2 – 449 z^2)
= y^2[(14X)2 – (27Z)2]
= y^2 (14X–27Z)(14X+27Z)
= y^2(x4 – 27Z)(x4 + 27Z)

Q.22) $75a^3b^2$ – $108ab^4$

Soln.:

$75a^3b^2 - 108ab^4$

$= 3ab^2(25a^2 - 36b^2)$

$= 3ab^2 [(5a)^2 - (6b)^2]$

$= 3ab^2(5a - 6b)(5a + 6b)$

Q.23) $x^5 - 16x^3$

Soln.:

$x^5 - 16x^3$

$= x^3(x^2 - 16)$

$= x^3(x^2 - 4^2)$

$= x^3(x - 4)(x + 4)$

Q.24) 50(x)2 – 2x281
Soln.:

50(x)2 – 2x281
= 2(25(x)2 – x281)
= 2{(5x)2 – (x9)2}
= 2(5x – x9)(5x + x9)

Q.25) $256x^5 - 81x$

Soln.:

$256x^5 - 81x$

$= x(256x^4 - 81)$

$= x[(16x^2)^2 - 9^2]$

$= x(16x^2 + 9)(16x^2 - 9)$

$= x(16x^2 + 9)[(4x)^2 - 3^2]$

$= x(16x^2 + 9)(4x + 3)(4x - 3)$

Q.26) $a^4 - (2b + c)^4$

Soln.:

$a^4 - (2b + c)^4$

$= (a^2)^2 - [(2b + c)^2]^2$

$= [a^2 + (2b + c)^2][a^2 - (2b + c)^2]$

$= [a^2 + (2b + c)^2]\{[a + (2b + c)][a - (2b + c)]\}$

$= [a^2 + (2b + c)^2](a + 2b + c)(a - 2b - c)$

Q.27) $(3x + 4y)^4 - x^4$

Soln.:

$(3x + 4y)^4 - x^4$

$= [(3x + 4y)^2]^2 - (x^2)^2$

$= [(3x + 4y)^2 + x^2][(3x + 4y)^2 - x^2]$

$= [(3x + 4y)^2 + x^2][(3x + 4y) + x]\ [(3x + 4y) - x]$

$= \{(3x + 4y)^2 + x^2\}\ (3x + 4y + x)(3x + 4y - x)$

$= \{(3x + 4y)^2 + x^2\}\ (4x + 4y)(2x + 4y)$

$= \{(3x + 4y)^2 + x^2\}\ 4(x + y)\ 2\ (x + 2y)$

$= 8\{(3x + 4y)^2 + x^2\}\ (x + y)(x + 2y)$

Q.28) $p^2q^2 - p^4q^4$

Soln.:

$p^2q^2 - p^4q^4$

$= p^2q^2(1 - p^2q^2)$

$= p^2q^2 [1 – (pq)^2]$

$= p^2q^2 (1 – pq)(1 + pq)$

Q.29) $3x^3y – 243xy^3$

Soln.:

$3x^3y – 243xy^3$

$= 3xy(x^2 – 81y^2)$

$= 3xy[x^2 – (9y)^2]$

$= 3xy(x – 9y)(x + 9y)$

Q.30) $a^4b^4 – 16c^4$

Soln.:

$a^4b^4 – 16c^4$

$= [(a^2b^2)^2 – (4c^2)^2]$

$= (a^2b^2 + 4c^2)(a^2b^2 – 4c^2)$

$= (a^2b^2 + 4c^2)[(ab)^2 – (2c)^2]$

$= (a^2b^2 + 4c^2)(ab + 2c)(ab – 2c)$

Q.31) $x^4 – 625$

Soln.:

$x^4 – 625$

$= (x^2)^2 – 25^2$

$= (x^2 + 25)(x^2 – 25)$

$= (x^2+ 25)(x^2 – 5^2)$

$= (x^2 + 25)(x + 5)(x – 5)$

Q.32) $x^4 – 1$

Soln.:

$x^4 – 1$

$= (x^2)^2 – 1$

$= (x^2 + 1)(x^2 – 1)$

$= (x^2 + 1)(x + 1)(x – 1)$

Q.33) $49(a – b)^2 – 25(a + b)^2$

Soln.:

$49(a – b)^2 – 25(a + b)^2$

$= [7(a – b)^2] – [5(a + b)]^2$

$= [7(a – b) – 5(a + b)] [7(a – b) + 5(a + b)]$

$= (7a – 7b – 5a – 5b)(7a – 7b + 5a + 5b)$

$= (2a – 12b)(12a – 2b)$

$= 2(a – 6b) 2(6a – b)$

$= 4(a – 6b)(6a – b)$

Q.34) $x – y – x^2 + y^2$

Soln.:

$x – y – x^2 + y^2$

$= (x – y) + (y^2 – x^2)$

$= (x – y) + (y + x)(y – x)$

$= (x – y) – (y + x)(x – y)$ [since, $(y – x) = -(x – y)$]

$= (x – y)[1 – (y + x)]$

$= (x – y)(1 – x – y)$

Q.35) $16(2x – 1)^2 – 25y^2$

Soln.:

$16(2x – 1)^2 – 25y^2$

$= [4(2x – 1)]^2 – (5y)^2$

$= [4(2x – 1) – 5y][4(2x – 1) + 5y]$

$= (8x – 4 – 5y)(8x -4 + 5y)$

$= (8x – 5y – 4)(8x + 5y – 4)$

Q.36) $4(xy + 1)^2 – 9(x – 1)^2$

Soln.:

$4(xy + 1)^2 – 9(x – 1)^2$

$= [2(xy + 1)]^2 – [3(x – 1)]^2$

$= [2(xy + 1) – 3(x – 1)] [2(xy + 1) + 3(x – 1)]$

$= (2xy + 2 – 3x + 3)(2xy + 2 + 3x – 3)$

$= (2xy – 3x + 5)(2xy + 3x – 1)$

Q.37) $(2x + 1)^2 – 9x^4$

Soln.:

$(2x + 1)^2 – 9x^4$

$= (2x + 1)^2 – (3x^2)^2$

$= [(2x + 1) – 3x^2][(2x +1) + 3x^2]$

$= (-3x^2 + 2x + 1)(3x^2 + 2x + 1)$

$= (-3x^2 + 3x – x + 1)(3x^2 + 2x + 1)$

$= \{3x(-x + 1) + 1(-x + 1)\}(3x^2 + 2x + 1)$

$= (-x + 1)(3x + 1)(3x^2 + 2x + 1)$

$= -(x – 1)(3x + 1)(3x^2 + 2x + 1)$

Q.38) $x^4 – (2y – 3z)^2$

Soln.:

$x^4 – (2y – 3z)^2$

$= (x^2)^2 – (2y – 3z)^2$

$= [x^2 – (2y – 3z)][x^2 + (2y – 3z)]$

$= (x^2 – 2y + 3z)(x^2 + 2y – 3z)$

Q.39) $a^2 – b^2 + a –b$

Soln.:

$a^2 - b^2 + a - b = (a^2 - b^2) + (a - b)$

$= (a + b)(a - b) + (a - b)$

$= (a - b)(a + b + 1)$

Q.40) $16a^4 - b^4$

Soln.:

$16a^4 - b^4$

$= (4a^2)^2 - (b^2)^2$

$= (4a^2 + b^2)(4a^2 - b^2)$

$= (4a^2 + b^2)[(2a)^2 - b^2]$

$= (4a^2 + b^2)(2a + b)(2a - b)$

Q.41) $a^4 - 16(b - c)^4$

Soln.:

$a^4 - 16(b - c)^4$

$= (a^2)^2 - [4(b - c)^2]^2$

$= [a^2 + 4(b - c)^2][a^2 - 4(b - c)^2]$

$= [a^2 + 4(b - c)^2]\ [a^2 - [2(b - c)]^2]$

$= [a^2 + 4(b - c)^2][a + 2(b - c)][a - 2(b - c)]$

$= [a^2 + 4(b - c)^2](a + 2b - 2c)(a - 2b + 2c)$

Q.42) $2a^5 - 32a$

Soln.:

$2a^5 - 32a$

$= 2a(a^4 - 16)$

$= 2a[(a^2)^2 - 4^2]$

$= 2a(a^2 + 4)(a^2 - 4)$

$= 2a(a^2 + 4)(a^2 - 2^2)$

$= 2a(a^2 + 4)(a + 2)(a - 2)$

$= 2a(a - 2)(a + 2)(a^2 + 4)$

Q.43) $a^4b^4 - 81c^4$

Soln.:

$a^4b^4 - 81c^4$

$= (a^2b^2)^2 - (9c^2)^2$

$= (a^2b^2 + 9c^2)(a^2b^2 - 9c^2)$

$= (a^2b^2 + 9c^2)[(ab)^2 - (3c)^2]$

$= (a^2b^2 + 9c^2)(ab + 3c)(ab - 3c)$

Q.44) $xy^9 - yx^9$

Soln.:

$xy^9 - yx^9$

$= xy(y^8 - x^8)$

$= xy[(y^4)^2 - (x^4)^2]$

$= xy(y^4 + x^4)[(y^2)^2 - (x^2)^2]$

$= xy(y^4 + x^4)(y^2 + x^2)(y^2 - x^2)$

$= xy(y^4 + x^4)(y^2 + x^2)(y + x)(y - x)$

Q.45) $x^3 - x$

Soln.:

$x^3 - x = x(x^2 - 1)$

$= x(x - 1)(x + 1)$

Q.46) $18a^2x^2 - 32$

Soln.:

$18a^2x^2 - 32$

$= 2(9a^2x^2 - 16)$

$= 2[(3ax)^2 - 4^2]$

$= 2(3ax - 4)(3ax + 4)$

Exercise 7.6

Solve:

Q1. $4x^2 + 12xy + 9y^2$

Soln.

$= (2x)^2 + 2 \times 2x \times 3y + (3y)^2$

$= (2x + 3y)^2$

$= (2x + 3y)(2x + 3y)$

Q2. $9a^2 - 24ab + 16b^2$

Soln.

$9a^2 - 24ab + 16b^2$

$= (3a)^2 - 2 \times 3a \times 4b + (4b)^2$

$= (3a - 4b)^2$

$= (3a - 4b)(3a - 4b)$

Q3. $p^2q2 - 6qr + 9r^2 = (pq)^2 - 2 \times pq \times 3r + (3r)^2$

Soln.

$p^2q2 - 6qr + 9r^2 = (pq)^2 - 2 \times pq \times 3r + (3r)^2$

$= (pq - 3r)^2$

$= (pq - 3r)(pq - 3r)$

Q4. $36a^2 + 36a + 9$

Soln.

$36a^2 + 36a + 9$

$= 9(4a^2 + 4a + 1) = 9\{(2a)^2 + 2 \times 2a \times 1 + 1^2\}$

$= 9(2a + 1)^2$

$= 9(2a + 1)(2a + 1)$

Q5. $a^2 + 2ab + b^2 — 16$

Soln.

$a^2 + 2ab + b^2 — 16$

$= a^2 + 2 x a x b + b^2 —16$

$=(a+b)^2 —4^2$

$= (a + b – 4) (a + b + 4)$

Q6. $9z^2 – x^2 + 4xy – 4y^2$

Soln.

$9z^2 – x^2 + 4xy – 4y^2$

$= 9z^2 – (x^2 – 4xy + 4y^2)$

$= 9z^2 – [x^2 – 2 \mathbf{x} x \mathbf{x} 2y+ (2y)^2]$

$= (3z)^2 – (x – 2y)^2$

$= [3z – (x – 2y)] [3z + (x – 2y)]$

$= (3z – x + 2y) (3x + x – 2y)$

$= (x – 2y + 3z) (-x + 2y + 3z)$

Q7. $9a^4 — 24a^2b^2 + 16b^4 — 256$

Soln.

$9a^4 — 24a^2b^2 + 16b^4 — 256$

$= (9a^4 — 24a^2b^2 + 16b^4) — 256$

$= [(3a^2)^2 — 2 x 3a^2 x 4b^2 + (4b^2)^2] —16^2$

$= (3a^2 —40^2)^2 — 16^2$

$= [(3a^2 – 4b^2) -16] [(3a^2 – 4^2) + 16]$

$= (3a^2 – 4b^2 -16) (3a^2 – 4b^2 + 16)$

Q8. $16 – a^6 + 4a^3b^3 – 4b^6$

Soln.

$16 – a^6 + 4a^3b^3 – 4b^6$

$= 16 – (a^6 – 4a^3b^3 + 4b^6)$

$= 4^2 – [(a^3)^2 – 2 \text{ x } a^3 \text{ x } 2b^3 + (2b^3)^2]$

$= 4^2 – (a^3 – 2b^3)^2$

$= [4 – (a^3 – 2b^3)] [4 + (a^3 – 2b^3)]$

$= (4 – a^3 – 2b^3) (4 + a^3 – 2b^3)$

$= (a^3 – 2b^3 + 4) (– a^3 – 2b^3 + 4)$

Q9. $a^2 – 2ab + b^2 – c^2$

Soln.

$a^2 – 2ab + b^2 – c^2$

$= (a^2 – 2ab + b^2) – c^2$

$= (a^2 – 2 \text{ x } a \text{ x } b + b^2) – c^2$

$= (a – b)^2 – c^2$

$= [(a – b) – c][(a – b) + c]$

$= (a – b – c) \ (a – b + c)$

Q10. $X^2 + 2X + 1 – 9Y^2$

Soln.

$X^2 + 2X + 1 – 9Y^2$

$= (X^2 + 2X + 1) – 9Y^2$

$= (X^2 + 2 \text{ x } X \text{ x } 1 + 1) – 9Y^2$

$= (X + 1)^2 – (3Y)^2$

$= [(X + 1) – 3Y] [(X + 1) -3Y]$

$= (X + 1 – 3Y) (X + 1 +3Y)$

$= (X + 3Y + 1) (X – 3Y + 1)$

Q11. $a^2 + 4ab + 3b^2$

Soln.

$a^2 + 4ab + 3b^2$

$= a^2 + 4ab + 4b^2 – b^2$

$= [a^2 + 2 \times a \times 2b + (2b)^2] – b^2$

$= (a + 2b)^2 – b^2$

$= [(a + 2b) – b] [(a + 2b) + b]$

$= (a + 2b – b)(a + 2b + b)$

$= (a + b) (a + 3b)$

Q12. $96 – 4x – x^2$

Soln:

$96 – 4x – x^2$

$= 100 – 4 – 4x – x^2$

$= 100 – (x^2 + 4x + 4)$

$= 100 – (x^2 + 2 \times x \times 2 + 2^2)$

$= 10^2 – (x + 2)^2$

$= [10 – (x + 2)] [10 + (x + 2)]$

$= (10 – x – 2)(10 + x + 2)$

$= (8 – x)(12 + x)$

$= (x + 12)(-x + 8)$

Q13. $a^4 + 3a^2 + 4$

Soln.

$a^4 + 3a^2 + 4$

$= a^4 + 4a^2 – a^2 + 4$

$= (a^4 + 4a^2 + 4) – a^2$

$= [(a^2)^2 + 2 \times a^2 \times 2 + 2^2] – a^2$

$= (a^2 + 2)^2 – a^2$

$= [(a^2 + 2) – a][(a^2 + 2) + a]$

$= (a^2 – a + 2)(a^2 + a + 2)$

Q14. $4x^4 + 1$

Soln.

$4x^4 + 1$

$= 4x^4 + 4x^2 + 1 – 4x^2$

$= [(2x^2)^2 + 2 \text{ x } 2x^2 \text{ x } 1 + 1] – 4x^2$

$= (2x^2 + 1)^2 – (2x)^2$

$= [(2x^2 + 1) – 2x] [(2x^2 + 1) + 2x]$

$= (2x^2 – 2x + 1)(2x^2 + 2x + 1)$

Q15. $4x^4 + y^4$

Soln.

$4x^4 + y^4$

$= 4x^4 + 4x^2 + y^4 – 4x^2y^2$

$= [(2x^2)^2 + 2 \text{ x } 2x^2 \text{ x } y + (y^2)^2] – (2xy)^2$

$= (2x^2 + y^2)^2 – (2xy)^2$

$= [(2x^2 + y^2) – 2xy] [(2x^2 + y^2) + 2xy]$

$= (2x^2 – 2xy + y^2)(2x^2 + 2xy + y^2)$

Q16. $(x + 2)^2 – 6(x + 2) + 9$

Soln.

$(x + 2)^2 – 6(x + 2) + 9$

$= (x + 2)^2 – 2 \text{ x } (x + 2) \text{ x } 3 + 3^2$

$= [(x + 2) – 3]^2$

$= (x + 2 – 3)^2$

$= (x – 1)^2$

$= (x – 1)(x – 1)$

Q17. $25 – p^2 – q^2 – 2pq$

Soln.

$25 – p^2 – q^2 – 2pq$

$= 25 – (p^2 + 2pq + q^2)$

$= 5^2 – (p^2 + 2 \text{ x } p \text{ x } q + q^2)$

$= 5^2 – (p + q)^2$

$= [5 – (p + q)] [5 + (p + q)]$

$= (5 – p + q) (5 + p + q)$

$= -(p + q – 5)(p + q + 5)$

Q18. $x^2 + 9y^2 – 6xy – 25a^2$

Soln.

$x^2 + 9y^2 – 6xy – 25a^2$

$=(x^2 – 6xy + 9y^2) – 25a^2$

$= [x^2 – 2 \times x \times 3y + (3y)^2] – 25a^2$

$= (x – 3y)^2 – (5a)^2$

$= [(x – 3y) – 5a][(x -3y) + 5a]$

$= (x – 3y – 5a)(x – 3y + 5a)$

Q19. $49 – a^2 + 8ab – 16b^2$

Soln.

$49 – a^2 + 8ab – 16b^2$

$= 49 – (a^2 – 8ab + 16b^2)$

$= 49 – [a^2 – 2 x a x 4b + (4b^2)]$

$= 7^2 – (a – 4b^2)$

$= [7 – (a – 4b)][7 + (a – 4b)]$

$= (7 – a + 4b)(7 + a – 4b)$

$= – (a – 4b – 7)(a – 4b + 7)$

$= – (a – 4b + 7)(a – 4b – 7)$

Q20. $a^2 – 8ab + 16b^2 – 25c^2$

Soln.

$a^2 – 8ab + 16b^2 – 25c^2$

$= (a^2 – 8ab + 16b^2) – 25c^2$

$= [a^2 – 2 x a x 4b + (4b)^2] – 25c^2$

$= (a - 4b)^2 - (5c)^2$

$= [(a - 4b) - 5c]\ [(a - 4b)^2 + 5c]$

$= (a - 4b - 5c)\ (a - 4b + 5c)$

Q21. $x^2 - y^2 + 6y - 9$

Soln.

$x^2 - y^2 + 6y - 9$

$= x^2 - (y^2 + 6y - 9)$

$= x^2 - (y^2 - 2 \text{ x } y \text{ x } 3 + 3^2)$

$= x^2 - (y - 3)^2$

$= [x - (y - 3)]\ [x + (y - 3)]$

$= (x - y + 3)(x + y - 3)$

Q22. $25x^2 - 10x + 1 - 36y^2$

Soln.

$25x^2 - 10x + 1 - 36y^2$

$= (25x^2 - 10x + 1) - 36y^2$

$= [(5x)^2 - 2 \textbf{ x } 5x \textbf{ x } 1 + 1] - 36y^2$

$= (5x - 1)^2 - (6y)^2$

$= [(5x - 1) - 6y]\ [(5x - 1) + 6y]$

$= (5x - 1 - 6y)(5x - 1 + 6y)$

$= (5x - 6y - 1)(5x + 6y - 1)$

Q23. $a^2 - b^2 + 2bc - c^2$

Soln.

$a^2 - b^2 + 2bc - c^2$

$= a^2 - (b^2 - 2bc + c^2)$

$= a^2 - (b^2 - 2 \text{ x } b \text{ x } c + c^2)$

$= a^2 - (b - c)^2$

$= [a - (b - c)][a + (b - c)]$

$= (a - b + c)(a + b - c)$

Q24. $a^2 + 2ab + b^2 - c^2$

Soln.

$a^2 + 2ab + b^2 - c^2$

$= (a^2 + 2ab + b^2) - c^2$

$= (a^2 + 2 \text{ x } a \text{ x } b + b^2) - c^2$

$= (a + b)^2 - c^2$

$= [(a + b) - c] [(a + b) + c]$

$= (a + b - c) (a + b + c)$

Q25. $49 - x^2 - y^2 + 2xy$

Soln.

$49 - x^2 - y^2 + 2xy$

$= 49 - (x^2 + 2xy - y^2)$

$= 7^2 - (x - y)^2$

$= [7 - (x - y)] [7 + (x - y)]$

$= (7 - x + y)(7 + x - y)$

$= (x - y + 7)(y - x + 7)$

Q26. $a^2 + 4b^2 - 4ab - 4c^2$

Soln.

$a^2 + 4b^2 - 4ab - 4c^2$

$= (a^2 + 4b^2 - 4ab) - 4c^2$

$= [a^2 - 2 \text{ x } a \text{ x } 2b + (2b)^2] - 4c^2$

$= (a - 2b)^2 - (2c)^2$

$= [(a - 2b) - 2c] [(a - 2b) + 2c]$

$= (a - 2b - 2c)(a - 2b + 2c)$

Q27. $x^2 - y^2 - 4xz + 4z^2$

Soln.

$x^2 - y^2 - 4xz + 4z^2$

$= (x^2 - 4xz + 4z^2) - y^2$

$= (x - 2z)^2 - y^2$

$= [(x - 2z) - y] [(x - 2z) + y]$

$= (x - 2z - y)(x - 2z + y)$

$= (x + y - 2z)(x - y - 2z)$

Exercise 7.7

Q1. $x^2 + 12x – 45$

Soln:

To factories $x^2 + 12x – 45$, we will find two numbers p and q such that p + q = 12 and pq = -45.

Now,

15 + (-3) = 12

And

15 x (-3) = -45

Splitting the middle term 12x in the given quadratic as -3x + 15x, we get:

$x^2 + 12x – 45$

$= x^2 – 3x + 15x – 45$

$= (x^2 – 3x) + (15x – 45)$

$= x(x – 3) + 15(x – 3)$

$= (x – 3) (x + 15)$

Q2. $40 + 3x – x^2$

Soln:

We have:

$40 + 3x – x^2$

$= -(x^2 – 3x – 40)$

To factories $(x^2 – 3x – 40)$, we fill find two number p and q such p + q = -3 and pq = -40

Now,

5 + (-8) = -3

And

5 x (-8) = -40

Splitting the middle term -3x in the given quadratic as 5x – 8x, we get:

$40 + 3x – x^2 = -(x^2 – 3x – 40)$

$= -(x^2 + 5x – 8x – 40)$

$= -[(x^2 + 5x) – (8x + 40)]$

$= -[x(x + 5) - 8(x + 5)]$

$= -(x - 8)(x + 5)$

$= (x + 5)(-x + 8)$

Q3. $a^2 + 3a - 88$

Soln:

To factories $a^2 + 3a - 88$, we will find two numbers p and q such that $p + q = 3$ and $pq = -88$.

Now, $11 + (-8) = 3$

And $11 \times (-8) = -88$

Splitting the middle term 3a in the given quadratic as $11a - 8a$, we get:

$a^2 + 3a - 88 = a^2 + 11a - 8a - 88$

$= (a^2 + 11a) - (8a + 88)$

$= a(a + 11) - 8(a + 11)$

$= (a - 8)(a + 11)$

Q4. $a^2 - 14a - 51$

Soln:

To factories $a^2 - 14a - 51$, we will find two numbers p and q such that $p + q = -14$ and $pq = -51$

Now,

$3 + (-17) = -14$

and

$3 \times (-17) = -51$

Splitting the middle term -14a in the given quadratic as $3a - 17a$, we get:

$a^2 - 14a - 51 = a^2 + 3a - 17a - 51$

$= (a^2 + 3a) - (17a + 51)$

$= a(a + 3) - 17(a + 3)3$

$= (a - 17)(a + 3)$

Q5. $x^2 + 14x + 45$

Soln:

To factories $x^2 + 14x + 45$, we will find two numbers p and q such that p + q = 14 and pq = 45

Now,

9 + 5 = 14

And

9 x 5 = 45

Splitting the middle term 14x in the given quadratic as 9x + 5x, we get:

$x^2 + 14x + 45 = x^2 + 9x + 5x + 45$

$= (x^2 + 9x) + (5x + 45)$

$= x(x + 9) + 5(x + 9)$

$= (x + 5)(x + 9)$

Q6. $x^2 – 22x + 120$

Soln:

To factories $x^2 – 22x + 120$, we will find two numbers p and q such that p + q = -22 and pq = 120

Now, (-12) + (-10) = -22

And

(-12) x (-10) = 120

Splitting the middle term -22x in the given quadratic as -12x – 10x, we get:

$x^2 – 22x + 12 = x^2 – 12x – 10x + 120$

$= (x^2 – 12x) + (-10x + 120)$

$= x(x – 12) – 10(x – 12)$

$= (x – 10)(x – 12)$

Q7. $x^2 – 11x – 42$

Soln:

To factories $x^2 – 11x – 42$, we will find two numbers p and q such that p + q = -11 and pq = -42

Now,

3 + (-14) = -22

And

3 x (-14) = 42

Splitting the middle term -11x in the given quadratic as -14x + 3x, we get:

$x^2 - 11x - 42 = x^2 - 14x + 3x - 42$

$= (x^2 - 14x) + (3x - 42)$

$= x(x - 14) + 3(x - 14)$

$= (x - 14)(x + 3)$

Q8. $a^2 - 2a - 3$

Soln:

To factories $a^2 - 2a - 3$, we will find two numbers p and q such that p + q = 2 and pq = -3

Now,

3 + (-1) = 2

And

3 x (-1) = -3

Splitting the middle terms 2a in the given quadratic as –a + 3a, we get:

$a^2 + 2a - 3 = a^2 - a + 3a - 3$

$= (a^2 - a) + (3a - 3)$

$= a(a - 1) + 3(a - 1)$

Q9. $a^2 + 14a + 48$

Soln:

To factories $a^2 + 14a + 48$, we will find two numbers p and q such that p + q = 14 and pq = 48

Now,

8 + 6 = 14

And

8 x 6 = 48

Splitting the middle terms 14a in the given quadratic as 8a + 6a, we get:

$a^2 + 14a + 48 = a^2 + 8a + 6a + 48$

$= (a^2 + 8a) + (6a + 48)$

$= a(a + 8) + 6(a + 8)$

$= (a + 6)(a + 8)$

Q10. $x^2 – 4x – 21$

Soln:

To factories $x^2 – 4x – 21$, we will find two numbers p and q such that $p + q = -4$ and $pq = -21$

Now,

$3 + (-7) = -4$

And

$3 \times (-7) = -21$

Splitting the middle terms -4x in the given quadratic as -7x + 3x, we get:

$x^2 – 4x – 21 = x^2 – 7x + 3x – 21$

$= (x^2 – 7x) + (3x – 21)$

$= x(x – 7) + 3(x – 7)$

$= (x – 7) (x + 3)$

Q11. $y^2 + 5y – 36$

Soln:

To factories $y^2 + 5y – 36$, we will find two numbers p and q such that $p + q = 5$ and $pq = -36$

Now,

$9 + (-4) = 5$

And

$9 \times (-4) = -36$

Splitting the middle terms 5y in the given quadratic as -7y + 9y, we get:

$y^2 + 5y – 36 = y^2 – 4y + 9y – 36$

$= (y^2 – 4y) + (9y – 36)$

$= y(y – 4) + 9(y – 4)$

$= (y – 4)(y – 4)$

Q12. $(a^2 — 54)^2 — 36$

Soln:

$(a^2 — 54)^2 — 36$

$= (a^2 — 5a)^2 — 6^2$

$= [(a^2 — 5a) — 6] [(a^2 – 5a) + 6]$

$= (a^2 — 5a — 6) (a^2 — 5a + 6)$

In order to factories $a^2 — 5a — 6$, we will find two numbers p and q such that p+ q = —5 and pq = —6

Now,

(-6) + 1 = —5

and

(-6) x 1 = —6

Splitting the middle term – 5 in the given quadratic as – 6a + a, we get :

$a^2 — 5a — 6 = a^2 — 6a + a — 6$

$= (a^2 — 6a) + (a — 6)$

$= a(a — 6) + (a — 6)$

$= (a + 1) (a — 6)$

Now, In order to factories $a^2 — 5a + 6$, we will find two numbers p and q such that p + q = —5 and pq = 6

Clearly,

(-2) + (-3) = -5

and

(-2) x (-3) = 6

Splitting the middle term – 5 in the given quadratic as – 2a – 3a, we get :

$a^2 – 5a + 6 = a^2 – 2a – 3a + 6$

$= (a^2 – 2a) – (3a – 6)$

$= a (a – 2) – 3(a – 2)$
$= (a – 3) (a – 2)$

$\therefore$ $(a^2 – 5a – 6) (a^2 – 5a + 6)$

$= (a – 6) (a + 1) (a – 3) (a – 2)$

$= (a + 1) (a – 2) (a – 3) (a – 6)$

Q13. (a + 7)(a – 10) + 16

Soln:

(a + 7)(a – 10) + 16

$= a^2 – 10a + 7a – 70 + 16$

$= a^2 - 3a – 54$

To factories $a^2 – 3a – 54$, we will find two numbers p and q such that $p + q = -3$ and $pq = -54$

Now,

$6 + (-9) = -3$

And $6 \times (-9) = -54$

Splitting the middle term -3a in the given quadratic as – 9a + 6a, we get:

$a^2 – 3a – 54 = a^2 – 9a + 6a – 54$

$= (a^2 -9a) + (6a – 54)$

$= a(a – 9) + 6(a – 9)$

$= (a + 6)(a – 9)$

Exercise 7.8

Resolve each of the following quadratic equation trinomials into factors:

Q-1. $2x^2 + 5x + 3$

Solution. The given expression is $2x^2 + 5x + 3$.

(Co-efficient of $x^2 = 2$, co-efficient of $x = 5$ and the constant term = 3)

We will split the co-efficient of x into two parts such that their sum is 5 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $2 \times 3 = 6$.

Now,

$2 + 3 = 5$

And

$2 \times 3 = 6$

Replacing the middle term 5x by 2x + 3x, we have:

$2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$

$= (2x^2 + 2x) + (3x + 3)$

$= 2x (x + 1) + 3 (x + 1)$

$= (2x + 3)(x + 1)$

Q-2. $2x^2 – 3x – 2$

Solution.

The given expression is $2x^2 – 3x – 2$.

(Co-efficient of $x^2 = 2$, co-efficient of $x = -3$ and the constant term = -2)

We will split the co-efficient of x into two parts such that their sum is -3 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $2 \times (-2) = -4$

Now,

$(-4) + 1 = -3$

And

$(-4) \times 1 = -4$

Replacing the middle term 3x by -4x + x, we have:

$2x^2 – 3x – 2 = 2x^2 – 4x + x – 2$

$= (2x^2 – 4x) + (x – 2)$

$= 2x(x – 2) + 1(x- 2)$

$= (x – 2)(2x + 1)$

Q-3. $3x^2 + 10x + 3$

Solution.

The given expression is $3x^2 + 10x + 3$.

(Co-efficient of $x^2 = 3$, co-efficient of $x = 10$ and the constant term = 3)

We will split the co-efficient of x into two parts such that their sum is 10 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $3 \times 3 = 9$

Now,

$9 + 1 = 10$

And

$9 \times 1 = 9$

Replacing the middle term 10x by 9x + x, we have:

$3x^2 + 10x + 3 = 3x^2 + 9x + x + 3$

$= (3x^2 + 9x) + (x + 3)$

$= 3x(x + 3) + 1(x + 3)$

$= (x + 3)(3x + 1)$

Q-4. $7x – 6 – 2x^2$

Solution.

The given expression is $7x – 6 – 2x^2$.

(Co-efficient of $x^2 = -2$, co-efficient of $x = 7$ and the constant term = -6)

We will split the co-efficient of x into two parts such that their sum is 7 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $(-2) \times (-6) = 12$

Now,

$4 + 3 = 7$

And

$4 \times 3 = 12$

Replacing the middle term 7x by 4x + 3x, we have:

$7x – 6 – 2x^2 = -2x^2 + 4x + 3x – 6$

$= (-2x^2 + 4x) + (3x – 6)$

$= 2x(2 – x) – 3(2 – x)$

$= (2x – 3)(2 – x)$

Q-5. $7x^2 – 19x – 6$

Solution.

The given expression is $7x^2 – 19x – 6$.

(Co-efficient of $x^2 = 7$, co-efficient of x = -19 and the constant term = -6)

We will split the co-efficient of x into two parts such that their sum is -19 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $7 \times (-6) = 9$

Now,

$(-21) + 2 = -19$

And

$(-21) \times 2 = -42$

Replacing the middle term -19x by -21x + 2x, we have:

$7x^2 – 19x – 6 = 7x^2 – 21x + 2x – 6$

$= (7x^2 – 21x) + (2x – 6)$

$= 7x(x – 3) + 2(x – 3)$

$= (x – 3)(7x + 2)$

Q-6. $28 – 31x – 5x^2$

Solution.

The given expression is $28 – 31x – 5x^2$.

(Co-efficient of $x^2 = -5$, co-efficient of x = -31 and the constant term = 28)

We will split the co-efficient of x into two parts such that their sum is -31 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., (-5) × (28) = -140

Now,

(-35) + 4 = -31

And

(-35) × 4 = -140

Replacing the middle term -31x by -35x + 4x, we have:

$28 - 31x - 5x^2 = -5x^2 - 35x + 4x + 28$

$= (-5x^2 - 35x) + (4x + 28)$

$= -5x(x + 7) + 4(x + 7)$

$= (4 - 5x)(x + 7)$

Q-7. $3 + 23y - 8y^2$

Solution.

The given expression is $3 + 23y - 8y^2$.

(Co-efficient of y^2 = -8, co-efficient of y = 23 and the constant term = 3)

We will split the co-efficient of x into two parts such that their sum is 23 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., (-8) × 3 = -24

Now,

(-1) + 24 = 23

And

(-1) × 24 = -24

Replacing the middle term 23y by -y + 24y, we have:

$3 + 23y - 8y^2 = -8y^2 - y + 24y + 3$

$= (-8y^2 - y) + (24y + 3)$

$= -y(8y + 1) + 3(8y + 1)$

$= (8y + 1)(y + 3)$

Q-8. $11x^2 - 54x + 63$

Solution.

The given expression is $11x^2 – 54x + 63$.

(Co-efficient of $x^2 = 11$, co-efficient of $x = -54$ and the constant term = 63)

We will split the co-efficient of x into two parts such that their sum is -19 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $11 \times 63 = 693$

Now,

$(-33) + (-21)= -54$

And

$(-33) \times (-21) = 693$

Replacing the middle term -54x by -33x – 21x, we have:

$11x^2 – 54x + 63 = 11x^2 – 33x – 21x + 63$

$= (11x^2 – 33x) + (-21x + 63)$

$= 11x(x – 3) – 21(x – 3)$

$= (x – 3)(11x – 21)$

Q-9. $7x – 6x^2 + 20$

Solution.

The given expression is $7x – 6x^2 + 20$.

(Co-efficient of $x^2 = -6$, co-efficient of $x = 7$ and the constant term = 20)

We will split the co-efficient of x into two parts such that their sum is -19 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $(-6) \times 20 = -120$

Now,

$(15) + (-8)= 7$

And

$(15) \times (-8) = -120$

Replacing the middle term 7x by 15x – 8x, we have:

$7x – 6x^2 + 20 = -6x^2 + 15x – 8x + 20$

$= (-6x^2 + 15x) + (-8x + 20)$

$= 3x(-2x + 5) + 4(-2x + 5)$

$= (-2x + 5)(3x + 4)$

Q-10. $3x^2 + 22x + 35$

Solution.

The given expression is $3x^2 + 22x + 35$.

(Co-efficient of $x^2 = 3$, co-efficient of $x = 22$ and the constant term = 35)

We will split the co-efficient of x into two parts such that their sum is -19 and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $3 \times 35 = 105$

Now,

$(15) + (7)= 22$

And

$(15) \times (7) = 105$

Replacing the middle term 22x by 15x + 7x, we have:

$3x^2 + 22x + 35 = 3x^2 + 15x + 7x + 35$

$= (3x^2 + 15x) + (7x + 35)$

$= 3x(x + 5) + 7(x + 5)$

$= (x + 5)(3x + 7)$

Q-11. $12x^2 – 17xy + 6y^2$

Solution.

The given expression is $12x^2 – 17xy + 6y^2$.

(Co-efficient of $x^2 = 12$, co-efficient of $x = -17y$ and the constant term $= 6y^2$)

We will split the co-efficient of x into two parts such that their sum is -17y and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $12 \times 6y^2 = 72y^2$

Now,

$(-9y) + (-8y)= -17y$

And

$(-9y) \times (-8y) = 72y^2$

Replacing the middle term -17xy by -9xy – 8xy, we have:

$12x^2 – 17xy + 6y^2 = 12x^2 – 9xy – 8xy + 6y^2$

$= (12x^2 – 9xy) – (8xy + 6y^2)$

$= 3x(4x – 3y) -2y(4x – 3y)$

$= (4x – 3y)(3x – 2y)$

Q-12. $6x^2 – 5xy – 6y^2$

Solution. The given expression is $6x^2 – 5xy – 6y^2$.

(Co-efficient of $x^2 = 6$, co-efficient of $x = -5y$ and the constant term $= -6y^2$)

We will split the co-efficient of x into two parts such that their sum is -17y and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $6 \times (-6y^2) = -36y^2$

Now,

$(-9y) + (4y)= -5y$

And

$(-9y) \times (4y) = -36y^2$

Replacing the middle term -5xy by -9xy + 4xy, we have:

$6x^2 – 5xy – 6y^2 = 6x^2 – 9xy + 4xy – 6y^2$

$= (6x^2 – 9xy) + (4xy – 6y^2)$

$= 3x(2x – 3y) + 2y(2x – 3y)$

$= (2x – 3y)(3x + 2y)$

Q-13. $6x^2 – 13xy + 2y^2$

Solution.

The given expression is $6x^2 – 13xy + 2y^2$.

(Co-efficient of $x^2 = 6$, co-efficient of $x = -13y$ and the constant term $= 2y^2$)

We will split the co-efficient of x into two parts such that their sum is -13y and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $6 \times (2y^2) = 12y^2$

Now,

$(-12y) + (-y)= -13y$

And

$(-12y) \times (-y) = 12y^2$

Replacing the middle term -13xy by -12xy – xy, we have:

$6x^2 – 13xy + 2y^2 = 6x^2 – 12xy – xy + 2y^2$

$= (6x^2 – 12xy) – (xy – 2y^2)$

$= 6x(x – 2y) – y(x – 2y)$

$= (x – 2y)(6x – y)$

Q-14. $14x^2 + 11xy – 15y^2$

Solution.

The given expression is $14x^2 + 11xy – 15y^2$.

(Co-efficient of $x^2 = 14$, co-efficient of $x = 11y$ and the constant term $= -15y^2$)

We will split the co-efficient of x into two parts such that their sum is 11y and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $14 \times (-15y^2) = -210y^2$

Now,

$(21y) + (-10y)= 11y$

And

$(21y) \times (-10y) = -210y^2$

Replacing the middle term -11xy by -10xy + 21 xy, we have:

$14x^2 + 11xy – 15y^2 = 14x^2 – 10xy + 21xy – 15y^2$

$= (14x^2 – 10xy) + (21 xy – 15y^2)$

$= 2x(7x – 5y) + 3y(7x – 5y)$

$= (7x – 5y)(2x + 3y)$

Q-15. $6a^2 + 17ab – 3b^2$

Solution.

The given expression is $6a^2 + 17ab – 3b^2$.

(Co-efficient of $a^2 = 6$, co-efficient of $a = 17b$ and the constant term $= -3b^2$)

We will split the co-efficient of x into two parts such that their sum is 17b and their product equals to the product of the co-efficient of a^2 and the constant term, i.e., $6 \times (-3b^2) = -18b^2$

Now,

$(18b) + (-b)= 17b$

And

$(18b) \times (-b) = -18b^2$

Replacing the middle term 17ab by -ab + 18ab, we have:

$6a^2 + 17ab – 3b^2 = 6a^2 -ab + 18ab – 3b^2$

$= (6a^2 – ab) + (18ab – 3b^2)$

$= a(6a – b) + 3b(6a – b)$

$= (a + 3b)(6a – b)$

Q-16. $36a^2 + 12abc – 15b^2c^2$

Solution.

The given expression is $36a^2 + 12abc – 15b^2c^2$.

(Co-efficient of $a^2 = 36$, co-efficient of a = 12bc and the constant term = $-15b^2 c^2$)

We will split the co-efficient of x into two parts such that their sum is 17b and their product equals to the product of the co-efficient of a^2 and the constant term, i.e., $36 \times (-15b^2 c^2) = -540b^2c^2$

Now,

$(-18bc) + 30bc= 12bc$

And

$(-18bc) \times (30bc) = -540b^2 c^2$

Replacing the middle term 12abc by -18abc + 30abc, we have:

$36a^2 + 12abc – 15b^2c^2 = 36a^2 -18abc + 30abc – 15b^2c^2$

$= (36a^2 -18abc) + (30abc – 15b^2c^2)$

$= 18a(2a – bc) + 15bc(2a – bc)$

$= 3(6a + 5bc)(2a – bc)$

Q-17. $15x^2 – 16xyz – 15y^2z^2$

Solution.

The given expression is $15x^2 – 16xyz – 15y^2z^2$.

(Co-efficient of $x^2 = 15$, co-efficient of x = -16yz and the constant term = $-15y^2 z^2$)

We will split the co-efficient of x into two parts such that their sum is -16yz and their product equals to the product of the co-efficient of x^2 and the constant term, i.e., $15 \times (-15y^2 z^2) = -225y^2z^2$

Now,

$(-25yz) + 9yz = -16yx$

And

$(-25yz) \times (9yz) = -225y^2 z^2$

Replacing the middle term -16xyz by -25xyz + 9xyz, we have:

$15x^2 – 16xyz – 15y^2z^2 = 15x^2 – 25xyz + 9xyz – 15y^2z^2$

$= (15x^2 -25xyz) + (9xyz – 15y^2z^2)$

$= 5x(3x – 5yz) + 3yz(3x – 5yz)$

$= (3x – 5yz)(5x + 3yz)$

Q-18. $(x – 2y)^2 – 5(x – 2y) + 6$

Solution.

The given expression is $a^2 – 5a + 6$.

Assuming a = x – 2y, we have:

$(x – 2y)^2 – 5(x – 2y) + 6 = a^2 – 5a + 6$

(Co-efficient of a^2 = 1, co-efficient of a = -5 and the constant term = 6)

Now, we will split the co-efficient of a into two parts such that their sum is -5 and their product equals to the product of the co-efficient of a^2 and the constant term, i.e., $1 \times 6 = 6$.

Clearly,

$(-2) + (-3) = -5$

And,

$(-2) \times (-3) = 6$

Replacing the middle term -5a by -2a – 3a, we have:

$a^2 – 5a + 6 = a^2 – 2a – 3a + 6$

$= (a^2 – 2a) – (3a – 6)$

$= a(a – 2) – 3 (a – 2)$

$= (a – 2)(a – 3)$

Replacing a by (x – 2y), we get:

$(a – 3)(a – 2) = (x – 2y – 3)(x – 2y – 2)$

Q-19. $(2a - b)^2 + 2(2a - b) - 8$

Solution.

Assuming $x = 2a - b$, we have:

$(2a - b)^2 + 2(2a - b) - 8 = x^2 + 2x - 8$

The given expression becomes $x^2 + 2x - 8$

(Co-efficient of $x^2 = 1$ and that of $x = 2$; constant term $= -8$)

Now, we will split the co-efficient of x into two parts such that their sum is 2 and their product equals the product of the co-efficient of x^2 and the constant term, i.e., $1 \times (-8) = -8$

Clearly,

$(-2) + 4 = 2$

And,

$(-2) \times 4 = -8$

Replacing the middle term 2x by $-2x + 4x$, we get:

$x^2 + 2x - 8 = x^2 - 2x + 4x - 8$

$= (x^2 - 2x) + (4x - 8)$

$= x(x - 2) + 4 (x - 2)$

$= (x - 2)(x + 4)$

Replacing x by $2a - b$, we get:

$(x + 4)(x - 2) = (2a - b + 4)(2a - b - 2)$

Exercise 7.9

Solve: Q1. $p^2 + 6p + 8$

Soln:

$p^2 + 6p + 8$

$= p^2 + 6p + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 8$ $[Adding\ and\ subtracting\ \left(\frac{6}{2}\right)^2,\ that\ is\ 3^2]$

$= p^2 + 6p + 3^2 - 3^2 + 8$

$= p^2 + 2 \times p \times 3 + 3^2 - 9 + 8$

$= p^2 + 2 \times p \times 3 + 3^2 - 1$

$= (p + 3)^2 - 1^2$ [Completing the square]

$= [(p + 3) - 1][(p + 3) + 1]$

$= (p + 3 - 1)(p + 3 + 1)$

$= (p + 2)(p + 4)$

Q2. $q^2 - 10q + 21$

Soln:

$q^2 - 10q + 21$

$= q^2 - 10q + \left(\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2 + 21$ $[Adding\ and\ subtracting\ \left(\frac{10}{2}\right)^2,\ that\ is\ 5^2]$

$= q^2 - 2 \times q \times 5 + 5^2 - 5^2 + 21$

$= (q - 5)^2 - 4$ [Completing the square]

$= [(q - 5) - 2][(q - 5) + 2]$

$= (q - 5 - 2)(q - 5 + 2)$

$= (q - 7)(q - 3)$

Q3. $4y^2 + 12y + 5$

Soln:

$4y^2 + 12y + 5$

$4\left(y^2+3y+\frac{5}{4}\right)$ [*Making the co – efficient of* y^2]

$4\left[y^2+3y+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\frac{5}{4}\right]$ $\left[\textit{Adding and subtracting } \left(\frac{3}{2}\right)^2\right]$

$=4\left[\left(y+\frac{3}{2}\right)^2-\frac{9}{4}+\frac{5}{4}\right]$

$=4\left[\left(y+\frac{3}{2}\right)^2-1\right]$ [*Completing the square*]

$=4\left[\left(y+\frac{3}{2}-1\right)\right]\left[\left(y+\frac{3}{2}+1\right)\right]$

$=4\left(y+\frac{3}{2}-1\right)\left(y+\frac{3}{2}-1\right)$

$=4\left(y+\frac{3}{2}\right)\left(y+\frac{3}{2}\right)$

$=(2y+1)(2y+5)$

Q4. $p^2 + 6p - 16$

Soln:

$p^2 + 6p - 16$

$p^2+6p+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2-16$ $\left[\textit{Adding and subtracting } \left(\frac{6}{2}\right)^2\textit{, that is } 3^2\right]$

$= p^2 + 6p + 3^2 - 9 - 16$

$= (p + 3)^2 - 25$ [Completing the square]

$= (p + 3)^2 - 5^2$

$= [(p + 3) - 5] [(p + 3) + 5]$

$= (p + 3 - 5) (p + 3 + 5)$

$= (p - 2)(p + 8)$

Q5. $x^2 + 12x + 20$

Soln:

$x^2 + 12x + 20$

$= x^2 + 12x + \left(\frac{12}{2}\right)^2 - \left(\frac{12}{2}\right)^2 + 20 \quad \left[Adding\ and\ subtracting\ \left(\frac{12}{2}\right)^2,\ that\ is\ 6^2\right]$

$= x^2 + 12x + 6^2 - 6^2 + 20$

$= (x + 6)^2 - 16$ [completing the square]

$= (x + 6)^2 - 4^2$

$= [(x + 6) - 4]\ [(x + 6) + 4]$

$= (x + 6 - 4)\ (x + 6 + 4)$

$= (x + 2)\ (x + 10)$

Q6. $a^2 - 14a - 51$

Soln:

$a^2 - 14a - 51$

$= a^2 - 14a + \left(\frac{14}{2}\right)^2 - \left(\frac{14}{2}\right)^2 - 51 \quad \left[Adding\ and\ subtracting\ \left(\frac{14}{2}\right)^2,\ that\ is\ 7^2\right]$

$= a^2 - 14a + 7^2 - 7^2 - 51$

$= (a - 7)^2 - 100$ [Completing the square]

$= (a - 7)^2 - 10^2$

$= [(a - 7) - 10]\ [(a - 7) + 10]$

$= (a - 7 - 10)\ (a - 7 + 10)$

$= (a - 17)(a + 3)$

Q7. $a^2 + 2a - 3$

Soln:

$a^2 + 2a - 3$

$= a^2 + 2a + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - 3$ $\left[\textit{Adding and subtracting } \left(\frac{2}{2}\right)^2, \textit{ that is } 1^2\right]$

$= a^2 + 2a + 1 - 1 - 3$

$= (a + 1)^2 - 4$ [Completing the square]

$= (a + 1)^2 - 2^2$

$= [(a + 1) - 2] [(a + 1) + 2]$

$= (a + 1 - 2) (a + 1 + 2)$

$= (a - 1)(a + 3)$

Q8. $4x^2 - 12x + 5$

Soln:

$4x^2 - 12x + 5$

$= 4(x^2 - 3x + \frac{5}{4})$ [Making the co-efficient of $x^2 = 1$]

$=$

$4\left[x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{5}{4}\right]$ $\left[\textit{Adding and subtracting}\left(\frac{3}{2}\right)^2\right]$

$= 4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{5}{4}\right]$ [*Completing the square*]

$= 4\left[\left(x - \frac{3}{2}\right)^2 - 1\right]$

$= 4\left[\left(x - \frac{3}{2}\right) - 1\right]\left[\left(x - \frac{3}{2}\right) + 1\right]$

$= 4\left(x - \frac{3}{2} - 1\right)\left(x - \frac{3}{2} + 1\right)$

$= 4\left(x - \frac{5}{2}\right)\left(x - \frac{1}{2}\right)$

$= (2x - 5)(2x - 1)$

Q9. (y − 3)(y − 4)

Soln:

$= y^2 - 7y + 12$

(Adding and subtracting $(\frac{7}{2})^2$)

$= y^2 - 7y + (\frac{7}{2})^2 - (\frac{7}{2})^2 + 12$

Completing the square

$= (y - (\frac{7}{2}))^2 - \frac{49}{4} + \frac{48}{4}$

$= (y - (\frac{7}{2}))^2 - (\frac{1}{4})$

$= (y - (\frac{7}{2})^2 - (\frac{1}{2}^2)$

$= [(y - (\frac{7}{2} - \frac{1}{2}) [(y - (\frac{7}{2} + \frac{1}{2})$

$= [(y - (\frac{7}{2} - \frac{1}{2})([(y - (\frac{7}{2} + \frac{1}{2})$

$= (y-4)(y-3)$

Q10. (z − 6)(z + 2)

Soln:

$= z^2 - 4z - 12$

(Adding and subtracting $(\frac{4}{2})^2$)

$= z^2 - 4z + (\frac{4}{2})^2 - (\frac{4}{2})^2 - 12$

$= z^2 - 4z + (2)^2 - (2)^2 - 12$

$= (z-2)^2 - 16$

Completing the squares

$= (z-2)^2 - (4)^2$

$= [(z-2)-4][(z-2)+4]$

$= (z-6)(z+2)$