

Ratio and Proportion

Exercise 8A

Ratio and Proportion

- A **ratio** is a comparison of two values expressed as a quotient
 - Example: A class has 12 girls and 18 boys. The ratio of girls to boys is $\frac{12}{18}$
 - This ratio can also be expressed as an equivalent fraction $\frac{2}{3}$
- A **proportion** is an equation stating that two ratios are equal.
 - Example: $\frac{12}{18} = \frac{2}{3}$

1. Ratio:

The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.
In the ratio $a : b$, we call a as the first term or antecedent and b , the second term or consequent.

Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. $4 : 5 = 8 : 10 = 12 : 15$. Also, $4 : 6 = 2 : 3$.

2. Proportion:

The equality of two ratios is called proportion.

If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called extremes, while b and c are called mean terms.

Product of means = Product of extremes.

Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. Fourth Proportional:

If $a : b = c : d$, then d is called the fourth proportional to a, b, c .

Third Proportional:

$a : b = c : d$, then c is called the third proportion to a and b .

Mean Proportional:

Mean proportional between a and b is \sqrt{ab} .

4. Comparison of Ratios:

We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$

Compounded Ratio:

The compounded ratio of the ratios: $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

5. Duplicate Ratios:

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

6. Variations:

We say that x is directly proportional to y , if $x = ky$ for some constant k and we write, $x \propto y$.

We say that x is inversely proportional to y , if $xy = k$ for some constant k and

we write, $x \propto \frac{1}{y}$.

Properties of proportions:**Convertendo:** If $a : b :: c : d$, then $a : (a - b) :: c : (c - d)$.**Invertendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$.**Alternendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$.**Componendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$.**Dividendo:** $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$ **Componendo and Dividendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$.

Q1

Answer :

(i) HCF of 24 and 40 is 8.

$$\therefore 24 : 40 = \frac{24}{40} = \frac{24 \div 8}{40 \div 8} = \frac{3}{5} = 3 : 5$$

Hence, 24 : 40 in its simplest form is 3 : 5.

(ii) HCF of 13.5 and 15 is 1.5.

$$\frac{13.5}{15} = \frac{135}{150}$$

The HCF of 135 and 150 is 15.

$$= \frac{135 \div 15}{150 \div 15} = \frac{9}{10}$$

Hence, 13.5 : 15 in its simplest form is 9 : 10.

$$(iii) \frac{20}{3} : \frac{15}{2} = 40 : 45$$

The HCF of 40 and 45 is 5.

$$\therefore 40 : 45 = \frac{40}{45} = \frac{40 \div 5}{45 \div 5} = \frac{8}{9} = 8 : 9$$

Hence, $6\frac{2}{3} : 7\frac{1}{2}$ in its simplest form is 8 : 9

(iv) 9 : 6

The HCF of 9 and 6 is 3.

$$\therefore 9 : 6 = \frac{9}{6} = \frac{9 \div 3}{6 \div 3} = 3 : 2$$

Hence, $\frac{1}{6} : \frac{1}{9}$ in its simplest form is 3 : 2.

(v) LCM of the denominators is 2.

$$\therefore 4 : 5 : \frac{9}{2} = 8 : 10 : 9$$

The HCF of these 3 numbers is 1.

 $\therefore 8 : 10 : 9$ is the simplest form.(vi) $2.5 : 6.5 : 8 = 25 : 65 : 80$

The HCF of 25, 65 and 80 is 5.

$$\therefore 25 : 65 : 80 = \frac{25}{80} = \frac{25 \div 5}{80 \div 5} = \frac{5}{16} = 5 : 13 : 16$$

Q2

Answer :

(i) Converting both the quantities into the same unit, we have:

$$75 \text{ paise} : (3 \times 100) \text{ paise} = 75 : 300$$

$$= \frac{75}{300} = \frac{75 \div 75}{300 \div 75} = \frac{1}{4} \quad (\because \text{HCF of 75 and 300} = 75)$$

$$= 1 \text{ paise} : 4 \text{ paise}$$

(ii) Converting both the quantities into the same unit, we have:

$$105 \text{ cm} : 63 \text{ cm} = \frac{105}{63} = \frac{105 \div 21}{63 \div 21} = \frac{5}{3} \quad (\because \text{HCF of 105 and 63} = 21)$$

$$= 5 \text{ cm} : 3 \text{ cm}$$

(iii) Converting both the quantities into the same unit

$$65 \text{ min} : 45 \text{ min} = \frac{65}{45} = \frac{65 \div 5}{45 \div 5} = \frac{13}{9} \quad (\because \text{HCF of 65 and 45} = 5)$$

$$= 13 \text{ min} : 9 \text{ min}$$

(iv) Converting both the quantities into the same unit, we get:

$$8 \text{ months} : 12 \text{ months} = \frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3} \quad (\because \text{HCF of 8 and 12} = 4)$$

$$= 2 \text{ months} : 3 \text{ months}$$

(v) Converting both the quantities into the same unit, we get:

$$2250 \text{ g} : 3000 \text{ g} = \frac{2250}{3000} = \frac{2250 \div 750}{3000 \div 750} = \frac{3}{4} \quad (\because \text{HCF of 2250 and 3000} = 750)$$

$$= 3 \text{ g} : 4 \text{ g}$$

(vi) Converting both the quantities into the same unit, we get:

$$1000 \text{ m} : 750 \text{ m} = \frac{1000}{750} = \frac{1000 \div 250}{750 \div 250} = \frac{4}{3} \quad (\because \text{HCF of 1000 and 750} = 250)$$

$$= 4 \text{ m} : 3 \text{ m}$$

Q3

Answer :

$$\frac{A}{B} = \frac{7}{5} \text{ and } \frac{B}{C} = \frac{9}{14}$$

Therefore, we have:

$$\frac{A}{B} \times \frac{B}{C} = \frac{7}{5} \times \frac{9}{14}$$

$$\frac{A}{C} = \frac{9}{10}$$

$$\therefore A : C = 9 : 10$$

Q4

Answer :

$$\frac{A}{B} = \frac{5}{8} \text{ and } \frac{B}{C} = \frac{16}{25}$$

$$\text{Now, we have : } \frac{A}{B} \times \frac{B}{C} = \frac{5}{8} \times \frac{16}{25} \Rightarrow \frac{A}{C} = \frac{2}{5}$$

$$\therefore A : C = 2 : 5$$

Q5

Answer :

$$A : B = 3 : 5$$

$$B : C = 10 : 13 = \frac{10 \div 2}{13 \div 2} = 5 : \frac{13}{2}$$

$$\text{Now, } A : B : C = 3 : 5 : \frac{13}{2}$$

$$\therefore A : B : C = 6 : 10 : 13$$

Q6

Answer :

We have the following:

$$A : B = 5 : 6$$

$$B : C = 4 : 7 = \frac{4 \times \frac{6}{4}}{7 \times \frac{6}{4}} = 6 : \frac{21}{2}$$

$$\therefore A : B : C = 5 : 6 : \frac{21}{2} = 10 : 12 : 21$$

Q7

Answer :

$$\text{Sum of the ratio terms} = 7 + 8 = 15$$

Now, we have the following:

$$\text{Kunal's share} = \text{Rs } 360 \times \frac{7}{15} = 24 \times 7 = \text{Rs } 168$$

$$\text{Mohit's share} = \text{Rs } 360 \times \frac{8}{15} = 24 \times 8 = \text{Rs } 192$$

Q8

Answer :

$$\text{Sum of the ratio terms} = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$$

Now, we have the following:

$$\text{Rajan's share} = \text{Rs } 880 \times \frac{\frac{1}{5}}{\frac{11}{30}} = \text{Rs } 880 \times \frac{6}{11} = \text{Rs } 80 \times 6 = \text{Rs } 480$$

$$\text{Kamal's share} = \text{Rs } 880 \times \frac{\frac{1}{6}}{\frac{11}{30}} = \text{Rs } 880 \times \frac{5}{11} = \text{Rs } 80 \times 5 = \text{Rs } 400$$

Q9

Answer :

Sum of the ratio terms is $(1 + 3 + 4) = 8$

We have the following:

$$\text{A's share} = \text{Rs } 5600 \times \frac{1}{8} = \text{Rs } \frac{5600}{8} = \text{Rs } 700$$

$$\text{B's share} = \text{Rs } 5600 \times \frac{3}{8} = \text{Rs } 700 \times 3 = \text{Rs } 2100$$

$$\text{C's share} = \text{Rs } 5600 \times \frac{4}{8} = \text{Rs } 700 \times 4 = \text{Rs } 2800$$

Q10

Answer :

Let x be the required number.

Then, $(9 + x) : (16 + x) = 2 : 3$

$$\begin{aligned}\Rightarrow \frac{9+x}{16+x} &= \frac{2}{3} \\ \Rightarrow 27 + 3x &= 32 + 2x \Rightarrow x = 5\end{aligned}$$

Hence, 5 must be added to each term of the ratio $9 : 16$ to make it $2 : 3$.

Q11

Answer :

Suppose that x is the number that must be subtracted.

Then, $(17 - x) : (33 - x) = 7 : 15$

$$\begin{aligned}\Rightarrow \frac{17-x}{33-x} &= \frac{7}{15} \\ \Rightarrow 255 - 15x &= 231 - 7x \Rightarrow 8x = 255 - 231 = 24 \Rightarrow x = 3\end{aligned}$$

Hence, 3 must be subtracted from each term of ratio $17 : 33$ so that it becomes $7 : 15$.

Q12

Answer :

Suppose that the numbers are $7x$ and $11x$.

Then, $(7x + 7) : (11x + 7) = 2 : 3$

$$\Rightarrow \frac{7x+7}{11x+7} = \frac{2}{3}$$

$$\Rightarrow 21x + 21 = 22x + 14$$

$$\Rightarrow x = 7$$

Hence, the numbers are $(7 \times 7 =) 49$ and $(11 \times 7 =) 77$.

Q13

Answer :

Suppose that the numbers are $5x$ and $9x$.

Then, $(5x - 3) : (9x - 3) = 1 : 2$

$$\Rightarrow \frac{5x-3}{9x-3} = \frac{1}{2}$$

$$\Rightarrow 10x - 6 = 9x - 3$$

$$\Rightarrow x = 3$$

Hence, the numbers are $(5 \times 3 =) 15$ and $(9 \times 3 =) 27$.

Q14

Answer :

Let the numbers be $3x$ and $4x$.

Their LCM is $12x$.

Then, $12x = 180$

$\Rightarrow x = 15$

\therefore The numbers are $(3 \times 15 =) 45$ and $(4 \times 15 =) 60$.

Q15

Answer :

Suppose that the present ages of A and B are $8x$ yrs and $3x$ yrs.

Then, $(8x + 6) : (3x + 6) = 9 : 4$

$$\Rightarrow \frac{8x+6}{3x+6} = \frac{9}{4}$$

$$\Rightarrow 32x + 24 = 27x + 54$$

$$\Rightarrow 5x = 30$$

$$\Rightarrow x = 6$$

Now, present age of A = 8×6 yrs = 48 yrs

Present age of B = 3×6 yrs = 18 yrs

Q16

Answer :

Suppose that the weight of zinc is x g.

Then, $48.6 : x = 9 : 5$

$$\Rightarrow x = \frac{48.6 \times 5}{9} = \frac{243}{9} = 27$$

Hence, the weight of zinc in the alloy is 27 g.

Q17

Answer :

Suppose that the number of boys is x .

Then, $x : 375 = 8 : 3$

$$\Rightarrow x = \frac{8 \times 375}{3} = 8 \times 125 = 1000$$

Hence, the number of girls in the school is 1000.

Q18

Answer :

Suppose that the monthly income of the family is Rs x .

Then, $x : 2500 = 11 : 2$

$$\Rightarrow x = \frac{11 \times 2500}{2} = 11 \times 1250$$

$$\Rightarrow x = \text{Rs } 13750$$

Hence, the income is Rs 13,750.

\therefore Expenditure = (monthly income – savings)

$$= \text{Rs } (13750 - 2500)$$

$$= \text{Rs } 11250$$

Q19

Answer :

Let the numbers one rupee, fifty paise and twenty-five paise coins be $5x$, $8x$ and $4x$, respectively.

$$\text{Total value of these coins} = (5x \times \frac{100}{100} + 8x \times \frac{50}{100} + 4x \times \frac{25}{100})$$

$$\begin{aligned} &\Rightarrow 5x + \frac{8x}{2} + \frac{4x}{4} \\ &= \frac{20x + 16x + 4x}{4} = \frac{40x}{4} = 10x \end{aligned}$$

However, the total value is Rs 750.

$$\therefore 750 = 10x$$

$$\Rightarrow x = 75$$

Hence, number of one rupee coins = $5 \times 75 = 375$

Number of fifty paise coins = $8 \times 75 = 600$

Number of twenty-five paise coins = $4 \times 75 = 300$

Q20

Answer :

$$(4x + 5) : (3x + 11) = 13 : 17$$

$$\begin{aligned} &\Rightarrow \frac{4x+5}{3x+11} = \frac{13}{17} \\ &\Rightarrow 68x + 85 = 39x + 143 \Rightarrow 29x = 58 \Rightarrow x = 2 \end{aligned}$$

Q21

Answer :

$$\begin{aligned} \frac{x}{y} &= \frac{3}{4} \\ \Rightarrow x &= \frac{3y}{4} \end{aligned}$$

Now, we have $(3x + 4y) : (5x + 6y)$

$$\begin{aligned} &= \frac{3x+4y}{5x+6y} = \frac{3 \times \frac{3y}{4} + 4y}{5 \times \frac{3y}{4} + 6y} \\ &= \frac{9y+16y}{15y+24y} = \frac{25y}{39y} = \frac{25}{39} \end{aligned}$$

$$= 25 : 39$$

Q22

Answer :

$$\begin{aligned} \frac{x}{y} &= \frac{6}{11} \\ \Rightarrow x &= \frac{6y}{11} \end{aligned}$$

Now, we have:

$$\begin{aligned} &\frac{8x-3y}{3x+2y} \\ &= \frac{8 \times \frac{6y}{11} - 3y}{3 \times \frac{6y}{11} + 2y} \\ &= \frac{48y-33y}{18y+22y} \\ &= \frac{15y}{40y} = \frac{3}{8} \end{aligned}$$

$$\therefore (8x - 3y) : (3x + 2y) = 3 : 8$$

Q23

Answer :

Suppose that the numbers are $5x$ and $7x$.

The sum of the numbers is 720.

$$\text{i.e., } 5x + 7x = 720$$

$$\Rightarrow 12x = 720$$

$$\Rightarrow x = 60$$

Hence, the numbers are $(5 \times 60 =) 300$ and $(7 \times 60 =) 420$.

Q24

Answer :

(i) The LCM of 6 and 9 is 18.

$$\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}$$

$$\frac{7}{9} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18} \quad \text{Clearly, } \frac{14}{18} < \frac{15}{18}$$

$$\therefore (7 : 9) < (5 : 6)$$

(ii) The LCM of 3 and 7 is 21.

$$\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$$

$$\frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

$$\text{Clearly, } \frac{12}{21} < \frac{14}{21}$$

$$\therefore (4 : 7) < (2 : 3)$$

(iii) The LCM of 2 and 7 is 14.

$$\frac{1 \times 7}{2 \times 7} = \frac{7}{14}$$

$$\frac{4 \times 2}{7 \times 2} = \frac{8}{14}$$

$$\text{Clearly, } \frac{7}{14} < \frac{8}{14}$$

$$\therefore (1 : 2) < (4 : 7)$$

(iv) The LCM of 5 and 13 is 65.

$$\frac{3}{5} = \frac{3 \times 13}{5 \times 13} = \frac{39}{65}$$

$$\frac{8}{13} = \frac{8 \times 5}{13 \times 5} = \frac{40}{65}$$

$$\text{Clearly, } \frac{39}{65} < \frac{40}{65}$$

$$\therefore (3 : 5) < (8 : 13)$$

Q25

Answer :

(i) We have $\frac{5}{6}$, $\frac{8}{9}$ and $\frac{11}{18}$.

$$\begin{array}{r} 2 \mid 6, 9, 18 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \mid 3, 9, 9 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \mid 1, 3, 3 \\ \hline \end{array}$$

$$\begin{array}{r} \mid 1, 1, 1 \\ \hline \end{array}$$

The LCM of 6, 9 and 18 is 18. Therefore, we have:

$$\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}$$

$$\frac{8}{9} = \frac{8 \times 2}{9 \times 2} = \frac{16}{18} \quad \frac{11}{18} = \frac{11}{18} \quad \text{Clearly, } \frac{11}{18} < \frac{15}{18} < \frac{16}{18}$$

$$\text{Hence, } (11 : 18) < (5 : 6) < (8 : 9)$$

(ii) We have $\frac{11}{14}$, $\frac{17}{21}$, $\frac{5}{7}$ and $\frac{2}{3}$.

$$2 \overline{) 14, 21, 7, 3}$$

$$7 \overline{) 7, 21, 7, 3}$$

$$3 \overline{) 1, 3, 1, 3}$$

$$\overline{) 1, 1, 1, 1}$$

The LCM of 14, 21, 7 and 3 is 42.

$$\frac{11}{14} = \frac{11 \times 3}{14 \times 3} = \frac{33}{42}$$

$$\frac{17}{21} = \frac{17 \times 2}{21 \times 2} = \frac{34}{42}$$

$$\frac{5}{7} = \frac{5 \times 6}{7 \times 6} = \frac{30}{42}$$

$$\frac{2}{3} = \frac{2 \times 14}{3 \times 14} = \frac{28}{42}$$

Clearly, $\frac{28}{42} < \frac{30}{42} < \frac{33}{42} < \frac{34}{42}$

Hence, $(2 : 3) < (5 : 7) < (11 : 14) < (17 : 21)$

Ratio and Proportion

Exercise 8B

Q1

Answer :

We have:

$$\text{Product of the extremes} = 30 \times 60 = 1800$$

$$\text{Product of the means} = 40 \times 45 = 1800$$

$$\text{Product of extremes} = \text{Product of means}$$

$$\text{Hence, } 30 : 40 :: 45 : 60$$

Q2

Answer :

We have:

$$\text{Product of the extremes} = 36 \times 7 = 252$$

$$\text{Product of the means} = 49 \times 6 = 294$$

$$\text{Product of the extremes} \neq \text{Product of the means}$$

Hence, 36, 49, 6 and 7 are not in proportion.

Q3

Answer :

$$\text{Product of the extremes} = 2 \times 27 = 54$$

$$\text{Product of the means} = 9 \times x = 9x$$

Since $2 : 9 :: x : 27$, we have:

$$\text{Product of the extremes} = \text{Product of the means}$$

$$\Rightarrow 54 = 9x$$

$$\Rightarrow x = 6$$

Q4

Answer :

Product of the extremes = $8 \times 35 = 280$

Product of the means = $16 \times x = 16x$

Since $8 : x :: 16 : 35$, we have:

Product of the extremes = Product of the means

$$\Rightarrow 280 = 16x$$

$$\Rightarrow x = 17.5$$

Q5

Answer :

Product of the extremes = $x \times 60 = 60x$

Product of the means = $35 \times 48 = 1680$

Since $x : 35 :: 48 : 60$, we have:

Product of the extremes = Product of the means

$$\Rightarrow 60x = 1680$$

$$\Rightarrow x = 28$$

Q6

Answer :

(i) Let the fourth proportional be x .

Then, $8 : 36 :: 6 : x$

$$8 \times x = 36 \times 6$$

[Product of extremes = Product of means]

$$\Rightarrow 8x = 216$$

$$\Rightarrow x = 27$$

Hence, the fourth proportional is 27.

(ii) Let the fourth proportional be x .

Then, $5 : 7 :: 30 : x$

$$\Rightarrow 5 \times x = 7 \times 30$$

[Product of extremes = Product of means]

$$\Rightarrow 8x = 216$$

$$\Rightarrow 5x = 210$$

$$\Rightarrow x = 42$$

Hence, the fourth proportional is 42.

(iii) Let the fourth proportional be x .

Then, $2.8 \times x = 14 \times 3.5$

[Product of extremes = Product of means]

$$\Rightarrow 8x = 216$$

$$\Rightarrow 2.8x = 49$$

$$\Rightarrow x = 17.5$$

Hence, the fourth proportional is 17.5.

Q7

Answer :

36, 54 and x are in continued proportion.

Then, $36 : 54 :: 54 : x$

$$\Rightarrow 36 \times x = 54 \times 54$$

[Product of extremes = Product of means]

$$\Rightarrow 36x = 2916$$

$$\Rightarrow x = 81$$

Q8

Answer :

27, 36 and x are in continued proportion.

Then, $27 : 36 :: 36 : x$

$$\Rightarrow 27 \times x = 36 \times 36 \quad [\text{Product of extremes} = \text{Product of means}]$$

$$\Rightarrow 27x = 1296$$

$$\Rightarrow x = 48$$

Hence, the value of x is 48.

Q9

Answer :

(i) Suppose that x is the third proportional to 8 and 12.

Then, $8 : 12 :: 12 : x$

$$\Rightarrow 8 \times x = 12 \times 12 \quad (\text{Product of extremes} = \text{Product of means})$$

$$\Rightarrow 8x = 144$$

$$\Rightarrow x = 18$$

Hence, the required third proportional is 18.

(ii) Suppose that x is the third proportional to 12 and 18.

Then, $12 : 18 :: 18 : x$

$$\Rightarrow 12 \times x = 18 \times 18 \quad (\text{Product of extremes} = \text{Product of means})$$

$$\Rightarrow 12x = 324$$

$$\Rightarrow x = 27$$

Hence, the third proportional is 27.

(iii) Suppose that x is the third proportional to 4.5 and 6.

Then, $4.5 : 6 :: 6 : x$

$$\Rightarrow 4.5 \times x = 6 \times 6 \quad (\text{Product of extremes} = \text{Product of means})$$

$$\Rightarrow 4.5x = 36$$

$$\Rightarrow x = 8$$

Hence, the third proportional is 8.

Q10

Answer :

The third proportional to 7 and x is 28.

Then, $7 : x :: x : 28$

$$\Rightarrow 7 \times 28 = x^2 \quad (\text{Product of extremes} = \text{Product of means})$$

$$\Rightarrow x = 14$$

Q11

Answer :

(i) Suppose that x is the mean proportional.

Then, $6 : x :: x : 24$

$$\Rightarrow 6 \times 24 = x \times x \quad (\text{Product of extremes} = \text{Product of means})$$

$$\Rightarrow x^2 = 144$$

$$\Rightarrow x = 12$$

Hence, the mean proportional to 6 and 24 is 12.

(ii) Suppose that x is the mean proportional.

Then, $3 : x :: x : 27$

$$\Rightarrow 3 \times 27 = x \times x \quad (\text{Product of extremes} = \text{Product of means})$$

$$\Rightarrow x^2 = 81$$

$$\Rightarrow x = 9$$

Hence, the mean proportional to 3 and 27 is 9.

(iii) Suppose that x is the mean proportional.

Then, $0.4 : x :: x : 0.9$

$$\begin{aligned}\Rightarrow 0.4 \times 0.9 &= x \times x && \text{(Product of extremes = Product of means)} \\ \Rightarrow x^2 &= 0.36 \\ \Rightarrow x &= 0.6\end{aligned}$$

Hence, the mean proportional to 0.4 and 0.9 is 0.6.

Q12

Answer :

Suppose that the number is x .

Then, $(5 + x) : (9 + x) :: (7 + x) : (12 + x)$

$$\begin{aligned}\Rightarrow (5 + x) \times (12 + x) &= (9 + x) \times (7 + x) \\ \text{(Product of extremes = Product of means)} \\ \Rightarrow 60 + 5x + 12x + x^2 &= 63 + 9x + 7x + x^2 \\ \Rightarrow 60 + 17x &= 63 + 16x \\ \Rightarrow x &= 3\end{aligned}$$

Hence, 3 must be added to each of the numbers: 5, 9, 7 and 12, to get the numbers which are in proportion.

Q13

Answer :

Suppose that x is the number that is to be subtracted.

Then, $(10 - x) : (12 - x) :: (19 - x) : (24 - x)$

$$\begin{aligned}\Rightarrow (10 - x) \times (24 - x) &= (12 - x) \times (19 - x) \\ \text{(Product of extremes = Product of means)} \\ \Rightarrow 240 - 10x - 24x + x^2 &= 228 - 12x - 19x + x^2 \\ \Rightarrow 240 - 34x &= 228 - 31x \\ \Rightarrow 3x &= 12 \\ \Rightarrow x &= 4\end{aligned}$$

Hence, 4 must be subtracted from each of the numbers: 10, 12, 19 and 24, to get the numbers which are in proportion.

Q14

Answer :

Distance represented by 1 cm on the map = 5000000 cm = 50 km

Distance represented by 3 cm on the map = 50×4 km = 200 km

\therefore The actual distance is 200 km.

Q15

Answer :

(Height of tree) : (height of its shadow) = (height of the pole) : (height of its shadow)

Suppose that the height of pole is x cm.

Then, $6 : 8 = x : 20$

$$\begin{aligned}\Rightarrow x &= \frac{6 \times 20}{8} = 15 \\ \therefore \text{Height of the pole} &= 15 \text{ cm}\end{aligned}$$

Ratio and Proportion

Exercise 8C

Q1

Answer :

The correct option is (d).

$$\frac{a}{c} = \frac{a}{b} \times \frac{b}{c} = \frac{3}{4} \times \frac{8}{9} \\ = \frac{2}{3}$$

Hence, $a : c = 2 : 3$

Q2

Answer :

(a) $15 : 8$

$$\frac{A}{B} = \frac{2}{3} \\ \frac{B}{C} = \frac{4}{5}$$

$$\text{Then, } \frac{A}{B} \times \frac{B}{C} = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

Hence, $C : A = 15 : 8$

Q3

Answer :

The correct option is (d).

$$A = \frac{3B}{2} \\ C = \frac{4B}{5}$$

$$\therefore A : C = \frac{A}{C} = \frac{\frac{3B}{2}}{\frac{4B}{5}} = \frac{15}{8}$$

Hence, $A : C = 15 : 8$

Q4

Answer :

The correct option is (b).

$$\frac{15}{100} A = \frac{20}{100} B$$
$$\Rightarrow \frac{A}{B} = \frac{4}{3}$$

Hence, $A : B = 4 : 3$

Q5

Answer :

(a) $1 : 3 : 6$

$$A = \frac{1}{3} B$$
$$C = 2B$$
$$\therefore A : B : C = \frac{1}{3} B : B : 2B = 1 : 3 : 6$$

Q6

Answer :

(b) $30 : 42 : 77$

$$\frac{A}{B} = \frac{5}{7}$$
$$\Rightarrow A = \frac{5B}{7} \quad \frac{B}{C} = \frac{6}{11} \Rightarrow C = \frac{11B}{6}$$
$$\therefore A : B : C = \frac{5B}{7} : B : \frac{11B}{6} = 30 : 42 : 77$$

Q7

Answer :

(c) $6 : 4 : 3$

$$2A = 3B = 4C$$
$$\text{Then, } A = \frac{3B}{2} \text{ and } C = \frac{3B}{4}$$
$$\therefore A : B : C = \frac{3B}{2} : B : \frac{3B}{4} = 6 : 4 : 3$$

Q8

Answer :

(a) $3 : 4 : 5$

$$A = \frac{3B}{4}$$
$$C = \frac{5B}{4}$$
$$\therefore A : B : C = \frac{3B}{4} : B : \frac{5B}{4}$$
$$= 3 : 4 : 5$$

Q9

Answer :

(b) $15 : 10 : 6$

$$\frac{1}{x} : \frac{1}{y} = 2 : 3$$
$$\text{Then, } y : x = 2 : 3 \text{ and } y = \frac{2}{3} x$$
$$\frac{1}{y} : \frac{1}{z} = 3 : 5$$
$$\text{Then, } z : y = 3 : 5 \text{ and } z = \frac{3}{5} y$$
$$\therefore x : y : z = x : \frac{2}{3} x : \frac{3}{5} y = x : \frac{2}{3} x : \frac{3}{5} \times \frac{2}{3} x$$
$$= x : \frac{2}{3} x : \frac{2}{5} x = 15 : 10 : 6$$

Q10

Answer :

$$\frac{x}{y} = \frac{3}{4}$$

$$x = \frac{3y}{4}$$

$$\begin{aligned}\therefore \frac{7x+3y}{7x-3y} &= \frac{7\frac{3y}{4}+3y}{7\frac{3y}{4}-3y} \\ &= \frac{21y+12y}{21y-12y} = \frac{33y}{9y} = \frac{11}{3}\end{aligned}$$

Hence, $(7x + 3y) : (7x - 3y) = 11 : 3$

The correct option is (c).

Q11

Answer :

(c) 5 : 2

$$\begin{aligned}\frac{3a+5b}{3a-5b} &= \frac{5}{1} \\ 3a+5b &= 15a-25b \\ 12a &= 30b \\ \frac{a}{b} &= \frac{30}{12} = \frac{5}{2}\end{aligned}$$

$$\therefore a : b = 5 : 2$$

Q12

Answer :

(c) 9

$$\begin{aligned}7 \times 45 &= x \times 35 \quad (\text{Product of extremes} = \text{Product of means}) \\ \Rightarrow 315 &= 35x \\ \Rightarrow x &= 9\end{aligned}$$

Q13

Answer :

(b) 7

Suppose that x is the number that is to be added.

$$\text{Then, } (3+x) : (5+x) = 5 : 6$$

$$\begin{aligned}\Rightarrow \frac{3+x}{5+x} &= \frac{5}{6} \\ \Rightarrow 18+6x &= 25+5x \\ \Rightarrow x &= 7\end{aligned}$$

Q14

Answer :

(d) 40

Suppose that the numbers are x and y .

$$\text{Then, } x : y = 3 : 5 \text{ and } (x+10) : (y+10) = 5 : 7$$

$$\begin{aligned}\frac{x}{y} &= \frac{3}{5} \\ x &= \frac{3y}{5} \\ \Rightarrow \frac{x+10}{y+10} &= \frac{5}{7} \Rightarrow 7x+70 = 5y+50 \Rightarrow 7 \times \frac{3y}{5} + 70 = 5y+50 \Rightarrow 5y - \frac{21y}{5} = \\ 20 \Rightarrow \frac{4y}{5} &= 20 \Rightarrow y = 25 \text{ Therefore, } x = \frac{3 \times 25}{5} = 15\end{aligned}$$

Hence, sum of numbers = $15 + 25 = 40$

Q15

Answer :

(a) 3

Suppose that x is the number that is to be subtracted.

Then, $(15 - x) : (19 - x) = 3 : 4$

$$\Rightarrow \frac{15-x}{19-x} = \frac{3}{4}$$

Cross multiplying, we get :

$$60 - 4x = 57 - 3x$$

$$\Rightarrow x = 3$$

Q16

Answer :

(a) Rs 180

$$\text{A's share} = \frac{3}{7} \times 420 = 180$$

Q17

Answer :

(d) 416

Let x be the number of boys.

Then, $8 : 5 = x : 160$

$$\Rightarrow \frac{8}{5} = \frac{x}{160}$$

$$\Rightarrow x = \frac{8 \times 160}{5} = 256$$

$$\therefore \text{Total strength of the school} = 256 + 160 = 416$$

Q18

Answer :

(a) (2 : 3)

LCM of 3 and 7 = $7 \times 3 = 21$

$$\frac{2 \times 7}{3 \times 7} = \frac{14}{21} \text{ and } \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

Clearly, $\frac{12}{21} < \frac{14}{21}$

Hence, $(4 : 7) < (2 : 3)$

Q19

Answer :

(c) 16

Suppose that the third proportional is x .

Then, $9 : 12 :: 12 : x$

$$\Rightarrow 9 \times x = 12 \times 12 \quad (\text{Product of extremes} = \text{Product of means})$$

$$\Rightarrow 9x = 144$$

$$\Rightarrow x = 16$$

Q20

Answer :

(b) 12

Suppose that the mean proportional is x .

Then, $9 : x :: x : 16$

$$9 \times 16 = x \times x \quad (\text{Product of extremes} = \text{Product of means})$$

$$\Rightarrow x^2 = 144$$

$$\Rightarrow x = 12$$

Q21

Answer :

(a) 18 years

Suppose that the present ages of A and B are $3x$ yrs and $8x$ yrs, respectively.

After six years, the age of A will be $(3x+6)$ yrs and that of B will be $(8x+6)$ yrs.

Then, $(3x+6) : (8x+6) = 4 : 9$

$$\Rightarrow \frac{3x+6}{8x+6} = \frac{4}{9}$$

$$\Rightarrow 27x + 54 = 32x + 24$$

$$\Rightarrow 5x = 30$$

$$\Rightarrow x = 6$$

Hence, the present ages of A and B are 18 yrs and 48 yrs, respectively.