RRB JE

RAILWAY RECRUITMENT BOARD

CIVIL

ENGINEERING

FLUID MECHANICS

SELF STUDY MATERIAL

CIVIL ENGINEERING FOR ALL

FLUID MECHANICS

Fluid: Fluid is a substance which has the property tendency to flow under the action of shear and tangential forces.
Liquids and gases both are fluids.
Ideal and Real fluids:

- In ideal fluids, there is no viscosity and no surface tension and are incompressible.
- In real fluids, viscosity, surface tension together exist and are compressible along with density.

FLUID PROPERTIES

1. ***Density*** **(ρ) :** It is defined as mass per unit volume of substance.

$$\rho = \frac{m}{V}$$

2. ***Specific Weight*** **(ω) :** It is defined as weight per unit volume of substance.

$$\omega = \frac{mg}{V} = \rho g$$

3. ***Relative density Specific gravity*** **(Sg) :** It is defined as ratio of density of fluid to the density of standard fluid.
It may also be defined as the ratio of specific weight of the fluid to the standard weight of fluid.

$$Sg = \frac{\text{weight of fluid}}{\text{weight of standard fluid}}$$

$$Sg = \frac{\text{Density of Fluid}}{\text{Density of standard Fluid}}$$

Ex: Oil of Sg of 0.8 $\Rightarrow$ $\rho_{oil} = 800\ kg/m^3$
Specific volume (v) :
It is expressed as the volume per unit mass of fluid.

$$v = \frac{V}{m} = \frac{1}{\rho}$$

4. ***Compressibility*** **(β)**
Hydrostatic law: It states that rate of increase of pressure in a vertical direction is equal to weight density of fluid at that point.

Mathematically, pressure head (h) = $\frac{\rho}{\rho g}$

$$\beta = \frac{-\frac{dV}{V}}{dp} = \frac{1}{\rho}\frac{d\rho}{dp}$$

Liquids are highly incompressible $\therefore$ $\frac{d\rho}{dp} = 0$

Gases are highly compressible as $P \propto \rho$.

5. ***Bulk Modulus of Elasticity*** **(κ)**
It is defined as reciprocal of compressibility.

VISCOSITY

It is the property of fluid by virtue of which one layer resists the motion of another adjacent layer. i.e. its resistence to shearing stresser.

Newton's Law of Viscosity

The viscous shear stress between two layers at a distance 'y' from the surface can be written as : $\tau \propto \frac{du}{dy}$

as $\tau = \mu \frac{du}{dy}$

- 'μ' is co-efficient of dynamic viscosity / viscosity.
- μ is a property of fluid called dynamic viscosity and is a function of temperature only.
- Fluids which obey Newton's law of viscosity are known as Newtonian fluids.
- If μ is high $\Rightarrow$ velocity gradient $\frac{du}{dy}$ is less $\Rightarrow$ highly viscous fluid.
- If μ is low $\Rightarrow$ velocity gradient $\frac{du}{dy}$ is high $\Rightarrow$ easy to flow fluid.

Kinematic Viscosity (ν)

It is expressed as the ratio of dynamic viscosity (μ) and density of fluid (ρ).

$$\nu = \frac{\mu}{\rho}$$

Units SI $\rightarrow m^2/s$
CgS $\rightarrow$ Stokes/cm^2/s
1 stokes = $10^{-4}\ m^2/s$

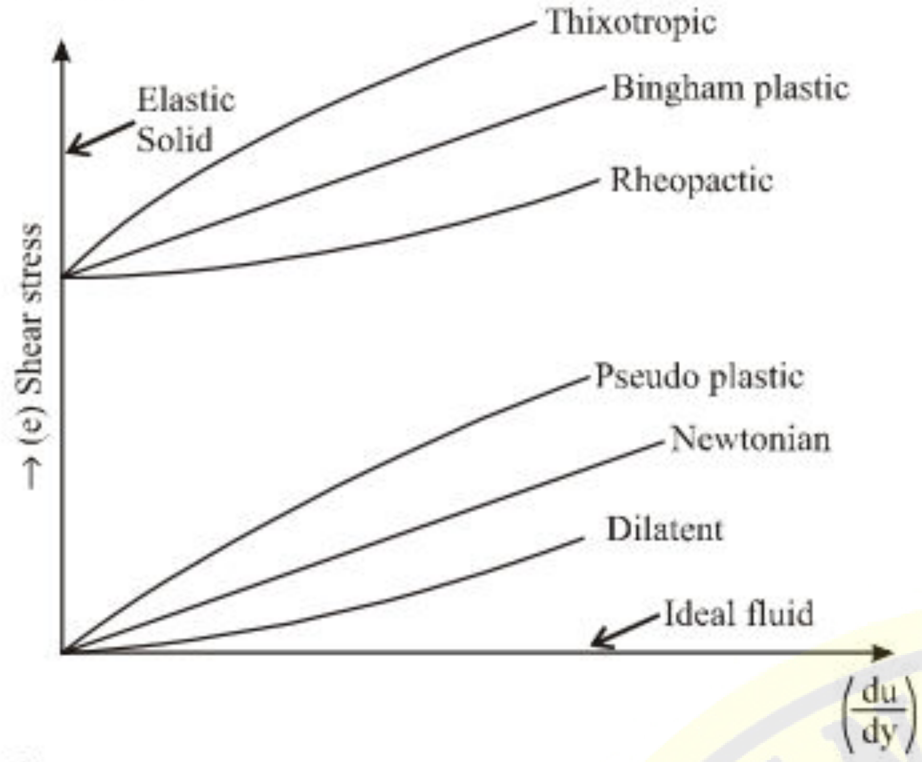
Effect of temperature and pressure on viscosity:

- Viscosity of liquids decrease but that of gases increase with increase in temperature.

- In ordinary situations, effect of pressure on viscosity is not so significant but in case of some oils, viscosity increase with increase in pressure.

RHEOLOGY

It is the branch of science in which we study about different types of fluids



Examples

- Newtonian : Water, air
- Dilatent : Butter, starch solution
- Psuedo plastic : Paints
- Bingham plastic : Gel, cream
- Thixotropic : Printer's ink and enamel

SURFACE TENSION (σ)

Cohesive and Adhesive forces:
Cohesive forces are intermolecular attraction of forever between molecular of same liquid/fluid.
Adhesive forces are attractive forces between the molecular of a liquid/fluid and the molecular of a solid boundary surface in contact.

- Property of a liquid.
- The basic cause of surface tension is the presence of cohesive forces.
- It is a property by virtue of which liquids want to mnimize their surface area upto maximum extent.

$$\boxed{\sigma = \frac{F}{e}\ \text{N/m}}$$

Wetting and Non-Wetting Liquids

- It is the mutual property of liquid-surface.
- If adhesion >>>> cohesion,
  Liquid wets the surface.
  If cohesion >>>> adhesion,
  No wetting
- For wetting, angle of contact (θ) should be acute and for non-wetting angle of contact (θ) should be obtuse.
- For pure water $\theta = 0°$.
- For Mercury-glass, $\theta = 130°$ to $140°$.

CAPILLARITY

When a tube of very fine diameter is immersed in a liquid, there will be rise or fall of liquid level in the tube depending upon whether the liquid is wetting with the tube or non-wetting.

The rise or fall of liquid level in the tube is a phenomenon known as capillarity.

h : rise of liquid level in tube
σ : surface tension
r : radius of capillary tube
ρ : density of liquid
θ : angle of contact

$$\boxed{h = \frac{2\sigma\cos\theta}{\rho g r}}$$

For an annular capillary having external radius r_2 and inner radius r_1,

$$\boxed{h = \frac{2\sigma\cos\theta}{\rho g(r_2 - r_1)}}$$

Pascal's law: It states that pressure intensity at any point in a liquid of rest, is same in all directions. If P_x, P_y and P_z are the pressure in x, y & z – direction acting on a fluid element, at rest, then,

$$p_x = p_y = p_z.$$

PRESSURE MEASUREMENT DEVICES

I. BAROMETER

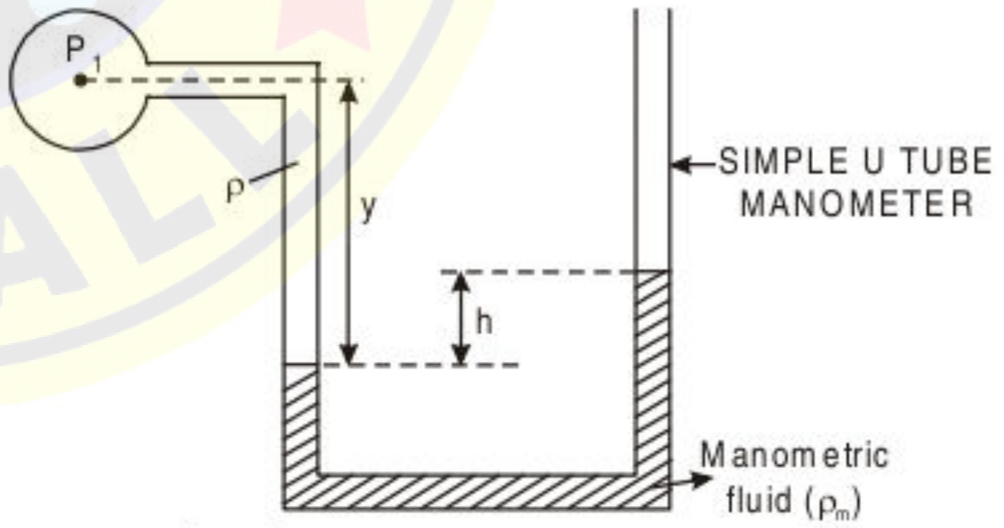
It is a device made by Torricelli and is used to measure local atmospheric pressure.

II. PIEZOMETER

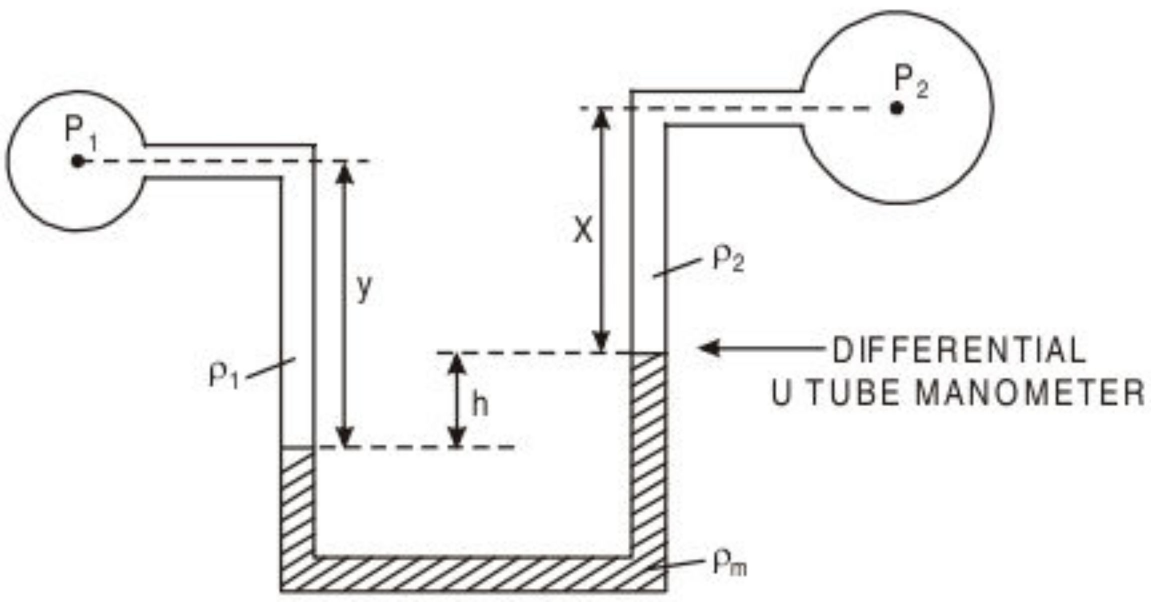
- It is a device used for measurements of moderate pressure (gauge) of liquids only.
- Piezometer cannot measure the pressure of gas.

III. MANOMETER

- used for measurement of high pressure.
- It makes the use of a manometric fluid.



$$P_1 + \rho g y - \rho_n g h = 0$$



$$P_1 + \rho_1 g y - \rho_m g h - \rho_2 g x = P_2$$

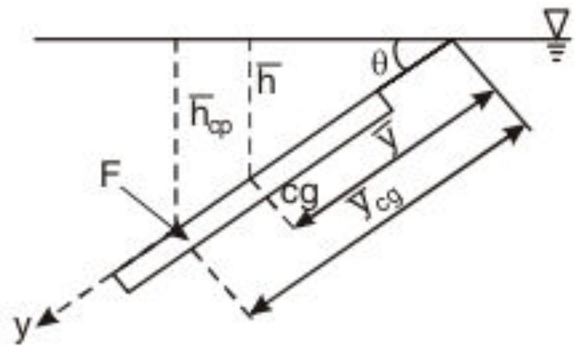
NOTE

(a) Absolute pressure = Atmospheric present + Gauge pressure

(b) Absolute pressure = Atmospheric pressure – Vacuum pressure

FLUID STATICS

Hydrostatic Force on a Plane Surface



cp : Centre of pressure

cg : Centre of gravity

F : hydrostatic force acting on the plane surface inclined to free surface.

$$\sin\theta = \frac{\bar{h}}{\bar{y}} = \frac{\bar{h}cp}{\bar{y}cp} = \frac{h}{y}$$

$$\boxed{\begin{aligned} F &= \rho g \bar{h} A \\ \bar{h}cp &= \bar{h} + \frac{I_{cg}\sin^2\theta}{\bar{h}A} \end{aligned}}$$

I_{cg} : moment of inertia of the plane surface about c.g

For a horizontal surface, $\theta = 0°$

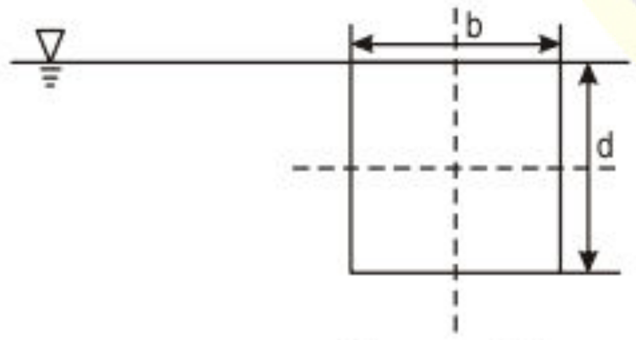
$\Rightarrow \quad \bar{h}_{cp} = \bar{h}$

For a vertical surface, $\theta = 90°$

$$\Rightarrow \quad \bar{h}_{cp} = \bar{h} + \frac{I_{cg}}{\bar{h}A}$$

Vertical Surfaces (θ = 90°)

(i)

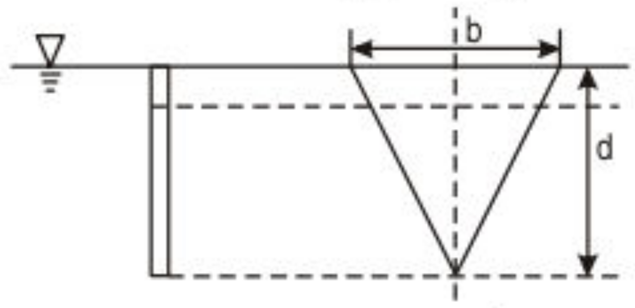


$\bar{h} = d/2$

$I_{cg} = \dfrac{bd^3}{12}$

$A = bd$

(ii)

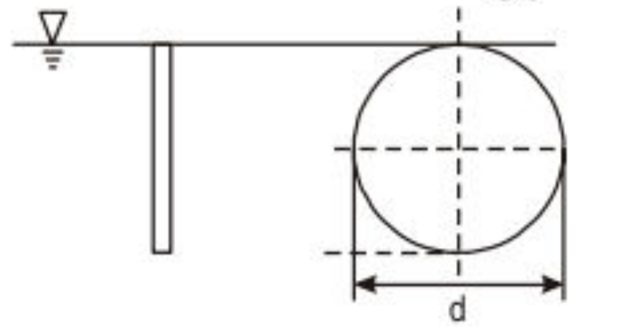


$\bar{h} = \dfrac{d}{3}$

$A = \dfrac{bd}{2}$

$I_{cg} = \dfrac{bd^2}{36}$

(iii)

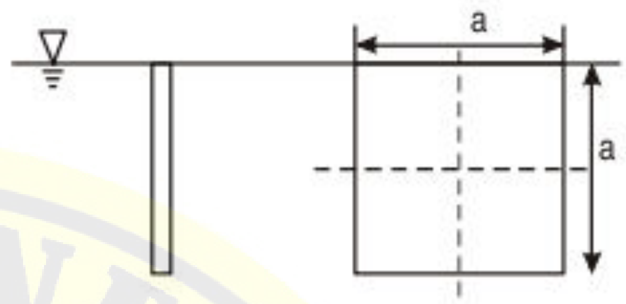


$\bar{h} = \dfrac{d}{2}$

$I_{cg} = \dfrac{\pi}{64}d^4$

$A = \dfrac{\pi}{4}d^2$

(iv)



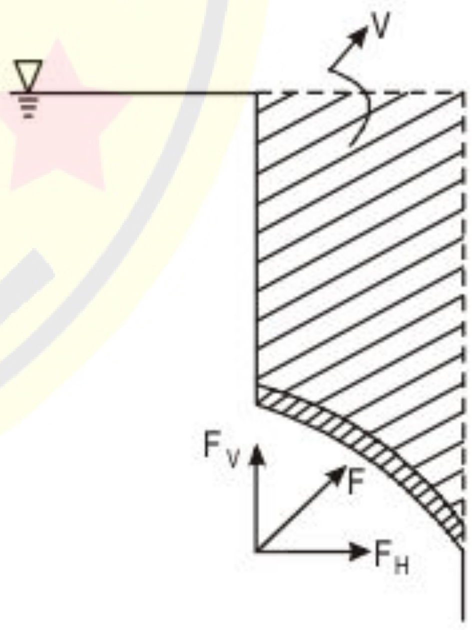
$\bar{h} = \dfrac{a}{2}$

$I_{cg} = \dfrac{a^4}{12}$

$A = a^2$

HYDROSTATIC FORCES ON CURVED SURFACES

Consider a curved surface as shown in the figure.



F : Hydrostatic force acting on the curved portion

F_H : Horizontal component of F

F_V : Vertical component of F

$$\boxed{\begin{aligned} F_H &= \rho g \bar{h} A \\ F_V &= \rho g V \\ F &= \sqrt{F_H^2 + F_V^2} \end{aligned}}$$

V = Volume till the free surface

STABILITY OF SUBMERGED BODY

- Centre of Buoyancy : B
  Centre of Gravity : G
- If B lies above G, the body is in stable equilibrium.

- If B and G coincide, the body is in neutral equilibrium.
- If B lies below G, the body is in unstable equilibrium.

Stability of Floating Body

Metacentric point (M): When a body is given a small angular displacement which is floating in a liquid in a state of equilibrium. It starts oscillating about some point (M), known as metacentric point.

- If M lies above G, the body is in stable equilibrium.
- If M and G coincide, the body is in neutral equilibrium.
- If M lies below G, the body is in unstable equilibrium.

Metacentric Height (GM)

$$GM = \frac{1}{V} - BG$$

I: Moment of inertia of the face of the body intersected by free surface

V: Volume of the fluid displaced.

BG: Distance between centre of buoyancy and centre of gravity.

GM: Metacentric height

For Stable equilibrium $GM > 0$

For neutral equilibrium $GM = 0$

For unstable equilibrium $GM < 0$

Buoyancy

When the bodies are immersed partially or fully in a fluid, the resultant hydrostatic force acts on the body in the vertical upward direction. This force is known as upthrust or buoyant force.

F_B : buoyant force

$F_B = \rho g V$

V = volume of the fluid displaced by body

Centre of Buoyancy

It is the point at which upthrust or buoyant force is acting on the body and is exactly same as the centre of gravity of displaced fluid.

Floatation

For floatation of body, the density of the body must be equal to or less than density of liquid i.e.

$$\rho s \le \rho$$

density of solid   density of liquid

NOTE:

For a completely submerged body, the centre of buoyancy doesn't change. However, for a floating body the centre of buoyancy changes when the orientation of body changes.

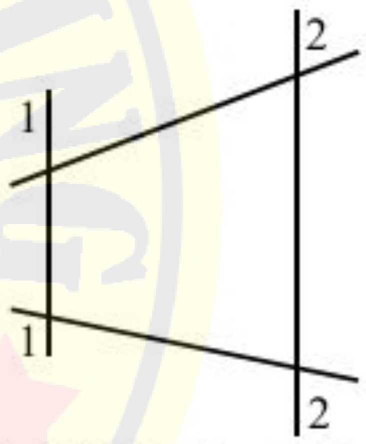
FLUID KINEMATICS

- There are two approaches to kinematics of a fluid flow i.e. Lagragrian approach and Eularian approach.
- In classical fluid mechanics, Eularian approach is considered.

Different Types of Flow

1. ***Steady flow***
   If the properties in the flow are not changing with respect to time, such a flow is known a steady flow.
2. ***Uniform flow***
   If the properties (velocity at any given time) is not changing with respect to space, such a flow is known as uniform flow.
3. ***Incompressible flow***
   If the density of the fluid doesn't change with respect to pressure, the flow is known as incompressible flow.
4. ***Rotational and Irrotational flow***
   If the fluid particles are rotating about their centre of mass, the flow is known as rotational flow. If the fluid particles aren't rotating about their centre of mass, the flow is known as irrotational flow.

- Laminas and turbulent flow: In Laminar flow, individual particles move in a zig-zag way.
  For Reynold's number (R_e).
  If $Re < 2000$, flow in laminar
  If $Re > 4000$, flow in turbulent
  If $2000 < Re < 4000$, flow may be laminar/turbulent
- Rate of flow / Discharge (Q):
  Q = Area × Average velocity
  $Q = A \times V$
- Continuity equation:
  If states if no fluid in added/removed from the pipe in any length then mass passing across different reactions will be equal. Mathematically, for reaction (1 – 1) and (2 – 2),
  $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$



for incompressible fluid, $A_1V_1 = A_2V_2$.

Continuity equation in cartesian - co-ordinates

$$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0}$$

Acceleration of A Fluid Particle

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$a = \frac{\partial \vec{V}}{\partial t} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\text{Convective acceleration}} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{temporal or local acceleration}}$$

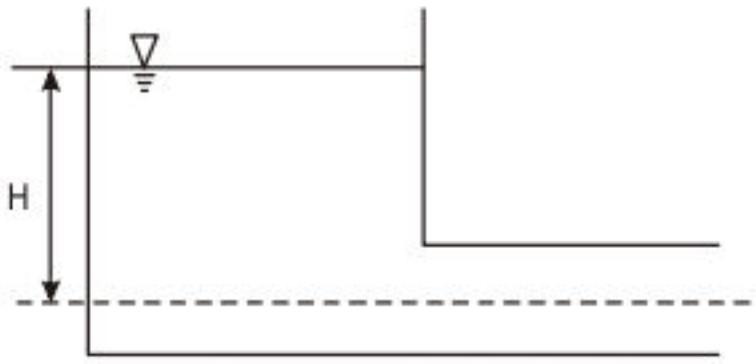
$$a_x = u\frac{\partial U}{\partial x} + v\frac{\partial U}{\partial y} + w\frac{\partial U}{\partial z} + \frac{\partial U}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$a=\sqrt{a^2x+a^2y+a^2z}$$

For uniform flow,
convective acceleration = 0
For steady flow



local/temporal acceleration = 0
For steady and uniform flow,
total acceleration = 0
Consider a tank as shown in figure
For the figure,
convective acceleration = 0
temporal acceleration = 0 (if H is constant)
temporal acceleration ≠ 0 (if H is verying)

Stream Line

It is an imaginary line drawn in such a way that the tangent drawn at any point on this line gives the direction of velocity vector of the fluid particle at that point.

$$\boxed{\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}}$$ Equation of streamline in differential form

PATH LINE
It is the actual path traced by a fluid particle.

STREAKLINE
It is the locus of all fluid particles at a moment which have passed through a given point.

Rotational components in flow

$$\vec{V}=u\hat{i}=v\hat{j}$$

$$w_z=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

where w_z is the net rotation of fluid particle about its own centre of mass.

If $w_z = 0 \Rightarrow$ flow is irrotational
If $w_z \neq 0 \Rightarrow$ flow is rotational

VORTICITY
It is defined as double of angular velocity. (Circulation per unit of enclosed area)
Vorticity = 2ω

CIRCULATION (Γ)
It is defined as the line integral of velocity vector along a closed loop.

$$\Gamma=\oint\vec{V}\cdot dr$$

$$\Gamma=(\text{Vorticity})\,\text{Area}$$

Velocity Potential Function (ϕ)

- Velocity potential function ϕ is a function of space and time.
- It is defined in such a way ϕ that

$$\boxed{\begin{aligned}u&=-\frac{\partial\phi}{\partial x}\\ v&=-\frac{\partial\phi}{\partial y}\\ w&=-\frac{\partial\phi}{\partial z}\end{aligned}}$$

where u, v and w are the components of velocity vector in x, y and z direction.

- ϕ only exists in irrotational flow. For this, ϕ must satisfy laplace equation i.e.

$$\boxed{\nabla^2\phi=0}$$

Equipotential Line

It is a line joining the points having same potential function values.

where $\frac{dy}{dx}=\boxed{\frac{dy}{dx}=-\frac{u}{v}}$

Slope of equipotential line

Stream Function (ψ)

- It is defined only for 2D flows and is a function of space and time.

$$\boxed{\begin{aligned}u&=-\frac{\partial\psi}{\partial y}\\ v&=\frac{\partial\psi}{\partial x}\end{aligned}}$$

- There is no boundation on ψ as it satisfies continuiting equation.

Equistream Line

It is a line obtained by joining points having same stream function values.

$$\frac{dy}{dx}=\frac{v}{u}$$

Slope of equistream line

$$\left(\frac{dy}{dx}\right)_{\phi=\text{constant}}\times\left(\frac{dy}{dx}\right)_{\psi=\text{constant}}=-\frac{u}{v}\times\frac{v}{u}=(-1)$$

∴ Equistream and Equipotential lines are orthogonal to each other.

Cauchy-Riemann Eq^n

In irrotational flows,

$$u=-\frac{\partial\phi}{\partial x}=-\frac{\partial\psi}{\partial y}\Rightarrow\boxed{\frac{\partial\phi}{\partial x}=\frac{\partial\psi}{\partial y}}\quad\ldots(1)$$

$$v=-\frac{\partial\phi}{\partial y}=\frac{\partial\psi}{\partial x}\Rightarrow\boxed{\frac{\partial\phi}{\partial y}=-\frac{\partial\psi}{\partial x}}\quad\ldots(2)$$

Equations (1) and (2) are known as Cauchy-Riemann equations.

FLUID DYNAMICS

Euler's Equation of Motion

The Euler's equation considers the following assumptions

- Flow is irrotational
- Flow is laminar

- Flow is invicid.

$$\boxed{\frac{dP}{\rho} + v\,dv + g\,dz = 0} \rightarrow \text{Euler's Eqn for steady flow}$$

Integrating the above equation. We obtain Bernoulli's equation

$$\boxed{\begin{array}{l} \dfrac{P}{\rho g} + \dfrac{V^2}{2g} + z = \text{constant (Head form)} \\ P + \dfrac{1}{2}\rho V^2 + \rho g z = \text{constant} \end{array}}$$

For Bernoulli's equation, there are two more assumptions i.e.

- flow is steady
- flow is incompressible

Under the five assumptions stated above, the summation of all energies (Pressure, Kinetic and Potential) per unit volume remains constant at each and every point in a flow.

Bernoulli's Equation for Real Fluid

In real fluids, viscous shear stresses are present due to which energy is not conserved.

The Bernoulli's Eq^n in such a case can be written as $\dfrac{P_1}{\rho g} + \dfrac{V_1^2}{2g} + z_1$

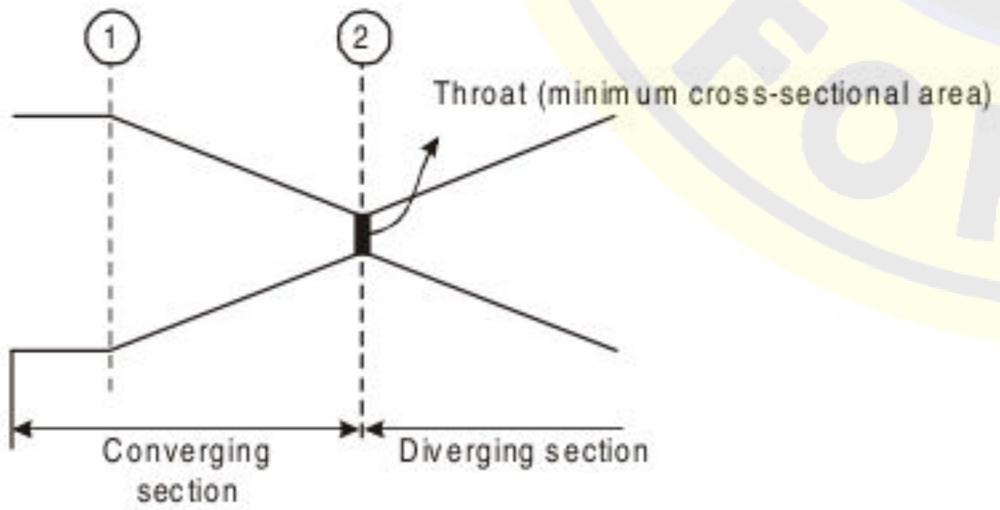
$$= \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

where h_f : head losses encountered as the fluid flows from point 1 to 2.

Flow Measurement Devices

Venturimeter

It is a highly accurate device used for measurement of discharge.



$$Q_{TH} = A_2 A_1 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

$$Q = C_d Q_{TH}$$

Coefficient of discharge (it's value vasics between 0.96 – 0.98)

$$C_d = \sqrt{\frac{h - h_L}{h}}$$

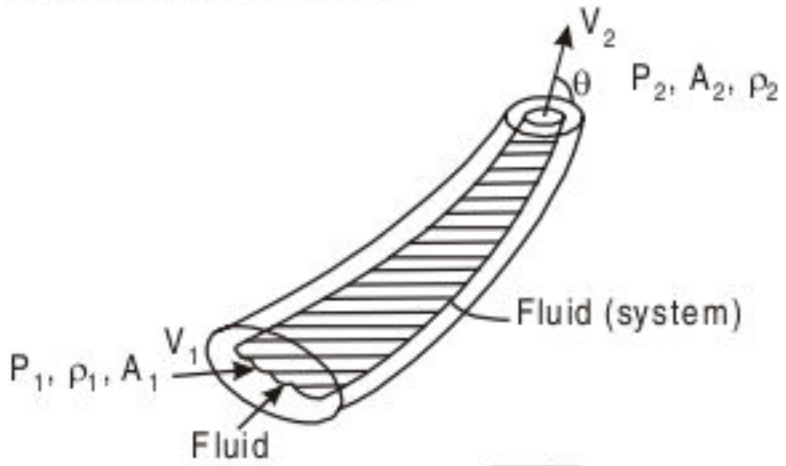
h : piezometric head difference between 1 and 2

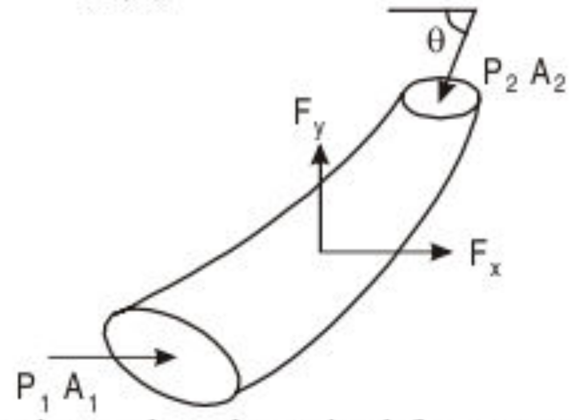
h_L : head loss

Flow Through Pipe Bends

- The main aim of this chapter is to determine the forces.
- The pipe bend is horizontal. Hence, there would be no effect of weight.

Consider a pipe bend as shown,





F_x, F_y are the horizontal and vertical forces acting on the fluid element.

∴ By momentum equation, F_x and F_y can be found

$$P_1A_1 - P_2A_2\cos\theta + F_x = \dot{m}V_2\cos\theta - \dot{m}V_1$$

$$F_y - P_2A_2\sin\theta = \dot{m}V_2\sin\theta$$

$$F = \sqrt{F_x^2 + F_y^2}$$

Vortex flows

- When a certain mass of fluid is rotating with respect to some different axis, such a flow is known as Vortex flow.
- There are 2 types of vortex flow
  - (i) Free vortex
  - (ii) Forced vortex

FREE VORTEX

- No external torque is required. Hence angular momentum remains conserved.

$$V \propto \frac{1}{r} \qquad \begin{array}{l} V : \text{velocity} \\ r : \text{radius} \end{array}$$

FORCED VORTEX

- External torque is required to maintain its angular velocity at a constant value.

$$w = \text{constant}$$

$$\therefore \quad V \propto r$$

NOTE :

- Free vortex flows are irrotational flows and thus, Bernoulli's equation can be applied.
- Forced vortex flows are rotational flows and hence, Bernoulli's equation cannot be applied.

Fundamental Equation of Vortex Flows

$$dp = \rho w^2 r\,dr - \rho g\,dZ$$

General equation and can be applied between any two points

For free surface, $dp = 0$

$$\Rightarrow \quad \rho w^2 r\,dr = \rho g\,dZ$$

Integrating the above equation we get,

$$\boxed{Z = \frac{w^2 r^2}{2g}}$$

- A pipe is a closed contour which carries fluid under pressure.
- When fluid flows through pipe, it encounters losses. These losses can be broadly categorized into
  (i) Major losses
  (ii) Minor losses

Major Losses

- These losses are due to friction. The losses are evaluated by Darcy-Weishback Equation.

$$\boxed{h_f = \frac{f\,LV^2}{2\,gd}} \qquad ...(1)$$

f : friction factors
L : length of pipe
V : velocity in pipe of fluid
d : diameter of pipe
$f = 4f'$
friction coefficient

The above equation (1) is valid for both laminar and turbulent flow.

NOTE :
Head loss is independent of pipe orientation. It depends only on details of the flow through the duct.
For fully developed laminar flow, $f = 64/Re$
where Re : Reynold's No.

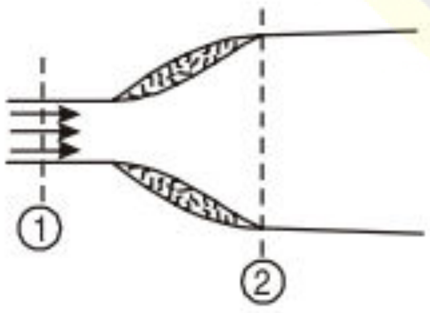
$$Re = \frac{\rho VD}{\mu}$$

V = velocity
D = diameter
m = dynamic Viscosity

Minor Losses

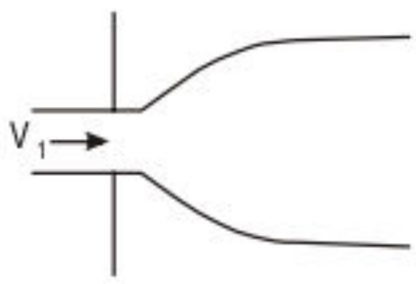
- Bernoulli's Equation, momentum Eq^n are used to determine these losses.
- The magnitude of minor losses is very loss.

(i) Losses Due to Sudden Enlargement



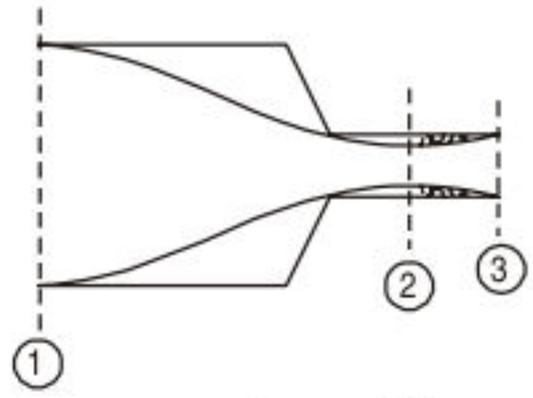
$$h_f = \frac{(V_1 - V_2)^2}{2g}$$

(ii) Exit Losses



$$h_f = \frac{V_1^2}{2g}$$

(iii) Losses Due to Sudden contraction

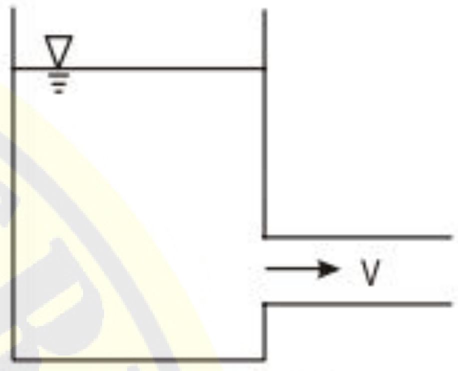


$$h_f = \frac{V_3^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$C_c = \frac{A_2}{A_3}$$

If C_c is not given, $h_f = 0.5\,V_3^2/2g$
Head loss occurs after Venacontracta as boundry layer separation occurs.

(iv) Entrance Losses



$$h_f = \frac{0.5\,V^2}{2g}$$

BEND LOSSES

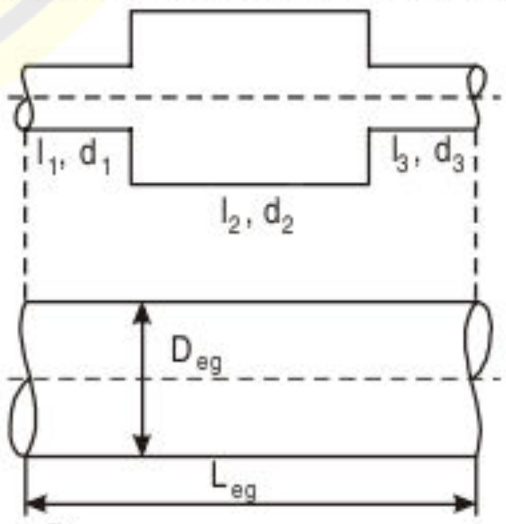
$$h_f = \frac{KV^2}{2g}$$

K = Constant which depends upon angle of bend and its radius of curvature.

FLOW THROUGH BRANCHED PIPES

PIPES IN SERIES

- In series, discharge (Q) remains same but head is divided.



$Q = Q_1 = Q_2 = Q_3$
$h_f = (h_f)_1 + (h_f)_2 + (h_f)_3,$

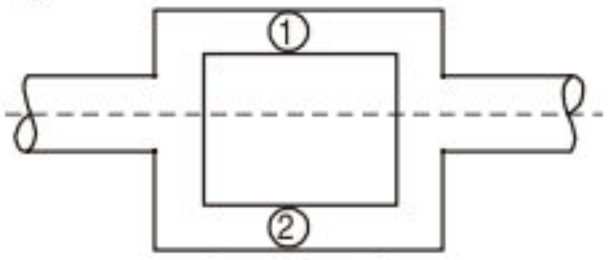
$$\Rightarrow h = \frac{4t}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

Dupit's Equation
A pipe of uniform diameter is said to be equivalent to compound pipe if it carries same discharge and encounters same losses.

$$\Rightarrow \boxed{\frac{L_{eg}}{D_\phi^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}}$$

PIPES IN PARALLEL

- In parallel arrangement, head losses remain same but discharge gets divided.

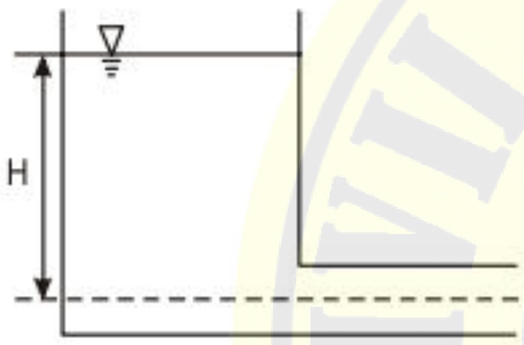


$$Q = Q_1 + Q_2$$
$$(h_f)_1 = (h_f)_2$$

SYPHON

- Siphon is a long bend pipe used in carrying water from a reservoir at higher level to another reservoir at lower level.
- The height point of siphon is called summit.
- No section of the pipe will be more than 7.6 m above the hydraulic gradient line.
- When absolute pressure of water becomes less than 2.7 m gases come out from water and get collected at the summit thereby providing an obstruction to flow.

POWER TRANSMISSION THROUGH PIPE



$$P_{theoretical} = \rho QgH$$
$$P_{actual} = \rho Q\,(H - h_f)$$

where h_f are the head losses in pipe.

$$\eta = \frac{\rho Q\,(H - h_f)}{\rho Q\, gH}$$

for maximum efficiency $\boxed{h_f = \frac{H}{3}}$.

Laminar Flow in Pipes

- At low velocity of real fluids, viscosity is dominant. The flow of fluid takes place in form of laminar. This laminated flow is known as laminar.

Features of Laminar Flow

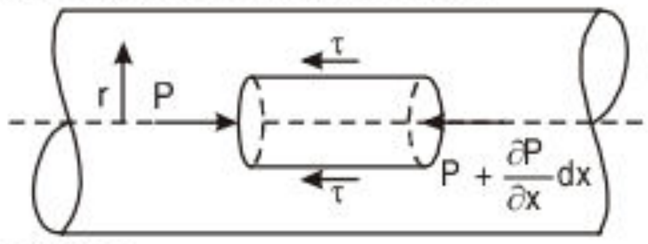
- No slip at boundary
- Flow is rotational
- No mixing of fluid layers
- Shear between fluid layers e = μ du/dy (x-dir.)

Entrance Length

The distance in downstream from the entrance to the location at which fully developed flow begins is called entrance length for laminar flow in pipes. $\frac{L_e}{D} = 0.06\, R_e$

L_e = entrance length
D = diameter of pipe

Steady Laminar Flow in Circular Pipes



τ : shear stress
R : radius of pipe
μ : dynamic viscosity of fluid
$\frac{\partial P}{\partial x}$: pressure gradient
u : velocity at a distance 'r' from cente

$$\tau = \left(-\frac{\partial P}{\partial x}\right)\frac{r}{2}$$

$$u = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)(R^2 - r^2)$$

from above expression of 'u', we can conclude that velocity is varying parabolically.

$$u_{max} = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2, \quad Q = \left(\frac{\pi R^2}{2}\right)U_{max}$$

$$\bar{u} = \frac{U_{max}}{2}$$

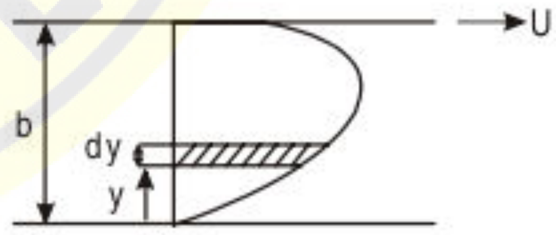
at $r = R/\sqrt{2}$, $= \bar{U} = U$ i.e. average velocity equals the local velocity.

Pressure drop $(P_1 - P_2)$ in a given finite length 'L'

$$P_1 - P_2 = \frac{32\,\mu\,\bar{U}\,L}{D^2}$$

LAMINAR FLOW BETWEEN TWO PARALLEL PLATES

Case I : One plate is moving with a velocity of 'U' while the other is stationary.



$$u = \frac{U_y}{b} - \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(by - y^2)$$

Case II : When both plates are at rest

$$u = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left(by - y^2\right) \text{ (Poiseuille flow)}$$

$$u_{max} = -\frac{1}{8\mu}\left(\frac{\partial P}{\partial x}\right)^{b^2}$$

$$\bar{U} = \frac{2}{3}U_{max}$$

TURBULENT FLOWS

- In turbulent flow, there is continuous mixing of fluid particles and hence velocity fluctuates continuously.
- u′ and v′ are fluctuating components of velocity

- $\tau = \rho u'v' = \rho l^2 \left(\frac{du}{dy}\right)^2$

  τ = turbulent shear stress
  l : mixing length, $l = 0.4$ y, y is distance from pipe wall
  Mixing length is the length in transverse direction where in fluid particles after colliding loose excess momentum and reach the momentum as of local environment.

- $\frac{U_{max} - u}{V_*} = 5.75 \log_{10}(R/y)$

  V_* : Shear velocity $V_* = \sqrt{\frac{\tau}{\rho}}$

- $\frac{u}{V_*} = 5.75 \log_{10}\left(\frac{y}{y'}\right)$

  $y' = \delta'/10^7$ (for smooth pipes)
  $y' = K/30$ (for rough pipes)

- Reynold's condition for rough & smooth pipes

  $R_{e_R} = \frac{V_* K}{\nu}$

  $R_{e_R} > 100 \Rightarrow$ rough pipe

  $R_{e_R} < 4 \Rightarrow$ smooth pipe

  $4 < R_{e_R} < 100 \Rightarrow$ transition

- In turbulent flow in pipes, average velocity equals local velocity at $y = 0.223$ R.

Thickness of Laminar Sublayer (δ')

$$\delta' = \frac{11.6\,\nu}{V_*}$$

Hydrodynamically Rough and Smooth Boundaries

From Nikuradsee's experiment,

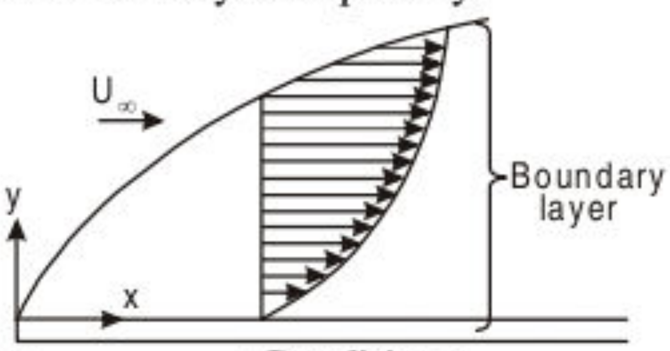
$\frac{K}{\delta'} < 0.25 \Rightarrow$ smooth boundary

$\frac{K}{\delta'} > 6 \Rightarrow$ rough boundary

$0.25 < \frac{K}{\delta'} < 6 \Rightarrow$ transition

BOUNDARY LAYER THEORY

- The concept of boundary layer was first introduced by L. Prandtl.
- Boundary layer is a layer in the vicinity of the surface with large velocity gradients existing in it.
- Velocity within the boundary layer increases from zero to main stream velocity asumptically.



Boundry	*Conditions*
at $y = 0$	$u = 0$
$y = \delta$	$u = 0.99U_\infty$
$y = \delta$	$\frac{\partial U}{\partial y} = 0$
$y = 0$	$\frac{\partial^2 U}{\partial y^2} = 0$

δ : boundary layer thickness
U_∞ : free stream velocity

- Nominal thickness is the thickness of boundary layer for which $\mu = 0.99\,U_\infty$
- In case of a converging flow ($\partial P/\partial x = -$ ve), the boundary layer growth is retarded.
- For separation of boundary layer, $\frac{\partial P}{\partial x} > 0$.
- $\delta \propto x^{1/2}$ 'x' is the distance from leading edge of the plate. As x increase, boundary layer thickness increases.
- The transition from laminar to turbulent flow is decided by Reynold's No.

  $R_e \le 5 \times 10^5 \Rightarrow$ flow is laminar
  $R_e > 6 \times 10^5 \Rightarrow$ flow is turbulent

Displacement thickness (δ^*)

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

Momentum thickness (θ)

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

Energy thickness (δ_E)

$$\delta_E = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right) dy$$

Shape factor (H) $= \frac{\delta^*}{\theta}$

Von Karman's Momentum Integral Equation

Assumptions

- Flow is 2D, incompressible and steady
- $\frac{dP}{dx} = 0$ $\boxed{\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx}}$

  where θ : momentum thickness
  τ_0 : plate shear stress
  ρ : density
  U_∞ : free stream velocity

Drag force (F_D)

It is the force exerted by the fluid in a direction parallel to relative motion.

A zero angle of incidence, of the plate the drag force is due to shear force.

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U_\infty^2} \qquad C_{fx} = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2}$$

C_D = average drag coefficient
C_{fx} = local drag coefficient

For air flow over a flat plate, velocity (U) and boundary layer thickness (δ) can be expressed as

$$\frac{U}{V_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

$$\delta = \frac{4.64\,x}{\sqrt{Re_x}}$$

EXERCISE

1. The velocity components in the x and y direction of a two-dimensional potential flow are u and v respectively, then $\frac{\partial u}{\partial x}$ is equal to
   (a) $\frac{\partial v}{\partial x}$ (b) $-\frac{\partial v}{\partial x}$
   (c) $\frac{\partial v}{\partial y}$ (d) $-\frac{\partial v}{\partial y}$
2. The velocity profile in fully developed laminar flow in a pipe of diameter D is given by $u = u_0(1 - 4r^2/D^2)$, where r is the radial distance from the centre. If the viscosity of the fluid is μ, the pressure drop across a length L of the pipe is
   (a) $\frac{\mu u_0 L}{D^2}$ (b) $\frac{4\mu u_0 L}{D^2}$
   (c) $\frac{8\mu u_0 L}{D^2}$ (d) $\frac{16\mu u_0 L}{D^2}$
3. A two-dimensional flow field has velocities along x and y directions given by $u = x^2t$ and $v = -2xyt$ respectively, where t is time. The equation of streamlines is
   (a) $x^2 y$ = constant
   (b) xy^2 = constant
   (c) xy = constant
   (d) not possible to determine
4. In a two-dimensional velocity field with velocities u and v along the x and y directions respectively, the convective acceleration along the x-direction is given by
   (a) $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$ (b) $u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}$
   (c) $u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial y}$ (d) $v\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y}$
5. For a Newtonian fluid
   (a) shear stress is proportional to shear strain
   (b) rate of sheer stress is proportional to shear strain
   (c) shear stress is proportional to rate of shear strain
   (d) rate of shear stress is proportional to rate of shear strain
6. For the continuity equation given $\nabla \cdot \vec{v} = 0$ to be valid, when $\vec{v}$ is the velocity vector, which one of the following is a necessary condition?
   (a) Steady flow (b) Irrotational flow
   (c) Invescid flow (d) Incompressible flow
7. Match the following:

P.	Compressive flow	U.	Reynolds number
Q.	Free surface flow	V.	Nusselt number
R.	Boundary layer flow	W.	Weber number
S.	Pipe flow	X.	Froude number
T.	Heat convection	Y.	Match number
		Z.	Skin friction coefficient

   (a) P-U; Q-X; R-V; S-Z; T-W
   (b) P-W; Q-X; R-Z; S-U; T-V
   (c) P-Y; Q-W; R-Z; S-U; T-X
   (d) P-Y; Q-W; R-Z; S-U; T-V
8. A hydraulic turbine develops 1000 kW power for a head of 40 m. If the head is reduced to 20 m, the power developed (in kW) is
   (a) 177 (b) 354
   (c) 500 (d) 707
9. A phenomenon is modelled using n dimensional variables with k primary dimensions. The number of non-dimensional variable is
   (a) k (b) n
   (c) n – k (d) n + k
10. The maximum velocity of a one-dimensional incompressible fully developed viscous flow, between two fixed parallel plates is 6 m/s. The mean velocity (in m/s) of the flow is
   (a) 2 (b) 3
   (c) 4 (d) 5
11. A pump handling a liquid raises its pressure from 1 bar to 30 bar. Take density of the liquid as 990 kg/m^3. The isentropic specific work done by the pump in kJ/kg is
   (a) 0.10 (b) 0.30
   (c) 2.50 (d) 2.93
12. A streamline and an equipotential line in a flow field
   (a) are parallel to each other
   (b) are perpendicular to each other
   (c) intersect at an acute angle
   (d) are identical
13. For steady, fully developed flow inside a straight pipe of diameter D, neglecting gravity effects, the pressure drop Δp over a length L and the wall shear stress τ_w are related by
   (a) $\tau_w = \frac{\Delta p\, D}{4L}$ (b) $\tau_w = \frac{\Delta p\, D^2}{4L^2}$
   (c) $\tau_w = \frac{\Delta p\, D}{2L}$ (d) $\tau_w = \frac{4\Delta p L}{D}$
14. Biot number signifies the ratio of
   (a) convective resistance in the fluid to conductive resistance in the solid
   (b) conductive resistance in the solid convective resistance in the fluid
   (c) inertia force to viscous force in the fluid
   (d) buoyancy force to viscous force in the fluid
15. A flow field which has only convective acceleration is
   (a) a steady uniform flow
   (b) an unsteady uniform flow
   (c) a steady non-uniform flow
   (d) an unsteady non-uniform flow

16. Match Group A with Group B:

Group A	Group B
P: Biot number	1 Ratio of buoyancy to viscous force
Q: Grashof number	2 Ratio of inertia force to viscous force
R: Prandtl number	3 Ratio of momentum to thermal diffusivities
S: Reynolds number	4 Ratio of internal thermal resistance to boundary layer thermal

(a) P-4, Q-1, R-3, S-2 (b) P-4, Q-3, R-1, S-2
(c) P-3, Q-2, R-1, S-4 (d) P-2, Q-1, R-3, S-4

17. Consider the following statements regarding streamline(s):
(i) It is a continuous line such that the tangent at any point on it shows the velocity vector at that point
(ii) There is no flow across streamlines
(iii) $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$ is the differential equation of a streamline, where u, v and w are velocities in directions x, y and z, respectively
(iv) In an unsteady flow, the path of a particle is a streamline. Which one of the following combinations of the statements is true?
(a) (i), (ii), (iv) (b) (ii), (iii), (iv)
(c) (i), (iii), (iv) (d) (i), (ii), (iii)

18. Consider a velocity field $\vec{V} = K\left(y\hat{i} + x\hat{k}\right)$, where K is a constant. The vorticity, Ω_Z, is
(a) $-K$ (b) K
(c) $-K/2$ (d) $K/2$

19. Length of mercury column at a place at an altitude will vary with respect to that at ground in a
(a) linear relation
(b) hyperbolic relation
(c) parabolic relation
(d) manner first slowly and then steeply

20. A type of flow in which the fluid particles while moving in the direction of flow rotate about their mass centre, is called
(a) steady flow (b) uniform flow
(c) laminar flow (d) rotational flow

21. A-2d flow having velocity $V = (x + 2y + 2)\,i + (4 - y)j$ will be
(a) compressible and irrotational
(b) compressible and not irrotational
(c) incompressible and irrotational
(d) incompressible and not irrotational

22. Buoyant force is
(a) resultant of upthrust and gravity forces acting on the body
(b) resultant force on the body due to the fluid surrounding it
(c) resultant of static weight of body and dynamic thrust of fluid
(d) equal to the volume of liquid displaced by the body

23. If cohesion between molecules of a fluid is greater than adhesion between fluid and glass, then the free level of fluid in a dipped glass tube will be
(a) higher than the surface of liquid
(b) same as the surface of liquid
(c) lower than the surface of liquid
(d) unpredictable

24. In a pipe pitot tube arrangement the static stagnation head is 20 m and static head is 5m. If the diameter of pipe is 400 mm. Find the velocity of flow of water in pipe
(a) 17.15 m/s (b) 22.22 m/s
(c) 38.76 m/s (d) 42.85 m/s

25. Depth of oil having specific gravity 0.6 to produce a pressure of 3.6 kg/cm^2 will be
(a) 40 cm (b) 36 cm
(c) 50 cm (d) 60 cm

26. The capillary rise in a narrow two-dimensional slit of width 'w' is
(a) half of that in a capillary tube of diameter 'w'
(b) two-third of that in a capillary tube of diameter 'w'
(c) one-third of that in a capillary tube of diameter 'w'
(d) one-fourth of that in a capillary tube of diameter 'w'

27. For a turbulent flow in pipe the value of y at which the point velocity is equal to the mean velocity of flow, is (y is measured form pipe axis).
(a) 0.772 R (b) 0.550 R
(c) 0.223 R (d) 0.314 R

28. Pressure in Pascals at a depth of 1 m below the free surface of a body of water will be equal to
(a) 1 Pa (b) 98.1 Pa
(c) 981 Pa (d) 9810 Pa

29. A circular disc of diameter 'd' is slowly rotated in a liquid of large viscosity μ at a small distance h from a fixed surface. The minimum torque required to maintain an angular velocity ω will be
(a) $\frac{\mu\pi d^4\omega}{32h}$ (b) $\frac{\mu\pi d^4\omega}{8h}$
(c) $\frac{\mu\pi d^3\omega}{32h}$ (d) $\frac{\mu\pi d^3\omega}{8h}$

30. Viscosity of a fluid with specific gravity 1.3 is measured to be 0.0034 Ns/m^2. Its kinematic viscosity, in m^2/s, is
(a) 2.6×10^{-6} (b) 4.4×10^{-6}
(c) 5.8×10^{-6} (d) 7.2×10^{-6}

31. The shear stress at a point in a glycerine mass in motion if the velocity gradient is 0.25 metre per sec/per meter, will be
(a) 0.0236 kg/m^2 (b) 0.02036 kg/m^2
(c) 0.0024 kg/m^2 (d) none of these

32. The general equation of continuity for three-dimensional flow of a incompressible fluid for steady flow is
(a) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{\partial w}{\partial z} = 0$
(c) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 1$ (d) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = u.v.w$
where u, v and w are components of velocity in x, y and z directions respectively.

33. The pressure in meters of oil (specific gravity 0.85) equivalent to 42.5 m of water is
(a) 42.5 m (b) 50 m
(c) 52.5 m (d) 85 m

34. The cause of turbulence in fluid flow may be
(a) high Reynold number
(b) abrupt discontinuity in velocity distribution
(c) critical Reynold number
(d) existence of velocity gradient without abrupt discontinuity

35. For an irrotational flow the equation $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}$ is known as
(a) Bernoulli's equation
(b) Cauchy Riemann's equation
(c) Euler's equation
(d) Laplace equation.

36. A control volume refers to
(a) a closed system (b) a specified mass
(c) an isolated system (d) a fixed region in space

37. The pressure coefficient may take the form
(a) $\frac{\Delta P}{\sigma\mu v}$ (b) $\frac{\Delta P}{(\sigma v^2/2)}$
(c) $\frac{\Delta P}{(\mu^2\ell^t v^2/\sigma)}$ (d) $\frac{\sigma\mu^2}{2\Delta P}$

38. Pressure force on the 15 cm dia head light of an automobile travelling at 0.25 m/s is
(a) 10.4 N (b) 6.8 N
(c) 4.8 N (d) 3.2 N

39. A fire engine supplies water to a hose pipe L m long and D mm in diameter at a pressure P kPa. The discharge end of the hose pipe has a nozzle of diameter d fixed to it. Determine the diameter d of nozzle so that the momentum of the issuing it may be maximum
(a) $d=\left(\frac{D^3}{\delta L}\right)^{1/4}$ (b) $d=\left(\frac{D^5}{\delta L}\right)^{1/4}$
(c) $d=\left(\frac{D^3}{\delta L}\right)^{1/5}$ (d) $d=\left(\frac{D^5}{\delta L}\right)^{1/5}$

40. When pressure p, flow rate Q, diameter D, and density d, a dimensionless group is represented by
(a) $\frac{pQ^2}{dD^4}$ (b) $\frac{p}{dQ^2D^4}$
(c) $\frac{pD^4d}{Q^2}$ (d) $\frac{pD^4}{dQ^2}$

41. Maximum wall shear stress for laminar flow in tube of diameter D with fluid properties μ and ρ will be
(a) $\frac{32000\mu^2}{\rho D^3}$ (b) $\frac{6400\mu^2}{\rho D^3}$
(c) $\frac{16000\mu^2}{\rho D^3}$ (d) $\frac{8000\mu^2}{\rho D^3}$

42. Air flows over a flat plate 1 m long at a velocity of 6 m/s. The shear stress at the middle of plate will be
[Take S = 1.226 kg/m^3, v = 0.15 × 10^{-4} m^3/s (0.15 stokes) for air]
(a) 84.84×10^{-3} N (b) 92.69×10^{-3} N
(c) 67.68×10^{-3} N (d) 103.45×10^{-3} N

43. The friction head lost due to flow of a viscous fluids through a circular pipe of length L and diameter d with a velocity v, and pipe friction factor 'f' is
(a) $\frac{4fL}{d}\cdot\frac{v^2}{2g}$ (b) $\frac{4fL}{\pi d^2}\cdot\frac{v^2}{2g}$
(c) $\frac{v^2}{2g}$ (d) $\frac{4fL}{\pi d}\cdot\frac{v^2}{2g}$

44. Value of coefficient of compressibility for water at ordinary pressure and temperature is
(a) 1000 kg/cm^3 (b) 2100 kg/cm^3
(c) 2700 kg/cm^3 (d) 21,000 kg/cm^3

45. Crude oil of kinematic viscosity 2.25 stokes flows through a 20 cm diameter pipe, the rate of flow being 1.5 litres/sec. The flow will be
(a) laminar (b) turbulent
(c) uncertain (d) None of these

46. Pseudo plastic is a liquid for which
(a) dynamic viscosity decreases as the rate of shear increases
(b) Newton's law of viscosity holds good
(c) dynamic viscosity increases as the rate of shear increases
(d) dynamic viscosity increases with the time for which shearing forces are applied.

47. Match List I with List II and select the correct answer using the codes given below the lists.

List I (Loss)	List II (Parameter responsible)
A. Leakage Loss	1. Zero at design point
B. Friction Loss	2. Proportional to head
C. Entrance Loss	3. Proportional to half of relative velocity square.

Codes

	A	B	C
(a)	1	2	3
(b)	2	3	1
(c)	1	3	2
(d)	2	1	3

48. The velocity potential in a flow field is $\phi = 2xy$. The corresponding value of stream function is
(a) $(y^2 - x^2)$ + constant (b) $(x^2 - y^2)$ + constant
(c) $\frac{1}{2}(x^2 - y^2)$ + constant (d) $2(x - y)$ + constant

49. Consider the following
The components of velocity u and v along X and Y directions in a two dimensional flow problem of an incompressible fluid are
(i) $u = x^2 \cos y$; $v = -2x \sin y$
(ii) $u = x + 2$; $v = 1 - 4$
(iii) $u = xyt$; $v = x^3 - y^{2t/2}$
(iv) $\ln u = xty$; $v = xy - y/x$
Which of that will satisfy the continuity equation ?
(a) 1, 2 and 3 (b) 1, 2 and 4
(c) 2, 3 and 4 (d) 1, 2, 3 and 4

50. Consider the following statements regarding bernaulli's theorom for fluid flow
1. Conservation of energy
2. Steady flow
3. Viscous flow
4. In compressible flow
Which of the above statements is/are correct?
(a) 1, 2 and 4 (b) 1 only
(c) 2, 3 and 4 (d) 1, 2, 3 and 4

ANSWER KEY															
1	(d)	9	(b)	17	(d)	25	(d)	33	(b)	41	(c)	49	(a)
2	(d)	10	(c)	18	(a)	26	(a)	34	(d)	42	(b)	50	(a)
3	(a)	11	(d)	19	(d)	27	(c)	35	(d)	43	(a)		
4	(a)	12	(b)	20	(d)	28	(d)	36	(d)	44	(d)		
5	(c)	13	(a)	21	(d)	29	(a)	37	(b)	45	(a)		
6	(d)	14	(b)	22	(d)	30	(a)	38	(b)	46	(a)		
7	(b)	15	(c)	23	(c)	31	(b)	39	(b)	47	(a)		
8	(b)	16	(a)	24	(a)	32	(a)	40	(a)	48	(a)		

HINTS AND EXPLANATIONS

1. **(d)** For two-dimensional flow, continuity equation has to be satisfied.

$$\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=0 \Rightarrow \frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$$

2. **(d)** Pressure drop across a straight pipe of length L is given by

$$\Delta p=\frac{32\mu v_{av}L}{D^2} \qquad \text{...(i)}$$

Here, $u_{av}=\frac{u_0}{2}$ ...(ii)

From Eqs. (i) and (ii),

$$\Delta p=\frac{16\mu u_0 L}{D^2}$$

3. **(a)** Given u x^2t and $v=-2xyt$

We know, $\frac{\partial\psi}{\partial x}=v=-2xyt$ ...(i)

and $\frac{\partial\psi}{\partial x}=-u=-x^2t$ ...(ii)

Integrating Eq. (i), we get

$\psi=x^2yt+f(y)$ ...(iii)

Differentiating Eq. (iii) w.r.t. y (iv) we get

$$\frac{\partial\psi}{\partial y}=-x^2t+f'(y) \qquad \text{...(iv)}$$

From equation (ii) putting value of $\frac{\partial\psi}{\partial y}$ in equation (iv) we get,

$-x^2t=-x^2t+f'(y)$

Since, $f'(y)=c$

$\Rightarrow \psi=-x^2yt+c$

c is a numerical constant taking it zero.

$\psi=-x^2yt$

For the equations of streamlines, ψ = constant

$\therefore\ -x^2yt$ = constant

For a particular instance, x^2y = constant

4. **(a)** In a two-dimensional velocity fluid with velocities u, v along x and y directions.

Acceleration along x direction

$a_x = a_{convective} + 3_{temporal\ or\ local}$

$$=\underbrace{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}}_{\text{convective acceleration}}+\underbrace{w\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial t}}_{\text{temporal acceleration}}$$

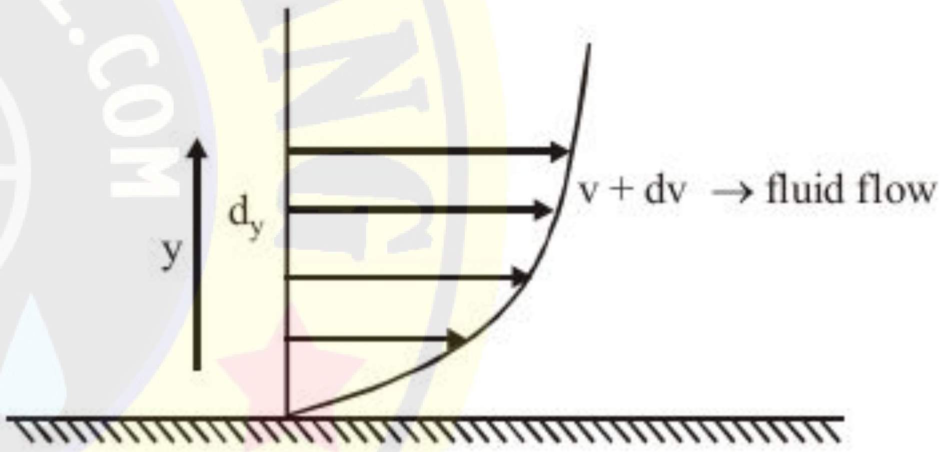
Since, $\frac{\partial u}{\partial z}=0$: for 2-dimensional flow.

$\therefore$ Convective acceleration $=u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}$.

5. **(c)** For Newtonian fluid,

Shear stress $\propto \frac{dy}{dy}$

where, $\frac{dy}{dy}$ = velocity gradient



Now, $\tau \propto \frac{dy}{dy}$

$\propto \frac{dx}{dt\,dy}$

$\propto \frac{(dx/dy)}{dt}$

where, $\frac{(dx/dy)}{dt}$ = rate of change of shear strain.

6. **(d)** Continuity equation

$\vec{\nabla}\cdot\vec{v}=0$

$$\Rightarrow \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Multiplying by density on both sides, we get,

$$\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0$$

This is the equation for compressible flow.

7. **(b)**

Compressive flow	Weber number
Free surface flow	Froude number
Boundary layer flow	Skin friction coefficient
Pipe flow	Reynolds number
Heat convection	Nusselt number

8. (b) The relation between head and power is given by

Pu = Power/Head $^{1.5}$

Thus, $\dfrac{\text{Power}_1}{(\text{Head}_1)^{1.5}} = \dfrac{\text{Power}_2}{(\text{Head}_2)^{1.5}}$ ... (i)

Given, $\text{power}_1 = 1000$ kW, $\text{Head}_1 = 40$ m

$\text{Head}_2 = 20$ m

Putting values in the Eq. (i) and solving, we get

$\text{Power}_2 = 353.6 = 354$ kW

10. (c) In case of two fixed parallel plates, when the flow is fully developed, the ratio v_{max} / v_{avg} is given by

$$\frac{v_{max}}{v_{avg}} = \frac{3}{2}$$

Thus, $v_{avg} = (2/3) \times v_{max}$. $= 2 \times 6/3 = 4$ m/s

11. (d) $p_1 = 1$ bar, $p_2 = 30$ bar

$\rho = 9900\ kg/m^3$.

Specific work done $W = Vdp = \dfrac{m}{\rho}dp$

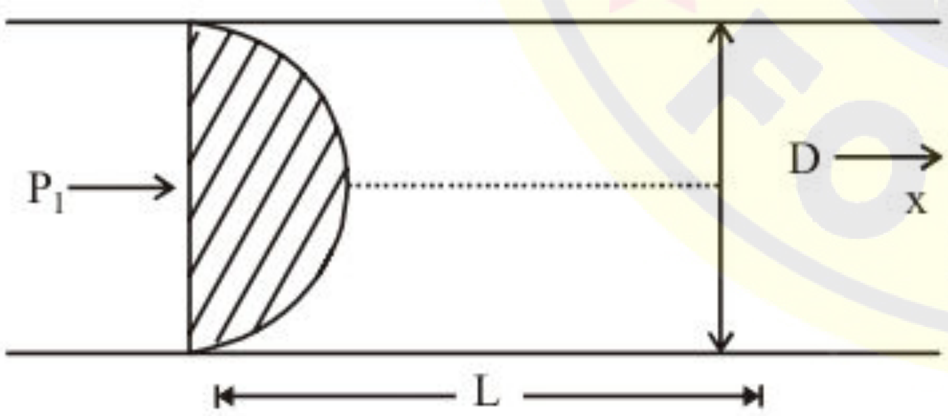
$$\frac{W}{m} = \frac{dp}{\rho} = \frac{(30-1)\times 10^5}{990\times 1000} = 2.93\ kJ/kg$$

12. (b) A streamline and equipotential line in a flow field are perpendicular to each other (because, when $(\text{slope})_1$ and $(\text{slope})_2$ are multiplied and we get, $(\text{slope})_1 \times (\text{slope})_2 = -1$

13. (a) Assumption :

(i) Flow is steadyy (i.e.) $\dfrac{\partial}{\partial t}() = 0$

(ii) Fully developed the $\dfrac{\partial}{\partial t}() - 0$; properties are not changing in the direction of the flow.



→ Pressure is constant along the vertical axis.

→ Pressure along horizontal axis does change.

$\Delta P \,.\, P_2 \,.\, P_1 < 0$

Apply N2M (2nd) over the length l

⇒ $P_1\pi r^2 - (P_1 - |\Delta P|)\ \pi r^2 - 2\pi r\ l_{ic}$

$$\frac{\Delta P}{L} = \frac{2\tau}{6L}$$

Neither P nor l depend as on r.

So, $\dfrac{\tau}{\varepsilon}$ is independ at then (t = r where (is on stat)

At center r = 0, $\tau = c \times 0 = 0$

$$\tau = \frac{2\tau\ wr}{D}$$

$$\frac{\Delta P}{L} = \frac{2}{r}\times\frac{\tau w\times r}{D} \Rightarrow TW = \frac{\Delta PD}{4L}$$

21. (d) $u = x + 2y + 2$

$v = 4 - y$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$$

As it satisfy the continuity equation for incompressible flow so this is incompressible

$$w_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}[0-2] = -1$$

and since rotational component is not zero so flow is not irrotational.

24. (a) stagnation head = 20 m

static head = 5 m

dynamic head = 20 – 5 = 15 m

Now, $\dfrac{u^2}{2g} = 15$

u = 17.15 m/s

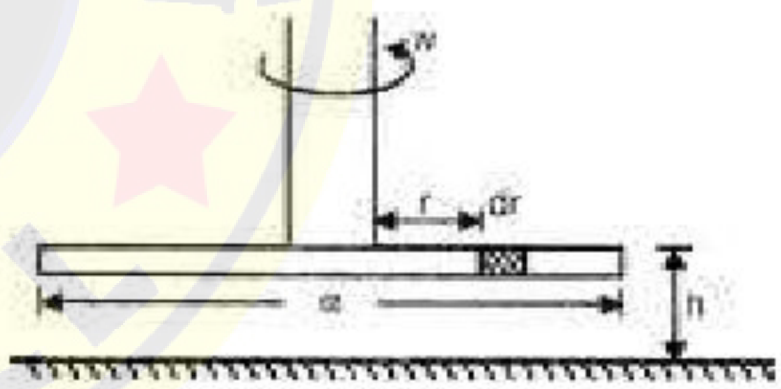
27. (c) $\dfrac{v-u}{u*} = 5.75\log_{10}\left(\dfrac{y}{k}\right) + 3.75$

for v = u

$$5.75 + \log_{10}\left(\frac{y}{k}\right) + 3.75 = 0$$

$$\frac{y}{R} = 0.223 \Rightarrow y = 0.223\ R$$

29. (a) Consider an element of disc at a distance r and having width dr.



Linear velocity at this radius = $r\omega$

Shear stress = $\mu\dfrac{du}{dy}$

torque = shear stress × area × r = $\tau\, 2\,\pi\, r\, dr \times r$

$$= \mu\frac{du}{dy}2\pi r^2 dr$$

assuming that gap h is small so that velocity distribution may be assumed linear

$$\frac{du}{dy} = \frac{r\omega}{h}$$

$$dT = \mu\frac{r\omega}{h}2\pi r^2 dr = \frac{2\pi\mu\omega}{h}r^3 dr$$

$$T = \int_0^{d/2}\frac{2\pi\mu\omega}{h}r^3 dr = \frac{\mu\pi d^4\omega}{32h}$$

31. **(b)** Velocity gradient $= \frac{du}{dy} = 0.25$ m/sec meter

Kinematic Viscosity, $\upsilon = 6.30 \times 10^{-4}\,m^2/s$

Shear stress $= \mu \frac{du}{dy} = \rho\nu \frac{du}{dy}$

$= 129.3 \times 6.30 \times 10^{-4} \times 0.25$ (As $\rho = 129.3$)

$= 0.02036\ kg/m^2$

39. **(b)** Momentum of issuing jet is $M = \frac{w}{g} QV_2$

$$M = \frac{w}{g}\frac{\pi d^2}{4}V_2^2$$

from continuity equation

$$Q = \frac{\pi}{4}D^2V_1 = \frac{\pi}{4}d^2V_2$$

If H is the head causing the flow, then

$$H = \frac{V_2^2}{2g} + \frac{fLV_1^2}{2gD}$$

$$2gH = V_2^2\left(1 + \frac{fLd^4}{D^5}\right)$$

$$M = \frac{w}{h}\frac{\pi}{4}d^2 \frac{2gH}{\left(1+\frac{fLd^4}{D^5}\right)}$$

$$M = w\frac{\pi}{4}2H\frac{D^5d^2}{(D^5 + fld^4)}$$

for maximum momentum

$$\frac{dM}{dd} = 0$$

$$\frac{dM}{dd} = w\frac{\pi}{4}2H = D^5\left[\frac{2d(D^5+fld^4) - d^2(4fLd^3)}{(D^5+fld^4)^2}\right] = 0$$

$$d = \left(\frac{D^5}{fL}\right)^{1/4}$$

41. **(c)** $u = \frac{-1}{4\mu}\left(\frac{\partial P}{\partial x}\right)(r_0^2 - r^2)$

$$\tau_0 = \frac{D}{4}\left(-\frac{\partial P}{\partial x}\right) \text{ at } r = \frac{D}{2}$$

$$\Delta P = \frac{32\mu VL}{D^2}$$

$$\frac{\Delta P}{L} = \frac{32\mu V}{D^2}$$

For τ_0 to be maximum $\frac{\Delta P}{L}$ should be maximum so V should be maximum. In laminar flow, maximum velocity will be

attained when $\frac{\rho VD}{\mu} = 2000$

$$\frac{\Delta P}{L} = \frac{32\mu \times 2000\mu^2}{\rho D \cdot D^2} = \frac{64000\mu^2}{\rho D^3}$$

$$\tau_0 = \frac{16000\mu^2}{\rho D^3}$$

42. **(b)** $R_e = \frac{VL}{\nu} = \frac{6\times 1}{0.15\times 10^{-4}} = 4\times 10^5$

Hence the boundary layer is laminar over the entire length of the plate.

$$\delta = 5\sqrt{1\times 0.15\times 10^{-4}} = 5\sqrt{\frac{x\nu}{v}} = 5\sqrt{\frac{1\times 0.15\times 10^{-4}}{6}}$$

$= 7.91 \times 10^{-3}$ m $= 7.91$ mm

$R_{ex} = \frac{6\times 0.5}{0.15\times 10^{-4}} = 2\times 10^5$ (for middle point of plate)

$$c_D = \frac{1.328}{\sqrt{Rc_L}} = \frac{1.328}{\sqrt{4\times 10^5}} = 2.1\times 10^{-3}$$

$$F_D = 2\times 1\times 1\times c_f\frac{\delta V^2}{2}$$

$$= 2\times 1\times 1\times 2.1\times 10^{-3}\times\frac{1226(6)^2}{2} = 92.69\times 10^{-3}\,N$$

45. **(a)** Kinematic viscosity, $\upsilon = 2.25$

dia of pipe, d = 20 cm

Rate of flow = 1.5 liters/sec

Now to find the flow we must know the reynolds number

$$R_e = \frac{vd}{\nu}$$

So, $\upsilon = \frac{15000}{\left(\frac{\pi}{2}\right)\times(20)^2} = 47.75$ cm/sec.

Now, $R_e = \frac{\upsilon d}{\nu} = \frac{20\times 47.75}{2.25} = 424.4$

$R_e = 424.4$, means Reynolds number of this flow is less then 2000 ($424.4 < 2000$)

Hence the flow is "Laminar"

48. **(a)** Given here, $\phi = 2xy$, conidering the following relation,

$$\Rightarrow 4 = \frac{-\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y}$$

$$\Rightarrow \frac{\partial\psi}{\partial x} = \left(-\frac{\partial\phi}{\partial y}\cdot\frac{\partial\psi}{\partial x}\right) = \frac{\partial\phi}{\partial x}$$

$$\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}(2xy) = 2y = -\frac{\partial\psi}{\partial y}$$

Similarly,

$$\frac{\partial \phi}{\partial y} = 2x = -\frac{\partial \psi}{\partial x}$$

On integrating, $\int \frac{\partial \psi}{\partial x} = -2x \Rightarrow \psi = -\frac{2x^2}{z} + c(y)$

$$\frac{\partial \psi}{\partial y} = 0 + c'(y)$$

$$\int c'(y) = \int 2y$$

$c(y) = \frac{2y^2}{2} + c_1$, 1then, $\Psi = \frac{-2x^2}{2} + \frac{2y^2}{2} + c_1$

$\psi = y^2 - x^2 + c_1$ or $y^2 - x^2 +$ constant

49. (a) If flow in 2D, continuity equation becomes,

$$\frac{\partial y}{\partial x} + \frac{\partial v}{\partial y} = 0$$

So, for (i), $u = x^2 \cos y$, $v = -2x \sin y$

$$\frac{\partial y}{\partial x} + \frac{\partial v}{\partial y} = 2x\cos y - 2x\cos y = 0$$

for (ii), $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$

for (iii) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = yt - \frac{2yt}{2} = yt - yt = 0$

for (iv) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{x} + x - \frac{1}{x} = x$