Now, net ultimate bearing capacity

(q_{nf}) = Ultimate bearing capacity $(q_f) - \gamma D$

$= 1100.4 - 21 \times 4.4$

$= \mathbf{1008\ kN/m^2}$

12. *A strip footing is to be designed to carry a load of 800 kN/m at a depth of 0.7 m in a gravelly sand. The appropriate shear strength parameters are c' = 0 and ϕ' = 40°. Determine the width of the footing if a factory of safety of 3 against shear-failure is specified and assuming that the water-lavel may rise to foundation level. Above the water-table, the unit weight of the sand is 17 kN/m³ and below the water-table the saturated unit weight is 20 kN/m³.*

Solution :

For $\phi' = 40°$,

$N_\gamma = 95$

and $N_q = 64$

Ultimate bearing capacity,

$$q_u = 0.5\,\gamma' BN_\gamma + \gamma DN_q$$
$$= [0.5 \times (20 - 9.8) \times B \times 95] + [17 \times 0.7 \times 64]$$
$$= 485\,B + 762$$

Net ultimate bearing capacity,

$$q_{nf} = q_u - \gamma D$$
$$= 485\,B + 762 - 17 \times 0.7$$
$$= 485\,B + 750$$

Net foundation pressure,

$$q_u = \frac{800}{B} - (17 \times 0.7)$$

For a factor of safety of

$$3, \frac{1}{3}(485\,B + 750) = \frac{800}{B} - 12$$

∴ $B = \mathbf{1.55\ m}$

13. *Using Rankine's formula, the minimum depth of foundation of a square footing 1 m × 1 m of a column carrying a load of 15 tons (inclusive of its own weight). The foundation consists of cohesionless soil weighing 1.85 tonnes/m³ with a shear angle of 35° approximately.*

Solution : According to Rankine's formula,

$$D = \frac{q}{r}\left[\frac{1-\sin\phi'}{1+\sin\phi'}\right]^2$$

where $q = 15$ tons/m²,

$\phi' = 35°$,

$\gamma = 1.85$ tons/m²

∴
$$D = \frac{15 \times 1000}{1.85 \times 1000}\left[\frac{1-\sin 35°}{1+\sin 35°}\right]$$

$= \mathbf{0.595\ m}$

14. *The gross safe bearing pressure of sand having ϕ = 36° and effective unit weight 1.8 tonnes/m³ under 1 m wide strip footing, will be*

(Consider the footings are placed at a depth of 1 m from ground surface and water table is at a great depth. Assume a factor of safety of 3.0 Use Terzaghi's theory.)

Given : for ϕ = 36°, from Terzaghi's chart $N_q = 47$ and $N_\gamma = 43$

Solution : As ϕ′ is equal to 36°, general shear failure occurs.

For strip footing,

$$q_f = \gamma DN_q + 0.5\,\gamma BN_\gamma$$
$$= \gamma D(N_q - 1) + 0.5\,\gamma BN_\gamma$$

Given, $\gamma = 1.8$ tonnes/m³;

$D = 1$ m;

$B = 1$m

Net safe bearing pressure,

$$q_{nf} = 1.8 \times 1 \times 47 + 0.5 \times 1.8 \times 1 \times 43$$
$$= 121.5\ t/m^2$$

Gross safe bearing pressure,

$$q_f = 1.8 \times 1 \times 47 + 0.5 \times 1.8 \times 1 \times 43$$
$$= 123.3\ t/m^2$$

Again
$$q_s = \frac{q_{nf}}{F} + \gamma D = \frac{121.5}{3} + 1.8 \times 1$$
$$= 42.3\ t/m^2$$

15. *Estimate the immediate settlement of a concrete footing, 1 m × 1.5 m in size, if it is founded at a depth of 1 m in silty soil whose compression modulus is found to be 90 kg/cm². The footing is expected to transmit a unit pressure of 20 tonnes/m².*

Solution : Immediate settlement,

$$S_i = qB\frac{(1-\mu)^2}{E_c}I_f$$

where, μ = Poisson's ratio, which can be taken as 0.3

I_f = Influence factor, which can be taken as 0.06

for $L/B = 1.5$ [from table]

and $q = 20$ tonnes/m²

$= 2.0$ kg/cm²

Therefore
$$S_i = \frac{2 \times 100(1 - 0.3^2)}{90} \times 1.06$$

$= \mathbf{2.12\ cms}$

16. *A footing of 1.5 metre diameter carries a load (including its self-weight) of 80 tonnes. The soil has angle of internal friction of 36° and effective unit weight of 1.2 t/m³. What will be the depth of foundation, if the desired factor of safety is 2.5 ? Use Terzaght's theory ?*

Solution : As the load of 80 tonnes includes the weight of footing, the gross bearing pressure,

$$Q_d = 80 = \frac{1}{F_s}\left(\gamma\, DN_q + 0.6\,\gamma\, RN\gamma\right)\pi R^2$$

If the soil is assumed to be dense enough to fail by general shear failure, the values of N_q and $N\gamma$ from table are,

$$N_q = 49.5,$$
$$N_\gamma = 54$$

for $\phi = 36°$.

$$\therefore\ 80 = \frac{1}{2.5}[1.2\times D\times 49.5 + 0.6\times 1.2\times 0.75\times 54] \times 3.14\times 0.75^2$$

or $D = \mathbf{1.41\ m}$

If the soil is in the mixed state of local and general shear failure condition, we have

$$N_q = 37,$$
$$N_\gamma = 40$$

$$\therefore\ 80 = \frac{1}{2.5}[1.2\times D\times 37 + 0.6\times 1.2\times 0.75\times 40] \times 3.14\times 0.75^2$$

or $D = \mathbf{2.2\ m}$

17. *A square footing, located at a depth of 1.5 m from the ground carries a column load of 15 tonnes. The soil is submerged having an effective unit weight of 1.1 gm/cm³ and angle of shearing resistance of 30°. What is the size of the footing, if the desired factor of safety is 3 ? Use Terzaghi's theory for general shear failure.*

Solution : As the column load of 15 tonnes does not include the weight of footing net bearing capacity equation will be considered, viz.

$$15 \text{ tonnes} = B^2\times\frac{1}{F}\left[\gamma\, DN_q + 0.4\,\gamma\, BN_\gamma\right] - B^2\gamma_b D$$

Since $\phi = 30°$, the soil is in a loose state and hence criterion of local shear failure should be considered.

For $\phi = 30°$,

$N = 10,$

and $N_\gamma = 6$

$$\therefore\ 15 = B^2\times\frac{1}{3}[1.1\times 1.5\times 10 + 0.4\times 1.1\times B\times 6] - B^2\times 1.1\times 1.5$$

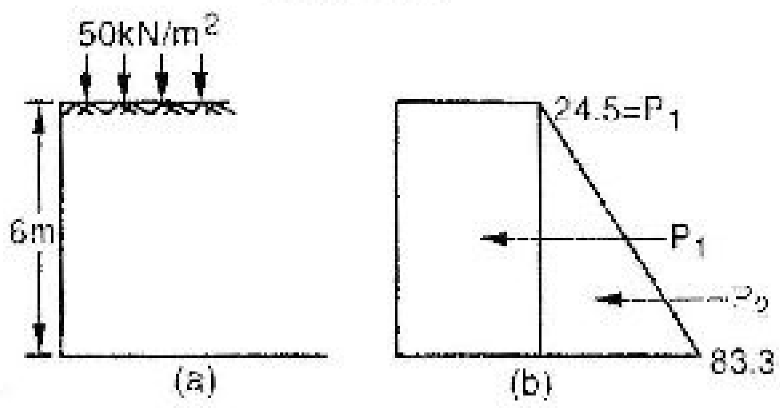
$$= B^2\times\frac{1}{3}(16.5 + 2.64\,B) - 1.64\,B^2$$

or $B^2 + 4.4\,B^2 - 17 = 0$

By trial and error we get, $B \approx \mathbf{1.7\ m}$

18. *A smooth backed vertical wall is 6 m high and retains a soil with a bulk unit weight of 20 kN/m³ and $\phi = 20°$. The top of the soil is level with the top of the wall and is horizontal. If the soil surface carries a u.d.l of 50 kN/m², find total thrust on the wall per metre run.*

Solution : $K_a = \dfrac{1-\sin 20°}{1+\sin 20°} = 0.49$



Pressure diagram

Equivalent head due to $u.d.l = \dfrac{50}{20} = 2.5$ m

$$p_1 = k_a\,\gamma\, h$$
$$= 0.49\times 20\times 2.5$$
$$= 24.5\ \text{kN/m}^2$$

Equivalent height of the wall at the base of the wall

$$= 6 + 2.5$$
$$= 8.5\ \text{m}$$

Pressure at base of wall $= 0.49\times 20\times 8.5$

$= 83.3$ kN/m²

$$\text{Total thrust} = 24.5\times 6 + \frac{1}{2}(83.3 - 24.6)\times 6$$

$$= \mathbf{323.4\ kN}$$

19. *What is the settlement of a raft 15 m × 20 m, resting on sand and carrying a net loading of 120 kN/m² at a depth of 13 m, due to consolidation of an underlying clay layer 2.0 m thick and 15 m below the surface ? The clay layer has γ_b = 10 kN/m², ω_n = 35, ω_d = 50.*

Solution :

$$\text{Settlement, } S_c = \frac{H}{1+e_o}C_c\cdot\log\frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_o}$$

Given, $H = 2.0$ m

$$e_o = \omega\, G = 0.35\,(2.7) = 0.946$$

$$C_c = 0.009\,(\omega_l - 10)$$
$$= 0.009\,(50 - 10)$$
$$= 0.36$$
$$\sigma_o = 15\,(10) + 1\,(10)$$
$$= 160 \text{ kN/m}^2$$
$$\Delta\bar{\sigma} = \frac{120(15\times 20)}{(15+13)(20+13)}$$
$$= 38.96 \text{ kN/m}^2$$
$$\therefore \quad S_c = \frac{2}{1+0.945}(0.36)\log\frac{160+38.96}{160}$$
$$= 0.035 \text{ m or } 35 \textbf{ mm}$$

20. *If a distortion of 1/300 is allowed between two columns 6 m apart, what is the differential settlement ?*

Solution :

Distortion, $\theta = \frac{\delta}{L} = \frac{S_1 - S_2}{L}$

$$= \frac{1}{300}$$
$$\delta = S_1 - S_2$$
$$= \frac{1}{300}(6) \text{ m}$$
$$= \textbf{20 mm}$$

PERMEABILITY

Permeability. It is a property of a soil which permits the flow of water through its voids.

Coefficient of Permeability

Darcy's Law. According to Darcy's law for laminar flow conditions in a saturated soil, the rate of flow or the discharge per unit time is proportional to the hydraulic gradient and is given by

$$q = k\,i\,A$$

or $$V = \frac{q}{A} = k\,i$$

Seepage velocity, $V_s = \frac{V}{n}$

Where, n = porosity

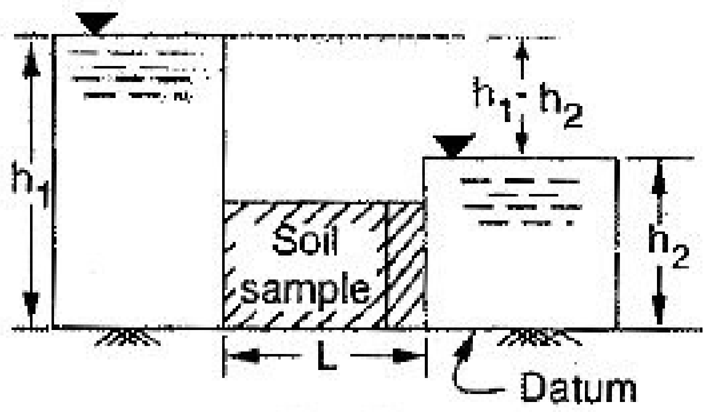
k = coefficient of permeability

A = total cross-sectional area of the soil mass, perpendicular to the direction of flow

V = average discharge velocity or velocity of flow

i = hydraulic gradient

If a soil sample of length L and cross-sectional area A, is subjected to differential head of water, the hydraulic gradient,



$$i = \frac{h_1 - h_2}{L}$$

Rate of flow, $$q = k\,\frac{h_1 - h_2}{L} \cdot A$$

Coefficient of permability (k). It is defined as the average velocity of flow that will occur through the total cross-sectional area of the soil under unit hydraulic gradient. It is usually expressed as cm/sec or m/day or feet/day.

Determination of Coefficient of Permeability,

It can be determined by following methods :

1. **Laboratory methods**
   (*a*) Constant head permeability method
   (*b*) Falling head permeability method
2. **Field methods**
   (*a*) Pumping-out tests
   (*b*) Pumping-in tests
3. **Indirect methods**
   (*a*) Computation from grain size of specific surface
   (*b*) Horizontal capillarity test
   (*c*) Consolidation data test

Classification of soil according to permeability 'k'

(*i*) **Permeable soil :** It is a soil containing continuous voids, which permits water to pass through the soil mass. Gravels are highly permeable.

(*ii*) **Impermeable soil :** It which does not permit water to pass through. Stiff clay is the least permeable soil and is usually regarded as impermeable for all practical purposes.

Sands, silts and medium clays falls between permeable and impermeable types of soils.

Flow of water through soils may be :

(*i*) Laminar (linear) flow in which all water particles move in parallel paths without crossing the paths of other particles.

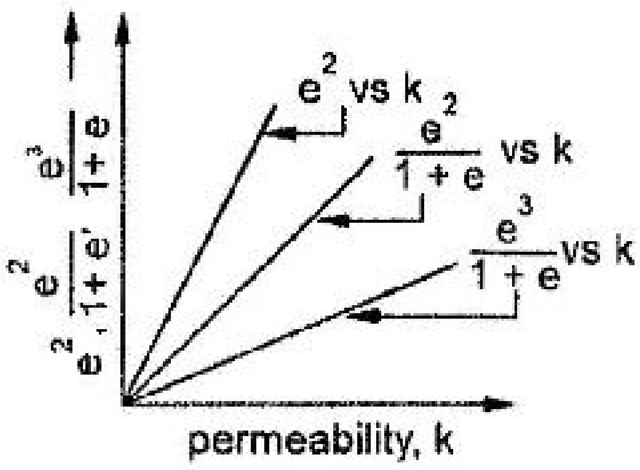
(*ii*) Turbulent (eddy) flow in which water particles move along zig zag paths and cross the path of other particles.

The nature of flow depends on the velocity of flow, size, shape and smoothness of the sides of the pipe. In laminar flow, the resistance to flow is mainly due to the viscosity of water.

The gravitational flow of water through soils is mostly laminar type of flow since the velocity of ground water rarely becomes high to produce turbulence.

Factors affecting coefficient of permeability.

On comparing Darcy's equation with the Poiseuille's equation, the coefficient of permeability



$$k = C\frac{\gamma_w}{\mu}D_s^2\frac{e^3}{1+e}$$

From the above equation, the factor k depends on the :

1. Properties of pore fluid (unit weight γ_w and viscosity η).
2. Void ratio.
3. Shape and arrangement of pores (structural arrangement) as represented by the factor C.
4. Second power of average grain size, D_s.
5. Thickness of absorbed water layer.
6. Presence of undissolved gas and other foreign matter.

Grain Size : Alien Hazen found that the coefficient of permeability can be expressed as

$$k \text{ (in cm)} = C \cdot D_{10}^2$$

where, D_{10} = effective diameter (cm), and

C = constant, approximately equal to 100.

Properties of pore fluid :

Permeability is directly proportional to the unit weight and inversely proportional to its viscosity. As compared to unit weight of water, the viscosity changes much with the change in temperature. Hence, when other factors remains constant, the effect of the property of water on the values of permeability can be expressed as

$$\frac{k_1}{k_2} = \frac{n_2}{n_1}$$

Void ratio : The relationship between permeability and void ratio are purely empirical and it can be expressed as

$$\frac{k_1}{k_2} = \frac{C_1e_1^3}{1+e_1}\Bigg/\frac{C_2e_2^3}{1+e_2}$$

The factor C changes very little with a change in void ratio of sand but varies appreciably for clays. Based on mean hydraulic radius for the soil, the following relationship is proposed

$$k_2 = k_1\left(\frac{e_2}{e_1}\right)^2$$

Structural Arrangements : Impending upon the method of compaction and condition existing during deposition, the structural arrangement of soil mass may vary at the same void ratio and consequently permeability also varies. It is defined by the factor C in equation,

$$k = C\frac{\gamma_w}{\eta}D_s^2\left(\frac{e^3}{1+e}\right)$$

For stratified deposits, the coefficient of permeability are different for flow parallel and perpendicular or stratification.

Degree of Saturation and Other Foreign Matter

The permeability is reduced if the soil is partially saturated and has entrapped air in the voids. The dissolved air in the pore fluid may get liberated, thus changing the permeability. Organic matter also has the tendency to move towards critical flow channels and choke them up, thus decreasing the permeability.

Adsorbed Water : The adsorbed water held on the surface of colloidal particles are highly cohesive and is immobile to normal hydrodynamic forces. Because of this the area of effective void space is reduced and thus permeability is reduced.

Steady Radial Flow into a Well : When a well penetrates into an extensive homogeneous aquifer, the water table remains initially horizontal in the well. If the well is pumped, water is removed from the aquifer and the piezometric surface depending upon the type of the aquifer is lowesred resulting in a circular depression in the water table or piezometric surface and this depression is known as the *drawdown curve* or *cone of depression. Consider two cases for the analysis of radial flow,*

(*i*) Well fully penetrating a confined aquifer

(*ii*) Well fully penetrating an unconfined aquifer

(*i*) Confined Aquifer.

$$Q = \frac{2\pi kH(h_2 - h_1)}{2.3\log_{10}\frac{r_2}{r_1}}$$

where T = coefficient of transmissibility

= kd, where d is saturated thickness

h_1 and h_2 = measured depth of water in two observation wells situated at distance r_1 and r_2 ($r_2 > r_1$) respectively.

(ii) Unconfined Aquifer.

$$Q = \frac{\pi k\left(H^2 - h^2\right)}{\log_e\left(\frac{R}{r}\right)}$$

$$= 1.36\,k\frac{\left(H^2 - h^2\right)}{\log 10\left(\frac{R}{r}\right)}$$

where H = thickness of aquifer, measured from the permeable layer to initial level of water table.

h = depth of water in the well measured above the impervious layer

R = radius of zero drawdown or maximum radius of influence

r = radius of the well

Pumping Test

The pumping out test can be used to measure the average k value of a stratum of soil below the water table and is effective upto depth of about 45 m. In this method, water is withdrawn from the well freely till the critical depression head or safe maximum head is created. Once this stage is reached, the rate of pumping is so adjusted as to maintain the constant water-level in the well. At this stage, the rate at which water is pumped out of the well will be equal to the rate at which water percolates into the well. This will be the yield of the well. Generally these tests are carried out in boreholes made for site investigations. These tests are classified as

1. Pumping-in-tests
2. Pumping-out-tests

1. Pumping-in-tests : Pumping-in tests are conducted to determine the coefficient of permeability of an individual stratum through which a hole is drilled. These tests are more economical than pumping out tests.

Pumping-in tests are of two types :

(*i*) Open end tests

(*ii*) Packer tests

(*i*) Open-end Tests : An open-end pipe is sunk in the strata and the soil is taken out of the pipe just to the bottom. Clean water having temperature just higher than the ground water is added through a metering system to maintain gravity flow under constant head. Water may be allowed to enter the hole under some pressure head. Then coefficient of permeability is calculated from,

$$k = \frac{q}{5.5\,rh}$$

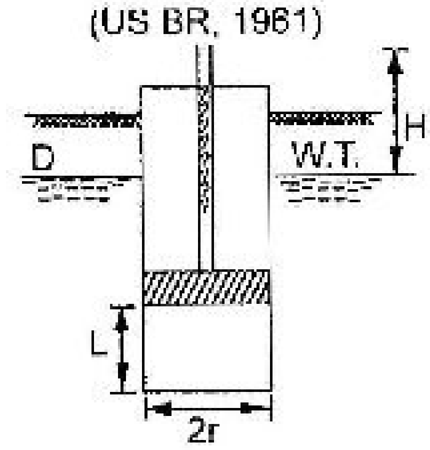
where r = radius of casing

q = constant rate of flow

h = differential head of water (gravity plus pressure, if any)

(*ii*) Packer Tests : A perforated portion of the casing or an uncased portion of the hole is used for performing this test. If the test is performed during drilling, a top packer is placed just inside or below the casing. Water is pumped in the lower portion of the hole. To perform the test after completion of the hole, which can stand without casing, two packers are set on a pipe or drill stem keeping the perforated portion of the pipe between the plugs. The bottom of the pipe is plugged. On expansion, the length of packer should be five times the diameter of the hole. From the bottom of the hole, testing is started and continued upwards. Coefficient of permeability can be computed by the following equation.

(US BR, 1961)



$$k = \frac{q}{2\pi L h}\log_e\frac{L}{r};\ L \geq 10r$$

and $$k = \frac{q}{2\pi L h}\sinh^{-1}\frac{L}{2r};\ 10r \geq L \geq r$$

where L = length of portion of the hole tested

r = radius of casing

q = constant rate of flow,

and h = differential head of water (gravity plus pressure, if any)

2. Pumping-out test (Field determination of coefficient of permeability, *k* and coefficient of transmissibility, T) : In pumping out tests drawdowns, corresponding to a steady discharge, q are observed at a number of observation wells. Pumping must continue at a uniform rate for a sufficient time to approach a steady state condition. Pumping out test give more reliable values than given by pumping-in tests

Pumping-out test in confined aquifer :

$$\text{Transmissibility, } T = \frac{q}{2.72\,\Delta h}\log_{10}\left(\frac{r_2}{r_1}\right)$$

where Δh = difference in drawdown at the two wells so selected that

$$r_2 = 10\, r_1$$

Radius of influence,

$$R = 3000\, d\sqrt{k}$$

where, d = radius of test well

k = coefficient of permeability (m/sec)

Pumping-out test in unconfined aquifer :

$$k = \frac{q}{1.36\left(H^2 - h^2\right)} \log_{10} \frac{R}{r}$$

and

$$k = \frac{q}{1.36\left(h_2^2 - h_1^2\right)} \log_{10} \frac{r_2}{r_1}$$

where H = depth of aquifer measured below the water table

h = depth of water in well

r = radius of the test well

Recuperation Test

In this type of test, at first water is drained from the well at a fast rate so as to cause sufficient drawdown. Then the pumping is stopped. Now the water level in the well will start rising. The time taken by the water to come back to its original level is then noted. Specific yield of the well can be calculated by,

$$\frac{K}{A} = \frac{2.303}{T} \log_{10} \frac{H_1}{H_2}$$

where H_1 and H_2 = water levels in the well at the time the pumping was stopped and at time T after the pumping is stopped.

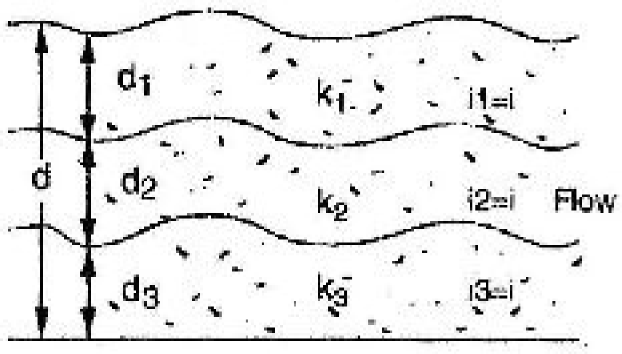
A = cross-sectional area of the well at bottom.

Permeability of stratified deposits

Soil mass may consist of several layers deposited one above the other in nature. Their bedding planes may be horizontal, vertical one inclined. Each layer, assumed to be homogeneous and isotropic, has its own value of coefficient of permeability. The average permeability of the whole deposit will depend upon the direction of flow with relation to the direction of the bedding planes.

Case I. *Average permeability parallel to the bedding planes*

Let $d_1, d_2, d_3 \ldots\ldots d_n$ are the thickness of layers, and k_1, k_2, k_3 ......, k_n are the permeabilities of the layers.

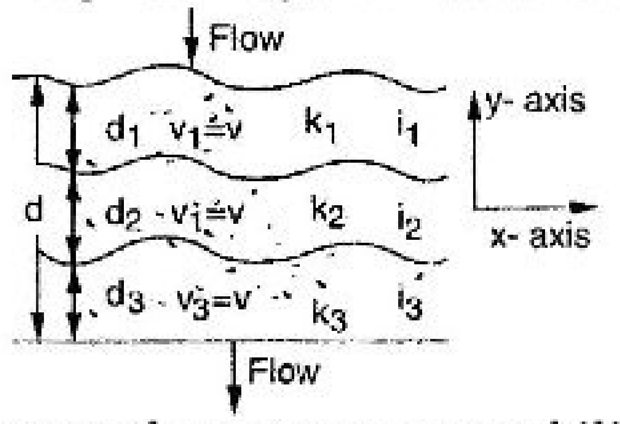


Let d is the total thickness of the deposit and k_x be the average permeability along x-axis, which is parallel to the bedding planes.

$$\therefore \qquad k_x = \frac{k_1 d_1 + k_2 d_2 + k_3 d_3 + \ldots..}{d_1 + d_2 + d_3 + \ldots...}$$

Case II. *Average permeability perpendicular to the bedding planes*

In this case, the hydraulic gradient, and hence the head loss through each layer will be different.



If k_y represents the average permability in flow direction (i.e, along y-axis) then

$$\therefore \qquad k_y = \left[\frac{d_1 + d_2 + d_3 + \ldots..}{\dfrac{d_1}{k_1} + \dfrac{d_2}{k_2} + \dfrac{d_3}{k_3} + \ldots..}\right]$$

In general, $\quad k_x > k_y$

The average flow parallel to the plane of stratification is governed by the most permeable layer and that perpendicular to the plane of stratification by the least permeable layer.

Determination of Coefficient of Permeability of Soil Samples : It can be determined by

1. Falling head permeameter, and
2. Constant head permeameter

1. Falling head permeameter method

Let $\qquad h$ = head of water at time t

According to Darcy's law,

$$Q = k \cdot \frac{h}{L} \cdot A \qquad \ldots(i)$$

where A = area of cross section of soil sample

dh = fall of water in time dt

a = area of cross section of vertical water pipe

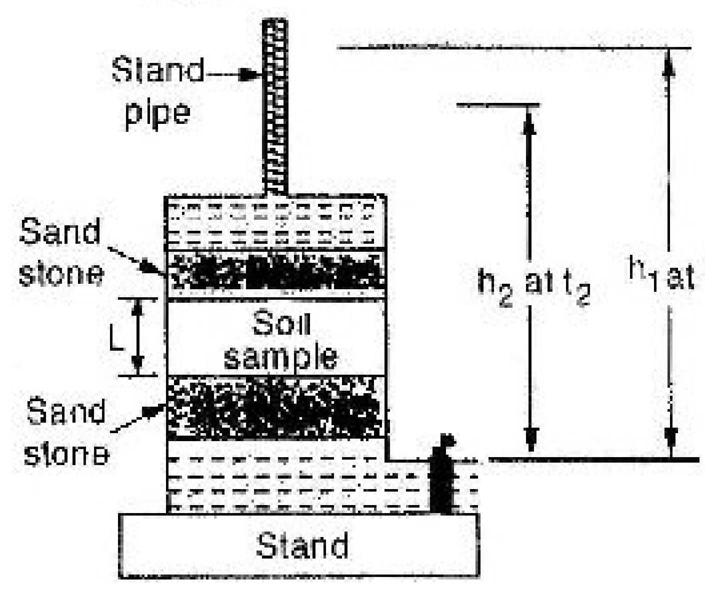


Fig. A falling head permeameter

Then, $$Q = \frac{-(dh \cdot a)}{dt} \quad ...(ii)$$

Equating equations (*i*) and equations (*ii*), we get

$$\frac{-(dh \cdot a)}{dt} = k \cdot \frac{h}{L} \cdot A$$

or $$\frac{kA}{aL} dt = \frac{-dh}{h} \quad ...(iii)$$

Integrating equations (*iii*), we get

$$\frac{kA}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2} \frac{dh}{h} = \int_{h_2}^{h_1} \frac{dh}{h}$$

or $$\frac{kA}{aL}(t_2 - t_1) = \log_e\left(\frac{h_1}{h_2}\right)$$

or $$k = \frac{aL}{A} \frac{\log_e\left(\frac{h_1}{h_2}\right)}{(t_2 - t_1)}$$

or $$k = \frac{2.3\ La}{A} \cdot \frac{\log_{10}\left(\frac{h_1}{h_2}\right)}{(t_2 - t_1)}$$

Knowing h_1, h_2, t_2, t_1 and L, a, A the coefficients of permeability of the soil may be determined.

In falling head permeameter a stand pipe containing water is attached to the top of the soil mass. As water percplates through the spil from top to the bottom, the water level in the stand pipe gradually falls down. The fall of water level in the stand pipe over a certain time interval t is measured.

2. Constant head permeameter method

In this type of permeameters, arrangements are made to keep the water levels at the top and bottom of the soil sample constant. Water fearing through the soil from top to bottom is collected in a graduated glass cylinder and its volume is measured.

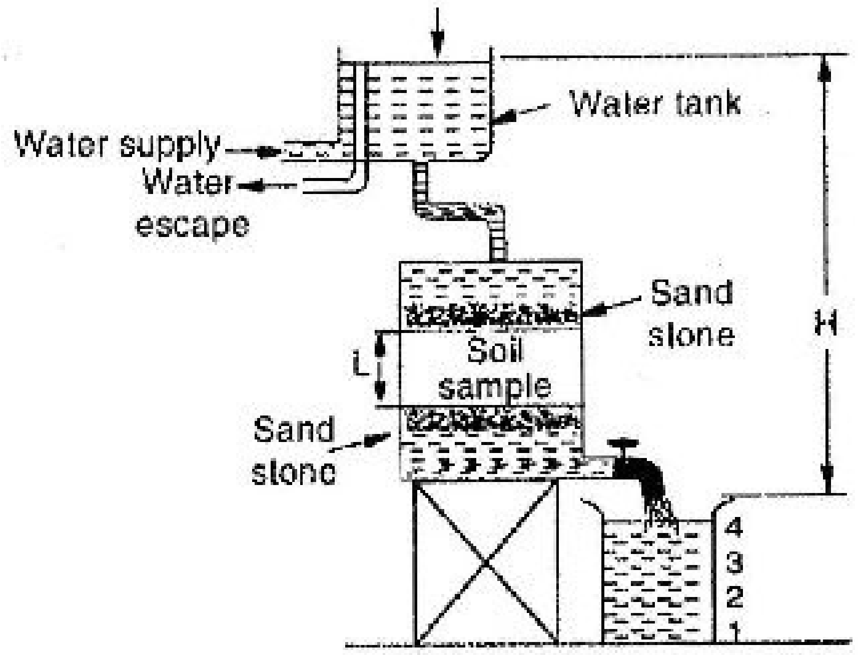


Fig. A constant head permeameter

Apparatus of constant head permeameter is shown in the figure.

Let H = constant head

L = length of solid sample

Q = discharge per unit time

then $$k = \frac{QL}{H.A}$$

Note : The constant head permeameter is suitable for coarse grained soils while the falling head permeameter is suitable for fine grained soils.

Example. *A block of soil is 12 cm long and 6 sq. cm in cross-section. The water level at one end of the block is 20 cm above a fixed plane, and at the other end it is 3 cm above the same plane. The flow rate is 2 cc per 1.5 min. Find the soil permeability.*

Solution : Given, L = 12 cm,

$$A = 6\ \text{cm}^2,$$
$$h_1 = 20\ \text{cm},$$
$$h_2 = 3\ \text{cm}$$
$$Q = \frac{2\text{cc}}{90}\text{sec} = \frac{2}{90}\text{cm}^3/\text{sec}$$
$$h_L = h_1 - h_2$$
$$= 20 - 3 = 17\ \text{cm}$$
$$i = \frac{h_L}{L} = \frac{17}{12}$$

Now $$Q = k.\ i\ A$$

$\therefore$ $$\frac{2}{90} = k \times \frac{17}{12} \times 6$$

or $$k = 0.0026\ \text{cm/sec}$$
$$= \mathbf{2.6 \times 10^{-3}\ cm/sec}$$

Example. *A soil sample, 20 sq cm in cross-sectional area and 10 cm long is tested for permeability in a variable head permeameter. The stand pipe has a cross-sectional area of 1 sq cm, and the head drops from 30 cm to 10 cm in 6 minutes and 20 seconds. The permeability of the soil, in cm/sec.*

Solution : We know,

$$k = \frac{23\ a.L}{A.\ t}\log_{10}\frac{h_1}{h_2}$$

where $A = 20\ \text{cm}^2$,

$L = 10$ cm,

$a = 1\ \text{cm}^2$,

$h_1 = 30$ cm,

$h_2 = 10$ cm,

$t = 380$ seconds

$\therefore$ $$k = \left[\frac{2.3 \times 1 \times 10}{20 \times 380}\log_{10}\frac{30}{10}\right] = 0.00144$$
$$= \mathbf{1.44 \times 10^{-3}\ cm/sec}$$

Example. *A sand sample is tested in a constant head permeameter 11.7 cm high and 10.2 cm in diameter. The quantity of water passing through the sample under an effective head of 10 cm, for a period of 90 seconds was measured to be 600 ml. Find coefficient of permeability in cm/sec.*

Solution : Given, L = 11.7 cm,

$$A = \frac{\pi}{4} \times (10.2)^2 \text{ cm}^2 = 81.67 \text{ cm}^2,$$

$$h = 10 \text{ cm}$$

$$\therefore \quad i = \frac{h}{L} = \frac{10}{11.7}$$

$$Q = \frac{\text{Volume}}{\text{Time}}$$

$$= \frac{600}{90} \text{cm}^3\text{sec} = 6.66 \text{ cm}^3\text{/sec}$$

From $Q = k.\, i.\, A$

$$6.66 = k \times \frac{10}{11.7} \cdot 81.67$$

or $k = 0.0955$

$$= \mathbf{9.55 \times 10^{-2}\ cm/sec}$$

Example. *A loose uniform sand with rounded grains has an effective grain size of 0.03 cm. Find coefficient of permeability.*

Solution : From Hazen's Formula,

$$k = 100\, D_{10}^2$$

where D is in cm and k in cm/sec

$$\therefore \quad k = 100 \times (0.03)^2 \text{ cm/sec}$$

$$= \mathbf{0.09\ cm/sec}$$

Example. *During a recuperation test, the water level was depressed by pumping 2.65 meters and is recuperated by an amount of 1.9 meters in 70 minutes. Then find*

(*i*) *yield from a well of 3.2 m diameter under a depression head of 3.7 metres.*

(*ii*) *diameter of the well to yield 12 litres/second under a depression head of 2.6 metres.*

Solution : Specific yield of a well,

$$\frac{K}{A} = \frac{2.303}{T} \log_{10} \frac{H_1}{H_2}$$

where H_1 = final drawdown = 2.65 m

H_2 = final drawdown = 2.65 – 1.90 = 0.75 m

T = time = 70 min = 70 × 60 = 4200 sec.

$$\therefore \quad \frac{K}{A} = \frac{2.303}{4200} \log_{10} \frac{2.65}{0.75}$$

$$= 0.3 \times 10^{-3}$$

(*i*) Yield from a well of 3.2 m diameter, under a depression head of 2.7 cm is

$$Q = \left(\frac{K}{A}\right) A \cdot H$$

$$= 0.03 \times 10^{-3} \times \left(\frac{\pi}{4} \times 3.2^2\right) \times 3.7$$

$$= 8.95 \times 10^{-3} \text{ m}^3\text{/sec}$$

$$= \mathbf{8.95\ litre\ sec}$$

(*ii*) Given, Q = 12 litres/second,

H = 2.6 m

From $Q = \left(\frac{K}{A}\right) A \cdot H$

$$12 \times 10^{-3} = (0.30 \times 10^{-3})\, A \times 2.6$$

or $A = 15.38 \text{ m}^2$

Now, $\frac{\pi}{4} \times d_w^2 = 15.38 \text{ m}^2$

$$\boldsymbol{d_w = 4.43\ \text{m}}$$

Example. *A constant head permeability test was carried out on a cylindrical sample of sand 10.2 cm diameter and 14 cm height. 162 cm³ of water was collected in 1.70 minutes under a head of 25 cm. Find the coefficient of permeability in m/year.*

Solution :

Given, $Q = 162 \text{ cm}^3$,

t = 1.70 minutes

= 102 seconds,

h = 25 cm

Diameter of the sample = 10.2 cm

Height of the sample = 14 cm

$$A = \frac{\pi}{4} \times (10.2)^2 = \frac{3.14}{4} \times (10.2)^2$$

$$= 81.67 \text{ cm}^2$$

Hydraulic gradient, $i = \frac{h}{L} = \frac{25}{14} = 1.786$

Now, $k = \frac{Q}{Ait} = \frac{162}{81.67 \times 1.786 \times 1.02}$

$$= 0.0108855 \text{ cm/sec}$$

$$= 1.08885 \times 10^{-4} \text{ m/sec}$$

$$= \mathbf{3386.76\ m/year}$$

Example. *In a falling head permeameter, a soil sample 75 mm in diameter and 55 mm in length was tested. At the commencement of the test, the initial head was 80 cm and after one hour, the head drops to 40 cm. Find The coefficient of permeability if the diameter of stand pipe is 1 cm, in cm/sec.*

Solution : In case of a falling head permeameter,

$$k = \frac{2.3La}{A(t_2 - t_1)} \log_{10}\left(\frac{h_1}{h_2}\right)$$

where L = 5.5 cm;

$$A = \frac{\pi(7.5)^2}{4} = 4418 \text{ cm}^3,$$

$$a = \frac{\pi}{4} \times 1^2 = 0.7854 \text{ cm}^2$$

h_1 = 80 cm;

$$A = \frac{\pi(7.5)^2}{4} = 4418 \text{ cm}^3.$$

$$a = \frac{\pi}{4} \times 1^2 = 0.7854 \text{ cm}^2$$

h_1 = 80 cm;

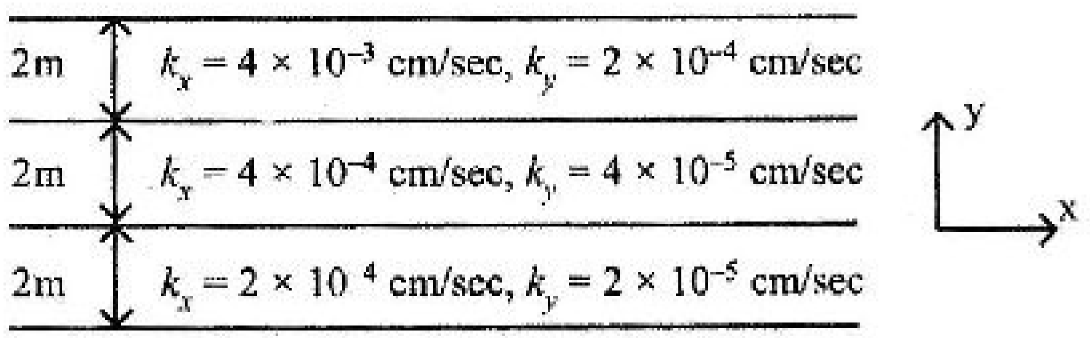
h_2 = 40 cm,

$t_2 - t_1$ = 60 minutes = 3600 secs

$$\therefore \quad k = \frac{2.3 \times 5.5 \times 0.7854}{44.18 \times 3600} \times \log_{10}\left(\frac{80}{40}\right)$$

$= \mathbf{1.88 \times 10^{-3}}$ **cm/sec**

Example. *The horizontal and vertical permeabilities for each layer are given in the figure. Find the equivalent coefficients of permeabilities in cm / sec in the X and Y directions.*



Solution : Equivalent coefficient of permeability in x-direction

$$(k_x)_{eq} = \frac{(k_x)_1 \times d_1 + (k_x)_2 \times d_2 + (k_x)_3 \times d_3}{d_1 + d_2 + d_3}$$

$$= \frac{(4 \times 10^{-3} \times 2) + (4 \times 10^{-4} \times 2) + (2 \times 10^{-4} \times 2)}{2+2+2}$$

$= \mathbf{1.53 \times 10^{-3}}$ **cm/sec**

Equivalent coefficient of permeability in y-direction

$$(k_y)_{eq} = \frac{d_1 + d_2 + d_3}{\frac{d_1}{(k_x)_1} + \frac{d_2}{(k_x)_2} + \frac{d_3}{(k_x)_3}}$$

$$= \frac{2+2+2}{\frac{2}{4 \times 10^{-3}} + \frac{2}{4 \times 10^{-4}} + \frac{2}{2 \times 10^{-4}}}$$

$= \mathbf{3.87 \times 10^{-4}}$ **cm/sec**

Check : $\boldsymbol{k_x > k_y}$

Example. *An undisturbed soil sample was tested in a falling head permeameter. Results were as follows: Initial head of water in stand pipe = 1500 mm. Final head of water in stane pipe = 605 mm, Test Duration = 281 s, Sample length = 150 mm Sample diameter = 100m, Stand-pipe diameter = 5 mm Find permeability of the soil in mm / s.*

Solution :

$$a = \frac{\pi \times 5^2}{4} = 19.6 \text{ mm}^2;$$

$$A = \frac{\pi \times 100^2}{4} = 7854 \text{ mm}^2$$

$$k = \frac{2.3\, al}{A.t} \log_{10} \frac{h_1}{h_2}$$

$$= \frac{2.3 \times 19.67 \times 150}{78.54 \times 281} \log_{10} 2.48$$

$= \mathbf{1.2 \times 10^{-3}}$ **mm/s**

Example. *A 9.15 m thick layer of sandy soil overlies an impermeable rock. Ground water level is at a depth of 1.22 m below the top of the soil. Water was pumped out of the soil from a central well at the rate of 5680 kg / min and the drawdown of the water-table was noted in two observation wells. These two well were on a radial line from the centre of the main well at distances of 3.05 and 30.5 m. During pumping the water level in the well nearest to the pump was 4.57 m below ground level and the farthest well was 2.13 m below the ground level.*

Find an average value of the permeability of the soil in m / s.

Solution : Given, q = 5680 kg/m

= 5.68 m³/min

= 0.0947 m³/s

h_1 = 9.15 – 4.67 = 4.58 m

h_2 = 9.15 – 2.13 = 7.02 m

$$\therefore \quad k = \frac{q \log_e\left(\frac{r_2}{r_1}\right)}{\left(h_1^2 - h_2^2\right)\pi}$$

$$= \frac{0.0947 \times 2.3026}{28.3 \times \pi}$$

$= \mathbf{2.45 \times 10^{-3}}$ **m/s**

Example. *The coefficient of permeability of a soil sample was found out in a soil mechanics laboratory by making use of a falling head permeameter. Data used and the test results obtained were as follows :*

Diameter of sample = 6 cm,

Height of sample = 15 cm,

Diameter of standpipe = 2 cm,

Initial head h_0 = 45 cm,

Final head h_1 after a time of 2 min = 30 cm

Find coefficient of permeability in m / day.

Solution : $k = \dfrac{2.3\,aL}{At}\log_{10}\dfrac{h_0}{h},$

where t is the time elapsed

$$\text{Area of stand pipe, } a = \frac{3.14\times 2\times 2}{4\times 100\times 100} = 3.14\times 10^{-4}\ \text{m}^2$$

$$\text{Area of sample} = \frac{3.14\times 6\times 6}{4\times 100\times 100} = 28.26\times 10^{-4}\ \text{m}^2$$

$$\text{Height of sample} = \frac{15}{100} = 15\times 10^{-2}\ \text{m}$$

$$\text{Head, } h_0 = \frac{45}{100} = 45\times 10^{-2}\ \text{m}$$

$$h_1 = \frac{30}{100} = 30\times 10^{-2}\ \text{m}$$

$$\text{Elapsed time, } t = 105\ \text{sec} = \frac{105}{60\times 60\times 24}\ \text{days} = 12.15\times 10^{-2}\ \text{days}$$

$$k = \frac{2.3\times 3.14\times 10^{-4}\times 10\times 10^{-2}}{28.26\times 10^{-4}\times 12.15\times 10^{-2}}\log_{10}\frac{45\times 10^{-2}}{30\times 10^{-2}}$$

$$= \mathbf{5.5\ m/day}$$

SEEPAGE

Percolation of water through the soil pores under an energy gradient is known as seepage. The pressure exerted on the soil due to seepage of water is known as *seepage force* or *seepage pressure*.

Seepage problem occurs in all earthen dams, retaining walls and foundations on permeable soil.

Flow Nets. The flow of water through a soil can be represented graphically by a flow net, a form of curvilinear net made up of a set of flow line intersected by a set of equipotential lines.

Flow Lines. The paths which water particle follow in the course of seepage are known as *flow lines.* Water flows from points of high head to points of low head, and makes smooth curves, representing the paths followed by moving water particles.

Equipotential Lines. As the water moves along the flow it experiences a continuous loss of head. If we can obtain the head causing flow at points along a flow line, then by joining up points of equal potential we obtain a second set of line known as **equipotential lines**.

Hydraulic Gradient. The potential drop between two adjacent equipotential lines divided by the distance between them is known as the hydraulic gradient. It attains a maximum along a path normal to the equipotentials and in isotropic soil the flow follows the path of the steepest gradients, so that flow lines cross equipotential lines at right angles.

Flow Path. The space between adjacent flow lines is called flow path or flow channel.

Piezometric Level. At all points along an equipotential line, the water would rise in a piezometric tube to a certain elevation known as piezometric level.

Two dimensional flow. To determine quantity of water flowing through saturated soil mass and the distribution of water pressure by the theory of flow of fluid through porous medium the following assumptions were made.

(*i*) Soil mass is homogeneous and isotropic.

(*ii*) The soil and water are incompressible.

(*iii*) The flow is assumed to be laminar so that Darcy's law is valid.

(*iv*) Quantity of water stored in soil pores is constant so that steady flow conditions have been established.

Laplace equation.

Flow of water through soils for a two dimensional flow conditions can be expressed mathematically as shown below.

$$k_x\frac{\partial^2\phi}{\partial x^2} + k_y\frac{\partial^2\phi}{\partial y^2} = 0$$

For steady flow and isotropic conditions, the above equation reduces to,

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad \left[k_x = k_y = 1\right]$$

This is known as *Laplace equation.*

Solutions of this equations give two sets of curves known as *stream lines* and *equipotential lines*.

The solution to Laplace equation may be found by

1. Direct mathematical solution
2. Numerical solution
3. Electrical analogy
4. Model Analysis
5. Graphical solution

Properties of Flow-Net

1. The flowlines and equipotential lines must intersect at right angles to each other.
2. The areas (field) bounded by flow lines and equipotential lines should form approximate squares.
3. The flow nets must satisfy the boundary conditions of flow field.
4. The quantity of water flowing through each flow channel is the same.
5. The potential drop in any two adjacent equipotential lines is equal (Total head loss/ Number of fields)
6. Flow lines and equipotential lines are smooth curves.

Application of Flow-Net

(i) Determination of Seepage : For the seepage through a flow net for isotropic condition

Total discharge, $q = kh\dfrac{N_f}{N_d}$

where, k = coefficient, of permeability,

h = head causing flow,

N_f = total number of flow channel

and N_d = total number of potential drops.

(ii) Determination of seepage pressure : The change in effective pressure due to flow of water is known as seepage pressure. The seepage pressure at any point can be determined by

$$p_s = H\gamma_w$$

where, H = Hydraulic potential at any point

$$= h - n\frac{h}{N_d} = (N_d - n)\frac{h}{N_d}$$

n = number of potential drops

Exit Gradient. The exit gradient is the hydraulic gradient where the percolating water leaves the soil mass and emerges into the free water.

Exit gradient, $i_e = \dfrac{\Delta h}{l}$

where, $\Delta h = \dfrac{h}{Nd}$ (potential drop)

and l = length of last field in the flow net at downstream end

Quick Sand and Piping

Quik sand is not a type of sand but it is a flow condition existing in fine sand when the flow takes place in upward direction. Due to upward flow, the seepage pressure acts in the upward direction and hence the effective pressure is reduced. When the soil pressure and the seepage pressure becomes equal, the effective stress is reduced to zero. In such a case, cohesionless soil losses its shear strength and soil particles have a tendency to move up in the direction of flow. The condition at which the soil particles are unstable or agitated is known as *Quick sand*.

$$\gamma' H - \gamma_w h = 0$$

where γ' = submerged weight of soil

γ'_w = unit weight of water

H = thickness of soil sample

h = head causing flow

Critical hydraulic gradient,

$$i_c = \frac{h}{H} = \frac{\gamma'}{\gamma_w} = \frac{(G-1)}{1+e}$$

For values of $G = 2.67$

and $e = 0.67$,

$$i_c = \frac{2.67-1}{1+0.67} \approx \text{unity}$$

Critical Hydraulic Gradient (i_c).

Hydraulic gradient at the critical condition when the soil particles just begin to move is known as *critical hydraulic gradient*. When the exit gradient is equal to or greater than critical hydraulic gradient, the soil is said to be in quick condition. Since, critical hydraulic gradient depends on the specific gravity and void ratio of the soil, sbch critical condition is more common in uniform sands and silts with high void ratios.

$$i_c = \frac{\gamma_{sub}}{\gamma_w} = \frac{G-1}{1+e}$$

where, G is specific gravity of the soil and e is its void-ratio.

Seepage Pressure (p_s). The seepage pressure can be defined as the pressure exerted by water on the soil through which it percolates. The total seepage force transmitted to the soil mass (F_s) can be easily obtained by multiplying the seepage pressure (P_s) by the total cross-sectional area of the soil mass (A).

The seepage pressure always acts in the direction of flow. In case of uniform flow conditions, it can be assumed to spread uniformly through the volume of the soil.

$$\text{Seepage force per unit volume} = \frac{\text{seepage force}}{\text{unit volume of soil}}$$

$$= \frac{h\cdot\gamma_w\cdot A}{A\times L} = i\,\gamma_w$$

Piping. The direct result of quick sand condition is that the soil particles are carried continuously along with the flowing water and the pipes are created at the downstream side, the undermining of foundation and increasing the loss due to seepage. This effect is described as piping. This phenomenon is common in dams.

Refraction of flow-lines at interface. An interface is the surface of boundary between two soils. If the flow lines across an interface are normal to it, then there will be no refraction and the flow net appears as shown in the figure. When the flow lines meet the interface at some acute angle to the normal, then the lines are bent as they pass into the second soil.

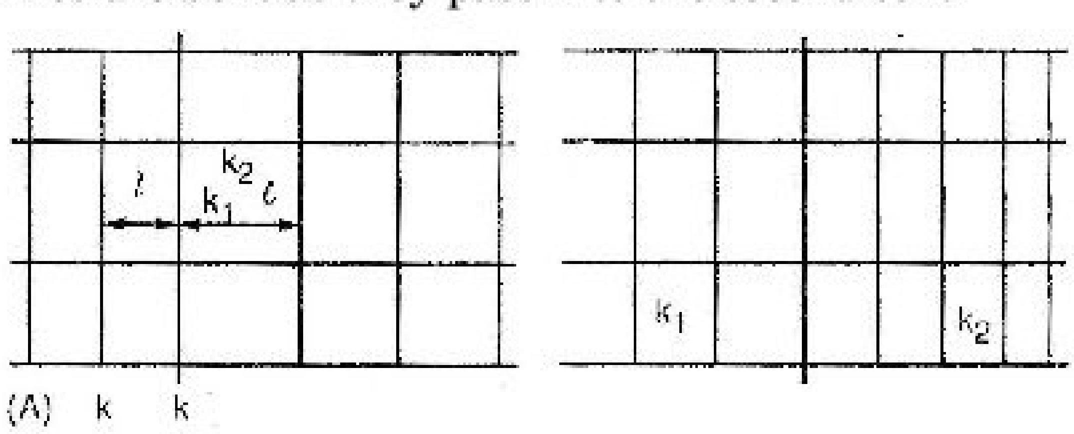


Fig. Effect of variation of permeability on a flow-net