Exercise 4.1

Q1. Find the cubes of the following numbers:

(i) 7 (ii) 12 (iii) 16 (iv) 21 (v) 40 (vi) 55 (vii) 100 (viii) 302 (ix) 301

Answer:

Cube of a number is given by the number raised to the power three.

(i) Cube of 7 = 7^3 = 7 x 7 x 7 = 343

(ii) Cube of 12 = 12^3 = 12 x 12 x 12 = 1728

(iii) Cube of 16 = 16^3 = 16 x 16 x 16 = 4096

(iv) Cube of 21 = 2^3 = 21 x 21 x 21 = 9261

(v) Cube of 40 = 40^3 = 40 x 40 x 40 = 64000

(vi) Cube of 55 = 553 = 55 x 55 x 55 = 166375

(vii) Cube of 100 = 100^3 = 100 x 100 x 100 = 1000000

(viii) Cube of 302 = 302^3 = 302 x 302 x 302 = 27543608

(ix) Cube of 301 = 3013 = 301 x 301 x 301 = 27270901

Q2. Write the cubes of all natural numbers between 1 and 0 and verify the following statements:

(i) Cubes of all odd natural numbers are odd. 6.0

(ii) Cubes of all even natural numbers are even, 3.

Answer:

The cubes of natural numbers between 1 and 10 are listed and classified in the following table.

We can classify all natural numbers as even or odd number; therefore, to check whether the cube of a natural number is even or odd, it is sufficient to check its divisibility by 2.

If the number is divisible by 2, it is an even number, otherwise, it will an odd number.

(i) From the above table, it is evident that cubes of all odd natural numbers are odd.

(ii) From the above table, it is evident that cubes of all even natural numbers are even.

Number	Cube	Classification
1	1	Odd

2	8	Even (Last digit is even, i.e., 0, 2, 4, 6, 8)
3	27	Odd (Not an even number)
4	64	Even (Last digit is even, i.e., 0, 2, 4, 6, 8)
5	125	Odd (Not an even number)
6	216	Even (Last digit is even, i.e., 0, 2, 4, 6, 8)
7	343	Odd (Not an even number)

8	512	Even (Last digit is even, i.e., 0, 2, 4, 6, 8)
9	729	Odd (Not an even number)
10	1000	Even (Last digit is even, i.e., 0, 2, 4, 6, 8)

Q3. Observe the following pattern:

$$1^3 = 1$$
$$1^3 + 2^3 = (1+2)^2$$
$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$

Write the next three rows and calculate the value of $1^3 + 2^3 + 3^3 + \ldots\ldots + 9^3 + 10^3$ by the above pattern.

Answer:

Extend the pattern as follows:

$$1^3 = 1$$
$$1^3 + 2^3 = (1+2)^2$$
$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$
$$1^3 + 2^3 + 3^3 + 4^3 = (1+2+3+4)^2$$
$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1+2+3+4+5)^2$$
$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = (1+2+3+4+5+6)^2$$

Now, from the ab

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3 = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)^2 =$

$55^2 = 3025$

Thus, the required value is 3025

Q4. Write the cubes of 5 natural numbers which are multiples of 3 and verify the followings: "The cube of a natural number which is a multiple of 3 is a multiple of 27".

Answer:

Five natural numbers, which are multiples of 3, are 3, 6, 9, 12 and 15.

Cubes of these five numbers are:

$3^3 = 3 \times 3 \times 3 = 27$

$6^3 = 6 \times 6 \times 6 = 216$

$9^3 = 9 \times 9 \times 9 = 729$

$12^3 = 12 \times 12 \times 12 = 1728$

$15^3 = 15 \times 15 \times 15 = 3375$

Now, let us write the cubes as a multiple of 27. We have:

$27 = 27 \times 1$

$216 = 27 \times 8$

$729 = 27 \times 27$

$1728 = 27 \times 64$

$3375 = 27 \times 125$

It is evident that the cubes of the above multiples of 3 could be written as multiples of 27. Thus, it is verified that the cube of a natural number, which is a multiple of 3, is a multiple of 27.

Q5. Write the cubes of 5 natural numbers which are of the form 3n + 1 (e.g. 4, 7, 10, ...) and verify the following:

'The cube of a natural number of the form 3n + I is a natural number of the same form i.e. when divided by 3 it leaves the remainder 1'.

Answer :

Five natural numbers of the form (3n + 1) could be written by choosing n = 1, 2, 3, ... etc.

Let five such numbers be 4, 7, 10, 13, and 16.

The cubes of these five numbers are: $4^3 = 64$, $7^3 = 343$, $10^3 = 1000$, $13^3 = 2197$ and $I6^3 = 4096$

The cubes of the numbers 4, 7, 10, 13, and 16 could be expressed as:

$64 = 3 \times 21 + 1$, which is of the form (3n + 1) for n = 21

$343 = 3 \times 114 + 1$, which is of the form (3n + 1) for n = 114

$1000 = 3 \times 333 + 1$, which is of the form (3n + 1) for n = 333

$2197 = 3 \times 732 + 1$, which is of the form (3n + 1) for n = 732

$4096 = 3 \times 1365 + 1$, which is of the form (3n + 1) for n = 1365

The cubes of the numbers 4, 7, 10, 13, and 16 could be expressed as the natural numbers of the form (3n + 1) for some natural number n; therefore, the statement is verified.

Q6. Write the cubes of 5 natural numbers of the form 3n + 2 (i.e. 5, 8, 11,...) and verify the following:

"The cube of a natural number of the form 3n + 2 is a natural number of the same form i.e. when it is divided by 3 the remainder is 2".

Answer :

Five natural numbers of the form (3n + 2) could be written by choosing n = 1, 2, 3... etc.

Let five such numbers be 5, 8, 11, 14, and 17.

The cubes of these five numbers are: $5^3 = 125$, $8^3 = 512$, $11^3 = 1331$, $14^3 = 2744$, and $17^3 = 4913$.

The cubes of the numbers 5, 8, 11, 14 and 17 could be expressed as:

125 = 3 x 41 + 2, which is of the form (3n + 2) for n = 41

512 = 3 x 170 + 2, which is of the form (3n + 2) for n = 170

1331 = 3 x 443 + 2, which is of the form (3n + 2) for n = 443

2744 = 3 x 914 + 2, which is of the form (3n + 2) for n = 914

4913 = 3 x 1637 + 2, which is of the form (3n + 2) for n = 1637

The cubes of the numbers 5, 8, 11, 14, and 17 can be expressed as the natural numbers of the form (3n + 2) for some natural number n. Hence, the statement is verified.

Q7. Write the cubes of 5 natural numbers of which are multiples of 7 and verify the following:

"The cube of a multiple of 7 is a multiple of 7^3"

Answer :

Five multiples of 7 can be written by choosing different values of a natural number n in the expression 7n.

Let the five multiples be 7, 14, 21, 28 and 35.

The cubes of these numbers are: $7^3 = 343$, $14^3 = 2744$, $21^3 = 9261$, $28^3 = 21952$, and $35^3 = 42875$

Now, write the above cubes as a multiple of 7^3. Proceed as follows:

$343 = 7^3 \times I$

$2744 = 14^3 = 14 \times 14 \times 14 = (7 \times 2) \times (7 \times 2) \times (7 \times 2) = (7 \times 7 \times 7) \times (2 \times 2 \times 2) = 7^3 \times 2^3$

$9261 = 21^3 = 21 \times 21 \times 21 = (7 \times 3) \times (7 \times 3) \times (7 \times 3) = 7^3 \times 3^3$

$21952 = 28^3 = 28 \times 28 \times 28 = (7 \times 4) \times (7 \times 4) \times (7 \times 4) = (7 \times 7 \times 7) \times (4 \times 4 \times 4) = 7^3 \times 4^3$

$42875 = 35^3 = 35 \times 35 \times 35 = (7 \times 5) \times (7 \times 5) \times (7 \times 5) = (7 \times 7 \times 7) \times (5 \times 5 \times 5) = 7^3 \times 5^3$

Hence, the cube of multiple of 7 is a multiple of 7^3.

Q8. Which of the following are perfect cubes?

(i) 64 (ii) 216 (iii) 243 (iv) 1000 (v) 1728 (vi) 3087 (vii) 4608 (viii) 106480 (ix) 166375 (x) 456533

Answer:

(i) On factorising 64 into prime factors, we get

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Group the factors in triples of equal factors as:

$64 = \{2 \times 2 \times 2\} \times \{2 \times 2 \times 2\}$

It is evident that the prime factors of 64 can be grouped into triples of equal factors and no factor is left over.

Therefore, 64 is a perfect cube.

(ii) On factorising 216 into prime factors, we get:

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Group the factors in triples of equal factors as:

216 = {2 x 2 x 2} x {3 x 3 x 3}

It is evident that the prime factors of 216 can be grouped into triples of equal factors and no factor is left over.

Therefore, 216 is a perfect cube.

(iii) On factorizing 243 into prime factors, we get:

243 = 3 x 3 x 3 x 3 x 3

Group the factors in triples of equal factors as:

243 = {3 x 3 x 3} x 3 x3

It is evident that the prime factors of 243 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243 is a not perfect cube.

(iv) On factorising 1000 into prime factors, we get:

1000 = 2 x 2 x 2 x 5 x 5 x 5

Group the factors in triples of equal factors as:

1000 = {2 x 2 x 2} x {5 x 5 x 5}

It is evident that the prime factors of 1000 can be grouped into triples of equal factors and no factor is left over. Therefore, 1000 is a perfect cube.

(v) On factorising 1728 into prime factors, we get:

1728 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3

Group the factors in triples of equal factors as:

1728 = {2 x 2 x 2} x {2 x 2 x 2} x {3 x 3 x 3}

It is evident that the prime factors of 1728 can be grouped into triples of equal factors and no factor is left over.

Therefore, 1728 is a perfect cube.

(vi) On factorizing 3087 into prime factors, we get:

3087 = 3 x 3 x 7 x 7 x 7

Group the factors in triples of equal factors as:

3087 = 3 x 3 x {7 x 7 x 7}

It is evident that the prime factors of 3087 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243 is a not perfect cube.

(vii) On factorising 4608 into prime factors, we get:

4608 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3

Group the factors in triples of equal factors as:

4608 = {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x 3 x 3

It is evident that the prime factors of 4608 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 4608 is a not perfect cube.

(viii) On factorising 106480 into prime factors, we get:

106480 = 2 x 2 x 2 x 2 x 5 x 11 x 11 x 11

Group the factors in triples of equal factors as:

106480 = {2 x 2 x 2} x 2 x 5 x {11 x 11 x 11}

It is evident that the prime factors of 106480 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 106480 is a not perfect cube.

(ix) On factorising 166375 into prime factors, we get:

166375 = 5 x 5 x 5 x 11 x 11 x 11

Group the factors in triples of equal factors as:

166375 = {5 x 5 x 5} x {11 x 11 x 11}

It is evident that the prime factors of 166375 can be grouped into triples of equal factors and no factor is left over.

Therefore, 166375 is a perfect cube.

(x) On factorizing 456533 into prime factors, we get:

456533 = 7 x 7 x 7 x 11 x 11 x11

Group the factors in triples of equal factors as:

456533={7 x 7 x 7} x {11 x 11 x 11}

It is evident that the prime factors of 456533 can be grouped into triples of equal factors and no factor is left over.

Therefore, 456533 is a perfect cube.

Q9. Which of the following are cubes of even natural numbers?

216, 512, 729, 1000, 3375, 13824

Answer :

We know that the cubes of all even natural numbers are even.

The numbers 216, 512, 1000 and 13824 are cubes of even natural numbers.

The numbers 216, 512, 1000 and 13824 are even and it could be verified by divisibility test of 2, i.e., a number is divisible by 2 if it ends with 0, 2, 4, 6 or 8.

Thus, the cubes of even natural numbers are 216, 512, 1000 and 13824.

Q10. Which of the following are cubes of odd natural numbers?

125, 343, 1728, 4096, 32768, 6859

Answer :

We know that the cubes of all odd natural numbers are odd.

The numbers 125, 343, and 6859 are cubes of odd natural numbers.

Any natural numbers could be either even or odd.

Therefore, if a natural number is not even, it is odd.

Now, the numbers 125, 343 and 6859 are odd (It could be verified by divisibility test of 2, i.e., a number is divisible by 2 if it ends with 0, 2, 4, 6 or 8).

None of the three numbers 125, 343 and 6859 are divisible by 2. Therefore, they are not even, they are odd. The numbers 1728, 4096 and 32768 are even.

Thus, cubes of odd natural numbers are 125, 343 and 6859.

Q11. What is the smallest number by which the following numbers must be multiplied, so that the products are perfect cubes?

(i) 675 (ii) 1323 (iii) 2560 (iv) 7803 (v) 107811 (vi) 35721

Answer :

(i) On factorising 675 into prime factors, we get:

$675 = 3 \times 3 \times 3 \times 5 \times 5$

On grouping the factors in triples of equal factors, we get:

$675 = \{3 \times 3 \times 3\} \times 5 \times 5$

It is evident that the prime factors of 675 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 675 is a not perfect cube. However, if the number is multiplied by 5, the factors can be grouped into triples of equal factors and no factor will be left over.

Thus, 675 should be multiplied by 5 to make it a perfect cube.

(ii) On factorising 1323 into prime factors, we get:

1323 = 3 x 3 x 3 x 7 x 7

On grouping the factors in triples of equal factors, we get:

675 = {3 x 3 x 3} x 5 x 5

It is evident that the prime factors of 1323 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 1323 is a not perfect cube.

However, if the number is multiplied by 7, the factors can be grouped into triples of equal factors and no factor will be left over.

Thus, 1323 should be multiplied by 7 to make it a perfect cube.

(iii) On factorising 2560 into prime factors, we get:

2560 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 5

On grouping the factors in triples of equal factors, we get:

2560 = {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x 5

It is evident that the prime factors of 2560 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 2560 is a not perfect cube.

However, if the number is multiplied by 5 x 5 = 25, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 2560 should be multiplied by 25 to make it a perfect cube.

(iv) On factorising 7803 into prime factors, we get:

7803 = 3 x 3 x 3 x 17 x 17

On grouping the factors in triples of equal factors, we get:

7803 = {3 x 3 x 3} x 17 x 17

It is evident that the prime factors of 7803 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 7803 is a not perfect cube.

However, if the number is multiplied by 17, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 7803 should be multiplied by 17 to make it a perfect cube.

(v) On factorising 107811 into prime factors, we get:

107811 = 3 x 3 x 3 x 3 x 11 x 11 x 11

On grouping the factors in triples of equal factors, we get:

107811 ={3 x 3 x 3} x 3 x{11 x 11 x 11}

It is evident that the prime factors of 107811 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 107811 is a not perfect cube.

However, if the number is multiplied by 3 x 3 = 9, the factors be grouped into triples of equal factors such that no factor is left over.

Thus, 107811 should be multiplied by 9 to make it a perfect cube.

(vi) On factorising 35721 into prime factors, we get:

35721 = 3 x 3 x 3 x 3 x 3 x 3 x 7 x 7

On grouping the factors in triples of equal factors, we get:

35721 = {3 x 3 x 3} x {3 x 3 x 3} x 7 x 7

It is evident that the prime factors of 35721 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 35721 is a not perfect cube.

However, if the number is multiplied by 7, the factors be grouped into triples of equal factors such that no factor is left over.

Thus, 35721 should be multiplied by 7 to make it a perfect cube.

Q12. By which smallest number must the following numbers be divided so that the quotient is a perfect cube?

(i) 675 (ii) 8640 (iii) 1600 (iv) 8788 (v) 7803 (vi) 107811 (vii) 35721 (viii) 243000

Answer :

(i) On factorising 675 into prime factors, we get: 675 = 3 x 3 x 3 x 5 x 5

On grouping the factors in triples of equal factors, we get:

675 = {3 x 3 x 3 x 5 x 5}

It is evident that the prime factors of 675 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 675 is a not perfect cube.

However, if the number is divided by 5 x 5 = 25, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 675 should be divided by 25 to make it a perfect cube.

(ii) On factorising 8640 into prime factors, we get:

8640 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 x 5

On grouping the factors in triples of equal factors, we get:

8640 = {2 x 2 x 2} x {2 x 2 x 2 } x {3 x 3 x 3} x 5

It is evident that the prime factors of 8640 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 8640 is a not perfect cube.

However, if the number is divided by 5, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 8640 should be divided by 5 to make it a perfect cube.

(iii) On factorising 1600 into prime factors, we get:

1600 = 2 x 2 x 2 x 2 x 2 x 2 x 5 x 5

On grouping the factors in triples of equal factors, we get:

1600 = {2 x 2 x 2} x {2 x 2 x 2} x 5 x 5

It is evident that the prime factors of 1600 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 1600 is a not perfect cube.

However, if the number is divided by (5 x 5= 25), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 1600 should be divided by 25 to make it a perfect cube.

(iv) On factorising 8788 into prime factors, we get:

8788 = 2 x 2 x 13 x 13 x 13

On grouping the factors in triples of equal factors, we get:

8788 = 2 x 2 x {13 x 13 x 13}

It is evident that the prime factors of 8788 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 8788 is a not perfect cube. However, if the number is divided by (2 x 2= 4), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 8788 should be divided by 4 to make it a perfect cube.

(v) On factorising 7803 into prime factors, we get:

7803 = 3 x 3 x 3 x 17 x 17

On grouping the factors in triples of equal factors, we get:

7803 = {3 x 3 x 3} x 17 x 17

It is evident that the prime factors of 7803 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 7803 is a not perfect cube. However, if the number is divided by 17 x 17 = 289), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 7803 should be divided by 289 to make it a perfect cube.

(vi) On factorising 107811 into prime factors, we get:

107811 = 3 x 3 x 3 x 3 x 11 x 11 x 11

On group the factors in triples of equal factors, we get: 107811 = {3 x 3 x 3} x 3 x {11 x 11 x 11}

It is evident that the prime factors of 107811 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 107811 is a not perfect cube.

However, if the number is divided by 3, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 107811 should be divided by 3 to make it a perfect cube.

(vii) On factorising 35721 into prime factors, we get:

35721 = 3 x 3 x 3 x 3 x 3 x 3 x 7 x 7

On grouping the factors in triples of equal factors, we get:

35721 = {3 x 3 x 3} x {3 x 3 x 3} x 7 x7

It is evident that the prime factors of 35721 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 35721 is a not perfect cube.

However, if the number is divided by (7 x 7= 49), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 35721 should be divided by 49 to make it a perfect cube.

(viii) On factorising 243000 into prime factors, we get:

243000 = 2 x 2 x 2 x 3 x 3 x 3 x 3 x 3 x 5 x 5 x 5

On grouping the factors in triples of equal factors, we get:

243000 = {2 x 2 x 2} x {3 x 3 x 3} x 3 x 3 x {5 x 5 x 5}

It is evident that the prime factors of 243000 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243000 is a not perfect cube. However, if the number is divided by (3 x 3= 9), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 243000 should be divided by 9 to make it a perfect cube.

Q13. Prove that if a number is trebled they its cube is 27 times the cube of the given number.

Answer:

Let us consider a number n. Then its cube be n^3.

If the number n is trebled, i.e., 3n, we get:

$(3n)^3 = 3^n \times n^3 = 27\ n^3$

It is evident that the cube of 3n is 27 times of the cube of n.

Hence, the statement is proved.

Q14. What happens to the cube of a number if the number is multiplied by

(i) 3? (ii) 4? (iii) 5?

Answer:

(i) Let us consider a number n. Its cube would be n^3. If n is multiplied by 3, it becomes 3n.

Let us now find the cube of 3n, we get: $(3n)^3 = 3^3 \times n^3 = 27n^3$ Therefore, the cube of 3n is 27 times of the cube of n.

Thus, if a number is multiplied by 3, its cube is 27 times of the cube of that number.

(ii) Let us consider a number n. Its cube would be n^3. If n is multiplied by 4, it becomes 4n. Let us now find the cube of 4n, we get: $(4n)^3 = 4^3 \times n^3 = 64n^3$ Therefore, the cube of 4n is 64 times of the cube of n.

Thus, if a number is multiplied by 4, its cube is 64 times of the cube of that number.

(iii) Let us consider a number n. Its cube would be n3. If the number n is multiplied by 5, it becomes 5n. Let us now find the cube of 4n, we get: $(5n)^3 = 5^3 \times n^3 = 125n^3$ Therefore, the cube of 5n is 125 times of the cube of n.

Thus, if a number is multiplied by 5, its cube is 125 times of the cube of that number.

Q15. Find the volume of a cube, one face of which has an area of 64 m^2.

Answer :

Area of a face of cube is given by:

$A = s^2$, where s = Side of the cube

Further, volume of a cube is given by:

$V = s^3$, where s = Side of the cube

It is given that the area of one face of the cube is 64 m^2. Therefore we have:

$s^2 = 64$

=> s = $\sqrt{64}$ = 8 m

Now, volume is given by:

V = s^3 = 8^3

V = 8 x 8 x 8 = 512 m^3

Thus, the volume of the cube is 512 m^3.

Q16. Find the volume of a cube whose surface area is 384 m^2.

Answer :

Surface area of a cube is given by: SA = 682, where s = Side of the cube

Further, volume of a cube is given by: V = s3, where s = Side of the cube

It is given that the surface area of the cube is 384 m2. Therefore, we have: 6s2 = 384 s = a = 1,/64 = 8 m Now, volume is given by: V = s3 = 83 V= 8 x 8 x 8 =512 m3 Thus, the required volume is 512 m3.

Q17. Evaluate the following:

(i) $\{(5^2 + 12^2)^{1/2}\}3$ (ii) $\{(6^2 + 8^2)^{1/2}\}^3$

Answer:

(i) To evaluate the value of the given expression, we can proceed as follows:

$\{(5^2 + 12^2)^{1/2}\}^3$

$= \{(25 + 144)^{1/2}\}^3 = \{(169)^{1/2}\}^3$

=

$$(\sqrt{169})^3$$
$$(\sqrt{13 \times 13})^3$$
$$13^3$$
$$13 \times 13 \times 13$$
$$= 2197$$

(ii) To evaluate the value of the given expression, we can proceed as follows:

$\{(6^2 + 8^2)^{1/2}\}^3$

$= \{(36 + 64)^{1/2}\}^3$

$= \{(100)^{1/2}\}^3$

=

$$(\sqrt{100})^3$$
$$= (\sqrt{10 \times 10})^3$$
$$= 10^3$$
$$= 10 \times 10 \times 10$$
$$= 1000$$

Q18. Write the units digit of the cube of each of the following numbers: 31, 109, 388, 833, 4276, 5922, 77774, 44447, 125125125

Answer :

Properties:

If numbers end with digits 1, 4, 5, 6 or 9, its cube will have the same ending digit.

If a number ends with 2, its cube will end with 8.

If a number ends with 8, its cube will end with 2.

If a number ends with 3, its cube will end with 7.

If a number ends with 7, its cube will end with 3.

From the above properties, we get:

Cube of the number 31 will end with 1.

Cube of the number 109 will end with 9.

Cube of the number 388 will end with 2.

Cube of the number 833 will end with 7.

Cube of the number 4276 will end with 6.

Cube of the number 5922 will end with 8.

Cube of the number 77774 will end with 4.

Cube of the number 44447 will end with 3.

Cube of the number 125125125 will end with 5.

Q19. Find the cubes of the following numbers by column method:

(i) 35 (ii) 56 (iii) 72

Answer:

(i) We have to find the cube of 35 using column method. We have:

a = 3 and b = 5

Column I a^3	Column II $3 \times a^2 \times b$	Column III $3 \times a \times b^2$	Column IV b^3
$3^3 = 27$	$3 \times a^2 \times b = 3 \times 3^2 \times 5 = 135$	$3 \times a \times b^2 = 3 \times 3 \times 5^2 = 225$	$5^3 = 125$
+ 15	+ 23	+ 12	12<u>5</u>
<u>42</u>	15<u>8</u>	23<u>7</u>	
42	8	7	5

Thus, the cube of 35 is 42875.

(ii) We have to find the cube of 56 using the column method. We have:

a = 5 and b = 6

Column I a^3	Column II $3 \times a^2 \times b$	Column III $3 \times a \times b^2$	Column IV b^3
$5^3 = 125$	$3 \times a^2 \times b = 3 \times 5^2 \times 6 = 450$	$3 \times a \times b^2 = 3 \times 5 \times 6^2 = 540$	$6^3 = 216$
+ 50	+ 56	+ 21	21$\underline{6}$
$\underline{175}$	50$\underline{6}$	56$\underline{1}$	
175	6	1	6

(iii) We have to find the cube of 72 using the column method. We have:

a = 7 and b = 2

Column I a^3	Column II $3 \times a^2 \times b$	Column III $3 \times a \times b^2$	Column IV b^3
7^3	$3 \times a^2 \times b = 3 \times 7^2 \times 2 = 294$	$3 \times a \times b^2 = 3 \times 7 \times 2^2 = 84$	$2^3 = 8$
+ 30	+ 8	+ 0	$\underline{8}$
$\underline{373}$	30$\underline{2}$	8$\underline{4}$	
373	2	4	8

Q20. Which of the following numbers are not perfect cubes?

(i) 64 (ii) 216 (iii) 243 (iv) 1728

Answer :

(i) On factorising 64 into prime factors, we get:

64 = 2 x 2 x 2 x 2 x 2 x 2

On grouping the factors in triples of equal factors, we get: 64 = {2 x 2 x 2} x {2 x 2 x 2}

It is evident that the prime factors of 64 can be grouped into triples of equal factors and no factor is left over.

Therefore, 64 is a perfect cube.

(ii) On factorising 216 into prime factors, we get:

216 = 2 x 2 x 2 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

216 = {2 x 2 x 2} x {3 x 3 x 3}

It is evident that the prime factors of 216 can be grouped into triples of equal factors and no factor is left over.

Therefore, 216 is a perfect cube.

(iii) On factorising 243 into prime factors, we get:

243 = 3 x 3 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

243 = {3 x 3 x 3} x 3 x 3

It is evident that the prime factors of 243 cannot be grouped into triples of equal factors such that no factor is left over. Therefore, 243 is a not perfect cube.

(iv) On factorising 1728 into prime factors, we get:

1728 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

1728 = {2 x 2 x 2} x {2 x 2 x 2} x {3 x 3 x 3}

It is evident that the prime factors of 1728 can be grouped into triples of equal factors and no factor is left over.

Therefore, 1728 is a perfect cube.

Thus, (iii) 243 is the required number, which is not a perfect cube.

Q21. For each of the non-perfect cubes in Q. No. 20 find the smallest number by which it must be:

(a) multiplied so that the product is a perfect cube.

(b) divided so that the quotient is a perfect cube.

Answer :

The only non-perfect cube in question number 20 is 243.

(a) On factorising 243 into prime factors, we get:

243 = 3 x 3 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

243 ={3 x 3 x 3}x 3 x 3

It is evident that the prime factors of 243 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243 is not a perfect cube.

However, if the number is multiplied by 3, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 243 should be multiplied by 3 to make it a perfect cube.

(b) On factorising 243 into prime factors, we get:

243 = 3 x 3 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

243 = {3 x 3 x 3} x 3 x 3

It is evident that the prime factors of 243 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243 is not a perfect cube.

However, if the number is divided by 3 x 3 = 9), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 243 should be divided by 9 to make it a perfect cube.

Q.22: By taking three different values of n verify the truth of the following statements:

(i) If n is even, then n^3 is also even.

(ii) if n is odd, then n^3 is also odd.

(iii) If n leaves remainder 1 when divided by 3, then n^3 also leaves 1 as the remainder when divided by 3.

(iv) If a natural number n is of the form 3p + 2 then n^3 also a number of the same type.

Answer :

(i) Let the three even natural numbers be 2, 4 and 8.

Cubes of these numbers are:

$2^3 = 8$, $4^3 = 64$, $8^3 = 512$

By divisibility test, it is evident that 8, 64 and 512 are divisible by 2.

Thus, they are even. This verifies the statement.

(ii) Let the three odd natural numbers be 3, 9 and 27. Cubes of these numbers are: $3^3 = 27$, $9^3 = 729$, $27^3 = 19683$

By divisibility test, it is evident that 27, 729 and 19683 are divisible by 3.

Thus, they are odd.

This verifies the statement.

(iii) Three natural numbers of the form $(3n + 1)$ can be written by choosing $n = 1,2,3...$ etc.

Let three such numbers be 4,7 and 10. Cubes of the three chosen numbers are: $4^3 = 64$, $7^3 = 343$ and $10^3 = 1000$

Cubes of 4,7 and 10 can expressed as:

$64 = 3 \times 21 + 1$, which is of the form $(3n + 1)$ for $n = 21$

$343 = 3 \times 114 + 1$, which is of the form $(3n + 1)$ for $n = 114$

$1000 = 3 \times 333 + 1$, which is of the form $(3n + 1)$ for $n = 333$

Cubes of 4, 7, and 10 can be expressed as the natural numbers of the form $(3n + 1)$ for some natural number n. Hence, the statement is verified.

(iv) Three natural numbers of the form $(3p + 2)$ can be written by choosing $p = 1,2,3...$ etc.

Let three such numbers be 5, 8 and 11.

Cubes of the three chosen numbers are: $5^3 = 125$, $8^3 = 512$ and $11^3 = 1331$

Cubes of 5, 8, and 11 can be expressed as:

$125 = 3 \times 41 + 2$, which is of the form $(3p + 2)$ for $p = 41$

$512 = 3 \times 170 + 2$, which is of the form $(3p + 2)$ for $p = 170$

$1331 = 3 \times 443 + 2$, which is of the form $(3p + 2)$ for $p = 443$

Cubes of 5, 8, and 11 could be expressed as the natural numbers of the form $(3p + 2)$ for some natural number p. Hence, the statement is verified.

Q23. Write true (T) or false (F) for the following statements:

(i) 392 is a perfect cube.

(ii) 8640 is not a perfect cube.

(iii) No cube can end with exactly two zeros.

(iv) There is no perfect cube which ends in 4.

(v) For an integer a, a^3 is always greater than a^2.

(vi) If a and b are integers such that $a^2 > b^2$, then $a^3 > b^3$.

(vii) if a divides b, then a^3 divides b^3

(viii) If a^2 ends in 9, then a^3 ends in 7.

(ix) If a^2 ends in 5, then a^3 ends in 25.

(x) I a^2 ends in an even number of zeros, then a^3 ends in an odd number of zeros.

Answer:

(i) False

On factorising 392 into prime factors, we get:

392 = 2 x 2 x 2 x 7 x 7

On grouping the factors in triples of equal factors, we get:

392 = {2 x 2 x 2} x 7 x 7

It is evident that the prime factors of 392 cannot be grouped into triples of equal factors such that no factor is left over. Therefore, 392 is not a perfect cube.

(ii) True

On factorising 8640 into prime factors, we get:

8640 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 x 5

On grouping the factors in triples of equal factors, we get:

8640 = {2 x 2 x 2} x {2 x 2 x 2} x {3 x 3 x 3} x 5

It is evident that the prime factors of 8640 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 8640 is not a perfect cube.

(iii) True

Because a perfect cube always ends with multiples of 3 zeros, e.g., 3 zeros, 6 zeros etc.

(iv) False.

64 is a perfect cube, and it ends with 4.

(v) False

It is not true for a negative integer Example: $(-5)^2 = 25$; $(-5)^3 = -125$

=> $(-5)^3 < (-5)^2$

(vi) False

It is not true for negative integers. Example: $(-5)^2 > (-4)^2$ but $(-5)^3 < (-4)^3$

(vii) True

a divides b

$$\frac{b^3}{a^3} = \frac{b\times b\times b}{a\times a\times a} = \frac{(ak)\times(ak)\times(ak)}{a\times a\times a}$$

a divides b

b = ak for some k

$$\frac{b^3}{a^3} = \frac{(ak)\times(ak)\times(ak)}{a\times a\times a} = k^3$$

$$=> b^3 = a^3(k^3)a^3\ divides\ b^3$$

(viii) False

a^3 ends in 7 if a ends with 3.

But for every a^2 ending in 9, it is not necessary that a is 3.

E.g., 49 is a square of 7 and cube of 7 is 343.

(ix) False

$35^2 = 1225$ but $35^3 = 42875$

(x) False

$100^2 = 10000$ and $100^3 = 100000$

Exercise 4.2

Q1. Find the cubes of:

(i) − 11 (ii) − 12 (iii) − 21

Answer:

(i) Cube of − 11 is given as: $(-11)^3 = -11 \times -11 \times -11 = -1331$

Thus, the cube of 11 is (-1331).

(ii) Cube of − 12 is given as: $(-12)^3 = -12 \times -12 \times -12 = -1728$

Thus, the cube of − 12 is (- 1728).

(iii) Cube of − 21 is given as:

$(-21)^3 = -21 \times -21 \times -21 = -9261$

Thus the cube of − 21 is (- 9261).

Q2. Which of the following numbers are cubes of negative integers?

(i) − 64

(ii) − 1056

(iii) − 2197

(iv) − 2744

(v) − 42875

Answer:

In order to check if a negative number is a perfect cube, first check if the corresponding positive integer is a perfect cube. Also, for any positive integer m, $-m^3$ is the cube of − m.

(i) On factorizing 64 into prime factors, we get:

64 = 2 x 2 x 2 x 2 x 2 x 2

On grouping the factors in triples of equal factors, we get:

64 = {2 x 2 x 2} x {2 x 2 x 2}

It is evident that the prime factors of 64 can be grouped into triples of equal factors and no factor is left over. Therefore, 64 is a perfect cube. This implies that − 64 is also a perfect cube.

Now, collect one factor from each triplet and multiply, we get: 2 x 2=4

This implies that 64 is a cube of 4. Thus, − 64 is the cube of -4.

(ii) On factorising 1056 into prime factors, we get:

1056 = 2 x 2 x 2 x 2 x 2 x 3 x 11

On grouping the factors in triples of equal factors, we get:

1056 ={2 x 2 x 2}x 2 x 2 x 3 x 11

It is evident that the prime factors of 1056 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 1056 is not a perfect cube. This implies that – 1056 is not a perfect cube as well.

(iii) On factorising 2197 into prime factors, we get:

2197 =13 x 13 x 13

On grouping the factors in triples of equal factors, we get:

2197 = {13 x 13 x13}

It is evident that the prime factors of 2197 can be grouped into triples of equal factors and no factor is left over. Therefore, 2197 is a perfect cube. This implies that – 2197 is also a perfect cube.

Now, collect one factor from each triplet and multiply, we get 13. This implies that 2197 is a cube of 13. Thus, -2197 is the cube of – 13.

(iv) On factorizing 2744 into prime factors, we get:

2744 = 2 x 2 x 2 x 7 x 7 x 7

On grouping the factors in triples of equal factors, we get: 2744 = {2 x 2 x 2} x {7 x 7 x 7}

It is evident that the prime factors of 2744 can be grouped into triples of equal factors and no factor is left over. Therefore, 2744 is a perfect cube. This implies that – 2744 is also a perfect cube.

Now, collect one factor from each triplet and multiply, we get:

2 x 7 = 14

This implies that 2744 is a cube of 14.

Thus, – 2744 is the cube of – 14.

(v) On factorizing 42875 into prime factors, we get:

42875 = 5 x 5 x 5 x 7 x 7 x 7

On grouping the factors in triples of equal factors, we get:

42875 = {5 x 5 x 5} x {7 x 7 x 7}

It is evident that the prime factors of 42875 can be grouped into triples of equal factors and no factor is left over. Therefore, 42875 is a perfect cube.

This implies that – 42875 is also a perfect cube.

Now, collect one factor from each triplet and multiply, we get: 5 x 7 = 35

This implies that 42875 is a cube of 35. Thus, – 42875 is the cube of – 35.

Q3. Show that the following integers are cubes of negative integers. Also find the integer whose cube is the given integer.

(i) – 5832   (ii) – 2744000

Answer:

In order to check if a negative number is a perfect cube, first check if the corresponding positive integer is a perfect cube. Also, for any positive integer in, $-m^3$ is the cube of $-m$.

(i) On factorising 5832 into prime factors, we get:

5832 = 2 x 2 x 2 x 3 x 3 x 3 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

5832 = 2 x 2 x 2 x 3 x 3 x 3 x 3 x 3 x 3

It is evident that the prime factors of 5832 can be grouped into triples of equal factors and no factor is left over. Therefore, 5832 is a perfect cube.

This implies that -5832 is also a perfect cube.

Now, collect one factor from each triplet and multiply, we get:

2 x 3 x 3 = 18

This implies that 5832 is a cube of 18.

Thus, − 5832 is the cube of − 18.

(ii) On factorising 2744000 into prime factors, we get:

2744000 = 2 x 2 x 2 x 2 x 2 x 2 x 5 x 5 x 5 x 7 x 7 x 7

On grouping the factors in triples of equal factors, we get:

2744000 = {2 x 2 x 2} x {2 x 2 x 2} x {5 x 5 x 5} x {7 x 7 x 7}

It is evident that the prime factors of 2744000 can be grouped into triples of equal factors and no factor is left over. Therefore, 2744000 is a perfect cube. This implies that − 2744000 is also a perfect cube.

Now, collect one factor from each triplet and multiply, we get: 2 x 2 x 5 x 7 = 140

This implies that 2744000 is a cube of 140. Thus, − 2744000 is the cube of − 140.

Q4. Find the cube of:

(i) $\frac{7}{9}$

(ii) $-\frac{8}{11}$

(iii) $\frac{12}{7}$

(iv) $-\frac{13}{8}$

(v) $2\frac{2}{5}$

(vi) $3\frac{1}{4}$

(vii) 0.3

(viii) 1.5

(ix) 0.08

(x) 2.1

Answer:

$(i)\ (\frac{m}{n})^3 = \frac{m^3}{n^3}$

$(\frac{7}{9})^3 = \frac{7^3}{9^3} = \frac{7\times7\times7}{9\times9\times9} = \frac{343}{729}$

$(ii)\ (-\frac{m}{n})^3 = \frac{-m^3}{n^3}$

$(-\frac{8}{11})^3 =-\frac{8^3}{11^3} =-(\frac{8\times8\times8}{11\times11\times11}) =-\frac{512}{1331}$

$(iii)\ (\frac{m}{n})^3 = \frac{m^3}{n^3}$

$(\frac{12}{7})^3 = \frac{12^3}{7^3} = (\frac{12\times12\times12}{7\times7\times7}) = \frac{1728}{343}$

$(iv)\ (-\frac{m}{n})^3 \ = - \frac{m^3}{n^3}$

$(-\frac{13}{8})^3 = \frac{13^3}{8^3} = -(\frac{13 \times 13 \times 13}{8 \times 8 \times 8}) = -\frac{2197}{512}$

(v) We have:

$2\frac{2}{5} = \frac{12}{5}$

$Also, (\frac{m}{n})^3 = \frac{m^3}{n^3}$

$(\frac{12}{5})^3 = \frac{12^3}{5^3} = \frac{12 \times 12 \times 12}{5 \times 5 \times 5} = \frac{1728}{125}$

(vi) We have:

$3\frac{1}{4} = \frac{13}{4}$

$Also, (\frac{m}{n})^3 = \frac{m^3}{n^3}$

$(\frac{13}{4})^3 = \frac{13^3}{4^3} = \frac{13 \times 13 \times 13}{4 \times 4 \times 4} = \frac{2197}{64}$

(vii) We have:

$0.3 = \frac{3}{10}$

$Also, (\frac{m}{n})^3 = \frac{m^3}{n^3}$

$(\frac{3}{10})^3 = \frac{3^3}{10^3} = \frac{3 \times 3 \times 3}{10 \times 10 \times 10} = \frac{27}{1000} = 0.027$

(viii) We have:

$$1.5 = \frac{15}{10}$$

$$Also, (\frac{m}{n})^3 = \frac{m^3}{n^3}$$

$$(\frac{15}{10})^3 = \frac{15^3}{10^3} = \frac{15\times15\times15}{10\times10\times10} = \frac{3375}{1000} = 3.375$$

(ix) We have:

$$0.08 = \frac{8}{100}$$

$$Also, (\frac{m}{n})^3 = \frac{m^3}{n^3}$$

$$(\frac{8}{100})^3 = \frac{8^3}{100^3} = \frac{8\times8\times8}{100\times100\times100} = \frac{512}{1000000} = 0.000512$$

(x) We have:

$$2.1 = \frac{21}{10}$$

$$Also, (\frac{m}{n})^3 = \frac{m^3}{n^3}$$

$$(\frac{21}{10})^3 = \frac{21^3}{10^3} = \frac{21\times21\times21}{10\times10\times10} = \frac{9261}{1000} = 9.261$$

Q5. Find which of the following numbers are cubes of rational numbers?

(i) $\frac{27}{64}$

(ii) $\frac{125}{128}$

(iii) 0.001331 (iv) 0.04

Answer:

(i) We have:

$\frac{27}{64} = \frac{3\times3\times3}{8\times8\times8} = \frac{3^3}{8^3} = (\frac{3}{8})^3$

Therefore, $\frac{27}{64}\ is\ a\ cube\ of\ \frac{3}{8}$

(ii) We have:

$\frac{125}{128} = \frac{5\times5\times5}{2\times2\times2\times2\times2\times2\times} = \frac{5^3}{2^3\times2^3\times2}$

It is evident that 128 cannot be grouped into triples of equal factors; Therefore, $\frac{125}{128}$ is not a cube of a rational number.

(iii) We have:

0.001331 = $\frac{1331}{1000000} = \frac{11\times11\times11}{2\times2\times2\times2\times2\times2\times5\times5\times5\times5\times5\times5} = \frac{11^3}{(2\times2\times5\times5)^3}$

(iv) We have:

0.04 = $\frac{4}{100} = \frac{2\times2}{5\times5\times5\times5}$

It is evident that 4 and 100 could not be grouped into triples of equal factors; therefore, 0.04 is not a cube of rational number.

Exercise 4.3

Q1. Find the cube roots of the following numbers by successive subtraction of numbers: 1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397,...

(i) 64 (ii) 512 (iii) 1728

Answer:

(i) We have:

64

$\underline{\ 1\ }$

63

$\underline{\ 7\ }$

56

$\underline{19}$

37

$\underline{37}$

0

Subtraction is performed 4 times.

$\sqrt[3]{64} = 4$

(ii) We have:

$$\begin{array}{r} 512 \\ \underline{-\ 1} \\ 511 \\ \underline{-\ 7} \\ 504 \\ \underline{-\ 19} \\ 485 \\ \underline{-\ 37} \\ 448 \\ \underline{-\ 61} \\ 387 \\ \underline{-\ 91} \\ 296 \\ \underline{-\ 127} \\ 169 \\ \underline{-\ 169} \\ 0 \end{array}$$

Subtraction is performed 8 times.

$\sqrt[3]{512} = 8$

(iii) We have:

1728

1

1727

7

1720

19

1701

37

1664

61

1603

91

1512

127

1385

169

1216

217

999

271

728

331

397

397

0

Subtraction is performed 12 times.

$\sqrt[3]{1728} = 12$

Q2. Using the method of successive subtraction examine whether or not the following numbers are perfect cubes:

(i) 130 (ii) 345 (iii) 792 (iv) 1331

Answer:

(i)

We have:

130

$\underline{\quad 1}$

129

$\underline{\quad 7}$

122

$\underline{\ 19}$

103

$\underline{\ 37}$

66

$\underline{61}$

5

Therefore, the next number to be subtracted is 91, which is greater than 5.

Hence, 130 is not a perfect cube.

(ii) We have:

345

$\underline{\quad 1}$

344

$\underline{\;\; 7}$

337

$\underline{\;\; 19}$

318

$\underline{\;\; 37}$

281

$\underline{\;\; 61}$

220

$\underline{\;\; 91}$

129

$\underline{127}$

2

Therefore, the next number to be subtracted is 161, which is greater than 2.

Hence, 345 is not a perfect cube

(iii) We have:

792

<u>1</u>

791

<u>7</u>

784

<u>19</u>

765

<u>37</u>

728

<u>61</u>

667

<u>91</u>

576

<u>127</u>

449

<u>169</u>

280

<u>217</u>

63

Therefore, the next number to be subtracted is 271, which is greater than 63.

Hence, 792 is not a perfect cube

(iv) We have:

1331

<u>   1</u>

1330

<u>   7</u>

1323

<u>   19</u>

1304

<u>   37</u>

1267

<u>   61</u>

1206

<u>   91</u>

1115

<u> 127</u>

988

<u>169</u>

819

<u>217</u>

602

271

331

331

0

The subtraction is performed 11 times.

Therefore, $\sqrt[3]{1331}$ = 11

Thus, 1331 is a perfect cube.

Q3. Find the smallest number that must be subtracted from those of the numbers in question 2 which are not perfect cubes, to make them perfect cubes. What are the corresponding cube roots?

Answer:

(i)

We have:

130

1

129

7

122

19

103

37

66

$\underline{\quad 61}$

5

The next number to be subtracted is 91, which is greater than 5.

130 is not a perfect cube.

However, if we subtract 5 from 130, we will get 0 on performing successive subtraction and the number will become a perfect cube.

If we subtract 5 from 125, we get 125. Now, find the cube root using successive subtraction.

We have:

125

$\underline{\quad 1\ }$

125

$\underline{\quad 7}$

117

$\underline{\quad 19}$

98

$\underline{\ 37\ }$

61

$\underline{\ 61\ }$

0

The subtraction is performed 5 times.

$\sqrt[3]{125}$ = 5

Thus, it is a perfect cube.

(ii) We have:

345

$\underline{\quad 1}$

344

$\underline{\quad 7}$

337

$\underline{\ 19}$

318

$\underline{\ 37}$

281

$\underline{\ 61}$

220

$\underline{\ 91}$

129

$\underline{127}$

2

Since, the next number to be subtracted is 161, which is greater than 2.

Thus, 345 is not a perfect cube.

However, if we subtract 2 from 345, we will get 0 on performing successive subtraction and the number will become a perfect cube.

If we subtract 2 from 345, we get 343. Now, find the cube root using successive subtraction.

343

$\underline{1}$

342

$\underline{7}$

335

$\underline{19}$

316

$\underline{37}$

279

$\underline{61}$

218

$\underline{91}$

127

$\underline{127}$

0

The subtraction is performed 7 times.

$\sqrt[3]{343}$ = 7

Thus, it is a perfect cube.

(iii) We have:

792

$\underline{\quad 1}$

791

$\underline{\quad 7}$

784

$\underline{\ 19}$

765

$\underline{\ 37}$

728

$\underline{\ 61}$

667

$\underline{\ 91}$

576

$\underline{127}$

449

$\underline{169}$

280

$\underline{217}$

63

The next number to be subtracted is 271, which is greater than 63.

792 is not a perfect cube.

However, if we subtract 63 from 792, we will get 0 on performing successive subtraction and the number will become a perfect cube.

If we subtract 63 from 792, we get 729.

Now, find the cube root using the successive subtraction. We have:

729

1

728

7

721

19

702

37

665

61

604

91

513

127

386

169

217

217

0

The subtraction is performed 9 times.

$\sqrt[3]{729} = 9$

Thus, it is a perfect cube.

Q4. Find the cube root of each of the following natural numbers:

(i) 343

(ii) 2744

(iii) 4913

(iv) 1728

(v) 35937

(vi) 17576

(vii) 134217728

(viii) 48228544

(ix) 74088000

(x) 157464

(xi) 1157625

(xii) 33698267

Answer :

(i) Cube root using units digit:

Let us consider 343.

The unit digit is 3; therefore, the unit digit in the cube root of 343 is 7.

There is no number left after striking out the units, tens and hundreds digits of the given number; therefore, the cube root of 343 is 7.

Hence, $\sqrt[3]{343}$ = 7

(ii) Cube root using units digit:

Let us consider 2744.

The unit digit is 4; therefore, the unit digit in the cube root of 2744 is 4.

After striking out the units, tens and hundreds digits of the given number, we are left with 2.

Now, 1 is the largest number whose cube is less than or equal to 2.

Therefore, the tens digit of the cube root of 2744 is 1.

Hence, $\sqrt[3]{2744}$ = 14

(iii) Cube root using units digit:

Let us consider 4913.

The unit digit is 3; therefore, the unit digit in the cube root of 4913 is 7.

After striking out the units, tens and hundreds digits of the given number, we are left with 4.

Now, 1 is the largest number whose cube is less than or equal to 4.

Therefore, the tens digit of the cube root of 4913 is 1.

Hence, $\sqrt[3]{4913}$ = 17

(iv) Cube root using units digit:

Let us consider 1728.

The unit digit is 8; therefore, the unit digit in the cube root of 1728 is 2.

After striking out the units, tens and hundreds digits of the given number, we are left with 1.

Now, 1 is the largest number whose cube is less than or equal to 1.

Therefore, the tens digit of the cube root of 1728 is 1.

Hence, $\sqrt[3]{1728}$ = 12

(v) Cube root using units digit:

Let us consider 35937.

The unit digit is 7; therefore, the unit digit in the cube root of 35937 is 3.

After striking out the units, tens and hundreds digits of the given number, we are left with 35.

Now, 3 is the largest number whose cube is less than or equal to 35 (33 < 35 < 43).

Therefore, the tens digit of the cube root of 35937 is 3.

Hence, $\sqrt[3]{35937}$ = 33

(vi) Cube root using units digit:

Let us consider the number 17576.

The unit digit is 6; therefore, the unit digit in the cube root of 17576 is 6.

After striking out the units, tens and hundreds digits of the given number, we are left with 17.

Now, 2 is the largest number whose cube is less than or equal to 17 (23 < 17 < 33).

Therefore, the tens digit of the cube root of 17576 is 2. Hence,' $\sqrt[3]{17576}$ = 26

(vii) Cube root by factors:

On factorising 134217728 into prime factors, we get:

134217728=2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

On grouping the factors in triples of equal factors, we get: 134217728 = {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x{2 x 2 x 2} x{2 x 2 x 2} x{2 x 2 x 2}

Now, taking one factor from each triple, we get: $\sqrt[3]{134217728}$ =2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = 512

(viii) Cube root by factors:

On factorising 48228544 into prime factors, we get:

48228544 = 2 x 2 x 2 x 2 x 2 x 2 x 7 x 7 x 7 x 13 x 13 x 13

On grouping the factors in triples of equal factors, we get:

48228544 = {2 x 2 x 2} x {2 x 2 x 2} x {7 x 7 x 7} x {13 x 13 x 13}

Now, taking one factor from each triple, we get: $\sqrt[3]{48228544}$ =2 x 2 x 7 x13=364

(ix) Cube root by factors:

On factorising 74088000 into prime factors, we get: 74088000 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 x 5 x 5 x 5 x 7 x 7 x 7

On grouping the factors in triples of equal factors, we get: 74088000 = {2 x 2 x 2} x {2 x 2 x 2} x {3 x 3 x 3} x {5 x 5 x 5} x {7 x 7 x 7}

Now, taking one factor from each triple, we get: $\sqrt[3]{74088000}$ = 2 x 2 x 3 x 5 x 7 = 420

(x) Cube root using units digit:

Let is consider 157464.

The unit digit is 4; therefore, the unit digit in the cube root of 157464 is 4.

After striking out the units, tens and hundreds digits of the given number, we are left with 157.

Now, 5 is the largest number whose cube is less than or equal to 157 (53 < 157 < 63).

Therefore, the tens digit of the cube root 157464 is 5. Hence, $\sqrt[3]{157464}$ = 54

(xi) Cube root by factors:

On factorising 1157625 into prime factors, we get: 1157625 = 3 x 3 x 3 x 5 x 5 x 5 x 7 x 7 x 7

On grouping the factors in triples of equal factors, we get:

1157625 = {3 x 3 x 3} x {5 x 5 x 5} x {7 x 7 x 7}

Now, taking one factor from each triple, we get:

$\sqrt[3]{1157625}$ = 3 x 5 x 7 = 105

(xii) Cube root by factors:

On factorising 33698267 into prime factors, we get: 33698267 = 17 x 17 x 17 x 19 x 19 x 19

On grouping the factors in triples of equal factors, we get: 33698267 = {17 x 17 x 17} x {19 x 19 x 19}

Now, taking one factor from each triple, we get:

$\sqrt[3]{33698267}$ = 17 x 19 = 323

Q5. Find the smallest number which when multiplied with 3600 will make the product a perfect cube. Further, find the cube root of the product.

Answer:

On factorising 3600 into prime factors, we get:

3600 = 2 x 2 x 2 x 2 x 3 x 3 x 5 x 5

On grouping the factors in triples of equal factors, we get:

3600 = {2 x 2 x 2} x 2 x 3 x 3 x 5 x 5

It is evident that the prime factors of 3600 cannot be grouped into triples of equal factors such that no factor is left over. Therefore, 3600 is not a perfect cube.

However, if the number is multiplied by (2 x 2 x 3 x 5 = 60), the factors can be grouped into triples of equal factors such that no factor is left over.

Hence, the number 3600 should be multiplied by 60 to make it a perfect cube.

Also, the product is given as:

3600 x 60 ={2 x 2 x 2} x 2 x 3 x 3 x 5 x 5 x 60

216000 = {2 x 2 x 2} x 2 x 3 x 3 x 5 x 5 x (2 x 2 x 3 x 5)

216000 ={2 x 2 x 2}x {2 x 2 x 2} x {3 x 3 x 3} x {5 x 5 x 5}

To get the cube root of the produce 216000, take one factor from each triple.

Cube root = 2 x 2 x 3 x 5 = 60

Q6. Multiply 210125 by the smallest number so that the product is a perfect cube. Also, find out the cube root of the product.

Answer:

On factorising 210125 into prime factors, we get: 210125 = 5 x 5 x 5 x 41 x 41

On grouping the factors in triples of equal factors, we get: 210125 = {5 x 5 x x 41 x 41

It is evident that the prime factors of 210125 cannot be grouped into triples of equal factors such that no factor is left over. Therefore, 210125 is not a perfect cube. However, if the number is multiplied by 41, the factors can be grouped into triples of equal factors such that no factor is left over.

Hence, the number 210125 should be multiplied by 41 to make it a perfect cube. Also, the product is given as: 210125 x 41 = {5 x 5 x 5} x {41 x 41 x 41} 8615125 = {5 x 5 x x {41 x 41 x 41} To get the cube root of the produce 8615125, take one factor from each triple. The cube root is 5 x 41 = 205. Hence, the required numbers are 41 and 205.

Q7. What is the smallest number by which 8192 must be divided so that quotient is a perfect cube? Also, find the cube root of the quotient so obtained.

Answer :

On factorising 8192 into prime factors, we get:

8192 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

On grouping the factors in triples of equal factors, we get:

8192 = {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x 2

It is evident that the prime factors of 8192 cannot be grouped into triples of equal factors such that no factor is left over. Therefore, 8192 is not a perfect cube. However, if the number is divided by 2, the factors can be grouped into triples of equal factors such that no factor is left over.

Hence, the number 8192 should be divided by 2 to make it a perfect cube.

Also, the quotient is given as:

$$\frac{8192}{2} = \frac{2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2}{2}$$

4096 = {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2}

To get the cube root of the quotient 4096, take one factor from each triple. We get:

Cube root = 2 x 2 x 2 x 2 = 16

Hence, the required numbers are 2 and 16.

Q8. Three numbers are in the ratio 1 : 2 : 3. The sum of their cubes is 98784. Find the numbers.

Answer:

Let the numbers be x, 2x and 3x

Therefore

$$x^3 + (2x)^3 + (3x)^3 = 98784$$
$$=> x^3 + 8x^3 + 27x^3 = 98784$$
$$=> 36x^3 = 98784$$
$$=> x^3 = \frac{98784}{36}$$
$$=> x^3 = 2744$$
$$=> x = \sqrt[3]{2744} = \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7} = 2 \times 7 = 14$$

Hence, the numbers are 14, (2 x 14 = 28) and (3 x 14 = 42)

Q9. The volume of a cube is 9261000 m3. Find the side of the cube.

Answer:

Volume of a cube is given by:

V = s^3, where s = Side of the cube

It is given that the volume of the cube is 9261000 m^3; therefore, we have:

s^3 = 9261000

Let us find the cube root of 9261000 using prime factorisation:

9261000 = 2 x 2 x 2 x 3 x 3 x 3 x 5 x 5 x 5 x 7 x 7 x 7 = {2 x 2 x 2} x {3 x 3 x 3} x {5 x 5 x 5} x {7 x 7 x 7}

9261000 could be written as triples of equal factors; therefore, we get: Cube root = 2 x 3 x 5 x 7 = 210

Therefore

s^3 = 9261000

s = $(9261000)^{(1/3)}$ = 210

Hence, the length of the side of cube is 210 m.

Exercise 4.4

Q1. Find the cube roots of each of the following integers:

(i) -125

(ii)-5832

(iii) -2744000

(iv) -753571

(v) -32768

Answer:

(i) We have:

$\sqrt[3]{-125}$=- $\sqrt[3]{125}$= $-\sqrt[3]{5 \times 5 \times 5}$= -5

(ii) We have:

$\sqrt[3]{-5832}$= $-\sqrt[3]{5382}$=-

To find the cube root of 5832, we use the method of unit digits.

Let us consider the number 5832. The unit digit is 2; therefore the unit digit in the cube root of 5832 will be 8. After striking out the units, tens and hundreds of digits of the given number, we are left with 5.

Now, 1 is the largest number whose cube is less than or equal to 5. Therefore, the tens digit of the cube root of 5832 is 1.

$\sqrt[3]{5832}$= 18

$\sqrt[3]{-5832}$= $-\sqrt[3]{5832}$= -18

(iii) We have:

$\sqrt[3]{-2744000}$= $-\sqrt[3]{2744000}$

To find the cube root of 2744000, we use the method of factorization.

On factorizing 2744000 into prime factors, we get:

2744000= $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$

On grouping the factors in triples of equal factors, we get:

2744000 = $(2 \times 2 \times 2)(2 \times 2 \times 2)(5 \times 5 \times 5)(7 \times 7 \times 7)$

It is evident that the prime factors of 2744000 can be grouped into triples of equal factors and no factor is left over.

Now, collect one factor from each triplet and multiply; we get: $(2 \times 2 \times 5 \times 7)$ = 140 This implies that 2744000 is a cube of 140.

Hence

$\sqrt[3]{-2744000}$= $-\sqrt[3]{2744000}$= -140

(iv) We have:

$\sqrt[3]{-753571}$= $-\sqrt[3]{753571}$

To find the cube root of 753571, we use the method of unit digits.

Let us consider the number 753571. The unit digit is 1; therefore the unit digit in the cube root of 753571 will be 1. After striking out the units, tens and hundreds digits of the given number, we are left with 753. Now, 9 is the largest number whose cube is less than or equal to 753 ($9^3 < 753 < 10^3$). Therefore, the tens digit of the cube root 753571 is 9.

$\sqrt[3]{753571}$=91

$\sqrt[3]{-753571}$=- $\sqrt[3]{753571}$= -91

(v) We have:

$\sqrt[3]{-32768}$= $-\sqrt[3]{32768}$

To find the cube root of 32768, we use the method of unit digits.

Let us consider the number 32768.

The unit digit is 8; therefore, the unit digit in the cube root of 32768 will be 2. After striking out the units, tens and hundreds digits of the given number, we are left with 32. Now, 3 is the largest number whose cube is less than or equal to 32 ($3^3 < 32 < 4^3$). Therefore, the tens digit of the cube root 32768 is 3.

$\sqrt[3]{32768}$=32

$\sqrt[3]{-32768}$=- $\sqrt[3]{32768}$= -32

Q2. Show that:

(i) $\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{27 \times 64}$

(ii) $\sqrt[3]{64 \times 729} = \sqrt[3]{64} \times \sqrt[3]{729}$

(iii) $\sqrt[3]{-125 \times 216} = \sqrt[3]{-125} \times \sqrt[3]{216}$

(iv) $\sqrt[3]{-125 \times -1000} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$

Answer:

(i)

LHS=$\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{3 \times 3 \times 3} \times \sqrt[3]{4 \times 4 \times 4} = 3 \times 4 = 12$

RHS=$\sqrt[3]{27 \times 64} = \sqrt[3]{3 \times 3 \times 3 \times 4 \times 4 \times 4} = 3 \times 4 = 12$

Because LHS is equal to RHS, the equation is true.

(ii)

LHS= $\sqrt[3]{64 \times 729} = \sqrt[3]{4 \times 4 \times 4 \times 9 \times 9 \times 9} = 4 \times 9 = 36$

RHS= $\sqrt[3]{64} \times \sqrt[3]{729} = \sqrt[3]{4 \times 4 \times 4} \times \sqrt[3]{9 \times 9 \times 9} = 4 \times 9 = 36$

Because LHS is equal to RHS, the equation is true.

(iii)

LHS= $\sqrt[3]{-125 \times 216} = \sqrt[3]{-5 \times -5 \times -5 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3} = -5 \times 2 \times 3 = -30$

RHS =
$\sqrt[3]{-125} \times \sqrt[3]{216} = \sqrt[3]{-5 \times -5 \times -5} \times \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} = -5 \times 2 \times 3 = -30$

Because LHS is equal to RHS, the equation is true.

(iv)

LHS= $\sqrt[3]{-125 \times -1000} = \sqrt[3]{-5 \times -5 \times -5 \times -10 \times -10 \times -10} = -5 \times -10 = 50$

RHS = $\sqrt[3]{-125} \times \sqrt[3]{-1000} = \sqrt[3]{-5 \times -5 \times -5} \times \sqrt[3]{-10 \times -10 \times -10} = -5 \times -10 = 50$

Because LHS is equal to RHS, the equation is true.

Q3. Find the cube root of the following numbers:

(i) 8×125

(ii) -1728×216

(iii) -27×2744

(iv) -729×-15625

Answer:

(i) From the above property, we have:

$\sqrt[3]{8 \times 125} = \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5} = 2 \times 5 = 10$

(ii) From the above property, we have:

$\sqrt[3]{-1728 \times 216}$
$=\sqrt[3]{-1728} \times \sqrt[3]{-216}$

$=-\sqrt[3]{1728} \times \sqrt[3]{-216}$

Cube root using units digit: Let us consider the number 1728. The unit digit is 8; therefore, the unit digit in the cube root of 1728 will be 2. After striking out the units, tens and hundreds digits of the given number, we are left with 1. Now, 1 is the largest number whose cube is less than or equal to 1. Therefore, the tens digit of the cube root of 1728 is 1.

$\sqrt[3]{-1728}$= 12

On factorizing 216 into prime factors, we get:

216 =$2 \times 2 \times 2 \times 3 \times 3 \times 3$

On grouping the factors in triples of equal factors, we get:

216 =$2 \times 2 \times 2 \times 3 \times 3 \times 3$

Now, taking one factor from each triple, we get:

216 =2×3

Thus

$\sqrt[3]{-1728} \times \sqrt[3]{-216}$ =12×6=-72

(iii) From the above property, we have:

$\sqrt[3]{-27 \times 2744}$
$=\sqrt[3]{-27} \times \sqrt[3]{2744}$

$=-\sqrt[3]{27} \times \sqrt[3]{2744}$

Cube root using units digit:

Let us consider the number 2744.

The unit digit is 4; therefore, the unit digit in the cube root of 2744 will be 4. After striking out the units, tens, and hundreds digits of the given number, we are left with 2. Now, 1 is the largest number whose cube is less than or equal to 2. Therefore, the tens digit of the cube root of 2744 is 1.

...12/2744 = 14

Thus

$\sqrt[3]{-27} \times \sqrt[3]{2744}$ =-3×14= -42

(iv) From the above property, we have:

$\sqrt[3]{-729 \times -15625}$
=$\sqrt[3]{-729} \times \sqrt[3]{15625}$

=-$\sqrt[3]{729} \times -\sqrt[3]{15625}$

Cube root using units digit:

Let us consider the number 15625.

The unit digit is 5; therefore, the unit digit in the cube root of 15625 will be 5. After striking out the units, tens and hundreds digits of the given number, we are left with 15. Now, 2 is the largest number whose cube is less than or equal to 15 ($2^3 < 15 < 3^3$). Therefore, the tens digit of the cube root of 15625 is 2.

$\sqrt[3]{15625}$= 25

Also

$\sqrt[3]{729}$=9

Thus

$\sqrt[3]{-729} \times \sqrt[3]{-15625}$ =-9×25= 225

Q4. Evaluate:

(i) $\sqrt[3]{4^3\ times 6^3}$

(ii) $\sqrt[3]{8\ times 17 \times 17 \times 17}$

(iii) $\sqrt[3]{700\ times 2 \times 49 \times 5}$

(iv) $125\sqrt[3]{a^6} - \sqrt[3]{125a^6}$

Answer:

For any two integers a and b,

$\sqrt[3]{ab}$= $\sqrt[3]{a} \times \sqrt[3]{b}$

(i) From the above property, we have:

$\sqrt[3]{4^3\ times 6^3}$= $\sqrt[3]{4^3} \times \sqrt[3]{6^3} = 4 \times 6 = 24$

(ii) Use above property and proceed as follows:

$\sqrt[3]{8\ times 17 \times 17 \times 17}$= $\sqrt[3]{2^3} \times \sqrt[3]{17^3} = 2 \times 17 = 34$

(iii) From the above property, we have:

$\sqrt[3]{700\ times 2 \times 49 \times 5}$
=$\sqrt[3]{2 \times 2\ times 5 \times 5 \times 7 \times \times 2 \times 7 \times 7 \times 5}$

=$\sqrt[3]{2^3\ times 5^3 \times 7^3}$

=$\sqrt[3]{700\ times 2 \times 49 \times 5}$

=$2 \times 5 \times 7$ =70

(iv) From the above property, we have:

$$125\sqrt[3]{a^6}-\sqrt[3]{125a^6}$$
$$=125\sqrt[3]{a^6}-\left(\sqrt[3]{125}\times\sqrt[3]{a^6}\right)$$
$$125\times a^2-\left(5\times a^2\right)$$
$$\sqrt[3]{a}^6=\sqrt[3]{(a\times a\times a)(a\times a\times a)}=a\times a=a^2\ \ and$$
$$\sqrt[3]{125}=\sqrt[3]{5\times 5\times 5}$$
$$=125a^2-5a^2$$
$$=120a^2$$

Q5. Find the cube root of each of the following rational numbers:

(i) $\frac{-125}{729}$

(ii) $\frac{10648}{12167}$

(iii) $\frac{-19683}{24389}$

(iv) $\frac{686}{-3456}$

(v) $\frac{-393304}{-42875}$

Answer:

(i) Let us consider the following rational number:

$\frac{-125}{729}$

Now

$$\sqrt[3]{\frac{-125}{729}}\ \frac{\sqrt[3]{-125}}{\sqrt[3]{729}}\ \frac{-\sqrt[3]{125}}{\sqrt[3]{729}}\ -\frac{5}{9}$$

(ii) Let us consider the following rational number:

$$\frac{10648}{12167}$$

Now

$$\sqrt[3]{\frac{10648}{12167}}\ \frac{\sqrt[3]{10648}}{\sqrt[3]{729}}$$

Cube root by factors:

On factorizing 10648 into prime factors, we get: 10648 = $2 \times 2 \times 2 \times 11 \times 11 \times 11$

On grouping the factors in triples of equal factors, we get: 10648 = $(2 \times 2 \times 2)(11 \times 11 \times 11)$

Now, taking one factor from each triple, we get: $\sqrt[3]{10648} = 2 \times 11$

Also

On factorizing

12167 into prime factors, we get: 12167 = $23 \times 23 \times 23$

On grouping the factors in triples of equal factors, we get: 12167 = $(23 \times 23 \times 23)$

Now, taking one factor from the triple, we get: 23

Now

$$\sqrt[3]{\frac{10648}{12167}}\ \frac{\sqrt[3]{10648}}{\sqrt[3]{729}}\ \frac{22}{23}$$

(iii) Let us consider the following rational number:

$$\frac{-19683}{24389}$$

Now,

$$\sqrt[3]{\frac{-19683}{24389}}$$

$$\frac{\sqrt[3]{-19683}}{\sqrt[3]{24389}}$$

$$\frac{-\sqrt[3]{19683}}{\sqrt[3]{24389}}$$

Cube root by factors:

On factorizing 19683 into prime factors, we get: 19683= $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

On grouping the factors in triples of equal factors, we get: 19683= $(3 \times 3 \times 3)(3 \times 3 \times 3)(3 \times 3 \times 3)$

Now, taking one factor from each triple, we get: $\sqrt[3]{19683} = 3 \times 3 \times 3 = 27$

Also

On factorising 24389 into prime factors, we get:

$\sqrt[3]{24389} = 29 \times 29 \times 29$

On grouping the factors in triples of equal factors, we get: $\sqrt[3]{24389} = 29 \times 29 \times 29$

Now, taking one factor from each triple, we get: $\sqrt[3]{24389} = 29$

Now

$$\sqrt[3]{\frac{-19683}{24389}} \sqrt[3]{\frac{-19683}{24389}} \frac{-\sqrt[3]{19683}}{\sqrt[3]{24389}} \frac{-27}{29}$$

(iv)

Let us consider the following rational number:

$$\frac{686}{-3456}$$

Now

$$\sqrt[3]{\frac{686}{-3456}} - \sqrt[3]{\frac{2\times 7^3}{2^7\times 3^3}} - \sqrt[3]{\frac{7^3}{2^6\times 3^3}} \ \frac{-\sqrt[3]{7^3}}{\sqrt[3]{2^6\times 3^3}} \ \frac{-7}{2\times 2\times 3} \ \frac{-7}{12}$$

(v) Let us consider the following rational number:

$$\frac{-393304}{-42875}$$

Now

$$\sqrt[3]{\frac{393304}{-42875}} \ \frac{\sqrt[3]{-393304}}{\sqrt[3]{-42875}} \ \frac{\sqrt[3]{393304}}{\sqrt[3]{42875}}$$

Cube root by factors:

On factorizing 39304 into prime factors, we get:

39304 = $2 \times 2 \times 2 \times 17 \times 17 \times 17$

On grouping the factors in triples of equal factors, we get:

39304 = $(2 \times 2 \times 2)(17 \times 17 \times 17)$

Now, taking one factor from each triple, we get:

$$\sqrt[3]{393304} = 2 \times 17 = 34$$

Also

On factorizing 42875 into prime factors, we get:

42875 =$5 \times 5 \times 5 \times 7 \times 7 \times 7$

On grouping the factors in triples of equal factors, we get:

42875 = $(5 \times 5 \times 5)(7 \times 7 \times 7)$

Now, taking one factor from each triple, we get:

$$\sqrt[3]{42875} = 5 \times 7 = 35$$

Now

$\sqrt[3]{\frac{-393304}{-42875}}$ $\frac{-\sqrt[3]{-393304}}{\sqrt[3]{-42875}}$ $\frac{-34}{-35}$ $\frac{34}{35}$

Q6. Find the cube root of each of the following rational numbers

(i) 0.001728

(ii) 0.003375

(iii) 0.001

(iv) 1.331

Answer:

(i)

We have:

0.001728=$\frac{1728}{1000000}$

$\sqrt[3]{\frac{1728}{1000000}}$ $\frac{\sqrt[3]{1728}}{\sqrt[3]{1000000}}$

Now

On factorizing 1728 into prime factors, we get:

1728=$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

On grouping the factors in triples of equal factors, we get:

1728=$(2 \times 2 \times 2)(2 \times 2 \times 2)(3 \times 3 \times 3)$

Now, taking one factor from each triple, we get:

$\sqrt[3]{1728}$ =$2 \times 2 \times 3 = 12$

Also

$\sqrt[3]{1000000}$ =$100 \times 100 \times 100 = 100$

0.001728=$\frac{1728}{1000000}$

$\sqrt[3]{\frac{1728}{1000000}}$ $\frac{\sqrt[3]{1728}}{\sqrt[3]{1000000}}$
$\frac{12}{100}$ =0.12

(ii)

We have:

0.003375=$\frac{3375}{1000000}$

$\sqrt[3]{\frac{3375}{1000000}}$ $\frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}}$
Now

On factorizing 3375 into prime factors, we get:

3375=$3 \times 3 \times 3 \times 5 \times 5 \times 5$

On grouping the factors in triples of equal factors, we get:

3375=$(3 \times 3 \times 3)(5 \times 5 \times 5)$

Now, taking one factor from each triple, we get:

$\sqrt[3]{3375}$ =$3 \times 5 = 15$

Also

$\sqrt[3]{1000000}$ =$100 \times 100 \times 100 = 100$

0.003375=$\frac{3375}{1000000}$

$\sqrt[3]{\frac{3375}{1000000}}$ $\frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}}$
$\frac{15}{100}$=0.15

(iii)

We have:

$0.001=\frac{1}{1000}$

$\sqrt[3]{\frac{1}{1000}}\ \frac{\sqrt[3]{1}}{\sqrt[3]{1000}}$

$\frac{1}{10}$ =0.1

(iv)

We have:

1.331 = $\frac{1331}{1000}$

$\sqrt[3]{\frac{1331}{1000}}\ \frac{\sqrt[3]{1331}}{\sqrt[3]{1000}}$

$\frac{11}{10}$ =1.1

Q7. Evaluate each of the following:

(i) $\sqrt[3]{27}+\sqrt[3]{0.008}+\sqrt[3]{0.064}$

(ii) $\sqrt[3]{1000}+\sqrt[3]{0.008}-\sqrt[3]{0.125}$

(iii) $\sqrt[3]{\frac{729}{216}}\times\frac{6}{9}$

(iv) $\sqrt[3]{\frac{0.027}{0.008}}\div\sqrt[3]{\frac{0.09}{0.04}}$

(v) $\sqrt[3]{0.1\times0.1\times0.1\times13\times13\times13}$

To evaluate the value of the given expression, we need to proceed as follows:

$\sqrt[3]{27}+ \sqrt[3]{0.008}+ \sqrt[3]{0.064}=$

$= \sqrt[3]{3 \times 3 \times 3}+ \sqrt[3]{\frac{8}{1000}} + \sqrt[3]{\frac{64}{1000}}$

$= \sqrt[3]{3 \times 3 \times 3}+ \frac{\sqrt[3]{8}}{\sqrt[3]{1000}} + \frac{\sqrt[3]{64}}{\sqrt[3]{1000}}$

$= \sqrt[3]{3 \times 3 \times 3}+ \frac{\sqrt[3]{2\times2\times2}}{\sqrt[3]{1000}} + \frac{\sqrt[3]{4\times4\times4}}{\sqrt[3]{1000}}$

$=3 + \frac{2}{10} + \frac{4}{10}$

=3+0.2+0.4 =3.6

Thus, the answer is 3.6.

(ii) To evaluate the value of the given expression, we need to proceed as follows:

$\sqrt[3]{1000}+ \sqrt[3]{0.008}- \sqrt[3]{0.125}=$

$= \sqrt[3]{10 \times 10 \times 10}+ \sqrt[3]{\frac{8}{1000}} - \sqrt[3]{\frac{125}{1000}}$

$= \sqrt[3]{10 \times 10 \times 10}+ \frac{\sqrt[3]{2^3}}{\sqrt[3]{5^3}} - \frac{\sqrt[3]{64}}{\sqrt[3]{1000}}$

$= \sqrt[3]{10 \times 10 \times 10}+ \frac{\sqrt[3]{2\times2\times2}}{\sqrt[3]{1000}} - \frac{\sqrt[3]{5\times5\times5}}{\sqrt[3]{1000}}$

$=10 + \frac{2}{10} - \frac{5}{10}$

=10+0.2+0.5 =9.7

Thus, the answer is 9.7.

(iii) To evaluate the value of the given expression, we need to proceed as follows:

$\sqrt[3]{\frac{729}{216}} \times \frac{6}{9}$

$\sqrt[3]{\frac{9\times9\times9}{2\times2\times2\times3\times3\times3}} \times \frac{6}{9}$

$\frac{9}{2\times3} \times \frac{6}{9} = 1$

Thus, the answer is 1.

(iv) To evaluate the value of the expression, we need to proceed as follows:

$$\sqrt[3]{\frac{0.027}{0.008}} \div \sqrt[3]{\frac{0.09}{0.04}} - 1$$

$$\sqrt[3]{\frac{\frac{27}{1000}}{\frac{8}{1000}}} \div \sqrt{\frac{\frac{9}{100}}{\frac{4}{100}}} - 1$$

$$\sqrt[3]{\frac{27}{8}} \div \sqrt{9}4 - 1$$

$$\frac{\sqrt[3]{27}}{\sqrt[3]{8}} \div \frac{\sqrt[3]{9}}{\sqrt{4}} - 1$$

$$\frac{3}{2} \div \frac{3}{2} - 1$$

$$\frac{3}{2} \times \frac{2}{3} - 1$$

$$1 - 1 = 0$$

(v) To evaluate the value of the expression, we need to proceed as follows:

$$\sqrt[3]{0.1 \times 0.1 \times 0.1 \times 13 \times 13 \times 13}$$

$$\sqrt[3]{\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times 13 \times 13 \times 13}$$

$$\frac{13}{10}$$

$$= 1.3$$

Thus, the answer is 1.3.

Q9. Fill in the blanks:

(i) $\sqrt[3]{125 \times 27} = 3 \times \ldots\ldots$

(ii) $\sqrt[3]{8 \times \ldots.}$

(iii) $\sqrt[3]{1728} = 12 = 4 \times \ldots\ldots$

(iv) $\sqrt[3]{480} = \sqrt[3]{3} \times 2 \times \sqrt[3]{\ldots}$

(v) $\sqrt[3]{\ldots.} = \sqrt[3]{7} \times \sqrt[3]{8}$

(vi) $\sqrt[3]{\ldots.} = \sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{5}$

(vii) $\frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{\ldots..}{5}$

(viii) $\frac{\sqrt[3]{729}}{\sqrt[3]{1331}} = \frac{9}{\ldots..}$

(ix) $\frac{\sqrt[3]{512}}{\sqrt[3]{\ldots.}} = \frac{8}{13}$

Answer:

(i) 5

$\sqrt[3]{125 \times 27} = 3 \times 5$
$\sqrt[3]{125 \times 27} = \sqrt[3]{125} \times \sqrt[3]{27} = \sqrt[3]{5 \times 5 \times 5} \times \sqrt[3]{3 \times 3 \times 3} = 5 \times 3$
$= 3 \times 5$

(ii)

$8 \times 8 = 64\ \sqrt[3]{8 \times 8 \times 8} = 8$

(iii)

3

$\sqrt[3]{1728} = 12 = 4 \times 3$

(iv)

$\sqrt[3]{480} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5} = 2\sqrt[3]{3} \times \sqrt[3]{5 \times 2 \times 2} = \sqrt[3]{3} \times 2 \times \sqrt[3]{20}$

(v)

$7 \times 8 = 56 \ \sqrt[3]{7 \times 8} = \sqrt[3]{7} \times \sqrt[3]{8}$

(vi)

$7 \times 8 = 56 \ \sqrt[3]{7 \times 8} = \sqrt[3]{7} \times \sqrt[3]{8}$

Q10. The volume of a cubical box is 474.552 cubic meters. Find the length of each side of the box.

Answer:

Volume of a cube is given by:

V = s^3, where s = side of the cube

Now s^3 = 474.552 cubic metres

= $\sqrt[3]{474.552} = \sqrt[3]{\frac{474552}{1000}} = \frac{\sqrt[3]{474552}}{\sqrt[3]{1000}}$

To find the cube root of 474552, we need to proceed as follows:

On factorising 474552 into prime factors, we get:

474552 = 2 x 2 x 2 x 3 x 3 x 3 x 13 x 13 x 13

On grouping the factors in triples of equal factors, we get:

474552 = (2 x 2 x 2) x (3 x 3 x 3) x (13 x 13 x 13)

Now, taking one factor from each triple, we get:

= $\sqrt[3]{474552}$ = $\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 13 \times 13 \times 13}$ = 2 x 3 x 13 = 78

Also $\sqrt[3]{1000} = 10$

s = $\frac{\sqrt[3]{474552}}{\sqrt[3]{1000}} = \frac{78}{10} = 7.8$

Thus the length of the side is 7.8 m.

Q11. Three numbers are to one another 2 : 3 : 4. The sum of their cubes is 0.334125. Find the length of each side of the box.

Answer:

$(2x)^3 + (3x)^3 + (4x)^3 = 0.334125$
$8x^3 + 27x^3 + 64x^3 = 0.334125$
$99x^3 = 0.334125$
$x^3 = \frac{0.334125}{99}$
$x = \sqrt[3]{\frac{0.334125}{99}}$
$x = \sqrt[3]{\frac{3375}{1000000}}$
$x = \frac{15}{100} = 0.15 Thus\ the\ numbers\ are : 2 \times 0.15 = 0.30$
$3 \times 0.15 = 0.45$
$4 \times 0.15 = 0.60$

Q12. Find the side of a cube whose volume is $\frac{24389}{216}\ m^3$.

Volume of a cube with side s is given by:

V = s^3

$s = \sqrt[3]{V}$
$= \sqrt[3]{\frac{24389}{216}}$
$= \frac{\sqrt[3]{24389}}{\sqrt[3]{216}}$
$= \frac{\sqrt[3]{29 \times 29 \times 29}}{\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}} (By\ prime\ factorisation)$
$= \frac{29}{2 \times 3}$
$= \frac{29}{6}$
$Thus,\ the\ length\ of\ the\ side\ is\ \frac{29}{6}\ m$

Q13. Evaluate:

(i) $\sqrt[3]{36}$ x $\sqrt[3]{384}$

(ii) $\sqrt[3]{96}$ x $\sqrt[3]{144}$

(iii) $\sqrt[3]{100}$ x $\sqrt[3]{270}$

(iv) $\sqrt[3]{121}$ x $\sqrt[3]{297}$

Answer:

(i)

36 and 384 are not perfect cubes; therefore, we use the following property:

$\sqrt[3]{ab}$ = $\sqrt[3]{a}$ x $\sqrt[3]{b}$ for any two integers a and b

$\sqrt[3]{36}$ x $\sqrt[3]{384}$

= $\sqrt[3]{36 \times 384}$

= $\sqrt[3]{2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3}$ (By prime factorisation)

= $\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$

= 2 x 2 x 2 x 3 = 24

Thus, the answer is 24.

(ii)

96 and 122 are not perfect cubes; therefore, we use the following property:

$\sqrt[3]{ab}$ = $\sqrt[3]{a}$ x $\sqrt[3]{b}$ for any two integers a and b

= $\sqrt[3]{96}$ x $\sqrt[3]{144}$

= $\sqrt[3]{96 \times 144}$

= $\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$ (By prime factorisation)

= $\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$

= 2 x 2 x 2 x 3 = 24

Thus, the answer is 24.

(iii) 100 and 270 are not perfect cubes; therefore, we use the following property:

$\sqrt[3]{ab}$ = $\sqrt[3]{a}$ x $\sqrt[3]{b}$ for any two integers a and b

= $\sqrt[3]{100}$ x $\sqrt[3]{270}$

= $\sqrt[3]{100 \times 270}$

= $\sqrt[3]{2 \times 2 \times 5 \times 5 \times 2 \times 3 \times 3 \times 3 \times 5}$ (By prime factorisation)

= $\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5}$

= 2 x 3 x 5 = 30

Thus, the answer is 30.

(iv) 121 and 297 are not perfect cubes; therefore, we use the following property:

$\sqrt[3]{ab}$ = $\sqrt[3]{a}$ x $\sqrt[3]{b}$ for any two integers a and b

= $\sqrt[3]{121}$ x $\sqrt[3]{297}$

= $\sqrt[3]{121 \times 297}$

= $\sqrt[3]{11 \times 11 \times 3 \times 3 \times 3 \times 11}$ (By prime factorisation)

= $\sqrt[3]{11 \times 11 \times 11 \times 3 \times 3 \times 3}$ = 11 x 3 = 33

Thus, the answer is 33.

Q14. Find the cube roots of the numbers 3048625, 20346417, 210644875, 57066625 using the fact that:

(i) 3048625 = 3375 x 729

(ii) 20346417 = 9261 x 2197

(iii) 210644875 = 42875 x 4913

(iv) 57066625 = 166375 x 343

Answer:

(i) To find the cube root, we use the following property:

$\sqrt[3]{ab}$ = $\sqrt[3]{a}$ x $\sqrt[3]{b}$ for any two integers a and b

Now

$\sqrt[3]{3048625}$

$= \sqrt[3]{3375 \times 729}$

$\sqrt[3]{3375} \times \sqrt[3]{729} (By\ the\ above\ property)$

$= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 9 \times 9 \times 9} (Byprime factorisation)$

$= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 9 \times 9 \times 9}$

$= 3 \times 5 \times 9 = 135$

Thus, the answer is 135.

(ii) To find the cube root, we use the following property:

$\sqrt[3]{ab}$ = $\sqrt[3]{a}$ x $\sqrt[3]{b}$ for any two integers a and b

Now

$\sqrt[3]{20346417}$
$= \sqrt[3]{9261 \times 2197}$
$\sqrt[3]{9261} \times \sqrt[3]{2197} (By\ the\ above\ property)$
$= \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7 \times 13 \times 13 \times 13} (Byprime factorisation)$
$= 3 \times 7 \times 13 = 273$
Thus, the answer is 273.

(iii) To find the cube root, we use the following property:

$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$ for any two integers a and b

Now

$\sqrt[3]{210644875}$
$= \sqrt[3]{42875 \times 4913}$
$\sqrt[3]{42875} \times \sqrt[3]{4913} (By\ the\ above\ property)$
$= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7 \times 17 \times 17 \times 17} (Byprime factorisation)$
$= 5 \times 7 \times 17 = 595$
Thus, the answer is 595.

(iv) To find the cube root, we use the following property:

$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$ for any two integers a and b

Now

$\sqrt[3]{57066625}$
$= \sqrt[3]{166375 \times 343}$
$\sqrt[3]{42875} \times \sqrt[3]{4913} (By\ the\ above\ property)$
$= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7 \times 11 \times 11 \times 11} (Byprime factorisation)$
$= 5 \times 7 \times 11 = 385$
Thus, the answer is 385.

Q15. Find the units digit of the cube root of the following numbers:

(i) 226981

(ii) 13824

(iii) 571787

(iv) 175616

Answer:

(i) Cube root using units digit:

Let us consider the number 226981. The unit digit is 1; therefore, the unit digit of the cube root of 226981 is 1.

(ii) Cube root using units digit:

Let us consider the number 13824. The unit digit is 4; therefore, the unit digit of the cube root of 13824 is 4.

(iii) Cube root using units digit:

Let us consider the number 571787. The unit digit is 7; therefore, the unit digit of the cube root of 571787 is 3.

(iv) Cube root using units digit:

Let us consider the number 175616. The unit digit is 6; therefore, the unit digit of the cube root of 175616 is 6.

Q16. Find the tens digit of the cube root of the each of the numbers in Q. No. 15.

Answer:

(i) Let us consider the number 226981.

The unit digit is 1; therefore, the unit digit of the cube root of 226981 is 1.

After striking out the units, tens and hundreds of digits of the given number, we are left with 226.

Now, 6 is the largest number, whose cube is less than or equal to 226 ($6^3 < 226 < 7^3$).

Therefore, the tens digit of the cube root of 226981 is 6.

(ii) Let us consider the number 13824.

The unit digit is 4; therefore, the unit digit of the cube root of 13824 is 4.

striking out the units, tens and hundreds of digits of the given number, we are left with 13.

Now, 2 is the largest number, whose cube is less than or equal to 13 ($2^3 < 13 < 3^3$).

Therefore, the tens digit of the cube root of 13824 is 2.

(iii) Let us consider the number 571787.

The unit digit is 7; therefore, the unit digit of the cube root of 571787 is 3.

After striking out the units, tens and hundreds of digits of the given number, we are left with 571.

Now, 8 is the largest number, whose cube is less than or equal to 571 ($8^3 < 571 < 9^3$).

Therefore, the tens digit of the cube root of 571787 is 8.

(iv) Let us consider the number 175616.

The unit digit is 6; therefore, the unit digit of the cube root of 175616 is 6.

After striking out the units, tens and hundreds of digits of the given number, we are left with 175.

Now, 5 is the largest number, whose cube is less than or equal to 175 ($5^3 < 175 < 6^3$).

Therefore, the tens digit of the cube root of 175616 is 5.

Exercise 4.5

Making use of the cube root table, find the table, find the cube roots of the following (correct to three decimal points):

1. 7
2. 70
3. 700
4. 7000
5. 1100
6. 780
7. 7800
8. 1346
9. 940
10. 5112
11. 9800
12. 732
13. 7342
14. 133100
15. 37800
16. 0.27
17. 8.6
18. 0.86
19. 8.65
20. 7532
21. 833
22. 34.2

Answer:

Q1. 7

Answer :

Because 7 lies between 1 and 100, we will look at the row containing 7 in the column of x.

By the cube root table, we have:

$\sqrt[3]{7}$ = 1.913

Thus, the answer is 1.913.

Q2. 70

Because 70 lies between 1 and 100, we will look at the row containing 70 in the column of x.

By the cube root table, we have:

$\sqrt[3]{70}$ = 4.121

Thus, the answer is 4.121

Q3. We have:

700 = 70 x 10

Cube root of 700 will be in the column of $\sqrt[3]{10x}$ against 70.

By the cube root table, we have:

$\sqrt[3]{700}$ = 8.879

Thus, the answer is 8.879

Q4. We have:

7000 = 70 x 100

$\sqrt[3]{7000}$ = $\sqrt[3]{7 \times 1000}$ = $\sqrt[3]{7}$ x $\sqrt[3]{1000}$

By the cube root table, we have:

$\sqrt[3]{7}$ = 1.913 and $\sqrt[3]{1000}$ = 10

$\sqrt[3]{7000}$ = $\sqrt[3]{7}$ = $\sqrt[3]{1000}$ = 1.913 x 10 = 19.13

Thus, the answer is 19.13

Q5. We have:

1100 = 11 x 100

Therefore,

$\sqrt[3]{1100}$ = $\sqrt[3]{11 \times 100}$ = $\sqrt[3]{11}$ x $\sqrt[3]{100}$

By the cube root table, we have:

$\sqrt[3]{11}$ = 2.224 and $\sqrt[3]{100}$ = 4.642

$\sqrt[3]{1100}$ = $\sqrt[3]{11}$ x $\sqrt[3]{100}$ = 2.224 x 4.642 = 10.323 (Up to three decimal places)

Thus, the answer is 10.323.

Q6. We have:

780 = 78 x 10

Therefore, Cube root of 780 will be in the column of $\sqrt[3]{10x}$ against 78.

By the cube root table, we have:

$\sqrt[3]{780}$ = 9.025

Thus, the answer is 9.025

Q7. 7800

7800 = 78 x 100

$\sqrt[3]{7800}$ = $\sqrt[3]{78 \times 100}$ = $\sqrt[3]{78}$ x $\sqrt[3]{100}$

By the cube root table, we have:

$\sqrt[3]{78}$ = 4.273 and $\sqrt[3]{100}$ = 4.642

$\sqrt[3]{7800}$ = $\sqrt[3]{78}$ x $\sqrt[3]{100}$ = 4.273 x 4.642 = 19.835 (up to three decimal places)

Thus, the answer is 19.835

Q8. 1346

Answer:

By prime factorisation, we have:

1346 = 2 x 673 => $\sqrt[3]{1346}$ = $\sqrt[3]{2}$ x $\sqrt[3]{673}$

Also 670 < 673 < 680 => $\sqrt[3]{670}$ < $\sqrt[3]{673}$ < $\sqrt[3]{680}$

From the cube root table, we have:

$\sqrt[3]{670}$ = 8.750 and $\sqrt[3]{680}$ = 8.794

For the difference (680 – 670), i.e., 10, the difference in the values

= 8.794 – 8.750 = 0. 044

For the difference of (673 – 670), i.e., 3, the difference in the values

= $\frac{0.044\times3}{10}$ = 0.0132 = 0.013 (up to three decimal places)

= 8.750 + 0.013 = 8.763

Now,

$\sqrt[3]{1346}$ = $\sqrt[3]{2}$ x $\sqrt[3]{8.763}$ = 1.260 x 8.763 = 11.041 (up to three decimal places)

Thus, the answer is 11.041

Q9. 940

Answer :

We have:

250 = 25 x 100

Cube root of 250 would be in the column of $\sqrt[3]{10x}$ against 25.

By the cube root table, we have:

$\sqrt[3]{250}$ = 6.3

Thus, the required cube root is 6.3.

Q10. 5112

Answer :

By prime factorisation, we have:

$5112 = 2^3 \times 3^2 \times 71 \Rightarrow \sqrt[3]{} = 2 \times \sqrt[3]{9} \times \sqrt[3]{71}$

By the cube root table, we have:

$\sqrt[3]{9} = 2.080$ and $\sqrt[3]{71} = 4.141$

$\sqrt[3]{5112} = 2 \times \sqrt[3]{9} \times \sqrt[3]{71} = 2 \times 2.080 \times 4.141 = 17.227$ (up to three decimal places)

Thus, the required cube root is 17.227.

Q11. We have:

$9800 = 98 \times 100$

$\sqrt[3]{9800} = \sqrt[3]{98 \times 100} = \sqrt[3]{98} \times \sqrt[3]{100}$

By the cube root table, we have:

$\sqrt[3]{98} = 4.610$ and $\sqrt[3]{100} = 4.642$

$\sqrt[3]{9800} = \sqrt[3]{98} \times \sqrt[3]{100} = 4.610 \times 4.642 = 21.40$ (up to three decimal places)

Thus, the required cube root is 21.40.

Q12. 732

Answer :

We have:

$730 < 732 < 740 \Rightarrow \sqrt[3]{730} < \sqrt[3]{732} < \sqrt[3]{740}$

From cube root table, we have:

$\sqrt[3]{730} = 9.004$ and $\sqrt[3]{740} = 9.045$

For the difference (740 − 730), i.e., 10, the difference in values

$= 9.045 - 9.004 = 0.041$

For the difference of (732 − 730), i.e., 2, the difference in values

$\frac{0.044 \times 2}{10} = 0.0082$

$\sqrt[3]{732} = 9.004 + 0.008 = 9.012$

Q13. 7342

Answer:

We have:

$7300 < 7342 < 7400 \Rightarrow \sqrt[3]{7300} < \sqrt[3]{7342} < \sqrt[3]{7400}$

From the cube root table, we have:

$\sqrt[3]{7300} = 19.39$ and $\sqrt[3]{7400} = 19.48$

For the difference (7400 − 7300), i.e., 100, the difference in values

$= 19.48 - 19.39 = 0.09$

For the difference of (7342 − 7300), i.e., 42, the difference in the values

$= \frac{0.09 \times 42}{100} = 0.0378 = 0.037$

$\sqrt[3]{7342} = 19.39 + 0.037 = 19.427$

Q14. We have:

133100 = 1331 x 100 => $\sqrt[3]{133100}$ = $\sqrt[3]{1331 \times 100}$ = 11 x $\sqrt[3]{100}$

From the cube root table, we have:

$\sqrt[3]{100}$ = 4.642

$\sqrt[3]{133100}$ = 11 x $\sqrt[3]{100}$ = 11 x 4.642 = 51.062

Q15. We have,

37800 = $2^3 \times 3^3 \times 175 => \sqrt[3]{37800} = \sqrt[3]{2^3 \times 3^3 \times 175} = 6 \times \sqrt[3]{175}$

Also 170 < 175 < 180 => $\sqrt[3]{170}$ < $\sqrt[3]{175}$ < $\sqrt[3]{180}$

From cube root table, we have:

$\sqrt[3]{170}$ = 5.540 and $\sqrt[3]{180}$ = 5.646

For the difference (180 − 170), i.e., 10, the difference in values

= 5.646 − 5.540 = 0.106

For the difference of (175 − 170), i.e., 5, the difference in values

$\frac{0.106 \times 5}{10}$ = 0.053

$\sqrt[3]{175}$ = 5.540 + 0.053 = 5.593

Now 37800 = 6 x $\sqrt[3]{175}$ = 6 x 5.593 = 33.558

Thus, the required cube root is 33.558.

Q16. 0.27

The number 0.27 can be written as $\frac{27}{100}$

Now,

$$\sqrt[3]{0.27} = \sqrt[3]{\frac{27}{100}} = \frac{\sqrt[3]{27}}{\sqrt[3]{100}} = \frac{3}{\sqrt[3]{100}}$$

From cube root table, we have:

$\sqrt[3]{100}$ = 4.642

$\sqrt[3]{0.27}$ = $\frac{3}{\sqrt[3]{100}} = \frac{3}{4.642} = 0.646$

Thus, the required cube root is 0.646.

Q17. 8.6

The number 8.6 can be written as $\frac{86}{10}$

Now

$$\sqrt[3]{8.6} = \sqrt[3]{\frac{86}{10}} = \frac{\sqrt[3]{86}}{\sqrt[3]{10}}$$

$$From\ cube\ root\ table,\ we\ have:$$

$$= \sqrt[3]{86} = 4.414\ and\ \sqrt[3]{10} = 2.154$$

$$= \sqrt[3]{8.6} = \frac{\sqrt[3]{86}}{\sqrt[3]{10}} = \frac{4.414}{2.154} = 2.049$$

Thus, the required cube root is 2.049.

Q18. 0.86

The number 0.86 can be written as $\frac{86}{100}$

Now

$$\sqrt[3]{0.86} = \sqrt[3]{\frac{86}{100}} = \frac{\sqrt[3]{86}}{\sqrt[3]{100}}$$
$$From\ cube\ root\ table,\ we\ have:$$
$$= \sqrt[3]{86} = 4.414\ and\ \sqrt[3]{100} = 4.342$$
$$= \sqrt[3]{0.86} = \frac{\sqrt[3]{86}}{\sqrt[3]{100}} = \frac{4.414}{4.642} = 0.951$$

Thus, the required cube root is 0.951.

Q19. 8.65

Answer :

The number 8.65 could be written as $\frac{865}{100}$

Now

$$\sqrt[3]{8.65} = \sqrt[3]{\frac{865}{100}} = \frac{\sqrt[3]{865}}{\sqrt[3]{100}}$$
$$Also,\ 860 < 865 < 870 => \sqrt[3]{860} < \sqrt[3]{865} < \sqrt[3]{870}$$
$$From\ cube\ root\ table,\ we\ have:$$
$$= \sqrt[3]{860} = 9.510\ and\ \sqrt[3]{870} = 9.546$$

For the difference (870 − 860), i.e., 10, the difference in values

= 9.546 − 9.510 = 0.036

For the difference of (865 − 860), i.e., 5, the difference in values

= $\frac{0.036 \times 5}{10}$ = 0.018 (up to three decimal places)

$\sqrt[3]{865}$ = 9.510 + 0.018 = 9.528 (up to three decimal places)

From cube root table, we also have:

$\sqrt[3]{100}$ 4.642

$= \sqrt[3]{8.65} = \frac{\sqrt[3]{865}}{\sqrt[3]{100}} = \frac{9.528}{4.642} = 2.053$ (up to three decimal places)

Thus, the required cube root is 2.053

Q20. We have, 7532

$$7500 < 7532 < 7600 => \sqrt[3]{7500} < \sqrt[3]{7532} < \sqrt[3]{7600}$$
$$From\ cube\ root\ table,\ we\ have:$$
$$= \sqrt[3]{7500} = 19.57\ and\ \sqrt[3]{7600} = 19.66$$

For the difference of (7600 − 7500), i.e., 100, the difference in values

= 19.66 − 19.57 = 0.09

For the difference of (7532 − 7500), i.e., 32, the difference in values,

= $\frac{0.009 \times 32}{100}$ = 0.0288 = 0.029 (up to three decimal places)

$\sqrt[3]{7532}$ = 19.57 + 0.029 = 19.599

Thus, the required cube root is 19.599

Q21. We have, 833

$$830 < 833 < 840 => \sqrt[3]{830} < \sqrt[3]{833} < \sqrt[3]{840}$$
$$From\ cube\ root\ table,\ we\ have:$$
$$= \sqrt[3]{830} = 9.398\ and\ \sqrt[3]{840} = 9.435$$

For the difference of (840 − 830), i.e., 10, the difference in values

= 9.435 − 9.398 = 0.037

For the difference of (833 − 830), i.e., 3, the difference in values

= $\frac{0.037 \times 3}{10}$ = 0.0111 = 0.011 (up to three decimal places)

$\sqrt[3]{833}$ = 9.398 + 0.011 = 9.409

Thus, the required cube root is 9.409

Q22. 34.2

The number 34.2 could be written as $\frac{342}{10}$

Now,

$$\sqrt[3]{34.2} = \sqrt[3]{\frac{342}{10}} = \frac{\sqrt[3]{342}}{\sqrt[3]{10}}$$

$$Also$$

$$340 < 342 < 350 => \sqrt[3]{340} < \sqrt[3]{342} < \sqrt[3]{350}$$

$$From\ cube\ root\ table,\ we\ have:$$

$$= \sqrt[3]{340} = 6.980\ and\ \sqrt[3]{350} = 7.047$$

For the difference of (350 − 340), i.e., 10, the difference in values

= 7.047 − 6.980 = 0.067

For the difference of (342 − 340), i.e., 2, the difference in values

= $\frac{0.067 \times 2}{10}$ (up to three decimal places)

From cube root table, we also have:

$\sqrt[3]{10}$ = 2.154

$\sqrt[3]{34.2}$ = $\frac{\sqrt[3]{342}}{\sqrt[3]{10}} = \frac{6.993}{2.154} = 3.246$

Thus, the required cube root is 3.246