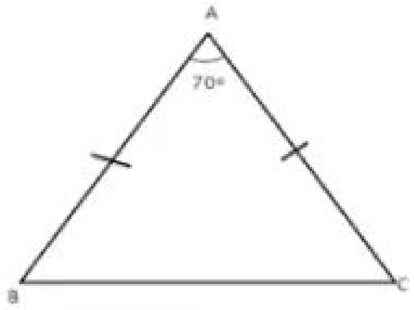
Congruence of Triangles and Inequalities in a Triangle

Exercise 5A

Question 1:

AB=AC implies their opposite angle are equal



$\Rightarrow \quad \angle B = \angle C$ [angles opposite to equal sides]

But $\angle A + \angle B + \angle C = 180°$

$\Rightarrow 70° + \angle B + \angle B = 180°$

$\Rightarrow 70° + 2\angle B = 180°$

$\Rightarrow 2\angle B = 180° - 70°$

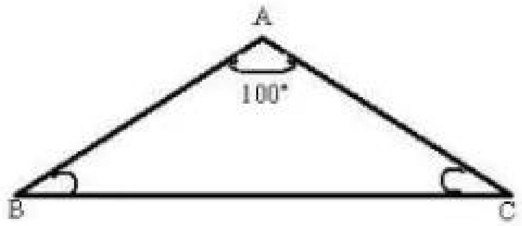
$\Rightarrow 2\angle B = 110°$

$\Rightarrow \angle B = \dfrac{110°}{2}$

$\Rightarrow \angle B = 55°$

$\Rightarrow \angle B = \angle C = 55°$

Question 2:



Consider the isosceles triangle ΔABC.
Since the vertical angle of ABC is 100° , we have, ∠A = 100°.
By angle sum property of a triangle, we have,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 100° + \angle B + \angle C = 180°$

$\Rightarrow 100° + 2\angle B = 180°$ [Since in an isosceles triangle base angles are equal, $\angle B = \angle C$]

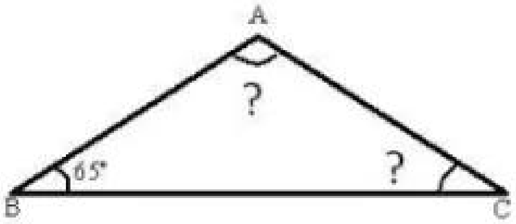
$\Rightarrow 2\angle B = 180° - 100° = 80°$

$\Rightarrow \angle B = \dfrac{80°}{2}$

$\Rightarrow \angle B = 40°$

$\Rightarrow \angle B = \angle C = 40°$

Question 3:



In $\triangle ABC$, if $AB = AC$

$\Rightarrow \triangle ABC$ is an isosceles triangle

$\Rightarrow$ Base angles are equal

$\Rightarrow \angle B = \angle C$

$\Rightarrow \angle C = 65°$ [Since $\angle B = 65°$]

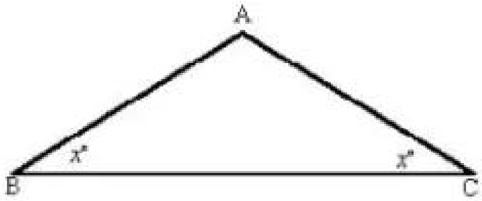
Also by angle sum property, we have

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 65° + 65° = 180°$ [$\angle B = \angle C = 65°$]

$\Rightarrow \angle A = 180° - 130° = 50°$

Question 4:



Let ABC be an isosceles triangle in which AB=AC.

Then we have $\angle B = \angle C$

Let $\angle B = \angle C = x$

Then vertex angle $A = 2(x+x) = 4x$

Now, $x + x + 4x = 180$

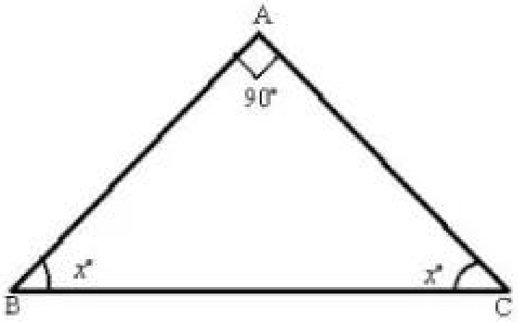
$\Rightarrow 6x = 180$

$\Rightarrow x = \frac{180}{6} = 30$

$\therefore$ Vertex $\angle A = 4 \times 30 = 120^0$

And, $\angle B = \angle C = 30^0$.

Question 5:



In a right angled isosceles triangle, the vertex angle is $\angle A = 90°$ and the other two base angles are equal.

Let x° be the base angle and we have, $\angle B = \angle C = 90°$.

By angle sum property of a triangle, we have

$$\angle A + \angle B + \angle C = 180^\circ$$
$$\Rightarrow 90^\circ + x^\circ + x^\circ = 180^\circ$$
$$\Rightarrow 90^\circ + 2x^\circ = 180^\circ$$
$$\Rightarrow 2x^\circ = 180^\circ - 90^\circ$$
$$\Rightarrow 2x^\circ = 90^\circ$$
$$\Rightarrow x^\circ = \frac{90^\circ}{2}$$
$$\Rightarrow x^\circ = 45^\circ$$

Thus, we have, $\angle B = \angle C = 45^\circ$

Question 6:

Given: ABC is an isosceles triangle in which AB=AC and BC
Is produced both ways,

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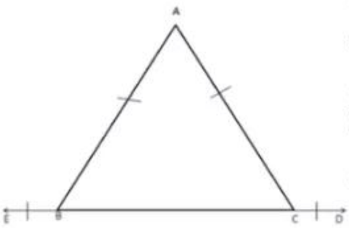


To Prove: $\angle EBA = \angle DCA$

Proof : In ΔABC we have,

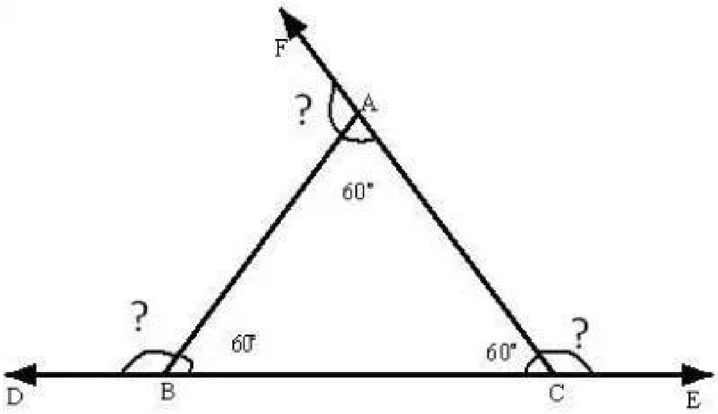
$$AB = AC$$
$$\Rightarrow \angle B = \angle C$$

Now exterior $\angle EBA = \angle A + \angle C = \angle A + \angle B$ $[\because \angle B = \angle C]$

and exterior $\angle DCA = \angle A + \angle B$

$\Rightarrow$ Exterior $\angle EBA$ = Exterior $\angle DCA$.

Question 7:



Let be an equilateral triangle.
Since it is an equilateral triangle, all the angles are equiangular and the measure of each angle is 60°
The exterior angle of ∠A is ∠BAF
The exterior angle of ∠B is ∠ABD
The exterior angle of ∠C is ∠ACE
We can observe that the angles ∠A and ∠BAF, ∠B and ∠ABD, ∠C and ∠ACE and form linear pairs.
Therefore, we have

$$\angle A + \angle BAF = 180^\circ$$
$$\Rightarrow 60^\circ + \angle BAF = 180^\circ$$
$$\Rightarrow \angle BAF = 180^\circ - 60^\circ$$
$$\Rightarrow \angle BAF = 120^\circ$$

Similarly, we have

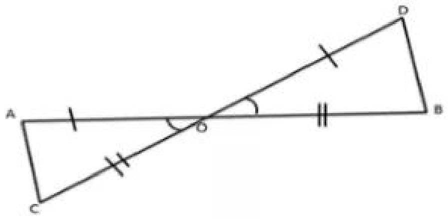
$$\angle B + \angle ABD = 180^\circ$$
$$\Rightarrow 60^\circ + \angle ABD = 180^\circ$$
$$\Rightarrow \angle ABD = 180^\circ - 60^\circ$$
$$\Rightarrow \angle ABD = 120^\circ$$

Also, we have

$$\angle C + \angle ACE = 180^\circ$$
$$\Rightarrow 60^\circ + \angle ACE = 180^\circ$$
$$\Rightarrow \angle ACE = 180^\circ - 60^\circ$$
$$\Rightarrow \angle ACE = 120^\circ$$

Thus, we have, ∠BAF = 120°, ∠ABD = 120°, ∠ACE = 120°
So, the measure of each exterior angle of an equilateral triangle is 120°.

Question 8:



Given: Two lines AB and CD intersect at O and O is the midpoint of AB and CD.

$\Rightarrow AO = OB$ and $CO = OD$

To prove: $AC = BD$ and $AC \parallel BD$

Proof: In $\triangle AOC$ and $\triangle BOD$, we have,

$AO = OB$ [Given: O is the midpoint of AB]

$\angle AOC = \angle BOD$ [Vertically opposite angles]

$CO = OD$ [Given: O is the mipoint of CD]

So, by Side−Angle−Side congruence, we have, $\triangle AOC \cong \triangle BOD$

The corresponding parts of the congruent triangles are equal.

Therefore, we have, $AC = BD$.

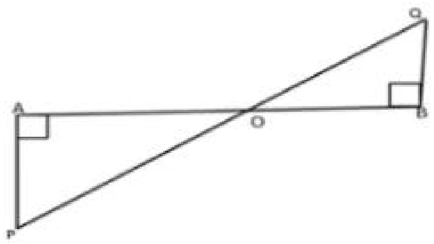
Similarly, by c.p.c.t, we have, $\angle ACO = \angle BDO$ and $\angle CAO = \angle DBO$

This implies that alternate angles formed by AC and BD with transversal CD are equal. This means that, $AC \parallel BD$.

Thus, $AC = BD$ and $AC \parallel BD$.

Question 9:

Given: $PA \perp AB$, $QB \perp AB$, and $PA = QB$
To Prove: $AO = OB$ and $PO = OQ$



Proof: In $\triangle APO$ and $\triangle BPO$,

$\angle PAO = \angle QBO = 90°$ [Given]

$PA = QB$ [Given]

$\angle PAO = \angle QBO$ [Since $PA \perp AB$, and $QB \perp AB$, $PA \parallel QB$, and thus PQ is a transversal, therefore, alternate angles are equal]

So, by Angle−Side−Angle criterion of congruence, we have

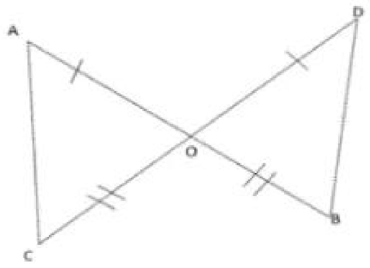
$\triangle APO \cong \triangle BPO$

$\Rightarrow AO = OB$ and $PO = OQ$ [Since corresponding parts of congruent triangles are equal]

Thus, we have
O is the midpoint of AB and PQ.

Question 10:

Given: Line segments AB and CD intersect at O such that OA = OD and OB = OC.



To prove: AC = BD

Proof: In $\triangle AOC$ and $\triangle BOD$, we have

$AO = OD$ [Given]

$\angle AOC = \angle BOD$ [Vertically opposite angles are equal]

$OC = OB$ [Given]

So, by Side−Angle−Side criterion of congruence, we have,

$\Rightarrow \triangle AOC \cong \triangle BOD$

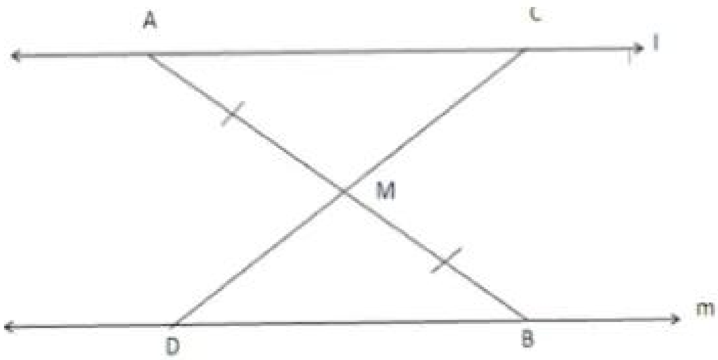
$\Rightarrow AC = BD$ [Since the corresponding parts of the congruent triangles are equal]

$\Rightarrow \angle CAO = \angle BDO$ [by c.p.c.t]

Thus, we have, AC = BD

In case $\angle ODB = \angle OBD$, then $\angle CAO = \angle OBD$ which means alternate angles made by lines AC and BD with transversal AB are equal and then lines AC and BD will be parallel.

Question 11:



Given: Two lines l and m are parallel to each other. M is the midpoint of segment AB. The line segment CD meets AB at M.

To prove: M is the midpoint of CD, that is CM = MD

Proof: In $\triangle AMC$ and $\triangle BMD$, we have

$\angle MAC = \angle MBD$ [Since l and m are parallel, AB is the transversal, and thus, alternate angles are equal]

$AM = MB$ [given]

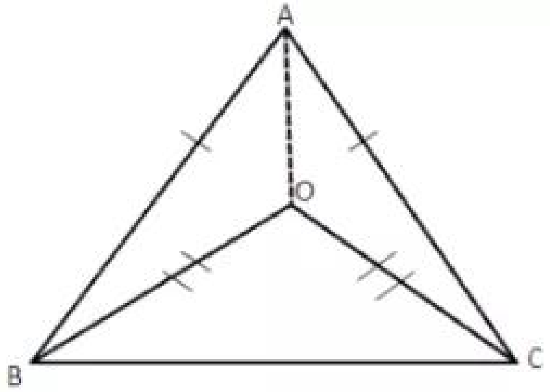
$\angle AMC = \angle BMD$ [vertically opposite angles are equal]

So, by Angle−Side−Angle criterion of congruence, we have

$\triangle AMC \cong \triangle BMD$

Therefore, by corresponding parts of the congruent triangles are equal, we have, CM = MD

Question 12:



Given: $AB = AC$ and O is an interior point of the triangle such that $OB = OC$

To prove: $\angle ABO = \angle ACO$

Construction: Join AO

Proof: In $\triangle AOB$ and $\triangle AOC$, we have

$AB = AC$ [Given]

$AO = AO$ [Common]

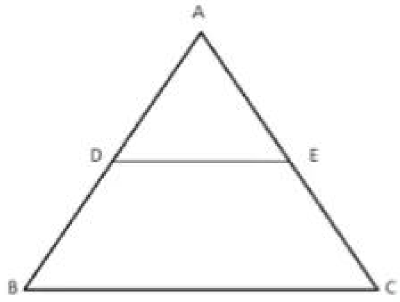
$OB = OC$ [Given]

So, by Side-Side-Side criterion of congruence, we have,

$\triangle ABO \cong \triangle ACO$

$\Rightarrow \angle ABO = \angle ACO$ [by corresponding parts of congruent triangles are equal]

Question 13:



Given: A ΔABC in which;

$AB = AC$

and, $DE \parallel BC$

To Prove: $AD = AE$

Proof: Since $DE \parallel BC$ and AB is a transversal.

So, $\angle ADE = \angle ABC$ ...(i)

[$\because$ These are corresponding angles]

Also $DE \parallel BC$ and AC is a transversal

So, $\angle AED = \angle ACB$ ...(ii)

[$\because$ these are corresponding angles]

But, $AB = AC$ [Given]

So, $\angle ABC = \angle ACB$ ...(iii)

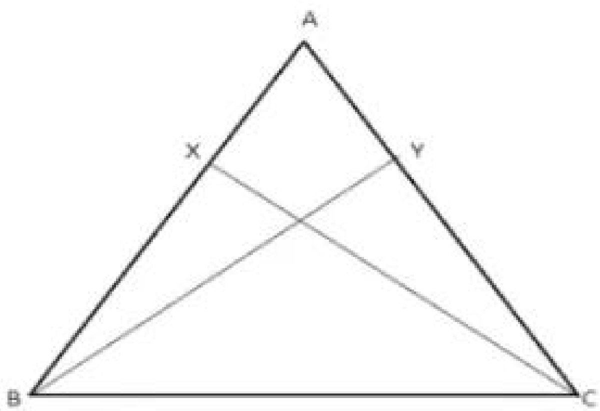
as oppsite angles are also equal in case sides are equal

So from (i), (ii) and (iii) we have

$\angle ADE = \angle AED$

and in ΔADE, this implies that $AD = AE$.

Question 14:



Given: $AX = AY$

To prove: $CX = BY$

Proof: In $\triangle AXC$ and $\triangle AYB$, we have

$AX = AY$ [Given]

$\angle A = \angle A$ [Common angle]

$AC = AB$ [Two sides are equal]

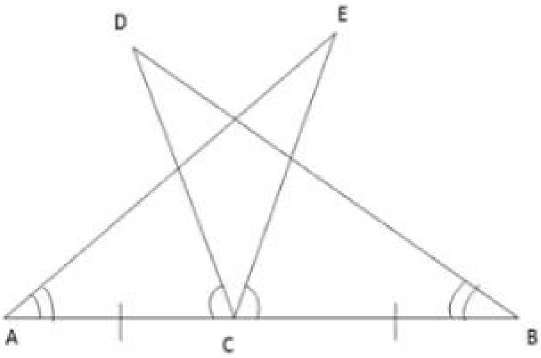
So, by Side-Angle-Side cirterion of congruence, we have

$\triangle AXC \cong \triangle AYB$

$\Rightarrow$ $XC = YB$ [Since corresponding parts of congruent triangles are equal]

Question 15:

Given: C is the mid point of a line segment AB, and D is point such that,



$\angle DCA = \angle ECB$

and $\angle DBC = \angle EAC$

Toprove : $DC = EC$

Proof: In ΔACE and ΔDCB we have;

$AC = BC$ [Given]

$\angle EAC = \angle DBC$ [Given]

Also, $\angle DCA = \angle CDB + \angle DBA$ because exterior $\angle DCA$ in ΔDCB is equal to sum of interior opposite angles.

Again in $\angle ACE$, we have

ext.$\angle BCE = \angle CAE + \angle AEC$

But, $\angle DCA = \angle BCE$ [Given]

$\Rightarrow \angle CDB + \angle DBA = \angle CAE + \angle AEC$

$\Rightarrow \angle CDB = \angle AEC$ [$\because \angle DBA = \angle CAE$ (given)

Thus in ΔACE and ΔDCB,

$\angle EAC = \angle DBC$

$AC = BC$

and, $\angle AEC = \angle CDB$

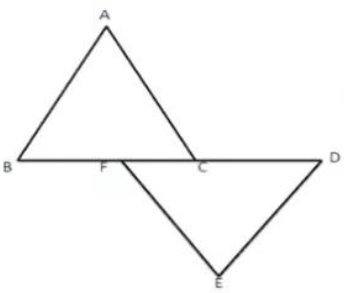
Thus by Angle-Side-Angle criterion of congruence, we have

$\Delta ACE \cong \Delta DCB$ (By ASA)

The corresponding parts of the congruent triangles are equal.

So, $DC = CE$ [by c.p.c.t]

Question 16:



Given: $AB \perp AC$ and $DE \perp FE$ such that ,

$AB = DE$ and $BF = CD$

To prove : $AC = EF$

Proof: In ΔABC, we have,

$BC = BF + FC$

and , in ΔDEF

$FD = FC + CD$

But, $BF = CD$ [Given]

So, $BC = BF + FC$

and, $FD = FC + BF$

$\Rightarrow BC = FD$

So, in ΔABC and ΔDEF, we have,

$\angle BAC = \angle DEF = 90^o$ [Given]

$BC = FD$ [Proved above]

$AB = DE$ [Given]

Thus, by Right angle-Hypotenuse-Side criterion of congruence, we have

$\Delta ABC \cong \Delta DEF$ [By RHS]

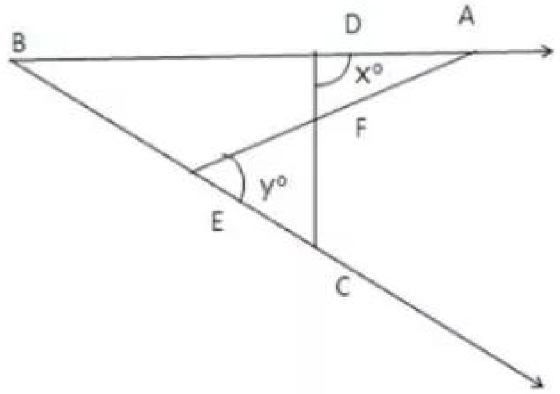
The corresponding parts of the congruent triangles are equal.

So, $AC = EF$ [C.P.C.T]

Question 17:

Given: AB = BC

and, $x^\circ = y^\circ$



To prove: AE = CD

Proof: In ΔABE, we have,

Exterior $\angle AEB = \angle EBA + \angle BAE$

$\Rightarrow$ $y^\circ = \angle EBA + \angle BAE$

Again, in ΔBCD we have

$x^\circ = \angle CBA + \angle BCD$

Since, $x = y$ [Given]

So, $\angle EBA + \angle BAE = \angle CBA + \angle BCD$

$\Rightarrow$ $\angle BAE = \angle BCD$

Thus in ΔBCD and ΔBAE, we have

$\angle B = \angle B$ [Common]

$BC = AB$ [Given]

and, $\angle BCD = \angle BAE$ [Proved above]

Thus by Angle-Side-Angle criterion of congruence, we have

$\Delta BCD \cong \Delta BAE$

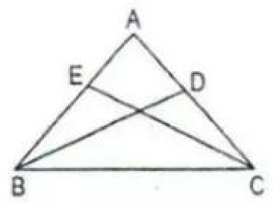
The corresponding parts of the congruent triangles are equal.

So, CD = AE [Proved]

Question 18:

Given: A ΔABC in which $AB = AC$ and

BD and CE are the bisectors of $\angle B$ and $\angle C$ respectively.



To prove: $BD = CE$

Proof: In ΔABD and ΔACE

$$\angle ABD = \frac{1}{2} \angle B$$

and $$\angle ACE = \frac{1}{2} \angle C$$

But $\angle B = \angle C$ as $AB = AC$ [In Isosceles triangle, base angles are equal]

$\Rightarrow$ $\angle ABD = \angle ACE$

$AB = AC$ [Given]

$\angle A = \angle A$ [Common]

Thus by Angle-Side-Angle criterion of congruence, we have

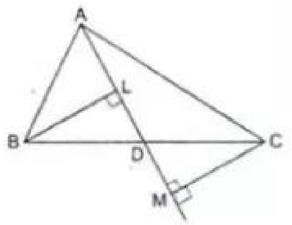
$\Delta ABD \cong \Delta ACE$ [By ASA]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $BD = CE$ [C.P.C.T]

Question 19:

Given: A Δ in which D is the mid point of BC and $BL \perp AD$ and $CM \perp AD$.



To Prove: $BL = CM$

Proof: In ΔBLD and ΔCMD

$\angle BLD = \angle CMD = 90^0$ [Given]

$\angle BDL = \angle MDC$ [Vertically opposite angles]

? $BD = DC$ [Given]

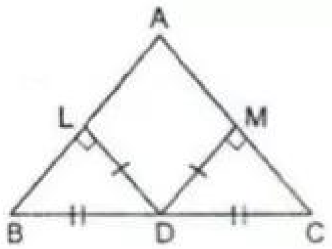
Thus by Angle-Angle-Side criterion of congruence, we have

$\Delta BLD = \Delta CMD$ [By AAS]

The corresponding parts of the congruent triangles are equal

So, $BL = CM$ [C.P.C.T]

Question 20:



Given: In a ΔABC, D is the mid point of BC and $DL \perp AB$ and $DM \perp AC$. Also, $DL = DM$

To prove: $AB = AC$

Proof: In right angled triangles ΔBLD and ΔCMD

$$\angle BLD = \angle CMD = 90^0$$

$$\text{Hypt.} BD = \text{Hypt.} CD \quad [\text{Given}]$$

$$DL = DM \quad [\text{Given}]$$

Thus, by Right Angle-Hypotenuse-Side criterion of congruence, we have

$$\Delta BLD \cong \Delta CMD \quad [\text{By RHS}]$$

The corresponding parts of the congruent triangles are equal.

$$\therefore \quad \angle ABD = \angle ACD \quad [\text{C.P.C.T}]$$

In ΔABC, we have

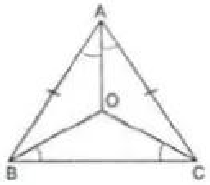
$$\angle ABD = \angle ACD$$

$$\Rightarrow \quad AB = AC$$

[$\therefore$ sides opposite to equal angles are equal]

Question 21:

Given: A ΔABC in which $AB = AC$, BO and CO are bisectors of $\angle B$ and $\angle C$



To Prove: In ΔBOC, we have,

$$\angle OBC = \frac{1}{2}\angle B$$

and, $$\angle OCB = \frac{1}{2}\angle C$$

But, $\angle B = \angle C$ $\quad$ [$\therefore$ AB= AC (given)]

So, $\angle OBC = \angle OCB$

Since base angles are equal, sides are equal

$\Rightarrow$ $OB = OC$ $\quad$ ....(1)

Since OB and OC are the bisectors of angles, $\angle B$ and $\angle C$ respectively, we have

$$\angle ABO = \frac{1}{2}\angle B$$

$$\angle ACO = \frac{1}{2}\angle C$$

$\Rightarrow$ $\angle ABO = \angle ACO$ $\quad$ .....(2)

Now, in ΔABO and ΔACO

$AB = AC$ $\quad$ [Given]

$\angle ABO = \angle ACO$ $\quad$ [from (2)]

$BO = OC$ $\quad$ [from (1)]

Thus, by Side-Angle-Side criterion of congruence, we have

$\Delta ABO \cong \Delta ACO$ $\quad$ [By SAS]

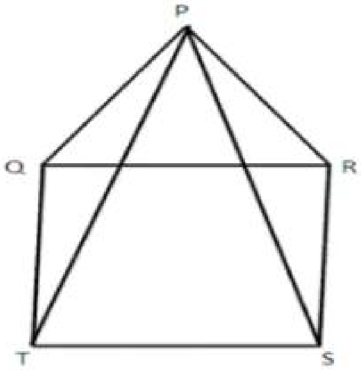
The corresponding parts of the congruent triangles are equal

$\therefore$ $\angle BAO = \angle CAO$ $\quad$ [By cpct]

i.e. AO bisects $\angle A$.

Question 22:

Given: PQR is an equilateral triangle and QRST is a square.



To Prove: PT = PS

and $\angle PSR = 15^0$

Proof: Since ΔPQR is an equilateral triangle,

$\angle PQR = 60^0$ and $\angle PRQ = 60^0$

Since QRTS is a square,

$\angle RQT = 90^0$ and $\angle QRS = 90^0$

In ΔPQT

$\angle PQT = \angle PQR + \angle RQT$

$= 60^0 + 90^0$

$= 150^0$

In ΔPRS

$\angle PRS = \angle PRQ + \angle QRS$

$= 60^0 + 90^0 = 150^0$......(1)

$\Rightarrow$ $\angle PQT = \angle PRS$ ......(2)

Thus, in ΔPQT and ΔPRS

PQ = PR [sides of equilateral triangle ΔPQR]

$\angle PQT = \angle PRS$ [from (2)]

QT= RS [sides of square □QRST]

Thus, by Side-Angle-Side criterion of congruence, we have

$\therefore$ $\Delta PQT \cong \Delta PRS$ [By SAS]

The corresponding parts of the congruent triangles are equal.

$\therefore$ PT = PS [C.P.C.T]

Now in Δ PRS , we have

PR = RS

$\Rightarrow$ $\angle RPS = \angle PSR$

But $\angle PRS = 150^0$ [from (1)]

So, by angle sum property in ΔPRS

$\angle PRS + \angle SPR + \angle PSR = 180^0$

$\Rightarrow$ $150^0 + \angle PSR + \angle PSR = 180^0$

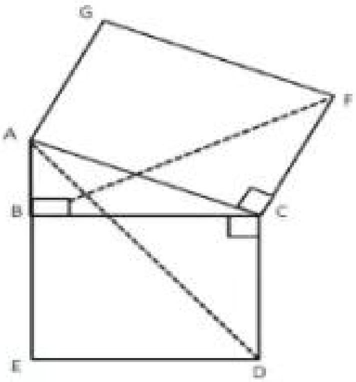
$\Rightarrow$ $2\angle PSR = 180^0 - 150^0$

$\Rightarrow$ $2\angle PSR = 30^0$

$\Rightarrow$ $\angle PSR = \frac{30}{2} = 15^0$

Question 23:

Given : ABC is atriangle , right angled at B. ACFG is a a square and BCDE is a square.



To prove: AD= EF

Proof: Since BCDE is a square,

$\angle BCD = 90^0$.....(1)

In ΔACD,

$\angle ACD = \angle ACB + \angle BCD$

$= \angle ACB + 90^0$ .....(2)

In ΔBCF,

$\angle BCF = \angle BCA + \angle ACF$

Since ACFG is a square,

$\angle ACF = 90^0$

Thus, we have

$\angle BCF = \angle BCA + 90^0$ ......(3)

From (2) and (3), we have

$\angle ACD = \angle BCF$ ......(4)

Thus in ΔACD and ΔBCF, we have

AC = CF	[sides of a square]
$\angle ACD = \angle BCF$	[from (4)]
CD=BC	[sides of a square]

Thus, by Side-Angle-Side criterion of congruence, we have

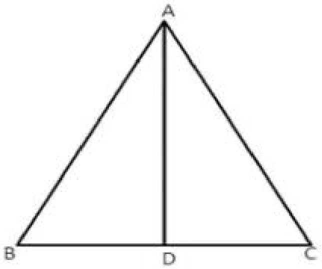
$\therefore$ $\Delta ACD \cong \Delta BCF$ [By SAS]

The corresponding parts of congruent triangles are equal.

So, AD= BF (C.P.C.T)

Question 24:

Given : ABC is an isosceles triangle in which AB = AC and AD is the median through A.



To prove: $\angle BAD = \angle DAC$

Proof: In ΔABD and ΔADC

AB = AC [Given]

BD = DC [Given]

AD = AD [Common]

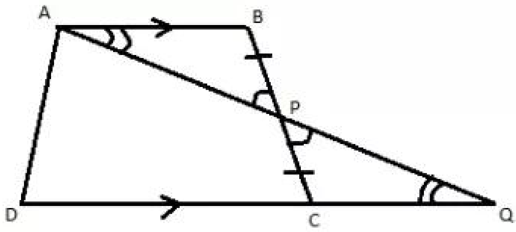
Thus by Side-Side-Side criterion of congruence, we have

$\Delta ABD \cong \Delta ADC$ [By SSS]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $\angle BAD = \angle DAC$ (Proved)

Question 25:



Given ABCD is a quadrilateral in which AB || DC

To Prove: (i) AB = CQ

(ii) DQ = DC + AB

Proof: In ΔABP and ΔPCQ we have

$\angle PAB = \angle PQC$ [alternate angles]

$\angle APB = \angle CPQ$ [Vertically opposite angles]

BP = PC [Given]

Thus by Angle-Angle-Side criterion of congruence, we have

$\Delta ABP \cong \Delta PCQ$

The corresponding parts of the congruent triangles are equal

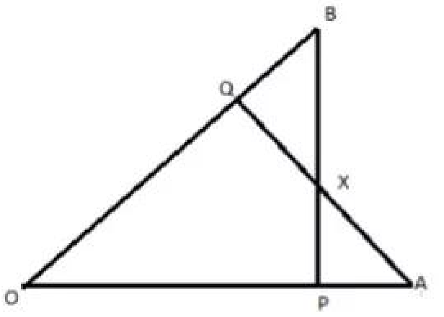
$\therefore$ AB = CQ .....(1)

Now, DQ = DC + CQ

= DC + AB [from (1)]

Question 26:

Given : OA = OB and OP = OQ



To Prove: (i) PX = QX

(ii) AX = BX

Proof: In ΔOAQ and ΔOPB, we have,

OA = OB [Given]

$\angle O = \angle O$ [Common]

OQ = OP [Given]

Thus by Side-Angle-Side criterion of congruence, we have

$\Delta OAQ = \Delta OPB$ [By SAS]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $\angle OBP = \angle OAQ$ ....(1)

Thus , in ΔBXQ and ΔPXA, we have

BQ = OB - OQ

and, PA = OA - OP

But, OP = OQ

and OA = OB [Given]

Therefore, we have, BQ = PA ....(2)

Now consider triangles ΔBXQ and ΔPXA.

$\angle BXQ = \angle PXA$ [Vertical opposite angles]

$\angle OBP = \angle OAQ$ [from (1)]

BQ = PA [from (2)]

Thus by Angle-Angle-Side criterion of congruence, we have,

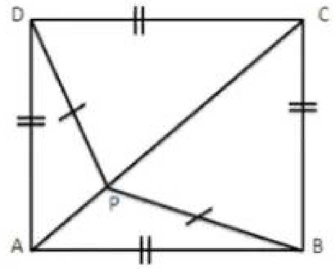
$\therefore$ $\Delta BXQ \cong \Delta PXA$

PX = QX [C.P.C.T]

AX = BX [C.P.C.T]

Question 27:

Given: ABCD is a sqaure and P is a point inside it such that PB =PD



To Prove: CPA is a straight line.

Proof : In ΔAPD and ΔAPB

DA= AB [∴ ABCD is a square]

AP =AP [Common]

and, PB =PD [Given]

Thus by Side-Side-Side criterion of congruence, we have

$\Delta APD \cong \Delta APB$

The corresponding parts of the congruent triangles are equal.

∴ $\angle APD = \angle APB$ ....(i)

Now consider the triangles, ΔCPD and ΔCPB.

CD = CB [∴ ABCD is a square]

CP = CP [Common]

and, PB = PD [Given]

Thus by Side-Side-Side criterion of congruence, we have

$\Delta CPD \cong \Delta CPB$

The corresponding parts of the congruent triangles are equal.

Hence we have

$\angle CPD = \angle CPB$ ......(ii)

Adding both sides of (i) and (ii) we get

$\angle APD + \angle CPD = \angle APB + \angle CPB$ ......(iii)

Angles around the point P add upto 360°,

$\Rightarrow \angle APD + \angle CPD + \angle APB + \angle CPB = 360°$

$\Rightarrow \angle APB + \angle CPB = 360° - (\angle APD + \angle CPD)$ ...(iv)

Substituting (iv) in (iii) we get,

$\angle APD + \angle CPD = 360° - (\angle APD + \angle CPD)$

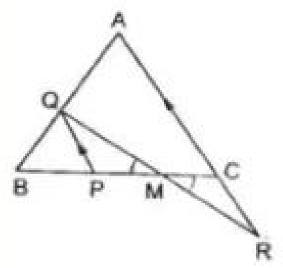
i.e $2(\angle APD + \angle CPD) = 360°$

$\angle APD + \angle CPD = \frac{360}{2} = 180°$

This proves that CPA is a straight line.

Question 28:

A ΔABC which is an equilateral triangle and $PQ||AC$.
AC is produced to R such that $CR=BP$



To Prove: $PM=MC$

Proof: Let QR intersects PC at M.

Since ΔABC is an equilateral triangle,

$\Rightarrow \angle A = \angle ACB = 60^\circ$

Since $PQ \parallel AC$ and corresponding angles are equal.

$\Rightarrow \angle BPQ = \angle ACB = 60^\circ$

In ΔBPQ, $\angle B = \angle ACB = 60^\circ$

$\Rightarrow \angle BQP = 60^\circ$

$\Rightarrow \Delta BPQ$ is an equilateral triangle

$\Rightarrow PQ = BP = BQ$

Since $BP = CR$, we have,

$PQ = CR$ ......(1)

Consider the triangles ΔPMQ and ΔCMR.

Since $PQ || AC$ and QR is a transversal

So, $\angle PQM = \angle CRM$ [alternate angles]

$\angle PMQ = \angle CMR$ [vertically opposite angles]

$PQ = CR$ [from (1)]

Thus by Angle-Angle-Side criterion of congruence, we have

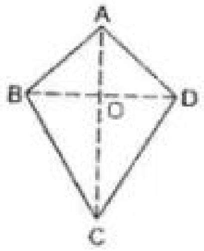
$\Delta PMQ \cong \Delta CMR$ [By AAS]

The corresponding parts of the congruent triangles are equal.

So, $PM = MC$ [C.P.C.T](proved)

Question 29:

Given: a quarilateral ABCD in which AB=AD and BC=DC



To Prove: (i) AC bisects $\angle A$ and $\angle C$

(ii) $AC \perp BD$ and AC bisects BD

Proof: In ΔABC and ΔADC, we have

$AB = AD$ [Given]

$BC = DC$ [Given]

$AC = AC$ [Common]

Thus by Side-Side-Side criterion of congruence,

$\Delta ABC \cong \Delta ADC$ [By SSS]

The corresponding parts of the congruent triangles are equal.

So, $\angle BAC = \angle DAC$ [C.P.C.T]

$\Rightarrow$ $\angle BAO = \angle DAO$ .....(1)

It means that AC bisects $\angle BAD$, that is $\angle A$

Also, $\angle BCA = \angle DCA$ [C.P.C.T]

It means that AC bisects $\angle BCD$, that is $\angle C$

(ii)

Now in ΔABO and ΔADO

$AB = AD$ [Given]

$\angle BAO = \angle DAO$ [from (1)]

$AO = AO$ [Common]

Thus, by Side-Angle-Side criterion of congruence, we have

$\Delta ABO \cong \Delta ADO$ [By SAS]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $\angle BOA = \angle DOA$

But $\angle BOA + \angle DOA = 180^\circ$

Or $2\angle BOA = 180^\circ$

$\Rightarrow$ $\angle BOA = \frac{180^\circ}{2} = 90^\circ$

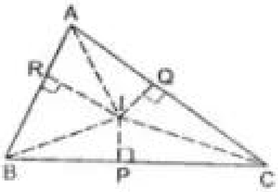
Also, as $\Delta ABO \cong \Delta ADO$

So, $BO = OD$

which means that AC bisects BD.

Question 30:

Given: A triangle ABC in which bisectors of $\angle B$ and $\angle C$ meet at I.



Also, we have $IP \perp BC$, $IQ \perp CA$ and $IR \perp AB$

To Prove:(i) $IP=IQ=IR$

(ii) $\angle IAR=\angle IAQ$

Proof:(i) In ΔBIP and ΔBIR we have,

$\angle PBI = \angle RBI$ [Given]

$\angle IRB = \angle IPB= 90^\circ$ [Given]

and, $IB = IB$ [Common]

Thus by Angle-Angle-Side criterion of congruence, we have

$\therefore$ $\Delta BIP \cong \Delta BIR$ [By AAS]

The corresponding parts of the congruent triangles are equal.

So, $IP = IR$

Similarly $IP = IQ$

$\therefore$ $IP = IQ = IR$

(ii) Now in ΔAIR and ΔAIQ we have

$IR = IQ$ [Proved above]

$IA = IA$ [Common]

and, $\angle IRA = \angle IQA = 90^\circ$

Thus by Side-Angle-Side criterion of congruence, we have

$\therefore$ $\Delta AIR \cong \Delta AIQ$ [By SAS]

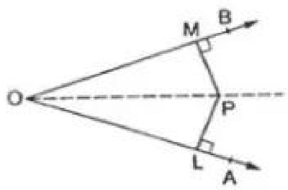
The corresponding parts of the congruent triangles are equal.

So, $\angle IAR = \angle IAQ$ [by c.p.c.t]

$\Rightarrow$ IA bisects $\angle A$

Question 31:

Given: An angle AOB and P is a point in the interior of $\angle AOB$ such that PL=PM. Also PL $\perp$ OA and PM $\perp$ OB



To Prove: $\angle POL = \angle POM$

Proof: In ΔOPL and ΔOPM, we have

$\angle OMP = \angle OLP = 90^\circ$ [Given]

OP = OP [Common

PL = PM [Given]

Thus, by Right angle-Hypotenuse-Side criterion of congruence, we have

$\Delta OPL \cong \Delta OPM$ [By R.H.S]

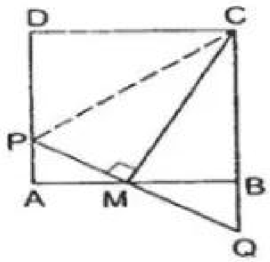
The corresponding parts of the congruent triangles are equal.

$\therefore$ $\angle POL = \angle POM$ [C.P.C.T]

$\Rightarrow$ OP is the bisector of $\angle LOM = \angle AOB$

Question 32:

Given M is the mid-point of side AB of a square ABCD and $CM \perp PQ$



To Prove : (i) $PA = BQ$

(ii) $CP = AB+PA$

Proof: (i) In ΔAMP and ΔBMQ

$\angle AMP = \angle BMQ$ [Vertically opposite angle]

$\angle PAM = \angle MBQ = 90^\circ$ [$\therefore$ ABCD is a square]

and $AM = MB$ [Given]

Thus by Angle-Angle-Side criterion of congruence, we have

$\Delta AMP \cong \Delta BMQ$ [By AAS]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $PA = BQ$ and $MP = MQ$ .....(1)

(ii) Now ΔPCM and ΔQCM, we have

$PM = QM$ [from (1)]

$\angle PMC = \angle QMC = 90^\circ$ [Given]

$CM = CM$ [Common]

Thus by Side-Angle-Side criterion of congruence we have

$\therefore$ $\Delta PCM \cong \Delta QCM$ [By SAS]

The corresponding parts of the congruent triangles are equal.

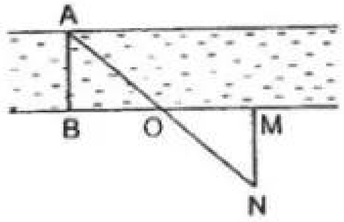
So, $PC = QC$ [C.P.C.T]

$\Rightarrow$ $PC = QB + CB$

$\Rightarrow$ $PC = AB + PA$ [$\because AB = CB$ and $PA = QB$]

Question 33:

Let AB be the breadth of a river. Now take a point M on that bank of the river where point B is situated. Through M draw a perpendicular and take point N on it such that point, A, O and N lie on a straight line where point O is the mid point of BM.



Now in ΔABO and ΔNMO we have,

$\angle OBA = \angle OMN = 90^0$

$OB = OM$ [$\therefore$ O is mid point of BM]

and $\angle BOA = \angle MON$ [Vertically opposite angles]

Thus, by Angle - Side - Angle criterion of congruence, we have,

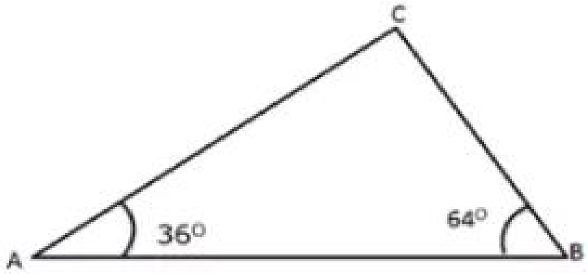
$\Delta ABO \cong \Delta NMO$ [By ASA]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $AB = NM$ [C.P.C.T]

Thus, we find that MN is the width of the river.

Question 34



We have $\angle A = 36°$ and $\angle B = 64°$

By the angle sum property in ΔABC, we have

$\angle A+\angle B+\angle C = 180°$

$\Rightarrow 36° + 64° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 100° = 80°$

Therefore, we have

$\angle A = 36°, \angle B = 64°$ and $\angle C = 80°$

$\therefore$ $\angle C$ is largest and $\angle A$ is shortest.

Side opposite to $\angle C$ is longest and hence AB is longest side.

Side opposite to $\angle A$ is shortest and hence BC is shortest side.

Question 35:

In a right angle triangle, greatest angle is $\angle A = 90°$.

And hence other angles are less than $90°$ because sum of the angles of a triangle is $180°$.

So, $\angle A$ is the greatest angle.

Therefore, side BC which is opposite to $\angle A$ is longest.

Question 36:

In ΔABC,

$$\angle A = \angle B = 45^\circ$$

So, $\angle C = 180^\circ - \angle A - \angle B$

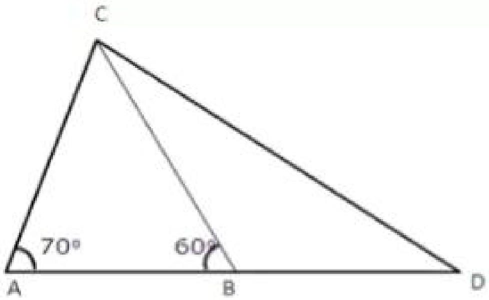
$= 180^\circ - 45^\circ - 45^\circ$

$= 180^\circ - 90^\circ = 90^\circ$

Thus we find that $\angle C$ is the greatest angle of ΔABC.

So, AB is the longest side which is opposite to $\angle C$.

Question 37:



In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 70^\circ + 60^\circ + \angle C = 180^\circ$

$\Rightarrow 130^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 130^\circ = 50^\circ$

Now in ΔBCD we have,

$\angle CBD = \angle DAC + \angle ACB$ [$\because$ $\angle CBD$ is the exterior angle of $\angle ABC$]

$= 70^\circ + 50^\circ = 120^\circ$

Since BC=BD [Given]

So, $\angle BCD = \angle BDC$

$\therefore \angle BCD + \angle BDC = 180^\circ - \angle CBD$

$= 180^\circ - 120^\circ = 60^\circ$

$\Rightarrow 2\angle BCD = 60^\circ$

$\Rightarrow \angle BCD = \angle BDC = 30^\circ$

Now in ΔACD we have

$\angle A = 70^\circ, \angle D = 30^\circ$

and $\angle ACD = \angle ACB + \angle BCD$

$= 50^\circ + 30^\circ = 80^\circ$

$\therefore$ $\angle ACD$ is the greatest angle.

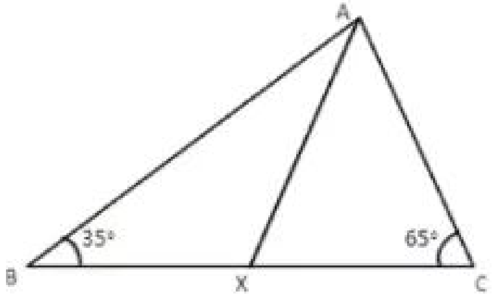
So the side opposite to $\angle ACD$, that is AD, is the longest side of ΔACD

$\therefore$ AD > CD

(ii) Since $\angle BDC$ is the smallest angle, the side opposite to $\angle BDC$, that is AC, is the shortest side of ΔACD

$\therefore$ AD > AC.

Question 38:



In ΔABC,

$$\angle A = 180^\circ - \angle B - \angle C$$
$$= 180^\circ - 35^\circ - 65^\circ$$
$$= 180^\circ - 100^\circ = 80^\circ$$

$$\therefore \quad \angle BAX = \frac{1}{2}\angle A$$
$$= \frac{1}{2} \times 80^\circ = 40^\circ$$

Now in ΔABX,

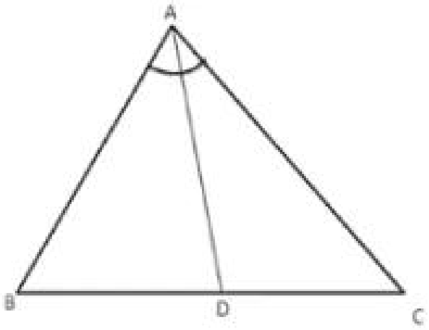
$$\angle B = 35^\circ$$
$$\angle BAX = 40^\circ$$
and $$\angle BXA = 180^\circ - 35^\circ - 40^\circ$$
$$= 180^\circ - 75^\circ = 105^\circ$$

So, in ΔABX,
$\angle B$ is smallest,so the side opposite to $\angle B$,
that is AX, is smallest
So $\quad AX < BX$ ....(i)
Now consider ΔAXC

$$\angle CAX = \frac{1}{2} \times \angle A$$
$$= \frac{1}{2} \times 80^\circ = 40^\circ$$
$$\angle AXC = 180^\circ - 40^\circ - 65^\circ$$
$$= 180^\circ - 105^\circ = 75^\circ$$

Therefore, in ΔAXC, we have,
$\angle CAX = 40^\circ, \angle C = 65^\circ$ and $\angle AXC = 75^\circ$
$\therefore \angle CAX$ is smallest in ΔAXC
So the side opposite to $\angle CAX$ is shortest.
$\Rightarrow$ CX is shortest
$\Rightarrow CX < AX$ ....(ii)
From (i) and (ii) ,we get
$BX > AX > CX$
This is the required descending order.

Question 39:



Given: ABC is a triangle in which AD is the bisector of $\angle A$.

Proof: (i) In ΔACD

Exterior $\angle ADB = \angle DAC + \angle ACD$

$= \angle BAD + \angle ACD$

[$\therefore \angle DAC = \angle BAD$(given)]

$\therefore \quad \angle ADB > \angle BAD$

The side opposite to angle $\angle ADB$ is the longest side

in ΔADB

So, $\quad AB > BD$

(ii) Again in ΔABD

Exterior $\angle ADC = \angle ABD + \angle BAD$

$= \angle ABD + \angle CAD$

$\therefore \quad \angle ADC > \angle CAD$

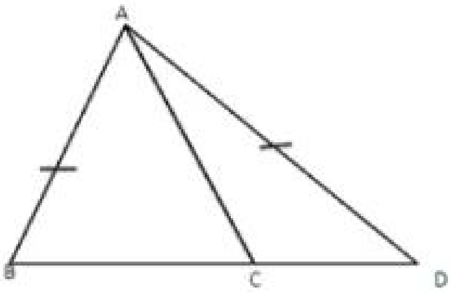
The side opposite to angle $\angle ADC$ is the longest side

in ΔACD

So, $\quad AC > DC$

Question 40:

Given :A ΔABC is which AB=AC side BC of ΔABC is produced to D.



To prove: $AD > AC$

Proof: In ΔABC

Ext.$\angle ACD = \angle B + \angle BAC$

$= \angle ACB + \angle BAC \quad [\because \angle B = \angle C$ as $AB = AC]$

$= \angle CAD + \angle CDA + \angle BAC$

$[\because$ Ext.$\angle ACB = \angle CAD + \angle CDA]$

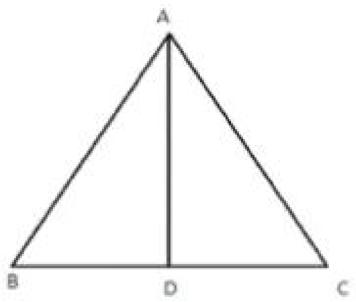
$\Rightarrow \angle ACD > \angle CDA$

So the side opposite to $\angle ACD$, is the longest.

$\therefore \quad AD > AC$

Question 41:

Given: A ΔABC in which $AC > AB$ and AD is a bisector of $\angle A$



To prove: $\angle ADC > \angle ADB$

Proof : Since $AC > AB$

$\Rightarrow \angle ABC > \angle ACB$

Adding $\frac{1}{2}\angle A$ on both sides of inequality.

$\angle ABC + \frac{1}{2}\angle A > \angle ACB + \frac{1}{2}\angle A$

$\Rightarrow \angle ABC + \angle BAD > \angle ACB + \angle DAC$

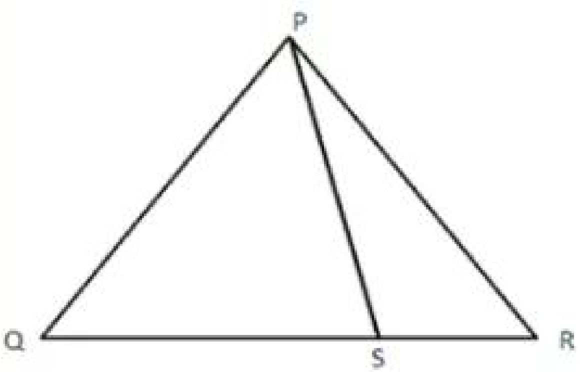
[$\because$ AD is a bisector of $\angle A$]

$\Rightarrow$ Exterior $\angle ADC >$ Exterior $\angle ADB$

$\therefore \angle ADC > \angle ADB$.

Question 42:

Given : A triangle PQR and S is a point on QR.



To prove: $PQ + QR + RP > 2PS$

Proof: Since in a triangle, sum of any two sides is always greater than the third side.

So in ΔPQS, we have

$PQ + QS > PS$ ....(i)

Similarly, in ΔPSR, we have

$PR + SR > PS$ ...(ii)

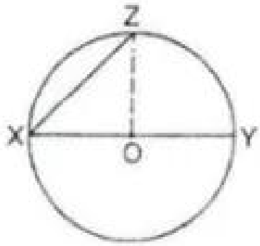
Adding both sides of (i) and (ii) , we get.

$PQ + QS + PR + SR > 2PS$

$\Rightarrow PQ + PR + QS + SR > 2PS$

$\Rightarrow PQ + PR + QR > 2PS$

Question 43:



Given : A circle with centre O is drawn in which XY is a diameter and XZ is a chord.

To prove : XY> XZ

Proof : In ΔXOZ, we have,

$OX+OZ > XZ$

[∴ sum of any two sides in a triangle is a greater than its third side]

$\Rightarrow$ $OX + OY > XZ$

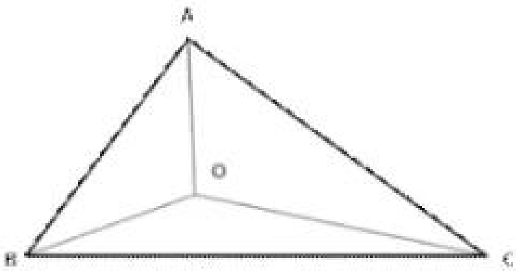
[∵ OZ = OY, radius of the circle]

∴ $XY > XZ$

[∵ OX+OY=XY]

Question 44:

Given : ABC is a triangle and O is appoint inside it.



To Prove : (i) AB+AC > OB +OC

(ii) AB+BC+CA > OA+OB+OC

(iii) OA+OB+OC > $\frac{1}{2}$ (AB+BC+CA)

Proof:

(i) In ΔABC,

AB+AC > BC ....(i)

And in , ΔOBC,

OB+OC > BC ....(ii)

Subtracting (i) from (i) we get

(AB+AC) – (OB+OC) > (BC-BC)

i.e. AB+AC>OB+OC

(ii) AB+AC > OB+OC [proved in (i)]

Similarly, AB+BC > OA+OC

And AC+BC > OA +OB

Adding both sides of these three inequalities, we get

(AB+AC) + (AC+BC) + (AB+BC) > OB+OC+OA+OB+OA+OC

i.e. 2(AB+BC+AC) > 2(OA+OB+OC)

Therefore, we have

AB+BC+AC > OA+OB+OC

(iii) In ΔOAB

OA+OB > AB ....(i)

In ΔOBC,

OB+OC > BC ....(ii)
And, in ΔOCA,
OC+OA > CA
Adding (i), (ii) and (iii) we get
(OA+OB) + (OB+OC) + (OC+OA) > AB+BC+CA
i.e 2(OA+OB+OC) > AB+BC+CA
$\Rightarrow$ OA+OB+OC > $\frac{1}{2}$(AB+BC+CA)

Question 45:
Since AB=3cm and BC=3.5 cm
$\therefore$ AB+BC=(3+3.5) cm =6.5 m
And CA=6.5 cm
So AB+BC=CA
A triangle can be drawn only when the sum of two sides is greater than the third side.
So, with the given lengths a triangle cannot be drawn.