

SOLUTIONS TO CONCEPTS CHAPTER – 2

1. As shown in the figure,

The angle between \vec{A} and $\vec{B} = 110^\circ - 20^\circ = 90^\circ$

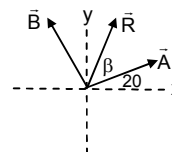
$|\vec{A}| = 3$ and $|\vec{B}| = 4\text{m}$

$$\text{Resultant } R = \sqrt{A^2 + B^2 + 2AB \cos \theta} = 5\text{ m}$$

Let β be the angle between \vec{R} and \vec{A}

$$\beta = \tan^{-1} \left(\frac{4 \sin 90^\circ}{3 + 4 \cos 90^\circ} \right) = \tan^{-1} (4/3) = 53^\circ$$

\therefore Resultant vector makes angle $(53^\circ + 20^\circ) = 73^\circ$ with x-axis.



2. Angle between \vec{A} and \vec{B} is $\theta = 60^\circ - 30^\circ = 30^\circ$

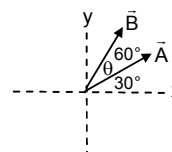
$|\vec{A}|$ and $|\vec{B}| = 10\text{ unit}$

$$R = \sqrt{10^2 + 10^2 + 2 \cdot 10 \cdot 10 \cdot \cos 30^\circ} = 19.3$$

β be the angle between \vec{R} and \vec{A}

$$\beta = \tan^{-1} \left(\frac{10 \sin 30^\circ}{10 + 10 \cos 30^\circ} \right) = \tan^{-1} \left(\frac{1}{2 + \sqrt{3}} \right) = \tan^{-1} (0.26795) = 15^\circ$$

\therefore Resultant makes $15^\circ + 30^\circ = 45^\circ$ angle with x-axis.



3. x component of $\vec{A} = 100 \cos 45^\circ = 100 / \sqrt{2}$ unit

$$\text{x component of } \vec{B} = 100 \cos 135^\circ = 100 / \sqrt{2}$$

$$\text{x component of } \vec{C} = 100 \cos 315^\circ = 100 / \sqrt{2}$$

$$\text{Resultant x component} = 100 / \sqrt{2} - 100 / \sqrt{2} + 100 / \sqrt{2} = 100 / \sqrt{2}$$

$$\text{y component of } \vec{A} = 100 \sin 45^\circ = 100 / \sqrt{2} \text{ unit}$$

$$\text{y component of } \vec{B} = 100 \sin 135^\circ = 100 / \sqrt{2}$$

$$\text{y component of } \vec{C} = 100 \sin 315^\circ = -100 / \sqrt{2}$$

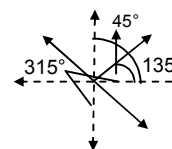
$$\text{Resultant y component} = 100 / \sqrt{2} + 100 / \sqrt{2} - 100 / \sqrt{2} = 100 / \sqrt{2}$$

Resultant = 100

$$\tan \alpha = \frac{\text{y component}}{\text{x component}} = 1$$

$$\Rightarrow \alpha = \tan^{-1} (1) = 45^\circ$$

The resultant is 100 unit at 45° with x-axis.



4. $\vec{a} = 4\vec{i} + 3\vec{j}$, $\vec{b} = 3\vec{i} + 4\vec{j}$

a) $|\vec{a}| = \sqrt{4^2 + 3^2} = 5$

b) $|\vec{b}| = \sqrt{9 + 16} = 5$

c) $|\vec{a} + \vec{b}| = |7\vec{i} + 7\vec{j}| = 7\sqrt{2}$

d) $\vec{a} - \vec{b} = (-3 + 4)\hat{i} + (-4 + 3)\hat{j} = \hat{i} - \hat{j}$

$$|\vec{a} - \vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}.$$

5. x component of $\vec{OA} = 2\cos 30^\circ = \sqrt{3}$

x component of $\vec{BC} = 1.5 \cos 120^\circ = -0.75$

x component of $\vec{DE} = 1 \cos 270^\circ = 0$

y component of $\vec{OA} = 2 \sin 30^\circ = 1$

y component of $\vec{BC} = 1.5 \sin 120^\circ = 1.3$

y component of $\vec{DE} = 1 \sin 270^\circ = -1$

$R_x = \text{x component of resultant} = \sqrt{3} - 0.75 + 0 = 0.98 \text{ m}$

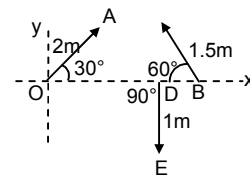
$R_y = \text{resultant y component} = 1 + 1.3 - 1 = 1.3 \text{ m}$

So, $R = \text{Resultant} = 1.6 \text{ m}$

If it makes an angle α with positive x-axis

$$\tan \alpha = \frac{\text{y component}}{\text{x component}} = 1.32$$

$$\Rightarrow \alpha = \tan^{-1} 1.32$$



6. $|\vec{a}| = 3\text{m}$ $|\vec{b}| = 4$

a) If $R = 1 \text{ unit} \Rightarrow \sqrt{3^2 + 4^2 + 2 \cdot 3 \cdot 4 \cdot \cos \theta} = 1$
 $\theta = 180^\circ$

b) $\sqrt{3^2 + 4^2 + 2 \cdot 3 \cdot 4 \cdot \cos \theta} = 5$
 $\theta = 90^\circ$

c) $\sqrt{3^2 + 4^2 + 2 \cdot 3 \cdot 4 \cdot \cos \theta} = 7$
 $\theta = 0^\circ$

Angle between them is 0° .

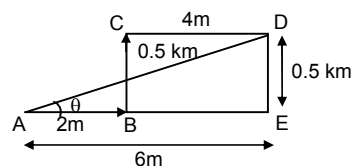
7. $\vec{AD} = 2\hat{i} + 0.5\hat{j} + 4\hat{k} = 6\hat{i} + 0.5\hat{j}$

$AD = \sqrt{AE^2 + DE^2} = 6.02 \text{ KM}$

$\tan \theta = DE / AE = 1/12$

$\theta = \tan^{-1} (1/12)$

The displacement of the car is 6.02 km along the distance $\tan^{-1} (1/12)$ with positive x-axis.



8. In $\triangle ABC$, $\tan \theta = x/2$ and in $\triangle DCE$, $\tan \theta = (2-x)/4$ $\tan \theta = (x/2) = (2-x)/4 = 4x$

$\Rightarrow 4 - 2x = 4x$

$\Rightarrow 6x = 4 \Rightarrow x = 2/3 \text{ ft}$

a) In $\triangle ABC$, $AC = \sqrt{AB^2 + BC^2} = \frac{2}{3}\sqrt{10} \text{ ft}$

b) In $\triangle CDE$, $DE = 1 - (2/3) = 1/3 \text{ ft}$

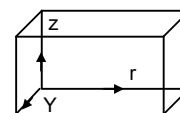
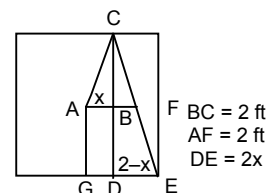
$CD = 4 \text{ ft}$. So, $CE = \sqrt{CD^2 + DE^2} = \frac{4}{3}\sqrt{10} \text{ ft}$

c) In $\triangle AGE$, $AE = \sqrt{AG^2 + GE^2} = 2\sqrt{2} \text{ ft}$.

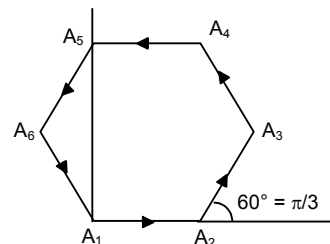
9. Here the displacement vector $\vec{r} = 7\hat{i} + 4\hat{j} + 3\hat{k}$

a) magnitude of displacement = $\sqrt{74} \text{ ft}$

b) the components of the displacement vector are 7 ft, 4 ft and 3 ft.



10. \vec{a} is a vector of magnitude 4.5 unit due north.
 a) $3|\vec{a}| = 3 \times 4.5 = 13.5$
 $3\vec{a}$ is along north having magnitude 13.5 units.
 b) $-4|\vec{a}| = -4 \times 1.5 = -6$ unit
 $-4\vec{a}$ is a vector of magnitude 6 unit due south.
11. $|\vec{a}| = 2$ m, $|\vec{b}| = 3$ m
 angle between them $\theta = 60^\circ$
 a) $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos 60^\circ = 2 \times 3 \times 1/2 = 3 \text{ m}^2$
 b) $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin 60^\circ = 2 \times 3 \times \sqrt{3}/2 = 3\sqrt{3} \text{ m}^2$.
12. We know that according to polygon law of vector addition, the resultant of these six vectors is zero.
 Here $A = B = C = D = E = F$ (magnitude)
 So, $R_x = A \cos \theta + A \cos \pi/3 + A \cos 2\pi/3 + A \cos 3\pi/3 + A \cos 4\pi/3 + A \cos 5\pi/3 = 0$
 [As resultant is zero. X component of resultant $R_x = 0$]
 $= \cos \theta + \cos \pi/3 + \cos 2\pi/3 + \cos 3\pi/3 + \cos 4\pi/3 + \cos 5\pi/3 = 0$
 Note : Similarly it can be proved that,



$$\sin \theta + \sin \pi/3 + \sin 2\pi/3 + \sin 3\pi/3 + \sin 4\pi/3 + \sin 5\pi/3 = 0$$

13. $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$; $\vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$$\vec{a} \cdot \vec{b} = ab \cos \theta \Rightarrow \theta = \cos^{-1} \frac{\vec{a} \cdot \vec{b}}{ab}$$

$$\Rightarrow \cos^{-1} \frac{2 \times 3 + 3 \times 4 + 4 \times 5}{\sqrt{2^2 + 3^2 + 4^2} \sqrt{3^2 + 4^2 + 5^2}} = \cos^{-1} \left(\frac{38}{\sqrt{1450}} \right)$$

14. $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$ (claim)

As, $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$

$AB \sin \theta \hat{n}$ is a vector which is perpendicular to the plane containing \vec{A} and \vec{B} , this implies that it is also perpendicular to \vec{A} . As dot product of two perpendicular vector is zero.

Thus $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$.

15. $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} + 2\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 3 & 2 \end{vmatrix} \Rightarrow \hat{i}(6-12) - \hat{j}(4-16) + \hat{k}(6-12) = -6\hat{i} + 12\hat{j} - 6\hat{k}.$$

16. Given that \vec{A} , \vec{B} and \vec{C} are mutually perpendicular

$\vec{A} \times \vec{B}$ is a vector which direction is perpendicular to the plane containing \vec{A} and \vec{B} .

Also \vec{C} is perpendicular to \vec{A} and \vec{B}

\therefore Angle between \vec{C} and $\vec{A} \times \vec{B}$ is 0° or 180° (fig.1)

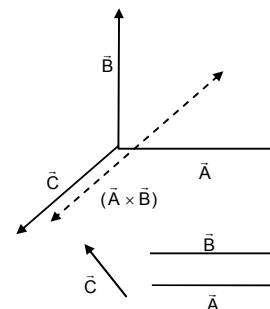
So, $\vec{C} \times (\vec{A} \times \vec{B}) = 0$

The converse is not true.

For example, if two of the vector are parallel, (fig.2), then also

$\vec{C} \times (\vec{A} \times \vec{B}) = 0$

So, they need not be mutually perpendicular.



17. The particle moves on the straight line PP' at speed v .

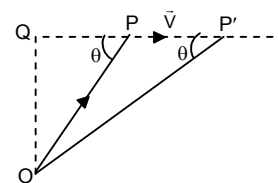
From the figure,

$$\vec{OP} \times \vec{v} = (OP)v \sin \theta \hat{n} = v(OP) \sin \theta \hat{n} = v(OQ) \hat{n}$$

It can be seen from the figure, $OQ = OP \sin \theta = OP' \sin \theta'$

So, whatever may be the position of the particle, the magnitude and direction of $\vec{OP} \times \vec{v}$ remain constant.

$\therefore \vec{OP} \times \vec{v}$ is independent of the position P.



18. Give $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = 0$

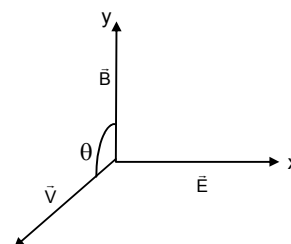
$$\Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$$

So, the direction of $\vec{v} \times \vec{B}$ should be opposite to the direction of \vec{E} . Hence, \vec{v} should be in the positive yz-plane.

$$\text{Again, } E = vB \sin \theta \Rightarrow v = \frac{E}{B \sin \theta}$$

For v to be minimum, $\theta = 90^\circ$ and so $v_{\min} = E/B$

So, the particle must be projected at a minimum speed of E/B along +ve z-axis ($\theta = 90^\circ$) as shown in the figure, so that the force is zero.



19. For example, as shown in the figure,

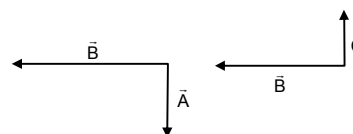
$\vec{A} \perp \vec{B}$ \vec{B} along west

$\vec{B} \perp \vec{C}$ \vec{A} along south

\vec{C} along north

$$\vec{A} \cdot \vec{B} = 0 \quad \therefore \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C}$$

$$\vec{B} \cdot \vec{C} = 0 \quad \text{But } \vec{B} \neq \vec{C}$$



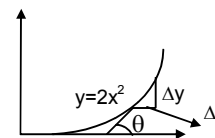
20. The graph $y = 2x^2$ should be drawn by the student on a graph paper for exact results.

To find slope at any point, draw a tangent at the point and extend the line to meet x-axis. Then find $\tan \theta$ as shown in the figure.

It can be checked that,

$$\text{Slope} = \tan \theta = \frac{dy}{dx} = \frac{d}{dx}(2x^2) = 4x$$

Where x = the x-coordinate of the point where the slope is to be measured.

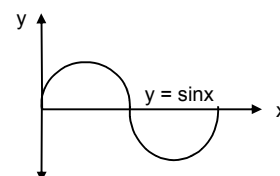


21. $y = \sin x$

$$\text{So, } y + \Delta y = \sin(x + \Delta x)$$

$$\Delta y = \sin(x + \Delta x) - \sin x$$

$$= \left(\sin \frac{\pi}{3} + \frac{\pi}{100} \right) - \sin \frac{\pi}{3} = 0.0157.$$



22. Given that, $i = i_0 e^{-t/RC}$

$$\therefore \text{Rate of change of current} = \frac{di}{dt} = \frac{d}{dt} i_0 e^{-t/RC} = i_0 \frac{d}{dt} e^{-t/RC} = \frac{-i_0}{RC} \times e^{-t/RC}$$

$$\text{When a) } t = 0, \frac{di}{dt} = \frac{-i_0}{RC}$$

$$\text{b) when } t = RC, \frac{di}{dt} = \frac{-i_0}{RCe}$$

$$\text{c) when } t = 10 RC, \frac{di}{dt} = \frac{-i_0}{RCe^{10}}$$

23. Equation $i = i_0 e^{-t/RC}$

$$i_0 = 2A, R = 6 \times 10^{-5} \Omega, C = 0.0500 \times 10^{-6} F = 5 \times 10^{-7} F$$

$$a) i = 2 \times e^{\left(\frac{-0.3}{6 \times 10^{-5} \times 5 \times 10^{-7}}\right)} = 2 \times e^{\left(\frac{-0.3}{0.3}\right)} = \frac{2}{e} \text{ amp.}$$

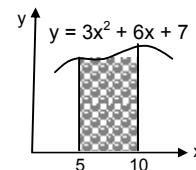
$$b) \frac{di}{dt} = \frac{-i_0}{RC} e^{-t/RC} \text{ when } t = 0.3 \text{ sec} \Rightarrow \frac{di}{dt} = -\frac{2}{0.30} e^{(-0.3/0.3)} = \frac{-20}{3e} \text{ Amp/sec}$$

$$c) \text{ At } t = 0.31 \text{ sec, } i = 2e^{(-0.3/0.3)} = \frac{5.8}{3e} \text{ Amp.}$$

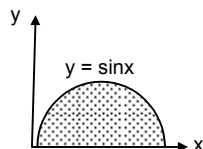
24. $y = 3x^2 + 6x + 7$

\therefore Area bounded by the curve, x axis with coordinates with $x = 5$ and $x = 10$ is given by,

$$\text{Area} = \int_0^y dy = \int_5^{10} (3x^2 + 6x + 7) dx = 3 \left[\frac{x^3}{3} \right]_5^{10} + 5 \left[\frac{x^2}{3} \right]_5^{10} + 7x \Big|_5^{10} = 1135 \text{ sq.units.}$$



$$25. \text{Area} = \int_0^y dy = \int_0^\pi \sin x dx = -[\cos x]_0^\pi = 2$$



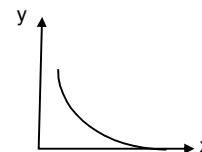
26. The given function is $y = e^{-x}$

$$\text{When } x = 0, y = e^{-0} = 1$$

x increases, y value decreases and only at $x = \infty, y = 0$.

So, the required area can be found out by integrating the function from 0 to ∞ .

$$\text{So, Area} = \int_0^\infty e^{-x} dx = -[e^{-x}]_0^\infty = 1.$$



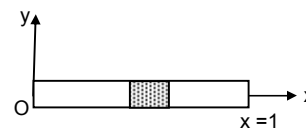
$$27. \rho = \frac{\text{mass}}{\text{length}} = a + bx$$

a) S.I. unit of ' a ' = kg/m and SI unit of ' b ' = kg/m² (from principle of homogeneity of dimensions)

b) Let us consider a small element of length ' dx ' at a distance x from the origin as shown in the figure.

$$\therefore dm = \text{mass of the element} = \rho dx = (a + bx) dx$$

$$\text{So, mass of the rod} = m = \int dm = \int_0^L (a + bx) dx = \left[ax + \frac{bx^2}{2} \right]_0^L = aL + \frac{bL^2}{2}$$



$$28. \frac{dp}{dt} = (10 \text{ N}) + (2 \text{ N/S})t$$

momentum is zero at $t = 0$

\therefore momentum at $t = 10$ sec will be

$$dp = [(10 \text{ N}) + 2 \text{Ns } t] dt$$

$$\int_0^p dp = \int_0^{10} 10 dt + \int_0^{10} (2t) dt = 10t \Big|_0^{10} + 2 \left[\frac{t^2}{2} \right]_0^{10} = 200 \text{ kg m/s.}$$

29. The change in a function of y and the independent variable x are related as $\frac{dy}{dx} = x^2$.

$$\Rightarrow dy = x^2 dx$$

Taking integration of both sides,

$$\int dy = \int x^2 dx \Rightarrow y = \frac{x^3}{3} + c$$

$\therefore y$ as a function of x is represented by $y = \frac{x^3}{3} + c$.

30. The number significant digits

- a) 1001 No. of significant digits = 4
 b) 100.1 No. of significant digits = 4
 c) 100.10 No. of significant digits = 5
 d) 0.001001 No. of significant digits = 4

31. The metre scale is graduated at every millimeter.

$$1 \text{ m} = 100 \text{ mm}$$

The minimum no. of significant digit may be 1 (e.g. for measurements like 5 mm, 7 mm etc) and the maximum no. of significant digits may be 4 (e.g. 1000 mm)

So, the no. of significant digits may be 1, 2, 3 or 4.

32. a) In the value 3472, after the digit 4, 7 is present. Its value is greater than 5.

So, the next two digits are neglected and the value of 4 is increased by 1.

\therefore value becomes 3500

b) value = 84

c) 2.6

d) value is 28.

33. Given that, for the cylinder

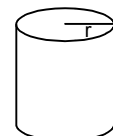
$$\text{Length} = l = 4.54 \text{ cm, radius} = r = 1.75 \text{ cm}$$

$$\text{Volume} = \pi r^2 l = \pi \times (4.54) \times (1.75)^2$$

Since, the minimum no. of significant digits on a particular term is 3, the result should have 3 significant digits and others rounded off.

$$\text{So, volume } V = \pi r^2 l = (3.14) \times (1.75) \times (1.75) \times (4.54) = 43.6577 \text{ cm}^3$$

Since, it is to be rounded off to 3 significant digits, $V = 43.7 \text{ cm}^3$.



34. We know that,

$$\text{Average thickness} = \frac{2.17 + 2.17 + 2.18}{3} = 2.1733 \text{ mm}$$

Rounding off to 3 significant digits, average thickness = 2.17 mm.

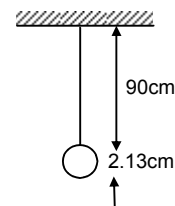
35. As shown in the figure,

$$\text{Actual effective length} = (90.0 + 2.13) \text{ cm}$$

But, in the measurement 90.0 cm, the no. of significant digits is only 2.

So, the addition must be done by considering only 2 significant digits of each measurement.

$$\text{So, effective length} = 90.0 + 2.1 = 92.1 \text{ cm.}$$



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