

## SOLVED EXAMPLES

### WATER DEMAND

1. The population of 5 decades from 1930 to 1970 are given in table 1. Below find out the population after one, two and three decades beyond the last known decade, by using arithmetic increase method.

Year	1930	1940	1950	1960	1970
Population	25,000	28,000	34,000	42,000	47,000

**Solution.** The given data in table 1 is extended in table below, so as to compute the increase in population ( $x$ ) for each decade (col. 3), the total increase, and average increase per decade ( $\bar{x}$ ), as shown.

Year	Population	Increase in population ( $x$ )
(1)	(2)	(3)
1930	25,000	
1940	28,000	3,000
1950	34,000	6,000
1960	42,000	8,000
1970	47,000	5,000
<b>Total</b>		<b>22,000</b>
Average increase per decade ( $\bar{x}$ )		$\bar{x} = \frac{22000}{4} = 5,500$

Future populations are calculated by the relation,

$$P_n = P_o + n \cdot \bar{x}$$

- (i) Population after 1 decade beyond 1970

$$\begin{aligned} P_{1980} &= P_1 \\ &= P_{1970} + 1 \cdot \bar{x} \\ &= 47,000 + 1 \times 5500 \\ &= \mathbf{52,500} \end{aligned}$$

- (ii) Population after 2 decades beyond 1970

$$\begin{aligned} P_{1990} &= P_2 = P_{1970} + 2 \cdot \bar{x} \\ &= 47,000 + 2 \times 5,500 \\ &= \mathbf{58,000} \end{aligned}$$

- (iii) Population after 3 decades beyond 1970,

$$\begin{aligned} P_{2000} &= P_3 = P_{1970} + 3 \cdot \bar{x} \\ &= 47,000 + 3 \times 5,500 \\ &= \mathbf{63,500.} \end{aligned}$$

2. Compute the population of the year 2000 and 2006 a city whose population in the year 1930 was 25,000 and in the year 1970 was 47,000. Make use of geometric increase method.

**Solution.** In this question, the intermediate census data between 1930 to 1970 is not given, and hence geometric mean method of all known decades is not possible.

Growth rate per decade,

$$\begin{aligned} r &= \sqrt[n]{\frac{P_2}{P_1}} - 1 \\ &= \sqrt[4]{\frac{47000}{25000}} - 1 \\ &= 0.17095 \\ &= \mathbf{17.095\% \text{ per decade.}} \end{aligned}$$

Now,  $P_n = P_o \left(1 + \frac{r}{100}\right)^n$

Hence,  $P_{2000} = P_3$  (after 3 decades from 1970 onward)

$$\begin{aligned} &= P_{1970} \left(1 + \frac{r}{100}\right)^3 \\ &= 47,000 (1 + 0.17095)^3 \\ &= 47,000 (1.17095)^3 \\ &= \mathbf{75,459} \end{aligned}$$

Population for the year 2006, means that it is after 36 years (3.6 decades) from 1970 onward

$$\begin{aligned} \therefore P_{2006} &= P_{3.6} \\ &= P_{1970} (1 + 0.17095)^{3.6} \\ &= 47000 (1.17095)^{3.6} \\ &= \mathbf{82,954} \end{aligned}$$

3. In a town, it has been decided to provide 200 l per head per day in the 21st century. Estimate the domestic water requirements of this town in the year AD 2000 by projecting the population of the town by incremental increase method:

Year	Population
1940	2,37,98,624
1950	4,69,78,325
1960	5,47,86,437
1970	6,34,67,823
1980	6,90,77,421

**Solution.** The given population data is analysed, as shown in table below.

Year	Population	Increase in population	Increment over the increase, i.e. incremental increase
1940	2,37,98,624	2,31,79,701	
1950	4,69,78,325	78,08,112	(-) 1,53,71,589
1960	5,47,86,437	86,81,386	(+) 8,73,274
1970	6,34,67,823	56,09,598	(-) 30,71,788
1980	6,90,77,421		
Total		4,52,78,797	(-) 1,75,70,103
Average per decade		$\bar{x} = 1,13,19,699$	$\bar{y} = (-) \frac{1,75,70,103}{3} = (-) 58,56,701$

Expected population at the end of year 2000 (i.e. after 2 decades from 1980)

$$\begin{aligned} P_2 &= P_0 + 2.\bar{x} + \frac{2 \times 3}{2}.\bar{y} \\ &= 6,90,77,421 + 2(1,13,19,699) - 3(58,56,701) \\ &= 7,41,46,216 \end{aligned}$$

∴ Water requirement in AD 2,000 @200 l/head/d

$$= \frac{200 \times 7,41,46,716}{10^6} \text{ M.l/day} = \mathbf{14,829 \text{ MI/day}}$$

4. Determine the future population of a satellite town by the Geometric increase method for the year 2011, given the following data:

Year	1951	1961	1971	1981	—	2011
Population in thousands	93	111	132	161	—	?

**Solution.**

Year	Population in thousands	Increase in population in thousands	Percentage increase in population = growth rate $= \frac{\text{Col.(3)}}{\text{Col.(2)}} \times 200$
1951	93		
		18	19.35
1961	111		
		21	18.92
1971	132		
		29	21.97
1981	161		

Assumed constant growth rate, for future

$$\begin{aligned} r &= \text{geometric mean of past growth rates} \\ &= \sqrt[3]{19.35 \times 18.92 \times 21.97} \\ &= 20.03\% \text{ per decade.} \end{aligned}$$

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Population after  $n$  decades,

$$P_n = P_0 \left(1 + \frac{r}{100}\right)^n$$

∴

$$P_{2011} = \text{Population after 3 decades from 1981}$$

$$= P_{1981} \left(1 + \frac{20.03}{100}\right)^3 = 1,61,000 (1.2003)^3 = \mathbf{2,78,417}$$

5. With the help of the following data; estimate by Incremental increase method, the population of a city for the year 2000 A.D.

<i>Year</i>	<i>Population</i>
1880	25,000
1890	27,500
1900	34,100
1910	41,500
1920	47,050
1930	54,500
1940	61,000

**Solution.** The given table below is extended to table below, so as to work out the average increase per decade and average of the net incremental increases, as shown.

<i>Year</i>	<i>Population</i>	<i>Increase in population</i>	<i>Increment over the increase, i.e. Incremental increase</i>
1880	25,000	2,500	
1890	27,500	6,600	(+) 4,100
1900	34,100	7,400	(+) 800
1910	41,500	5,500	(-) 1,850
1920	47,050	7,450	(+) 1,900
1930	54,500	6,500	(-) 950
1940	61,000		
<b>Total</b>		<b>36,000</b>	<b>6,800 – 2,800 = 4,000</b>
Average per decade		$\bar{x} = \frac{36,000}{6} = 6,000$	$\bar{y} = \frac{4,000}{5} = (+) 800$

Expected population at the end of year 2000

$$\begin{aligned} P_6 &= P_0 + 6.\bar{x} + \frac{6 \times 7}{2} \bar{y} \\ &= 61,000 + 6(6,000) + 21(800) \\ &= \mathbf{1,13,800} \end{aligned}$$

6. Given the following data, calculate the population at the end of next three decades by decreasing rate method.

<i>Year</i>	<i>Population</i>
1940	80,000
1950	1,20,000
1960	1,68,000
1970	2,28,580



**Solution.**

Year	Population	Increase in population	Percentage increase in population	Decrease in the percentage increase
1940	80,000	40,000	$\frac{40,000}{80,000} \times 100 = 50\%$	10%
1950	1,20,000		$\frac{48,000}{1,20,000} \times 100 = 40\%$	
1960	1,68,000		$\frac{60,580}{1,68,000} \times 100 = 36\%$	
1970	2,28,580	60,580		
<b>Total</b>				<b>14%</b>
Average per decade				$\frac{14}{2} = 7\%$

(i) Expected population at the end of year 1980

$$\begin{aligned} &= 2,28,580 + \left[ \frac{36-7}{100} \right] 2,28,580 \\ &= \mathbf{2,94,870} \end{aligned}$$

(ii) Expected population at the end of year 1990

$$\begin{aligned} &= 2,94,870 + \frac{29-7}{100} \times 2,94,870 \\ &= \mathbf{3,59,740} \end{aligned}$$

(iii) Expected population at the end of year 2000

$$\begin{aligned} &= 3,59,740 + \frac{22-7}{100} \times 3,59,740 \\ &= \mathbf{4,13,700} \end{aligned}$$

**SURFACE AND GROUND WATER**

7. Three wells, each having a diameter of 10 cm are installed at the vertices of an equilateral triangle 12 m apart in a confined aquifer. The radius of influence of each well is 400 m, and coefficient of permeability  $k$  is 20 m/day. The drawdown in each well is 2 m. The thickness of the confined aquifer is 15 m. Find the discharge of each well, and the percentage decrease in discharge because of well interference.

**Solution.**

Given,  $r_w = 10/2 = 5 \text{ cm} = 0.05\text{m};$   
 $B = 12 \text{ m};$   
 $R = 400 \text{ m}; k = 20 \text{ m/day}; H = 15 \text{ m}$

Drawdown in well,  $s = (D - h_w) = 2 \text{ m}$

Here, we have the discharge of each of the three interfering artesian wells, arranged in a pattern of equilateral triangle at distances  $B$  apart, as :

$$\begin{aligned} Q_1 = Q_2 = Q_3 &= \frac{2\pi k H (D - h_w)}{2.3 \log_{10} \left( \frac{R^3}{r_w \cdot B^2} \right)} \\ &= \frac{2\pi \times 20 \times 15 \times 2}{2.3 \log_{10} \left\{ \frac{400^3}{0.05 \times 12^2} \right\}} \text{ m}^3/\text{day} \\ &= 235.76 \text{ m}^3/\text{day} \end{aligned}$$

Discharge of each such well without interference is given by

$$\begin{aligned} Q_1 &= \frac{2\pi k H (D - h_w)}{2.3 \log_{10} \left( \frac{R}{r_w} \right)} \\ &= \frac{2\pi \times 20 \times 15 \times 2}{2.3 \log_{10} \left\{ \frac{400}{0.05} \right\}} \\ &= 419.73 \text{ m}^3/\text{day} \end{aligned}$$

$\therefore$  Reduction in discharge due to interference

$$= \left[ \frac{419.73 - 235.76}{419.73} \right] \times 100\% = \mathbf{43.83}$$

8. A pumping test was made in a medium sand and gravel to a depth of 15 m where a bed of clay was encountered. The normal ground water level was at surface. Observation holes were located at distances of 3 m and 7.5 m from the pumping well. At a discharge of 3.6 litres/sec. From the pumping well, a steady state was attained in about 24 hrs. The drawdown of 3 m was 1.65 m and at 7.5 m was 0.36 m. Compute the coefficient of permeability of the soil.

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**Solution.** Given,  $Q = 3.6$  litres/sec.

$$r_1 = 3 \text{ m}; \quad s_1 = 1.65 \text{ m}; \quad h_1 = 15 - 1.65 = 13.35 \text{ m}$$

$$r_2 = 7.5 \text{ m}; \quad s_2 = 0.36 \text{ m}; \quad h_2 = 15 - 0.36 = 14.64 \text{ m}$$

Now,

$$Q = \frac{\pi k(h_2^2 - h_1^2)}{2.3 \log_{10} \left( \frac{r_2}{r_1} \right)}$$

$$\therefore \frac{3.6}{1000} \text{ m}^3/\text{sec.} = \frac{\pi k}{2.3} \left[ \frac{(14.64)^2 - (13.35)^2}{\log_{10} \left( \frac{7.5}{3} \right)} \right]$$

$$= \frac{\pi k}{2.3} \left[ \frac{27.99 \times 1.29}{0.398} \right]$$

or  $k = \left[ \frac{2.3 \times 0.398}{\pi \times 27.99 \times 1.29} \right] \times \frac{3.6}{1000} \times 100 \text{ cm/sec}$

$$= \mathbf{0.00289 \text{ cm/sec.}}$$

9. A 30 cm diameter well penetrates 25 m below the static water table. After 24 hours of pumping at 5400 litres/minute, the water-level in a test well at 90 m is lowered by 0.53 m, and in a well 30 m away, the drawdown is 1.11 m. Find the transmissibility of the aquifer; and the drawdown in the main well

**Solution.** For unconfined aquifer, from Thiem's formula,

$$Q = \frac{\pi k(h_2^2 - h_1^2)}{2.3 \log_{10} \frac{r_2}{r_1}}$$

where  $Q = 5400 \text{ lit/minute} = 5.4 \text{ m}^3/\text{min}$

$$r_1 = 30 \text{ m}$$

$$r_2 = 90 \text{ m}$$

$$s_1 = 1.11 \text{ m}$$

$$s_2 = 0.53 \text{ m}$$

$$h_1 = 25 - 1.11 = 23.89 \text{ m}$$

$$h_2 = 25 - 0.53 = 24.47 \text{ m}$$

$$\therefore k =$$

$$\frac{2.3 \times 5.4 \times \log_{10} 90/30}{\pi(24.47^2 - 23.89^2)} = 0.067 \text{ m/min.}$$

$$\therefore T = kd = 0.067 \times 25 \text{ m} = \mathbf{1.675 \text{ m}^2/\text{min}}$$

Hence, coefficient of transmissibility

$$= \mathbf{1.675 \text{ m}^2/\text{minute.}}$$

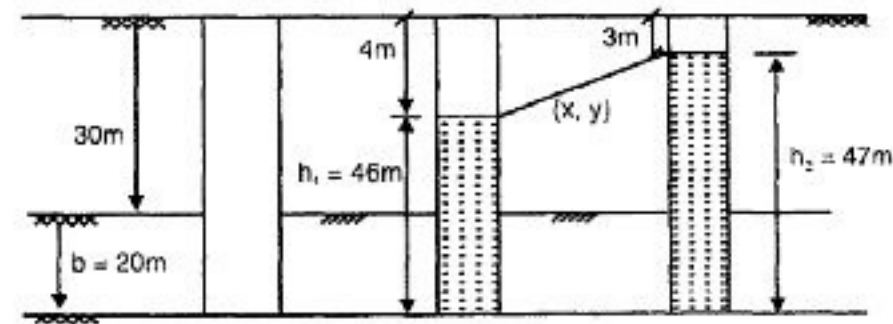
10. An aquifer of 20 m average thickness is overlain by an impermeable layer of 30 m thickness. A test well of 0.5 m diameter and two observation wells at a distance of 10 m and 60 m from the test well are drilled through the aquifer. After pumping at a rate of 0.1 m<sup>3</sup>/sec for a long time, the following drawdowns are stabilized in these wells: first

observation well, 4 m; second observation well, 3 m. Show the arrangement in a diagram. Determine the coefficient of permeability and the drawdowns in the test well.

**Solution.** Given,

$$d = 0.5 \text{ m} \quad Q = 0.1 \text{ cumec} \quad r_1 = 10 \text{ m}$$

$$h_1 = 46 \text{ m} \quad r_2 = 60 \text{ m} \quad h_2 = 47 \text{ m}$$



Now,  $q = Aki = (2\pi xb)k \frac{dy}{dx}$

$$\therefore q \int_{r_1}^{r_2} \frac{dx}{x} = \int_{h_1}^{h_2} 2\pi b k dy$$

$$q \log_e \frac{r_2}{r_1} = 2\pi b k (h_2 - h_1)$$

$$\therefore k = \frac{2.3q \log_{10} \frac{r_2}{r_1}}{2\pi b (h_2 - h_1)} = \frac{2.3 \times 0.1 \times \log_{10} \frac{60}{10}}{2\pi \times 20 (47 - 46)}$$

$$= 1.426 \times 10^{-3} \text{ m/sec.}$$

Drawdown in test well =  $s$

Now,  $q \log_e \frac{r_2}{r} = 2\pi b k (h_2 - h)$

or  $2.30q \log_e \frac{r_2}{r} = 2\pi b k (h_2 - h)$

or  $2.303 \times 0.1 \times \log_{10} \frac{60}{0.25} = 2\pi \times 20 \times 1.426$

$$\times 10^{-3} (47 - h)$$

$$\therefore h = 43.9 \text{ m}$$

$$\therefore \text{Draw down, } s = 50 - 43.9 = \mathbf{6.1 \text{ m}}$$

11. 60 cm diameter well is being pumped at a rate of 1360 litres/minute. Measurements in a nearby test well were made at the same time as follows. At a distance of 6 m from the well being pumped, the drawdown was 6 m, and at 15 m, the drawdown was 1.5 m. The bottom of the well is 90 m below the ground watertable:

- Determine the coefficient of permeability.
- If all the observed pts. were on the Dupuit curve, what was the drawdown in the well during pumping?
- What is the specific capacity of the well?
- What is the maximum rate at which water can be drawn from this well?

**Solution.** For unconfined aquifers, from Thiem's

$$\text{formula, } Q = \frac{\pi k [h_2^2 - h_1^2]}{2.3 \log_{10} \left( \frac{r_2}{r_1} \right)}$$

where  $r_1 = 6 \text{ m}$   $r_2 = 15 \text{ m}$ ;

$s_1 = 6 \text{ m}$   $s_2 = 1.5 \text{ m}$

$d = 90 \text{ m}$ ,

$Q = 1,360 \text{ litres/minute} = 1.36 \text{ m}^3/\text{min.}$

$h_1 = 90 - 6 = 84 \text{ m}$ ,

$h_2 = 90 - 1.5 = 88.5 \text{ m}$

(i) Putting values,

$$1.36 = \frac{\pi k [(88.5)^2 - (84)^2]}{2.3 \log_{10} \left( \frac{15}{6} \right)}$$

$$\text{or } \pi k = \frac{1.36 \times 2.3 \times 0.398}{172.5 \times 4.5} \text{ m/min}$$

$$= 1.603 \times 10^{-3} \text{ m/min}$$

$$\therefore k = 0.51 \times 10^{-3} \text{ m/min.}$$

(ii) Now  $r_w = 0.3 \text{ m}$ ,  $r_2 = 15 \text{ m}$ ,

$h_2 = 88.5 \text{ m}$

$$\therefore Q = \frac{\pi k (d^2 - h_w^2)}{2.3 \log_{10} \left( \frac{r_2}{r_w} \right)} = \frac{\pi k (88.5^2 - h_w^2)}{2.3 \log_{10} \left( \frac{15}{0.3} \right)}$$

But  $Q = 1.36$ ,

and  $\pi k = 1.603 \times 10^{-3}$

$$\therefore 1.36 = \frac{1.603 \times 10^{-3} (88.5^2 - h_w^2)}{2.3 \log_{10} (50)}$$

$$\text{or } \frac{1.36 \times 2.3 \times 1.69}{1.603 \times 10^{-3}} = (88.5)^2 - h_w^2$$

$$\text{or } 3,290 = 7,820 - h_w^2$$

$$\text{or } h_w^2 = 7,820 - 3,290 = 4,530$$

$$\text{or } h_w = 67.4 \text{ m.}$$

$\therefore$  Drawdown in the pumped well

$$= 90 - 67.4 = 22.6 \text{ m}$$

(iii) Specific capacity of the well is the discharge for a unit (i.e. 1 m) drawdown in the pumped well.

At first determine the value of R

$$1.36 = \frac{\pi k (d^2 - h_w^2)}{2.3 \log_{10} \left( \frac{R}{r_w} \right)}$$

$$= \frac{\pi k (90^2 - 67.4^2)}{2.3 \log_{10} \left( \frac{R}{0.3} \right)}$$

$$= \frac{1.603 \times 10^{-3} \times 157.4 \times 22.6}{2.3 \log_{10} \left( \frac{R}{0.3} \right)}$$

$$\text{or } \log_{10} \left( \frac{R}{0.3} \right) = \frac{1.603 \times 157.4 \times 22.6}{2.3 \times 1,360} = 1.824$$

Taking antilog, we get

$$\frac{R}{0.3} = 66.7$$

$$\text{or } R = 20.01 \approx 20 \text{ m}$$

Now Specific capacity

$$= Q_{\text{unit drawdown}}$$

$$= \frac{\pi k (90^2 - 89^2)}{2.3 \log_{10} \left( \frac{20}{0.3} \right)} = \frac{1.603 \times 10^{-3} \times 179 \times 1}{2.3 \times 1.824}$$

$$= 68.3 \times 10^{-3} \text{ m}^3/\text{min.}$$

$$= 68.3 \text{ litres/min.}$$

(iv) Maximum discharge will occur when  $h_w = 0$

$\therefore$  Maximum rate of discharge,

$$Q_{\text{max}} = \frac{1.603 \times 10^{-3} \times 8100}{2.3 \times 1.824}$$

$$= 3.09 \text{ m}^3/\text{min.}$$

$$= 3,090 \text{ litres/minute}$$

**12.** A well penetrates into an unconfined aquifer having a saturated depth of 100 metres. The discharge is 250 litres per minute at 12 metres drawdown. Assuming equilibrium flow conditions and a homogeneous aquifer, estimate the discharge at 18 metres drawdown. The distance from the well where the drawdown influenced are not appreciable may be taken to be equal for both the cases.

**Solution.** Given,  $d = 100 \text{ m}$ ;

$Q_1 = 250 \text{ litres/minute}$ ;

$s_1 = 12 \text{ m}$ ;

$s_2 = 18 \text{ m}$ ;

$$\text{Now } Q = \frac{\pi k (d^2 - h_w^2)}{2.3 \log_{10} \left( \frac{R}{r_w} \right)}$$

**In the First case**

Drawdown = 12 m; and  $h_w = 100 \text{ m} - 12 \text{ m} = 88 \text{ m}$

Since here R and  $r_w$  are the same for both the cases, therefore

$$250 \text{ litres/minute} = \frac{\pi k (100^2 - 88^2)}{2.3 \log_{10} \left( \frac{R}{r_w} \right)}$$



$$\text{or } \frac{\pi k}{2.3 \log_{10} \left( \frac{R}{r_w} \right)} = \frac{250}{(100)^2 - (88)^2} = \frac{250}{188 \times 12}$$

**Second case :** Drawdown = 18 m,  
and  $h_w = 100 - 18 = 82$  m

$$Q_2 = \left[ \frac{\pi K \times (100^2 - 82^2)}{2.3 \log_{10} \left( \frac{R}{r_w} \right)} \right]$$

$$= \left[ \frac{250}{188 \times 12} \right] (100^2 - 82^2)$$

$$= \mathbf{363 \text{ litres/minute.}}$$

### TREATMENT OF WATER

- The maximum daily demand at a water purification plant has been estimated as 12 million litres per day. Design the dimensions of a suitable sedimentation tank (fitted with mechanical sludge removal arrangements) for the raw supplies, assuming a detention period of 6 hours and the velocity of flow as 20 cm per minute.

**Solution.** Quantity of water to be treated in 24 hours =  $12 \times 10^6$  litres

Quantity of water to be treated during the

$$\text{detention period of 6 hours} = \frac{12 \times 10^6}{24} \times 6 = 3 \times 10^6$$

litres = 3000 cubic metres

$\therefore$  Capacity of the tank required = **3000 cu. m.**

Velocity of flow to be maintained through the tank = 20 cm/min. = 0.2 m/minute

Length of the tank required

$$= \text{Velocity of flow} \times \text{Detention period}$$

$$= 0.2 \times (6 \times 60) = 72 \text{ m.}$$

Cross-sectional area of the tank required

$$= \frac{\text{Capacity of the tank}}{\text{Length of the tank}} = \frac{3000}{72}$$

$$= 41.67 \text{ m}^2, \text{ say } 41.7 \text{ m}^2.$$

Assume the water depth in the tank as 4 m, then width of the tank required

$$= \frac{41.7}{4} = 10.42 \text{ m, say } 10.5 \text{ m.}$$

Use a free-board of 0.5 m overall depth

$$= 0.5 + 4.0 = 4.5 \text{ m.}$$

Hence, a rectangular sedimentation tank with an overall size of **72 m  $\times$  10.5 m  $\times$  4.5 m** can be used.

*Alternatively:* Assume an overflow rate, say as 600 litres/hr/m<sup>2</sup>

$$\therefore \frac{Q}{BL} = 600$$

$$\text{But } Q = \frac{12 \times 10^6}{24} \text{ litres/hr}$$

$$= 0.5 \times 10^6 \text{ litres/hr.}$$

$$\therefore B = \frac{Q}{600} = \frac{0.5 \times 10^6}{600} = 833 \text{ m}^2$$

$$\text{or } BL = \frac{833}{72} = 11.6 \text{ m.}$$

$$\therefore \text{Depth} = \frac{3000}{72 \times 11.6} = 3.6 \text{ m}$$

$\therefore$  Dimensions of the tank

$$= 72 \text{ m} \times 11.6 \text{ m} (3.6 + 0.5) \text{ m overall depth.}$$

$$= 72 \text{ m} \times 11.6 \text{ m} \times 4.1 \text{ m size}$$

- In a continuous flow settling tank, 3.5 m deep and 65 m long, if the flow velocity of water observed is 1.22 cm/s, what size of the particles of specific gravity 2.65 may be effectively removed? Assume temperature 25°C and kinematic viscosity of water as 0.01 cm<sup>2</sup>/s.

**Solution.** We know,  $\frac{V}{V_s} = \frac{L}{H} = \frac{65}{3}$ ,

where  $V = 1.22$  cm/s

Assume, water depth in tank = 3 m

$$\therefore \frac{1.22}{V_s} = \frac{65}{3}$$

$$\text{or } V_s = \frac{3}{65} \times 1.22 = 0.0563 \text{ cm/s}$$

$$\text{For } d < 0.1 \text{ mm, } V_s = \frac{g}{18} (S_s - 1) \frac{d^2}{\nu}$$

$$\text{or } 0.0563 = \frac{981}{18} (1.65) \frac{d^2}{0.01}$$

$$\text{or } d = 2.5 \times 10^{-3} \text{ cm}$$

$$= \mathbf{0.025 \text{ mm}} \text{ (which is } < 0.1 \text{ mm)}$$

Hence, the particles of size 0.025 mm and above shall be effectively removed.

- Two million litres of water per day is passing through a sedimentation tank which is 6 m wide, 15 m long and having a water depth of 3 m. Find
  - Detention time for the tank.
  - Average flow velocity through the tank

- (iii) If 60 ppm is the concentration of suspended solids present in turbid raw water, how much dry solids will be deposited per day in the tank, assuming 70% removal in the basin, and average specific gravity of the deposit as 2.
- (iv) The overflow rate.

**Solution.**

$$\begin{aligned} \text{(i) Capacity of the tank} &= \text{L.B.D.} \\ &= 15 \text{ m} \times 6 \text{ m} \times 3 \text{ m} \\ &= 270 \text{ m}^3 \end{aligned}$$

Discharge passing through the tank,

$$\begin{aligned} Q &= 2 \text{ million litres per day} \\ &= 2 \times 10^6 \text{ litres/day} \\ &= \frac{2 \times 10^6}{24} \text{ litres/hr} \\ &= 83.33 \times 10^3 \text{ litres/hr.} \\ &= 83.33 \text{ cu.m./hrs.} \end{aligned}$$

$$\text{Detention time} = \frac{\text{Capacity of the tank}}{\text{Discharge}}$$

$$= \frac{270}{83.33} \text{ hours} = \mathbf{3.24 \text{ hours.}}$$

- (ii) Average velocity of flow through the tank

$$= \frac{\text{Discharge}}{\text{Cross-sectional area, i.e. BH}}$$

$$= \frac{83.33}{6 \times 3} \text{ m/hr}$$

$$= \frac{83.33}{6 \times 3} \times \frac{100}{60} \text{ cm/minute}$$

$$= \mathbf{7.72 \text{ cm/minute.}}$$

- (iii) Quantity of water passing per day = 2 million litres =  $2 \times 10^6$  litres

Concentration of suspended solids = 60 ppm.

$\therefore$  Quantity of suspended solids entering the tank per day

$$= 2 \times 10^6 \times \frac{60}{10^6} \text{ litres}$$

$$= 120 \text{ litres} = 0.12 \text{ cu.m.}$$

Given, average specific gravity of the deposited material = 2

Density of deposited material

$$= 2000 \text{ kg/m}^3$$

$$= [0.12 \times 0.7][2000] \text{ kg}$$

$$= \mathbf{168 \text{ kg.}}$$

- (iv) Overflow rate

= Discharge per unit plan area

$$= \frac{Q}{\text{B.L}} = \frac{83.33 \times 10^3 \text{ litres/hr}}{6 \times 15 \text{ m}^2}$$

$$= \mathbf{926 \text{ litres/hr/m}^2}$$

4. Find the settling velocity of a discrete particle in water under conditions when Reynold's number is less than 0.5. The diameter and specific gravity of the particle is  $5 \times 10^{-3}$  cm and 2.65, respectively. Water temperature is  $20^\circ\text{C}$  (kinematic viscosity  $\nu$  of water at  $20^\circ\text{C} = 1.01 \times 10^{-2} \text{ cm}^2/\text{sec}$ ).

**Solution.** Using Stoke's equation and c.g.s. units, we have

Settling velocity (in cm/sec),

$$V_s = \frac{g}{18}(S_s - 1) \cdot \frac{d^2}{\nu}; \text{ when } d < 0.1 \text{ mm}$$

where  $S_s = 2.65$ ,  $\nu = 1.01 \times 10^{-2} \text{ cm}^2/\text{sec}$ ,

$$g = 981 \text{ cm}^2/\text{sec}$$

$$d = 5 \times 10^{-3} \text{ cm} = 0.05 \text{ mm,}$$

which is  $< 0.1 \text{ mm}$

$$\therefore V_s = \frac{981}{18}(2.65 - 1) \cdot \frac{(5 \times 10^{-3})^2}{1.01 \times 10^{-2}} \text{ cm/sec}$$

$$= \mathbf{0.22 \text{ cm/sec.}}$$

5. In a continuous flow settling tank 3 m deep and 60 m long, what flow velocity of water would you recommend for effective removal of 0.025 mm particles at  $25^\circ\text{C}$ . The specific gravity of particles is 2.65, and kinematic viscosity  $\nu$  for water may be taken as  $0.01 \text{ cm}^2/\text{sec}$ .

**Solution.** Settling velocity for particles of 0.025 mm (i.e.,  $< 0.1 \text{ mm}$ ) diameter is given by

$$V_s = \frac{g}{18}(S_s - 1) \frac{d^2}{\nu}$$

where  $g = 981 \text{ cm/sec}^2$ ,

$$S_s = 2.65 \text{ (given)}$$

$$d = 0.025 \text{ mm} = 0.0025 \text{ cm,}$$

$$\nu = 0.01 \text{ cm}^2/\text{sec}$$

$$V_s = \frac{981}{18}(2.65 - 1) \frac{(0.0025)^2}{0.01} \text{ cm/sec}$$

$$= \mathbf{0.0562 \text{ cm/sec.}}$$

$$\text{Now, we have } \frac{V}{V_s} = \frac{L}{H}$$

where  $L$  = length of the tank = 60 m,

$H$  = height of water in the tank

Assuming 0.5 m free-board from out of the total depth of 3 m of the tank,



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we have the water depth in the tank,  $H = 2.5 \text{ m}$

$$\therefore V = V_s \cdot \frac{L}{H} = 0.0562 \times \frac{60}{2.5} \text{ cm/sec} \\ = 1.35 \text{ cm/sec}$$

Hence, in order to ensure effective removal of particles upto  $0.025 \text{ mm}$ , the flow velocity in the settling tank should not be more than  $1.35 \text{ cm/sec}$ .

6. A settling basin is designed to have a surface overflow rate of  $32.6 \text{ m/day}$ . Determine the overall removal obtained for a suspension with size distribution given below. The specific gravity of the particles is  $1.2$  and the water temperature is  $20^\circ\text{C}$ , at which the dynamic viscosity is  $1.027 \text{ centipoise}$  and the density  $0.997 \text{ g/cm}^3$ .

Particle size, mm	0.10	0.08	0.07	0.06	0.04	0.02	0.01
Weight fraction greater than size (per cent)	10	18	40	70	93	99	100

**Solution.** In a sedimentation basin, the overflow rate  $Q/BL$  represents the settling velocity of particles of size ' $d$ ' which get removed, i.e. all particles whose settling velocity equals or exceeds  $Q/BL$  will settle down.

Hence, when settling velocity,  $V_s = \frac{Q}{BL}$  and the corresponding particles size is  $d$ , then all the particles up to size ' $d$ ' will get removed.

$$\text{Here, } \frac{Q}{BL} = V_s = 32.6 \text{ m/day} \\ = \frac{32.6}{24 \times 60 \times 60} \text{ m/s} \\ = 3.773 \times 10^{-4} \text{ m/s} \quad \dots(i)$$

$$\text{Also, we have } V_s = \frac{g}{18} (S_s - 1) \cdot \frac{d^2}{\nu},$$

where  $S_s = 1.2$  (given)

Kinematic viscosity  $\nu$  in the above equation has units of  $\text{m}^2/\text{s}$

$$\text{Given : } \mu = 1.027 \text{ centipoise} \\ = 1.027 \times 10^{-2} \text{ poise} \\ = \frac{1.027 \times 10^{-2}}{10} \text{ Ns/m}^2 \\ (\because 10 \text{ poise} = 1 \text{ Ns/m}^2)$$

$$\frac{\mu}{\rho\omega} = \frac{1.027 \times 10^{-2}}{10} \left( \frac{\text{Ns}}{\text{m}^2} \right) \times \frac{1}{997 (\text{kg/m}^3)}$$

$$= 1.032 \times 10^{-6} \frac{\text{kgm.s m}^3}{\text{s}^2 \cdot \text{m}^3 \text{kg}} \\ (\because 1 \text{ N} = \text{kg m/s}^2) \\ = 1.03 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\therefore V_s = \frac{9.81}{18} (1.2 - 1) \frac{d^2}{1.03 \times 10^{-6}} \text{ m/s} \quad \dots(ii)$$

where  $d$  is in  $\text{m}$ .

From equations (i) and (ii), we get

$$3.773 \times 10^{-4} = \frac{9.81}{18} (0.2) \frac{d^2}{1.03 \times 10^{-6}}$$

$$\text{or } d^2 = \frac{3.773 \times 10^{-4} \times 1.03 \times 10^{-6} \times 18}{9.81 \times 0.2}$$

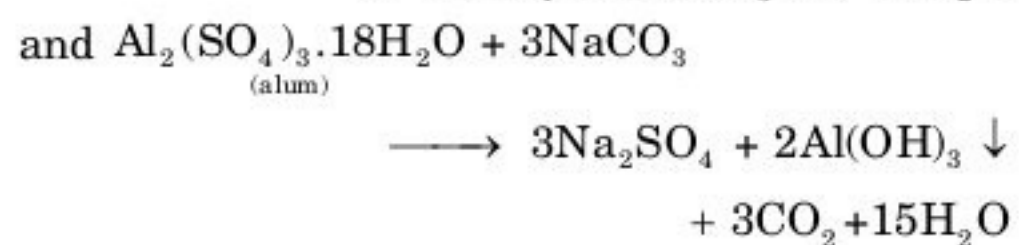
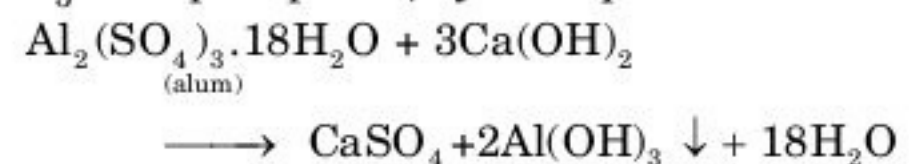
$$\text{or } d = \sqrt{3.565 \times 10^{-3} \sqrt{(10^{-6})}} \\ = 0.0597 \times 10^{-3} \text{ m} \\ = 0.0597 \text{ mm} \approx 0.06 \text{ mm (say).}$$

From the given table, the percentage of particles in suspension equal to or heavier than  $0.06 \text{ mm}$  size =  $70\%$ . Hence,  $70\%$  removal will occur, because all particles above and up to this size will get removed in this basin.

7. A coagulation treatment plant with a flow of  $0.5 \text{ m}^3/\text{sec}$  is dosing alum at  $23 \text{ mg/l}$ . No other chemicals are being added. The raw water suspended solids concentration is  $37 \text{ mg/l}$ . The effluent suspended solids is measured as  $12 \text{ mg/l}$ . The sludge content is  $1$  percent and the specific gravity of sludge solids is  $3.01$ . What volume of sludge must be disposed of each day?

**Solution.** Alum added =  $23 \text{ mg/l}$

Alum reacts with alkali in raw water to produce  $\text{Al}(\text{OH})_3$  solid precipitate, by the equations:



It eventually means that  $1 \text{ mol}$  of Alum produces  $2 \text{ moles}$  of  $\text{Al}(\text{OH})_3$ . The Molecular weight of Alum is  $666 \text{ gm}$  and that of  $\text{Al}(\text{OH})_3$  is  $78 \text{ gm}$ .

Hence,  $666 \text{ gm}$  of alum produces =  $2 \times 78 \text{ gm}$  of  $\text{Al}(\text{OH})_3 \downarrow$  as solid sludge

$\therefore 1 \text{ gm}$  of alum will produce

$$= \frac{2 \times 78}{666} = 0.24 \text{ gm of } \text{Al}(\text{OH})_3 \downarrow$$

Hence,  $1 \text{ mg/l}$  of Alum will produce  $0.24 \text{ mg/L}$  of solid sludge.

$$\begin{aligned}\therefore 23 \text{ mg/L of Alum will produce} \\ &= 23 \times 0.24 \text{ mg/L of solid sludge} \\ &= \mathbf{5.52 \text{ mg/l}} \text{ of solid sludge} \quad \dots(i)\end{aligned}$$

$$\begin{aligned}\text{Now Suspended solids (turbidity) removed} \\ &= 37 \text{ mg/l} - 12 \text{ mg/l} \\ &= \mathbf{25 \text{ mg/l}} \quad \dots(ii)\end{aligned}$$

$$\begin{aligned}\therefore \text{Total dry sludge (solids) removed} &= (i) + (ii) \\ &= 5.52 + 25 = \mathbf{30.52 \text{ mg/l}}\end{aligned}$$

$$\begin{aligned}\text{Total flow in the plant} \\ &= 0.05 \text{ m}^3/\text{s} \\ &= 0.5 \times 24 \times 60 \times 60 \text{ m}^3/\text{day} \\ &= \mathbf{43,200 \text{ m}^3/\text{day}}\end{aligned}$$

$$\begin{aligned}\text{Total dry sludge solids produced per day} &= 43,200 \\ \text{m}^3/\text{d} \times 30.52 \text{ mg/l} \\ &= 43,200 \text{ m}^3/\text{d} \times 30.52 \text{ gm/m}^3 \\ &= \mathbf{1.318 \text{ t/day}} \quad \dots(iii)\end{aligned}$$

$$\begin{aligned}\text{Given, specific gravity of sludge solids} &= 3.01 \\ \therefore \text{Volume of sludge solids produced} \\ &= \frac{1.318 \text{ t/d}}{\text{Unit wt. of sludge solids}} \\ &= \frac{1.318 \text{ t/d}}{3.01 \text{ t/m}^3} = \mathbf{0.44 \text{ m}^3/\text{day}} \quad \dots(iv)\end{aligned}$$

Since the sludge content is stated to be 1%, the produced wet sludge will contain 1% solids and 99% water, by weight.

i.e. 1 tonne of sludge solids will contain 99 tonnes of water;

$$\begin{aligned}\therefore 1.318 \text{ t/day of sludge solids will contain water} \\ &= 1.318 \text{ t/day} \times 99 = \mathbf{130.48 \text{ t/day}}.\end{aligned}$$

$$\begin{aligned}\text{Since unit weight of water} &= 1 \text{ t/m}^3, \text{ therefore} \\ \therefore \text{Volume of water contained in the wet sludge} \\ &= \frac{130.48 \text{ t/day}}{1.0 \text{ t/m}^3} = \mathbf{130.48 \text{ m}^3/\text{day}} \quad \dots(v)\end{aligned}$$

$$\begin{aligned}\text{Total volume of wet sludge} \\ &= (iv) + (v) \\ &= (0.44 + 130.48) \text{ m}^3/\text{day} \\ &= \mathbf{130.92 \text{ m}^3/\text{day}}.\end{aligned}$$

8. Design a coagulation-cum-sedimentation tank with continuous flow for a population of 60,000 persons with a daily per capita water allowance of 120 litres. Make suitable assumptions where needed.

**Solution.** At first, design the settling tank, and then the floc chamber.

*Design of the Settling Tank*

$$\begin{aligned}\text{Average daily consumption} \\ &= \text{Population} \times \text{Per capita demand} \\ &= 60,000 \times 120 \\ &= \mathbf{7.2 \times 10^6 \text{ litres}}\end{aligned}$$

$$\begin{aligned}\text{Assume, maximum daily demand} \\ &= 1.8 \times \text{average daily demand} \\ &= 1.8 \times (7.2 \times 10^6) \text{ lit.} \\ &= \mathbf{12.96 \times 10^6 \text{ litres}}.\end{aligned}$$

$\therefore$  Quantity of water to be treated during an assumed detention period of 4 hours

$$\begin{aligned}&= \frac{12.96 \times 10^6}{24} \times 4 = 2.16 \times 10^6 \text{ litres} \\ &= \mathbf{2.16 \times 10^3 \text{ cu-m}}\end{aligned}$$

$$\begin{aligned}\text{Hence, required capacity of the tank} \\ &= \mathbf{2.16 \times 10^3 \text{ cu-m}}.\end{aligned}$$

Assume an overflow rate of 1000 litres/hr/m<sup>2</sup> of plan area

$$\frac{Q}{\text{B.L}} = 1000$$

$$\begin{aligned}\text{where } Q &= \frac{2.16 \times 10^6}{4} \text{ litres/hr} \\ &= \mathbf{540 \times 10^3 \text{ litres/hr}}.\end{aligned}$$

$$\begin{aligned}\therefore \text{Plan area} = \text{B.L.} &= \frac{Q}{1000} \\ &= \frac{540 \times 10^3}{1000} = \mathbf{540 \text{ m}^2}.\end{aligned}$$

Using the width of the tank as 12 m, we get

$$\text{Length of the tank} = \frac{540}{12} = \mathbf{45 \text{ m}}.$$

*Alternatively:* Adopt the water depth as 4 m

$$\text{Plan area} = \frac{\text{Capacity}}{\text{Depth}} = \frac{2.16 \times 10^3}{4} = 540 \text{ m}^2$$

Hence, use a tank of 45 m  $\times$  12 m  $\times$  4 m.

Provide an extra depth for sludge storage, say use 4.5 m depth at the starting end, and  $4.5 + \frac{45}{50} = 5.4 \text{ m}$  at the d/s end (using 1 in 50 slope).

Use a freeboard of 0.5 m above the water level.

*Design of the Floc Chamber*

In addition to 45 m length of the settling tank, the floc chamber at the entry has to be provided.

Assume, effective depth in the floc chamber

$$\begin{aligned}&= \frac{1}{2} \times \text{depth in the tank near the floc chamber,} \\ &= \frac{4.5}{2} = \mathbf{2.25 \text{ m}}\end{aligned}$$



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Assume the period of flocculation or detention period = 20 minutes.

Capacity of the chamber

$$\begin{aligned}
 &= \text{Flow required in 20 minutes} \\
 &= \frac{12.96 \times 10^3}{24} \times \frac{20}{60} \text{ cu. m} \\
 &= 180 \text{ cu. m.}
 \end{aligned}$$

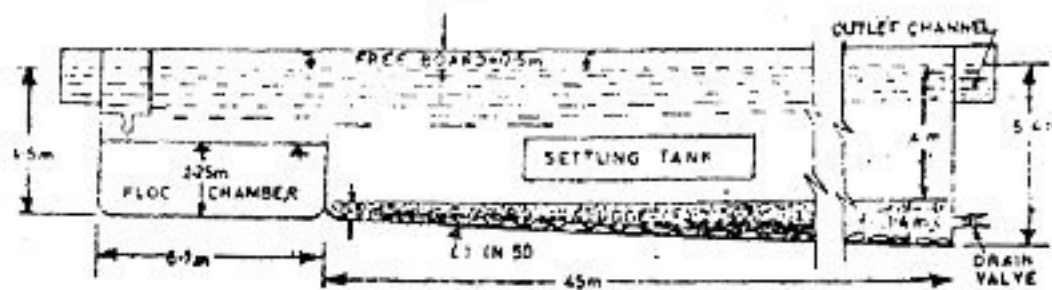
Required plan area

$$\begin{aligned}
 &= \frac{\text{Capacity}}{\text{Depth}} = \frac{180}{2.25} \text{ m}^2 = 80 \text{ m}^2
 \end{aligned}$$

Use the same width as 12 m,

Length of flocculation chamber

$$\begin{aligned}
 &= \frac{80}{12} = 6.7 \text{ m (say 6.7 m)}
 \end{aligned}$$



**Fig. Dimensions of the Coagulation-cum-Sedimentation tank**

9. A rectangular sedimentation basin is to handle 10 million litres/day of raw water. A detention basin of width to length ratio of 1/3 is proposed to trap all particles larger than 0.04 mm in size. Assuming a relative density of 2.65 for the particles of 20°C as the average temperature, compute the basin dimensions. If the depth of the tank is 3.5 m, calculate the detention time.

**Solution.** Settling velocity,

$$\begin{aligned}
 V_s &= 418 (S_s - 1) d^2 \left( \frac{3T + 70}{100} \right) \text{ for } d < 0.1 \text{ mm} \\
 &= 418 (2.65 - 1) \times (0.01)^2 \left( \frac{130}{100} \right) \text{ mm/sec.} \\
 &= 1.45 \text{ mm/sec} = 0.1435 \text{ cm/sec.}
 \end{aligned}$$

$$\text{But, } \frac{V}{V_s} = \frac{L}{H}, \text{ therefore } V = 0.1435 \frac{L}{H} \text{ cm/sec.}$$

where  $V$  = Maximum flow velocity in the tank.

Also  $L = \text{Flow velocity} \times \text{Detention time } (t)$

$$\begin{aligned}
 &= \left[ 0.1435 \frac{L}{H} (t \times 60 \times 60) \right] \text{ cm} \\
 &= 5.164 \frac{Lt}{H}, \text{ where } t \text{ is in hr.}
 \end{aligned}$$

$$\text{or } t = \frac{H}{5.164}$$

Capacity of the tank of  $t$  hr detention period

$$\begin{aligned}
 &= \frac{10 \times 10^6}{10^3} \times \frac{t}{24} \text{ m}^3 = 416.67 t
 \end{aligned}$$

$$\text{or } B.L.H. = 416.67 t, \quad \text{where } L = 3B \text{ (given)}$$

$$\therefore 3.B^2.H = 416.67 t$$

From equation (i)

$$3B^2H = 416.67 \times \frac{H}{5.164}$$

$$\therefore B = 5.19 \text{ m; say 5.2 m;}$$

$$\text{and } L = 15.6 \text{ m.}$$

$$\begin{aligned}
 \text{When } H &= \text{Depth, i.e. Height of the tank} \\
 &= 3.5 \text{ m (given)}
 \end{aligned}$$

$$t = \frac{H}{5.164} = \frac{3.5}{5.164}$$

$$= 0.68 \text{ hr.} = 41 \text{ minutes.}$$

10. A rectangular sedimentation tank following coagulation-flocculation is to treat a flow of 3000 m<sup>3</sup>/day with a detention time of 6 hours. It is to be hand cleaned of sludge at 6 week intervals. The suspended solids concentration of the water is reduced from 250 mg/l by coagulation-flocculation. The settled sludge includes 40 mg/l (based on water flow) of metallic precipitate and has a moisture content of 85% and specific gravity of 1.24. Determine the volume of sludge produced between cleanings and the basic dimensions of the tank if the water depth just before cleaning is 3 m and its length is twice its width.

**Solution.** Initial suspended solids concentration of water = 250 mg/l

Final suspended solids concentration of water = 5 mg/l

Suspended solids removed in the tank

$$\begin{aligned}
 &= 250 \text{ mg/l} - 5 \text{ mg/l} \\
 &= 245 \text{ mg/l}
 \end{aligned}$$

These removed suspended solids will settle down as sludge along with the metallic precipitates formed during coagulation.

Given concentration of such metallic precipitates in sludge = 40 mg/l.

Hence, total solids concentration in sludge

$$= 245 \text{ mg/l} + 40 \text{ mg/l} = 285 \text{ mg/l.}$$

Discharge or water flow in the tank

$$= 3000 \text{ m}^3/\text{day.}$$

$\therefore$  Total solids removed as sludge per day

$$\begin{aligned}
 &= 285 \text{ mg/l} \times 3000 \text{ m}^3/\text{day} \\
 &= 285 \text{ mg/l} \times \frac{(3000 \times 1000) \text{ l}}{\text{day}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{285}{10^6} \times (3000 \times 1000) \text{ kg/day} \\
 &= 855 \text{ kg/day.}
 \end{aligned}$$