10. An aeroplane weights 24 kn and has an area of 35 m^2. If the coefficient of lift varies linearly from 0.3 to 0.8 while the angle of attack varies from 0° to 8°, calculate the angle of attack for the aeroplane speed of 150 km/h. Assume : $\rho_{air} = 1.1$ kg/m^3

Solutions:

$$U_0 = \frac{150 \times 1000}{3600} 41.66 \text{ m/s}$$

Since the weigh of aeroplane should be balanced by the lift force, therefore

$$F_L = W = \frac{C_L A \rho U_0^2}{2}$$

or $$24 \times 100 = \frac{C_L \times 35 \times 1.1 \times 41.66^2}{2}$$

$\therefore$ $$C_L = 0.717$$

Since C_L change by 0.5 for 8° change in α_0, hence for 0.717 = 6.67°

11. A flat disc of 10 cm diameter rotates on a table separated by an oil film of 1.5 mm thickness. If the torque to rotate the disc at 50 RPM is 3×10^{-5} kg-m, find the viscosity of the oil in centipoise. Assume suitable velociy profile.

Solution: Diameter of disc,

$$d = 10 \text{ cm, Area, } A = \frac{\pi}{4}(d)^2 = 25\pi \text{ cm}^2$$

Thickness of oil film $y = 1.5$ mm,

$n = 50$ rpm,

and $$\omega = 2\pi \times \frac{50}{60} \text{sec}^{-1}$$

$$v = r.\omega = \frac{10\pi}{6} \times 5 = \frac{50\pi}{6} \text{ cm/sec.}$$

Now, $$\tau = \mu \frac{\delta v}{\delta y}$$

$\therefore$ Force $F = \mu . A$

and Torque $= \mu \frac{\delta v}{\delta y} A.r$

$$\therefore (3 \times 10^{-5}) \times 10^7 = \mu \times \frac{50\pi}{6 \times 25 \times 15} \times 25\pi \times 15$$

or, $\mu = 0.042B$ dyne-sec/m^3 = 0.0429 Poise.

TURBULENT FLOW

1. Find the shear stress at a point in a glycerine mass in motion if the velocity gradient is 0.25 metre per sec/per metre. The density and kinematic viscosity of glycerine are 129.3 slug/metre and 6.30×10^{-4} metre per second respectively.

Solution:

Velocity gradient $= \frac{du}{dy} = 0.25$ metre/sec metre,

Density, $\rho = 129.3$ slug/metre^3

Kinematic Viscosity, $\nu = 6.30 \times 10^4$ metre/sec

$$\text{Shear stress} = \mu \frac{du}{dy} = \rho\, \nu \frac{du}{dy}$$

$$= 129.3 \times 6.30 \times 10^{-4} \times 0.25$$

$$= 0.02036 \text{ kg/m}^2$$

2. Glycerine ($\mu = 1.5$ Pa. sec. and $\rho = 1260$ kg/m^3) flows at a mean velocity of 5 m/sec in a 10 cm diameter pipe. Estimate the power expended by the flow in a distance of 12 m.

Solution:

$$Q = A.V. = \frac{\pi}{4} d^2 \times V$$

$$= \frac{\pi}{4} \times (0.1)^2 \times 5$$

$$= 0.0329 \text{ m}^2/\text{sec}$$

$$h = \frac{f.l.Q^2}{3d^5}$$

$$= \frac{0.02 \times 12 \times (0.0329)^2}{3 \times (0.1)^5} = 8.65 \text{ m}$$

Net Head $= H - h = 12 - 8.65 = 3.35$ m

$\therefore$ Power transmitted through pipe,

$$P = \frac{wQ(H-h)}{75}$$

$$= \frac{1260 \times 0.0329 \times 3.35}{75}$$

$$= 1.85 \text{ H.P.}$$

3. A G.I. pipe 45 cm in diameter ($e = 0.15$) carries 25000 lpm of water ($\mu = 0.001$ Pas) from a reservoir to tank a 12 m. A pressure of 196.2 kPa is to be maintained at the discharge end. What should be the H.P. input to the pump if the length of the pipe is 360 m and pump efficiency is 80% ?

Solution: $$h_f = \frac{128\, \mu\, QL}{\pi\, \gamma\, d^4}$$

$$= \frac{128 \times 0.001 \times 25 \times 360}{\pi \times 9810 \times 0.45^4}$$

$$= 0.911 \text{ m}$$

$$\text{Total head, } H = 12 + 0.911 + \frac{(196.2 \times 10)^3}{9810}$$

$$= 32.911 \text{ m}$$

$$\therefore \text{ Power of the pump} = \frac{\gamma Q H}{\eta}$$

$$= \frac{910 \times 25 \times 32.911}{0.8}$$

= 10089449 W or 10.089 MW

= 13524 HP ...since 1 H.P. = 746 Waston

4. What power is needed per km length of pipe to maintain a flow of 600 liters in a 60 cm diameter pipe (rough). Take height of roughness prostrusions as 0.30 cms.

Solution:

Given, $k = 0.30 \times 10^{-2}$, $Q = 0.6\ m^3/s$

$$\frac{1}{\sqrt{f}} = \log_{10}\frac{r_0}{k} + 1.74$$

$$= 2\log_{10}\frac{0.3}{0.3\times10^{-2}} + 1.74 = 5.74,$$

Solving, we get $f = 0.03035$

$$V = \frac{0.6}{\left(\frac{\pi}{4}\right)\times 0.6^2} = 2.122$$

$$h_f = \frac{fLV^2}{2gd}$$

$$= \frac{0.03035\times1000\times2.122^2}{2\times9.81\times0.6}$$

$$= 11.61 \text{ m of water}$$

$$\text{Power} = \gamma Q h_f = 9810 \times 0.6 \times 11.61$$

$$= 68336.46\ W = 91.6\ H.P.$$

5. A 25 cm diameter pipe carries air ($\rho = 1.22\ kg/m^3$, $\upsilon = 1.5 \times 10^{-5}\ m^2/s$) at an average velocity of 8.0 m/s. The equivalent and grain roughness on the pipe surface is 0.8 mm. The flow is detected to be fully mesh turbulent, (*a*) What is friction factor and boundary shear stress ?

Solution: $R = \frac{Vd}{\nu}$

$$= \frac{8\times0.25}{1.25\times10^{-5}} = 1.33 \times 10^{-3}$$

For Rough pipe and Turbulent flow, we have

$$\frac{V}{u_*} = 5.75\log_{10}\left(\frac{r_0}{k}\right) + 4.75$$

or $$\frac{8}{u_*} = 5.75\log_{10}\left(\frac{0.125}{0.8\times10^{-3}}\right) + 4.75 \text{ or}$$

Solving, we get $u^* = 0.461$

From $$u^* = V\sqrt{\frac{f}{8}}, 0.461$$

$$= 8\sqrt{\frac{f}{8}} \text{ or } f = 0.0265$$

From $$u^* = \sqrt{\frac{\tau_0}{\rho}}, 0.461$$

$$= \sqrt{\frac{\tau_0}{1.22}}$$

Solving, we get $\tau_0 = 0.2589$ or $0.26\ N/m^2$ or 0.26 Pa

6. For turbulent flow in a pipe, determine the distance from pipe wall at which local velocity is equal to average velocity.

Solution: From Velocity defect law,

$$\frac{V-u}{u_*} = 5.75\log_{10}\left(\frac{r_0}{y}\right) - 3.75$$

When $u = V$, L.H.S. = 0,

$$\therefore \quad \log_{10}\left(\frac{r_0}{y}\right) = 0.652$$

or $$\left(\frac{r_0}{y}\right) = 4.489 \text{ say } 4.5$$

$$\therefore \quad y = 0.223\, r_0 \text{ from wall,}$$

or distance from center, $r = 0.776\, r_0$

7. A test for determining the equivalent sand grain roughness of a certain pipe gave the following data:

Diameter of pipe = 300 mm, Discharge = 0.47 m^3/s

Head loss in 10 m = 1.9 meters, Kinematic viscosity = 1.0 centistoke

Determine the equivalent sand grain roughness and also the maximum roughness in order that the pipe may act as hydrodynamically smooth pipe at the given discharge.

Solution: $$V = \frac{Q}{A} = \frac{0.47}{\frac{\pi}{4}\times0.3^2} = 6.65 \text{ m/s}$$

$$R = \frac{Vd}{\nu} = \frac{6.65\times0.3}{10^{-6}} = 1.995\times10^{-6}$$

$$f = \frac{2gd\,h_f}{V^2L}$$

$$= \frac{2\times9.81\times0.3\times1.9}{6.65^2\times10} = 0.0253$$

and $$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{r_0}{k}\right) + 1.74$$

or $$\frac{1}{\sqrt{(0.0253)}} = 2\log\left(\frac{0.15}{k}\right) + 1.74$$

$$\therefore \quad k = 7.986\times10^{-4} = 8\times10^{-4}\text{ m } 0.8\text{ mm}$$

Now $$\frac{u_*}{V} = \sqrt{\frac{f}{8}}$$

$$\Rightarrow \quad u_* = 6.65\sqrt{\frac{0.0253}{8}} = 0.374 \text{ m/s}$$

From $$\frac{u_*\delta}{\upsilon} = 11.6, \text{ thickness of laminar sublayer,}$$

$$\delta' = \frac{11.6\nu}{u_*}$$

$$= \frac{11.6\times10^{-6}}{0.374} = 3.10\times10^{-5}$$

Now, $\frac{k}{\delta'} = \frac{8 \times 10^{-4}}{3.10 \times 10^{-5}}$

$= 25.79 < 0.25$, hence present pipe is rough

Now for smooth pipe $k = \frac{\delta'}{4}$

$$= \frac{3.10 \times 10^{-5}}{4}$$

$$= 7.84 \times 10^{-6} \text{ m} \approx 8 \text{ } \mu\text{m}$$

8. The wind velocity was measured at two points 2m and 4m above ground. The values obtained being 2 m/s and 2.3 m/s respectively. Compute shear velocity u^*. Assume Karman's constant K = 0.4, what is probable laminar sublayer thickness? What is the velocity at 8m above the ground? Assume the boundary as smooth. Take $\upsilon = 0.145$ stoke

Solution: For turbulent flow (for $k = 0.4$),

$$\frac{u}{u_*} = 25 \log_e \left(\frac{y}{y'}\right)$$

But at $y = 2$ m, $u = 2$ m/s and at $y = 4$, $u = 2.3$ m/s

$$\therefore \quad 2 = u^* \left(2.5 \log_e \left(\frac{2}{y'}\right)\right) \quad ...(i)$$

and $$2.3 = u^* \left(2.5 \log_e \left(\frac{4}{y'}\right)\right) \quad ...(ii)$$

Solving equations (*i*) and (*ii*) we get, $u^* = 0.173$ m/s, and $y' = 0.0196$ m.

$\frac{u_* y}{\upsilon}$ for the two points of measurement are respectively 23,862 and 47,724 and the corresponding value of $\frac{u_* y'}{\upsilon}$ is 233.

We know, for $\frac{u_* y}{\upsilon} < 5$, velocity distribution is given by

$$\frac{u}{u_*} = \frac{u_* y}{\nu}$$

and for $\frac{u_* y}{\upsilon} > 70$, velocity distribution is given by

$$\frac{u}{u_*} = 5.75 \log_{10} \frac{u_* y}{\upsilon} + 55$$

Laminar sublayer thickness,

$$\delta' = \frac{5\nu}{u_*}$$

$$= \frac{5 \times 0.145 \times 10^{-4}}{0.173} = 4.19 \times 10^{-4} \text{ m.}$$

For $y = 8$ m, $u = u^* \left(2.5 \log_e \left(\frac{8}{y'}\right)\right)$

$$= 0.173 \times 25 \log \frac{8}{0.0196}$$

$$= 2.6 \text{ m/s}$$

9. For a cast iron pipe 500 mm in diameter with a maximum velocity of flow of 4 m/s has a friction factor of 0.02, Determine mean velocity, discharge and velocity at half the radius.

Solution: For rough and smooth pipes,

$$\frac{u - V}{u_*} = 5.75 \log_{10} \left(\frac{y}{r_0}\right) + 3.75$$

Substituting U_{max} and corresponding distance of center, we get

$$\frac{4 - V}{V\sqrt{\frac{f}{8}}} = 5.75 \log_{10} \left(\frac{250}{250}\right) + 3.75$$

or $$\frac{4 - V}{V\sqrt{\frac{0.02}{8}}} = 3.75$$

Solving, we get $V = 3.368$ m/s

$$Q = \frac{\pi}{4} d^2 V$$

$$= \frac{\pi}{4} \times 0.5^2 \times 3.368$$

$$= 0.66 \text{ m}^3/\text{s}$$

Hence, velocity at half the radius i.e., at $y = 125$ mm can be determined as follows :

$$\frac{u - V}{V\sqrt{\frac{f}{8}}} = 5.75 \log_{10} \left(\frac{y}{r_0}\right) + 3.75,$$

or $$\frac{u - 3.368}{3.368\sqrt{\frac{0.02}{8}}} = 5.75 \log_{10} \left(\frac{125}{250}\right) + 3.75$$

Solving, we get $u = 3.71$ m/s

10. In the case of a 300 mm diameter rough pipe carrying water the velocity at 20 mm from the wall is 2.67 m/s and the velocity gradient at same point is 25.8/s. Determine the average height of roughness protrusions. Also determine the average shear at the wall, the friction factor and the mean velocity of flow.

Solution:

Given, $d = 0.3$m,

$$u_{0.02} = 2.67 \text{ m/s}, \left(\frac{du}{dy}\right)$$

$$= 25.8/\text{sec}$$

For rough pipe, $\dfrac{u}{u^*} = 5.75 \log_{10} \dfrac{y}{k} + 8.5$

$$= 2.5 \log_e \frac{30y}{k}$$

$$\frac{1}{u^*}\frac{du}{dy} = \frac{2.5}{y}$$

$\Rightarrow$ $25.8 = 125\, u^*$

and $\left(\dfrac{du}{dy}\right)_{0.02} = \dfrac{2.5}{0.02} \times u^*$

$\Rightarrow$ $u^* = 0.206$ m/s

$\therefore$ $125\, u^* = 25.8$/sec

Now, $\dfrac{u}{u_*} = 5.75 \log_{10} \dfrac{y}{k} + 8.5$

or $\dfrac{2.67}{0.2064} = 5.75 \log_{10} \dfrac{0.02}{k} + 8.5$

Solving, we get $k = 3.3849 \times 10^{-3}$ m or 3.3849 mm.

'V' can found out from velocity defect law as follows:

$$\frac{u - V}{u_*} = 5.75 \log_{10}\left(\frac{v}{r_0}\right) + 3.75$$

or $\dfrac{2.67 - V}{0.2064} = 5.75 \log_{10}\left(\dfrac{0.02}{0.15}\right) + 3.75$

Solving, we get $V = 2.9345$

and $u^* = 0.2064 = V\sqrt{\dfrac{f}{8}} = 2.9345\sqrt{\dfrac{f}{8}}$

Solving, we get $f = 0.0396$

Now $\tau_0 = \rho\, u^{*2} = 1000 \times (0.2064)^2 = 42.6$ N/m^2

11. After 10 years of service an asphalted cast iron 450 mm in diameter is found to require 30% more power to deliver 240 l/s for which it was originally designed. Determine the corresponding magnitude of rate of roughness increase for water is 0.014 stokes ($k_0 = 0.12$ mm)

Solution: For the same rate of flow, the power 'P' is increased by 30% and $P \propto h_f$ and $h_f \propto f$, hence friction factor, is also increased by 30%.

Thus if f_{10} is the value of f after 10 years, we have

$f_{10} = 1.3 f$,

k_6 = value of k for a new asphalted iron pipe = 0.12 mm

At the time of installation, relative roughness

$$= \frac{k_0}{D}$$

$$= \frac{0.12 \text{ mm}}{450} = 0.000267$$

$$Q = 0.24 \text{ m}^3/\text{s}$$

$\therefore$ $V = \dfrac{Q}{A} = \dfrac{0.24}{\frac{\pi}{4} \times (0.45)^2} = 1.51$ m/s,

$$R = \frac{VD}{\nu} = \frac{1.51 \times 0.45}{0.014 \times 10^{-4}}$$

$$= 4.85 \times 10^5$$

Also $\dfrac{1}{\sqrt{f}} = 2 \log_{10}\left(\dfrac{r_0}{k}\right) + 1.74 = 0.0146$

$\therefore$ $f_{10} = 1.3 f = 1.3 \times 0.0146 = 0.019$

and from $\dfrac{1}{\sqrt{f_{10}}} = 2 \log_{10}\left(\dfrac{r_0}{k_{10}}\right) + 1.74$,

we get $k_{10} = 0.39$ mm

$\therefore$ Relative roughness $= \left(\dfrac{k}{D}\right)_{10}$

$$= \frac{0.39}{450} = 0.000867$$

$\therefore$ Rate of increase of roughness height

$$= \frac{k_{10} - k_0}{t}$$

$$= \frac{0.39 - 0.12}{10} = 0.027 \text{ mm/yr}$$

12. The pipe wall roughness height k for a new cast iron pipe is 0.26 mm. After three years of service, it becomes 0.41 mam. What will be its value after 18 years ?

Solution:

From $k = k_0 + \alpha t$

$0.41 = 0.26 + \alpha \times 0.3$,

or $\alpha = 0.05$ mm/year

After 18 years, $k = k_0 + \alpha t$

$= 0.26 + 18 \times 0.05$

$= 1.16$ mm

BOUNDARY LAYER THEORY

1. Determine the Displacement, Momentum and Energy thickness of the boundary layer for the following velocity distribution:

$$\frac{u}{U} = 2\frac{y}{\delta} - \frac{y^2}{\delta^2} \text{ (usual notations)}$$

Solution:

Put $\dfrac{y^2}{\delta^2} = \eta$, we get $dy = \delta\, d\eta$ ...(*i*)

When $y = 0$, $\eta = 0$ and when $y = \delta$, $\eta = 1$

Displacement thickness, $\delta^* = \displaystyle\int_0^\delta \left(1 - \frac{u}{U}\right) dy$ ...(*ii*)

Now, $\frac{u}{U} = 2\eta - \eta^2$ ...(*iii*)

From equations (*i*), (*ii*) and (*iii*),

$$\delta^* = \int_0^1 (1 - 2\eta + \eta^2)\delta\eta$$

$$= \left(\eta - \frac{2\eta^2}{2} + \frac{\eta^2}{3}\right)_0^1$$

$$\therefore \quad \delta^* = \frac{\delta}{3}$$

Momentum thickness,

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy$$

$$= \int_0^1 \delta \times (1 - 2\eta + \eta^2)(2\eta - \eta^2)\delta\eta$$

$$= \int_0^1 \delta \times (2\eta + 5\eta^2 + 4\eta^3 - \eta^4)\delta\eta$$

$$= \frac{2}{15}\delta$$

Energy thickness,

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right)dy$$

$$= \delta\int_0^1 (2\eta - \eta^2)\left[1 - \left(2\eta - \eta^2\right)\right]\delta\eta$$

$$= \delta\int_0^1 \left[2\eta - \eta^2 - \left(2\eta - \eta^2\right)\right]\delta\eta$$

$$= \delta\int_0^1 \left[2\eta - \eta^2 - \left(8\eta^3 - 12\eta^4 + 6\eta^5 - \eta^6\right)\right]\delta\eta$$

$$= \delta\int_0^1 \left[2\eta - \eta^2 - 8\eta^3 - 12\eta^4 + 6\eta^5 - \eta^7\right]\delta\eta$$

$$= \delta\left(\frac{2}{2} - \frac{1}{3} - \frac{8}{4} + \frac{12}{5} - \frac{6}{6} + \frac{1}{7}\right)$$

$$= \frac{22}{105}\delta$$

2. Calculate the loss of momentum due to formation of boundary layer. Given

Free stream velocity = 10 m/s, Boundary layer thickness = 25 mm

Mass density of fluid = 1.25 kg/m^3, $\frac{\theta}{\delta} = 0.097$.

Solution:

$$\theta = 25 \times 10^{-3} \times 0.097$$

$$= 2.425 \times 10^{-3} \text{ m}$$

Loss of Momentum per meter width of plate

$$= \rho U^2 \theta$$

$$= 1.25 \times 10^2 \times 2.425 \times 10^{-3}$$

$$= 0.303 \text{ N/m}$$

3. Show that in case of laminar boundary layer in a pipe the percentage loss of mass, momentum and energy per second are 50,16.66 and 25% respectively.

Solution: For laminar flow velocity distribution

$$\frac{u}{U} = 1 - \frac{r^2}{r_0^2}$$

$\therefore$ Energy thickness $= 0.134\, r_o = \delta^{**}$

Momentum thickness, $0 = 0.087\, r_o$

and displacement thickness $\delta^* = 0.293\, r_o$

(*i*) Loss of Mass rate of flow

$$= \rho U [\pi r_o^2 - (r_o - 0.293\, r_o)^2]$$

$$= \rho U (0.5\, \pi r_o^2) = 0.5\, \rho U \pi r_o^2$$

Free stream discharge $= \rho U \pi r_o^2$

$\therefore$ **Percentage loss = 50%**

(*ii*) Loss of Momentum rate

$$= \rho U^2 [\pi r_o^2 - \pi(r_o - 0.087\, r_o^2)]$$

$$= \rho U^2 \pi r_o^2 \times 0.1666$$

$$= 0.1666\, \rho U^2 \pi r_o^2$$

$\therefore$ **Percentage loss = 16.66%**

(*iii*) Energy loss $= \frac{1}{2}\rho U^3 [\pi r_o^2 - (r_o - 0.134\, r_o)^2]$

$$= \frac{1}{2} 0.25\, \rho U^3 \pi r_o^3$$

$\therefore$ **Percentage loss = 25%**

4. Smooth flat plate 1 m wide and 3 m long moves through stationary air of specific weight 0.0115 kN/m^3 at 1 m/s. Calculate the drag force on one side of the plate when

(*i*) the boundary layer is entirely laminar

(*ii*) when the boundary layer is entirely turbulent.

What is the thickness of the boundary layer at the trailing edge for both cases ?

Take $\upsilon = 1.5 \times 10^{-5}$ m^2/s.

Solution:

$$R_e = \frac{U_0 L}{\nu} = \frac{1 \times 3}{1.5 \times 10^{-5}} = 2 \times 10^5$$

(*i*) When the boundary layer is entirely laminar and assuming it follows Balsius results,

$$F_D = \frac{1.328}{\sqrt{R_e}}$$

$$= \frac{\rho BLU_0^2}{2}$$

$$= \frac{1.382}{\sqrt{2 \times 10^5}} \times \frac{0.0115}{0.81} \times \frac{1 \times 3 \times (1)^2}{2}$$

$$= 0.52 \times 10^{-5} \text{ kN}$$

(*ii*) For turbulent boundary layer, assuming the velocity distribution to be one seventh power law,

$$F_D = \frac{0.072}{(R_e)^{\frac{1}{5}}}$$

$$= \frac{\rho BLU_0^2}{2}$$

$$= \frac{0.072}{\left(2\times10^5\right)^{\frac{1}{5}}}\times\frac{0.0115}{9.81}\times\frac{1\times3\times(1)^2}{2}$$

$$= 1.07\times10^{-5}\ \text{kN}$$

(*iii*) Thicksess of laminar boundary layer for the velocity distribution used above is given

$$\frac{\delta}{x} = \frac{5}{\sqrt{R_e}}$$

$$\therefore \quad \delta = \frac{5x}{\sqrt{R_e}} = \frac{5\times3}{\sqrt{2\times10^5}} = 0.0335\ \text{m}$$

Thickness of turbulent boundary layer is given by,

$$\frac{\delta}{x} = \frac{0.37}{(R_e)^{\frac{1}{5}}}$$

$$\therefore \quad \delta = \frac{0.37\times3}{(2\times10^5)^{\frac{1}{5}}} = 0.0966\ \text{m}$$

It is evident that the thickness of turbulent boundary layer should be more as compared to laminar boundary layer.

5. A roughened thin board 30 cm wide, 240 cm long moves at 3 mis through water. Each boundary layer is 7.5 cm thick at the reatrend of the board and the velocity distribution is given by $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{4}}$. Find the resistance and express in Newtons and as a pure number independent of δ.

Solution: From $\frac{1}{n}$th power law,

$$\frac{\theta}{\delta} = \frac{n}{(n+1)(2n+1)}$$

$$= \frac{\frac{1}{4}}{\left(\frac{1}{4}+1\right)\left(2\times\frac{1}{4}+1\right)} = \frac{2}{15}$$

$$\therefore \quad \theta = \frac{2}{15}\delta = \frac{2}{15}\times7.5 = 1\ \text{cm}$$

Frictional resistances $= 2\times\rho(b\times\theta)\,U^2$

$$= 2\times1000\times(0.30\times0.01)\times9$$

$$= 54\ \text{N}$$

From $\quad F = 2\left(\frac{1}{2}C_f\rho\, A\, U^2\right)$

$$C_f = \frac{F}{2\times\left(\frac{1}{2}\rho U^2 A\right)}$$

$$= \frac{54}{1000\times3^2\times(2.4\times0.3)}$$

$$= 8.33\times10^{-3}$$

6. Water is flowing over a thin smooth plate of length 5 m and width 2 m at a velocity if 1.0 m/s. If the boundary layer flow changes from laminar to turbulent at a Reynolds number of 5×10^5. Find the distance from leading edge upto which boundary layer is laminar. Assume $\mu = 0.01$ poise.

Solution: Let the boundary layer remains laminar for distance x from the leading edge.

From $\quad R_{ex} = \frac{\rho Ux}{\mu}$

$$5\times10^5 = \frac{1000\times1\times x}{(0.01\times0.1)}$$

Solving, we get $\quad x = 0.5$ m

7. A flat plate of 2.0m width and 4.0m length is kept parallel to air flowing at 5.0 m/s velocity at 15°C the length of plate over which the boundary layer is laminar, shear at the location which boundary layer ceases to be laminar, and total force on both sides on that portion of plate where the boundary layer is laminar. Take $r = 1.208$ kg/m³ and $\upsilon = 1.47\times10^{-5}$ m²/s.

Solution: $R_e = \frac{UL}{\nu} = \frac{5\times4}{1.47\times10^5} = 1.361\times10^6$

Hence on the front portion, boundary layer is laminar and on the rear, it is turbulent.

From $\quad \frac{Ux}{\nu} = 5\times10^5,$

$$x = \frac{5\times10^5\times1.47\times10^{-5}}{5} = 1.47\ \text{m}$$

Hence boundary layer is laminar on 1.47 m length of the plate.

$$\delta = \frac{5\times1.47}{\sqrt{5\times10^5}}$$

$$= 0.01039\ \text{m or } 1.093\ \text{cm},$$

$$C_f = \frac{0.664}{\sqrt{5\times10^5}} = 0.000939$$

$$\therefore \quad \tau_0 = 0.000939\times\frac{1.208\times5^2}{2}$$

$$= 0.01418\ \text{N/m}^2,$$

$$C_f = \frac{1.328}{\sqrt{5\times10^5}} = 0.01878$$

$\therefore$ Force on 1.47 m length (on both sides)

$$F = 2\times(2\times1.47)\,C_f\frac{\rho U^2}{2}$$

$$= 2\times(2\times1.47)\times0.001878\times1.208\times\frac{5^2}{2}$$

$$= 0.1667\text{ N}$$

8. Find the ratio of friction drag on the front half and rear half of the flat plate kept at zero incidence in a stream of uniform velocity, if the boundary layer is laminar over the whole plate,

Solution: Reynolds number for whole plate

$$= \frac{UL}{\nu}$$

and for front half of the plate Reynolds number,

$$R_e = \frac{UL}{2\nu}$$

C_f for total plate $= 1.328/\left(\frac{UL}{\nu}\right)^{\frac{1}{2}}$,

$$\therefore\text{ for front half} = \frac{1.328}{\left(\frac{UL}{2\nu}\right)^{\frac{1}{2}}} = \frac{0.664}{\left(\frac{UL}{\nu}\right)^{\frac{1}{2}}}$$

$$\text{Force on front half } F_1 = \frac{BL}{2}\times\frac{\rho U^2}{2}\times\frac{1.878}{\left(\frac{UL}{\nu}\right)^{\frac{1}{2}}}$$

$$= 0.4695\frac{BL\rho U^2}{\left(\frac{UL}{\nu}\right)^{\frac{1}{2}}}$$

$\therefore$ Force on rear half, $F_2 = F - F_1$

$$= (0.6640 - 0.4695)\frac{BL\rho U^2}{\left(\frac{UL}{\nu}\right)^{\frac{1}{2}}}$$

$$= 0.1945\frac{BL\rho U^2}{\left(\frac{UL}{\nu}\right)^{\frac{1}{2}}}$$

$$\therefore\quad \frac{F_1}{F_2} = \frac{0.4695}{0.1945} = 2.414.$$

OPEN CHANNEL FLOW

1. A rectangular channel is to carry 1.3 m^3/s at a slope of 0.009. If the channel is lined with galvanized iron, $n = 0.0011$, what is the minimum of square meters of , metal needed for each 100 m of channel ? Neglect free board.

Solution: For most economic section of a rectangular channel,

$$b = 2y$$

Thus for most economical channel,

$$A = 2y^2,\ P = 4y,\ R = \frac{y}{2}$$

and

$$Q = \frac{1}{n}AR^{2/3}S^{1/2},$$

or

$$1.13 = \frac{1}{0.011}2y^2\left(\frac{(y}{2)}\right)^{\frac{2}{3}}(0.009)^{1/2}$$

Solving, we get $y = 0.4279$ m

Area of wetted channel surface of 100 m length

$$= P \times L$$

$$= (0.4279 \times 4) \times 100$$

$$= 171.17\text{ m}^2$$

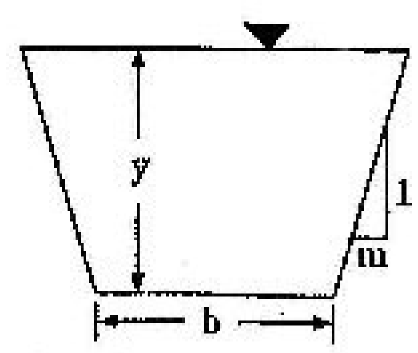
2. A trapezoidal channel made out of brick with bottom width 2 m, bottom slope 0.001 is to carry 17 m^3/s. Calculate the dimensions of the channel such that least number of bricks are required, $n = 0.0125$.

Solution: When width is kept constant, the most economical trapezoidal channel section requires

$$m = \frac{\sqrt{3}}{3} = 0.557$$

$$P = b + 2y\sqrt{1+m^2}$$

$$= 2 + 2y\sqrt{1+0.577^2} = 2 + 2.309y$$



$$A = by + my^2 = 2y + 0.577y^2$$

$$R = \frac{A}{P} = 2y + \frac{0.577y^{\frac{2}{2}}}{2+2.309y}$$

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$

or

$$17 = \frac{1}{0.0125}\times(2y + 0.577y^2)\times\left(\frac{2y+0.577y^2}{2+2.309y}\right)\times(0.001)^{1/2}$$

or,

$$6.72 = \frac{(2y+0.577y^2)^{\frac{5}{3}}}{(2+2.309y)^{\frac{2}{3}}}$$

By trial, we get $y = 2.11$ and therefore $m = \frac{\sqrt{3}}{3}$

3. Calculate the radius required by a semicircular corrugated metal channel 1 km long to convey 2.5 m³/s with a head loss of 1.326 m n = 0.022. Also find another cross-section that requires less perimeter.

Solution: $S = \dfrac{1.326}{1000} = 1.326 \times 10^{-3}$

$$Q = \frac{1}{n} AR^{2/3}S^{1/2}$$

or $$2.5 = \frac{1}{0.022} \times \frac{\pi r^2}{2} \times \left(\frac{r}{2}\right)^{\frac{2}{3}} \times (1.326 \times 10^{-3})^{1/2}$$

Solving, we get $r = 1.172$m.

Area of this section, $A = \dfrac{\pi r^2}{2} = 2.157\text{m}^2$,

and $P = 3.682$,

$$R = \frac{A}{P} = 0.586 \text{ m}.$$

Another circular section requiring less perimeter is governed by theory of most economical sections.

For this, $A = \dfrac{r^2}{2}(\theta - \sin\theta)$ and $P = r\theta$

where θ for most economical section based on Manning's formula

$$= 302° \, 20' = 5.2767 \text{ radians}$$

$$A = \frac{r^2}{2}(5.2767 - \sin 302° \, 20')$$

$$= 3.0608\, r^2, \; P = 5.2767\, r$$

$$R = \frac{A}{P} = \frac{3.0608 r^2}{5.2767 r} = 0.58 r$$

and $$Q = 2.5 = \frac{1}{0.022} \times (3.0608 r^2)(0.58 r)^{2/3}(1.326 \times 10^{-3})^{1/2}$$

Solving, we get $r = 0.88$ m

$\therefore$ $A = 2.37 \text{ m}^2$,

$P = 4.614$ m and $R = 0.51$

The channel has to be an arc of a circle of radius 0.88 m, and subtending an angle of 302°20′ at the centre such that depth of flow in the channel is,

$$y = 1.876 r$$

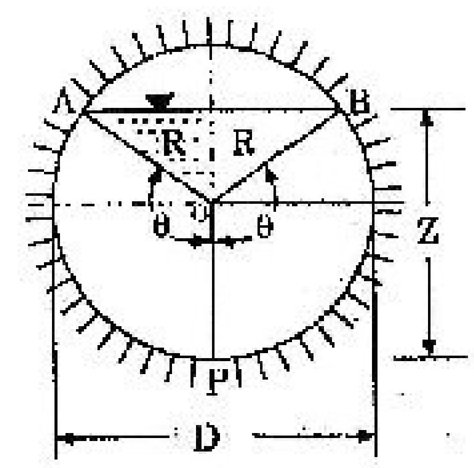
$$= 1.876 \times 0.88 = 1.65 \text{ m}$$

4. Determine the expression for the most economical depth of flow of water in terms of the diameter of a channel of circular cross section

(*a*) for maximum velocity and

(*b*) for maximum discharge.

Solution:



Area of flow,

A = area of sector AOBA +area of triangle AOBN

$$= \frac{1}{2} R^2 . 2\theta + \frac{1}{2} . 2R \sin(180 - \theta) R \cos(180 - \theta)$$

$$= R^2\theta + R^2 \sin\theta \cos\theta$$

$$= R^2 (\theta + \frac{1}{2} \sin 2\theta)$$

Wetted perimeter, $P = 2R\theta$

Hydraulic mean depth, $m = \dfrac{A}{P}$

$$= \frac{R\left(\theta + \frac{1}{2}\sin 2\theta\right)}{2R\theta}$$

(*a*) For maximum velocity, hydraulic mean depth should be maximum, i.e.,

$$\frac{dm}{d\theta} = 0$$

or, $$-\frac{1}{4\theta}\left[2\theta\left(R + \frac{R}{2} . 2\cos 2\theta\right) - R\left(\theta + \frac{1}{2}\sin 2\theta\right)2\right] = 0$$

or, $\theta \cos 2\theta - \dfrac{1}{2} \sin 26 = 0$,

or, $\tan 2\theta = 2\theta$,

$\therefore$ $\theta = 128.75°$

Corresponding depth of flow,

$$z = R + R\cos(180 - \theta)$$

$$= R + R\cos(180 - 128.75)$$

$$= R(1 + \cos 51.25°)$$

$$= 1.62\, R = 0.81\, D$$

(*b*) For maximum discharge,

$Q = AC\sqrt{mi}$ should be maximum

Since $m = \dfrac{A}{P}$, therefore discharge will be the maximum when $\dfrac{A^3}{P}$ is the maximum. Then

$$\frac{d\left(\frac{A^3}{P}\right)}{d\theta} = \frac{1}{P^2}\left(3PA^2\frac{dA}{d\theta} - A^3\frac{dP}{d\theta}\right)$$

$$= 0$$

or $3PA\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$

Substituting in the expression for P,

$$P = 2R\theta$$

or, $\frac{dA}{d\theta} = R^2(1+\cos 2\theta);$

and $\frac{dP}{d\theta} = 2R$

The condition becomes, $6R^3\theta(1-\cos 2\theta) - 2R^3$

$$\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

or $4\theta - 6\theta\cos 2\theta + \sin 2\theta = 0$

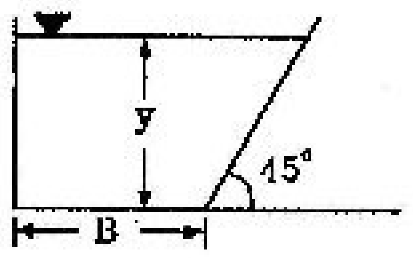
which gives $\theta = 154°$

The corresponding depth of flow,

$$Z = R[1+\cos(180-154)°]$$
$$= R(1+0.90)$$
$$= 1.90\,R = 0.95\,D$$

5. A certain stretch of a lined channel has one vertical side and the other 45° sloping wall. If it is to deliver water at 30 m³/s with a velocity of 1 m/s, compute width and flow depth for minimum lining area.

Solution: Refer adjoining diagram,



$$A = y(B + 0.5y) \quad ...(i)$$

and $P = B + 2.4142y \quad ...(ii)$

But $A = \frac{Q}{V} = \frac{30}{1} = 30\text{ m}^2$

From equation (*i*),

$$B = \frac{30}{y} - 0.5y$$

and from equation (*ii*),

$$P = \frac{30}{y} + 1.9142y$$

For minimum lining area,

$$\frac{dP}{dy} = 0,$$

or $(-)\frac{30}{y^2} + 1.9142 = 0$

Solving, we get $y = 3.96$ m and $B = 5.6$ m

6. Determine the best hydraulic trapezoidal section to convey 85 m³/s with a bottom slope of 0.001. The lining is finished concrete; $n = 0.012$.

Solution: $P = 2\sqrt{3}\,y$, $A = \sqrt{3}$ and $b = 2\frac{\sqrt{3}}{3}y$

$\therefore$ $R = \frac{A}{P} = \frac{y}{2}$

From $Q = \frac{1}{n}AR^{2/3}S^{1/2}$,

$$85 = \frac{1}{0.012} \times \sqrt{3}y^2 \times \left(\frac{y}{2}\right)^{\frac{2}{3}}(0.001)^{\frac{1}{2}}$$

Solving, we get $y = 3.56$ m and $b = 4.11$ m

7. An irrigation channel of trapezoidal section having side slope of 3H to 2V, is required to carry a flow of 10 m³/s on a longitudinal slope of 1 in 5000. The channel is to be lined for which manning's 'n' = 0.012. Find dimensions of most economic section.

Solution:

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$
$$= \frac{1}{n}AR^{5/3}P^{-2/3}S^{1/2}$$

or $10 = \frac{1}{0.012}A^{5/3}P^{-2/3}\left(\frac{1}{5000}\right)^{\frac{1}{2}}$

or $10 = 1.18 \times A^{5/3}P^{-2/3} \quad ...(i)$

For most economical section,

each of the side slopes = Half the top width

or $y\sqrt{1+m^2} = \frac{b+2my}{2}$

or $2y\sqrt{1+m^2} = b + 3y$

or $2y\sqrt{1+1.5^2} = b + 3y$

or $b = 0.6055y$

and sum of side slope, $T = b + 3y$

$$P = 2b + 3y$$
$$= 2 \times 0.6055y + 3y$$
$$= 4.2ly \quad ...(ii)$$

and $A = y(b + my)$

$$= y(0.6055y + 1.5y)$$
$$= 2.104y^2 \quad ...(iii)$$

From equation (*i*), (*ii*) and (*iii*), we get

$$1.18 \times (2.10y^2)^{5/3} = 10$$

Solving, we get $y = 2.00$ meters

and $b = 0.6055 \times 2 = 1.21$ m.

EXERCISE - I

1. The line of action of force exerted by a liquid on a plane area passes through the
(*a*) centre of pressure (*b*) buoyancy centre
(*c*) centre of gravity (*d*) centre of inertia

2. A fluid is a substance that
(*a*) always expands until it fills any container
(*b*) has the same shear stress at a point regardless of its motion
(*c*) cannot remain at rest under action of any shear force
(*d*) cannot be subjected to shear forces

3. Centre of buoyancy is
(*a*) the point of intersection of buoyant force and centre line of the body
(*b*) centre of gravity of the body
(*c*) centroid of displaced volume fluid
(*d*) mid point between C.G. and metacentre.

4. Length of mercury column at a place at an altitude will vary with respect to that at ground in a
(*a*) linear relation
(*b*) hyperbolic relation
(*c*) parabolic relation
(*d*) manner first slowly and then steeply

5. In isentropic flow, the temperature
(*a*) cannot exceed the reservoir temperature
(*b*) cannot drop and again increase downstream
(*c*) is independent of Match number
(*d*) is a function of Match number only

6. A stream line is
(*a*) the line of equal velocity in a flow
(*b*) the line along which the rate of pressure drop is uniform
(*c*) the line along the geometrical centre of the flow
(*d*) fixed in space in steady flow.

7. A rotameter is a device used to measure
(*a*) velocity of fluid in pipes
(*b*) velocity of gauges
(*c*) votex flow
(*d*) flow of fluids

8. An ideal fluid
(*a*) is very viscous
(*b*) obeys Newton's law of viscous
(*c*) is assumed in conduit flow
(*d*) frictionless and incompressible

9. Reynolds number for pipe flow is given by
(*a*) $\frac{vD}{\nu}$ (*b*) $\frac{vD\mu}{\delta}$
(*c*) $\frac{vD\rho}{\mu}$ (*d*) $\frac{vD}{\mu}$

10. With rise in gas temperature, dynamic viscosity of most of the gases
(*a*) increases
(*b*) decreases
(*c*) does not change significantly.
(*d*) none of the above

11. The flow of water in a pipe of diameter 3000 mm can be measured by
(*a*) Venturimeter (*d*) Rotameter
(*c*) Pilot tube (*d*) Orifice plate.

12. The continuity equation
(*a*) expresses relationship between hydraulic parameters of flow
(*b*) expresses the relationship between work and energy
(*c*) is based on Bernouli's theorem
(*d*) relates the mass rate of flow along a stream line.

13. Weber number is the ratio of
(*a*) inertial forces to surface tension
(*b*) inertial forces to viscous forces
(*c*) elastic forces to pressure forces
(*d*) viscous forces to gravity

14. One dimensional flow
(*a*) restricted to flow in a straight line
(*b*) neglects changes in a transverse direction
(*c*) steady uniform flow
(*d*) a uniform flow

15. Steady flow occurs when
(*a*) pressure does not change along the flow
(*b*) velocity does not change
(*c*) conditions change gradually with time
(*d*) conditions do not change with time at any point.

16. A flow in which each liquid particle has a definite path and their paths do not cross each other, is called
(*a*) Steady flow
(*b*) Uniform flow
(*c*) Streamline flow
(*d*) Turbulent flow

17. Buoyant force is

(*a*) resultant of upthrust and gravity forces acting on the body

(*b*) resultant force on the body due to the fluid surrounding it

(*c*) resultant of static weight of body and dynamic thrust of fluid

(*d*) equal to the volume of liquid displaced by the body

18. In a rectangular notch, the ratio of per-centage error in $\dfrac{\text{discharge}}{\text{measurement of head}}$ is

(*a*) 1 (*b*) $\frac{1}{2}$

(*c*) $\frac{3}{4}$ (*d*) $\frac{3}{2}$

19. Cavitation is caused by

(*a*) high velocity

(*b*) low barometric pressure

(*c*) high pressure

(*d*) low pressure

20. For a pipe not running full, the hydraulic mean depth, m is given by

(*a*) $\dfrac{\frac{r^2}{2}(\theta - \sin\theta)}{r\theta}$ (*b*) $\dfrac{(\theta - \sin\theta)}{r\theta}$

(*c*) $\dfrac{r(\theta - \sin\theta)^2}{\theta^2}$ (*d*) $\dfrac{r(\tan\theta - \theta)}{\theta^2}$

21. The general energy equation is applicable to

(*a*) Steady flow (*b*) Unsteady flow

(*c*) Non-uniform flow (*d*) Turbulent flow

22. In a turbulent flow in a pipe

(*a*) Reynolds number is greater than 10000

(*b*) fluid particles move in straight lines

(*c*) head loss varies linearly with flow rate

(*d*) shear stress varies linearly with radius

23. The friction resistance in pipe is proportional to V^2, according to

(*a*) Froude-number (*b*) Reynolds-Weber

(*c*) Darcy-Reynolds (*d*) Weber-Froude

24. In laminar flow, maximum velocity at the centre of pipe is how many times to the average velocity

(*a*) Two (*b*) Three

(*c*) Four (*d*) none of these

25. Pilot tube is used to measure the velocity head of

(*a*) still fluid (*b*) laminar flow

(*c*) turbulent flow (*d*) flowing fluid

26. In equilibrium condition, fluids are not able to sustain

(*a*) shear force (*b*) resistance to viscosity

(*c*) surface tension (*d*) geometric similitude

27. When Reynold's number is greater than 4200, flow in a pipe will be

(*a*) laminar (*b*) ideal

(*c*) turbulent (*d*) sonic

28. Flow occurring in a pipeline when a valve is being opened is

(*a*) steady (*d*) unsteady

(*c*) laminar (*d*) vortex

29. Coefficient of discharge in comparison to coefficient of velocity is

(*a*) more (*b*) less

(*c*) same (*d*) not necessary

30. A large Reynold number is indication of

(*a*) smooth and streamline flow

(*b*) laminar flow

(*c*) steady flow

(*d*) highly turbulent flow

31. In steady flow of a fluid, the acceleration of any fluid particle is

(*a*) constant (*b*) variable

(*c*) zero (*d*) never zero

32. For measuring flow by a venturimeter, it should be installed in

(*a*) vertical line

(*b*) horizontal line

(*c*) inclined line with upward flow

(*d*) in any direction and in any location

33. The fluid forces considered in the Navier Stokes equation are

(*a*) gravity, pressure and viscous

(*b*) gravity, pressure and turbulent

(*c*) pressure, viscous and turbulent

(*d*) gravity, viscous and turbulent

34. The flow in venturiflume takes place at

(*a*) atmospheric pressure

(*b*) vacuum

(*c*) high pressure

(*d*) any pressure

35. The depth of centre of pressure in rectangular lamina of height h with one side in the liquid surface is at

(*a*) h (*b*) $\frac{h}{3}$

(*c*) $\frac{2h}{3}$ (*d*) $\frac{h}{2}$