Elementary Particles (Part - 1)

Q.291. Calculate the kinetic energies of protons whose momenta are 0.10, 1.0, and 10 GeVic, where c is the velocity of light.

Ans. The formula is

$$T = \sqrt{c^2 p^2 + m_0^2 c^4} - m_0 c^2$$

Thus T = 5.3 MeV for $p = 0{\cdot}10 \frac{GeV}{c} = 5{\cdot}3 \times 10^{-3}\,\text{GeV}$

T = 0.433 GeV for $p = 1{\cdot}0 \frac{GeV}{c}$

T = 9.106 GeV for $p = 10 \frac{GeV}{c}$

Here we have used $m_o c^2$ = 0.938 GeV

Q.292. Find the mean path travelled by pions whose kinetic energy exceeds their rest energy η = 1.2 times. The mean lifetime of very slow pions is ζ_o = 25.5 ns.
Ans. Energy of pions is $(1 + \eta)\, m_o c^2$ so

$$(1 + \eta)\, m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \beta^2}}$$

Hence $\frac{1}{\sqrt{1 - \beta^2}} = 1 + \eta$ or $\beta = \frac{\sqrt{\eta\,(2 + \eta)}}{1 + \eta}$

Here $\beta = \frac{v}{c}$ of pion. Hence time dilation factor is 1 + η and the distance traversed by the pion in its lifetime will be

$$\frac{c\,\beta\,\tau_0}{\sqrt{1 - \beta^2}} = c\,\tau_0 \sqrt{\eta\ (2 + \eta)} = 15.0 \text{ metres}$$

on substituting the values of various quantities. (Note. The factor $\frac{1}{\sqrt{1 - \beta^2}}$ can be looked at as a time dilation effect in the laboratory frame or as length contraction factor brought to the other side in the proper frame of the pion).

Q.293. Negative pions with kinetic energy T = 100 MeV travel an average distance l = 11 m from their origin to decay. Find the proper lifetime of these pions.

Ans. From the previous problem

$$l = c\,\tau_0 \sqrt{\eta\,(\eta + 2)}$$

where $\eta = \frac{T}{m_\pi c^2}$, m_π is the rest mass of pions.

substitution gives $\tau_0 = \frac{l}{c\sqrt{\eta\,(2+\eta)}} = 2{\cdot}63 \text{ ns}$

$$= \frac{l\,m_\pi c}{\sqrt{T\,(T + 2\,m_\pi c^2)}}$$

where we have used $\eta = \frac{100}{139{\cdot}6} = 0{\cdot}716$

Q.294. There is a narrow beam of negative pions with kinetic energy T equal to the rest energy of these particles. Find the ratio of fluxes at the sections of the beam separated by a distance l = = 20 m. The proper mean lifetime of these pions is ζₒ= 25.5 ns.

Ans. here $\eta = \frac{T}{m c^2} = 1$ so the life time of the pion in the laboratory frame is

$$\eta = (1 + \eta)\,\tau_0 = 2\,\tau_0$$

The law of radioactive decay implies that the flux decrease by the factor.

$$\frac{J}{J_0} = e^{-t/\tau} = e^{-l/v\tau} = e^{-l/c\tau_0\sqrt{\eta\,(2+\eta)}}$$

$$= \exp\left(-\frac{m\,c\,l}{\tau_0\sqrt{T\,(T + 2\,m\,c^2)}}\right) = 0{\cdot}221$$

Q.295. A stationary positive pion disintegrated into a muon and a neutrino. Find the kinetic energy of the muon and the energy of the neutrino.

Ans. Energy-momentum conservation implies

$$O = \vec{p_\mu} + \vec{p_\nu}$$

$$m_\pi c^2 = E_\mu + E_\nu \quad \text{or} \quad m_\pi c^2 - E_\nu = E_\mu$$

But $E_\nu = c\,|\vec{p_\nu}| = c\,|p_\mu|$. Thus

$$m_\pi^2 c^4 - 2\,m_\pi c^2 \cdot c\,|\vec{p_\mu}| + c^2 p_\mu^2 = E_\mu^2 = c^2 p_\mu^2 + m_\mu^2 c^4$$

Hence $c\,|\vec{p_\mu}| = \dfrac{m_\pi^2 - m_\mu^2}{2\,m_\pi} \cdot c^2$

So $T_\mu = \sqrt{c^2 p_\mu^2 + m_\mu^2 c^4} - m_\mu c^2 = \sqrt{\dfrac{(m_\pi^2 - m_\mu^2)^2}{4\,m_\pi^2} + m_\mu^2} \cdot c^2 - m_\mu c^2$

$$= \frac{m_\pi^2 + m_\mu^2}{2\,m_\pi} c^2 - m_\mu c^2 = \frac{(m_\pi - m_\mu)^2}{2\,m_\pi} \cdot c^2$$

Substituting $m_\pi c^2 = 139{\cdot}6$ MeV

$m_\mu c^2 = 105{\cdot}7$ MeV we get

$$T_\mu = 4{\cdot}12 \text{ MeV}$$

Also $E_\nu = \dfrac{m_\pi^2 - m_\mu^2}{2\,m_\pi} c^2 = 29{\cdot}8$ MeV

Q.296. Find the kinetic energy of a neutron emerging as a result of the decay of a stationary ∑ - hyperon (∑ - →n + π-).

Ans. We have

$$O = \vec{p_n} + \vec{p_\pi} \qquad (1)$$

$$m_\Sigma c^2 = E_n + E_\pi$$

or $(m_\Sigma c^2 - E_n)^2 = E_\pi^2$

or $m_\Sigma^2 c^4 - 2\,m_\Sigma c^2 E_n = E_\pi^2 - E_n^2 = c^4 m_\pi^2 - c^4 m_n^2$

because (1) implies $E_\pi^2 - E_n^2 = m_\pi^2 c^4 - m_n^2 c^4$

Hence $E_n = \frac{m_\Sigma^2 + m_n^2 - m_\pi^2}{2\, m_\Sigma} c^2$

and $T_n = \left(\frac{m_\Sigma^2 + m_n^2 - m_\pi^2}{2\, m_\Sigma} - m_n \right) c^2 = \frac{(m_\Sigma - m_n)^2 - m_\pi^2}{2\, m_\Sigma} c^2 .$

Substitution gives $T_n = 19.55$ MeV

Q.297. A stationary positive muon disintegrated into a positron and two neutrinos. Find the greatest possible kinetic energy of the positron.

Ans. The reaction is

$$\mu^+ \rightarrow e^+ + \bar{\nu}_e + \bar{\nu}_\mu$$

The neutrinoes are massless. The positron will carry largest momentum if both

neutriones $(\nu_e \,\&\, \bar{\nu}_\mu)$ move in the same direction in the rest frame of the nuon. Then the

final product is effectively a two body system and we get from problem (295)

$$(T_{e^+})_{max} = \frac{(m_\mu - m_e)^2}{2\, m_\mu} c^2$$

Substitution gives $(T_{e^+})_{max} = 52{\cdot}35$ MeV

Q.298. A stationary neutral particle disintegrated into a proton with kinetic energy T = 5.3 MeV and a negative pion. Find the mass of that particle. What is its name?

Ans. By conservation of energy-momentum

$$M c^2 = E_p + E_\pi$$
$$O = \vec{p}_p + \vec{p}_\pi$$

Then $m_\pi^2 c^4 = E_\pi^2 - \vec{p}_\pi^{\,2} c^2 = (M c^2 - E_p)^2 - c^2 \vec{p}_p^{\,2}$

$$= M^2 c^4 - 2 M c^2 E_p + m_p^2 c^4$$

This is a quadratic equation in M $M^2 - 2\frac{E_p}{c^2}M + m_p^2 - m_\pi^2 = 0$

or using $E_p = m_p c^2 + T$; and solving

$$\left(M - \frac{E_p}{c^2}\right)^2 = \frac{E_p^2}{c^4} - m_p^2 + m_\pi^2$$

Hence, $M = \frac{E_p}{c^2} + \sqrt{\frac{E_p^2}{c^4} - m_p^2 + m_\pi^2}$

taking the positive sign. Thus

$$M = m_p + \frac{T}{c^2} + \sqrt{m_\pi^2 + \frac{T}{c^2}\left(2\,m_p + \frac{T}{c^2}\right)}$$

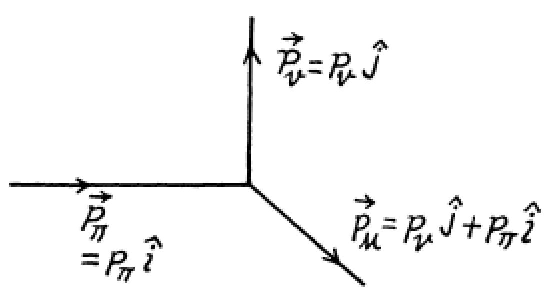
Substitution gives

$$M = 1115{\cdot}4\,\frac{\text{MeV}}{c^2}$$

From the table of masses we identify the particle as a ∧ particle

Q.299. A negative pion with kinetic energy T = 50 MeV disintegrated during its flight into a muon and a neutrino. Find the energy of the neutrino outgoing at right angles to the pion's motion direction.

Ans. See the diagram. By conservation of eneigy



$$\sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} = c\,p_\nu + \sqrt{m_\mu^2 c^4 + p_\pi^2 c^2 + c^2 p_\nu^2}$$

or

$$\left(\sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} - c\,p_\nu\right)^2 = m_\mu^2 c^4 + c^2 p_\pi^2 + c^2 p_\nu^2$$

or $m_\pi^2 c^4 - 2\,c\,p_\nu \sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} = m_\mu^2 c^4$

Hence the eneigy of the neutrino is

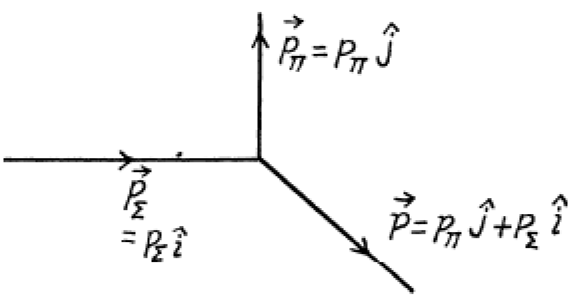
$$E_\nu = c\,p_\nu = \frac{m_\pi^2 c^4 - m_\mu^2 c^4}{2\,(m_\pi c^2 + T)}$$

on writing $\sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} = m_\pi c^2 + T$

Substitution gives E_ν = 21.93 MeV

Q.300. A $\sum^+$ hyperon with kinetic energy T_Σ = 320 MeV disintegrated during its flight into a neutral particle and a positive pion outgoing with kinetic energy T_π = 42 MeV at right angles to the hyperon's motion direction. Find the rest mass of the neutral particle (in MeV units).

Ans. By eneigy conservation



$$\sqrt{m_\Sigma^2 c^4 + c^2 p_\Sigma^2} = \sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} + \sqrt{m_n^2 c^4 + c^2 p_\pi^2 + c^2 p_\Sigma^2}$$

or $\left(\sqrt{m_\Sigma^2 c^4 + c^2 p_\Sigma^2} - \sqrt{m_\pi^2 c^4 + c^2 p_\pi^2}\right)^2 = m_n^2 c^4 + c^2 p_\pi^2 + c^2 p_\Sigma^2$

or $m_\Sigma^2 c^4 + c^2 p_\Sigma^2 + m_\pi^2 c^4 + c^2 p_\pi^2 - 2\sqrt{m_\pi^2 c^4 + c^2 p_\Sigma^2}\sqrt{m_\pi^2 c^2 + c^2 p_\pi^2}$

$$= m_n^2 c^2 + c^2 p_\pi^2 + c^2 p_\Sigma^2$$

or using the K.E. of $\sum$ & π

$$m_n^2 = m_\Sigma^2 + m_\pi^2 - 2\left(m_\Sigma + \frac{T_\Sigma}{c^2}\right)\left(m_\pi + \frac{T_\pi}{c^2}\right)$$

and $$m_n = \sqrt{m_\Sigma^2 + m_\pi^2 - 2\left(m_\Sigma + \frac{T_\Sigma}{c^2}\right)\left(m_\pi + \frac{T_\pi}{c^2}\right)} = 0{\cdot}949\,\frac{\text{GeV}}{c^2}$$

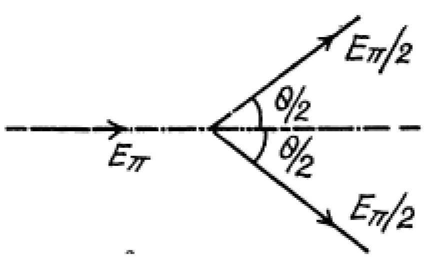
Q.301. A neutral pion disintegrated during its flight into two gamma quanta with equal energies. The angle of divergence of gamma quanta is $\Theta = 60^\circ$ Find the kinetic energy of the pion and of each gamma quantum.

Ans. Here by conservation of momentum

$$p_\pi = 2 \times \frac{E_\pi}{2\,c} \times \cos\frac{\theta}{2}$$

$$\text{or} \quad c\,p_\pi = \bar{E}_\pi \cos\frac{\theta}{2}$$

Thus $$E_\pi^2 \cos^2\frac{\theta}{2} = E_n^2 - m_\pi^2 c^4$$



or $$E_\pi = \frac{m_\pi c^2}{\sin\frac{\theta}{2}}$$

and $$T_\pi = m_\pi c^2\left(\operatorname{cosec}\frac{\theta}{2} - 1\right)$$

substitution gives $T_\pi = m_\pi c^2 = 135\text{ MeV for } \theta = 60^\circ$.

Also

$$E_\gamma = \frac{m_\pi c^2 + T_\pi}{2} = \frac{m_\pi c^2}{2}\operatorname{cosec}\frac{\theta}{2}$$

$= m_x c^2$ in this case ($\theta = 60°$)

Q.302. A relativistic particle with rest mass m collides with a stationary particle of mass M and activates a reaction leading to formation of new particles: $m + M \rightarrow m_1 + m_2 \ldots$, where the rest masses of newly formed particles are written on the right-hand side. Making use of the invariance of the quantity $E^2 - p^2c^2$, demonstrate that the threshold kinetic energy of the particle m required for this reaction is defined by Eq. (6.7c).

Ans. With particle masses standing for the names of the particles, the reaction is

$$m + M \rightarrow m_1 + m_2 + \ldots$$

On R.H.S. let the energy momenta be $(E_1, c\vec{p_1})$, $(E_2, c\vec{p_2})$ etc. On the left the energy momentum of the particle m is $(E, c\vec{p})$ and that of the other particle is $(Mc^2, \vec{0})$ where, of course, the usual relations

$$E^2 - c^2\vec{p}^2 = m^2c^4 \text{ etc}$$

hold. From the conservation of energy momentum we see that

$$(E + Mc^2)^2 - c^2\vec{p}^2 = (\Sigma E_i)^2 - (\Sigma c\vec{p_i})^2$$

Left hand side is

$$m^2c^4 + M^2c^4 + 2Mc^2E$$

We evaluate the R.H.S. in the frame where $\Sigma\vec{p_i} = 0$ (CM frame of the decay product).

Then $$R.H.S. = (\Sigma E_i)^2 \geq (\Sigma m_i c^2)^2$$

because all energies are +ve. Therefore we have the result

$$E \geq \frac{(\Sigma m_i)^2 - m^2 - M^2}{2M}c^2$$

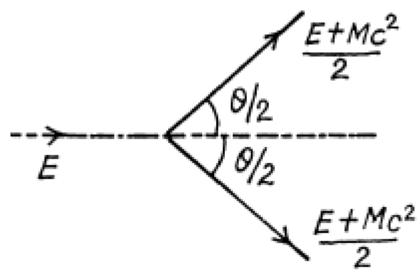
or since $E = mc^2 + T$, we see that $T \leq T_{th}$ where

$$T_{th} = \frac{(\Sigma\, m_i)^2 - (m + M)^2}{2M} c^2$$

Q.303. A positron with kinetic energy T = 750 keV strikes a stationary free electron. As a result of annihilation, two gamma quanta with equal energies appear. Find the angle of divergence between them.

Ans. By momentum conservation

$$\sqrt{E^2 - m_e^2 c^4} = 2\frac{E + m_e c^2}{2} \cos\frac{\theta}{2}$$



or $$\cos\frac{\theta}{2} = \sqrt{\frac{E - m_e c^2}{E + m_e c^2}} = \sqrt{\frac{T}{T + 2 m_e c^2}}$$

Substitution gives

$\theta = 98.8^\circ$

Q.304. Find the threshold energy of gamma quantum required to form (a) an electron-positron pair in the field of a stationary electron; (b) a pair of pions of opposite signs in the field of a stationary proton.

Ans. The formula of problem 3.02 gives

$$E_{th} = \frac{(\Sigma\, m_i)^2 - M^2}{2M} c^2$$

when the projectile is a photon

(a) For $\gamma + e^- \rightarrow e^- + e^- + e^+$

$$E_{th} = \frac{9\, m_e^2 - m_e^2}{2\, m_e} c^2 = 4\, m_e c^2 = 2{\cdot}04 \text{ MeV}$$

(b) For $\gamma + p \rightarrow p + \pi^+ \pi^-$

$$E_{th} = \frac{(M_p + 2\, m_\pi)^2 - M_p^2}{2\, M_p} c^2 = \frac{4\, m_\pi M_p + 4\, m_\pi^2}{2\, M_p} c^2 = 2\left(m_\pi + \frac{m_\pi^2}{M_p}\right) c^2 = 320{\cdot}8 \text{ MeV}$$

Q.305. Protons with kinetic energy T strike a stationary hydrogen target. Find the threshold values of T for the following reactions:
(a) $p + p \to p + p + p + \tilde{p}$; **(b)** $p + p \to p + p + \pi^0$.

Ans. (a) For $p + p \to p + p + p + \bar{p}$

$$T \geq T_{th} = \frac{16\, m_p^2 - 4\, m_p^2}{2\, m_p} c^2 = 6\, m_p c^2 = 5{\cdot}63 \text{ GeV}$$

(b) For $p + p \to p + p + \pi^\circ$

$$T \geq T_{th} = \frac{(2\, m_p + m_{\pi^\circ})^2 - 4\, m_p^2}{2\, m_p} c^2$$

$$= \left(2\, m_\pi + \frac{m_{\pi^\circ}^2}{2\, m_p}\right) c^2 = 0{\cdot}280 \text{ GeV}$$

Q.306. A hydrogen target is bombarded by pions. Calculate the threshold values of kinetic energies of these pions making possible the following reactions: (a) $\pi^- + \bar{p} \to K^+ + \Sigma^-$; (b) $\pi^0 + p \to K^+ + \Lambda^0$.

Ans. (a) Here

$$T_{th} = \frac{(m_K + m_\Sigma)^2 - (m_\pi + m_p)^2}{2\, m_p} c^2$$

Substitution gives $T_{th} =$ 0.904 GeV

$$T_{th} = \frac{(m_{K^+} + m_\Lambda)^2 - (m_{\pi^\circ} + m_p)^2}{2\, m_p} c^2$$

Substitution gives $T_{th} =$ 0.77 GeV.

Q.307. Find the strangeness S and the hypercharge Y of a neutral elementary particle whose isotopic spin projection is T_z = +1/2 and baryon charge B = +1. What particle is this?

Ans. From the Gell-Mann Nishijima formula

$$Q = T_Z + \frac{Y}{2}$$

we get

$$O = \frac{1}{2} + \frac{Y}{2} \text{ or } Y = -1$$

Also $Y = B + S \Rightarrow S = -2.$. Thus the particle is =° 0.

Q.308. Which of the following processes are forbidden by the law of conservation of lepton charge:

(1) $n \to p + e^- + \nu$;
(2) $\pi^+ \to \mu^+ + e^- + e^+$;
(3) $\pi^- \to \mu^- + \nu$;
(4) $p + e^- \to n + \nu$;
(5) $\mu^+ \to e^+ + \nu + \tilde{\nu}$;
(6) $K^- \to \mu^- + \tilde{\nu}$?

Ans. (1) The process $n \to p + e^- + \nu_e$ cannot occur as there are 2 more leptons (e^-, ν_e) on the right comopared to zero on the left

(2) The process $\pi^+ \to \mu^+ + e^- + e^+$ is forbidden because this corresponds to a change of lepton number by, (0 on the left - 1 on the right)

(3) The process $\pi^- \to \mu^- + \nu_\mu$ is forbidden because μ^-, ν_μ being both leptons ΔL = 2hre.

(4) , (5), (6) are allowed (except that one must distinguish between muon neutrinoes and electron neutrinoes). The correct names would be

(4) $p + e^- \to n + \nu_e$
(5) $\mu^+ \to e^+ + \nu_e + \tilde{\nu}_\mu$
(6) $K^- \to \mu^- + \tilde{\nu}_\mu$.

Q.309. Which of the following processes are forbidden by the law of conservation of strangeness:

(1) $\pi^- + p \to \Sigma^- + K^+$;
(2) $\pi^- + p \to \Sigma^+ + K^-$;
(3) $\pi^- + p \to K^+ + K^- + n$;
(4) $n + p \to \Lambda^0 + \Sigma^+$;
(5) $\pi^- + n \to \Xi^- + K^+ + K^-$;
(6) $K^- + p \to \Omega^- + K^+ + K^0$?

Ans. (1) $\pi^- + p \to \Sigma^- + K^+$

$$\begin{matrix} 0 & 0 & -1 & 1 \end{matrix}$$

SO $\Delta S = 0$. allowed

(2) $\pi^- + p \to \Sigma^+ + K^-$

$0 \quad 0 \quad -1 \quad 1$

SO $\Delta S = -2$. forbidden

(3) $\pi^- + p \to K^- + K^+ + n$

$0 \quad 0 \to -1 \quad 1 \quad 0$

SO $\Delta S = 0$, allowed.

(4) $n + p \to \Lambda^\circ + \Sigma^+$

$0 \quad 0 \quad -1 \quad -1$

SO $\Delta S = -2$. forbidden

(5) $\pi^- + n \to \Xi^- + K^+ + K^-$

$0 \quad 0 \to -2 \quad 1 \quad -1$

SO $\Delta S = -2$. forbidden.

(6) $K^- + p \to \Omega^- + K^+ K^\circ$

$-1 \quad 0 \quad -3 \quad +1 \quad +1$

SO $\Delta S = 0$. allowed.

Q.310. Indicate the reasons why the following processes are forbidden:

(1) $\Sigma^- \to \Lambda^0 + \pi^-$;
(2) $\pi^- + p \to K^+ + K^-$;
(3) $K^- + n \to \Omega^- + K^+ + K^0$;
(4) $n + p \to \Sigma^+ + \Lambda^0$;
(5) $\pi^- \to \mu^- + e^+ + e^-$;
(6) $\mu^- \to e^- + \nu_e + \nu_\mu$.

Ans. (1) $\Sigma^- \to \Lambda^\circ + \pi^-$

is forbidden by energy conservation. The mass difference

$$M_{\Sigma^-} - M_{\Lambda^\circ} = 82 \frac{\text{MeV}}{c^2} < m_{\pi^-}$$

(The process $1 \to 2 + 3$ will be allowed only if $m_1 > m_2 + m_3$.)

(2) $\pi^- + p \to K^+ + K^-$
is disallowed by conservation of baryon number.

(3) $K^- + n \to \Omega^- + K^+ + K^\circ$
is forbidden by conservation of charge

(4) $n + p \to \Sigma^+ + \Lambda^\circ$

is forbidden by strangeness conservation.

(5) $\pi^- \rightarrow \mu^- + e^- + e^+$
is forbidden by conservation of muon number (or lepton number).

(6) $\mu^- \rightarrow e^- + \nu_e + \tilde{\nu}_\mu$
is forbidden by the separate conservation of muon number as well as lepton number.