Velocity of jet, $U = \dfrac{0.075}{0.785 \times 0.050^2} = 38.217$ m/s

Discharging striking the value

$= 0.785 \times 0.050^2 (U - u)$

$= 0.785 \times 0.050^2 (38.217 - 10.0)$

$= 0.0554$ m³/s

Relative velocity of jet with respect to vane,

$U_{rf} = (38.217 - 10.000)$

$= 28.217$ m/s $= U_{r2}$

$U_{f2} = 28.217 \sin 10° = 4.90$ m/s

$U_{w2} = U_{r2} \cos 10° - u^2$

..... (where $u_2 = u_1 = 10$ m/s)

$= 27.788 - 10 = 17.788$ m/s

$\therefore \quad U_2 = \sqrt{U_{w2}^2 + U_{f2}^2}$

$= \sqrt{17.788^2 + 4.900^2}$

$= 18.451$ m/s

$F_{Bx} = 0.0554 \times 988 (-17.788 - 28217)$

$= -2543.580$ N

$\therefore \quad F_{fx} = 2543.580$ N

$F_{By} = 0.0554 \times 998 (4.90 - 0)$

$= -270.917$ N,

$\therefore \quad F_{fy} = -270.917$ N

Power developed $= \dfrac{2543.58 \times 10}{1000}$

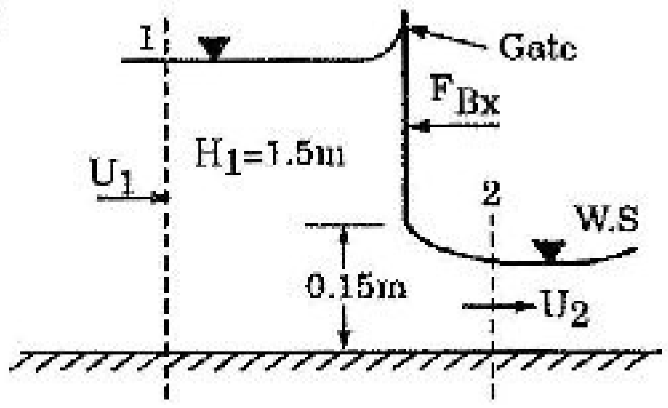
$= 25.436$ kW

Absolute velocity of jet as it leaves vane,

$U_2 = 18.451$ m/s

8. For the sluice gate shown in the figure, determine the force required to hold the gate in place. Assume gate width to be 2.0 m.

Solution:



Assume $C_c = 0.60$

$$C_d = \frac{0.60}{\sqrt{1 + 0.6\dfrac{0.15}{1.50}}} = 0.583$$

$q = C_d a \sqrt{2gh}$

$= 0.583 \times 0.15 \times \sqrt{2 \times 0.806 \times 1.5}$

$= 0.474$ m³/sm

$U_1 = \dfrac{0.474}{1.50} = 0.316$ m/s

$H_2 = 0.150 \times 6.0 = 0.090$ m

and $\quad U_2 = \dfrac{0.474}{0.09} = 5.267$ m/s

Apply moment equation between sections 1 and 2. The boundary frictional force F_t is assumed to be very small and hence neglected.

$$\therefore \quad \frac{1}{2}\gamma H_1^2 B - \frac{1}{2}\gamma H_2^2 B - F_{BX} = Q\rho(U_2 - U_1)$$

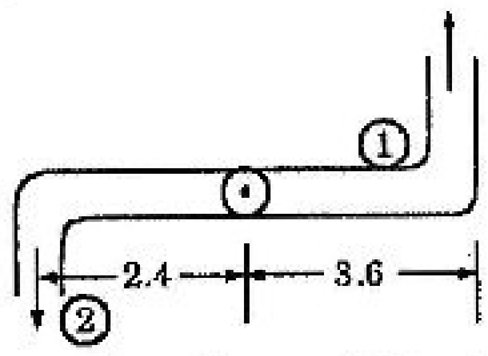
or $\quad \dfrac{1}{2} \times 2 \times 9787(1.5^2 - 0.09^2) - F_{Bx}$

$= 0.474 \times 2 \times 998(5.267 - 0.316)$

$\therefore \quad F_{Bx} = 17255.94$ N or 17.256 kN

9. The sprinkler given in the figure discharges 0.283 litres/s through each horizontal nozzle. Neglecting friction find its speed of rotation. The area of each nozzle of opening is 93 mm².

Solution:



The fluid entering the sprinkler has no moment of momentum, and no torque is exerted on the system externaly. Hence, the moment of momentm of fluid leaving must be zero. Let the speed of rotation be ω radians/sec, then

Moment of Momentm leaving

$$= \frac{\gamma Q}{g} V_{a1} r1 + \frac{\gamma Q}{g} V_{a2} r2$$

where V_{a1} and V_{a2} are absolute velocities at sections (1) and (2) respectively given by

$V_1 = \omega \cdot r_1$

and $\quad V_{r2} = \omega r_2$

Now, $\quad V_1 = \dfrac{0.283 \times 10^{-3}}{93 \times 10^{-6}}$

$= 3.4$m/s $= V_2$

$\therefore \quad V_{a1} = 3.04 - 3.6\,\omega$

and $\quad V_{a2} = 3.04 - 2.4\,\omega$

For Moment of momentum to be zero, we have

$$\rho Q(V_{a1} r_1 + V_{a2} r_2) = 0$$

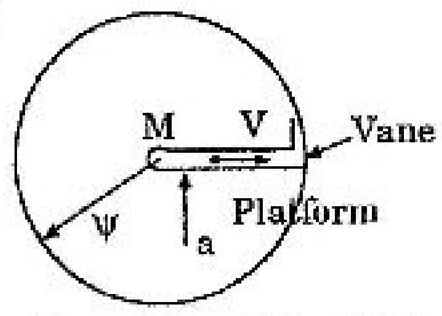
or $\quad 1000 \times 0.283 \times 10^{-3} [(3.04 - 3.6\omega) \times 3.6 + (3.4 - 2.4\,\omega) \times 2.4] = 0$

or $\quad 3.097 - 3.668\omega + 2.65 - 1.63\omega = 0$

or $\quad \omega = 0.97$ radians/second or 9.30 r.p.m,

10. The given figure indicates a platform which can rotate about an axis through M (normal to the plane of the figure). A jet of water is directed out from the center of the platform while it is stationary and strikes a vane at the periphery which turns the jet through 90° as shown. Calculate the torque developed.

Solution:



$$Q = aV = 6.5 \times 10^{-4} \times 3$$
$$= 19.5 \times 10^{-4}\ \text{m}^3/\text{s}$$

Force of jet on the vane in Y-direction

$$= \frac{\gamma Q}{g}(Y_{y2} - V_{y1})$$
$$= \frac{9810 \times 19.5 \times 10^{-4}}{9.81}(3 - 0)$$
$$= 5.85\text{N}$$

$\therefore$ Torque $= 5.85 \times r = 5.85 \times 0.6 = 3.51$ Nm.

KINEMATICS OF FLOW

1. Calculate velocity components, if $\phi = \log(x + y)$?

Solution: $u = -\frac{\partial \phi}{\partial x} = -\left(\frac{1}{x+y}\right)$

$$v = -\frac{\partial \phi}{\partial y} = -\left(\frac{1}{x+y}\right)$$

2. If $u = x^2 + y^2 = 2z^2$ and $v = -x^2y - yz - xy$, calculate w ?

Solution:

$$\frac{\partial \phi}{\partial x} = 3x \text{ and } \frac{\partial \phi}{\partial y} = -x^2 - z - x$$

From continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

or, $$\frac{\partial w}{\partial z} = -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = -x + z + x^2$$

$\therefore$ $$w = -xz + x^2z + \frac{z^2}{2} + f(x, y)$$

3. If for 2-D incompressible flow, $u = \frac{y^3}{3} + 2x - x^2y$ and $v = xy^2 - 2y - \frac{x^3}{3}$, calculate Ψ?

Solution:

$$\frac{\partial \Psi}{\partial x} = -v = -xy^2 + 2y + \frac{x^3}{3}$$

Integrating w.r.t. x we get,

$$\Psi = \frac{x^2y^2}{2} + 2xy + \frac{x^4}{12} + f(y) \quad ...(i)$$

Also $$\frac{\partial \Psi}{\partial y} = u = \frac{y^3}{3} + 2x - x^2y$$

Integrating w.r.t. y we get,

$$\Psi = \frac{y^4}{12} + 2xy - \frac{x^2y^2}{2} + f(x) \quad ...(ii)$$

From equations (i) and (ii),

$$\Psi = \frac{x^2y^2}{2} + 2xy + \frac{x^4}{12} + \frac{y^4}{12}$$

4. For 2-D flow, if $u = ax$, calculate v ?

Solution: Given, $u = ax$,

therefore $\frac{\partial u}{\partial x} = a$.

From continuity equation, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$,

or, $$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -a$$

$\therefore$ $$v = -\int a.dy$$

or, $$v = -ay + f(x)$$

5. Which of following functions represent the velocity potential of a fluid flow

(a) $\phi = x^2 + y^2$ (b) $\phi = x^2 - y^2$

(c) $\phi = 2x^2y^2$ (d) $x = x^3 - y^3$

Solution: For velocity potential to represent a fluid flow, the condition is $\nabla^2 \phi = 0$, i.e., the continuity equation must be satisfied or

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

In case of option (b) only, this condition is satisfied

$$\frac{\partial \phi}{\partial x} = 2x, \ \frac{\partial^2 \phi}{\partial x^2} = 2$$

and $$\frac{\partial \phi}{\partial y} = -2y, \ \frac{\partial^2 \phi}{\partial y^2} = -2$$

$\therefore$ $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 2 - 2 = 0$$

6. The velocity field in a field is given by,

$$V_s = (3x + 2y)\,i + (2_z + 3x^2)j + (2t - 3z)\,k.$$

(i) What are the velocity components u, v, w at any point in the flow field ?

(ii) Determine the speed at the point (1, 1, 1)

(iii) Determine the speed at time $t = 2$ s at point (0, 0, 2)

Solution: The velocity components at any point (x, y, z) are

(*i*) $u = (3x + 2y)$, $v = (2z + 3x^2)$, $w = (2t - 3z)$

(*ii*) Substitute $x = 1, y = 1, z = 1$ in the expressions for u, v and w, we get

$u = (3 + 2) = 5$, $v = (2 + 3) = 5$, $w = (2t - 3)$

$$\begin{aligned} V^2 &= u^2 + v^2 + w^2 \\ &= 5^2 + 5^2 + (2t-3)^2 \\ &= 50 + 4t^2 \\ &= 9 - 12t \\ &= 4t^2 - 12t \\ &= 59 \end{aligned}$$

$$\therefore \quad V_{(1,1,1)} = \sqrt{(4t^2 - 12t + 59)}$$

(*iii*) Putting $t = 2$, $x = 0$, $y = 0$, $z = 1$, we get

$u = 0$, $v = (4 + 0) = 4$, $10 = (4 - 6) = -2$

$$\therefore \quad V_{(0,0,2)} = \sqrt{(0 + 16 + 4)} = \sqrt{20}$$

7. Determine whether the continuity equation is satisfied by the following velocity components for incompressible fluid.

$$u = x^3 - y^3 - z^2x,\ v = y^3 - z^3,\ w = -3x^2z - 3y^2z + \frac{z^3}{3}$$

Solution:

$$\frac{\partial u}{\partial x} = 3x^2 - z^2,\ \frac{\partial v}{\partial y} = 3y^2,\ \frac{\partial w}{\partial z} = -3x^2 - 3y^2 + z^2$$

$$\therefore \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 3x^2 - z^2 + 3y^2 - 3x^2 - 3y^2 + z^2 = 0$$

Therefore, these velocity components satisfy the continuity equation

8. Determine the missing component of velocity distribution such that they satisfy continuity equation $u = ?$, $v = ax^3 - by^2 + cz^2$, $w = bx^3 - cy^2 + az^2x$

Solution:

Since $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0$,

$$\therefore \quad \frac{\partial u}{\partial x} = \left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$= -(-2by = 2azx) = 2by - 2azx$$

$$\therefore \quad u = \int \frac{\partial y}{\partial x} dx = \int (2by - 2axz) dx$$

$$= 2byz - 2az\frac{x^2}{2} = f(y, z)$$

The exact nature of $f(y, z)$ will be known if the boundary conditions are known

9. Determine the equation of a streamline passing through point (2, 3) if velocity components for two dimensional flow are given by $u = a$ and $v = a$, where a is non□zero constant.

Solution: Equation of stream line is,

$$\frac{u}{dx} = \frac{v}{dy}$$

$$\frac{a}{dx} = \frac{a}{dy}$$

or $\quad dy = dx$

On integration we get, $y = x + c$

the value of the constant of integration can be found from the condition that the stream line must pass through

Hence equation of stream line is, $y = x + 1$

which represents a straight line with slope of 45° and intersecting the y axis at 1.

10. If the velocity components for two dimensional flow are given by $u = \dfrac{x}{x^2 + y^2}$ and $v = \dfrac{y}{x^2 + y^2}$ determine the acceleration components a_x and a_y.

Solution: $a = u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y}$

$$= \frac{x}{(x^2 + y^2)}\frac{(x^2 - y^2)}{(x^2 + y^2)} - \frac{2y^2x}{(x^2 + y^2)^3}$$

$$= \frac{xy^2 - x^3 - 2xy^2}{(x^2 + y^2)^3} = \frac{x}{(x^2 - y^2)^3}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= \frac{x(-2xy)}{(x^2 + y^2)^3} + \frac{y(x^2 - y^2)}{(x^2 + y^2)^3}$$

$$= \frac{-2x^2y + x^2y - y^3}{(x^2 + y^2)^3}$$

$$= \frac{-y(x^2 + y^2)}{(x^2 + y^2)^3} = \frac{-y}{(x^2 + y^2)^2}$$

11. If the velocity field is given by $V_x = 0$, $V_r = 0$ and $V_0 = a, r$, show that the flow is rotational.

Solution: $\omega_x = \dfrac{1}{2}\left[\dfrac{1}{r}\cdot\dfrac{\partial}{\partial r}({}_r V_\theta) - \dfrac{1}{r}\dfrac{\partial}{\partial \theta}(V_r)\right]$

$$= \frac{1}{2}\left[\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\right]$$

$$= \frac{1}{2}[a + a] = a$$

Hence the flow is rotational.

12. Check whether the following functions represent possible flow phenomenon of irrotational type

(*i*) $\phi = x^2 - y^2 + y$

(*ii*) $\phi = \sin(x + y + z)$

Solution:

(*i*) $\dfrac{\partial \phi}{\partial x} = 2x,$

$\Rightarrow \dfrac{\partial^2 \phi}{\partial x^2} = 2$

and $\dfrac{\partial \phi}{\partial y} = -2y + 1,$

$\Rightarrow \dfrac{\partial^2 \phi}{\partial y^2} = -2$

$\therefore \dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} = 2 - 2 = 0.$

Hence, $\phi = x^2 - y^2 + y$ satisfies Laplace's equation and can represent irrotational flow.

(*ii*) $\phi = \sin(x + y + z)$

$\Rightarrow \dfrac{\partial \phi}{\partial x} = \cos(x + y + z)$

$\Rightarrow \dfrac{\partial^2 \phi}{\partial x^2} = -\sin(x + y + z)$

Similarly, $\dfrac{\partial^2 \phi}{\partial y^2} = -\sin(x + y + z)$

and $\dfrac{\partial^2 \phi}{\partial z^2} = -\sin(x + y + z)$

$\therefore \dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} + \dfrac{\partial^2 \phi}{\partial z^2} = -3\sin(x + y + z) \neq 0$

Hence, $\phi = \sin(x + y + z)$ does not satisfy Laplace's equation and thus can not represent irotational flow.

13. If $\phi = 2xy$, determine Ψ.

Solution:

$\dfrac{-\partial \phi}{\partial x} = u = -\dfrac{\partial \Psi}{\partial y} = -2y$

and $\dfrac{-\partial \phi}{\partial y} = v = +\dfrac{\partial \Psi}{\partial x} = -2y$

$\therefore \dfrac{\partial \phi}{\partial x} = 2y = \dfrac{\partial \Psi}{\partial y}$

Integrating we get, $\Psi = y^2 + f(x)$

$\therefore \dfrac{\partial \Psi}{\partial x} = f(x) = \dfrac{\partial \phi}{\partial y} = -2x$

and $\dfrac{\partial f}{\partial x} = -2x$

Hence $\Psi = (y^2 - x^2) + C$, where C is a numerical constant.

LAMINAR AND TURBULENT FLOW

1. For laminar flow in a pipe, prove that $V = 0.5\,U_{max}$

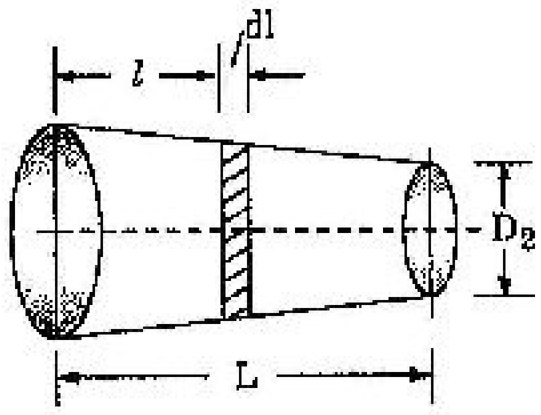
Solution:

$$U_{max} = (-)\frac{1}{4\mu}\cdot\frac{dp}{dl}\cdot r_0^2$$

and
$$V = \frac{1}{8\mu}\cdot\frac{dp}{dl} r_0^2$$
$$= \frac{1}{2}\cdot(-)\frac{1}{4\mu}\cdot\frac{dp}{dl} r_0^2 = 0.5\,U_{max}$$

2. For steady laminar flow, determine an expression for pressure loss across a conical conatraction.

Solution: From the figure,



$$D = D_2 + (D_1 - D_2)\frac{(L-1)}{l}$$

or
$$dD = \frac{-(D_1 - D_2)}{l} dL$$

or
$$dL = \frac{l\,.dD}{(D_1 - D_2)}$$

From Hagen-Poiseulle's equation, pressure loss in length dl

$$\Delta p = \frac{128\mu Q}{\pi D^4}\,.\,dl = \frac{-128\mu Q\,l}{\pi D^4 (D_1 - D_2)} dD$$

Integrating we get, pressure loss

$$\int_{p_1}^{p_2} \Delta P = (p_2 - p_1)$$
$$= \frac{128\mu Q}{3\pi}\frac{l}{(D_1 - D_2)}\cdot\left[\frac{1}{D^3}\right]_{D_1}^{D_2}$$
$$= \frac{-128\mu Q\,l}{3\pi(D_1 - D_2)}\cdot\left[\frac{1}{D_2^3} - \frac{1}{D_1^3}\right]$$

3. Water is flowing through a 20 cm diameter pipe with friction factor, $f = 0.04$. The shear stress at a point 4 cm from the pipe axis is 0.0098 N/cm². Calculate the shear stress at the pipe wall.

Solution: $f = \dfrac{16}{R_e}$, or $0.04 = \dfrac{16}{R_e}$,

$R_e = 400.$

$\therefore$ Since $R_e <$ is less than 2000, the flow is viscous.

For viscous flow, shear stress, $\tau = -\dfrac{\delta p}{\delta x} \times \dfrac{r}{2}.$

But $\frac{\delta p}{\delta x}$ = constant, therefore $\tau \propto r$

Let τ_0 be the shear stress at pipe wall, therefore

$$\frac{\tau_0}{\tau} = \frac{r_0}{r}$$

or, shear stress at wall, $\tau_0 = \frac{r_0}{r} \times \tau$

$$= 0.0098 \times \frac{10}{4}$$

$$= 0.0245 \text{ N/cm}^2$$

4. Convert 1 kg/s-m in dynamic viscosity in poise.

Solution: In C.G.S. units, the unit of viscosity is 'POISE'

$$1 \text{ Poise} = \frac{dyne - \text{sec}}{\text{cm}^2}$$

$$= \frac{\frac{1}{981} gm - \text{sec}}{10^{-4}} \text{m}^2$$

$$= \frac{\frac{1}{981} \times \frac{1}{100}}{10^{-4}}$$

$$= \frac{10}{981} \frac{kgf - \text{sec}}{m^2}$$

$$= \frac{10}{981} \times 9.81 \frac{\text{kg}}{\text{sec} - \text{m}}$$

$$= 0.1 \frac{\text{kg}}{\text{sec} - \text{m}}$$

5. Find the flow of oil $\mu = 0.5$ poise, density 800 kg/m^3 through a pipe of diameter 9 mm and 12 m long when the head lost is 0.75 m.

Solution: $\mu = 0.5$ Poise = 0.05 Pas

$$R = \frac{\rho VD}{\mu}$$

$$= \frac{800 \times V \times 0.009}{0.05} = 144\,V$$

and $f = \frac{64}{R} = \frac{64}{144V} = \frac{0.444}{V}$

From $h_f = \frac{fLV^2}{2gd}$

$$0.75 = \frac{0.444}{V} \times 12 \times V^2 \times \frac{1}{2} \times \frac{1}{9.81} \times \frac{1}{9 \times 10^{-3}}$$

Solving, we get V = 0.0248 m/s or 24.8 mm/s

6. Find the critical velocity of water at 20°C flowing through a 20 cm diameter pipe. Take kinematic viscosity of water at 20°C = 0.0101 stoke. Assume that the changes from laminar to turbulent at $R_e = 2320$.

Solution: $R = \frac{dv_c}{\upsilon} = 2320,$

$$\therefore \quad v_c = 2320 \frac{\upsilon}{d}$$

$$= 2320 \times \frac{0.0101}{20} = 1.117 \text{ cm/sec.}$$

7. Crude oil of kinematic viscosity 2.25 stokes flows through a 20 cm diameter pipe, the rate of flow being 1.5 litres/sec. Find the type of flow.

Solution: Q = 15 × 1000 ce/sec, d = 20 cm

$$\therefore \quad v = \frac{15000}{\frac{\pi}{4} \times (20)^2} = 47.75 \text{ cm/sec.}$$

Reynolds number, $R_e = \frac{dv}{\upsilon}$

$$= \frac{20 \times 47.75}{2.25}$$

$$= 424.4 < 2000,$$

Hence the flow is laminar

8. Water flows through a 8 mm diameter 350 m long pipe at a velocity of 25 cm per second. Find the maximum temperature of water for laminar flow. Specific weight may be assumed to be constant. Take the flow to be laminar when the Reynolds number does not exceed 2000.

Solution: $R_e = \frac{Dv\rho}{\mu} = 2000,$

$$\therefore \quad \mu = \frac{Dv\rho}{200} = \frac{0.8 \times 25 \times 1}{2000} = 0.01 \text{ poise}$$

Let the viscosity of water be 0.01 poise at t°C,

$$\mu = \frac{0.01776}{1 + 0.03368\,t + 0.000221\,t^2}$$

$$= 0.01$$

or $0.000221\,t^2 + 0.03368\,t + 1 = \frac{0.01776}{0.01}$

or $0.0002211\,t^2 + 0.03368\,t - 0.776 = 0$

or $t^2 + 152.398t - 3511.3122 = 0$

$\therefore \; t = 20.32$ °C

9. A 0.20 m diameter pipe 20 km long transports oil at a flow rate of 0.01 m^3/s. Calculate the power required to maintain the flow if the dynamic viscosity and density of oil are 0.08 P and 900 kg/m^3, respectively.

Solution:

Velocity of flow, $v = \frac{0.01}{\frac{\pi}{4}(0.2)^2} = \frac{1}{\pi}$ m/s

Reynolds number, $R = \frac{\rho v d}{v}$

$$= \frac{900 \times \frac{1}{\pi} \times 0.2}{0.08}$$

$$= 716 < 2000,$$

Hence the flow is laminar.

Loss of head, $h_f = \frac{32\mu LV}{wd^2}$

$$= \frac{32 \times 0.8(20 \times 1000) \times \frac{1}{\pi}}{900 \times (0.2)^2}$$

$$= 4527 \text{ metres}$$

power required to maintain the flow,

$$P = \frac{WQh_f}{75}$$

$$= \frac{900 \times 0.01 \times 4527}{75}$$

$$= 543 \text{ HP or } 405 \text{ kW.}$$

10. From long horizontal pipe is to deliver 900 kg of oil ($S = 0.9$, $v = 0.002$ m²/s) per minute. If the head loss is not to exceeds 8 m of oil, find the pipe diameter.

(Factor in laminar flow, $f = 64/R$).

Solution:

$$l = 400 \text{ m}; Q = \frac{900}{0.9} = 1000 \text{ litres}$$

$$= 1 \text{ m}^3\text{/minutes} = \frac{1}{60} \text{ m}^3\text{/s}, v = 0.0002 \text{ m}^2\text{/s}$$

$$v = \frac{Q}{A} = \frac{\frac{1}{60}}{\frac{\pi}{4}d^2} = \frac{1}{15\pi d^2} \text{ m/s},$$

where d is diameter of pipe in metres.

Now, $h_f = \frac{4flv^2}{2gD}$

where $f = \frac{64}{R_e} = \frac{64}{\frac{vd}{v}}$

$$\therefore \ 8 = \frac{4 \times (64 \times 0.0002)}{\frac{1}{15}\pi d^2 \times d} \times \frac{400}{2 \times 9.81 \times d} \times \frac{1}{(15\pi d^2)}$$

$$= \frac{0.022}{d^4}$$

or $d = 0.2728$ m

11. Oil (specific gravity = 0.85) is flowing in a 150 long smooth pipe line at the rate of 80 liters per sec. The loss of head in the line is 67 m. If the kinematic viscosity of the oil is 0.1 cm/sec, calculate the diameter of the pipe.

Solution:

$$Q = 80 \text{ lit/sec} = 80 \times 10^{-3} \text{ m}^3\text{/sec.}$$

Let D be the required diameter of the pipe.

$$\therefore \quad V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

$$= \frac{4 \times 80 \times 10^{-3}}{3.14\, D^2} = \frac{101.91 \times 10^{-3}}{D^2}$$

$$\therefore \quad R_e = \frac{vD}{\nu}$$

$$= \frac{101.91 \times 10^{-3} D \times 10^4}{D^2 \times 0.1} = \frac{1019.1}{D}$$

From $h_f = 4f\frac{L}{D}\frac{v^2}{2g}$

$$67 = 4f \times \frac{150}{D} \times \frac{(101.91)^2 \times 10^{-6}}{D^4} \times \frac{1}{19.62}$$

$$= (4f)\frac{794 \times 10^{-4}}{D^5}$$

$$\therefore \quad D^5 = (4f) \times \frac{794 \times 10^{-4}}{67}$$

$$= (4f) \times 118.5 \times 10^{-5}$$

or $\quad D = 10^{-1} \times \sqrt[5]{(118.5(4f))}$

Assume, $\quad 4f = 0.02$

thus $\quad D = 10^{-1} \times \sqrt[5]{118.5 \times 0.02}$

$$= 0.118836 \text{ metre.}$$

But, $\quad R_e = \frac{101.91 \times 10^2}{0.118836} = 8.575 \times 10^4$

From Balsius equation,

$$4f = \frac{0.3164}{(R_e)^{\frac{1}{4}}}$$

$$= \frac{0.3164}{(8.575)^{\frac{1}{4}} \times 10} \times 0.0185$$

$$\neq 0.02 \text{ (assumed value)}$$

Again assume, $4f = 0.0185$ and repeat the above method, we get,

$$4f = 0.0184$$

Hence, $\quad D = 10^{-1} \times \sqrt[5]{(118.5 \times 0.0185)}$

$$= 0.117 \text{ cm}$$

12. Glycerine ($\mu = 1.5$ Pas and $\rho = 1260$ kg/m^3) flows at a mean velocity of 5 m/s in a100 mm diameter pipe. Estimate the power expended by the flow in a distance of 12 m.

Solution: $$\Delta p = \frac{32\mu VL}{d^2}$$

$$= \frac{32 \times 1.5 \times 5 \times 12}{(100 \times 10^{-3})}$$

$$= 288 \text{ kN/m}^2 \text{ or } 288000 \text{ N/m}^2$$

Power expended $= \Delta p \times Q$

$$= 288000 \times \left[\frac{\pi}{4} \times (100 \times 10^{-3}) \times 5\right]$$

$$= 11307.73 \text{ watts}$$

Superfluous data: ρ

13. Oil of specific gravity 0.82 and dynamic viscosity 12.0663×10^{-2} Pas is pumped at the rate of 0.02 m^3/s through a 150 mm. dia, 300 meter long pipe. Calculate the pressure drop and the power required to maintain flow

(*a*) if the pipe is horizontal

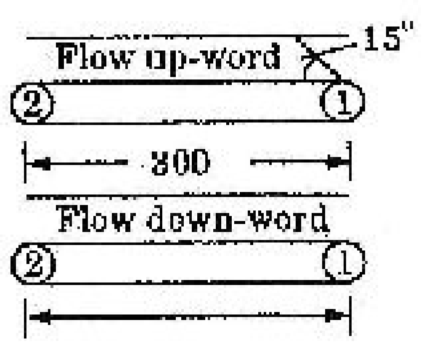
(*b*) if the pipe is inclined at 15. with the horizontal and the flow is

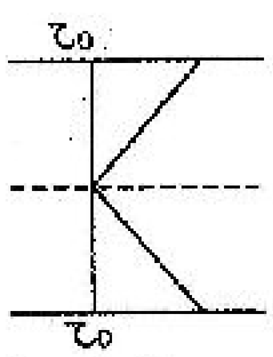
(*i*) in upward direction

(*ii*) downward direction.

Also determine the slope of the pipe and direction of flow so that the pressure gradient along the pipe is zero.

Solution:





Specific weight of oil,

$$\gamma_{oil} = 0.82 \times 9810$$

$$= 8044.2 \text{ N/m}^3$$

$$\mu = 12.0663 \times 10^{-2}$$

$$Q = 0.02 \text{ m}^3\text{/s}, d = 0.15 \text{ m},$$

$$\therefore \quad \text{Area} = \frac{\pi d^2}{4} = 0.01767 \text{ m}^2$$

(*a*) Pipe is Horizontal:

$$V = \frac{Q}{a} = \frac{0.02}{0.01767} = 1.132 \text{ m/s}$$

$$h_f = \frac{p_1 - p_2}{\gamma}$$

$$= \frac{32\mu Vl}{\gamma d^2}$$

$$= \frac{32 \times 0.120663 \times 1.132 \times 300}{8044.2 \times 0.15^2}$$

$$= 7.2448 \text{ m. of oil.}$$

$$\therefore \quad p_1 - p_2 = 8044.2 \times 7.2448$$

$$= 58278.62 \text{ N/m}^2$$

Shear stress, $\tau_0 = -\dfrac{r_0}{2} \cdot \dfrac{dp}{dl}$

$$= -\frac{0.15}{4} \times \frac{(-)58278.62}{300}$$

$$= 7.285 \text{ N/m}^2$$

Power required $= \gamma \,.\, Q \,.\, h_f$

$$= 8044.2 \times 0.02 \times 7.2448$$

$$= 1165.57 \text{ watts.}$$

(*b*) Pipe is inclined

(*i*) Flow in upward direction. Power is same i.e., 1165.57. W- Shear stress is 7.258 N/m^2 and pressure drop is different and given by 300 sin 15°, i.e. 77.65 m

$$h_f = \frac{32\mu VL}{gd^2} = 7.2448 \text{ m of oil}$$

Assuming datum at (2), we have flow from (2) to (1)

$$\therefore \quad \frac{p_2}{\gamma} = \frac{p_1}{\gamma} + z_1 + h_f$$

or, Pressure drop $= \dfrac{p_2 - p_1}{\gamma}$

$$= Z_1 + h_f$$

$$= 77.65 + 7.2448$$

$$= 84.8949 \text{ m}$$

$$\therefore \quad p_2 - p_1 = 84.8948 \times 8044.2$$

$$= 682911 \text{ N/m}^2 \text{ or } 0.683 \text{ MPa.}$$

(*ii*) Flow in downward directions, i.e., from (I) to (II) and datum is assumed again at 2

From $\dfrac{p_1}{\gamma} + z_1 = \dfrac{p_2}{\gamma} + h_f$

Pressure drop $= \dfrac{p_1 - p_2}{\gamma}$

$$= h_f - z_1$$

$$= 7.0448 - 77.65$$

$$= -70.4052 \text{ m}$$

$$p_2 - p_1 = -70.4052 \times 8044.2$$

$$= -0.566 \text{ MPa}$$

(c) When pressure gradient is zero :

$$p_1 = p_2'$$

and $$\frac{p_1 - p_2}{\gamma} = 0$$

In either case, $z = h_f = 7.2448,$

and $$\sin\theta = \frac{z}{l} = \frac{7.2448}{300} = 0.024$$

Hence, inclination of the pipe to horizontal, $0 = 1° \, 23' \, 2''$

14. Determine kinetic energy correction factor and momentum correction factor for laminar flow through a pipe ?

Solution:

For laminar flow,

$$u = (-)\frac{1}{4\pi}\cdot\frac{dp}{dl}\cdot\left(r_0^2 - r^2\right)$$

and $$V = (-)\frac{1}{8\pi}\cdot\frac{dp}{dl}\cdot r_0^2$$

$$\left(\frac{u}{V}\right) = 2\left(1 - \frac{r^2}{r_0^2}\right)$$

K.E. correction factor,

$$\alpha = \int_A \left(\frac{u}{V}\right)^3 dA$$

$$= \frac{1}{\pi r_0^2}\int_0^{r_0} 8\left(1 - \frac{r^2}{r_0^2}\right).2\pi r\,.\,dr$$

$$= \frac{16}{r_0^2}\int_0^{r_0}\left(1 - \frac{3r^2}{r_0^2} + \frac{3r^4}{r_0^4} - \frac{r^6}{r_0^6}\right) r\,.\,dr$$

$$= 16\left[\frac{1}{2} - \frac{3}{4} + \frac{3}{6} - \frac{1}{8}\right]$$

$$= 2$$

Momentum correction factor,

$$\beta = \frac{1}{A}\int_A \left(\frac{v}{V}\right)^2 dA$$

$$= \frac{1}{\pi r_0^2}\int_0^{r_0} 4\left(1 - \frac{r^2}{r_0^2}\right).2\pi r\,.\,dA$$

$$= \frac{8}{r_0^2}\int_0^{r_0}\left(1 - \frac{2r^2}{r_0^2} + \frac{r^4}{r_0^4}\right) dr$$

$$= 8\left[\frac{1}{2} - \frac{1}{2} + \frac{1}{6}\right]$$

$$= 1.33$$

15. Evalute the momentum correction factor if the velocity profile satisfies the equation

$$v = v_{\max}\left[\frac{\left(r_0^2 - r^2\right)}{r_0^2}\right]$$

Solution: Average velocity,

$$v = \frac{v_{\max}}{2}$$

$$\beta = \frac{1}{A}\int_A \left(\frac{v}{V}\right)^2 dA$$

$$= \frac{1}{\pi r_0^2}\int_0^{r_0} 4\left(\frac{U_{\max}\dfrac{\left(r_0^2 - r^2\right)}{r_0^2}}{\dfrac{1}{2}U_{\max}}\right).2\,\pi r.dr$$

$$= \frac{8}{r_0^6}\left(\frac{1}{2}r_0^6 - \frac{1}{2}r_0^6 + \frac{1}{6}r_0^6\right) = 1.33$$

FORCES ON IMMERSED BODIES

1. Find ratio of the skin friction drage on the front half and rear half of a flat plate kept in a uniform, stream at zero incidence, assume the boundary layer to be turbulent over the entrie plate.

Solution: For the front half length,

$$R_{eL} = \frac{U_0(0.5L)}{\nu}$$

$$= \frac{0.5_0}{\nu}$$

On the front half portion of the plate,

$$F_{DI} = C_f A\frac{\pi U_0^2}{2}$$

where $$C_f = \frac{0.074}{R_e^{\frac{1}{5}}}$$

$$\therefore \quad F_{DI} = \frac{0.074}{\left(\dfrac{0.5U_0L}{\nu}\right)^{1/5}} \times B \times 0.15L \times \frac{\rho U_0^2}{2}$$

$$= \frac{0.074 \times 0.5}{(0.5)^{\frac{1}{5}}} \times \frac{\rho U_0^2}{2}$$

$$= 0.0425BL\frac{\rho U_0^2}{\left(2R_{eL}^{\frac{1}{5}}\right)} \qquad \text{...}(i)$$

On the whole length of the plate,

$$F_D = \frac{0.74}{\left(\dfrac{U_0L}{\nu}\right)} BL \times \frac{\rho U_0^2}{2}$$

∴ Drage on the rear half portion of the plate

$$F_D = F_D - F_{DI}$$

$$= \left[0.074 - \frac{0.074 \times 0.5}{(0.5)^{\frac{1}{5}}}\right] \frac{BL \times \rho U_0^2}{\left(2R_{eL}^{\frac{1}{5}}\right)}$$

$$= 0.0315 \frac{BL \times \rho U_0^2}{2} \quad ...(ii)$$

From equations (*i*) and (*ii*), we get $\frac{F_{D1}}{F_{D2}} = 1.35$

2. A smooth plate 1m. wide moves through stationary air of specific weight 11.28 N/m³ at 1 m/s. Calculate drag on one side of the plate when the boundary layer is

(*i*) entirely laminar;

(*ii*) turbulent.

Assume plate to be 3m long, what is the boundary layer thickness at the trailing edge for both cases $\nu = 0.15 \times 10^{-2}$ m²/s

Solution: Reynolds Number at the trailing edge,

$$R_e = \frac{Ul}{\nu} = \frac{1 \times 3}{0.15 \times 10^{-4}} = 2 \times 10^5$$

Hence the boundary layer can be assumed to be fully laminar (2×10^5) is the beginning of turbulence

Drag on one side of the plate,

$$C_f = \frac{1.46}{\sqrt{R_1}} = \frac{1.46}{\sqrt{2 \times 10^5}} = 3.265 \times 10^{-3}$$

Drag on one side of the plate,

$$C = \frac{1}{2} \times 3.265 \times 10^{-5} \times \frac{11.28}{9.81} \times (3 \times 1) \times 1^2$$

$$= 5.631 \times 10^{-3} N$$

If the boundary layer is assumed fully turbulent, then for turbulent boundary layer

$$C_f = \frac{0.079}{R_1^{\frac{1}{5}}}$$

$$= \frac{0.079}{(2 \times 10^5)^{\frac{1}{5}}}$$

$$= 6.877 \times 10^{-3}$$

$$\text{Drag} = \frac{1}{2} C_f \rho AV^2$$

$$= \frac{1}{2} \times 6.877 \times 10^{-3} \times \frac{11.28}{9.81} \times 3 \times 1^2$$

$$= 0.01186 N$$

Thickness of laminar boundary layer at the trailing edge,

$$\delta = \frac{5.48l}{\sqrt{R_1}}$$

$$= \frac{5.48 \times 3}{\sqrt{2 \times 10^5}} = 0.037\text{m. or } 37 \text{ mm}$$

and thickness of turbulent boundary layer

$$= \frac{0.371}{\sqrt{R_e}}$$

$$= \frac{0.37 \times 3}{\sqrt{2 \times 10^5}} = 0.0966\text{m or } 96.6 \text{ mm}$$

3. If in the above question the free stream velocity is 2m/s and the boundary layer is not completely turbulent, but is turbulent at the trailing edge. Find the drag. The Length of the plate is 6m in this case and the upper critical Rynolds number $= 4 \times 10^5$

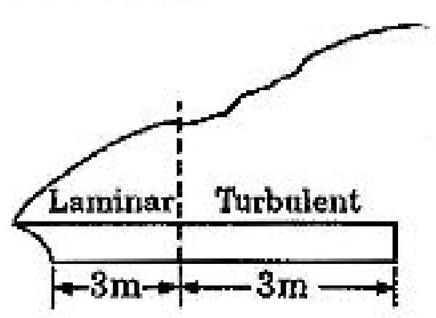
Solution:

From $\quad R_x = \frac{U.x}{\nu},$

$$\Rightarrow \quad 4 \times 10^5 = \frac{2 \times x}{0.15 \times 10^4},$$

where x is the distance of the point where the boundary layer becomes turbulent Solving, we get

$$x = 3 \text{ m.}$$



The boundary layer is laminar upto the middle point of the plate and then becomes turbulent

Now, $\quad R_6 = \frac{U \times 6}{\nu}$

$$= \frac{2 \times 6}{0.15} \times 10^{-4} = 8 \times 10^5$$

$$\text{Total drag, } F_D = \left(\frac{1.46}{\sqrt{R_2}} + \frac{0.079}{\sqrt{R_6^{\frac{1}{5}}}}\right) \frac{11.28}{9.81} \times \frac{3 \times 2^2}{2}$$

$$= 0.052 N$$

$$= \left(\frac{1.46}{\sqrt{1 \times 10^5}} + \frac{0.079}{\sqrt{(8 \times 10^5)^{\frac{1}{5}}}}\right) \frac{11.28}{9.81} \times \frac{3 \times 2^2}{2}$$

$$= 0.052 N$$

Drag on second half of the plate or Turbulent Drag

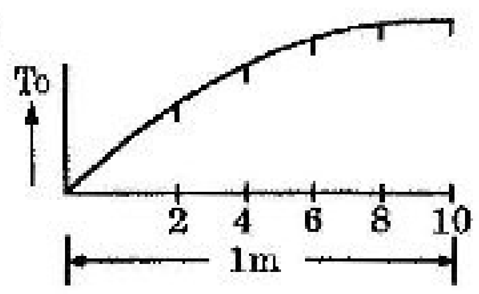
= Total Drag – laminar Drag

$$= 0.052 - \frac{1.46}{\sqrt{4 \times 10^5}} \times \frac{11.28}{9.81} \times \frac{3 \times 2^2}{2}$$

$$= 0.036 \text{ N}$$

4. A thin rectangular plate 15 cm wide and 1m long is immersed in glycerine (l = 12.36 kN/m³, $\mu = 80.44 \times 10^{-2}$ pas) and is towed in the direction of its length at a constant velocity of 1 m/s. Determine the coefficient of drag, boundary thickness at the trailing edge, and shear stress at the trailing edge. Plot the shear stress variation along the plate.

Solution:



For glycerin, $\rho = \dfrac{12.36 \times 10^3}{9.81} = 1260 \text{ kg/m}^3$

At the trailing edge, $R_1 = \dfrac{Ul}{\nu}$

$$= \frac{1 \times 1 \times 12.36 \times 10^3}{80.44 \times 10^{-2} \times 9.81}$$

$$= 1566$$

Local skin friction coefficient,

$$c_f = \frac{0.73}{\sqrt{R_x}} = \frac{0.73}{\sqrt{1566}} = 0.018$$

$$\tau_0 = \frac{1}{2} c_f \rho U^2$$

$$= \frac{1}{2} \times 0.018 \times 1260 \times 1^2$$

$$= 11.34 \text{N/m}^2$$

$$C_f = \frac{1.46}{\sqrt{R_1}} = \frac{1.46}{\sqrt{1566}} = 0.037$$

$$\delta l = \frac{5.481}{\sqrt{R_1}} = \frac{5.48 \times 1}{\sqrt{1566}}$$

$$= 0.138 = 138 \text{ mm}$$

If x is the distance fom the leading edge, we have the following table for τ_0

x_m	0	0.2	0.4	0.6	0.8	1.0	Formulae
c_f	0	8.25×10^{-3}	0.0117	0.0143	0.0165	0.0184	$c_f = 0.0184\sqrt{x}$
τ_0	0	5.2	7.4	9	10.4	11.34	$\tau_0 = \frac{1}{2}\rho U^2 \times c_f \text{N/m}^2$

5. A rectangular plate 1m long and 30 cm wide is dragged lengthwise in a stream with a velocity 4 m/s. calculate the drage on both the sides of the plate. Specific weight of liquid = 11.8 N/m³ and kinematic viscosity = 1.49×10^{-5} m²/s

Solution:

$$R_1 = \frac{Ul}{\nu} = \frac{4 \times 1}{1.49 \times 10^{-5}}$$

$$= 2.68 \times 10^{-2} = 2.68 \times 10^5$$

Coefficient of drag,

$$C_f = \frac{1.46}{\sqrt{2.68 \times 10^5}}$$

$$= 0.282 \times 10^{02}$$

Drag on both sides of plate

$$= 2\left(\frac{1}{2}\rho U^2 A \times C_f\right)$$

$$= 2\left(\frac{1}{2} \times \frac{11.8}{9.81} \times 4^2 \times (1 \times 0.3) \times 0.282 \times 10^{-2}\right)$$

$$= 1.62 \times 10^{-2}$$

6. In the above example, what will be the drag if the plate is towed breadth wise?

Solution:

$$R_1 = \frac{Ul}{\nu}$$

$$= \frac{4 \times 0.3}{1.49 \times 10^{-5}}$$

$$= 0.81 \times 10^5.$$

$$C_f = \frac{1.46}{\sqrt{R_1}}$$

$$= \frac{1.46}{\sqrt{0.8 \times 10^5}}$$

$$= 0.51 \times 10^{-2}$$

$$\therefore \quad F_D = 2 \times \left(\frac{1}{2} c_f \rho U^2 A\right)$$

$$= 0.51 \times 10^{-2} \times \frac{118}{9.81} \times 4^2 \times 0.3 \times 1$$

$$= 2.94.10^{-2} \text{ N}$$

7. A truck having a projected area of 6.5 m^2 travelling at 80 km/h experiences a total resistance of 2 kN. Out of this, 25% is due to rolling friction, 15% due to surface friction and the remaining 60% due to form drage. Calculate the coefficient of form drage. Assume specific weight of air as 12 N/m^2.

Solution: Let the total resistance to the motion of the truck be R.

Rolling resistance = 0.25 R = 0.5 kN,

Skin friction = 0.3 kN,

Form drag = 0.6 R = 1.2 kN

$$\rho_{air} = \frac{12}{9.81} = 1.22 \text{ kg/cm}^2$$

$$U_0 = \frac{80 \times 1000}{3600} = 22.2 \text{ m/s}$$

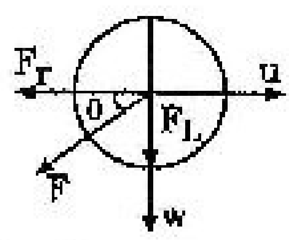
From $$F_D = \frac{C_D.A\rho U_0^2}{2}$$

$$1.2 \times 1000 = C_D \times \frac{6.5 \times 1.22 \times (22.2)^2}{2}$$

$$\therefore \quad C_D = 0.74$$

8. A ping pong ball weighing 0.025 N and having a diameter of 3.5 cm is served in such a manner that it moves over the net in a horizontal direction with a velocity of 15 m/s and forward spin of 90 revolutions/second. Determine the instantaneous magnitude and direction of its acceleration.

Solution:



Circulatory velocity,

$$U_c = \omega r = 2\pi N r$$

$$= 2\pi \times 90 \times \frac{1.75}{100} = 0.98 \text{ m/s}$$

Hence $$\frac{U_c}{U_0} = \frac{9.89}{15} = 0.66$$

$$C_D = 1.0 \text{ and } C_L = 0.8$$

$$F_L = \frac{C_L A\rho U_0^2}{2}$$

$$= 0.8 \times \frac{\pi}{4} \times \frac{3.5}{100^2} \times 1.15 \times \frac{15^2}{2}$$

$$= 0.9953$$

$$F_D = \frac{C_D F_L}{C_L}$$

$$= \frac{1.0 \times 0.09953}{0.8} = 0.1244 \text{ N}$$

$\therefore$ Upward force, $$F_1 = W + F_2$$

$$= 0.025 + 0.09953$$

$$= 0.1245 \text{ N}$$

$$F = \sqrt{F_D^2 + F_1^2}$$

$$= 0.1593\text{N}$$

and $$\theta = \tan^{-1}\frac{F_1}{F_D}$$

$$= \tan^{-1}\left(\frac{0.1245}{0.1244}\right) = 45°$$

Acceleration of the ball $= -\dfrac{F}{\frac{W}{g}} = \dfrac{0.1593}{\frac{0.025}{9.81}} = 62.5 \text{ m/s}^2$

9. Electric transmission towers, 10m high, are fixed 400 m apart to support 10 cables, each of 2 cm diameter, Determine the moment acting at the base of each tower when a wind flows with a velocity of 100 km/h. assume r_{air} = 1.2 kg/m^3 and m(air) = 1.6. 10^{-5} kg/m s. Also calculate the frequency of vortex shedding.

Solution: $$U_0 = \frac{100 \times 1000}{36} = 27.7 \text{ m/s}$$

$$R_e = \frac{\rho U_0 D}{\mu}$$

$$= \frac{1.2 \times 27.7 \times 0.2}{1.6 \times 10^{-5}}$$

$$= 4.1 \times 10^3$$

$$\therefore \quad C_D = 0.95$$

and $$A = L \times D$$

$$= 400 \times 0.02 = 8 \text{ m}^2$$

$\therefore$ Drag on each wire 400 m long,

$$F_D = \frac{C_D A\rho U_0^2}{2}$$

$$= \frac{0.95 \times 8 \times 1.2 \times 27.7^2}{2} = 3.5 \text{ kN}$$

Moments at the base $= F_D \times \dfrac{10}{2}$

$$= 3500 \times 5$$

$$= 17.5 \text{ kN m}$$

Frequency of vortex shedding,

$$f = 0.1989 \frac{U_0}{D}\left(1 - \frac{19.7}{R}\right)$$

$$= 90.198 \times \frac{27.7}{0.02}\left(1 - \frac{197}{4.18 \times 10^3}\right)$$

$$= 272.9\text{Hz}$$