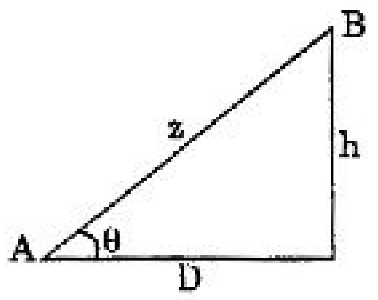
$$Cn = (l_1 + l_2) - (l_1 \cos\theta_1 + l_2 \cos\theta_2)$$
$$= l_1(1 - \cos\theta_1) + l_2(1 - \cos\theta_2)$$

It is always substractive

If A and B are not intervisible, then

$$AB = \sqrt{AC^2 + BC^2 - 2AC.BC\cos\alpha}$$

**(3) Correction for slope**



$$C_S = L - D = L - (L^2 - b^2)l/2$$

$$C_S = \frac{h^2}{2L}$$

It is always substractive 2L

orslope correction = $L - L\cos\theta$

$= L(1 - \cos\theta)$

**(4) Correction for Tension :**

$$C_t = \frac{PL}{AE} - \frac{P_0L}{AE} = \frac{(P - P_0)^2}{AE}$$

where, $P_0$ = standard pull

$P$ = pull applied during measurement

If applied pull in more, tension correction is +ve, and if it is less, tension correction is '–' ve

**Triangulation computation**

(1) Checking the means of the observed angles

(2) Checking the triangular errors

(3) Checking the total round of each station

(4) Computation of the length of the base line

(5) Computation of the sides of the main triangle

(6) Satellite computation if any.

(7) Sag correction $= \frac{L}{24}\left(\frac{W}{P}\right)^2$

where, W = Total weight of the tape

L = Horizontal distance between

and $P = \frac{wL^2}{8h}$ [Since $W = wL$]

(8) **Reduction to M.S.L.** The difference in the length of the measured line and its equivalent length at sea–level, is known as error due to reduction to M.S.L.

Correction due to reduction to M.S.L.

$$L = L - L' = L - \frac{R}{R+h}\cdot L$$

$$= L \times \frac{h}{R+h} \approx L\frac{h}{R}$$

**Reconnance**

Preliminary field inspection of the entire area to be concerned by triangulation is known as *reconnance*.

**It includes the following operations :**

(*i*) Proper examination of the terrian

(*ii*) Selection of suitable position for base lines

(*iii*) Selection of suitable positions of triangulation stations.

(*iv*) Determination of intervisibility of triangulation stations.

**Determination of intervisibility of Stations**

If there is no obstruction due to intervening ground, the distance of visible horizon from the station of known elevation above mean sea level, may be calculated with the formula.

$$h = 0.06735\,D^2$$

- while deciding the intervisibility of various stations, the line of sight should be at least 3 metres above the point of tangency of the earth's surface to avoid grazing sights
- Intervisibility of stations intervened by ground profiles

$$h = \frac{1}{2}(h_2 + h_1) + \frac{1}{2}(h_2 - h_1)\frac{d}{S} - (S^2 - d^2) \times 0.06735$$

**Atmospheric calibaration of instruments**

A typical set of standard conditions used in calibrating an infrared instruments of wave length 0.875 mm are pressure of 760 mm of mercury and a temperature of 12°C. The value of standardising index ($n_s$) is obtained as

$$(n_s - 1) = (n_0 - 1)\left(\frac{273}{T}\right)\left(\frac{P}{360}\right)$$

$$= 10 - 6\left(287.604 + \frac{4.8864}{(0.857)^2} + \frac{0.068}{(0.575)^4}\right)\left(\frac{273}{273+12}\right)\left(\frac{760}{760}\right)$$

$$n_s = 1.000282$$

**Determination of correct distances**

If a line is measure with an E.D.M instruments under atmospheric conditions which are not the same as the standard conditions, the measured distance will be incorrect. The correct distance (D) is obtained from the relation

$$D = D'\left(\frac{n_s}{n}\right)$$

where $D'$ = recorded distance

$n_s$ = standarding refractive index, and

$n$ = refractive inded at the time of measurement

### Slope and Height Correction

Slope correction, $\quad C_g = \dfrac{h^2}{2D}$

where $h$ = elevation differences between the two ends.

Height correction, $\quad C_h = \dfrac{-Dh_m}{R}$

where Hm = mean elevation of the instrument station.

### Effect of Atmospheric conditions

$$\lambda = \frac{V}{f}\left(V = \frac{c}{n}\right)$$

when measuring distances with an E.D.M. instruments, the effect of atmospheric on the wave length should be considered and a suitable correction is applied.

- In E.D.M instruments, the distance is measured in terms of the wave length; whereas in correction in taping, it is measured in terms of the tape length.

### Determination of *n*

(*a*) For instruments using carrier waves

$$(n-1) = (n_0 - 1)\left(\frac{273}{T}\right)\left(\frac{P}{760}\right)$$

where $\quad$ P = atmospheric pressure in mm of Hg.

T = absolute temperature in degree kelvin = 273 + T°

and $\quad n_0$ = refractive index ratio of air at 0°C and 760 mm of Hg.

The value of $n_0$ is obtained as

where $\lambda$ is the wave length of the carrier wave in $\mu$ m.

The above equation are applicable to the E.D.M. instruments using visible light, laser and enforced waves.

(*b*) For instruments using microwave

$$(n-1)\times 10^6 = \frac{103.49}{T}(P-e) + \frac{86.25}{T}\left(1+\frac{5748}{T}\right)\times e$$

where $t_1$ is the total time from A to B and back to A at the transmitter end.

At the receiver end, the reverse process of demodulation occurs in which the meaning wave is separated from the carrier wave and the phase difference of the meaning wave is determined.

The carrier waves used in E.D.M instrument are basically of two forms

(1) Visible or infrared light waves

(2) Microwaves

### Method of Modulation

The meaning wave is superimposed on the carrier wave and this process is called modulation. The following two methods of modulation are used, depending upon the nature of carrier wave.

(*i*) Amplitude modulation (AM.) or intensity modulation

(*ii*) Frequency modulation (F.M.)

**(*i*) Amplitude modulation :** Amplitude modulation is used when light waves are the carrier waves. In this method, the amplitude of the carrier waves is varied in direct proportion to the amplitude of the measuring wave (modulating wave). However the frequency remains constant.

**(*ii*) Frequency modulation :** Frequency modulation is used in microwave instruments. In frequency modulation, the frequency of the carrier wave is varied in proportion to the frequency of the measuring wave. However the amplitude of the carrier wave remains constant,

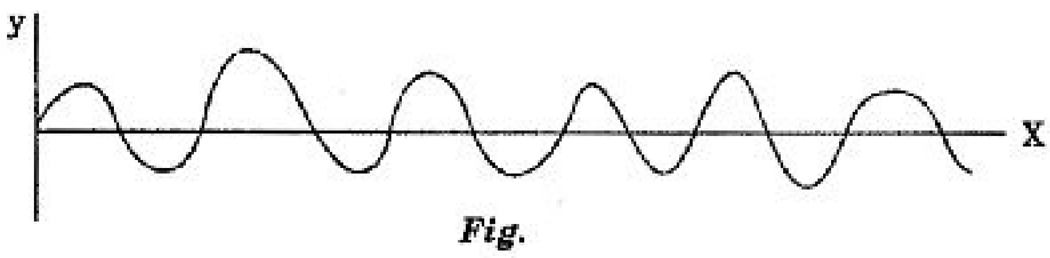



*Fig.*

In general, $\quad 2D = n\lambda + \Delta\lambda$

or $\quad D = \dfrac{1}{2}(n\lambda + 4\alpha)$

where $n$ = number of whole waves travelled by the wave from A to B and back from B to A

$\Delta\lambda$ = fraction of the wave length travelled by the wave from A to B and back from B to A

The value of A X depends upon the phase difference of the wave transmitted and that received back

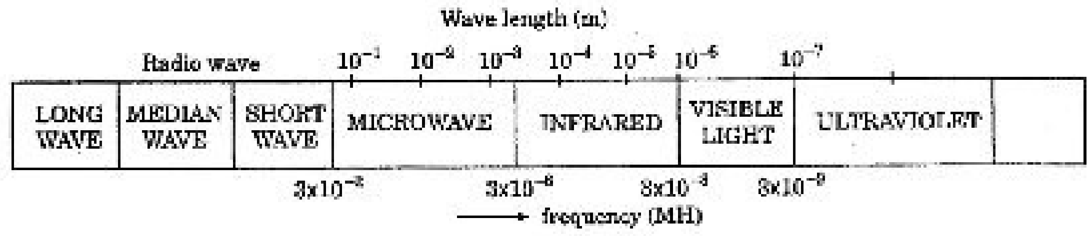
$$\Delta\lambda = \left(\frac{\Delta\phi}{360}\right)\times\lambda$$

where $\Delta\phi$ = phase difference

### Carrier Waves

The electromagnetic waves whose wave lengths are suitable for measuring the distance have the frequency in the range of 500m Hz to 7.5 MHz. This corresponds to the wave lengths in the range of 0.6 to 40 m. These waves are called measuring waves. The measuring waves are also known as *modulation waves.*

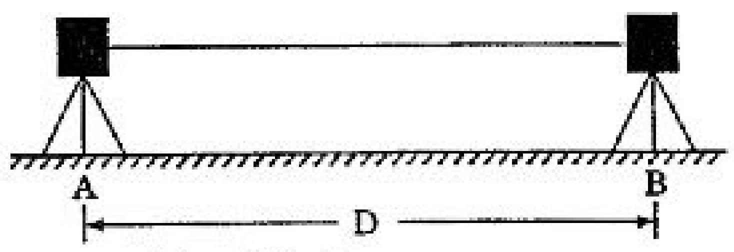
### Types of waves



### Distance from measurement of transit time

Let us consider a survey line AB whose length is to be measured with an E.D.M. instrument. The unit at A is a transmitter which propagets an electromagnetic wave in the direction towards the receiver B.

When the wave is generated at A, an electronic time starts at Bt and as soon as the wave reaches B, the timer stay. Thus the transit time of the wave from A to B is determined. The distance (D) travelled from A to B can be found as




$$D = V \times T$$

where $V$ = velocity of the curve

$t$ = transit time

When the unit at A acts both as a transmitter and a receiver,

$$D = \frac{V \times t_1}{2}$$

### Phase of the wave

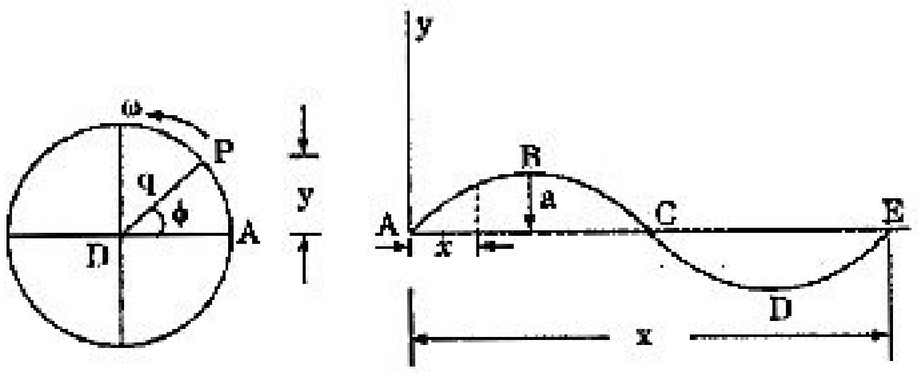



*Fig.*

We assumed that the vector OP is at the initial line when $t = 0$ i.e., $\phi = 0$.

Let us assume that the wave starts from the position A. It sweeps out on angle \$ = rot with respect to the initial line OA in time.

The displacement y along the $y$-axis is given by

$$y = a \sin \phi = a \sin (\omega t) \qquad ...(i)$$

The displacement x along the $x$-axis is given by

$$x = Vt \qquad ...(ii)$$

From equations (*i*) and (*ii*)

$$y = a \sin\left(\omega \times \frac{x}{V}\right) \qquad ...(iii)$$

We know that equation of the electromagnetic wave

$$y = a \sin\left(\frac{2\pi x}{\lambda}\right) \qquad ...(iv)$$

Let AB is a survey line. The wave is transmitted from the unit at A towards B. It is instantly reflected by the reflector at B and then received back to A. Fig (*a*) and (*c*) shows the details of the electromagnetic wave from A to B and back from B to A

From A to B $\quad \Delta\phi = 90° - 0° = 90° = \frac{\lambda}{4} \qquad ...(i)$

Again from B to A, $\Delta\phi = 180 - 90° = 90° = \frac{\lambda}{4}$

$\therefore$ Distance between B and A = $D = 2\lambda \frac{\lambda}{4} \qquad ...(ii)$

The phase difference between the wave at A when transmitted and when received back is

$$\Delta\phi = 180° - 0° = 180° = \frac{\lambda}{4}$$

Unfortunately an electromagnetic wave of the frequency range 500 MHz is to 7.5 MHz to not suitable for direct transmission through the atmosphere. This is due to the effect of interference, reflection, fading and scatter. To overcome the difficulty of direct transmission, the measuring wave is electronically superimposed on one wave called the *carrier wave*. The carrier wave is of much high frequency.

### Infrared E.D.M instrument

In infrared E.D.M. instruments the near infrared radiation bond of wave length $0.9 \times 10^{-6}$ is used as a carrier wave. As the carrier were length is close to the visible light spectrum, the infrared instruments are sometimes classified in *electro-optical instruments*.

This type of E.D.M. instrument can be used upto range limited to 2 to 3 km. The accuracy is usually ± 10 mm

### Microwave E.D.M. Instruments

- The Tellurometer was the first instrument which used microwave in the measurement of distance for surveying purposes.
- The carrier wave frequencies used in the microwave E.D.M. instruments are in the range of $3 \times 10^3$ MHz to $3 \times 10^4$ MHz ($\lambda$ = 0.1 m to 0.01 m). These waves can be transmitted over long distances upto 100 km. The carrier waves are frequency modulated.

The Tellurometer system consists of two identical units, called the master instrument and the remote instrument. As the units are indentical each unit can be switched to operate either of a master unit or remote unit. (This is unlike a geodimeter which has only one instrument) but function is same.

The range of the microwave E.D.M. instruments is generally between 30 to 80 km. The accuracy is of the order of ± 15 mm plus 5mm/km. These instruments are mainly for establishing control for very large projects.

### Electro optical E.D.M. instruments or light wave E.D.M. instruments

The electro optical E.D.M. instruments use light waves as carrier waves. The instruments was called *geodimeter*.

The geodimeter transmits a light beam of wave length $0.55 \times 10^{-6}$ m ($f = 5.5 \times 10^{8}$ MHz) to which a modulation of about 30 MHz ($\lambda$ = 10 m) is applied. The light wave is amplitude modulated. The light beam is directed on to a reflector at the other end of the line which is to be measured. The reflector reflects back the light beam to the instruments. The incoming light beam is received by a photo electric cell which measures the difference between the transmitted and received modulation.

**The geodimeter consists of the following 5 main components**

(*i*) Light source

(*ii*) Electro–optical shutter

(*iii*) Oscillator or modulation generator

(*iv*) Transmitting and receiving optical system.

(*v*) Photo-electric cell.

**Working of a Geodimeter :** A continuous waves carrier beam of light is generated in the transmitter. It is modulated by an electro–optical shutter. It then enters the transmitting optical system and is transmitted to the far end of the line. It is reflected back to the instruments by the reflector. It is received by the receiving optical system and relayed to the photo cell detector for phase comparison.

$$\frac{\omega}{V} = \frac{2\pi}{\lambda}$$

or
$$\omega = \frac{2\pi V}{\lambda}$$

As one revolution is completed in time T, we have

$$\omega T = \left(\frac{2\pi V}{\lambda}\right) T = \frac{2\pi \times \lambda}{\lambda} = 2T \qquad [\lambda = VT]$$

Thus when the radial vector OP completes one revolution the wave travels through a distance equal to the wave length ($\lambda$).

**Meaurement of distance from phase difference**

The phase difference is equal to the difference of the phase angle of the reflected signal and the phase angle of the transmitted signal. Thus

$$\Delta\phi = \phi_2 - \phi_1$$

where $\phi_2$ = phase difference

$\phi_1$ = phase angle of the transmitted signal

$\phi_2$ = phase angle of the reflected signal

**ELECTRONIC DISTANCE MEASUREMENT (E.D.H)**

Electronic distance measuring instrument or electromagnetic distance measuring instrument (called E.D.M. instrument) have been recently developed which give a very high accuracy in distance measurement. An accuracy of 1 in $10^5$ can be achieved without much difficulty for sights upto 100 km. These methods are extremely useful for indirect measurement of distance over difficult terrains and where a high accuracy is erquired, e.g., base line measurment in triangulation, trilateration and precise traverse.

- **The electronics distance measuring instruments can be broad classified into 3-types**

1. Light waves instruments
2. Infra-red waves instruments
3. Micro waves instruments

   The basic theory of all the three types is essentially the same

**Electromagnetic waves**

- The electromagnetic waves require no medium, and they can travel even in vaccum
- The velocity of light in vaccum = $3 \times 10^8$ n/s
- The ratio of the velocity of electromagnetic waves in vaccum to that in atmosphere is called the refractive index ratio ($n$)

Thus
$$n = \frac{C}{V}$$

or
$$V = \frac{C}{n}$$

The value of the refractive index ratio ($n$) is greater than unity. It depends on air temperature, atmospheric condition and relative humidity.

The quantity $\left(\frac{2\pi x}{\alpha} + \phi_0\right)$ is called the *phase angle* or simply *phase of the motion*

At point B phase angle is 90° At point $c$, 180° and at D = 270°, at E phase angle is 360°. As the cycle repeates after E, the phase angle at E may not be taken as 0°.

The distance travelled by the wave corresponding to different values of the phase angles are :

| $\phi$ | 0° | 90° | 180° | 270 | 360 |
|---|---|---|---|---|---|
| Distance | 0 | $\frac{\lambda}{4}$ | $\frac{\lambda}{2}$ | $\frac{3\lambda}{4}$ | $\lambda$ |

As the cycle repeated after $\phi$ = 360°, the total distance travelled can be represented as

$$D = n\alpha + d$$

where $n$ = number of completed waves

$d$ = part of the wave

The port of the wave d can be written in terms of the wave length ($\lambda$) if the phase angle § is known. Because angle of 360° corresponding to one year length ($\lambda$)

$$d = \lambda\left(\frac{\phi}{360}\right)$$

Thus
$$D = n\lambda + \lambda\left(\frac{\phi}{360}\right)$$

**Units**

- The wave length is measured in metres ($m$.)
- The frequency is measured in hertz (Hz) which corresponds to a frequency of 1 cycle per second.
- The unit of time is second ($s$).

(c) If $n$ is the number of observations of an angle and V is the residual error i.e., differences between the mean observed value of the angle and its observation value, then weight of the observation of the angle is given by $\frac{\frac{1}{2}n^2}{\Sigma V^2}$ This rule is known as Gauss's rule.

**Figure adjustment**

Determination of the most probable value of the angles of any geometrical figure to fulfill the geometerical conditions, is known as the figure adjustments. The geometrical figures generally used in a triangulation system are the following.

(i) Triangles

(ii) Quadrilaterals

(iii) centred polygons

**Triangle adjustment :** The distribution of triangular error may be made by one of the following rules :

(a) If the angles of the triangle are of equal wights, then distribute the triangular error equally among the three angles

(b) If the anlges of the triangle are of unequal weights distribute, then the triangular error among the three angles inversely to their respective weights.

$$\text{Correction of angle} \quad x = \frac{\frac{e}{w_1}}{\frac{1}{w_1}+\frac{1}{w_2}+\frac{1}{w_3}+\frac{1}{w_4}}$$

$$\text{Correction of angle} \quad \beta = \frac{\frac{e}{w_2}}{\frac{1}{w_2}+\frac{1}{w_2}+\frac{1}{w_3}+\frac{1}{w_4}}$$

$$\text{Correction of angle} \quad \gamma = \frac{\frac{e}{w_2}}{\left(\frac{1}{w_1}+\frac{1}{w_2}+\frac{1}{w_s}+\frac{1}{w_2}\right)}$$

$$\text{Correction of angles} \quad \delta = \frac{\frac{e}{w_4}}{\frac{1}{w_1}+\frac{1}{w_2}+\frac{1}{w_s}+\frac{1}{w_4}}$$

**(iii) Part horizon having individual and combined angles.**

The probable error of a quantity which is a function of a observed quantity is obtained by multiplying the probable error of the observed quantity by the differentiation of the quantity with respect to the observed quantity.

Let $x$ = observed quantity with $e_x$ p.e.

$y$ = observed quantity with $e_y$ p.e

and $y = f(x)$

then $e_y = \frac{dt}{dx} e_x$

(E) The probable error of the quantity which is a function of more than two observed quantities is equal to the sqaure of the summation of the squares of the probable errors of observed quantity multiplied by its differentiation with respect to quantity.

Let $x_1, x_2, x_3 \ldots\ldots x_n$ be the observed quantities

$y$ = computed quantity

$y = f(x_1, x_2, x_3 \ldots\ldots x_n)$

$$\text{Then } e_y = \sqrt{\left(e_{x1}\frac{dy}{dx}\right)^2+\left(e_{x2}\frac{dy}{dx}\right)^2+\ldots+\left(e_{x_n}\frac{dy}{dx}\right)}$$

whose $e_{x1}, e_{x2}, e_{x3}, \ldots\ldots e_{x_n}$ are the problem error of the observed quantities and ey is the probable error ol the computed quantity.

**Triangulation Adjustment**

The adjustment of triangulation is generally carried out two steps :

(a) Station adjustment

(b) Figure adjustment

**Probable error**

While determining the probable error of observation the following cases may arise

**(A) Direct observations of equal weights**

(a) The probable error of single observations of unit weight

(b) The probable error of any observation of weight of, we

(c) The probable error of arithmetic mean observation

**(B) Direct observations of unequal weights**

**(C) Computed quantities**

**Direct observations of equal weights**

(a) The probable error of single observation weight unit of may be calculated from the following formula.

$$E_s = \pm 0.6745 \sqrt{\frac{\Sigma V^2}{n-1}}$$

where $\Sigma V^2 = V_1^2 + V_2^2 + V_3^2 + \ldots + V_n^2$

V = Residual errors

= Observed value of the quantity – Probable value of the quantity

$n$ = total number of observations

(*b*) The probable error of any observation of weight, we may be calculated from the following formula.

**Station Adjustment :** Depending upon the method of observation the following the cases may arise.

(*i*) A close horizon with angles of equal weights

(*ii*) A close horizon with angles of unequal weights.

(*iii*) Part horizon with angles observed individually and also in combination.

**(*i*) Closed horizon with angles of equal weights.**

Let α, β, γ and δ be the observed angles of

To get the most probable values of the angles

the error of closure is distributed equally

**(*ii*) Closed horizon with angles of unequal wts.**

In this case, the error of closure is distributed among the angles inversely as their weights.

Let $e$ be the closure error

$$\text{Correction of angle } \alpha = \frac{\frac{e}{w_1}}{\frac{1}{w_1}+\frac{1}{w_2}+\frac{1}{w_3}+\frac{1}{w_4}}$$

$$\text{Correction of angle } \beta = \frac{\frac{e}{w_2}}{\frac{1}{w_2}+\frac{1}{w_2}+\frac{1}{w_3}+\frac{1}{w_4}}$$

$$\text{Correction of angle } \gamma = \frac{\frac{e}{w_2}}{\left(\frac{1}{w_1}+\frac{1}{w_2}+\frac{1}{w_s}+\frac{1}{w_2}\right)}$$

**Conditioned quantities**

When the observation are a combination of observation equations, and conditioned equation the M.P.Vs may be obtained by the following two methods.

**(1)** By avoding the conditioned equation and forming the normal equations of the unknowns.

**(2)** By avoiding the observation equations and finding the values of unknown by the correlated.

1. **Methods of Normal equation**
2. **Method of difference :** Let $k_1$, $k_2$, $k_3$ .... be the most probable corrections to the observed angle A, B, C.

The most probable values of A, B and C are

A = observed value + $k_1$

B = observed value + $k_2$

C = observed value + $k_3$

So $k_1$, $k_2$, $k_3$ be the residual error

∴ Residual error = M.P.V.– Observed value

3. **Methods of correlates.**

**Direct observations of unequal weight**

(*a*) The probable error of a single observation of unit wt.

$$E_s = \pm 0.6795\sqrt{\frac{\Sigma w V^2}{n-1}}$$

(*b*) The probable error of any observation of weight $w$

$$E = \frac{E_s}{\sqrt{w}}$$

(*c*) The probable error of weighted arithematic mean

$$E = 1.06745\sqrt{\frac{\Sigma w V^2}{\Sigma w(n-1)}}$$

where, $n$ = number of observations

$w$ = weight of an observation

I$w$ = sum of the weights

I$wr^2$ = sum of the weighted squares of the residual

**(3) The computed quantities.** The probable errors of computed quantities may be calculated by the following laws

(*a*) The probable error of any computed quantities which is either the sum or the difference of the observed quantities plus or minus a constant is always the same as that of observed quantity.

(*b*) The probable error of the multiple of the observed quantity and a constant is obtained by multiplying the computed quantity by the same constant.

(*c*) The probable error of the sum of the observed quantities is equal to the square root of the sum of the squares of the probable errors in the observed quantities.

**Weight of an observation**

The relative precision and trust worthiness of an observation as compared to the precision of other quantities is known as *weight of the observations*. It is always expressd in number. The higher number indicates higher precision as compared to lower number.

**Observation equation.**

The relation between the observed quantity and its numerical values is known as *observation equation*.

**Normal Equation**

The equation which is obtained by multiplying each equation by the coefficient of the unknown whose normal equation is to be found and by adding the equation thus formed is known as normal equation. The number of normal equation is always the same as the number of the unknowns.

**Conditioned equation**

The equation which express the relation existing between the several dependent quantities is known as conditioned equation.

**Laws of weight**

The methods of Least squares for errors adopts the following laws of weights

1. The weight of the arithmetic mean of a number of observation of unit weight is equal to the no. of the observations.
2. The weight of the weighted arithmetic mean of a numbers of observation is equal to the sum of the individual weight of observations

**In this method, following steps are involved**

(1) Assume suitable correction for each observed quantities $e_1, e_2, e_3 \ldots e_n$

(2) Enter all the conditions equation i.e.,

$$e_1 + e_2 + e_3 + e_4 = E$$
$$e_2 + e_3 = F$$

(3) Add the equation of conditions as suggested by the theory of least squares

i.e., $w_1e_1^2 + w_2e_2^2 + \ldots = a$ minimum

(4) Differentiate each conditioned equation and the equation of least squares separately.

(5) Multiply each differentiated equation of conditions by correlates $-\alpha_1, -\alpha_2, -\alpha_3, \ldots \lambda_n$ etc and add the result to the differentiated equation of the least squares.

(6) Obtain the co-efficients of $\delta e_1, \delta e_2, \ldots \delta e_n$ and equate each to zero to get the values of $e_1, e_2, e_3, \ldots e_n$.

(7) Substitute the values of $e_1, e_2, e_3$ ... etc in the condition equation and solve the simultaneous equation thus formed to obtain the values of correlates.

(8) Knowing the values of correlates and the weight, the values of $e_1, e_2, e_3$ ... etc can be calculated

**UNEQUAL WEIGHTS**

To form the normal equations, "multiplied each observation equation by the product of the algebraic coefficient of that unknown quantity in the equation and the weight of that observation and add the equation thus formed"

(*a*) It may be left of C

(*b*) It may lie right of C

(*c*) It may be lie below C

(*d*) It may lie above C

**(7) Computation of Latitude and Departures.**

Let the bearing of side CA is knwon

$\therefore$ Bearing of side AC = Bearing of CA – 180

Bearing of side AB = Bearing of AC + $\angle$CAB

Bearing of BC = Bearing of BA + $\angle$ABC

Knowing the co-ordinate of A, the co-ordinate of B and C be computed as under

Latitude of C = AC × cosine of the R.B of the (Northing or Southing)

Departure of C = AC × sine of the R.B of AC (Easting or Westing)

(8) The weight of the quotient of any quantity divided by a constant is equal to the weight of the quantity multiplied by the square of that constant.

(9) The weight of an equation by multiplying the weight of the given equation is equal to the reciprocal of the weight of the given equation.

(10) The weight of an equation remain unchanged if all the signs of the equation are changed or if the equation is added to an subtracted from a constant.

- The arithmatic mean is the most probable value of number of observations of equal wts.

**Most probable values of directly observed quantities.**

***(i)*** **Equal weights.** The most probable of the observed quantity is equal to the arithmetic mean of the observed values of the quantity.

$$\text{M.P.V(X)} = \frac{X_1 + X_2 + X_3 \ldots + X_n}{n}$$

where $X_1, X_2, X_3$ ... are the observed values of a quantity

***(ii)*** **Different weights.** The most probable value of the observed quantity is equal to the weighted arithmetic mean of the observed values of the quantity.

$$\text{M.P.V(X)} = \frac{w_1X_1 + w_2X_2 + w_3X_3 \ldots + w_nX_n}{w_1 + w_2 + w_3 + \ldots + w_n}$$

**Residual errors**

The difference between any measured quantity and the most probable value of that quantity is known of residual error.

**Normal tension**

For normal tension correction due to pull and correction due to sag is neutralises normal tension P can be found

$$\frac{(P - P_0)L}{AE} = \frac{L\,W^2}{24\,P^2}$$

### Measurement of horizontal angles

The horizontal angle of a triangulation system can be observed by the following methods

(1) The Repetition method

(2) The Reiteration method

## THEORY OF ERRORS AND ADJUSTMENT

**Broadly.** We may divide various errors into the following three main categories.

**(1) Mistakes.** Arise due to carelessness in attention and confusion by inexperienced workers.

**(2) Systemtie errors.** These errors which always follow some definite mathematical and physical laws are termed as systematic errors.

*e.g.,* wrong length of a chain, atmospheric refraction

**(3) Accidental or random errors or compensating erros.** Those errors which remain even after eliminating mistakes and systematic errors are caused due to a combination of various reasons beyond the control of the observers are termed as accidental errors. Random errors are generally small and can never be avoided.

### Definition

**True value.** The value of a quantity which is absolutely free from all types of errors is termed as the value. Which is never known.

**Observed value.** The value of a quantity which is obtained after correcting it for known errors is known as observed value. Observed quantity may further classified in

(*a*) independent quantity

e.g., R.L of each mark

(*b*) conditioned quantity

**True error.** True Value– Observed value.

**Most probable value.** It is that value of a quantity which has more chances of being true.

**Most probable error.** It may be defined as the quantity which is subtracted from or added to the most probable value.

**Satellite computation.** Observation are taken at the satellite station

According to the position of satelite station (S) with regard to the side of AC and EC, the following four cases may arises

1. The triangulation stations which are used only to provide additional rays to intersect points are known as subsidiary stations.
2. The stations which are selected close to the main triangular stations to avoid intervening obstructor, are known as satellite stations or eccentric or false stations.
3. The stations at which no obstructions are made but the angles of these stations are used for the continuity of a triangular series, are known as pivot stations.

### Triangulation computation

(1) Checking the means of the observed angles

(2) Checking the triangular errors

(3) Checking the total round of each station

(4) Computation of the length of the base line

(5) Computation of the sides of the main triangle

(6) Satellite computation if any.

(7) Sag correction $= \dfrac{L}{24}\left(\dfrac{W}{P}\right)^2$

where, W = Total weight of the tape

L = Horizontal distance between

and $P = \dfrac{wL^2}{8h}$ [Since W = wL]

(8) **Reduction ot M.S.L.** The difference in the length of the measured line and its equivalent length at sea–level, is known as error due to reduction to M.S.L.

Correction due to reduction to M.S.L.

$$L = L - L' = L - \frac{R}{R+h}\cdot L$$

$$= L \times \frac{h}{R+h} = L\frac{h}{R}$$

**(4) Correction for Tension :**

$$C_t = \frac{PL}{AE} - \frac{P_0 L}{AE} = \frac{(P - P_0)^2}{AE}$$

where, $P_0$ = standard pull

P = pull applied during measurement

If applied pull in more, tension correction is +ve, and if it is less, tension correction is '–'ve

**II. When the observation is made on the Bright line**

$$\angle SCO = 180 - (\theta - \beta)$$

$$\angle PCO = 180 - 90 + \frac{\theta - \beta}{2} = 90 + \frac{\theta - \beta}{2}$$

$$\angle CPO = 180 - \left(\beta + 90 + \frac{\theta - \beta}{2}\right) = 90 - \left(\frac{\theta + \beta}{2}\right)$$

$$= 90 - \frac{\theta}{2}, \text{ignoring } \beta$$

$$\tan\beta = \frac{r\sin\left(\dfrac{90 - \theta}{2}\right)}{D - r\cos\left(\dfrac{90^\circ - \theta}{2}\right)} = \frac{\dfrac{r\cos\theta}{2}}{D} \text{ radians}$$

$$\beta = \frac{\dfrac{r\cos\theta}{2}}{D\sin 1''} \text{ seconds}$$

The phase correction is applied algebraically to the observed angle according to the relative positions of the sun and the signal, i.e., it is the +ve and zero.

**Measurement of Base line**

The accuracy of any order of triangulation depends upon the accuracy of the measurement of its base line. Hence in triangulation a base line is of prime importance.

**Selection of the site for a Base line**

1. The ground of the sight should be fairly level or uniformal.

**Number of zeros :** To eliminate the error due to inaccurate graduations of the horizontal circle, the measures of the horizontal angles are taken in two or three different zones,

i.e., Face $\angle$ 90°, Face L 180° Face L 270°.

**Types of Triangulation stations**

(*i*) Main station

(*ii*) Subsidiary stations

(*iii*) Satellite stations or Eccentric or false stations

(*iv*) Pivot stations

**Signals and Towers**

The signals may be classified as

(*a*) Luminous signal, and

(*b*) Opaque signals or non–luminous signal

- Base line is measured by standard tape and after that following connections are applied to the measured base length.

**(1) Correction for standard length**

$$\frac{C_a}{L} = \frac{C}{L}$$

$$\Rightarrow \quad C_a = \frac{CL}{l}$$

Where, L = measured length of the line

$l$ = designed length of the tape

$Ca$ = correction for absolute length

C = correction to be applied to the tape.

The sign of the correction $C_a$ will be the same as that of C.

**(2) Correction of Alignment**

$$Cn = (l_1 + l_2) - (l_1 \cos\theta_1 + l_2 \cos\theta_2)$$
$$= l_1(1-\cos\theta_1) + l_2(1-\cos\theta_2)$$

It is always substractive If A and B are not intervisible, then

$$AB = \sqrt{AC^2 + BC^2 - 2AC.BC\cos\alpha}$$

**(3) Correction for slope**

$$C_S = L - D = L - \left(L^2 - b^2\right)\frac{1}{2}$$

$$C_S = \frac{h^2}{2L}$$

It is always substractive

or slope correction = L – L cos θ

$$= L(1-\cos\theta)$$

$$\Delta\,\text{OAP}, \quad \tan\alpha_2 = \frac{r}{D}$$

$$\text{or} \quad \alpha_2 = \frac{r}{D}\text{ radians}$$

$$\text{and} \quad \alpha_1 = \frac{r\sin(90-\theta)}{D} = \frac{r\cos\theta}{D}$$

$$\therefore \quad B = \frac{1}{2}\left[\frac{r}{D} + \frac{r\cos D}{D}\right]$$

$$= \frac{r}{2D}(1+\cos\theta)$$

$$= \frac{\frac{r\cos^2\theta}{2}}{D}\text{ radians}$$

$$B = \frac{\frac{r\cos^2\theta}{2}}{DHn1''}\text{ seconds}$$

where, $r$ = radius of the signal

θ = angle between the sun and the line OP.

**Reconnance**

Preliminary field inspection of the entire area to be concerned by triangulation is known as reconnance.

**It includes the following operations :**

(*i*) Proper examination of the terrian

(*ii*) Selection of suitable position for base lines

(*iii*) Selection of suitable positions of triangulation stations.

(*iv*) Determination of intervisibility of triangulation stations.

**Determination of intervisibility of stations**

If there is no obstruction due to intervening ground, the distance of visible horizon from the station of known elevation above mean sea level, may be calculated with the formula.

$$h = 0.06735\,D^2$$

- while deciding the intervisibility of various stations, the line of sight should be at least 3 metres above the point of tangency of the earth's surface to avoid grazing sights
- **Intervisibility of stations intervened by ground profiles**

$$h = \frac{1}{2}(h_2 + h_1) + \frac{1}{2}(h_2 - h_1)\frac{d}{S} - \left(S^2 - d^2\right)\times 0.06735$$

# SOLVED EXAMPLES

**1.** The following bearings were observed with a compass. Calculate the interior angles :

| Line | Fore Bearing |
|---|---|
| AB | 60°30′ |
| BC | 122V |
| CD | 46°0′ |
| DE | 205 °30′ |
| EA | 300° |

**Solution.** Included angle = Bearing of previous line – Bearing of next line

$\angle$A = Bearing of AE – Bearing of AB
= (300° – 180°) – 60° 30′ = 59° 30′

$\angle$B = Bearing of BA – Bearing of BC
= (60° 30′ + 180°) – 122° = 118° 30′

$\angle$C = Bearing of CB – Bearing of CD
= (122° + 180°) – 46° = 256°

$\angle$D = Bearing of DC – Bearing of DE
= (46° + 180°) – 205° 30′ = 20° 30′

$\angle$E = Bearing of ED – Bearing of EA
= (205° 30′ – 180°) – 300° + 360°
= 85° 30′

**Check :** $(2n - 4)90° = (10–4)90° = 540°$ = sum of all the angles, hence correct

**2.** In a closed travers, "latitudes" and "departures" of the sides were calculated and it was observed that

$\Sigma$ latitude =1.39;

$\Sigma$ Departure = – 2.17

Calculate the length and bearing of the closing error.

**Solution.** From $\Sigma$ latitude and Z Departure readings,

$$\therefore \text{ Closing error, } e = \sqrt{\left(\Sigma L^2\right)+\left(\Sigma D^2\right)}$$

$$= \sqrt{(1.39)^2 + (-2.17)^2} = 2.577 \text{ m}$$

Reduced bearing of closing line

$$= \tan^{-1}\left(\frac{2.17}{1.39}\right) = 57°21'22.82''$$

As the latitude and departure of closing line are + ve, and – ve, the closing line will lie in fourth quardant

R.B. of closing line = N 57° 21′ 22.82″ W

**3.** For a railway project, straight tunnel is to be run between two points P and Q whose coordinates are given below :

| Point | Coordinates | |
|---|---|---|
| | N | F |
| P | 0 | 0 |
| Q | 4020 | 800 |
| R | 2110 | 1900 |

It is desired to sink a shaft at S, the midpoint of PQ. S is to be fixed from R, the third known point. Calculate.

(*i*) the coordinates of S

(*ii*) length of RS

(*iii*) the bearing of RS.

**Solution.** The coordinates of the shaft S

$$\text{Northing} = \frac{\text{North of P} + \text{Northing of Q}}{2}$$

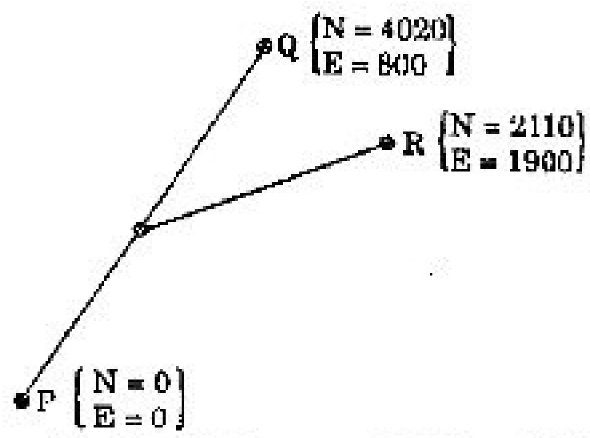
$$= \frac{0 + 4020}{2} = 2010 \text{ m}$$

$$\text{Easting} = \frac{\text{Easting of P} + \text{Easting of Q}}{2}$$

$$= \frac{0 + 800}{2} = 400 \text{ m}$$

Coordinates of S are: N = **2010 m,** E = **400 m**

**Length of RS**




Difference in Northings = 2110 – 2010 = 100 m

Difference in Easting = 1900 – 240 = 1500 m

$$\therefore \quad \text{Distance RS} = \sqrt{100^2 + 1500^2} = 1503.33 \text{ m}$$

Let reduced bearing of RS be 0, then

$$\tan\theta = \frac{\Delta E}{\Delta N} = 15$$

or, $\theta = \tan^{-1} 15 = 86°11'09$

$\therefore$ Whole circle bearing of RS
= 180° + 86° 11′09
= **266° 11′09″**

**4.** The following consecutive readings were taken with a level and 5 metre levelling staff on continuously sloping ground at a common interval of 25 metres :

0.450, 1.120, 2.905, 3.685, 0.520, 2.150, 3.205 and 4.485.

The reduced level of the change point was 250.000. Calculate the reduced levels of the points by rise and fall method and also the gradient of the line joining the first and last point in a regular format of the level field book.

**Solution.**

| B.S. | I.S. | F.S. | Rise | Full | R.I. | Remarks |
|---|---|---|---|---|---|---|
| 0.450 | | | | | 250.050 | |
| | 1.20 | | | 0.670 | 253.380 | |
| | 1.875 | | | 0.755 | 252.625 | |
| | 2.905 | | | 1.030 | 251.595 | |
| | 3.685 | | | 0.780 | 250.815 | |
| 0.520 | | 4.500 | | 1.815 | 250.000 | Change point |
| | 2.150 | | | 1.630 | 248.370 | |
| | | | | 1.055 | 247.315 | |
| | | 4.485 | | 1.280 | 246.035 | |
| Sum = 0.970 | | 8.985 | 0.000 | 8.015 | | |

**Arithmetical Check :**

$$\Sigma\text{B.S} - \Sigma\text{F.S.} = \Sigma\text{ Rise} - \Sigma\text{ Fall}$$

$$= \text{R.L. of last point} - \text{R.L. of first point}$$

$$= 0.970 - 8.895 = -8.015$$

$$= 246.035 - 250.0500$$

$$= -8.015$$

**5.** A road embankment 40 m wide at formation level with side slopes 1 to 1 and with an average height of 15 m is constructed with an average gradient of 1 in 40 from contour 150 m to 590 m. The ground has an average slope of 10 to 1 in direction transverse to the centre line. Find :

(*i*) Length of the road

(*ii*) Volume of embankment in cubic metres, and

(*iii*) Cost of the earth work © ₹ 100 per 1000 cubic metres

**Solution.**

(*i*) Along the length of the road, the gradient = 1 in 40

$$\text{Contours difference} = 590 - 150 = 440 \text{ m}$$

$$\therefore \quad \text{Length of road} = 440 \times 40 = \mathbf{17600\ m}$$

(*ii*) The section is two level section

$$\text{Area,} \quad A = \left[\frac{r^2 s}{r^2 - s}\left(H + \frac{B}{2S}\right) - \frac{B^2}{4S}\right]$$

where r = 10, S = 1, H = 15 m, $b$ = 40 m

$$A = \left[\frac{10^2 \times 1}{10^2 - 1}\left(15 + \frac{40}{2 \times 1}\right) - \frac{40^2}{4 \times 1}\right]$$

$$= 837.374\ m^2$$

$\therefore$ Volume of embankment,

$$V = \text{Length} \times \text{Area}$$

$$= 17600 \times 837.374$$

$$= \mathbf{14737782\ m^3}$$

(*iii*) Cost of earth work @ ₹ 100 per 1000

$$\text{cubic metres} = \frac{100 \times 14737782}{1000}$$

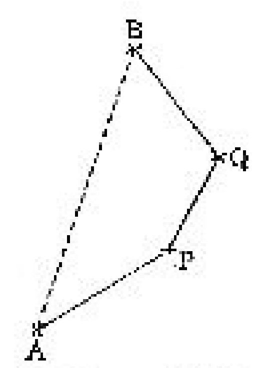
$$= ₹\ 1473778.2$$

**6.** The following observations were taken from stations P and Q .

| Line | Length (m) | Bearings |
|---|---|---|
| PA | 125.0 | S60°30'W |
| PQ | 200.0 | N30°30'E |
| QB | 150.5 | N50°15'W |

Calculate the length and bearing of AB, and also the angles ∠PAB and ∠QBA.

**Solution.** Let the figure be rearranged as a closed traverse BQPAB. The reduced bearings of BQ and QP will be numerically equal to that of QB and PQ, but their respective quandrants will be opposite, i.e.,




RB of BQ = S 50°15′ E

RB of QP = S 30° 30′ W