Exercise 3.1

1.) Which of the following numbers are perfect squares?

(i) 484

$484 = 22^2$

(ii) 625

$625 = 25^2$

(iii)576

$576 = 24^2$

(iv) 941

Perfect squares closest to 941 are 900 (30^2) and 961 (31^2). Since 30 and 31 are consecutive numbers, there are no perfect squares between 900 and 961. Hence, 941 is not a perfect square.

(v) 961

$961 = 31^2$

(vi)2500

$2500 = 50^2$

Hence, all numbers except that in (iv), i.e. 941, are perfect squares.

2.) Show that each of the following numbers is a perfect square. Also, find the number whose square is the given number in each case:

Answer:

In each problem, factorize the number into its prime factors.

(i) **1156** = 2 x 2 x 17 x 17

Grouping the factors into pairs of equal factors, we obtain:

1156 = (2 x 2) x (17 x 17) No factors are left over. Hence, 1156 is a perfect square. Moreover, by grouping 1156 into equal factors:

1156 = (2 x 17) x (2 x 17) = $(2 \times 17)^2$

Hence, 1156 is the square of 34, which is equal to 2 x 17.

(ii) **2025** =3x3x3x3x5x5

Grouping the factors into pairs of equal factors, we obtain:

2025 = (3 x 3) x (3 x 3) x (5 x 5)

No factors are left over. Hence, 2025 is a perfect square. Moreover, by grouping 2025 into equal factors:

2025 = (3 x 3 x 5) x (3 x 3 x 5) = $(3 \times 3 \times 5)^2$

Hence, 2025 is the square of 45, which is equal to 3 x 3 x 5.

(iii) **14641** = 11 x 11 x 11 x 11

Grouping the factors into pairs of equal factors, we obtain:

14641 = (11 x 11) x (11 x 11)

No factors are left over. Hence, 14641 is a perfect square. The above expression is already grouped into equal factors:

14641 = (11 x 11) x (11 x 11) = (11 x 11)2 Hence, 14641 is the square of 121, which is equal to 11 x 11.

(iv) **4761** = 3 x 3 x 23 x 23

Grouping the factors into pairs of equal factors, we obtain:

4761 = (3 x 3) x (23 x 23)

No factors are left over. Hence, 4761 is a perfect square. The above expression is already grouped into equal factors:

4761 = (3 x 23) x (3 x 23) = $(3 \times 23)^2$

Hence, 4761 is the square of 69, which is equal to 3 x 23.

3.) Find the smallest number by which of the following number must be multiplied so that the product is a perfect square:

Answer:

Factorize each number into its factors

(i) **23805** = 3 x 3 x 5 x 23 x 23

3	23805
3	7935
5	2645
23	529
23	23
	1

Grouping 23805 into pairs of equal factors:

23805 = (3 x 3) x (23 x 23) x 5

Here, the factor 5 does not occur in pairs. To be a perfect square, every prime factor has to be in pairs. Hence, the smallest number by which 23805 must be multiplied is 5.

(ii) 12150 = 2 x 3 x 3 x 3 x 3 x 3 x 5 x 5

2	12150
3	6075
3	2025
3	675
3	225
3	75
5	25
5	5
	1

Grouping 12150 into pairs of equal factors:

12150 = (3 x 3 x 3 x 3) x (5 x 5) x 2 x 3

Here, 2 and 3 do not occur in pairs. To be a perfect square, every prime factor has to be in pairs.

Hence, the smallest number by which 12150 must be multiplied is 2 x 3, i.e. by 6.

(iii) 7688 = 2 x 2 x 2 x 31 x 31

2	7688
2	3844
2	1922
31	961
31	31
	1

Grouping 7688 into pairs of equal factors:

7688 = (2 x 2) x (31 x 31) x 2

Here, 2 do not occur in pairs. To be a perfect square, every prime factor has to be in pairs. Hence the smallest number by which 7688 must be multiplied is 2.

4.) Find the smallest number by which the given number must be divided so that the resulting number is a perfect square:

Answer:

For each question, factorize the number into its prime factors.

(i) 14283 = 3 x 3 x 3 x 23 x x23

3	14283
3	4761
3	1587
23	529
23	23
	1

Grouping the factors into pairs:

14283 = (3 x 3) x (23 x 23) x 3

Here, the factor 3 does not occur in pairs. To be a perfect square, all the factors have to be in pairs. Hence, the smallest number by which 14283 must be divided for it to be a perfect square is 3.

(ii) **1800** = 2 x 2 x 2 x 3 x 3 x 5 x 5

2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

Grouping the factors into pairs:

1800 = (2 x2) x (3 x 3) x (5x 5) x2

Here the factor 2 does not occur in pairs. To be a perfect square, all the factors have to be in pairs. Hence, the smallest number by which 1800 must be divided for it to be a perfect square is 2.

(iii) **2904** = 2 x 2 x 2 x3 x 11 x 11

2	2904
2	1452
2	726
3	363
11	121
11	11
	1

Grouping the factors into pairs:

2904= (2 x2) x (11 x 11) x 2 x 3

Here the factor 2 and 3 does not occur in pairs. To be a perfect square, all the factors have to be in pairs. Hence, the smallest number by which 2304 must be divided for it to be a perfect square is 2 x 3, i.e. 6.

5.) Which of the following numbers are perfect squares?

Answer:

11: The perfect squares closest to 11 are 9 ($9 = 3^2$) and 16 ($16 = 4^2$). Since 3 and 4 are consecutive numbers, there are no perfect squares between 9 and 16, which mean that 11 is not a perfect square.

12: The perfect squares closest to 12 are 9 ($9 =3^2$) and 16 ($16 = 4^2$). Since 3 and 4 are consecutive numbers, there are no perfect squares between 9 and 16, which mean that 12 is not a perfect square.

$16 = 4^2$

32: The perfect squares closest to 32 are 25 (25 = 52) and 36 (36 = 62). Since 5 and 6 are consecutive numbers, there are no perfect squares between 25 and 36, which means that 32 is not a perfect square.

$36 = 6^2$

50: The perfect squares closest to 50 are 49 (49 = 72) and 64 (64 = 82). Since 7 and 8 are consecutive numbers, there are no perfect squares between 49 and 64, which means that 50 is not a perfect square. $64 = 8^2$

79: The perfect squares closest to 79 are 64 (64 = 82) and 81 (81 = 92). Since 8 and 9 are consecutive numbers, there are no perfect squares between 64 and 81, which mean that 79 is not a perfect square.

81 = 92

111: The perfect squares closest to 111 are 100 (100 = 102) and 121 (121 = 112). Since 10 and 11 are consecutive numbers, there are no perfect squares between 100 and 121, which means that 111Is not a perfect square.

$121 = 11^2$

Hence, the perfect squares are 16, 36, 64, 81 and 121.

6.) Using prime factorization method, find which of the following numbers are perfect squares?

(i) 189 = 3 x 3 x 3 x 7

3	189
3	63
3	21
7	7
	1

Grouping them into pairs of equal factors:

189 = (3 x 3) x 3 x 7

The factors 3 and 7 cannot be paired. Hence, 189 is not a perfect square.

(ii) 225 = 3 x 3 x 5 x 5

3	225
3	75
5	25
5	5
	1

Grouping them into pairs of equal factors:

225 = (3 x 3) x (5 x 5)

There are no left out of pairs. Hence, 225 is a perfect square.

(iii) 2048 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Grouping them into pairs of equal factors:

2048 = (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x 2

The last factor, 2 cannot be paired.. Hence, 2048 is a perfect square.

(iv) 343 = 7 x 7 x 7

7	343
7	49
7	7
	1

Grouping them into pairs of equal factors:

343 = (7 x 7) x 7

The last factor, 7 cannot be paired. Hence, 343 is not a perfect square.

(v) 441 = 3 x 3 x 7 x 7

3	441
3	147
7	49
7	7
	1

Grouping them into pairs of equal factors:

441 = (3 x 3) x (7 x 7)

There are no left out of pairs. Hence, 441 is a perfect square.

(vi) 2916 = 2 x 2 x 3 x 3 x 3 x 3 x 3 x 3 x 3

2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Grouping them into pairs of equal factors:

2916 = (2 x 2) x (3 x 3) x (3 x 3) x (3 x 3)

There are no left out of pairs. Hence, 2916 is a perfect square.

(vii) 11025= 3 x 3 x5 x7 x 7

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

Grouping them into pairs of equal factors:

11025= (3 x 3) x (5 x 5) x (7 x 7)

There are no left out of pairs. Hence, 11025 is a perfect square.

(viii) 3549 = 3 x 7 x 13 x 13

3	3549
7	1183
13	169
13	13
	1

Grouping them into pairs of equal factors:

3549 = (13 x 13) x 3 x 7

The last factors, 3 and 7 cannot be paired. Hence, 3549 is not a perfect square.

Hence, the perfect squares are 225, 441, 2916 and 11025.

7.) By what number should each of the following numbers be multiplied to get a perfect square in each case? Also, find the number whose square is the new number.

Factorizing each number

(i) 8820 = 2 x 2 x 2 x 3 x 5 x 7 x 7

2	8820
2	4410
3	2205
3	735
5	245
7	49
7	7
	1

Grouping them into pairs of equal of equal factors:

8820= (2 x 2) x (3 x 3) x (7 x 7) x 5

The factor, 5 is not paired. For a number to be a perfect square, each prime factor has to be paired.

Hence, 8820 must be multiplied by 5 for it to be a perfect square.

The new number would be (2×2) x (3 x 3) x (7 x7) x (5 x 5).

Furthermore, we have:

(2 x 2) x (3 x 3) x (7 x 7) x (5 x 5)= (2 x 3 x 5 x 7) x (2 x 3 x 5 x 7)

Hence, the number whose square is the new number is:

2 x 3 x5 x 7 = 210

(ii) 3675 = 3 x 5 x 5 x 7 x 7

3	3675
5	1225
5	245
7	49
7	7
	1

Grouping them into pairs of equal factors:

3675 = (5 x 5) x (7 x 7) x 3

The factor 3 is not paired. For a number to be the perfect square, each prime factor has to be paired.

Hence, 3675 must be multiplied by 3 for it to be a perfect square.

The new number would be (5 x 5) x (7 x 7) x (3 x 3).

Furthermore, we have:

(5 x 5) x (7 x 7) x (3 x 3) = (3 x 5 x 7) x (3 x 5 x 7)

Hence, the number whose square is the new number is:

3 x 5 x 7 = 105

(iii) 605 = 5 x 11 x 11

5	605
11	121
11	11
	1

Grouping them into pairs of equal factors:

605 =5 x (11 x 11)

The factor 5 is not paired. For a number to be perfect square, each prime factor has to be paired.

Hence, 605 must be multiplied by 5 for it to be a perfect square.

The new number would be (5 x 5) x (11 x 11)

Furthermore, we have:

(5 x 5) x (11 x 11) = (5 x 11) x (5x 11)

Hence, the number whose square is the new number is:

5 x 11 = 55

(iv) 2880 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 5

2	2880
2	1440
2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

Grouping them into pairs of equal factors:

2880 = (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) x 5

There is a 5 as the leftover. For a number to be a perfect square, each prime factor has to be paired.

Hence, 2880 must be multiplied by 5 to be a perfect square.

The new number would be (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) x (5 x 5).

Furthermore, we have:

(2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) x (5 x 5) = (2 x2x2x3x 5) x (2 x2x2x3x 5) Hence, the number whose square is the new number is:

2x2x2x3x5= 120

(v) 4056 =2 x 2 x 2 x 3 x 13 x 13

2	4056
2	2028
2	1014
3	507
13	169
13	13

Grouping them into pairs of equal factors:

4056 = (2 x 2) x(13 x 13) x 2 x 3

The factors at the end, 2 and 3 are not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 4056 must me multiplied by 6 (2 x 3) for it to be a perfect square.

The new number would be (2 x 2) x (2 x2) x (3 x 3) x (13 x 13).

Furthermore, we have

(2 x 2) x (2 x2) x (3 x 3) x (13 x 13) = (2 x 2 x 3 x 13) x (2 x 2 x 3 x13)

Hence, the number whose square is the new number is:

2 x 2 x 3 x13 = 156

(vi) 3468 = 2 x 2 x 3 x 17 x 17

2	3468
2	1734
3	864
17	289
17	17
	1

Grouping them into pairs of equal factors:

3468 = (2 x 2) x (17 x 17) x 3

The factor at the end, 3 is not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 3468 must me multiplied by 3 for it to be a perfect square.

The new number would be (2 x 2) x (17 x 17) x (3 x 3).

Furthermore, we have

(2 x 2) x (17 x17) x (3 x3) = (2 x 3 x 17) x (2 x 3 x17)

Hence, the number whose square is the new number is:

2 x 3 x17 = 102

(viii) 7776 = 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 x 3

2	7776
2	3888
2	1944
2	972
2	486
3	243
3	81
3	27
3	9
3	3
	1

Grouping them into pairs of equal factors:

7776= (2 x 2) x (2 x 2) x (3 x 3) x (3 x 3) x 2 x 3

The factor at the end, 2 and 3 are not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 7776 must me multiplied by 6 (2 x 3) for it to be a perfect square.

The new number would be (2 x 2) x (2 x 2) x (2 x 2)(3 x 3) x (3 x 3) x (3 x 3) .

Furthermore, we have

(2 x 2) x (2 x 2) x (2 x 2) (3 x 3) x (3 x 3) x (3 x 3)= (2 x 2 x 2 x 3 x 3 x 3) x (2 x 2 x 2 x 3 x 3 x 3)

Hence, the number whose square is the new number is:

2 x 2 x 2 x 3 x 3 x 3 = 216

8.) By what numbers should each of the following be divided to get a perfect square in each case? Also, find the number whose square is the new number.

Answer:

Factorizing each number

(i) 16562 = 2 x 7 x 7 x 13 x13

2	16562
7	8281
7	1183
13	169
13	13
	1

Grouping them into pairs of equal factors:

16562= 2 x(7 x 7) x (13 x 13)

The factor at the end, 2 is not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 16652 must me multiplied by 2 for it to be a perfect square.

The new number would be (7x 7) x (13 x13).

Furthermore, we have

(7x 7) x (13 x13) = (7 x 13) x (7 x 13)

Hence, the number whose square is the new number is:

7 x 13= 91

(ii) 3698 = 2 x 43 x 43

2	3698
43	1849
43	43
	1

Grouping them into pairs of equal factors:

3698 = 2 x (43 x 43)

The factor at the end, 2 is not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 3698 must me multiplied by 2 for it to be a perfect square.

The new number would be (43 x43)

Hence, the number whose square is the new number is 43.

(iii) 5103 = 3 x 3 x 3 x 3 x 3 x 3 x 7

3	5103
3	1701
3	567
3	189
3	63
3	21
7	7
	1

Grouping them into pairs of equal factors: 5103 = (3 x 3) x (3 x 3) x (3 x 3) x 7

The factor, 7 is not paired. For a number to be a perfect square, each prime factor has to be paired.

Hence, 5103 must be divided by 7 for it to be a perfect square. The new number would be (3 x 3) x (3 x 3) x (3 x 3).

Furthermore, we have: (3 x 3) x (3 x 3) x (3 x 3) = (3 x 3 x 3) x (3 x 3 x 3) Hence, the number whose square is the new number is:

3 x 3 x 3=27

(iv) 3174= 2 x 3 x 23 x 23

2	3174
	1587
	529
	23
	1

Grouping them into pairs of equal factors:

3174 = 2 x 3 x (23 x 23)

The factors, 2 and 3 are not paired.

For a number to be a perfect square, each prime factor has to be paired.

Hence, 3174 must be divided by 6 (2 x 3) for it to be a perfect square.

The new number would be (23 x 23).

Hence, the number whose square is the new number is 23.

(v) 1575= 3 x3 x 5 x 7

3	1575
3	525
5	175
5	35
7	7
	1

Grouping them into pairs of equal factors:

1575 = (3 x 3) x (5 x 5) x 7

The factor, 7 is not paired. For a number to be a perfect square, each prime factor has to be paired.

Hence, 1575 must be divided by 7 for it to be a perfect square.

The new number would be (3 x 3) x (5 x 5).

Furthermore, we have:

(3 x 3) x (5 x 5) = (3 x 5) x (3 x 5)

Hence, the number whose square is the new number is: 3 x 5 = 15

9.) Find the greatest number of two digits which is a perfect square.

Answer:

We know that 102 is equal to 100 and 9^2 is equal to 81.

Since 10 and 9 are consecutive numbers, there is no perfect square between 100 and 81.

Since 100 is the first perfect square that has more than two digits, 81 is the greatest two-digit perfect square.

10.) Find the least number of three digits which is a perfect square.

Answer:

Let us make a list of the squares starting from 1.

$1^2=1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2=25$

$6^2 = 36$

$7^2= 49$

$8^2 = 64$

$9^2 = 81$

$10^2=100$

The square of 10 has three digits. Hence, the least three-digit perfect square is 100.

11.) Find the smallest number by which 4851 must be multiplied so that the product becomes a perfect square.

Answer:

4581 = 3 x 3 x 7 x 7 x11

3	4851
3	1617
7	539
7	77
11	11
	1

Grouping them into pairs of equal factors:

4851 = (3 x 3) x (7 x 7) x 11

The factor, 11 is not paired. The smallest number by which 4851 must be multiplied such that the resulting number is a perfect square is 11.

12.) Find the smallest number by which 28812 must be divided so that the quotient becomes a perfect square.

Prime factorization of 28812:

28812 = 2 x 2 x 3 x 7 x 7 x 7 x7

2	22812
2	14406
3	7203
7	2401
7	343
7	49
7	7
	1

Grouping them into pairs of equal factors:

28812 = (2 x2) x (7 x 7) x (7x 7) x 3

The factor, 3 is not paired. Hence, the smallest number by which 28812 must be divided such that the resulting number is a perfect square is 3.

13.) Find the smallest number by which 1152 must be divided so that it becomes a perfect square. Also, find the number whose square is the resulting number.

Answer:

Prime factorization of 1152:

1152 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3

2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Grouping them into pairs of equal factors:

1152=(2 x 2)x(2 x 2)x(2 x 2)x(3 x 3)x 2

The factor, 2 at the end is not paired.

For a number to be a perfect square, each prime factor has to be paired.

Hence, 1152 must be divided by 2 for it to be a perfect square.

The resulting number would be (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3).

Furthermore, we have:

(2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) = (2 x 2 x 2 x 3) x (2 x 2 x 2 x 3)

Hence, the number whose square is the resulting number is: 2 x 2 x 2 x 3 = 24

Exercise 3.2

1.) The following number are not perfect squares. Give reason:

A number ending with 2, 3, 7 or 8 cannot be a perfect square.

(i) 1547

Its last digit is 7. Hence, 1547 cannot be a perfect square.

(ii) 45743

Its last digit is 3. Hence, 45743 cannot be a perfect square.

(iii) 22453

Its last digit is 8. Hence, 8948 cannot be a perfect square.

(iv) 333333

Its last digit is 3. Hence, 333333 cannot be a perfect square.

2.) Show that the following numbers are not perfect squares:

A number ending with 2, 3, 7 or 8 cannot be a perfect square.

(i) 9327

Its last digit is 7. Hence, 9327 is not a perfect square.

(ii) 4058

Its last digit is 8. Hence, 4058 is not a perfect square.

(iii) 22453

Its last digit is 3. Hence, 22453 is not a perfect square.

(iv)743522

Its last digit is 2. Hence, 743522 is not a perfect square.

3.) The square of which of the following numbers would be an odd number?

The square of an odd number is always odd.

(i) 731

731 is an odd number. Hence, its square will be an odd number.

(ii)3456

3456 is an even number. Hence, its square will not be an odd number.

(iii) 5559

5559 is an odd number. Hence, its square will not be an odd number.

(iv) 42008

42008 is an even number. Hence, its square will not be an odd number.

Hence, only the squares of 731 and 5559 will be odd numbers.

4.) What will be the unit digit of the squares of the following numbers?

The unit's digit is affected only by the last digit of the number.

Hence, for each question, we only need to examine the square of its last digit.

(i) 52

Its last digit is 2. Hence, the unit's digit is 2^2, which is equal to 4.

(ii) 977

Its last digit is 7. Hence, the unit's digit is the last digit of 49 (49 = 72), which is 9.

(iii) 4583

Its last digit is 3. Hence, the unit's digit is 3^2, which is equal to 9.

(iv) 78367

Its last digit is 7. Hence, the unit's digit is the last digit of 49 (49 = 72), which is 9.

(v) 52698

Its last digit is 8. Hence, the unit's digit is the last digit of 64 (64 = 82), which is 4.

(vi) 99880

Its last digit is 0. Hence, the unit's digit is 02, which is equal to 0.

(vii 12796

Its last digit is 6. Hence, the unit's digit is the last digit of 36 (36 = 62), which is 6.

(viii) 55555

Its last digit is 5. Hence, the unit's digit is the last digit of 25 (25 = 52), which is 5.

(ix) 53924

Its last digit is 4. Hence, the unit's digit is the last digit of 16 (16 = 42), which is 6.

5.) Observe the following pattern:

$1 + 3 = 2^2$

$1 + 3 + 5 = 3^2$

$1 + 3 + 5 + 7 = 4^2$ **and write the value of** $1 + 3 + 5 + 7 + 9 + \ldots$ **up to n terms.**

From the pattern, we can say that the sum of the first n positive odd numbers is equal to the square of the n-th positive number. Putting that into formula:

$1 + 3 + 5 + 7 + n = n^2$, where the left hand side consists of n terms.

6.) Observe the following pattern:

$2^2 - 1^2 = 2 + 1$

$3^2 - 2^2 = 3 + 2$

$4^2 - 3^2 = 4 + 3$

$5^2 - 4^2 = 5 + 4$

From the pattern, we can say that the difference between the squares of two consecutive numbers is the sum of the numbers itself. In a formula:

$(n+1)^2 - (n)^2 = (n+1) + n$

Using this formula, we get:

(i) $100^2 - 99^2 = (99 + 1) + 99 = 199$

(ii) $111^2 - 109^2 = 111^2 - 110^2 + 110^2 - 109^2 = (111 + 110) + (110 + 109) = 440$

(iii) $99^2 - 96^2 = 99^2 - 98^2 + 98^2 - 97^2 + 97^2 - 96^2$

$= 99 + 98 + 98 + 97 + 97 + 96 = 585$

7.) Which of the following triplets is Pythagorean?

Only (i), (ii), (iv) and (v) are Pythagorean triplets.

A triplet (a, b, c) is called Pythagorean if the sum of the squares of the two smallest numbers is equal to the square of the biggest number.

(i) (8, 15, 17)

The two smallest numbers are 8 and 15. The sum of their squares is:

$8^2 + 15^2 = 289 = 17^2$

Hence, (8, 15, 17) is a Pythagorean triplet.

(ii) (18, 80, 82)

The two smallest numbers are 18 and 80. The sum of their squares is: $18^2 + 80^2 = 6724 = 82^2$

Hence, (18, 80, 82) is a Pythagorean triplet.

(iii) (14, 48, 51)

The two smallest numbers are 14 and 48. The sum of their squares is:

$14^2 + 48^2 = 2500$, this is not equal to $512 = 2601$

Hence, (14, 48, 51) is not a Pythagorean triplet.

(iv) (10, 24, 51)

The two smallest numbers are 10 and 24. The sum of their squares is:

$10^2 + 24^2 = 676 = 26^2$

Hence, (10, 24, 26) is a Pythagorean triplet.

(v) (16, 63, 65)

The two smallest numbers are 16 and 63. The sum of their squares is:

$16^2 + 63^2 = 4225 = 65^2$ Hence, (16, 63, 65) is a Pythagorean triplet.

(vi) (12, 35, 38)

The two smallest numbers are 12 and 35. The sum of their squares is:

$12^2 + 35^2 = 1369$, which is not equal to $38^2 = 1444$ Hence, (12, 35, 38) is not a Pythagorean triplet.

8.) Observe the following pattern:

$(1 \times 2) + (2 \times 3) = \frac{2\times3\times4}{3}$

$(1 \times 2) + (2 \times 3) + (3 \times 4) = \frac{3\times4\times5}{3}$

$(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) = \frac{4\times5\times6}{3}$ and find the value of $(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + (5 \times 6)$

The RHS of the three equalities is a fraction whose numerator is the multiplication of three consecutive numbers and whose denominator is 3.

If the biggest number (factor) on the LHS is 3, the multiplication of the three numbers on the RHS begins with 2.

If the biggest number (factor) on the LHS is 4, the multiplication of the three numbers on the RHS begins with 3.

If the biggest number (factor) on the LHS is 5, the multiplication of the three numbers on the RHS begins with 4.

Using this pattern, (1 x 2) + (2 x 3) + (3 x 4) + (4 x 5) + (5 x 6) has 6 as the biggest number. Hence, the multiplication of the three numbers on the RHS will begin with 5. Finally, we have:

$1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 = \frac{5\times6\times7}{3} = 70$

9.) Observe the following pattern:

$1 = \frac{1}{2}\{1 \times (1+1)\}$

$1 + 2 = \frac{1}{2}\{2 \times \{2+2)\}$

$1 + 2 + 3 = \frac{1}{2}\{3 \times (3+1)\}$

$1 + 2 + 3 + 4 = \frac{1}{2}\{4 \times (4+1)\}$ and find the values of each of the following :

(i) 1+ 2 + 3 + 4 + 5 ... + 50

(ii) 31 + 32+ ... + 50

Observing the three numbers for right hand side of the equalities:

The first equality, whose biggest number on the LHS is 1, has 1, 1 and 1 as the three numbers.

The second equality, whose biggest number on the LHS is 2, has 2, 2 and 1 as the three numbers

The third equality, whose biggest number on the LHS is 3, has 3, 3 and 1 as the three numbers.

The fourth equality, whose biggest number on the LHS is 4, has 4, 4 and 1 as the three numbers.

Hence, if the biggest number on the LHS is n, the three numbers on the RHS will be n, n and 1.

Using this property, we can calculate the sums for (i) and (ii) as follows:

(i) $1 + 2 + 3 + \ldots\ldots + 50 = \frac{1}{2} \times 50 \times (50 + 1) = 1275$

(ii) The sum can be expressed as the difference of the two sums as follows:

$31 + 32 + \ldots. + 50 = (1 + 2 + 3 + \ldots. + 50) - (1 + 2 + 3 + \ \ +30)$ The result of the first bracket is exactly the same as in part (i).

$1 + 2 + \ldots.+ 50 = 1275$

Then, the second bracket:

$1 + 2 + \ldots. +30 = \frac{1}{2} \ (30 \times (30 + 1)) = 465$

Finally, we have: $31 + 32 + + 50 = 1275 - 465 = 810$

11.) Which of the following numbers are squares of even numbers: 121, 225, 256, 324, 1296, 6561, 5476, 4489, 373758

The numbers whose last digit is odd can never be the square of even numbers. So, we have to leave out 121, 225, 6561 and 4489, leaving only 256, 324, 1296, 5476 and 373758.

For each number, use prime factorization method and make pairs of equal factors.

(i) 256 =2x2x2x2x2x2x2x2 = (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2)

There are no factors that are not paired. Hence, 256 is a perfect square. The square of an even number is always even. Hence, 256 is the square of an even number.

(ii) 324 =2x2x3x3x3x3 = (2 x 2) x (3 x 3) x (3 x 3) There are no factors that are not paired. Hence, 324 is a perfect square. The square of an even number is always even. Hence, 324 is the square of an even number.

(iii) 1296 =2x2x2x2x3x3x3x3 = (2 x 2) x (2 x 2) x (3 x 3) x (3 x 3) There are no factors that are not paired. Hence, 1296 is a perfect square. The square of an even number is always even. Hence, 1296 is the square of an even number.

(iv) 5476 = 2 x 2 x 37 x 37 = (2 x 2) x (37 x 37) There are no factors that are not paired. Hence, 5476 is a perfect square. The square of an even number is always even. Hence, 5476 is the square of an even number.

(v) 373758 =2 x 3 x 7 x 11 x 809 Here, each factor appears only once, so grouping them into pairs of equal factors is not possible. It means that 373758 is not the square of an even number.

Hence, the numbers that are the squares of even numbers are 256, 324, 1296 and 5476.

12.) By just examining the units digit, can you tell which of the following cannot be whole squares?

If the unit's digit of a number is 2, 3, 7 or 8, the number cannot be a whole square.

(i) 1026

1026 has 6 as the unit's digit, so it is possibly a perfect square.

(ii) 1028

1028 has 8 as the unit's digit, so it cannot be a perfect square.

(iii) 1024

1024 has 4 as the unit's digit, so it is possibly a perfect square.

(iv) 1022

1022 has 2 as the unit's digit, so it cannot be a perfect square.

(v) 1023

1023 has 3 as the unit's digit, so it cannot be a perfect square.

(vi) 1027

1027 has 7 as the unit digit, so it cannot be a perfect square.

Hence, by examining the unit's digits, we can be certain that 1028, 1022, 1023 and 1027 cannot be whole squares.

13.) Write five numbers which you cannot decide whether they are squares.

Answer:

A number whose unit digit is 2, 3, 7 or 8 cannot be a perfect square.

On the other hand, a number whose unit digit is 1, 4, 5, 6, 9 or 0 might be a perfect square (although we will have to verify whether it is a perfect square or not).

Applying the above two conditions, we cannot quickly decide whether the following numbers are squares of any numbers:

1111, 1444, 1555, 1666, 1999

14.) Write five numbers which you cannot decide whether they are square just be looking at the units digit.

Answer:

A number whose unit digit is 2, 3, 7 or 8 cannot be a perfect square.

On the other hand, a number whose unit digit is 1, 4, 5, 6, 9 or 0 might be a perfect square although we have to verify that.

Applying these two conditions, we cannot determine whether the following numbers are squares just by looking at their unit digits:

1111, 1001, 1555, 1666 and 1999

15.) Write True (T) and false (F) for the following statements.

(i) The number of digits in a square number is even.

False

Example: 100 is the square of a number but its number of digits is three, which is not an even number.

(ii) The square of a prime number is prime.

False

If p is a prime number, its square is p2, which has at least three factors: 1, p and p2. Since it has more than two factors, it is not a prime number.

(iii) The sun of two square numbers is a square number.

False

1 is the square of a number (1 = 12). But 1 + 1 = 2, which is not the square of any number.

(iv) The difference of two square numbers is a square number.

False

4 and 1 are squares (4 = 22, 1 = 12). But 4 — 1 = 3, which is not the square of any number.

(v) The product of two square numbers is a square number.

True

If a2 and b2 are two squares, their product is a2 x b2 = (a x b)2, which is a square.

(vi) No square number is negative.

True

The square of a negative number will be positive because negative times negative is positive.

(vii) There is no square number between 50 and 60

True 72 = 49 and 82 = 64. 7 and 8 are consecutive numbers and hence there are no square numbers between 50 and 60.

(viii) There are fourteen square number up to 200.

True 142 is equal to 196, which is below 200. There are 14 square numbers below 200.

Exercise 3.3

1.) Find the squares of the following numbers using column method. Verify the result finding the square using the usual multiplication.

(i) 25

Here a = 2, b = 5

Step: 1 Make 3 columns and write the values of a^2, 2 x a x b, and b^2 in these columns.

Column I	Column II	Column III
a^2	2 x a x b	b^2
4	20	25

Step: 2 Underline the unit digit of b^2 (in Column III) and add its tens digit, if any, with 2 x a x b (in column II)

Column I	Column II	Column III
a^2	2 x a x b	b^2
4	20 + 2	2<u>5</u>
	22	

Step: 3 Underline the unit digit in Column II and add the number formed by the tens and other digits if any, with a^2 in Column I.

Column I	Column II	Column III
a^2	2 x a x b	b
4 + 2	20 + 2	2<u>5</u>
6	2<u>2</u>	

Step 4: Underline the number in Column I.

Column I	Column II	Column III
a^2	2 x a x b	b^2
4 +2	20 + 2	2<u>5</u>
6	2<u>2</u>	

Step: 5 write the underlined digits at the bottom of each column to obtain the square of the given number.

In this case, we have:

$25^2 = 625$

Using Multiplication:

25 x 25 = 625

This matches with the result obtained by the column method:

(ii) 37

Here, a = 3, b = 7

Step: 1 Make 3 columns and write the values of a^2, 2 x a x b, and b^2 in these columns.

Column I	Column II	Column III
a^2	2 x a x b	b^2
9	42	49

Step: 2 Underline the unit digit of b^2 (in Column III) and add its tens digit, if any, with 2 x a x b (in Column II)

Column I	Column II	Column III
A^2	2 x a x b	B^2
9 +4	42 + 4	49
13	46	

Step: 3 Underline the unit digit in Column II and add the number formed by tens and others digits if any, with a^2 in Column I.

Column I	Column II	Column III
A^2	2 x a x b	B^2
9 + 4	42 + 4	49
13	46	

Step 5: Write the underlined digits at the bottom of each column to obtain the square of the given number.

In this case, we have:

$37^2 = 1369$

Using multiplication:

$37 \times 37 = 1369$

This matches with the result obtained using the column method.

(iii) 54

Here, a= 5, b= 4

Step 1: make 3 columns and write the values of a^2, $2 \times a \times b$ and b^2 in these columns.

Column I	Column II	Column III
a^2	$2 \times a \times b$	b^2
25	40	

Step: 2 Underline the unit digit of b^2 (in Column III) and add its tens digit, if any, with $2x\ a \times b$ (in Column II)

Column I	Column II	Column III
a^2	$2 \times a \times b$	b^2
25	40 +1	$1\underline{6}$
	41	

Step: 3 Underline the digit in Column II and add the number formed by the tens and other digits if any, with a^2 in Column I.

Column I	Column II	Column III
a^2	2 x a x b	b^2
25 +4	40 +1	$1\underline{6}$
29	$4\underline{1}$	

Step: 4 underline the number in Column I.

Column I	Column II	Column III
a^2	2 x a x b	b^2
25 +4	40 +1	$1\underline{6}$
$\underline{29}$	$4\underline{1}$	

Step: 5 write the underlined digits at the bottom of each column to obtain the square of the given number.

In this case, we have:

$54^2 = 2916$

Using multiplication:

$54 \times 54 = 2916$

This matches with the result obtained using the column method.

(iv) 71

Here, $a = 7$, $b = 1$

Step: 1 Make 3 columns and write the values of a^2, $2 \times a \times b$ and b^2 in these columns.

Column I	Column II	Column III
a^2	$2 \times a \times b$	b^2
49	14	1

Step: 2 Underline the unit digit of b^2 (in column III) and add its ten digit, if any with $2 \times a \times b$ (in column II)

Column I	Column II	Column III
a^2	$2 \times a \times b$	b^2
49	14 + 0	$\underline{1}$
	14	

Step: 3 Underline the unit digit in Column II and add the number formed by the tens and other digits, if any, with a^2 in column I.

Column I	Column II	Column III
a^2	2 x a x b	b^2
49 + 1	14 + 0	$\underline{1}$
50	$1\underline{4}$	

Step: 4 underline the number in column I.

Column I	Column II	Column III
a^2	2 x a x b	b^2
49 + 1	14 + 0	$\underline{1}$
$\underline{50}$	$1\underline{4}$	

Step: 5 write the underlined digits at the bottom of each column to obtain the square of the given number:

In this case, we have:

$71^2 = 5041$

Using multiplication:

71 x 71 = 5041

This matches with the result obtained using the column method.

(v) 96

Here, a = 9, b = 6

Step: 1 Make 3 columns and write the values of a^2 , 2 x a x b and b^2 in these columns.

Column I	Column II	Column III
a^2	2 x a x b	b^2
81	108	36

Step: 2 Underline the unit digit of b^2 (in column III) and add its tens digit, if any with 2 x a x b (in column II)

Column I	Column II	Column III
a^2	2 x a x b	b^2
81	108 + 3	$3\underline{6}$
	111	

Step: 3 Underline the unit digit in Column II and add the number formed by the tens and other digits if any, with a^2 in column I.

Column I	Column II	Column III
a^2	2 x a x b	b^2
81 + 11	108 + 3	$3\underline{6}$
92	$11\underline{1}$	

Step: 4 underline the number in Column I

Column I	Column II	Column III
a^2	2 x a x b	b^2
81 + 11	108 + 3	$3\underline{6}$
$\underline{92}$	$11\underline{1}$	

Step: 5 write the underlined digits at the bottom of each column to obtain the square of the given number.

In this case, we have:

$96^2 = 9216$

Using multiplication:

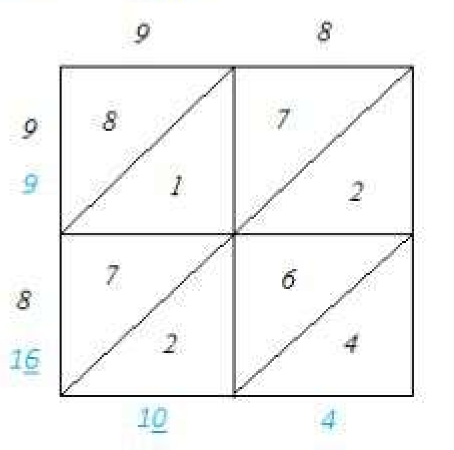
96 x 96 = 2916

This matches with the result obtained using the column method.

2.) Find the squares of the following numbers using diagonal method:

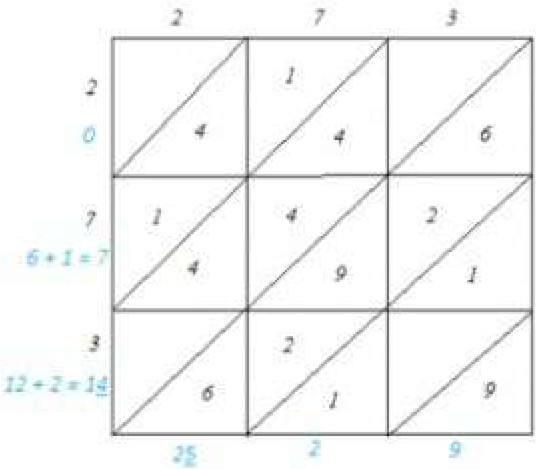
(i) 98

$\therefore 98^2 = 9604$



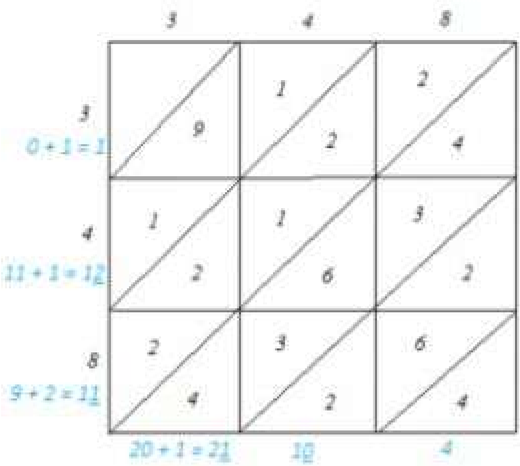
(ii) 273

$\therefore 273^2 = 74529$



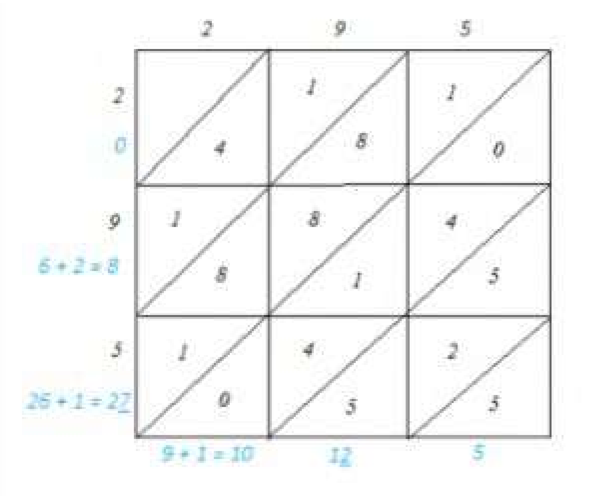
(iii) 348

$\therefore 348^2 = 121104$



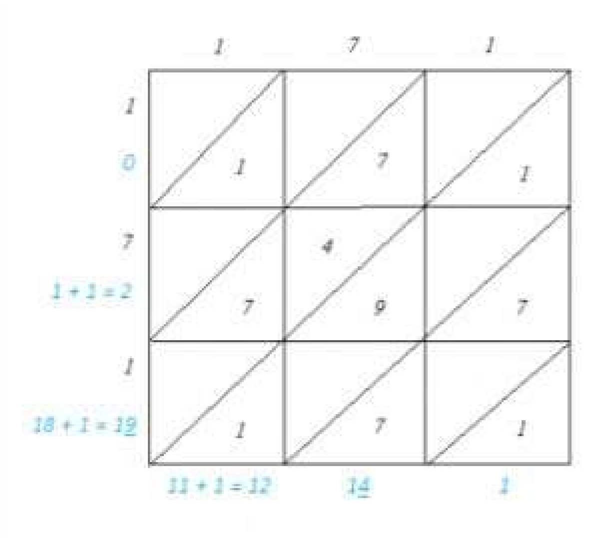
(iv) 295

$\therefore 295^2 = 87025$



(v) 171

$\therefore 171^2 = 29241$



3.) Find the squares of the following numbers:

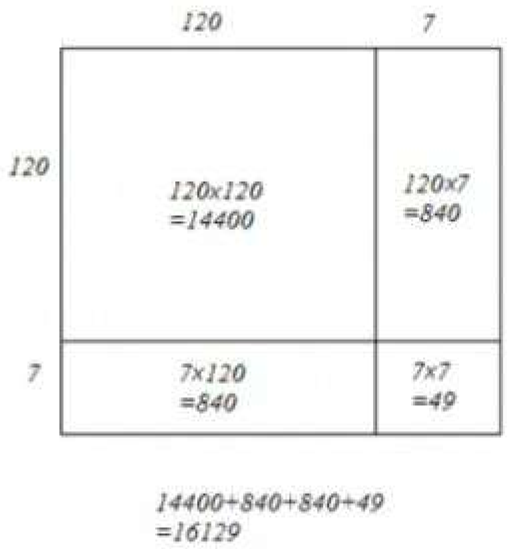
Answer:

We will use visual method as it is the efficient method to solve this problem.

(i) We have:

127 = 120 + 7

Hence, let us draw a square having side 127 units. Let us split it into 120 units and 7 units.

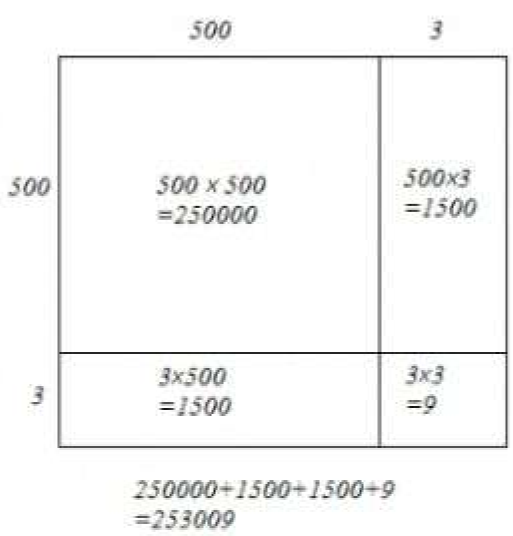


Hence, the square of 127 is 16129.

(ii) We have:

503 = 500 + 3

Hence, let us draw a square having side 503 units. Let us split it into 500 units and 3 units.

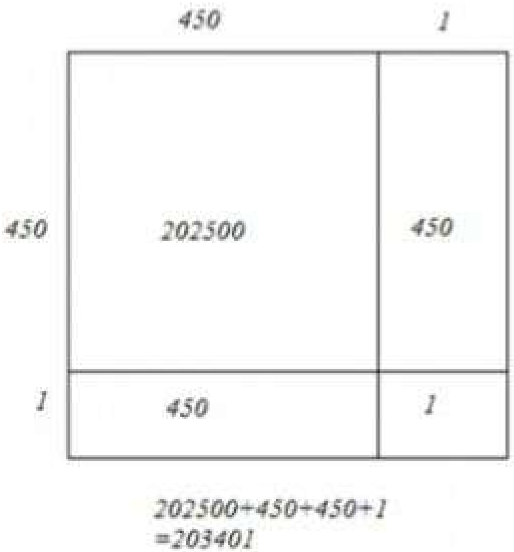


Hence, the square of 503 is 253009.

(iii) We have:

451 = 450 + 1

Hence, let us draw a square of having side 451 units. Let us split it into 450 units and 1 units.

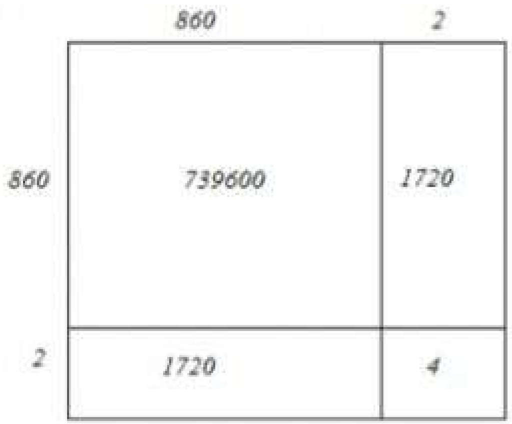


Hence, the square of 451 is 203401.

(iv) We have:

862 = 860 + 2

Hence, let us draw a square having side 862 units. Let us split it into 860 units and 2 units.



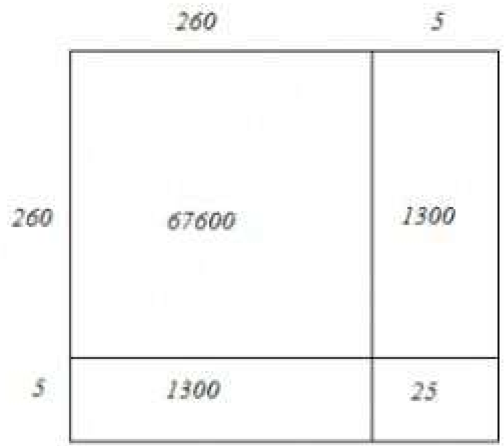
739600+1720+1720+4
=743044

Hence, the square of 862 is 743044.

(v) We have:

265= 260 +5

Hence, let us draw a square having 265 units. Let us split it into 260 units and 5 units.



67600+1300+1300+25
=70225

Hence, the square of 265 units is 70225.

4.) Find the squares of the following numbers:

Notice that all numbers except the one in question (vii) has 5 as their respective unit digits. We know that the square of a number with the form n5 is a number ending with 25 and has the number n(n + 1) before 25.

(i) 425

Here, n = 42

∴ n(n + 1) = (42)(43) = 1806

∴ 4252 = 180625

(ii) 575

Here, n = 57

∴n(n + 1) = (57)(58) = 3306

∴ 5752 = 330625

(iii) 405

Here n = 40

∴n(n + 1) = (40)(41) = 1640

∴ 4052 = 164025

(iv) 205

Here n = 20

∴ n(n + 1) = (20)(21) = 420

∴ 2052 = 42025

(v) 95

Here n = 9

∴ n(n + 1) = (9)(10) = 90

∴ 952 = 9025

(vi) 745

Here n = 74

∴ n(n + 1) = (74)(75) = 5550

∴ 7452 = 555025

(vii) 512

We know: The square of a three-digit number of the form 5ab = (250 + ab) 1000 + $(ab)^2$

∴ 512^2 = (250+12)1000 + (12)2 = 262000 + 144 = 262144

(viii) 995

Here n = 99

Here, n = 99

$\therefore$ n(n + 1) = (99)(100) = 9900

$\therefore 995^2 = 990025$

5.) Find the squares of the following numbers using the identity $(a + b)^2 = a^2 + 2ab + b^2$:

(i) 405

On decomposing:

405 = 400 + 5

Here, a = 400x and b = 5

Using the identity (a + b)2 = a2 + 2ab + b2:

$405^2 = (400 + 5)^2 = 400^2 + 2(400)(5) + 5^2 = 160000 + 4000 + 25 = 164025$

(ii) 510

On decomposing:

510 = 500 + 10 Here, a = 500 and b = 10

Using the identity (a + b)2 = a2 + 2ab + b2:

$510^2 = (500 + 10)^2 = 500^2 + 2(500)(10) + 10^2 = 250000 + 10000 + 100 = 260100$

(iii) 1001

On decomposing:

1001 = 1000 + 1

Here, a = 1000 and b = 1

Using the identity (a + b)2 = a2 + 2ab + b2:

$1001^2 = (1000 + 1)^2 = 1000^2 + 2(1000)(1) + 12 = 1000000 + 2000 + 1 = 1002001$

(iv) 209

On decomposing:

209 = 200 + 9

Here, a = 200 and b = 9

Using the identity $(a + b)^2 = a^2 + 2ab + b^2$:

$209^2 = (200 + 9)^2 = 200^2 + 2(200)(9) + 9^2 = 40000 + 3600 + 81 = 43681$

(v) 605

On decomposing:

605 = 600 + 5

Here, a = 600 and b = 5

Using the identity $(a + b)^2$= a2 + 2ab + b2:

$605^2 = (600 + 5)^2 = 6002 + 2(600)(5) + 5^2 = 360000 + 6000 + 25 = 366025$

6.)Find the squares of the following numbers using the identity $(a - b)^2 = a^2 - 2ab + b^2$:

(i) 395

Decomposing: $395 = 400 - 5$

Here, $a = 400$ and $b = 5$

Using the identity $(a - b)^2 = a^2 - 2ab + b^2$:

$395^2 = (400 - 5)^2 = 400^2 - 2(400)(5) + 5^2 = 160000 - 4000 + 25 = 156025$

(ii) 995

Decomposing:

$995 = 1000 - 5$

Here, $a = 1000$ and $b = 5$

Using the identity $(a - b)^2 = a^2 - 2ab + b^2$:

$995^2 = (1000 - 5)^2 = 1000^2 - 2(1000)(5) + 5^2 = 1000000 - 10000 + 25 = 990025$

(iii) 495

Decomposing: $495 = 500 - 5$ Here, $a = 500$ and $b = 5$ Using the identity $(a - b)^2 = a^2 - 2ab + b^2$:

$495^2 = (500 - 5)^2 = 500^2 - 2(500)(5) + 5^2 = 250000 - 5000 + 25 = 245025$

(iv) 498

Decomposing: $498 = 500 - 2$ $250000 - 5000 + 25 = 245025$

Here, $a = 500$ and $b = 2$

Using the identity $(a - b)^2 = a^2 - 2ab + b^2$:

$4982 = (500 - 2)2 = 5002 - 2(500)(2) + 22 = 250000 - 2000 + 4 = 248004$

(v) 99

Decomposing: 99 = 100 − 1 Here, a = 100 and b = 1 Using the identity (a − b)2 = a2 − 2ab + b2: 992 = (100 − 1)2 = 1002 − 2(100)(1) + 12 = 10000 − 200 + 1 = 9801

(vi) 999

Decomposing: 999 = 1000 − 1

Here, a = 1000 and b = 1

Using the identity $(a - b)^2 = a^2 - 2ab + b^2$:

$999^2 = (1000 - 1)^2 - 1000^2 - 2(1000)(1)+1^2 = 1000000 - 2000 + 1 = 998001$

(vii) 599

Decomposing: 599 = 600 − 1

Here, a = 600 and b = 1

Using the identity $(a - b)^2 = a^2 - 2ab + b^2$:

$5992 = (600 - 1)^2 = 600^2 - 2(600)(1) + 12 = 360000 - 1200 + 1 = 358801$

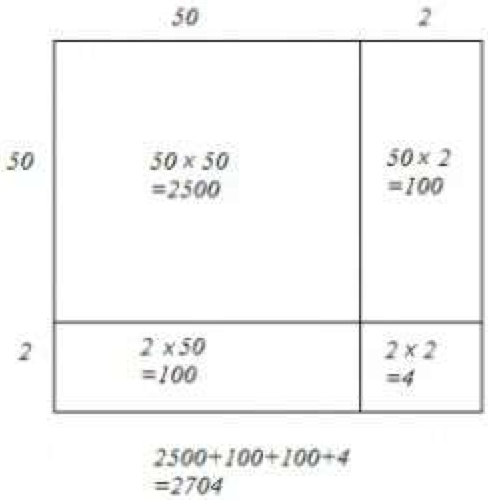
7.) Find the squares of the following numbers by visual method:

(i) 52

We have:

52 = 50 +2

Let us draw a square having side 52 units. Let us split it into 50 units and 2 units.



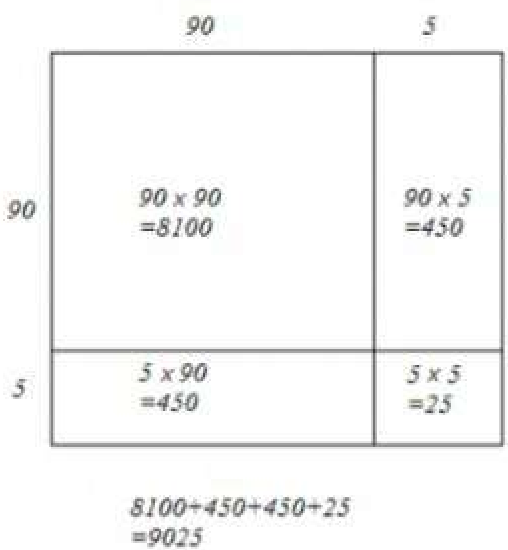
The sum of the areas of these four parts is the square of 52. Thus, the square of 52 is 2704.

(ii) 95

We have:

95 = 90 +5

Let us draw a square having side 95 units. Let us split it into 90 units and 5 units.



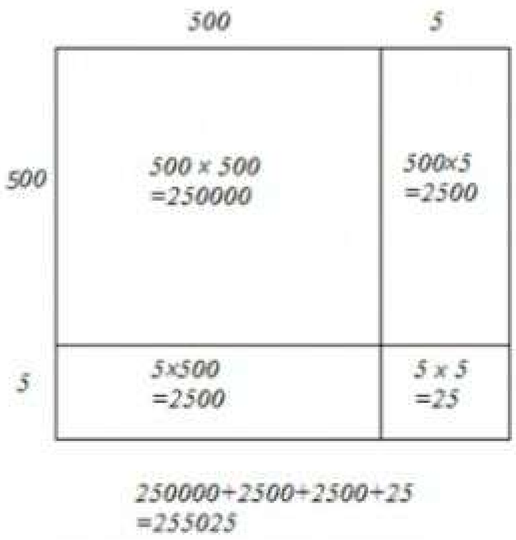
The sum of the areas of these four parts is the square of 95. Thus, the square of 95 is 9025.

(iii) 505

We have:

505 = 500 +5

Let us draw a square having side 505 units. Let us split it into 500 units and 5 units.



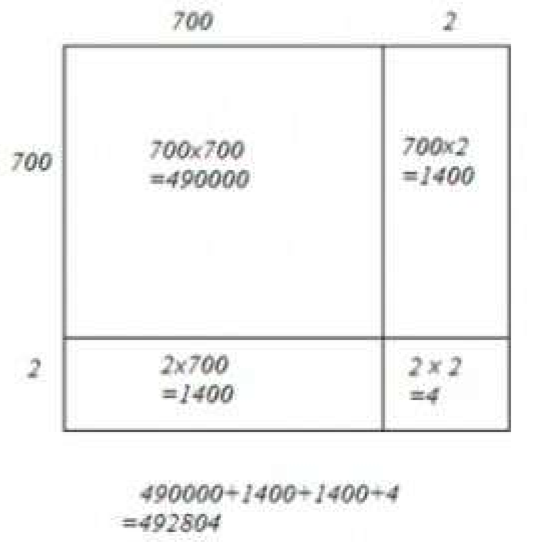
The sum of the areas of these four parts is the square of 505. Thus, the square of 505 is 255025.

(iv) 702

We have:

702 = 700 + 2

Let us draw a square of having side 702 units. Let us split it into 700 units and 2 units.

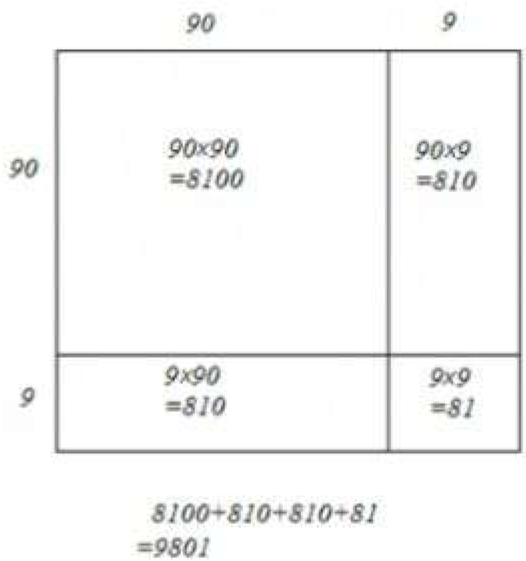


The sum of the areas of these four parts is the square of 702. Thus, the square of 702 is 492804.

(v) 99

We have:
99 = 90 +9

Let us draw a square of having side 99 units. Let us split it into 90 units and 9 units.



The sum of the areas of these four parts is the square of 99. Thus, the square of 99 is 9801.

Exercise 3.4

Q-1. Write the possible unit's digits of the square root of the following numbers. Which of these numbers are odd square roots?

(i) 9801 (ii) 99856 (iii) 998001 (iv) 657666025

Solution.

(i) The unit digit of the number 9801 is 1. So, the possible unit digits are 1 or 9 (Table 3.4). Note that 9801 is equal to 99^2. Hence, the square root is an odd number.

(ii) The unit digit of the number 99856 is 6. So, the possible unit digits are 4 or 6 (Table 3.4). Since its last digit is 6 (an even number), it cannot have an odd number as its square root.

(iii) The unit digit of the number 998001 is 1. So, the possible unit digits are 1 or 9. Note that 998001 is equal to $\left(3^3 \times 37\right)^2$. Hence, the square root is an odd number.

(iv) The unit digit of the number 657666025 is 5. So, the only possible unit digit is 5. Note that 657666025 is equal to $(5 \times 23 \times 223)^2$. Hence, the square root is an odd number.

Hence, among the given numbers, (i), (iii) and (iv) have odd numbers as their square roots.

Q-2. Find the square root of each of the following by prime factorization:

(i) 441

(ii) 196

(iii) 529

(iv) 1764

(v) 1156

(vi) 4096

(vii) 7056

(viii) 8281

(ix) 11664

(x) 47089

(xi) 24336

(xii) 190969

(xiii) 586756

(xiv) 27225

(xv) 3013696

Solution:

(i) Resolving 441 into prime factors:

441=3 x 3 x 7 x 7

3	441
3	147
7	49
7	7
	1

Grouping the factors into pairs of equal factors:

441 = (3 x 3) x (7 x 7)

Taking one factor for each pair, we get the square root of 441:

3 x 7 = 21

(ii) Resolving 196 into prime factors:

196 = 2 x 2 x 7 x 7

2	196
2	98
7	49
7	7
	1

Grouping the factors into pairs of equal factors:

196 = (2 x 2) x (7 x 7)

Taking one factor for each pair, we get the square root of 196:

$2 \times 7 = 14$

(iii) Resolving 529 into prime factors:

529 = 23 x 23

23	529
23	23
	1

Grouping the factors into pairs of equal factors:

529= (23 x 23)

Taking one factor for each pair, we get the square root of 529 as 23.

(iv) Resolving 1764 into prime factors:

1764 =2 x 2 x 3 x 3 x 7 x 7

2	1764
2	882
3	441
3	49
7	7
	1

Grouping the factors into pairs of equal factors:

1764 = (2 x 2) x (3 x 3) x (7 x 7)

Taking one factor for each pair, we get the square root of 1764:

2 x 3 x 7 = 42

(v) Resolving 1156 into prime factors:

1156 = 2 x 2 x 17 x 17

2	1156
2	578
17	289
17	17
	1

Grouping the factors into pairs of equal factors:

1156 = (2 × 2) × (17 × 17)

Taking one factor for each pair, we get the square root of 1156:

2 × 17 = 34

(vi) Resolving 4096 into prime factors:

4096 =(2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2)

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Grouping the factors into pairs of equal factors:

4096 = (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2)

Taking one factor for each pair, we get the square root of 4096:

(2 x 2) x (2 x 2) x (2 x 2) = 64

(vii) Resolving 7056 into prime factors:

7056 = (2 x 2) x (2 x 2) x (3 x 3) x (7 x 7)

2	7056
2	3528
2	1764
2	882
3	441
3	147
7	49
7	7
	1

Grouping the factors into pairs of equal factors:

$7056 = (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (7 \times 7)$

Taking one factor for each pair, we get the square root of 7056:

$2 \times 2 \times 3 \times 7 = 84$

(viii) Resolving 8281 into prime factors:

$8281 = 7 \times 7 \times 13 \times 13$

7	8281
7	1183
13	169
13	13
	1

Grouping the factors into pairs of equal factors:

8281 = (7 × 7) × (13 × 13)

Taking one factor for each pair, we get the square root of 8281:

7 × 13 = 91

(ix) Resolving 11664 into prime factors:

11664 = 2 × 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3

2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Grouping the factors into pairs of equal factors:

11664 = (2 × 2) × (2 × 2) × (3 × 3) × (3 × 3) × (3 × 3)

Taking one factor for each pair, we get the square root of 11664:

2 × 2 × 3 × 3 × 3 = 108

(x) Resolving 47089 into prime factors:

47089 = 7 × 7 × 31 × 31

7	47089
7	6727
31	961
31	31
	1

Grouping the factors into pairs of equal factors:

47089 = (7 × 7) × (31 × 31)

Taking one factor for each pair, we get the square root of 47089:

7 × 31 = 217

(xi) Resolving 24336 into prime factors:

24336 = 2 × 2 × 2 × 2 × 3 × 3 × 13 × 13

2	24336
2	12168
2	6084
2	3042
3	1521
3	507
13	169
13	13
	1

Grouping the factors into pairs of equal factors:

24336 = (2 × 2) × (2 × 2) × (3 × 3) × (13 × 13)

Taking one factor for each pair, we get the square root of 24336:

2 × 2 × 3 × 13 = 156

(xii) Resolving 190969 into prime factors:

190969 = 19 × 19 × 23 × 23

19	190969
19	10051
23	529
23	23
	1

Grouping the factors into pairs of equal factors:

190969 = (19 × 19) × (23 × 23)

Taking one factor for each pair, we get the square root of 190969:

19 × 23 = 437

(xiii) Resolving 586756 into prime factors:

568756 = 2 × 2 × 383 × 383

2	568756
2	293378
383	146689
383	383
	1

Grouping the factors into pairs of equal factors:

568756 = (2 × 2) × (383 × 383)

Taking one factor for each pairs, we get the square root of 568756;

2 × 383 = 766

(xiv) Resolving 27225 into prime factors:

27225 = 3 × 3 × 5 × 5 × 11 × 11

3	27225
3	9075
5	3025
5	605
11	121
11	11
	1

Grouping the factors into pairs of equal factors:

27225 = (3 × 3) × (5 × 5) × (11 × 11)

Taking one factor for each pair, we get the square root of 27225:

3 × 5 × 11 = 165

(xv) Resolving 3013696 into prime factors:

3013696 = 2 × 2 × 2 × 2 × 2 × 2 × 7 × 7 × 31 × 31

2	3013696
2	1506848
2	753424
2	376712
2	188356
2	94178
7	47089
7	6727
31	961
31	31
	1

Grouping the factors into pairs of equal factors:

$3013696 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (7 \times 7) \times (31 \times 31)$

Taking one factor for each pairs, we get the square root of 3013696;

$2 \times 2 \times 2 \times 7 \times 31 = 1736$

Q-3. Find the smallest number by which 180 must be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square so obtained.

Sol:

The prime factorization of 180:

$180 = 2 \times 2 \times 3 \times 3 \times 5$

Grouping the factors into pairs of equal factors, we get:

$180 = (2 \times 2) \times (3 \times 3) \times 5$

The factor, 5 does not have a pair.

Therefore, we must multiply 180 by 5 to make a perfect square. The new number is:

$(2 \times 2) \times (3 \times 3) \times (5 \times 5) = 900$

Taking one factor from each pair on the LHS, the square root of the new number is $2 \times 3 \times 5$, which is equal to 30.

Q-4. Find the smallest number by which 147 must be multiplied so that it becomes a perfect square. Also, find the square root of the number so obtained.

Solution:

The prime factorization of 147:

$147 = 3 \times 7 \times 7$

Grouping the factors into pairs of equal factors, we get:

147 = 3 x (7 x 7)

The factor, 3 does not have a pair. Therefore, we must multiply 147 by 3 to make a perfect square. The new number is:

(3 x 3) x (7 x 7) = 441

Taking one factor from each pair on the LHS, the square root of the new number is 3 x 7, which is equal to 21.

Q-5. Find the smallest number by which 3645 must be divided so that it becomes a perfect square. Also, find the square root of the resulting number.

Solution:

The prime factorization of 3645:

3645=3 x3x3x3x3x3x5

Grouping the factors into pairs of equal factors, we get:

3645 = (3 x 3) x (3 x 3) x (3 x 3) x 5

The factor, 5 does not have a pair. Therefore, we must divide 3645 by 5 to make a perfect square. The new number is:

(3 x 3) x (3 x 3) x (3 x 3) = 729

Taking one factor from each pair on the LHS, the square root of the new number is 3 x 3 x 3, which is equal to 27.

Q-6. Find the smallest number by which 1152 must be divided so that it becomes a perfect square. Also, find the square root of the number so obtained.

Solution:

The prime factorization of 1152:

1152 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3

Grouping the factors into pairs of equal factors, we get:

1152 = (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) x 2

The factor, 2, at the end, does not have a pair. Therefore, we must divide 1152 by 2 to make a perfect square. The new number is:

(2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) = 576

Taking one factor from each pair on the LHS, the square root of the new number is 2 x 2 x 2 x 3, which is equal to 24.

Q-7. The product of two numbers is 1296. If one of the numbers is 16 times the other, find the numbers.

Solution:

Let the two numbers be a and b.

From the first statement, we have:

a x b = 1296

If one number is 16 times the other, then we have:

b = 16 x a.

Substituting this value in the first equation, we get:

a x (16 x a) = 1296

By simplifying both sides, we get:

$a^2 = \frac{1296}{16} = 81$

Hence, a is the square root of 81, which is 9.

To find b, use equation b = 16 x a.

Since a = 9 ; b = 16 x 9 = 144

So, the two numbers satisfying the question are 9 and 144.

Q-8. A welfare association collected Rs. 202500 as donation from the residents. If each paid as many rupees as there were residents, find the number of residents.

Solution:

Let R be the number of residents.

Let r be the money in rupees donated by each resident.

Total donation = R x r = 202500

Since the money received as donation is the same as the number of residents:

r = R.

Substituting this in the first equation, we get:

R x R = 202500 R2 = 202500

R^2 = (2 x 2) x (5 x 5) x (5 x 5) x $(3 \times 3)^2$

R=2 x 5 x 5 x 3 x 3 = 450

So, the number of residents is 450.

Q-9. A society collected Rs. 92.16. Each member collected as many paise as there were members. How many members were there and how much did each contribute?

Solution:

Let M be the number of members.

Let r be the amount in paise donated by each member.

The total contribution can be expressed as follows:

M x r = Rs 92.16 = 9216 paise

Since the amount received as donation is the same as the number of members:

r = M

Substituting this in the first equation, we get:

M x M = 9216

M^2 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3

M^2 = (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3)

M=2 x 2 x 2 x 2 x 2 x 3 = 96

To find r, we can use the relation r = M.

Let M be the number of members.

Let r be the amount in paise donated by each member.

The total contribution can be expressed as follows:

M x r = Rs 92.16 = 9216 paise

Since the amount received as donation is the same as the number of members:

r = 96

So, there are 96 members and each paid 96 paise.

Q-10. A school collected Rs. 2304 as fees from its students. If each student paid as many paise as there were students in the school, how many students were there in the school?

Solution:

Let S be the number of students.

Let r be the money donated by each student.

The total contribution can be expressed by (S)(r) = Rs 2304

Since each student paid as many paise as the number of students, then r = S.

Substituting this in the first equation, we get:

S x S = 2304

S^2= 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3

S^2 = (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3)

S =2 x 2 x 2 x 2 x 3 = 48

So, there are 48 students in total in the school.

Q-11. The area of a square field is 5184 m^2. A rectangular field, whose length is twice its breadth has its perimeter equal to the perimeter of the square field. Find the area of the rectangular field.

Solution:

First, we have to find the perimeter of the square.

The area of the square is r^2, where r is the side of the square.

Then, we have the equation as follows:

r^2 = 5184 = (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) x (3 x 3)

Taking the square root, we get r = 2 x 2 x 2 x 3 x 3 = 72

Hence the perimeter of the square is 4 x r = 288 m

Now, let L be the length of the rectangular field.

Let W be the width of the rectangular field.

The perimeter is equal to the perimeter of square.

Hence, we have:

2(L + W) = 288

Moreover, since the length is twice the width:

L = 2 x W.

Substituting this in the previous equation, we get:

2 x (2 x W+ W) = 288 3 x W= 144 W= 48

To find L: L = 2 x W = 2 x 48 = 96

Area of the rectangular field = L x W = 96 x 48 = 4608 m^2

Q-12. Find the least square number, exactly divisible by each one of the numbers:

(i) 6, 9, 15 and 20 **(ii) 8, 12, 15 and 20**

Solution:

(i) The smallest number divisible by 6, 9, 15 and 20 is their L.C.M., which is equal to 60.

Factorizing 60 into its prime factors:

60 =2 x2 x 3 x 5

Grouping them into pairs of equal factors:

60 =(2 x 2)x 3 x 5

The factors 3 and 5 are not paired.

To make 60 a perfect square, we have to multiply it by 3 x 5, i.e . by 15.

The perfect square is 60 x 15, which is equal to 900.

(ii) The smallest number divisible by 8, 12, 15 and 20 is their L.C.M., which is equal to 120.

Factorizing 120 into its prime factors:

120 =2 x 2 x 2 x 3 x 5

Grouping them into pairs of equal factors:

120 = (2 x 2) x 2 x 3 x 5

The factors 2, 3 and 5 are not paired.

To make 120 into a perfect square, we have to multiply it by 2 x 3 x 5, i.e. by 30.

The perfect square is 120 x 30, which is equal to 3600.

Q-13. Find the square roots of 121 and 169 by the method of repeated subtraction.

Solution:

To find the square root of 121:

121 − 1 = 120

120 − 3 = 117

117 − 5 = 112

112 − 7 = 105

105 − 9 = 96

96 − 11 = 85

85 − 13 = 72

72 − 15 = 57

$57 - 17 = 40$

$40 - 19 = 21$

$21 - 21 = 0$

In total, there are 11 numbers to subtract from 121. Hence, the square root of 121 is 11.

To find the square root of 169:

$169 - 1 = 168$

$168 - 3 = 165$

$165 - 5 = 160$

$160 - 7 = 153$

$153 - 9 = 144$

$144 - 11 = 133$

$133 - 13 = 120$

$120 - 15 = 105$

$105 - 17 = 88$

$88 - 19 = 69$

$69 - 21 = 48$

$48 - 23 = 25$

$25 - 25 = 0$

In total, there are 13 numbers to subtract from 169. Hence, the square root of 169 is 13.

Q-14. Write the prime factorization of the following numbers and hence find their square roots:

(i) 7744 **(ii) 9604** **(iii) 5929** **(iv) 7056**

Solution:

(i) The prime factorization of 7744:

$7744 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 11$

Grouping them into one pair of equal factors, we get:

7744 = (2 × 2) × (2 × 2) × (2 × 2) × (11 × 11)

Taking one factor from each pairs, we will get:

$\sqrt{7744}$ = 2 × 2 × 2 × 11 = 88

(ii) The prime factorization of 9604:

9604 = 2 × 2 × 7 × 7 × 7 × 7

Grouping them into one pair of equal factors, we get:

9604 = (2 × 2) × (7 × 7) × (7 × 7)

Taking one factor from each pairs, we will get:

$\sqrt{9604}$ = 2 × 7 × 7 = 98

(iii) The prime factorization of 5929:

5929 = 7 × 7 × 11 × 11

Grouping them into one pair of equal factors, we get:

5929 = (7 × 7) × (11 × 11)

Taking one factor from each pairs, we will get:

$\sqrt{5929}$ = 7 × 11 = 77

(iv) The prime factorization of 7056:

7056 = 2 × 2 × 2 × 2 × 3 × 3 × 7 × 7

Grouping them into one pair of equal factors, we get:

7056 = (2 × 2) × (2 × 2) × (3 × 3) × (7 × 7)

Taking one factor from each pairs, we will get:

$\sqrt{7056}$ = 2 × 2 × 3 × 7 = 84

Q-15. The student of class VIII of a school donated Rs. 2401 for PM's National Relief Fund. Each student denoted as many rupees as the number of students in the class. Find the number of students in the class.

Solution:

Let S be the number of students.

Let r be the amount in rupees denoted by each student.

The total donation can be expressed by:

S × r = Rs. 2401

Since, the total amount in rupees is equal to the number of students

So, r is equal to S.

Substituting this in the first equation:

S × S = 2401

S^2 = (7 × 7) × (7 × 7)

S = 7 × 7 = 49

So, there are 49 students in the class.

Q-16. A PT teacher wants to arrange maximum possible number of 6000 students in a field such that the number of rows is equal to the number of columns. Find the number of rows if 71 were left out after arrangement.

Solution:

Since, 71 students were left out, there are only 5929, i.e., (6000 − 72) students remaining.

Hence, the number of rows and columns is simply the square root of 5929.

Factorizing 5929 into its prime factors:

5929 = 7 × 7 × 11 × 11

Grouping them into pairs of equal factors:

5929 = (7 × 7) × (11 × 11)

The square root of 5929 = $\sqrt{5929}$ = 7 × 11 = 77

Hence, in the arrangement, there were 77 rows of students.

Exercise 3.5

Q-1. Find the square root of each of the following by long division method:

(i) 12544

(ii) 97344

(iii) 286225

(iv) 390625

(v) 363609

(vi) 974169

(vii) 120409

(viii) 1471369

(ix) 291600

(x) 9653449

(xi) 1745041

(xii) 4008004

(xiii) 20657025

(xiv) 152547201

(xv) 20421361

(xvi) 62504836

(xvii) 82264900

(xviii) 3226694416

(xix) 6407522209

(xx) 3915380329

Solution:

(i)

	112
1	12544
1	1
21	025
1	21
222	444
2	444
	0

Hence, the square root of 12544 is 112.

(ii)

	312
3	97344
3	9
61	073
1	63
622	1244
2	1244
	0

Hence, the square root of 97344 is 312.

(iii)

	535
5	286225
5	25
103	362
3	303
1065	5325
5	5325
	0

Hence, the square root of 286225 is 535.

(iv)

	625
6	390625
6	36
122	306
2	244
1245	6225
5	6225
	0

Hence, the square root of 390625 is 625.

(v)

	603
6	363609
6	36
1203	3609
3	3609
	0

Hence, the square root of 363609 is 603.

(vi)

	987
9	974169
9	81
188	1641
8	1504
1967	13769
7	13769
	0

Hence, the square root of 974169.

(vii)

	347
3	120409
3	9
64	304
4	256
687	4809
7	4809
	0

Hence, the square root of 120409 is 347.

(viii)

	1213
1	1471369
1	1
22	47
2	44
241	313
1	241
2423	7269
3	7269
	0

Hence, the square root of 1471369 is 1213.

(ix)

	540
5	291600
5	25
104	416
4	416
1080	000
0	000
	0

Hence, the square root of 291600 is 540

(x)

	3107
3	9653449
3	9
61	65
1	61
6207	43449
7	43449
	0

Hence, the square root of 9653449 is 3107.

(xi)

	1321
1	1745041
1	1
23	74
3	69
262	550
2	524
2641	2641
1	2641
	0

Hence, the square root of 1745041 is 1321.

(xii)

	2002
2	4008004
2	4
40	00
0	00
400	080
0	00
4002	8004
2	8004
	0

Hence, the square root 4008004 is 2002

(xiii)

	4545
4	20657025
4	16
85	465
5	425
904	4070
4	3616
9085	45425
5	45425
	0

Hence, the square root of 20657025 is 4545

(xiv)

	12351
1	152547201
1	1
22	52
2	44
243	854
3	729
2465	12572
5	12325
24701	24701
1	24701
	0

Hence, the square root of 152547201 is 12351.

(xv)

	4519
4	20421361
4	16
85	442
5	425
901	1713
1	901
9089	81261
9	81261
	0

Hence, the square root of 20421361 is 4519.

(xvi)

	7906
7	62504836
7	49
149	1350
9	1341
1580	948
0	0
15806	94836
6	94836
	0

Hence, the square root of 6250486 is 7906.

(xvii)

	9070
9	82264900
9	81
180	126
0	0
1807	12649
7	12649
180140	000
0	0
	0

Hence, the square root of 82264900 is 9070.

(xviii)

	56804
5	3226694416
5	25
106	726
6	636
1128	9069
8	9024
11360	4544
0	0
113604	454416
4	454416
	0

Hence, the square root of 3226694416 is 56804.

(xix)

	80047
8	6407522209
8	64
160	007
0	0
1600	752
0	0
16004	75222
4	64016
160087	1120609
7	1120609
	0

Hence, the square root of 6407522209 is 80047

(xx)

	625763
6	3915380329
6	36
122	315
2	244
1245	7138
5	6225
124507	91303
7	87549
125143	375429
3	375429
	0

Hence, the square root of 3915380329 is 625763.

Q-2. Find the least number which must be subtracted from the following numbers to make them a perfect square:

(i) 2361 **(ii) 194491** **(iii) 26535** **(iv) 16160** **(v) 4401624**

Solution.

(i) Using the long division method:

	48
4	2361
4	16
88	761
8	704
	57

We can see that 2361 is 57 more than 47^2. Hence, 57 must be subtracted from 2361 to get a perfect square.

(ii) Using the long division method:

	441
4	194491
4	16
84	344
4	336
881	891
1	881
	10

We can see that 194491 is 10 more than 441^2. Hence, 10 must be subtracted from 194491 to get a perfect square.

(iii) Using the long division method:

	162
1	26535
1	1
26	165
6	156
322	935
2	644
	291

We can see that 26535 is 291 more than 162^2. Hence, 291 must be subtracted from 26535 to get a perfect square.

(iv) Using the long division method:

	127
1	16160
1	1
22	061
2	44
247	1760
7	1729
	31

We can see that 16160 is 31 more than 127^2. Hence, 31 must be subtracted from 16160 to get a perfect square.

(v) Using the long division method;

	2098
2	4401624
2	4
40	40
0	4016
409	3681
9	33524
4188	33524
8	33504
	20

We can see that 4401624 is 20 more than 2098^2. Hence, 20 must be subtracted from 4401624 to get a perfect square.

Q-3. Find the least number which must be added from the following numbers to make them a perfect square:

(i) 5607 (ii) 4931 (iii) 4515600 (iv) 37460 (v) 506900

Solution.

(i) Using the long division method:

	75
7	5607
7	49
145	707
5	725
	-18

We can see that 5607 is 18 more than 75^2. Hence, we have to add 18 to 5607 to get a perfect square.

(ii) Using the long division method:

	71
7	4931
7	49
141	031
1	141
	-110

We can see that 4931 is 110 more than 71^2. Hence, we have to add 110 to 4931 to get a perfect square.

(iii) Using the long division method:

	2125
2	4515600
2	4
41	051
1	41
422	1056
2	844
4245	21200
5	21225
	-25

We can see that 4515600 is 25 more than 2125^2. Hence, we have to add 25 to 4515600 to get a perfect square.

(iv) Using the long division method:

	194
1	37460
1	1
29	274
9	261
384	1360
4	1536
	-176

We can see that 37460 is 176 more than 194^2. Hence, we have to 176 to 37460 to get a perfect square.

(v) Using the long division method:

	712
7	506900
7	49
141	169
1	141
1422	2800
2	2844
	-44

We can see that 506900 is 44 more than 712^2. Hence, we have to add 44 to 506900 to get a perfect square.

Q-4. Find the greatest number of 5 digits which is a perfect square.

Solution.

The greatest number with five digits is 99999.

To find the greatest square number with five digits, we must find the smallest number that must be subtracted from 99999 in order to make a perfect square.

For that, we have to find the square root of 99999 by the long division method as follows:

	316
3	99999
3	9
61	099
1	61
626	3899
6	3756
	143

Hence, we must subtract 143 from 99999 to get a perfect square.

99999 − 143 = 99856

Q-5. Find the least number of 4 digits which is a perfect square .

Solution.

The least number with four digits is 1000. To find the square number with four digits, we must find the smallest number that must be added to 1000 in order to make a perfect square. For that, we have to find the square root of 1000 by the long division method as shown below:

	32
3	1000
3	9
62	100
2	124
	-24

1000 is 24 (124 − 100) less than the nearest square number 32^2. Thus, 24 must be added to 1000 to be a perfect square.

1000 + 24 = 1024

Hence, the smallest perfect square number with four digits is 1024.

Q-6. Find the least number of six digits which is a perfect square.

Solution.

The least number with six digits is 100000. To find the least square number with six digits. we must find the smallest number that must be added to 100000 in order to make a perfect square. For that, we have to find the square root of 100000 by the long division method as follows:

	317
3	100000
3	9
61	100
1	61
627	3900
7	4389
	-489

100000 is 489 (4389 − 3900) less than 3172.

Hence, to be a perfect square, 489 should be added to 100000.

100000 + 489 = 100489

Hence, the least number of six digits that is a perfect square is 100489.

Q-7. Find the greatest number of 4 digits which is a perfect square.

Solution.

The greatest number with four digits is 9999.

To find the greatest perfect square with four digits. we must find the smallest number that must be subtracted from 9999 in order to make a perfect square. For that, we have to find the square root of 9999 by the long division method as shown below:

	99
9	1000
9	9
189	1899
9	1701
	198

We must subtract 198 from 9999 to make a perfect square:

9999 — 198 = 9801

Hence, the greatest perfect square with four digits is 9801.

Q-8. A General arranges his soldiers in rows to form a perfect square. He finds that in doing so, 60 soldiers are left out. If the total number of soldiers be 8160, find the number of soldiers in each row.

Solution.

60 soldiers are left out.

So, Remaining soldiers = 8160 — 60 = 8100

The number of soldiers in each row to form a perfect square would be the square root of 8100.

We have to fund the square root of 8100 by the long division method as shown below:

	90
9	8100
9	81
	000

Hence, the number of soldiers in each row to form a perfect square is 90.

Q-9. The area of a square field is 60025 m^2. A man cycles along its boundary at 18 km/hr. In how much time will he return at the starting point?

Solution.

Area of the square field = 60025 m^2

The length of the square field would be the square root of 60025.

Using the long division method:

	245
2	60025
2	4
44	200
4	176
485	2425
5	2425
	0

Hence, the length of the square field is 245 m.

The square has four sides, so the number of boundaries of the field is 4.

The distance s covered by the man = 245 m x 4 = 980 m = 0.98 km

If the velocity v is 18 km/hr, the required time t can be calculated using the following formula:

$t = \frac{s}{v}$

$t = \frac{0.98}{18}$ = 0.054 hr = 3 minutes, 16 seconds

So, the man will return to the starting point after 3 minutes and 16 seconds.

Q-10. The cost of leveling and turfing a square lawn at Rs. 2.50 per m^2 is Rs. 13322.50. Find the cost of fencing it at Rs. 5 per metres.

Solution.

First, we have to find the area of the square lawn, which the total cost divided by the cost of leveling and turfing per square metre:

Area of a square = $\frac{13322.5}{2.5}$ = 5329 m^2

The length of one side of the square is equal to the square root of the area. We will use the long division method to find it as shown below:

	73
7	5329
7	49
143	429
3	429
	0

Therefore, the length of one side of the square = 73 m

The circumference of the square is 73 × 4 = 292 m

Hence, the cost of fencing the lawn at Rs. 5 per metre = 292 × 5 = Rs. 1460.

Q-11. Find the greatest number of three digits which is a perfect square.

Solution.

The greatest number with three digits is 999.

To find the greatest perfect square with three digits, we must find the smallest number that must be subtracted from 999 in order to get a perfect square. For that, we have to find the square root by the long division method as shown below:

	31
3	999
3	9
61	99
1	61
	38

So, 38 must be subtracted from 999 to get a perfect square.

999 − 38 = 961

961 = 31^2.

Hence, the greatest perfect square with three digits is 961.

Q-12. Find the smallest number which must be added to 2300 so that it becomes a perfect square.

Solution.

To find the square root of 2300, we use the long division method:

	48
4	2300
4	16
88	700
8	704
	-4

2300 is 4 (704 − 700) less than 48^2.

Hence, 4 must be added to 2300 to get a perfect square.

Exercise 3.6

1.) Find the square root of:

(i) $\frac{441}{961}$

We know:

$$\sqrt{\frac{441}{961}} = \sqrt{\frac{441}{961}}$$

Now, let us complete the square roots of the numerator and denominator separately.

$\sqrt{441} = \sqrt{(3 \times 3) \times (7 \times 7)}$ = 3 x 7 =21

$\sqrt{961} = \sqrt{31 \times 31}$ = 31

$$\therefore \sqrt{\frac{441}{961}} = \frac{21}{31}$$

(ii) $\frac{324}{841}$

We know:

$$\sqrt{\frac{324}{841}} = \sqrt{\frac{324}{841}}$$

Now, let us complete the square roots of the numerator and denominator separately

$\sqrt{324} = \sqrt{2 \times 2 \times 3 \times 3 \times 3}$ = 2 x 3 x 3 =18

$\sqrt{841} = \sqrt{29x29}$ = 29

$$\therefore \frac{324}{841} = \frac{18}{29}$$

(iii) 4 $\frac{29}{29}$

By looking at the book's answer key, the fraction should be $\sqrt{4\frac{29}{49}}$, not $\sqrt{4\frac{29}{29}}$

We know:

$\sqrt{4\frac{29}{49}} = \sqrt{\frac{225}{49}}$

$\therefore \sqrt{4\frac{29}{49}} = \frac{15}{7}$

(iv) $2\ \frac{14}{25}$

We know:

$\sqrt{2\frac{14}{25}} = \sqrt{\frac{64}{25}} = \frac{8}{5}$

(v) $2\frac{137}{196}$

We know

$\sqrt{2\frac{137}{196}} = \sqrt{\frac{529}{196}}$

Now, let us complete the square roots of the numerator and the denominator separately.

$\sqrt{529} = \sqrt{23 \times 23} = 23$

$\sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7} = 2 \times 7 = 14$

$\sqrt{2\frac{137}{196}} = \frac{23}{14}$

(vii) $25\ \frac{54}{729}$

We know:

$$\sqrt{25\frac{544}{729}} = \sqrt{\frac{18769}{729}}$$

Now, let us compute the square roots of the numerator and denominator separately.

$$\sqrt{25\frac{544}{729}} = \frac{137}{27}$$

	137
1	18769
1	1
23	87
3	69
267	1869
7	1869
	0

(viii) 75 $\frac{46}{49}$

We know,

$$\therefore \sqrt{75\frac{46}{49}} = \sqrt{\frac{3721}{49}}$$

Now, let us compute the square roots of the numerator and denominator separately.

$$\therefore \sqrt{75\frac{46}{49}} = [latex]\frac{61}{7}$$

	61
6	3721
6	36
121	121
1	121
	0

(ix) $3\frac{942}{2209}$

We know:

$$\sqrt{3\frac{942}{2209}} = \sqrt{3\frac{942}{2209}}$$

Now, let us compute the square roots of the numerator and the denominator separately.

$$\sqrt{3\frac{942}{2209}} = \frac{87}{47}$$

	87
8	7569
8	64
167	1169
7	1169
	0

	47
4	2209
4	16
87	609
7	609
	0

(x) $3\frac{334}{3025}$

We know:

$$\sqrt{3\frac{334}{3025}} = \sqrt{\frac{73441}{3364}}$$

Now, let us compute the square roots of the numerator and denominator separately.

$$\therefore \sqrt{3\frac{334}{3025}} = \frac{97}{55}$$

$$
\begin{array}{r|l}
 & 97 \\
\hline
9 & 9409 \\
9 & 81 \\
\hline
187 & 1309 \\
7 & 1309 \\
\hline
 & 0
\end{array}
\qquad
\begin{array}{r|l}
 & 55 \\
\hline
5 & 3025 \\
5 & 25 \\
\hline
105 & 525 \\
5 & 525 \\
\hline
 & 0
\end{array}
$$

(xi) 21 $\frac{2797}{3364}$

We know:

$$\therefore \sqrt{21\frac{2797}{3364}} = \frac{73441}{3364}$$

Now, let us compute the square roots of the numerator and denominator separately.

$$\therefore \sqrt{21\frac{2797}{3364}} = \frac{271}{58}$$

$$
\begin{array}{r|l}
 & 271 \\
\hline
2 & 73441 \\
2 & 4 \\
\hline
47 & 334 \\
7 & 329 \\
\hline
541 & 541 \\
1 & 541 \\
\hline
 & 0
\end{array}
\qquad
\begin{array}{r|l}
 & 58 \\
\hline
5 & 3364 \\
5 & 25 \\
\hline
108 & 864 \\
8 & 864 \\
\hline
 & 0
\end{array}
$$

(xii) 38 $\frac{11}{25}$

We know:

$$\sqrt{38\frac{11}{25}} = \sqrt{\frac{961}{25}}$$

Now, let us compute the square roots of the numerator and the denominator separately.

$\therefore \sqrt{38\frac{11}{25}} = \frac{31}{5}$

(xiii) 23 $\frac{394}{729}$

We know:

$\sqrt{23\frac{394}{729}} = \sqrt{\frac{17161}{729}}$

Now, let us compute the square roots of the numerator and the denominator separately.

$\therefore \sqrt{23\frac{394}{729}} = \frac{131}{24} = 4\frac{23}{27}$

	131
1	17161
1	1
23	71
3	69
261	261
1	261
	0

(xiv) 21 $\frac{51}{169}$

We know:

$\therefore \sqrt{21\frac{51}{169}} = \frac{3600}{169} =$

Now, let us compute the square roots of the numerator and denominator separately.

$\therefore \sqrt{21\frac{51}{169}} = \frac{60}{13} = 4\frac{8}{13}$

(xv) 10 $\frac{151}{225}$

We know:

$\sqrt{10\frac{151}{225}} = \sqrt{\frac{2401}{225}}$

Now let us compute the square roots of the numerator and denominator separately.

$\sqrt{2401} = \sqrt{7 \times 7 \times 7 \times 7}$ = 7 x 7 = 49

$\sqrt{225} = \sqrt{3 \times 3 \times 5 \times 5}$ = 3 x 5 =15

$\therefore \sqrt{10\frac{151}{225}} = \frac{49}{15} = 3\frac{4}{15}$

2.) Find the value of:

(i) $\frac{\sqrt{80}}{\sqrt{405}}$

We have:

$\frac{\sqrt{80}}{\sqrt{405}} = \sqrt{\frac{80}{405}} = \sqrt{\frac{16}{81}} = \frac{4}{9}$

(ii) $\frac{\sqrt{441}}{\sqrt{625}}$

Comparing the square roots:

$\sqrt{441} = \sqrt{(3 \times 3) \times (7 \times 7)}$ = 3 x 7 =21

$\sqrt{625} = \sqrt{(5 \times 5) \times (5 \times 5)}$ = 5 x 5 =25

$\therefore \frac{\sqrt{441}}{\sqrt{625}} = \frac{21}{25}$

(iii) $\frac{\sqrt{1587}}{\sqrt{1728}}$

We have:

$\frac{\sqrt{1587}}{\sqrt{1728}} = \sqrt{\frac{529}{576}}$ (by dividing both numbers by 3)

Computing the square roots of the numerator and the denominator:

$\sqrt{529} = \sqrt{23 \times 23}$ = 23

$\sqrt{576} = \sqrt{24 \times 24}$ = 24

$\therefore \frac{\sqrt{1587}}{\sqrt{1728}} = \frac{23}{24}$

(iv) $\sqrt{72} \times \sqrt{338}$

We have:

$\sqrt{72} \times \sqrt{328} = \sqrt{72 \times 338} = \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 13 \times 13}$
= $\sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 13 \times 13}$ = 2 x 2 x 3 x 13

= 156

(v) $\sqrt{45} \times \sqrt{20}$

We have:

$\sqrt{45} \times \sqrt{20} = \sqrt{3 \times 3 \times 5 \times 2 \times 2 \times 5}$ = 30

3.) The area of a square is 80 $\frac{244}{729}$ square metres. Find the length of each side of the field.

The length of one side is the square root of the area of the field. Hence, we need to calculate the value of $\sqrt{80\frac{244}{729}}$

We have

$\sqrt{80\frac{244}{729}} = \sqrt{\frac{58564}{729}}$

Now, to calculate the square of the numerator and the denominator:

We know that:

$\sqrt{729} = 27$

Therefore, length of one side of the field = $\frac{242}{27} = 8\frac{26}{27}$ m

	242
2	58564
2	4
44	185
4	176
482	964
2	964
	0

4.) The area of a square field is $30\frac{1}{4}$ m^2. Calculate the length of the side of the square.

Answer 4:

The length of one side is equal to the square root of the area of the field. Hence, we just need to calculate the value of $\frac{242}{27} = 8\frac{26}{27}m$

We have:

$$\sqrt{30\frac{1}{4}} = \frac{\sqrt{121}}{\sqrt{4}}$$

Now, calculating the square root of the numerator and the denominator

$\sqrt{121} = \sqrt{11 \times 11}$ = 11

$\sqrt{4} = 2$

Therefore, the length of the side of the square= $30\frac{1}{4} = \frac{11}{2} = 5\frac{1}{2}$m

5.) Find the length of a side of a square playground whose area is equal to the area of a rectangular field of dimensions 72 m and 338 m.

Answer 5:

The area of the playground = 72 x 338 = 24336 m^2

The length of one side of a square is equal to the square root of its area. Hence, we just need to find the square root of 24336.

Hence, the length of one side of the playground is 156 meters.

	156
1	24336
1	1
25	143
5	125
306	1836
6	1836
	0

Exercise 3.7

Find the square root of the following numbers in the decimal form:

1.) 84.8241

Answer:

	9.21
9	84.8241
9	81
182	382
2	364
1841	1841
1	1841
	0

Hence, the square root of 84.821 is 9.21.

2.) 0.7225

Answer:

	0.85
8	0.7225
8	64
165	825
5	825
	0

Hence, the square root of 0.7225 is 0.85.

3.) 0.813604

Answer:

	0.902
9	0.813604
9	81
1802	3604
2	3604
	0

Hence, the square root of 0813604 is 0.902

4.) 0.00002025

Answer:

	0.0045
4	0.00002025
4	16
85	425
5	425
	0

Hence, the square root of 0.00002025 is 0.0045.

5.) 150.0625

Answer:

	12.25
1	150.0625
1	1
22	050
2	44
242	606
2	484
2445	12225
5	12225
	0

Hence, the square root of 150.0625 is 12.25

6.) 225.6004

Answer:

	15.02
1	225.6004
1	1
25	125
5	125
3002	6004
2	6004
	0

Hence, the square root of 225.6004 is 15.02

7.) 3600.720036

Answer:

	60.006
6	3600.720036
6	36
120006	000720036
6	720036
	0

Hence, the square root of 3600.720036 is 60.006

8.) 236.144689

Answer:

	15.367
1	236.144689
1	1
25	136
5	125
303	1114
3	909
3066	20546
6	18396
30727	215089
7	215089
	0

Hence, the square root of 236.144869 is 15.367 .

9.).00059049

Answer:

	0.0243
2	0.00059049
2	4
44	190
4	176
483	1449
3	1449
	0

Hence, the square of 0.0059049 is 0.0243

10.) 176.252176

Answer:

	13.276
1	176.252176
1	1
23	076
3	69
262	725
2	524
2647	20121
7	18529
26546	159276
6	159276
	0

Hence, the square root of 176.252176 is 13.276

11.) 9998.0001

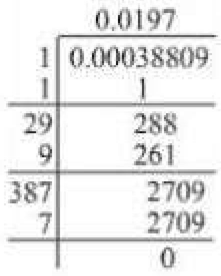
Answer:

	99.99
9	9998.0001
9	81
189	1898
9	1701
1989	19700
9	17901
19989	179901
9	179901
	0

Hence, the square root of 9998.0001 is 99.99.

12.) 0.00038809

Answer:

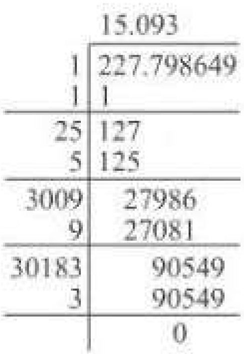


Hence, the square root of 0.00038809 is 0.0197.

13.) What is that fraction which when multiplied by itself gives 227.798649?

Answer:

We have to find the square root of the given number:



Hence, the fraction, which when multiplied by itself, gives 227.798649 is 15.093.

14.) The area of a square playground is 256.6404 square meters. Find the length of one side of the playground.

Answer:

The length of one side of the playground is the square root of its area.

	16.02
1	256.6404
1	1
26	156
6	156
3202	6404
2	6404
	0

So, the length of one side of the playground is 16.02 meters.

15.) What is the fraction which when multiplied by itself gives 0.00053361?

Answer:

We have to find the square root of the given number:

	0.0231
2	0.00053361
2	4
43	133
3	129
461	461
1	461
	0

Hence, the fraction, which when multiplied by itself, gives 0.00053361 is 0.0231.

16.) Simplify:

(i) $\frac{\sqrt{59.29}-\sqrt{5.29}}{\sqrt{59.29}+\sqrt{5.29}}$

(ii) $\frac{\sqrt{0.2304}+\sqrt{0.1764}}{\sqrt{0.2304}-\sqrt{0.1764}}$

Answer:

(i) We have:

$\sqrt{59.29}=\sqrt{\frac{5929}{100}}=\frac{\sqrt{7\times7\times11\times11}}{10}=\frac{7\times11}{10}=7.7\ \sqrt{5.29}=\sqrt{\frac{592}{100}}=\frac{\sqrt{529}}{\sqrt{100}}=\frac{23}{10}=2.3$

$\frac{\sqrt{59.29}-\sqrt{5.29}}{\sqrt{59.29}+\sqrt{5.29}}=\frac{7.7-2.3}{7.7+2.3}=\frac{5.4}{10}=.54$

(ii) We have:

$\sqrt{0.2304}=\sqrt{\frac{2304}{10000}}$

= $\frac{\sqrt{2\times2\times2\times2\times2\times2\times2\times2\times3\times3}}{\sqrt{10000}}$

= $\frac{2\times2\times2\times2\times3}{100}$ = 0.42

$\frac{\sqrt{0.2304}+\sqrt{0.1764}}{\sqrt{0.2304}-\sqrt{0.1764}}=\frac{0.48+0.42}{0.48-0.42}=\frac{0.9}{0.06}$ = 15

17.) Evaluate $\sqrt{50625}$ and hence find the value of $\sqrt{506.25}+\sqrt{5.0625}$

Answer:

We have:

$\sqrt{50625} = \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5} = 3 \times 3 \times \times 5 \times 5$ = 225

Next, we calculate $\sqrt{506.25}$ and $\sqrt{5.0625}$

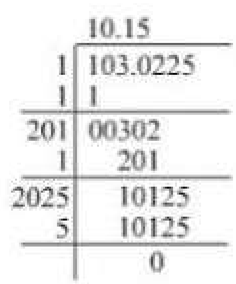
$\sqrt{506.25} = \sqrt{\frac{50625}{100}} = \frac{\sqrt{52625}}{\sqrt{100}} = \frac{225}{10} = 22.5$ $\sqrt{5.0625} = \sqrt{\frac{50625}{10000}} = \frac{\sqrt{52625}}{\sqrt{10000}} = \frac{225}{100} = 2.25$
$\sqrt{506.25} + \sqrt{5.0625} = 22.5 + 2.25 = 24.75$

18.) Find the value of $\sqrt{103.0225}$ and hence find the value of:

(i) $\sqrt{10302.25}$

(ii) $\sqrt{1.030225}$

Answer:



The value of 103.0225 is:

Hence, the square root of 103.0225 is 10.15

Now, we can solve the following questions as shown below:

(i) $\sqrt{10302.25} = \sqrt{103.0225 \times 100} = \sqrt{103.0225} \times \sqrt{100} = 10.15 \times 10 = 101.5.$

(ii) $\sqrt{1.030225} = \sqrt{\frac{103.0225}{100}} = \frac{\sqrt{103.0225}}{\sqrt{100}} = \frac{10.15}{10} = 1.015$

Exercise 3.8

find the square root of following correct to three places of decimal.

1)

5

	2.236
2	5.00 00 00 00
	4
42	100
	84
443	1600
	1329
4466	27100
	26796
	304

= 2.236

2)

7

	2.646
2	7.00 00 00
	4
46	300
	276
524	2400
	2096
5286	30400
	30316
	84

= 2.646

3)

17

	4.123
4	17.00 00 00
	16
81	1.00
	81
822	1900
	1644
8243	25600
	24729
	871

= 4.123

4)

20

	4.472
4	20.00 00 00
	16
84	400
	336
887	6400
	6209
8942	19100
	17884
	1216

= 4.472.

5)

66

	8.124
8	66.00 00 00
	64
161	200
	161
1622	3900
	3244
16244	65600
	64976
	624

= 8.124.

6)

427

	20.664
	427. 00 00 00 00
2	4
40	027
	0
406	2700
	2436
4126	26400
	24756
41324	164400
	164296
	104.

= 20.664

7)

1.7

	1.304
1	1.70 00 00
	1
23	0.70
	69
260	1 00
	0
2604	10000
	10416
	−416.

= 1.304.

8)

23.1

4.806

4 | 23.10 00 00
16

88 | 710
704

960 | 600
0

9606 | 60000
57636

2364

= 4.806.

9)

2.5

1.581

1 | 2.50 00 00
1

25 | 150
125

308 | 2500
2464

3161 | 3600
3161

~~1136~~ 439

= 1.581

10)

237.615

15.415

$\overline{2}\overline{37}.\overline{61}\overline{50}\overline{00}$

1 | 1

25 | 137
125

304 | 1261
1216

3081 | 4550
3081

30825 | 146900
154125

7225

= 15.415

11)

15.3215

3.914

$\overline{15}.\overline{32}\overline{15}\overline{00}$

3 | 9

69 | 632
621

781 | 1115
781

7824 | 33400
31296

2104

= 3.914.

12)

0.9

0.949

$\overline{0}.\overline{90}\overline{00}\overline{00}$

0 | 0

9 | 090
81

184 | 900
736

1889 | 16400
17001

601

= 0.949.

Exercise 3.9

Using square root table, find the square roots of the following:

1.) 7

Answer:

From the table, we directly find that square root of 7 is 2.646.

2.) 15

Answer:

Using the table to find $\sqrt{3}$ and $\sqrt{5}$

$\sqrt{15}$ = $\sqrt{3}$ x $\sqrt{5}$

= 1.732 x 2.236 = 3.873

3.) 74

Answer:

Using the table to find $\sqrt{2}$ and $\sqrt{37}$

$\sqrt{74}$ = $\sqrt{2}$ x $\sqrt{37}$

= 1.414 x 6.083 = 8.602

4.) 82

Answer:

Using the table to find $\sqrt{2}$ and $\sqrt{41}$

$\sqrt{82}$ = $\sqrt{2}$ x $\sqrt{41}$

= 1.414 x 6.403 = 9.055

5.) 198

Answer:

Using the table to find $\sqrt{2}$ and $\sqrt{11}$

$\sqrt{198}$ = $\sqrt{2}$ x 9 x $\sqrt{41}$

= 1.414 x 3 x 6.403 = 14.070

6.) 540

Answer:

Using the table to find $\sqrt{3}$ and $\sqrt{5}$

$\sqrt{540}$ = $\sqrt{54}$ x $\sqrt{10}$

= 2 x 3$\sqrt{3}$ x $\sqrt{5}$ = 23.24

7.) 8700

Answer:

Using the table to find $\sqrt{3}$ and $\sqrt{29}$

$\sqrt{8700}$ = $\sqrt{3}$ x $\sqrt{29}$ x $\sqrt{100}$

= 1.414 x 6.403 x 10 = 93.27

8.) 3509

Answer:

Using the table to find $\sqrt{29}$

$\sqrt{3509}$ = $\sqrt{121}$ x $\sqrt{29}$

= 11 x 5.3851 = 59.235

9.) 6929

Answer:

Using the table to find $\sqrt{41}$

$\sqrt{6929}$ = $\sqrt{169}$ x 9 x $\sqrt{41}$

= 13 x 6.403 = 83.239

10.) 25725

Answer:

Using the table to find $\sqrt{3}$ and $\sqrt{7}$

$\sqrt{25725}$ = $\sqrt{3 \times 5 \times 5 \times 7 \times 7 \times 7}$

= 1.732 x 5 x 7 x 2.646 = 160.41

11.) 1312

Answer

Using the table to find $\sqrt{2}$ and $\sqrt{41}$

$\sqrt{1312}$ = $\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 41}$

= 2 x 2 x 1.414 x 6.4031 = 36.222

12.) 4192

Answer:

$\sqrt{4192} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 131}$
= $2 \times 2\sqrt{2} \times \sqrt{131}$

The square root of 131 is not listed in the table. Hence, we have to apply long division to find it.

	11.4455
1	131
1	1
21	31
1	21
224	1000
4	896
2284	10400
4	9136
22885	126400
5	114425
	52975

Substituting the values:

= 2 x 2 x 11.4455 (using the table to find $\sqrt{2}$) = 64.75

13.) 4955

Answer:

On prime factorization:

4955 is equal to 5 x 991, which means that $\sqrt{4955} = \sqrt{5}$

The square root of 991 is not listed in the table; it lists the square roots of all the numbers below 100. Hence, we have to manipulate the number such that we get the square root of a number less than 100. This can be done in the following manner:

$\sqrt{4955} = \sqrt{49.55 \times 100}$ = $\sqrt{49.55}$ x 10

Now, we have to find the square root of 49.55.

We have: $\sqrt{49} = 7\, and\, \sqrt{50} = 7.071$.

Their difference is 0.071. Thus, for the difference of 1 (50 − 49), the difference in the values of the square roots is 0.071.

For the difference of 0.55, the difference in the values of the square roots is:

0.55 x 0.0701 = 0.03905

$\therefore \sqrt{49.55} = 7 + 0.03905 = 7.03905$

Finally, we have:

$\sqrt{49.55} = \sqrt{49.55} \times 10 = 7.03905x10 = 70.3905$

14.) $\frac{99}{144}$

Answer:

$\sqrt{\frac{99}{144}} = \frac{\sqrt{3\times3\times11}}{\sqrt{144}}$

= $\frac{3\sqrt{11}}{12}$ = $\frac{3\times3.3166}{12}$ (using the square root to find $\sqrt{11}$) = 0.829

15.) $\frac{57}{169}$

Answer:

$\sqrt{\frac{57}{169}} = \frac{\sqrt{3\times19}}{\sqrt{169}}$

= $\frac{1.732\times4.3589}{13}$ (using the square root to find $\sqrt{3}$ and $\sqrt{19}$) = 0.581

16.) $\frac{101}{169}$

$$\sqrt{\frac{101}{169}}=\frac{\sqrt{101}}{\sqrt{169}}$$

The square of 101 is not listed in the table. This is because the table lists the square roots of all the numbers below 100.

Hence, we have to manipulate the number such that we get the square root of a number less than 100.

This can be done in the following manner:

$$\sqrt{101}=\sqrt{1.01\times100}=\sqrt{1.01}\times10$$

Now, we have to find the square root of 1.01.

We have:

$$\sqrt{1}=1 and \sqrt{2}=1.414$$

Their difference is .414.

Thus, for the difference of 1(2 -1), the difference in the values of the square roots is .414.

For the difference of .01, the difference in the values of the square roots is:

0.1 x 0.414 = 0.00414

$$\therefore\sqrt{1.01}=1+.00414=1.00414\ \sqrt{101}=\sqrt{1.01}\times10=1.00414\times10=10.0414$$

Finally,

$\sqrt{\frac{101}{169}}$ = 0.772

This value is really close to the one from the key answer.

17.) 13.21

Answer:

From the square root table, we have:

$\sqrt{13}$= 3.606 and $\sqrt{14} = \sqrt{2} \times \sqrt{7} = 3.742$

Their difference is 0.136.

Thus, for the difference of 1 (14 – 13), the difference in the values of the square roots is 0.136.

For the difference of 0.21, the difference in the values of their square roots is:

0.136 x 0.21 = 0.2856

$\therefore \sqrt{13.21} = 3.606 + 0.02856 \approx 3.635$

18.) 21.97

Answer:

We have to find $\sqrt{21.97}$

From the square root table, we have:

$\sqrt{21} = \sqrt{3} \times \sqrt{7} = 4.583 and \sqrt{22} = \sqrt{2} \times \sqrt{11} = 4.690$

Their difference is 0.107.

Thus, for the difference of 1 (22 – 21), the difference in the values of the square roosts is 0.107.

For the difference of 0.97, the difference in the values of their square roots is:

0.107 x 0.97 = 0.104

$\therefore \sqrt{21.97} = 4.583 + 0104 \approx 4.687$

19.) 110

Answer:

$$\sqrt{110} = \sqrt{2} \times \sqrt{5} \times \sqrt{11}$$
= 1.414 x 2.236 x 3.317 (Using the square root table to find all the square roots) = 10.488

20.) 1110

Answer:

$$\sqrt{1110} = \sqrt{2} \times \sqrt{3} \times \sqrt{5} \times \sqrt{37}$$
= 1.414 x 1.732 x 2.236 x 6.083 (using the table to find all the square roots) = 33.312

21.) 11.11

Answer:

We have:

$$\sqrt{11} = 3.317 and \sqrt{12} = 3.464$$
Their difference is 0.1474.

Thus, for the difference of 1 (12 – 11), the difference in the values of the square roots is 0.1474.

For the difference of 0.11, the difference in the values of the square roots is:

0.11 x 0.1474 = 0.0162

$$\therefore \sqrt{11.11} = 3.3166 + 0.0162 = 3.328 \approx 3.333$$

22.) The area of a square field is 325 m^2. Find the appropriate length of one side of the field.

Answer:

The length of one side of the square field will be the square root of 325.

$\therefore \sqrt{325} = \sqrt{5 \times 5 \times 13}$

= 5 x $\sqrt{13}$

= 5 x 3.605 = 18.030

Hence, the length of one side of the field is 18.030 m.

23.) Find the length of a side of a square, whose area is equal to the area of a rectangle with sides 240m and 70 m.

Answer:

The area of the rectangle = 240 m x 70 m =16800m^2

Given that the length of the square is equal to the area of the rectangle.

Hence, the area of the square will also be 16800 m^2.

The length of one side of a square is the square root of its area.

$\therefore \sqrt{16800} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 7}$

= 2 x2 x 5 $\sqrt{2 \times 3 \times 7}$

= 20 $\sqrt{42}$ = 129.60 m

Hence, the length of one side of the square is 129.60 m.