CENTRE OF MASS

Mass Moment : $\vec{M} = m\vec{r}$

CENTRE OF MASS OF A SYSTEM OF 'N' DISCRETE PARTICLES

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots\ldots + m_n} \; ; \; \vec{r}_{cm}$$

$$= \frac{\sum\limits_{i=1}^{n} m_i \vec{r}_i}{\sum\limits_{i=1}^{n} m_i} \quad \vec{r}_{cm} = \frac{1}{M} \sum\limits_{i=1}^{n} m_i \vec{r}_i$$
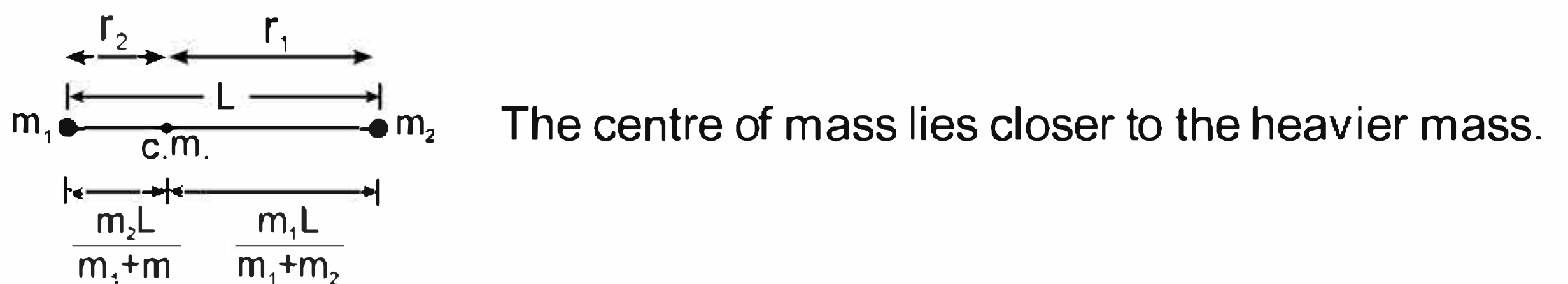
CENTRE OF MASS OF A CONTINUOUS MASS DISTRIBUTION

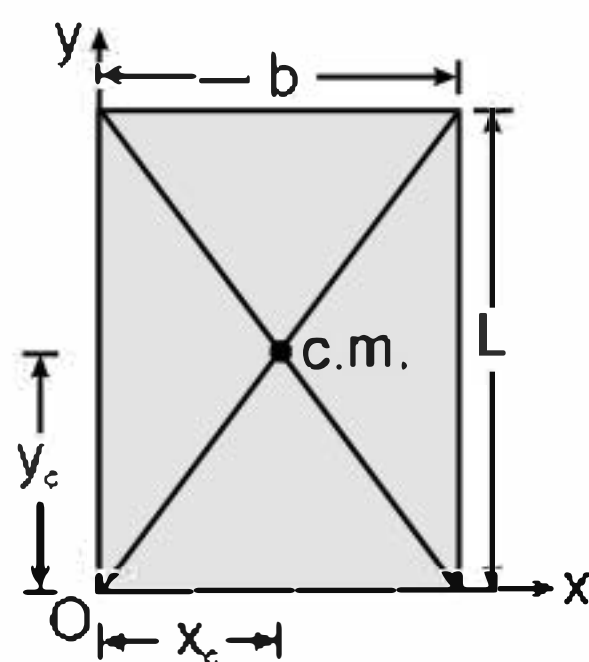$$x_{cm} = \frac{\int x\,dm}{\int dm} \, , \; y_{cm} = \frac{\int y\,dm}{\int dm} \, , \; z_{cm} = \frac{\int z\,dm}{\int dm}$$

$\int dm = M$ (mass of the body)

CENTRE OF MASS OF SOME COMMON SYSTEMS

$\Rightarrow$ A system of two point masses $m_1 r_1 = m_2 r_2$

The centre of mass lies closer to the heavier mass.
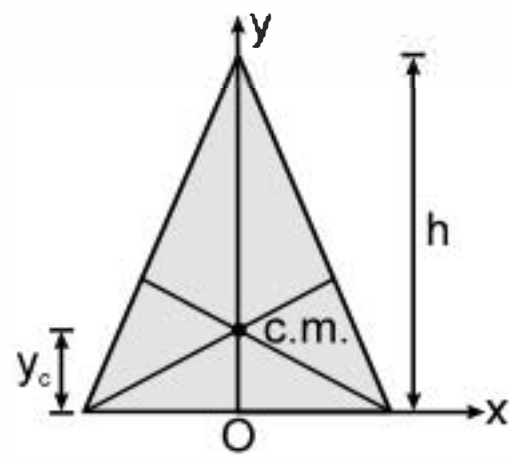
$\Rightarrow$ Rectangular plate (By symmetry)

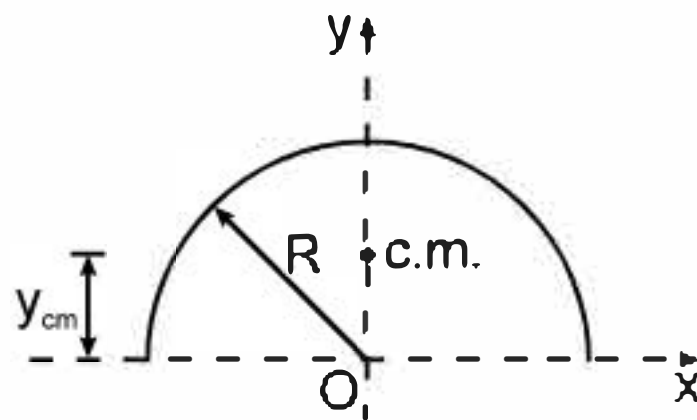$$x_c = \frac{b}{2} \qquad\qquad y_c = \frac{L}{2}$$

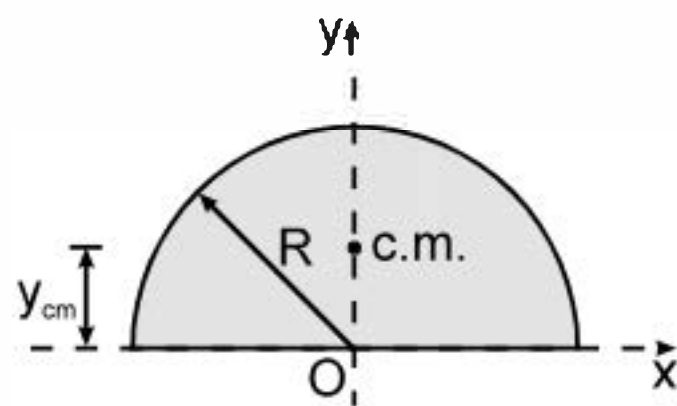⇒ A triangular plate (By qualitative argument)

at the centroid : $y_c = \dfrac{h}{3}$
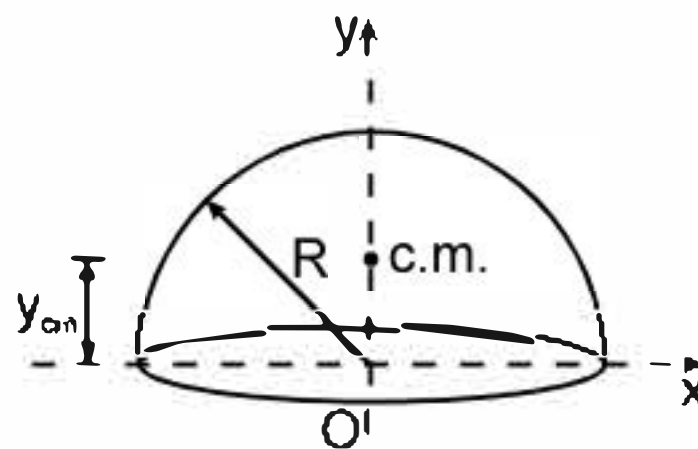
⇒ A semi-circular ring

$y_c = \dfrac{2R}{\pi} \qquad x_c = 0$
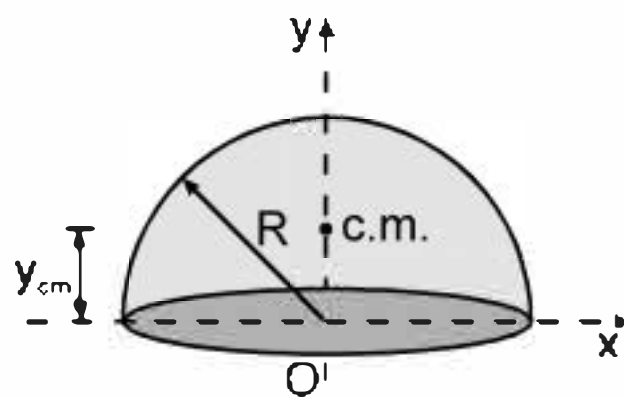
⇒ A semi-circular disc

$y_c = \dfrac{4R}{3\pi} \qquad x_c = 0$
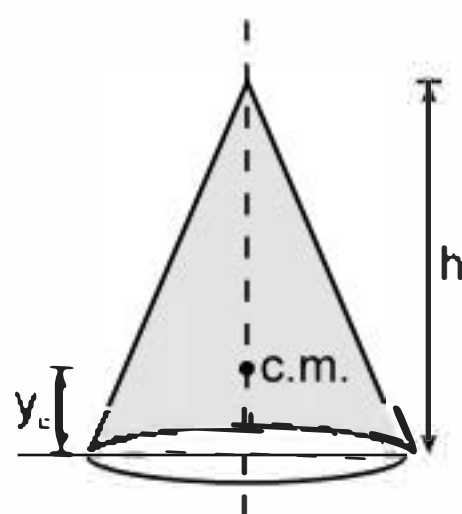
⇒ A hemispherical shell

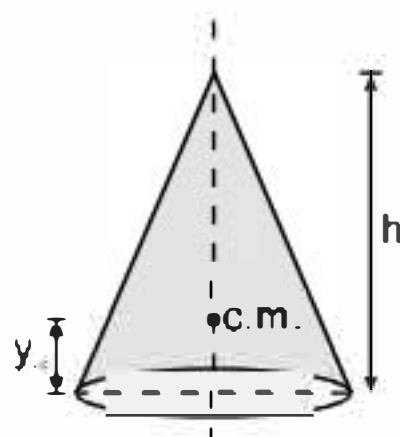

$y_c = \dfrac{R}{2} \qquad x_c = 0$

⇒ A solid hemisphere

$y_c = \dfrac{3R}{8} \qquad x_c = 0$

⇒ A circular cone (solid)

$y_c = \dfrac{h}{4}$

⇒ A circular cone (hollow)

$y_c = \dfrac{h}{3}$

MOTION OF CENTRE OF MASS AND CONSERVATION OF MOMENTUM:
Velocity of centre of mass of system

$$\vec{v}_{cm} = \frac{m_1 \dfrac{\overrightarrow{dr_1}}{dt} + m_2 \dfrac{\overrightarrow{dr_2}}{dt} + m_3 \dfrac{\overrightarrow{dr_3}}{dt} \ldots\ldots\ldots\ldots + m_n \dfrac{\overrightarrow{dr_n}}{dt}}{M}$$

$$= \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 \ldots\ldots\ldots + m_n \vec{v}_n}{M}$$

$$\vec{P}_{System} = M\vec{v}_{cm}$$

Acceleration of centre of mass of system

$$\vec{a}_{cm} = \frac{m_1 \dfrac{\overrightarrow{dv_1}}{dt} + m_2 \dfrac{\overrightarrow{dv_2}}{dt} + m_3 \dfrac{\overrightarrow{dv_3}}{dt} \ldots\ldots\ldots\ldots + m_n \dfrac{\overrightarrow{dv_n}}{dt}}{M}$$

$$= \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 \ldots\ldots\ldots + m_n\vec{a}_n}{M}$$

$$= \frac{Net\ force\ on\ system}{M} = \frac{Net\ External\ Force + Net\ internal\ Force}{M}$$

$$= \frac{Net\ External\ Force}{M}$$

$$\vec{F}_{ext} = M\vec{a}_{cm}$$

IMPULSE
Impulse of a force F action on a body is defined as :-

$$\vec{J} = \int_{t_i}^{t_f} F\,dt \qquad \vec{J} = \Delta\vec{P} \qquad \textit{(impulse - momentum theorem)}$$

Important points :

1.   *Gravitational force and spring force are always non-impulsive.*

2.   *An impulsive force can only be balanced by another impulsive force.*

COEFFICIENT OF RESTITUTION (e)

$$e = \frac{Impulse\ of\ reformation}{Impulse\ of\ deformation} = \frac{\int F_r\,dt}{\int F_d\,dt}$$

$$= s\frac{Velocity\ of\ separation\ along\ line\ of\ impact}{Velocity\ of\ approach\ along\ line\ of\ impact}$$

(a)   *e = 1*   ⇒ *Impulse of Reformation = Impulse of Deformation*
                                      ⇒ *Velocity of separation = Velocity of approach*
                                      ⇒ *Kinetic Energy may be conserved*
                                      ⇒ *Elastic collision.*
(b)   *e = 0*   ⇒ *Impulse of Reformation = 0*
                                      ⇒ *Velocity of separation = 0*
                                      ⇒ *Kinetic Energy is not conserved*
                                      ⇒ *Perfectly Inelastic collision.*
(c)   *0 < e < 1*   ⇒ *Impulse of Reformation < Impulse of Deformation*
                                      ⇒ *Velocity of separation < Velocity of approach*

                                      ⇒ *Kinetic Energy is not conserved*
                                      ⇒ *Inelastic collision.*

VARIABLE MASS SYSTEM :

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity $\vec{v}_{rel}$ (w.r.t. the system), then the force exerted by this mass on the system has magnitude $\mu \left| \vec{v}_{rel} \right|$.

Thrust Force ($\vec{F}_t$)

$$\vec{F}_t = \vec{v}_{rel} \left(\frac{dm}{dt} \right)$$

Rocket propulsion :

If gravity is ignored and initial velocity of the rocket u = 0;

$$v = v_r \ln \left(\frac{m_0}{m} \right).$$