Exercise – 8A

1. (i) $\left(1-\cos^2\theta\right)\operatorname{cosec}^2\theta=1$

(ii) $\left(1+\cot^2\theta\right)\sin^2\theta=1$

Sol:

(i) $LHS = (1-\cos^2\theta)\, cosec^2\theta$

$= \sin^2\theta\ \ cosec^2\theta \qquad (\because \cos^2\theta+\sin^2\theta=1)$

$= \frac{1}{cosec^2\theta}\times cosec^2\theta$

$= 1$

Hence, LHS = RHS

(ii) $LHS = (1+\cot^2\theta)\sin^2\theta$

$= cosec^2\theta\sin^2\theta \qquad (\because cosec^2\theta-\cot^2\theta=1)$

$= \frac{1}{\sin^2\theta}\times\sin^2\theta$

$= 1$

Hence, LHS = RHS

2. (i) $\left(\sec^2\theta-1\right)\cot^2\theta=1$

(ii) $\left(sec^2\theta-1\right)\left(\operatorname{cosec}^2\theta-1\right)=1$

(iii) $\left(1-\cos^2\theta\right)sec^2\theta=\tan^2\theta$

Sol:

(i) $LHS = (\sec^2\theta-1)\cot^2\theta$

$= \tan^2\theta\times\cot^2\theta \qquad (\because sec^2\theta-\tan^2\theta=1)$

$= \frac{1}{\cot^2\theta}\times\cot^2\theta$

$= 1$

= RHS

(ii) $LHS = (\sec^2\theta-1)(cosec^2\theta-1)$

$= \tan^2\theta\times\cot^2\theta \qquad (\because \sec^2\theta-\tan^2\theta=1 and\ cosec^2\theta-\cot^2\theta=1)$

$= \tan^2\theta\times\frac{1}{\tan^2\theta}$

$= 1$

=RHS

(iii) $LHS = (1-\cos^2\theta)\sec^2\theta$

$= \sin^2\theta\times\sec^2\theta \qquad (\because \sin^2\theta+\cos^2\theta=1)$

$= \sin^2\theta\times\frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

= RHS

3. (i) $\sin^2\theta + \frac{1}{\left(1+\tan^2\theta\right)} = 1$

(ii) $\frac{1}{\left(1+\tan^2\theta\right)} + \frac{1}{\left(1+\cot^2\theta\right)} = 1$

Sol:

(i) $LHS = \sin^2\theta + \frac{1}{(1+\tan^2\theta)}$

$= \sin^2\theta + \frac{1}{\sec^2\theta}$ $\quad (\because \sec^2\theta - \tan^2\theta = 1)$

$= \sin^2\theta + \cos^2\theta$

= 1

= RHS

(ii)$LHS = \frac{1}{(1+\tan^2\theta)} + \frac{1}{(1+\cot^2\theta)}$

$= \frac{1}{\sec^2\theta} + \frac{1}{cosec^2\theta}$

$= \cos^2\theta + \sin^2\theta$

= 1

= RHS

4. (i) $\left(1+\cos\theta\right)\left(1-\cos\theta\right)\left(1+\cos^2\theta\right) = 1$

(ii) $\cos ec\theta\left(1+\cos\theta\right)\left(cosec\theta - \cot\theta\right) = 1$

Sol:

(i) $LHS = (1+\cos\theta)\,(1-\cos\theta)\ \ (1+\cot^2\theta)$

$= (1-\cos^2\theta)cosec^2\theta$

$= \sin^2\theta \times cosec^2\theta$

$= \sin^2\theta \times \frac{1}{\sin^2\theta}$

= 1

= RHS

(ii) $LHS = cosec\theta(1+\cos\theta)(cosec\,\theta - \cot\theta)$

$= (cosec\,\theta + cosec\,\theta \times \cos\theta)\,(cosec\,\theta - \cot\theta)$

$= \left(cosec\theta + \frac{1}{\sin\theta} \times \cos\theta\right)(cosec\,\theta - \cot\theta)$

$= (cosec\,\theta + \cot\theta)\,(cosec\theta - \cot\theta)$

$= cosec^2\theta - \cot^2\theta$ $\quad (\because cosec^2\theta - \cot^2\theta = 1)$

= 1

= RHS

5. (i) $\cot^2\theta - \frac{1}{\sin^2\theta} = -1$

(ii) $\tan^2\theta - \frac{1}{\cos^2\theta} = -1$

(iii) $\cos^2\theta + \frac{1}{(1+\cot^2\theta)} = 1$

Sol:

(i) $LHS = \cot^2\theta - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta - 1}{\sin^2\theta}$

$= \frac{-\sin^2\theta}{\sin^2\theta}$

= -1

=RHS

(ii) $LHS = \tan^2\theta - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta - 1}{\cos^2\theta}$

$= \frac{-\cos^2\theta}{\cos^2\theta}$

= -1

= RHS

(iii) $LHS = \cos^2\theta + \frac{1}{(1+\cot^2\theta)}$

$= \cos^2\theta + \frac{1}{cosec^2\theta}$

$= \cos^2\theta + \sin^2\theta$

=1

= RHS

6. $\frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)} = 2\sec^2\theta$

Sol:

$LHS = \frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)}$

$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)\,(1-\sin\theta)}$

$= \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}$

$= 2\sec^2\theta$

= RHS

7. (i) $\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)=1$

(ii) $\sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)=(\sec\theta+\cos ec\theta)$

Sol:

(i) $LHS = \sec\theta(1-\sin\theta)\ (\sec\theta+\tan\theta)$

$= (\sec\theta - \sec\theta\sin\theta)\ (\sec\theta+\tan\theta)$

$=(\sec\theta - \frac{1}{\cos\theta}\times\sin\theta)\ (\sec\theta+\tan\theta)$

$= (\sec\theta-\tan\theta)\ (\sec\theta+\tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$= 1$

$=$ RHS

(ii) $LHS = \sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)$

$=\sin\theta+\sin\theta\times\frac{\sin\theta}{\cos\theta}+\cos\theta+\cos\theta\times\frac{\cos\theta}{\sin\theta}$

$=\frac{\cos\theta\sin^2\theta+\sin^3\theta+\cos^2\theta\sin\theta+\cos^3\theta}{\cos\theta\sin\theta}$

$=\frac{(\sin^3\theta+\cos^3\theta)+(\cos\theta\sin^2\theta+\cos^2\theta\sin\theta)}{\cos\theta\sin\theta}$

$=\frac{(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)+\sin\theta\cos\theta(\sin\theta+\cos\theta)}{\cos\theta\sin\theta}$

$=\frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta+\sin\theta\cos\theta)}{\cos\theta\sin\theta}$

$=\frac{(\sin\theta+\cos\theta)\ (1)}{\cos\theta\sin\theta}$

$=\frac{\sin\theta}{\cos\theta\sin\theta}+\frac{\cos\theta}{\cos\theta\sin\theta}$

$=\frac{1}{\cos\theta}+\frac{1}{\sin\theta}$

$=\sec\theta+cosec\ \theta$

$=$RHS

8. (i) $1+\frac{\cot^2\theta}{(1+\cos ec\theta)}=\cos ec\theta$

(ii) $1+\frac{\tan^2\theta}{(1+sec\theta)}=sec\theta$

Sol:

(i) $LHS = 1+\frac{\cot^2\theta}{(1+cosec\theta)}$

$= 1+\frac{(cosec^2\theta-1)}{(cosec\theta+1)}$ $(\because cosec^2\theta-\cot^2\theta=1)$

$= 1+\frac{(cosec\theta+1)(cosec\ \theta-1)}{(cosec\ \theta+1)}$

$=1+(cosec\ \theta-1)$

$= cosec\ \theta$

= RHS

(ii) $LHS = 1 + \frac{\tan^2\theta}{(1+\sec\theta)}$

$= 1 + \frac{(\sec^2\theta - 1)}{(\sec\theta + 1)}$

$= 1 + \frac{(\sec\theta+1)(\sec\theta-1)}{(\sec\theta+1)}$

$= 1 + (\sec\theta - 1)$

$= \sec\theta$

= RHS

9. $1 + \frac{(\tan^2\theta)\cot\theta}{\cos ec^2\theta} = \tan\theta$

Sol:

$LHS = \frac{(1+\tan^2\theta)\cot\theta}{cosec^2\theta}$

$= \frac{\sec^2\theta\cot\theta}{cosec^2\theta}$

$= \frac{\frac{1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}}$

$= \frac{1}{cos\,\theta sin\theta} \times sin^2\,\theta$

$= \frac{sin\,\theta}{cos\,\theta}$

$= tan\,\theta$

=RHS

Hence, LHS = RHS

10. $\frac{\tan^2\theta}{(1+\tan^2\theta)} + \frac{\cot^2\theta}{(1+\cot^2\theta)} = 1$

Sol:

LHS = $\frac{\tan^2\theta}{(1+\tan^2\theta)} + \frac{\cot^2\theta}{(1+\cot^2\theta)}$

$= \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{cosec^2\theta}$ $(\because \sec^2\theta - \tan^2\theta = 1 and\ cosec^2\theta - \cot^2\theta = 1)$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$

$= \sin^2\theta + \cos^2\theta$

= 1

= RHS

Hence, LHS = RHS

11. $\frac{\sin\theta}{(1+\cos\theta)}+\frac{(1+\cos\theta)}{\sin\theta}=2\cos ec\theta$

Sol:

LHS = $\frac{\sin\theta}{(1+\cos\theta)}+\frac{(1+\cos\theta)}{\sin\theta}$

$=\frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$

$=\frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$=\frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$=\frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$=\frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$=\frac{2}{\sin\theta}$

= 2cosec θ

= RHS

Hence, L.H.S = R.H.S.

12. $\frac{\tan\theta}{(1-\cot\theta)}+\frac{\cot\theta}{(1-\tan\theta)}=(1+\sec\theta\cos ec\theta)$

Sol:

LHS = $\frac{\tan\theta}{(1-\cot\theta)}+\frac{\cot\theta}{(1-\tan\theta)}$

$=\frac{\tan\theta}{(1-\frac{\cos\theta}{\sin\theta})}+\frac{\cot\theta}{(1-\frac{\sin\theta}{\cos\theta})}$

$=\frac{\sin\theta\tan\theta}{(\sin\theta-\cos\theta)}+\frac{\cos\theta\cot\theta}{(\cos\theta-\sin\theta)}$

$=\frac{\sin\theta\times\frac{\sin\theta}{\cos\theta}-\cos\theta\times\frac{\cos\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$

$=\frac{\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$

$=\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$=\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$=\frac{1+\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$=\frac{1}{\cos\theta\sin\theta}+\frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$=\frac{1}{\cos\theta\sin\theta}+\frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$=\sec\theta cosec\ \theta+1$

$=1+\sec\theta cosec\ \theta$

= RHS

13. $\frac{\cos^2\theta}{(1-\tan\theta)}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}=(1+\sin\theta\cos\theta)$

Sol:

$\frac{\cos^2\theta}{(1-\tan\theta)}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}=(1+\sin\theta\cos\theta)$

$LHS=\frac{\cos^2\theta}{(1-\tan\theta)}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$

$=\frac{\cos^2\theta}{(1-\frac{\sin\theta}{\cos\theta})}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$

$=\frac{\cos^3\theta}{(\cos\theta-\sin\theta)}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$

$=\frac{\cos^3\theta-\sin^3\theta}{(\cos\theta-\sin\theta)}$

$=\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\cos\theta sin+\sin^2\theta)}{(\cos\theta-\sin\theta)}$

$=(\sin^2\theta+\cos^2\theta+\cos\theta\sin\theta)$

$=(1+\sin\theta\cos\theta)$

= RHS

Hence, L.H.S = R.H.S.

14. $\frac{\cos\theta}{(1-\tan\theta)}+\frac{\sin^2\theta}{(\cos\theta-\sin\theta)}=(\cos\theta+\sin\theta)$

Sol:

LHS = $\frac{\cos\theta}{(1-\tan\theta)}-\frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$

$=\frac{\cos\theta}{\left(1-\frac{\sin\theta}{\cos\theta}\right)}-\frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$

$=\frac{\cos^2\theta}{(\cos\theta-\sin\theta)}-\frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$

$=\frac{\cos^2\theta-\sin^2\theta}{(\cos\theta-\sin\theta)}$

$=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)}$

$=(\cos\theta+\sin\theta)$

= RHS

Hence, LHS = RHS

15. $(1+\tan^2\theta)(1+\cot^2\theta)=\frac{1}{(\sin^2\theta-\sin^4\theta)}$

Sol:

$LHS=(1+\tan^2\theta)\,(1+\cot^2\theta)$

$=\sec^2\theta.cosec^2\theta$ $\quad(\because \sec^2\theta-\tan^2\theta=1\ and\ cosec^2\ -\cot^2\theta=1)$

$= \frac{1}{\cos^2\theta . \sin^2\theta}$

$= \frac{1}{(1-\sin^2\theta)\sin^2\theta}$

$= \frac{1}{\sin^2\theta - \sin^4\theta}$

=RHS

Hence, LHS = RHS

16. $\frac{\tan\theta}{\left(1+\tan^2\theta\right)^2} + \frac{\cot\theta}{\left(1+\cot^2\theta\right)^2} = \sin\theta\cos\theta$

Sol:

$LHS = \frac{\tan\theta}{(1+\tan^2\ \theta\ )^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

$= \frac{\tan\theta}{(\sec^2\theta)^2} + \frac{\cot\theta}{(cosec^2\theta)^2}$

$= \frac{\tan\theta}{\sec^4\theta} + \frac{\cot\theta}{cosec^4\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \cos^4\theta + \frac{\cos\theta}{\sin\theta} \times \sin^4\theta$

$= \sin\theta\cos^3\theta + \cos\theta\sin^3\theta$

$= \sin\theta\cos\theta(\cos^2\theta + \sin^2\theta)$

$= \sin\theta\cos\theta$

= RHS

17. (i) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

(ii) $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$

(iii) $\cos ec^4\theta + \cos ec^2\theta = \cot^4\theta + \cot^2\theta$

Sol:

(i) $LHS = \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)\ (\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta)$

$= 1 \times \{(\sin^2\theta)^2 + 2\sin^2\theta\cos^2\theta + (\cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta\}$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$

$= (1)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

= RHS

Hence, LHS = RHS

(ii)$LHS = \sin^2\theta + \cos^4\theta$

$= \sin^2\theta + (\cos^2\theta)^2$

$= \sin^2\theta + (1 - \sin^2\theta)^2$

$= \sin^2\theta + 1 - 2\sin^2\theta + \sin^4\theta$

$= 1 - \sin^2\theta + \sin^4\theta$

$= \cos^2\theta + \sin^4\theta$

$= \text{RHS}$

Hence, LHS = RHS

(iii) $LHS = cosec^4\theta - cosec^2\theta$

$= cosec^2\theta(cosec^2\theta - 1)$

$= cosec^2\theta \times \cot^2\theta \qquad (\because cosec^2\theta - \cot^2\theta = 1)$

$= (1 + \cot^2\theta) \times \cot^2\theta$

$= \cot^2\theta + \cot^4\theta$

$= \text{RHS}$

Hence, LHS = RHS

18. (i) $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \left(\cos^2\theta - \sin^2\theta\right)$

(ii) $\frac{1-\tan^2\theta}{\cot^2 - 1} = \tan^2\theta$

Sol:

(i) $LHS = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{1}$

$= \cos^2\theta - \sin^2\theta$

$= \text{RHS}$

(ii) $LHS = \frac{1-\tan^2\theta}{\cot^2\theta - 1}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta} - 1}$

$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

$= \text{RHS}$

19. (i) $\frac{\tan\theta}{(sec\ \theta-1)}+\frac{\tan\theta}{(sec\ \theta+1)}=2\cos ec\theta$

(ii) $\frac{\cot\theta}{(\cos ec\theta+1)}+\frac{(\cos ec\theta+1)}{\cot\theta}=2sec\theta$

Sol:

(i) $LHS=\frac{\tan\theta}{(\sec\theta-1)}+\frac{\tan\theta}{(\sec\theta+1)}$

$=\tan\theta\,\{\frac{\sec\theta+1+\sec\theta-1}{(\sec\theta-1)(\sec\theta+1)}\}$

$=\tan\theta\,\{\frac{2\sec\theta}{(\sec^2\theta-1)}\}$

$=\tan\theta\times\frac{2\sec\theta}{\tan^2\theta}$

$=2\frac{\sec\theta}{\tan\theta}$

$=2\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$

$=2\frac{1}{\sin\theta}$

$=2cosec\ \theta$

= RHS

Hence, LHS = RHS

(ii) $LHS=\frac{\cot\theta}{(cosec\ \theta+1)}+\frac{(cosec\theta+1)}{\cot\theta}$

$=\frac{\cot^2\theta+(cosec\ \theta+1)^2}{(cosec\ \theta+1)\cot\theta}$

$=\frac{\cot^2\theta+cosec^2\theta+2cosec\ \theta+1}{(cosec\ \theta+1)\cot\theta}$

$=\frac{\cot^2\theta+cosec^2\theta+2cosec\ \theta+cosec^2\theta-\cot^2\theta}{(cosec\theta+1)\cot\theta}$

$=\frac{2cosec^2\theta+2cosec\ \theta}{(cosec\ \theta+1)\cot\theta}$

$=\frac{2cosec\theta(cosec\theta+1)}{(cosec\theta+1)\cot\theta}$

$=\frac{2cosec\theta}{\cot\theta}$

$=2\times\frac{1}{\sin\theta}\times\frac{\sin\theta}{\cos\theta}$

$=2\sec\theta$

= RHS

Hence, LHS = RHS

20. (i) $\frac{\sec\theta-1}{\sec\theta+1}=\frac{\sin^2\theta}{(1+\cos\theta)^2}$

(ii) $\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}=\frac{\cos^2\theta}{(1+\sin\theta)^2}$

Sol:

(i) $LHS=\frac{\sec\theta-1}{\sec\theta+1}$

$=\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}$

$=\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$

$=\frac{1-\cos\theta}{1+\cos\theta}$

$=\frac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1+\cos\theta)}$ $\left\{\begin{array}{l}\text{Dividing the numerator and}\\ deno\min ator\ by(1+\cos\theta)\end{array}\right\}$

$=\frac{1-\cos^2\theta}{(1+\cos\theta)^2}$

$=\frac{\sin^2\theta}{(1+\cos\theta)^2}$

= RHS

(ii) $LHS=\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}$

$=\frac{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}$

$=\frac{\frac{1-\sin\theta}{\cos\theta}}{\frac{1+\sin\theta}{\cos\theta}}$

$=\frac{1-\sin\theta}{1+\sin\theta}$

$=\frac{(1-\sin\theta)(1+\sin\theta)}{(1+\sin\theta)(1+\sin\theta)}$ $\left\{\begin{array}{l}\text{Dividing the numerator and}\\ deno\min ator\ by(1+\cos\theta)\end{array}\right\}$

$=\frac{(1-\sin^2\theta)}{(1+\sin\theta)^2}$

$=\frac{\cos^2\theta}{(1+\sin\theta)^2}$

= RHS

21. (i) $\sqrt{\frac{1+sin\theta}{1-sin\theta}}=(\sec\theta+\tan\theta)$ (ii) $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=(\cos ec\theta-\cot\theta)$

(iii) $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=2\cos ec\theta$

Sol:

(i) $LHS = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= (\sec\theta + \tan\theta)$

= RHS

(ii) $LHS = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)} \times \frac{(1-\cos\theta)}{(1-\cos\theta)}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$= \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$= (cosec\ \theta - \cot\theta)$

= RHS

(iii) $LHS = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}} + \sqrt{\frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos^2\theta)}} + \sqrt{\frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$= \frac{(1+\cos\theta)}{\sin\theta} + \frac{(1-\cos\theta)}{\sin\theta}$

$= \frac{1+\cos\theta+1-\cos\theta}{\sin\theta}$

$= \frac{2}{\sin\theta}$

$= 2\cos ec\theta$

= RHS

22. $\dfrac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\dfrac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}=2$

Sol:

$LHS=\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

$=\frac{(\cos\theta+\sin\theta)(\cos^2\theta-\cos\theta\sin\theta+\sin^2\theta)}{(\cos\theta+\sin\theta)}+\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\cos\theta\sin\theta+\sin^2\theta)}{(\cos\theta-\sin\theta)}$

$=(\cos^2\theta+\sin^2\theta-\cos\theta\sin\theta)+(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)$

$=(1-\cos\theta\sin\theta)+(1+\cos\theta\sin\theta)$

= 2

= RHS

Hence, LHS = RHS

23. $\dfrac{\sin\theta}{(\cot\theta+\operatorname{cosec}\theta)}-\dfrac{\sin\theta}{(\cot\theta-\operatorname{cosec}\theta)}=2$

Sol:

$LHS=\frac{\sin\theta}{(\cot\theta+cosec\ \theta)}-\frac{\sin\theta}{(\cot\theta-cosec\ \theta)}$

$=\sin\theta\left\{\frac{(\cot\theta-cosec\ \theta)-(\cot\theta+cosec\ \theta)}{(\cot\theta+cosec\ \theta)(\cot\theta-cosec\ \theta)}\right\}$

$=\sin\theta\left\{\frac{-2cosec\theta}{-1}\right)$ $\quad(\because cosec^2\theta-\cot^2\theta=1)$

$=\sin\theta.2\operatorname{cosec}\theta$

$=\sin\theta\times2\times\frac{1}{\sin\theta}$

= 2

= RHS

24. (i) $\dfrac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=\dfrac{2}{(2\sin^2\theta-1)}$

(ii) $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\dfrac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\dfrac{2}{(1-2\cos^2\theta)}$

Sol:

(i) $LHS=\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$

$=\frac{(\sin\theta-\cos\theta)^2+(\sin\theta+\cos\theta)^2}{(\sin\theta+cos\ \theta)(\sin\theta-\cos\theta)}$

$=\frac{\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

$=\frac{1+1}{\sin^2\theta-(1-\sin^2\theta)}$ $\quad(\because\sin^2\theta+\cos^2\theta=1)$

$=\frac{2}{\sin^2\theta-1+\sin^2\theta}$

$=\frac{2}{\sin^2\theta-1}$

= RHS

(ii) $LHS = \frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} + \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$= \frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{(\sin^2\theta-\cos^2\theta)}$

$= \frac{1+1}{(1-\cos^2\theta)-\cos^2\theta}$ $(\because \sin^2\theta+\cos^2\theta=1)$

$= \frac{2}{1-2\cos^2\theta}$

= RHS

25. $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$

Sol:

$LHS = \frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{(1+\cos\theta)-(1-\cos^2\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

= RHS

Hence, L.H.S. = R.H.S.

26. (i) $\frac{\cos ec\theta+\cot\theta}{\cos ec\theta-\cot\theta} = (\cos ec\theta+\cot\theta)^2 = 1+2\cot^2\theta+2\cos ec\theta\cot\theta$

(ii) $\frac{sec\theta+\tan\theta}{\sec\theta-\tan\theta} = (s ec\theta+\tan\theta)^2 = 1+2\tan^2\theta+2sec\theta\tan\theta$

Sol:

(i) Here, $\frac{cosec\theta+\cot\theta}{cosec\theta-\cot\theta}$

$= \frac{(cosec\theta+\cot\theta)(cosec\theta+\cot\theta)}{(cosec\theta-\cot\theta)(cosec\theta+\cot\theta)}$

$= \frac{(cosec\theta+\cot\theta)^2}{(cosec^2\theta-\cot^2\theta)}$

$= \frac{(cosec\theta+\cot\theta)^2}{1}$

$= (cosec\theta+\cot\theta)^2$

Again, $(cosec\theta+\cot\theta)^2$

$= cosec^2\theta+\cot^2\theta+2cosec\theta\cot\theta$

$= 1 + \cot^2\theta + \cot^2\theta + 2cosec\theta\cot\theta \quad (\because cosec^2\theta - \cot^2\theta = 1)$

$= 1 + 2\cot^2\theta + 2cosec\theta\cot\theta$

(ii) Here, $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$

$= \frac{(\sec\theta+\tan\theta)(\sec\theta+\tan\theta)}{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}$

$= \frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}$

$= \frac{(\sec\theta+\tan\theta)^2}{1}$

$= (\sec\theta + \tan\theta)^2$

Again, $(\sec\theta + \tan\theta)^2$

$= \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$= 1 + 2\tan^2\theta + 2\sec\theta\tan\theta$

27. (i) $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

(ii) $\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

Sol:

(i) LHS $= \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-sin\theta}$

$= \frac{\{(1+\cos\theta)+\sin\theta\}\{(1+\cos\theta)+\sin\theta\}}{\{(1+\cos\theta)-\sin\theta\}\{(1+\cos\theta)+\sin\theta\}}$ $\left\{\begin{matrix}\text{Multiplying the numerator and} \\ \text{denominator by } (1+\cos\theta+\sin\theta)\end{matrix}\right\}$

$= \frac{\{(1+\cos\theta)+\sin\theta\}^2}{\{(1+\cos\theta)^2-\sin^2\theta\}}$

$= \frac{1+\cos^2\theta+2\cos\theta+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-\sin^2\theta}$

$= \frac{2+2\cos\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-(1-\cos^2\theta)}$

$= \frac{2(1+\cos\theta)+2\sin\theta(1+\cos\theta)}{2\cos^2\theta+2\cos\theta}$

$= \frac{2(1+\cos\theta)(1+\sin\theta)}{2\cos\theta(1+\cos\theta)}$

$= \frac{1+\sin\theta}{\cos\theta}$

= RHS

(ii)$LHS = \frac{\sin\theta+1\cos\theta}{\cos\theta-1+\sin\theta}$

$= \frac{(\sin\theta+1-\cos\theta)(\sin\theta+\cos\theta+1)}{(\cos\theta-1+\sin\theta)(\sin\theta+\cos\theta+1)}$ $\left\{\begin{matrix}\text{Multiplying the numerator and} \\ \text{denominator by } (1+\cos\theta+\sin\theta)\end{matrix}\right\}$

$= \frac{(\sin\theta+1)^2-\cos^2\theta}{(\sin\theta+\cos\theta)^2-1^2}$

$= \frac{\sin^2\theta+1+2\sin\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}$

$= \frac{\sin^2\theta+\sin^2\theta+\cos^2\theta+2\sin\theta-\cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{2\sin^2\theta+2sin\,\theta}{2\sin\theta\cos\theta}$

$= \frac{2\sin\theta(1+\sin\theta)}{2\sin\theta\cos\theta}$

$= \frac{1+\sin\theta}{\cos\theta}$

= RHS

28. $\frac{\sin\theta}{(\sec\theta+\tan\theta-1)}+\frac{\cos\theta}{(\cos ec\theta+\cot\theta-1)}=1$

Sol:

$LHS = \frac{\sin\theta}{(\sec\theta+\tan\theta-1)} + \frac{\cos\theta}{(cosec\,\theta+\cot\theta-1)}$

$= \frac{\sin\theta\cos\theta}{1+\sin\theta-\cos\theta} + \frac{\cos\theta\sin\theta}{1+\cos\theta-\sin\theta}$

$= \sin\theta\cos\theta\ [\frac{1}{1+(\sin\theta-\cos\theta)} + \frac{1}{1-(\sin\theta-\cos\theta)}]$

$= \sin\theta\cos\theta\left[\frac{1-(\sin\theta-\cos\theta)+1+(\sin\theta-\cos\theta)}{\{1+(\sin\theta-\cos\theta)\}\{1-(\sin\theta-\cos\theta)\}}\right]$

$= \sin\theta\cos\theta[\frac{1-\sin\theta+\cos\theta+1+\sin\theta-\cos\theta}{1-(\sin\theta-\cos\theta)^2}$

$= \frac{2\sin\theta cos\,\theta}{1-(\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta)}$

$= \frac{2\sin\theta\cos\theta}{2\sin\theta\cos\theta}$

= 1

= RHS

Hence, LHS = RHS

29. $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2}{(\sin^2\theta-\cos^2\theta)}=\frac{2}{(2\sin^2\theta-1)}$

Sol:

We have $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} + \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$= \frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$= \frac{\sin^2\theta+\cos^2\theta+2sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

$= \frac{1+1}{\sin^2\theta-\cos^2\theta}$

$= \frac{2}{\sin^2\theta-\cos^2\theta}$

Again, $\frac{2}{\sin^2\theta-\cos^2\theta}$

$= \frac{2}{\sin^2\theta-(1-\sin^2\theta)}$

$= \frac{2}{2\sin^2\theta-1}$

30. $\dfrac{\cos\theta\cos ec\theta-\sin\theta\sec\theta}{\cos\theta+\sin\theta}=\cos ec\theta-\sec\theta$

Sol:

$$LHS=\frac{\cos\theta cosec\theta-\sin\theta sec\theta}{\cos\theta+\sin\theta}$$
$$=\frac{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}}{\cos\theta+\sin\theta}$$
$$=\frac{\cos^2\theta-\sin^2\theta}{\cos\theta\sin\theta(\cos\theta+\sin\theta)}$$
$$=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta\sin\theta(\cos\theta+\sin\theta)}$$
$$=\frac{(\cos\theta-\sin\theta)}{\cos\theta\sin\theta}$$
$$=\frac{1}{\sin\theta}-\frac{1}{\cos\theta}$$
$$=cosec\ \theta-\sec\theta$$
$= \text{RHS}$

Hence, LHS = RHS

31. $(1+\tan\theta+\cot\theta)(\sin\theta-\cos\theta)=\left(\dfrac{sec\,\theta}{\cos ec^2\theta}-\dfrac{\cos ec\theta}{sec^2\theta}\right)$

Sol:

$$LHS=(1+\tan\theta+\cot\theta)(\sin\theta-\cos\theta)$$
$$=\sin\theta+tan\theta\sin\theta+\cot\theta\sin\theta-\cos\theta-\tan\theta\cos\theta-\cot\theta\cos\theta$$
$$=\sin\theta+\tan\theta\sin\theta+\frac{\cos\theta}{\sin\theta}\times\sin\theta-\cos\theta-\frac{\sin\theta}{\cos\theta}\times\cos\theta-\cot\theta\cos\theta$$
$$=\sin\theta+\tan\theta\sin\theta+\cos\theta-\cos\theta-\sin\theta-\cot\theta\cos\theta$$
$$=\tan\theta\sin\theta-\cot\theta\cos\theta$$
$$=\frac{\sin\theta}{\cos\theta}\times\frac{1}{cosec\theta}-\frac{\cos\theta}{\sin\theta}\times\frac{1}{\sec\theta}$$
$$=\frac{1}{cosec\theta}\times\frac{1}{cosec\theta}\times\sec\theta-\frac{1}{\sec\theta}\times\frac{1}{\sec\theta}\times cosec\theta$$
$$=\frac{\sec\theta}{cosec^2\theta}-\frac{cosec\theta}{\sec^2\theta}$$
$= \text{RHS}$

Hence, LHS = RHS

32. $\dfrac{\cot^2\theta(sec\,\theta-1)}{(1+\sin\theta)}+\dfrac{\sec^2\theta(\sin\theta-1)}{(1+\sec\theta)}=0$

Sol:

$$LHS=\frac{\cot^2\theta(\sec\theta-1)}{(1+\sin\theta)}+\frac{\sec^2\theta(\sin\theta-1)}{(1+\sec\theta)}$$
$$=\frac{\frac{\cos^2\theta}{\sin^2\theta}(\frac{1}{\cos\theta}-1)}{(1+\sin\theta)}+\frac{\frac{1}{\cos^2\theta}(\sin\theta-1)}{(1+\frac{1}{\cos\theta})}$$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta}(\frac{1-\cos\theta}{\cos\theta})}{(1+\sin\theta)} + \frac{\frac{(\sin\theta-1)}{\cos^2\theta}}{(\frac{\cos\theta+1}{\cos\theta})}$$
$$= \frac{\cos^2\theta(1-\cos\theta)}{\sin^2\theta\cos\theta(1+\sin\theta)} + \frac{(\sin\theta-1)\cos\ \theta}{(\cos\theta+1)\cos^2\theta}$$
$$= \frac{\cos\theta(1-\cos\theta)}{(1-\cos^2\theta)(1+\sin\theta)} + \frac{(\sin\theta-1)\cos\ \theta}{(\cos\theta+1)(1-\sin^2\theta)}$$
$$= \frac{\cos\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)(1+\sin\theta)} + \frac{-(1\sin\theta)\cos\theta}{(\cos\theta+1)(1-\sin\theta)(1+\sin\theta)}$$
$$= \frac{\cos\ \theta}{(1+\cos\theta)(1+\sin\theta)} - \frac{\cos\theta}{(\cos\theta+1)(1+\sin\theta)}$$
$$= \theta$$
= RHS

33. $$\left\{\frac{1}{\left(sec^2\theta-\cos^2\theta\right)} + \frac{1}{\left(cosec^2\theta-\sin^2\theta\right)}\right\}\left(\sin^2\theta\cos^2\theta\right) = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$$

Sol:

$$LHS = \{\frac{1}{\sec^2\theta-\cos^2\theta} + \frac{1}{cosec^2\theta-\sin^2\theta}\}(\sin^2\theta\cos^2\theta)$$
$$= \{\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\}(\sin^2\theta\cos^2\theta)$$
$$= \left\{\frac{\cos^2\theta}{(1-\cos^2\theta)(1+\cos^2\theta)} + \frac{\sin^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right\}(\sin^2\theta\cos^2\theta)$$
$$= \left[\frac{\cot^2\theta}{1+\cos^2\theta} + \frac{\tan^2\theta}{1+\sin^2\theta}\right]\sin^2\theta\cos^2\theta$$
$$= \frac{\cos^4\theta}{1+\cos^2\theta} + \frac{\sin^4\theta}{1+\sin^2\theta}$$
$$= \frac{\left(\cos^2\theta\right)^2}{1+\cos^2\theta} + \frac{(\sin^2\theta)^2}{1+\sin^2\theta}$$
$$= \frac{(1-\sin^2\theta)}{1+\cos^2\theta} + \frac{(1-\cos^2\theta)^2}{1+\sin^2\theta}$$
$$= \frac{(1-\sin^2\theta)^2(1+\sin^2)+(1-\cos^2\theta)^2(1+\cos^2\theta)}{(1+\sin^2\theta)(1+\cos^2\theta)}$$
$$= \frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$$
$$= \frac{\cos^4\theta\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{1+1\sin^2\theta\cos^2\theta}$$
$$= \frac{\cos^4\theta+\sin^4\theta+\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)}{2+\sin^2\theta\cos^2\theta}$$
$$= \frac{(\cos^2\theta)^2+(\sin^2\theta)^2+\sin^2\theta\cos^2\theta(1)}{2+\sin^2\theta\cos^2\theta}$$
$$= \frac{(\cos^2\theta+\sin^2\theta)^2-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta(1)}{2+\sin^2\theta\cos^2\theta}$$
$$= \frac{1^2+\cos^2\theta\sin^2\theta-2\cos^2\theta\sin^2\theta}{2+\sin^2\theta\cos^2\theta}$$
$$= \frac{1-\cos^2\theta\sin^2\theta}{2+\sin^2\theta\cos^2\theta}$$
= RHS

34. $\frac{(\sin A - \sin B)}{(\cos A + \cos B)} + \frac{(\cos A - \cos B)}{(\sin A + \sin B)} = 0$

Sol:

$LHS = \frac{(sinA - sinB)}{(cosA + cosB)} + \frac{(cosA - cosB}{(sinA + SinB)}$

$= \frac{(sinA - sinB)(sinA + sinB) + (cosA - cosB)(cosA - cosB)}{(cosA + cosB)(sinA + sinB)}$

$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(cosA + cosB)(sinA + sinB)}$

$= \frac{0}{(cosA + cosB)(sinA + sinB)}$

= 0

= RHS

35. $\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$

Sol:

$LHS = \frac{tanA + tanB}{cotA + cotB}$

$= \frac{tanA + tanB}{\frac{1}{tanA} + \frac{1}{tanB}}$

$= \frac{tanA + tanB}{\frac{tanA + tanB}{tanA\ tanB}}$

$= \frac{tanA\ tanB(tanA + tanB)}{(\tan A + \tan B)}$

$= tanA\ tanB$

= RHS

Hence, LHS = RHS

36. Show that none of the following is an identity:

(i) $\cos^2\theta + \cos\theta = 1$

(ii) $\sin^2\theta + \sin\theta = 2$

(iii) $tan^2\theta + \sin\theta = \cos^2\theta$

Sol:

$(i)\ \cos^2\theta + \cos\theta = 1$

$LHS = \cos^2\theta + \cos\theta$

$= 1 - \sin^2\theta + \cos\theta$

$= 1 - (\sin^2\theta - \cos\theta)$

Since LHS ≠ RHS, this not an identity.

(ii) $\sin^2\theta + \sin\theta = 1$

$LHS = \sin^2\theta + \sin\theta$

$= 1 - \cos^2\theta + \sin\theta$

$= 1 - (\cos^2\theta - \sin\theta)$

Since LHS ≠ RHS, this is not an identity.

(iii) $\tan^2\theta + \sin\theta = \cos^2\theta$

$LHS = \tan^2\theta + \sin\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} + \sin\theta$

$= \frac{1-\cos^2\theta}{\cos^2\theta} + \sin\theta$

$= \sec^2\theta - 1 + \sin\theta$

Since LHS ≠ RHS, this is not an identity.

37. Prove that $\left(\sin\theta - 2\sin^3\theta\right) = \left(2\cos^3\theta - \cos\theta\right)\tan\theta$

Sol:

$RHS = (2\cos^3\theta - \cos\theta)\tan\theta$

$= (2\cos^2\theta - 1)\cos\ \theta \times \frac{\sin\theta}{\cos\theta}$

$= [2(1-\sin^2\theta) - 1]\sin\ \theta$

$= (2 - 2\sin^2\theta - 1)\sin\ \theta$

$= (1 - 2\sin^2\theta)\sin\theta$

$= (\sin\theta - 2\sin^3\theta)$

= LHS

Exercise – 8B

1. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that, $\left(m^2+n^2\right) = \left(a^2+b^2\right)$

Sol:

We have $m^2 + n^2 = [(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2]$

$= (a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta)$

$+ (a^2\sin^2\theta + b^2\cos^2\theta - 2abcos\theta\sin\theta)$

$= a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$

$= (a^2\cos^2\theta + b^2\sin^2\theta) + (b^2\cos^2\theta + b^2\sin^2\theta)$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$= a^2 + b^2 \qquad [\because \sin^2 + \cos^2 = 1]$

Hence, $m^2 + n^2 = a^2 + b^2$

2. If $x = a\ sec\ \theta + b\tan\theta$ and $y = a\tan\ \theta + b\ sec\theta$, prove that $\left(x^2 - y^2\right) = \left(a^2 - b^2\right)$.

Sol:

We have $x^2 - y^2 = [(\text{a}\sec\theta + btan\ \theta)^2 - (\text{a}\tan\theta + bsec\ \theta)^2]$

$= (a^2\sec^2\theta + b^2\tan^2\theta + 2absec\theta\tan\theta)$

-$(a^2\tan^2\theta + b^2\sec^2\theta + 2abtan\theta\sec\theta)$
$= a^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta - b^2\sec^2\theta$
$= (a^2\sec^2\theta - a^2\tan^2\theta) - (b^2\sec^2\theta - b^2\tan^2\theta)$
$= a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$
$= a^2 - b^2 \quad [\because \sec^2\theta - \tan^2\theta = 1]$

Hence, $x^2 - y^2 = a^2 - b^2$

3. If $\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right) = 1$ and $\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right) = 1$, prove that $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 2.$

Sol:

We have$(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta) = 1$

Squaring both side, we have:

$(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta)^2 = (1)^2$

$\Rightarrow (\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{x}{a}\times\frac{y}{b}\sin\theta\cos\theta) = 1 \quad ...(i)$

Again, $(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta) = 1$

$Squaring\ both\ side, we\ get$:

$(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta)^2 = (1)^2$

$\Rightarrow(\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{x}{a}\times\frac{y}{a}\sin\theta\cos\theta) = \quad ...(ii)$

Now, adding (i) and (ii), we get:

$(\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{x}{a}\times\frac{y}{b}\sin\theta\cos\theta) + (\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{x}{a}\times \frac{y}{b}\sin\theta\cos\theta)$

$\Rightarrow\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta + \frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta = 2$

$\Rightarrow(\frac{x^2}{a^2}\sin^2\theta + \frac{x^2}{a^2}\cos^2\theta) + (\frac{y^2}{b^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta) = 2$

$\Rightarrow\frac{x^2}{a^2}(\sin^2\theta + \cos^2\theta) + \frac{y^2}{b^2}(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

4. If $(\sec\theta + \tan\theta) = m$ and $(\sec\theta - \tan\theta) = n$, show that $mn = 1.$

Sol:

We have $(\sec\theta + \tan\theta) = m \quad ...(i)$

Again, $(\sec\theta - \tan\theta) = n \quad ...(ii)$

Now, multiplying (i) and (ii), we get:

$(\sec\theta + \tan\theta)\times(\sec\theta - \tan\theta) = mn$

$=> \sec^2\theta - \tan^2\theta = mn$
$=> 1 = mn \quad [\because \sec^2\theta - \tan^2\theta = 1]$
$\therefore mn = 1$

5. If $(\operatorname{cosec}\theta + \cot\theta) = m$ and $(\operatorname{cosec}\theta - \cot\theta) = n$, show that $mn = 1$.

Sol:

We have $(cosec\ \theta + \cot\theta) = m \quad \ldots.(i)$
$Again, (cosec\ \theta - \cot\theta) = n \quad \ldots(ii)$
$Now, multiplying\ (i) and\ (ii), we\ get:$
$(cosec\theta + \cot\theta)\times(cosec\theta - \cot\theta) = mn$
$=> cosec^2\theta - \cot^2\theta = mn$
$=> 1 = mn \quad [\because cosec^2\theta - \cot^2\theta = 1]$
$\therefore mn = 1$

6. If $x = a\cos^3\theta$ and $y = b\sin^3\theta$, prove that $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

Sol:

$We\ have\ x = a\cos^3\theta$
$=> \frac{x}{a} = \cos^3\theta \quad \ldots(i)$
$Again, y = bsin^3\theta$
$=> \frac{y}{b} = \sin^3\theta \quad \ldots(ii)$
$Now, LHS = (\frac{x}{a})^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}}$
$= (\cos^3\theta)^{\frac{2}{3}} + (\sin^3\theta)^{\frac{2}{3}} \quad [From\ (i) and\ (ii)]$
$= \cos^2\theta + \sin^2\theta$
$= 1$
$Hence, LHS = RHS$

7. If $(\tan\theta + \sin\theta) = m$ and $(\tan\theta - \sin\theta) = n$, prove that $(m^2 - n^2)^2 = 16\ mn$.

Sol:

$We\ have\ (\tan\theta + \sin\theta) = m\ and\ (\tan\theta - \sin\theta) = n$
$Now, LHS = (m^2 - n^2)^2$
$= [(\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2]^2$
$= [(\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta) - (\tan^2\theta + \sin^2\theta - 2\tan\theta\sin\theta)]^2$
$= [(\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta)]^2$
$= (4\tan\theta\sin\theta)^2$
$= 16\tan^2\theta\sin^2\theta$

$= 16\frac{\sin^2\theta}{\cos^2\theta}\sin^2\theta$
$= 16\frac{(1-\cos^2\theta)\sin^2\theta}{\cos^2\theta}$
$= 16[\tan^2\theta(1-\cos^2\theta)]$
$= 16(\tan^2\theta - \tan^2\theta\cos^2\theta)$
$= 16(\tan^2\theta - \frac{\sin^2\theta}{\cos^2\theta}\times\cos^2\theta)$s
$= 16(\tan^2\theta - \sin^2\theta)$
$= 16(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$
$= 16\,mn$ $\qquad [(\tan\theta+\sin\theta)(\tan\theta-\sin\theta) = mn]$
$\therefore (m^2-n^2)(m^2-n^2)^2 = 16mn$

8. If $(\cot\theta+\tan\theta)=m$ and $(\sec\theta-\cos\theta)=n$ prove that $(m^2n)^{2/3}-(mn^2)^{2/3}=1$

Sol:

$We\ have\ (\cot\theta+\tan\theta) = m\ and\ (\sec\theta-\cos\theta) = n$

Now, $m^2n = [(\cot\theta+\tan\theta)^2(\sec\theta-\cos\theta)]$
$= [(\frac{1}{\tan\theta}+\tan\theta)^2(\frac{1}{\cos\theta}-\cos\theta)]$
$= \frac{(1+\tan^2\theta)^2}{\tan^2\theta}\times\frac{(1-\cos^2\theta)}{\cos\theta}$
$= \frac{\sec^4\theta}{\tan^2\theta}\times\frac{\sin^2\theta}{\cos\theta}$
$= \frac{\sec^4\theta}{\frac{\sin^2\theta}{\cos^2\theta}}\times\frac{\sin^2\theta}{\cos\theta}$
$= \frac{\cos^2\theta\times\sec^4\theta}{\cos\theta}$
$= \cos\theta\sec^4\theta$
$= \frac{1}{\sec\theta}\times\sec^4\theta = \sec^3\theta$

$\therefore (m^2n)^{\frac{2}{3}} = (\sec^3\theta)^{\frac{2}{3}} = \sec^2\theta$

$Again, mn^2 = [(\cot\theta+\tan\theta)(\sec\theta-\cos\theta)^2]$
$= [(\frac{1}{\tan\theta}+\tan\theta).(\frac{1}{\cos\theta}-\cos\theta)^2]$
$= \frac{(1+\tan^2\theta)}{\tan\theta}\times\frac{(1-\cos^2\theta)^2}{\cos^2\theta}$
$= \frac{\sec^2\theta}{\tan\theta}\times\frac{\sin^4\theta}{\cos^2\theta}$
$= \frac{\sec^2\theta}{\frac{\sin\theta}{\cos\theta}}\times\frac{\sin^4\theta}{\cos^2\theta}$
$= \frac{\sec^2\theta\times\sin^3\theta}{\cos\theta}$
$= \frac{1}{\cos^2\theta}\times\frac{\sec^3\theta}{\cos\theta} = \tan^3\theta$

$\therefore (mn^2)^{\frac{2}{3}} = (\tan^3\theta)^{\frac{2}{3}} = \tan^2\theta$

$Now, (m^2n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}}$

$= \sec^2\theta - \tan^2\theta = 1$

= RHS

Hence proved.

9. If $(\cos ec\theta - \sin\theta) = a^3 and (\sec\theta - \cos\theta) = b^3$, prove that $a^2b^2(a^2+b^2)=1$

Sol:

$We\ have\ (cosec\ \theta - \sin\theta) = a^3$

$=> a^3 = (\frac{1}{\sin\theta} - \sin\theta)$

$=> a^3 = \frac{(1-\sin^2\theta)}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$

$\therefore$ a$=\dfrac{\cos^{\frac{2}{3}}\theta}{\sin^{\frac{1}{3}}\theta}$

$Again, (\sec\theta - \cos\theta) = b^3$

$=> b^3 = (\frac{1}{\cos\theta} - \cos\theta)$

$= \frac{(1-\cos^2\theta)}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$\therefore b = \frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta}$

$Now, LHS = a^2b^2(a^2+b^2)$

$= a^4b^2 + a^2b^4$

$= a^3(ab^2) + (a^2b^2)b^3$

$= \frac{\cos^2\theta}{\sin\theta} \times \left[\frac{\cos^{\frac{2}{3}}\theta}{\sin^{\frac{1}{3}}\theta} \times \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\right] + \left[\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} \times \frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta}\right] \times \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta}{\sin\theta} \times \sin\theta + \cos\theta \times \frac{\sin^2\theta}{\cos\theta}$

$= \cos^2\theta + \sin^2\theta = 1$

= RHS

Hence, proved.

10. If $\left(2\sin\theta + 3\cos\theta\right) = 2,$ prove that $\left(3\sin\theta - 2\cos\theta\right) = \pm 3.$

Sol:

$Given, (2\sin\theta + 3\cos\theta) = 2 \quad ... (i)$

$We\ have\ (2\sin\theta + 3\cos\theta)^2 + (3\sin\theta - 2\cos\theta)^2$

$= 4\sin^2\theta + 9\cos^2\theta + 12\sin\theta\cos\theta + 9\sin^2\theta + 4\cos^2\theta - 12\sin\theta\cos\theta$

$= 4(\sin^2\theta + \cos^2\theta) + 9(\sin^2\theta + \cos^2\theta)$

= 4+9

= 13

i.e., $(2\sin\theta + 3\cos\theta)^2 + (3\sin\theta - 2\cos\theta)^2 = 13$
$=> 2^2 + (3\sin\theta - 2\cos\theta)^2 = 13$
$=> (3\sin\theta - 2\cos\theta)^2 = 13 - 4$
$=> (3\sin\theta - 2\cos\theta)^2 = 9$
$=> (3\sin\theta - 2\cos\theta) = \pm 3$

11. If $(\sin\theta + \cos\theta) = \sqrt{2}$, prove that $\cot\theta = (\sqrt{2} + 1)$.

Sol:

We have, $(\sin\theta + \cos\theta) = \sqrt{2}\cos\theta$
Dividing both sides by $\sin\theta$ *, we get*
$\frac{\text{Sin}\,\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{2}\cos\theta}{\sin\theta}$
$\Longrightarrow 1 + \cot\theta = \sqrt{2}\cot\theta$
$\Longrightarrow \sqrt{2}\cot\theta - \cot\theta = 1$
$\Longrightarrow (\sqrt{2} - 1)\cot\theta = 1$
$\Longrightarrow \cot\theta = \frac{1}{(\sqrt{2}-1)}$
$\Longrightarrow \cot\theta = \frac{1}{(\sqrt{2}-1} \times \frac{(\sqrt{2}+1)}{(\sqrt{2+1})}$
$\Longrightarrow \cot\theta = \frac{(\sqrt{2}+1)}{2-1}$
$\Longrightarrow \cot\theta = \frac{(\sqrt{2}+1)}{1}$
$\therefore \cot\theta = (\sqrt{2} + 1)$

12. If $(\cos\theta + \sin\theta) = \sqrt{2}\sin\theta$, prove that $(\sin\theta - \cos\theta) = \sqrt{2}\cos\theta$.

Sol:

Given: $\cos\theta + \sin\theta = \sqrt{2}\sin\theta$
We have $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2(\sin^2\theta + \cos^2\theta)$
$=> (\sqrt{2}\sin\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$
$=> 2\sin^2\theta + (\sin\theta - \cos\theta)^2 = 2$
$=> (\sin\theta - \cos\theta)^2 = 2 - 2\sin^2\theta$
$=> (\sin\theta - \cos\theta)^2 = 2(1 - \sin^2\theta)$
$=> (\sin\theta - \cos\theta)^2 = 2\cos^2\theta$
$=> (\sin\theta - \cos\theta) = \sqrt{2}\cos\theta$
Hence proved.

13. If $\sec\theta + \tan\theta = p$, prove that

(i) $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$ (ii) $\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$ (iii) $\sin\theta = \frac{p^2-1}{p^2+1}$

Sol:

(i) We have, $\sec\theta+\tan\theta=p$ ……………………………..(1)

$$\Rightarrow \frac{\sec\theta+\tan\theta}{1}\times\frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}=p$$

$$\Rightarrow \frac{\sec^2\theta-\tan^2\theta}{\sec\theta-\tan\theta}=p$$

$$\Rightarrow \frac{1}{\sec\theta-\tan\theta}=p$$

$$\Rightarrow \sec\theta-\tan\theta=\frac{1}{p} \quad ....(2)$$

Adding (1) and (2), we get

$$2\sec\theta=p+\frac{1}{p}$$

$$\Rightarrow \sec\theta=\frac{1}{2}\left(p+\frac{1}{p}\right)$$

(ii) Subtracting (2) from (1), we get

$$2\tan\theta=\left(p-\frac{1}{p}\right)$$

$$\Rightarrow \tan\theta=\frac{1}{2}\left(p-\frac{1}{p}\right)$$

(iii) Using (i) and (ii), we get

$$\sin\theta=\frac{\tan\theta}{\sec\theta}$$

$$=\frac{\frac{1}{2}\left(p-\frac{1}{p}\right)}{\frac{1}{2}\left(p+\frac{1}{p}\right)}$$

$$=\frac{\left(\frac{p^2-1}{p}\right)}{\frac{\left(p^2+1\right)}{p}}$$

$$\therefore\ \sin\theta=\frac{p^2-1}{p^2+1}$$

14. If tan A = n tan B and sin A =m sin B, prove that $\cos^2 A=\frac{\left(m^2-1\right)}{\left(n^2-1\right)}$.

Sol:

We have $\tan A=n\tan B$

$$\Rightarrow \cot B = \frac{n}{\tan A} \quad ......(i)$$

Again, sin A = m sin B

$$\Rightarrow \cos ecB = \frac{m}{\sin A} \quad ....(ii)$$

Squaring (i) and (ii) and subtracting (ii) form (i), we get

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = \cos ec^2 B - \cot^2 B$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2 \cos}{\sin^2 A} = 1$$

$$\Rightarrow m^2 - n^2 \cos^2 A = \sin^2 A$$

$$\Rightarrow m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$\Rightarrow n^2 \cos^2 A - \cos^2 A = m^2 - 1$$

$$\Rightarrow \cos^2 A\left(n^2 - 1\right) = \left(m^2 - 1\right)$$

$$\Rightarrow \cos^2 A = \frac{\left(m^2 - 1\right)}{\left(n^2 - 1\right)}$$

$$\therefore \cos^2 A = \frac{\left(m^2 - 1\right)}{\left(n^2 - 1\right)}$$

15. 15. if $m = (\cos\theta - \sin\theta)$ and $n = (\cos\theta + \sin\theta)$ then show that $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{2}{\sqrt{1 - \tan^2\theta}}$.

Sol:

$$LHS = \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}$$

$$= \frac{\sqrt{m}}{\sqrt{n}} + \frac{\sqrt{n}}{\sqrt{m}}$$

$$= \frac{m+n}{\sqrt{mn}}$$

$$= \frac{(\cos\theta - \sin\theta) + (\cos\theta + \sin\theta)}{\sqrt{\left(\cos\theta - \sin\theta\right)\left(\cos\theta + \sin\theta\right)}}$$

$$= \frac{2\cos\theta}{\sqrt{\cos^2\theta - \sin^2\theta}}$$

$$= \frac{2\cos\theta}{\sqrt{\cos^2\theta - \sin^2\theta}}$$

$$= \frac{\left(\frac{2\cos\theta}{\cos\theta}\right)}{\left(\frac{\sqrt{\cos^2\theta - \sin^2\theta}}{\cos\theta}\right)}$$

$$= \frac{2}{\sqrt{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}}$$

$$= \frac{2}{\sqrt{1 - \tan^2\theta}}$$

$= RHS$

Exercise – 8C

1. Write the value of $(1-\sin^2\theta)\sec^2\theta$.

Sol:

$(1 - \sin^2\theta)\sec^2\theta$

$= \cos^2\theta \times \frac{1}{\cos^2\theta}$

$= 1$

2. Write the value of $(1-\cos^2\theta)co\sec^2\theta$.

Sol:

$(1 - \cos^2\theta)cosec^2\theta$

$= \sin^2\theta \times \frac{1}{\sin^2\theta}$

$= 1$

3. Write the value of $(1+\tan^2\theta)\cos^2\theta$.

Sol:

$(1 + \tan^2\theta)\cos^2\theta$

$= \sec^2\theta \times \frac{1}{\sec^2\theta}$

$= 1$

4. Write the value of $(1+\cot^2\theta)\sin^2\theta$.

Sol:

$= (1 + \cot^2\theta)\sin^2\theta$

$= cosec^2\theta \times \frac{1}{cosec^2\theta}$

$= 1$

5. Write the value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\theta}\right)$.

Sol:

$(\sin^2\theta + \frac{1}{1+\tan^2\theta})$

$= (\sin^2\theta + \frac{1}{\sec^2\theta})$

$= (\sin^2\theta + \cos^2\theta)$

$= 1$

6. Write the value of $\left(\cot^2\theta - \frac{1}{\sin^2\theta}\right)$.

Sol:

$\left(\cot^2\theta - \frac{1}{\sin^2\theta}\right)$

$= (\cot^2\theta - cosec^2\theta)$

$= -1$

7. Write the value of $\sin\theta\cos(90° - \theta) + \cos\theta\sin(90° - \theta)$.

Sol:

$\text{Sin}\,\theta\cos(90^0 - \theta) + \cos\theta\sin(90^0 - \theta)$

$= \sin\theta\sin\theta + \cos\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

8. Write the value of $\text{cosec}^2(90° - \theta) - \tan^2\theta$.

Sol:

$cosec^2(90^0 - \theta) - \tan^2\theta$

$= \sec^2\theta - \tan^2\theta$

$= 1$

9. Write the value of $\sec^2\theta(1+\sin\theta)(1-\sin\theta)$.

Sol:

$\text{Sec}^2\theta(1 + \sin\theta)(1 - \sin\theta)$

$= \sec^2\theta(1 - \sin^2\theta)$

$= \frac{1}{\cos^2\theta} \times \cos^2\theta$

$= 1$

10. Write the value of $cosec^2\theta(1+\cos\theta)(1-\cos\theta)$.

Sol:

$cosec^2\theta(1+\cos\theta)(1-\cos\theta)$

$= cosec^2\theta(1-\cos^2\theta)$

$= \frac{1}{\sin^2\theta} \times \sin^2\theta$

$= 1$

11. Write the value of $\sin^2\theta\cos^2\theta(1+\tan^2\theta)(1+\cot^2\theta)$.

Sol:

$\text{Sin}^2\theta\cos^2\theta(1+\tan^2\theta)(1+\cot^2\theta)$

$= \sin^2\theta\cos^2\theta\sec^2\theta cosec^2\theta$

$= \sin^2\theta \times \cos^2\theta \times \frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta}$

$= 1$

12. Write the value of $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)$.

Sol:

$(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)$

$= \sec^2\theta(1-\sin^2\theta)$

$= \frac{1}{\cos^2\theta} \times \cos^2\theta$

$= 1$

13. Write the value of $3\cot^2\theta - 3cosec^2\theta$.

Sol:

$3\cot^2\theta - 3cosec^2\theta$

$= 3(\cot^2\theta - cosec^2\theta)$

$= 3(-1)$

$= -3$

14. Write the value of $4\tan^2\theta - \frac{4}{\cos^2\theta}$.

Sol:

$4\tan^2\theta - \frac{4}{\cos^2\theta}$

$= 4\tan^2\theta - 4\sec^2\theta$

$= 4(\tan^2\theta - \sec^2\theta)$

$= 4(-1)$

$= -4$

15. Write the value of $\frac{\tan^2\theta-\sec^2\theta}{\cot^2\theta-\cos ec^2\theta}$.

Sol:

$\frac{\tan^2\theta-\sec^2\theta}{\cot^2\theta-cosec^2\theta}$

$= \frac{-1}{-1}$

$= 1$

16. If $\sin\theta=\frac{1}{2}$, write the value of $\left(3\cot^2\theta+3\right)$.

Sol:

$As, \sin\theta = \frac{1}{2}$

$So, cosec\theta = \frac{1}{\sin\theta} = 2 \qquad ..... (i)$

Now,

$3\cot^2\theta + 3$

$= 3(\cot^2\theta + 1)$

$= 3cosec^2\theta$

$= 3(2)^2 \qquad [Using\ \ (i)]$

$= 3(4)$

$= 12$

17. If $\cos\theta=\frac{2}{3}$, write the value of $\left(4+4\tan^2\theta\right)$.

Sol:

$4 + 4\tan^2\theta$

$= 4(1+\tan^2\theta)$

$= 4\sec^2\theta$

$= \frac{4}{\cos^2\theta}$

$= \frac{4}{\left(\frac{2}{3}\right)^2}$

$= \frac{4}{\left(\frac{4}{9}\right)}$

$= \frac{4\times 9}{4}$

$= 9$

18. If $\cos\theta=\frac{7}{25}$, write the value of $\left(\tan\theta+\cot\theta\right)$.

Sol:

$As\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{7}{25}\right)^2$
$= 1 - \frac{49}{625}$
$= \frac{625-49}{625}$
$\Longrightarrow \sin^2\theta = \frac{576}{625}$
$\Longrightarrow \sin\theta = \sqrt{\frac{576}{625}}$
$\Longrightarrow \sin\theta = \frac{24}{25}$

Now,

$\tan\theta + \cot\theta$
$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$
$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$
$= \frac{1}{\left(\frac{7}{25}\times\frac{24}{25}\right)}$
$= \frac{1}{\left(\frac{168}{625}\right)}$
$= \frac{625}{168}$

19. If $\cos\theta = \frac{2}{3}$, write the value of $\frac{(\sec\theta-1)}{(\sec\theta+1)}$.

Sol:

$\frac{\text{Sec}\,\theta-1}{\sec\theta+1}$
$= \frac{\left(\frac{1}{\cos\theta}-\frac{1}{1}\right)}{\left(\frac{1}{\cos\theta}+\frac{1}{1}\right)}$
$= \frac{\left(\frac{1-\cos\theta}{\cos\theta}\right)}{\left(\frac{1+\cos\theta}{\cos\theta}\right)}$
$= \frac{1-\cos\theta}{1+\cos\theta}$
$= \frac{\left(\frac{1}{1}-\frac{2}{3}\right)}{\left(\frac{1}{1}+\frac{2}{3}\right)}$
$= \frac{\left(\frac{1}{3}\right)}{\left(\frac{5}{3}\right)}$
$= \frac{1}{5}$

20. If $5\tan\theta = 4$, write the value of $\frac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)}$.

Sol:

We have,

$5\tan\theta = 4$

$\Rightarrow \tan\theta = \frac{4}{5}$

Now,

$$\frac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)}$$

$$= \frac{(\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta})}{(\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta})} \qquad (Dividing\ numerator\ and\ denominator\ by\ \cos\theta)$$

$$= \frac{(1 - \tan\theta)}{(1 + \tan\theta)}$$

$$= \frac{(\frac{1}{1} - \frac{4}{5})}{(\frac{1}{1} + \frac{4}{5})}$$

$$= \frac{\left(\frac{1}{5}\right)}{\left(\frac{9}{5}\right)}$$

$$= \frac{1}{9}$$

21. If $3\cot\theta = 4$, write the value of $\frac{(2\cos\theta - \sin\theta)}{(4\cos\theta - \sin\theta)}$.

Sol:

We have,

$3\cot\theta = 4$

$\Rightarrow \cot\theta = \frac{4}{3}$

Now,

$$\frac{(2\cos\theta + \sin\theta)}{(4\cos\theta - \sin\theta)}$$

$$= \frac{\left(\frac{2\cos\theta}{\sin\theta} + \frac{\sin\theta}{\sin\theta}\right)}{\left(\frac{4\cos\theta}{\sin\theta} - \frac{\sin\theta}{\sin\theta}\right)} \qquad (Dividing\ numerator\ and\ denominator\ by\ \sin\theta)$$

$$= \frac{(2\cot\theta + 1)}{(4\cot\theta - 1)}$$

$$= \frac{\left(2\times\frac{4}{3} + 1\right)}{\left(4\times\frac{4}{3} - 1\right)}$$

$$= \frac{\left(\frac{8}{3} + \frac{1}{1}\right)}{\left(\frac{16}{3} - \frac{1}{1}\right)}$$

$$= \frac{\left(\frac{8+3}{3}\right)}{\left(\frac{16-3}{3}\right)}$$

$$= \frac{\left(\frac{11}{3}\right)}{\left(\frac{13}{3}\right)}$$

$= \frac{11}{13}$

22. If $\cot\theta = \frac{1}{\sqrt{3}}$, write the value of $\frac{(1-\cos^2\theta)}{(2-\sin^2\theta)}$.

Sol:
We have,
$\text{Cot}\,\theta = \frac{1}{\sqrt{3}}$
$\Longrightarrow \cot\theta = \cot\left(\frac{\pi}{3}\right)$
$\Longrightarrow \theta = \frac{\pi}{3}$
Now,

$$\frac{(1-\cos^2\theta)}{(2-\sin^2\theta)}$$
$$= \frac{1-\cos^2\left(\frac{\pi}{3}\right)}{2-\sin^2\left(\frac{\pi}{3}\right)}$$
$$= \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$$
$$= \frac{\left(\frac{1}{1}-\frac{1}{4}\right)}{\left(\frac{2}{1}-\frac{3}{4}\right)}$$
$$= \frac{\left(\frac{3}{4}\right)}{\left(\frac{5}{4}\right)}$$
$$= \frac{3}{5}$$

23. If $\tan\theta = \frac{1}{\sqrt{5}}$, write the value of $\frac{(\cos ec^2\theta - \sec^2\theta)}{(\cos ec^2\theta - \sec^2\theta)}$.

Sol:

$$\frac{(cosec^2\theta-\sec^2\theta)}{(cosec^2\theta+\sec^2\theta)}$$
$$= \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$$
$$= \frac{\left(1+\frac{1}{\tan^2\theta}\right)-(1+\tan^2\theta)}{\left(1+\frac{1}{\tan^2\theta}\right)+(1+\tan^2\theta)}$$
$$= \frac{(1+\frac{1}{\tan^2\theta}-1-\tan^2\theta)}{(1+\frac{1}{\tan^2\theta}+1+\tan^2\theta)}$$
$$= \frac{(\frac{1}{\tan^2\theta}-\tan^2\theta)}{(\frac{1}{\tan^2\theta}+\tan^2\theta+2)}$$

$$= \frac{\left(\frac{\sqrt{5}}{1}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{5}}{1}\right)^2 + \left(\frac{1}{\sqrt{5}}\right)^2 + 2}$$

$$= \frac{\left(\frac{5}{1} - \frac{1}{5}\right)}{\left(\frac{5}{1} + \frac{1}{5} + \frac{2}{1}\right)}$$

$$= \frac{\left(\frac{24}{5}\right)}{\left(\frac{36}{5}\right)}$$

$$= \frac{24}{36}$$

$$= \frac{2}{3}$$

24. If $\cot A = \frac{4}{3}$ *and* $(A + B) = 90°$, what is the value of tan B?

Sol:

We have,

$cotA = \frac{4}{3}$

$\Longrightarrow \cot(90^0 - B) = \frac{4}{3} \quad (As, A + B = 90^0)$

$\therefore tanB = \frac{4}{3}$

25. If $\cos B = \frac{3}{5}$ *and* $(A + B) = 90°$, find the value of sin A.

Sol:

We have,

$cosB = \frac{3}{5}$

$\Longrightarrow \cos(90^0 - A) = \frac{3}{5} \quad (As, A + B = 90^0)$

$\therefore sinA = \frac{3}{5}$

26. If $\sqrt{3}\sin\theta = \cos\theta$ *and* θ is an acute angle, find the value of θ.

Sol:

We have,

$\sqrt{3}\sin\theta = \cos\theta$

$\Longrightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$

$\Longrightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Longrightarrow \tan\theta = tan30^0$

$\therefore \theta = 30^0$

27. Write the value of $\tan 10° \tan 20° \tan 70° \tan 80°$.

Sol:

$tan 10^0\ tan 20^0\ tan 70^0\ tan 80^0$

$= \cot(90^0 - 10^0) \cot(90^0 - 20^0)\ tan70^0\ tan80^0$

$= cot80^0\ cot70^0\ tan70^0\ tan80^0$

$= \frac{1}{tan\ 80^0} \times \frac{1}{\tan 70^0} \times \tan 70^0 \times \tan 80^0$

$= 1$

28. Write the value of $\tan 1° \tan 2° .... \tan 89°$.

Sol:

$\text{Tan } 1^0 \tan 2^0 \ldots \tan 89^0$

$= \tan 1^0 \tan 2^0 \tan 3^0\ \ldots\ \tan 45^0\ \ldots \tan 87^0\ \tan 88^0 \tan 89^0$

$= \tan 1^0 \tan 2^0 \tan 3^0\ \ldots\ \tan 45^0\ \ldots \cot(90^0 - 87^0) \cot(90^0 - 88^0) \cot(90^0 - 89^0)$

$= \tan 1^0 \tan 2^0 \tan 3^0\ \ldots\ \tan 45^0\ \ldots \cot 3^0\ \cot 2^0 \cot 1^0$

$= \tan 1^0 \times \tan 2^0 \times \tan 3^0 \times \ldots \times 1 \times \ldots \times \frac{1}{\tan 3^0} \times \frac{1}{\tan 2^0} \times \frac{1}{\tan 1^0}$

$= 1$

29. Write the value of $\cos 1° \cos 2° .... \cos 180°$.

Sol:

$\text{Cos } 1^0\ \cos 2^0\ \ldots\ \cos 180^0$

$= \cos 1^0\ \cos 2^0\ \ldots \cos 90^0\ \ldots \cos 180^0$

$= \cos 1^0 \cos 2^0 \ldots 0 \ldots \cos 180^0$

$= 0$

30. If $\tan A = \frac{5}{12}$, find the value of $(\sin A + \cos A)\sec A$.

Sol:

$(sinA + cosA)secA$

$= (sinA + cosA)\ \frac{1}{cosA}$

$= \frac{sinA}{cosA} + \frac{cosA}{cosA}$

$= tanA + 1$

$= \frac{5}{12} + \frac{1}{1}$

$= \frac{5+12}{12}$

$= \frac{17}{12}$

31. If $\sin\theta = \cos(\theta - 45°)$, where θ is acute, find the value of θ.

Sol:

We have,

$\text{Sin } \theta = \cos(\theta - 45^0)$

$\Rightarrow \cos(90^0 - \theta) = \cos(\theta - 45^0)$

Comparing both sides, we get

$90^0 - \theta = \theta - 45^0$
$\Rightarrow \theta + \theta = 90^0 + 45^0$
$\Rightarrow 2\theta = 135^0$
$\Rightarrow \theta = \left(\frac{135}{2}\right)^0$
$\therefore \theta = 67.5^0$

32. Find the value of $\frac{\sin 50°}{\cos 40°} + \frac{\cos ec40°}{\sec 50°} - 4\cos 50° \cos ec40°$.

Sol:
$\frac{\text{Sin } 50^0}{\cos 40^0} + \frac{cosec40^0}{\sec 50^0} - 4\cos 50^0 \ cosec\ 40^0$
$= \frac{\cos(90^0-50^0)}{\cos 40^0} + \frac{\sec(90^0-40^0)}{\sec 50^0} - 4\sin(90^0 - 50^0)\ cosec\ 40^0$
$= \frac{cos\ 40^0}{\cos 40^0} + \frac{\sec 50^0}{\sec 50^0} - 4\sin 40^0 \times \frac{1}{\sin 40^0}$
$= 1 + 1 - 4$
$= -2$

33. Find the value of $\sin 48° \sec 42° + \cos 48° \cos ec42°$.
Sol:
$\text{Sin } 48^0 \sec 42^0 + \cos 48^0\ cosec\ 42^0$
$= \sin 48^0\ cosec(90^0 - 42^0) + \cos 48^0 \sec(90^0 - 42^0)$
$= \sin 48^0\ cosec\ 48^0 + \cos 48^0 \sec 48^0$
$= \sin 48^0 \times \frac{1}{\sin 48^0} + \cos 48^0 \times \frac{1}{\cos 48^0}$
= 1+1
=2

34. If $x = a\sin\theta\ and\ y = b\cos\theta,$ write the value of $\left(b^2x^2 + a^2y^2\right)$.

Sol:
$(b^2x^2 + a^2y^2)$
$= b^2(\text{a}\sin\theta)^2 + a^2(bcos\ \theta)^2$
$= b^2a^2\sin^2\theta + a^2b^2\cos^2\theta$
$= a^2b^2(\sin^2\theta + \cos^2\theta)$
$= a^2b^2(1)$
$= a^2b^2$

35. If $5x = \sec\theta\ and\ \frac{5}{x} = \tan\theta,$ find the value of $5\left(x^2 - \frac{1}{x^2}\right)$.

Sol:
$5\left(x^2 - \frac{1}{x^2}\right)$
$= \frac{25}{5}\left(x^2 - \frac{1}{x^2}\right)$

$= \frac{1}{5}\left(25x^2 - \frac{25}{x^2}\right)$
$= \frac{1}{5}\left[(5x)^2 - \left(\frac{5}{x}\right)^2\right]$
$= \frac{1}{5}\ [(sec\ \theta)^2 - (tan\ \theta)^2]$
$= \frac{1}{5}\ (sec^2\ \theta - tan^2\ \theta)$
$= \frac{1}{5}\ (1)$
$= \frac{1}{5}$

36. If $\cos ec\theta = 2x\ and\ \cot\theta = \frac{2}{x}$, find the value of $2\left(x^2 - \frac{1}{x^2}\right)$.

Sol:

$2\left(x^2 - \frac{1}{x^2}\right)$
$= \frac{4}{2}\left(x^2 - \frac{1}{x^2}\right)$
$= \frac{1}{2}\left(4x^2 - \frac{4}{x^2}\right)$
$= \frac{1}{2}\left[(2x)^2 - \left(\frac{2}{x}\right)^2\right]$
$= \frac{1}{2}\ [(cosec\ \theta)^2 - (\sec\theta)^2]$
$= \frac{1}{2}\ (cosec^2\theta - \sec^2\theta)$
$= \frac{1}{2}\ (1)$
$= \frac{1}{2}$

37. If $sec\theta + \tan\theta = x$, find the value of $sec\theta$.

Sol:

$We\ have,$
$\text{Sec}\,\theta + \tan\theta = x \quad \ldots\ldots(i)$
$\Longrightarrow \frac{\sec\theta + \tan\theta}{1} \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = x$
$\Longrightarrow \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = x$
$\Longrightarrow \frac{1}{\sec\theta - \tan\theta} = \frac{x}{1}$
$\Longrightarrow \sec\theta - \tan\theta = \frac{1}{x} \qquad \ldots..(ii)$
$Adding\ (i)and\ (ii), we\ get$
$2\sec\theta = x + \frac{1}{x}$
$\Longrightarrow 2\sec\theta = \frac{x^2+1}{x}$
$\therefore \sec\theta = \frac{x^2+1}{2x}$

38. Find the value of $\dfrac{\cos 38^\circ \operatorname{cos}ec 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$.

Sol:

$\dfrac{\text{Cos}\, 38^0\ cosec\ 52^0}{\tan 18^0 \tan 35^0 \tan 60^0 \tan 72^0 \tan 55^0}$

$= \dfrac{\cos 38^0 \sec(90^0 - 52^0)}{\cot(90^0-18^0)\cot(90^0-35^0)\tan 60^0 \tan 72^0 \tan 55^0}$

$= \dfrac{\cos 38^0 \sec 38^0}{\cot 72^0 \cot 55^0 \tan 60^0 \tan 72^0 \tan 55^0}$

$= \dfrac{\cos 38^0 \times \frac{1}{\cos 38^0}}{\frac{1}{\tan 72^0} \times \frac{1}{\tan 55^0} \times \sqrt{3} \times \tan 72^0 \times \tan 55^0}$

$= \dfrac{1}{\sqrt{3}}$

39. If $\sin\theta = x$, write the value of $\cot\theta$.

Sol:

$\text{Cot}\,\theta = \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$

$= \dfrac{\sqrt{1-x^2}}{2}$

40. If $\sec\theta = x$, write the value of $\tan\theta$.

Sol:

As, $\tan^2\theta = \sec^2\theta - 1$

So, $\tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{x^2 - 1}$

Formative Assessment

1. $\dfrac{\cos^2 56^\circ + \cos^2 34^\circ}{\sin^2 56^\circ + sin^2 34^\circ} + 3\tan^2 56^\circ \tan^2 34^\circ = ?$

(a) $3\frac{1}{2}$ (b) 4

(c) 6 (d) 5

Answer: (b) 4

Sol:

$\dfrac{\cos^2 56^0 + \cos^2 34^0}{\sin^2 56^0 + \sin^2 34^0} + 3\tan^2 56^0 \tan^2 34^0\backslash$

$= \dfrac{\{\cos(90^0-34^0)\}^2 + \cos^2 34^0}{\{\sin(90^0-34^0)\}^2 + \sin^2 34^0} + 3\{\tan(90^0 - 34^0)\}^2 \tan^2 34^0$

$= \dfrac{\sin^2 34^0 + \cos^2 34^0}{\cos^2 34^0 + \sin^2 34^0} + 3\cot^2 34^0 \tan^2 34^0$ $\left[\because \cos(90^\circ - \theta) = \sin\theta, \sin(90^\circ - \theta) = \cos\theta\ and\ \tan(90^\circ - \theta) = \cot\theta\right]$

$=\frac{1}{1}+3\times1 \quad \left[\because \cot\theta=\frac{1}{\tan\theta} \text{ and } \sin^2\theta+\cos^2\theta=1\right]$
$= 4$

2. The value of $\left(\sin^2 30°\cos^2 45°+4\tan^2 30°+\frac{1}{2}\sin^2 90°+\frac{1}{8}\cot^2 60°\right)=?$

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$
(c) 6 (d) 2

Answer: (d) 2

Sol:

$(\sin^2 30^0 \cos^2 45^0)+4\tan^2 30^0+\frac{1}{2}\sin^2 90^0+\frac{1}{8}\cot^2 60^0$

$=\frac{1}{2^2}\times\frac{1}{(\sqrt{2})^2}+4\times\frac{1}{(\sqrt{3})^2}+\frac{1}{2}\times1^2+\frac{1}{8}\times\frac{1}{(\sqrt{3})^2} \quad \left[\begin{array}{l}\because \sin 30°=\frac{1}{2} \text{ and } \cos 45°=\frac{1}{\sqrt{2}} \\ \text{and } \tan 30°=\frac{1}{2} \text{ and } \cot 60°=\frac{1}{\sqrt{3}}\end{array}\right]$

$=\frac{1}{4}\times\frac{1}{2}+4\times\frac{1}{3}+\frac{1}{2}+\frac{1}{24}$
$=\frac{1}{8}+\frac{4}{3}+\frac{1}{2}+\frac{1}{24}$
$=\frac{3+32+12+1}{24}$
$=\frac{48}{24}$
$=2$

3. If $\cos A+\cos^2 A=1$ then $\left(\sin^2 A+\sin^4 A\right)=?$

(a) $\frac{1}{2}$ (b) 2
(c) 1 (d) 4

Answer: (c) 1

Sol:

$\cos^2 A+A=1$
$\Rightarrow \cos A=\sin^2 A \quad \ldots(i)$
Squaring both sides of (i), *we get*:
$\cos^2 A=\sin^4 A \quad \ldots(ii)$
Adding (i) *and* (ii), *we get*:
$\sin^2 A+\sin^4 A=\cos A+\cos^2 A$
$\Rightarrow \sin^2 A+\sin^4 A=1 \quad [\because \cos A+\cos^2 A=1]$

4. If $\sin\theta = \frac{\sqrt{3}}{2}$ then $\left(\cos ec\theta + \cot\theta\right) = ?$

(a) $\left(2+\sqrt{3}\right)$ (b) $2\sqrt{3}$

(c) $\sqrt{2}$ (d) $\sqrt{3}$

Answer: (d) $\sqrt{3}$

Sol:

$Given: \sin\theta = \frac{\sqrt{3}}{2}\ and \cos ec\theta = \frac{2}{\sqrt{3}}$

$\cos ec^2\theta - \cot^2\theta = 1$

$=> \cot^2\theta = \cos ec^2\theta - 1$

$=> \cot^2\theta = \frac{4}{3} - 1 \quad [Given]$

$=> \cot\theta = \frac{1}{\sqrt{3}}$

$\therefore \cos ec\theta + \cot\theta = \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}$

$= \frac{3}{\sqrt{3}}$

$= \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$

$= \sqrt{3}$

5. If $\cot A = \frac{4}{5}$, prove that $\frac{(\sin A + \cos A)}{(\sin A - \cos A)} = 9.$

Sol:

$Given : \cot A = \frac{4}{5}$

$Writing \cot A = \frac{\cos A}{\sin A}\ and\ sqauring\ the\ equation, we\ get :$

$\frac{\cos^2 A}{\sin^2 A} = \frac{16}{25}$

$=> 25\cos^2 A = 16\sin^2 A$

$=> 25\cos^2 A = 16 - 16\cos^2 A$

$=> \cos^2 A = \frac{16}{41}$

$=> \cos A = \frac{4}{\sqrt{41}}$

$\therefore \sin^2 A = 1 - \cos^2 A$

$= 1 - \frac{16}{41}$

$Now, \sin A = \sqrt{\frac{25}{41}}$

$=> \sin A = \frac{5}{\sqrt{41}}$

$\therefore LHS = \frac{\sin A + \cos A}{\sin A - \cos A}$

$$= \frac{\frac{5}{\sqrt{41}}+\frac{4}{\sqrt{41}}}{\frac{5}{\sqrt{41}}-\frac{4}{\sqrt{41}}}$$
$$= \frac{9}{1}$$
$$= 9 = RHS$$

6. If $2x = \sec A \, and \, \frac{2}{x} = \tan A,$ prove that $\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{4}.$

Sol:

$Given: 2x = \sec A$

$=> x = \frac{\sec A}{2} \quad ... (i)$

$and \frac{2}{x} = \tan A$

$=> \frac{1}{x} = \tan A \quad ... (ii)$

$\therefore x + \frac{1}{x} = \frac{\sec A}{2} + \frac{\tan A}{2} \quad [\because From\ (i) and\ (ii)]$

$Also, x - \frac{1}{x} = \frac{\sec A}{2} - \frac{\tan A}{2}$

$\therefore \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = \left(\frac{\sec A}{2} + \frac{\tan A}{2}\right)\left(\frac{\sec A}{2} - \frac{\tan A}{2}\right)$

$=> x^2 - \frac{1}{x^2} = \frac{1}{4}\left(\sec^2 A - \tan^2 A\right)$

$\therefore x^2 - \frac{1}{x^2} = \frac{1}{4} \times 1 \qquad (\because \sec^2 A - \tan^2 A = 1)$

$= \frac{1}{4}$

$Hence\ proved.$

7. If $\sqrt{3}\tan\theta = 3\sin\theta,$ prove that $\left(\sin^2\theta - \cos^2\theta\right) = \frac{1}{3}.$

Sol:

$Given: \sqrt{3}\tan\theta = 3\sin\theta$

$=> \frac{\sqrt{3}}{\cos\theta} = 3 \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$=> \cos\theta = \frac{\sqrt{3}}{3}$

$=> \cos^2\theta = \frac{3}{9}$

$\therefore \sin^2\theta = 1 - \frac{3}{9}$

$=> \sin^2\theta = \frac{6}{9}$

$\therefore LHS = \sin^2\theta - \cos^2\theta$

$= \frac{6}{9} - \frac{3}{9} \qquad \left[\therefore \sin^2\theta = \frac{6}{9}, \cos^2\theta = \frac{3}{9}\right]$

$= \frac{3}{9}$

$= \frac{1}{3}$

$=RHS$

Hence Proved.

8. Prove that $\frac{(\sin^2 73° + \sin^2 17°)}{(\cos^2 28° + \cos^2 62°)} = 1.$

Sol:

$\frac{(\sin^2 73^0 + \sin^2 17^0)}{(\cos^2 28^0 + \cos^2 62^0)} = 1.$

$LHS = \frac{\sin^2 73^0 + \sin^2 17^0}{\cos^2 28^0 + \cos^2 62^0}$

$= \frac{[\sin(90^0 - 17^0)]^2 + \sin^2 17^0}{[\cos(90^0 - 62^0)]^2 + \cos^2 62^0}$

$= \frac{\cos^2 17^0 + \sin^2 17^0}{\sin^2 62^0 + \cos^2 62^0}$

$= \frac{1}{1} \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 = RHS$

9. If $2\sin 2\theta = \sqrt{3}$, prove that $\theta = 30°$.

Sol:

$2\sin(2\theta) = \sqrt{3}$

$=> \sin(2\theta) = \frac{\sqrt{3}}{2}$

$=> \sin(2\theta) = \sin(60^0)$

$=> 2\theta = 60^0$

$=> \theta = \frac{60^0}{2}$

$=> \theta = 30^0$

10. Prove that $\sqrt{\frac{1+\cos A}{1-\cos A}} = (\cos ec\, A + \cot A).$

Sol:

$\sqrt{\frac{1+\cos A}{1-\cos A}} = (cosecA + \cot A).$

$LHS = \sqrt{\frac{1+\cos A}{1-\cos A}}$

Multiplying the numerator and denominator by $(1 + \cos A)$, *we have*:

$\sqrt{\frac{(1+\cos A)^2}{(1-\cos A)(1+\cos A)}}$

$= \sqrt{\frac{(1+\cos A)^2}{1\cos^2 A}}$

$= \frac{1+\cos A}{\sqrt{\sin^2 A}}$

$= \frac{1+\cos A}{\sin A}$

$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$
$= cosecA + \cot A = RHS$
$Hence\ proved.$

11. If $\cos ec\,\theta + \cot\theta = p$, prove that $\cos\theta = \frac{(p^2-1)}{(p^2+1)}$.

Sol:
$\cos ec\theta + \cot\theta = p$
$=> \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = p$
$=> \frac{1+\cos\theta}{\sin\theta} = p$
$Squaring\ both\ sides, we\ get:$
$\left(\frac{1+\cos\theta}{\sin\theta}\right)^2 = p^2$
$=> \frac{(1+\cos\theta)^2}{\sin^2\theta} = p^2$
$=> \frac{(1+\cos\theta)^2}{1-\cos^2\theta} = p^2$
$=> \frac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)} = p^2$
$=> \frac{(1+\cos\theta)}{(1-\cos\theta)} = p^2$
$=> 1 + \cos\theta = p^2(1-\cos\theta)$
$=1 + \cos\theta = p^2 - p^2\cos\theta$
$=> \cos\theta(1+p^2) = p^2 - 1$
$=> \cos\theta = \frac{p^2-1}{p^2+1}$
$Hence\ proved.$

12. Prove that $(\cos ec\,A - \cot A)^2 = \frac{(1-\cos A)}{(1+\cos A)}$.

Sol:
$(cosecA - \cot A)^2 = \frac{(1-\cos A)}{(1+\cos A)}.$
$LHS = (cosecA - \cot A)^2$
$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$
$= \left(\frac{1-\cos A}{\sin A}\right)^2$
$= \frac{(1-\cos A)^2}{\sin^2 A}$
$= \frac{(1-\cos A)^2}{1-\cos^2 A}\quad [\because \sin^2\theta + \cos^2\theta = 1]$
$= \frac{(1-\cos A)(1-\cos A)}{(1-\cos A)(1+\cos A)}$
$= \frac{(1-\cos A)}{(1+\cos A)} = RHS$
$Hence\ proved.$

13. If $5\cot\theta = 3$, show that the value of $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$ is $\frac{16}{29}$.

Sol:

$Given: 5\cot\theta = 3$

$=> \frac{5\cos\theta}{\sin\theta} = 3 \quad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$=> 5\cos\theta = 3\sin\theta$

Squaring both sides, we get:

$25\cos^2\theta = 9\sin^2\theta$

$=> 25\cos^2\theta = 9 - 9\cos^2\theta \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$=> 34\cos^2\theta = 9$

$=> \cos\theta = \sqrt{\frac{9}{34}}$

$=> \cos\theta = \frac{3}{\sqrt{34}}$

$Again, \sin^2\theta = 1 - \cos^2\theta$

$=> \sin^2\theta = \frac{34-9}{34} = \frac{25}{34}$

$=> \sin\theta = \frac{5}{\sqrt{34}}$

$\therefore LHS = \left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$

$= \frac{5\times\frac{5}{\sqrt{34}} - 3\times\frac{3}{\sqrt{34}}}{4\times\frac{5}{\sqrt{34}} + 3\times\frac{3}{\sqrt{34}}} \quad \left[\because \cos\theta = \frac{3}{\sqrt{34}}, \sin\theta = \frac{5}{\sqrt{34}}\right]$

$= \frac{25-9}{20+9}$

$= \frac{16}{29}$

14. Prove that $(\sin 32^\circ \cos 58^\circ + \cos 32^\circ \sin 58^\circ) = 1$.

Sol:

$(\sin 32^0 \cos 58^0 + \cos 32^0 \sin 58^0) = 1$

$LHS = \sin 32^0 \cos 58^0 + \cos 32^0 \sin 58^0$

$= \sin(90^0 - 58^0)\cos 58^0 + \cos(90^0 - 58^0)\sin 58^0$

$= \cos 58^\circ \times \cos 58^\circ + \sin 58^\circ \times \sin 58^\circ \quad \begin{bmatrix} \because \sin(90^\circ - \theta) = \cos\theta, \\ \cos(90^\circ - \theta) = \cos\theta \end{bmatrix}$

$= \cos^2 58^\circ + \sin^2 58^\circ$

$= 1 \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= RHS$

15. If $x = a\sin\theta + b\cos\theta\, and\; y = a\cos\theta - b\sin\theta$, prove that $x^2 + y^2 = a^2 + b^2$.

Sol:

$Given: x = \text{a}\sin\theta + bcos\,\theta$

$Squaring\ both\ sides, we\ get:$

$x^2 = a^2\sin^2\theta + 2absin\theta\cos\theta + b^2\cos^2\theta \qquad ...(i)$

$Also, y = \text{a}\cos\theta - bsin\,\theta$

$Squaring\ both\ sides, we\ get:$

$y^2 = a^2\cos^2\theta - 2absin\theta\cos\theta + b^2\sin^2\theta \qquad ...(ii)$

$\therefore LHS = x^2 + y^2$

$= a^2\sin^2 + 2absin\theta\cos\theta + b^2\cos^2\theta + a^2\cos^2\theta - 2absin\theta\cos\theta + b^2\sin^2\theta \qquad [using\ (i) and\ (ii)]$

$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= a^2 + b^2 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$

$= RHS$

$Hence\ proved.$

16. Prove that $\left(\frac{1+\sin\theta}{1-\sin\theta}\right) = (\sec\theta + \tan\theta)^2$.

Sol:

$\frac{(1+\sin\theta)}{(1-\sin\theta)} = (\sec\theta + \tan\theta)^2$

$LHS = \frac{(1+\sin\theta)}{(1-\sin\theta)}$

$Multiplying\ the\ numerator\ and\ denominator\ by\ (1+\sin\theta), we\ get:$

$\frac{(1+\sin\theta)^2}{1-\sin^2\theta}$

$= \frac{1+2\sin\theta+\sin^2\theta}{\cos^2\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$

$= \sec^2\theta + 2\times\frac{\sin\theta}{\cos\theta}\times\sec\theta + \tan^2\theta$

$= \sec^2\theta + 2\times\tan\theta\times\sec\theta + \tan^2\theta$

$= (\sec\theta + \tan\theta)^2$

$= RHS$

$Hence\ proved.$

17. Prove that $\frac{1}{(\sec\theta - \tan\theta)} - \frac{1}{cos\theta} = \frac{1}{cos\theta} - \frac{1}{(\sec\theta + \tan\theta)}$.

Sol:

$\frac{1}{(\sec\theta-\tan\theta)} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{(\sec\theta+\tan\theta)}$

$LHS = \frac{1}{\sec\theta-\tan\theta} - \frac{1}{\cos\theta}$

$=\frac{\sec\theta+\tan\theta}{\sec^2\theta-\tan^2\theta}-\sec\theta$

$=\sec\theta+\tan\theta-\sec\theta \quad [\because \sec^2\theta-\tan^2\theta=1]$

$=\tan\theta$

$RHS=\frac{1}{\cos\theta}-\frac{1}{\sec\theta+\tan\theta}$

$=\sec\theta-\frac{(\sec\theta-\tan\theta)}{\sec^2\theta-\tan^2\theta}$ $(Multipying\ the\ numerator\ and\ denomenator\ by\ (\sec\theta-\tan\theta))$

$=\sec\theta+\tan\theta-\sec\theta \quad [\because \sec^2\theta-\tan^2\theta=1]$

$=\tan\theta$

$\therefore LHS=RHS$

Hence Proved

18. Prove that $\frac{(\sin A-2\sin^3 A)}{(2\cos^3 A-\cos A)}=\tan A.$

Sol:

$LHS=\frac{(\sin A-2\sin^3 A)}{(2\cos^3 A-\cos A)}$

$=\frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A-1)}$

$=\tan A\left\{\frac{(\sin^2 A+\cos^2 A-2\sin^2 A)}{2\cos^2 A-\sin^2 A-\cos^2 A}\right\} \quad [\because \sin^2 A+\cos^2 A=1]$

$=\tan A\left\{\frac{(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)}\right\}$

$=\tan A$

$=RHS$

19. Prove that $\frac{\tan A}{(1-\cot A)}+\frac{\cot A}{(1-\tan A)}=(1+\tan A+\cot A).$

Sol:

$LHS=\frac{\tan A}{(1-\cot A)}+\frac{\cot A}{(1-\tan A)}$

$=\frac{\tan A}{(1-\cot A)}+\frac{\cot^2 A}{(\cot A-1)} \quad \left[\because \tan A=\frac{1}{\cot A}\right]$

$=\frac{\tan A}{(1-\cot A)}-\frac{\cot^2 A}{(1-\cot A)}$

$=\frac{\tan A-\cot^2 A}{(1-\cot A)}$

$=\frac{\left(\frac{1}{\cot A}\right)-\cot^2 A}{(1-\cot A)}$

$=\frac{1-\cot^3 A}{\cot A(1-\cot A)}$

$=\frac{(1-\cot A)(1+\cot A+\cot^2 A)}{\cot A(1-\cot A)} \quad [\because a^3-b^3=(a-b)(a^2+ab+b^2)]$

$= \frac{1}{\cot A} + \frac{\cot^2 A}{\cot A} + \frac{\cot A}{\cot A}$
$= 1 + \tan A + \cot A$
$= RHS$
Hence proved

20. If $\sec 5A = \cos ec\left(A - 36°\right) and\, 5A$ is an acute angle , show that $A = 21°$.

Sol:
$Given: sec5A = \cos ec(A - 36^0)$
$=> cosec(90^0 - 5A) = \cos ec(A - 36^0)$ $[\because \cos ec(90^0 - \theta) = \sec\theta]$
$=> 90^0 - 5A = A - 36^0$
$=> 6A = 90^0 + 36^0$
$=> 6A = 126^0$
$=> A = 21^0$

Multiple Choice Question

1. $\frac{\sec 30°}{\cos ec 60°} = ?$

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\sqrt{3}$ (d) 1

Answer: (d) 1
Sol:

$\frac{\text{Sec } 30^0}{cosec\ 60^0} = \frac{\sec 30^0}{\sec(90^0 - 60^0)} = \frac{\sec 30^0}{\sec 30^0} = 1$

2. $\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°} = ?$

(a) 0 (b) 1
(c) 2 (d) none of these

Answer: (c) 2
Sol:
We have,

$\frac{\text{Tan } 35^0}{\cot 55^0} + \frac{\cot 78^0}{\tan 12^0}$
$= \frac{\tan 35^0}{\cot(90^0 - 35^0)} + \frac{\cot(90^0 - 12^0)}{\tan 12^0}$
$= \frac{\tan 35^0}{\tan 35^0} + \frac{\tan 12^0}{\tan 12^0}$ $[\because \cot(90^0 - \theta) = \tan\theta]$
$= 1 + 1 = 2$

3. $\tan 10° \tan 15° \tan 75° \tan 80° = ?$

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) -1 (d) 1

Answer: (d) 1

Sol:

We have,

$\tan 10^0 \tan 15^0 \tan 75^0 \tan 80^0$

$= \tan 10^0 \times \tan 15^0 \times \tan(90^0 - 15^0) \times \tan(90^0 - 10^0)$

$= \tan 10^0 \times \tan 15^0 \times \cot 15^0 \times \cot 10^0$ $[\because \tan(90^0 - \theta) = \cot\theta]$

$= 1$

4. $\tan 5° \tan 25° \tan 30° \tan 65° \tan 85° = ?$

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) 1 (d) none of these

Answer: (b) $\frac{1}{\sqrt{3}}$

Sol:

We have:

$\tan 5^0 \tan 25^0 \tan 30^0 . \tan 65^0 \tan 85^0$

$= \tan 5^0 \tan 25^0 \tan 30^0 \tan(90^0 - 25^0) \tan(90^0 - 5^0)$

$= \tan 5^0 \tan 25^0 \times \frac{1}{\sqrt{3}} \times \cot 25^0 \cot 5^0$ $\left[\because \tan(90^0 - \theta) = \cot\theta \ and \ \tan 30^0 = \frac{1}{\sqrt{3}}\right]$

$= \frac{1}{\sqrt{3}}$

5. $\cos 1° \cos 2° \cos 3° .... \cos 180° = ?$

(a) -1 (b) 1

(c) 0 (d) $\frac{1}{2}$

Answer: (c) 0

Sol:

$\text{Cos } 1^0 \cos 2^0 \cos 3^0 \ldots \cos 180^0$

$= \cos 1^0 \cos 2^0 \cos 3^0 \ldots \cos 90^0 \ldots \cos(180)^0$

$= 0$ $[\because \cos 90^0 = 0]$

6. $\frac{2\sin^2 63° + 1 + 2\sin^2 27°}{3\cos^2 17° - 2 + 3\cos^2 73°} = ?$

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$
(c) 2 (d) 3
Answer: (d) 3
Sol:
$Given: \frac{2\sin^2 63^0+1+2\sin^2 27^0}{3\cos^2 17^0-2+3\cos^2 73^0}$
$= \frac{2(\sin^2 63^0+\sin^2 27^0)+1}{3(\cos^2 17^0+\cos^2 73^0)-2}$
$= \frac{2[\sin^2 63^0+\sin^2(90^0-63^0)]+1}{3[\cos^2 17^0+\cos^2(90^0-17^0)]-2}$
$= \frac{2(\sin^2 63^0+\cos^2 63^0)+1}{3(\cos^2 17^0+\sin^2 17^0)-2}$ $[\because \sin(90^0-\theta) = \cos\theta \ and \cos(90^0-\theta) = \sin\theta]$
$= \frac{2\times1+1}{3\times1-2}$ $[\because \sin^2\theta + \cos^2\theta = 1]$
$= \frac{2+1}{3-2}$
$= \frac{3}{1} = 3$

7. $\sin 47° \cos 43° + \cos 47° \sin 43° = ?$
(a) $\sin 4°$ (b) $\cos 4°$
(c) 1 (d) 0
Answer: (c) 1
Sol:
We have:
$(\sin 43^0 \cos 47^0 + \cos 43^0 \sin 47^0)$
$= \sin 43^0 \cos(90^0 - 43^0) + \cos 43^0 \sin(90^0 - 43^0)$
$= \sin 43^0 \sin 43^0$
$+ \cos 43^0 \cos 43^0$ $[\because \cos(90^0-\theta) = \sin\theta \ and \sin(90^0-\theta) = \cos\theta]$
$= \sin^2 43^0 + \cos^2 43^0$
$= 1$

8. $\sec 70° \sin 20° + \cos 20° \cos ec 70° = ?$
(a) 0 (b) 1
(c) -1 (d) 2
Answer: (d) 2
Sol:
We have:
$\text{Sec } 70^0 \sin 20^0 + \cos 20^0 \ cosec \ 70^0$
$= \frac{\sin 20^0}{\cos 70^0} + \frac{\cos 20^0}{\sin 70^0}$
$= \frac{\sin 20^0}{\cos(90^0-20^0)} + \frac{\cos 20^0}{\sin(90^0-20^0)}$
$= \frac{\sin 20^0}{\sin 20^0} + \frac{\cos 20^0}{\cos 20^0}$ $[\because \cos(90^0-\theta) = \sin\theta \ and \sin(90^0-\theta) = \cos\theta]$

= 1 + 1
= 2
OR
Sec $70^0 \sin 20^0 + \cos 20^0\, cosec\ 70^0$
= $cosec(90^0 - 70^0) \sin 20^0 + \cos 20^0 \sec(90^0 - 70^0)$
= $cosec\ 20^0 \sin 20^0 + \cos 20^0 \sec 20^0$
= $\frac{1}{\sin 20^0} \times \sin 20^0 + \cos 20^0 \times \frac{1}{\cos 20^0}$
= 1 + 1
= 2

9. If $\sin 3A = \cos(A - 10°)$ *and* $3A$ is acute then $\angle A = ?$

(a) $35°$ (b) $25°$
(c) $20°$ (d) $45°$

Answer: (b) $25°$

Sol:
We have:
$[\sin 3A = \cos(A - 10^0)]$
$=> \cos(90^0 - 3A) = \cos(A - 10^0)$ $[\because \sin\theta = \cos(90^0 - \theta)]$
$=> 90^0 - 3A = A - 10^0$
$=> -4A = -100$
$=> A = \frac{100}{4}$
$=> A = 25^0$

10. If $\sec 4A = \operatorname{cosec}(A - 10°)$ *and* $4A$ is acute then $\angle A = ?$

(a) $20°$ (b) $30°$
(c) $30°$ (d) $50°$

Answer: (a) $20°$

Sol:
We have,
Sec $4A = cosec(A - 10^0)$
$\Rightarrow cosec\ (90^0 - 4A) = cosec(A - 10^0)$
$Comparing\ both\ sides, we\ get$
$90^0 - 4A = A - 10^0$
$\Rightarrow 4A + A = 90^0 + 10^0$
$\Rightarrow 5A = 100^0$
$\Rightarrow A = \frac{100^0}{5}$
$\therefore A = 20^0$

11. If A and B are acute angles such that sin A = cos B then (A +B) =?

(a) $45°$ (b) $60°$

(c) $90°$ (d) $180°$

Answer: (c) $90°$

12. If $\cos(\alpha+\beta)=0$ then $\sin(\alpha-\beta)=?$

(a) $\sin\alpha$ (b) $\cos\beta$

(c) $\sin 2\alpha$ (d) $\cos 2\beta$

Answer: (d) $\cos 2\beta$

Sol:

We have:

$\cos(\alpha+\beta)=0$

=> $\cos(\alpha+\beta)=\cos 90^0$

=> $\alpha+\beta=90^0$

=> $\alpha=90^0-\beta$ ... (i)

Now, $\sin(\alpha-\beta)$

= $\sin[(90^0-\beta)-\beta]$ $[Using\ (i)]$

= $\sin(90^0-2\beta)$

= $\cos 2\beta$ $[\because \sin(90^0-\theta)=\cos\theta]$

13. $\sin(45°+\theta)-\cos(45°-\theta)=?$

(a) $2\sin\theta$ (b) $2\cos\theta$

(c) 0 (d) 1

Answer: (c) 0

Sol:

We have:

$[\sin(45^0+\theta)-\cos(45^0-\theta)]$

=$[\sin\{90^0-(45^0-\theta)\}-\cos(45^0-\theta)]$

=$[\cos(45^0-\theta)-\cos(45^0-\theta)]$ $[\because \sin(90^0-\theta)=\cos\theta]$

= 0

14. $\sec^2 10°-\cot^2 80°=?$

(a) 1 (b) 0

(c) $\frac{3}{2}$ (d) $\frac{1}{2}$

Answer: (a) 1

Sol:

We have: $(\sin 79^0 \cos 11^0+\cos 79^0 \sin 11^0)$

$= \sin 79^0 \cos(90^0-79^0)+\cos 79^0 \sin(90^0-79^0)$

$= \sin 79^0 \sin 79^0 + \cos 79^0 \cos 79^0 [\because \cos(90^0 - \theta) = \sin\theta \; and \; \sin(90^0 - \theta) = \cos\theta]$

$= \sin^2 79^0 + \cos^2 79^0$

$= 1$

15. $cosec^2 57° - \tan^2 33° = ?$

(a)0 (b) 1

(c) -1 (d) 2

Answer: (b) 1

Sol:

We have:

$\left(\cos ec^2 57° - \tan^2 33°\right)$

$= [cosec^2(90^0 - 33^0) - \tan^2 33^0]$

$= (\sec^2 33^0 - \tan^2 33^0) \quad [\because \cos ec(90^0 - \theta) = \sec\theta]$

$= 1 \quad [\because \sec^2\theta - \tan^2\theta = 1]$

16. $\dfrac{2\tan^2 30° \sec^2 52° \sin^2 38°}{\cos ec^2 70° - \tan^2 20°} = ?$

(a) 2 (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Answer: (c) $\frac{2}{3}$

Sol:

We have:

$\left[\dfrac{2\tan^2 30^0 \sec^2 52^0 \sin^2 38^0}{\cos ec^2 70^0 - \tan^2 20^0}\right]$

$= \left[\dfrac{2\times\left(\frac{1}{\sqrt{3}}\right)^2 \sec^2 52^0 \{\sin^2(90^0 - 52^0)\}}{\{\cos ec^2(90^0 - 20^0)\} - \tan^2 20^0}\right]$

$= \left[\dfrac{2}{3} \times \dfrac{\sec^2 52^0 . \cos^2 52^0}{\sec^2 20^0 - \tan^2 20^0}\right] \quad [\because \sin(90^0 - \theta) = \cos\theta \; and \; cosec(90^0 - \theta) = \sec\theta]$

$= \frac{2}{3} \times \frac{1}{1} \quad [\because \sec^2\theta - \tan^2\theta = 1]$

$= \frac{2}{3}$

17. $\left\{\dfrac{\left(\sin^2 22° + \sin^2 68°\right)}{\left(\cos^2 22° + \cos^2 68°\right)} + \sin^2 63° + \cos 63° \sin 27°\right\} = ?$

(a) 0 (b) 1

(c) 2 (d)3

Answer: (c) 2

Sol:

We have:

$\left[\frac{\text{Sin}^2 22^0+\sin^2 68^0}{\cos^2 22^0+\cos^2 68}+\sin^2 63^0+\cos 63^0 \sin 27^0\right]$

$=\left[\frac{\sin^2 22^0+\sin^2(90^0-22^0)}{\cos^2(90^0-68^0)+\cos^2 68^0}+\sin^2 63^0+\cos 63^0\{\sin(90^0-63^0)\}\right]$

$=\left[\frac{\sin^2 22^0+\cos^2 22^0}{\sin^2 68^0+\cos^2 68^0}+\sin^2 63^0+\cos 63^0 \cos 63^0\right]$ $[\because \sin(90^0-\theta))$

$=\cos\theta \; and \cos(90^0-\theta)=\sin\theta]$

$=\left[\frac{1}{1}+\sin^2 63^0+\cos^2 63^0\right]$ $[\because \sin^2\theta+\cos^2\theta=1]$

$=1+1=2$

18. $\frac{\cot(90°-\theta).\sin(90°-\theta)}{\sin\theta}+\frac{\cot 40°}{\tan 50°}-\left(\cos^2 20°+\cos^2 70°\right)=?$

(a) 0 (b)1

(c)-1 (d)none of these

Answer: (b)1

Sol:

We have:

$\left[\frac{\cot(90^0-\theta).\sin(90^0-\theta)}{\sin\theta}+\frac{\cot 40^0}{\tan 50^0}-(\cos^2 20^0+\cos^2 70^0)\right]$

$=[\frac{\tan\theta.\cos\theta}{\sin\theta}+\frac{\cot(90^0-50^0)}{\tan 50^0}-\{\cos^2(90^0-70^0)+\cos^2 70^0\}]$ $[\because \cot(90^0-\theta)=$ $\tan\theta \; and \sin(90^0-\theta)=\cos\theta]$

$=[\frac{\sin\theta}{\sin\theta}+\frac{\tan 50^0}{\tan 50^0}-(\sin^2 70^0+\cos^2 70^0)]$ $[\because \cos(90^0-\theta)=\sin\theta]$

$=\left(\frac{\sin\theta}{\sin\theta}+1-1\right)$

$=1+1-1=1$

19. $\frac{\cos 38°\cos ec52°}{\tan 18°\tan 35°\tan 60°\tan 72°\tan 55°}=?$

(a) $\sqrt{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

Answer: (c) $\frac{1}{\sqrt{3}}$

Sol:
We have:

$$\left[\frac{\text{Cos}\,38^0 \cos ec52^0}{\tan 18^0 \tan 35^0 \tan 60^0 \tan 72^0 \tan 55^0}\right]$$

$$=\left[\frac{\cos 38^0 \cos ec(90^0-38^0)}{\tan 18^0 \tan 35^0\times\sqrt{3}\times \tan(90^0-18^0)\tan(90^0-35^0)}\right]$$ $[\because \cos ec(90^0-\theta)=\sec\theta \ and \ \tan(90^0-\theta)=\cot\theta]$

$$=\left[\frac{\cos 38^0 \sec 38^0}{\tan 18^0 \tan 35^0\times\sqrt{3}\times \cot 18^0 \cot 35^0}\right]$$

$$=\left[\frac{\frac{1}{\sec 38^0}\times \sec 38^0}{\frac{1}{\cot 18^0}\frac{1}{\cot 35^0}\times\sqrt{3}\cot 18^0 \cot 35^0}\right]$$

$$=\frac{1}{\sqrt{3}}$$

20. If $2\sin 2\theta=\sqrt{3}$ then $\theta=?$

(a) $30°$ (b) $45°$
(c) $60°$ (d) $90°$

Answer: (a) $30°$

Sol:

$2\sin 2\theta=\sqrt{3}$

$\Rightarrow \sin 2\theta=\frac{\sqrt{3}}{2}=\sin 60^0$

$\Rightarrow \sin 2\theta=\sin 60^0$

$\Rightarrow 2\theta=60^0$

$\Rightarrow \theta=30^0$

21. If $2\cos 3\theta=1$ then $\theta=?$

(a) $10°$ (b) $15°$
(c) $20°$ (d) $30°$

Answer: (c) $20°$

Sol:

$2\cos 3\theta=1$

$\Rightarrow \cos 3\theta=\frac{1}{2}$

$\Rightarrow \cos 3\theta=\cos 60^0$ $\left[\because \cos 60^0=\frac{1}{2}\right]$

$\Rightarrow 3\theta=60^0$

$\Rightarrow \theta=\frac{60^0}{3}=20^0$

22. If $\sqrt{3}\tan 2\theta-3=0$ then $\theta=?$

(a) $15°$ (b) $30°$

(c) $45°$ (d) $60°$

Answer: (b) $30°$

Sol:

$\sqrt{3}\tan 2\theta - 3 = 0$

$\Rightarrow \sqrt{3}\tan 2\theta = 3$

$\Rightarrow \tan 2\theta = \frac{3}{\sqrt{3}}$

$\Rightarrow \tan 2\theta = \sqrt{3}$ $[\because \tan 60^0 = \sqrt{3}]$

$\Rightarrow \tan 2\theta = \tan 60^0$

$\Rightarrow 2\theta = 60^0$

$\Rightarrow \theta = 30^0$

23. If $\tan x = 3\cot x$ then x=?

(a) $45°$ (b) $60°$

(c) $30°$ (d) $15°$

Answer: (b) $60°$

Sol:

$\text{Tan}\, x = 3\cot x$

$\Rightarrow \frac{\tan x}{\cot x} = 3$

$\Rightarrow \tan^2 x = 3$ $\left[\because \cot x = \frac{1}{\tan x}\right]$

$\Rightarrow \tan x = \sqrt{3} = \tan 60^0$

$\Rightarrow x = 60^0$

24. If $x\tan 45°\cos 60° = \sin 60°$ then x=?

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{3}$

Answer: (a) 1

Sol:

$x\tan 45^0 \cos 60^0 = \sin 60^0 \cot 60^0$

$\Rightarrow x\,(1)\left(\frac{1}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow x\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)$

$\Rightarrow x = 1$

25. If $t\text{an}^2 45° - \cos^2 30° = x\sin 45°\cos 45°$ then x=?

(a) 2 (b) -2

(c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

Answer: (c) $\frac{1}{2}$

Sol:

$(\tan^2 45^0 - \cos^2 30^0) = x \sin 45^0 \cos 45^0$

$\Rightarrow x = \frac{(\tan^2 45^0 - \cos^2 30^0)}{\sin 45^0 \cos 45^0}$

$= \frac{\left[(1)^2 - \left(\frac{\sqrt{3}}{2}\right)^2\right]}{\left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right)}$

$= \frac{\left(1 - \frac{3}{4}\right)}{\left(\frac{1}{2}\right)}$

$= \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}$

$= \frac{1}{4} \times 2 = \frac{1}{2}$

26. $\sec^2 60° - 1 = ?$

(a) 2 (b) 3

(c) 4 (d) 0

Answer: (b) 3

Sol:

$\text{Sec}^2 60^0 - 1$

$= (2)^2 - 1$

$= 4 - 1$

$= 3$

27. $(\cos 0° + \sin 30° + \sin 45°)(\sin 90° + \cos 60° - \cos 45°) = ?$

(a) $\frac{5}{6}$ (b) $\frac{5}{8}$

(c) $\frac{3}{5}$ (d) $\frac{7}{4}$

Answer: (d) $\frac{7}{4}$

Sol:

$(\cos 0^0 + \sin 30^0 + \sin 45^0)(\sin 90^0 + \cos 60^0 - \cos 45^0)$

$= \left(1 + \frac{1}{2} + \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{2} - \frac{1}{\sqrt{2}}\right)$

$= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)$

$= \left[\left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right] = \left(\frac{9}{4}\right) - \left(\frac{1}{2}\right) = \left(\frac{9-2}{4}\right) = \frac{7}{4}$

28. $\sin^2 30° + 4\cot^2 45° - \sec^2 60° = ?$

(a) 0 (b) $\frac{1}{4}$

(c) 4 (d) 1

Answer: (b) $\frac{1}{4}$

Sol:

$(\text{Sin}^2 30^0 + 4\cot^2 45^0 - \sec^2 60^0)$

$= \left[\left(\frac{1}{2}\right)^2 + 4\times(1)^2 - (2)^2\right]$

$= \left(\frac{1}{4} + 4 - 4\right) = \frac{1}{4}$

29. $3\cos^2 60° + 2\cot^2 30° - 5\sin^2 45° = ?$

(a) $\frac{13}{6}$ (b) $\frac{17}{4}$

(c) 1 (d) 4

Answer: (b) $\frac{17}{4}$

Sol:

$(3\cos^2 60^0 + 2\cot^2 30^0 - 5\sin^2 45^0)$

$= \left[3\times\left(\frac{1}{2}\right)^2 + 2\times\left(\sqrt{3}\right)^2 - 5\times\left(\frac{1}{\sqrt{2}}\right)^2\right]$

$= \left[\frac{3}{4} + 6 - \frac{5}{2}\right]$

$= \frac{3+24-10}{4} = \frac{17}{4}$

30. $\cos^2 30° \cos^2 45° + 4\sec^2 60° + \frac{1}{2}\cos^2 90° - 2\tan^2 60° = ?$

(a) $\frac{73}{8}$ (b) $\frac{75}{8}$

(c) $\frac{81}{8}$ (d) $\frac{83}{8}$

Answer: (d) $\frac{83}{8}$

Sol:

$(\cos^2 30^0 \cos^2 45^0 + 4\sec^2 60^0 + \frac{1}{2}\cos^2 90^0 - 2\tan^2 60^0)$

$= \left[\left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\times(2)^2 + \frac{1}{2}\times(0)^2 - 2\times\left(\sqrt{3}\right)^2\right]$

$= \left[\left(\frac{3}{4}\times\frac{1}{2}\right)+16-6\right]$

$= \left[\frac{3}{8}+10\right]$

$= \frac{3+80}{8} = \frac{83}{8}$

31. If $\cos ec\theta = \sqrt{10}$ then $s\,ec\theta = ?$

(a) $\frac{3}{\sqrt{10}}$ (b) $\frac{\sqrt{10}}{3}$

(c) $\frac{1}{\sqrt{10}}$ (d) $\frac{2}{\sqrt{10}}$

Answer: (b) $\frac{\sqrt{10}}{3}$

Sol:

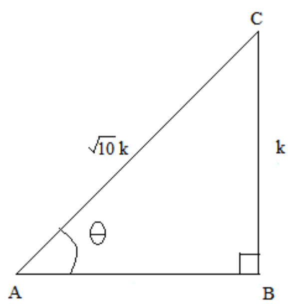
Let us first draw a right ΔABC right angled at B and $\angle A = \theta$.

Given: cosec $\theta = \sqrt{10}, but \sin\theta = \frac{1}{\cos eco} = \frac{1}{\sqrt{10}}$

Also, $\sin\theta = \frac{Perpendicular}{Hypotenuse} = \frac{BC}{AC}$

So, $\frac{BC}{AC} = \frac{1}{\sqrt{10}}$

Thus, BC = k and AC = $\sqrt{10}k$



Using Pythagoras theorem in triangle ABC, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2$

$\Rightarrow AB^2 = \left(\sqrt{10}k\right)^2 - (k)^2$

$\Rightarrow AB^2 = 9k^2$

$\Rightarrow AB = 3k$

$\therefore \sec\theta = \frac{AC}{AB} = \frac{\sqrt{10}k}{3k} = \frac{\sqrt{10}}{3}$

32. If $\tan\theta = \frac{8}{15}$ then $\cos ec\theta = ?$

(a) $\frac{17}{8}$ (b) $\frac{8}{17}$

(c) $\frac{17}{15}$ (d) $\frac{15}{17}$

Answer: (a) $\frac{17}{8}$

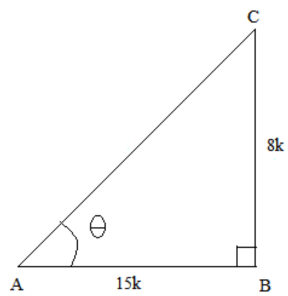
Sol:

Let us first draw a right ΔABC right angled at B and $\angle A = \theta$.

Give: $\tan\theta = \frac{8}{5}, but \tan\theta = \frac{BC}{AB}$

So, $\frac{BC}{AB} = \frac{8}{15}$

Thus, BC = 8k and AB = 15k



Using Pythagoras theorem in triangle ABC, we have:

$AC^2 = AB^2 + BC^2$

$\Longrightarrow AC^2 = (15k)^2 + (8k)^2$

$\Longrightarrow AC^2 = 289k^2$

$\Longrightarrow AC = 17k$

$\therefore cosec\ \theta = \frac{AC}{BC} = \frac{17k}{8k} = \frac{17}{8}$

33. If $\sin\theta = \frac{b}{a}$ then $\cos\theta = ?$

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{\sqrt{b^2 - a^2}}{b}$

(c) $\frac{a}{\sqrt{b^2 - a^2}}$ (d) $\frac{b}{a}$

Answer: (b) $\frac{\sqrt{b^2 - a^2}}{b}$

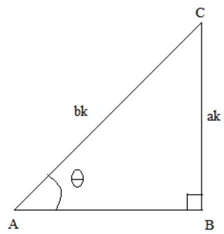
Sol:

Let us first draw a right ΔABC right angled at B and $\angle A = \theta$.

Given: $\sin\theta = \frac{a}{b}, but \sin\theta = \frac{BC}{AC}$

So, $\frac{BC}{AC} = \frac{a}{b}$

Thus, BC = ak and AC =bk



Using Pythagoras theorem in triangle ABC, we have:

$AC^2 = AB^2 + BC^2$

$\Longrightarrow AB^2 = AC^2 - BC^2$

$\Longrightarrow AB^2 = (bk)^2 - (ak)^2$

$\Longrightarrow AB^2 = (b^2 - a^2)k^2$

$\Longrightarrow AB = (\sqrt{b^2 - a^2)k}$

$\therefore \cos\theta = \frac{AB}{AC} = \frac{\sqrt{b^2-a^2k}}{bk} = \frac{\sqrt{b^2-a^2}}{b}$

34. If $\tan\theta = \sqrt{3}$ then $\sec\theta = ?$

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) 2

Answer: (d) 2

Sol:

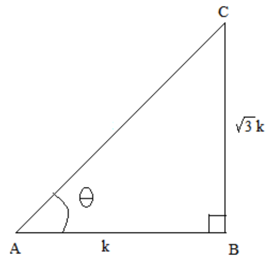
Let us first draw a right ΔABC right angled at B and $\angle A = \theta$.

Give: $\tan\theta = \sqrt{3}$

But $\tan\theta = \frac{BC}{AB}$

So, $\frac{BC}{AB} = \frac{\sqrt{3}}{1}$

Thus, BC = $\sqrt{3}k$ and $AB = k$



Using Pythagoras theorem, we get:
$AC^2 = AB^2 + BC^2$
$\Longrightarrow AC^2 = \left(\sqrt{3}k\right)^2 + (k)^2$
$\Longrightarrow AC^2 = 4k^2$
$\Longrightarrow AC = 2k$
$\therefore \sec\theta = \frac{AC}{AB} = \frac{2k}{k} = \frac{2}{1}$

35. If $\sec\theta = \frac{25}{7}$ then $\sin\theta = ?$

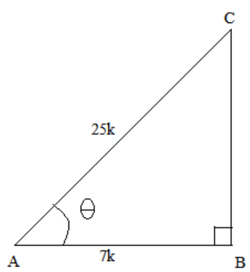
(a) $\frac{7}{24}$ (b) $\frac{24}{7}$

(c) $\frac{24}{25}$ (d) none of these

Answer: (c) $\frac{24}{25}$

Sol:

Let us first draw a right ΔABC right angled at B and $\angle A = \theta$.
Given $\sec\theta = \frac{25}{7}$
But $\cos\theta = \frac{1}{sec0} = \frac{AB}{AC} = \frac{7}{25}$
Thus, $AC = 25k \ and \ AB = 7k$



Using Pythagoras theorem, we get:
$AC^2 = AB^2 + BC^2$
$\Longrightarrow BC^2 = AC^2 - AB^2$
$\Longrightarrow BC^2 = (25k)^2 - (7k)^2$
$\Longrightarrow BC^2 = 576k^2$

$\Rightarrow BC = 24k$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{24k}{25k} = \frac{24}{25}$

36. If $\sin\theta = \frac{1}{2}$ then $\cot\theta = ?$

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

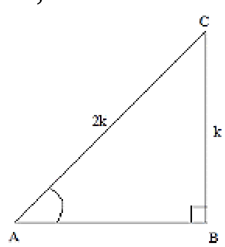
Answer: (b) $\sqrt{3}$

Sol:

$Given: \sin\theta = \frac{1}{2}, but \sin\theta = \frac{BC}{AC}$

$So, \frac{BC}{AC} = \frac{1}{2}$

Thus, BC = k and AC = 2k



Using Pythagoras theorem in triangle ABC, we have:

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2$

$AB^2 = (2k)^2 - (k)^2$

$AB^2 = 3k^2$

AB = $\sqrt{3}k$

So, $\tan\theta = \frac{BC}{AB} = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$

$\therefore \cot\theta = \frac{1}{\tan\theta} = \sqrt{3}$

37. If $\cos\theta = \frac{4}{5}$ then $\tan\theta = ?$

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) $\frac{3}{5}$ (d) $\frac{5}{3}$

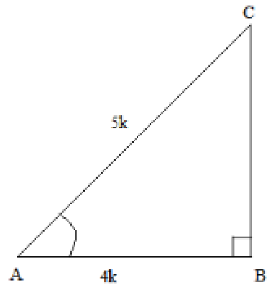
Answer: (a) $\frac{3}{4}$

Sol:

Since $\cos\theta = \frac{4}{5}$ $but \cos\theta = \frac{AB}{AC}$

So, $\frac{AB}{AC} = \frac{4}{5}$

Thus, AB = 4k and AC = 5k



Using Pythagoras theorem in triangle ABC, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

$\Rightarrow BC^2 = (5k)^2 - (4K)^2$

$\Rightarrow BC^2 = 9k^2$

$\Rightarrow$ BC=3k

$\therefore \tan\theta = \frac{BC}{AB} = \frac{3}{4}$

38. If $3x = \cos ec\theta$ and $\frac{3}{x} = \cot\theta$ then $\left(x^2 - \frac{1}{x^2}\right) = ?$

(a) $\frac{1}{27}$ (b) $\frac{1}{81}$

(c) $\frac{1}{3}$ (d) $\frac{1}{9}$

Answer: (c) $\frac{1}{3}$

Sol:

$Given: 3\times = cosec\ \theta and \frac{3}{x} = \cot\theta$

$Also, we\ can\ deduce\ that\ x = \frac{\cos ec\theta}{3}\ and \frac{1}{x} = \frac{\cot\theta}{3}$

So, substituting the values of x and $\frac{1}{x}$ $in\ the\ given\ expression, we\ get$:

$3\left(x^2 - \frac{1}{x^2}\right) = 3\left(\left(\frac{\cos ec\,\theta}{3}\right)^2 - \left(\frac{\cot\theta}{3}\right)^2\right)$

$= 3\left(\left(\frac{\cos ec^2\theta}{9}\right) - \left(\frac{\cot^2\theta}{9}\right)\right)$

$= \frac{3}{9}\ (\cos ec^2\theta - \cot^2\theta)$

$=\frac{1}{3}$ $\quad [By\ using\ the\ identity: (\cos ec^2\theta - \cot^2\theta = 1)]$

39. If $2x = \sec A\, and\, \frac{2}{x} = \tan A$ then $2\left(x^2 - \frac{1}{x^2}\right) = ?$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

Answer: (a) $\frac{1}{2}$

Sol:

$Given: 2x = \sec A\ and\ \frac{2}{x} = \tan A$

$Also, we\ can\ deduce\ that\ x = \frac{\sec A}{2}\ and\ \frac{1}{x} = \frac{\tan A}{2}$

So, substituting the values of x and $\frac{1}{x}$ $in\ the\ given\ expression, we\ get$:

$2\left(x^2 - \frac{1}{x^2}\right) = 2\left(\left(\frac{\sec A}{2}\right)^2 - \left(\frac{\tan A}{2}\right)^2\right)$

$= 2\left(\left(\frac{\sec^2 A}{4}\right) - \left(\frac{\tan^2 A}{4}\right)\right)$

$= \frac{2}{4}\ (\sec^2 A - \tan^2 A)$

$= \frac{1}{2}$ $\quad [By\ using\ the\ identity: \ (\sec^2\theta - \tan^2\theta = 1)\ ]$

40. If $\tan\theta = \frac{4}{3}$ then $(\sin\theta + \cos\theta) = ?$

(a) $\frac{7}{3}$ (b) $\frac{7}{4}$

(c) $\frac{7}{5}$ (d) $\frac{5}{7}$

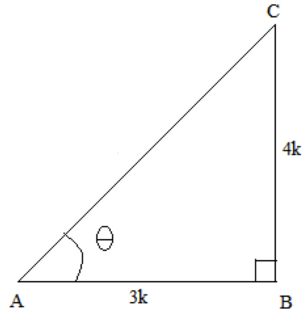
Answer: (c) $\frac{7}{5}$

Sol:

Let us first draw a right ΔABC right angled at B and $\angle A = \theta$.

$\text{Tan}\,\theta = \frac{4}{3} = \frac{BC}{AB}$

So, AB = 3k and BC = 4k



Using Pythagoras theorem, we get:

$AC^2 = AB^2 + BC^2$

$\Longrightarrow AC^2 = (3k)^2 + (4k)^2$

$\Longrightarrow AC^2 = 25k^2$

$\Longrightarrow AC = 5k$

Thus, $\sin\theta = \frac{BC}{AC} = \frac{4}{5}$

And $\cos\theta = \frac{AB}{AC} = \frac{3}{5}$

$\therefore (\sin\theta + \cos\theta) = \left(\frac{4}{5} + \frac{3}{5}\right) = \frac{7}{5}$

41. If $\left(\tan\theta + \cot\theta\right) = 5$ then $\left(\tan^2\theta + \cot^2\theta\right) = ?$

(a) 27 (b) 25

(c) 24 (d) 23

Answer: (d) 23

Sol:

We have $(\tan\theta + \cot\theta) = 5$

Squaring both sides, we get:

$(\text{Tan}\,\theta + \cot\theta)^2 = 5^2$

$\Longrightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 25$

$\Longrightarrow \tan^2\theta + \cot^2\theta + 2 = 25 \quad \left[\because \tan\theta = \frac{1}{\cot\theta}\right]$

$\Longrightarrow \tan^2\theta + \cot^2\theta = 25 - 2 = 23$

42. If $\left(\cos\theta + \sec\theta\right) = \frac{5}{2}$ then $\left(\cos^2\theta + \sec^2\theta\right) = ?$

(a) $\frac{21}{4}$ (b) $\frac{17}{4}$

(c) $\frac{29}{4}$ (d) $\frac{33}{4}$

Answer: (b) $\frac{17}{4}$

Sol:

We have $(\cos\theta + \sec\theta) = \frac{5}{2}$

Squaring both sides, we get:

$(\text{Cos}\,\theta + \sec\theta)^2 = \left(\frac{5}{2}\right)^2$

$\Longrightarrow \cos^2\theta + \sec^2\theta + 2\theta = \frac{25}{4}$

$\Longrightarrow \cos^2\theta + \sec^2\theta + 2 = \frac{25}{4} \quad \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$

$\Longrightarrow \cos^2\theta + \sec^2\theta = \frac{25}{4} - 2 = \frac{17}{4}$

43. If $\tan\theta = \frac{1}{\sqrt{7}}$ then $\frac{\left(\cos ec^2\theta + \sec^2\theta\right)}{\left(\cos ec^2\theta + \sec^2\theta\right)} = ?$

(a) $\frac{-2}{3}$ (b) $\frac{-3}{4}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Answer: (d) $\frac{3}{4}$

Sol:

$= \frac{cosec^2\theta - \sec^2\theta}{cosec^2\theta + \sec^2\theta}$

$= \frac{\sin^2\theta\left(\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}\right)}{\sin^2\theta\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)} \quad [Multipying\ the\ numerator\ and\ denominator\ by \sin^2\theta]$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{1}{7}}{1+\frac{1}{7}} = \frac{6}{8} = \frac{3}{4}$

44. If $7\tan\theta = 4$ then $\frac{(7\sin\theta - 3\cos\theta)}{(7\sin\theta + 3\cos\theta)} = ?$

(a) $\frac{1}{7}$ (b) $\frac{5}{7}$

(c) $\frac{3}{7}$ (d) $\frac{5}{14}$

Answer: (a) $\frac{1}{7}$

Sol:

$7\tan\theta = 4$

$Now, dividing\ the\ numerator\ and\ denomintor\ of\ the\ given\ expression\ by \cos\theta,$

We get:

$\frac{\frac{1}{\cos\theta}(7\sin\theta - 3\cos\theta)}{\frac{1}{\cos\theta}(7\sin\theta + 3\cos\theta)}$

$= \frac{7\tan\theta - 3}{7\tan\theta + 3}$

$= \frac{4-3}{4+3} \quad [\because 7\tan\theta = 4]$

$= \frac{1}{7}$

45. If $3\cot\theta = 4$ then $\frac{(5\sin\theta + 3\cos\theta)}{(5\sin\theta - 3\cos\theta)} = ?$

(a) $\frac{1}{3}$ (b) 3

(c) $\frac{1}{9}$ (d) 9

Answer: (d) 9

Sol:

$We\ have\ \frac{(5\sin\theta + 3\cos\theta)}{(5\sin\theta - 3\cos\theta)}$

$Dividing\ the\ numerator\ and\ denominator\ of\ the\ given\ expression\ by \sin\theta,\ we\ get:$

$\frac{\frac{1}{\sin\theta}(5\sin\theta + 3\cos\theta)}{\frac{1}{\sin\theta}(5\sin\theta - 3\cos\theta)}$

$= \frac{5+3\cot\theta}{5-3\cot\theta}$

$= \frac{5+4}{5-4} = 9 \quad [\because 3\cot\theta = 4]$

46. If $\tan\theta = \frac{a}{b}$ then $\frac{(a\sin\theta - b\cos\theta)}{(a\sin\theta + b\cos\theta)} = ?$

(a) $\frac{(a^2+b^2)}{(a^2-b^2)}$ (b) $\frac{(a^2-b^2)}{(a^2+b^2)}$

(c) $\frac{a^2}{(a^2+b^2)}$ (d) $\frac{a^2}{(a^2+b^2)}$

Answer: (b) $\frac{(a^2-b^2)}{(a^2+b^2)}$

Sol:

$We\ have \tan\theta = \frac{a}{b}$

$Now, dividing\ the\ numerator\ and\ denominator\ of\ the\ given\ expression\ by \cos\theta$

We get:

$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

$= \frac{\frac{1}{\cos\theta}(a\sin\theta - b\cos\theta)}{\frac{1}{\cos\theta}(a\sin\theta + b\cos\theta)} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{\frac{a^2}{b} - b}{\frac{a^2}{b} + b} = \frac{a^2 - b^2}{a^2 + b^2}$

47. If $\sin A + \sin^2 A = 1$ then $\cos^2 A + \cos^4 A = ?$

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) 3

Answer: (b) 1

Sol:

$\text{Sin}\, A + \sin^2 A = 1$

$=> \sin A = 1 - \sin^2 A$

$=> \sin A = \cos^2 A \quad (\because 1 - \sin^2 A)$

$=> \sin^2 A = \cos^4 A \quad (Squaring\ both\ sides)$

$=> 1 - \cos^2 A = \cos^4 A$

$=> \cos^4 A + \cos^2 A = 1$

48. If $\cos A + \cos^2 A = 1$ then $\sin^2 A + \sin^4 A = ?$

(a) 1 (b) 2

(c) 4 (d) 3

Answer: (a) 1

Sol:

$\cos A + \cos^2 A = 1$

$=> \cos A = 1 - \cos^2 A$

$=> \cos A = \sin^2 A \quad (\because 1 - \cos^2 A = sin^2)$

$=> \cos^2 A = \sin^4 A \quad (Aquaring\ both\ sides)$

$=> 1 - \sin^2 A = \sin^4 A$

$=> \sin^4 A + \sin^2 A = 1$

49. $\frac{\sqrt{1-\sin A}}{\sqrt{1+\sin A}} = ?$

(a) $\sec A + \tan A$ (b) $s\text{ec}\, A - \tan A$

(c) $\sec A \tan A$ (d) none of these

Answer: (b) $s\text{ec}\, A - \tan A$

Sol:

$\sqrt{\frac{1-\sin A}{1+\sin A}}$

$=\sqrt{\frac{(1-\sin A)}{(1+\sin A)}\times\frac{(1-\sin A)}{(1-\sin A)}}$ $[Multiplying\ the\ denominator\ and\ numerator\ by\ (1-\sin A)]$

$=\frac{(1-\sin A)}{\sqrt{1-\sin^2 A}}$

$=\frac{(1+\sin A)}{\sqrt{\cos^2 A}}$

$=\frac{(1-\sin A)}{\cos A}$

$=\frac{1}{\cos A}-\frac{\sin A}{\cos A}$

$=\sec A-\tan A$

50. $\frac{\sqrt{1+\cos A}}{\sqrt{1-\cos A}}=?$

(a) $\cos\sec A-\cot A$ (b) $\cos\sec A+\cot A$

(c) $\operatorname{cosec} A\cot A$ (d) none of these

Answer: (b) $\cos\sec A+\cot A$

Sol:

$\sqrt{\frac{1-\cos A}{1+\cos A}}$

$=\sqrt{\frac{(1-\cos A)}{(1+\cos A)}\times\frac{(1-\cos A)}{(1-\cos A)}}$ $[Multiplying\ the\ numerator\ and\ denominator\ by\ (1-\cos A)]$

$=\sqrt{\frac{(1-\cos A)(1-\cos A)}{1-\cos^2 A}}$

$=\frac{1-\cos A}{\sqrt{\sin^2 A}}$

$=\frac{1-\cos A}{\sin A}$

$=\frac{1}{\sin A}-\frac{\cos A}{\sin A}$

$=\cos ecA-\cot A$

51. If $\tan\theta=\frac{a}{b}$ then $\frac{(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)}=?$

(a) $\frac{a+b}{a-b}$ (b) $\frac{a+b}{a-b}$

(c) $\frac{b+a}{b-a}$ (d) $\frac{b-a}{b+a}$

Answer: (c) $\frac{b+a}{b-a}$

Sol:

$Given: \tan\theta=\frac{a}{b}$

$Now, \frac{(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)}$

$= \frac{(1+\tan\theta)}{(1-\tan\theta)}$ [*Dividing the numerator and denominator by* $\cos\theta$]

$= \frac{\left(1+\frac{a}{b}\right)}{\left(1-\frac{a}{b}\right)}$

$= \frac{\left(\frac{b+a}{b}\right)}{\left(\frac{b-a}{b}\right)}$

$= \frac{(b+a)}{(b-a)}$

52. $(\operatorname{cosec}\theta-\cot\theta)^2=?$

(a) $\frac{1+\cos\theta}{1-\cos\theta}$ (b) $\frac{1-\cos\theta}{1+\cos\theta}$

(c) $\frac{1+\sin\theta}{1-\sin\theta}$ (d) none of these

Answer: (b) $\frac{1-\cos\theta}{1+\cos\theta}$

Sol:

$(cosec\ \theta-\cot\theta)^2$

$= \left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$

$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{(1-\cos\theta)}{(1+\cos\theta)}$

53. $(\sec A+\tan A)(1-\sin A)=?$

(a) $\sin A$ (b) $\cos A$

(c) $\sec A$ (d) $\operatorname{cosec} A$

Answer: (b) $\cos A$

Sol:

$(\sec A+\tan A)(1-\sin A)$

$=\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$

$=\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$

$=\left(\frac{1-\sin^2 A}{\cos A}\right)$

$=\left(\frac{\cos^2 A}{\cos A}\right)$

$=\cos A$