Exercise 5.1

1. Without performing actual addition and division write the quotient when the sum of 69 and 96 is divided by:

(i) 11 **(ii)15**

Soln:

(i) Clearly, 69 and 96 are two numbers such that one can be obtained by reversing the digits of the other. Therefore, when the sum of 69 and 96 is divided by 11, we get 15 (sum of the digits) as the quotient.

(ii) Clearly, 69 and 96 are two numbers such that one can be obtained by reversing the digits of the other. Therefore, when the sum of 69 and 96 is divided by 15(sum of the digits), we get 11 as the quotient.

2.Without performing actual computations, find the quotient when 94 -49 is divided by:

(i) 9 **(ii) 5**

Soln:

(i) We know that when $\overline{ab} - \overline{ba}$ is divided by 9, the quotient is a − b. Therefore, when (94 − 49) is divided by 9, the quotient is (9 − 4 = 5).

(ii) We know that when $\overline{ab} - \overline{ba}$ is divided by (a − b), the quotient is 9. Therefore, when (94 − 49) is divided by (9 − 4 = 5), the quotient is 9

3.If sum of the number 985 and two other number obtained by arranging the digits of 985 in cyclic order is divided by 111, 22 and 37 respectively. Find the quotient in each case.

Soln:

The sum of (985+ 859 + 598) when divided by:

(i) 111 *Quotient* = (9 + 8 + 5) = 22

(ii) 22 i.e. (9 + 8 + 5) *Quotient* = 111

(iii) 37 (= $\frac{111}{3}$) *Quotient* = 3(9 + 8 + 5) = 66

4. Find the quotient when the difference of 985 and 958 is divided by 9.

Soln:

If $\overline{abc} - \overline{acb}$ is divided by 9, the quotient is (b − c).

∴ If (985 − 958) is divided by 9

Quotient = (8 − 5) = 3

Exercise 5.2

Q.1: Given that the number $\overline{35a64}$ is divisible by 3, where a is a digit, what are the possible values of a.

Soln:

It is given that $\overline{35a64}$ is a multiple of 3. ∴ (3 + 5 + a + 6 + 4) is a multiple of 3.

∴ (a + 18) is a multiple of 3. ∴(a + 18) = 0, 3, 6, 9, 12, 15, 18, 21...

But a is digit of number $\overline{35a64}$.

So, a can take value 0, 1, 2, 3, 4.... 9, a + 18 = 18

=> a = 0a + 18 = 21

=> a = 3a + 18 = 24

=> a = 6a + 18 = 27

=> a = 9 ∴ a = 0, 3, 6, 9.

Q.2: If x is a digit of the number $\overline{18 \times 17}$ is divisible by 3, find possible values of x.

Soln:

It is given that $\overline{18 \times 71}$ is a multiple of 3.

(1 + 8 + x + 7 +1) is a multiple of 3.

∴(17 + x) is a multiple of 3.

∴ 17 + x = 0, 3, 6, 9, 12, 15, 18, 21 . . . But x is a digit. So, x can take value 0, 1, 2, 3, 4 .... 9.

17+ x = 18

=>> x = 117 + x = 21

=>> x = 417 + x = 24

=>> x = 7x = 1, 4, 7

3.If x is a digit of the number $\overline{66784x}$ such that it is divisible by 9, find possible values of x.

Soln:

It is given that $\overline{66784x}$ is a multiple of 9.

∴ (6 + 6 + 7 + 8 + 4 + x) is a multiple of 9. And (31 + x) is a multiple of 9.

Possible values of (31 + x) are 0, 9, 18, 27, 36, 45...

But x is a digit, so, x can only take value 0, 1, 2, 3, 4, . . . 9.

∴ 31 + x = 36

=> x = 36 − 31

=> x = 5

4.Given that the number $\overline{67y19}$ is divisible by 9, where y is a digit, what are the possible values of y?

Soln:

it is given that $\overline{67y19}$ is a multiple of 9.

∴ (6 + 7 + y + 1 +9) is a multiple of 9.

∴ (23 + y) is a multiple of 9. 23 + y = 0, 9, 18, 27, 36 ... But x is a digit. So, x can take values 0, 1, 2, 3, 4, ... 9. 23 + y = 27 => y = 4

5. If $\overline{3 \times 2}$ is a multiple of 11, where x is a digit, what is the value of x?

Soln: Sum of the digits at odd places = 3 + 2 = 5

Sum of the digits at even places = x

∴ sum of the digits at even place − sum of the digits at odd places = (x − 5)

∵(x − 5) must be multiple by 11. ∴

Possible values of (x − 5) are 0, 11, 22, 33 ... But x is a digit:

∴ x must be 0, 1, 2, 3, .. 9 ∴ x − 5 = 0 => x = 5

6.If $\overline{98215 \times 2}$ is a number with x as its tens digit such that it is divisible by 4. Find all possible values of x

Soln: A natural number is divisible by 4 if the number formed by its digits in units and tens places in divisible by 4.

∴ $\overline{98215 \times 2}$ will be divisible by 4 if $\overline{x2}$
is divisible by 4.

∴ $\overline{x2}$= 10x + 2x is a digit; therefore possible values of x are 0, 1, 2, 3... 9.

$\overline{x2}$ = 2, 12, 22, 32, 42, 52, 62, 72, 82, 92.

The numbers that are divisible by 4 are 12, 32, 52, 72, 92. Therefore, the values of x are 1, 3, 5, 7, 9.

7. If x denotes the digit at hundreds place of the number $\overline{67 \times 19}$ such that the number is divisible by 11. Find all possible values of x

Soln: A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places is either 0 or a multiple of 11.

Sum of digits at odd places − sum of digits at even places − (6 + x + 9) − (7 + 1) = (15 + x) − 8 = x + 7

∴ x + 7 = 11 => x = 4

8.Find the remainder when 981547 is divided by 5. Do this without doing actual division.

Soln: If a natural number is divided by 5, it has the same remainder when its unit digit is divided by 5.

Here, the unit digit of 981547 is 7. When 7 is divided by 5, remainder is 2.

Therefore, remainder will be 2 when 981547 is divided by 5.

9.Find the remainder when 51439786 is divided by 3. Do this without performing actual division.

Soln:sum of the digits of the number 51439786 = 5 + 1 + 4 + 3 + 9 + 7 + 8 + 6 = 43.

The remainder of 51439786, when divided by 3, is the same as the remainder when the sum of the digits is divided by 3.

When 43 is divided by 3, the remainder is 1.

Therefore, when 51439786 is divided by 3, the remainder will be 1.

10. Find the remainder, without performing actual division, when 798 is divided by.

Soln: 798 = A multiple of 11 + (sum of its digits at odd places − sum of its digits at even places) 798

= A multiple of 11 + (7 + 8 − 9) 798

= A multiple of 11 + (15 − 9)798

= A multiple of 11 + 6

Therefore, the remainder is 6.

11.Without performing actual division, find the remainder when 928174653 is divided by 11.

Soln:

928174653 = A multiple of 11 + (Sum of its digits at odd places − sum if its digits at even places) 928174653

= A multiple of 11 + {(9 + 8 + 7 + 6 + 3) − (2 + 1 + 4 + 5)} 928174653

= A multiple of 11 + (33 − 12)928174653

= A multiple of 11 + 21928174653

= A multiple of 11 + (11 x 1 + 10)928174653

= A multiple of 11 + 10.

Therefore, the remainder is 10.

12.Given an example of a number which is divisible by:

(i) 2 but not by 4

(ii) 3 but not by 6

(iii) 4 but not by 8

(iv) both 4 and 8 but not by 32.

Soln:

(i) 10 Every number with the structure $(4n + 2)$ is an example of a number that is divisible by 2 but not by 4.

(ii) 15 Every number with the structure $(6n + 3)$ is an example of a number that is divisible by 3 but not by 6.

(iii) 28 Every number with the structure $(8n + 4)$ is an example of a number that is divisible by 4 but not by 8.

(iv) 8 Every number with the structure $(32n + 8)$, $(32n + 16)$ or $(32n + 24)$ is an example of a number that is divisible by 4 and 8 but not by 32.

13.Which of the following statements are true?

(i) If a number is divided by 3, it must be divisible by 9.

Ans: False

Every number with the structure $(9n + 3)$ or $(9n + 6)$ is divisible by 3 but not by 9.

(ii) If a number is divisible by 9, it must be divisible by 3.

Ans: True

(iii) If a number is divisible by 4, it must be divisible by 8.

Ans: False

Every number with the structure $(8n + 4)$ is divisible by 4 but not by 8.

(iv) If a number is divisible by 8, it must be divisible by 4

Ans: True

(v) A number is divisible by 18, If it is divisible by both 3 and 6.

Ans: False

(vi) If a number is divisible by both 9 and 10, it must be divisible by 90

Ans: True

(vii) If a number exactly divides the sum of two numbers, it must exactly divides the numbers separately.

Ans: False

(viii) If a number divides three numbers exactly, it must divide their sum exactly.

Ans: True

(ix) if two numbers are co-prime, at least one of them must be a prime number.

Ans: False

(x) The sum of two consecutive odd numbers is always divisible by 4

Ans: True

Exercise 5.3

Solve each of the following Cryptarithms:

Q1.

$$\begin{array}{r} 3\ \ 7 \\ +A\ \ B \\ \hline 9\ \ A \end{array}$$

Soln:

Two possible values of A are :

(i) If $7 + B \leq 9$ $3 + A = 9$

$\therefore A = 6$

But if A = 6, 7 + B must be larger than 9.

Hence, it is impossible.

(ii) If $7 + B \geq 9$

$\therefore 1 + 3 + A = 9$

=> A = 5

1f A = 5 and 7 + B = 5,

B must be 8

$\therefore$ A = 5, B = 8

Q2.

$$\begin{array}{rr} A & B \\ +3 & 7 \\ \hline 9 & A \end{array}$$

Soln:

Two possibilities of A are :

(i) If B + 7 < 9,

A = 6

But clearly, if A = 6,

B + 7 $\geq$ 9;

it is impossible

(ii) If $B + 7 \geq 9$,

$A = 5$ and $B + 7 = 5$

Clearly, $B = 8$

$\therefore A = 5, B = 8$

Q3.

$$\begin{array}{rr} A & 1 \\ +1 & B \\ \hline B & 0 \end{array}$$

Soln:If $1 + B = 0$ Surely, $B = 9$

If $1 + A + 1 = 9$ Surely, $A = 7$

Q4.

$$\begin{array}{rrr} 2 & A & B \\ +A & B & 1 \\ \hline A & B & 1 \end{array}$$

Soln:

B + 1 = 8, B = 7A + B = 1, A + 7 = 1, A = 4

So, A = 4, B = 7

Q5.

$$\begin{array}{r} 1\ \ 2\ \ A \\ +6\ \ A\ \ B \\ \hline A\ \ 0\ \ 9 \end{array}$$

Soln:

A + B = 9 as the sum of two digits can never be 192 + A = 0, A must be 8A + B = 9, 8 + B = 9, B = 1

So, A = 8, B = 1

Q6.

$$\begin{array}{r} A \quad B \quad 7 \\ +7 \quad A \quad B \\ \hline 9 \quad 8 \quad A \end{array}$$

Soln:

If A + B = 8, A + B $\geq$ 9 is possible only if A = B = 9 But from 7 + B = A, A = B = 9 is impossible. Surely, A + B = 8, A + B $\leq$ 9

So, A + 7 = 9, Surely A = 27 + B = A, 7 + B = 2, B = 5

So, A = 2, B = 5

Q7. Show that the Cryptarithm $4 \times \overline{AB} = \overline{CAB}$ does not have any solution.

Soln:

0 is the only unit digit number, which gives the same 0 at the unit digit when multiplied by 4. So, the possible value of B is 0. Similarly, for A also, 0 is the only possible digit. But then A, B and C will all be 0, and if A, B and C become 0, these numbers cannot be of two – digit or three – digit. Therefore, both will become a one – digit number. Thus, there is no solution possible.