Exercise 15.1

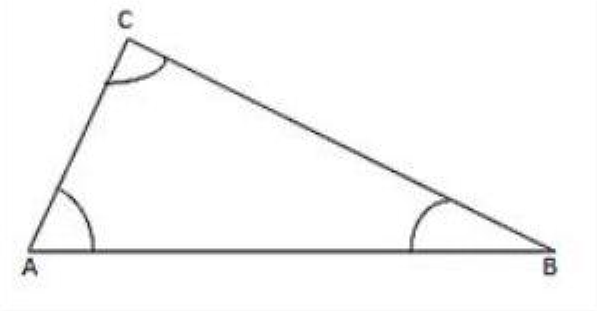
1. Take three non-collinear points A. B and C on a page of your notebook. Join AB, BC and CA. What figure do you get? Name the triangle. Also, name

(i) The side opposite to $\angle B$

(ii) The angle opposite to side AB

(iii) The vertex opposite to side BC

(iv) The side opposite to vertex B.



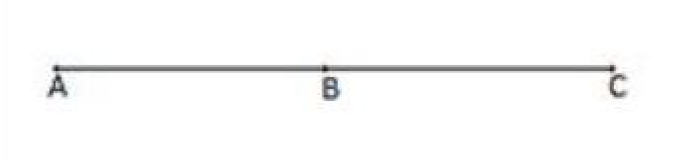
(i) AC

(ii) $\angle B$

(iii) A

(iv) AC

2. Take three collinear points A, B and Con a page of your note book. Join AB. BC and CA. Is the figure a triangle? If not, why?

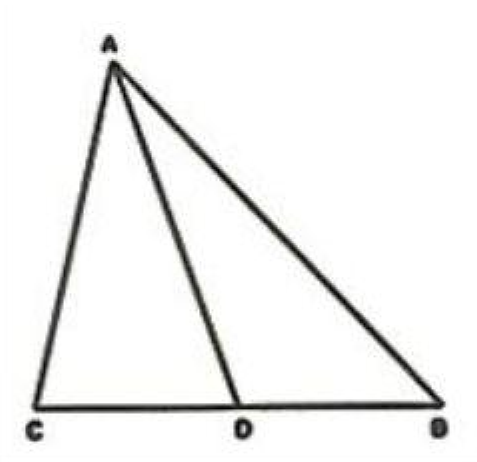


No, the figure is not a triangle. By definition a triangle is a plane figure formed by three non-parallel line segments

3. Distinguish between a triangle and its triangular region.

A triangle is a plane figure formed by three non-parallel line segments, whereas, its triangular region includes the interior of the triangle along with the triangle itself.

4. D is a point on side BC of a $\triangle$. AD is joined. Name all the triangles that you can observe in the figure. How many are they?



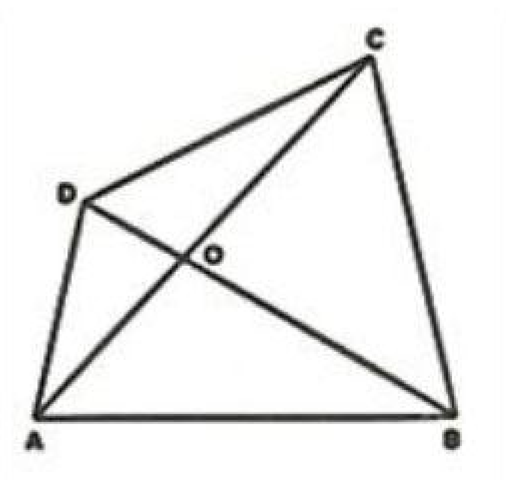
We can observe the following three triangles in the given figure

(i) $\triangle ABC$

(ii) $\triangle ACD$

(iii) $\triangle ADB$

5. A, B. C and Dare four points, and no three points are collinear. AC and BD intersed at 0. There are eight triangles that you can observe. Name all the triangles



(i) $\triangle ABC$

(ii) $\triangle ABD$

(iii) $\triangle ABO$

(iv) $\triangle BCD$

(v) $\triangle DCO$

(vi) $\triangle AOD$

(vii) $\triangle ACD$

(viii) $\triangle BCD$

6. What is the difference between a triangle and triangular region?

A plane of figure formed by three non-parallel line segments is called a triangle whereastriangular region is the interior of $/triangle ABC$ together with the $/triangle ABC$ itself is called the triangular region ABC

7. Explain the following terms:

(i) Triangle

(a) Parts or elements of a triangle

(iii) Scalene triangle

(iv) Isosceles triangle

(v) Equilateral triangle

(vi) Acute triangle

(vii) Right triangle

(viii) Obtuse triangle

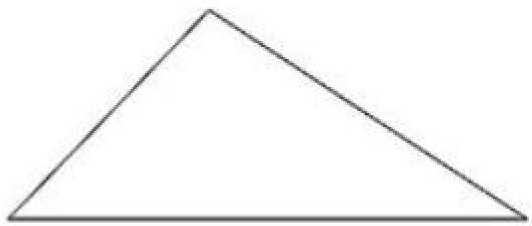
(ix) Interior of a triangle

(x) Exterior of a triangle.

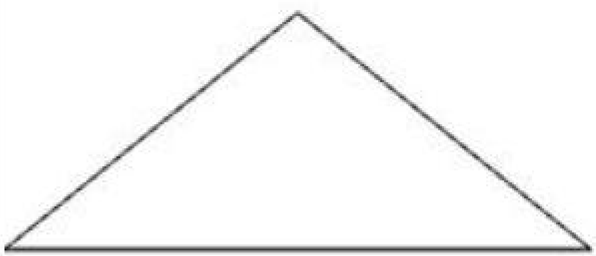
(i) A triangle is a plane figure formed by three non-parallel line segments.

(ii) The three sides and the three angles of a triangle are together known as the parts or elements of that triangle.

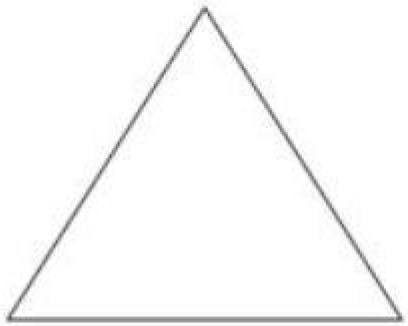
(iii) A scalene triangle is a triangle in which no two sides are equal.



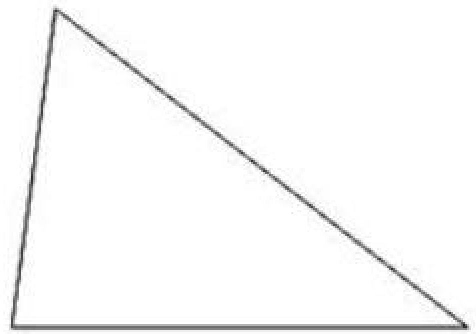
(iv) An isosceles triangle is a triangle in which two sides are equal. Isosceles triangle



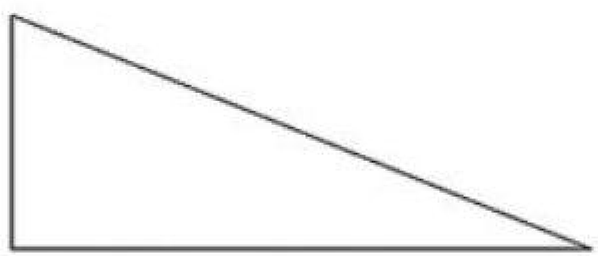
(v) An equilateral triangle is a triangle in which all three sides are equal. Equilateral triangle



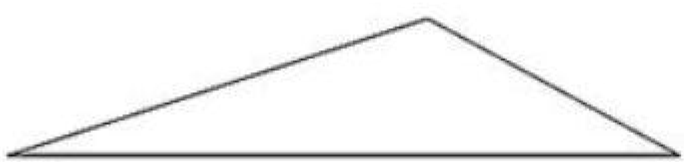
(vi) An acute triangle is a triangle in which all the angles are acute (less than 90°).



(vii) A right angled triangle is a triangle in which one angle is right angled, i.e. 90°.



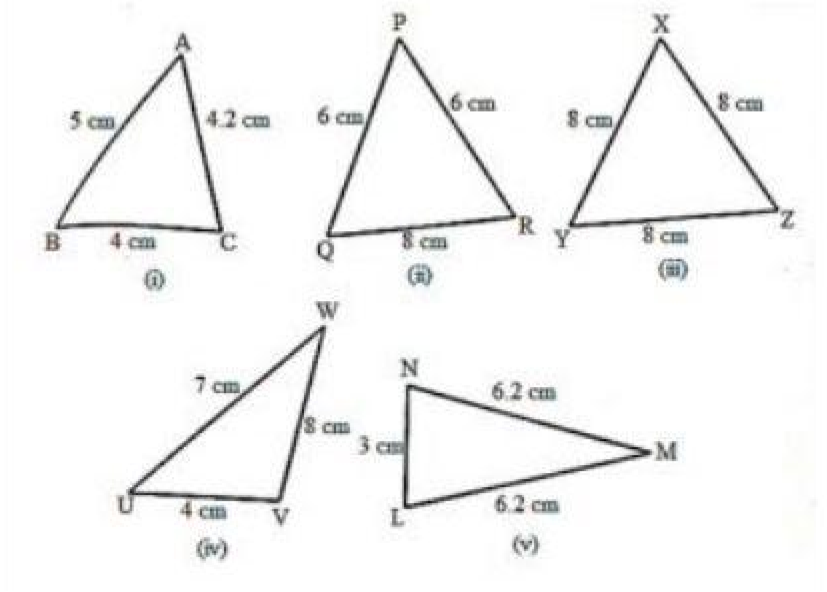
(viii) An obtuse triangle is a triangle in which one angle is obtuse (more than 90°).



(ix) The interior of a triangle is made up of all such points that are enclosed within the triangle.

(x) The exterior of a triangle is made up of all such points that are not enclosed within the triangle.

8. In Figure, the length (in cm) of each side has been indicated along the side. State for each triangle angle whether it is scalene, isosceles or equilateral:



(i) This triangle is a scalene triangle because no two sides are equal.

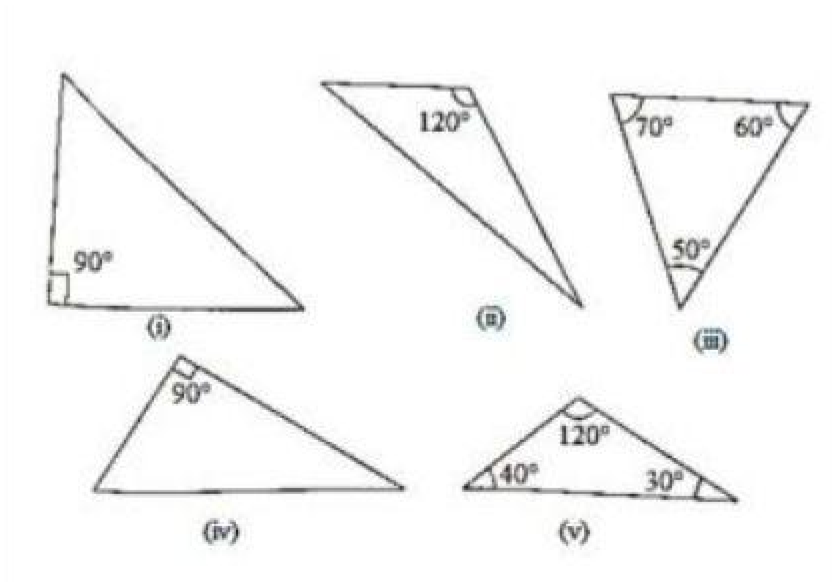
(ii) This triangle is an isosceles triangle because two of its sides, viz. PQ and PR, are equal.

(iii) This triangle is an equilateral triangle because all its three sides are equal.

(iv) This triangle is a scalene triangle because no two sides are equal.

(v) This triangle is an isosceles triangle because two of its sides are equal.

9. There are five triangles. The measures of some of their angles have been indicated. State for each triangle whether it is acute, right or obtuse.



(i) This is a right triangle because one of its angles is 90°.

(ii) This is an obtuse triangle because one of its angles is 120°, which is greater than 90°.

(iii) This is an acute triangle because all its angles are acute angles (less than 90°).

(iv) This is a right triangle because one of its angles is 90°.

(v) This is an obtuse triangle because one of its angles is 110°, which is greater than 90°.

10. Fill in the blanks with the correct word/symbol to make it a true statement:

(i) A triangle has_____ sides.

(ii) A triangle has_____ vertices.

(iii) A triangle has_____ angles.

(iv) A triangle has_______ parts.

(v) A triangle whose no two sides are equal is known as_______

(v0 A triangle whose two sides are equal is known as_______

(vii) A triangle whose all the sides are equal is known as_______

(viii) A triangle whose one angle is a right angle is known as________

(ix) A triangle whose all the angles are of measure less than 90′ is known as________

(x) A triangle whose one angle is more than 90′ is known as________

(i) three

(ii) three

(iii) three

(iv) six (three sides + three angles)

(v) a scalene triangle

(vi) an isosceles triangle

(vii) an equilateral triangle

(viii) a right triangle

(ix) an acute triangle

(x) an obtuse triangle

11. In each of the following, state if the statement is true (T) or false (F):

(i) A triangle has three sides.

(ii) A triangle may have four vertices.

(iii) Any three line-segments make up a triangle.

(iv) The interior of a triangle includes its vertices.

(v) The triangular region includes the vertices of the corresponding triangle.

(vi) The vertices of a triangle are three collinear points.

(vii) An equilateral triangle is isosceles also.

(viii) Every right triangle is scalene.

(ix) Each acute triangle is equilateral.

(x) No isosceles triangle is obtuse.

(i) True.

(ii) False. A triangle has three vertices.

(iii) False. Any three non-parallel line segments can make up a triangle.

(iv) False. The interior of a triangle is the region enclosed by the triangle and the vertices are not enclosed by the triangle.

(v) True. The triangular region includes the interior region and the triangle itself.

(vi) False. The vertices of a triangle are three non-collinear points.

(vii) True. In an equilateral triangle, any two sides are equal.

(viii) False. A right triangle can also be an isosceles triangle.

(ix) False. Each acute triangle is not an equilateral triangle, but each equilateral triangle is an acute triangle.

(x) False. An isosceles triangle can be an obtuse triangle, a right triangle or an acute triangle

Exercise 15.2

Q1. Two angles of a triangle are of measures 150° and 30°. Find the measure of the third angle.

Let the third angle be x

Sum of all the angles of a triangle=180°

105°+ 30°+x= 180°

135°+x=180°

x=$180^\circ - 135^\circ$

x=45°

Therefore the third angle is 45°

Q2. One of the angles of a triangle is 130°, and the other two angles are equal What is the measure of each of these equal angles?

Let the second and third angle be x

Sum of all the angles of a triangle=180°

130°+ x+x= 180°

130°+2x=180°

2x=$180^\circ - 130^\circ$

2x=50°

x=$\frac{50}{2}$

x=25°

Therefore the two other angles are 25° each

Q3. The three angles of a triangle are equal to one another. What is the measure of each of the angles?

Let the each angle be x

Sum of all the angles of a triangle=180°

x+x+x= 180°

3x=180°

x=$\frac{180}{3}$

x=60°

Therefore angle is 60° each

Q4. If the angles of a triangle are in the ratio 1 : 2 : 3, determine three angles.

If angles of the triangle are in the ratio 1:2:3 then take first angle as ‘x’, second angle as ‘2x’ and third angle as ‘3x’

Sum of all the angles of a triangle=180°

x+2x+3x= 180°

6x=180°

x=$\frac{180}{6}$

x=30°

2x=$30^\circ \times 2$=60°

3x=$30^\circ \times 3$=90°

Therefore the first angle is 30°, second angle is 60° and third angle is 90°

Q5. The angles of a triangle are $(x-40)^{\circ}$, $(x-20)^{\circ}$ and $\left(\frac{1}{2}-10\right)^{\circ}$. Find the value of x.

Sum of all the angles of a triangle=180°

$(x-40)^{\circ}+(x-20)^{\circ}+\left(\frac{x}{2}-10\right)^{\circ}=180^{\circ}$

$x+x+\frac{x}{2}-40^{\circ}-20^{\circ}-10^{\circ}=180^{\circ}$

$x+x+\frac{x}{2}-70^{\circ}=180^{\circ}$

$x+x+\frac{x}{2}=180^{\circ}+70^{\circ}$

$\frac{5x}{2}=250^{\circ}$

$x=\frac{2}{5}\times 250^{\circ}$

$x=100^{\circ}$

Hence we can conclude that x is equal to 100°

Q6. The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10°. Find the three angles.

Let the first angle be x

Second angle be x+10°

Third angle be x+10°+ 10°

Sum of all the angles of a triangle=180°

x+ x+10°+ x+10°+ 10°= 180°

3x+30=180

3x=180-30

3x=150

x=$\frac{150}{3}$

x=50°

First angle is 50

Second angle x+10°=50+10= 60°

Third angle x+10°+ 10°=50+10+10= 70°

Q7. Two angles of a triangle are equal and the third angle is greater than each of those angles by 30°. Determine all the angles of the triangle

Let the first and second angle be x

The third angle is greater than the first and second by 30°=x+30°

The first and the second angles are equal

Sum of all the angles of a triangle=180°

x+x+x+30°= 180°

3x+30=180

3x=180-30

3x=150

x=$\frac{150}{3}$

x=50°

Third angle=x+30°= 50°+ 30°= 80°

The first and the second angle is 50° and the third angle is 80°

Q8. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.

One angle of a triangle is equal to the sum of the other two

x=y+z

Let the measure of angles be x,y,z

x+y+z=180°

x+x=180°

2x=180°

x=$\frac{180^\circ}{2}$

x=90°

If one angle is 90° then the given triangle is a right angled triangle

Q9. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

Each angle of a triangle is less than the sum of the other two

Measure of angles be x,y and z

x>y+z

y<x+z

z<x+y

Therefore triangle is an acute triangle

Q10. In each of the following, the measures of three angles are given. State in which cases the angles can possibly be those of a triangle:

(i) $63^\circ, 37^\circ, 80^\circ$

(ii) $45^\circ, 61^\circ, 73^\circ$

(iii) $59^\circ, 72^\circ, 61^\circ$

(iv) $45^\circ, 45^\circ, 90^\circ$

(v) $30^\circ, 20^\circ, 125^\circ$

(i) $63^\circ, 37^\circ, 80^\circ = 180^\circ$

Angles form a triangle

(ii) $45^\circ, 61^\circ, 73^\circ$ is not equal to 180°

Therefore not a triangle

(iii) $59^\circ, 72^\circ, 61^\circ$ is not equal to 180°

Therefore not a triangle

(iv) $45^\circ, 45^\circ, 90^\circ = 180$

Angles form a triangle

(v) $30^\circ, 20^\circ, 125^\circ$ is not equal to 180°

Therefore not a triangle

Q11. The angles of a triangle are in the ratio 3: 4 : 5. Find the smallest angle

Given that

Angles of a triangle are in the ratio: 3: 4: 5

Measure of the angles be 3x, 4x, 5x

Sum of the angles of a triangle=180°

3x+4x+5x=180°

12x=180°

x=$\frac{180^\circ}{12}$

x=15°

Smallest angle=3x

=3 x 15°

=45°

Q12. Two acute angles of a right triangle are equal. Find the two angles.

Given acute angles of a right angled triangle are equal

Right triangle: whose one of the angle is a right angle

Measured angle be x,x,90°

x+x+180° =180°

2x=90°

x=$\frac{90^\circ}{2}$

x=45°

The two angles are 45° and 45°

Q13. One angle of a triangle is greater than the sum of the other two. What can you say about the measure of this angle? What type of a triangle is this?

Angle of a triangle is greater than the sum of the other two

Measure of the angles be x,y,z

x>y+z or

y>x+z or

z>x+y

x or y or z>90° which is obtuse

Therefore triangle is an obtuse angle

Q14. AC, AD and AE are joined. Find

$\angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA$

$\angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA$

We know that sum of the angles of a triangle is 180°

Therefore in $\triangle ABC$,we have

$\angle CAB + \angle ABC + \angle BCA = 180^{\circ}$−(i)

In $\triangle ACD$,we have

$\angle DAC + \angle ACD + \angle CDA = 180^{\circ}$−(ii)

In $\triangle ADE$,we have

$\angle EAD + \angle ADE + \angle DEA = 180^\circ$–(iii)

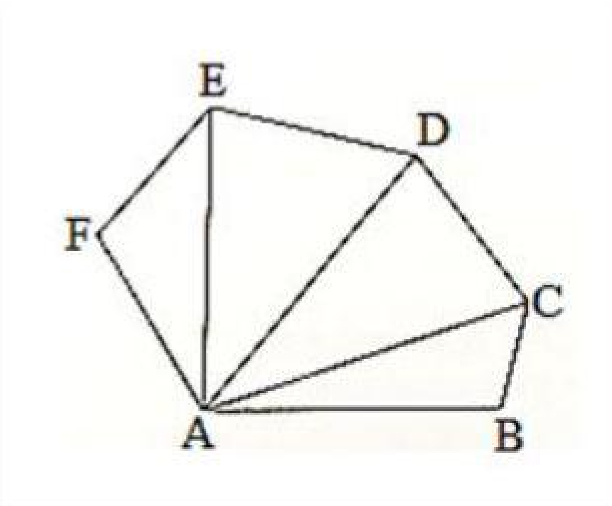
In $\triangle AEF$,we have

$\angle FAE + \angle AEF + \angle EFA = 180^\circ$–(iv)

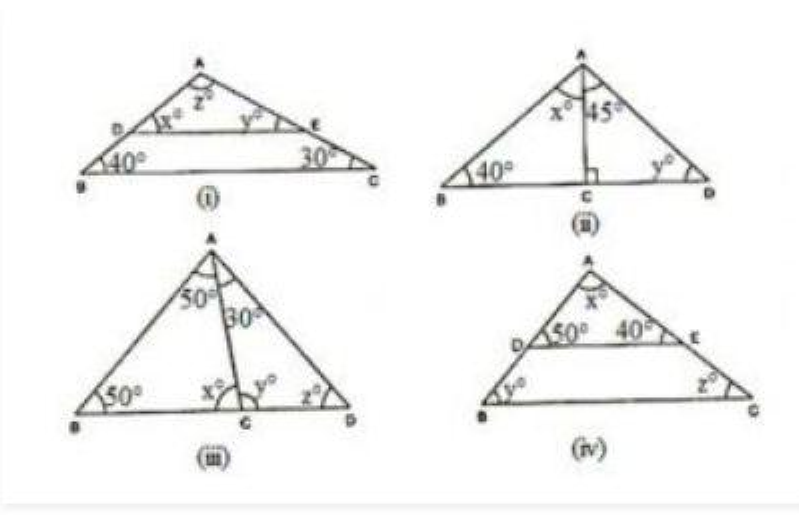
Adding (i),(ii),(iii),(iv) we get

$\angle CAB + \angle ABC + \angle BCA + \angle DAC + \angle ACD + \angle CDA + \angle EAD + \angle ADE + \angle DEA$
$+ \angle FAE + \angle AEF + \angle EFA$
= 720°

Therefore $\angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA$= 720°



Q15.Find x,y,z(whichever is required) from the figures given below



(i)

In $\triangle ABC$ and $\triangle ADE$ we have :

$\angle ADE = \angle ABC$ (corresponding angles)

x=40°

$\angle AED = \angle ACB$ (corresponding angles)

y=30°

We know that the sum of all the three angles of a triangle is equal to 180°

x +y +z = 180° (Angles of A ADE)

Which means : 40° +30° + z = 180°

z= $180^\circ - 70^\circ$

z=110°

Therefore, we can conclude that the three angles of the given triangle are 40°, 30° and 110°.

(ii) We can see that in $\triangle ADC, \angle ADC$ is equal to 90°.

($\triangle ADC$ is a right triangle)

We also know that the sum of all the angles of a triangle is equal to 180°.

Which means : 45°+ 90°+y=180° (Sum of the angles of $\triangle ADC$)

135° + y = 180°

y = 180° – 135°.

y $=45^\circ$.

We can also say that in $\triangle ABC$, $\angle ABC + \angle ACB + \angle BAC$ is equal to 180°.

(Sum of the angles of A ABC)

$40^\circ + y + (x + 45^\circ) = 180^\circ$

$40^\circ + 45^\circ + x + 45^\circ = 180^\circ$ (y = 45°)

x= $180^\circ - 130^\circ$

x= 50°

Therefore, we can say that the required angles are 45° and 50°.

(iii) We know that the sum of all the angles of a triangle is equal to 180°.

Therefore, for $\triangle ABD$:

$\angle ABD + \angle ADB + \angle BAD = 180^\circ$ (Sum of the angles of $\triangle ABD$)

$50^\circ + x + 50^\circ = 180^\circ$

$100^\circ + x = 180^\circ$

$x = 180^\circ - 100^\circ$

$x = 80^\circ$

For $\triangle ABC$:

$\angle ABC + \angle ACB + \angle BAC = 180^\circ$ (Sum of the angles of $\triangle ABC$)

$50^\circ + z + (50^\circ + 30^\circ) = 180^\circ$

$50^\circ + z + 50^\circ + 30^\circ = 180^\circ$

$z = 180^\circ - 130^\circ$

$z = 50^\circ$

Using the same argument for $\triangle ADC$:

$\angle ADC + \angle ACD + \angle DAC = 180^\circ$ (Sum of the angles of $\triangle ADC$)

y +z $+30^\circ$ $=180^\circ$

y + 50° + 30° $=180^\circ$ (z $=50^\circ$)

y = $180^\circ - 80^\circ$

y = 100°

Therefore, we can conclude that the required angles are 80°, 50° and 100°.

(iv) In $\triangle ABC$ and $\triangle ADE$ we have :

$\angle ADE = \angle ABC$ (Corresponding angles)

y = 50°

Also, $\angle AED = \angle ACB$ (Corresponding angles)

z = 40°

We know that the sum of all the three angles of a triangle is equal to 180°.

Which means : x$+50^\circ$ $+40^\circ$ $=180^\circ$ (Angles of $\triangle ADE$)

x = $180^\circ - 90^\circ$

x = 90°

Therefore, we can conclude that the required angles are 50°, 40° and 90°.

Q16. If one angle of a triangle is 60° and the other two angles are in the ratio 1 :2, find the angles

We know that one of the angles of the given triangle is 60°. (Given)

We also know that the other two angles of the triangle are in the ratio 1 : 2.

Let one of the other two angles be x.

Therefore, the second one will be 2x.

We know that the sum of all the three angles of a triangle is equal to 180°.

$60^\circ + x + 2x = 180^\circ$

$3x = 180^\circ - 60^\circ$

$3x = 120^\circ$

$x = \frac{120^\circ}{3}$

$x = 40^\circ$

2x = 2 x 40

$2x = 80^\circ$

Hence, we can conclude that the required angles are 40° and 80°.

Q17. It one angle of a triangle is 100° and the other two angles are in the ratio 2 : 3. find the angles.

We know that one of the angles of the given triangle is 100°.

We also know that the other two angles are in the ratio 2 : 3.

Let one of the other two angles be 2x.

Therefore, the second angle will be 3x.

We know that the sum of all three angles of a triangle is 180°.

$100^\circ + 2x + 3x = 180^\circ$

$5x = 180^\circ - 100^\circ$

$5x = 80^\circ$

$x = \frac{80^\circ}{5}$

2x = 2 x 16

$2x = 32^\circ$

3x = 3 x 16

$3x = 48^\circ$

Thus, the required angles are 32° and 48°.

Q18. In $\triangle ABC$, if $3\angle A = 4\angle B = 6\angle C$, calculate the angles.

We know that for the given triangle, $3\angle A = 6\angle C$

$\angle A = 2\angle C$–(i)

We also know that for the same triangle, $4\angle B = 6\angle C$

$\angle B = \frac{6}{4}\angle C$–(ii)

We know that the sum of all three angles of a triangle is 180°.

Therefore, we can say that:

$\angle A + \angle B + \angle C = 180^\circ$ (Angles of $\triangle ABC$)–(iii)

On putting the values of $\angle A \ and \ \angle B$ in equation (iii), we get :

$2\angle C + \frac{6}{4}\angle C + \angle C = 180^\circ \ \frac{18}{4} \ angleC = 180^\circ \ \ angleC = 40^\circ$

From equation (i), we have:

$angleA = 2\angle C = 2 \times 40 \ \ angleA = 80^\circ$

From equation (ii), we have:

$angleB = \frac{6}{4}\angle C = \frac{6}{4} \times 40^\circ \ \ angleB = 60^\circ$

$angleA = 80^\circ, \ angleB = 60^\circ, \ angleC = 40^\circ$

Therefore, the three angles of the given triangle are 80°, 60°, and 40°.

Q19. Is it possible to have a triangle, in which

(i) Two of the angles are right?

(ii) Two of the angles are obtuse?

(iii) Two of the angles are acute?

(iv) Each angle is less than 60°*?*

(v) Each angle is greater than 60°*?*

(vi) Each angle is equal to 60°

Give reasons in support of your answer in each case.

(i) No, because if there are two right angles in a triangle, then the third angle of the triangle must be zero, which is not possible.

(ii) No, because as we know that the sum of all three angles of a triangle is always 180°. If there are two obtuse angles, then their sum will be more than 180°, which is not possible in case of a triangle.

(iii) Yes, in right triangles and acute triangles, it is possible to have two acute angles.

(iv) No, because if each angle is less than 60°, then the sum of all three angles will be less than 180°, which is not possible in case of a triangle.

Proof:

Let the three angles of the triangle be $\angle A$, $\angle B$ and $\angle C$.

As per the given information,

$\angle A < 60^\circ$ ... (i)

$\angle B < 60^\circ$ ...(ii)

$\angle C < 60^\circ$ ... (iii)

On adding (i), (ii) and (iii), we get :

$\angle A + \angle B + \angle C < 60^\circ + 60^\circ + 60^\circ$

$\angle A + \angle B + \angle C < 180^\circ$

We can see that the sum of all three angles is less than 180°, which is not possible for a triangle.

Hence, we can say that it is not possible for each angle of a triangle to be less than 60°.

(v) No, because if each angle is greater than 60°, then the sum of all three angles will be greater than 180°, which is not possible.

Proof:

Let the three angles of the triangle be $\angle A$, $\angle B$ and $\angle C$. As per the given information,

$\angle A > 60^\circ$ ... (i)

$\angle B > 60^\circ$ ...(ii)

$\angle C > 60^\circ$ ... (iii)

On adding (i), (ii) and (iii), we get:

$\angle A + \angle B + \angle C > 60^\circ + 60^\circ + 60^\circ$

$\angle A + \angle B + \angle C > 180^\circ$

We can see that the sum of all three angles of the given triangle are greater than 180°, which is not possible for a triangle.

Hence, we can say that it is not possible for each angle of a triangle to be greater than 60°.

(vi) Yes, if each angle of the triangle is equal to 60°, then the sum of all three angles will be 180°, which is possible in case of a triangle.

Proof:

Let the three angles of the triangle be $\angle A$, $\angle B$ and $\angle C$. As per the given information,

$\angle A = 60^\circ$ ... (i)

$\angle B = 60^\circ$ ...(ii)

$\angle C = 60^\circ$ ... (iii)

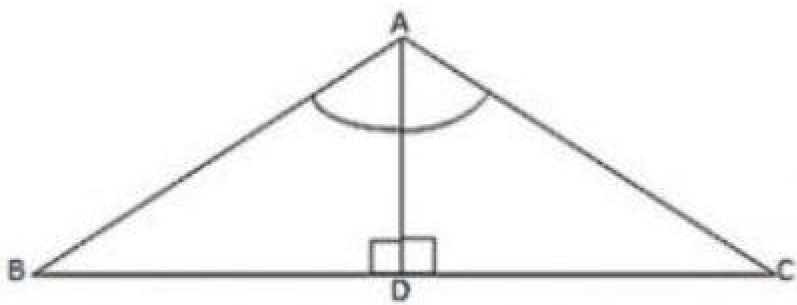
On adding (i), (ii) and (iii), we get:

$\angle A + \angle B + \angle C = 60^\circ + 60^\circ + 60^\circ$

$\angle A + \angle B + \angle C = 180^\circ$

We can see that the sum of all three angles of the given triangle is equal to 180° , which is possible in case of a triangle. Hence, we can say that it is possible for each angle of a triangle to be equal to 60° .

Q20. In $\triangle ABC, \angle A = 100^\circ$, AD bisects $\angle A$ and AD perpendicular BC. Find $\angle B$



Consider $\triangle ABD$

$\angle BAD$ = $\frac{100}{2}$ (AD bisects $\angle A$)

$\angle BAD$ = 50°

$\angle ADB$ = 90° (AD perpendicular to BC)

We know that the sum of all three angles of a triangle is 180°.

Thus,

$\angle ABD + \angle BAD + \angle ADB$= 180° (Sum of angles of $\triangle ABD$)

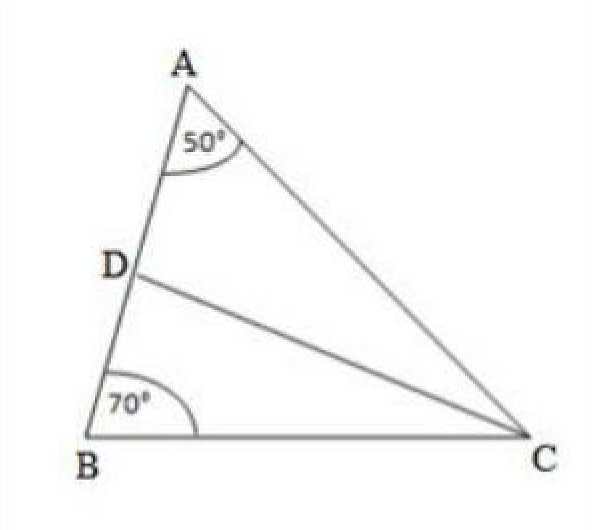
Or,

$\angle ABD$ + 50° + 90° =180°

$\angle ABD$ =180° – 140°

$\angle ABD$= 40°

Q21. In $\triangle ABC$, $\angle A = 50^\circ, \angle B = 100^\circ$ and bisector of $\angle C$ meets AB in D. Find the angles of the triangles ADC and BDC



We know that the sum of all three angles of a triangle is equal to 180°.

Therefore, for the given $\triangle ABC$, we can say that :

$\angle A + \angle B + \angle C = 180^\circ$ (Sum of angles of $\triangle ABC$)

$50^\circ + 70^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 120^\circ$

$\angle C = 60^\circ$

$\angle ACD = \angle BCD = \frac{\angle C}{2}$ (CD bisects $\angle C$ and meets AB in D.)

$\angle ACD = \angle BCD = \frac{60^\circ}{2} = 30^\circ$

Using the same logic for the given $\triangle ACD$, we can say that :

$\angle DAC + \angle ACD + \angle ADC = 180^\circ$

$50^\circ + 30^\circ + \angle ADC = 180^\circ$

$\angle ADC = 180^\circ - 80^\circ$

$\angle ADC = 100^\circ$

If we use the same logic for the given $\triangle BCD$, we can say that

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$

$70^\circ + 30^\circ + \angle BDC = 180^\circ$

$\angle BDC = 180^\circ - 100^\circ$

$\angle BDC = 80^\circ$

Thus,

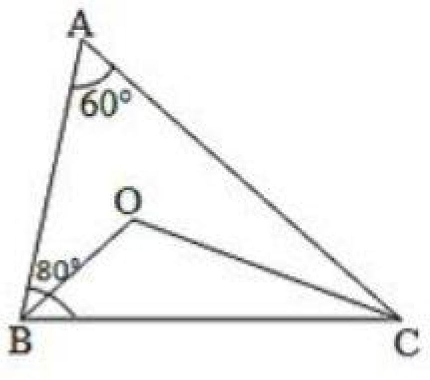
For $\triangle ADC$: $\angle A = 50^\circ, \angle D = 100^\circ \quad \angle C = 30^\circ$

$\triangle BDC$: $\angle B = 70^\circ, \angle D = 80^\circ \quad \angle C = 30^\circ$

Q22. In $\triangle ABC, \angle A = 60^\circ, \angle B = 80^\circ$, and the bisectors of $\angle B$ and $\angle C$, meet at O. Find

(i) $\angle C$

(ii) $\angle BOC$



We know that the sum of all three angles of a triangle is 180°.

Hence, for $\triangle ABC$, we can say that :

$\angle A + \angle B + \angle C = 180^\circ$ (Sum of angles of $\triangle ABC$)

$60^\circ + 80^\circ + \angle C = 180^\circ$.

$\angle C = 180^\circ - 140^\circ$

$\angle C = 140^\circ$.

For $\triangle OBC$,

$\angle OBC = \frac{\angle B}{2} = \frac{80^\circ}{2}$ (OB bisects $\angle B$)

$\angle OBC = 40^\circ$

$\angle OCB = \frac{\angle C}{2} = \frac{40^\circ}{2}$ (OC bisects $\angle C$)

$\angle OCB = 20^\circ$

If we apply the above logic to this triangle, we can say that :

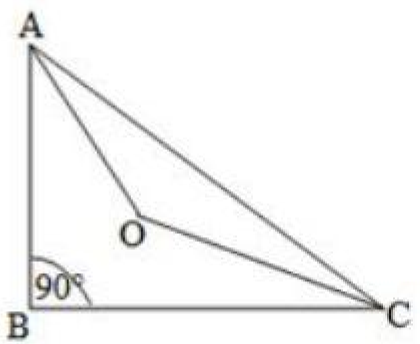
$\angle OCB + \angle OBC + \angle BOC = 180^\circ$ (Sum of angles of $\triangle OBC$)

$20^\circ + 40^\circ + \angle BOC = 180^\circ$

$\angle BOC = 180^\circ - 60^\circ$

$\angle BOC = 120^\circ$

Q23. The bisectors of the acute angles of a right triangle meet at O. Find the angle at O between the two bisectors.



We know that the sum of all three angles of a triangle is 180°.

Hence, for $\triangle ABC$, we can say that :

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A \ + 90^\circ \ + \angle C \ = 180^\circ$

$\angle A \ + \angle C \ = 180^\circ - 90^\circ$

$\angle A \ + \angle C \ = 90^\circ$

For $\triangle OAC$:

$\angle OAC \ = \frac{\angle A}{2}$ (OA bisects LA)

$\angle OCA \ = \frac{\angle C}{2}$ (OC bisects LC)

On applying the above logic to $\triangle OAC$, we get :

$\angle AOC + \angle OAC + \angle OCA \ = 180^\circ$ (Sum of angles of $\triangle AOC$)

$\angle AOC + \frac{\angle A}{2} + \frac{\angle C}{2} = 180^\circ$

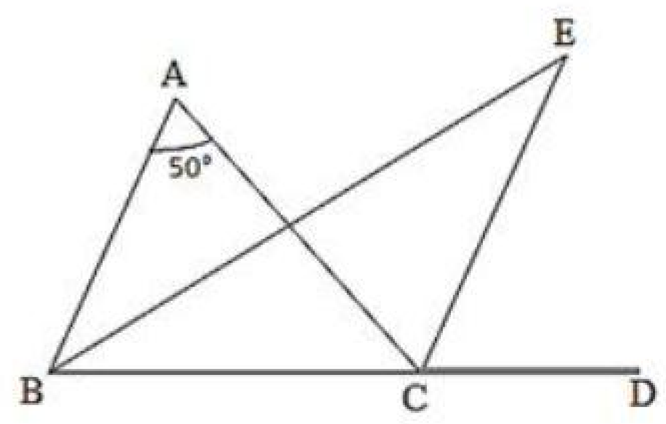
$\angle AOC + \frac{\angle A + \angle C}{2} = 180^\circ$

$\angle AOC + \frac{90^\circ}{2} = 180^\circ$

$\angle AOC = 180^\circ - 45^\circ$

$\angle AOC = 135^\circ$

Q24. In $\triangle ABC$, $\angle A = 50^\circ$ and BC is produced to a point D. The bisectors of $\angle ABC$ and $\angle ACD$ meet at E. Find $\angle E$.



In the given triangle,

$\angle ACD$= $\angle A$ + $\angle B$. (Exterior angle is equal to the sum of two opposite interior angles.)

We know that the sum of all three angles of a triangle is 180° .

Therefore, for the given triangle, we can say that :

$\angle ABC$+ $\angle BCA$ + $\angle CAB$ = 180° (Sum of all angles of $\triangle ABC$)

$\angle A$ + $\angle B$ + $\angle BCA$= 180°

$\angle BCA$=180°- ($\angle A$ + $\angle B$)

$\angle ECA$= $\frac{\angle ACD}{2}$ (EC bisects $\angle ACD$)

$\angle ECA$= $\frac{\angle A+\angle B}{2}$ ($\angle ACD = \angle A + \angle B$)

$\angle EBC$= $\frac{\angle ABC}{2} = \frac{\angle B}{2} (EB bisects \angle ABC)$

$\angle ECB$= $\angle ECA$ + $\angle BCA$

$\angle ECB$= $\frac{\angle A+\angle B}{2} + 180^\circ - (\angle A + \angle B)$

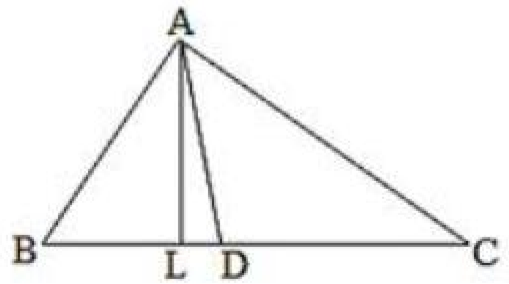
If we use the same logic for $\triangle EBC$, we can say that :

$\angle EBC$+ $\angle ECB$ + $\angle BEC$ = 180° (Sum of all angles of $\triangle EBC$)

$\frac{\angle B}{2} + \frac{\angle A+\angle B}{2} + 180^\circ - (\angle A + \angle B) + \angle BEC = 180^\circ$

$\angle BEC = \angle A + \angle B - (\frac{\angle A+\angle B}{2} - \frac{\angle B}{2} \angle BEC = \frac{\angle A}{2} \angle BEC = \frac{50^\circ}{2} = 25^\circ$

Q25. In $\triangle ABC, \angle B = 60^\circ, \angle C = 40^\circ$, AL perpendicular BC and AD bisects $\angle A$ such that L and D lie on side BC. Find $\angle LAD$



We know that the sum of all angles of a triangle is 180°

Therefore, for $\triangle ABC$, we can say that :

$\angle A + \angle B + \angle C = 180^\circ$

Or,

$\angle A + 60^\circ + 40^\circ = 180^\circ$

$\angle A = 80^\circ$

$\angle DAC = \frac{\angle A}{2}$ (AD bisects $\angle A$)

$\angle DAC = \frac{80^\circ}{2}$

If we use the above logic on $\triangle ADC$, we can say that :

$\angle ADC + \angle DCA + \angle DAC = 180^\circ$ (Sum of all the angles of $\triangle ADC$)

$\angle ADC + 40^\circ + 40^\circ = 180^\circ$

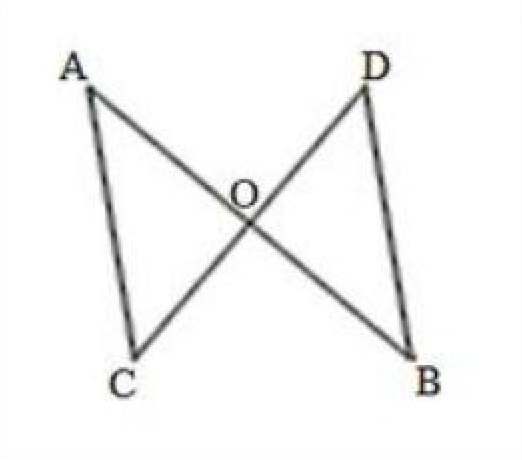
$\angle ADC = 180^\circ + 80^\circ$

$\angle ADC = \angle ALD + \angle LAD$ (Exterior angle is equal to the sum of two Interior opposite angles.)

$100^\circ = 90^\circ + \angle LAD$ (AL perpendicular toBC)

$\angle LAD = 90^\circ$

Q26. Line segments AB and CD intersect at O such that AC perpendicular DB. It $\angle CAB = 35^\circ$ and $\angle CDB = 55^\circ$. Find $\angle BOD$.



We know that AC parallel to BD and AB cuts AC and BD at A and B, respectively.

$\angle CAB$= $\angle DBA$ (Alternate interior angles)

$\angle DBA = 35^\circ$

We also know that the sum of all three angles of a triangle is 180°.

Hence, for $\triangle OBD$, we can say that :

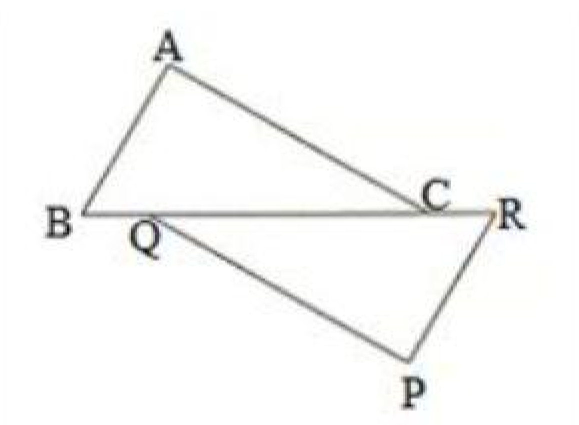
$\angle DBO$ + $\angle ODB$+ $\angle BOD$= 180°

35°+ 55°+ $\angle BOD$ = 180° ($\angle DBO = \angle DBA$ and $\angle ODB = \angle CDB$)

$\angle BOD$ =$180^\circ - 90^\circ$

$\angle BOD$ = 90°

Q27. In Fig. 22, $\triangle ABC$ is right angled at A, Q and R are points on line BC and P is a point such that QP perpendicular to AC and RP perpendicular to AB. Find $\angle P$



In the given triangle, AC parallel to QP and BR cuts AC and QP at C and Q, respectively.

$\angle QCA = \angle CQP$ (Alternate interior angles)

Because RP parallel to AB and BR cuts AB and RP at B and R, respectively,

$\angle ABC = \angle PRQ$ (alternate interior angles).

We know that the sum of all three angles of a triangle is 180°.

Hence, for $\triangle ABC$, we can say that :

$\angle ABC + \angle ACB + \angle BAC = 180^\circ$

$\angle ABC + \angle ACB + 90^\circ = 180^\circ$ (Right angled at A)

$\angle ABC + \angle ACB = 90^\circ$

Using the same logic for $\triangle PQR$, we can say that :

$\angle PQR + \angle PRQ + \angle QPR = 180^\circ$

$\angle ABC + \angle ACB + \angle QPR = 180^\circ$ ($\angle ABC = \angle PRQ$ and $\angle QCA = \angle CQP$)

Or,

$90^\circ + \angle QPR = 180^\circ$ ($\angle ABC + \angle ACB = 90^\circ$)

$\angle QPR = 90^\circ$

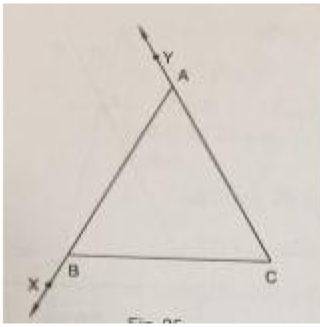
Exercise 15.3

1. $\angle CBX$ is an exterior angle of $\triangle ABC$ at B. Name

(i) the interior adjacent angle

(ii) the interior opposite angles to exterior $\angle CBX$

Also, name the interior opposite angles to an exterior angle at A.

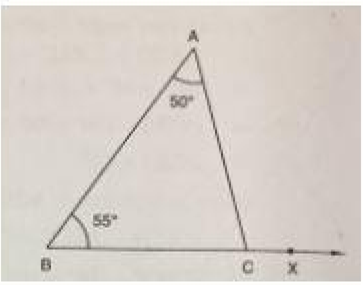


(i) $\angle ABC$

(ii) $\angle BAC and \angle ACB$

Also the interior angles opposite to exterior are $\angle ABC$ and $\angle ACB$

2. In the fig, two of the angles are indicated. What are the measures of $\angle ACX$ and $\angle ACB$?



In $\triangle ABC$, $\angle A = 50^\circ$ and $\angle B = 55^\circ$

Because of the angle sum property of the triangle, we can say that

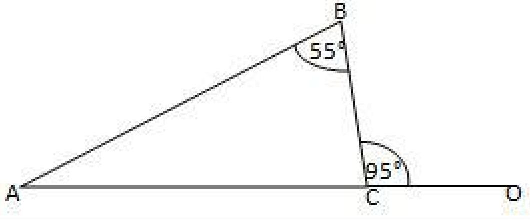
$\angle A + \angle B + \angle C = 180^\circ$

$50^\circ + 55^\circ + \angle C = 180^\circ$

Or

$\angle C = 75^\circ \; \angle ACB = 75^\circ \; \angle ACX = 180^\circ - \angle ACB = 180^\circ - 75^\circ = 105^\circ$

3. In a triangle, an exterior angle at a vertex is 95° and its one of the interior opposite angles is 55°. Find all the angles of the triangle.



We know that the sum of interior opposite angles is equal to the exterior angle.

Hence, for the given triangle, we can say that :

$\angle ABC + \angle BAC = \angle BCO$

$55^\circ + \angle BAC = 95^\circ$

Or,

$\angle BAC = 95^\circ - 95^\circ$

$= \angle BAC = 40^\circ$

We also know that the sum of all angles of a triangle is 180°.

Hence, for the given $\triangle ABC$, we can say that :

$\angle ABC + \angle BAC + \angle BCA = 180^\circ$

$55^\circ + 40^\circ + \angle BCA = 180^\circ$

Or,

$\angle BCA = 180^\circ - 95^\circ$

$= \angle BCA = 85^\circ$

4. One of the exterior angles of a triangle is 80°, and the interior opposite angles are equal to each other. What is the measure of each of these two angles?

Let us assume that A and B are the two interior opposite angles.

We know that $\angle A$ is equal to $\angle B$.

We also know that the sum of interior opposite angles is equal to the exterior angle.

Hence, we can say that :

$\angle A + \angle B = 80^\circ$

Or,

$\angle A + \angle A = 80^\circ$ ($\angle A = \angle B$)

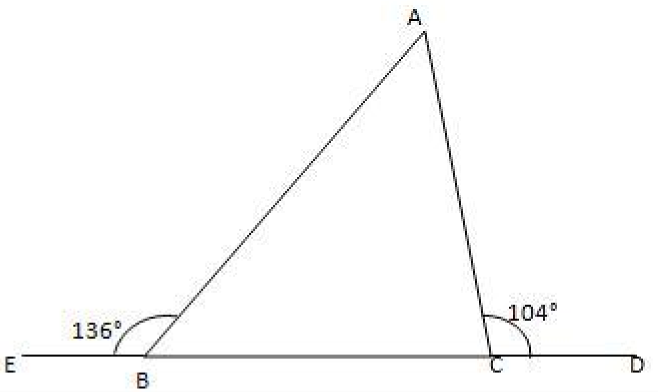
$2\angle A = 80^\circ$

$\angle A = \frac{40^\circ}{2} = 40^\circ$

$\angle A = \angle B = 40^\circ$

Thus, each of the required angles is of 40°.

5. The exterior angles, obtained on producing the base of a triangle both ways are 104° and 136°. Find all the angles of the triangle.



In the given figure, $\angle ABE$ and $\angle ABC$ form a linear pair.

$\angle ABE + \angle ABC = 180^\circ$

$\angle ABC = 180^\circ - 136^\circ$

$\angle ABC = 44^\circ$

We can also see that $\angle ACD$ and $\angle ACB$ form a linear pair.

$\angle ACD + \angle ACB = 180^\circ$

$\angle AUB = 180^\circ - 104^\circ$

$\angle ACB = 76^\circ$

We know that the sum of interior opposite angles is equal to the exterior angle.

Therefore, we can say that :

$\angle BAC + \angle ABC = 104^\circ$

$\angle BAC = 104^\circ - 44^\circ = 60^\circ$

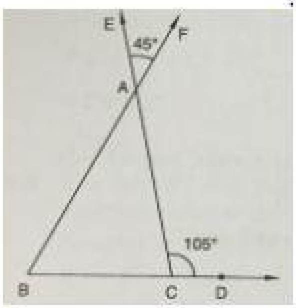
Thus,

$\angle ACE = 76^\circ$

and

$\angle BAC = 60^\circ$

6. In Fig, the sides BC, CA and BA of a $\triangle ABC$ have been produced to D, E and F respectively. If $\angle ACD = 105^\circ$ and $\angle EAF = 45^\circ$; find all the angles of the $\triangle ABC$



In a $\triangle ABC$, $\angle BAC$ and $\angle EAF$ are vertically opposite angles.

Hence, we can say that :

$\angle BAC = \angle EAF = 45^\circ$

Considering the exterior angle property, we can say that :

$\angle BAC + \angle ABC = \angle ACD = 105^\circ$

$\angle ABC = 105^\circ - 45^\circ = 60^\circ$

Because of the angle sum property of the triangle, we can say that :

$\angle ABC + \angle ACS + \angle BAC = 180^\circ$

$\angle ACB = 75^\circ$

Therefore, the angles are 45°, 65° and 75°.

7. In Fig, AC perpendicular to CE and C $\angle A : \angle B : \angle C$=3:2:1. Find the value of $\angle ECD$.

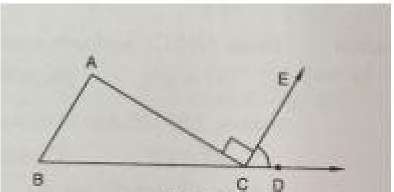


Fig. 38

In the given triangle, the angles are in the ratio 3 : 2 : 1.

Let the angles of the triangle be 3x, 2x and x.

Because of the angle sum property of the triangle, we can say that :

3x+2x+x = 180°

6x = 180°

Or,

x = 30° ... (i)

Also, $\angle ACB + \angle ACE + \angle ECD = 180^{\circ}$

x+ 90° + $\angle ECD = 180^{\circ}$ ($\angle ACE = 90^{\circ}$)

$\angle ECD$= 60° [From (i)]

8 A student when asked to measure two exterior angles of $\triangle ABC$ observed that the exterior angles at A and B are of 103° and 74° respectively. Is this possible? Why or why not?

Here,

Internal angle at A+ External angle at A = 180°

Internal angle at A + 103° =180°

Internal angle at A = 77°

Internal angle at B + External angle at B = 180°

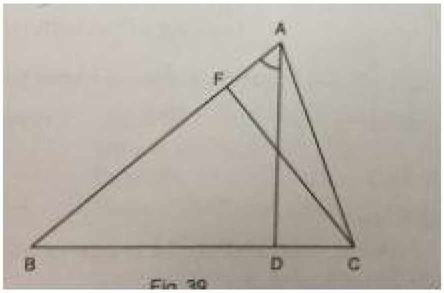
Internal angle at B + 74° = 180°

Internal angle at B = 106°

Sum of internal angles at A and B = 77° + 106° =183°

It means that the sum of internal angles at A and B is greater than 180°, which cannot be possible.

9. In Fig, AD and CF are respectively perpendiculars to sides BC and AB of $\triangle ABC$. If $\angle FCD = 50^\circ$, find $\angle BAD$



We know that the sum of all angles of a triangle is 180°

Therefore, for the given $\triangle FCB$, we can say that :

$\angle FCB + \angle CBF + \angle BFC = 180^\circ$

$50^\circ + \angle CBF + 90^\circ = 180^\circ$

Or,

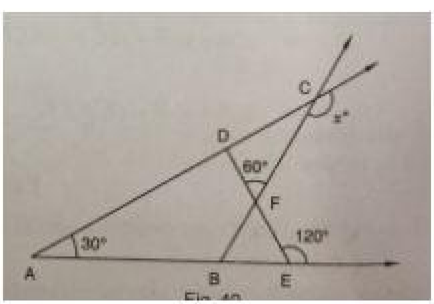
$\angle CBF = 180^\circ - 50^\circ - 90^\circ = 40^\circ$ ... (i)

Using the above rule for $\triangle ABD$, we can say that :

$\angle ABD + \angle BDA + \angle BAD = 180^\circ$

$\angle BAD = 180^\circ - 90^\circ - 40^\circ = 50^\circ$ [from (i)]

10. In Fig, measures of some angles are indicated. Find the value of x.



Here,

$\angle AED + 120^\circ = 180^\circ$ (Linear pair)

$\angle AED = 180^\circ - 120^\circ = 60^\circ$

We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle ADE$, we can say that :

$\angle ADE + \angle AED + \angle DAE = 180^\circ$

$60^\circ + \angle ADE + 30^\circ = 180^\circ$

Or,

$\angle ADE = 180^\circ - 60^\circ - 30^\circ = 90^\circ$

From the given figure, we can also say that :

$\angle FDC + 90^\circ = 180^\circ$ (Linear pair)

$\angle FDC = 180^\circ - 90^\circ = 90^\circ$

Using the above rule for $\triangle CDF$, we can say that :

$\angle CDF + \angle DCF + \angle DFC = 180^\circ$

$90^\circ + \angle DCF + 60^\circ = 180^\circ$

$\angle DCF = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

Also,

$\angle DCF + x = 180^\circ$ (Linear pair)

$30^\circ + x = 180^\circ$

Or,

$x = 180^\circ - 30^\circ = 150^\circ$

11. In Fig, ABC is a right triangle right angled at A. D lies on BA produced and DE perpendicular to BC intersecting AC at F. If $\angle AFE = 130^\circ$, find

(i) $\angle BDE$

(ii) $\angle BCA$

(iii) $\angle ABC$

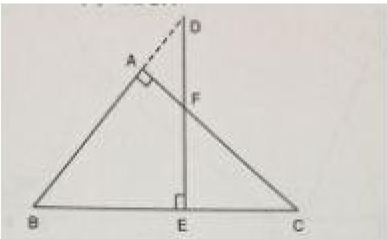


Fig. 41

(i)

Here,

$\angle BAF + \angle FAD = 180^\circ$ (Linear pair)

$\angle FAD$ =180°-$\angle BAF$ =$180^\circ - 90^\circ = 90^\circ$

Also,

$\angle AFE = \angle ADF + \angle FAD$ (Exterior angle property)

$\angle ADF + 90^\circ = 130^\circ$

$\angle ADF = 130^\circ - 90^\circ = 40^\circ$

(ii) We know that the sum of all the angles of a triangle is 180°.

Therefore, for $\triangle BDE$, we can say that :

$\angle BDE + \angle BED + \angle DBE = 180^\circ$.

$\angle DBE$= $180^\circ - \angle BDE \angle BED = 180^\circ - 90^\circ - 40^\circ = 50^\circ$ −−(i)

Also,

$\angle FAD = \angle ABC + \angle ACB$ (Exterior angle property)

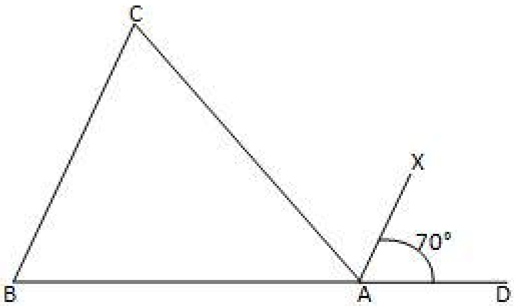
$90^\circ = 50^\circ + \angle ACB$

Or,

$\angle ACB = 90^\circ - 50^\circ = 40^\circ$

(iii) $\angle ABC = \angle DBE$ =50° [From (i)]

12. ABC is a triangle in which $\angle B = \angle C$ and ray AX bisects the exterior angle DAC. If $\angle DAX = 70^\circ$. Find $\angle ACB$.



Here,

$\angle CAX$ = $\angle DAX$ (AX bisects $\angle CAD$)

$\angle CAX$ =70°

$\angle CAX$ +$\angle DAX$ + $\angle CAB$ =180°

70°+ 70° + $\angle CAB$ =180°

$\angle CAB$ =180° −140°

$\angle CAB$ =40°

$\angle ACB$ +$\angle CBA$ + $\angle CAB$ =180° (Sum of the angles of $\triangle ABC$)

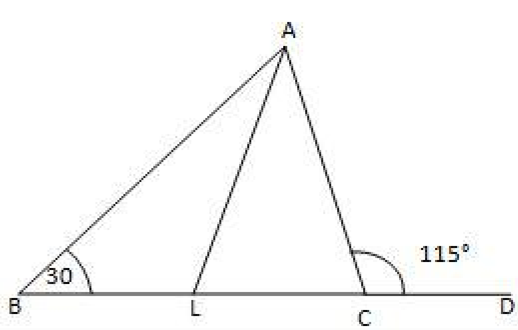
$\angle ACB$ +$\angle ACB$+ 40° =180° ($\angle C$= $\angle B$)

2$\angle ACB$= 180°− 40°

$\angle ACB$ = $\frac{140^\circ}{2}$

$\angle ACB$= 70°

13. The side BC of $\triangle ABC$ is produced to a point D. The bisector of $\angle A$ meets side BC in L. If $\angle ABC = 30^\circ$ and $\angle ACD = 115^\circ$, find $\angle ALC$



$\angle ACD$ and $\angle ACL$ make a linear pair.

$\angle ACD$+ $\angle ACB$ = 180°

115° + $\angle ACB$ =180°

$\angle ACB$ = 180°− 115°

$\angle ACB$= 65°

We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle ABC$, we can say that :

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$

$30^\circ + \angle BAC + 65^\circ = 180^\circ$

Or,

$\angle BAC = 85^\circ$

$\angle LAC = \frac{\angle BAC}{2} = \frac{85^\circ}{2}$

Using the above rule for $\triangle ALC$, we can say that :

$\angle ALC + \angle LAC + \angle ACL = 180^\circ$

$\angle ALC + \frac{82^\circ}{2} + 65^\circ = 180^\circ$ ($\angle ACL = \angle ACB$)

Or,

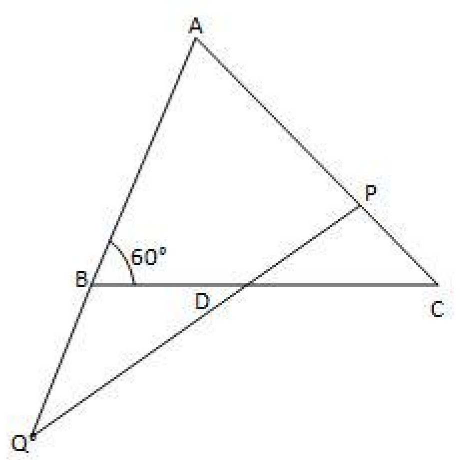
$\angle ALC = 180^\circ - \frac{85^\circ}{2} - 65^\circ$

$\angle ALC = \frac{145^\circ}{2} = 72\frac{1}{2}^\circ$

14. D is a point on the side BC of $\triangle ABC$. A line PDQ through D, meets side AC in P and AB produced at Q. If $\angle A = 80^\circ$, $\angle ABC = 60^\circ$ and $\angle PDC = 15^\circ$, find

(i) $\angle AQD$

(ii) $\angle APD$



$\angle ABD$ and $\angle QBD$ form a linear pair.

$\angle ABC + \angle QBC = 180^\circ$

$60^\circ + \angle QBC = 180^\circ$

$\angle QBC = 120^\circ$

$\angle PDC = \angle BDQ$ (Vertically opposite angles)

$\angle BDQ = 75^\circ$

In $\triangle QBD$:

$\angle QBD + \angle QDB + \angle BDQ = 180^\circ$ (Sum of angles of $\triangle QBD$)

$120^\circ + 15^\circ + \angle BQD = 180^\circ$

$\angle BQD = 180^\circ - 135^\circ$

$\angle BQD = 45^\circ$
$\angle AQD = \angle BQD = 45^\circ$

In $\triangle AQP$:

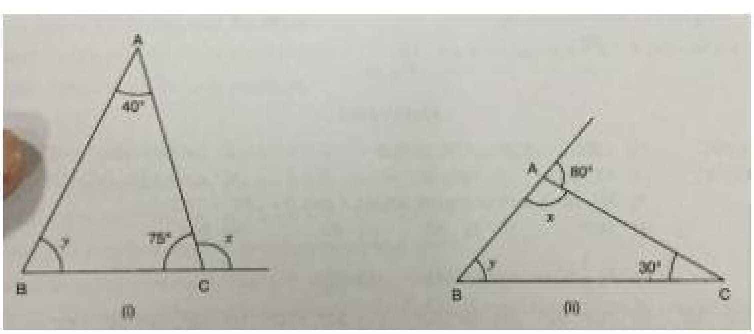
$\angle QAP + \angle AQP + \angle APQ = 180^\circ$ (Sum of angles of $\triangle AQP$)

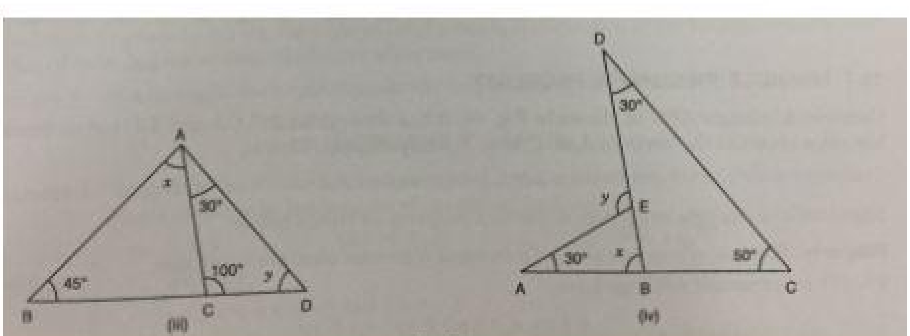
$80^\circ + 45^\circ + \angle APQ = 180^\circ$

$\angle APQ = 55^\circ$

$\angle APD = \angle APQ$

15. Explain the concept of interior and exterior angles and in each of the figures given below. Find x and y





The interior angles of a triangle are the three angle elements inside the triangle.

The exterior angles are formed by extending the sides of a triangle, and if the side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Using these definitions, we will obtain the values of x and y.

(I) From the given figure, we can see that:

$\angle ACB$ + x = 180° (Linear pair)

75°+ x = 180°

Or,

x = 105°

We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle ABC$, we can say that :

$\angle BAC$+ $\angle ABC$ +$\angle ACB$ = 180°

40°+y+ 75° = 180°

Or,

y = 65°

(ii)

x +80° = 180° (Linear pair)

= x = 100°

In $\triangle ABC$:

x+ y+ 30° = 180° (Angle sum property)

100° + 30° + y = 180°

= y = 50°

(iii)

We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle ACD$, we can say that :

$30^\circ + 100^\circ + y = 180^\circ$

Or,

$y = 50^\circ$

$\angle ACB + 100^\circ = 180^\circ$

$\angle ACB = 80^\circ$ ... (i)

Using the above rule for $\triangle ACD$, we can say that :

$x + 45^\circ + 80^\circ = 180^\circ$

$= x = 55^\circ$

(iv)

We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle DBC$, we can say that :

$30^\circ + 50^\circ + \angle DBC = 180^\circ$

$\angle DBC = 100^\circ$

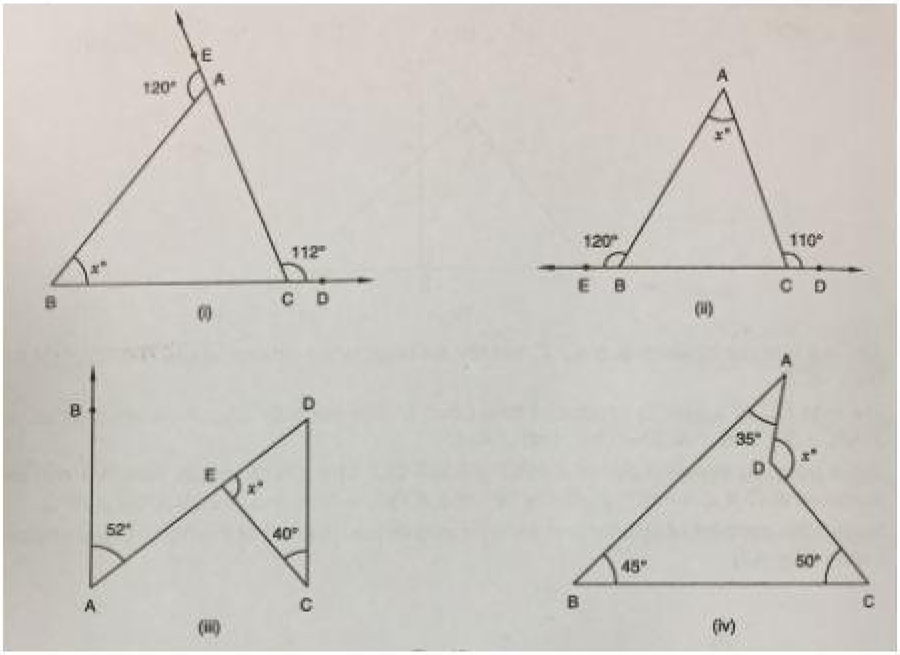
$x + \angle DBC = 180^\circ$ (Linear pair)

$x = 80^\circ$

And,

$y = 30^\circ + 80^\circ = 110^\circ$ (Exterior angle property)

16. Compute the value of x in each of the following figures



(i) From the given figure, we can say that :

$\angle ACD + \angle ACB = 180^\circ$ (Linear pair)

Or,

$\angle ACB = 180^\circ - 112^\circ = 68^\circ$

We can also say that :

$\angle BAE + \angle BAC = 180^\circ$ (Linear pair)

Or,

$\angle BAC = 180^\circ - 120^\circ = 60^\circ$

We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle ABC$:

x+ $\angle BAC + \angle ACB = 180^\circ$

x = $180^\circ - 60^\circ - 68^\circ = 52^\circ$

= x = 52°

(ii) From the given figure, we can say that :

$\angle ABC + 120^\circ = 180^\circ$ (Linear pair)

$\angle ABC = 60^\circ$

We can also say that :

$\angle ACB + 110^\circ = 180^\circ$ (Linear pair)

$\angle ACB = 70^\circ$

We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle ABC$:

x+ $\angle ABC + \angle ACB = 180^\circ$

= x = 50°

(iii)

From the given figure, we can see that :

$\angle BAD = \angle ADC = 52^\circ$ (Alternate angles)

We know that the sum of all the angles of a triangle is 180°.

Therefore, for $\triangle DEC$:

x + $40^\circ + 52^\circ = 180^\circ$

= x = 88°

(iv) In the given figure, we have a quadrilateral whose sum of all angles is 360°.

Thus,

$35^\circ + 45^\circ + 50^\circ$ + reflex$\angle ADC = 360^\circ$

Or,

reflex$\angle ADC = 230^\circ$

230° + x = 360° (A complete angle)

= x = 130°

Exercise 15.4

Q1. In each of the following, there are three positive numbers. State if these numbers could possibly be the lengths of the sides of a triangle:

(i) 5, 7, 9

(ii) 2, 10.15

(iii) 3, 4, 5

(iv) 2, 5, 7

(v) 5, 8, 20

(i) Yes, these numbers can be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side. Here, 5+7>9, 5+9>7, 9+7>5

(ii) No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.

(iii) Yes, these numbers can be the lengths of the sides of a triangle because the sum of any two sides of triangle is always greater than the third side. Here, 3+4 >5, 3+5> 4, 4+5> 3

(iv) No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case. Here, 2 + 5 = 7

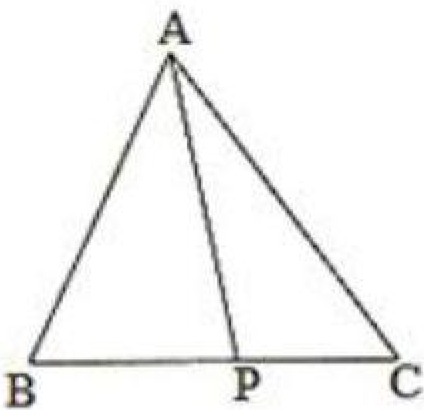
(v) No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case. Here, 5 + 8 <20

Q2. In Fig, P is the point on the side BC. Complete each of the following statements using symbol ' =',' > 'or ' < 'so as to make it true:

(i) AP... AB+ BP

(ii) AP... AC + PC

(iii) AP.... $\frac{1}{2}(AB + AC + BC)$



(i) In triangle APB, AP < AB + BP because the sum of any two sides of a triangle is greater than the third side.

(ii) In triangle APC, AP < AC + PC because the sum of any two sides of a triangle is greater than the third side.

(iii) AP < $\frac{1}{2}(AB + AC + BC)$ In triangles ABP and ACP, we can see that:

AP < AB + BP...(i) (Because the sum of any two sides of a triangle is greater than the third side)

AP < AC + PC...(ii) (Because the sum of any two sides of a triangle is greater than the third side)

On adding (i) and (ii), we have:

AP + AP < AB + BP + AC + PC

2AP < AB + AC + BC (BC = BP + PC)

AP < (AB-FAC+BC)

Q3. P is a point in the interior of $\triangle ABC$ as shown in Fig. State which of the following statements are true (T) or false (F):

(i) AP+ PB< AB

(ii) AP+ PC> AC

(iii) BP+ PC = BC

(i) False

We know that the sum of any two sides of a triangle is greater than the third side: it is not true for the given triangle.

(ii) True

We know that the sum of any two sides of a triangle is greater than the third side: it is true for the given triangle.

(iii) False

We know that the sum of any two sides of a triangle is greater than the third side: it is not true for the given triangle.

Q4. O is a point in the exterior of $\triangle ABC$. What symbol '>','<' or '=' will you see to complete the statement OA+OB....AB? Write two other similar statements and show that

OA+OB+OC>$\frac{1}{2}(AB+BC+CA)$

Because the sum of any two sides of a triangle is always greater than the third side, in triangle OAB, we have:

OA+OB> AB –(i)

OB+OC>BC –-(ii)

OA+OC > CA –-(iii)

On adding equations (i), (ii) and (iii) we get :

OA+OB+OB+OC+OA+OC> AB+BC+CA

2(OA+OB+OC) > AB+BC +CA

OA+ OB + OC > $\frac{AB+BC+CA}{2}$

Q5. In $\triangle ABC$, $\angle B=30^\circ$, $\angle C=50^\circ$. Name the smallest and the largest sides of the triangle.

Because the smallest side is always opposite to the smallest angle, which in this case is 30°, it is AC. Also, because the largest side is always opposite to the largest angle, which in this case is 100°, it is BC.

Exercise 15.5

Q1. State Pythagoras theorem and its converse.

The Pythagoras Theorem: In a right triangle, the square of the hypotenuse is always equal to the sum of the squares of the other two sides.

Converse of the Pythagoras Theorem: If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle, with the angle opposite to the first side as right angle.

Q2. In right $\triangle ABC$, the lengths of the legs are given. Find the length of the hypotenuse

(i) a= 6 cm, b= 8 cm

(ii) a=8cm, b= 15 cm

(iii) a= 3 cm, b= 4 cm

(iv) a=2cm,b=1.5cm

According to the Pythagoras theorem,

$$(Hypotenuse)^2 = (Base)^2 + (Height)^2$$

(i)

$$c^2 = a^2 + b^2\ c^2 = 6^2 + 8^2\ c^2 = 36 + 64 = 100$$

c = 10 cm

(ii)

$$c^2 = a^2 + b^2\ c^2 = 8^2 + 15^2\ c^2 = 64 + 225 = 289$$

c=17cm

(iii)

$$c^2 = a^2 + b^2\ c^2 = 3^2 + 4^2\ c^2 = 9 + 16 = 25$$

c=5cm

(iv)

$$c^2 = a^2 + b^2\ c^2 = 2^2 + 1.5^2\ c^2 = 4 + 2.25 = 6.25$$

c=2.5cm

Q3. The hypotenuse of a triangle is 2.5 cm. If one of the sides is 1.5 cm. find the length of the other side.

Let the hypotenuse be " c " and the other two sides be " b " and " c".

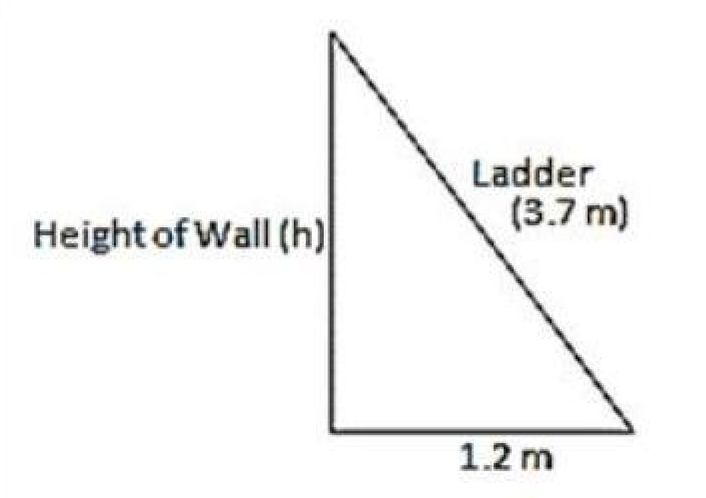
Using the Pythagoras theorem, we can say that :

$c^2 = a^2 + b^2 \; 2.5^2 = 1.5^2 + b^2 \; b^2 = 6.25 - 2.25 = 4$

c=2cm

Hence, the length of the other side is 2 cm.

Q4. A ladder 3.7 m long is placed against a wall in such a way that the foot of the ladder is 1.2m away from the wall. Find the height of the wall to which the ladder reaches.



Let the hypotenuse be h.

Using the Pythagoras theorem, we get :

$3.7^2 = 1.2^2 + h^2 \; h^2 = 13.69 – 1.44 = 12.25$

h=3.5m

Hence, the height of the wall is 3.5 m.

Q5. If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is right-angled triangle.

In the given triangle, the largest side is 6 cm.

We know that in a right angled triangle, the sum of the squares of the smaller sides should be equal to the square of the largest side.

Therefore,

$3^2 + 4^2 = 9 + 16 = 25$
But,

$6^2 = 36$
$3^2 + 4^2$ not equal to 6^2

Hence, the given triangle is not a right angled triangle.

Q6. The sides of certain triangles are given below. Determine which of them are right triangles.

(i) a = 7 cm, b = 24 cm and c= 25 cm

(ii) a = 9 cm, b = 16 cm and c= 18cm

(i) We know that in a right angled triangle, the square of the largest side is equal to the sum of the squares of the smaller sides.

Here, the larger side is c, which is 25 cm.

$c^2 = 625$

We have :

$a^2 + b^2 = 7^2 + 24^2 = 49 + 576 = 625 = c^2$
Thus, the given triangle is a right triangle.

(ii) We know that in a right angled triangle, the square of the largest side is equal to the sum of the squares of the smaller sides.

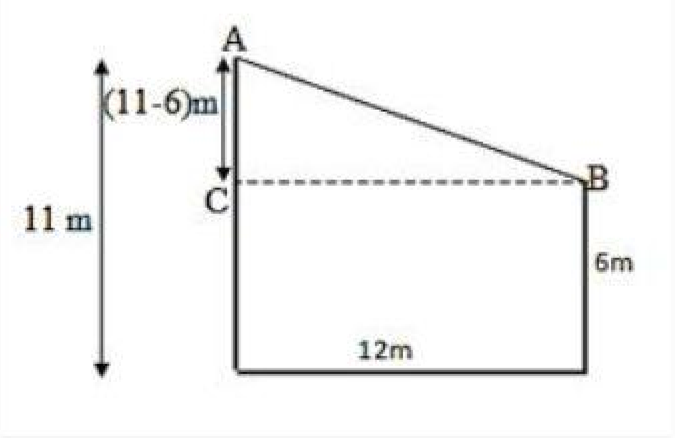
Here, the larger side is c, which is 18 cm.

$c^2 = 324$
We have :

$a^2 + b^2 = 9^2 + 16^2 = 81 + 256 = 337\ not\ equal\ to\ c^2$
Thus, the given triangle is not a right triangle.

Q7. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m. Find the distance between their tops.

(Hint: Find the hypotenuse of a right triangle having the sides (11 − 6) m = 5m and 12m)



The distance between the tops of the poles is the distance between points A and B.

We can see from the given figure that points A, B and C form a right triangle, with AB as the hypotenuse.

On using the Pythagoras Theorem in $\triangle ABC$, we get :

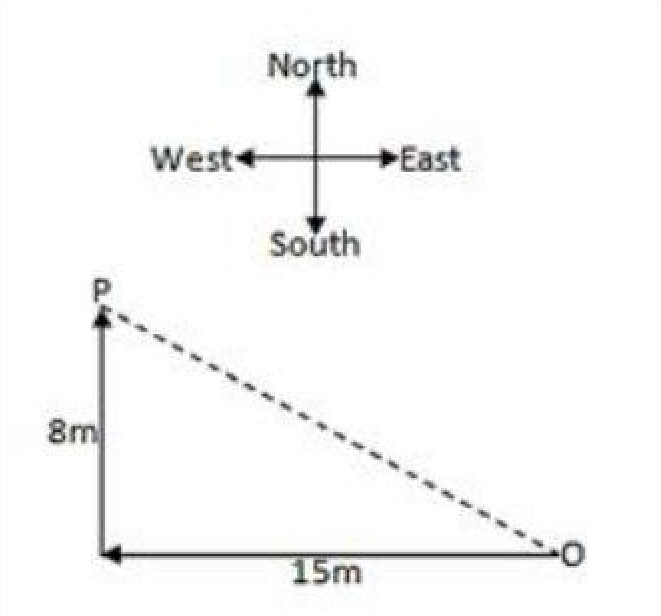
$(11-6)^2 + 12^2 = AB^2$
AB^2 =25+144

AB^2 =169

AB=13

Hence, the distance between the tops of the poles is 13 m.

Q8. A man goes 15 m due west and then 8 m due north. How far is he from the starting point?



Let O be the starting point and P be the final point.

By using the Pythagoras theorem, we can find the distance OP.

$OP^2 = 15^2 + 8^2$

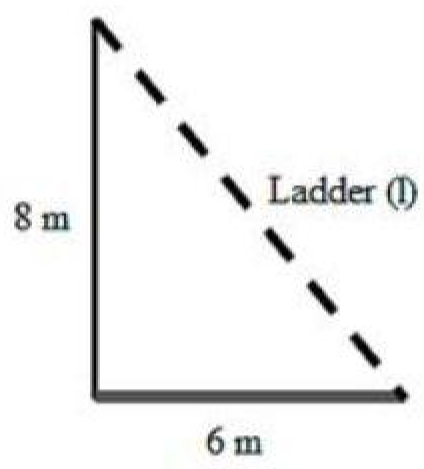
OP^2=225+64

OP^2=289

OP=17

Hence, the required distance is 17 m.

Q9. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its top reach?

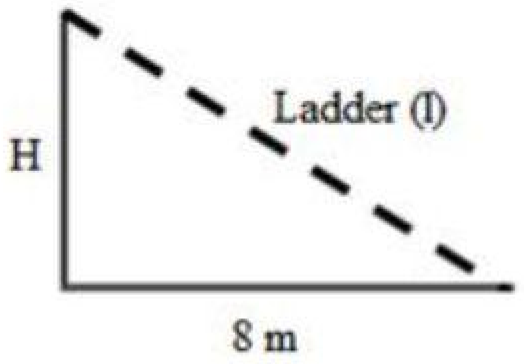


Given Let the length of the ladder be L m.

By using the Pythagoras theorem, we can find the length of the ladder.

$6^2 + 8^2 = L^2\ L^2 = 36 + 64 = 100$
L = 10

Thus, the length of the ladder is 10 m.



When the ladder is shifted:

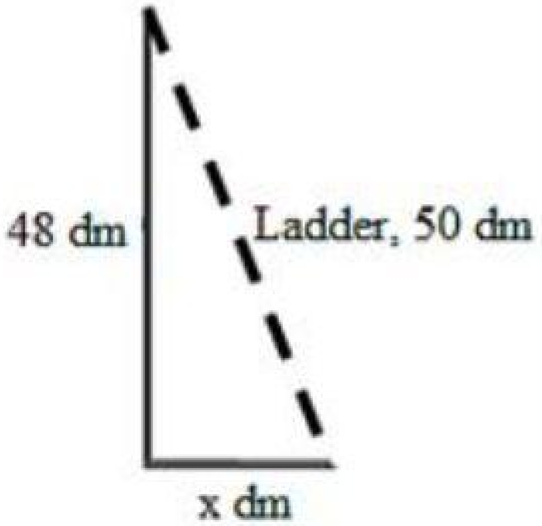
Let the height of the ladder after it is shifted be H m.

By using the Pythagoras theorem, we can find the height of the ladder after it is shifted.

$8^2 + H^2 = 10^2\ H^2 = 100 - 64 = 36$
H = 6

Thus, the height of the ladder is 6 m.

Q10. A ladder 50 dm long when set against the wall of a house just reaches a window at a height of 48 dm. How far is the lower end of the ladder from the base of the wall?



Let the distance of the lower end of the ladder from the wall be X m.

On using the Pythagoras theorem, we get:

$$X^2 + 48^2 = 50^2 \; X^2 = 50^2 - 48^2 = 2500 - 2304 = 196$$

H = 14dm

Hence, the distance of the lower end of the ladder from the wall is 14 dm.

Q11 The two legs of a right triangle are equal and the square of the hypotenuse is 50. Find the length of each leg.

Let the length of each leg of the given triangle be x units.

Using the Pythagoras theorem, we get :

$$X^2 + X^2 = (Hypotenuse)^2 \; X^2 + X^2 = 50 \; 2X^2 = 50 \; X^2 = 25$$

X=5

Hence, we can say that the length of each leg is 5 units.

Q12. Verity that the following numbers represent Pythagorean triplet:

(i) 12, 35, 37

(ii) 7, 24, 25

(iii) 27, 36, 45

(iv) 15, 36, 39

We will check for a Pythagorean triplet by checking if the square of the largest side is equal to the sum of the squares of the other two sides.

(i) $37^2 = 1369$

$12^2 + 35^2 = 144 + 1225 = 1369\ 12^2 + 35^2 = 37^2$

Yes, they represent a Pythagorean triplet.

(ii) $25^2 = 625$

$7^2 + 24^2 = 49 + 576 = 625\ 7^2 + 24^2 = 25^2$

Yes, they represent a Pythagorean triplet.

(iii) $45^2 = 2025$

$27^2 + 36^2 = 729 + 1296 = 2025\ 27^2 + 36^2 = 45^2$

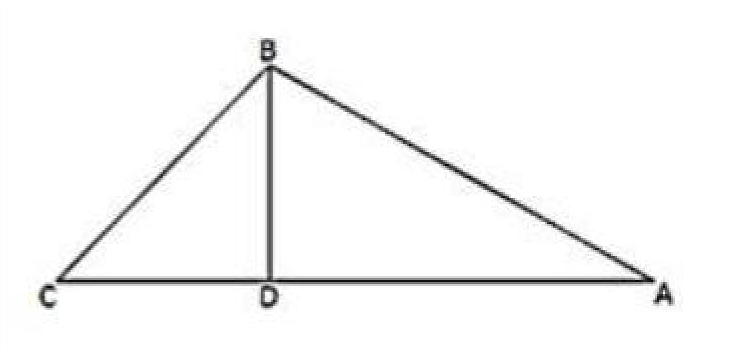
Yes, they represent a Pythagorean triplet.

(iv) $39^2 = 1521$

$15^2 + 36^2 = 225 + 1296 = 1521\ 15^2 + 36^2 = 39^2$

Yes, they represent a Pythagorean triplet.

Q13. In $\triangle ABC$, $\angle ABC = 100^\circ$, $\angle BAC = 35^\circ$ and BD perpendicular to AC meets side AC in D. If BD = 2 cm, find $\angle C$, and length DC.



We know that the sum of all angles of a triangle is 180°

Therefore, for the given $\triangle ABC$, we can say that :

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$

$100^\circ + 35^\circ + \angle ACB = 180^\circ$

$\angle ACB = 180^\circ - 135^\circ$

$\angle ACB = 45^\circ$

$\angle C = 45^\circ$

If we apply the above rule on $\triangle BCD$, we can say that :

$\angle BCD + \angle BDC + \angle CBD = 180^\circ$

$45^\circ + 90^\circ + \angle CBD = 180^\circ$ ($\angle ACB = \angle BCD$ and BD parallel to AC)

$\angle CBD = 180^\circ - 135^\circ$

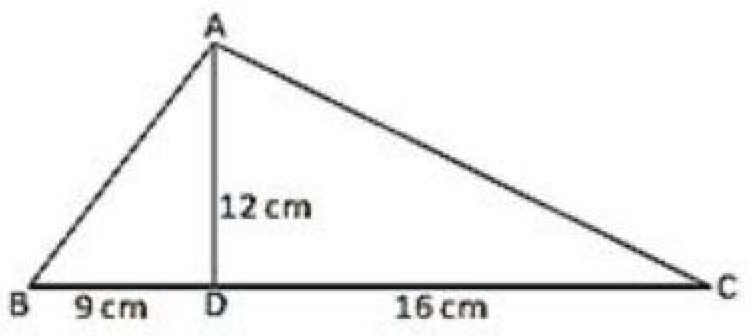
$\angle CBD = 45^\circ$

We know that the sides opposite to equal angles have equal length.

Thus, BD = DC

DC = 2 cm

Q14. In a $\triangle ABC$, AD is the altitude from A such that AD = 12 cm. BD = 9 cm and DC = 16 cm. Examine if $\triangle ABC$ is right angled at A.



In $\triangle ADC$,

$\angle ADC$ = 90° (AD is an altitude on BC)

Using the Pythagoras theorem, we get:

$12^2 + 16^2 = AC^2 \; AC^2 = 144 + 256 = 400$
AC=20cm

In $\triangle ADB$,

$\angle ADB$ = 90° (AD is an altitude on BC)

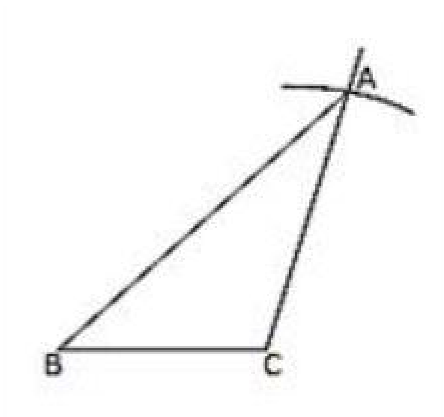
Using the Pythagoras theorem, we get:

$12^2 + 9^2 = AB^2 \; AB^2 = 144 + 81 = 225$
AB=15cm

In $\triangle ABC$,

$BC^2 = 25^2 = 625 \; AB^2 + AC^2 = 15^2 + 20^2 = 625 \; AB^2 + AC^2 = BC^2$
Because it satisfies the Pythagoras theorem, we can say that $\triangle ABC$ is right angled at A.

Q15. Draw a triangle ABC, with AC= 4 cm, BC= 3 cm and $\angle C = 105^\circ$. Measure AB. Is $(AC)^2 + (BC)^2$? If not which one of the following is true: $(AB)^2 > (AC)^2 + (BC)^2$ or $(AB)^2 < (AC)^2 + (BC)^2$



Draw $\triangle ABC$.

Draw a line BC = 3 cm.

At point C, draw a line at 105° angle with BC.

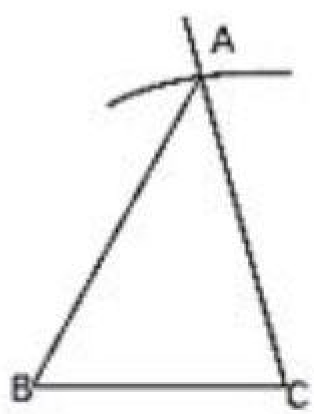
Take an arc of 4 cm from point C, which will cut the line at point A.

Now, join AB, which will be approximately 5.5 cm.

$$AC^2 + BC^2 = 4^2 + 3^2 = 9 + 16 = 25 \; AB^2 = 5.5^2 = 30.25$$
$$AB^2 \text{ not equal to } AC^2 + BC^2$$
Here,

$$AB^2 > AC^2 + BC^2$$

Q16. Draw a triangle ABC, with AC = 4 cm, BC=3 cm and $\angle C = 80^\circ$. Measure AB. Is $(AC)^2 + (BC)^2$? If not which one of the following is true: $(AB)^2 > (AC)^2 + (BC)^2$ or $(AB)^2 < (AC)^2 + (BC)^2$



Draw $\triangle ABC$.

Draw a line BC = 3 cm.

At point C, draw a line at 80° angle with BC.

Take an arc of 4 cm from point C, which will cut the line at point A.

Now, join AB; it will be approximately 4.5 cm.

$$AC^2 + BC^2 = 4^2 + 3^2 = 9 + 16 = 25 \; AB^2 = 4.5^2 = 20.25$$
$$AB^2 \text{ not equal to } AC^2 + BC^2$$
Here,

$$AB^2 < AC^2 + BC^2$$