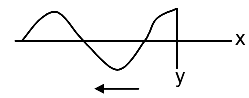
SOLUTIONS TO CONCEPTS
CHAPTER 15

1. $v = 40$ cm/sec
   As velocity of a wave is constant location of maximum after 5 sec
   $= 40 \times 5 = 200$ cm along negative x-axis.



2. Given $y = Ae^{-[(x/a)+(t/T)]^2}$
   a) $[A] = [M^0L^1T^0]$, $[T] = [M^0L^0T^1]$
      $[a] = [M^0L^1T^0]$
   b) Wave speed, $v = \lambda/T = a/T$ [Wave length $\lambda = a$]
   c) If $y = f(t - x/v) \rightarrow$ wave is traveling in positive direction
      and if $y = f(t + x/v) \rightarrow$ wave is traveling in negative direction
      So, $y = Ae^{-[(x/a)+(t/T)]^2} = Ae^{-(1/T)\left[\frac{x}{a/T}+t\right]^2}$
      $= Ae^{-(1/T)\left[\frac{x}{v}+t\right]^2}$
      i.e. $y = f\{t + (x/v)\}$
   d) Wave speed, $v = a/T$
      $\therefore$ Max. of pulse at $t = T$ is $(a/T) \times T = a$ (negative x-axis)
      Max. of pulse at $t = 2T = (a/T) \times 2T = 2a$ (along negative x-axis)
      So, the wave travels in negative x-direction.
3. At $t = 1$ sec, $s_1 = vt = 10 \times 1 = 10$ cm
   $t = 2$ sec, $s_2 = vt = 10 \times 2 = 20$ cm
   $t = 3$ sec, $s_3 = vt = 10 \times 3 = 30$ cm
4. The pulse is given by, $y = [(a^3) / \{(x - vt)^2 + a^2\}]$
   $a = 5$ mm $= 0.5$ cm, $v = 20$ cm/s
   At $t = 0$s, $y = a^3 / (x^2 + a^2)$
   The graph between y and x can be plotted by taking different values of x.
   (left as exercise for the student)
   similarly, at $t = 1$ s, $y = a^3 / \{(x - v)^2 + a^2\}$
   and at $t = 2$ s, $y = a^3 / \{(x - 2v)^2 + a^2\}$
5. At $x = 0$, $f(t) = a \sin(t/T)$
   Wave speed = v
   $\Rightarrow \lambda$ = wavelength = vT (T = Time period)
   So, general equation of wave
   $Y = A \sin[(t/T) - (x/vT)]$ [because $y = f((t/T) - (x/\lambda))$
6. At $t = 0$, $g(x) = A \sin(x/a)$
   a) $[M^0L^1T^0] = [L]$
      $a = [M^0L^1T^0] = [L]$
   b) Wave speed = v
      $\therefore$ Time period, $T = a/v$ (a = wave length = λ)
   $\therefore$ General equation of wave
   $y = A \sin\{(x/a) - t/(a/v)\}$
   $= A \sin\{(x - vt) / a\}$
7. At $t = t_0$, $g(x, t_0) = A \sin(x/a)$ …(1)
   For a wave traveling in the positive x-direction, the general equation is given by
   $y = f\left(\frac{x}{a} - \frac{t}{T}\right)$
   Putting $t = -t_0$ and comparing with equation (1), we get
   $\Rightarrow g(x, 0) = A \sin\{(x/a) + (t_0/T)\}$
   $\Rightarrow g(x, t) = A \sin\{(x/a) + (t_0/T) - (t/T)\}$

As T = a/v (a = wave length, v = speed of the wave)

$$\Rightarrow y = A\sin\left(\frac{x}{a} + \frac{t_0}{(a/v)} - \frac{t}{(a/v)}\right)$$

$$= A\sin\left(\frac{x + v(t_0 - t)}{a}\right)$$

$$\Rightarrow y = A\sin\left[\frac{x - v(t - t_0)}{a}\right]$$

8. The equation of the wave is given by

$y = (0.1\ mm)\sin[(31.4\ m^{-1})x + (314\ s^{-1})t]$ $\quad y = r\sin\{(2\pi x/\lambda)\} + \omega t)$

a) Negative x-direction

b) $k = 31.4\ m^{-1}$

$\Rightarrow 2\lambda/\lambda = 31.4 \Rightarrow \lambda = 2\pi/31.4 = 0.2$ mt = 20 cm

Again, $\omega = 314\ s^{-1}$

$\Rightarrow 2\pi f = 314 \Rightarrow f = 314 / 2\pi = 314 / (2 \times (3/14)\} = 50\ sec^{-1}$

$\therefore$ wave speed, $v = \lambda f = 20 \times 50 = 1000$ cm/s

c) Max. displacement = 0.10 mm

Max. velocity = $a\omega = 0.1 \times 10^{-1} \times 314 = 3.14$ cm/sec.

9. Wave speed, v = 20 m/s

A = 0.20 cm

λ = 2 cm

a) Equation of wave along the x-axis

$y = A\sin(kx - wt)$

$\therefore$ $k = 2\pi/\lambda = 2\pi/2 = \pi\ cm^{-1}$

$T = \lambda/v = 2/2000 = 1/1000\ sec = 10^{-3}$ sec

$\Rightarrow \omega = 2\pi/T = 2\pi \times 10^{-3}\ sec^{-1}$

So, the wave equation is,

$\therefore$ $y = (0.2\ cm)\sin[(\pi\ cm^{-1})x - (2\pi \times 10^{3}\ sec^{-1})t]$

b) At x = 2 cm, and t = 0,

$y = (0.2\ cm)\sin(\pi/2) = 0$

$\therefore$ $v = r\omega\cos\pi x = 0.2 \times 2000\ \pi \times \cos 2\pi = 400\ \pi$

$= 400 \times (3.14) = 1256$ cm/s

$= 400\ \pi$ cm/s $= 4\pi$ m/s

10. $Y = (1\ mm)\sin\pi\left[\frac{x}{2cm} - \frac{t}{0.01sec}\right]$

a) $T = 2 \times 0.01 = 0.02$ sec = 20 ms

$\lambda = 2 \times 2 = 4$ cm

b) $v = dy/dt = d/dt\,[\sin 2\pi\,(x/4 - t/0.02)] = -\cos 2\pi\,\{x/4) - (t/0.02)\} \times 1/(0.02)$

$\Rightarrow v = -50\cos 2\pi\,\{(x/4) - (t/0.02)\}$

at x = 1 and t = 0.01 sec, $v = -50\cos 2^*\,[(1/4) - (1/2)] = 0$

c) i) at x = 3 cm, t = 0.01 sec

$v = -50\cos 2\pi\,(3/4 - ½) = 0$

ii) at x = 5 cm, t = 0.01 sec, v = 0 (putting the values)

iii) at x = 7 cm, t = 0.01 sec, v = 0

at x = 1 cm and t = 0.011 sec

$v = -50\cos 2\pi\,\{(1/4) - (0.011/0.02)\} = -50\cos(3\pi/5) = -9.7$ cm/sec

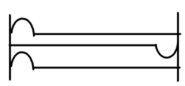
(similarly the other two can be calculated)

11. Time period, $T = 4 \times 5\ ms = 20 \times 10^{-3} = 2 \times 10^{-2}$ s

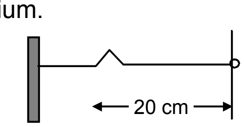
$\lambda = 2 \times 2$ cm = 4 cm

frequency, $f = 1/T = 1/(2 \times 10^{-2}) = 50\ s^{-1} = 50$ Hz

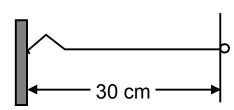
Wave speed = $\lambda f = 4 \times 50$ m/s = 2000 m/s = 2 m/s

12. Given that, $v = 200$ m/s
   a) Amplitude, $A = 1$ mm
   b) Wave length, $\lambda = 4$ cm
   c) wave number, $n = 2\pi/\lambda = (2 \times 3.14)/4 = 1.57\ \text{cm}^{-1}$ (wave number = k)
   d) frequency, $f = 1/T = (26/\lambda)/20 = 20/4 = 5$ Hz
   (where time period $T = \lambda/v$)
13. Wave speed = $v = 10$ m/sec
   Time period = $T = 20$ ms $= 20 \times 10^{-3} = 2 \times 10^{-2}$ sec
   a) wave length, $\lambda = vT = 10 \times 2 \times 10^{-2} = 0.2$ m = 20 cm
   b) wave length, $\lambda = 20$ cm
   $\therefore$ phase diff^n = $(2\pi/\lambda)\, x = (2\pi / 20) \times 10 = \pi$ rad
   $y_1 = a \sin(\omega t - kx) \Rightarrow 1.5 = a \sin(\omega t - kx)$
   So, the displacement of the particle at a distance x = 10 cm.
   [$\phi = \dfrac{2\pi x}{\lambda} = \dfrac{2\pi \times 10}{20} = \pi$] is given by
   $y_2 = a \sin(\omega t - kx + \pi) \Rightarrow -a \sin(\omega t - kx) = -1.5$ mm
   $\therefore$ displacement = -1.5 mm
14. mass = 5 g, length l = 64 cm
   $\therefore$ mass per unit length = m = 5/64 g/cm
   $\therefore$ Tension, $T = 8\text{N} = 8 \times 10^5$ dyne
   $V = \sqrt{(T/m)} = \sqrt{(8 \times 10^5 \times 64)/5} = 3200$ cm/s = 32 m/s
15. 

   a) Velocity of the wave, $v = \sqrt{(T/m)} = \sqrt{(16 \times 10^5)/0.4} = 2000$ cm/sec
   $\therefore$ Time taken to reach to the other end = 20/2000 = 0.01 sec
   Time taken to see the pulse again in the original position = $0.01 \times 2 = 0.02$ sec
   b) At t = 0.01 s, there will be a 'though' at the right end as it is reflected.
16. The crest reflects as a crest here, as the wire is traveling from denser to rarer medium.
   $\Rightarrow$ phase change = 0

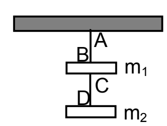


   a) To again original shape distance travelled by the wave S = 20 + 20 = 40 cm.
   Wave speed, $v = 20$ m/s $\Rightarrow$ time = s/v = 40/20 = 2 sec
   b) The wave regains its shape, after traveling a periodic distance = $2\times30 = 60$ cm
   $\therefore$ Time period = 60/20 = 3 sec.

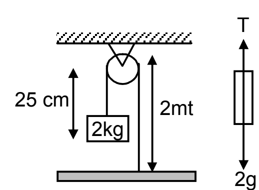


   c) Frequency, $n = (1/3\ \text{sec}^{-1})$
   $n = (1/2l)\sqrt{(T/m)}$ m = mass per unit length = 0.5 g/cm
   $\Rightarrow 1/3 = 1/(2 \times 30)\ \sqrt{(T/0.5)}$
   $\Rightarrow T = 400 \times 0.5 = 200$ dyne $= 2 \times 10^{-3}$ Newton.
17. Let v_1 = velocity in the 1st string
   $\Rightarrow v_1 = \sqrt{(T/m_1)}$
   Because m_1 = mass per unit length = $(\rho_1 a_1 l_1 / l_1) = \rho_1 a_1$ where a_1 = Area of cross section
   $\Rightarrow v_1 = \sqrt{(T/\rho_1 a_1)}$ …(1)
   Let v_2 = velocity in the second string
   $\Rightarrow v_2 = \sqrt{(T/m^2)}$
   $\Rightarrow v_2 = \sqrt{(T/\rho_2 a_2)}$ …(2)
   Given that, $v_1 = 2v_2$
   $\Rightarrow \sqrt{(T/\rho_1 a_1)} = 2\sqrt{(T/\rho_2 a_2)} \Rightarrow (T/a_1\rho_1) = 4(T/a_2\rho_2)$
   $\Rightarrow \rho_1/\rho_2 = 1/4 \Rightarrow \rho_1 : \rho_2 = 1 : 4$ (because $a_1 = a_2$)

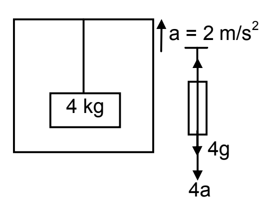
18. m = mass per unit length = 1.2×10^{-4} kg/mt
    $Y = (0.02m) \sin [(1.0\ m^{-1})x + (30\ s^{-1})t]$
    Here, $k = 1\ m^{-1} = 2\pi/\lambda$
    $\omega = 30\ s^{-1} = 2\pi f$
    $\therefore$ velocity of the wave in the stretched string
    $v = \lambda f = \omega/k = 30/I = 30$ m/s
    $\Rightarrow v = \sqrt{T/m} \Rightarrow 30\sqrt{(T/1.2)\times 10^{-4} N)}$
    $\Rightarrow T = 10.8 \times 10^{-2}\ N \Rightarrow T = 1.08 \times 10^{-1}$ Newton.
19. Amplitude, A = 1 cm, Tension T = 90 N
    Frequency, f = 200/2 = 100 Hz
    Mass per unit length, m = 0.1 kg/mt
    a) $\Rightarrow V = \sqrt{T/m} = 30$ m/s
       $\lambda = V/f = 30/100 = 0.3$ m = 30 cm
    b) The wave equation y = (1 cm) cos 2π (t/0.01 s) – (x/30 cm)
       [because at x = 0, displacement is maximum]
    c) $y = 1 \cos 2\pi(x/30 - t/0.01)$
       $\Rightarrow v = dy/dt = (1/0.01)2\pi \sin 2\pi \{(x/30) - (t/0.01)\}$
       $a = dv/dt = -\{4\pi^2 / (0.01)^2\} \cos 2\pi \{(x/30) - (t/0.01)\}$
       When, x = 50 cm, t = 10 ms = 10×10^{-3} s
       $x = (2\pi / 0.01) \sin 2\pi \{(5/3) - (0.01/0.01)\}$
       $= (p/0.01) \sin (2\pi \times 2 / 3) = (1/0.01) \sin (4\pi/3) = -200\ \pi \sin (\pi/3) = -200\ \pi x\ (\sqrt{3}/2)$
       = 544 cm/s = 5.4 m/s
       Similarly
       $a = \{4\pi^2 / (0.01)^2\} \cos 2\pi \{(5/3) - 1\}$
       $= 4\pi^2 \times 10^4 \times ½ \Rightarrow 2 \times 10^5\ cm/s^2 \Rightarrow 2\ km/s^2$
20. l = 40 cm, mass = 10 g
    $\therefore$ mass per unit length, m = 10 / 40 = 1/4 (g/cm)
    spring constant K = 160 N/m
    deflection = x = 1 cm = 0.01 m
    $\Rightarrow T = kx = 160 \times 0.01 = 1.6\ N = 16 \times 10^4$ dyne
    Again $v = \sqrt{(T/m)} = \sqrt{(16 \times 10^4 /(1/4)} = 8 \times 10^2$ cm/s = 800 cm/s
    $\therefore$ Time taken by the pulse to reach the spring
    t = 40/800 = 1/20 = 0/05 sec.
21. $m_1 = m_2 = 3.2$ kg
    mass per unit length of AB = 10 g/mt = 0.01 kg.mt
    mass per unit length of CD = 8 g/mt = 0.008 kg/mt
    for the string CD, $T = 3.2 \times g$
    $\Rightarrow v = \sqrt{(T/m)} = \sqrt{(3.2\times 10)/0.008} = \sqrt{(32 \times 10^3)/8} = 2 \times 10\sqrt{10} = 20 \times 3.14 = 63$ m/s
    for the string AB, $T = 2 \times 3.2\ g = 6.4 \times g = 64$ N
    $\Rightarrow v = \sqrt{(T/m)} = \sqrt{(64/0.01)} = \sqrt{6400} = 80$ m/s



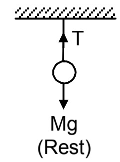
22. Total length of string 2 + 0.25 = 2.25 mt
    Mass per unit length $m = \dfrac{4.5 \times 10^{-3}}{2.25} = 2 \times 10^{-3}$ kg/m
    T = 2g = 20 N
    Wave speed, $v = \sqrt{(T/m)} = \sqrt{20}/(2 \times 10^{-3}) = \sqrt{10^4} = 10^2$ m/s = 100 m/s
    Time taken to reach the pully, t = (s/v) = 2/100 = 0.02 sec.



23. $m = 19.2 \times 10^{-3}$ kg/m
    from the freebody diagram,
    T – 4g – 4a = 0
    $\Rightarrow T = 4(a + g) = 48$ N
    wave speed, $v = \sqrt{(T/m)} = 50$ m/s



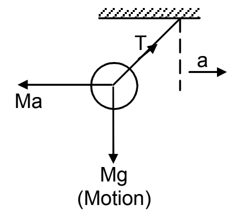
24. Let M = mass of the heavy ball
(m = mass per unit length)
Wave speed, $v_1 = \sqrt{(T/m)} = \sqrt{(Mg/m)}$ (because T = Mg)
$\Rightarrow 60 = \sqrt{(Mg/m)} \Rightarrow Mg/m = 60^2$ …(1)



From the freebody diagram (2),
$v_2 = \sqrt{(T'/m)}$

$\Rightarrow v_2 = \dfrac{[(Ma)^2 + (Mg)^2]^{1/4}}{m^{1/2}}$ (because T' = $\sqrt{(Ma)^2 + (Mg)^2}$)

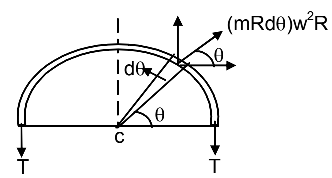
$\Rightarrow 62 = \dfrac{[(Ma)^2 + (Mg)^2]^{1/4}}{m^{1/2}}$



$\Rightarrow \dfrac{\sqrt{(Ma)^2 + (Mg)^2}}{m} = 62^2$ …(2)

Eq(1) + Eq(2) $\Rightarrow$ (Mg/m) $\times$ [m / $\sqrt{(Ma)^2 + (Mg)^2}$] = 3600 / 3844
$\Rightarrow g / \sqrt{(a^2 + g^2)} = 0.936 \Rightarrow g^2 / (a^2 + g^2) = 0.876$
$\Rightarrow (a^2 + 100)\ 0.876 = 100$
$\Rightarrow a^2 \times 0.876 = 100 - 87.6 = 12.4$
$\Rightarrow a^2 = 12.4 / 0.876 = 14.15 \Rightarrow a = 3.76\ m/s^2$
$\therefore$ Acce^n of the car = 3.7 m/s^2

25. m = mass per unit length of the string
R = Radius of the loop
ω = angular velocity, V = linear velocity of the string
Consider one half of the string as shown in figure.
The half loop experiences cetrifugal force at every point, away from centre, which is balanced by tension 2T.
Consider an element of angular part $d\theta$ at angle θ. Consider another element symmetric to this centrifugal force experienced by the element = $(mRd\theta)\omega^2R$.

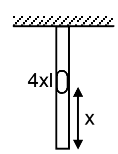


(…Length of element = $Rd\theta$, mass = $mRd\theta$)
Resolving into rectangular components net force on the two symmetric elements,
DF = $2mR^2\, d\theta\omega^2 \sin\theta$ [horizontal components cancels each other]

So, total F = $\int_0^{\pi/2} 2mR^2\omega^2 \sin\theta\, d\theta = 2mR^2\omega^2\, [-\cos\theta] \Rightarrow 2mR^2\omega^2$

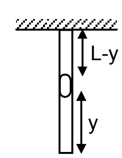
Again, $2T = 2mR^2\omega^2$ $\Rightarrow T = mR^2\omega^2$
Velocity of transverse vibration $V = \sqrt{T/m} = \omega R = V$
So, the speed of the disturbance will be V.

26. a) m $\rightarrow$ mass per unit of length of string
consider an element at distance 'x' from lower end.
Here wt acting down ward = (mx)g = Tension in the string of upper part
Velocity of transverse vibration = v = $\sqrt{T/m} = \sqrt{(mgx/m)} = \sqrt{(gx)}$



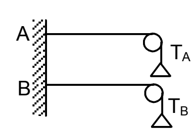
b) For small displacement dx, dt = dx / $\sqrt{(gx)}$

Total time T = $\int_0^L dx/\sqrt{gx} = \sqrt{(4L/g)}$



c) Suppose after time 't' from start the pulse meet the particle at distance y from lower end.

t = $\int_0^y dx/\sqrt{gx} = \sqrt{(4y/g)}$



$\therefore$ Distance travelled by the particle in this time is (L – y)

$\therefore$ S – ut + 1/2 gt^2

$\Rightarrow$ L – y (1/2)g $\times$ $\{\sqrt{(4y/g)^2}\}$ {u = 0}

$\Rightarrow$ L – y = 2y $\Rightarrow$ 3y = L

$\Rightarrow$ y = L/3. So, the particle meet at distance L/3 from lower end.

27. $m_A = 1.2 \times 10^{-2}$ kg/m, T_A = 4.8 N

$\Rightarrow V_A = \sqrt{T/m}$ = 20 m/s

$m_B = 1.2 \times 10^{-2}$ kg/m, T_B = 7.5 N

$\Rightarrow V_B = \sqrt{T/m}$ = 25 m/s

t = 0 in string A

$t_1 = 0 + 20$ ms $= 20 \times 10^{-3} = 0.02$ sec

In 0.02 sec A has travelled 20 $\times$ 0.02 = 0.4 mt

Relative speed between A and B = 25 – 20 = 5 m/s

Time taken for B for overtake A = s/v = 0.4/5 = 0.08 sec

28. r = 0.5 mm = 0.5×10^{-3} mt

f = 100 Hz, T = 100 N

v = 100 m/s

$v = \sqrt{T/m} \Rightarrow v^2 = (T/m) \Rightarrow m = (T/v^2) = 0.01$ kg/m

$P_{ave} = 2\pi^2 mvr^2f^2$

$= 2(3.14)^2(0.01) \times 100 \times (0.5 \times 10^{-3})^2 \times (100)^2 \Rightarrow 49 \times 10^{-3}$ watt = 49 mW.

29. A = 1 mm = 10^{-3} m, m = 6 g/m = 6×10^{-3} kg/m

T = 60 N, f = 200 Hz

$\therefore V = \sqrt{T/m}$ = 100 m/s

a) $P_{average} = 2\pi^2$ mv A^2f^2 = 0.47 W

b) Length of the string is 2 m. So, t = 2/100 = 0.02 sec.

Energy = $2\pi^2$ mvf^2A^2t = 9.46 mJ.

30. f = 440 Hz, m = 0.01 kg/m, T = 49 N, r = 0.5×10^{-3} m

a) $v = \sqrt{T/m}$ = 70 m/s

b) $v = \lambda f \Rightarrow \lambda = v/f$ = 16 cm

c) $P_{average} = 2\pi^2 mvr^2f^2$ = 0.67 W.

31. Phase difference $\phi = \pi/2$

f and λ are same. So, ω is same.

y_1 = r sin wt, y_2 = rsin(wt + π/2)

From the principle of superposition

$y = y_1 + y_2$ $\rightarrow$ = r sin wt + r sin (wt + π/2)

= r[sin wt + sin(wt + π/2)]

= r[2sin{(wt + wt + π/2)/2} cos {(wt – wt – π/2)/2}]

$\Rightarrow$ y = 2r sin (wt + π/4) cos (–π/4)

Resultant amplitude = $\sqrt{2}$ r = 4$\sqrt{2}$ mm (because r = 4 mm)

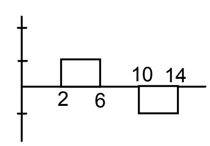
32. The distance travelled by the pulses are shown below.

t = 4 ms = 4×10^{-3} s s = vt = $50 \times 10 \times 4 \times 10^{-3}$ = 2 mm

t = 8 ms = 8×10^{-3} s s = vt = $50 \times 10 \times 8 \times 10^{-3}$ = 4 mm

t = 6 ms = 6×10^{-3} s s = 3 mm

t = 12 ms = 12×10^{-3} s s = $50 \times 10 \times 12 \times 10^{-3}$ = 6 mm



The shape of the string at different times are shown in the figure.

33. f = 100 Hz, λ = 2 cm = 2×10^{-2} m

$\therefore$ wave speed, v = fλ = 2 m/s

a) in 0.015 sec 1st wave has travelled

x = 0.015 $\times$ 2 = 0.03 m = path diff^n

$\therefore$ corresponding phase difference, $\phi = 2\pi x/\lambda = \{2\pi / (2 \times 10^{-2})\} \times 0.03 = 3\pi$.

b) Path different x = 4 cm = 0.04 m

$\Rightarrow \phi = (2\pi/\lambda)x = \{(2\pi/2 \times 10^{-2}) \times 0.04\} = 4\pi$.

c) The waves have same frequency, same wavelength and same amplitude.
Let, $y_1 = r \sin wt$, $y_2 = r \sin (wt + \phi)$
$\Rightarrow y = y_1 + y_2 = r[\sin wt + (wt + \phi)]$
$= 2r \sin (wt + \phi/2) \cos (\phi/2)$
$\therefore$ resultant amplitude $= 2r \cos \phi/2$
So, when $\phi = 3\pi$, $r = 2 \times 10^{-3}$ m
$R_{res} = 2 \times (2 \times 10^{-3}) \cos (3\pi/2) = 0$
Again, when $\phi = 4\pi$, $R_{res} = 2 \times (2 \times 10^{-3}) \cos (4\pi/2) = 4$ mm.

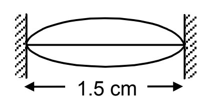
34. $l = 1$ m, $V = 60$ m/s
$\therefore$ fundamental frequency, $f_0 = V/2l = 30\ \text{sec}^{-1} = 30$ Hz.

35. $l = 2$m, $f_0 = 100$ Hz, $T = 160$ N
$f_0 = 1/2l\sqrt{(T/m)}$
$\Rightarrow m = 1$ g/m. So, the linear mass density is 1 g/m.

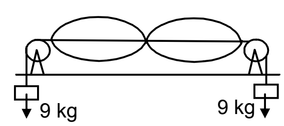
36. $m = (4/80)$ g/ cm $= 0.005$ kg/m
$T = 50$ N, $l = 80$ cm $= 0.8$ m
$v = \sqrt{(T/m)} = 100$ m/s
fundamental frequency $f_0 = 1/2l\sqrt{(T/m)} = 62.5$ Hz
First harmonic = 62.5 Hz
f_4 = frequency of fourth harmonic $= 4f_0 = F_3 = 250$ Hz
$V = f_4 \lambda_4 \Rightarrow \lambda_4 = (v/f_4) = 40$ cm.

37. $l = 90$ cm $= 0.9$ m
$m = (6/90)$ g/cm $= (6/900)$ kg/mt
$f = 261.63$ Hz
$f = 1/2l\sqrt{(T/m)} \Rightarrow T = 1478.52\ \text{N} = 1480$ N.

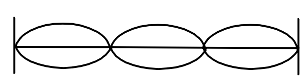
38. First harmonic be f_0, second harmonic be f_1
$\therefore f_1 = 2f_0$
$\Rightarrow f_0 = f_1/2$
$f_1 = 256$ Hz
$\therefore 1^{st}$ harmonic or fundamental frequency
$f_0 = f_1/2 = 256 / 2 = 128$ Hz
$\lambda/2 = 1.5\ \text{m} \Rightarrow \lambda = 3$m (when fundamental wave is produced)
$\Rightarrow$ Wave speed $= V = f_0Ql = 384$ m/s.



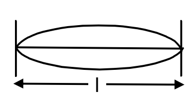
39. $l = 1.5$ m, mass – 12 g
$\Rightarrow m = 12/1.5\ \text{g/m} = 8 \times 10^{-3}$ kg/m
$T = 9 \times g = 90$ N
$\lambda = 1.5$ m, $f_1 = 2/2l\sqrt{T/m}$
[for, second harmonic two loops are produced]
$f_1 = 2f_0 \Rightarrow 70$ Hz.



40. A string of mass 40 g is attached to the tuning fork
$m = (40 \times 10^{-3})$ kg/m
The fork vibrates with $f = 128$ Hz
$\lambda = 0.5$ m
$v = f\lambda = 128 \times 0.5 = 64$ m/s
$v = \sqrt{T/m} \Rightarrow T = v^2m = 163.84\ \text{N} \Rightarrow 164$ N.



41. This wire makes a resonant frequency of 240 Hz and 320 Hz.
The fundamental frequency of the wire must be divisible by both 240 Hz and 320 Hz.
a) So, the maximum value of fundamental frequency is 80 Hz.
b) Wave speed, $v = 40$ m/s
$\Rightarrow 80 = (1/2l) \times 40 \Rightarrow 0.25$ m.



42. Let there be 'n' loops in the 1st case

$\Rightarrow$ length of the wire, $l = (n\lambda_1)/2$ [$\lambda_1 = 2 \times 2 = 4$ cm]

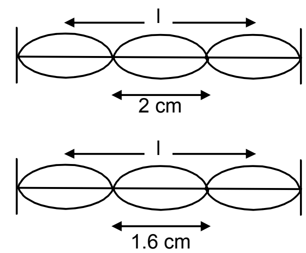
So there are (n + 1) loops with the 2nd case

$\Rightarrow$ length of the wire, $l = \{(n+1)\lambda_2/2$ [$\lambda = 2 \times 1.6 = 3.2$ cm]

$\Rightarrow n\lambda_1/2 = \dfrac{(n+1)\lambda_2}{2}$

$\Rightarrow n \times 4 = (n + 1)(3.2) \Rightarrow n = 4$

$\therefore$ length of the string, $l = (n\lambda_1)/2 = 8$ cm.



43. Frequency of the tuning fork, f = 660 Hz

Wave speed, v = 220 m/s $\Rightarrow \lambda = v/f = 1/3$ m

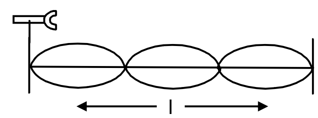
No.of loops = 3

a) So, $f = (3/2l)v \Rightarrow l = 50$ cm

b) The equation of resultant stationary wave is given by

$y = 2A \cos(2\pi x/Ql) \sin(2\pi vt/\lambda)$

$\Rightarrow y = (0.5 \text{ cm}) \cos(0.06\,\pi \text{ cm}^{-1}) \sin(1320\,\pi s^{-1} t)$



44. $l_1 = 30$ cm = 0.3 m

$f_1 = 196$ Hz, $f_2 = 220$ Hz

We know $f \propto (1/l)$ (as V is constant for a medium)

$\Rightarrow \dfrac{f_1}{f_2} = \dfrac{l_2}{l_1} \Rightarrow l_2 = 26.7$ cm

Again $f_3 = 247$ Hz

$\Rightarrow \dfrac{f_3}{f_1} = \dfrac{l_1}{l_3} \Rightarrow \dfrac{0.3}{l_3}$

$\Rightarrow l_3 = 0.224$ m = 22.4 cm and $l_3 = 20$ cm

45. Fundamental frequency $f_1 = 200$ Hz

Let l_4 Hz be nth harmonic

$\Rightarrow F_2/F_1 = 14000/200$

$\Rightarrow NF_1/F_1 = 70 \Rightarrow N = 70$

$\therefore$ The highest harmonic audible is 70th harmonic.

46. The resonant frequencies of a string are

$f_1 = 90$ Hz, $f_2 = 150$ Hz, $f_3 = 120$ Hz

a) The highest possible fundamental frequency of the string is f = 30 Hz

[because f_1, f_2 and f_3 are integral multiple of 30 Hz]

b) The frequencies are $f_1 = 3f$, $f_2 = 5f$, $f_3 = 7f$

So, f_1, f_2 and f_3 are 3rd harmonic, 5th harmonic and 7th harmonic respectively.

c) The frequencies in the string are f, 2f, 3f, 4f, 5f, ……….

So, 3f = 2nd overtone and 3rd harmonic

5f = 4th overtone and 5th harmonic

7f = 6th overtone and 7th harmonic

d) length of the string is l = 80 cm

$\Rightarrow f_1 = (3/2l)v$ (v = velocity of the wave)

$\Rightarrow 90 = \{3/(2\times80)\} \times K$

$\Rightarrow K = (90 \times 2 \times 80) / 3 = 4800$ cm/s = 48 m/s.

47. Frequency $f = \dfrac{1}{lD}\sqrt{\dfrac{T}{\pi\rho}} \Rightarrow f_1 = \dfrac{1}{l_1D_1}\sqrt{\dfrac{T_1}{\pi\rho_1}} \Rightarrow f_2 = \dfrac{1}{l_2n_2}\sqrt{\dfrac{T_2}{\pi\rho_2}}$

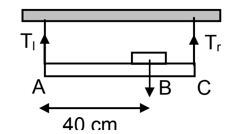
Given that, $T_1/T_2 = 2$, $r_1 / r_2 = 3 = D_1/D_2$

$\dfrac{\rho_1}{\rho_2} = \dfrac{1}{2}$

So, $\dfrac{f_1}{f_2} = \dfrac{l_2D_2}{l_1D_1}\sqrt{\dfrac{T_1}{T_2}}\sqrt{\dfrac{\pi\rho_2}{\pi\rho_1}}$ ($l_1 = l_2$ = length of string)

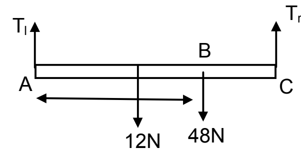
$\Rightarrow f_1 : f_2 = 2 : 3$

48. Length of the rod = L = 40 cm = 0.4 m
Mass of the rod m = 1.2 kg
Let the 4.8 kg mass be placed at a distance 'x' from the left end.
Given that, $f_l = 2f_r$



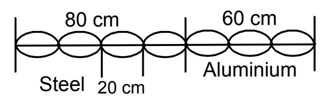
$\therefore \frac{1}{2l}\sqrt{\frac{T_l}{m}} = \frac{2}{2l}\sqrt{\frac{T_r}{m}}$

$\Rightarrow \sqrt{\frac{T_l}{T_r}} = 2 \Rightarrow \frac{T_l}{T_r} = 4$ …(1)



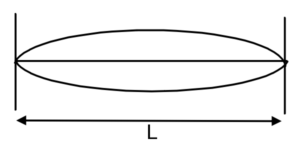
From the freebody diagram,
$T_l + T_r = 60$ N
$\Rightarrow 4T_r + T_r = 60$ N
$\therefore T_r = 12$ N and $T_l = 48$ N
Now taking moment about point A,
$T_r \times (0.4) = 48x + 12\,(0.2) \Rightarrow x = 5$ cm
So, the mass should be placed at a distance 5 cm from the left end.

49. $\rho_s = 7.8$ g/cm^3 , $\rho_A = 2.6$ g/cm^3
$m_s = \rho_s A_s = 7.8 \times 10^{-2}$ g/cm (m = mass per unit length)
$m_A = \rho_A A_A = 2.6 \times 10^{-2} \times 3$ g/cm $= 7.8 \times 10^{-3}$ kg/m



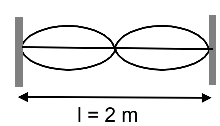
A node is always placed in the joint. Since aluminium and steel rod has same mass per unit length, velocity of wave in both of them is same.
$\Rightarrow v = \sqrt{T/m} \Rightarrow 500/7$ m/x
For minimum frequency there would be maximum wavelength for maximum wavelength minimum no of loops are to be produced.
$\therefore$ maximum distance of a loop = 20 cm
$\Rightarrow$ wavelength = $\lambda = 2 \times 20 = 40$ cm = 0.4 m
$\therefore f = v/\lambda = 180$ Hz.

50. Fundamental frequency
$V = 1/2l\ \sqrt{T/m} \Rightarrow \sqrt{T/m} = v2l$ [$\sqrt{T/m}$ = velocity of wave]



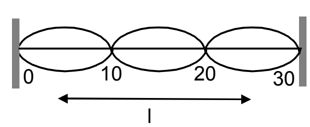
a) wavelength, λ = velocity / frequency = $v2l / v = 2l$
and wave number = $K = 2\pi/\lambda = 2\pi/2l = \pi/l$
b) Therefore, equation of the stationary wave is
$y = A\cos(2\pi x/\lambda)\sin(2\pi Vt / L)$
$= A\cos(2\pi x / 2l)\sin(2\pi Vt / 2L)$
$v = V/2L$ [because $v = (v/2l)$]

51. V = 200 m/s, 2A = 0.5 m
a) The string is vibrating in its 1st overtone
$\Rightarrow \lambda = 1 = 2$m
$\Rightarrow f = v/\lambda = 100$ Hz



b) The stationary wave equation is given by
$y = 2A\ \cos\frac{2\pi x}{\lambda}\sin\frac{2\pi Vt}{\lambda}$
$= (0.5\text{ cm})\cos[(\pi m^{-1})x]\sin[(200\ \pi s^{-1})t]$

52. The stationary wave equation is given by
$y = (0.4\text{ cm})\sin[(0.314\text{ cm} - 1)x]\cos[(6.00\ \pi s^{-1})t]$
a) $\omega = 600\ \pi \Rightarrow 2\pi f = 600\ \pi \Rightarrow f = 300$ Hz
wavelength, $\lambda = 2\pi/0.314 = (2 \times 3.14) / 0.314 = 20$ cm



b) Therefore nodes are located at, 0, 10 cm, 20 cm, 30 cm
c) Length of the string = $3\lambda/2 = 3 \times 20/2 = 30$ cm
d) $y = 0.4\sin(0.314\,x)\cos(600\ \pi t) \Rightarrow 0.4\sin\{(\pi/10)x\}\cos(600\ \pi t)$
since, λ and v are the wavelength and velocity of the waves that interfere to give this vibration λ = 20 cm

$v = \omega/k = 6000$ cm/sec = 60 m/s

53. The equation of the standing wave is given by

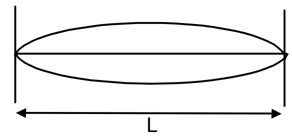
$y = (0.4 \text{ cm}) \sin [(0.314 \text{ cm}^{-1})x] \cos [(6.00 \pi s^{-1})t]$

$\Rightarrow k = 0.314 = \pi/10$

$\Rightarrow 2\pi/\lambda = \pi/10 \Rightarrow \lambda = 20$ cm

for smallest length of the string, as wavelength remains constant, the string should vibrate in fundamental frequency

$\Rightarrow l = \lambda/2 = 20 \text{ cm} / 2 = 10$ cm



54. L = 40 cm = 0.4 m, mass = 3.2 kg = 3.2×10^{-3} kg

$\therefore$ mass per unit length, m = (3.2)/(0.4) = 8×10^{-3} kg/m

change in length, $\Delta L = 40.05 - 40 = 0.05 \times 10^{-2}$ m

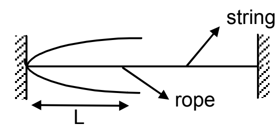
strain = $\Delta L/L = 0.125 \times 10^{-2}$ m

f = 220 Hz

$$f = \frac{1}{2l'}\sqrt{\frac{T}{m}} = \frac{1}{2 \times (0.4005)}\sqrt{\frac{T}{8 \times 10^{-3}}} \Rightarrow T = 248.19 \text{ N}$$

Strain = 248.19/1 mm^2 = 248.19×10^6

Y = stress / strain = 1.985×10^{11} N/m^2



55. Let, $\rho \rightarrow$ density of the block

Weight ρ Vg where V = volume of block

The same turning fork resonates with the string in the two cases

$$f_{10} = \frac{10}{2l}\sqrt{\frac{T - \rho_w Vg}{m}} = \frac{11}{2l}\sqrt{\frac{(\rho - \rho_w)Vg}{m}}$$

As the f of tuning fork is same,

$$f_{10} = f_{11} \Rightarrow \frac{10}{2l}\sqrt{\frac{\rho Vg}{m}} = \frac{11}{2l}\sqrt{\frac{(\rho - \rho_w)Vg}{m}}$$

$$\Rightarrow \frac{10}{11} = \sqrt{\frac{\rho - \rho_w}{m}} \Rightarrow \frac{\rho - 1}{\rho} = \frac{100}{121} \qquad \text{(because, } \rho_w = 1 \text{ gm/cc)}$$

$\Rightarrow 100\rho = 121\rho - 121 \Rightarrow 5.8 \times 10^3$ kg/m^3

56. l = length of rope = 2 m

M = mass = 80 gm = 0.8 kg

mass per unit length = m = 0.08/2 = 0.04 kg/m

Tension T = 256 N

Velocity, V = $\sqrt{T/m}$ = 80 m/s

For fundamental frequency,

$l = \lambda/4 \Rightarrow \lambda = 4l = 8$ m

$\Rightarrow f = 80/8 = 10$ Hz

a) Therefore, the frequency of 1st two overtones are

1st overtone = 3f = 30 Hz

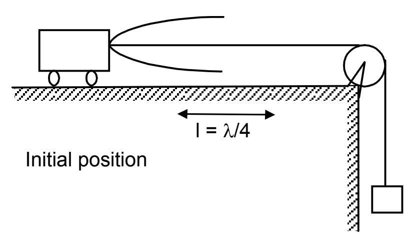
2nd overtone = 5f = 50 Hz

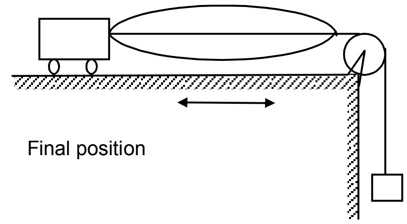
b) $\lambda_1 = 4l = 8$ m

$\lambda_1 = V/f_1 = 2.67$ m

$\lambda_2 = V/f_2 = 1.6$ mt

so, the wavelengths are 8 m, 2.67 m and 1.6 m respectively.





57. Initially because the end A is free, an antinode will be formed.

So, $l = Ql_1 / 4$

Again, if the movable support is pushed to right by 10 m, so that the joint is placed on the pulley, a node will be formed there.

So, $l = \lambda_2 / 2$

Since, the tension remains same in both the cases, velocity remains same.

As the wavelength is reduced by half, the frequency will become twice as that of 120 Hz i.e. 240 Hz.

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