

(ii) Structural resistance : The structural arrangement of soil particles also affects the shear stress. The same soil may exhibit remarkably different shear strength at different void ratios and also at different rates of loading.

(iii) Friction : The internal friction between soil particles resists the shearing friction of soil mass. The friction may be either sliding friction or a rolling friction or a combination of the two.

Angle of Internal Friction

The measure of the resistance of soil to sliding along a plane, is termed as *angle of internal friction* or *friction angle* ϕ .

The angle of internal friction of soil mass depends upon,

- (i) Shape of particles
- (ii) Surface roughness
- (iii) Type of interlocking
- (iv) Lateral pressure, and
- (v) State of packing

Mohr's Circle for Cohesion and Angle of Internal Friction

The value of cohesion and angle of internal friction can be determined graphically by drawing Mohr's circle of stresses as under :

Minor principal stress,

$$\sigma_3 = \text{horizontal water pressure}$$

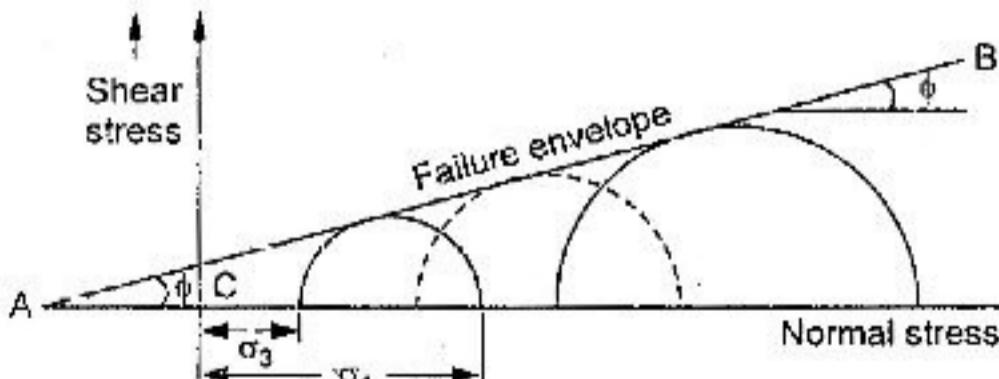
Major principal stress,

$$\sigma_1 = \text{deviator stress} + \sigma_3$$

where deviator stress is the additional axial load divided by the corrected area.

By performing three experiments on three specimens of the given soil by triaxial apparatus, three different values of σ_1 and σ_3 can be obtained.

Now, plot the normal stresses on the X-axis and shear stress τ and Y-axis and draw three Mohr's circles for three sets of the values as shown in the figure below.



Draw a common tangent AB to the three circles of the normal stresses. The intercept c on the Y-axis is the required values of the cohesion and the slope of the tangent with the horizontal is the angle of internal friction, ϕ .

Measurement of shear strength

Shear strength can be determined in the laboratory by the following methods.

- (a) Direct shear test
- (b) Triaxial shear test
- (c) Unconfined compression test
- (d) Vane shear test.

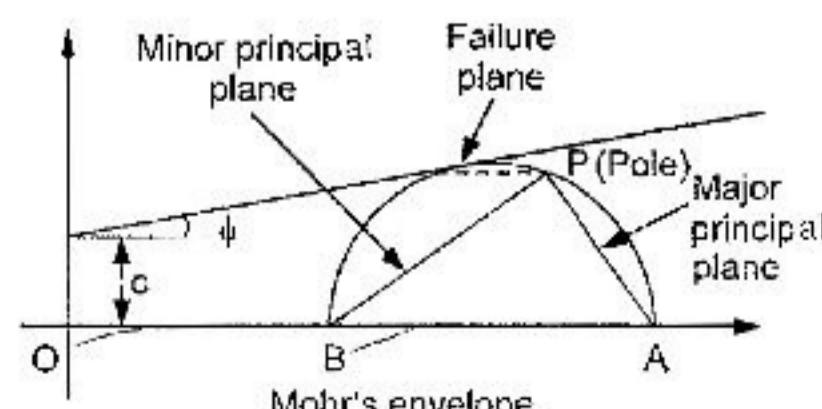
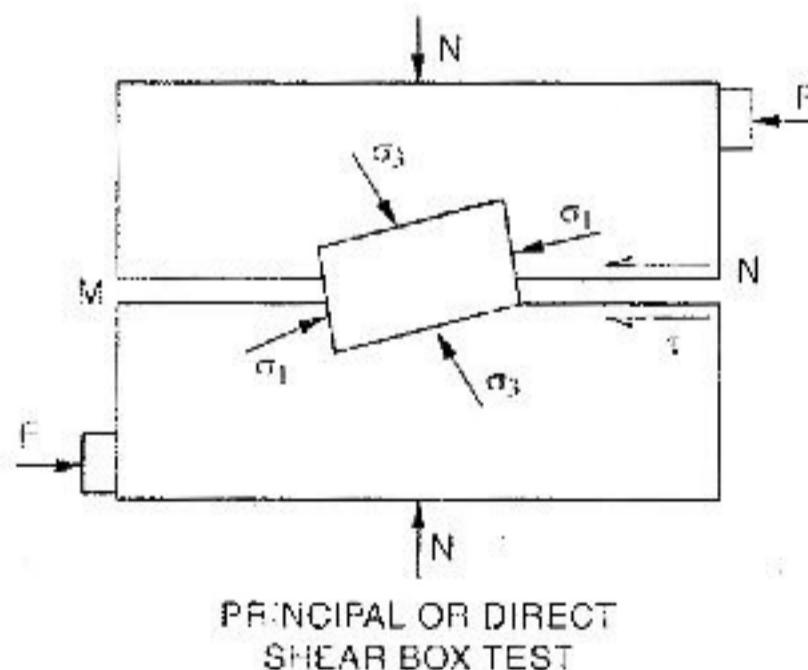
Again depending upon the drainage conditions, three types of shear tests, have been developed.

(i) Unconsolidated : Undrained test in Quick test [No drainage is permitted during consolidated and shear stage].

(ii) Consolidated : Undrained tests, Rapid test (R-test) [drainage is permitted during consolidation stage and not during shear stage].

(iii) Consolidated : Drained test (S-test) slow test.

(a) Direct shear test : A vertical force (N) is applied to the specimen through a loading plate and shear stress is gradually applied on a horizontal plane by causing the two halves of the box to move relative to each other. The shear force (τ) being measured together with the corresponding shear displacement (Δl). Normally the change in thickness (Δh) of the specimen is also measured. A number of specimens of the soils are tested, each under a different vertical force, and the value of shear stress at failure is plotted against the normal stress for each test. The shear strength parameters are then obtained from the best line fitting the plotted points.



Equation of this line is,

$$\sigma = c + \tan \phi$$

where, s = horizontal force divided by the area A of the cross-section of the soil specimen, i.e., the unit shear resistance,

c = cohesion per unit area-the horizontal shear force under no vertical load. Cohesion for a granular soil (dry sand) is zero. Can be read off from the graph :

N = vertical normal load per unit area,
 (ϕ) = angle of shearing resistance or angle of internal friction which can be read off from the graph.

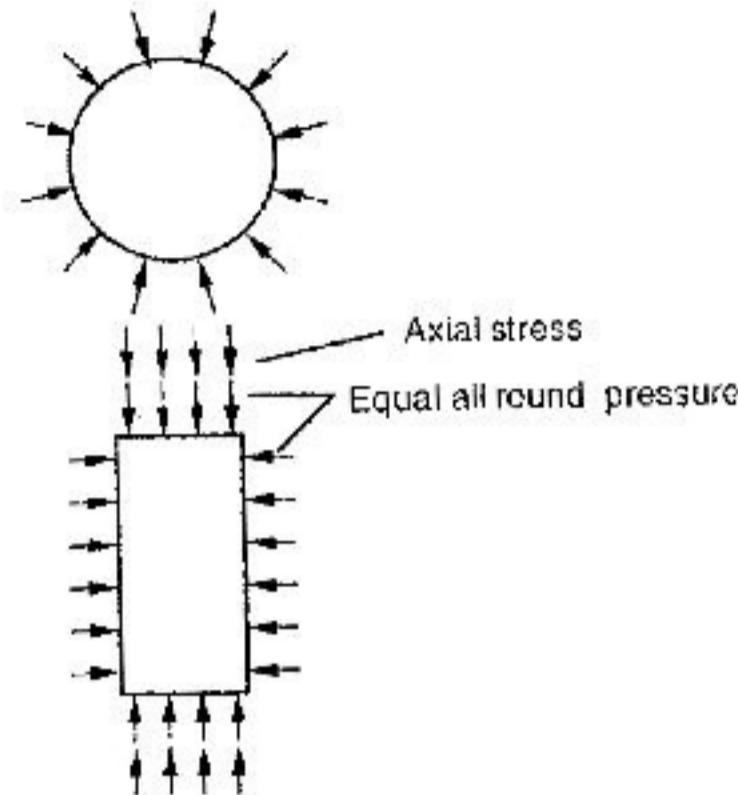
Advantages of direct shear test

- (i) Test is simple and convenient.
- (ii) Sample preparation is easy.
- (iii) As the thickness of the sample is relatively small, the drainage is quick and pore pressure dissipates very rapidly. Consequently, the consolidated-drained and the consolidated-undrained tests take relatively small period.
- (iv) It is identically suited for conducting drained tests on cohesionless soils.
- (v) Apparatus is relatively cheap.

Disadvantages of direct shear test

- (i) The side walls of the shear box cause lateral restraint on the specimen and do not allow it to deform laterally.
- (ii) The stress conditions are known only at failure. The conditions prior to failure are indeterminate and, therefore, the Mohr circle cannot be drawn.
- (iii) The area under shear gradually decreases as the test progress. But the corrected area cannot be determined and, therefore, original area is taken for the computation of stresses.
- (iv) The orientation of the plane is fixed. The plane may not be the weakest plane.
- (v) The measurement of pore water pressure is not possible.
- (vi) The stress distribution on the failure is not uniform. The stresses are more at the edges and lead to the progressive failure. Consequently, the full strength of the soil is not mobilised simultaneously on the entire failure plane.
- (vii) Control on the drainage conditions is very difficult. Consequently, only drainage tests can be conducted on highly permeable soils.

(b) Triaxial Shear Strength Test: This is the most widely used and is suitable for all types of soils. A cylindrical specimen, generally having a length/diameter ratio of 2, is used in the test and is stressed under conditions of axial symmetry in the manner shown in the figure :



The specimen is subjected to an all-round fluid pressure in the cell, consolidation is allowed to take place if appropriate, then the axial stress is gradually increased by the application of compressive load through the ram until failure of the specimen takes place, usually on a diagonal plane.

The length of the soil sample is kept about 2 to 2½ times its diameter. As the cell pressure $\sigma_3 = \sigma_2$, all round the specimen, net major stress on the top of the specimen is

$$\sigma_1 - \sigma_3 + \sigma_3 = \sigma_1$$

During first observation, a particular fluid pressure σ_3 is applied and corresponding value of σ_1 is obtained at failure. With the help of each set of the values (σ_1, σ_3) different Mohr's circles corresponding to failure conditions are drawn. A tangent to these circles gives the failure envelope for the soil under the given drainage condition of the test.

Deviator stress,

$$\sigma_d = \frac{\text{additional axial load}}{\text{changed area of cross section}}$$

$$= \frac{\text{additional axial load}}{A_2}$$

where $A_2 = \frac{V_1 \pm \Delta V}{L_1 - \Delta L}$ [For drained triaxial tests];

$$A_2 = \frac{A_0}{1 - e} \text{ (for undrained test).}$$

where, A_0 = Initial area

$$\epsilon = \text{Axial strain} = \frac{\Delta L}{L}$$

V_1 = initial volume of the specimen

L_1 = initial length of the specimen

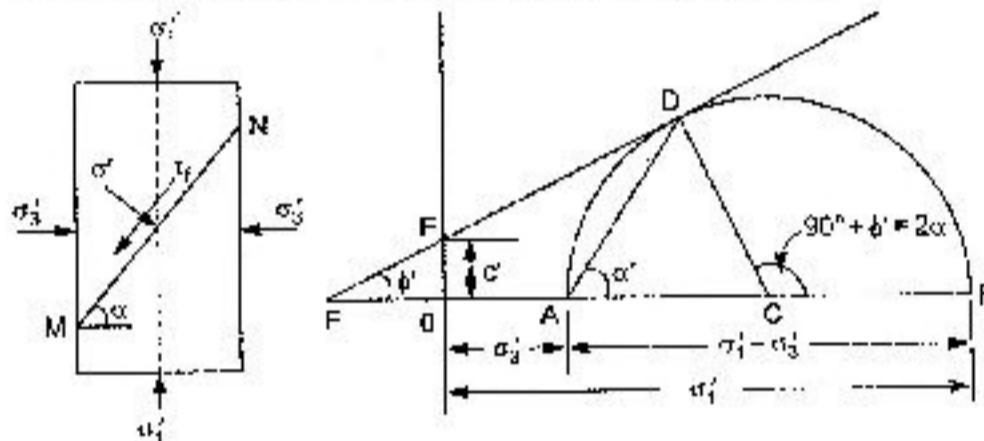
ΔV = change in the volume of specimen

ΔL = change in the length of specimen

Stress conditions in soil specimen during triaxial testing :

The effective stresses acting on the soil specimen during testing are shown in the figure. In this case, effective minor principal stress is equal to the cell pressure (fluid pressure) minus the pore pressure. The major principal stress is equal to deviator stress plus the cell pressure. Hence, the effective major principal stress σ'_1 is equal to the major principal stress minus the pore pressure.

Let the stress components on the failure plane MN be σ' and τ_f , and the failure plane is inclined at an angle α to the major principal plane.



Let the failure envelope DF cut the abscissa at angle ϕ' . C be the centre of Mohr's circle. From ΔADC and ΔFDC , we get

$$\begin{aligned}\alpha' &= \frac{\angle DCB}{2} = \frac{90^\circ + \phi'}{2} \\ &= 45^\circ + \frac{\phi'}{2}\end{aligned}$$

Principal stresses relationship at failure

$DC = CB = \text{radius of Mohr's circles}$

$$= \frac{1}{2}(\sigma'_1 - \sigma'_3)$$

$$OC = \frac{1}{2}(\sigma'_1 + \sigma'_3),$$

$$OF = c' \cot \phi'$$

Again, from ΔCFD ,

$$\begin{aligned}\sin \phi &= \frac{DC}{FC} = \frac{DC}{OC + OF} \\ &= \frac{\frac{1}{2}(\sigma'_1 - \sigma'_3)}{\frac{1}{2}(\sigma'_1 + \sigma'_3) + c \cot \phi'} \\ &= \frac{(\sigma'_1 - \sigma'_3)}{(\sigma'_1 + \sigma'_3) + 2c \cot \phi'}\end{aligned}$$

$$\text{or } \sigma'_1 - \sigma'_3 = 2c' \cos \phi' + (\sigma'_1 + \sigma'_3) \sin \phi'$$

$$\text{or } \sigma'_1(1 - \sin \phi') = \sigma'_3(1 + \sin \phi') + 2c' \cos \phi'$$

$$\begin{aligned}\text{or } \sigma'_1 &= \frac{3(1 + \sin \phi')}{1 - \sin \phi'} + \frac{2c' \cos \phi'}{1 - \sin \phi'} \\ &= \sigma'_3 \tan^2 \left(45^\circ + \frac{\phi}{2} \right) + 2c' \tan \left(45^\circ + \frac{\phi}{2} \right)\end{aligned}$$

$$\text{But } \alpha' = 45^\circ + \frac{\phi}{2}$$

$$\therefore \sigma'_1 = \sigma'_3 \tan^2 \alpha' + 2c' \tan \alpha' \\ = \sigma'_3 N_\phi^2 + 2c' \sqrt{N_\phi}$$

$$\text{where } N_\phi = \tan^2 \alpha = \tan^2 \left(45^\circ + \frac{\phi}{2} \right)$$

\therefore In terms of total stresses,

$$\sigma_1 = \sigma_3 \tan^2 2\alpha + 2c_u \tan \alpha$$

Pore water pressure measurement :

Pore water pressure must be measured under conditions of no flow either out of or into the specimen, otherwise the correct pressure gets modified. It is possible to measure pore water pressure at one end of the specimen while drainage is taking place at the other end. The no flow condition is maintained by the use of the null indicator, essentially a U-tube partly filled with mercury.

Advantages of triaxial test

- (i) The stress distribution on the failure plane is uniform.
- (ii) The specimen is free to fail on the weakest/ plane.
- (iii) There is complete control over the drainage conditions.
- (iv) Pore pressure changes and the volumetric changes can be measured directly.
- (v) The state of stress at all intermediate stages upto failure is known. The Mohr circle can be drawn at any stage of shear.
- (vi) This test is suitable for accurate research work and the apparatus adaptable to special requirements such as extension test and tests for different stress paths.

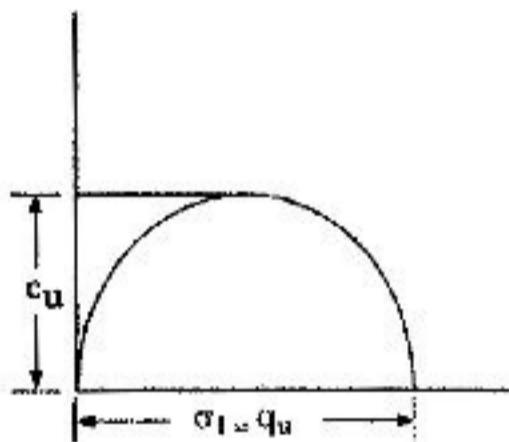
Disadvantages of triaxial test

- (i) The apparatus is elaborate, bulky and costly.
- (ii) The drained test takes a longer period in comparison with a direct shear test.

4.26 Soil Mechanics

- (iii) It is not possible to determine the cross-sectional area of the specimen at larger strains, as the assumption that the specimen remains cylindrical does not hold good.
- (iv) The strain conditions in the specimen are not uniform due to frictional restraint produced by the loading cap and the pedestal disc. This leads to the formation of the dead zones at each end of the specimen.
- (v) The consolidation of the specimen in the test is isotropic, whereas in the field, the consolidation is generally anisotropic.

(c) Unconfined Compression test: This is a modified test in which σ_2 , and σ_3 each is kept zero, leaving only axial stress σ_1 on the cylindrical specimen, till the soil fails due to shearing along a critical plane of failure. During this test, it is assumed that no moisture is lost and hence, it is considered undrained shear test.



Fig, Mohr circle for unconfined compression test

$$\begin{aligned} \text{We know } \sigma_1 &= \sigma_3 \tan^2 \alpha + 2c_u \tan \alpha \\ &= 2c_u \tan \alpha \text{ (because } \sigma_3 = 0) \\ &= 2c_u \left(45^\circ + \frac{\phi_u}{2} \right) \end{aligned}$$

$$\begin{aligned} \text{or } \sigma_1 &= 2c_u \text{ (} \phi_u = 0 \text{ for saturated clays soils)} \\ \therefore c_u &= \frac{1}{2}\sigma_1 = \frac{1}{2}q_u \end{aligned}$$

where q_u is unconfined compressive strength at failure.

Compressive strength is calculated for the changed dimensions of the cylinder.

Suitability. This test is most suitable for intact saturated clays for which angle of shearing resistance ϕ_u is zero.

Advantages of the test

- (i) The test is convenient, simple and quick.
- (ii) It is ideally suited for measuring the unconsolidated-undrained shear strength of intact, saturated clays.

- (iii) The sensitivity of the soil may be easily determined by conducting the test on an undisturbed sample and then on the remoulded sample.

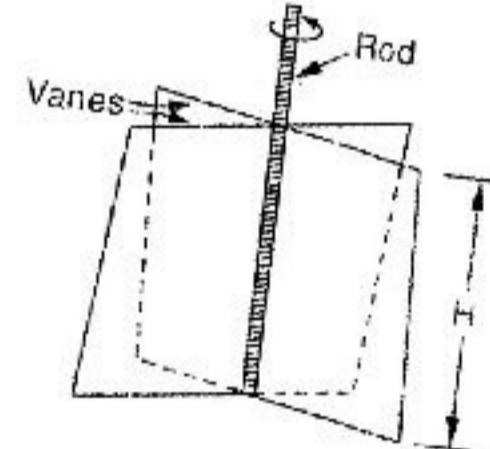
$$\text{Sensitivity, } S_t = \frac{(q_n) \text{ undisturbed}}{(q_r) \text{ remoulded}}$$

Disadvantage

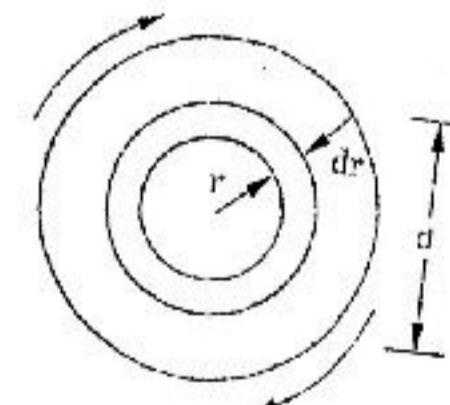
- (i) The test cannot be conducted on fissured clays.
- (ii) The test may be misleading for soils for which the angle of shearing resistance is not zero. For such soils, the shear strength is not equal to half the compressive strength.

Vane shear test

The vane shear test apparatus consists of four stainless steel blades fixed at right angle to each other and firmly attached to a high tensile steel rod. The length of the vane is usually kept equal to twice its overall width. The diameters and length of the stainless steel rod were limited to 2.5 mm and 60 mm respectively.



Let c_u = unit shear strength of the soil
 T = maximum torque at failure in kg. cm
 H = height of vanes in cm.
 d = diameter of vane tester in cm.



Shear strength at failure along the cylindrical surface
 $= \pi d H c_u \quad \dots (i)$

Consider the sheared surface to be composed of a number of elementary rings of thickness dr .

Shearing, resistance on an elementary ring of radius ' r ' $= 2 \pi r dr \cdot c_u$.

Total shear resistance on the top and bottom surfaces of the cylinder

$$= \int_0^{d/2} 2 \pi r dr \cdot c_u \quad \dots (ii)$$

Total shear strength $= (i) + (ii)$

Equating the torque T to the total shear resistance, we get

$$\begin{aligned} T &= (\pi dH \cdot c_u) \frac{d}{2} + \int_0^{d/2} (2\pi r dr \cdot c_u)_r \\ &= \pi \cdot c_u \left[\frac{d^2 H}{2} + \frac{d^3}{6} \right] \\ &= \pi \cdot c_u \cdot d^2 \left[\frac{H}{2} + \frac{d}{6} \right] \end{aligned}$$

or

$$c_u = \frac{T}{\pi d^2 \left[\frac{H}{2} + \frac{d}{6} \right]}$$

If the top of the vane is above the soil surface and depth of vane inside the sample is H_1 , then

$$c_u = \frac{T}{\pi d^2 \left[\frac{H_1}{2} + \frac{d}{12} \right]}$$

Procedure. The following steps are involved.

- Push into the clay, the vane and rod below the bottom of the bore hole, ensuring the verticality of the central rod.

SOLVED EXAMPLES

- Two samples of a soil were tested on a triaxial machine. The all-round pressure maintained for the first was 2.0 kg/cm^2 , and failure occurred at additional axial stress of 7.7 kg/cm^2 , while for the second these values were 5.0 kg/cm^2 and 13.7 kg/cm^2 respectively. The find value of c and ϕ of the soil.

Solution :

Let σ_1 = major principal stress,
 σ_3 = cell pressure
 α = angle of failure envelope with major principal plane

and c = cohesion

then, $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$

where, $\sigma_3 = 2 \text{ kg/cm}^2$,

$$\sigma_1 = 2 + 7.7 = 9.7 \text{ kg/cm}^2$$

$$\therefore 9.7 = 2 \tan^2 \alpha + 2c \tan \alpha \quad \dots (i)$$

$$\text{and } 18.7 = 5 \tan^2 \alpha + 2c \tan \alpha \quad \dots (ii)$$

From equations (i) and (ii), we get

$$9.7 - 2 \tan^2 \alpha = 18.7 - 5 \tan^2 \alpha$$

$$\text{or } 3 \tan^2 \alpha = 9$$

$$\text{or } \tan \alpha = \sqrt{3}$$

$$\therefore \alpha = 60^\circ$$

$$\text{But } \alpha = 45^\circ + \frac{\phi}{2}$$

- Rotate the vanes at a constant speed of 1° per minute by a suitable equipment.
- The test can be conveniently used to determine the sensitivity of the soil.

Advantage of Vane Shear Test

- The test is simple and quick.
- It is ideally suited for the determination of the undrained shear strength of non-fissured, fully saturated clay.
- The test can be conveniently used to determine the sensitivity of the soil.
- The test can be conducted in soft clays situated at a great depth, samples of which are difficult to obtain.

Disadvantages of Vane Shear Test

- The test cannot be conducted on the clay containing sand or silt laminations or the fissured clay.
- The test does not give accurate results when the failure envelope is not horizontal.

$$\therefore 60^\circ = 45^\circ + \frac{\phi}{2}$$

$$\text{or } \phi = 30^\circ$$

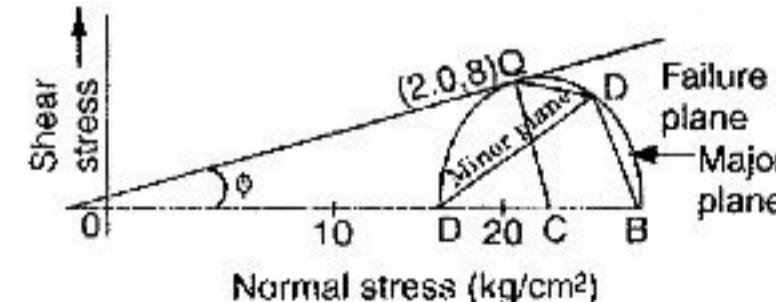
Substituting the values of $\tan \alpha$ and $\tan^2 \alpha$ in equation (i), we get

$$9.7 = 2.3 + 2c\sqrt{3}$$

$$\text{or } c = 1.07 \text{ kg/cm}^2$$

- A cohesive soil has an angle of shearing of 15° and a cohesion of 35 kN/m^2 . If the specimen of this soil is subjected to a triaxial compression test, then what is value of the lateral pressure in the cell for failure to occur at a total stress of 300 kN/m^2 ?

Solution :



$$\text{Given, } \sigma_1 = 300 \text{ kN/m}^2$$

$$\phi = 15^\circ$$

$$c = 35 \text{ kN/m}^2$$

$$\text{From } \sigma_1 = \sigma_3 \tan^2(45^\circ + \phi/2) + 2c \tan(45^\circ + \phi/2)$$

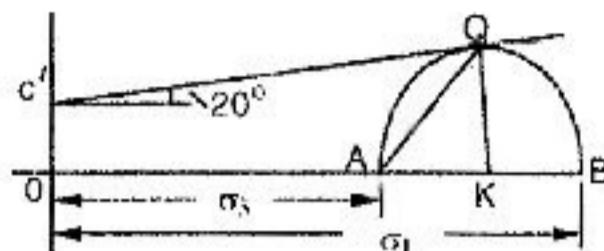
$$300 = \sigma_3 \tan^2(45^\circ + 7.5^\circ) + 2 \cdot 35 \tan(45^\circ + 7.5^\circ)$$

$$\text{or } \sigma_3 = \frac{300 - 70 \tan 52.5^\circ}{\tan^2 52.5^\circ} = 122.924 \text{ kN/m}^2$$

4.28 Soil Mechanics

3. A cylindrical sample soil, having cohesion of 0.8 kg/cm^2 and angle of internal friction of 20° , is subjected to a cell pressure of 1.0 kg/cm^2 . Calculate the maximum deviator stress at which the sample will fail and the angle made by the failure plane with the axis of the sample.

Solution :



Given, $\sigma_3 = 1.0 \text{ kg/cm}^2$

$$\phi = 20^\circ$$

$$c = 0.8 \text{ kg/cm}^2$$

Now,
$$\begin{aligned}\sigma_1 &= \sigma_3 \tan^2(45^\circ + \phi/2) + 2c \tan(45^\circ + \phi/2) \\ &= 1.0 \tan^2(45^\circ + 10^\circ) + 2 \\ &\quad \times 0.8 \tan(45^\circ + 10^\circ)\end{aligned}$$

or $\sigma_1 = 4.3246 \text{ kg/cm}^2$

But, deviator stress,

$$\begin{aligned}\sigma_d &= \sigma_1 - \sigma_3 \\ &= 4.3246 - 1.0 \\ &= 3.3246 \text{ kg/cm}^2\end{aligned}$$

Angle made by the failure plane with major principle plane

$$\begin{aligned}\alpha &= 45^\circ + \frac{\phi}{2} \\ &= 45 + 10^\circ = 55^\circ\end{aligned}$$

Angle made by the failure plane with the axis of sample

$$\begin{aligned}&= 90^\circ - \alpha \\ &= 90^\circ - 55^\circ = 35^\circ\end{aligned}$$

4. Two samples of soil were tested in a triaxial machine. The all-round pressure maintained for the first was 2.5 kg/cm^2 and failure occurred at additional axial stress of 8 kg/cm^2 , while for the second these values were 7 kg/cm^2 and 15 kg/cm^2 respectively. Find value of ϕ of the soil,

Solution : Given, $\sigma'_3 = 2.5 \text{ kg/cm}^2$

For first sample $\sigma_1 = 2.5 + 8 = 10.5 \text{ kg/cm}^2$

For second sample, $\sigma'_3 = 7 \text{ kg/cm}^2$

$$\sigma_1 = 7 + 15 = 22 \text{ kg/cm}^2$$

From $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$
 $10.5 = 2.5 \tan^2 \alpha + 2c \tan \alpha \quad \dots(i)$

and $22 = 7 \tan^2 \alpha + 2c \tan \alpha \quad \dots(ii)$

From equations (i) and (ii), we get

$$\alpha = 58^\circ$$

But $\alpha = 45^\circ + \frac{\phi}{2}$,

hence $58^\circ = 45^\circ + \frac{\phi}{2}$

or $\phi = 26^\circ$

5. An embankment being constructed of soil whose properties are $c' = 50 \text{ kN/m}^2$, $\phi' = 21^\circ$ and $\rho = 1.6 \text{ mg/m}^3$. The pore pressure parameters as found from triaxial tests are $A = 0.5$ and $B = 0.9$. Calculate the shear strength of the soil at the base of the embankment just after the height of fill has been raised from 3 m to 6 m . Assume that the dissipation of pore pressure during this stage of construction is negligible, and the lateral pressure at any point is one half of the vertical pressure.

Solution : Relation between increase of pore pressure and increase of principal stress is given by

$$\Delta u = B[\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)]$$

where $\Delta \sigma_1$ = increase in vertical pressure due to 3 m of fill

$$= 3 \times 1.6 \times 9.81 = 47.09 \text{ kN/m}^2$$

By assumption

$$\begin{aligned}\Delta \sigma_3 &= \frac{\Delta \sigma_1}{2} \\ &= 23.55 \text{ kN/m}^2 \\ \Delta u &= 0.9[23.55 + 0.5(47.09 - 23.55)] \\ &= 31.79 \text{ kN/m}^2\end{aligned}$$

Original pressure,

$$\begin{aligned}\sigma_1 &= 3 \times 1.6 \times 9.81 \\ &= 47.09 \text{ kN/m}^2\end{aligned}$$

Effective stress,

$$\begin{aligned}\sigma' &= \sigma_1 + \Delta \sigma_1 - \Delta u \\ &= 47.09 + 47.09 - 31.79 \\ &= 62.39 \text{ kN/m}^2\end{aligned}$$

$$\begin{aligned}\text{Shear strength} &= c' + \sigma' \tan \phi' \\ &= 50 + 62.39 \tan 21^\circ \\ &= 73.25 \text{ kN/m}^2\end{aligned}$$

6. At a depth of 6 m from the ground level at a site, a shear test gave a torque of 604 kg/cm . The vane was 10 cm high and 7 cm across the blades. Then find cohesive strength of the soil,

Solution : Resisting moment

$$\begin{aligned}&= 2\pi r L c \times r + (2\pi r^2 c) \frac{2}{3} r \\ &= 2\pi r^2 c \left[L + \frac{2}{3} r \right]\end{aligned}$$

where c = cohesive strength,
and r = radius of the vane
But Resisting moment = Torque T applied

$$\therefore c = \frac{T}{2\pi r^2 \left[L + \frac{2}{3}r \right]}$$

$$= \frac{604}{2\pi \left(\frac{7}{2} \right)^2 \left[10 + \frac{2}{3} \times \frac{7}{2} \right]}$$

$$= 0.64 \text{ kg/cm}^2$$

COMPACTION, COMPRESSIBILITY AND CONSOLIDATION OF SOILS

Compaction is defined as the process of increasing the unit weight of soil by forcing the soil solids to move closer due to the expulsion of air from the voids and this is accomplished by rolling or tamping,

Compaction Theory : This theory gives a relationship between soil density, water content and compactive effort applied on a specimen when water (moisture) is added to a dry soil, the soil grains get surrounded by a film of absorbed water. If more water is added, this film becomes thicker, and soil grains surrounded by such films, can slide over each other more easily. The grains thus becomes capable to occupy closer spacing more easily on application of external loading. The water thus acts only as a lubricant in the compaction process, and a closer packing of soil grains is achieved on rolling, by the expulsion of air from the voids.

If more water is continued to be added, then a stage comes when water becomes excess, and starts occupying the space which otherwise could have been occupied by soil grains under compaction. Evidently, water then starts hindering the closer packing of grains. In the limiting stage, there occurs a particular amount of water, which causes maximum lubrication, without becoming excess to cause hinderance in compaction by occupying space for its own molecules. This limiting moisture, which is most useful to compaction, is called the optimum moisture content (OMC).

At OMC, maximum density or unit weight obtained is called Proctor density.

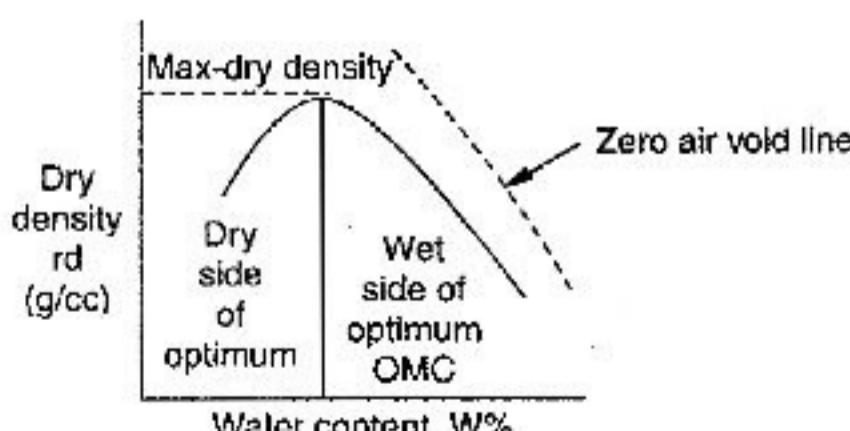


Fig. Compaction Curve

Methods of Laboratory determination of OMC

1. Standard proctor (Light compaction)
2. Modified proctor (Heavy compaction)
3. Abbot compaction
4. Harvard miniature compaction
5. Jodhpur mini compaction

Standard Proctor Test : A cylindrical mould of 1000 ml volume is filled with a soil sample in three layers, each layer being compacted by 25 blows of a free fall of standard dimension hammer of 2.6 kg mass and drop height of 310 mm. The excess material projecting outside the mould is then trimmed and weighed, hence giving bulk density of soil. The moisture content of the soil is determined and hence the dry density. Soil with different water content is compacted and the dry density corresponding moisture content in each case is plotted (Figure above). From the plot O.M.C. and corresponding maximum dry density can be found.

The bulk density, γ and dry density γ_d for the compacted soil are determined by using the following equations :

$$\gamma = \frac{W}{V} \text{ (g/cm}^3\text{)}$$

where, W = weight of compacted soil (g)

V = volume of mould (cm^3) = 944 ml.

Also, $\gamma = \frac{M_2 - M_1}{V}$

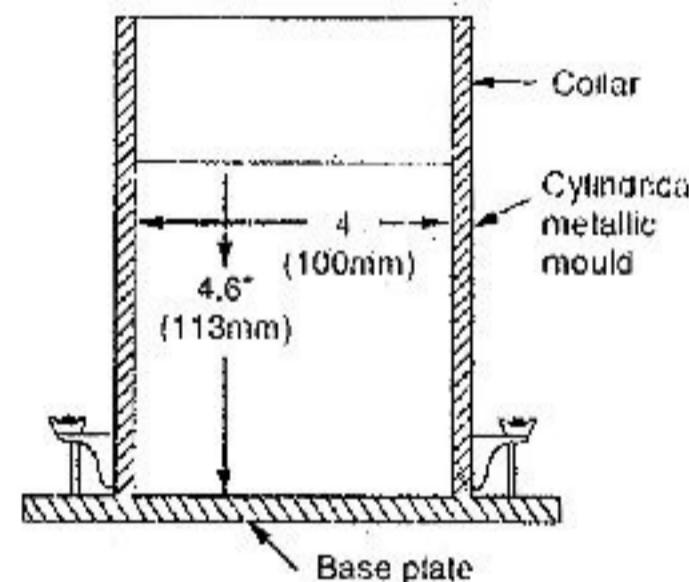


Fig. The Standard Proctor mould

where, M_1 = mass of empty mould
 M_2 = mass of mould + compacted soil
 w = moisture content

$$\gamma_d = \frac{\gamma}{(1+w)} \text{ (g/cm}^3\text{)}$$

Zero Air Void Line : It is a plot between dry unit weight and water content corresponding to degree of saturation of 100% or zero air voids. This curve represents an upper bound for dry unit weight. The moisture density line cannot cross this line. The zero air void density for any moisture content can be obtained from the expression :

$$\gamma_d = \frac{G\gamma_w}{\left(1 + \frac{wG}{S_r}\right)} = \frac{G\gamma_w(1 - n_a)}{1 + wG}$$

where G = specific gravity

n_a = % air voids

γ_w = density of water = 1g/cm^3

w = water content of compacted soil

and S_r = degree of saturation = 100%

It may be noted that 10% air void lines and 90% saturation lines are not identical. While zero air void line and 100% saturation line are identical.

Relative Compaction : The results of laboratory compaction are not directly applicable to field compaction because the equipment used in field is different as well as no lateral confinement is possible in field compaction. It is very difficult to obtain minimum compaction in the field. The dry density achieved by field compaction expressed as a percentage of the maximum dry density in a particular laboratory test is defined as the relative compaction.

Relative compaction

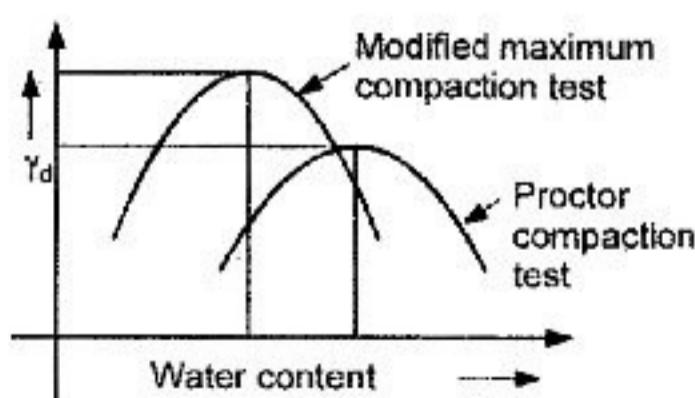
$$= \frac{\text{Field dry density, } \gamma_d}{\text{Maximum dry density in laboratory test, } \gamma_d} \times 100$$

Usually 90 to 97% relative compaction in cohesive soils can be obtained by sheep foot roller or pneumatic typed roller.

FACTOR AFFECTING COMPACTION

(i) **Water content of soil :** At low moisture content, the cohesion is more and the particles do not move easily when energy is supplied and hence the high air void and low density. The increase in water content results in increase in density (inter particle repulsion is more) till an OMC is reached. After OMC is reached further addition of water reduces the dry density because the extra water displace the soil particle occupying that space.

(ii) **Mode and type of compaction :** The minimum dry density and OMC depends upon the amount of energy supplied to the soil. The increase in compactive energy results in an increase in maximum dry density and decrease in optimum moisture content.



(iii) **Type of soil :** The coarse grained soil attains a higher density and lower OMC as compared to fine grains soils for the same amount of energy transmitted. Because the specific surface of fine grain particles is more and they have more affinity towards water because of particles having negative charge.

(iv) **Addition of admixtures :** The compaction properties of soil can often be improved by the use of admixtures. Some of the more widely used admixtures are lime and cement.

(v) **Amount of compaction :** At a water content less than the optimum, the effect of increased compaction is more predominant. At a water content more than optimum, the effect of increased compaction is not significant as volume of air voids becomes constant.

ENGINEERING PROPERTIES OF COMPACTED SOILS

1. Shear strength

- (i) Shear strength will increase with increasing compactive effort until a critical degree of saturation is reached and then decrease rapidly with further increase in water content.
- (ii) The sample compacted on dry side of optimum will exhibit more strength than soils compacted on wet side of optimum.
- (iii) On the wet side of optimum, the static compaction tends to cause more shear strength than kneading compaction.

2. Permeability : The permeability of a soil decreases with an increase in water content on the dry side of the optimum water content. There is an improved orientation of the particles and a corresponding reduction in size of voids. The minimum permeability occurs at or slightly above the optimum water content. After that stage, the permeability slightly increase, but it always remains much less than on the dry side of the optimum. The slight increase in the permeability occurs because the effect of the decrease in the dry unit weight is more pronounced than the effect of the improved orientation.

If the compactive effort is increased, the permeability of the soil decreased due to increased unit dry weight and better orientation of particles.

3. Compressibility and volume change

- (i) Soil compacted on dry side of optimum tends to shrink less and swell more as compared to soil compacted on wet side of optimum.
- (ii) Type of compaction plays major role when samples compacted on wet side of optimum. Kneading compaction exhibits higher compressibility than those with other methods.

(iii) The compressibility of a compacted cohesive soil increases with an increase in degree of saturation and with a decrease in dry density.

(iv) Compressibility of compacted cohesive soil increase with an increase in liquid limit.

Compaction Control in field : The field compaction control consists of the determination of

- water content at which the soil has been compacted; and
- the dry density

Normally the water content and dry density are determined by the help of Proctor needle method.

Example. An earth embankment is compacted at water content of 17% to a bulk density of 1.90 gm/cc. If the specific gravity of soil grains is 2.65, then what is degree of saturation of the compacted embankment?

$$\text{Solution : } \gamma = \frac{\rho}{\rho_w} \gamma_w = \frac{1.90 \text{ gm/cc}}{1 \text{ gm/cc}} \times 9.81 \\ = 1.90 \times 9.81 \text{ kN/m}^3$$

$$\text{From, } \gamma = \frac{S_s(1+w)\gamma_w}{1+e}$$

$$1.90 \times 9.81 = \frac{2.65(1+0.17)}{1+e} \times 9.81$$

$$\text{or } e = 0.63$$

$$\text{From } e = \frac{G \cdot w}{S}$$

Degree of saturation,

$$S = \frac{G \cdot w}{e} = \frac{2.65 \times 0.17}{0.63} = 0.715$$

Example. A core cutter 13 cm in height and 10 cm in diameter, and having a mass of 1078 gm when empty, is used to determine the insitu density of an embankment. Mass of the core cutter full of soil is 2968 gm. If the water content of the soil is 8%, what is the insitu dry unit weight? Specific gravity of soil particles is 2.65.

Solution :

$$\text{Volume of core cutter} = \frac{\pi}{4} \times D^2 \cdot H \\ = \frac{\pi}{4} \times (10)^2 \times 13 \\ = 1020.5 \text{ cm}^3$$

Mass of empty core cutter = 1078 gm

Mass of core cutter with soil sample = 2968 gm

$$\therefore \text{Mass of soil sample} = 2968 - 1078 = 1890 \text{ gm}$$

$$\text{Volume of this sample} = 1020.5 \text{ cm}^3$$

\therefore Bulk density of soil,

$$\rho = \frac{M}{V} = \frac{1890}{1020.5} = 1.85 \text{ gm/cm}^3$$

From

$$\rho = \rho_d (1 + w)$$

$$1.85 = \rho_d (1 + 0.08)$$

or

$$\rho_d = 1.76 \text{ gm/cc}$$

$$\gamma_d = \frac{\rho_d}{\rho_w} \cdot \gamma_w$$

$$= \frac{1.76 \text{ gm/cc}}{1 \text{ gm/cc}} \times 9.81 \text{ kN/m}^3$$

$$= 17.27 \text{ kN/m}^3$$

Example. A soil deposit has a void ratio of 0.9. If the void ratio is reduced to 0.6 by find the compaction, then percentage volume loss due to compaction.

Solution : Initial void ratio is given by,

$$\frac{V_v}{V_s} = 0.9$$

$$\text{or } V_v = 0.9 V_s$$

$$\text{Total initial volume, } V_i = V_v + V_s \\ = V_s + 0.9 V_s = 1.9 V_s$$

$$\text{Final void ratio} = 0.6$$

$$\frac{V_v}{V_s} = 0.6,$$

$$V_v = 0.6 V_s$$

$$\text{Total final volume, } V_f = 0.6 V_s + V_s = 1.6 V_s$$

$$\therefore \% \text{ volume loss} = \frac{1.9V_s - 1.6V_s}{1.9V_s} \times 100 = 15.79\%$$

COMPRESSIBILITY AND CONSOLIDATION

The property of the soil mass pertaining to the susceptibility to decrease in volume under pressure is called *compressibility*.

The process of the gradual reduction in the volume of a partly or fully saturated soil mass due to expulsion of water from the pores of the soil, under a long term static load, is termed as *consolidation*. It may be clearly understood that the compaction is different from consolidation because compaction is done by mechanical means whereas consolidation takes place due to expulsion of water from the soil in course of time.

Consolidation of Clay : The delay in compression of clayey soil is due to

(i) **Hydrodynamic lag :** The static load acting on the soil mass develops higher pore pressure compared to surrounding soil mass which is away from loading area and thus causes the movement of water through drained surface. The movement

4.32 Soil Mechanics

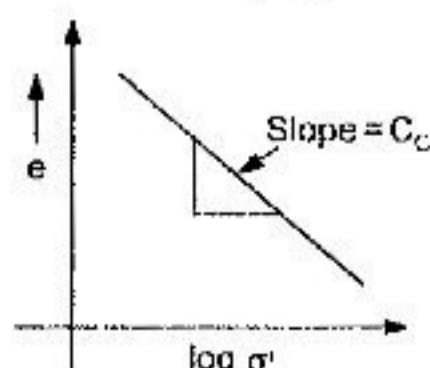
however is slow because of low permeability of clay soil and the rate of settlement is a function of permeability. The delay caused in settlement by the slow drainage of water is known as *hydrodynamic lag*.

(ii) **Viscous lag** : The compression of clayey soil is not only due to expulsion of free water but also due to shear deformation of double layer water surrounding the clay particles. The double layer water is highly viscous, and bounded well with the clay particles. Because of its viscous nature, the shear deformation or expulsion of double layer water takes more time. The time lag associated with this viscous behaviour is called *viscous lag*.

Consolidation Test : The consolidation test is performed on an undisturbed saturated clayey soil. The sample is trimmed to fit in a 75 mm diameter cylindrical container. The diameter to thickness ratio of sample should be a minimum of 3. Weight of the soil sample filled in the cylindrical container is determined and its initial water content also determined from the cuttings. The porous stones are placed both on top and bottom of the sample so as to drain the water both in upward and downward directions. Initially a seating load of intensity 0.1 kg/cm² is applied on the sample. After reaching equilibrium under the seating load, the test is started by adding first increment of load. The sample is allowed to consolidate until it reaches equilibrium (generally each increment of load is kept for 24 hours on the sample). The change in thickness of soil sample is measured using compression gauge. Next increment of load is applied on the sample (each load increment is equal to the total of proceeding load). The test is carried out until the total load intensity on the sample is about 8 to 10 kg/cm². Finally release the pressure and allow the sample to expand. The results obtained from this test are used to determine the following.

Compression Index (C_c) : The slope of the linear portion of the pressure void ratio, which remains constant within a fairly large range of pressure, is called compression index and is given by

$$C_c = \frac{e_o - e}{\log_{10} \left(\frac{\sigma'_1}{\sigma'_0} \right)}$$



where C_c = compression index (dimensionless)
 e_o = initial void ratio (i.e. at starting point)
 e = final void ratio (i.e. at any other point)
 σ'_0 = initial pressure or stress
 σ'_1 = final pressure or stress

Note : The recompression index C_r is very small as compared with the compression index C_c .

Compression index can be determined by

$$C_c = 0.009 (W_l - 10) \text{ for undisturbed sample}$$

$$C_c = 0.007 (W_l - 10) \text{ for remoulded sample}$$

where, w_l = liquid limit of soil

Coefficient of compressibility (a_v) : The decrease in void ratio per unit increase of pressure, is defined as coefficient of compressibility

$$a_v = \frac{e_o - e}{p - p_o}$$

where e_o = initial void ratio
 e = final void ratio
 p_o = initial pressure
 p = final pressure

Coefficient of volume compressibility (m_v) : The change in volume of a soil per unit of initial volume due to unit increase in effective stress, is known as coefficient of volume compressibility or coefficient of volume change and is given by

$$m_v = \frac{a_v}{1 + e_o}$$

$$\text{Also, } m_v = \frac{\Delta e}{(1 + e_o)} \times \frac{1}{\Delta \sigma'} \\ = -\frac{\Delta H}{H_o} \times \frac{1}{\Delta \sigma'}$$

$$\text{and } \Delta H = m_v H_o \Delta \sigma' \\ = \frac{C_c}{(1 + e_o)} H_o \log_{10} \left(\frac{\sigma'_o + \Delta \sigma'}{\sigma'_o} \right)$$

Pre-consolidation pressure : The temporary overburden pressure to which a soil has been subjected to during the geologic past and under which it got consolidated, is known as *pre-consolidation pressure*.

Normally consolidated soil : The soil which has never been subjected to an effective pressure more than the existing overburden pressure and which is otherwise completely consolidated by the existing overburden, is called *normally consolidated soil*.

Under consolidated soil : The soil which is not fully consolidated under the existing overburden pressure, is called the *under-consolidated soil*.

Pre-consolidated soil : The soil which was subjected to pre-consolidation pressure during geologic past, is known as *pre-consolidated soil*.

Over-consolidated soil : If soil had been subjected in the past to a pressure in excess of the present pressure. The maximum pressure to which an over consolidated soil had been subjected in the past divided by the present pressure is known as *over consolidation ratio* (O.C.R).

Terzaghi's one-dimensional consolidation theory

Assumptions

1. The soil is homogenous.
2. The soil is fully saturated.
3. The drainage and consolidation both are one dimensional.
4. The compressibility of soil grains and water is practically nil.
5. The soil is laterally confined.
6. The Darcy's law is valid
7. The coefficient of permeability is constant during the process of consolidation.
8. The time lag of consolidation is entirely due to low permeability of the soil.
9. The change in the effective pressure in the soil causes corresponding change in the void ratio.
10. The change in the thickness of the layer during consolidation is insignificant.
11. Constant values for certain soil properties remain constant.

Terzaghi theory describes the time rate of consolidation of saturated clayey soils.

The differential equation for consolidation is,

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2}$$

where, C_v = coefficient of consolidation

$$= \frac{K(1+e)}{a_v \cdot \gamma_w} = \frac{K}{\gamma_w \cdot m_v}$$

Solution of this equation can be expressed in terms of degree of consolidation, U and time factor, T_v .

Degree of compression. It is defined as the ratio between compression at any time ' t ' to compression at the end of consolidation.

The dimensionless constant,

$$T_v = \frac{C_v l}{H^2}$$

where C_v = coefficient of consolidation,

and t = time (variable)

H = drainage path (for double drainage, the drainage path is equal to half the thickness)

The time factor T_v for different percentage of consolidation can be determined by

$$\text{For } U < 60\%, \quad T_v = \frac{\pi}{4} \left(\frac{U}{100} \right)^2$$

$$\text{For } U > 60\%, \quad T_v = -0.932 \log_{10} \left(1 - \frac{U}{100} \right)$$

Determination of Coefficient of Consolidation

The results of one-dimensional consolidation test on undisturbed soil samples are used to determine coefficient of consolidation C_v . The value of C_v is determined using curve fitting procedures applied to the elapsed time-degree of consolidation curve. The characteristics of the theoretical relationship between degree of consolidation U and time factor T_v were used to fit the laboratory curve to determine well defined point on the time compression curve corresponding to given degree of consolidation.

Curve fitting methods commonly used

(i) **Square root time fit method :** The theoretical curve between $\sqrt{T_v}$ and U , indicates that the curve is a straight line upto 60% of consolidation. It is also observed that the time factor $\sqrt{T_v}$ value for 90% consolidation is about 1.15 times the abscissa of extension of the straight line portion. Thus characteristic of the theoretical curve is utilised to locate the 90% consolidation point on the laboratory curve.

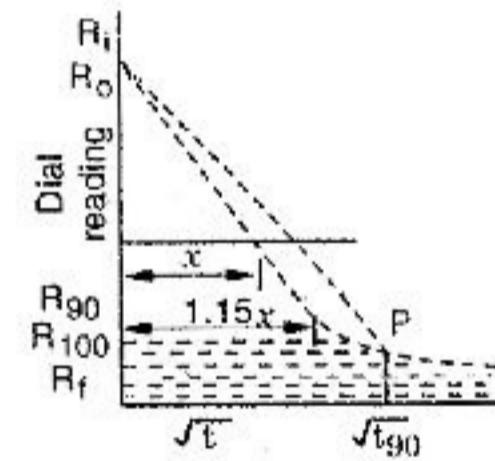


Fig. (a) Laboratory curve \sqrt{t} Vs dial reading

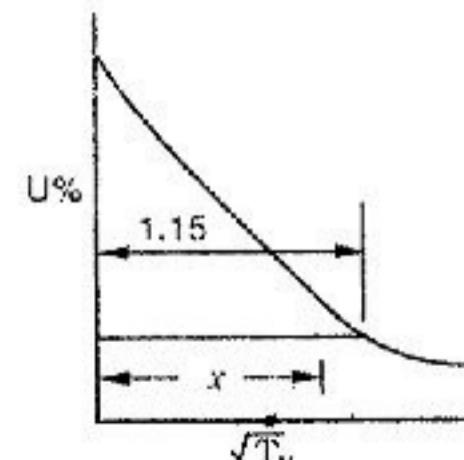


Fig. (b) Theoretical curve T_v Vs U

The curve is drawn on an ordinary graph sheet by selecting time \sqrt{t} in x-axis and dial reading on y-axis. A correction is applied to the consolidation curve to obtain the point