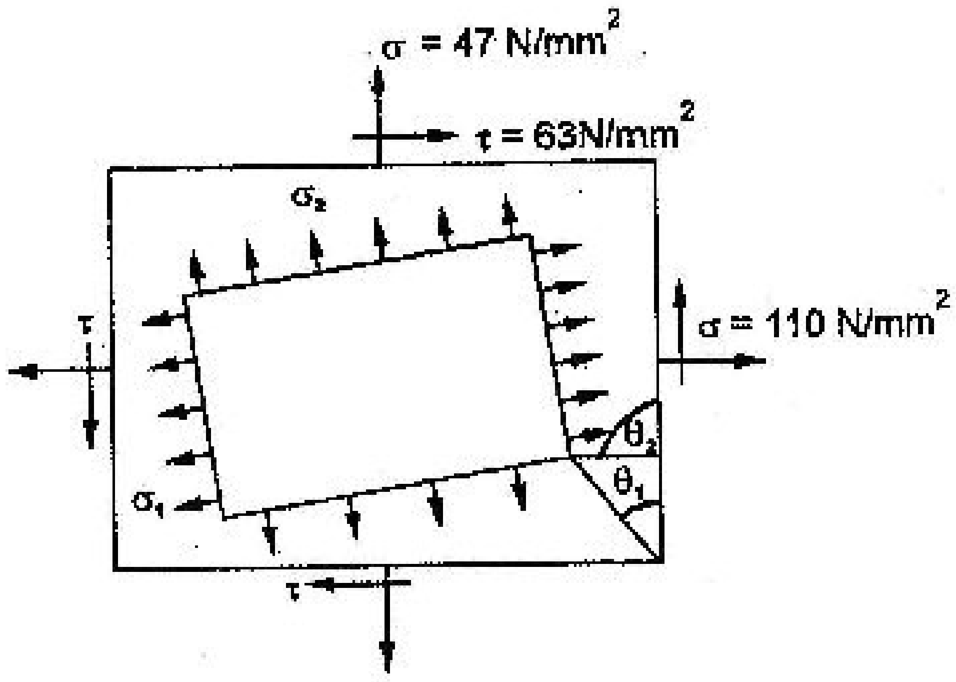
(*ii*) **Major principal stress :**



$$\sigma_1 = \frac{\sigma+\sigma'}{2} + \sqrt{\left(\frac{\sigma-\sigma'}{2}\right) + \tau^2}$$

$$= \frac{110+47}{2} + \sqrt{\left(\frac{110-47}{2}\right)^2 + 63^2}$$

$= 78.5 + 704 = 148.9$ N/mm² (tensile)

Minor Principal Stress:

$$\sigma_2 = \frac{\sigma+\sigma'}{2} - \sqrt{\left(\frac{\sigma-\sigma'}{2}\right) + \tau^2}$$

$= 78.5 - 70.4 = 8.1$ N/mm² (tensile)

(*iii*) **Maximum shear stress :**

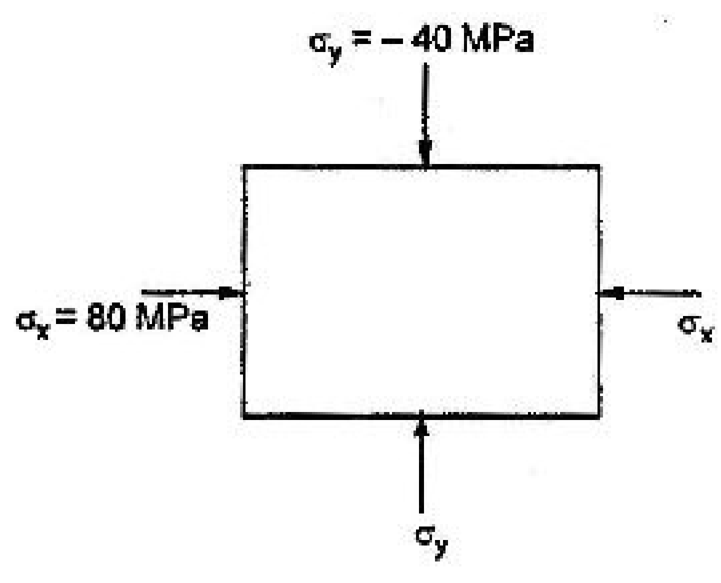
$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{148.9 - 8.1}{2}$$

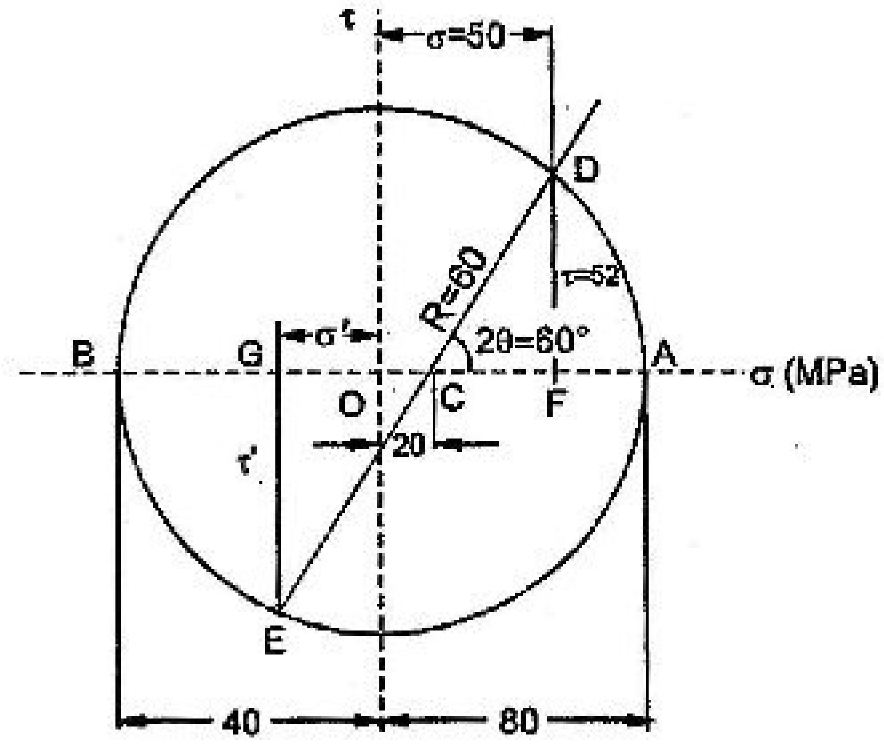
$= 70.4$ N/mm²

This will occur at planes at 31° 43′ + 45° = 76° 43′ and 76° 43′ + 90° = 166° 43′ with the plane AB carrying the normal stress of 110 N/mm².

3. At a certain point in a stressed body, the principal stresses are σ_x = 80 MPa and σ_y = − 40 MPa. Determine σ and τ on the planes whose normals are at + 30° and + 120° with the X-axis. Show your results on a sketch of a differential element.

Solution :





Co-ordinates of point D represent the required stress components on the 30° face.

From the geometry of Mohr's circle

σ = OF = OC + CF = 20 + 60 cos 60° = 50 MPa

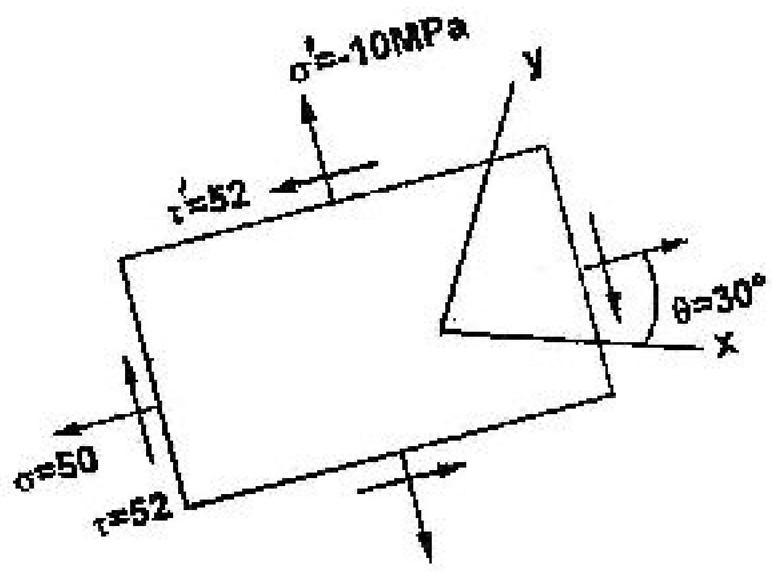
τ = DF = 60 sin 60° = 52.0 MPa

On the perpendicular 120° face, we have

σ' = OG = OC − CG = 20 − 50 cos 60° = −10 MPa

τ' = GE = − 60 sin 60° = − 52.0 MPa.

Both the stress components are shown on the differential element



4. A rectangular block 250 mm × 100 mm × 80 mm is subjected to axial load as follows :

480 kN tensile in the direction of its length.

900 kN tensile on the 250 mm × 80 mm faces.

1000 kN compressive on the 250 mm × 100 mm faces.

(*i*) Assuming Poisson's ratio as 0.25, find volumetric strain

(*ii*) The change in the volume of the block due to the application of the loading specified above

(*iii*) If E = 2 × 10⁻⁵ N/mm², the value of mouduls of rigidity and bulk modulus for the material respectively

Solution :

(*i*) The stresses in the direction of these axis are

$$\sigma_x = \frac{480 \times 1000}{100 \times 80} = 60 \text{ N/mm}^2 \text{ (tensile)}$$

$$\sigma_y = \frac{900 \times 1000}{250 \times 80} = 45 \text{ N/mm}^2 \text{ (tensile)}$$

$$\sigma_z = \frac{100 \times 1000}{250 \times 100} = 40 \text{ N/mm}^2 \text{ (compressive)}$$

The strains along the three principal directions are :

$$\in_x = \frac{60}{E} - \frac{45}{mE} + \frac{40}{mE}$$

$$= \frac{1}{E}\left(60 - \frac{45}{m} + \frac{40}{m}\right) = \frac{1}{E}\left(60 - \frac{5}{m}\right)$$

$$= \frac{1}{E}(60 - 5 \times 0.25) = \frac{58.75}{E}$$

$$\in_y = \frac{45}{E} - \frac{60}{mE} + \frac{40}{mE}$$

$$= \frac{1}{E}\left(45 - \frac{20}{m}\right) = \frac{1}{E}(45 - 20 \times 0.25) = +\frac{40}{E}$$

$$\in_z = \frac{40}{E} - \frac{60}{mE} - \frac{45}{mE}$$

$$= \frac{1}{E}\left(40 + \frac{105}{m}\right) = -\frac{1}{E}(40 + 105 \times 0.25)$$

$$= -\frac{66.25}{E}$$

Volumetric strain, $\in_v = \in_x + \in_y + \in_z$

$$= \frac{58.75}{E} + \frac{40}{E} - \frac{66.25}{E}$$

$$= \frac{32.5}{E} = \frac{325}{2 \times 10^5} = 16.25 \times 10^{-5}$$

(*ii*) Increase in volume = $\in_v \times V$

$$= \frac{32.5}{2 \times 10^5}(250 \times 100 \times 80) \text{ mm}^2 = 325 \text{ mm}^2$$

(*iii*) $E = 2C\left(1 + \frac{1}{m}\right)$ and $E = 3K\left(1 - \frac{2}{m}\right)$

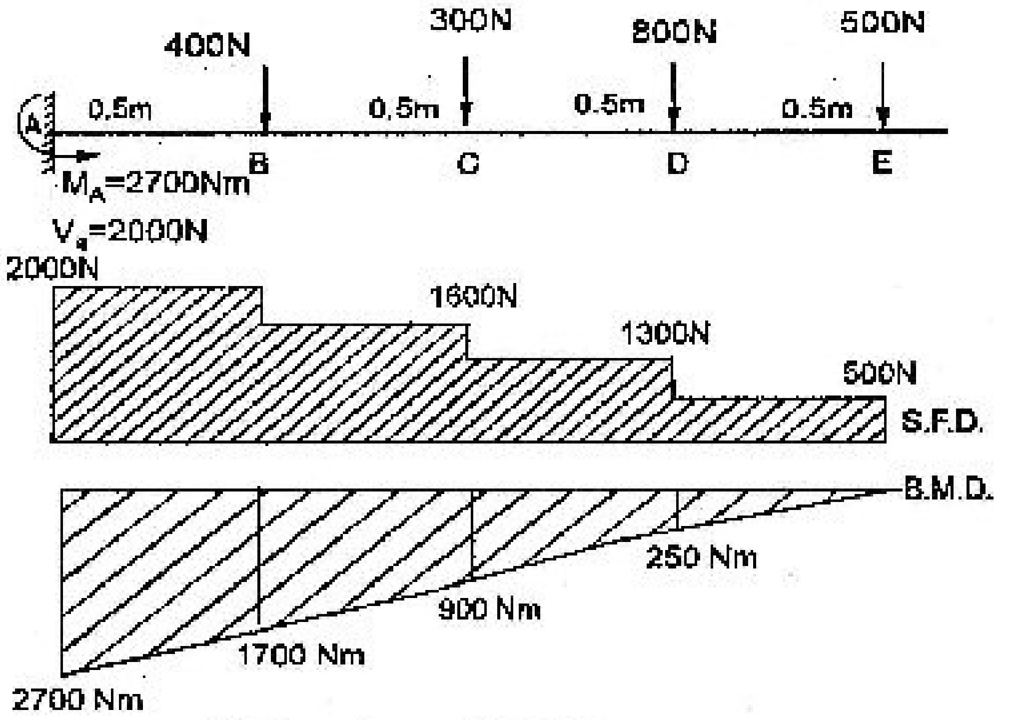
$\therefore\ 2 \times 10^5 = 2C\,(1 + 0.25) = 3K\,(1 - 2 \times 0.25)$

$C = 0.8 \times 10^5$ N/mm², and

$K = 1.33 \times 10^5$ N/mm²

5. Figure below shows a cantilever subjected to a system of loads. Draw shearing force and bending force diagrams.

Solution : At any section between D and E, a distant x from E



S.F. = S = + 500 N

B.M = M_x = – 500 x

At $x = 0$, $M_x = 0$

and at $x = 0.5$ m,

M = – 250 Nm

At any section between C and D, distant x from E

S.F. = S_x = + 500 + 800 = + 1300 N

B.M = M_x = – 500x – 800 (x – 0.5)

$= -1300\,x + 400$

At $x = 0.5$, $M_x = -1300 \times 0.5 + 400 = -250$ Nm

At $x = 1$ m, $M_x = -1300 + 400 = -900$ Nm

At any section between B and C distant x from E

S.F. = S_x = + 500 + 800 + 300 = 1600 N

B.M. = M_x

$= -500x - 800\,(x - 0.5) - 300\,(x - 1)$ Nm

$= -1600\,x + 700$ Nm

At $x = 1$ m, $M_x = -1600 + 700 = -900$ Nm

At $x = 1.5$ m,

$M_x = -1600 \times 1.5 + 700 = -1700$ Nm

At any section between A and B distant x from E

S.F. = S_x = 500 + 800 + 300 + 400 = 2000 N

B.M. = M_x = – 500 x – 800 (x – 0.5)

– 300 (x – 1) – 400 (x – 1.5)

= – 2000 x + 1300 Nm

At $x = 1.5$m,

$M_x = -2000 \times 1.5 + 1300 = -1700$ Nm

At $x = 2$m, $M_x = -2000 \times 2 + 1300$

= – 2700 Nm

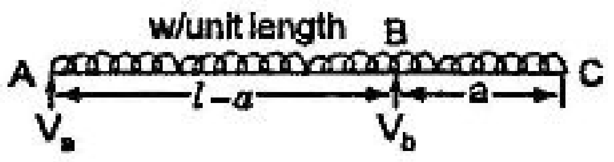
6. A beam of length l carries a uniformly distributed load of 'w' per unit run on its whole length. It has one support at its left end and the other support is at a distance 'a' from the other end.

(*i*) Find the value of length 'a', so that the maximum bending moment for the beam is as small as possible.

(*ii*) Find also the maximum bending moment for this condition.

Solution : Let ABC be the beam of length l with supports at A and B, so that overhang BC = a

Taking moments about A, we have



$$V_b(l-a) = \frac{wl^2}{2}$$

or $$V_b = \frac{wl^2}{2(l-a)}$$

$\therefore$ $$V_a = wl - \frac{wl^2}{2(l-a)} = \frac{wl(l-2a)}{2(l-a)}$$

$$\text{Hogging moment at B} = -\frac{wa^2}{2}$$

Maximum positive bending moment will occur at a section in AB where the shear force is zero. Let, this section be at a distance x from A.

Equating the shear force to zero, we have,

$$V_a - wx = 0$$

or $$\sqrt{\frac{wl(l-2a)}{2(l-a)}} - wx = 0,$$

thus $$x = \frac{l(l-2a)}{2(l-a)}$$

Maximum sagging moment at the above section

$$= V_a x - \frac{wx^2}{2}$$

$$= \frac{wl(l-2a)}{2(l-a)} \cdot \frac{l(l-2a)}{2(l-a)} - \frac{w}{2} \cdot \frac{l^2(l-2a)^2}{4(l-a)^2}$$

$$= \frac{wl^2(l-2a)^2}{8(l-a)^2}$$

For the condition that the maximum bending moment shall be as small as possible, the hogging moment at B and the maximum sagging moment should be numerically equal.

or $$\frac{wl^2(l-2a)^2}{8(l-a)^2} = \frac{wa^2}{2}$$

or $$\frac{l^2(l-2a)^2}{4(l-a)^2} = a^2$$

or $$\frac{l(l-2a)}{2(l-a)} = a$$

or $$l^2 - 2al = 2al$$

or $$2a^2 - 4al + l^2 = 0$$

or $$a = \frac{4l \pm \sqrt{16l^2 - 8l^2}}{4}$$

or $$a = \left(\frac{4-2\sqrt{2}}{4}\right)l = \left(\frac{2-\sqrt{2}}{2}\right)l$$

$$= 0.293l$$

(*ii*) Maximum bending moment with the above value of a

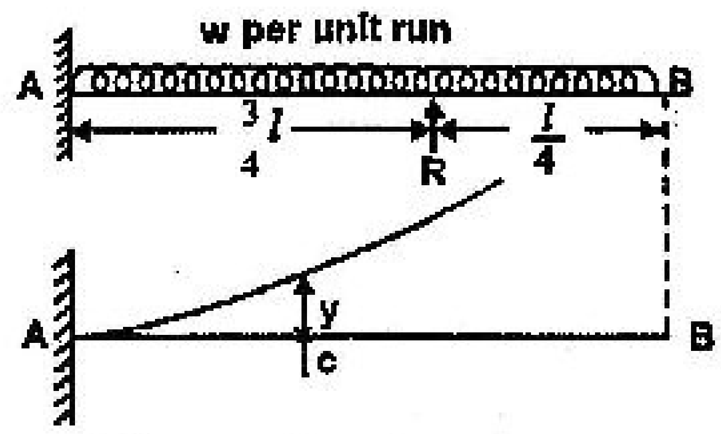
$$= \frac{wa^2}{2} = \frac{w}{2}(0.293\,l)^2 = 0.04292$$

$$wl^2 = \frac{wl^2}{23} \text{ (nearly)}$$

7. A horizontal cantilever of length l supports a uniformly distributed load of w per unit run along its length. The cantilever is propped to the level of the fixed end at a distance $\frac{3}{4}l$ from the fixed end. What is the reaction of the prop?

Solution : $$EI\frac{d^2y}{dx^2} = -\frac{w}{2}(l-x)^2$$

Integrating, we get $$EI\frac{dy}{dx} = \frac{w(l-x)^3}{6} + C_1$$



Slope at A is zero, *i.e.* at $x = 0$

$$\frac{dy}{dx} = 0$$

$\therefore$ $$0 = \frac{wl^3}{6} + C_1$$

or $$C_1 = \frac{wl^3}{6}$$

$\therefore$ $$EI\frac{dy}{dx} = \frac{w(l-x)^3}{6} - \frac{wl^3}{6}$$

On integration again, we get

$$EIy = \frac{w(l-x)^4}{6} - \frac{wl^3}{6}x + C_2$$

At $x = 0, y = 0$

$\therefore$ $$0 = \frac{wl^4}{24} + C_2$$

or $\qquad C_2 = \dfrac{wl^4}{24}$

$\therefore \qquad EIy = \dfrac{-w(l-x)^4}{24} \dfrac{wl^3}{6}x + \dfrac{wl^4}{24}$

Let, the deflection at C be y_c

At $x = \dfrac{3}{4}l,\ y = y_c$

$$\therefore \qquad EIy_c = \frac{w}{24}\left(\frac{l}{4}\right)^4 - \frac{wl^3}{6} \times \frac{3}{4}l + \frac{wl^4}{24}$$

$$= -\frac{171}{2048}wl^4$$

$$\therefore \qquad y_c = -\frac{171}{2048}\frac{wl^4}{EI}$$

Hence, the downward deflection at C when the prop is absent

$$y_c = \frac{171}{2048}\frac{wl^4}{EI}$$

But upward deflection at C due to R alone

$$\frac{R\left(\frac{3}{4}l\right)^3}{3\ EI} = \frac{9}{64}\frac{Rl^3}{EI}$$

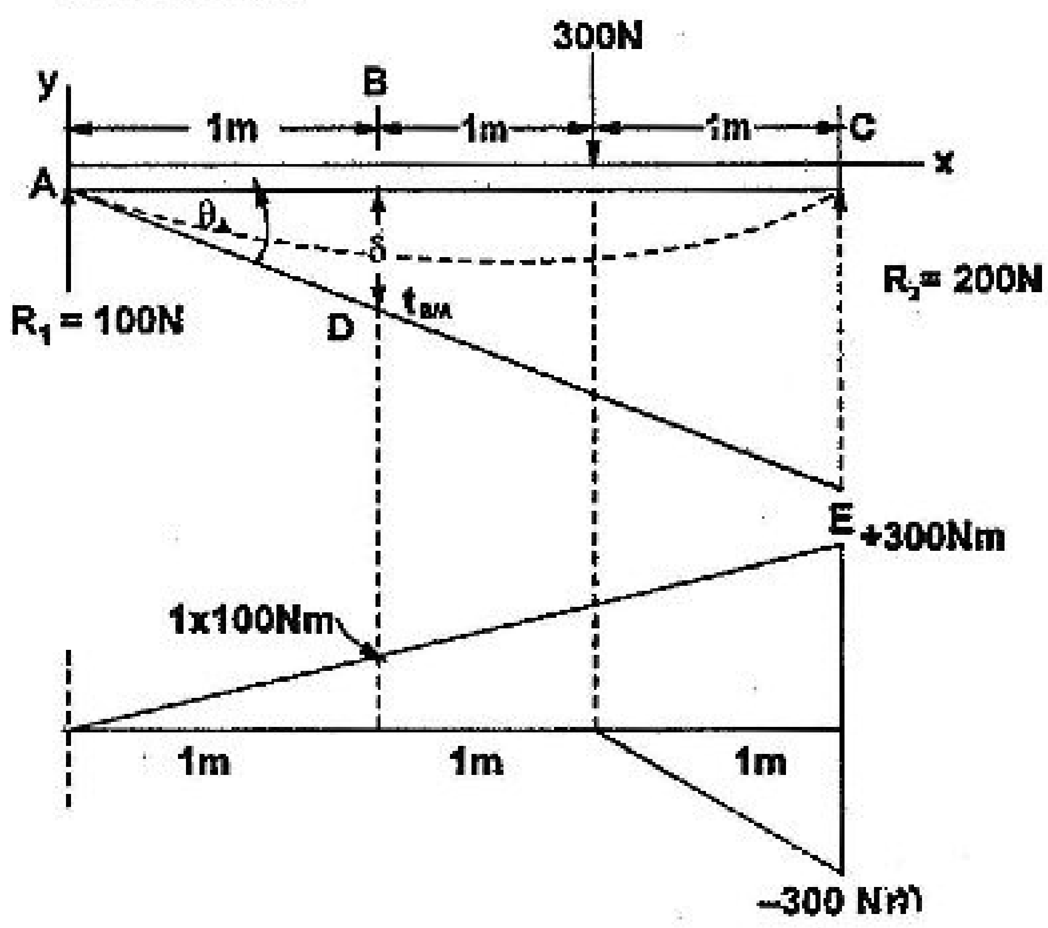
Since, the net deflection at C is zero, there fore

$$\frac{9}{64}\frac{Rl^3}{EI} = \frac{171}{2048}\frac{wl^4}{EI}$$

or $\qquad R = \dfrac{19}{32}wl$

8. The simple beam in the figure below supports a concentrated load of 300 N at 2 m from the left support. What is the value of EIδ at point B which is 1 m from the left support ?

Solution :



$$t_{C/A} = \frac{1}{EI}(\text{area})_{CA} \cdot \bar{x}_c$$

$$= \frac{1}{EI}\left[\left(\frac{3 \times 300}{2}\right)\left(\frac{1}{3} \times 3\right) - \left(\frac{1 \times 300}{2}\right)\left(\frac{1}{3} \times 1\right)\right]$$

or $\qquad t_{C/A} = \dfrac{400}{EI}$

From similar triangles ABD and ACE, we have

$$BD = \frac{1}{3} \times t_{C/A} = \frac{400}{3EI}$$

Now, $\qquad t_{B/A} = \left(\dfrac{1}{EI}(\text{area})_{BA} \cdot \bar{x}_B\right)$

$$\therefore \qquad t_{B/A} = \frac{1}{EI}\left[\left(\frac{1 \times 100}{2}\right)\left(\frac{1}{3} \times 1\right)\right] = \frac{100}{6\ EI}$$

Finally, $\qquad \delta = BD - t_{B/A} = \dfrac{1}{EI}\left(\dfrac{400}{3} - \dfrac{100}{6}\right)$

or $\qquad EI\delta = 116.7\ \text{N-m}^3$

9. (i) Determine the maximum allowable internal pressure to which a cylinder with an internal diameter to 10 cm and external diameter 20 cm may be subjected, if a cylinder is to have a factor of safety 3. Use maximum shear stress theory of failure. Yield point stress determined by actual test for the material is $\sigma_{y.p} = 3000\ \text{kg/cm}^2$

(ii) What will be the internal pressure if maximum stress theory of failure is used ?

Solution : Inner radius, $a = \dfrac{10}{2} = 5$ cm

Outer radius, $b = \dfrac{20}{2} = 10$ cm

For a cylinder subjected to internal pressure p only, we have

At $r = 5,\ \sigma_r = -p$

At $r = 10,\ \sigma_r = 0$

Now, $\qquad \sigma_r = A + \dfrac{B}{r^2}$

We have, $\qquad A = \dfrac{p}{3}$ and $B = -\dfrac{100}{3}p$

$$\therefore \qquad \sigma_r = \frac{p}{3} - \frac{100p}{3} \cdot \frac{1}{r^2}$$

Since the radial stress for this case is maximum at inner surface of the cylinder, we get

$$\sigma_r = \frac{p}{3} - \frac{200\ p}{3} \cdot \frac{1}{25} = -p$$

Circumferential stress, $\sigma_\theta = \frac{p}{3} + \frac{100p}{3} \cdot \frac{1}{r^2}$

$$= \frac{p}{3} + \frac{100\,p}{3} \cdot \frac{1}{25} = \frac{5p}{3}$$

(*i*) Now if, $\sigma_r = -p$ and $\sigma_\theta = \frac{5p}{3}$ are the principal stresses, then failure according to maximum shear stress theory,

$$\tau_{\max} = \frac{\sigma_\theta - \sigma_r}{2}$$

$$(\tau_{\max})_{\text{allowable}} = \frac{\sigma_{y.p}}{2 \times \text{F.S}}$$

$$= \frac{300}{2 \times 3} = 500 \text{ kg/cm}^2$$

or $$500 = \frac{5}{6}p + \frac{p}{2}$$

or $$p = \frac{1500}{4} = 375 \text{ kg/cm}^2$$

So, maximum internal pressure is 375 kg/cm^2.

(*ii*) If maximum principal stress theory of failure is used for computing the safe internal pressure for given cylinder, we have

$$\frac{\sigma_{y.p}}{\text{F} \cdot \text{S}} = (\sigma_\theta)$$

or $$\frac{3000}{3} = \frac{5p}{3}$$

$\therefore$ $$p = 600 \text{ kg/cm}^2$$

10. Compare the permissible diameter of the steel circular shaft, subjected to torsion, according to following theories of failure, $\mu = 0.3$

(*i*) Maximum stress theory;

(*ii*) Maximum strain theory;

(*iii*) Maximum shearing stress theory; and

(*iv*) Maximum strain energy theory.

Solution : Let, the material has same yield point in tension and compression, then according to

Stress theory, $\sigma_1 = \sigma_{yp}$

Strain theory, $\sigma_1 - \mu\,\sigma_2 = \sigma_{yp}$

Shear stress theory, $\sigma_1 - \sigma_2 = \sigma_{yp}$

Maximum strain energy theory,

$$\sigma_1^2 + \sigma_2^2 - 2\,\mu\,\sigma_1\sigma_2 = \sigma_{yp}^2$$

Thus, it is a case of pure shear, we have

$$\sigma_1 = -\sigma_2 = \tau$$

So, above equations can be written as

$$\tau_{yp} = \sigma_{yp}$$

and $$\tau_{yp} = \frac{\sigma_{yp}}{1+\mu};\ \tau_{yp} = \frac{\sigma_{yp}}{2};\ \tau_{yp} = \frac{\sigma_{yp}}{\sqrt{2(1+\mu)}}$$

Putting $\mu = 0.3$ for steel, we get from

Maximum stress theory, $\tau_{yp} = \sigma_{yp}$

Maximum strain theory $\tau_{yp} = 0.77\,\sigma_{yp}$

Maximum shear stress theory $\tau_{yp} = 0.5\,\sigma_{yp}$

Maximum strain energy theory $\tau_{yp} = 0.62\,\sigma_{yp}$

or $$\tau_w = \frac{16\,\text{M}_t}{\pi\,d^3}$$

Using maximum stress theory, we have

$$\tau_w = \frac{\tau_{yp}}{\text{F.S}} = \frac{16\,\text{M}_t}{\pi d_1^3} = \frac{\sigma_{yp}}{\text{F.S}} \quad ...(i)$$

Using maximum strain theory, we have

$$\frac{16\,\text{M}_t}{\pi d_2^2} = \frac{0.77\,\sigma_{yp}}{\text{F.S}}$$

or $$\frac{\sigma_{yp}}{\text{F.S}} = \frac{16\,\text{M}_t}{0.77\,\pi\,d_2^3} \quad ...(ii)$$

Comparing equations (i) and (ii), we have

$$d_1 : d_2 = 1 : 1.09$$

Using maximum shear stress theory, we get

$$\frac{16\,\text{M}_t}{\pi\,d_2^3} = \frac{0.5\sigma_{yp}}{\text{F.S.}}$$

$$\frac{\sigma_{yp}}{\text{F.S.}} = \frac{16\,\text{M}_t}{0.5\pi\,d_3^3} \quad ...(iii)$$

Comparing equations (*i*) and (*iii*), we get

$$d_1 : d_3 = 1:1.26$$

By strain energy theory, we have

$$\frac{16\,\text{M}_t}{\pi d_4^3} = \frac{0.62\,\sigma_{yp}}{\text{F.S.}}$$

or $$\frac{\sigma_{yp}}{\text{F.S.}} = \frac{16\,\text{M}_t}{0.62\,\pi d_4^3} \quad ...(iv)$$

Comparing equations (*i*) and (*iv*), we have

$$d_1 : d_4 = 1 : 1.17$$

Hence, consisting the four theories of failure, we obtained diameters as

$$1 : 1.09 : 1.26 : 1.77$$

11. A round steel rod of diameter 15 mm and length 2 metre is subjected to a gradually increasing axial compressive load.

Take E = 2.1×10^5 N/mm^2 and the yield stress of steel = 250 N/mm^2.

(i) Using Euler's formula, what is the buckling?

(ii) Find maxium lateral deflection corresponding to the buckling condition

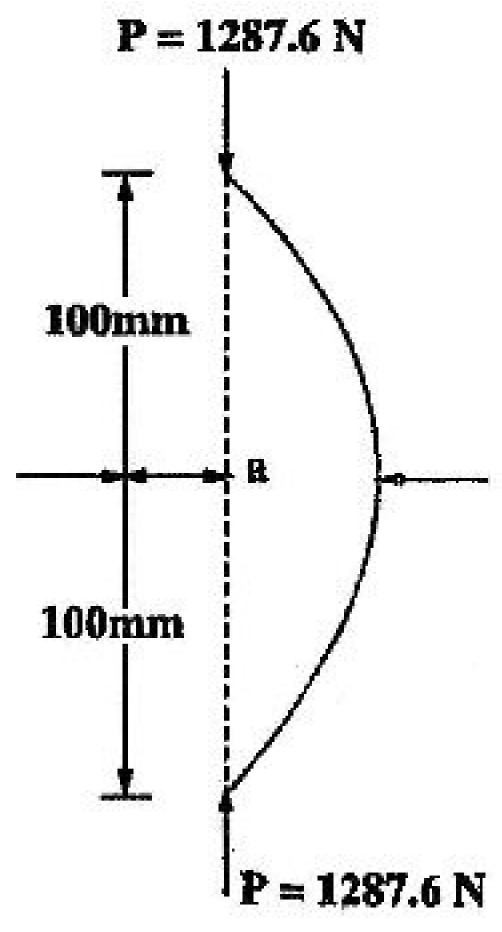
Solution : Area of the rod,

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (15)^2 = 176.71 \text{ mm}^2$$

M.I. of this section,

$$I = \frac{\pi d^4}{64} = \frac{\pi \times 15^4}{64} = 2485.05 \text{ mm}^2$$

Since, both ends of the member are hinged, hence effective length, L = 2000 mm



(*i*) Buckling load (using Biller's formula),

$$P = \frac{\pi^2 \times 2.1 \times 10^5 \times 248.05}{(2000)^2} \text{ N} = 1287.6 \text{ M}$$

(*ii*) Direct compressive stress,

$$\sigma_0 = \frac{P}{A} = \frac{1287.6}{176.71} \text{ N/mm}^2 = 7.29 \text{ N/mm}^2$$

Let, the maximum bending stress corresponding to buckling condition be p_b

$\therefore \quad p_b + p_c =$ yield stress

$\therefore \quad p_b = 250 - 7.29 = 242.71 \text{ N/mm}^2$

Let, M be the maximum bending moment which occurs as the centre

$$\therefore \quad \frac{M}{I} = \frac{p_b}{\left(\frac{d}{2}\right)}$$

$$\text{or} \quad M = \frac{242.71}{7.5} \times 2485.05 \text{ Nm} = 80420 \text{ N-mm}$$

Let, the maximum critical deflection be a mm

$\therefore \quad M = P.a = 1287.6\, a = 80420$

$$\text{or} \quad a = \frac{80420}{12876} = 62.5 \text{ mm}$$

12. A solid shaft is of 100 mm diameter. It transmits 120 NW at 200 rpm.

(Take G = 8×10^4 N mm^2.)

(i) Find the maximum intensity of shear stress induced

(ii) Find the angle of twist for a length of 6 meters

Solution : Power transmitted $= \dfrac{2\,\pi NT}{60000}$

$$\text{or} \quad 120 = \frac{2\pi \times 200T}{60000}$$

$$T = \frac{120 \times 20000}{2\pi \times 200} = 5729 \text{ Nm}$$

$$= 5729 \times 1000 \text{ Nmm}$$

Polar moment of inertia, $I_P = \dfrac{\pi d^4}{32}$, $R = \dfrac{d}{2}$

$$\therefore \quad \frac{T}{I_P} = \frac{f_s}{R}$$

(*ii*)

$$f_s = \frac{T}{I_P}.\, R = \frac{16T}{\pi d^3}$$

$$= \frac{16 \times 5729 \times 1000}{\pi \times 100^3} = 29.17 \text{ N/mm}^2$$

$$\frac{T}{I_P} = \frac{C\theta}{l}$$

$$\text{or} \quad \theta = \frac{T}{I_P}.\frac{l}{C}$$

$$= \frac{5729 \times 1000 \times 32}{\pi \times 100^4} \times \frac{6000}{8 \times 10^4} \text{ radians}$$

$$= 0.04375 \text{ radians} = 2°30'$$

13. A solid shaft of diameter 30 mm is fixed at one end. It is subjected to a tensile force of 10 kN and a torque of 60 Nm. At a point on the surface of the shaft, determine the principal stresses and the maximum shear stress.

Solution : Given : D = 30 mm = 0.03 m;

P = 10 kN; T = 60 Nm.

Principal stresses (σ_1, σ_2) and maximum shear stress (τ_{max}) :

$$\text{Tensile stress, } \sigma_1 = \sigma_x = \frac{10 \times 10^3}{\frac{\pi}{4} \times 0.03^2}$$

$$= 14.15 \times 10^6 \text{ N/m}^2 \text{ or } 14.15 \text{ MN/m}^2$$

As per torsion equation, $\dfrac{T}{I_P} = \dfrac{\tau}{R}$

$$\therefore \quad \text{Shear stress, } \tau = \frac{TR}{I_P} = \frac{TR}{\frac{\pi}{32} D^4}$$

$$= \frac{60 \times 0.015}{\frac{\pi}{32} \times (0.03)^4}$$

$= 11.32 \times 10^6$ N/m² or 11.32 MN/m²

The principal stresses are calculated by using the relations:

$$\sigma_1, \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Here, $\sigma_x = 14.15\text{MN/m}^2$, $\sigma_y = 0$;

$\tau_{xy} = t = 11.32$ MN/m²

$$\therefore \quad \sigma_1, \sigma_2 = \frac{14.15}{2} \pm \sqrt{\left(\frac{14.15}{2}\right)^2 + (1132)^2}$$

$= 7.075 \pm 13.35$

$= 20.425$ NM/m², -6.275 MN/m²

Hence, major principal stress

$\sigma_1 = 20.425$ MN/m² (tensile)

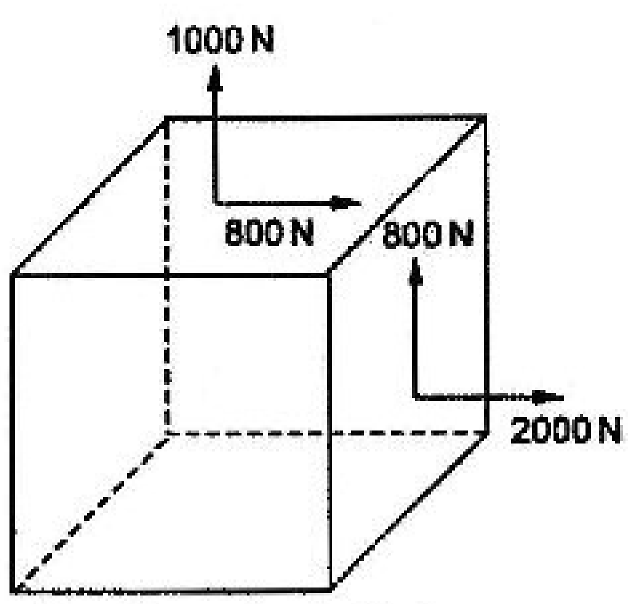
Minor principal stress,

$\sigma_2 = 6.275$ MN/m² (compressive)

Maximum shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$$= \frac{20.425 - (-6.275)}{2} = 13.35 \text{ MN/m}^2$$

14. A cube of 5 mm side is loaded as shown in the figure.



(*i*) Determine the principal stresses.

(*ii*) Will the cube yield if the yield strength of the material is 70 MPa ? Uses Yen Mises theory.

Solution : For loading, $s_x = \frac{2000}{5 \times 5} = 80$ N/mm²;

$$s_y = \frac{1000}{5 \times 5} = 40 \text{ N/mm}^2;$$

$$t_{xy} = \frac{\text{shear force}}{\text{area}}$$

$$= \frac{800}{5 \times 5} = 32 \text{ N/mm}^2;$$

Principal stresses, σ_1, σ_2 and σ_3 :

Principal stresses (maximum and minimum normal stresses)

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80 + 40}{2} \pm \sqrt{\left(\frac{80 - 40}{2}\right)^2 + (32)^2}$$

$= 60 \pm 37.73 = 97.73, 22.27$

$\therefore \quad \sigma_1 = 97.73$ N/mm² and $\sigma_2 = 22.27$ N/mm²

According to Von Mises theory, the material will yield if $(\sigma_1 - \sigma_2)^2 \geq 2$,

where σ_y = yield strength of material

$= 70$ MPa $= 70$ N/mm²

Now, $(\sigma_1 - \sigma_2)^2 = (97.73 - 22.27)^2 = 5694.21$

and, $2\sigma_y^2 = 2 \times 70^2 = 9800$

Therefore, $(\sigma_1 - \sigma_2)^2 < 2\sigma_y^2$

hence, material will not yield.

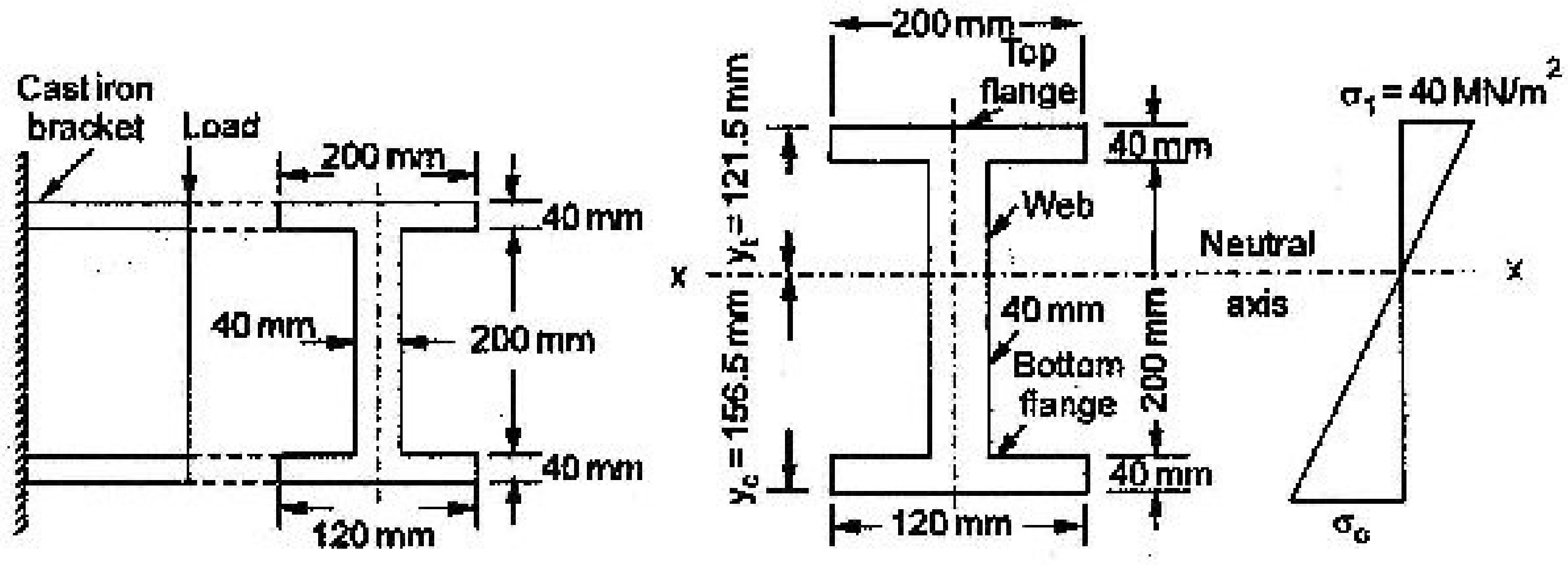
15. Given figure shows a cast iron bracket of cross-section of I-from. Find:

(i) Position of the neutral axis and the moment of the section about the neutral axis; and

(ii) Determine the maximum bending moment that should be imposed on this section if the tensile stress in the top flanges is not to exceed 40 MN/m². What is then the value of the compressive stress in the bottom flange ?

Solution :

(*i*) **Position of the neutral axis :** The section may be split into three rectangle components. The table of calculations is given below:



Component	Area 'a' (mm^2)	Centroidal distance from Bottom edge 'y' (mm)	Ay (mm^3)
Top flange	200 × 40 = 8000	260 mm	2080000
Web	200 × 40 = 8000	140 mm	1120000
Bottom flange	120 × 40 = 4800	20 mm	9600
Total	$(\Sigma a) = 20800$		$(\Sigma ay) = 3296000$

∴ Distance of the neutral axis from the bottom edge,

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{3296000}{20800} = 158.5 \text{ mm}$$

Hence, $y_c = 158.5$ mm

and, $y_t = (40 + 200 + 40) - 158.5 = 121.5$mm

M.O.I.

$$I_{xx} = ?,\ I_{xx} = I_{\text{top flange}} + I_{\text{web}} + I_{\text{bottom flange}}$$

$$= \left[\frac{200 \times 40^3}{12} + 200 \times 40 \times (1215 - 20)^2\right] + \left[\frac{40 \times 200^3}{12} + 40 \times 200 \times (1585 - 140)^2\right]$$

$$+ \left[\frac{120 \times 40^3}{12} + 120 \times 40 \times (1585 - 20)^2\right]$$

$$= 83484667 + 29404667 + 92714800 = 2.056 \times 10^8 \text{ mm}^4 = 2.056 \times 10^{-4} \text{ m}^4$$

(*ii*) **Maximum bending moment, M:**

Using the relation, $\frac{M}{I} = \frac{\sigma}{y}$

we get, $\frac{M}{I} = \frac{\sigma_t}{y_t}$

or $M = \frac{\sigma_t}{y_t} \times I = \frac{40 \times 10^6 \times 2.056 \times 10^{-4}}{1215 \times 10^{-3}}$

$= 67687.2$ Nm $= 67.7$ kNm (say)

Compressive stress in the bottom flange, σ_c:

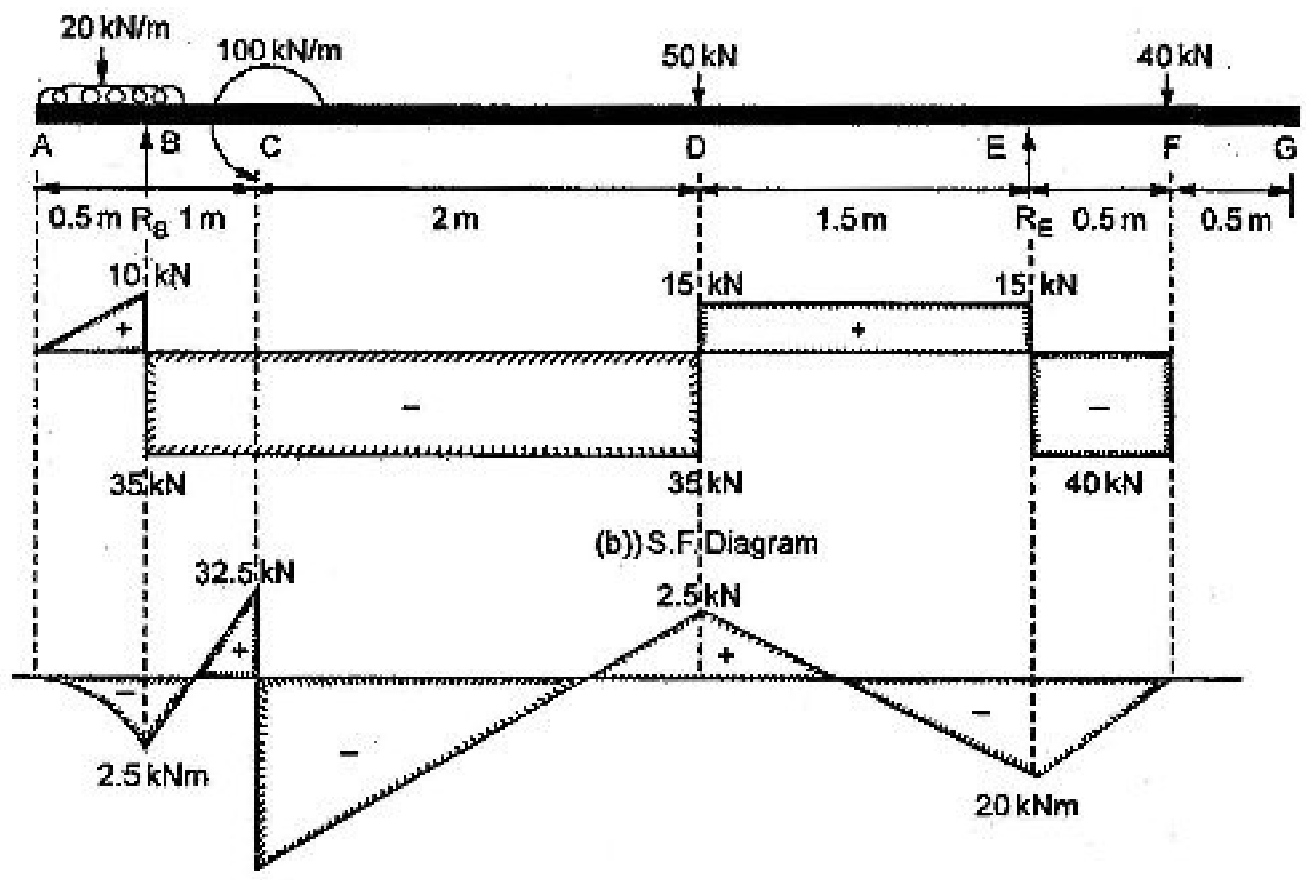
Using the relation, $\frac{M}{I} = \frac{\sigma_c}{y_c}$

or $\sigma_c = \frac{M \cdot y_c}{I}$

$$= \frac{(67.7 \times 10^3) \times (158.5 \times 10^{-3})}{2.056 \times 10^{-4}}$$

$= 52.19 \times 10^6$ N/m^2 $= 52.19$ MN/m^2

16. Construct the bending moment and shearing force diagrams for the beam shown in the figure.



(c) B.M. Diagram

Solution : Reactions at B and E:

Taking moment about B, we get

$$R_E \times 4.5 + 20 \times 0.5 \times \frac{0.5}{2} + 100 = 50 \times 3 + 40 \times 5$$

or, $\quad 4.5\, R_E + 2.5 + 100 = 150 + 200$

$\quad R_E = 55$ kN

Also, $\quad R_B + R_E = 20 \times 0.5 + 50 + 40$

or, $\quad R_B + 55 = 10 + 50 + 40$

$\therefore \quad R_B = 45$ kN

S.F. Calculations: $S_F = -40$ kN

$S_E = -40 + 55 = 15$ kN

$S_D = 15 - 50 = -35$ kN

$S_B = -35 + 45 = 10$ kN

S.F. diagram is shown in (*b*).

B.M. Calculations: $M_G = 0$

$M_F = 0$

$M_E = -40 \times 0.5 = -20$ kNm

$M_D = -40 \times 2 + 55 \times 1.5 = 2.5$ kNm

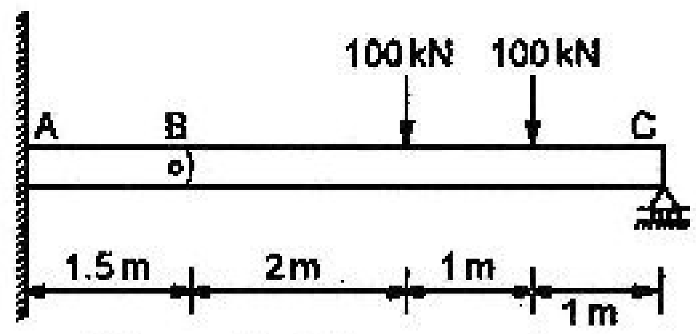
$M_C = -40 \times 4 + 55 \times 3.5 - 50 \times 2 = -67.5$ kNm

The bending moment increases from – 67.5 kNm is 100 kNm

$$M_B = -20 \times 0.5 \times \frac{0.5}{2} = -2.5 \text{ kNm}$$

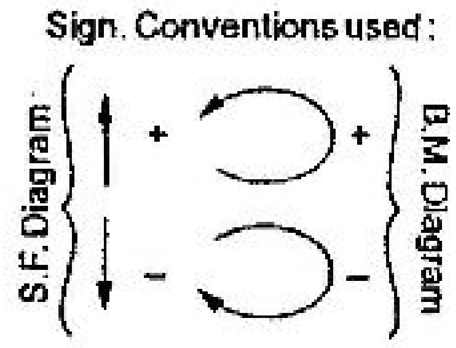
B.M. diagram is shown in the figure.

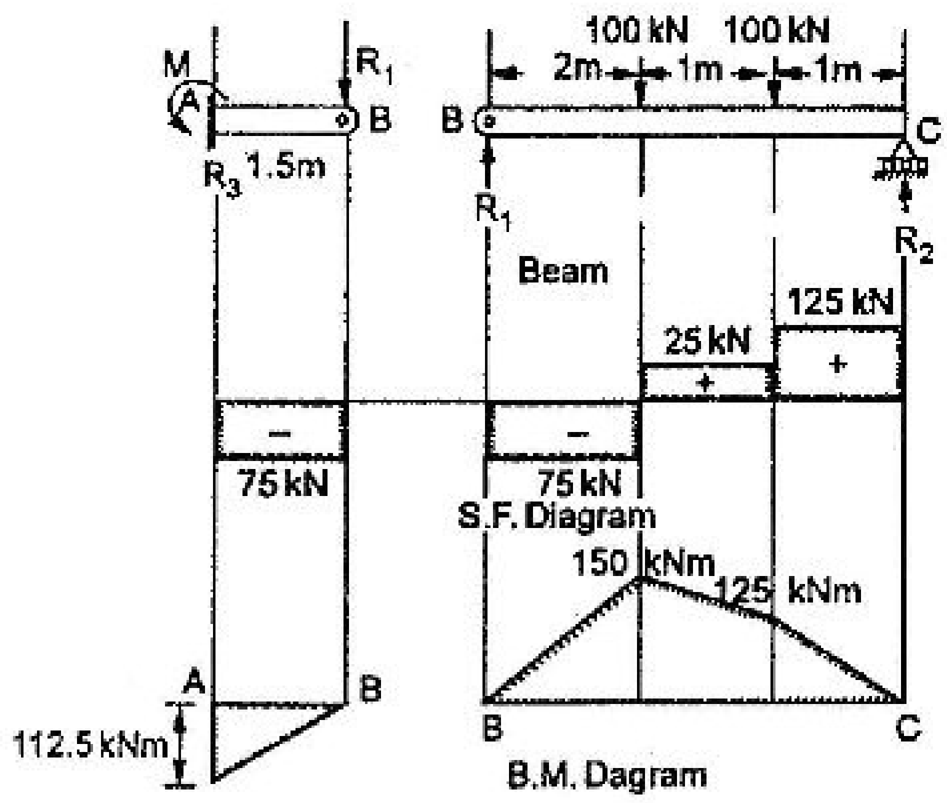
17. Two bars AB and BC are connected by a frictionless hinge at B. The assembly is supported and loaded as shown in the figure. Draw the shear force and bending moment diagrams for the combined beam AC, clearly labelling the important values. Also indicate your sign convections.



Solution : There shall be a vertical reaction at hinge B and we can split the problem in two parts.

Sign. Conventions used :





$F_y = 0$

From $\quad R_1 + R_2 = 200$ kN

From $\Sigma M_B = 0, \quad 100 \times 2 + 100 \times 3 - R_2 \times 4 = 0$

or, $R_2 = \frac{500}{4} = 125\text{kN}$

$\therefore$ $R_1 = 200 - 125 = 75$ kN

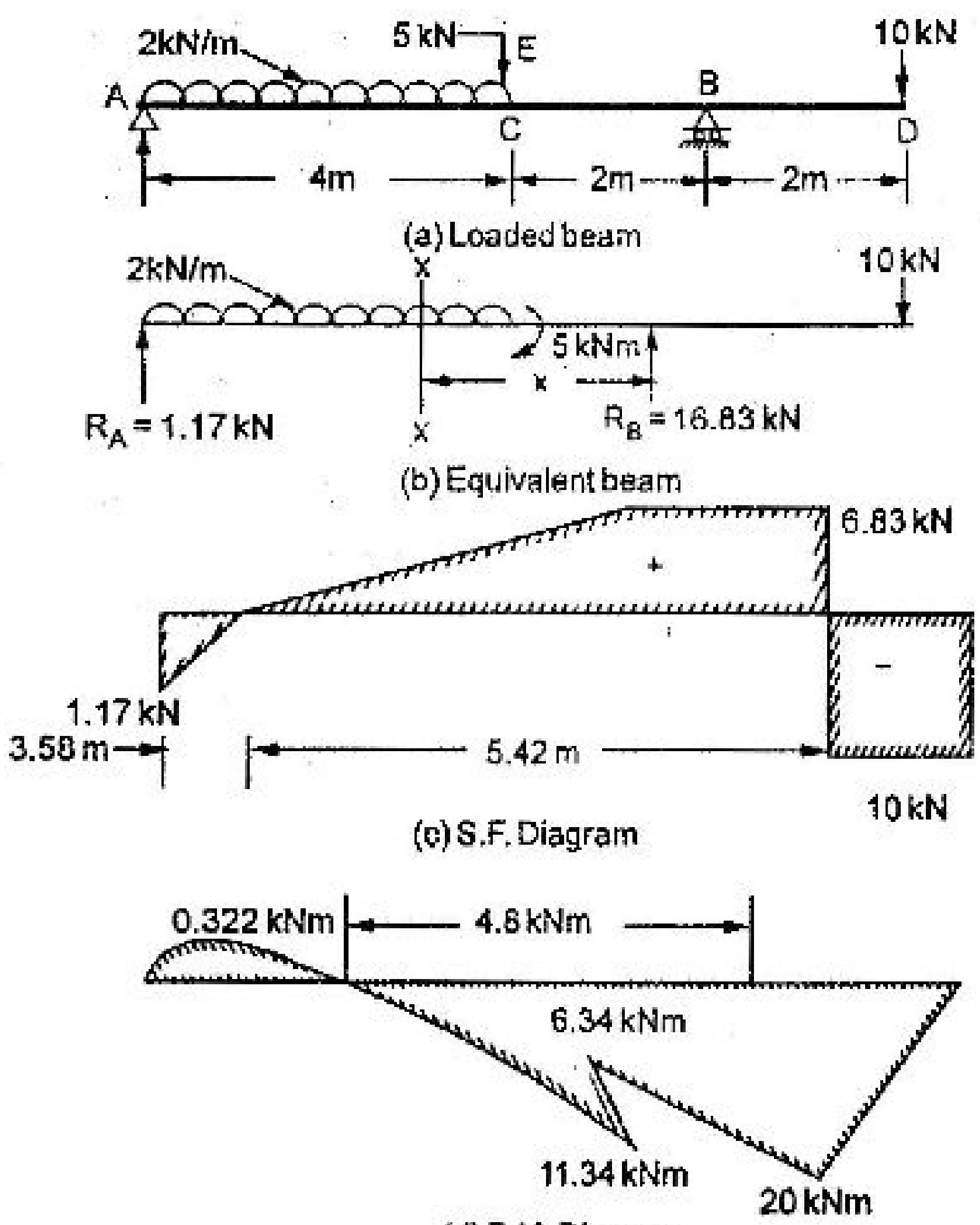
From figure, $R_3 = R_1 = 75$ kN

and $M = 75 \times 1.5 = 112.4$ kNm.

The important values are labelled on S.F. diagram and B.M. diagram.

18. Draw the shearing force and bending moment diagrams for a beam shown in the figure.

Solution : The horizontal force of 5 kN acting on the top of aim lever at C causes a clockwise moment of 5 kNm at C and a horizontal thrust of 5 kN in the beam. Since, the horizontal thrust in beam effects neither the shear force in the beam nor the B.M. in it, for purpose of analysing the shear and the bending moment in the beam the given force of 5 kN can be replaced by a clockwise movement of 5 kNm Fig. (*b*) acting at C.



(a) Loaded beam

(b) Equivalent beam

(c) S.F. Diagram

(d) B.M. Diagram

To determine reactions R_A and R_B, taking moments about A, we get,

$$R_B \times 6 = 2 \times 4 \times \frac{4}{2} + 5 + 10(4 + 2 + 2) = 16 + 5 + 80$$

or $R_B = \frac{101}{6} = 16.83$ kN

But $R_A + R_B = 2 \times 4 + 10 = 18,$

$\therefore$ $R_A = 18 - R_B = 18 - 16.83 = 1.17$ kN

S.F Calculations:

$$S_{D-B} = -10 \text{ kN}$$

$$S_{B-C} = -10 + 16.83 = 6.83 \text{ kN}$$

$$S_A = 6.83 - 2 \times 4 = -1.17 \text{ kN}$$

S.F. at a distance x from B in AC is

$$S_x = 10 + 16.83 - 2(x - 2)$$

To locate the point of zero shear we have, $S_x = 0$

i.e., $-10 + 16.83 - 2(x - 2) = 0$

or $+6.83 - 2x + 4 = 0$

or $x = 5.415$ m say 5.42 m

or $(6 - 5.42) = 0.58$ m from A

S.F. diagram is shown in Fig.(*c*)

B.M. Calculations: $M_D = 0$

$M_B = -10 \times 2 = -20$ kNm

$M_C = -10 \times 4 + 16.83 \times 2 - 5 = -11.34$ kNm

$$M_{max} \text{ (5.42 m from B)} = -10 \times (2 + 5.42) + 16.83 \times 5.42 - 5 - \frac{2 \times (5.42 - 2)^2}{2}$$

$$= -74.2 + 91.218 - 5 - 11.696 = 0.322 \text{ kNm}$$

$M_A = 0$

To locate the point of contra flexure between A and C, find out the bending moment at the section XX at a distance x from B in AC and equate it to zero as given below :

$$M_x = -10(2 + x) + 16.83 \times x - 5 - \frac{2 \times (x - 2)^2}{2}$$

or, $-20 - 10x + 16.38x - 5 - (x - 2)^2 = 0$

or $-25 + 6.83x - (x^2 - 4x + 4) = 0$

or $-25 + 6.83x - x^2 + 4x - 4 = 0$

or $x^2 - 10.83x + 29 = 0$

$$\therefore \quad x = \frac{+10.83 \pm \sqrt{(10.83)^2 - 4 \times 29}}{2}$$

$$= \frac{+10.83 \pm 1.135}{2} = 5.98 \text{ m or } 4.84 \text{ m}$$

The value of $x = 5.98$ m is discarded being impossible since the beam is only 6 m long to the left of B. The B.M. is thus zero at 4.84 m from B.

19. A beam of span 8 metres is simply supported at the ends. It carries a uniformly distributed load of 30 kN/m over its entire length and a concentrated load of 60 kN at 3 metres from the support A. Determine the maximum deflection in the beam and the location where the deflection occurs. Take $E = 200 \times 10^6$ kN/m², $I = 80 \times 10^{-6}$ m⁴

Solution : Length of span of the beam, $l = 8$ m

Moment of inertia, $I = 80 \times 10^{-4}$ m⁴

Young's modulus, $E = 200 \times 10^6$ kN/m²

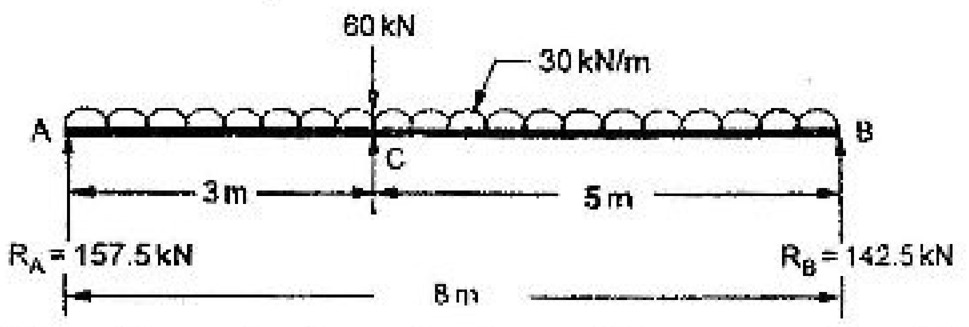
To calculate reaction at B, taking moments about A, we get,

$$R_B \times 8 = 60 \times 3 + 30 \times 8 \times \frac{8}{2} = 1140,$$

$\therefore$ $R_B = 142.5$ kN

Also, $R_A + R_B = 60 + 30 \times 8 = 300$

$R_A = 300 - 142.5 = 157.5$ kN



Using Macaulay's method consider any section XX at a distance x from the end A; the bending moment at the section XX,

$$M_x = EI\frac{d^2y}{dx^2} = 157.5x - \frac{30x^2}{2} - 60(x-3) \quad ...(i)$$

On successively integrating the above equation, we have,

$$EIy\frac{dy}{dx} = \frac{157.5x^2}{2} - 5x^3 + C_1 - 30(x-3)^2 \quad ...(ii)$$

$$EIy = \frac{157.5x^2}{6} - \frac{5x^4}{4} + C_1 - 30(x-3)^3 \quad ...(iii)$$

When, $x = 0$, y therefore

$\therefore$ $C_2 = 0$

When, $x = 8\ m, y = 0$

$$\therefore\ 0 = \frac{157.5 \times 8^3}{6} - \frac{5 \times 8^4}{4} + 8C_1 - 10(8-3)^3$$

$$= 13440 - 5120 + 8\,C_1 - 1250 = 7070 + 8C_1$$

$C_1 = -883.75$

Hence, the slope and deflection equations are

$$EI\frac{dy}{dx} = \frac{157.5x^2}{2} - 5x^3 - 883.75 - 30\ (x-3)^2 \ ...$$

Slope equation

$$EIy = \frac{157.5x^3}{6} - \frac{5x^4}{4} - 883.75x - 10\,(x-3)^3$$

The maximum deflection and its location :

For maximum deflection, equating the slope at the section to zero, we get

$$EI\frac{dy}{dx} = \frac{157.5x^2}{6} - 5x^3 - 883.75 - 30(x-3)^2 = 0$$

By trial and error, $x = 3.92$ m satisfies it.

$\therefore$ Deflection is maximum at a distance of 3.92 metres from A.

To get maximum deflection (y_{max}) putting this value of x in the deflection equation, we have

$$EIy_{max} = \frac{157.5 \times 3.92^3}{6} - \frac{5 \times 3.92^4}{4}$$

$$- 883.75 \times 3.92 - 10 \times (3.92 - 3)^3$$

$$= 1581.2 - 295.16 - 3464.3 - 7.78 = -2186$$

$$\therefore \quad y_{max} = \frac{2186}{EI}$$

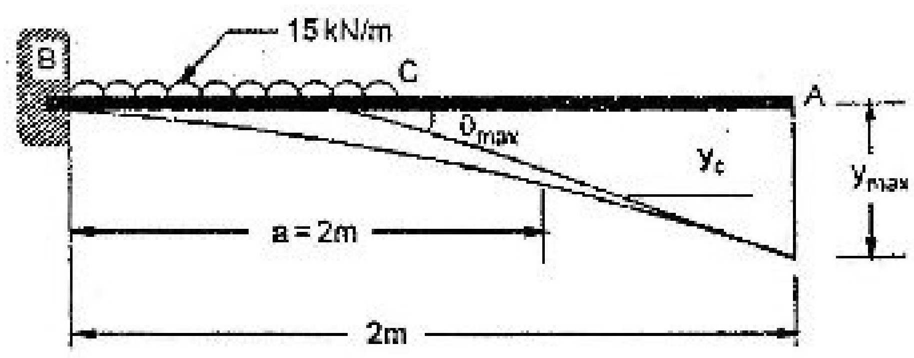
$$= -\frac{2186}{200 \times 10^6 \times 80 \times 10^{-4}} \times 10^3 \text{ mm}$$

$$= -1.366 \text{ mm}$$

Hence, $y_{max} = 1.366$ mm (downward)

20. A cantilever 3 m long is loaded with a uniformly distributed load of 15 kN/m over a length of 2 m from the fixed end. Determine the slope and deflection at the free end of the cantilever. Take $E = 2.1 \times 10^8$ kN/m² and $I = 0.000095$ m⁴.

Solution :



Given : Length, $l = 3$ m

U.D.L., $w = 15$kN/m

Loaded length, $a = 2$ m

Tbtal load, $W = 15\,a = 15 \times 2 = 30$ kN

Moment of inertia, $I = 0.000095$ m⁴

Young's modulus, $E = 2.1 \times 10^8$ kN/m²

Slope at the free end, ϕ_{max}:

$$\theta_{max} = \frac{Wa^2}{6EI}$$

$$= \frac{30 \times 2^2}{6 \times 2.1 \times 10^8 \times 0.000095}$$

$$= 0.01 \text{ radian}$$

Deflection at the free end, y_{max} :

$$y_{max} = \frac{Wa^3}{8\,EI} + \frac{Wa^3}{6\,EI}(l - a)$$ with usual notations, we have,

$$= \frac{30 \times 2^3}{8 \times 2.1 \times 10^8 \times 0.000095} + \frac{30 \times 2^2}{6 \times 2.1 \times 10^8 \times 0.000095}(3-2)$$

$$= 0.0015 + 0.001$$

$$= 0.0025 \text{ m}$$

$$= 2.5 \text{ mm}$$