

Properties of Parallel Lines

Q1

Answer :

Given : $l \parallel m$

t is a transversal.

$$\angle 5 = 70^\circ$$

$$\angle 5 = \angle 3 = 70^\circ \quad (\text{alternate interior angles})$$

$$\angle 5 + \angle 8 = 180^\circ \quad (\text{linear pair})$$

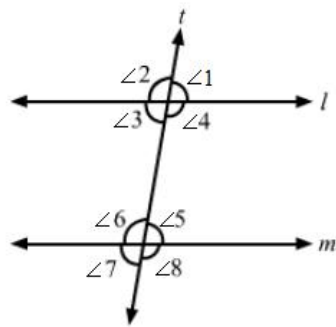
$$\text{or } 70^\circ + \angle 8 = 180^\circ$$

$$\angle 8 = 110^\circ$$

$$\angle 1 = \angle 3 = 70^\circ \quad (\text{vertically opposite angles})$$

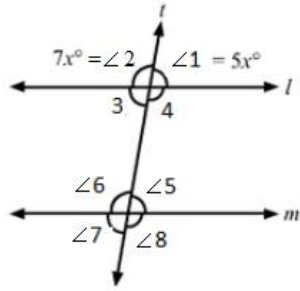
$$\angle 3 + \angle 4 = 180^\circ \quad (\text{linear pair})$$

$$\text{or } \angle 4 = 180 - \angle 3 = 180 - 70 = 110^\circ$$



Q2

Answer :



Given : $l \parallel m$

t is a transversal.

$$\angle 1 : \angle 2 = 5 : 7$$

Let the angles measure $5x$ and $7x$.

$$\angle 1 + \angle 2 = 180^\circ \quad (\text{linear pair})$$

$$\therefore 5x + 7x = 180$$

$$\text{or } 12x = 180$$

$$\text{or } x = 15$$

$$\therefore \angle 1 = 5x = 5(15) = 75^\circ$$

$$\text{and } \angle 2 = 7x = 7(15) = 105^\circ$$

$$\angle 2 + \angle 3 = 180^\circ \quad (\text{linear pair})$$

$$\angle 3 = 180 - 105 = 75^\circ$$

$$\angle 3 + \angle 6 = 180 \quad (\text{interior angles on the same side of the transversal are supplementary})$$

$$\angle 6 = 180 - \angle 3 = 105^\circ$$

$$\text{and } \angle 6 = \angle 8 = 105^\circ \quad (\text{vertically opposite angles})$$

$$\therefore \angle 1 = 75^\circ$$

$$\angle 2 = 105^\circ$$

$$\angle 3 = 75^\circ$$

$$\angle 8 = 105^\circ$$

Q3

Answer :

Given : $l \parallel m$

t is a transversal.

Let :

$$\angle 1 = (2x - 8)^\circ$$

$$\angle 2 = (3x - 7)^\circ$$

We know that the consecutive interior angles are supplementary.

$$\therefore \angle 1 + \angle 2 = 180^\circ$$

$$\text{or } (2x - 8) + (3x - 7) = 180$$

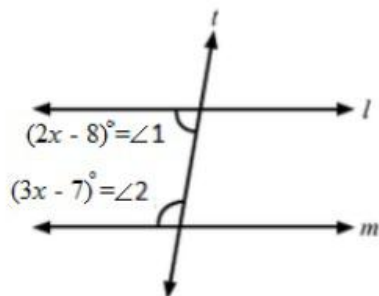
$$\text{or } 5x - 15 = 180$$

$$\text{or } 5x = 195$$

$$\text{or } x = 39$$

$$\angle 1 = (2x - 8) = (2 \times 39 - 8) = 70^\circ$$

$$\angle 2 = (3x - 7) = (3 \times 39 - 7) = 110^\circ$$



Q4

Answer :

From the given figure:

$$\angle 1 = \angle 3 = 50^\circ \text{ (corresponding angles)}$$

$$\text{and } \angle 1 + x^\circ = 180^\circ \text{ (linear pair)}$$

$$\text{or } x^\circ = 180^\circ - 50^\circ = 130^\circ$$

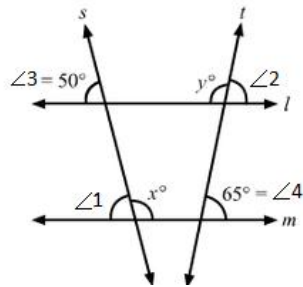
$$\text{or } x = 130$$

$$\angle 2 = \angle 4 = 65^\circ \text{ (corresponding angles)}$$

$$\text{and } \angle 2 + y^\circ = 180^\circ \text{ (linear pair)}$$

$$\text{or } y^\circ = 180^\circ - 65^\circ = 115^\circ$$

$$\text{or } y = 115$$



Q5

Answer :

Given :

$$\angle B = 65^\circ$$

$$\angle C = 45^\circ$$

DAE \parallel BC

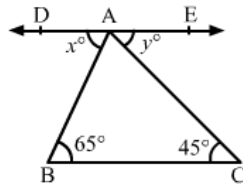
The given lines are parallel.

$$\therefore x^\circ = \angle B = 65^\circ \quad (\text{alternate angles when AB is taken as the transversal})$$

$$y^\circ = \angle C = 45^\circ \quad (\text{alternate angles when AC is taken as the transversal})$$

$$\therefore x = 65$$

$$y = 45$$



Q6

Answer :

Given : CE \parallel BA

$$\angle BAC = 80^\circ, \angle ECD = 35^\circ$$

$$(i) \angle BAC = \angle ACE = 80^\circ \quad (\text{alternate angles with AC as a transversal})$$

$$(ii) \angle ACB + \angle ACD = 180^\circ \quad (\text{linear pair})$$

$$\text{or } \angle ACB + \angle ACE + \angle ECD = 180^\circ$$

$$\text{or } \angle ACB + 80^\circ + 35^\circ = 180^\circ$$

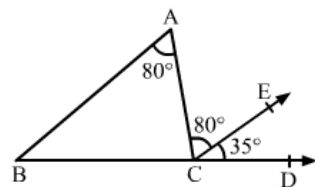
$$\text{or } \angle ACB = 65^\circ$$

(iii) In $\triangle ABC$:

$$\angle BAC + \angle ACB + \angle ABC = 180^\circ \quad (\text{angle sum property})$$

$$80^\circ + 65^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 35^\circ$$



Q7

Answer :

Given : $AO \parallel CD$

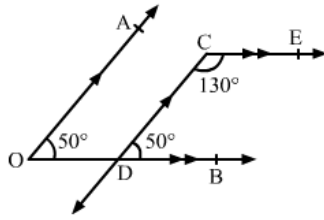
$OB \parallel CE$

$$\angle AOB = 50^\circ$$

$$\angle AOD = \angle CDB = 50^\circ \quad (\text{when } AO \parallel CD \text{ and } OB \text{ is the transversal})$$

$$\angle ECD + \angle CDB = 180^\circ \quad (\text{consecutive interior angles are supplementary, } DB \parallel CE \text{ and } CD \text{ is the transversal})$$

$$\angle ECD = 180^\circ - 50^\circ = 130^\circ$$



Q8

Answer :

Given : $AB \parallel CD$

$$\angle ABO = 50^\circ$$

$$\angle CDO = 40^\circ$$

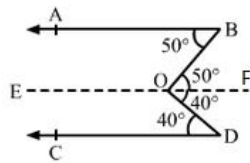
Construction : Through O, draw $EOF \parallel AB$.

$$\angle ABO = \angle BOF = 50^\circ \quad (\text{alternate angles, when } AB \parallel EF \text{ and } OB \text{ is a transversal})$$

$$\angle FOD = \angle ODC = 40^\circ \quad (\text{alternate angles, when } CD \parallel EF \text{ and } OD \text{ is a transversal})$$

$$\angle BOD = \angle BOF + \angle FOD$$

$$\angle BOD = 50^\circ + 40^\circ = 90^\circ$$



Q9

Answer :

Given : $AB \parallel CD$

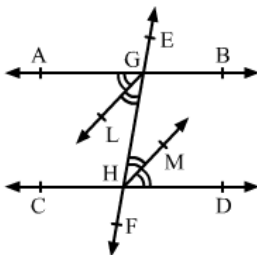
GL and HM are angle bisectors of $\angle AGH$ and $\angle GHD$, respectively.

$$\angle AGH = \angle GHD \quad (\text{alternate angles})$$

$$\text{or } \frac{1}{2} \angle AGH = \frac{1}{2} \angle GHD$$

$$\text{or } \angle LGH = \angle GHM \quad (\text{given})$$

Therefore, $GL \parallel HM$ as we know that if the angles of any pair of alternate interior angles are equal, then the lines are parallel.



Q10

Answer :

Given : $AB \parallel CD$

$$\angle ABE = 120^\circ$$

$$\angle ECD = 100^\circ$$

$$\angle BEC = x^\circ$$

Construction : $FEG \parallel AB$

Now, since $AB \parallel FEG$ and $AB \parallel CD$, $FEG \parallel CD$

$\therefore EFG \parallel AB \parallel CD$

$$\angle ABE = \angle BEG = 120^\circ \text{ (alternate angles)}$$

$$\text{or } x^\circ + y^\circ = 120^\circ \dots (i)$$

$$\angle DCE = \angle CEF = 100^\circ \text{ (alternate angles)}$$

$$\text{or } x^\circ + z^\circ = 100^\circ \dots (ii)$$

$$\text{Also, } x^\circ + y^\circ + z^\circ = 180^\circ \text{ (FEG is a straight line)} \dots (iii)$$

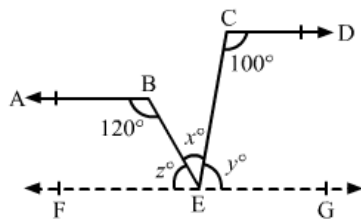
Adding (i) and (ii) :

$$2x^\circ + y^\circ + z^\circ = 220^\circ$$

$$\text{or, } x^\circ + 180^\circ = 220^\circ \text{ (substituting (iii))}$$

$$x^\circ = 40^\circ$$

$$\therefore x = 40$$



Q11

Answer :

Given : $AB \parallel CD$

$AD \parallel BC$

$$\angle 1 + \angle 2 = 180^\circ \text{ (AB \parallel CD and AD is the transversal)} \dots (i)$$

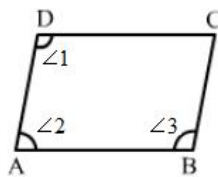
$$\angle 2 + \angle 3 = 180^\circ \text{ (AD \parallel BC and AB is the transversal)} \dots (ii)$$

From (i) and (ii) :

$$\angle 1 + \angle 2 = 180^\circ = \angle 2 + \angle 3$$

$$\angle 1 = \angle 3$$

$$\angle ADC = \angle ABC$$



Q12

Answer :

Given :

$$l \parallel m$$

$$p \parallel q$$

$$\angle 1 = 65^\circ$$

$$\therefore \angle 1 = \angle a = 65^\circ \quad (\text{vertically opposite angles})$$

$$\angle a + \angle d = 180^\circ \quad (\text{consecutive interior angles on the same side of a transversal are supplementary})$$

$$\text{or } \angle d = 180^\circ - 65^\circ = 115^\circ$$

$$\angle c + \angle d = 180^\circ \quad (\text{consecutive interior angles on the same side of a transversal are supplementary})$$

$$\text{or } \angle c = 180^\circ - 115^\circ = 65^\circ$$

$$\angle c + \angle b = 180^\circ \quad (\text{consecutive interior angles on the same side of a transversal are supplementary})$$

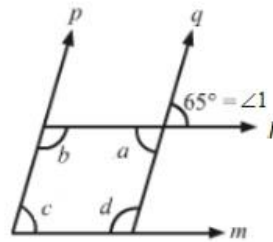
$$\text{or } \angle b = 180^\circ - 65^\circ = 115^\circ$$

$$\therefore \angle a = 65^\circ$$

$$\angle b = 115^\circ$$

$$\angle c = 65^\circ$$

$$\angle d = 115^\circ$$



Q13

Answer :

Given :

$$AB \parallel DC$$

$$AD \parallel BC$$

$$\angle BAC = 35^\circ$$

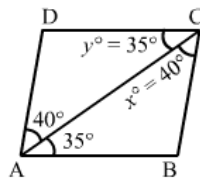
$$\angle CAD = 40^\circ$$

$$\therefore \angle BAC = y = 35^\circ \quad (\text{alternate angles when } AB \parallel DC)$$

$$\angle CAD = x = 40^\circ \quad (\text{alternate angles when } AD \parallel BC)$$

$$\therefore x = 40$$

$$y = 35$$



Q14

Answer :

Given :

$$AB \parallel CD$$

$$\angle BAE = 125^\circ$$

$$\angle CAB + \angle BAE = 180^\circ$$

$$\text{or } 125^\circ + x^\circ = 180^\circ$$

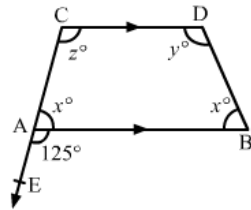
$$\text{or } x = 55$$

$x + z = 180^\circ$ (consecutive interior angles on the same side of transversal are supplementary)

$$z = 180 - x = 180 - 55 = 125$$

$y + x = 180^\circ$ (consecutive interior angles on the same side of transversal are supplementary)

$$y = 180 - x = 180 - 55 = 125$$



Q15

Answer :

$$(i) \angle 1 + \angle 2 = 180 \text{ (linear pair)}$$

$$\text{or } 130^\circ + \angle 2 = 180^\circ$$

$$\text{or } \angle 2 = 50^\circ \neq 40^\circ = \angle 3$$

$$\therefore l \nparallel m$$

$$(ii) \angle 2 + \angle 3 = 180^\circ \text{ (linear pair)}$$

$$35^\circ + \angle 3 = 180^\circ$$

$$\angle 3 = 145^\circ = 145^\circ = \angle 1$$

$$\therefore l \parallel m$$

$$(iii) \angle 2 + \angle 3 = 180 \text{ (linear pair)}$$

$$\angle 3 = 180^\circ - 125^\circ = 55^\circ$$

$$\angle 3 = 55^\circ \neq 60^\circ = \angle 1$$

$$\therefore l \nparallel m$$

