1. Without using trigonometric tables, evaluate:

(i) $\frac{\sin 16^0}{\cos 74^0}$ (ii) $\frac{\sec 11^0}{\operatorname{cosec} 79^0}$ (iii) $\frac{\tan 27^0}{\cot 63^0}$

(iv) $\frac{\cos 35^0}{\sin 55^0}$ (v) $\frac{\operatorname{cosec} 42^0}{\sec 48^0}$ (vi) $\frac{\cot 38^0}{\tan 52^0}$

Sol:

(i) $\frac{\sin 16^0}{\cos 74^0}$

$= \frac{\sin (90^0 - 74^0)}{\cos 74^0}$

$= \frac{\cos 74^0}{\cos 74^0}$ [$\because \sin (90 - \theta) = \cos \theta$]

$= 1$

(ii) $\frac{\sec 11^0}{\operatorname{cosec} 79^0}$

$= \frac{\sec (90^0 - 79^0)}{\operatorname{cosec} 79^0}$

$= \frac{\operatorname{cosec} 79^0}{\operatorname{cosec} 79^0}$ [$\because \sec (90 - \theta) = \operatorname{cosec} \theta$]

$= 1$

(iii) $\frac{\tan 27^0}{\cot 63^0}$

$= \frac{\tan (90^0 - 63^0)}{\cot 63^0}$

$= \frac{\cos 63^0}{\cos 63^0}$ [$\because \tan (90 - \theta) = \cot \theta$]

$= 1$

(iv) $\frac{\cos 35^0}{\sin 55^0}$

$= \frac{\cos (90^0 - 55^0)}{\sin 55^0}$

$= \frac{\sin 55^0}{\sin 55^0}$ [$\because \sin (90 - \theta) = \cos \theta$]

$= 1$

(v) $\frac{\operatorname{cosec} 42^0}{\sec 48^0}$

$= \frac{\operatorname{cosec} (90^0 - 48^0)}{\sec 48^0}$

$= \frac{\sec 48^0}{\sec 48^0}$ [$\because \sec (90 - \theta) = \operatorname{cosec} \theta$]

$= 1$

(vi) $\frac{\cot 38^0}{\tan 52^0}$

$= \frac{\cot (90^0 - 52^0)}{\tan 52^0}$

$= \frac{\tan 52^0}{\tan 52^0}$ [$\because \tan(90 - \theta) = \cot \theta$]

$= 1$

2. Without using trigonometric tables, prove that:

(i) $\cos 81^0 - \sin 9^0 = 0$ (ii) $\tan 71^0 - \cot 19^0 = 0$

(iii) $\text{cosec } 80^0 - \sec 10^0 = 0$ (iv) $\text{cosec}^2 72^0 - \tan^2 18^0 = 1$

(v) $\cos^2 75^0 + \cos^2 15^0 = 1$ (vi) $\tan^2 66^0 - \cot^2 24^0 = 0$

(vii) $\sin^2 48^0 + \sin^2 42^0 = 1$ (viii) $\cos^2 57^0 - \sin^2 33^0 = 0$

(ix) $(\sin 65^0 + \cos 25^0)(\sin 65^0 - \cos 25^0) = 0$

Sol:

(i) LHS $= \cos 81^0 - \sin 9^0$

$= \cos(90^0 - 9^0) - \sin 9^0$

$= \sin 9^0 - \sin 9^0$

$= 0$

$=$ RHS

(ii) LHS $= \tan 71^0 - \cot 19^0$

$= \tan(90^0 - 19^0) - \cot 19^0$

$= \cot 19^0 - \cot 19^0$

$= 0$

$=$ RHS

(iii) LHS $= \text{cosec } 80^0 - \sec 10^0$

$= \text{cosec } (90^0 - 10^0) - \sec 10^0$

$= \sec 10^0 - \sec 10^0$

$= 0$

$=$ RHS

(iv) LHS $= \text{cosec}^2 72^0 - \tan^2 18^0$

$= \text{cosec}^2(90^0 - 18^0) - \tan^2 18^0$

$= \sec^2 18^0 - \tan^2 18^0$

$= 1$

$=$ RHS

(v) LHS $= \cos^2 75^0 + \cos^2 15^0$

$= \cos^2(90^0 - 15^0) + \cos^2 15^0$

$= \sin^2 15^0 + \cos^2 15^0$

$= 1$

$=$ RHS

(vi) LHS $= \tan^2 66^0 - \cot^2 24^0$

$= \tan^2(90^0 - 24^0) - \cot^2 24^0$

$= \cot^2 24^0 - \cot^2 24^0$

$= 0$

$=$ RHS

(vii) LHS = $\sin^2 48^0 + \sin^2 42^0$
$= \sin^2(90^0 – 42^0) + \sin^2 42^0$
$= \cos^2 42^0 + \sin^2 42^0$
$= 1$
= RHS
(viii) LHS = $\cos^2 57^0 - \sin^2 33^0$
$= \cos^2(90^0 – 33^0) - \sin^2 33^0$
$= \sin^2 33^0 - \sin^2 33^0$
$= 0$
= RHS
(ix) LHS = $(\sin 65^0 + \cos 25^0)(\sin 65^0 - \cos 25^0)$
$= \sin^2 65^0 - \cos^2 25^0$
$= \sin^2(90^0 - 25^0) - \cos^2 25^0$
$= \cos^2 25^0 - \cos^2 25^0$
$= 0$
= RHS

3. Without using trigonometric tables, prove that:
(i) $\sin 53^0 \cos 37^0 + \cos 53^0 \sin 37^0 = 1$
(ii) $\cos 54^0 \cos 36^0 - \sin 54^0 \sin 36^0 = 0$
(iii) $\sec 70^0 \sin 20^0 + \cos 20^0 \operatorname{cosec} 70^0 = 2$
(iv) $\sin 35^0 \sin 55^0 – \cos 35^0 \cos 55^0 = 0$
(v) $(\sin 72^0 + \cos 18^0)(\sin 72^0 - \cos 18^0) = 0$
(vi) $\tan 48^0 \tan 23^0 \tan 42^0 \tan 67^0 = 1$
Sol:
(i) LHS = $\sin 53^0 \cos 37^0 + \cos 53^0 \sin 37^0$
$= \sin(90^0 – 37^0)\cos 37^0 + \cos(90^0 – 37^0)\sin 37^0$
$= \cos 37^0 \cos 37^0 + \sin 37^0 \sin 37^0$
$= \cos^2 37^0 + \sin^2 37^0$
$= 1$
= RHS
(ii) LHS = $\cos 54^0 \cos 36^0 - \sin 54^0 \sin 36^0$
$= \cos(90^0 – 36^0)\cos 36^0 - \sin(90^0 – 36^0)\sin 36^0$
$= \sin 36^0 \cos 36^0 - \cos 36^0 \sin 36^0$
$= 0$
= RHS
(iii) LHS = $\sec 70^0 \sin 20^0 + \cos 20^0 \operatorname{cosec} 70^0$
$= \sec(90^0 – 20^0)\sin 20^0 + \cos 20^0 \operatorname{cosec}(90^0 – 20^0)$
$= \operatorname{cosec} 20^0 . \frac{1}{cosec\ 20^0} + \frac{1}{sec\ 20^0} . \sec 20^0$
$= 1 + 1$

$= 2$
$= RHS$

(iv) LHS = $\sin 35^0 \sin 55^0 - \cos 35^0 \cos 55^0$
$= \sin 35^0 \cos(90^0 - 55^0) - \cos 35^0 \sin(90^0 - 55^0)$
$= \sin 35^0 \cos 35^0 - \cos 35^0 \sin 35^0$
$= 0$
$= RHS$

(v) LHS = $(\sin 72^0 + \cos 18^0)(\sin 72^0 - \cos 18^0)$
$= (\sin 72^0 + \cos 18^0)[\cos(90^0 - 72^0) - \cos 18^0]$
$= (\sin 72^0 + \cos 18^0)(\cos 18^0 - \cos 18^0)$
$= (\sin 72^0 + \cos 18^0)(0)$
$= RHS$

(vi) LHS = $\tan 48^0 \tan 23^0 \tan 42^0 \tan 67^0$
$= \cot(90^0 - 48^0) \cot(90^0 - 23^0) \tan 42^0 \tan 67^0$
$= \cot 42^0 \cot 67^0 \tan 42^0 \tan 67^0$
$= \frac{1}{\tan 42^0} \times \frac{1}{\tan 67^0} \times \tan 42^0 \times \tan 67^0$
$= 1$
$= RHS$

4. Without using trigonometric tables, prove that:

(i) $\frac{\sin 70^0}{\cos 20^0} + \frac{\operatorname{cosec} 20^0}{\sec 70^0} - 2\cos 70^0 \operatorname{cosec} 20^0 = 0$

(ii) $\frac{\cos 80^0}{\sin 10^0} + \cos 59^0 \operatorname{cosec} 31^0 = 2$

(iii) $\frac{2\sin 68^0}{\cos 22^0} - \frac{2\cot 15^0}{5\tan 75^0} - \frac{3\tan 45^0 \tan 20^0 \tan 40^0 \tan 50^0 \tan 70^0}{5} = 1$

(iv) $\frac{\sin 18^0}{\cos 72^0} + \sqrt{3}(\tan 10^0 \tan 30^0 \tan 40^0 \tan 50^0 \tan 80^0) = 2$

(v) $\frac{7\cos 55^0}{3\sin 35^0} - \frac{4(\cos 70^0 \operatorname{cosec} 20^0)}{3(\tan 5^0 \tan 25^0 \tan 45^0 \tan 65^0 \tan 85^0)} = 1$

Sol:

(i) LHS = $\frac{\sin 70^0}{\cos 20^0} + \frac{\operatorname{cosec} 20^0}{\sec 70^0} - 2\cos 70^0 \operatorname{cosec} 20^0$
$= \frac{\sin 70^0}{\sin(90^0 - 20^0)} + \frac{\sec(90^0 - 20^0)}{\sec 70^0} - 2\cos 70^0 \sec(90^0 - 20^0)$
$= \frac{\sin 70^0}{\sin 70^0} + \frac{\sec 70^0}{\sec 70^0} - 2\cos 70^0 \sec 70^0$
$= 1 + 1 - 2 \times \cos 70^0 \times \frac{1}{\cos 70^0}$
$= 2 - 2$
$= 0$
$= RHS$

(ii) LHS = $\frac{\cos 80^0}{\sin 10^0}$ + cos59^0 cosec31^0

= $\frac{\cos 80^0}{\sin(90^0 - 10^0)}$ + sin(90^0 - 59^0) cosec31^0

= $\frac{\cos 80^0}{\cos 80^0}$ + sin31^0 cosec31^0

= 1 + sin31^0 $\times$ $\frac{1}{\sin 31^0}$

= 1 + 1

= 2

= RHS

(iii) LHS = $\frac{2\sin 68^0}{\cos 22^0}$ - $\frac{2\cot 15^0}{5\tan 75^0}$ - $\frac{3\tan 45^0 \tan 20^0 \tan 40^0 \tan 50^0 \tan 70^0}{5}$

= $\frac{2\sin 68^0}{\sin(90^0 - 22^0)}$ - $\frac{2\cot 15^0}{5\tan(90^0 - 75^0)}$ - $\frac{3 \times 1 \times \cot(90^0 - 20^0) \times \cot(90^0 - 40^0) \times \tan 50^0 \times \tan 70^0}{5}$

= $\frac{2\sin 68^0}{\sin 68^0)}$ - $\frac{2\cot 15^0}{5\cot\ 15^0}$ - $\frac{3 \times\ \cot 70^0\ \ \cot 50^0 \tan 50^0\ \ \tan 70^0}{5}$

= 2 - $\frac{2}{5}$ - $\frac{3 \times \frac{1}{\tan 70^0} \times \frac{1}{\tan 50^0} \times\ \tan 50^0 \times \tan 70^0}{5}$

= 2 - $\frac{2}{5}$ - $\frac{3}{5}$

= $\frac{10-2-3}{5}$

= $\frac{5}{5}$

= 1

= RHS

(iv) LHS = $\frac{\sin 18^0}{\cos 72^0}$ + $\sqrt{3}$ (tan 10^0 tan 30^0 tan 40^0 tan 50^0 tan 80^0)

= $\frac{\sin 18^0}{\sin(90^0 - 72^0)}$ + $\sqrt{3}$ [cot (90^0 – 10^0) $\times$ $\frac{1}{\sqrt{3}}$ $\times$ cot(90^0 – 40^0) $\times$ tan 50^0 $\times$ tan 80^0]

= $\frac{\sin 18^0}{\sin 18^0}$ + $\sqrt{3}$ $\left(\frac{\cot 80^0 \times\ \cot 50^0 \times\ \tan 50^0\ \times \tan 80^0}{\sqrt{3}}\right)$

= 1 + $\left(\frac{1}{\tan 80^0} \times \frac{1}{\tan 50^0} \times \tan 50^0 \times \tan 80^0\right)$

= 1 + 1

= 2

= RHS

(v) LHS = $\frac{7\cos 55^0}{3\sin 35^0}$ - $\frac{4\ (\cos 70^0 cosec\ 20^0)}{3(\tan 5^0 \tan 25^0 \tan 45^0 \tan 65^0 \tan 85^0)}$

= $\frac{7\cos 55^0}{3\cos(90^0 - 35^0)}$ - $\frac{4\ [\sin(90^0 - 70^0) cosec\ 20^0]}{3\ [\cot(90^0 - 5^0) \times \cot(90^0 - 25^0) \times 1 \times \tan 65^0 \times \tan 85^0}$

= $\frac{7\cos 55^0}{3\cos 55^0}$ - $\frac{4\ (\sin 20^0\ cosec\ 20^0)}{3\ (\cot 85^0 \cot 65^0 \tan 65^0\ \ \tan 85^0)}$

= $\frac{7}{3}$ - $\frac{4\left(\sin 20^0 \times \frac{1}{\sin 20^0}\right)}{3\left(\frac{1}{\tan 85^0} \times \frac{1}{\tan 65^0} \times \tan 65^0 \times \tan 85^0\right)}$

= $\frac{7}{3}$ - $\frac{4}{3}$

$= \frac{3}{3}$
$= 1$
$=$ RHS

5. Prove that:

(i) $\sin\theta \cos(90^0 - \theta) + \sin(90^0 - \theta)\cos\theta = 1$

(ii) $\frac{\sin\theta}{\cos(90^0 - \theta)} + \frac{\cos\theta}{\sin(90^0 - \theta)} = 2$

(iii) $\frac{\sin\theta\cos(90^0 - \theta)\cos\theta}{\sin(90^0 - \theta)} + \frac{\cos\theta\sin(90^0 - \theta)\sin\theta}{\cos(90^0 - \theta)} = 1$

(iv) $\frac{\cos(90^0 - \theta)\sec(90^0 - \theta)\tan\theta}{cosec\,(90^0 - \theta)\sin\,(90^0 - \theta)\cot\,(90^0 - \theta)} + \frac{\tan(90^0 - \theta)}{\cot\theta} = 2$

(v) $\frac{\cos(90^0 - \theta)}{1+\sin(90^0 - \theta)} + \frac{1+\sin(90^0 - \theta)}{\cos(90^0 - \theta)} = 2\,\mathrm{cosec}\,\theta$

(vi) $\frac{\sec(90^0 - \theta)cosec\,\theta - \tan(90^0 - \theta)\cot\theta + cos^2 25^0 + cos^2 65^0}{3\tan 27^0 \tan 63^0} = \frac{2}{3}$

(vii) $\cot\theta\tan(90^0 - \theta) - \sec(90^0 - \theta)\,\mathrm{cosec}\,\theta + \sqrt{3}\tan 12^0 \tan 60^0 \tan 78^0 = 2$

Sol:

(i) LHS $= \sin\theta\cos(90^0 - \theta) + \sin(90^0 - \theta)\cos\theta$

$= \sin\theta\sin\theta + \cos\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

$=$ RHS

Hence proved.

(ii) LHS $= \frac{\sin\theta}{\cos(90^0 - \theta)} + \frac{\cos\theta}{\sin(90^0 - \theta)}$

$= \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\cos\theta}$

$= 1 + 1$

$= 2$

$=$ RHS

Hence proved.

(iii) LHS $= \frac{\sin\theta\cos(90^0 - \theta)\cos\theta}{\sin(90^0 - \theta)} + \frac{\cos\theta\sin(90^0 - \theta)\sin\theta}{\cos(90^0 - \theta)}$

$= \frac{\sin\theta\sin\theta\cos\theta}{\cos\theta} + \frac{\cos\theta\cos\theta\sin\theta}{\sin\theta}$

$= \sin^2\theta + \cos^2\theta$

$= 1$

$=$ RHS

Hence proved.

(iv) LHS = $\frac{\cos(90^0 - \theta)\sec(90^0 - \theta)\tan\theta}{cosec\ (90^0 - \theta)\sin\ (90^0 - \theta)\cot\ (90^0 - \theta)} + \frac{\tan(90^0 - \theta)}{\cot\theta}$

$= \frac{\sin\theta\ cosec\ \theta\tan\theta}{\sec\theta\cos\theta\tan\theta} + \frac{\cot\theta}{\cot\theta}$

= 1 + 1

= 2

= RHS

Hence proved.

(v) LHS = $\frac{\cos(90^0 - \theta)}{1+\sin(90^0 - \theta)} + \frac{1+\sin(90^0 - \theta)}{\cos(90^0 - \theta)}$

$= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$

$= \frac{sin^2\theta+1+ cos^2\theta+2\cos\theta}{(1+cos\theta)\sin\theta}$

$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= 2\,\frac{1}{\sin\theta}$

= 2 cosec θ

= RHS

Hence proved.

(vi) LHS = $\frac{\sec(90^0 - \theta)cosec\ \theta - \tan(90^0 - \theta)\cot\theta + cos^2 25^0 + cos^2 65^0}{3\tan 27^0\tan 63^0}$

$= \frac{cosec\ \theta\ cosec\ \theta - \cot\theta\cot\theta + sin^2(90^0 - 25^0) + cos^2 65^0}{3\tan 27^0\cot(90^0 - 63^0)}$

$= \frac{cosec^2\theta - cot^2\theta + sin^2 65^0 + cos^2 65^0}{3\tan 27^0\cot 27^0}$

$= \frac{1+1}{3\times\tan 27^0\times\frac{1}{\tan 27^0}}$

$= \frac{2}{3}$

= RHS

(vii) LHS = cot θ tan $(90^0 – θ)$ – sec $(90^0 – θ)$ cosec θ + $\sqrt{3}$ tan 12^0 tan 60^0 tan 78^0

= cot θ cot θ - cosec θ cosec θ + $\sqrt{3}$ tan $12^0 \times \sqrt{3} \times$ cot $(90^0 – 78^0)$

= $\cot^2\theta$ – $cosec^2\theta$ + 3 tan 12^0 cot 12^0

= -1 + 3 $\times$ tan $12^0 \times \frac{1}{\tan 12^0}$

= -1 + 3

= 2

= RHS

6. Without using trigonometric tables, prove that:

(i) $\tan 5^0 \tan 25^0 \tan 30^0 \tan 65^0 \tan 85^0 = 1$

(ii) $\cot 12^0 \cot 38^0 \cot 52^0 \cot 60^0 \cot 78^0 = \frac{1}{\sqrt{3}}$

(iii) $\cos 15^0 \cos 35^0 \operatorname{cosec} 55^0 \cos 60^0 \operatorname{cosec} 75^0 = \frac{1}{2}$

(iv) $\cos 1^0 \cos 2^0 \cos 3^0 \ldots\ldots \cos 180^0 = 0$

(v) $\left(\frac{\sin 49^0}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\sin 49^0}\right) = 2$

Sol:

(i) LHS = $\tan 5^0 \tan 25^0 \tan 30^0 \tan 65^0 \tan 85^0$

$= \tan(90^0 - 85^0) \tan(90^0 - 65^0) \times \frac{1}{\sqrt{3}} \times \frac{1}{\cot 60^0} \frac{1}{\cot 85^0}$

$= \cot 85^0 \cot 65^0 \frac{1}{\sqrt{3}} \frac{1}{\cot 60^0} \frac{1}{\cot 85^0}$

$= \frac{1}{\sqrt{3}}$ = RHS

(ii) LHS = $\cot 12^0 \cot 38^0 \cot 52^0 \cot 60^0 \cot 78^0$

$= \tan(90^0 - 12^0) \times \tan(90^0 - 38^0) \times \cot 52^0 \times \frac{1}{\sqrt{3}} \times \cot 78^0$

$= \frac{1}{\sqrt{3}} \times \tan 78^0 \times \tan 52^0 \times \cot 52^0 \times \cot 78^0$

$= \frac{1}{\sqrt{3}} \times \tan 78^0 \times \tan 52^0 \times \frac{1}{\tan 52^0} \times \frac{1}{\tan 78^0}$

$= \frac{1}{\sqrt{3}}$

= RHS

(iii) LHS = $\cos 15^0 \cos 35^0 \operatorname{cosec} 55^0 \cos 60^0 \operatorname{cosec} 75^0$

$= \cos(90^0 - 75^0) \cos(90^0 - 55^0) \frac{1}{\sin 55^0} \times \frac{1}{2} \times \frac{1}{\sin 75^0}$

$= \sin 75^0 \sin 55^0 \quad \frac{1}{\sin 55^0} \times \frac{1}{2} \times \frac{1}{\sin 75^0}$

$= \frac{1}{2}$ = RHS

(iv) LHS = $\cos 1^0 \cos 2^0 \cos 3^0 \ldots . \cos 180^0$

$= \cos 1^0 \times \cos 2^0 \times \cos 3^0 \times \ldots\ldots \times \cos 90^0 \times \ldots \times \cos 180^0$

$= \cos 1^0 \times \cos 2^0 \times \cos 3^0 \times \ldots\ldots \times 0 \times \ldots \times \cos 180^0$

= 0

= RHS

(v) LHS = $\left(\frac{\sin 49^0}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\sin 49^0}\right)$

$= \left(\frac{\cos(90^0 - 49^0)}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\cos(90^0 - 49^0)^0}\right)^2$

$= \left(\frac{\cos 41^0}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\cos 41^0}\right)^2$

$= 1^2 + 1^2$

= 1 + 1
= 2
= RHS
Disclaimer: The RHS of (v) given in textbook is incorrect. There should be 2 instead 1. The same has been corrected in the solution here.

7. Prove that:
(i) $\sin(70^0 + \theta) - \cos(20^0 - \theta) = 0$
(ii) $\tan(55^0 - \theta) - \cot(35^0 + \theta) = 0$
(iii) $\operatorname{cosec}(67^0 + \theta) - \sec(20^0 - \theta) = 0$
(iv) $\operatorname{cosec}(65^0 + \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot(35^0 + \theta) = 0$
(v) $\sin(50^0 + \theta) - \cos(40^0 - \theta) + \tan 1^0 \tan 10^0 \tan 80^0 \tan 89^0 = 1$

Sol:

(i) LHS = $\sin(70^0 + \theta) - \cos(20^0 - \theta)$
= $\sin\{90^0 - (20^0 - \theta)\} - \cos(20^0 - \theta)$
= $\cos(20^0 - \theta) - \cos(20^0 - \theta)$
= 0
= RHS

(ii) LHS = $\tan(55^0 - \theta) - \cot(35^0 + \theta)$
= $\tan\{90^0 - (35^0 + \theta)\} - \cot(35^0 + \theta)$
= $\cot(35^0 + \theta) - \cot(35^0 + \theta)$
= 0
= RHS

(iii) LHS = $\operatorname{cosec}(67^0 + \theta) - \sec(23^0 - \theta)$
= $\operatorname{cosec}\{90^0 - (23^0 - \theta)\} - \sec(23^0 - \theta)$
= $\sec(23^0 - \theta) - \sec(23^0 - \theta)$
= 0
= RHS

(iv) LHS = $\operatorname{cosec}(65^0 + \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot(35^0 + \theta)$
= $\operatorname{cosec}\{90^0 - (25^0 - \theta)\} - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot\{90^0 - (55^0 - \theta)\}$
= $\sec(25^0 - \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \tan(55^0 - \theta)$
= 0
= RHS

(v) LHS = $\sin(50^0 + \theta) - \cos(40^0 - \theta) + \tan 1^0 \tan 10^0 \tan 80^0 \tan 89^0$
= $\sin\{90^0 - (40^0 - \theta)\} - \cos(40^0 - \theta) + \{\tan 1^0 \tan(90^0 - 1^0)\}\{\tan 10^0 \tan(90^0 - 10)\}$
= $\cos(40^0 - \theta) - \cos(40^0 - \theta) + (\tan 1^0 \cot 1^0)(\tan 10^0 \cot 10^0)$

$= \left(\frac{1}{\cot 1^0} \times \cot 1^0\right)\left(\tan 10^0 \times \frac{1}{\tan 10^0}\right)$
$= 1 \times 1$
$= 1$
= RHS

8. Express each of the following in terms of trigonometric ratios of angles lying between 0^0 and 45^0:
(i) $\sin 67^0 + \cos 75^0$
(ii) $\cot 65^0 + \tan 49^0$
(iii) $\sec 78^0 + \text{cosec } 56^0$
(iv) $\text{cosec } 54^0 + \sin 72^0$

Sol:
(i) $\sin 67^0 + \cos 75^0$
$= \cos(90^0 - 67^0) + \sin(90^0 - 75^0)$
$= \cos 23^0 + \sin 15^0$
(ii) $\cot 65^0 + \tan 49^0$
$= \cos(90^0 - 65^0) + \cot(90^0 - 49^0)$
$= \cos 25^0 + \cot 41^0$
(iii) $\sec 78^0 + \text{cosec } 56^0$
$= \sec(90^0 - 12^0) + \text{cosec}(90^0 - 34^0)$
$= \text{cosec } 12^0 + \sec 34^0$
(iv) $\text{cosec } 54^0 + \sin 72^0$
$= \sec(90^0 - 54^0) + \cos(90^0 - 72^0)$
$= \sec 36^0 + \cos 18^0$

9. If A, B, C are the angles of a ΔABC, prove that $\tan\left(\frac{C+A}{2}\right) = \cot \frac{B}{2}$.

Sol:
In ΔABC,
$A + B + C = 180^0$
$\Rightarrow A + C = 180^0 - B$ ……..(i)
Now,
LHS $= \tan\left(\frac{C+A}{2}\right)$
$= \tan\left(\frac{180^0 - B}{2}\right)$ [Using (i)]
$= \tan\left(90^0 - \frac{B}{2}\right)$
$= \cot \frac{B}{2}$
= RHS

10. If $\cos 2\theta = \sin 4\theta$ and 2θ is acute, then find the value of θ.

Sol:

We have,

$\cos 2\theta = \sin 4\theta$

$\Rightarrow \sin (90^0 - 2\theta) = \sin 4\theta$

Comparing both sides, we get

$90^0 - 2\theta = 4\theta$

$\Rightarrow 2\theta + 4\theta = 90^0$

$\Rightarrow 6\theta = 90^0$

$\Rightarrow \theta = \frac{90^0}{6}$

$\therefore \theta = 15^0$

Hence, the value of θ is 15^0.

11. If $\sec 2A = \text{cosec}\,(A - 42^0)$, where 2A is an acute angle, find the value of A.

Sol:

We have,

$\sec 2A = \text{cosec}\,(A - 42^0)$

$\Rightarrow \text{cosec}\,(90^0 - 2A) = \text{cosec}\,(A - 42^0)$

Comparing both sides, we get

$90^0 - 2A = A - 42^0$

$\Rightarrow 2A + A = 90^0 + 42^0$

$\Rightarrow 3A = 132^0$

$\Rightarrow A = \frac{132^0}{3}$

$\therefore A = 44^0$

Hence, the value of A is 44^0.

12. If $\sin 3A = \cos (A - 26^0)$, where 3A is an acute angle, find the value of A.

Sol:

$\sin 3A = \cos (A - 26^0)$

$\Rightarrow \cos (90^0 - 3A) = \cos (A - 26^0)$ $[\because \sin\theta = \cos(90^0 - \theta)]$

$\Rightarrow 90^0 - 3A = A - 26^0$

$\Rightarrow 116^0 = 4A$

$\Rightarrow A = \frac{116^0}{4} = 29^0$

13. If $\tan 2A = \cot (A - 12^0)$, where 2A is an acute angle, find the value of A.

Sol:

$\tan 2A = \cot (A - 12^0)$

$\Rightarrow \cot (90^0 - 2A) = \cot (A - 12^0)$ [$\because \tan \theta = \cot (90^0 - \theta)$]

$\Rightarrow (90^0 - 2A) = (A - 12^0)$

$\Rightarrow 102^0 = 3A$

$\Rightarrow A = \frac{102^0}{3} = 34^0$

14. If $\sec 4A = \operatorname{cosec} (A - 15^0)$, where 4A is an acute angle, find the value of A.

Sol:

$\sec 4A = \operatorname{cosec} (A - 15^0)$

$\Rightarrow \operatorname{cosec} (90^0 - 4A) = \operatorname{cosec} (A - 15^0)$ [$\because \sec \theta = \operatorname{cosec} (90^0 - \theta)$]

$\Rightarrow (90^0 - 4A) = (A - 15^0)$

$\Rightarrow 105^0 = 5A$

$\Rightarrow A = \frac{105^0}{5} = 21^0$

15. Without using trigonometric tables, evaluate the following:

Sol:

$\frac{2}{3} \operatorname{cosec}^2 58^0 - \frac{2}{3} \cot 58^0 \tan 32^0 - \frac{5}{3} \tan 13^0 \tan 37^0 \tan 45^0 \tan 53^0 \tan 77^0$

$= \frac{2}{3} (cosec^2\ 58^0 - \cot 58^0 \tan 32^0) - \frac{5}{3} \tan 13^0 \tan (90^0 - 13^0) \tan 37^0 \tan(90^0 - 37^0)$ $(\tan 45^0)$

$= \frac{2}{3} \{cosec^2\ 58^0 - cot\ 58^0\ tan(90^0 - 58^0)\} - \frac{5}{3} \tan 13^0 \cot 13^0 \tan 37^0 \cot 37^0\ (1)$

$= \frac{2}{3} (cosec^2\ 58^0 - cot\ 58^0\ tan\ 58^0) - \frac{5}{3} \tan 13^0 \frac{1}{\tan 13^0} \tan 37^0 \frac{1}{\tan 37^0}$

$= \frac{2}{3} (cosec^2\ 58^0 - cot^2 58^0) - \frac{5}{3}$

$= \frac{2}{3} - \frac{5}{3}$

$= -1$

Hence proved.