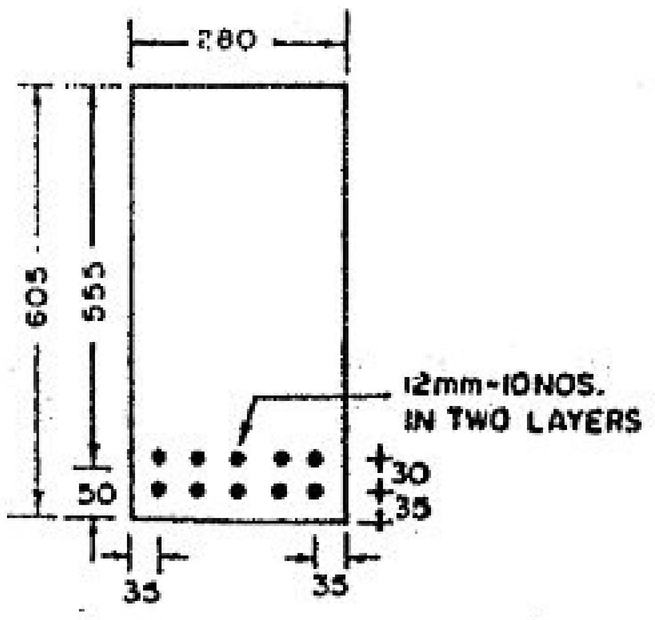
Area of tension reinforcement



$$A_{st} = \frac{175.71 \times 10^6}{0.87 \times 415(55 - 0.42 \times 266.4)}$$

$$= 1098.29 \text{ mm}$$

Provide 12 mm diameter bars-10 Nos. These should be provided in two layers as shown in the figure given.

5. A singly concrete beam has rectangular section 230 mm wide × 460 mm effective depth. At a section under consideration there are two 16 mm diameter bent-up bars at 45° and 6 mm diameter two legged stirrups at a spacing of 200 mm centre to centre. The grade of concrete is M 20. The reinforcement is HYSD 415. The design shear strength of concrete is 0.6 N/mm² and maximum shear stress is 2.8 N/mm². Using limit state, determine the value of factored shear force to be permitted at the section.

Solution: Given,

$b = 230$ mm, $d = 460$ mm

$f_{ck} = 20$ N/mm², $f_y = 415$ N/mm²

$S_V = 200$ mm c/c, $\tau_c = 0.6$ N/mm²

$\tau_{c,max} = 2.8$ N/mm²

$$A_{sv1} = 2 \times \frac{22}{7} \times \frac{16^2}{4} \text{ (inclined at 45°)}$$

$$A_{sv2} = 2 \times \frac{22}{7} \times \frac{6^2}{4} \text{(vertical)}$$

For vertical stirrups,

$$V_{us1} = \frac{0.87 f_y A_{sv2} d}{S_v}$$

$$= \frac{0.87 \times 415 \times 56.57 \times 460}{200}$$

$$= 46976.6 \text{ N}$$

For bent up bars,

$$V_{us2} = \frac{0.87 f_y A_{sv1} d}{S_v} \sin\alpha$$

$$= \frac{0.87 \times 415 \times 402.28 \times 460}{200}$$

$$\sin 45° = 283192.2 \text{ N}$$

$$V_{us} = V_{us1} + V_{us2}$$

Now,

$$V_{us} = V_u - \tau_c bd$$

or

$$V_u = V_{us} + \tau_c bd$$

$$= 283192.2 + 0.6 \times 230 \times 460$$

$$= 346672.2 \text{ N}$$

$$= 346.67 \text{ kN}$$

Factored shear force permissible at the given section is 346.67 kN

$$\therefore \quad \tau_c = \frac{346.67}{bd} \times 10^3$$

$$= \frac{346.67}{230 \times 460} \times 10^3$$

$$= 3.27 \text{ N/mm}^2 > \tau_{c,max}$$

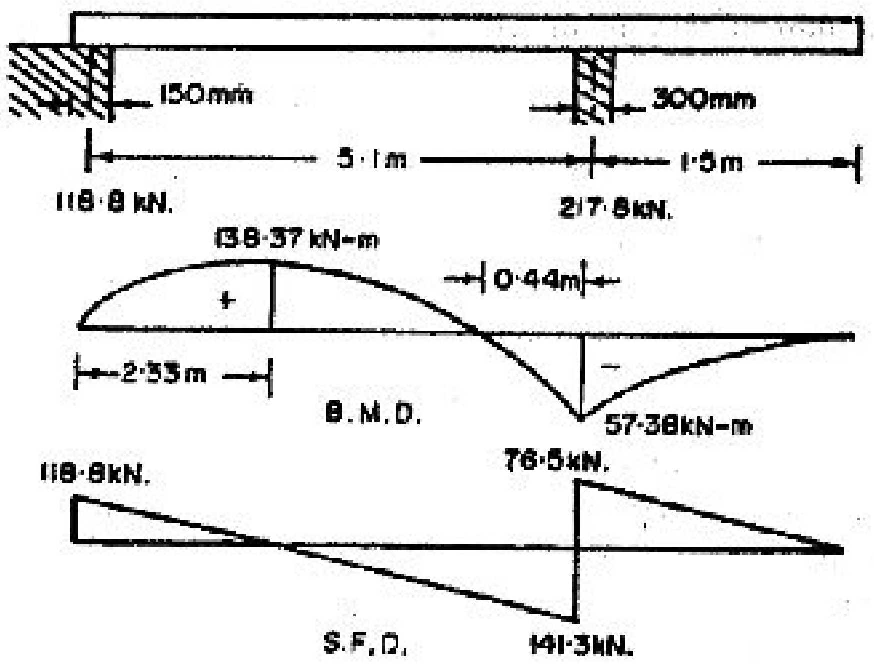
Hence, factored shear force to be permitted at the section

$$= \tau_{c,max} bd = \frac{2.8 \times 230 \times 460}{10^3} = 296.24 \text{ kN}$$

6. An overhanging beam shown in the figure carries a super-imposed dead load of 18kN/m and live load of 12 kN/m.

Concrete grade: M15 and steel grade: Fe 415.

Calculate the size of beam and the tension reinforcement. Ratio b/d = 0.5.



Solution:

Beam loads are as given below:

Superimposed dead load = 18 kN/m

Estimated self weight = 4 kN/m

Total dead load, DL = 22 kN/m

Live load, LL = 12 kN/m

∴ Design collapse load,

$$w = 1.5 \text{ DL} + 1.5 \text{ LL}$$

$$= 1.5 \times 2.2 + 1.5 \times 12$$

$$= 51 \text{ kN/m}$$

The bending moment and shear force diagrams are shown in the figure.

Maximum bending is positive and it occurs within the main span and its value is 138.37kN-m

We know, $0.36 f_{ck} \dfrac{X_u}{d}\left(1-0.42\dfrac{X_u}{d}\right) bd^2 = M_u$

where, $f_{ck} = 15$ N/mm², $b = 0.5d$,

$$\frac{X_u}{d} = 0.48 \text{ for balanced design, and}$$

$$M_u = 138.37 \text{ kN-m}$$

Substituting and solving, we get

$$d = 511.3 \text{ mm}$$

$$b = \frac{1}{2} \times 511.3 = 255.65 \text{ mm}$$

Adopt $b = 260$ mm and $d = 515$ mm.

∴ Tension reinforcement,

$$A_{st} = \frac{138.37 \times 10^6}{0.87 \times 415(515 - 0.42 \times 247.2)}$$

$$= 932.1 \text{ mm}^2$$

Provide 16 mm dia. bars-5 numbers on the lower face with in the main span. (Figure below) of these 5 bars it is proposed to cut-off 2 bars towards the supports. As such, only 3 bars will then remain beyond the cut-off points and the same will enter into the supports.

Hence, after curtailment, $A_{st} = 3 \times 201.06$

$$= 603.18 \text{ mm}^2$$

Bending strength of the beam,

$$M_u = 0.87 \times 415 \times 603.18\left(515 - \frac{415 \times 603.18}{15 \times 260}\right)$$

$$= 98.18 \times 10^6 \text{ N-mm}$$

$$= 98.18 \text{ kN-m}$$

Bending moment at any point within the main span distant x from the left hand support is given by

$$M_x = 118.8\,x - \frac{51x^2}{2}$$

To find the theoretical point of cut-off, equate bending moment to the bending strength.

$$\therefore \quad 118.8x - \frac{51x^2}{2} = 98.18$$

or $$x^2 - 4.659\,x + 3.585 = 0$$

$$\therefore \quad x = 1.074 \text{ m and } 3.585 \text{ m.}$$

As per the IS Code: 456 the bars to be cut-off should extend beyond the point at which they are no longer required, for a distance not less than 12 times the diameter or the effective depth whichever is greater.

As such, the actual points of curtailment of the bars, from the left hand support will be

(*i*) 1.704 – 0.515 = 0.559 m say 0.555 m and

(*ii*) 3.585 + 0.515 = 4.1 m.

Over the support, negative bending moment = 57.38 kN-m exists.

For this, the reinforcement is calculated as follows

$$57.38 \times 10^6 = 0.87 \times 415 A_{st}\left(515 - \frac{415 A_{st}}{15 \times 260}\right)$$

or $$A_{st}^2 - 4840 A_{st} + 1.494 \times 10^6 = 0$$

$$\therefore \quad A_{st} = 331.4 \text{ mm}^2$$

Provide 12 mm dia. bars – 3 numbers on the upper face.

Now $\quad d = 515$ mm

$$12\phi = 12 \times 12 = 144 \text{ mm}$$

$$\frac{1}{16} \text{ of clear span} = \frac{4875}{16} = 305 \text{ mm}$$

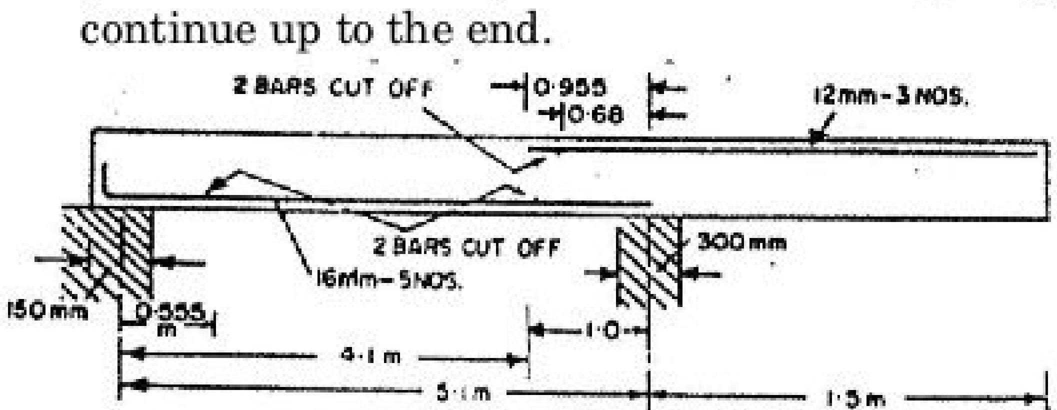
The largest of these is 515 mm.

Development length of 12 mm bars,

$$l_d = 56.39 \times 12$$

$$= 676.68 \text{ mm}$$

This is greater than the distance of the point of contraflexure from the right support which is only 440 mm. The extension of the negative reinforcement into the main span should not be less than the development length of the bar. It is proposed to extend two bars up to 680m, and one bar up to 440 + 515 = 955 mm into the main span from the right support. In the overhanging portion of the beam, the three bars at top may continue up to the end.



LAYOUT OF TENSION REINFORCEMENT

At the point of contraflexure, considering the positive reinforcement only, the bending strength of beam is

$$M_1 = 98.18 \text{ kN-m}$$

Shear force at the same point,

$$V = 141.3 - 0.44 \times 51$$

$$= 118.86 \text{ kN}.$$

Length of bar beyond the point of contraflexure,

$$l_o = 440 \text{ mm}$$

As the support reaction induces compression in the region of the beam over the support, M_1 should be increased by 30 %.

$$\therefore \quad \frac{M_1}{V} + l_o = \frac{1.3 \times 98.18 \times 10^6}{118.16 \times 10^3} + 440 = 1513.8 \text{ mm}$$

Development length of 16 mm diameter bar in tension,

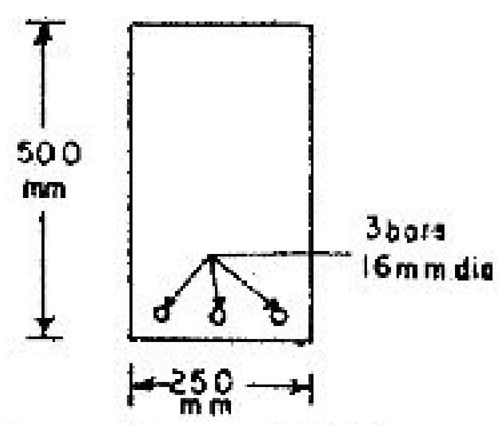
$$l_d = 56.39 \times 16 = 902.2 \text{ mm}$$

Evidently $l_d \not> \frac{M_1}{V} + l_0$. Hence, the anchorage of the bottom reinforcement bars beyond the point of contraflexure is adequate. As such, the diameter of the bars need not be reduced.

7. A beam cross-section as shown has to carry a shear force of 60 kN under service load conditions. Calculate the spacing of 8 mm diameter, 2-legged shear stirrups to be provided in the beam. The concrete of grade M 15 and HYSD bars (Fe 415) are being used. The design shear strength of concrete, τ_c is given as

$\frac{100A_s}{bd}$	τ_c(N/mm^2) for M 15 concrete
0.25	0.85
0.50.	0.46
0.75	0.54
1.00	0.60

The maximum shear stress, τ_c for M 15 concrete is 2.5 N/mm^2. Use partial safety factor for load as 1.5.



Solution: Shear force = 60 kN

Partial safety factor for load = 1.5

Factored shear force,

$$V_u = 1.5 \times 60 = 90 \text{ kN}$$

$$b = 250 \text{ mm}, D = 500 \text{ mm}$$

$$d = D - \text{clear cover} - \frac{\phi}{2}$$

$$= 500 - 25 - \frac{16}{2}$$

$$= 467 \text{ mm}$$

Nominal shear stress,

$$\tau_v = \frac{V_u}{bd} = \frac{90 \times 10^3}{250 \times 467} = 0.771 \text{ N/mm}^2$$

Area of steel reinforcement present,

$$A_s = 3 \times \frac{22}{7} \times \frac{16^2}{4} = 603.43 \text{ mm}^2$$

$$\frac{100A_s}{bd} = \frac{100 \times 603.43}{250 \times 467} = 0.517$$

As per given table, the design shear stress of concrete,

$$\tau_c = 0.46 + \frac{(0.54 - 0.46)}{(0.75 - 0.5)} \times (0.517 - 0.5)$$

$$= 0.465 \text{ N/mm}^2$$

Maximum shear stress,

$$\tau_{c,max} = 2.5 \text{ N/mm}^2$$

Nominal shear stress, τ_v is less than maximum shear stress, $\tau_{c,max}$ and greater than τ_c

Shear reinforcement shall be provided to carry a shear,

$$V_{us} = V_u - \tau_c bd$$

$$= 90 \times 10^3 - 0.465 \times 250 \times 467$$

$$= 35711.25 \text{ N}$$

For vertical stirrups, $V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$

where, A_{sv} = total cross sectional area of stirrup legs.

$$= 2 \times \frac{\pi}{4} \times 8^2 = 100.57 \text{ mm}^2$$

$$f_y = 415 \text{ N/mm}^2, \text{ and } d = 467 \text{ mm}$$

Hence, Spacing of stirrups, $S_v = \frac{0.87 f_y S_{sv} d}{V_{us}}$

$$= \frac{0.87 \times 415 \times 100.57 \times 467}{35711.25}$$

$$= 474.5 \text{ mm, say } 475 \text{ mm}$$

Minimum shear reinforcement shall be such that

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{f_y}$$

$$S_v \leq \frac{A_{sv} f_y}{0.4b}$$

$$\leq \frac{100.57 \times 415}{0.4 \times 250}, \text{ i.e. } 417.36 \text{ mm}$$

Maximum spacing of shear reinforcement shall be 0.75 d subject to maximum of 450 mm

$$S_v < 0.75 \times 467, \text{ i.e. } 350.25 \text{ mm}$$

Hence, provide 2L – 8 ϕ @ 350 mm c/c of shear reinforcement.

8. Design a singly reinforced rectangular beam for a clear span of 5.7 m. Width of beam = 280 mm. Bearing at each end = 200 mm. Superimposed dead load =15 kN/m. Live load =12kN/m. Concrete grade: M15 and steel grade: Fe 415.

Check the beam for shear, deflection and lateral stability.

Solution:

(*i*) Limit state of collapse in flexure.

Loads on the beam are :

Estimated self weight = 5 kN/m

Superimposed dead load = 15 kN/m·

Total dead load, DL = 20 kN/m

Live load, LL = 12 kN/m

Design load on beam w =1.5 DL + 1.5 LL

$$=1.5 \times 20 + 1.5 \times 12$$
$$= 48 \text{ kN/m}$$

Effective span,

$$l = 5.7 + 0.2 = 5.9 \text{ m}$$

Design bending moment,

$$M = \frac{wl^2}{8}$$
$$= \frac{48 \times 5.9^2}{8} = 208.86 \text{ kN-m}$$

Bending strength

$$M_u = 0.36 f_{ck} \frac{X_u}{d}\left(1 - 0.42\frac{X_u}{d}\right) bd^2$$

Here $f_{ck} = 15 \text{ N/mm}^2$

and $\frac{X_u}{d} = 0.48$ for balanced design

$b = 280$ mm

Substituting these values, we get

$$208.86 \times 10^6 = 0.36 \times 15 \times 0.48\,(1 - 0.42 \times 0.48)\, 280 \times d^2$$

$d = 600$ mm

Tension reinforcement,

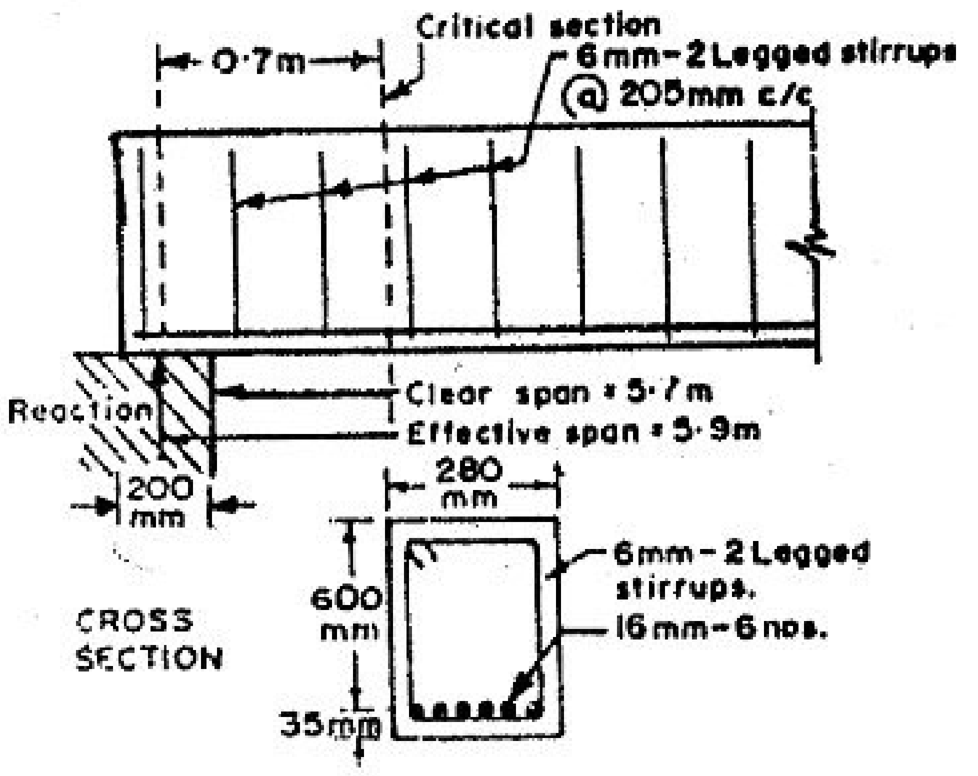
$$A_{st} = \frac{M_u}{0.87 f_y (d - 0.42 X_u)}$$
$$= \frac{208.86 \times 10^6}{0.87 \times 415 \times (600 - 0.42 \times 0.48 \times 600)}$$
$$= 1207.6 \text{ mm}^2$$

Provide 16 mm diameter bars, 6 numbers

(*ii*) Limit state of collapse in shear.

Critical section occurs at an effective depth away from the face of support. Its distance from the centre of support = 0.1 + 0.6 = 0.7 m.

Shear force at critical section



$$V_u = 48\left(\frac{1}{2} \times 5.9 - 0.7\right) = 108 \text{ kN}$$

Nominal Shear stress

$$\tau_v = \frac{108 \times 10^3}{280 \times 600} = 0.643 \text{ N/mm}^2$$

Now $\frac{100 A_{st}}{bd} = \frac{100 \times 6 \times 201.06}{280 \times 600} = 0.718$

For concrete grade : M 15

$$\tau_c = 0.53 \text{ N/mm}^2$$

As $\tau_v > \tau_c$, the beam is unsafe in shear

Shear strength of concrete,

$$V_{uc} = \tau_c bd$$
$$= 0.53 \times 280 \times 600 \times 10^{-3}$$
$$= 89.04 \text{ kN}$$

Shear to be taken by the shear reinforcement

$$V_{us} = V_u - V_{uc}$$
$$= 108 - 89.04$$
$$= 18.96 \text{ kN}$$

Provide 6 mm diameter two-legged stirrups against shear.

Now $$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

or $$18.96 \times 10^3 = \frac{0.87 \times 415 \times 2 \times 28.27 \times 600}{S_v}$$

$\therefore$ $S_v = 646$ mm c/c

Nominal shear reinforcement is given by

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{f_y}$$

or $$\frac{2 \times 28.27}{280 \times S_v} \geq \frac{0.4}{415}$$

$S_v \leq 209.5$ mm, say 205 mm c/c.

Limits on stirrup spacing are

$$S_v \ngtr 0.75 \times 600 \text{ i.e. } 450 \text{ mm}$$

and $\ngtr$ 450 mm

Evidently, the nominal shear reinforcement requires the least spacing for the stirrups, i.e. 205 mm c/c.

Hence, shear stirrups at a spacing of 205 mm C/C may be provided throughout the length of the beam.

(*iii*) **Limit state of Serviceability for deflection.**

Short term deflection :

n = depth of neutral axis factor for cracked beam

= 0.289

$x = 0.289 \times 600 = 173.4$ mm

$$j = 1 - \frac{0.289}{3} \text{ the lever arm factor} = 0.904$$

$a = 0.904 \times 600 = 542.4$ mm

$D = 600 + 35 = 635$ mm

$$I_g = \frac{bD^3}{12} = \frac{280 \times 635^3}{12} = 5.97 \times 10^9 \text{ mm}^4$$

$$I_{cr} = \frac{bX^3}{3} + (m-1)A_{st}(d-x)^2$$

$$= \frac{280 \times 173.4^3}{3} + (18.67 - 1) \times 1206.36 \times (600 - 173.4)^2$$

$= 4.366 \times 10^9$ mm^4

Modulus of rupture of concrete,

$$f_{cr} = 0.7\sqrt{15} = 2.71 \text{ N/mm}^2$$

$$y_t = D - x = 635 - 173.4 = 461.6 \text{ mm}$$

$$M_{cr} = \frac{2.71 \times 5.97 \times 10^9}{461.6}$$

$= 35.05 \times 10^6$ N-mm

w = DL + LL for serviceability limit state

= 20 + 12 = 32 kN/m

$$M = \frac{wl^2}{8} = \frac{32 \times 5.9^2}{8} = 139.24 \text{ kN-m}$$

$$I_e = \frac{4.366 \times 10^9}{1.2 - \frac{35.05}{139.24} \times \frac{542.4}{600}(1-289) \times 1}$$

$= 4.205 \times 10^9$ mm^4

Take $I_c = I_{cr} = 4.366 \times 10^9$ mm^4

$$E_c = 5700\sqrt{15} = 22076 \text{ N/mm}^2$$

Short term deflection at mid-span:

$$d_s = \frac{5}{384} \times \frac{wl^4}{E_c I_c}$$

$$= \frac{5}{384} \times \frac{32 \times 5900^4}{22076 \times 4.366 \times 10^9}$$

= 5.24 mm

Hence, the beam satisfies the limit state of serviceability in deflection.

Alternative indirect check for the limit state of serviceability for deflection :

Basic value of span/effective depth ratio = 20

$$\frac{100A_{st}}{bd} = 0.718$$

Modification factor for steel grade:

Fe 415 = 1.05.

Hence, permissible value of span/effective depth ratio $= 1.05 \times 20 = 21$

Actual value of span/effective depth ratio

$$= \frac{5900}{600} = 9.83 < 21$$

Hence, the beam satisfies the serviceability limit state of deflection.

(*iv*) **Check for lateral stability :**

$$60\ b = 60 \times 280 = 16800 \text{ mm}$$

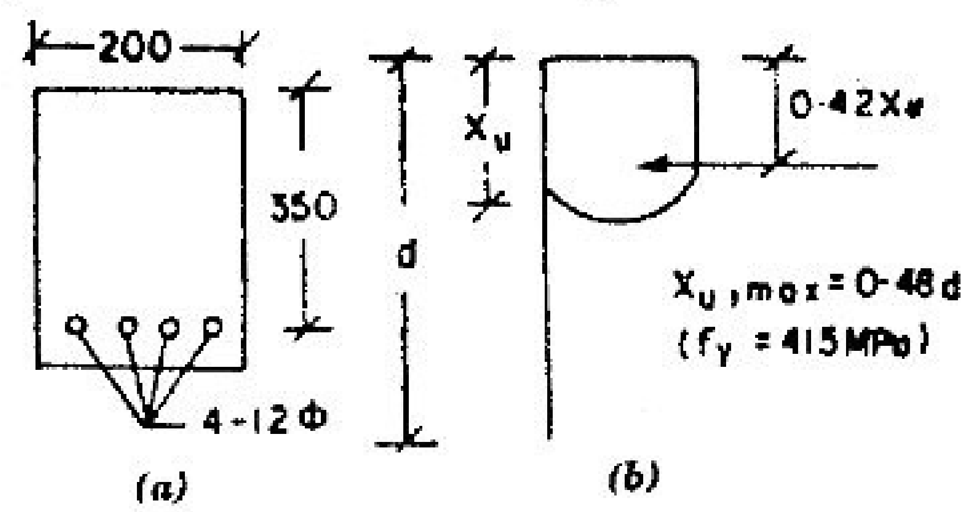
$$\text{or } \frac{250 \times b^2}{d} = \frac{250 \times 280^2}{600} = 32667 \text{ mm}$$

$$l = 5900 \text{ mm}$$

Evidently, $l < 60\,b$ and also $l = \dfrac{250 \times b^2}{d}$

Hence, the beam is safe against lateral bucking.

9. A rectangular R.C. beam of concrete grade M 15 is 200 mm wide and 350 mm deep (effective depth). It is provided with 4 numbers of 12 mm dia HYSD steel rods as tension reinforcement. Determine the moment of resistance of the beam by limit state method. The stress block parameters from IS 456 are given below.



Solution: Area of steel provided,

$$A_{st} = 4 \times \frac{\pi}{4} \times 12^2 = 452.4 \text{ mm}^2$$

$$x_{u,max} = 0.48 \text{ d} = 0.48 \times 350 = 168 \text{ mm}$$

Total compressive force,

$$C = 0.36 f_{ck} b x_u$$
$$= 0.36 \times 15 \times 200 \times 168$$
$$= 181440 \text{ N}.$$

Total tensile force, $T = 0.87 f_y A_{st}$

$$= 0.87 \times 415 \times 452.4$$
$$= 163335.2 \text{ N}$$

Since, C > T, it is under reinforced.

Depth of actual neutral axis is determined by,

$$x_a = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 452.4}{0.36 \times 15 \times 200} = 151.24 \text{ mm}$$

∴ Actual lever arm, $jd = d - 0.416\, x_a$

$$= 350 - 0.416 \times 151.24$$
$$= 287.1 \text{ mm}$$

Moment of resistance of the given section

$$= 0.87 - f_y A_{st}(d - 0.416 x_a)$$
$$= 0.87 \times 415 \times 452.4(350 - 0.416 \times 151.24)$$
$$= 46.891 \times 10^6 \text{ N.mm}$$
$$= 46.891 \text{ kNm}$$

10. Design a beam of effective span 6 m. to support a total design load of 12 kN/m including self weight of the beam using limit state method of design. Load factor for live load and dead load is 1.5. M 15 grade concrete and Fe 415 grade steel are used. Width of beam is limited to 250 mm.

Solution: Factored load = 1.5 × 12 = 18 kN/m

Maximum factored bending moment

$$= \frac{wl^2}{8} = \frac{18 \times 6^2}{8} = 81 \text{ kNm}$$

Depth of neutral axis, for balanced section

$$x_{u,\text{Lim}} = \frac{0.0035}{0.0055 + 0.87 f_y / E_x}.d$$

$$= \frac{0.0035}{0.0055 + \frac{0.87 \times 415}{200{,}000}}.d$$

$$= 0.479\, d.$$

Lever arm, $jd = (d - 0.416\, x_u)$

For balanced section,

$$M = 0.36 f_{ck} b . x_{u,\text{Lim}}(d - 0.416 x_{u.\text{Lim}})$$

or $\quad 81 \times 10^6 = 0.36 \times 15 \times 250$

$$\times d^2(1 - 0.416 \times 0.479)\, 0.479$$

$$d = 395.5$$

From deflection criteria,

$$d = \frac{l}{20} = \frac{6000}{20} = 300 \text{ mm}$$

So adopt a beam of overall depth 450 mm with effective depth

$$= 450 - 25 - 10 = 415 \text{ mm}$$

For balanced section,

$$M_{u,\text{ Lim}} = 0.36 f_{ck} b x_{u,\text{Lim}} (d - 0.416_{u,\text{Lim}})$$
$$= 0.36 \times 15 \times 250 \times 0.479 \times 415 (415 - 0.416 \times 0.479 \times 415)$$
$$= 89.18 \times 10^6 \text{ Nmm}$$
$$= 89.18 \text{ kNm}$$

Since $M_u < M_{u,\text{Lim}}$, it is a singly reinforced section

$$A_{st} = \frac{Mu, \text{Lim}}{0.87 f_y jd} = \frac{81 \times 10^6}{0.87 \times 415 \times 0.479 \times 415}$$

$$= 1128.6 \text{ mm}^2 = 11.29 \text{ cm}^2$$

Provide 4 number 20 mm ϕ bars.

11. Design a reinforced concrete slab over a clear opening in plan 3.75m × 9m. Plain concrete tile flooring on top of slab is 50 mm thick. Live load is 4 kN/m². Bearing of slab along all the edges is 150 mm. Concrete grade: M15 and steel grade: Fe 415.

Solution:

Length to width ratio of slab = $\frac{9}{3.75}$ = 2.4

As the ratio is greater than 2, the slab may be designed spanning one-way over the shorter span.

To estimate the self weight of slab assume its thickness to be 160 mm. Hence, the load per unit area of slab will be as follows :

$$\text{Self weight} = 0.16 \times 1 \times 1 \times 25$$
$$= 4.0 \text{ kN/m}^2$$
$$\text{Weight of tiles} = 0.05 \times 1 \times 1 \times 24$$
$$= 1.2 \text{ kN/m}^2$$

Total dead load, DL = 5.2kN/m²

Live load, LL = 4.0 kN/m²

Design load on slab, w = 1.5 DL + 1.5 LL

$$= 1.5 \times 5.2 + 1.5 \times 4.0$$
$$= 13.8 \text{ kN/m}^2$$

Consider a 1 m wide strip of slab. The strip may be viewed as a simple beam carrying a load of 13.8 kN/m.

Effective span, $l = 3.75 + 0.15 = 3.90$ m

Bending moment at mid-span,

$$M_u = \frac{13.8 \times 3.9^2}{8} = 26.24 \text{ kN-m/m}$$

Now $0.36 f_{ck} \dfrac{x_u}{d}\left(1 - 0.42\dfrac{x_u}{d}\right) bd^2 = M_u$

Here $f_{ck} = 15$ N/mm² $b = 1000$ mm for the strip

and $\dfrac{x_u}{d} = 0.48$ for balanced design

Substituting all the values in the above expression, we get

$$0.36 \times 15 \times 0.48\,(1 - 0.42 \times 0.48)\, 1000 \times d^2 = 26.24 \times 10^6$$

$$\therefore \quad d = 112.6\text{mm}$$

Adopt an effective depth of 115 mm.

Tension reinforcement

$$A_{st} = \frac{26.24 \times 10^6}{0.87 \times 415(115 - 0.42 \times 115)} = 1030.2 \text{ mm}^2\text{/m}$$

Now $$\frac{100A_{st}}{bd} = \frac{100 \times 1030.2}{1000 \times 115} = 0.896$$

Modification factor = 0.99 (from IS code 456)

$\therefore$ Permitted span/effective depth ratio $= 20 \times 0.99 = 19.8$

Actual value of span/effective depth ratio

$$= \frac{3900}{115} = 33.9$$

This is greater than the permitted value. Therefore, the effective depth of the slab should be increased so that the span/effective depth ratio is decreased.

Try an effective depth = 145 mm. The corresponding value of tension reinforcement is obtained from

$$M_u = 0.87 f_y A_{st}\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

or $$26.24 \times 10^6 = 0.87 \times 415\, A_{st} = \left(145 - \frac{415 \times A_{st}}{15 \times 1000}\right)$$

$$A_{st} = 477.65 \text{ mm}^2$$

Now $$\frac{100A_{st}}{bd} = \frac{100 \times 477.65}{1000 \times 145} = 0.329$$

Modification factor = 1.36 (From IS code 456)

$\therefore$ Permitted value of span/effective depth ratio

$$= 20 \times 1.36 = 27.2$$

Actual value of span/effective depth ratio

$$= \frac{3900}{145} = 26.9$$

Here the actual value of span/effective depth ratio is less than the permitted value. Hence, the slab satisfies the limit state of serviceability of deflection.

Provide 10 mm diameter bars for reinforcement.

Adopt a spacing of 160 mm c/c.

As a practice, alternate main reinforcement bars may be bent up at a distance of 0.15 of span from the supports.

Limits on bar spacing are as follows:

Spacing $\not> 3 \times 145$, i.e. 435 mm

$\not>$ 450mm

The actual spacing is within the above limits.

Cover on reinforcement $\not>$ 15 mm

$\not<$ bar diameter, i.e. 12 mm.

Adopt an overall thickness of slab = 165 mm. This will ensure a cover of 15 mm on the bars.

At right angles to the main reinforcement bars, temperature and shrinkage reinforcement bars should be provided.

The sectional area of such reinforcement,

$$A_{st} = 12\% \text{ of main steel area.}$$

$$A_{st} = \frac{0.12 \times 165 \times 1000}{100} = 198 \text{ mm}^2 \text{ per meter width}$$

Provide 6 mm diameter bars.

$$\text{Spacing of bar} = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times 28.27}{198} = 142.78 \text{ mm c/c}$$

Adopt a spacing 140 mm/cc.

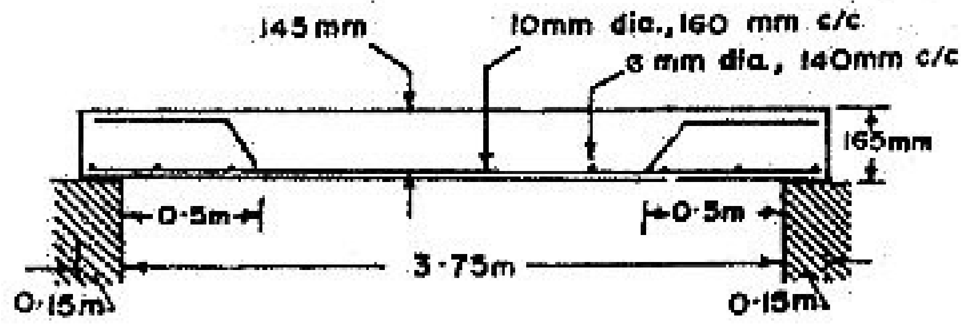
Limits on bar spacing are as follows:

$\not> 5 \times 145$ *i.e.* 725 mm

$\not>$ 450 mm

Actual spacing is within the above limits.

A section of slab giving details of reinforcement is shown in the figure below:



Sectional area of tension reinforcement per unit width of slab

$$= \frac{78.54 \times 1000}{160}$$

$$= 490.9 \text{ mm}^2/\text{m}$$

$$\therefore \quad \frac{100 A_{st}}{bd} = \frac{100 \times 490.9}{1000 \times 145} = 0.339$$

For concrete grade : M15, $\tau_c = 0.39$ N/mm²

For a slab of 130 mm thickness

$$k = 1.27 \text{ (From IS 456)}$$

Design shear of concrete $= k\tau_c$

$$= 1.27 \times 0.39$$

$$= 0.495 \text{ N/mm}^2$$

Shear strength of slab per metre width

$$V_{ue} = 0.495 \times 165 \times 1000 \text{ N/m}$$

$$= 81675 \text{ N/m}$$

$$= 81.675 \text{ kN/m}$$

Critical section for Shear occurs at an effective depth, *i.e.* 145 mm away from the face of support. At this section, shear force

$$V_u = \left(\frac{1}{2} \times 3.75 - 0.145\right) \times 13.80$$

$$= 23.87 \text{ kN/m}$$

As shear force is much less than the shear strength of the slab, the slab is safe in shear.

DOUBLY REINFORCED MEMBERS

12. For the beam in the above problem, determine the main reinforcement required by limit state method, if the overall depth of beam is 925 mm and the partial factor of safety is 1.5 for both dea.d load and live load.

Solution:

D.L. of beam $= 0.3 \times 0.925 \times 25$

$= 6.9375$ kN/m

L.L. on beam $= 12$ kN/m

Total load $= 18.9375$ kN/m

Total factored load $= 1.5 \times 18.9375$

$= 28.406$ kN/m

Maximum factored bending moment

$$= \frac{28.406 \times 8^2}{8}$$

$$= 227.25 \text{ kNm}$$

$$M_{u,\text{Lim}} = 0.138 f_{ck} bd^2$$

$$= 0.138 \times 15 \times 300 \times 925^2$$

$$= 531.3 \text{ kNm} > 227.25 \text{ kNm}$$

Thus area of steel can be obtained by equating the bending moment to the moment of resistance with respect to tension steel,

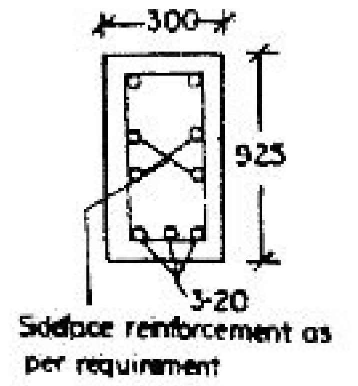
$$M_u = 0.87$$

where, $\quad d = 925 - 25 - 12.5 = 887.5$ mm

$$227.25 \times 10^6 = 0.87 \times 415 \times A_{st} \left(d - \frac{415 \times A_{st}}{15 \times 300}\right)$$

or $\quad A_{st} = 771$ mm²

Provide 20 mm diameter bars

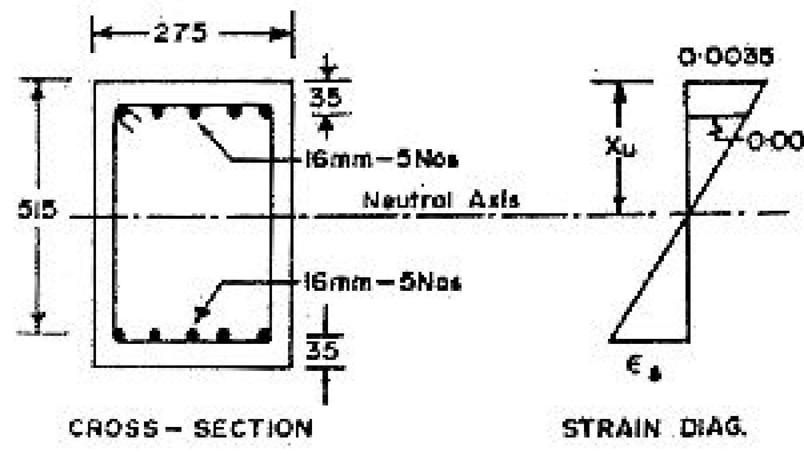


$$\text{Number of bars required} = \frac{771}{\frac{\pi}{4}(20)^2}$$

$$= 2.46 = 3 \text{ nos}$$

Side face reinforcement as per requirement is provided.

13. Calculate the bending, strength of the doubly reinforced beam section shown in the figure.



Solution:

Given, $b = 275$ mm, $d = 515$ mm

$$A_{st} = 5 \times 201.06 = 1005.3 \text{ mm}^2$$

$$A_{sc} = 5 \times 201.06 = 1005.3 \text{ mm}^2$$

Let the depth of neutral axis be X_u. Then compressive forces are

$$C_1 = 0.36 \times 15 \times X_u \times 275$$

$$= 1485 X_u \text{ N}$$

$$C_2 = 1005.3 f_{sc} \text{ N}$$

Tensile force in reinforcement,

$$T = 0.87 \times 415 \times 1005.3$$

$$= 362.96 \times 10^3 \text{ N}$$

Equating compressive force to the tensile force, we get

$$1485 X_u + 1005.3 f_{sc} = 362.96 \times 10^3 \qquad \ldots(i)$$

Assuming that the stress in the compression reinforcement is within the elastic range, the magnitude of stress is

$$f_{sc} = 0.0035 \frac{(X_u - 35)}{X_u} \times 200 \times 10^3$$

$$= 700 \frac{(X_u - 35)}{X_u}$$

Substituting the value of f_{sc} in equation (*i*), we get

$$1485\, X_u + 1005.3 \times 700 \times \frac{(X_u - 35)}{X_u} = 362.96 \times 10^3$$

or $X_u^2 + 229.46\, X_u - 16585.8 = 0$

Solving we get, $X_u = 57.75$ mm

$$f_{sc} = \frac{700(57.75 - 35)}{57.75}$$

$$= 275.76 \text{ N/mm}^2$$

Stress at the limit of proportionality $= \dfrac{0.8 f_y}{1.15}$

$$= \frac{0.8 \times 415}{1.15} = 288.7 \text{ N/mm}^2$$

It is evident that the calculated stress in the compression reinforcement is within the elastic range only. Therefore, the assumption in this respect was right.

Values of the compressive forces are

$C_1 = 1485 \times 57.75 = 85.76 \times 10^3$ N

and $C_2 = 1005.3 \times 275.76 = 277.22 \times 10^3$ N

Bending strength of the section

$$M_u = C_1 (d - 0.42\, X_u) + C_2 (d - d_c)$$

$$= 85.76 \times 10^3 (515 - 0.42 \times 57.75) + 277.22 \times 10^3 (515 - 35)$$

$$= 175.16 \times 10^6 \text{ N-mm}$$

$$= 175.16 \text{ kN-m}$$

14. Design a reinforced concrete beam with balanced section for flexure by working stress method for the data given below;

Effective span (simply supported) = 8 m; Live load = 12 kN/m

Breadth of beam = 300 mm; Concrete grade M 15

Reinforcement steel grade Fe 415

Solution: As per IS 456 ; for simply supported beams, deflection criteria is,

$$\frac{\text{span}}{\text{effective depth}} = 20, \text{ or } \frac{8000}{d} = 20$$

$\therefore$ $d = 400$ mm

Adopting a beam of over all depth 450 mm

Effective depth = (450 – 25 –10) = 415 mm

(assuming main longitudinal steel of 20 mm diameter)

For M15 grade concrete,

$\sigma_{cbc} = 5$ N/mm² and for Fe 415, $\sigma_{st} = 230$ N/mm²

So, neutral axis depth factor for balanced section,

$$k = \frac{1}{1 + \dfrac{\sigma_{st}}{m\sigma_{cbc}}}$$

where, $m = \dfrac{280}{3\sigma_{cbc}} = 18.67$

$$\therefore \quad k = \frac{1}{1 + \dfrac{230}{18.67 \times 5}} = 0.289$$

$\therefore$ Neutral axis depth = k.d.

= 0.289 × 415 =119.8 mm

D.L. of beam per meter run = 0.3 × 0.45 × 25

= 3.375 kN/m

L.L. = 12 kN/m (given)

Total (D.L. + L.L.) = 15.375 kN/m

Maximum bending moment,

$$M = \frac{wl^2}{8} = \frac{15.375 \times 8^2}{8} = 123 \text{ kNm}$$

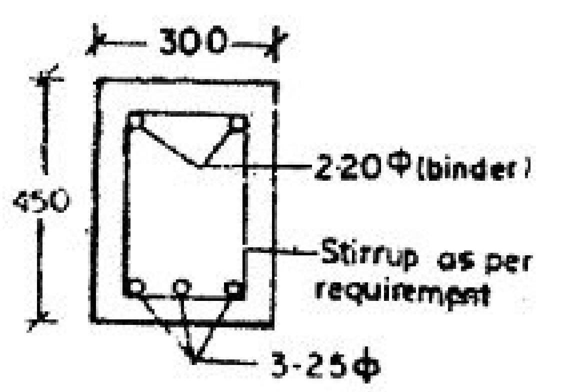
We know,

$$M = \sigma_{st} A_{st} \left(d - \frac{x}{3} \right)$$

So, required area of steel to be provided for a balanced design

$$A_{st} = \frac{M}{\sigma_{st}(d - x/3)}$$

$$= \frac{123 \times 10^6}{230\left(415 - \dfrac{119.8}{3}\right)}$$



$$= 1425.8 \text{ mm}^2$$

$$= 14.26 \text{ cm}^2$$

Providing 25 mm diameter main steel.

Number of bars required $= \dfrac{14.26}{\dfrac{\pi}{4}(2.5)^2} = 2.9 \approx 3$

15. A rectangular concrete beam 800 mm wide is subjected to a sagging moment of 20 kN-*m* and *a* torsional moment of 30 kN-m. Design a suitable depth for the beam and the longitudinal reinforcement required. Allowable stresses are, σ_{cbc} = 5 N/mm², σ_{sc} = 140 N/mm² and m = 19.

Solution: Given, M = 20 kNm, T = 30 kNm

We know, $M_t = T\dfrac{(1+D/b)}{1.7}$

Assume an overall depth of 500 mm, we have

$$M_t = \frac{30(1+500/300)}{1.7} = 47.1 \text{ m}$$

Equivalent moment,

$$M_e = M + M_t$$
$$= 20 + 47.1 = 67.1 \text{ kNm}$$

Now $\quad Qbd^2 = 67.1$

or $$d = \sqrt{\frac{67.1\times10^6}{0.867\times300}} = 507.9 \text{ mm}$$

So, adopt an overall depth of 650 mm, then

$$M_t = \frac{30(1+650/300)}{1.7} = 55.9 \text{ kNm}$$

Equivalent moment, $M_{e1} = M + M_t$
$$= 20 + 55.9$$
$$= 75.9 \text{ kNm}$$

$$d_{reqd} = \sqrt{\frac{75.9\times10^6}{0.867\times300}} = 540 \text{ mm}$$

Longitudinal reinforcement shall be provided to resist eqivalent bending moment of 75.9 kNm.

$$A_{st1} = \frac{M_{e_1}}{\sigma_{st}jd}$$
$$= \frac{75.9\times10^6}{140\times0.867\times610}$$
$$= 1025.1 \text{ mm}^2$$

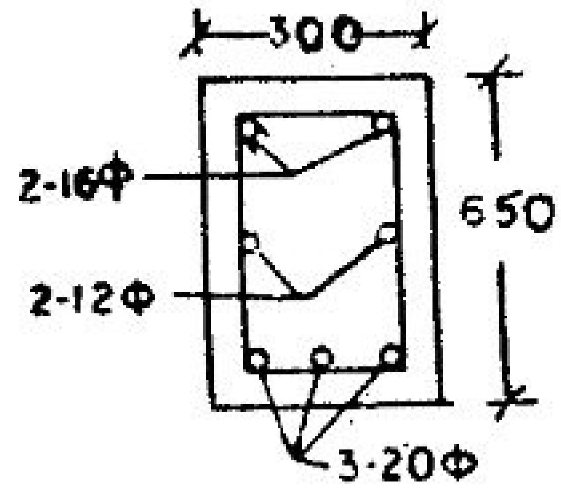
Providing 20 mm diameter bars,

So, number of bars required as tension reinforcement $= \dfrac{1025.1}{314} = 3.3 \simeq 4$ (say)

Also, since $M_t > M$, Longitudinal compression steel to be provided for an equivalent moment,

$$M_{e2} = M_t - M$$
$$= 55.9 - 20 = 30 \text{ kNm}$$

$$A_{st2} = \frac{M_t - M}{\sigma_{st}(d-d')}$$
$$= \frac{30\times10^6}{140[610-40]} = 376 \text{ mm}^2$$



Provide 2 numbers of 16 mm diameter as compression steel.

So, the cross-section of beam after detailing will be as per figure.

Since $d > 450$ mm, side face reinforcement of 0.1 % is provided on each face equally

$$= \frac{1}{2}\times0.1\times\frac{300}{100}\times610$$
$$= 91.5 \text{ mm}^2 \text{ on each face.}$$

So provide 1 bar of 12 mm diameter on each face.

16. The cross-section of a T-beam is shown in the figure below. Calculate the bending strength of the beam in the limit state of collapse. Concrete grade: M15 and steel grade: Fe 415.

Solution: A_{st} = 10 × 314.16 = 3141.6 mm²,

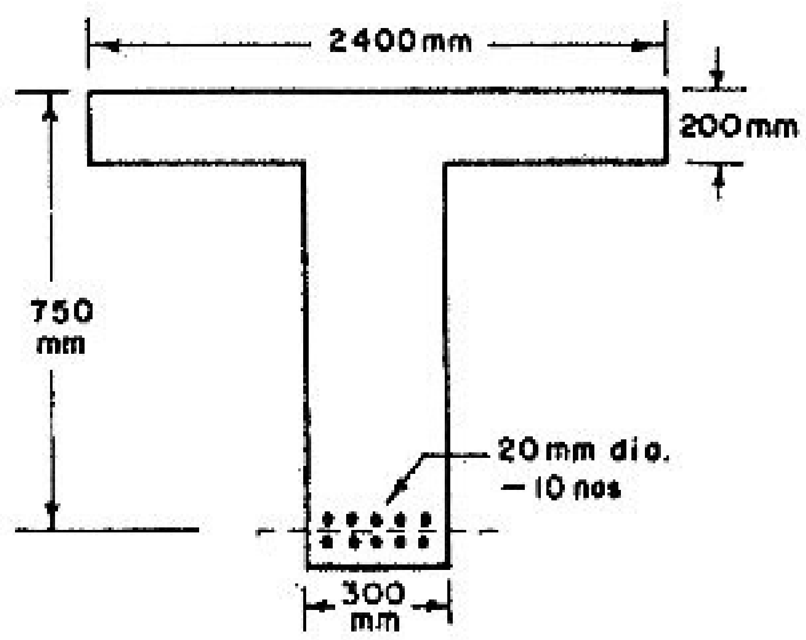
f_y = 415 N/mm²,

f_{ck} = 15 N/mm²,

b_f = 2400 N/mm,

D_f = 200 mm,

d = 750 mm.



As a trial, assume that the neutral axis is within the flange *i.e.* $X_u \le D_f$, then from the relations

$$C = 0.36\times15\times2400\,X_u$$
$$= 12960\,X_u \qquad ...(i)$$

and $$T = 0.87\times415\times3141.6$$
$$= 1134275 \text{ N} \qquad ...(ii)$$

Equating equations (*i*) and (*ii*), we get

$$C = T$$

or $\quad 12960\,X_u = 1134275$

$\therefore \quad X_u = 87.52$ mm

$< D_f$, *i.e.* 200 mm.

Hence, the assumption that $X_u \le D_f$ is right.

Bending strength of beam

$$M_u = 1134275\,(750 - 0.42 \times 87.52)$$

$$b = 809 \times 10^6 \text{ N-mm} = 809 \text{ kN-m}$$

17. A reinforced concrete beam of rectangular section is 550 mm wide and has an overall depth of 750 mm. It is subjected to an ultimate bending moment of 150 kNm and ultimate twising moment of 50 kN-m. M 15 grade concrete and Fe 415 grade steel are used. Determine the necessary longitudinal reinforcement

Given relevant IS codal provisions.

Solution: Given, M_u = 150 kNm = 550 mm

T_u = 50 kNm = 750 mm

Bending moment due to torsion,

$$M_t = \frac{T_u(1+D/b)}{1.7}$$

$$= \frac{50(1+750/500)}{1.7} = 69.52 \text{ kNm}$$

Equivalent bending moment,

$$M_{e1} = M_u + M_t$$

$$= 150 + 69.52 = 219.52 \text{ kNm}$$

Depth of balanced neutral axis;

$$x_{u.\text{Lim}} = 0.48 \text{ d}$$

$$= 0.48 \times 710 = 553.8 \text{ mm}$$

$$M_{u,\text{ Lim}} = 0.36 f_{ck}.b.x_{u,\text{ lim}}(d - 0.416\, X_{u,\text{ lim}})$$

$$= 0.36 \times 15 \times 550 \times 553.8\,[710 - 0.416 \text{ X } 553.8]$$

$$= 7.89 \times 10^8 \text{ N.mm} = 789 \text{ kNm}$$

Since $M_{e1} < M_{u,\text{lim}}$, it is an under reinforced section, then depth of actual neutral axis, x_a

$$M_{e1} = 0.36 f_{ck} b x_a\,(d - 0.416 x_a)$$

or $219.52 \times 10^6 = 0.36 \times 15 \times 550 \times x_a\,(710 - 0.416 x_a)$

$$\frac{x_a}{d} = 1.202 \pm \left(1.4446 - \frac{6.67735 \times 219.52 \times 10^6}{15 \times 550 \times 710^2}\right)^{1/2}$$

$$= 0.157$$

$$x_a = 0.157 \times 710 = 111.43 \text{ mm}$$

Lever arm, $jd = d - 0.416\, x_a$

$$= 710 - 0.416 \times 111.43$$

$$= 663.64 \text{ mm}$$

Area of steel, $A_{st} = \dfrac{M_{e_1}}{0.87 f_y jd}$

$$= \frac{219.52 \times 10^6}{0.87 \times 415 \times 663.64}$$

$$= 916.2 \text{ mm}^2$$

Provide 3 bars of 20 mm diameter, (so that area provides A_{st} = 942 mm² > 916.2 mm²).

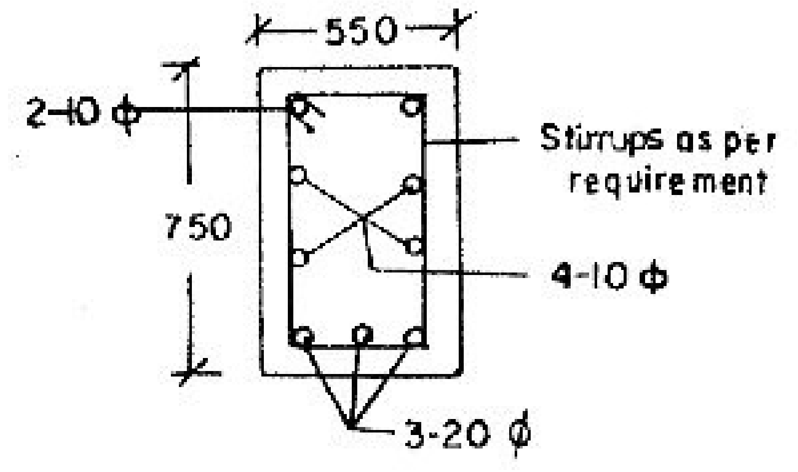
Since $M_1 < M_u$ longitudinal reinforcement in the flexural compression face is not required for quivalent moment $(M_t - M_u)$

As per IS 456, minimum tension reinforcement required is given by,

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y}$$

$$\therefore \quad A_{st} = \frac{0.85 \times 550 \times 710}{415}$$

= 819 mm² < 942 mm² hence, O.K.



Side face reinforcement to be provided @ 0.1% of sectional area of beam equally on each face *i.e.*

$$= \frac{1}{2} \times \frac{0.1 \times 550 \times 710}{100}$$

$$= 195.25 \text{ mm}^2$$

So provide 2 number of 10 mm diameter side face reinforcement on each face.

18. For the beam in the above question, determine the transverse reinforcement required, if it carries in addition, an ultimate shear force of 130 kN.

Solution: Given, T_u = 50 kNm

and V_u = 130 kN

Equivalent shear force, $V_e = V_u + 1.6\dfrac{T_u}{b}$

$$= 130 + 1.6 \times \frac{50}{0.55}$$

$$= 275.45 \text{ kN}$$

Equivalent shear stress, $\tau_{ve} = \dfrac{V_e}{bd} = \dfrac{275.45 \times 10^3}{550 \times 710}$

$$= 0.705 \text{ N/m}^2$$

Longitudinal steel provided in the beam is 3 bars of 20 mm diameter.

Hence area provided = 942.5 mm²

$$\therefore \quad p = \frac{100 A_{st}}{bd} = \frac{100 \times 942.5}{550 \times 710} = 0.241$$

For $p = 0.241$, τ_c = 0.35 N/mm²