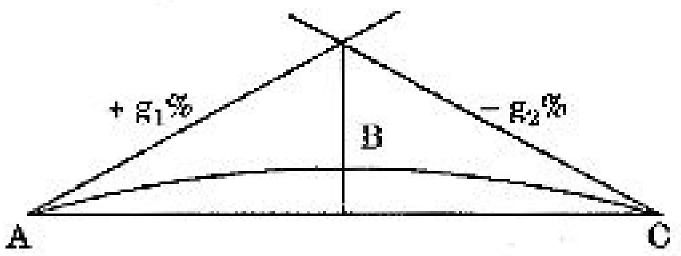
Slopes of vertical curves :

Depending upon the different combination of different grades, the following six types of vertical curve, are generally used while executing a rail or highway project.

An up grade ($+ g_1\%$) followed by a downgrade ($— g_2\%$)

A down grade ($- g_1$) followed



Length of the transition curve,

$$L = \frac{g_1 + g_2}{r} \cdot \frac{g_1 - (g_2)}{r}$$

where g_1 = % upgrade

g_2 = % down grade

r = rate of change of grade

If both rising then,

$$L = \frac{g_1 - (g_2)}{r}$$

CONTOURING

Contour

It is an imaginary line on the ground joining points of equal elevations. Maps representing horizontal as well as vertical relative positions of points are called *topographic maps*.

Contour interval

Vertical distance between any two consecutive contours is called *contour interval*.

It depends upon following :

(1) Scale and purpose of the map

(2) Nature of ground

(3) Availability of fund and time.

Horizontal Equivalent. It is the shortest horizontal distance between two consecutive contours.

Contour gradient.

It is an imaginary line lying on the surface of the earth and maintaining a constant inclination to the horizontal.

Characteristics of Contours

(1) Two contours of different elevations do not cross each other except in the case of an overhanging cliff.

(2) Contours of different elevations, do not combine to from one contour except in the case of a vertical cliff.

(3) Contours drawn closer depict a *steep slope* and if drawn far apart, it represent a *gentle slope*.

(4) Contours equally spaced depict a *uniform slope*. When contours are parallel, equidistant and straight, they represent an *inclined plane surface*.

(5) A contour at any point is perpendicular to the line of the steepest slope at the point.

(6) A contour must close itself or go out of the limit of the map.

(7) A set of ring contours with higher values inside, depict *hill* whereas a set of ring contours with lower values inside, depict *pond* or *depression*.

(8) When contours cross a ridge or V - Shaped valley, these form sharp V - shaped across them.

Contours represent a ridge line, if concavity of higher value contour lies towards next lower value contour and on the other hand these represent a valley if concavity of the lower value contour, lies towards higher value contour.

(9) The same contour must appear on both the sides of a ridge or a valley.

Uses of Contour Maps

(1) With the knowledge of characteristics of contours, it is easy to visualise whether countary is flat, undulating or mountainous.

(2) To decide most economical and suitable sites for engineering works such as canal, sewer reservoir, road,railway, etc.

(3) To determine catchment area of the drainage basin and hence capacity of the proposed reservoir.

(4) To compute earth work required for filling or cutting along the linear alignment of projects such as canals, roads, etc.

(5) To ascertain intervisibility of the points.

(6) To trace a contour gradient for road alignments.

(7) To draw longitudinal sections and cross - sections to as certain nature of the ground.

(8) To calculate water capacities of reservoirs.

Minor Instruments

Following minor instruments are convenient in undertaking rough-type of survey which do not require precision :

1. **Hand Level**

   It is a simple compact instrument used for locating contours, taking cross -sections in reconnaissance surveys.

2. **Clinometer**

   It is a light compact hand instrument used for measuring vertical angles, finding out slope of the ground and for locating points on a given grade.

There are three commonly used forms of clinometers :

(***i***) Abney's level

(***ii***) Tangent clinometer

(***iii***) Ceylon Ghat tracer

3. Box Sextant

It is a reflecting instrument capable of measuring upto 120° with an accuracy of one minute. It is one of the most precise hand instruments.

4. Pantagraph

It is used to reduce or enlarge a given figure.

5. Planimeter

It is used to measure the area of a given figure.

TACHEOMETRIC SURVEYING

Tacheometry is the branch of surveying in which both horizontal and vertical distances between stations are determined from instrumental observations.

Horizontal distances obtained by tacheometric observations do not require slope correction, tension correction etc. This method is very rapid and convenient. Though accuracy of tachometric distances is low as compared to direct chaining on flat ground but the accuracy achievable by tacheometry is better as compared to chaining in broken grounds, deep ravines or across large water bodies.

The instrument employed for tacheometric purpose, is generally known as a tacheometer which is similar to theodolite having diaphragm fitted with two additional horizontal wires, called **stadia hairs.**

Whenever surveys of higher accuracy are carried out, tacheometry provides a good check on distances measured with a tape or a chain.

Instruments Used for Tachometric Surveying :

1. **Tacheometer :**
2. **Stadia rods :** For short distances (say upto 100 metres) ordinary levelling staves may be used. For greater distances, the stadia rods 3 to 5 – metres in length, are generally used.

SYSTEMS OF TACHEDMETRIC MEASUREMENTS :

(*i*) Stadia hair system

(*a*) Fixed hair method ,

(*b*) Movable hair method

(*ii*) Tangential system

(*iii*) Substance bar (movable hair)

Principle of Techometry Surveying : In isoscles triangle, the ratio of the perpendiculars from the vertex on their bases is constant.

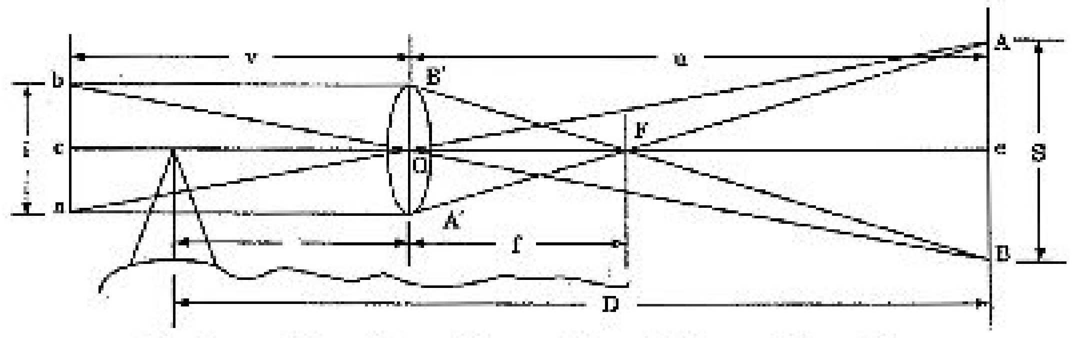
Let ABC and AB'C' be two isoscles triangles then

$$\frac{AO}{BC} = \frac{AO}{B^1C'} = \frac{1}{2}\cot\frac{\alpha}{2} = k$$

THE STADIA METHOD

1. Distance and Elevation Formulae for Horizontal Sights by Fixed Hair Method.

(*a*) Horizontal distance of the staff position :



Let f = focal length of the object lens

i = stadia hair interval ab,

S = staff intercept AB.

In ΔAFB and ΔA'B'F,

$$\frac{CF}{OF} = \frac{AB}{A'B'}$$

$$\therefore \quad CF = \frac{OF \times AB}{A'B'}$$

$$= \frac{f}{i} \times S$$

$$\text{But} \quad D = CF + d + f$$

$$\therefore \quad D = \frac{f}{i}S + d + f$$

This is known as the tacheometric distance equation in which f, i and d are constants for a particular theodolite. The tacheometric distance formula may be stated as

$$D = KS + C$$

where K and C are generally known as tacheometric constants of a theodolife,

K is known as multiplying constant, is usually kept 100 and

C is known as additive constant is generally kept between 0.3 to 0.5 m.

Note :

(*i*) The distance formula $D = \frac{f}{i}S + (f + d)$ is applicable only if the line of the sight is horizontal and the staff is held truly vertical.

(*ii*) The point F, the vertex of measuring triangle is sometimes known as analytical point.

(*b*) Elevation of the staff station : Since the line of sight is kept horizontal and the staff is held vertical, the elevation of the staff station is obtained exactly in a similar way as in ordinary levelling.

Elevation of the staff station = Elevation of the inst. axis — central hair reading

2. DISTANCE AND ELEVATION FORMULATE FOR INCLINED SIGHT BY FIXED HAIR METHOD

(I) Distance and Elevation formula for inclined sights with staff vertical.

(*i*) Horizontal distance formula :

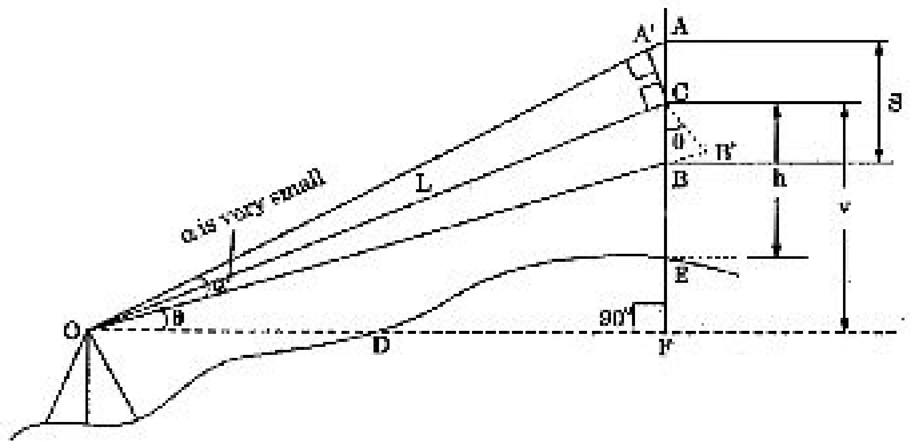


Fig. (a) Elevated line of sight with staff vertical

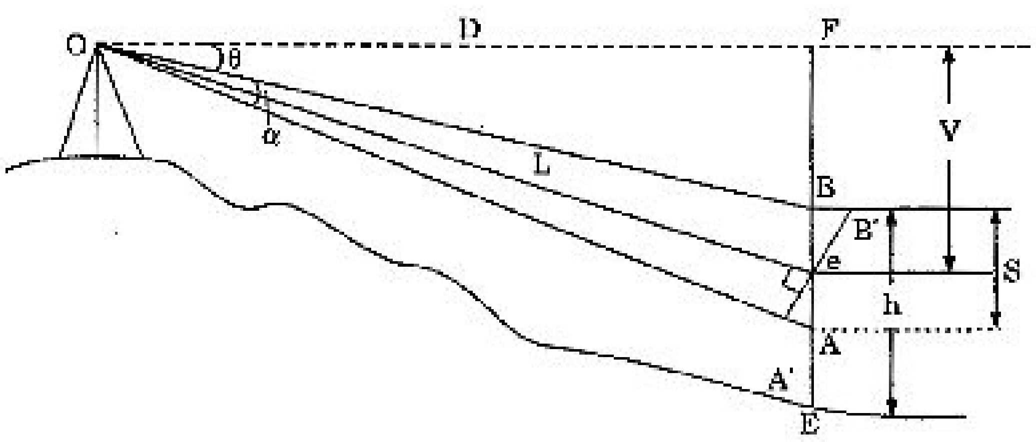


Fig. (b) Depressed line of sight with staff vertical

Let θ be the angle of elevation or depression of the line of sight from the horizontal. As the staff is held vertical, the staff intercept AB is not normal to the line of sight OC.

Draw a line A'B' passing through C and perpendicular to OC cutting OA at A′ and OB produced at B[1]

From ΔAA′C and BBC,

$$\cos\theta = \frac{CA^1}{CA}$$

$$\Rightarrow \quad CA' = CA\cos\theta$$

$$\text{and} \quad B'C = BC\cos\theta$$

$$A'C + B'C = AC\cos\theta + BC\cos\theta$$

$$= AB\cos\theta \quad ...(\because AC = BC = A)$$

$$A'B' = AB\cos\theta$$

$$= S\cos\theta$$

But, the inclined distance,

$$OC\,(L) = \frac{f}{i}\,A'B' + (f + d)$$

$$L = K\,S\cos\theta + C$$

Horizontal distance,

$$D = L\cos(KS\cos + C)\cos$$

$$D = KS(\cos^2 + C\cos\theta)$$

where K and C are tacheometric constant.

(*ii*) Election formula :

$$V = L\sin\theta$$

$$= (KS\cos\theta + c)\sin\theta$$

$$= KS\sin\theta\cos\theta + c\sin\theta$$

$$V = \frac{KS\sin 2\theta}{2} + C\sin\theta$$

(*a*) Elevation of the staff station for an angle of elevation.

Let the central hair reading oh the staff be h.

The difference in level between O and E. FE = V – h

R.L. of staff station E = R.L. of horizontal line of sight + V – b

(*b*) Elevation for the staff station for an angle of depression

Let the central hair reading on the staff be h. Then difference in level between O and E

$$FE = V + h$$

so R.L. of staff station E = R.L. of line of sight – FE

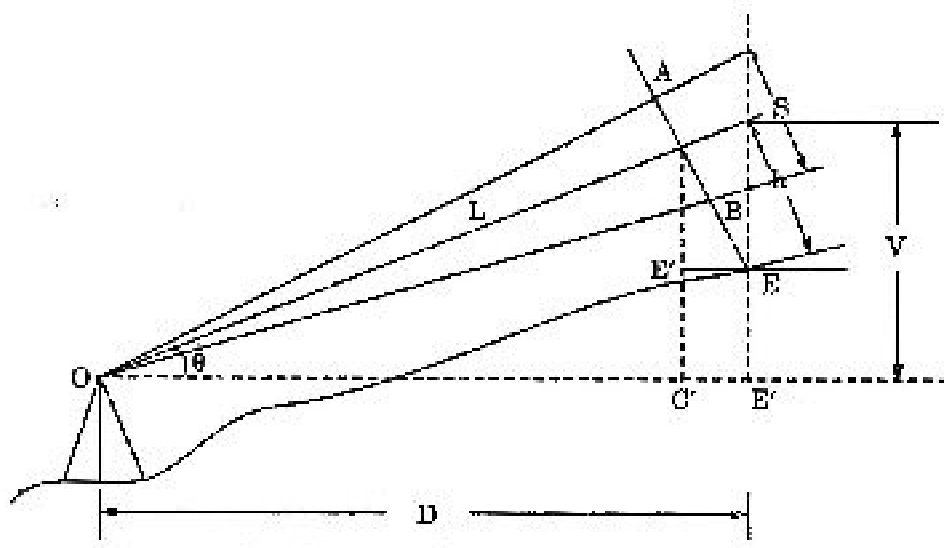
R.L. of Staff station E = R.L. of line of horizontal sight ± V – h

Remember : R.L. of staff station = R.L. of line of horizontal sight ± V –h

(3) DISTANCE AND ELEVATION FORMULAE FOR INCLINED SIGHT WITH STAFF NORMAL

Case I : Line of sight at an angle of elevation

(*a*) Horizontal distance formula :



Let AB = S, the staff intercept

CE = central hair reading = h

θ = angle of elevation for central wire

L = inclined distance = OC

We know, L = KS + C

$$OC' = L\cos\theta = (KS + C)\cos\theta$$

But $$D = OC' + C'E'$$

$$...[\because C'E' = EE_1' = h\sin\theta]$$

$$D = (KS + C)\cos\theta + h\cos\theta)$$

(b) Elevation of the staff station :

$$V = L \sin\theta = (KS + C)\sin\theta$$

R.L. of E = R.L. of line of horizontal sight

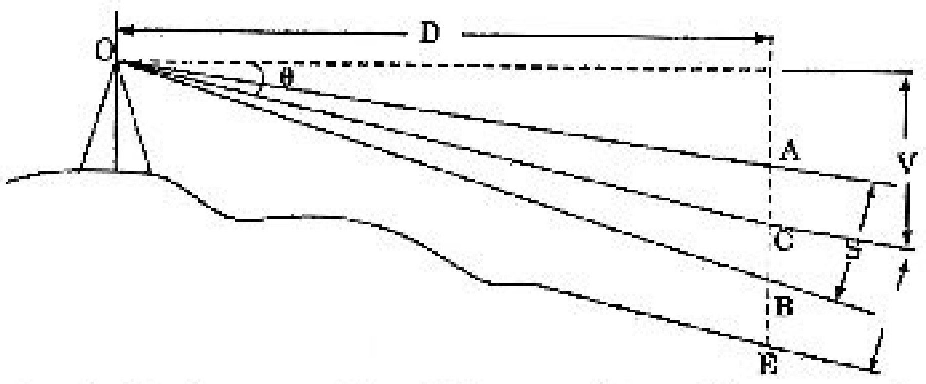
$$+ \text{ve} - C'E_1'$$

$$\text{R.L. of E} = \text{H.I.} + (KS + C)\sin\theta - h\cos\theta$$

$$...[\because C_1E_1{}^1 = h\cos\theta]$$

Case II : Line of sight at an angle of depression

(a) Horizontal distance formula



Apply tacheometric distance formula, we get

$$L = KS + C$$

$$OC' = L\cos\theta = (KS + C)\cos\theta$$

But $\quad D = OC' - C'E'$

$$D = (KS + C)\cos\theta - h\sin\theta$$

$$[C'E' = CP\ h\sin\theta]$$

(b) Elevation–of the staff station

$$V = L\sin\theta = (KS + C)\sin\theta$$

R.L. of E = height of trunnion axis – V – PE

$$\text{R.L. of E} = \text{H.I.} - (KS + C)\sin\theta - h\cos\theta$$

Remember : R.L. of staff station

$$= \text{H.I} + (KS + C)\sin\theta - h\cos\theta$$

TYPE OF TELESCOPE FITTED IN STADIA THEODOLITE :

The telescope used in stadia theodolites are of the following three types :

(i) Internal focusing telescope

(ii) External focusing telescope

(iii) Internal focusing telescope fitted with an analytic lens. (C = 0)

The theodolite fitted with the second type is known as stadia theodolite whereas that fitted with third type is known as a tacheometer. In a tacheometer, additive constant is usually kept zero.

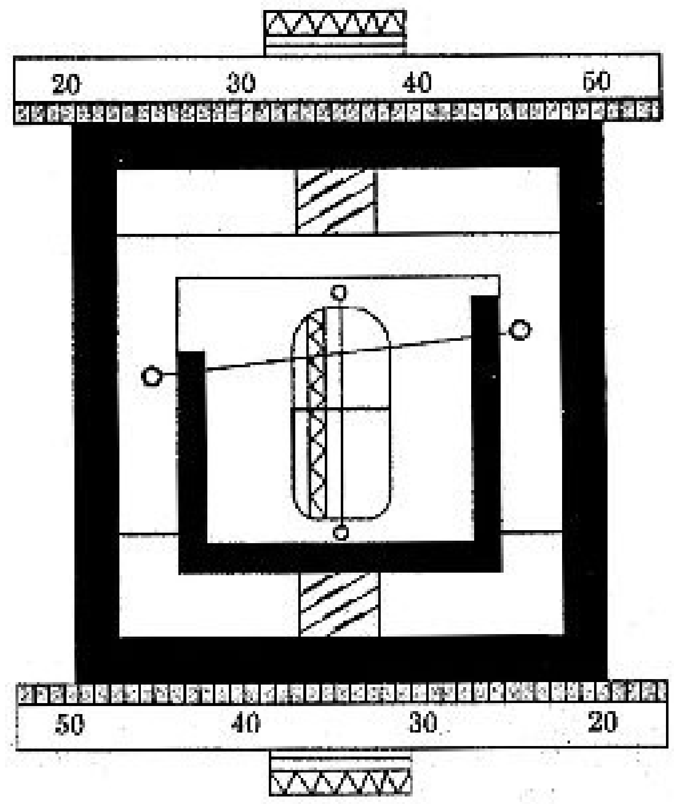
A TACHEOMETER MUST MEET THE FOLLOWING REQUIREMENTS :

(i) The multiplying constant (K) must be 100 and the error in the computed distance should not exceed 1 in 1000.

(ii) The central hair should be exactly midway between the stadia hair.

(iii) The telescope must be truly analytic (c = 0). That is analytic constant (C) must be zero.

SUBSTANCE THEODOLITE (MOVABLE HAIR METHOD) : In this method, the staff intercept is kept constant whereas the distance between stadia hairs is variable. Instrument used in this method are a theodolite with a special type of diaphragm and a staff provided with two targets at a known distance.



Tacheometric formulae for substance theodolite

Let Number of turn of upper screw = n_1

Number of turn of bottom screw = n_2

$\therefore$ Total number of turn = n

$$= n_1 + n_2 \text{ of the micrometer}$$

If $\quad$ pitch = p

then $\quad i = n\text{P}$

we know that $\quad D = \dfrac{f}{s}S + C = \dfrac{\phi}{2} + C$

$$D = \frac{KS}{n} + C$$

where $K = \dfrac{f}{p}$ is know from 600 to 1000

For inclined sight staff held vertical is

$$D = \frac{KS\cos^2\theta}{n} + C\cos\theta$$

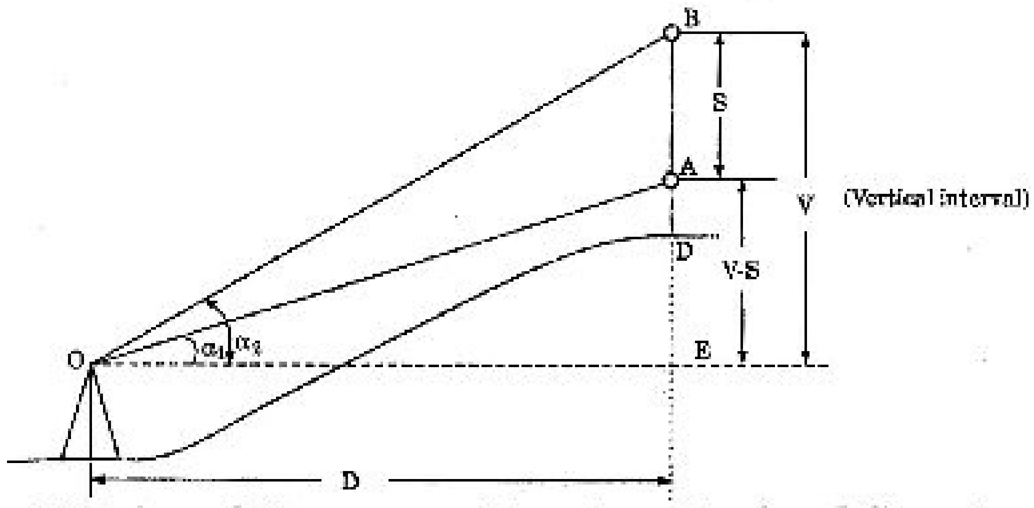
Tangential Method of Tacheometry :

In tangential tacheometry, horizontal and vertical distances from the instrument to the staff position, are computed from the observed vertical angle of two targets fixed at a known distance S on the staff.

DEPENDING UPON THE VERTICAL ANGLES, THREE CASE MAY BE ARISE :

1. Both vertical angles may be elevation angles.
2. Both vertical angle may be depression angles.
3. One of the angle may be an elevation angle and the other may be a depression angle.

Case 1 : Distance and elevation angle :



Let A and B represent two targets fixed S metres apart on a staff held vertical at D. Let O represent the trunnion axis of the theodolite.

$$\tan\alpha_2 = \frac{V}{D}$$

$$\Rightarrow \quad V = D\tan\alpha_2 \quad \ldots(i)$$

and

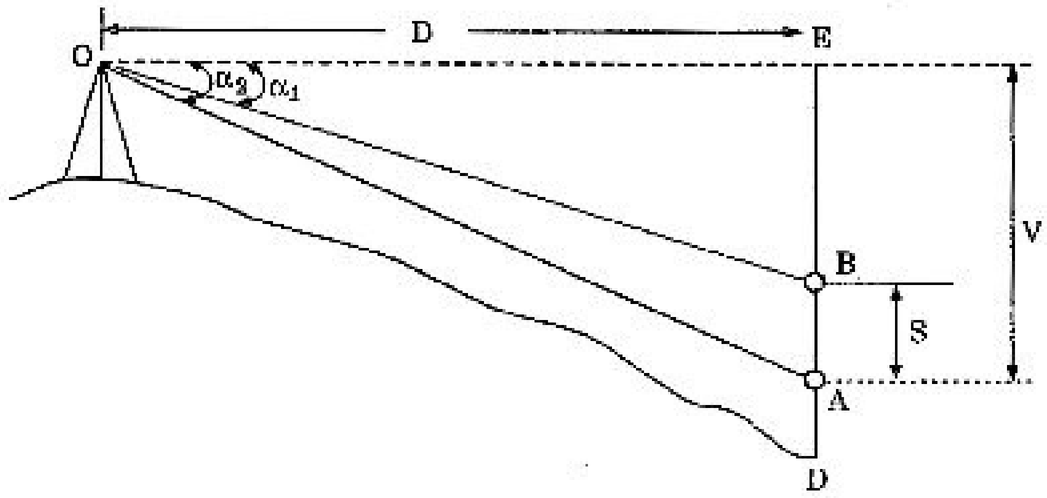
$$\tan\alpha_1 = \frac{V}{D}$$

$$\Rightarrow \quad V - S = D\tan\alpha_1 \quad \ldots(ii)$$

Substracting equation (*ii*) from equation (*i*), we get

$$S = D(\tan\alpha_2 - \tan\alpha_1)$$

$$\therefore \quad D = \frac{S}{\tan\alpha_2 - \tan\alpha_1}$$



Case 2 : Distance and depression angle

$$\tan\alpha_2 = \frac{V}{D}$$

$$\Rightarrow \quad V = D\tan\alpha_2 \quad \ldots(i)$$

And

$$\tan\alpha_1 = \frac{V-S}{D}$$

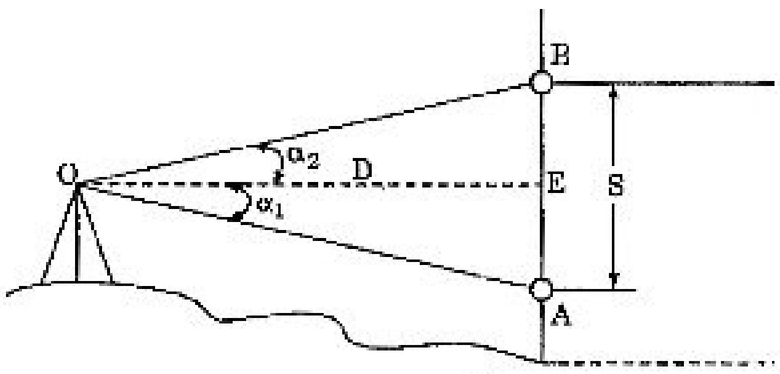
$$\Rightarrow \quad V - S = D\tan\alpha_1 \quad \ldots(ii)$$

Substracting equation (*ii*) from equation (*i*), we get

$$S = D(\tan\alpha_2 - \tan\alpha_1)$$

$$D = \frac{S}{\tan\alpha_2 - \tan\alpha_1}$$

Case 3 :

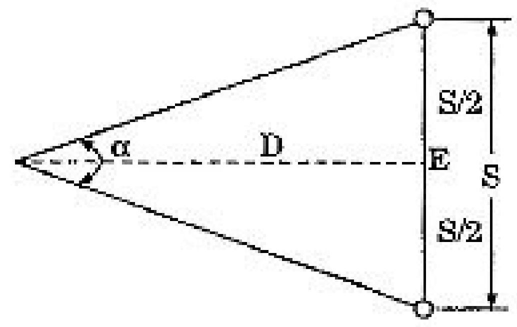


$$D = \frac{S}{\tan\alpha_2 + \tan\alpha_1}$$

Note : The most common method of tacheometry is a fixed hair stadia method, using a staff held vertically.

THE SUBSTANCE BAR : The substance bar is an instrument used for measuring horizontal distances in the area where direct chaining becomes difficult due to undulations or other obstructions.

Computation of Substance bar distance



$$\tan\frac{\alpha}{2} = \frac{S/2}{D}$$

$$D = \frac{S}{2\tan\alpha_2}$$

$$D = \frac{S}{2}\alpha\frac{L}{\alpha/2} \text{ (where } \alpha \text{ is very small)}$$

$$D = \frac{S\times 206265}{\alpha}, \text{ where } \alpha \text{ is in seconds}$$

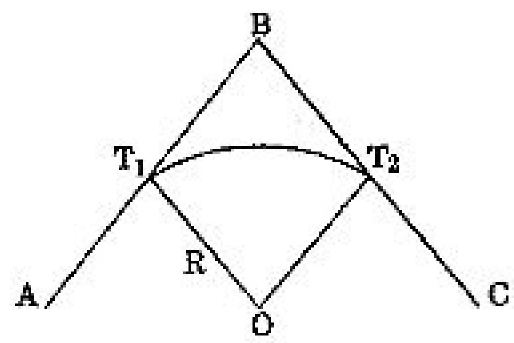
SIMPLE CURVES

Curve : A regular curved path followed by a railway or highway alignment is called a curve. A curve may be either circular, parabolic or spiral and is always tangential to the two straight directions at its ends.

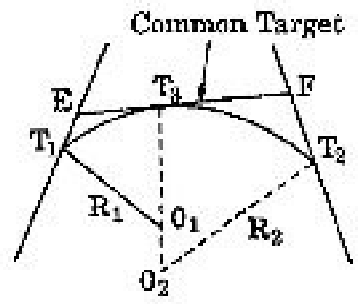
Types of Curve :

1. **Simple curve :** This is a type of circular curve
2. **Compound curve :** This is a type of circular curve
3. **Reverse curve :** This is a type of circular curve
4. **Transition curve :**

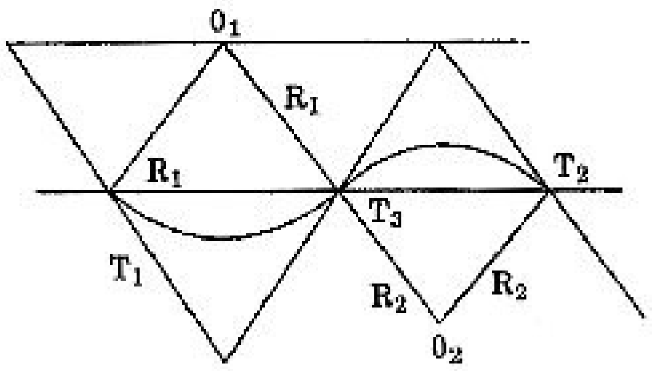
1. **SIMPLE CURVE :** The curve which is a single are of a circle is known as simple curve or a simple circular curve. It is tangential to both the straights AT_1 and CT_2.



2. **COMPOUND CURVE :** A curve which consists of two or more arce of different circles with different radii having different centres lying on the same side of the common tangent is known as compound curve.

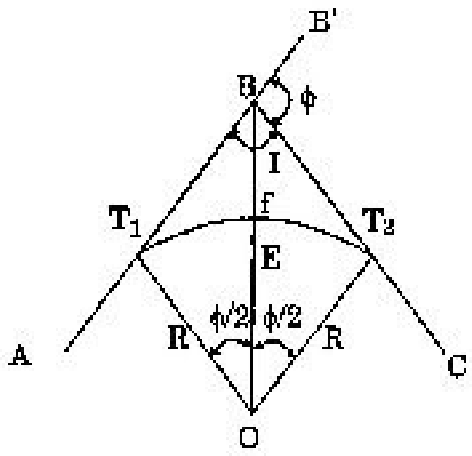


3. **REVERSE CURVE :** A curve which consists of two areas of different circles of same or different radii, is known as a reverse curve. In such curves the centre areas are on the opposite sides of the curve. The two areas turn in opposite directions with a common tangent at the junction.



4. **Transition Curve :** A non–circular curve introduced betw< curve is known as a transition curve

NOTATION FOR CIRCULAR CURVE :



TANGENTS : The straight line AB and BC, which are connected by the curve are called the tangents or straights to the curve.

Point of intersection or vertex : The point (B) at which the two tangent lines AB and BC intersect is known as the point of intersection (P.I.) or the vertex.

Right and Left Hand Curve : If the curve deflects to the right of the direction of the progress of survey (AB), is called a right–hand curve, if to the left, it is called a *left hand curve.*

Back Tangents : The tangent AB at T_1, the point of commencement of the curve, is called 'back tangent' or first tangent.

Forward Tangent : The tangent BE at T_2 the end point of the curve is called *forward tangent or second tangent*.

Angle of Deflection : The angle through which forward tangent deflects, is called *angle of deflection of the curve*. It may be either to the right or to the left.

Tangent Distance : The distance from the point of intersection to the tangent point (T.P) is called the tangent distance or Tangent length (BT_1 and BT_2).

Long Chord : The chord joining the point of curve and the point of tangency is called long chord.

Length of the Curve : The total length of the curve from the tangent curve to the curve tangent or the total length of the curve from tengent point to point of tengeny is called length of the curve.

MID Ordinate : The ordinate joining the mid p1 oint of the curve and long chord, is called mid–ordinate it is also known as versed sign.

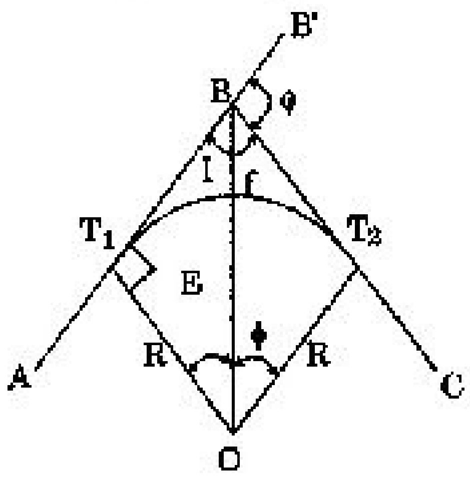
Normal Chord : A chord between two successive regular pegs on the curve, is called a normal chord.

Sub–Chord : When a chord is shorter than the normal chord, it is called a sub–chord. These sub–chords generally occur at the beginning and at the end of the curve.

Elements of the Simple Curve :

Tangent length $= BT_1 = BT_2 = ?$

In ΔOBT, $\tan\frac{\phi}{2} = \frac{BT_1}{R}$



$$\Rightarrow \quad BT, = R\tan\frac{\phi}{2}$$

$$\therefore \quad \text{Tangent length } BT_1 = BT_2 = R\tan\frac{\phi}{2} \quad ...(i)$$

Length of the long chord $= T_1T_2 = 2T_1E$

$$= 2R\sin\frac{\phi}{2} \quad ...(ii)$$

Length of the curve, $(l) = \phi \times R$ (in radian)

$$= \frac{\pi R\phi}{180} \quad ...(iii)$$

Apex distance $= BF = OB–OF$

where, $\cos\frac{\phi}{2} = \frac{R}{OB}$

$$\Rightarrow \quad OB = R\sec\frac{\phi}{2}$$

$$= R\sec\frac{\phi}{2} - R$$

$$BF = R\left(\sec\frac{\phi}{2}\right) \quad ...(iv)$$

Versed sine of the curve = EF = OF – OE,

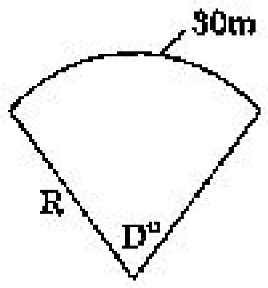
$$\cos\frac{\phi}{2} = \frac{DE}{R} = R - R\cos\frac{\phi}{2}$$

$$DE = R\cos\frac{\phi}{2}$$

$$EF = R\left(1-\cos\frac{\phi}{2}\right)$$

Degree of Curve : Degree of curve may be defined either with respect to a fixed length of an are of the curve or with respect to a fixed length of a normal chord of the curve.

Fixed length of an are : The degree of curve may be defined as the central angle of the curve that is subtended by an arc of 30 metres (or 100 ft). It is adopted for railway curves.

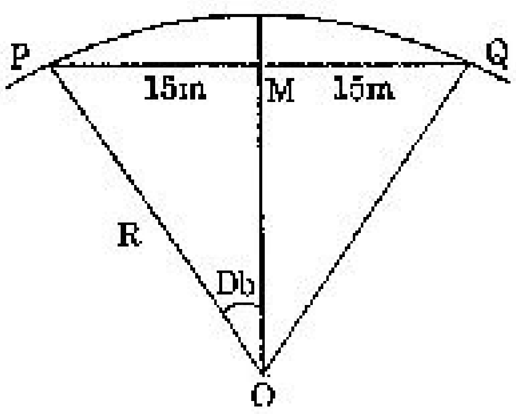


$$\frac{30}{2\pi R} = \frac{D}{360}$$

$$\Rightarrow \quad D^\circ = \frac{30 \times 360}{2\pi R} = \frac{1718.9}{R}$$

$$D^\circ = \frac{1718.9}{R}$$

Based on Fixed Length of a Chord : The degree of a curve may be defined a fixed central angle of the curve that is subtended by a chord of 30 metres length. It is adopted for road curve.



$$\sin\frac{D}{2} = \frac{15}{R}$$

$$\Rightarrow \quad \frac{D}{2} = \frac{15}{R}$$

$$\therefore \quad D^\circ = \frac{30}{D} = \frac{30}{D \times 5} = \frac{30 \times 180}{\pi D}$$

$$R = \frac{1718.9}{D^\circ}$$

METHOD OF CURVE RANGING :

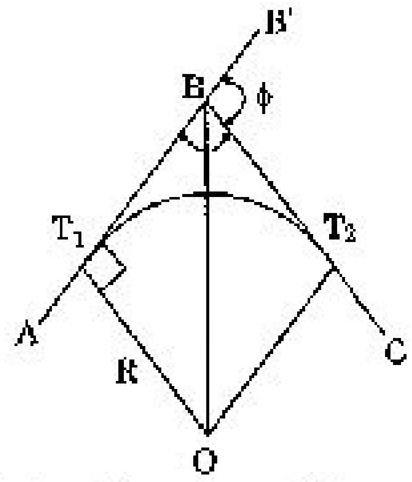
1. Linear or chain and tape method and
2. Angular or instrumental methods.

- Linear methods is those in which the curve is set out with a chain and tape only.
- Instrumental methods are those in which a theodolite with or without a chain is employed to set out the curve.

PEG INTERVAL : The length of unit chord (peg interval) is, therefore, 30m for flat curve 20m for sharp curves and 10m or less for very sharp curves.

LOCATION OF TANGENT POINTS :

To locate the tangent points T_1 and T_2, proceeds as follows :



(*i*) Having fixed the direction of the tangents, produce them so as to meet at the point B

(*ii*) Find the deflection angle by theodolite.

(*iii*) Calculate the tangent length

$$BT_1 = R \tan OB$$

$$= R \sec\frac{\phi}{2}$$

(*iv*) Locate the point T_1 and T_2 on back tangent and forward tangent.

$\Rightarrow$ Chainage of point T_1 = chainage of point of intersection B — BT_1

Chainage of point T_2 = chainage of point T_1 + length of the curve

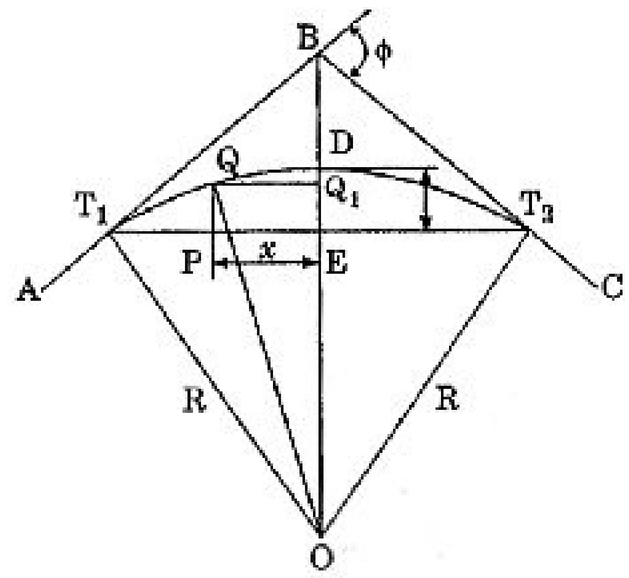
Chain and Tape (or linear) methods of setting out curves

1. By offsets or ordinates from the long chord (Linear method)
2. Offsets from the tangents (Linear method)
3. Successive bisection of the chords (Linear method)

Angular or Instrumental Methods

1. Offsets from Chord produced (Linear method)
2. Deflection angles from the point of curve and normal chord (Angular method)
3. Deflection angles from the point of curve and point of tangency, using two theodolites (Angular method)

1. OFFSETS FROM THE LONG CHORD :



Let AB and BC = tangents to the curve T_1DT_2

T_1 and T_2 are tangent points

T_1T_2 = Long chord of length L

$ED = O_0$

= offset at the midpoint of T_1T_2 (the versed sine)

$PQ = Q_x$

= offset at a distance x from E,

so that $EP = x$

$OT_1 = OT_2 = OD = R$

= radius of the curve

In ΔOT_1E, $OT = R_1T_1E = L/2$

$$OT = OD - ED = R - O_0$$

$$OT_1^2 = T_1E^2 + OE^2$$

$$\Rightarrow \quad R^2 = (L_2)^2 + (R - O_O)^2$$

$$\therefore \quad OO = R - \sqrt{R^2 - (L/2)^2}$$

In ΔOQQ_1 $OQ^2 = QQ_1^2 + OQ_1^2$

$$\Rightarrow \quad R^2 = x^2 + (OE + O_x)^2$$

$$\Rightarrow \quad OE + O_x = \sqrt{R^2 - x^2}$$

$$\Rightarrow \quad O_x = \sqrt{R^2 - x^2} - OE$$

$$\therefore \quad O_x = \sqrt{R^2 - x^2} - (R - O_O)$$

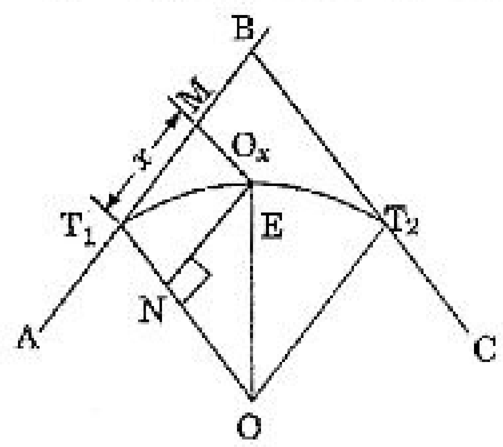
[since OE = OD – ED = R – O]

2. OFFSET FROM THE TANGENTS :

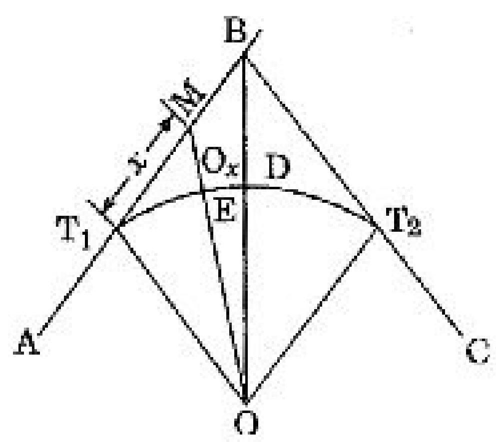
(*i*) Perpendicular offsets

(*ii*) Radial offsets

(i) Perpendicular off sets



(ii) Radial offset

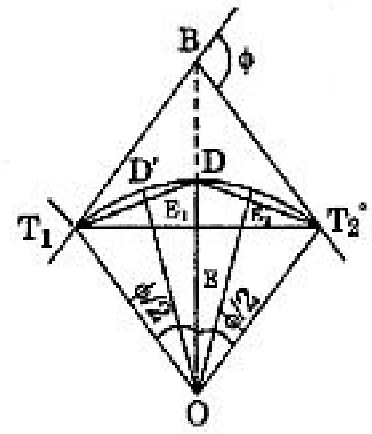


$$DE^2 = NE^2 + ON^2$$

$$\Rightarrow \quad R^2 = x^2 + (R - O_x)^2$$

$$\therefore \quad O_x = R - \sqrt{R^2 - x^2}$$

3. SUCCESSIVE BISECTION OF CHORDS



Let T_1T_2 be the long chord of a curve whose angle of deflection is ϕ.

Divide T_1T_2 at C. join OE and produce it to intersect the curve at

Now $DE = OD - OE$

$$= R - R\cos\frac{\phi}{2}$$

$$O_O = DE = R\left(1 - \cos\frac{\phi}{2}\right)$$

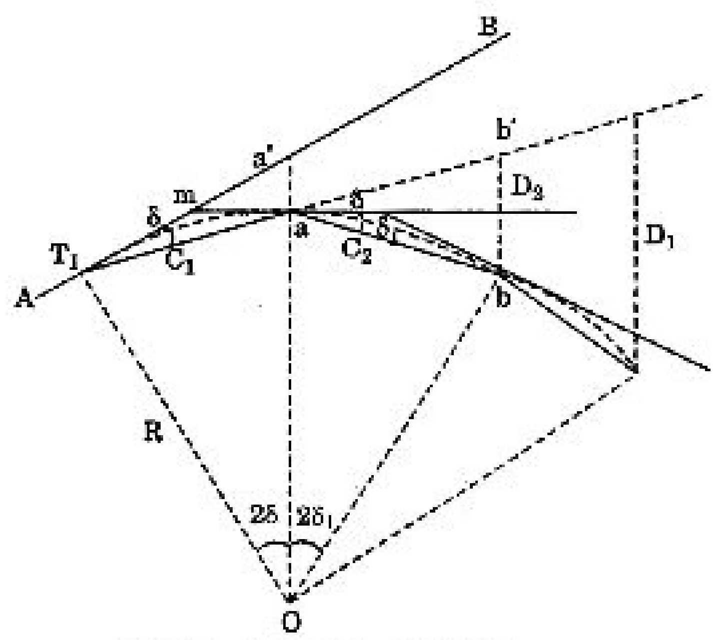
Now consider T_1D and T_2D independent portions of the curve having T_1D and T_2D as long chord. It can be proved that offset E_1D_1 and E_2D_2 are each equal to R (1 — cos ϕ/4).

By further successive bisection of the chords T_1D_1, D_1D_1, DD_2 and D_2T_2 we may obtain the locations of other points on the curve.

4. OFFSET FROM CHORD PRODUCED :

This method is commonly adopted when a theodolie is not available and it is necessary to set out a curve only with a chain or a tape. The curve is divided into a number of chords normally 20 or 30 m in length. As continuous chainage is required along the curve, two sub–chords generally occurs one at the beginning and the other at the end of the curve.

Offsets from chords produced may be computed with the help of the formula derived under :



$$MO^2 = T_1O^2 + T_1M^2$$

$$(R + O_x)^2 = R^2 + x^2$$

$$O_x = \sqrt{R^2 - x^2} - R$$

Let AB be the back tangent, $T_1a = C_1$ be the first sub-chord and $\angle BT_1a = \delta$

$$OT_1 = Oa = R$$

The chord T_1a being very newly equal to are T_1a

$$2\delta = \frac{T_1a}{R}$$

$$\Rightarrow \quad \delta = \frac{C_1}{2R} \quad ...(i)$$

Similarly the chord aa' is very neurly equal to the arc aa'

$$\text{or} \quad \phi = \frac{aa'}{C_1}$$

$$\Rightarrow \quad aa' = \delta C_1 \quad ...(ii)$$

From equations (*i*) and (*ii*)

$$aa' = \frac{C_1^2}{2R}$$

$$\Rightarrow \quad O_1 = \frac{C_1^2}{2R} \quad ...(iii)$$

Now mT_1 and ma both being tangents

$$\angle mT_1a = \angle maT_1 = \delta = \angle b'an$$

(opposite angles)

$$\text{Now} \quad \delta + \delta_1 = \frac{O_1}{C_2} \quad ...(iv)$$

$$2\delta_1 = \frac{C_2}{R}$$

$$\Rightarrow \quad \delta_1 = \frac{C_2}{R} \quad ...(v)$$

Putting the value of δ and δ_1 in equation (*iii*)

$$\frac{C_1}{2R} + \frac{C_2}{2R} = \frac{O_2}{C_2}$$

$$\therefore \quad O_2 = \frac{C_2(C_1 + C_2)}{C_2} \quad ...(vi)$$

Similarly we get the value of third offset O_3

$$\text{i.e,} \quad O_3 = \frac{C_3(C_2 + C_3)}{2R} \quad ...(vii)$$

All the chords, excepting the sub-chords are generally equal

$$\text{i.e.} \quad C_2 = C_3 = C_4 = \text{.........} \; C_{n-1}$$

$$\therefore \quad O_{2 \text{ to } n-1} = \frac{C^2}{R},$$

where C is the length of the normal chord R

The offsets for the last sub chord C_n

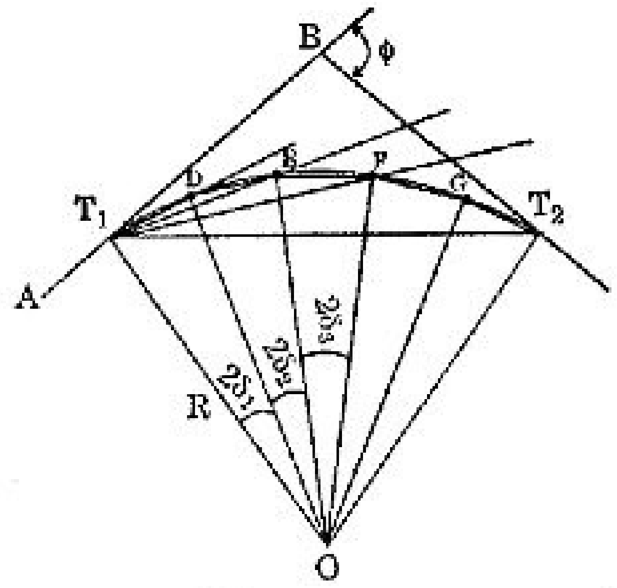
$$O_n = \frac{C_n(C_{n-1} + C_n)}{2R}$$

5. RANKING METHOD OF TRNGENTIAL DEFLECTION ANGLES :

- Deflection angle (δ) and angle of deflection (ϕ) are different things.

  In this method the curve is set out by the tangential angle called the deflection angle with a theodolite and a chain or tope.

Derivation of the Formula :



Let AB = rear tangent of the curve

T_1 and T_2 = tangent points

D, E, F etc = successive points on the curve

$\delta_1, \delta_2, \delta_3$ = tangential angles, which each of the successive chords T_1D, DE, EF etc makes with the respective tangents at T_1, D, E, etc.

D_1, D_2, D_3, etc = total tangential or deflection angles (between the rear tangent AB and each of the line T_1, D, T_1E, T_1F etc) for the chord T_1D, DE, EF etc.

C_1, C_2, C_3 etc = length of the subchord and normal chord T_1D, DE, EF etc.

R = radius of the curve

$$\text{Chord} \quad T_1D = \text{are } T_1D \approx C_1$$

$$\delta_1 = \angle BT_1D = \frac{1}{2}\angle T_1OP,$$

$$\text{i.e.,} \quad \angle T_1OD = 2\delta_1$$

$$\text{Now} \quad 2\delta_1 = \frac{C_1}{R},$$

$$\Rightarrow \quad \frac{2\delta_1 \times \Pi}{180} = \frac{C_1}{R}$$

$$\Rightarrow \quad \delta_1 = \frac{90C_1}{TR} \times 60 \text{ minutes}$$

$$= \frac{1718.99}{R} \text{ minutes}$$

Similarly, $\delta_2 = \dfrac{1718.9C_2}{R}$,

$\delta_3 = \dfrac{1718.9C_3}{R}$ and so on

$\delta_n = \dfrac{1718.9C_n}{R}$

Since each of chord lengths $C_2, C_3 \ldots\ldots C_{n-1}$ is equal to the length of the unit chord,

$$\delta_2 = \delta_3 = \delta, \ldots\ldots = \delta_{n-1}$$

Hence the total tengential angle (deflection angle)

$$\Delta_1 = \delta_1$$
$$\Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2$$
$$\Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3$$
$$\vdots$$
$$\Delta_n = \delta_1 + \delta_2 + \delta_3 \ldots + \delta_n = \Delta_{n-1} + \delta_n$$

Check : total deflection angle $(BT_1T_2) = \Delta n = \dfrac{\phi}{2}$, where ϕ is the deflection angle of the curve.

$$\Rightarrow \quad c_1 = \frac{\text{Percentage of up grade}}{\text{rate of change of grade}} = \frac{g_1}{r}$$

$$c_2 = \frac{\text{Percentage of down}}{\text{gredient}} = \frac{g_2}{r}$$

$\therefore$ Total length of the vertical curves ,

$$L = \frac{g_1}{r} - \frac{(-g_2)}{r}$$

$$= \frac{g_1 + g_2}{r}$$

$$\Rightarrow \quad h = kN^2$$

= Tangent correction

where k is a constant and N is the distance measured from O.

TRIANGULATION

The process of measuring the angles of a chosen networks of triangles formed by a number

of stations marked on the surface of the earth is called *triangulation.*

Principle of Triangulation

If all the three angles and the length of one its sides of a triangles are known, then the

lengths of the remaining sides of the triangle can be calculated trignometrically.

- The sides of the first triangle, whose length is predetermined is called the base line and vertices of the individual triangles are known as *triangulation stations* and the whose figure is called the *triangulation system* or *triangulation figure.*
- The triangulation stations at which astronomical observations for azimuth are made are called *Laplace stations.*

Classification of Triangulations

(1) *Primary triangulation of first order triangulation:* Highest grade of triangulations.

(2) *Secondary triangulation or Second order triangulation*

(3) Tertiary triangulation or third order triangulation or Topo triangulation

Specification of primary secondary and Tertiary triangulation

	Item	Primary	Secondary	Tertiary
(1)	Length of the base links	8 to 12 km	2 to 5 km	100 to 500 km
(2)	Length of the sides	16 km to 150 km	10 to 25 km	2 to 10 km
(3)	Averagetriangulation error	less than 3"	12"	
(4)	Maximum station closure	not more than 3"	8"	15"
(5)	Actual error of the base	1 : 50,000	1 in 25000	I in 10,000
(6)	Probable error of the base	1 : 1000000	1 in 500000	I in 250000

Phase of a Signal

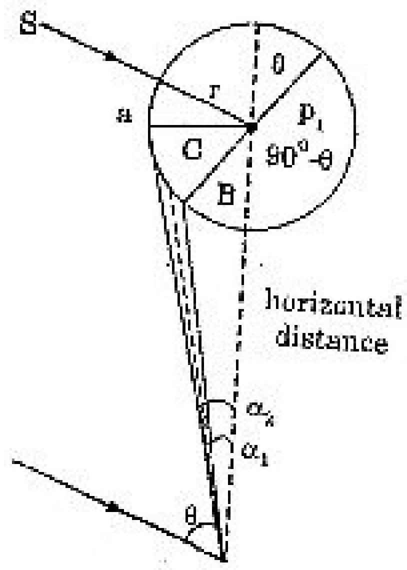
When a cylindrical opaque signal which is partly illuminated by the sun and partly remain in shadow, is observed, the observer sees only illuminated portion from his station and thus bisects the centre of the illuminated portion. The error of bisection thus introduced is known as phase of the signal.

Two cases may arise

(I) An observation is made on the bright portion

(I) An observation is made on the bright line

(I) When the observation is made on the bright portion



$$\beta = \text{phase correction angle COP}$$

$$= \alpha_1 + \frac{\alpha_1 + \alpha_2}{2}$$

$$\beta = \frac{\alpha_1 + \alpha_2}{2}$$

ΔOAP, $\tan \alpha_2 = \frac{r}{D}$,

or $\alpha_2 = \frac{r}{D}$ radius

and $\alpha_1 = \frac{r\sin(90-\theta)}{D} = \frac{r\cos\theta}{D}$

$$\therefore \quad \beta = \frac{1}{2}\left[\frac{r}{D} + \frac{r\cos\theta}{D}\right]$$

$$= \frac{r}{2D}[1+\cos\theta]$$

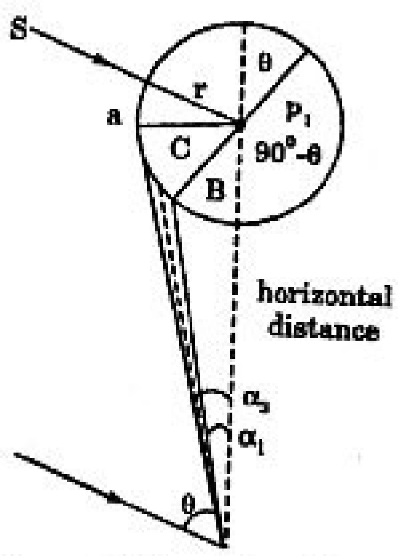
$$= \frac{2\cos^2\theta/2}{D} \text{radians}$$

$$\beta = \frac{r\cos^2\theta/2}{D\sin 1}\text{seconds}$$

where, r = radius of the signal

β = angle between sun and the line OP

II. When the observation is made on the Bright line



$$\angle SCO = 180 - (\theta - \beta)$$

$$\angle PCO = 180 - 90 + \frac{\theta-\beta}{2} = 90 + \frac{\theta-\beta}{2}$$

$$\angle CPO = 180 - \left(\beta + 90 + \frac{\theta-\beta}{2}\right)$$

$$= 90 - \left(\frac{\theta+\beta}{2}\right)$$

$$= 90 - \frac{\theta}{2}, \text{ ignoring } \beta$$

$$\tan\beta = \frac{r\sin\left(\frac{90-\theta}{2}\right)}{D - r\cos\left(\frac{90^\circ-\theta}{2}\right)}$$

$$= \frac{\frac{r\cos\theta}{2}}{D}\text{radians}$$

$$\beta = \frac{\frac{r\cos\theta}{2}}{D\sin 1''}\text{seconds}$$

The phase correction is applied algebraically to the observed angle according to the relative positions of the sun and the signal, i.e., it is the +ve and zero.

Measurement of Base line

The accuracy of any order of triangulation depends upon the accuracy of the measurement of its base line. Hence in triangulation a base line is of prime importance.

Selection of the site for a Base line

1. The ground of the sight should be fairly level or uniformal.

Number of zeros : To eliminate the error due to inaccurate graduations of the horizontal circle, the measures of the horizontal angles are taken in two or three different zones, i.e., Face $\angle$ 90°, Face L 180° Face L 270°.

Types of Triangulation stations

(*i*) Main station

(*ii*) Subsidiary stations

(*iii*) Satellite stations or Eccentric or false stations

(*iv*) Pivot stations

Signals and Towers

The signals may be classified as

(*a*) Luminous signal, and

(*b*) Opaque signals or non–luminous signal

- Base line is measured by standard tape and after that following connections are applied to the measured base length.

(1) Correction for standard length

$$\frac{C_a}{L} = \frac{C}{L}$$

$$\Rightarrow \quad C_a = \frac{CL}{l}$$

Where, L = measured length of the line

l = designed length of the tape

Ca = correction for absolute length

C = correction to be applied to the tape.

The sign of the correction C_a will be the same as that of C.

(2) Correction of Alignment

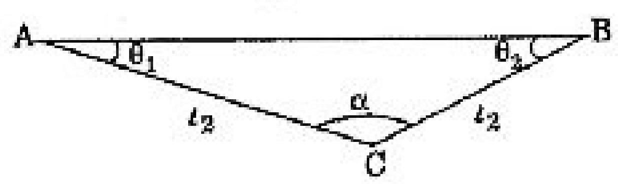


Fig.