Example. *A wall with a smooth vertical back, 10 m high, supports a purely cohesive soil with c = 9.81 kN/m² and γ = 17.66 kN/m². Then find total Rankine's active pressure against the wall will be*

Solution : When soil is purely cohesive,

then $\phi = 0$

$$\therefore \quad K_A = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 0^\circ}{1+\sin 0^\circ} = 1$$

Given, $c = 9.81$ kN/m²

Base pressure at depth H,

$$P_A = \sigma_z \cdot K_A - 2c \cdot \sqrt{K_A}$$

where $\sigma_z = \gamma \cdot H = 17.66 \times 10 = 177.6$ kN/m²

$$\therefore \quad P_A = 177.6 \times 1 - 2 \times 9.81 \times \sqrt{1}$$

$$= 157.98 \text{ kN/m}^2$$

Example. *A vertical excavation was made in a clay deposit having weight of 19 kN/m³. It caved in after the depths of digging reached 5 m. Taking the angle of internal friction to be zero, compute the value of cohesion. If the same clay is used as a backfill against a retaining wall, upto a height of 10m, then find total active earth pressure and total passive earth pressure.*

(Assume that the wall yields far enough to allow Rankine deformation conditions to establish.)

Solution : In cohesive soil, the critical height of an unsupported vertical cuts,

$$H_c = \frac{4c}{\gamma}\tan\alpha$$

But
$$\tan\alpha = \tan\left(45^\circ + \frac{\phi}{2}\right) = \left(45^\circ + \frac{0}{2}\right) = 1$$

$$\therefore \quad c = \frac{\gamma H_c}{4} = \frac{19 \times 5}{4} = 23.75 \text{ kN/m}^2$$

(*i*) Total active earth pressure,

$$P_a = \left[\frac{1}{2}\gamma H^2 \cot^2\alpha\right] - (2\,cH\cot\alpha)$$

$$= \left[\frac{1}{2} \times 19 \times 10^2 \times 1\right] - (2 \times 23.75 \times 10 \times 1)$$

= 475 kN/m

(*ii*) Total passive earth pressure,

$$P_p = \left[\frac{1}{2}\gamma H^2 \tan^2\alpha\right] - [2c\,H\tan\alpha]$$

$$= \left[\frac{1}{2} \times 19 \times 10^2 \times 1\right] - [2 \times 23.75 \times 10 \times 1]$$

= 1425 kN/m

Relationship between Principal Stresses at Failure

The relationship between major principal stress and minimum principal stress during plastic equilibrium state in cohesive soils is given by Bell.

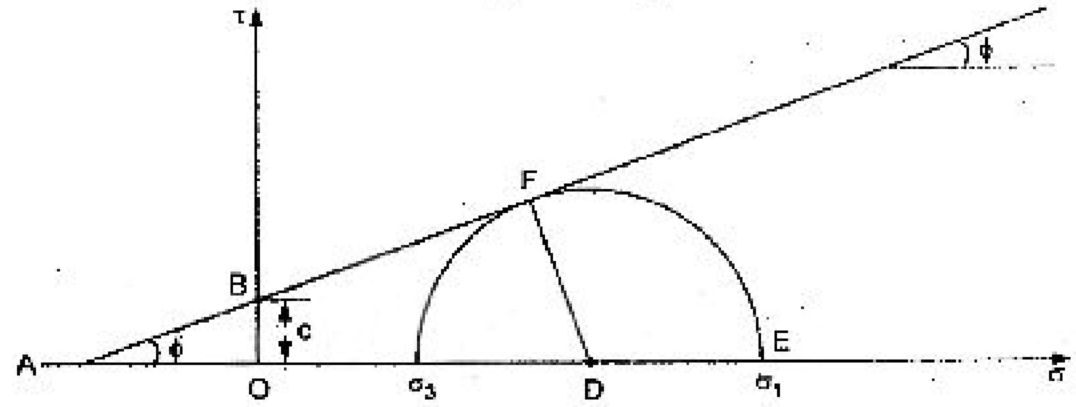


Fig. Mohr-Coulomb envelope (Bells solution)

Figure shows the Mohr circle drawn with principal stresses σ_1 and σ_3 and failure envelopes.

From the geometry of the Mohr circle it can be shown that

$$\frac{\sigma_1 - \sigma_3}{2} = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin\phi + c\cos\phi$$

or
$$\frac{\sigma_1}{2}(1-\sin\phi) = \frac{\sigma_3}{2}(1+\sin\phi) + c\cos\phi$$

or
$$\sigma_1 = \sigma_3\frac{(1+\sin\phi)}{(1-\sin\phi)} + 2c\frac{\cos\phi}{(1-\sin\phi)}$$

$$= \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\sigma_1 = \sigma_3\tan^2\alpha + 2c\tan\alpha$$

where
$$\alpha = \left(45 + \frac{\phi}{2}\right)$$

$$\therefore \quad \sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

This equation gives relationship between principal stresses in case of cohesive soils. For the cohesionless soils $c = 0$ and the strength envelope passes through the origin. Therefore, the above equation reduced to

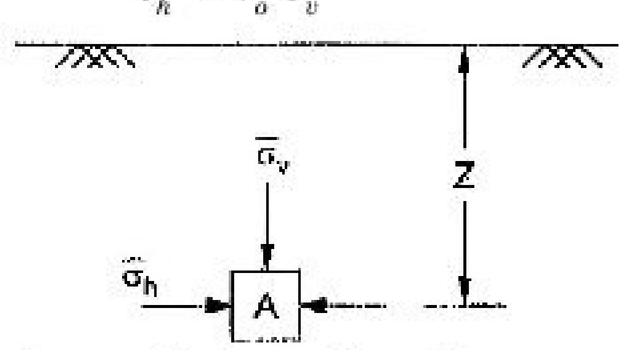
$$\sigma_1 = \sigma_3 N_\phi$$

which gives the relationship between principal stresses for cohesionless soils.

Earth Pressure at Rest

Consider element A of figure in a natural or artifically placed soil deposit that has not been subjected to lateral yielding. The effective vertical pressure, α_v is a function of depth of the element and specific weight of the material and the position of water table. The horizontal pressure, is related to vertical pressure by

$$\sigma_h = k_o\,\sigma_v$$



where k_o is the coefficient of earth pressure at rest.

The magnitude of k_o depends on

(*i*) the relative density of the soil for sands,

(*ii*) the over consolidation ratio of the soil for clays and

(*iii*) the process by which soil deposits was formed.

This k_o values were computed on the basis of Poisson's ratio and the assumption of zero lateral yielding during deposition.

$$k_o = \frac{\mu}{1-\mu}$$

where, μ = Poisson's ratio of soil,

Alphen has suggested the following equation for a normally consolidated clay.

$$k_o = 0.19 + 0.2333 \log \text{PI}$$

where, PI = Plasticity Index.

Active State. The stress condition within the soil mass during plastic equilibrium state is represented by

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

when the soil is in the active state of plastic equilibrium, minor principal stress is equal to horizontal intergranular stress and major principal stress is equal to vertical intergranular stress.

i.e., $\bar{\sigma}_h = \sigma_3$

and $\bar{\sigma}_v = \sigma_1$

$$\bar{\sigma}_v = \bar{\sigma}_h N_\phi + 2c\sqrt{N_\phi}$$

$$p_a = \bar{\sigma}_h$$

$$= \frac{\bar{\sigma}_v}{N_\phi} - 2c\frac{\sqrt{N_\phi}}{N_\phi}$$

$$= \frac{\bar{\sigma}_v}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}$$

For $c = 0$, $\bar{\sigma}_h = \dfrac{\bar{\sigma}_v}{N_\phi} = K_a\bar{\sigma}_v$

where active earth pressure coefficient.

$$K_a = \cot^2\left(45 + \frac{\phi}{2}\right) \text{ or } \left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

Passive State. When the soil is in the passive state of plastic equilibrium, minor principal, stress is equal to vertical intergranular stress and major principal stress is equal to horizontal intergranular stress, *i.e.*,

$$p_p = \bar{\sigma}_h = \sigma_1$$

and $\bar{\sigma}_v = \sigma_3$

$\therefore$ $p_p = \bar{\sigma}_h = \bar{\sigma}_v N_\phi + 2c\sqrt{N_\phi}$

For $c = 0$, $\bar{\sigma}_h = \bar{\sigma}_v N_\phi$

and $\dfrac{\bar{\sigma}_h}{\bar{\sigma}_v} = N_\phi = K\phi$

where passive earth pressure coefficient.

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right) \text{ or } \left(\frac{1+\sin\phi}{1-\sin\phi}\right)$$

Yield Condition :

The magnitude of lateral earth pressure depends upon magnitude of deformation (horizontal strain) of the retaining-structures. The yield conditions required to attain the active state are much less than those required to obtain the passive state. Lambe and Whitman suggested the relationship based on the analysis of results obtained from triaxial test on dense sand.

The following conclusions are drawn :

1. Very little horizontal strain, less than 0.5% is required to reach the full active pressure.
2. The horizontal strain about 0.5% is required to reach 50% of the maximum passive pressure.
3. More horizontal strain, about 2% is required to reach the full passive resistance. For loose sand, the first two conditions remain valid but the horizontal strain required, to reach full passive resistance is about 15%.

Example. *A 12 m retaining wall with a smooth vertical back retains a mass of moist cohesionless sand with a horizontal surface. The sand weighs 14 kN/m² and has an angle of internal friction equal to 32°. Then find total lateral earth pressure at rest.*

Solution : Since the unit weight of sand filled behind the wall is 14 kN/m³. it can be considered as loose sand; and the value of K_o for loose sand may be taken as 0.5.

Given, $\gamma = 14 \text{ kN/m}^3$.

$\therefore$ $\sigma_z = \gamma \cdot H = 14 \times 12 = 168 \text{ kN/m}^2$

Now $\sigma_h = K_o \sigma_z = 0.5 \times 168 = 84 \text{ kN/m}^2$

Total lateral pressure (thrust) at rest $= \dfrac{1}{2}\sigma_h \cdot H$

$$= \frac{1}{2} \times 84 \times 12 = 504 \text{ kN/m}$$

EFFECTIVE STRESS PRINCIPLE

Consider a prism of soil with a cross-sectional area A.

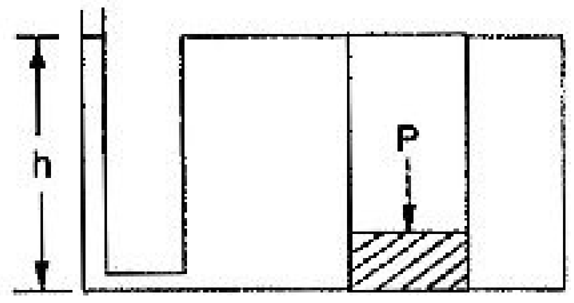


Fig. Saturated soil mass

Weight of the soil in the prism,

$$P = \gamma_{sat}\, h \cdot A$$

where h = height of the prism

γ_{sat} = saturated weight of the soil

Total stress on the base of the prism,

$$\bar{\sigma} = \text{force per unit area}$$

$$= \frac{P}{A} = \gamma_{sat} \cdot h$$

Pore Water Pressure (u)

It is the pressure due to pore water filling the voids of the soil and is given by

$$u = \gamma_w \cdot h$$

where γ_w = unit weight of water

Pore pressure is also called as *neutral stress* or *neutral pressure* because it cannot resist shear stresses.

Also effective stress at a point in the mass,

$$\bar{\sigma} = \sigma - u$$

For saturated soils, $\bar{\sigma} = \gamma_{sat} \cdot h - \gamma_w h$

$$= h(\gamma_{sat} - \gamma_w) = h \cdot \gamma'$$

where, γ' = submerged unit weight

Effective stress is an abstract quantity as it cannot be measured directly in the laboratory.

The effective stress controls the engineering properties of soils. Compression and shear strength of a soil are dependent on the effective stress,

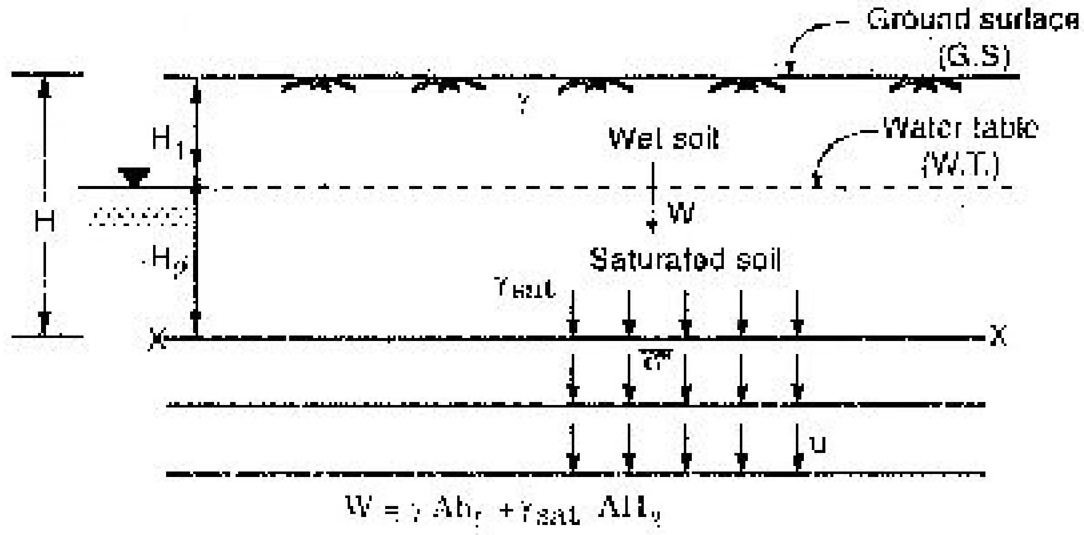
i.e., Compression $= f(\bar{\sigma})$

and shear strength $= \phi(\bar{\sigma})$

Where f and ϕ represent some functions.

As the effective stress in a soil increases, the compression of the soil occurs. This causes settlement of structures built in soils. The shear strength of soil depends upon its effective stress. As the effective stress is changed, the shear strength changes. The bearing capacity of soils, the stability of slopes and the earth pressures against retaining structure depend upon the shear strength of soil and hence, the effective stress.

Effect of Water Table Fluctuations on Effective Stress



Let depth of water table (W.T.) below the ground surface is H_1. The soil above the water table is assumed to be wet, with a bulk unit weight of y and the soil below the water table is saturated, with a saturated unit weight of γ_{sat}.

Total downward force at section

X–X, P = weight of the soil W

$$= \gamma H_1 A + \gamma_{sat} H_2 A$$

where A = area of cross-section of the soil mass

Also $\sigma = \gamma H_1 A + \gamma_{sat} H_2$

Pore water pressure, $u = \gamma_w H_2$

Effective stress, $\bar{\sigma} = \gamma H_1 + \gamma' H_2$

If the water table rises to the ground surface, the whole of the soil is saturated, then

$$\bar{\sigma} = \gamma' H$$

where γ' = submerged unit weight

As $\gamma' = \gamma,$

the effective stress is reduced due to rise of water table.

If the water table is depressed below the section X-X, then $\bar{\sigma} = \gamma H$

If the water table is depressed below the section X-X, then the effective stress increases.

From this, it is observed that the fluctuation in water table level cause changes in the pore water pressure with corresponding changes in the effective stress.

Effective stress in a soil mass under hydrostatic conditions

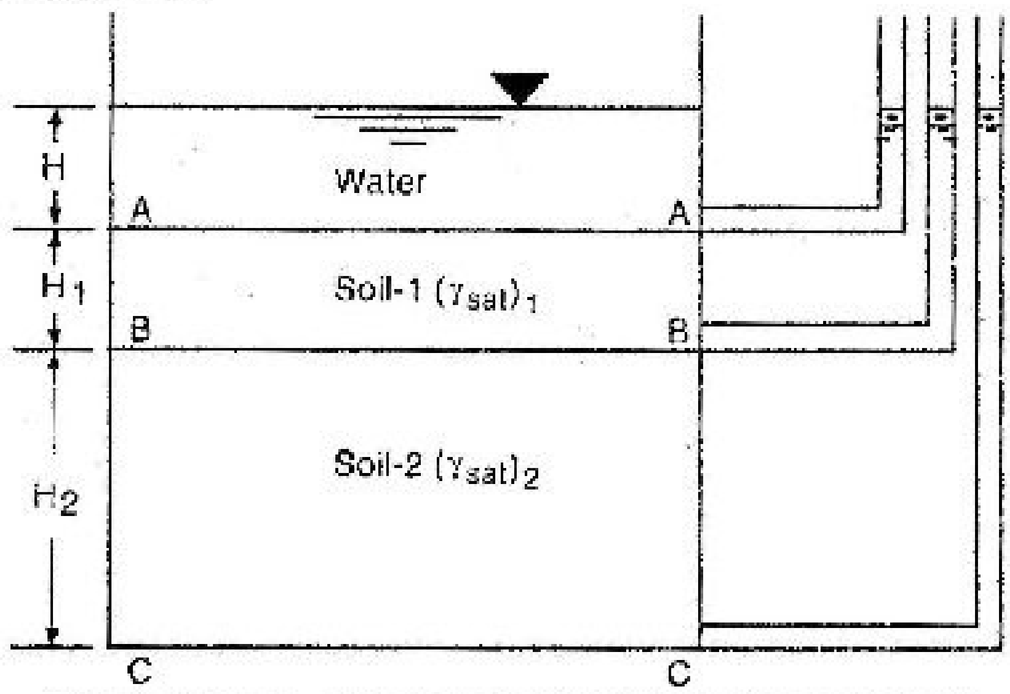


Fig. Soil mass, when water level remains constant

(*i*) Water table is above Soil Surface

At section A-A :

$$\bar{\sigma} = \sigma - u = \gamma_w \cdot H - \gamma_w \cdot H = 0$$

At section B-B :

$$\bar{\sigma} = \left[\gamma_w H + (\gamma_{sat})_1 H_1\right] - \gamma_w (H + H_1) = \gamma_1' \cdot H$$

where $\gamma_1' = (\gamma_{sat})_1 - \gamma_w$ = submerged unit weight of soil 1.

At section C-C :

$$\bar{\sigma} = \gamma_1' \cdot H_1 - \gamma_2' \cdot H_2$$

where γ_2' = submerged unit weight of soil 2.

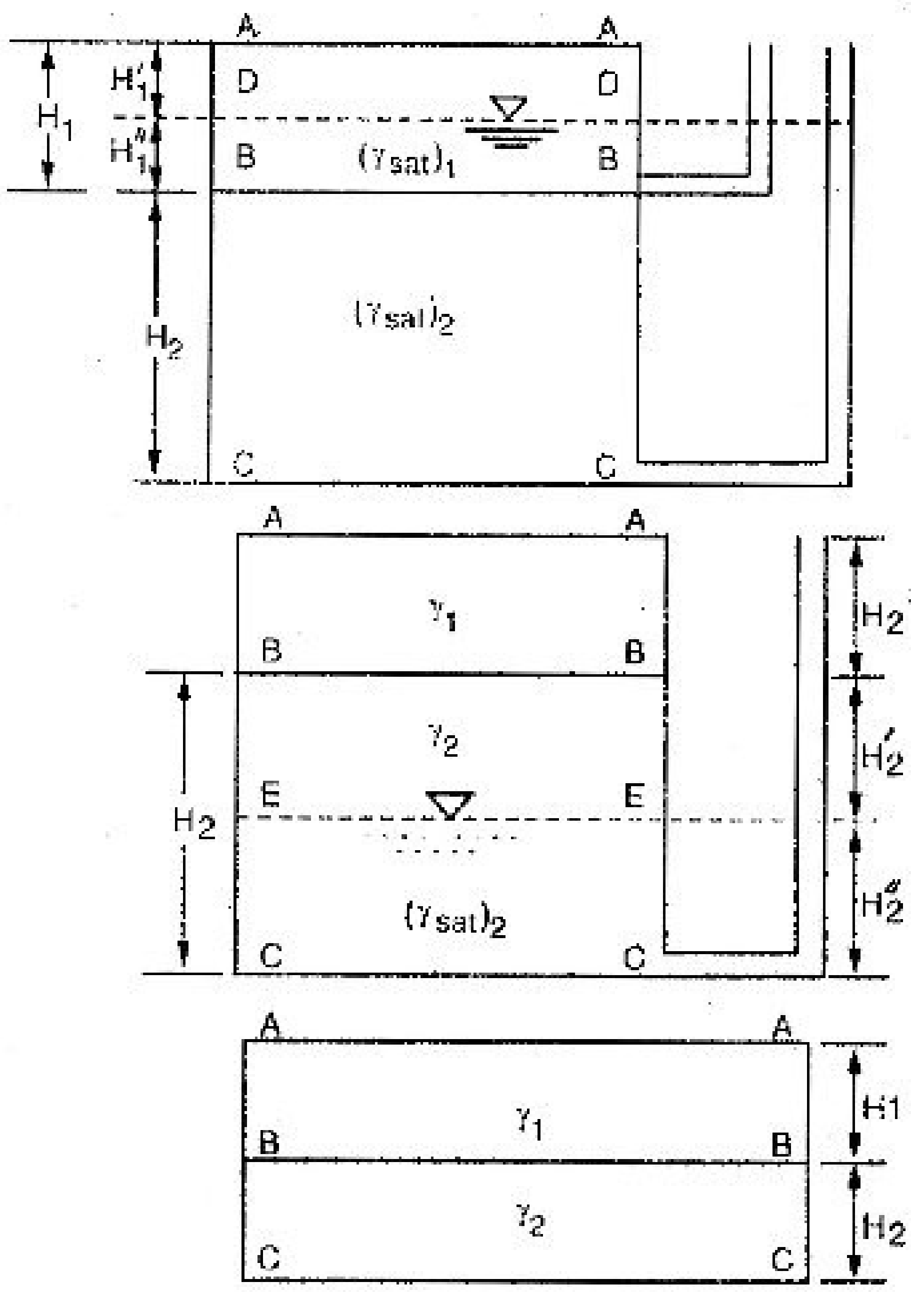


Fig. Water table below soil

(ii) Water table is below the soil surface

At section A-A :

$$\sigma = u = \bar{\sigma} = 0$$

At section B-B :

$$\bar{\sigma} = (\gamma_{sat})H_1 - \gamma_w H_1 = \gamma_1' H_1$$

At section C-C :

$$\bar{\sigma} = \gamma_1' \cdot H_1 + \gamma_2' H_2$$

All the above expressions shows that the depth (H) of water above the soil surface does not contribute to the effective stress at all, hence the effective stress in the soil mass is independent of the depth of water table above the soil surface.

(iii) Water table in soil-1

At section A-A :

$$\sigma = u = \bar{\sigma} = 0$$

At section D-D :

$$\bar{\sigma} = \gamma_1 H_1' \text{ as } u = 0$$

where γ_1 = unit weight of soil above D-D

At section B-B :

$$\bar{\sigma} = \left[\gamma_1 H_1' + (\gamma_{sat})_1 H_1''\right] - \gamma_w H_1''$$

$$= \gamma_1 H_1' + \gamma_1 H_1''$$

At section C-C :

$$\bar{\sigma} = \gamma_1 H_1' + \gamma_1' H_1'' + \gamma_2' H_2$$

(iv) Water table in soil 2

At section A-A :

$$\sigma = u = \bar{\sigma} = 0$$

At section B-B :

$$\bar{\sigma} = \gamma_1 H_1 \text{ .... as } u = 0$$

At section E-E :

$$\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2' \text{..... as } u = 0$$

At section C-C :

$$\bar{\sigma} = \left[\gamma_1 H_1 + \gamma_2 H_2' + (\gamma_{sat})_2 H_2''\right] - \left(\gamma_2 H_2''\right)$$

$$= \gamma_1 H_1 + \gamma_2 H_2' + \gamma_2 H_2''$$

(v) Water below table C-C : As the pore water pressure is zero everywhere, the effective stresses are equal to the total stresses.

At sectin B-B :

$$\sigma = \bar{\sigma} = \gamma_1 H_1$$

At section C-C :

$$\sigma = \bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2$$

Note :

(i) The effective stresses in a soil mass can be determined from the basic definitions, without memorising any formula.

(ii) The effective stress at any section goes on increasing as the water table goes down.

(iii) The effective stress depends upon the bulk unit weight above the water table and the submerged unit weight below the water level.

Increase in effective stresses due to surcharge

At section A-A :

$$\bar{\sigma} = q \text{ .... as } u = 0$$

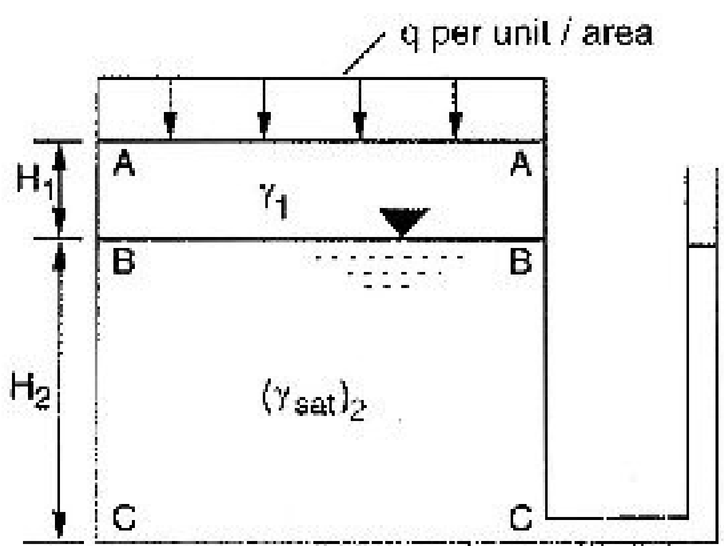
At section B-B :

$$\bar{\sigma} = q + \gamma_1 H_1 \text{ ...... as } u = 0$$

At section C-C :

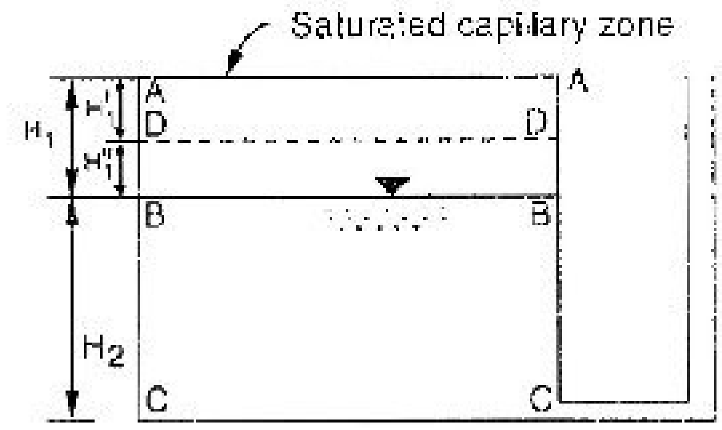
$$\bar{\sigma} = \left[q + \gamma_1 H_1 + (\gamma_{sat})_2 H_2\right] - \left[\gamma_2 H_2\right]$$

$$= q + \gamma_1 H_1 + \gamma_2 H_2$$



Effective stresses in soils saturated by capillary action

(*i*) Soil saturated upto surface level A-A



At section A-A :

$$\bar{\sigma} = 0-(-\gamma_w H_1) = \gamma_w H_1$$

When the soil was not saturated with capillary action, the effective stress at section A-A would have been zero. Thus, the capillary action has increased the effective struess by γ_w H_1

At section D-D :

$$\bar{\sigma} = \gamma_1' H_1' + \gamma_w H_1$$

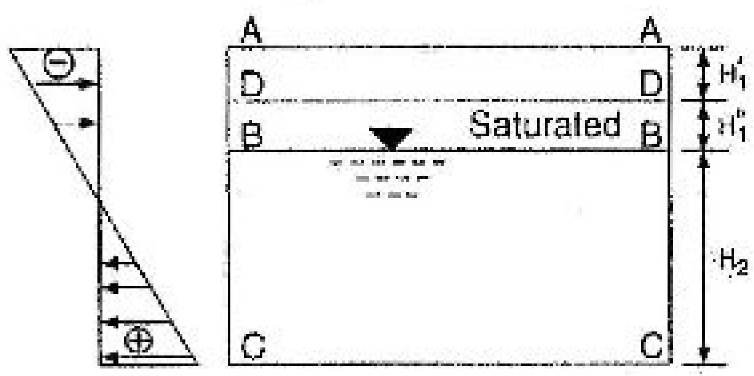
At section B-B :

$$\bar{\sigma} = (\gamma_{sat}), H_1 \text{...... as } u = 0$$

At section C-C :

$$\bar{\sigma} = (\gamma_{sat}), H_1 + \gamma_2' H_2$$

(*ii*) Soil saturated upto level D-D



At section A-A :

$$\sigma = u = \bar{\sigma} = 0$$

i.e., there is no effect of capillary water

At section D-D :

$$\bar{\sigma} = \gamma_1 H_1' - [-\gamma_w H_1'']$$

$$= \gamma_1 H_1 + \gamma_w H_1''$$

The effective stress due to capillary pressure is increased by γ_w H_1

At section B-B :

$$\bar{\sigma} = (\gamma_1 H_1' + \gamma_1 H_1') + \gamma_w H_1'' \text{ ...... as } u = 0$$

Note :

(*i*) If the soil is saturated due to rise in water table, the effective stress depends upon the submerged unit weight; whereas for the soil saturated with capillary water, the effective stress depends upon the saturated unit weight.

(*ii*) Equation $\bar{\sigma} = \sigma - u$, is applicable in all cases. However, it should be remembered that the pore water pressure in the capillary zone is negative.

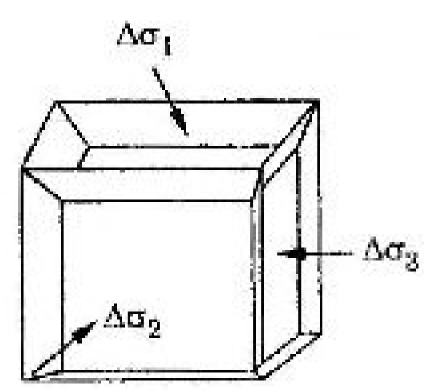
(*iii*) The capillary water above the water table causes a negative pressure γ_w H, where H = capillary rise. This negative pressure causes an increase in the effective stresses at all levels below the saturation level. The increase is equal to γ_w H. The capillary action is equivalent to a surcharge, $q = \gamma_w$ H.

(*iv*) If the water table rises to the top of the soil surface, the meniscus is destroyed and the capillary water changes to the free water and the effective stress is reduced throughout.

PORE WATER PRESSURE PARAMETERS

Skempton's pore pressure parameters : The change in pore pressure due to change in the applied stress, during an undrained shear, may be explained in terms of empirical coefficients, called a *pore pressure parameter*.

Consider a soil mass or skeleton subjected to increase in the three principal stresses $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\sigma_3$ resulting in a volume decrease ΔV, and a consequent increase in pore pressure of Δu. The increase in the effective stresses will be



$$\Delta \sigma_1' = \Delta \sigma_1 - \Delta u$$

$$\Delta \sigma_2' = \Delta \sigma_2 - \Delta u$$

and $$\Delta \sigma_3' = \Delta \sigma_3 - \Delta u$$

Let e_1, e_2 and e_3 denote the strains in the three directions, then

$$Ee_1 = \Delta \sigma_1' - \mu(\Delta \sigma_2' + \Delta \sigma_3')$$

$$Ee_2 = \Delta \sigma_2' - \mu(\Delta \sigma_3' + \Delta \sigma_1')$$

and $$Ee_3 = \Delta \sigma_3' - \mu(\Delta \sigma_1' + \Delta \sigma_2')$$

$$\therefore \quad E(e_1 + e_2 + e_3) = E\frac{\Delta V}{V}$$

$$= (1-2\mu)(\Delta \sigma_1' + \Delta \sigma_2 + \Delta \sigma_3')$$

or $$\frac{\Delta V}{V} = \frac{3(1-2\mu)}{E}\frac{1}{3}(\Delta \sigma_1' + \Delta \sigma_2 + \Delta \sigma_3')$$

Putting $3\left(\frac{1-2\mu}{E}\right)$ = Cc = Compressibility of the soil skeleton, we get

$$\frac{\Delta V}{V} = Cc\left\{\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta u\right\}$$

If 'n' is the porosity, the volume of voids or the volume of pore fluids equals nV, the coefficient of volume compressibility is C_v, change of volume in the pore fluid ΔV_w due to the increase in the pore pressure Δu under the condition of no drainage is given by

$$\Delta V_w = nVC_v \cdot \Delta u$$

The decrease in the volume of the soil skeleton is almost entirely due to the decrease in volume of the voids.

As the change in volume of the soil mass is equal to the reduction in volume of voids

$$\therefore \quad \frac{Cc}{3}(\Delta\sigma_d + 3\Delta u) = C_v\,(u\ V)\,\Delta u$$

$$\Delta u = \left(\frac{C_c}{nC_v + C_c}\right)\left(\frac{\Delta\sigma_d}{3}\right)$$

$$= \frac{1}{1+\frac{nC_v}{C_c}} \times \left(\frac{1}{3}\right)(\Delta\sigma_d)$$

Because a soil is not perfectly elastic, the constant $\left(\frac{1}{3}\right)$ is replaced by A in the above expression.

Thus, $$\Delta u = \left(\frac{A}{1+\frac{nC_v}{C_c}}\right)(\Delta\sigma_d)$$

$$\therefore \quad \Delta u = AB\,\Delta\sigma_d$$

where, $$B = \frac{1}{1+\frac{nC_v}{C_c}}$$

For fully saturated soil, **B = 1.**

$$\therefore \quad \Delta u = B\left[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

Example. *A foundation trench is to be excavated in a stratum of still clay 10 m thick underlain by a bed of sand. In a trial bore hole, the ground water is observed to rise to an elevation of 3.5 m below the surface of ground.*

(i) Calculate the depth to which an excavation can be safely carried out without the danger of the bottom becoming unstable under uplift pressure of ground water. The specific gravity of clay particles is 2.75 and the void ratio 0.83.

(ii) If the excavation is to be carried out safely to a depth of 8m, how much should the water be lowered in the vicinity of the trench.

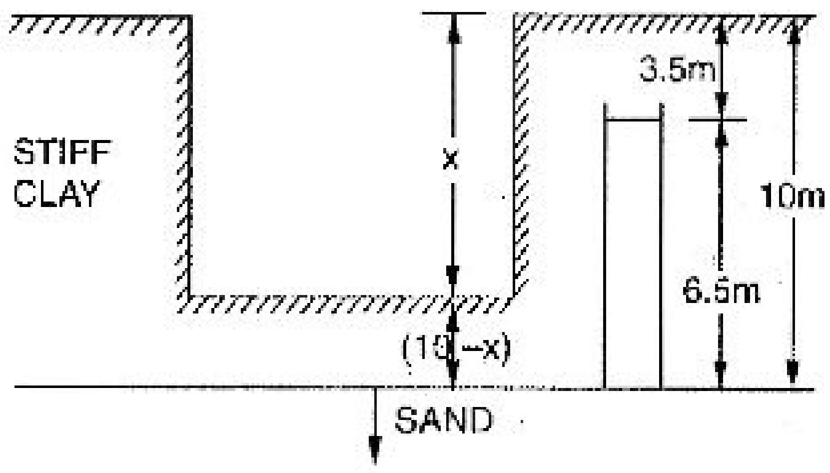
Solution : $$\gamma = \frac{(G + S_r e)}{1+e}\gamma_w$$

$$= \frac{2.75 \times 1}{1+0.83} \quad (\because S = 0)$$

$= 1.50$ t/m³ (taken as zero saturation)

(i) $$\sigma = 1.50\,(10 - x) = (15 - 1.5\,x)\ \text{t/m}^2$$

$$u = 6.5 \times 1 = 6.5\ \text{t/m}^2$$



Effective stress $\bar{\sigma} = \sigma - u$

where σ = total stress and

$$\therefore \quad \bar{\sigma} = (15 - 1.5x) - 6.5 = 0$$

or $$1.5x = 15 - 6.5$$

or $$x = \mathbf{5.67\ m}$$

(ii) $$\sigma = 1.5\,(x - 8) = 1.5\,x - 8 \times 1.5\ \text{t/m}^2$$

$$u = (x - 3.5) \times 1 = (x - 3.5)\ \text{t/m}^2$$

$$\bar{\sigma} = \sigma - u = (1.5x - 8\times 1.5) - (x - 3.5) = 0$$

or $$0.5x = 8.5$$

or $$x = \mathbf{17\ m}$$

STRESS DISTRIBUTION IN SOILS

Stresses are imposed on the soil by the weight of the overburden or by structural load. The stresses due to overburden increases with depth but the stresses due to structural load decreases with depth and distance from the point of application of the load. The stress distribution depends on physical properties of soil, rigidity of foundation, shape of foundation and type of loading.

Boussincsq Theory

Boussinesq (1855) developed a theory to calculate stresses and deformation due to point load applied at the surface of semi-infinite soil mass. The following assumptions are made :

1. The soil mass is elastic, homogeneous, isotropic and semi-infinite and obeys Hooke's law.
2. The soil is weightless.
3. The soil is unstressed before the application of load.

4. The stress distribution due to applied load is independent of the type of soil.
5. The law of linear stress distribution is valid.
6. There exists continuity of stress.
7. The stresses are symmetrical with respect to the vertical axis.

According to Boussinesq's theory, the vertical stress at any point in the semi-infinite medium due to various loads is given by the following equations :

(a) Point load (Q) on the surface

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(r/z\right)^2}\right]^{5/2}$$

where
$$K = \frac{3}{2\pi}\left[\frac{1}{1+\left(r/z\right)^2}\right]^{5/2}$$

is the Boussinesq vertical stress coefficient.

Q = magnitude of concentrated load;

z = depth;

r = radial distance

(b) Vertical pressure under a uniformly loaded circular area

$$\sigma_z = q\left\{1-\left[\frac{1}{1+\left(R/z\right)^2}\right]\right\}$$

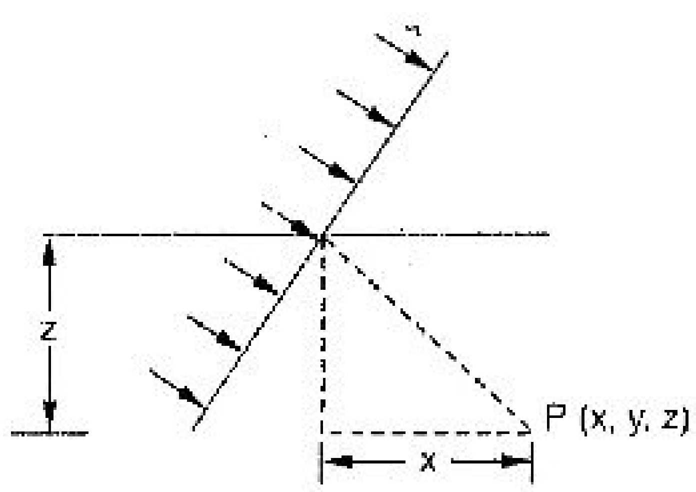
where q = intensity of load;

R = radius of foundation;

z = depth

(c) Line load

$$\sigma_z = \frac{2q'}{\pi z}\frac{1}{\left[1+\left(x/2\right)^2\right]^2}$$



where q' = line load

x = horizontal distance

z = depth

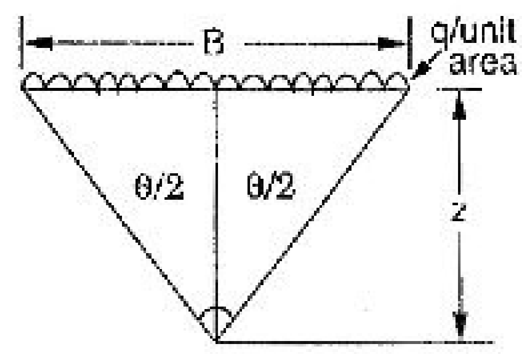
(d) Strip load

$$\sigma_z = \frac{q}{\pi}(\theta+\sin\theta)$$

where q = uniformly distributed load intensiry per unit area

B = width of the strip

(e) Circular Area



$$\sigma_z = \frac{3}{2\pi}Q\cdot z^3\left[\frac{1}{\left(r^2+z^2\right)^{5/2}}\right]$$

$$= q\left[1-\left\{\frac{1}{1+\frac{R^2}{z^2}}\right\}^{3/2}\right]$$

where (r, z) = co-ordinates of the point

R = radius of the circular area, and

q = uniform pressure on the area

Boussinesq Applied Equations (methods) for Uniform Load on Rectangular or Square Areas.

In actual field problems, the load is never applied at a point, but is applied through footings of certain finite sizes (generally rectangular or square). The total load is thus spread over the area of the footing. It is, therefore, necessary to determine the stress distribution in soils beneath such loadings.

There are three methods of solving this problem :

(i) Equivalent point load method : Vertical stress at a depth z due to uniformly distributed load on any area of known geometry can be estimated by equivalent point load method. The accuracy of the results computed by this technique depends upon the area chosen. Smaller the area chosen, the error involved in the result is negligible.

The given agea is divided into number of smaller areas such that the largest dimension of this unit becomes equal or less than 0.3z, where z is the depth at which the stress is required and the distributed load over this area is replaced by a point load of equal magnitude acting at the centroid of the area chosen. Like this the distributed load on entire area is replaced by point load at the centroid of each area unit. The vertical stress at any point due to the uniformly disturbted load is given by algebraic sum of stresses due to point load acting at the centroid of various area units.

$$\sigma_z = \frac{1}{z^2}[Q_1K_1 + Q_2K_2 + Q_3K_3 + ...]$$

If all the points load are of equal magnitude Q, then

$$\sigma_z = \frac{Q'}{z^2}\Sigma K$$

where K_1, K_2 and K_3 ..... are the individual Boussinesq vertical stress influence factor for various area units.

Q_1, Q_2 and Q_3 ..... are the intensity of load acting on various area units.

ΣK = sum of the individual influence factors for the various area unit.

(ii) **Using Integral form of Boussinesq Equation for Specific Shapes :** In this method, a chart is used in which values of influence factor (I_R) for different values of log *m* (for a particular value of (*n*) are plotted. From the value of I_R, the value of σ_z can be calculated.

(iii) **Using Newmark's Influence charts :** Newmarks chart is a graphical representation of Boussinesq's equation. It is a more accurate method of determining the vertical stresses under uniformly distributed loaded area of any shape. This chart is so developed that each area on the chart when loaded with a uniform stress, will produce the same increment of vertical stress at a depth z beneath the centre of this diagram. To use the influence chart, a plan of the uniformly loaded area is drawn on a tracing sheet to a scale such that the depth z at which stress is to be determined equals the length marked on the influence chart. The drawing sheet is then placed on the Newmarks chart with the point where stresses are to be found located over the centre of the influence chart. Count the number of squares (n) covered by the plan diagram. Then

Total intensity of stress at point,

$$\Delta\sigma_z = q\left[(i_f)N_1 + \left(\frac{i_f}{2}\right)N_2 + \left(\frac{i_f}{3}\right)N_3\right]$$

where N_1 = Number of elements fully covered

N_2 = Number of elements half covered

N_3 = Number of elements of which one third is covered

i_f = influence factor

$$= \frac{1}{\text{number of elements of the chart}}$$

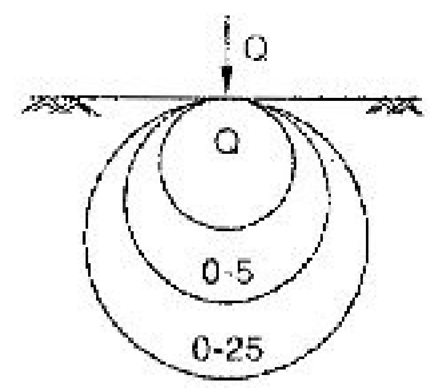
q = intensity of loading

Pressure Distribution Diagrams

By using the Boussinesq vertical stress distribution equation, the following stress distribution diagram can be developed :

1. Pressure bulb (stress isobar)
2. Vertical stress distribution on horizontal plane
3. Vertical stress distribution on vertical plane

Stress Isobars



An isobar or pressure bulb is a line which connects the points of equal intensity of stresses. For any one load system many isobars corresponding to various intensity of vertical pressure can be drawn on one diagram as shown in the figure.

The zone in a loaded soil mass bounded by an isobar of given vertical pressure is called as pressure zone.

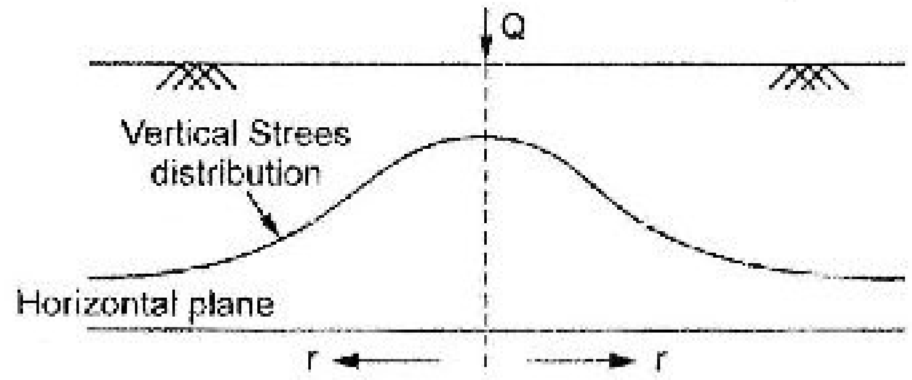
Vertical Pressure on Horizontal Plane

The vertical stress distribution on a horizontal plane at a depth z due to concentrated load Q can be determined by

$$\sigma_z = K\frac{Q}{z^2}$$

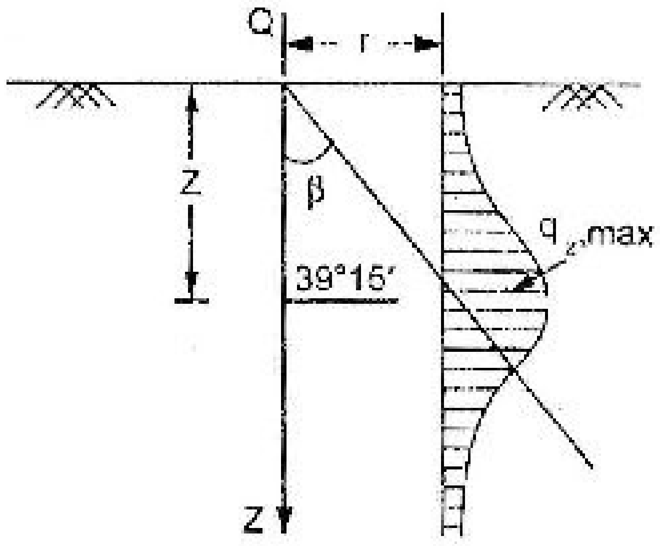
where, K = Boussinesq influence factor which can be found by varying the horizontal distance *r*.

The vertical stress distribution diagram on the horizontal plane is bell shaped i.e., the stress distribution is non-uniform and that the maximum stress is on the line of action of load. The stress decreases with the increase in value of horizontal distance *r*.



Vertical Stress Distribution along a Vertical Plane

Figure below shows the variation of vertical stress with depth z on a vertical plane at constant distance r from the axis of the load. The variation of σ_z can be computed from the relation given by,



$$\sigma_z = K\frac{Q}{z^2}$$

In which for the given radial distance the factor K is constant.

Westergaard's Theory

Westergaard (1938) developed a solution to determine distribution of stress due to point load in soils composed of thin layer of granular material that partially prevent lateral deformation of the soil. He made the following assumptions;

1. The soil is elastic and semi-infinite.
2. Soil is composed of numerous closely spaced horizontal layers of negligible thickness of an infinite rigid material.
3. The rigid material permits only the downward deformation of mass in which horizontal deformation is zero.

Point Load : $$\sigma_z = \frac{Q}{2\pi z^2}\frac{\sqrt{\left(\frac{1-2\mu}{2-2\mu}\right)}}{\left[\left(\frac{1-2\mu}{2-2\mu}\right)+\left(\frac{r}{cz}\right)^2\right]^{3/2}}$$

where μ is Poisson's Ratio

where $$e = \frac{\sqrt{(1-2\mu)}}{(2-2\mu)}$$

When $\mu = 0$, then $$\sigma_z = \frac{Q}{\pi z^2}\frac{1}{\left[1+2\left(\frac{r}{2}\right)^2\right]^{3/2}} = \frac{Q}{z^2}K$$

where K = Westergaard's influence coefficient

$$= \frac{1}{\pi\left[1+2\left(\frac{r}{2}\right)^2\right]^{3/2}}$$

Note : The Westergaard's influence coefficient is $\frac{2}{3}$ of Boussinesq values for small values of $\left(\frac{r}{2}\right)$. But for $\frac{r}{2}$ more than 2.0, Westergaard's values are slightly greater.

The stress σ_z for rectangular uniform pressure has been worked out at a corner of a rectangle with sixes mz and nz as

$$\sigma_z = \frac{q}{2\pi}\cot^{-1}\left(\sqrt{\frac{1-2\mu}{2-3\mu}\left(\frac{1}{m^2}+\frac{1}{n^2}\right)+\left(\frac{1-2\mu}{2-2\mu}\right)^2\frac{1}{m^2n^2}}\right)$$

When $\mu = 0$, then

$$\sigma_z = \frac{q}{2\pi}\cot^{-1}\left(\sqrt{\frac{1}{2m^2}+\frac{1}{2n^2}+\frac{1}{4m^2n}}\right)$$

Vertical pressure under uniformly loaded circular area is given by

$$\sigma_z = q\left[1-\left\{\frac{1}{1+\left(\frac{R}{cz}\right)^2}\right\}^{1/2}\right]$$

where q = intensity of load

R = radius of circular footing

z = depth below surface of ground

$$c = \sqrt{\frac{1-2\mu}{2(1-\mu)}}$$

SHEAR STRENGTH

Shear Strength of Soil. It is the resistance to deformation by continuous shear displacement of soil particles on masses upon the action of a shear stress. The failure conditions for a soil may be expressed in terms of limiting shear stress, called *shear strength* or as a function of the principal stresses.

The shearing resistance of a soil is constituted basically of the following compounds :

(*i*) The structural resistance to displacement of the soil because of the interlocking of the particles.

(*ii*) The frictional resistance to translocation between the individual soil particles at their contact point, and

(*iii*) Cohesion or adhesion between the surfaces of the soil particles.

The shear strength in cohesionless soil results from intergrangular friction alone, while in all other soils it results both from internal friction as well as cohesion.

Mohr-Coulomb's Failure Theory

If at a point on any plane within a solid mass, the shear stress becomes equal to the shear strength of the soil, failure will occur at that point. The shear strength (T) of a soil at a point on a particular plane was expressed by Coulomb as a linear function of the normal stress (a) on the plane at that point.

$$\tau = c + \alpha \tan \phi \qquad ......(i)$$

where c = apparant cohesion

ϕ = angle of shearing resistance.

In accordance with Terzaghi's fundamental concept, the shear stress in a soil can be resisted only by the skeleton of solid particles. Shear strength is expressed as a function of effective normal stress

$$\tau = c + \sigma' \tan \phi' \qquad ...... (ii)$$

where c = effective cohesion,

ϕ' = effective angle of internal friction,

σ' = effective stress = $(\sigma - \mu)$

The Mohr-Coulomb theory shows a reasonably good agreement with the observed failures in the field and in the laboratory.

Limitations of Mohr-Coulomb Theory :

(*i*) For some clayey soils, there is no fixed relationship between the normal and shear stresses on the plane of failure. For such type of soils, this theory cannot be used.

(*ii*) It neglects the effect of the intermediate principal stress.

(*iii*) It approximates the curved failure envelop by a straight line, which may can give correct results.

(*iv*) When the Mohr envelope is curved the actual obliquity of the failure plane is slightly smaller than the maximum obliquity. Therefore, the angle of failure plane, as found, is not correct.

Again, in terms of total stress,

$$\tau_f = c_u + \sigma \tan \phi$$

where, c_u= apparent cohesion,

ϕ_u= apparent angle of shearing resistance

Now, normal stress σ' and shear stress τ on any plane inclined at angle α to the major principal plane, can be expressed in terms of effective major principal stress σ_1' and effective minor principal stress σ_3' as

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\alpha$$

and
$$\tau = \frac{\sigma_1' - \sigma_3'}{2}\sin 2\alpha$$

These equations for the normal and shear stress developed on a given particular plane can be represented graphically by a figure given below called Mohr stress circle.

Substituting the value of σ' in equation (*ii*), we get

$$\tau = c' + \tan\phi'\left[\frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' + \sigma_3'}{2}\cos 2\alpha\right]$$

To locate the most stressed plane, determine the minimum difference ($\tau_f - \tau$) between the shear strength and shear stress.

$$(\tau_f - \tau) = c' + \frac{\sigma_1' + \sigma_3'}{2}\tan\phi' + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\alpha \cdot \tan\phi' - \frac{\sigma_1' - \sigma_3'}{2}\sin 2\alpha$$

Differentiating with respect to α, we get

$$\frac{d}{d\alpha}\left(\overline{\tau}_f - \tau\right) = -\left(\sigma_1' - \sigma_3'\right)\sin 2\alpha \cdot \tan\phi' - \left(\sigma_1' - \sigma_3'\right)\cos 2\alpha$$

For minimum $(\tau_f - \tau)$,

$$\frac{d}{d\alpha}\left(\tau_f - \tau\right) = 0$$

or $$\cos 2\alpha = -\sin 2\alpha \tan\phi'$$

or $$\cot 2\alpha = -\tan\phi' = \cot(90° + \phi')$$

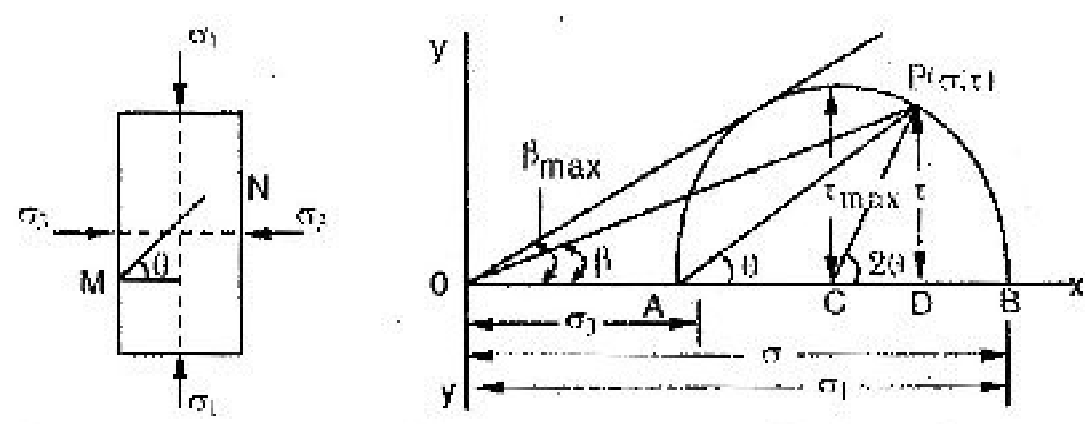
or $$2\alpha = 90° + \phi'$$

or $$\alpha = 45° + \frac{\phi'}{2}$$

Mohr's Circle

Through every point within a mass subjected to a three dimensional stress system, three mutually perpendicular planes can be drawn, which experience only normal stress free from shear or tangential stress. These planes are called the principal planes and their normal stresses are called principal stress. In case of soil, only two principal planes passing through any point of the stressed mass are considered. According to the magnitude of normal stress, these are termed as major principal plane and minor principal plane.

Consider an element which is subjected to two dimensional stress system shown in the figure below.



Axis XX represents the major principal plane, having normal stress σ_1.

Axis YY represents the minor principal plane, having normal stress σ_3.

Let the given plane MN pass through the point of intersection of the normal stress σ_1 and σ_3 making an angle θ to the major principal plane. Analytically it can be proved that normal stress σ and shear stress (τ) on the inclined plane MN are :

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

and $$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$$

Mohr gave a graphical solution for the determination of the normal and shear stresses on the given inclined plane as explained below.

He conveniently assumed compressive normal stresss as positive and tensile normal stresses as negative. The complete solution may be obtained from the following steps.

1. Along X-axis, plot the normal stress σ_1 = OB and σ_3 = OA (compressive stress positive and tensile stress negative).
2. Divide AB at C.
3. Draw a semicircle, having its centre at C and radius equal to AC or BC.
4. Through A, draw AP parallel to the inclined plane i.e. making angle PAB = 0.
5. Now coordinate of P is the normal stress and Y-coordinate of P is the shear stress on the given plane.

Proof : Join P and C and drop a perpendicular PD to AB,

From ΔPCD, we get

$$\angle PCD = 20$$

Normal stress, σ = OD

$$= OA + AC = CD$$

$$= \sigma_3 + \frac{\sigma_1 - \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

or $$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

Shear stress, τ = PD

$$= PC \sin 2\theta$$

or $$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$$

Resultant of normal stress and shear stress on the given plane.

(*i*) Joint O and P so that OP is the resultant of the stresses σ and τ

i.e. resultant stress = $\sqrt{\sigma^2 + \tau^2}$.

The angle between the resultant OP and X-axis is called the angle of obliquity denoted by β,

i.e., $$\tan \beta = \frac{\tau}{\sigma}$$

(*ii*) To get maximum angle of obliquity β_{max}, draw a tangent to the circle from the origin of stress coordinates.

Note :

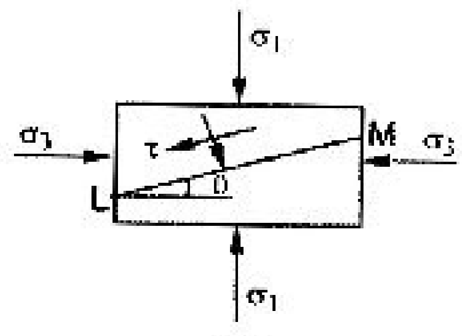
(*i*) Maximum shear stress,

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

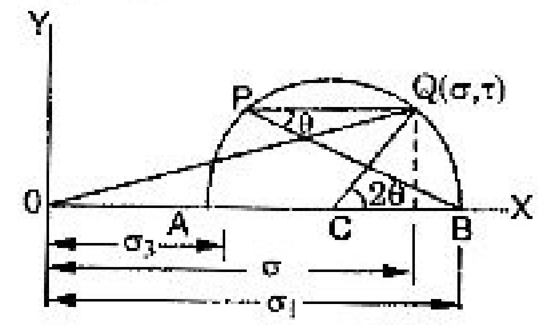
when $\theta = 45°$

(*ii*) Stress on the plane of maximum obliquity, is always less than the maximum shear stress, τ.

Drawing Mohr's circle when major principal plane is not parallel to X-axis.



(*a*)



(*b*)

Let the principal palnes be oriented as shown in figure (*a*).

LM be the given inclined plane on which stresses are required.

Construction :

1. Plot OB = σ_1 on X-axis.
2. Plot OA = σ_3 on X-axis.
3. Draw a semicircle with AB as diameter.
4. Draw BP parallel to the major principal plane, where P lies on the semicircle.
5. Join PA which represents the minor principal plane.
6. Draw PQ parallel to the given inclined plane LM where Q lies on the semicircle.
7. Now, X co-ordinate of Q is the required normal stress (σ) and Y-co-ordinate of Q is the required shear stress (τ) on the inclined plane LM.
8. To obtain the resultant of σ and τ, join OQ.

Shear Strength of Soils

1. The shearing resistance of soil is due to

   (***i***) **Cohesion :** The property of the soil holding its soil particle together, is called *cohesion.* The soil which does not possess any cohesion, is called *cohesionless soil,* e.g. dry sand.

   The soil which possesses cohesion is called *cohesive soil,* e.g. clays. The cohesion in an important property of fine grained soils. Coarse grained soil do not exhibit any cohesion.