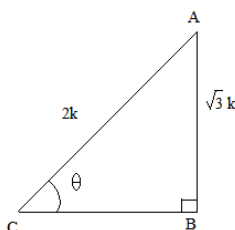**1.** If $\sin \theta = \dfrac{\sqrt{3}}{2}$, find the value of all T- ratios of θ

Sol:

Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$

Now, we know that $\sin \theta = \dfrac{Prependicular}{hypotenuse} = \dfrac{AB}{AC} = \dfrac{\sqrt{3}}{2}$

So, if $AB = \sqrt{3}k, then\ AC = 2k, where\ k\ is\ a\ positive\ number.$

Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2 = (2k)^2 - \left(\sqrt{3}k\right)^2$

$\Rightarrow BC^2 = 4k^2 - 3k^2 = k^2$

$\Rightarrow BC = k$

Now, finding the other T-rations using their definitions, we get:

$Cos\ \theta = \dfrac{BC}{AC} = \dfrac{k}{2k} = \dfrac{1}{2}$

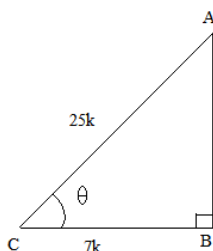$Tan\ \theta = \dfrac{AB}{BC} = \dfrac{\sqrt{3}k}{k} = \sqrt{3}$

$\therefore \cot \theta = \dfrac{1}{\tan \theta} = \dfrac{1}{\sqrt{3}}, cosec\ \theta = \dfrac{1}{\sin \theta} = \dfrac{2}{\sqrt{3}}\ and \sec \theta = \dfrac{1}{\cos \theta} = 2$

2. If $\cos \theta = \dfrac{7}{25}$, find the value of all T-ratios of θ

Sol:

Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$.

Now, we know that $\cos \theta = \dfrac{Base}{hypotenuse} = \dfrac{BC}{AC} = \dfrac{7}{25}$

So, if BC = 7k, then AC = 25k, were k is a positive number.

Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2 = (25k)^2 - (7k)^2$

$\Rightarrow AB^2 = 625k^2 - 49k^2 = 576k^2$

$\Rightarrow AB = 24k$

Now, finding the trigonometric ratios using their definitions, we get:

Sin $\theta = \dfrac{AB}{AC} = \dfrac{24k}{25k} = \dfrac{24}{25}$

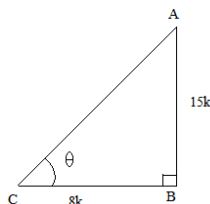Tan $\theta = \dfrac{AB}{BC} = \dfrac{24k}{7k} = \dfrac{24}{7}$

$\therefore \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{7}{24}, cosec\,\theta = \dfrac{1}{\sin\theta} = \dfrac{25}{24} \, and \, sec\theta = \dfrac{1}{\cos\theta} = \dfrac{25}{7}$

3.   If $\tan\theta = \dfrac{15}{8}$, find the values of all T-ratios of θ

   Sol:

   Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$

   Now, we know that $\tan\theta = \dfrac{Perpendicular}{Base} = \dfrac{AB}{BC} = \dfrac{15}{8}$

   So, if BC = 8k, then AB = 15k where k is positive number.

   Now, using Pythagoras theorem, we have:

   $AC^2 = AB^2 + BC^2 = (15k)^2 + (8k)^2$

   $\Rightarrow AC^2 = 225k^2 + 64k^2 = 289k^2$

   $\Rightarrow AC = 17k$

   Now, finding the other T-ratios using their definitions, we get:

   Sin $\theta = \dfrac{AB}{AC} = \dfrac{15k}{17k} = \dfrac{15}{17}$

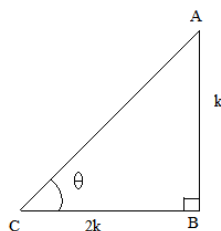   Cos $\theta = \dfrac{BC}{AC} = \dfrac{8k}{17k} = \dfrac{8}{17}$

   $\therefore \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{8}{15}, cosec\,\theta = \dfrac{1}{\sin\theta} = \dfrac{17}{15} \, and \, sec\theta = \dfrac{1}{\cos\theta} = \dfrac{17}{8}$

4.   If $\cot\theta = 2$ find all the values of all T-ratios of θ

   Sol:

   Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$

   Now, we know that $\cot\theta = \dfrac{Base}{Perpendicular} = \dfrac{BC}{AB} = 2$

So, if BC = 2k, then AB = k, is a positive number.

Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2 = (2k)^2 + (k)^2$

$\Rightarrow AC^2 = 4k^2 + k^2 = 5k^2$

$\Rightarrow AC = \sqrt{5}k$

Now, finding the other T-ratios using their definitions, we get:

$\text{Sin } \theta = \frac{AB}{AC} = \frac{5}{\sqrt{5}k} = \frac{1}{\sqrt{2}}$

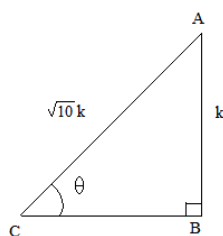$\text{Cos } \theta = \frac{BC}{AC} = \frac{2k}{\sqrt{5}k} = \frac{2}{\sqrt{5}}$

$\therefore \tan \theta = \frac{1}{\cot \theta} = \frac{1}{2}, cosec\ \theta = \frac{1}{\sin \theta} = \sqrt{5}\ and\ sec\theta = \frac{1}{\cos \theta} = \frac{\sqrt{5}}{2}$

5. If cosec $\theta = \sqrt{10}$ find all the values of all T-ratios of θ

Sol:

Let us first draw a right ΔABC, right angled at B and $\angle C = \theta$

Now, we know that cosec $\theta = \frac{Hypotenuse}{Perpendicular} = \frac{AC}{AB} = \frac{\sqrt{10}}{1}$

So, if AC = $(\sqrt{10})k,$ then $AB = k$ is a positive number.

Now, by using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 + BC^2$

$\Rightarrow BC^2 = 9k^2$

$\Rightarrow$ BC = 3k

Now, finding the other T-ratios using their definitions, we get:

$\tan \theta = \frac{AB}{BC} = \frac{k}{3k} = \frac{1}{3}$

$\cos \theta = \frac{BC}{AC} = \frac{3k}{\sqrt{10}k} = \frac{3}{\sqrt{10}}$

$\therefore \sin \theta = \frac{1}{cos\ ec\ \theta} = \frac{1}{\sqrt{10}}, \cot \theta = \frac{1}{\tan \theta} = 3\ and\ sec\theta = \frac{1}{\cos \theta} = \frac{\sqrt{10}}{3}$

6. If $\sin \theta = \dfrac{a^2 - b^2}{a^2 + b^2}$ find all the values of all T-ratios of θ

Sol:

We have $\sin \theta = \dfrac{a^2 - b^2}{a^2 + b^2}$

As,

$\cos^2 \theta = 1 - \sin^2 \theta$

$= 1 - \left(\dfrac{a^2 - b^2}{a^2 + b^2}\right)^2$

$= \dfrac{1}{1} - \dfrac{(a^2 - b^2)^2}{(a^2 + b^2)^2}$

$= \dfrac{(a^2 + b^2)^2 - (a^2 - b^2)^2}{(a^2 + b^2)}$

$= \dfrac{[(a^2 + b^2) - (a^2 - b^2)][(a^2 + b^2) + (a^2 - b^2)]}{(a^2 + b^2)^2}$

$= \dfrac{[a^2 + b^2 - a^2 + b^2][a^2 + b^2 + a^2 - b^2]}{(a^2 + b^2)^2}$

$= \dfrac{[2b^2][2a^2]}{(a^2 + b^2)^2}$

$\Rightarrow \cos^2 \theta = \dfrac{4a^2 b^2}{(a^2 + b^2)^2}$

$\Rightarrow \cos\theta = \sqrt{\dfrac{4a^2 b^2}{(a^2 + b^2)^2}}$

$\Rightarrow \cos\theta = \dfrac{2ab}{(a^2 + b^2)}$

Also,

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\left(\dfrac{a^2 - b^2}{a^2 + b^2}\right)}{\left(\dfrac{2ab}{a^2 + b^2}\right)}$

$= \dfrac{a^2 - b^2}{2ab}$

Now,

$\csc\theta = \dfrac{1}{\sin\theta}$

$= \dfrac{1}{\left(\dfrac{a^2 - b^2}{a^2 - b^2}\right)}$

$= \dfrac{a^2 + b^2}{a^2 - b^2}$

Also,

$\sec\theta = \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\left(\dfrac{2ab}{a^2 + b^2}\right)}$

$= \dfrac{a^2 + b^2}{2ab}$

And,

$$cot\theta = \frac{1}{tan\theta}$$

$$= \frac{1}{\left(\frac{a^2-b^2}{2ab}\right)}$$

$$= \frac{2ab}{a^2-b^2}$$

7. If 15 cot A = 8 find all the values of sin A and sec A

Sol:

We have,

$$15 \cot A = 8$$

$$\Rightarrow \cot A = \frac{8}{15}$$

As,

$$cosec^2 A = 1 + cot^2 A$$

$$= 1 + \left(\frac{8}{15}\right)^2$$

$$= 1 + \frac{64}{225}$$

$$= \frac{225+64}{225}$$

$$\Rightarrow cosec^2 A = \frac{289}{225}$$

$$\Rightarrow cosec\, A = \sqrt{\frac{289}{225}}$$

$$\Rightarrow cosec\, A = \frac{17}{15}$$

$$\frac{1}{\sin A} = \frac{17}{15}$$

$$\text{Sin A} = \frac{15}{17}$$

Also,

$$Cos^2 A = 1 - \sin^2 A$$

$$= 1 - \left(\frac{15}{17}\right)^2$$

$$= 1 - \frac{225}{289}$$

$$= \frac{289-225}{289}$$

$$\Rightarrow cos^2 A = \frac{64}{289}$$

$$\Rightarrow \cos A = \sqrt{\frac{64}{289}}$$

$$\Rightarrow \cos A = \frac{8}{17}$$

$$\Rightarrow \frac{1}{\sec A} = \frac{8}{17}$$

$$\Rightarrow \sec A = \frac{17}{8}$$

8. If $\sin A = \dfrac{9}{41}$ find all the values of cos A and tan A

Sol:

We have $\sin A = \dfrac{9}{41}$

As,

$\text{Cos}^2 A = 1 - \sin^2 A$

$= 1 - \left(\dfrac{9}{41}\right)^2$

$= 1 - \dfrac{81}{1681}$

$= \dfrac{1681 - 81}{1681}$

$\Rightarrow \cos^2 A = \dfrac{1600}{1681}$

$\Rightarrow \cos A = \sqrt{\dfrac{1600}{1681}}$

$\Rightarrow \cos A = \dfrac{40}{41}$

Also,

$\text{Tan } A = \dfrac{sinA}{cosA}$

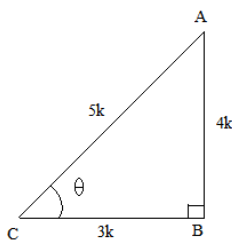$= \dfrac{\left(\dfrac{9}{41}\right)}{\left(\dfrac{40}{41}\right)}$

$= \dfrac{9}{40}$

9. If cos θ=0.6 show that (5sin θ -3tan θ) = 0

Sol:

Let us consider a right ΔABC right angled at B.

Now, we know that $\cos \theta = 0.6 = \dfrac{BC}{AC} = \dfrac{3}{5}$

So, if BC = 3k, then AC = 5k, where k is a positive number.

Using Pythagoras theorem, we have:

$Ac^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2$

$\Rightarrow AB^2 = (5k)^2 - (3k)^2 = 25k^2 - 9k^2$

$\Rightarrow AB^2 = 16k^2$

$\Rightarrow AB = 4k$

Finding out the other T-rations using their definitions, we get:

$\text{Sin}\,\theta = \dfrac{AB}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5}$

$\tan\theta = \dfrac{AB}{BC} = \dfrac{4k}{3k} = \dfrac{4}{3}$

Substituting the values in the given expression, we get:

$5\sin\theta - 3\tan\theta$

$\Rightarrow 5\left(\dfrac{4}{5}\right) - 3\left(\dfrac{4}{3}\right)$

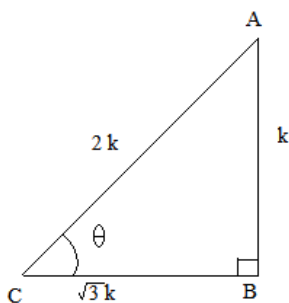$\Rightarrow 4 - 4 = 0 = RHS$

i.e., LHS = RHS

Hence, Proved.

10. If cosec θ= 2 show that $\left(\cot\theta + \dfrac{\sin\theta}{1+\cos\theta}\right) = 2$

Sol:

Let us consider a right ΔABC, right angled at B and $\angle C = \theta$.

Now, it is given that cosec θ = 2.

Also, $\sin\theta = \dfrac{1}{\cos ec\theta} = \dfrac{1}{2} = \dfrac{AB}{AC}$

So, if AB =k, then AC =2k, where k is a positive number.

Using Pythagoras theorem, we have:

$\Rightarrow AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

$\Rightarrow BC^2 (2k)^2 - (k)^2$

$\Rightarrow BC^2 = 3k^2$

$\Rightarrow BC = \sqrt{3}k$

Finding out the other T-ratios using their definitions, we get:

$\cos\theta = \dfrac{BC}{AC} = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2}$

$\text{Tan}\,\theta = \dfrac{AB}{BC} = \dfrac{k}{\sqrt{3}k} = \dfrac{1}{\sqrt{3}}$

$\text{Cot}\,\theta = \dfrac{1}{\tan\theta} = \sqrt{3}$

Substituting these values in the given expression, we get:

$Cot\,\theta + \dfrac{sin\,\theta}{1+cos\,\theta}$

$= \sqrt{3} + \dfrac{\left(\frac{1}{2}\right)}{1+\frac{\sqrt{3}}{2}}$

$= \sqrt{3} + \dfrac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}$

$= \sqrt{3} + \dfrac{1}{2+\sqrt{3}}$

$= \dfrac{\sqrt{3}(2+\sqrt{3})+1}{2+\sqrt{3}}$

$= \dfrac{2\sqrt{3}+3+1}{2+\sqrt{3}}$

$= \dfrac{2(2+\sqrt{3})}{2+\sqrt{3}} = 2$

i.e., LHS = RHS

Hence proved.

11.   If $tan\ \theta = \dfrac{1}{\sqrt{7}}$ show that $\dfrac{\left(cos\,ec^2\theta - sec^2\,\theta\right)}{cos\,ec^2\theta + sec^2\,\theta} = \dfrac{3}{4}$

Sol:

Let us consider a right ΔABC, right angled at B and $\angle C = \theta$.

Now it is given that $tan\,\theta = \dfrac{AB}{BC} = \dfrac{1}{\sqrt{7}}$
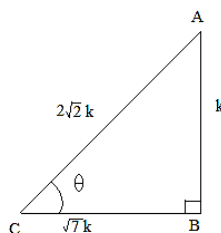
So, if AB = k, then BC = $\sqrt{7}k,$ *wher k is a positive number.*

Using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (k)^2 + \left(\sqrt{7}k\right)^2$

$\Rightarrow AC^2 = k^2 + 7k^2$

$\Rightarrow AC = 2\sqrt{2}k$

Now, finding out the values of the other trigonometric ratios, we have:

$Sin\,\theta = \dfrac{AB}{AC} = \dfrac{k}{2\sqrt{2}k} = \dfrac{1}{2\sqrt{2}}$

$Cos\,\theta = \dfrac{BC}{AC} = \dfrac{\sqrt{7}k}{2\sqrt{2}k} = \dfrac{\sqrt{7}}{2\sqrt{2}}$

$\therefore cosec\,\theta = \dfrac{1}{sin\,\theta} = 2\sqrt{2}\ and\ sec\,\theta = \dfrac{1}{cos\,\theta} = \dfrac{2\sqrt{2}}{\sqrt{7}}$

Substituting the values of cosec θ and sec θ in the give expression, we get:

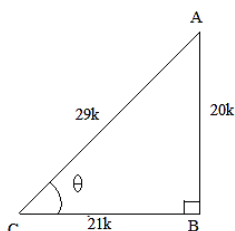$\dfrac{cosec^2\theta - sec^2\theta}{cosec^2\theta + sec^2\theta}$

$= \dfrac{\left(2\sqrt{2}\right)^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(2\sqrt{2}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$

$= \dfrac{8 - \left(\frac{8}{7}\right)}{8 + \left(\frac{8}{7}\right)}$

$= \dfrac{\frac{56-8}{7}}{\frac{56+8}{7}}$

$= \dfrac{48}{64} = \dfrac{3}{4} = RHS$

i.e., LHS = RHS

Hence proved.

12. If $\tan\theta = \dfrac{20}{21}$, show that $\dfrac{\left(1 - \sin\theta + \cos\theta\right)}{\left(1 + \sin\theta + \cos\theta\right)} = \dfrac{3}{7}$

Sol:

Let us consider a right ΔABC right angled at B and $\angle C = \theta$

Now, we know that $\tan\theta = \dfrac{AB}{BC} = \dfrac{20}{21}$

So, if AB = 20k, then BC = 21k, where k is a positive number.

Using Pythagoras theorem, we get:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (20k)^2 + (21k)^2$

$\Rightarrow AC^2 = 841k^2$

$\Rightarrow AC = 29k$

Now. $\sin\theta = \dfrac{AB}{AC} = \dfrac{20}{29}$ and $\cos\theta = \dfrac{BC}{AC} = \dfrac{21}{29}$

Substituting these values in the give expression, we get:

$LHS = \dfrac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta}$

$= \dfrac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}}$

$= \dfrac{\frac{29-20+21}{29}}{\frac{29+20+21}{29}} = \dfrac{30}{70} = \dfrac{3}{7} = RHS$

$\therefore$ LHS = RHS

Hence proved.

13. If $\sec\theta = \dfrac{5}{4}$ show that $\dfrac{(\sin\theta - 2\cos\theta)}{(\tan\theta - \cot\theta)} = \dfrac{12}{7}$

Sol:

We have,

$\operatorname{Sec}\theta = \dfrac{5}{4}$

$\Rightarrow \dfrac{1}{\cos\theta} = \dfrac{5}{4}$

$\Rightarrow \cos\theta = \dfrac{4}{5}$

Also,

$\operatorname{Sin}^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\dfrac{4}{5}\right)^2$

$= 1 - \dfrac{16}{25}$

$= \dfrac{9}{25}$

$\Rightarrow \sin\theta = \dfrac{3}{5}$

Now,

$LHS = \dfrac{(\sin\theta - 2\cos\theta)}{(\tan\theta - \cot\theta)}$

$= \dfrac{(\sin\theta - 2\cos\theta)}{\left(\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta}\right)}$

$= \dfrac{(\sin\theta - 2\cos\theta)}{\left(\dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta\,\cos\theta}\right)}$

$\dfrac{\sin\theta\cos\theta\left(\sin\theta - 2\cos\theta\right)}{\left(\sin^2\theta - \cos^2\theta\right)}$

$= \dfrac{\dfrac{3}{5}\times\dfrac{4}{5}\left(\dfrac{3}{5} - 2\times\dfrac{4}{5}\right)}{\left(\dfrac{3}{5}\right)^2 - \left(\dfrac{4}{5}\right)^2}$

$= \dfrac{\dfrac{12}{25}\left(\dfrac{3}{5} - \dfrac{8}{5}\right)}{\left(\dfrac{9}{25} - \dfrac{16}{25}\right)}$

$= \dfrac{\dfrac{12}{25}\times\left(\dfrac{-5}{5}\right)}{\left(\dfrac{-7}{25}\right)}$

$= \dfrac{12}{7}$

$= RHS$

14. If $\cot\theta = \dfrac{3}{4}$, show that $\sqrt{\dfrac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \dfrac{1}{\sqrt{7}}$

Sol:

$LHS = \sqrt{\dfrac{\sec\theta - cosec\,\theta}{\sec\theta + cosec\,\theta}}$

$= \sqrt{\dfrac{\left(\frac{1}{\cos\theta} - \frac{1}{\sin\theta}\right)}{\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)}}$

$= \sqrt{\dfrac{\left(\frac{\sin\theta - \cos\theta}{\sin\theta\cos\theta}\right)}{\left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)}}$

$= \sqrt{\dfrac{\left(\frac{\sin\theta - \cos\theta}{\sin\theta}\right)}{\left(\frac{\sin\theta + \cos\theta}{\sin\theta}\right)}}$

$= \sqrt{\dfrac{\left(\frac{\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)}}$

$= \sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}}$

$= \sqrt{\dfrac{\left(1 - \frac{3}{4}\right)}{\left(1 + \frac{3}{4}\right)}}$

$= \sqrt{\dfrac{\left(\frac{1}{4}\right)}{\left(\frac{7}{4}\right)}}$

$= \sqrt{\dfrac{1}{7}}$

$= \dfrac{1}{\sqrt{7}}$

$= \text{RHS}$

15. If $\sin\theta = \dfrac{3}{4}$, show that $\sqrt{\dfrac{\cos ec^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \dfrac{\sqrt{7}}{3}$

Sol:

$LHS = \sqrt{\dfrac{cosec^2\theta - cot^2\theta}{sec^2\,2 - 1}}$

$= \sqrt{\dfrac{1}{tan^2\theta}}$

$= \sqrt{cot^2\theta}$

$= \cot\theta$

$= \sqrt{cosec^2\theta - 1}$

$$= \sqrt{\left(\frac{1}{\left(\frac{3}{4}\right)}\right)^2 - 1}$$

$$= \sqrt{\left(\frac{4}{3}\right)^2 - 1}$$

$$= \sqrt{\frac{16}{9} - 1}$$

$$= \sqrt{\frac{16-9}{9}}$$

$$= \sqrt{\frac{7}{9}}$$

$$= \frac{\sqrt{7}}{3}$$

$= $ RHS

16. If $\sin \theta = \dfrac{a}{b}$, show that $\left(\sec \theta + \tan \theta\right) = \sqrt{\dfrac{b+a}{b-a}}$

Sol:

$LHS = (\sec \theta + \tan \theta)$

$$= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1+\sin \theta}{\cos \theta}$$

$$= \frac{1+\sin \theta}{\sqrt{1-\sin^2 \theta}}$$

$$= \frac{\left(1+\frac{a}{b}\right)}{\sqrt{1-\left(\frac{a}{b}\right)^2}}$$

$$= \frac{\left(\frac{1}{1}+\frac{a}{b}\right)}{\sqrt{\frac{1}{1}-\frac{a^2}{b^2}}}$$

$$= \frac{\left(\frac{b+a}{b}\right)}{\sqrt{\frac{b^2-a^2}{b^2}}}$$

$$= \frac{(b+a)}{\sqrt{(b+a)}\sqrt{(b-a)}}$$

$$= \frac{\sqrt{(b+a)}}{\sqrt{(b-a)}}$$

$$= \sqrt{\frac{b+a}{b-a}}$$

$= $ RHS

17. If $\cos\theta = \dfrac{3}{5}$, show that $\dfrac{(\sin\theta - \cot\theta)}{2\tan\theta} = \dfrac{3}{160}$

Sol:
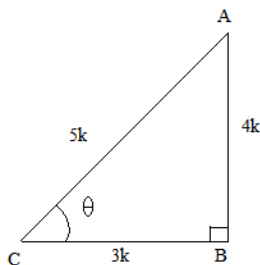
$LHS = \dfrac{(\sin\theta - \cot\theta)}{2\tan\theta}$

$= \dfrac{\sin\theta - \frac{\cos\theta}{\sin\theta}}{2\left(\frac{\sin\theta}{\cos\theta}\right)}$

$= \dfrac{\frac{\sin^2\theta - \cos\theta}{\sin\theta}}{\left(\frac{2\sin\theta}{\cos\theta}\right)}$

$= \dfrac{\cos\theta(\sin^2\theta - \cos\theta)}{2\sin^2\theta}$

$= \dfrac{\cos\theta(1 - \cos^2\theta - \cos\theta}{2(1 - \cos^2\theta)}$

$= \dfrac{\frac{3}{5}[1 - \left(\frac{3}{5}\right)^2 - \frac{3}{5}]}{2[1 - \left(\frac{3}{5}\right)^2]}$

$= \dfrac{\frac{3}{5}\left(\frac{1}{1} - \frac{9}{25} - \frac{3}{5}\right)}{2\left(1 - \frac{9}{25}\right)}$

$= \dfrac{\frac{3}{5}\left(\frac{25 - 9 - 15}{25}\right)}{2\left(\frac{25 - 9}{25}\right)}$

$= \dfrac{\frac{3}{5}\left(\frac{1}{25}\right)}{2\left(\frac{16}{25}\right)}$

$= \dfrac{3}{5 \times 2 \times 16}$

$= \dfrac{3}{160}$

$= RHS$

18. If $\tan\theta = \dfrac{4}{3}$, show that $(\sin\theta + \cos\theta) = \dfrac{7}{5}$

Sol:

Let us consider a right $\triangle ABC$, right angled at B and $\angle C = \theta$

Now, we know that $\tan\theta = \dfrac{AB}{BC} = \dfrac{4}{3}$

So, if BC = 3k, then AB = 4k, where k is a positive number.

Using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2$
$\Rightarrow AC^2 = 16k^2 + 9k^2 = 25k^2$
$\Rightarrow AC = 5k$

Finding out the values of $\sin\theta$ and $\cos\theta$ using their definitions, we have:

$\text{Sin}\,\theta = \dfrac{AB}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5}$

$\text{Cos}\,\theta = \dfrac{BC}{AC} = \dfrac{3k}{5k} = \dfrac{3}{5}$

Substituting these values in the given expression, we get:

$(\sin\theta + \cos\theta) = \left(\dfrac{4}{5} + \dfrac{3}{5}\right) = \left(\dfrac{7}{5}\right) = RHS$

i.e., LHS = RHS

Hence proved.

19. If $\tan\theta = \dfrac{a}{b}$, show that $\dfrac{(a\sin\theta - b\cos\theta)}{(a\sin\theta + b\cos\theta)} = \dfrac{(a^2 - b^2)}{a^2 + b^2}$

Sol:

It is given that $\tan\theta = \dfrac{a}{b}$

$LHS = \dfrac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

Dividing the numerator and denominator by $\cos\theta$, we get:

$\dfrac{a\tan\theta - b}{a\tan\theta + b}$ $\left(\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right)$

Now, substituting the value of $\tan\theta$ in the above expression, we get:

$\dfrac{a\left(\frac{a}{b}\right) - b}{a\left(\frac{a}{b}\right) + b}$

$= \dfrac{\frac{a^2}{b} - b}{\frac{a^2}{b} + b}$
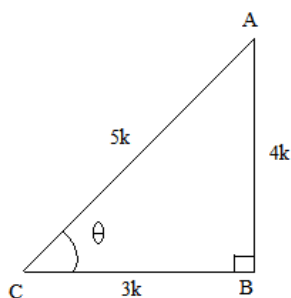
$= \dfrac{a^2 - b^2}{a^2 + b^2} = RHS$

i.e., LHS = RHS

Hence proved.

20. If $3\tan\theta = 4$, show that $\dfrac{(4\cos\theta - \sin\theta)}{(4\cos\theta + \sin\theta)} = \dfrac{4}{5}$

Sol:

Let us consider a right $\triangle ABC$ right angled at B and $\angle C = \theta$.

We know that $\tan\theta = \dfrac{AB}{BC} = \dfrac{4}{3}$

So, if BC = 3k, then AB = 4k, where k is a positive number.

Using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 16k^2 + 9k^2$

$\Rightarrow AC^2 = 25k^2$

$\Rightarrow$ AC = 5k

Now, we have:

$Sin\,\theta = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

$Cos\,\theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

Substituting these values in the given expression, we get:

$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

$= \frac{4\left(\frac{3}{5}\right) - \frac{4}{5}}{2\left(\frac{3}{5}\right) + \frac{4}{5}}$

$= \frac{\frac{12}{5} - \frac{4}{5}}{\frac{6}{5} + \frac{4}{5}}$

$= \frac{\frac{12-4}{5}}{\frac{6+4}{5}}$

$= \frac{8}{10} = \frac{4}{5} = RHS$

i.e., LHS = RHS

Hence proved.

21.   If $3\cot\theta = 2$, show that $\dfrac{\left(4\sin\theta - 4\cos\theta\right)}{\left(2\sin\theta + 6\cos\theta\right)} = \dfrac{1}{3}$

Sol:

It is given that $\cos\theta = \frac{2}{3}$

$LHS = \frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$

Dividing the above expression by $\sin\theta, we\ get$:

$\frac{4 - 3\cot\theta}{2 + 6\cot\theta}$ $[\because \cot\theta = \frac{\cos\theta}{\sin\theta}]$

Now, substituting the values of $\cot\theta$ in the above expression, we get:

$$\frac{4-3\left(\frac{2}{3}\right)}{2+6\left(\frac{2}{3}\right)}$$

$$= \frac{4-2}{2+4} = \frac{2}{6} = \frac{1}{3}$$

i.e., LHS = RHS

Hence proved.

22. If $3\cot\theta = 4$, show that $\dfrac{\left(1-\tan^2\theta\right)}{\left(1+\tan^2\theta\right)} = \left(\cos^2\theta - \sin^2\theta\right)$

Sol:

$LHS = \dfrac{(1-\tan^2\theta)}{(1+\tan^2\theta)}$

$= \dfrac{\left(1-\frac{1}{\cot^2\theta}\right)}{\left(1+\frac{1}{\cot^2\theta}\right)}$

$= \dfrac{\frac{\cot^2\theta-1}{\cot^2\theta}}{\left(\frac{\cot^2\theta+1}{\cot^2\theta}\right)}$

$= \dfrac{\cot^2\theta-1}{\cot^2\theta+1}$

$= \dfrac{\left(\frac{4}{3}\right)^2-1}{\left(\frac{4}{3}\right)^2+1}$ $\qquad \left(As, 3\cot\theta = 4 \ or \ \cot\theta = \frac{4}{3}\right)$

$= \dfrac{\frac{16}{9}-1}{\frac{16}{9}+1}$

$= \dfrac{\left(\frac{16-9}{9}\right)}{\left(\frac{16+9}{9}\right)}$

$= \dfrac{\left(\frac{7}{9}\right)}{\left(\frac{25}{9}\right)}$

$= \dfrac{7}{25}$

$RHS = (\cos^2\theta - \sin^2\theta)$

$= \dfrac{(\cos^2\theta-\sin^2\theta)}{1}$

$= \dfrac{\left(\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}\right)}{\left(\frac{1}{\sin^2\theta}\right)}$

$= \dfrac{\frac{\cos^2\theta}{\sin^2\theta}-\frac{\sin^2\theta}{\sin^2\theta}}{cosec^2\theta}$

$= \dfrac{(\cot^2\theta-1)}{(\cot^2\theta+1)}$

$= \dfrac{\left[\left(\frac{4}{3}\right)^2-1\right]}{\left[\left(\frac{4}{3}\right)^2+1\right]}$

$$= \frac{\left(\frac{16}{9} - \frac{1}{1}\right)}{\left(\frac{16}{9} + \frac{1}{1}\right)}$$

$$= \frac{\left(\frac{16-9}{9}\right)}{\left(\frac{16+9}{9}\right)}$$

$$= \frac{\left(\frac{7}{9}\right)}{\left(\frac{25}{9}\right)}$$

$$= \frac{7}{25}$$

Since, LHS = RHS

Hence, verified.

23. If $\sec \theta = \dfrac{17}{8}$ verify that $\dfrac{\left(3 - 4\sin^2 \theta\right)}{\left(4\cos^2 \theta - 3\right)} = \dfrac{\left(3 - \tan^2 \theta\right)}{\left(1 - \tan^3 \theta\right)}$

Sol:

It is given that $\sec \theta = \dfrac{17}{8}$

Let us consider a right $\triangle ABC$ right angled at B and $\angle C = \theta$

We know that $\cos \theta = \dfrac{1}{\sec \theta} = \dfrac{8}{17} = \dfrac{BC}{AC}$
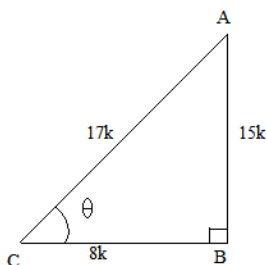
So, if BC = 8k, then AC = 17k, where k is a positive number.

Using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2 = (17k)^2 - (8k)^2$

$\Rightarrow AB^2 = 289k^2 - 64k^2 = 225k^2$

$\Rightarrow AB = 15k.$

Now, $\tan \theta = \dfrac{AB}{BC} = \dfrac{15}{8}$ and $\sin \theta = \dfrac{AB}{AC} = \dfrac{15k}{17k} = \dfrac{15}{17}$

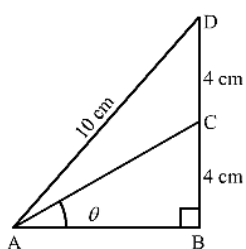The given expression is $\dfrac{3 - 4\sin^2 \theta}{4\cos^2 \theta - 3} = \dfrac{3 - \tan^2 \theta}{1 - 3\tan^2 \theta}$

Substituting the values in the above expression, we get:

$$LHS = \frac{3 - 4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2 - 3}$$

$$= \frac{3 - \frac{900}{289}}{\frac{256}{289} - 3}$$
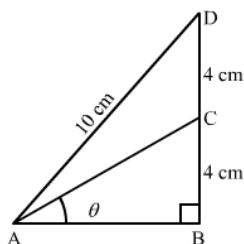
$$= \frac{867 - 900}{256 - 867} = -\frac{33}{-611} = \frac{33}{611}$$

$$RHS = \frac{3 - \left(\frac{15}{8}\right)^2}{1 - 3\left(\frac{15}{8}\right)^2}$$

$$= \frac{3 - \frac{225}{64}}{1 - \frac{675}{64}}$$

$$= \frac{192 - 255}{64 - 675} = -\frac{33}{-611} = \frac{33}{611}$$

$\therefore$ LHS = RHS

Hence proved.

24. In the adjoining figure, $\angle B = 90°$, $\angle BAC = \theta°$, $BC = CD = 4cm$ and AD = 10 cm. find (i) $\sin\theta$
and (ii) $\cos\theta$

Sol:

In $\triangle$ABD,

Using Pythagoras theorem, we get

$AB = \sqrt{AD^2 - BD^2}$

$= \sqrt{10^2 - 8^2}$

$= \sqrt{100 - 64}$

$= \sqrt{36}$

$= 6$ cm

Again,

In $\triangle$ABC,

Using Pythagoras therem, we get

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{6^2 + 4^2}$

$= \sqrt{36 + 16}$

$= \sqrt{52}$

$= 2\sqrt{13} \; cm$

Now,

(i) $\sin \theta = \dfrac{BC}{AC}$

$= \dfrac{4}{2\sqrt{13}}$

$= \dfrac{2}{\sqrt{13}}$

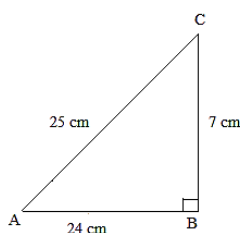$= \dfrac{2\sqrt{13}}{13}$

(ii) $\cos \theta = \dfrac{AB}{AC}$

$= \dfrac{6}{2\sqrt{13}}$

$= \dfrac{3}{\sqrt{13}}$

$= \dfrac{3\sqrt{13}}{13}$

25.   In a $\triangle ABC$, $\angle B = 90°$, AB= 24 cm and BC = 7 cm find (i) sin A (ii) cos A (iii) sin C
(iv) cos C

Sol:

Using Pythagoras theorem, we get:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (24)^2 + (7)^2$

$\Rightarrow AC^2 = 576 + 49 = 625$

$\Rightarrow AC = 25\ cm$

Now, for T-Ratios of $\angle A$, base $= AB$ and perpendicular $= BC$

(i) $\sin A = \dfrac{BC}{AC} = \dfrac{7}{25}$

(ii) $\cos A = \dfrac{AB}{AC} = \dfrac{24}{25}$

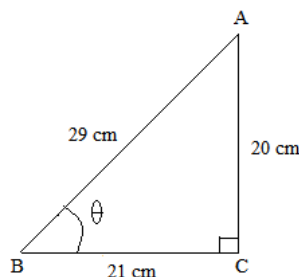   Similarly, for T-Ratios of $\angle C$, base $= BC$ and perpendicular $= AB$

(iii) $\sin C = \dfrac{AB}{AC} = \dfrac{24}{25}$

(iv) $\cos C = \dfrac{BC}{AC} = \dfrac{7}{25}$

---

26. In $\triangle ABC$, $\angle C = 90°$ $\angle ABC = \theta°$ BC $= 21$ units . and AB$= 29$ units. Show that $\left(\cos^2\theta - \sin^2\theta\right) = \dfrac{41}{841}$

Sol:

Using Pythagoras theorem, we get:

$AB^2 = AC^2 + BC^2$

$\Rightarrow AC^2 = AB^2 - BC^2$

$\Rightarrow AC^2 = (29)^2 - (21)^2$

$\Rightarrow AC^2 = 841 - 441$

$\Rightarrow AC^2 = 400$

$\Rightarrow AC = \sqrt{400} = 20\ units$

Now, $\sin\theta = \dfrac{AC}{AB} = \dfrac{20}{29}$ $and \cos\theta = \dfrac{BC}{AB} = \dfrac{21}{29}$

$Cos^2\theta - \sin^2\theta = \left(\dfrac{21}{29}\right)^2 - \left(\dfrac{20}{29}\right)^2 = \dfrac{441}{841} - \dfrac{400}{841} = \dfrac{41}{841}$

Hence proved.

27. In a $\triangle ABC$, $\angle B = 90°$, AB = 12 cm and BC = 5 cm Find

(i) cos A (ii) cosec A (iii) cos C (iv) cosec C

Sol:
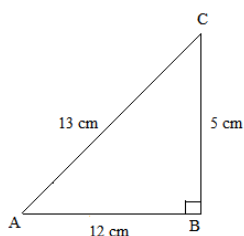
Using Pythagoras theorem, we get:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 12^2 + 5^2 = 144 + 25$

$\Rightarrow AC^2 = 169$

$\Rightarrow AC = 13\ cm$

Now, for T-Ratios of $\angle A, base = AB$ $and\ perpendicular = BC$

(i)$\cos A = \dfrac{AB}{AC} = \dfrac{12}{13}$

(ii)$cosec\ A = \dfrac{1}{\sin A} = \dfrac{AC}{BC} = \dfrac{13}{5}$

Similarly, for T-Ratios of $\angle C, base = BC\ and\ perpendicular = AB$

(iii)$\cos C = \dfrac{BC}{AC} = \dfrac{5}{13}$

(iv)$cosec\ C = \dfrac{1}{\sin C} = \dfrac{AC}{AB} = \dfrac{13}{12}$

28. If $\sin \alpha = \dfrac{1}{2}$ prove that $\left(3\cos\alpha - 4\cos^2 \alpha\right) = 0$
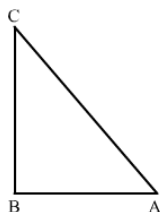
Sol:

$LHS = (3cosa - 4\cos^3 a)$

$= \cos a(3 - 4\cos^2 a)$

$= \sqrt{1 - \sin^2 a}\ [3 - 4(1 - \sin^2 a)]$

$= \sqrt{1 - \left(\tfrac{1}{2}\right)^2}\ \left[3 - 4\left(1 - \left(\tfrac{1}{2}\right)^2\right)\right]$

$= \sqrt{\tfrac{1}{1} - \tfrac{1}{4}}\left[3 - 4\left(\tfrac{1}{1} - \tfrac{1}{4}\right)\right]$

$= \sqrt{\tfrac{3}{4}}\left[3 - 4\left(\tfrac{3}{4}\right)\right]$

$= \sqrt{\tfrac{3}{4}}\ [3 - 3]$

$= \sqrt{\tfrac{3}{4}}\ [0]$

$= 0$

$= \text{RHS}$

29. IF $\triangle ABC, \angle B = 90°$ AND Tan $A = \dfrac{1}{\sqrt{3}}$. Prove that

(i) Sin A. cos C+ cos A. Sin c = 1

(ii) cos A. cos C -sin A. sin C = 0

Sol:

In $\triangle$ABC, $\angle B = 90^0$,

As, $\tan A = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let $BC = x$ and $AB = x\sqrt{3}$

Using Pythagoras the get

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{\left(x\sqrt{3}\right)^2 + x^2}$

$= \sqrt{3x^2 + x^2}$

$= \sqrt{4x^2}$

$= 2x$

Now,

(i)LHS $= \sin A.\cos C + \cos A.\sin C$

$= \frac{BC}{AC}.\frac{BC}{AC} + \frac{AB}{AC}.\frac{AB}{AC}$

$= \left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2$

$= \left(\frac{x}{2x}\right)^2 + \left(\frac{x\sqrt{3}}{2x}\right)^2$

$= \frac{1}{4} + \frac{3}{4}$

$= 1$

$= $ RHS

(ii)LHS $= \cos A.\cos C - \sin A.\sin C$

$= \frac{AB}{AC}.\frac{BC}{AC} - \frac{BC}{AC}.\frac{AB}{AC}$

$= \frac{x\sqrt{3}}{2x}.\frac{x}{2x} - \frac{x}{2x}.\frac{x\sqrt{3}}{2x}$
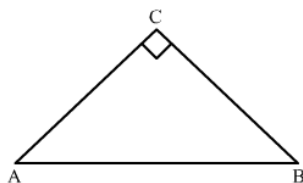
$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$= 0$

$= $ RHS

30. If $\angle A$ and $\angle B$ are acute angles such that $\sin A = \sin B$ prove that $\angle A = \angle B$

Sol:

In $\triangle ABC$, $\angle C = 90^0$

$\sin A = \frac{BC}{AB}$ *and*

$\sin B = \frac{AC}{AB}$

As, $\sin A = \sin B$
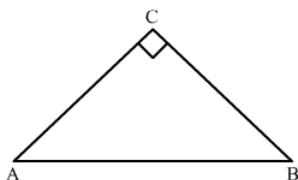
$\Rightarrow \frac{BC}{AB} = \frac{AC}{AB}$

$\Rightarrow BC = AC$

So, $\angle A = \angle B$ (*Angles opposite to equal sides are equal*)

31. If $\angle A$ and $\angle B$ are acute angles such that tan A= Tan B then prove that $\angle A = \angle B$

Sol:

In $\triangle ABC, \angle C = 90^0$

$\operatorname{Tan} A = \frac{BC}{AC}$ and

$\operatorname{Tan} B = \frac{AC}{BC}$

As, $\tan A = \tan B$

$\Rightarrow \frac{BC}{AC} = \frac{AC}{BC}$

$\Rightarrow BC^2 = AC^2$

$\Rightarrow BC = AC$

So, $\angle A = \angle B$ (*Angles opposite to equal sides are equal*)

32. If a right $\triangle ABC$, right-angled at B, if tan A=1 then verify that $2\sin A \cdot \cos A = 1$

Sol:

We have,

$\operatorname{Tan} A = 1$

$\Rightarrow \frac{sinA}{cosA} = 1$

$\Rightarrow \sin A = \cos A$

$\Rightarrow \sin A - \cos A = 0$

Squaring both sides, we get

$(sinA - cosA)^2 = 0$

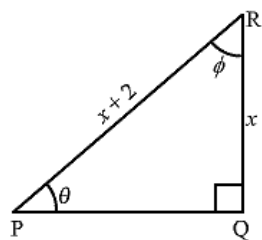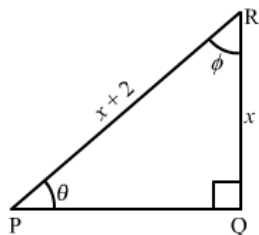$\Rightarrow \sin^2 A + \cos^2 A - 2\sin A \cdot \cos A = 0$

$\Rightarrow 1 - 2\sin A \cdot \cos A = 0$

$\therefore 2\sin A \cdot cosA = 1$

33. In the figure of $\triangle PQR$, $\angle P = \theta°$ and $\angle R = \phi°$ find

(i) $\sqrt{X+1}\cot\phi$

(ii) $\sqrt{x^3 + x^2}\tan\theta$

(iii) $\cos \theta$

Sol:

In $\Delta PQR, \angle Q = 90^0$,

Using Pythagoras theorem, we get

$PQ = \sqrt{PR^2 - QR^2}$

$= \sqrt{(x+2)^2 - x^2}$

$= \sqrt{x^2 + 4x + 4 - x^2}$

$= \sqrt{4(x+1)}$

$= 2\sqrt{x+1}$

       Now,

(i) $\left(\sqrt{x+1}\right) \cot \emptyset$

$= \left(\sqrt{x+1}\right) \times \frac{QR}{PQ}$

$= \left(\sqrt{x+1}\right) \times \frac{x}{2\sqrt{x+1}}$

$= \frac{x}{2}$

(ii) $\left(\sqrt{x^3 + x^2}\right) \tan \theta$

$= \left(\sqrt{x^2(x+1)}\right) \times \frac{QR}{PQ}$

$= x\sqrt{(x+1)} \times \frac{x}{2\sqrt{x+1}}$

$= \frac{x^2}{2}$

(iii)$\cos \theta$

$= \frac{PQ}{PR}$ $\theta = \frac{2\sqrt{x+1}}{x+2}$

34. If x = cosec A +cos A and y = cosec A – cos A then prove that $\left(\dfrac{2}{x+y}\right)^2+\left(\dfrac{x-y}{2}\right)^2=1$

Sol:

$LHS = \left(\dfrac{2}{x+y}\right)^2+\left(\dfrac{x-y}{2}\right)^2-1$

$= \left[\dfrac{2}{(cosec\ A+\cos A)+(cosecA-\cos A)}\right]^2+\left[\dfrac{(cosecA+\cos A)-(cosecA-\cos A)}{2}\right]^2-1$

$= \left[\dfrac{2}{cosecA+\cos A+cosecA-\cos A}\right]^2+\left[\dfrac{cosecA+\cos A-cosecA+\cos A}{2}\right]^2-1$

$= \left[\dfrac{2}{2\ cosecA}\right]^2+\left[\dfrac{2\cos A}{2}\right]^2-1$

$= \left[\dfrac{1}{cosecA}\right]^2+[cosA]^2-1$

$= [sinA]^2+[cosA]^2-1$

$= \sin^2 A + \cos^2 A - 1$

$= 1\text{-}1$

$= 0$

$= RHS$

35. If x = cot A + cos A and y = cot A – cos A then prove that $\left(\dfrac{x-y}{x+y}\right)^2+\left(\dfrac{x-y}{2}\right)^2=1$

Sol:

$LHS = \left(\dfrac{x-y}{x+y}\right)^2+\left(\dfrac{x-y}{2}\right)^2$

$= \left[\dfrac{(cotA+cosA)-(cotA-cosA)}{(cotA+cosA)+(cotA-cosA)}\right]^2+\left[\dfrac{(cotA+cosA)-(cotA-cosA)}{2}\right]^2$

$= \left[\dfrac{cotA+cosA-cotA+cosA}{cotA+cosA+cotA-cosA}\right]^2+\left[\dfrac{cotA+cosA-cotA+cosA}{2}\right]^2$

$= \left[\dfrac{2cosA}{2cotA}\right]^2+\left[\dfrac{2cosA}{2}\right]^2$

$= \left[\dfrac{cosA}{\left(\dfrac{cosA}{sinA}\right)}\right]^2+[cosA]^2$

$= \left[\dfrac{sinA\ cosA}{cosA}\right]^2+[cosA]^2$

$= [sinA]^2+[cosA]^2$

$= \sin^2 A + \cos^2 A$

$= 1$

$= RHS$