ELECTROMAGNETIC WAVES
CHAPTER - 40

1. $\frac{\epsilon_0\, d\phi_E}{dt} = \frac{\epsilon_0\, EA}{dt\; 4\pi\, \varepsilon_0 r^2}$

$= \frac{M^{-1}L^{-3}T^4A^2}{M^{-1}L^{-3}A^2} \times \frac{A^1T^1}{L^2} \times \frac{L^2}{T} = A^1$

= (Current) (proved).

2. $E = \frac{Kq}{x^2}$, [from coulomb's law]

$\phi_E = EA = \frac{KqA}{x^2}$

$I_d = \epsilon_0 \frac{d\phi E}{dt} = \epsilon_0 \frac{d}{dt}\frac{kqA}{x^2} = \epsilon_0\, KqA = \frac{d}{dt}x^{-2}$

$= \epsilon_0 \times \frac{1}{4\pi\, \epsilon_0} \times q \times A \times -2 \times x^{-3} \times \frac{dx}{dt} = \frac{qAv}{2\pi x^3}$.

3. $E = \frac{Q}{\epsilon_0\, A}$ (Electric field)

$\phi = E.A. = \frac{Q}{\epsilon_0\, A}\frac{A}{2} = \frac{Q}{\epsilon_0\, 2}$

$i_0 = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0\, 2}\right) = \frac{1}{2}\left(\frac{dQ}{dt}\right)$

$= \frac{1}{2}\frac{d}{dt}(EC\, e^{-t/RC}) = \frac{1}{2}EC - \frac{1}{RC}e^{-t/RC} = \frac{-E}{2R}e^{\frac{-td}{RE_0\lambda}}$

4. $E = \frac{Q}{\epsilon_0\, A}$ (Electric field)

$\phi = E.A. = \frac{Q}{\epsilon_0\, A}\frac{A}{2} = \frac{Q}{\epsilon_0\, 2}$

$i_0 = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0\, 2}\right) = \frac{1}{2}\left(\frac{dQ}{dt}\right)$

5. $B = \mu_0 H$

$\Rightarrow H = \frac{B}{\mu_0}$

$\frac{E_0}{H_0} = \frac{B_0/(\mu_0\, \epsilon_0\, C)}{B_0/\mu_0} = \frac{1}{\epsilon_0\, C}$

$= \frac{1}{8.85 \times 10^{-12} \times 3 \times 10^8} = 376.6\ \Omega = 377\ \Omega.$

Dimension $\frac{1}{\epsilon_0\, C} = \frac{1}{[LT^{-1}][M^{-1}L^{-3}T^4A^2]} = \frac{1}{M^{-1}L^{-2}T^3A^2} = M^1L^2T^{-3}A^{-2} = [R]$.

6. $E_0 = 810$ V/m, $B_0 = ?$

We know, $B_0 = \mu_0\, \epsilon_0\, C\, E_0$

Putting the values,

$B_0 = 4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 3 \times 10^8 \times 810$

$= 27010.9 \times 10^{-10} = 2.7 \times 10^{-6}\ T = 2.7\ \mu T.$

7. $B = (200\ \mu T)\ \text{Sin}\ [(4 \times 10^{15}\ 5^{-1})\ (t - x/C)]$

a) $B_0 = 200\ \mu T$

$E_0 = C \times B_0 = 200 \times 10^{-6} \times 3 \times 10^8 = 6 \times 10^4$

b) Average energy density = $\frac{1}{2\mu_0} B_0^2 = \frac{(200 \times 10^{-6})^2}{2 \times 4\pi \times 10^{-7}} = \frac{4 \times 10^{-8}}{8\pi \times 10^{-7}} = \frac{1}{20\pi}$ = 0.0159 = 0.016.

8. $I = 2.5 \times 10^{14}\ W/m^2$

We know, $I = \frac{1}{2} \in_0 E_0^2 C$

$\Rightarrow\ E_0^2 = \frac{2I}{\in_0 C}$ or $E_0 = \sqrt{\frac{2I}{\in_0 C}}$

$E_0 = \sqrt{\frac{2 \times 2.5 \times 10^{14}}{8.85 \times 10^{-12} \times 3 \times 10^8}} = 0.4339 \times 10^9 = 4.33 \times 10^8$ N/c.

$B_0 = \mu_0 \in_0 C\ E_0$

$= 4 \times 3.14 \times 10^{-7} \times 8.854 \times 10^{-12} \times 3 \times 10^8 \times 4.33 \times 10^8 = 1.44$ T.

9. Intensity of wave = $\frac{1}{2} \in_0 E_0^2 C$

$\in_0 = 8.85 \times 10^{-12}$; $E_0 = ?$; $C = 3 \times 10^8$, $I = 1380\ W/m^2$

$1380 = 1/2 \times 8.85 \times 10^{-12} \times E_0^2 \times 3 \times 10^8$

$\Rightarrow\ E_0^2 = \frac{2 \times 1380}{8.85 \times 3 \times 10^{-4}} = 103.95 \times 10^4$

$\Rightarrow\ E_0 = 10.195 \times 10^2 = 1.02 \times 10^3$

$E_0 = B_0 C$

$\Rightarrow\ B_0 = E_0/C = \frac{1.02 \times 10^3}{3 \times 10^8} = 3.398 \times 10^{-5} = 3.4 \times 10^{-5}$ T.

♠ ♠ ♠