CHAPTER – 23
HEAT AND TEMPERATURE
EXERCISES

1. Ice point = 20° (L_0) L_1 = 32°
   Steam point = 80° (L_{100})
   $$T = \frac{L_1 - L_0}{L_{100} - L_0} \times 100 = \frac{32-20}{80-20} \times 100 = 20°C$$
2. P_{tr} = 1.500 × 10^4 Pa
   P = 2.050 × 10^4 Pa
   We know, For constant volume gas Thermometer
   $$T = \frac{P}{P_{tr}} \times 273.16\,K = \frac{2.050 \times 10^4}{1.500 \times 10^4} \times 273.16 = 373.31$$
3. Pressure Measured at M.P = 2.2 × Pressure at Triple Point
   $$T = \frac{P}{P_{tr}} \times 273.16 = \frac{2.2 \times P_{tr}}{P_{tr}} \times 273.16 = 600.952\ K \approx 601\ K$$
4. P_{tr} = 40 × 10^3 Pa, P = ?
   T = 100°C = 373 K, $T = \frac{P}{P_{tr}} \times 273.16\,K$
   $$\Rightarrow P = \frac{T \times P_{tr}}{273.16} = \frac{373 \times 49 \times 10^3}{273.16} = 54620\ Pa = 5.42 \times 10^3\ pa \approx 55\ K\ Pa$$
5. P_1 = 70 K Pa, P_2 = ?
   T_1 = 273 K, T_2 = 373K
   $T = \frac{P_1}{P_{tr}} \times 273.16$ $\Rightarrow 273 = \frac{70 \times 10^3}{P_{tr}} \times 273.16$ $\Rightarrow P_{tr}\ \frac{70 \times 273.16 \times 10^3}{273}$
   $T_2 = \frac{P_2}{P_{tr}} \times 273.16$ $\Rightarrow 373 = \frac{P_2 \times 273}{70 \times 273.16 \times 10^3}$ $\Rightarrow P_2 = \frac{373 \times 70 \times 10^3}{273}$ = 95.6 K Pa
6. $P_{ice\ point} = P_{0°}$ = 80 cm of Hg
   $P_{steam\ point} = P_{100°}$ 90 cm of Hg
   P_0 = 100 cm
   $$t = \frac{P - P_0}{P_{100} - P_0} \times 100° = \frac{80-100}{90-100} \times 100 = 200°C$$
7. $T' = \frac{V}{V - V'} T_0$ T_0 = 273,
   V = 1800 CC, V′ = 200 CC
   $$T' = \frac{1800}{1600} \times 273 = 307.125 \approx 307$$
8. $R_t = 86\Omega$; $R_{0°} = 80\Omega$; $R_{100°} = 90\Omega$
   $$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{86-80}{90-80} \times 100 = 60°C$$
9. R at ice point (R_0) = 20Ω
   R at steam point (R_{100}) = 27.5Ω
   R at Zinc point (R_{420}) = 50Ω
   $R_\theta = R_0 (1 + \alpha\theta + \beta\theta^2)$
   $\Rightarrow R_{100} = R_0 + R_0\,\alpha\theta + R_0\,\beta\theta^2$
   $$\Rightarrow \frac{R_{100} - R_0}{R_0} = \alpha\theta + \beta\theta^2$$

$\Rightarrow \frac{27.5-20}{20} = \alpha \times 100 + \beta \times 10000$

$\Rightarrow \frac{7.5}{20} = 100\,\alpha + 10000\,\beta$

$R_{420} = R_0\,(1+ \alpha\theta + \beta\theta^2) \Rightarrow \frac{50-R_0}{R_0} = \alpha\theta + \beta\theta^2$

$\Rightarrow \frac{50-20}{20} = 420 \times \alpha + 176400 \times \beta$ $\Rightarrow \frac{3}{2} = 420\,\alpha + 176400\,\beta$

$\Rightarrow \frac{7.5}{20} = 100\,\alpha + 10000\,\beta$ $\Rightarrow \frac{3}{2} = 420\,\alpha + 176400\,\beta$

10. L_1 = ?, L_0 = 10 m, $\alpha = 1 \times 10^{-5}$/°C, t= 35
$L_1 = L_0\,(1 + \alpha t) = 10(1 + 10^{-5} \times 35) = 10 + 35 \times 10^{-4} = 10.0035$m

11. t_1 = 20°C, t_2 = 10°C, L_1 = 1cm = 0.01 m, L_2 =?
$\alpha_{steel} = 1.1 \times 10^{-5}$ /°C
$L_2 = L_1\,(1 + \alpha_{steel}\Delta T) = 0.01(1 + 101 \times 10^{-5} \times 10) = 0.01 + 0.01 \times 1.1 \times 10^{-4}$
$= 10^4 \times 10^{-6} + 1.1 \times 10^{-6} = 10^{-6}\,(10000 + 1.1) = 10001.1$
$=1.00011 \times 10^{-2}$ m = 1.00011 cm

12. L_0 = 12 cm, $\alpha = 11 \times 10^{-5}$ /°C
tw = 18°C ts = 48°C
$Lw = L_0(1 + \alpha tw) = 12\,(1 + 11 \times 10^{-5} \times 18) = 12.002376$ m
$Ls = L_0\,(1 + \alpha ts) = 12\,(1 + 11 \times 10^{-5} \times 48) = 12.006336$ m
$\Delta L = 12.006336 - 12.002376 = 0.00396$ m ≈ 0.4cm

13. $d_1 = 2$ cm $= 2 \times 10^{-2}$
t_1 = 0°C, t_2 = 100°C
$\alpha_{al} = 2.3 \times 10^{-5}$ /°C
$d_2 = d_1\,(1 + \alpha\Delta t) = 2 \times 10^{-2}\,(1 + 2.3 \times 10^{-5}\;10^2)$
= 0.02 + 0.000046 = 0.020046 m = 2.0046 cm

14. $L_{st} = L_{Al}$ at 20°C $\alpha_{Al} = 2.3 \times 10^{-5}$/°C
So, $Lo_{st}\,(1 - \alpha_{st} \times 20) = Lo_{Al}\,(1 - \alpha_{Al} \times 20)$ $\alpha_{st} = 1.1 \times 10^{-5}$/°C

(a) $\Rightarrow \frac{Lo_{st}}{Lo_{Al}} = \frac{(1-\alpha_{Al}\times 20)}{(1-\alpha_{st}\times 20)} = \frac{1-2.3\times 10^{-5}\times 20}{1-1.1\times 10^{-5}\times 20} = \frac{0.99954}{0.99978} = 0.999$

(b) $\Rightarrow \frac{Lo_{40st}}{Lo_{40Al}} = \frac{(1-\alpha_{Al}\times 40)}{(1-\alpha_{st}\times 40)} = \frac{1-2.3\times 10^{-5}\times 20}{1-1.1\times 10^{-5}\times 20} = \frac{0.99954}{0.99978} = 0.999$

$= \frac{Lo_{Al}}{Lo_{st}} \times \frac{1+2.3\times 10^{-5}\times 10}{273} = \frac{0.99977\times 1.00092}{1.00044} = 1.0002496 \approx 1.00025$

$\frac{Lo_{100Al}}{Lo_{100St}} = \frac{(1+\alpha_{Al}\times 100)}{(1+\alpha_{st}\times 100)} = \frac{0.99977\times 1.00092}{1.00011} = 1.00096$

15. (a) Length at 16°C = L
L = ? T_1 =16°C, T_2 = 46°C
$\alpha = 1.1 \times 10^{-5}$/°C
$\Delta L = L\alpha\Delta\theta = L \times 1.1 \times 10^{-5} \times 30$
% of error = $\left(\frac{\Delta L}{L}\times 100\right)\% = \left(\frac{L\alpha\Delta\theta}{2}\times 100\right)\% = 1.1 \times 10^{-5} \times 30 \times 100\% = 0.033\%$

(b) T_2 = 6°C
% of error = $\left(\frac{\Delta L}{L}\times 100\right)\% = \left(\frac{L\alpha\Delta\theta}{L}\times 100\right)\% = -1.1 \times 10^{-5} \times 10 \times 100 = -0.011\%$

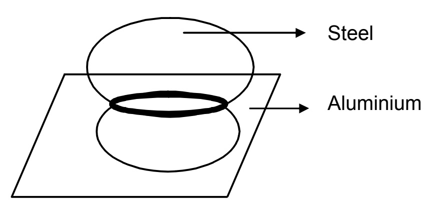
16. T_1 = 20°C, ΔL = 0.055mm = 0.55 × 10^{-3} m
t_2 = ? α_{st} = 11 × 10^{-6}/°C
We know,
$\Delta L = L_0 \alpha \Delta T$
In our case,
$0.055 \times 10^{-3} = 1 \times 1.1\ \mathrm{I}\ 10^{-6} \times (T_1 + T_2)$
$0.055 = 11 \times 10^{-3} \times 20 \pm 11 \times 10^{-3} \times T_2$
$T_2 = 20 + 5 = 25°C$ or $20 - 5 = 15°C$
The expt. Can be performed from 15 to 25°C

17. $f_{0°C}$=0.098 g/m³, $f_{4°C}$ = 1 g/m³
$$f_{0°C} = \frac{f_{4°C}}{1+\gamma\Delta T} \Rightarrow 0.998 = \frac{1}{1+\gamma \times 4} \Rightarrow 1 + 4\gamma = \frac{1}{0.998}$$
$$\Rightarrow 4 + \gamma = \frac{1}{0.998} - 1 \Rightarrow \gamma = 0.0005 \approx 5 \times 10^{-4}$$
As density decreases $\gamma = -5 \times 10^{-4}$

18. Iron rod Aluminium rod
L_{Fe} L_{Al}
$\alpha_{Fe} = 12 \times 10^{-8}$ /°C $\alpha_{Al} = 23 \times 10^{-8}$ /°C
Since the difference in length is independent of temp. Hence the different always remains constant.
$L'_{Fe} = L_{Fe}(1 + \alpha_{Fe} \times \Delta T)$ …(1)
$L'_{Al} = L_{Al}(1 + \alpha_{Al} \times \Delta T)$ …(2)
$L'_{Fe} - L'_{Al} = L_{Fe} - L_{Al} + L_{Fe} \times \alpha_{Fe} \times \Delta T - L_{Al} \times \alpha_{Al} \times \Delta T$
$$\frac{L_{Fe}}{L_{Al}} = \frac{\alpha_{Al}}{\alpha_{Fe}} = \frac{23}{12} = 23 : 12$$

19. g_1 = 9.8 m/s², g_2 = 9.788 m/s²
$$T_1 = 2\pi\frac{\sqrt{l_1}}{g_1} \qquad T_2 = 2\pi\frac{\sqrt{l_2}}{g_2} = 2\pi\frac{\sqrt{l_1(1+\Delta T)}}{g}$$
$\alpha_{Steel} = 12 \times 10^{-6}$ /°C
T_1 = 20°C T_2 = ?
$T_1 = T_2$
$$\Rightarrow 2\pi\frac{\sqrt{l_1}}{g_1} = 2\pi\frac{\sqrt{l_1(1+\Delta T)}}{g_2} \qquad \Rightarrow \frac{l_1}{g_1} = \frac{l_1(1+\Delta T)}{g_2}$$
$$\Rightarrow \frac{1}{9.8} = \frac{1+12\times10^{-6}\times\Delta T}{9.788} \qquad \Rightarrow \frac{9.788}{9.8} = 1 + 12 \times 10^{-6} \times \Delta T$$
$$\Rightarrow \frac{9.788}{9.8} - 1 = 12 \times 10^{-6}\,\Delta T \qquad \Rightarrow \Delta T = \frac{-0.00122}{12\times10^{-6}}$$
$\Rightarrow T_2 - 20 = -101.6$ $\Rightarrow T_2 = -101.6 + 20 = -81.6 \approx -82°C$

20. Given
d_{St} = 2.005 cm, d_{Al} = 2.000 cm
$\alpha_S = 11 \times 10^{-6}$ /°C $\alpha_{Al} = 23 \times 10^{-6}$ /°C
$d'_s = 2.005\,(1 + \alpha_s\,\Delta T)$ (where ΔT is change in temp.)
$\Rightarrow d'_s = 2.005 + 2.005 \times 11 \times 10^{-6}\,\Delta T$
$d'_{Al} = 2(1 + \alpha_{Al}\,\Delta T) = 2 + 2 \times 23 \times 10^{-6}\,\Delta T$
The two will slip i.e the steel ball with fall when both the diameters become equal.



So,
$\Rightarrow 2.005 + 2.005 \times 11 \times 10^{-6}\,\Delta T = 2 + 2 \times 23 \times 10^{-6}\,\Delta T$
$\Rightarrow (46 - 22.055)10^{-6} \times \Delta T = 0.005$
$$\Rightarrow \Delta T = \frac{0.005\times10^{6}}{23.945} = 208.81$$

Now $\Delta T = T_2 - T_1 = T_2 - 10°C$ [$\therefore$ $T_1 = 10°C$ given]

$\Rightarrow T_2 = \Delta T + T_1 = 208.81 + 10 = 281.81$

21. The final length of aluminium should be equal to final length of glass.

Let the initial length o faluminium = l

$l(1 - \alpha_{Al}\Delta T) = 20(1 - \alpha_0\Delta\theta)$

$\Rightarrow l(1 - 24 \times 10^{-6} \times 40) = 20\ (1 - 9 \times 10^{-6} \times 40)$

$\Rightarrow l(1 - 0.00096) = 20\ (1 - 0.00036)$

$\Rightarrow l = \dfrac{20 \times 0.99964}{0.99904} = 20.012$ cm

Let initial breadth of aluminium = b

$b(1 - \alpha_{Al}\Delta T) = 30(1 - \alpha_0\Delta\theta)$

$\Rightarrow b = \dfrac{30 \times (1 - 9 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40)} = \dfrac{30 \times 0.99964}{0.99904} = 30.018$ cm

22. $V_g = 1000$ CC, $T_1 = 20°C$

$V_{Hg} = ?$ $\gamma_{Hg} = 1.8 \times 10^{-4}$ /°C

$\gamma_g = 9 \times 10^{-6}$ /°C

ΔT remains constant

Volume of remaining space = $V'_g - V'_{Hg}$

Now

$V'_g = V_g(1 + \gamma_g\Delta T)$ …(1)

$V'_{Hg} = V_{Hg}(1 + \gamma_{Hg}\Delta T)$ …(2)

Subtracting (2) from (1)

$V'_g - V'_{Hg} = V_g - V_{Hg} + V_g\gamma_g\Delta T - V_{Hg}\gamma_{Hg}\Delta T$

$\Rightarrow \dfrac{V_g}{V_{Hg}} = \dfrac{\gamma_{Hg}}{\gamma_g} \Rightarrow \dfrac{1000}{V_{Hg}} = \dfrac{1.8 \times 10^{-4}}{9 \times 10^{-6}}$

$\Rightarrow V_{HG} = \dfrac{9 \times 10^{-3}}{1.8 \times 10^{-4}} = 500$ CC.

23. Volume of water = $500 cm^3$

Area of cross section of can = $125\ m^2$

Final Volume of water

$= 500(1 + \gamma\Delta\theta) = 500[1 + 3.2 \times 10^{-4} \times (80 - 10)] = 511.2\ cm^3$

The aluminium vessel expands in its length only so area expansion of base cab be neglected.

Increase in volume of water = $11.2\ cm^3$

Considering a cylinder of volume = $11.2\ cm^3$

Height of water increased = $\dfrac{11.2}{125} = 0.089$ cm

24. $V_0 = 10 \times 10 \times 10 = 1000$ CC

$\Delta T = 10°C$, $V'_{HG} - V'_g = 1.6\ cm^3$

$\alpha_g = 6.5 \times 10^{-6}$/°C, $\gamma_{Hg} = ?$, $\gamma_g = 3 \times 6.5 \times 10^{-6}$/°C

$V'_{Hg} = v_{HG}(1 + \gamma_{Hg}\Delta T)$ …(1)

$V'_g = v_g(1 + \gamma_g\Delta T)$ …(2)

$V'_{Hg} - V'_g = V_{Hg} - V_g + V_{Hg}\gamma_{Hg}\,\Delta T - V_g\gamma_g\,\Delta T$

$\Rightarrow 1.6 = 1000 \times \gamma_{Hg} \times 10 - 1000 \times 6.5 \times 3 \times 10^{-6} \times 10$

$\Rightarrow \gamma_{Hg} = \dfrac{1.6 + 6.3 \times 3 \times 10^{-2}}{10000} = 1.789 \times 10^{-4} \approx 1.8 \times 10^{-4}$/°C

25. $f_\omega = 880$ Kg/m^3, $f_b = 900$ Kg/m^3

$T_1 = 0°C$, $\gamma_\omega = 1.2 \times 10^{-3}$ /°C,

$\gamma_b = 1.5 \times 10^{-3}$ /°C

The sphere begins t sink when,

$(mg)_{sphere}$ = displaced water

$\Rightarrow V f'_{\omega}\, g = V f'_{b}\, g$

$\Rightarrow \dfrac{f_{\omega}}{1+\gamma_{\omega}\Delta\theta} = \dfrac{f_{b}}{1+\gamma_{b}\Delta\theta}$

$\Rightarrow \dfrac{880}{1+1.2\times10^{-3}\Delta\theta} = \dfrac{900}{1+1.5\times10^{-3}\Delta\theta}$

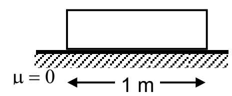
$\Rightarrow 880 + 880 \times 1.5 \times 10^{-3}\ (\Delta\theta) = 900 + 900 \times 1.2 \times 10^{-3}\ (\Delta\theta)$

$\Rightarrow (880 \times 1.5 \times 10^{-3} - 900 \times 1.2 \times 10^{-3})\ (\Delta\theta) = 20$

$\Rightarrow (1320 - 1080) \times 10^{-3}\ (\Delta\theta) = 20$

$\Rightarrow \Delta\theta = 83.3°C \approx 83°C$

26. $\Delta L = 100°C$



A longitudinal strain develops if and only if, there is an opposition to the expansion.
Since there is no opposition in this case, hence the longitudinal stain here = Zero.

27. $\theta_1 = 20°C$, $\theta_2 = 50°C$

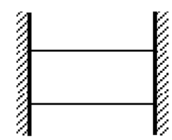
$\alpha_{steel} = 1.2 \times 10^{-5}$ /°C

Longitudinal stain = ?

Stain = $\dfrac{\Delta L}{L} = \dfrac{L\alpha\Delta\theta}{L} = \alpha\Delta\theta$

$= 1.2 \times 10^{-5} \times (50 - 20) = 3.6 \times 10^{-4}$

28. $A = 0.5 mm^2 = 0.5 \times 10^{-6}\ m^2$



$T_1 = 20°C$, $T_2 = 0°C$

$\alpha_s = 1.2 \times 10^{-5}$ /°C, $Y = 2 \times 2 \times 10^{11}\ N/m^2$

Decrease in length due to compression = $L\alpha\Delta\theta$ …(1)

$Y = \dfrac{Stress}{Strain} = \dfrac{F}{A} \times \dfrac{L}{\Delta L} \Rightarrow \Delta L = \dfrac{FL}{AY}$ …(2)

Tension is developed due to (1) & (2)

Equating them,

$L\alpha\Delta\theta = \dfrac{FL}{AY} \Rightarrow F = \alpha\Delta\theta AY$

$= 1.2 \times 10^{-5} \times (20 - 0) \times 0.5 \times 10^{-5}\ 2 \times 10^{11} = 24$ N

29. $\theta_1 = 20°C$, $\theta_2 = 100°C$

$A = 2 mm^2 = 2 \times 10^{-6}\ m^2$

$\alpha_{steel} = 12 \times 10^{-6}$ /°C, $Y_{steel} = 2 \times 10^{11}\ N/m^2$

Force exerted on the clamps = ?

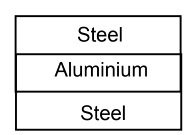
$\dfrac{\left(\dfrac{F}{A}\right)}{Strain} = Y \Rightarrow F = \dfrac{Y \times \Delta L}{L} \times L = \dfrac{YL\alpha\Delta\theta A}{L} = YA\alpha\Delta\theta$

$= 2 \times 10^{11} \times 2 \times 10^{-6} \times 12 \times 10^{-6} \times 80 = 384$ N

30. Let the final length of the system at system of temp. 0°C = ℓ_θ

Initial length of the system = ℓ_0

When temp. changes by θ.



Strain of the system = $\ell_1 - \dfrac{\ell_0}{\ell_\theta}$

But the total strain of the system = $\dfrac{\text{total stress of system}}{\text{total young's modulusof of system}}$

Now, total stress = Stress due to two steel rod + Stress due to Aluminium

$= \gamma_s\alpha_s\theta + \gamma_s\ ds\ \theta + \gamma_{al}\ at\ \theta = 2\%\ \alpha_s\ \theta + \gamma 2\ A\ell\ \theta$

Now young'' modulus of system = $\gamma_s + \gamma_s + \gamma_{al} = 2\gamma_s + \gamma_{al}$

$\therefore$ Strain of system = $\dfrac{2\gamma_s\alpha_s\theta + \gamma_s\alpha_{al}\theta}{2\gamma_s + \gamma_{al}}$

$\Rightarrow \dfrac{\ell_\theta - \ell_0}{\ell_0} = \dfrac{2\gamma_s\alpha_s\theta + \gamma_s\alpha_{al}\theta}{2\gamma_s + \gamma_{al}}$

$\Rightarrow \ell_\theta = \ell_0 \left[\dfrac{1 + \alpha_{al}\gamma_{al} + 2\alpha_s\gamma_s\theta}{\gamma_{al} + 2\gamma_s}\right]$

31. The ball tries to expand its volume. But it is kept in the same volume. So it is kept at a constant volume. So the stress arises

$\dfrac{P}{\left(\dfrac{\Delta V}{v}\right)} = B \Rightarrow P = B\dfrac{\Delta V}{V} = B \times \gamma\Delta\theta$

$= B \times 3\alpha\Delta\theta = 1.6 \times 10^{11} \times 10^{-6} \times 3 \times 12 \times 10^{-6} \times (120 - 20) = 57.6 \times 19^7 \approx 5.8 \times 10^8$ pa.

32. Given

I_0 = Moment of Inertia at 0°C

α = Coefficient of linear expansion

To prove, $I = I_0 = (1 + 2\alpha\theta)$

Let the temp. change to θ from 0°C

$\Delta T = \theta$

Let 'R' be the radius of Gyration,

Now, $R' = R(1 + \alpha\theta)$, $I_0 = MR^2$ where M is the mass.

Now, $I' = MR'^2 = MR^2 (1 + \alpha\theta)^2 \approx = MR^2 (1 + 2\alpha\theta)$

[By binomial expansion or neglecting $\alpha^2\,\theta^2$ which given a very small value.]

So, $I = I_0 (1 + 2\alpha\theta)$ (proved)

33. Let the initial m.I. at 0°C be I_0

$T = 2\pi\sqrt{\dfrac{I}{K}}$

$I = I_0 (1 + 2\alpha\Delta\theta)$ (from above question)

At 5°C, $T_1 = 2\pi\sqrt{\dfrac{I_0(1+2\alpha\Delta\theta)}{K}} = 2\pi\sqrt{\dfrac{I_0(1+2\alpha 5)}{K}} = 2\pi\sqrt{\dfrac{I_0(1+10\alpha)}{K}}$

At 45°C, $T_2 = 2\pi\sqrt{\dfrac{I_0(1+2\alpha 45)}{K}} = 2\pi\sqrt{\dfrac{I_0(1+90\alpha)}{K}}$

$\dfrac{T_2}{T_1} = \sqrt{\dfrac{1+90\alpha}{1+10\alpha}} = \sqrt{\dfrac{1+90\times 2.4\times 10^{-5}}{1+10\times 2.4\times 10^{-5}}}\ \sqrt{\dfrac{1.00216}{1.00024}}$

% change = $\left(\dfrac{T_2}{T_1} - 1\right)\times 100 = 0.0959\% = 9.6 \times 10^{-2}\%$

34. $T_1 = 20°C$, $T_2 = 50°C$, $\Delta T = 30°C$

$\alpha = 1.2 \times 10^5$ /°C

ω remains constant

(I) $\omega = \dfrac{V}{R}$ (II) $\omega = \dfrac{V'}{R'}$

Now, $R' = R(1 + \alpha\Delta\theta) = R + R \times 1.2 \times 10^{-5} \times 30 = 1.00036R$

From (I) and (II)

$\dfrac{V}{R} = \dfrac{V'}{R'} = \dfrac{V'}{1.00036R}$

$\Rightarrow V' = 1.00036\ V$

% change = $\dfrac{(1.00036V - V)}{V} \times 100 = 0.00036 \times 100 = 3.6 \times 10^{-2}$

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