# SOLUTIONS TO CONCEPTS
## CHAPTER 14

1. F = mg

   Stress = $\frac{F}{A}$

   Strain = $\frac{\Delta L}{L}$

   Y = $\frac{FL}{A\Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{YA}$

2. $\rho$ = stress = mg/A

   e = strain = $\rho$/Y

   Compression $\Delta L$ = eL

3. y = $\frac{F}{A}\frac{L}{\Delta L} \Rightarrow \Delta L = \frac{FL}{AY}$

4. $L_{steel} = L_{cu}$ and $A_{steel} = A_{cu}$

   a) $\frac{\text{Stress of cu}}{\text{Stress of st}} = \frac{F_{cu}}{A_{cu}}\frac{A_g}{F_g}$ = $\frac{F_{cu}}{F_{st}} = 1$

   b) Strain = $\frac{\Delta Lst}{\Delta lcu} = \frac{F_{st}L_{st}}{A_{st}Y_{st}} \cdot \frac{A_{cu}Y_{cu}}{F_{cu}l_{cu}}$ ($\because$ $L_{cu} = l_{st}$ ; $A_{cu} = A_{st}$)

5. $\left(\frac{\Delta L}{L}\right)_{st} = \frac{F}{AY_{st}}$

   $\left(\frac{\Delta L}{L}\right)_{cu} = \frac{F}{AY_{cu}}$

   $\frac{\text{strain steel wire}}{\text{Strain om copper wire}} = \frac{F}{AY_{st}} \times \frac{AY_{cu}}{F}(\because A_{cu} = A_{st}) = \frac{Y_{cu}}{Y_{st}}$

6. Stress in lower rod = $\frac{T_1}{A_1} \Rightarrow \frac{m_1g + \omega g}{A_1}$ $\Rightarrow$ w = 14 kg

   Stress in upper rod = $\frac{T_2}{A_u} \Rightarrow \frac{m_2g + m_1g + wg}{A_u} \Rightarrow$ w = .18 kg

   For same stress, the max load that can be put is 14 kg. If the load is increased the lower wire will break first.

   $\frac{T_1}{A_1} = \frac{m_1g + \omega g}{A_1}$ = 8 $\times$ $10^8$ $\Rightarrow$ w = 14 kg

   $\frac{T_2}{A_u} \Rightarrow \frac{m_2g + m_1g + \omega g}{A_u}$ = 8 $\times$ $10^8$ $\Rightarrow$ $\omega_0$ = 2 kg

   The maximum load that can be put is 2 kg. Upper wire will break first if load is increased.

7. $Y = \frac{F}{A}\frac{L}{\Delta L}$

8. $Y = \frac{F}{A}\frac{L}{\Delta L} \Rightarrow F = \frac{YA\ \Delta L}{L}$

9. $m_2g$ – T = $m_2a$ …(1)

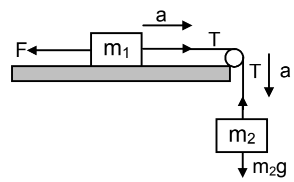
   and T – F = $m_1a$ …(2)

   $\Rightarrow$ a = $\frac{m_2g - F}{m_1 + m_2}$

From equation (1) and (2), we get $\frac{m_2 g}{2(m_1+m_2)}$

Again, T = F + $m_1$a

$\Rightarrow T = \frac{m_2 g}{2} + m_1 \frac{m_2 g}{2(m_1+m_2)} \Rightarrow \frac{m_2^2 g + 2m_1 m_2 g}{2(m_1+m_2)}$




Now Y = $\frac{FL}{A\ \Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{AY}$

$\Rightarrow \frac{\Delta L}{L} = \frac{(m_2^2 + 2m_1 m_2)g}{2(m_1+m_2)AY} = \frac{m_2 g(m_2+2m_1)}{2AY(m_1+m_2)}$

10. At equilibrium $\Rightarrow$ T = mg

When it moves to an angle θ, and released, the tension the T′ at lowest point is

$\Rightarrow T' = mg + \frac{mv^2}{r}$

The change in tension is due to centrifugal force $\Delta T = \frac{mv^2}{r}$ …(1)

$\Rightarrow$ Again, by work energy principle,

$\Rightarrow \frac{1}{2}mv^2 - 0 = mgr(1 - \cos\theta)$

$\Rightarrow v^2 = 2gr\ (1 - \cos\theta)$ …(2)

So, $\Delta T = \frac{m[2gr(1-\cos\theta)]}{r} = 2mg(1-\cos\theta)$

$\Rightarrow F = \Delta T$

$\Rightarrow F = \frac{YA\ \Delta L}{L} = 2mg - 2mg\cos\theta \Rightarrow 2mg\cos\theta = 2mg - \frac{YA\ \Delta L}{L}$

$= \cos\theta = 1 - \frac{YA\ \Delta L}{L(2mg)}$

11. From figure $\cos\theta = \frac{x}{\sqrt{x^2+l^2}} = \frac{x}{l}\left[1+\frac{x^2}{l^2}\right]^{-1/2}$

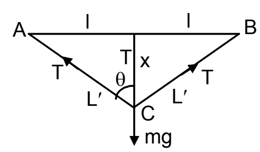
$= x / l$ … (1)

Increase in length $\Delta L$ = (AC + CB) – AB

Here, AC = $(l^2 + x^2)^{1/2}$

So, $\Delta L = 2(l^2 + x^2)^{1/2} - 100$ …(2)

$Y = \frac{F}{A}\frac{l}{\Delta l}$ …(3)




From equation (1), (2) and (3) and the freebody diagram,

2l cosθ = mg.

12. $Y = \frac{FL}{A\Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{Ay}$

$\sigma = \frac{\Delta D/D}{\Delta L/L} \Rightarrow \frac{\Delta D}{D} = \frac{\Delta L}{L}$

Again, $\frac{\Delta A}{A} = \frac{2\Delta r}{r}$

$\Rightarrow \Delta A = \frac{2\Delta r}{r}$

13. B = $\frac{Pv}{\Delta v} \Rightarrow$ P = $B\left(\frac{\Delta v}{v}\right)$

14. $\rho_0 = \frac{m}{V_0} = \frac{m}{V_d}$

so, $\frac{\rho_d}{\rho_0} = \frac{V_0}{V_d}$ …(1)

vol.strain = $\frac{V_0 - V_d}{V_0}$

B = $\frac{\rho_0 gh}{(V_0 - V_d)/V_0} \Rightarrow 1 - \frac{V_d}{V_0} = \frac{\rho_0 gh}{B}$

$\Rightarrow \frac{vD}{v_0} = \left(1 - \frac{\rho_0 gh}{B}\right)$ …(2)

Putting value of (2) in equation (1), we get

$\frac{\rho_d}{\rho_0} = \frac{1}{1-\rho_0 gh/B} \Rightarrow \rho_d = \frac{1}{(1-\rho_0 gh/B)} \times \rho_0$

15. $\eta = \frac{F}{A\theta}$

Lateral displacement = $l\theta$.

16. F = T l

17. a) $P = \frac{2T_{Hg}}{r}$ b) $P = \frac{4T_g}{r}$ c) $P = \frac{2T_g}{r}$

18. a) F = $P_0A$

b) Pressure = $P_0$ + (2T/r)

F = P′A = ($P_0$ + (2T/r)A

c) P = 2T/r

F = PA = $\frac{2T}{r}A$

19. a) $h_A = \frac{2T\cos\theta}{r_A - \rho g}$ b) $h_B = \frac{2T\cos\theta}{r_B \rho g}$ c) $h_C = \frac{2T\cos\theta}{r_C \rho g}$

20. $h_{Hg} = \frac{2T_{Hg}\cos\theta_{Hg}}{r\rho_{Hg} g}$

$h_\omega = \frac{2T_\omega \cos\theta_\omega}{r\rho_\omega g}$ where, the symbols have their usual meanings.

$\frac{h_\omega}{h_{Hg}} = \frac{T_\omega}{T_{Hg}} \times \frac{\rho_{Hg}}{\rho_\omega} \times \frac{\cos\theta_\omega}{\cos\theta_{Hg}}$

21. $h = \frac{2T\cos\theta}{r\rho g}$

22. P = $\frac{2T}{r}$

P = F/r

23. A = $\pi r^2$

24. $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \times 8$

$\Rightarrow$ r = R/2 = 2

Increase in surface energy = TA′ – TA

25. h = $\frac{2T\cos\theta}{r\rho g}$, h′ = $\frac{2T\cos\theta}{r\rho g}$

$\Rightarrow \cos\theta = \frac{h'r\rho g}{2T}$

So, $\theta = \cos^{-1}(1/2) = 60°$.

26. a) h = $\frac{2T\cos\theta}{r\rho g}$

b) $T \times 2\pi r \cos\theta = \pi r^2 h \times \rho \times g$

$\therefore \cos\theta = \frac{hr\rho g}{2T}$

27. $T(2l) = [1 \times (10^{-3}) \times h]\rho g$

28. Surface area = $4\pi r^2$

29. The length of small element = $r\,d\theta$

$dF = T \times r\,d\theta$

considering symmetric elements,

$dF_y = 2T\,rd\theta \,.\, \sin\theta$ $[dF_x = 0]$

so, $F = 2Tr\int_0^{\pi/2}\sin\theta d\theta = 2Tr[\cos\theta]_0^{\pi/2} = T \times 2r$

Tension $\Rightarrow 2T_1 = T \times 2r \Rightarrow T_1 = Tr$

30. a) Viscous force = $6\pi\eta rv$

b) Hydrostatic force = B = $\left(\frac{4}{3}\right)\pi r^3\sigma g$

c) $6\pi\eta\, rv + \left(\frac{4}{3}\right)\pi r^3\sigma g = mg$

$$v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta} \Rightarrow \frac{2}{9}r^2\frac{\left(\frac{m}{(4/3)\pi r^3}-\sigma\right)g}{n}$$

31. To find the terminal velocity of rain drops, the forces acting on the drop are,

i) The weight $(4/3)\pi r^3 \rho g$ downward.

ii) Force of buoyancy $(4/3)\pi r^3 \sigma g$ upward.

iii) Force of viscosity $6\pi\eta r v$ upward.

Because, $\sigma$ of air is very small, the force of buoyancy may be neglected.

Thus,

$$6\pi\eta r v = \left(\frac{4}{3}\right)\pi r^2\rho g \quad \text{or} \quad v = \frac{2r^2\rho g}{9\eta}$$

32. $v = \frac{R\eta}{\rho D} \Rightarrow R = \frac{v\rho D}{\eta}$

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