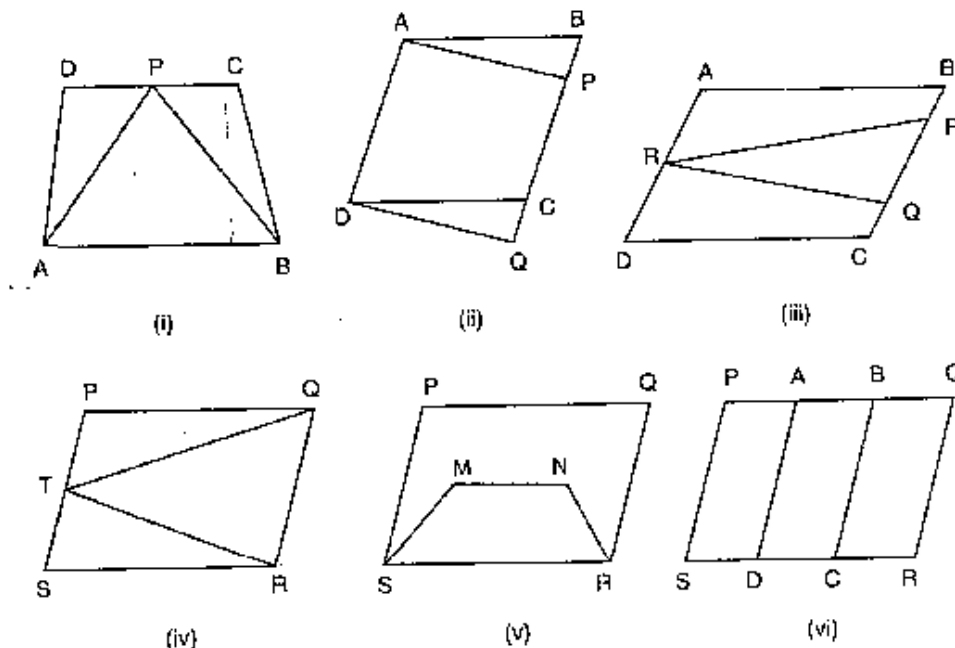


## Exercise – 15.1

1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and two parallels.

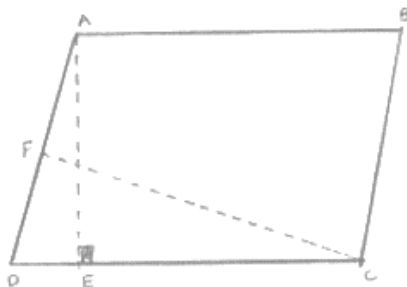


**Sol:**

- (i)  $\triangle PCD$  and trapezium ABCD are on the same base CD and between the same parallels AB and DC.
- (ii) Parallelogram ABCD and APQD are on the same base AD and between the same parallels AD and BQ.
- (iii) Parallelogram ABCD and  $\triangle PQR$  are between the same parallels AD and BC but they are not on the same base.
- (iv)  $\triangle QRT$  and parallelogram PQRS are on the same base QR and between the same parallels QR and PS.
- (v) Parallelogram PQRS and trapezium SMNR are on the same base SR but they are not between the same parallels.
- (vi) Parallelograms PQRS, AQRD, BCQR and between the same parallels also, parallelograms PQRS, BPSC and APSD are between the same parallels.

## Exercise – 15.2

1. In fig below, ABCD is a parallelogram,  $AE \perp DC$  and  $CF \perp AD$ . If  $AB = 16$  cm,  $AE = 8$  cm and  $CF = 10$  cm, find AD.



**Sol:**

Given that,

In a parallelogram  $ABCD$ ,  $CD = AB = 16$  cm [Opposite sides of a parallelogram are equal]

We know that,

Area of parallelogram = base  $\times$  corresponding altitude

Area of parallelogram  $ABCD = CD \times AE = AD \times CF$

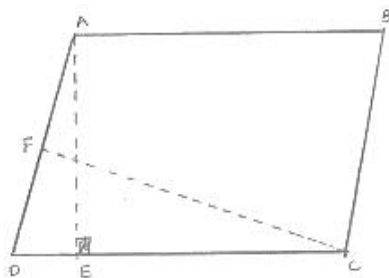
$$16 \text{ cm} \times 8 \text{ cm} = AD \times 10 \text{ cm}$$

$$AD = \frac{16 \times 8}{10} \text{ cm} = 12.8 \text{ cm}$$

Thus, the length of AD is  $12.8 \text{ cm}$

2. In Q. No 1, if  $AD = 6$  cm,  $CF = 10$  cm, and  $AE = 8$  cm, find AB.

**Sol:**



We know that,

$$\text{Area of parallelogram } ABCD = AD \times CF \quad \dots\dots(1)$$

$$\text{Again area of parallelogram } ABCD = DC \times AE \quad \dots\dots(2)$$

Compare equation (1) and equation (2)

$$AD \times CF = DC \times AE$$

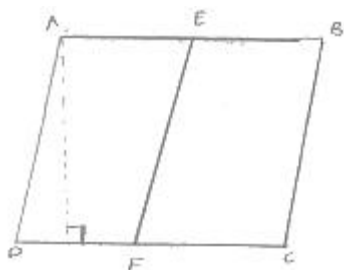
$$\Rightarrow 6 \times 10 = DC \times 8$$

$$\Rightarrow DC = \frac{60}{8} = 7.5 \text{ cm}$$

$$\therefore AB = DC = 7.5 \text{ cm} \quad [\because \text{Opposite sides of a parallelogram are equal}]$$

3. Let ABCD be a parallelogram of area  $124 \text{ cm}^2$ . If E and F are the mid-points of sides AB and CD respectively, then find the area of parallelogram AEFD.

**Sol:**



Given,

$$\text{Area of parallelogram } ABCD = 124 \text{ cm}^2$$

Construction: draw  $AP \perp DC$

Proof:

$$\text{Area of parallelogram } AFED = DF \times AP \quad \dots\dots(1)$$

$$\text{And area of parallelogram } EBCF = FC \times AP \quad \dots\dots(2)$$

$$\text{And } DF = FC \quad \dots\dots(3) \quad [\text{F is the midpoint of DC}]$$

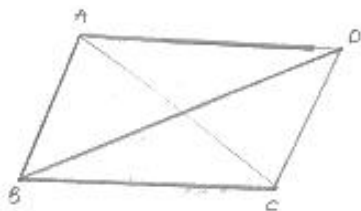
Compare equation (1), (2) and (3)

$$\text{Area of parallelogram } AEFD = \text{Area of parallelogram } EBCF$$

$$\begin{aligned} \therefore \text{Area of parallelogram } AEFD &= \frac{\text{Area of parallelogram } ABCD}{2} \\ &= \frac{124}{2} = 62 \text{ cm}^2 \end{aligned}$$

4. If ABCD is a parallelogram, then prove that  
 $\text{ar}(\triangle ABD) = \text{ar}(\triangle BCD) = \text{ar}(\triangle ABC) = \text{ar}(\triangle ACD) = \frac{1}{2} \text{ar}(\text{||}^{\text{gm}} ABCD)$

**Sol:**



Given: ABCD is a parallelogram

To prove:  $\text{area}(\triangle ABD) = \text{ar}(\triangle ABC) = \text{are}(\triangle ACD)$

$$= \frac{1}{2} \text{ar}(\text{||}^{\text{gm}} ABCD)$$

Proof: we know that diagonals of a parallelogram divides it into two equilaterals.

Since, AC is the diagonal.

$$\text{Then, } ar(\triangle ABC) = ar(\triangle ACD) = \frac{1}{2} ar(\text{||}^{\text{gm}} ABCD) \dots\dots(1)$$

Since, BD is the diagonal

$$\text{Then, } ar(\triangle ABD) = ar(\triangle BCD) = \frac{1}{2} ar(\text{||}^{\text{gm}} ABCD) \dots\dots(2)$$

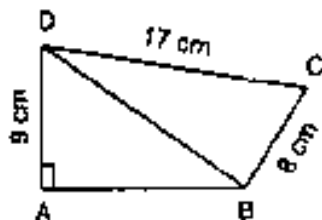
Compare equation (1) and (2)

$$\therefore ar(\triangle ABC) = ar(\triangle ACD)$$

$$= ar(\triangle ABD) = ar(\triangle BCD) = \frac{1}{2} ar(\text{||}^{\text{gm}} ABCD)$$

### Exercise – 15.3

1. In the below figure, compute the area of quadrilateral ABCD.



**Sol:**

Given that

$$DC = 17\text{ cm}$$

$$AD = 9\text{ cm and } BC = 8\text{ cm}$$

In  $\triangle BCD$  we have

$$CD^2 = BD^2 + BC^2$$

$$\Rightarrow (17)^2 = BD^2 + (8)^2$$

$$\Rightarrow BD^2 = 289 - 64$$

$$\Rightarrow BD = 15$$

In  $\triangle ABD$ , we have

$$AB^2 + AD^2 = BD^2$$

$$\Rightarrow (15)^2 = AB^2 + (9)^2$$

$$\Rightarrow AB^2 = 225 - 81 = 144$$

$$\Rightarrow AB = 12$$

$$ar(\text{quad, } ABCD) = ar(\triangle ABD) + ar(\triangle BCD)$$

$$\Rightarrow ar(\text{quad, } ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17) = 54 + 68$$

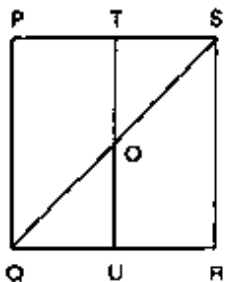
$$= 112 \text{ cm}^2$$

$$\Rightarrow \text{ar (quad, } ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 15)$$

$$= 54 + 60 \text{ cm}^2$$

$$= 114 \text{ cm}^2$$

2. In the below figure, PQRS is a square and T and U are respectively, the mid-points of PS and QR. Find the area of  $\triangle OTS$  if  $PQ = 8 \text{ cm}$ .



**Sol:**

From the figure

T and U are the midpoints of PS and QR respectively.

$$\therefore TU \parallel PQ$$

$$\Rightarrow TO \parallel PQ$$

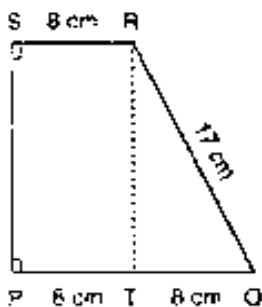
Thus, in  $\triangle PQS$ , T is the midpoint of PS and  $TO \parallel PQ$

$$\therefore TO = \frac{1}{2}PQ = 4 \text{ cm}$$

$$\text{Also, } TS = \frac{1}{2}PS = 4 \text{ cm}$$

$$\therefore \text{ar}(\triangle OTS) = \frac{1}{2}(TO \times TS) = \frac{1}{2}(4 \times 4) \text{ cm}^2 = 8 \text{ cm}^2$$

3. Compute the area of trapezium PQRS is Fig. below.



**Sol:**

We have

$$ar(\text{trap } PQRS) = ar(\text{rect } PSRT) + ar(\Delta QRT)$$

$$\Rightarrow ar(\text{trap } PQRS) = PT \times RT + \frac{1}{2}(QT \times RT)$$

$$= 8 \times RT + \frac{1}{2}(8 \times RT) = 12 \times RT$$

In  $\Delta QRT$ , we have

$$QR^2 = QT^2 + RT^2$$

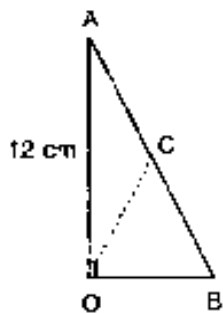
$$\Rightarrow RT^2 = QR^2 - QT^2$$

$$\Rightarrow (RT)^2 = 17^2 - 8^2 = 225$$

$$\Rightarrow RT = 15$$

$$\text{Hence, } ar(\text{trap } PQRS) = 12 \times 15 \text{ cm}^2 = 180 \text{ cm}^2$$

4. In the below fig.  $\angle AOB = 90^\circ$ ,  $AC = BC$ ,  $OA = 12$  cm and  $OC = 6.5$  cm. Find the area of  $\Delta AOB$ .



**Sol:**

Since, the midpoint of the hypotenuse of a right triangle is equidistant from the vertices

$$\therefore CA = CB = OC$$

$$\Rightarrow CA = CB = 6.5 \text{ cm}$$

$$\Rightarrow AB = 13 \text{ cm}$$

In a right angle triangle OAB, we have

$$AB^2 = OB^2 + OA^2$$

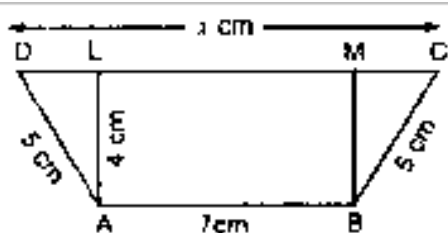
$$\Rightarrow 13^2 = OB^2 + 12^2$$

$$\Rightarrow OB^2 = 13^2 - 12^2 = 169 - 144 = 25$$

$$\Rightarrow OB = 5$$

$$\therefore ar(\Delta AOB) = \frac{1}{2}(OA \times OB) = \frac{1}{2}(12 \times 5) = 30 \text{ cm}^2$$

5. In the below fig. ABCD is a trapezium in which  $AB = 7$  cm,  $AD = BC = 5$  cm,  $DC = x$  cm, and distance between  $AB$  and  $DC$  is 4cm. Find the value of  $x$  and area of trapezium ABCD.



**Sol:**

Draw  $AL \perp DC, BM \perp DC$  Then,

$$AL = BM = 4\text{ cm and } LM = 7\text{ cm}$$

In  $\triangle ADL$ , we have

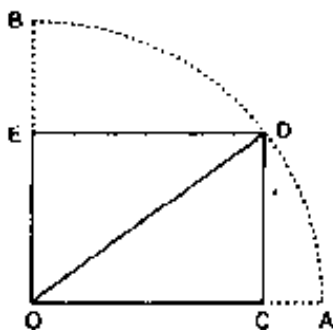
$$AD^2 = AL^2 + DL^2 \Rightarrow 25 = 16 + DL^2 \Rightarrow DL = 3\text{ cm}$$

$$\text{Similarly } MC = \sqrt{BC^2 - BM^2} = \sqrt{25 - 16} = 3\text{ cm}$$

$$\therefore x = CD = CM + ML + LD = 3 + 7 + 3 = 13\text{ cm}$$

$$\begin{aligned} \text{ar}(\text{trap. } ABCD) &= \frac{1}{2}(AB + CD) \times AL = \frac{1}{2}(7 + 13) \times 4\text{ cm}^2 \\ &= 40\text{ cm}^2 \end{aligned}$$

6. In the below fig. OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If  $OE = 2\sqrt{5}$ , find the area of the rectangle.



**Sol:**

$$\text{Given } OD = 10\text{ cm and } OE = 2\sqrt{5}\text{ cm}$$

By using Pythagoras theorem

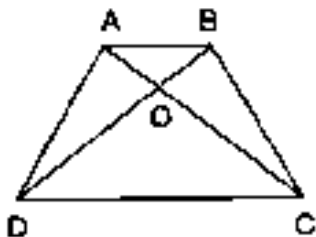
$$\therefore OD^2 = OE^2 + DE^2$$

$$\Rightarrow DE = \sqrt{OD^2 - OE^2} = \sqrt{(10)^2 - (2\sqrt{5})^2} = 4\sqrt{5}\text{ cm}$$

$$\therefore \text{ar}(\text{rect } DCDE) = OE \times DE = 2\sqrt{5} \times 4\sqrt{5}\text{ cm}^2$$

$$= 40\text{ cm}^2 \quad \left[ \because \sqrt{5} \times \sqrt{5} = 5 \right]$$

7. In the below fig. ABCD is a trapezium in which  $AB \parallel DC$ . Prove that  $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$ .



**Sol:**

Given: ABCD is a trapezium with  $AB \parallel DC$

To prove:  $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$

Proof:

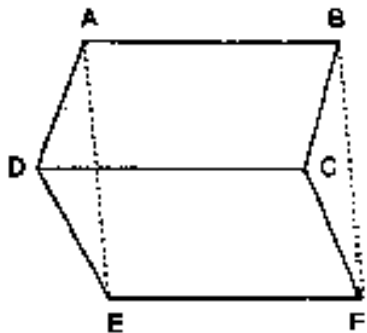
Since  $\triangle ADC$  and  $\triangle BDC$  are on the same base DC and between same parallels AB and DC

Then,  $\text{ar}(\triangle ADC) = \text{ar}(\triangle BDC)$

$$\Rightarrow \text{ar}(\triangle AOD) + \text{ar}(\triangle DOC) = \text{ar}(\triangle BOC) + \text{ar}(\triangle DOC)$$

$$\Rightarrow \text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

8. In the given below fig. ABCD, ABFE and CDEF are parallelograms. Prove that  $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$



**Sol:**

Given that,

ABCD is a parallelogram  $\Rightarrow AD = BC$

CDEF is a parallelogram  $\Rightarrow DE = CF$

ABFE is a parallelogram  $\Rightarrow AE = BF$

Thus, in  $\triangle ADE$  and  $\triangle BCF$ , we have

$AD = BC, DE = CF$  and  $AE = BF$

So, by SSS criterion of congruence, we have

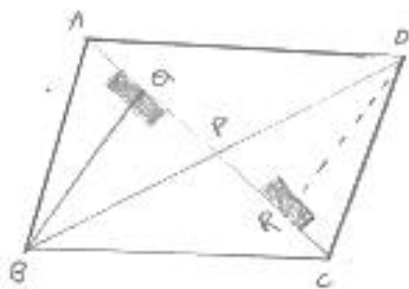
$\triangle ADE \cong \triangle BCF$

$$\therefore \text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$$



9. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that:  
 $\text{ar}(\triangle APB) \times \text{ar}(\triangle CPD) = \text{ar}(\triangle APD) \times \text{ar}(\triangle BPC)$

**Sol:**



Construction: Draw  $BQ \perp AC$  and  $DR \perp AC$

Proof:

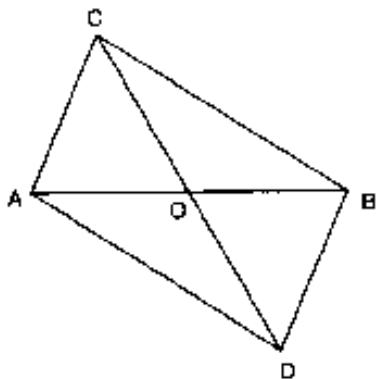
L.H.S

$$\begin{aligned}
 &= \text{ar}(\triangle APB) \times \text{ar}(\triangle CPD) \\
 &= \frac{1}{2} \left[ (AP \times BQ) \right] \times \left( \frac{1}{2} \times PC \times DR \right) \\
 &= \left( \frac{1}{2} \times PC \times BQ \right) \times \left( \frac{1}{2} \times AP \times DR \right) \\
 &= \text{ar}(\triangle BPC) \times \text{ar}(\triangle APD) \\
 &= \text{RHS}
 \end{aligned}$$

$\therefore \text{LHS} = \text{RHS}$

Hence proved.

10. In the below Fig, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that  $\text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$



**Sol:**

Given that  $CD$  is bisected at O by AB

To prove:  $\text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$

Construction: Draw  $CP \perp AB$  and  $DQ \perp AB$

Proof:-

$$ar(\triangle ABC) = \frac{1}{2} \times AB \times CP \quad \dots\dots(i)$$

$$ar(\triangle ABC) = \frac{1}{2} \times AB \times DQ \quad \dots\dots(ii)$$

In  $\triangle CPO$  and  $\triangle DQO$

$$\angle CPQ = \angle DQO \quad [Each\ 90^\circ]$$

Given that  $CO = DO$

$$\angle COP = \angle DOQ \quad [vertically\ opposite\ angles\ are\ equal]$$

Then,  $\triangle CPO \cong \triangle DQO$  [By AAS condition]

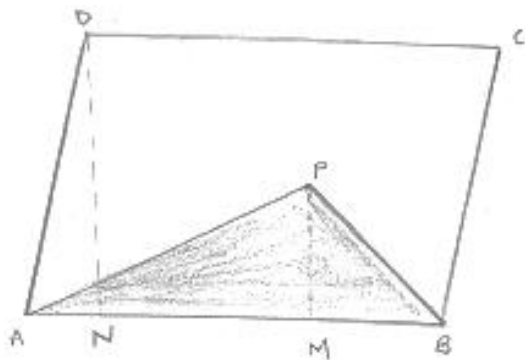
$$\therefore CP = DQ \quad \dots\dots(3) \quad [CP.C.T]$$

Compare equation (1), (2) and (3)

$$Area(\triangle ABC) = area\ of\ \triangle ABD$$

11. If P is any point in the interior of a parallelogram ABCD, then prove that area of the triangle APB is less than half the area of parallelogram.

**Sol:**



Draw  $DN \perp AB$  and  $PM \perp AB$ .

Now,

$$Area(\text{||}^{\text{gm}} ABCD) = AB \times DN, ar(\triangle APB) = \frac{1}{2}(AB \times PM)$$

Now,  $PM < DN$

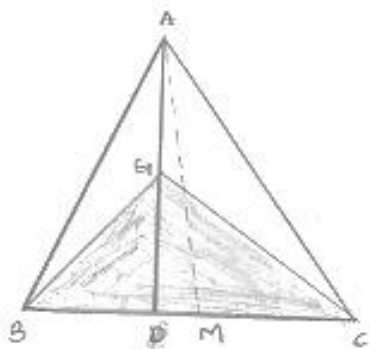
$$\Rightarrow AB \times PM < AB \times DN$$

$$\Rightarrow \frac{1}{2}(AB \times PM) < \frac{1}{2}(AB \times DN)$$

$$\Rightarrow area(\triangle APB) < \frac{1}{2} ar(\text{Parragram } ABCD)$$

12. If AD is a median of a triangle ABC, then prove that triangles ADB and ADC are equal in area. If G is the mid-point of median AD, prove that  $\text{ar}(\triangle BGC) = 2 \text{ar}(\triangle AGC)$ .

**Sol:**



Draw  $AM \perp BC$

Since, AD is the median of  $\triangle ABC$

$$\therefore BD = DC$$

$$\Rightarrow BD \times AM = DC \times AM$$

$$\Rightarrow \frac{1}{2}(BD \times AM) = \frac{1}{2}(DC \times AM)$$

$$\Rightarrow \text{ar}(\triangle ABD) = \text{ar}(\triangle ACD) \quad \dots(i)$$

In  $\triangle BGC$ , GD is the median

$$\therefore \text{ar}(\triangle BGD) = \text{ar}(\triangle CGD) \quad \dots(ii)$$

In  $\triangle ACD$ , CG is the median

$$\therefore \text{ar}(\triangle AGC) = \text{ar}(\triangle CGD) \quad \dots(iii)$$

From (i) and (ii), we have

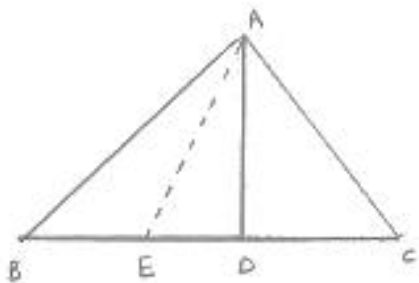
$$\text{Area}(\triangle BGD) = \text{ar}(\triangle AGC)$$

$$\text{But, } \text{ar}(\triangle BGC) = 2\text{ar}(\triangle BGD)$$

$$\therefore \text{ar}(\triangle BGC) = 2\text{ar}(\triangle AGC)$$

13. A point D is taken on the side BC of a  $\triangle ABC$  such that  $BD = 2DC$ . Prove that  $\text{ar}(\triangle ABD) = 2\text{ar}(\triangle ADC)$ .

**Sol:**



Given that,

In  $\triangle ABC$ ,  $BD = 2DC$

To prove:  $ar(\triangle ABD) = 2ar(\triangle ADC)$

Construction: Take a point E on BD such that  $BE = ED$

Proof: Since,  $BE = ED$  and  $BD = 2DC$

Then,  $BE = ED = DC$

We know that median of  $\triangle ABD$  divides it into two equal  $\triangle AED$

$\therefore$  In  $\triangle ABD$ , AE is a median

Then, area  $(\triangle ABD) = 2ar(\triangle AED)$  .....(i)

In  $\triangle AEC$ , AD is a median

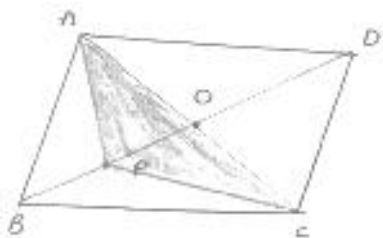
Then area  $(\triangle AED) = area(\triangle ADC)$  .....(ii)

Compare equation (i) and (ii)

Area  $(\triangle ABD) = 2ar(\triangle ADC)$ .

- 14.** ABCD is a parallelogram whose diagonals intersect at O. If P is any point on BO, prove that: (i)  $ar(\triangle ADO) = ar(\triangle CDO)$  (ii)  $ar(\triangle ABP) = ar(\triangle CBP)$

**Sol:**



Given that ABCD is a parallelogram

To prove: (i)  $ar(\triangle ADO) = ar(\triangle CDO)$

(ii)  $ar(\triangle ABP) = ar(\triangle CBP)$

Proof: We know that, diagonals of a parallelogram bisect each other

$\therefore AO = OC$  and  $BO = OD$

(i) In  $\triangle DAC$ , since DO is a median

Then area  $(\triangle ADO) = area(\triangle CDO)$

(ii) In  $\triangle BAC$ , Since BO is a median

Then; area  $(\triangle BAO) = area(\triangle BCO)$  .....(1)

In a  $\triangle PAC$ , Since PO is a median

Then, area  $(\triangle PAO) = area(\triangle PCO)$  .....(2)

Subtract equation (2) from equation (1)

$$\Rightarrow \text{area}(\triangle BAO) - \text{ar}(\triangle PAO) = \text{ar}(\triangle BCO) - \text{area}(\triangle PCO)$$

$$\Rightarrow \text{Area}(\triangle ABP) = \text{Area of } \triangle CBP$$

15. ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F.

(i) Prove that  $\text{ar}(\triangle ADF) = \text{ar}(\triangle ECF)$

(ii) If the area of  $\triangle DFB = 3 \text{ cm}^2$ , find the area of  $\parallel^{\text{gm}} \text{ABCD}$ .

**Sol:**

In triangles  $ADF$  and  $ECF$ , we have

$$\angle ADF = \angle ECF \quad [\text{Alternative interior angles, Since } AD \parallel BE]$$

$$AD = EC \quad [\text{Since } AD = BC = CE]$$

$$\text{And } \angle DFA = \angle CFA \quad [\text{vertically opposite angles}]$$

So, by AAS congruence criterion, we have

$$\triangle ADF \cong \triangle ECF$$

$$\Rightarrow \text{area}(\triangle ADF) = \text{area}(\triangle ECF) \text{ and } DF = CF.$$

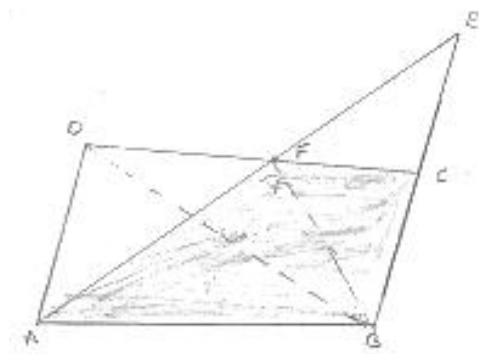
Now,  $DF = CF$

$$\Rightarrow BF \text{ is a median in } \triangle BCD$$

$$\Rightarrow \text{area}(\triangle BCD) = 2\text{ar}(\triangle BDF)$$

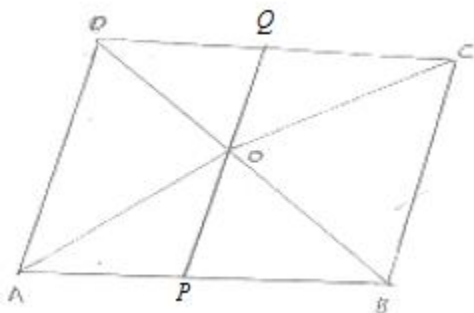
$$\Rightarrow \text{area}(\triangle BCD) = 2 \times 3 \text{ cm}^2 = 6 \text{ cm}^2$$

$$\begin{aligned} \text{Hence, } \text{ar}(\parallel^{\text{gm}} \text{ABCD}) &= 2\text{ar}(\triangle BCD) = 2 \times 6 \text{ cm}^2 \\ &= 12 \text{ cm}^2 \end{aligned}$$



16. ABCD is a parallelogram whose diagonals AC and BD intersect at O. A line through O intersects AB at P and DC at Q. Prove that  $\text{ar}(\triangle POA) = \text{ar}(\triangle QOC)$ .

**Sol:**



In triangles  $POA$  and  $QOC$ , we have

$$\angle AOP = \angle COQ \quad [\text{vertically opposite angles}]$$

$$OA = OC \quad [\text{Diagonals of a parallelogram bisect each other}]$$

$$\angle PAC = \angle QCA \quad [AB \parallel DC; \text{alternative angles}]$$

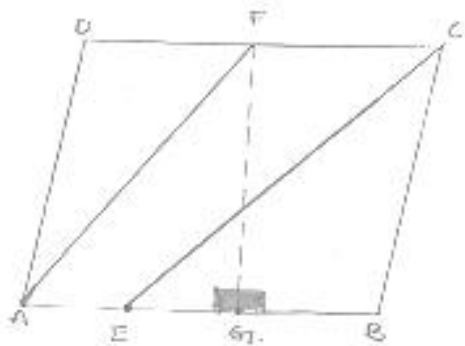
So, by ASA congruence criterion, we have

$$\Delta POA \cong \Delta QOC$$

$$\text{Area}(\Delta POA) = \text{area}(\Delta QOC).$$

17. ABCD is a parallelogram. E is a point on BA such that  $BE = 2EA$  and F is a point on DC such that  $DF = 2FC$ . Prove that AEFC is a parallelogram whose area is one third of the area of parallelogram ABCD.

**Sol:**



Construction: Draw  $FG \perp AB$

Proof: We have

$$BE = 2EA \text{ and } DF = 2FC$$

$$\Rightarrow AB - AE = 2EA \text{ and } DC - FC = 2FC$$

$$\Rightarrow AB = 3EA \text{ and } DC = 3FC$$

$$\Rightarrow AE = \frac{1}{3}AB \text{ and } FC = \frac{1}{3}DC \quad \dots\dots(1)$$

$$\text{But } AB = DC$$

$$\text{Then, } AE = FC \quad [\text{opposite sides of } \parallel^{\text{gm}}]$$

$$\text{Then, } AE \parallel FC.$$

Thus,  $AE = FC$  and  $AE \parallel FC$ .

Then,  $AECF$  is a parallelogram

Now  $ar(\parallel^{\text{gm}} AECF) = AE \times FG$

$$\Rightarrow ar(\parallel^{\text{gm}} AECF) = \frac{1}{3} AB \times FG \text{ from } (1)$$

$$\Rightarrow 3ar(\parallel^{\text{gm}} AECF) = AB \times FG \text{ ....(2)}$$

$$\text{and } area[\parallel^{\text{gm}} ABCD] = AB \times FG \text{ ....(3)}$$

Compare equation (2) and (3)

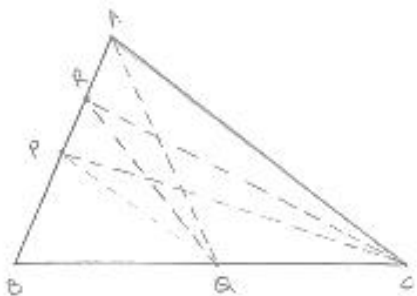
$$\Rightarrow 3 ar(\parallel^{\text{gm}} AECF) = area(\parallel^{\text{gm}} ABCD)$$

$$\Rightarrow area(\parallel^{\text{gm}} AECF) = \frac{1}{3} area(\parallel^{\text{gm}} ABCD)$$

- 18.** In a  $\triangle ABC$ , P and Q are respectively the mid-points of AB and BC and R is the mid-point of AP. Prove that :

- (i)  $ar(\triangle PBQ) = ar(\triangle ARC)$
- (ii)  $ar(\triangle PRQ) = \frac{1}{2} ar(\triangle ARC)$
- (iii)  $ar(\triangle RQC) = \frac{3}{8} ar(\triangle ABC)$ .

**Sol:**



- (i) We know that each median of a  $\triangle$  divides it into two triangles of equal area  
Since, OR is a median of  $\triangle CAP$

$$\therefore ar(\triangle CRA) = \frac{1}{2} ar(\triangle CAP) \text{ ....(i)}$$

Also,  $CP$  is a median of  $\triangle CAB$

$$\therefore ar(\triangle CAP) = ar(\triangle CPB) \text{ ....(ii)}$$

From (i) and (ii) we get

$$\therefore area(\triangle ARC) = \frac{1}{2} ar(\triangle CPB) \text{ ....(iii)}$$

$PQ$  is the median of  $\triangle PBC$

$$\therefore area(\triangle CPB) = 2area(\triangle PBQ) \text{ ....(iv)}$$

From (iii) and (iv) we get

$$\therefore \text{area}(\triangle ARC) = \text{area}(\triangle PBQ) \quad \dots\dots(v)$$

- (ii) Since QP and QR medians of  $\triangle QAB$  and  $\triangle QAP$  respectively.

$$\therefore \text{ar}(\triangle QAP) = \text{area}(\triangle QBP) \quad \dots(vi)$$

$$\text{And area}(\triangle QAP) = 2\text{ar}(\triangle QRP) \quad \dots(vii)$$

From (vi) and (vii) we have

$$\text{Area}(\triangle PRQ) = \frac{1}{2} \text{ar}(\triangle PBQ) \quad \dots(viii)$$

From (v) and (viii) we get

$$\text{Area}(\triangle PRQ) = \frac{1}{2} \text{area}(\triangle ARC)$$

- (iii) Since,  $\angle R$  is a median of  $\triangle CAP$

$$\therefore \text{area}(\triangle ARC) = \frac{1}{2} \text{ar}(\triangle CAP)$$

$$= \frac{1}{2} \times \frac{1}{2} \cdot \text{ar}(\triangle ABC)$$

$$= \frac{1}{4} \text{area}(\triangle ABC)$$

Since RQ is a median of  $\triangle RBC$

$$\therefore \text{ar}(\triangle RQC) = \frac{1}{2} \text{ar}(\triangle RBC)$$

$$= \frac{1}{2} [\text{ar}(\triangle ABC) - \text{ar}(\triangle ARC)]$$

$$= \frac{1}{2} \left[ \text{ar}(\triangle ABC) - \frac{1}{4} (\triangle ABC) \right]$$

$$= \frac{3}{8} (\triangle ABC)$$

- 19.** ABCD is a parallelogram, G is the point on AB such that  $AG = 2 GB$ , E is a point of DC such that  $CE = 2DE$  and F is the point of BC such that  $BF = 2FC$ . Prove that:

(i)  $\text{ar}(\triangle DEG) = \text{ar}(\triangle GCE)$

(ii)  $\text{ar}(\triangle EGB) = \frac{1}{6} \text{ar}(\triangle ABCD)$

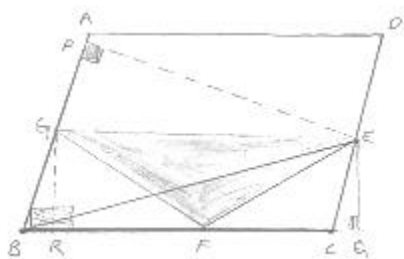
(iii)  $\text{ar}(\triangle EFC) = \frac{1}{2} \text{ar}(\triangle EBF)$

(iv)  $\text{ar}(\triangle EBG) = \text{ar}(\triangle EFC)$

- (v) Find what portion of the area of parallelogram is the area of  $\triangle EFG$ .



**Sol:**



Given,

$ABCD$  is a parallelogram

$AG = 2GB, CE = 2DE$  and  $BF = 2FC$

To prove:

(i)  $ar(ADEG) = ar(GBCE)$

(ii)  $ar(\triangle EGB) = \frac{1}{6} area(ABCD)$

(iii)  $ar(\triangle EFC) = \frac{1}{2} area(\triangle EBF)$

(iv)  $area(\triangle EBG) = \frac{3}{2} area(\triangle EFC)$

(v) Find what portion of the area of parallelogram is the area of  $\triangle EFG$ .

Construction: draw  $EP \perp AB$  and  $EQ \perp BC$

Proof : we have,

$AG = 2GB$  and  $CE = 2DE$  and  $BF = 2FC$

$\Rightarrow AB - GB = 2GB$  and  $CD - DE = 2DE$  and  $BC - FC = 2FC$

$\Rightarrow AB - GB = 2GB$  and  $CD - DE = 2DE$  and  $BC - FC = 2FC$ .

$\Rightarrow AB = 3GB$  and  $CD = 3DE$  and  $BC = 3FC$

$\Rightarrow GB = \frac{1}{3}AB$  and  $DE = \frac{1}{3}CD$  and  $FC = \frac{1}{3}BC$  ....(i)

(i)  $ar(ADEG) = \frac{1}{2}(AG + DE) \times EP$

$\Rightarrow ar(ADEG) = \frac{1}{2} \left( \frac{2}{3}AB + \frac{1}{3}CD \right) \times EP$  [By using (1)]

$\Rightarrow ar(ADEG) = \frac{1}{2} \left( \frac{2}{3}AB + \frac{1}{3}AB \right) \times EP$  [ $\because AB = CD$ ]

$\Rightarrow ar(ADEG) = \frac{1}{2} \times AB \times EP$  ....(2)

And  $ar(GBCE) = \frac{1}{2}(GB + CE) \times EP$

$$\Rightarrow ar(GBCE) = \frac{1}{2} \left[ \frac{1}{3} AB + \frac{2}{3} CD \right] \times EP \quad [\text{By using (1)}]$$

$$\Rightarrow ar(GBCE) = \frac{1}{2} \left[ \frac{1}{3} AB + \frac{2}{3} AB \right] \times EP \quad [\because AB = CD]$$

$$\Rightarrow ar(GBCE) = \frac{1}{2} \times AB \times EP \quad \dots(1)$$

Compare equation (2) and (3)

$$(ii) \quad ar(\triangle EGB) = \frac{1}{2} \times GB \times EP$$

$$= \frac{1}{6} \times AB \times EB$$

$$= \frac{1}{6} ar(1)^{9m} ABCD \Bigg].$$

$$(iii) \quad \text{Area}(\triangle EFC) = \frac{1}{2} \times FC \times EQ \quad \dots(4)$$

$$\text{And area}(\triangle EBF) = \frac{1}{2} \times BF \times EQ$$

$$\Rightarrow ar(\triangle EBF) = \frac{1}{2} \times 2FC \times EQ \quad [BF = 2FC \text{ given}]$$

$$\Rightarrow ar(\triangle EBF) = FC \times EQ \quad \dots(5)$$

Compare equation 4 and 5

$$\text{Area}(\triangle EFC) = \frac{1}{2} \times \text{area}(\triangle EBF)$$

(iv) From (i) part

$$ar(\triangle EGB) = \frac{1}{6} ar(11^{5m} ABCD) \quad \dots(6)$$

From (iii) part

$$ar(\triangle EFC) = \frac{1}{2} ar(\triangle EBF)$$

$$\Rightarrow ar(\triangle EFC) = \frac{1}{3} ar(\triangle EBC)$$

$$\Rightarrow ar(\triangle EFC) = \frac{1}{3} \times \frac{1}{2} \times CE \times EP$$

$$= \frac{1}{2} \times \frac{1}{3} \times \frac{2}{3} CD \times EP$$

$$= \frac{1}{6} \times \frac{2}{3} \times ar(11^{5m} ABCD)$$

$$\Rightarrow ar(\triangle EFC) = \frac{2}{3} \times ar(\triangle EGB) \quad [\text{By using}]$$

$$\Rightarrow ar(\triangle EGB) = \frac{3}{2} ar(\triangle EFC).$$

$$(v) \text{ Area } (\triangle EFG) = ar(\text{Trap} \cdot BGEC) = -ar(\triangle BGF) \rightarrow (1)$$

$$\text{Now, area (trap BGEC)} = \frac{1}{2}(GB + EC) \times EP$$

$$= \frac{1}{2} \left( \frac{1}{3} AB + \frac{2}{3} CD \right) \times EP$$

$$= \frac{1}{2} AB \times EP$$

$$= \frac{1}{2} ar(11^{5m} ABCD)$$

$$\text{Area } (\triangle EFC) = \frac{1}{9} \text{area}(11^{5m} ABCD) \quad [\text{From iv part}]$$

$$\text{And area}(\triangle BGF) = \frac{1}{2} BF \times GR$$

$$= \frac{1}{2} \times \frac{2}{3} BC \times GR$$

$$= \frac{2}{3} \times \frac{1}{2} BC \times GR$$

$$= \frac{2}{3} \times ar(\triangle GBC)$$

$$= \frac{2}{3} \times \frac{1}{2} GB \times EP$$

$$= \frac{1}{3} \times \frac{1}{3} AB \times EP$$

$$= \frac{1}{9} AB \times EP$$

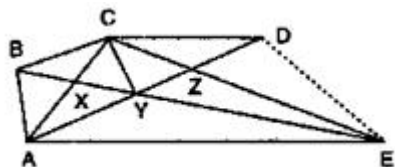
$$= \frac{1}{9} ar(11^{5m} ABCD) \quad [\text{From (1)}]$$

$$ar(\triangle EFG) = \frac{1}{2} ar(11^{5m} ABCD) = \frac{1}{9} ar(11^{5m} ABCD) = \frac{1}{9} ar(11^{5m} ABCD)$$

$$= \frac{5}{18} ar(11^{5m} ABCD).$$

**20.** In Fig. below,  $CD \parallel AE$  and  $CY \parallel BA$ .

- (i) Name a triangle equal in area of  $\triangle CBX$
- (ii) Prove that  $ar(\triangle ZDE) = ar(\triangle CZA)$
- (iii) Prove that  $ar(BCZY) = ar(\triangle EDZ)$



**Sol:**

Since,  $\triangle BCA$  and  $\triangle BYA$  are on the same base BA and between same parallels BA and CY

Then area  $(\triangle BCA) = ar(BYA)$

$$\Rightarrow ar(\Delta CBX) + ar(\Delta BXA) = ar(\Delta BXA) + ar(\Delta AXY)$$

$$\Rightarrow ar(\Delta CBX) = ar(\Delta AXY) \dots\dots(1)$$

Since,  $\triangle ACE$  and  $\triangle ADE$  are on the same base AE and between same parallels CD and AE

Then,  $ar(\Delta ACE) = ar(\Delta ADE)$

$$\Rightarrow ar(\Delta CLA) + ar(\Delta AZE) = ar(\Delta AZE) + ar(\Delta DZE)$$

$$\Rightarrow ar(\Delta CZA) = (\Delta DZE) \quad \dots (2)$$

Since  $\triangle CBY$  and  $\triangle CAY$  are on the same base  $CY$  and between same parallels  $BA$  and  $CY$

Then  $ar(\Delta CBY) = ar(\Delta CAY)$

Adding  $ar(\Delta CYG)$  on both sides, we get

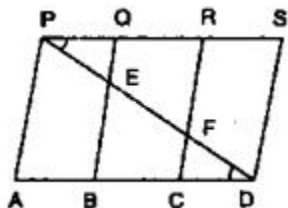
$$\Rightarrow ar(\Delta CBX) + ar(\Delta CYZ) = ar(\Delta CAY) + ar(\Delta CYZ)$$

$$\Rightarrow ar(BCZX) = ar(\Delta CZA) \dots (3)$$

Compare equation (2) and (3)

$$ar(BCZY) = ar(\Delta DZE)$$

- 21.** In below fig., PSDA is a parallelogram in which  $PQ = QR = RS$  and  $AP \parallel BQ \parallel CR$ . Prove that  $\angle PQE = \angle CRD$ .



**Sol:**

Given that PSDA is a parallelogram

Since,  $AP \parallel BQ \parallel CR \parallel DS$  and  $AD \parallel PS$

$$\therefore PQ = CD \quad \dots\dots(i)$$

In  $\triangle BED$ , C is the midpoint of BD and  $CF \parallel BE$

$\therefore F$  is the midpoint of  $ED$

$$\Rightarrow EF = PE$$

Similarly

$$EF = PE$$

$$\therefore PE = FD \quad \dots(2)$$

In  $\triangle SPQE$  and  $CFD$ , we have

$$PE = FD$$

$$\angle EDQ = \angle FDC,$$

[Alternative angles]

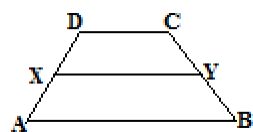
$$\text{And } PQ = CD$$

So by SAS congruence criterion, we have  $\triangle PQE \cong \triangle DCF$ .

22. In the below fig. ABCD is a trapezium in which  $AB \parallel DC$  and  $DC = 40$  cm and  $AB = 60$  cm. If X and Y are respectively, the mid-points of AD and BC, prove that:

- (i)  $XY = 50$  cm
- (ii) DCYX is a trapezium
- (iii)  $\text{ar (trap. DCYX)} = \frac{9}{11} \text{ ar (trap. (XYBA))}$

**Sol:**



- (i) Join DY and produce it to meet AB produced at P

In  $\triangle$ 's  $BYP$  and  $CYD$  we have

$$\angle BYP = (\angle CYD)$$

[Vertical opposite angles]

$$\angle DCY = \angle PBY$$

[ $\because DC \parallel AP$ ]

$$\text{And } BY = CY$$

So, by ASA congruence criterion, we have

$$\triangle BYP \cong \triangle CYD$$

$$\Rightarrow DY = YP \text{ and } DC = BP$$

$$\Rightarrow Y \text{ is the midpoint of DP}$$

Also, X is the midpoint of AD

$$\therefore XY \parallel AP \text{ and } XY = \frac{1}{2} AD$$

$$\Rightarrow xy = \frac{1}{2} (AB + DC)$$

$$\Rightarrow xy = \frac{1}{2} (60 + 40) \Rightarrow xy = \frac{1}{2} (100)$$

- (ii) We have

$$XY \parallel AP$$

$$\Rightarrow XY \parallel AB \text{ and } AB \parallel DC \quad [\text{As proved above}]$$

$$\Rightarrow XY \parallel DC$$

$$\Rightarrow DCY \text{ is a trapezium}$$

(iii) Since  $x$  and  $y$  are the midpoint of  $DA$  and  $CB$  respectively

$\therefore$  Trapezium  $DCXY$  and  $ABYX$  are of the same height say  $h$

Now

$$ar(\text{Trap } DCXY) = \frac{1}{2}(DC + XY) \times h$$

$$= \frac{1}{2}(50 + 40)h \text{ cm}^2 = 45h \text{ cm}^2$$

$$\Rightarrow ar(\text{trap } ABXY) = \frac{1}{2}(AB + XY) \times h = \frac{1}{2}(60 + 50)h \text{ cm}^2$$

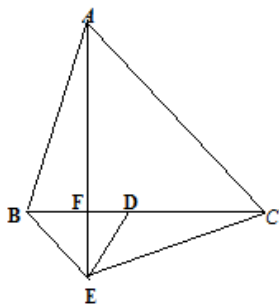
$$\Rightarrow ar(\text{trap } ABYX) = \frac{1}{2}(AB + XY) \times h = \frac{1}{2}(60 + 50)h \text{ cm}^2$$

$$= 55h \text{ cm}^2$$

$$\frac{ar \text{ trap}(YX)}{ar \text{ trap}(ABYX)} = \frac{45h}{55h} = \frac{9}{11}$$

$$\Rightarrow ar(\text{trap } DCYX) = \frac{9}{11} ar(\text{trap } ABXY)$$

23. In Fig. below,  $ABC$  and  $BDE$  are two equilateral triangles such that  $D$  is the mid-point of  $BC$ .  $AE$  intersects  $BC$  in  $F$ . Prove that



- (i)  $ar(\triangle BDE) = \frac{1}{4} ar(\triangle ABC)$
- (ii)  $area(\triangle BDE) = \frac{1}{2} ar(\triangle BAE)$
- (iii)  $ar(\triangle BEF) = ar(\triangle AFD)$ .
- (iv)  $area(\triangle ABC) = 2area(\triangle BEC)$
- (v)  $ar(\triangle FED) = \frac{1}{8} ar(\triangle AFC)$
- (vi)  $ar(\triangle BFE) = 2ar(\triangle EFD)$

**Sol:**

Given that,

 $ABC$  and  $BDE$  are two equilateral triangles.Let  $AB = BC = CA = x$ . Then  $BD = \frac{x}{2} = DE = BE$ 

(i) We have

$$ar(\triangle ABC) = \frac{\sqrt{3}}{4} x^2$$

$$ar(\triangle ABC) = \frac{\sqrt{3}}{4} \left(\frac{x}{2}\right)^2 = \frac{1}{4} \times \frac{\sqrt{3}}{4} x^2$$

$$\Rightarrow ar(\triangle BDE) = \frac{\sqrt{3}}{4} \left(\frac{x}{2}\right)^2$$

(ii) It is given that triangles  $ABC$  and  $BED$  are equilateral triangles

$$\angle ACB = \angle DBE = 60^\circ$$

 $\Rightarrow BE \parallel AC$  (Since alternative angles are equal)Triangles  $BAF$  and  $BEC$  are on the same base $BE$  and between the same parallel  $BE$  and  $AC$ 

$$\therefore ar(\triangle BAE) = area(\triangle BEC)$$

$$\Rightarrow ar(\triangle BAE) = 2ar(\triangle BDE)$$

$$[\because ED \text{ is a median of } \triangle BEC; ar(\triangle BEC) = 2ar(\triangle BDE)]$$

$$\Rightarrow area(\triangle BDE) = \frac{1}{2} ar(\triangle BAE)$$

(iii) Since  $\triangle ABC$  and  $\triangle BDE$  are equilateral triangles

$$\therefore \angle ABC = 60^\circ \text{ and } \angle BDE = 60^\circ$$

$$\angle ABC = \angle BDE$$

 $\Rightarrow AB \parallel DE$  (Since alternative angles are equal)Triangles  $BED$  and  $AED$  are on the same base  $ED$  and between the same parallels  $AB$  and  $DE$ .

$$\therefore ar(\triangle BED) = area(\triangle AED)$$

$$\Rightarrow ar(\triangle BED) - area(\triangle EFD) = area(\triangle AED) - area(\triangle EFD)$$

$$\Rightarrow ar(\triangle BEF) = ar(\triangle AFD).$$

(iv) Since  $ED$  is the median of  $\triangle BEC$ 

$$\therefore area(\triangle BEC) = 2ar(\triangle BDE)$$

$$\Rightarrow ar(\triangle BEC) = 2 \times \frac{1}{4} ar(\triangle ABC) \quad [\text{from (i)}]$$

$$\Rightarrow ar(\triangle BEC) = \frac{1}{2} area(\triangle ABC)$$

$$\Rightarrow area(\triangle ABC) = 2area(\triangle BEC)$$

- (v) Let  $h$  be the height of vertex E, corresponding to the side BD on triangle BDE  
 Let  $H$  be the height of the vertex A corresponding to the side BC in triangle ABC  
 From part (i)

$$ar(\triangle BDE) = \frac{1}{4} ar(\triangle ABC)$$

$$\Rightarrow \frac{1}{2} \times BD \times h = \frac{1}{4} ar(\triangle ABC)$$

$$\Rightarrow BD \times h = \frac{1}{4} \left( \frac{1}{2} \times BC \times H \right)$$

$$\Rightarrow h = \frac{1}{2} H \quad \dots\dots\dots(1)$$

From part .....(iii)

$$Area(\triangle BFE) = ar(\triangle AFD)$$

$$= \frac{1}{2} \times FD \times H$$

$$= \frac{1}{2} \times FD \times H$$

$$= 2 \left( \frac{1}{2} \times FD \times 2h \right)$$

$$= 2ar(\triangle EFD)$$

- (vi)  $area(\triangle AFC) = area(\triangle AFD) + area(\triangle ADC)$

$$\Rightarrow ar(\triangle BFE) + \frac{1}{2} ar(\triangle ABC)$$

[using part (iii); and AD is the median  $\triangle ABC$ ]

$$= ar(\triangle BFE) + \frac{1}{2} \times 4ar(\triangle BDE) \text{ using part (i)}$$

$$= ar(\triangle BFE) = 2ar(\triangle FED) \quad \dots\dots(3)$$

$$Area(\triangle BDE) = ar(\triangle BFE) + ar(\triangle FED)$$

$$\Rightarrow R ar(\triangle FED) + ar(\triangle FED)$$

$$\Rightarrow 3 ar(\triangle FED) \quad \dots\dots(4)$$

From (2), (3) and (4) we get

$$Area(\triangle AFC) = 2area(\triangle FED) + 2 \times 3ar(\triangle FED)$$

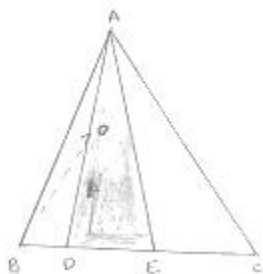
$$= 8 ar(\triangle FED)$$

$$\text{Hence, } area(\triangle FED) = \frac{1}{8} area(\triangle AFC)$$



24. D is the mid-point of side BC of  $\triangle ABC$  and E is the mid-point of BD. if O is the mid-point of AE, prove that  $\text{ar}(\triangle BOE) = \frac{1}{8} \text{ar}(\triangle ABC)$ .

**Sol:**



Given that

D is the midpoint of side BC of  $\triangle ABC$ .

E is the midpoint of BD and

O is the midpoint of AE

Since AD and AE are the medians of  $\triangle ABC$  and  $\triangle ABD$  respectively

$$\therefore \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC) \quad \dots(i)$$

$$\text{ar}(\triangle ABE) = \frac{1}{2} \text{ar}(\triangle ABD) \quad \dots(ii)$$

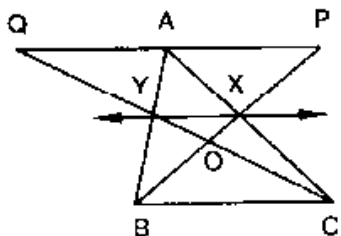
OB is a median of  $\triangle ABE$

$$\therefore \text{ar}(\triangle BOE) = \frac{1}{2} \text{ar}(\triangle ABE)$$

From i, (ii) and (iii) we have

$$\text{ar}(\triangle BOE) = \frac{1}{8} \text{ar}(\triangle ABC)$$

25. In the below fig. X and Y are the mid-points of AC and AB respectively, QP  $\parallel$  BC and CYQ and BXP are straight lines. Prove that  $\text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$ .



**Sol:**

Since x and y are the midpoint AC and AB respectively

$$\therefore XY \parallel BC$$

Clearly, triangles  $BYC$  and  $BXC$  are on the same base BC and between the same parallels XY and BC

$$\therefore \text{area}(\triangle BYC) = \text{area}(\triangle BXC)$$

$$\Rightarrow \text{area}(\triangle BYC) = \text{ar}(\triangle BOC) = \text{ar}(\triangle BXC) - \text{ar}(\triangle BOC)$$

$$\Rightarrow \text{ar}(\triangle BOY) = \text{ar}(\triangle COX)$$

$$\Rightarrow \text{ar}(\triangle BOY) + \text{ar}(\triangle XOY) = \text{ar}(\triangle COX) + \text{ar}(\triangle XOY)$$

$$\Rightarrow \text{ar}(\triangle BXY) = \text{ar}(\triangle CXY)$$

We observe that the quadrilateral XYAP and XYAQ are on the same base XY and between the same parallel XY and PQ.

$$\therefore \text{area}(\text{quad } XYAP) = \text{ar}(\text{quad } XYPA) \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\text{ar}(\triangle BXY) + \text{ar}(\text{quad } XYAP) = \text{ar}(\triangle CXY) + \text{ar}(\text{quad } XYQA)$$

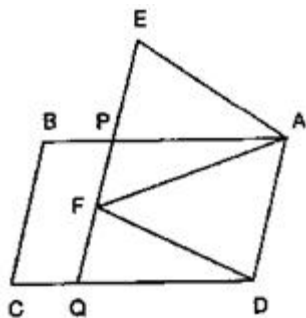
$$\Rightarrow \text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$$

**26.** In the below fig. ABCD and AEFD are two parallelograms. Prove that

(i)  $PE = FQ$

(ii)  $\text{ar}(\triangle APE) : \text{ar}(\triangle PFA) = \text{ar}(\triangle QFD) : \text{ar}(\triangle PFD)$

(iii)  $\text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$



**Sol:**

Given that,  $ABCD$  and  $AEFD$  are two parallelograms

To prove: (i)  $PE = FQ$

$$(ii) \frac{\text{ar}(\triangle APE)}{\text{ar}(\triangle PFA)} = \frac{\text{ar}(\triangle QFD)}{\text{ar}(\triangle PFD)}$$

$$(iii) \text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$$

Proof: (i) In  $\triangle EPA$  and  $\triangle FQD$

$$\angle PEA = \angle QFD \quad [\because \text{Corresponding angles}]$$

$$\angle EPA = \angle FQD \quad [\text{Corresponding angles}]$$

$$PA = QD \quad [\text{opp. sides of } \parallel^{\text{gm}}]$$

Then,  $\triangle EPA \cong \triangle FQD$  [By. AAS condition]

$$\therefore EP = FQ \quad [c.p.c.t.]$$

(ii) Since,  $\triangle PEA$  and  $\triangle QFD$  stand on the same base  $PE$  and  $FQ$  lie between the same parallels  $EQ$  and  $AD$

$$\therefore ar(\triangle PEA) = ar(\triangle QFD) \rightarrow (1)$$

$$AD \quad \therefore ar(\triangle PFA) = ar(\triangle PFD) \quad \dots(2)$$

Divide the equation (i) by equation (2)

$$\frac{\text{area of } (\triangle PEA)}{\text{area of } (\triangle PFA)} = \frac{ar\triangle(QFD)}{ar\triangle(PFD)}$$

(iii) From (i) part  $\triangle EPA \cong \triangle FQD$

$$\text{Then, } ar(\triangle EDA) = ar(\triangle FQD)$$

- 27.** In the below figure, ABCD is parallelogram. O is any point on AC.  $PQ \parallel AB$  and  $LM \parallel AD$ . Prove that  $ar(\text{11}^{gm} DLOP) = ar(\text{11}^{gm} BMOQ)$

**Sol:**

Since, a diagonal of a parallelogram divides it into two triangles of equal area

$$\therefore area(\triangle ADC) = area(\triangle ABC)$$

$$\Rightarrow area(\triangle APO) + area(\text{11}^{gm} DLOP) + area(\triangle OLC)$$

$$\Rightarrow area(\triangle AOM) + ar(\text{11}^{gm} DLOP) + area(\triangle OQC) \quad \dots(i)$$

Since,  $AO$  and  $OC$  are diagonals of parallelograms  $AMOP$  and  $OQCL$  respectively.

$$\therefore area(\triangle APO) = area(\triangle AMO) \quad \dots(ii)$$

$$\text{And, } area(\triangle OLC) = Area(\triangle OQC) \quad \dots(iii)$$

Subtracting (ii) and (iii) from (i), we get

$$Area(\text{11}^{gm} DLOP) = area(\text{11}^{gm} BMOQ)$$

- 28.** In a  $\triangle ABC$ , if  $L$  and  $M$  are points on  $AB$  and  $AC$  respectively such that  $LM \parallel BC$ . Prove that:

$$(i) \quad ar(\triangle LCM) = ar(\triangle LBM)$$

$$(ii) \quad ar(\triangle LBC) = ar(\triangle MBC)$$

$$(iii) \quad ar(\triangle ABM) = ar(\triangle ACL)$$

$$(iv) \quad ar(\triangle LOB) = ar(\triangle MOC)$$

**Sol:**

- (i) Clearly Triangles  $LMB$  and  $LMC$  are on the same base  $LM$  and between the same parallels  $LM$  and  $BC$ .

$$\therefore ar(\triangle LMB) = ar(\triangle LMC) \quad \dots\dots(i)$$

- (ii) We observe that triangles  $LBC$  and  $MBC$  are on the same base  $BC$  and between the same parallels  $LM$  and  $BC$

$$\therefore ar(\triangle LBC) = ar(\triangle MBC) \quad \dots\dots(ii)$$

- (iii) We have

$$ar(\triangle LMB) = ar(\triangle LMC) \quad [\text{from (1)}]$$

$$\Rightarrow ar(\triangle ALM) + ar(\triangle LMB) = ar(\triangle ALM) + ar(\triangle LMC)$$

$$\Rightarrow ar(\triangle ABM) = ar(\triangle ACM)$$

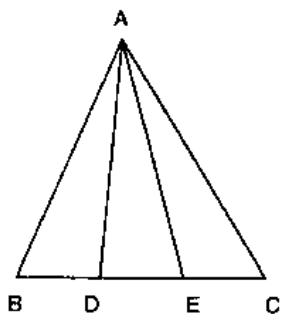
- (iv) We have

$$ar(\triangle OBC) = ar(\triangle OMC) \quad \therefore [\text{from (1)}]$$

$$\Rightarrow ar(\triangle OBC) = ar(\triangle OMC) = ar(\triangle OBC) - ar(\triangle OMC)$$

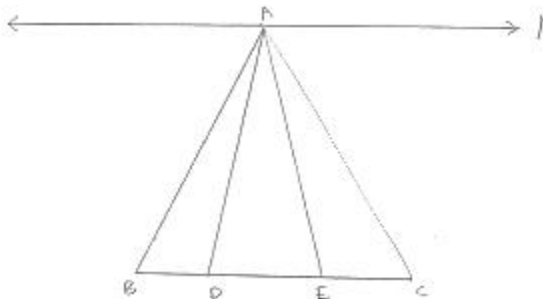
$$\Rightarrow ar(\triangle OAB) = ar(\triangle OAC)$$

29. In the below fig.  $D$  and  $E$  are two points on  $BC$  such that  $BD = DE = EC$ . Show that  $ar(\triangle ABD) = ar(\triangle ADE) = ar(\triangle AEC)$ .



**Sol:**

Draw a line through  $A$  parallel to  $BC$



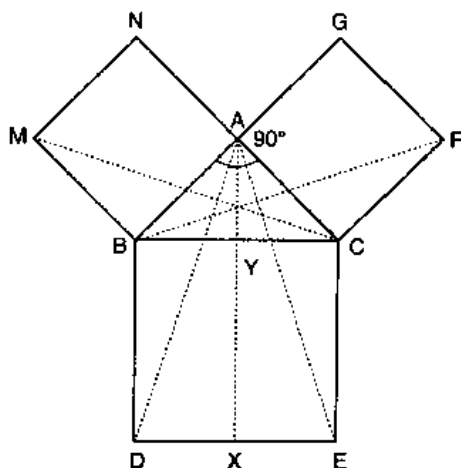
Given that,  $BD = DE = EC$

We observe that the triangles  $ABD$  and  $AEC$  are on the equal bases and between the same parallels  $l$  and  $BC$ . Therefore, Their areas are equal.

Hence,  $ar(\triangle ABD) = ar(\triangle ADE) = ar(\triangle AEC)$

30. If below fig. ABC is a right triangle right angled at A, BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment  $AX \perp DE$  meets BC at Y. Show that:

- (i)  $\triangle MBC \cong \triangle ABD$
- (ii)  $ar(BYXD) = 2 ar(\triangle MBC)$
- (iii)  $ar(BYXD) = ar(\triangle ABMN)$
- (iv)  $\triangle FCB \cong \triangle ACE$
- (v)  $ar(CYXE) = 2 ar(\triangle FCB)$
- (vi)  $ar(CYXE) = ar(ACFG)$
- (vii)  $ar(BCED) = ar(ABMN) + ar(ACFG)$



**Sol:**

- (i) In  $\triangle MBC$  and  $\triangle ABD$ , we have

$$MB = AB$$

$$BC = BD$$

$$\text{And } \angle MBC = \angle ABD$$

[ $\because \angle MBC$  and  $\angle ABD$  are obtained by adding  $\angle ABC$  to a right angle]

So, by SAS congruence criterion, We have

$$\triangle MBC \cong \triangle ABD$$

$$\Rightarrow ar(\triangle MBC) = ar(\triangle ABD) \dots\dots(1)$$

- (ii) Clearly,  $\triangle ABC$  and  $BYXD$  are on the same base BD and between the same parallels AX and BD

$$\therefore Area(\triangle ABD) = \frac{1}{2} Area(rect BYXD)$$

$$\Rightarrow ar(rect BYXD) = 2ar(\triangle ABD)$$

$$\Rightarrow are(rect BYXD) = 2ar(\triangle MBC) \dots\dots(2)$$

$$[\because ar(\triangle ABD) = ar(\triangle MBC) \dots\dots from (i)]$$

- (iii) Since triangle  $M \cdot BC$  and square  $MBAN$  are on the same Base MB and between the same parallels MB and NC

$$\therefore 2ar(\triangle MBC) = ar(MBAN) \quad \dots(3)$$

From (2) and (3) we have

$$ar(sq \cdot MBAN) = ar(rect BYXD).$$

- (iv) In triangles  $FCB$  and  $ACE$  we have

$$FC = AC$$

$$CB = CF$$

$$\text{And } \angle FCB = \angle ACE$$

[ $\because \angle FCB$  and  $\angle ACE$  are obtained by adding  $\angle ACB$  to a right angle]

So, by SAS congruence criterion, we have

$$\triangle FCB \cong \triangle ACE$$

- (v) We have

$$\triangle FCB \cong \triangle ACE$$

$$\Rightarrow ar(\triangle FCB) = ar(\triangle ECA)$$

Clearly,  $\triangle ACE$  and rectangle  $CYXE$  are on the same base CE and between the same parallels CE and AX

$$\therefore 2ar(\triangle ACE) = ar(CYXE) \quad \dots(4)$$

- (vi) Clearly,  $\triangle FCB$  and rectangle  $FCAG$  are on the same base FC and between the same parallels FC and BG

$$\therefore 2ar(\triangle FCB) = ar(FCAG) \quad \dots(5)$$

From (4) and (5), we get

$$\text{Area}(CYXE) = ar(ACFG)$$

- (vii) Applying Pythagoras theorem in  $\triangle ACB$ , we have

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC \times BD = AB \times MB + AC \times FC$$

$$\Rightarrow \text{area}(BCED) = \text{area}(ABMN) + ar(ACFG)$$