

## CHAPTER – 27

### SPECIFIC HEAT CAPACITIES OF GASES

1.  $N = 1$  mole,  $W = 20$  g/mol,  $V = 50$  m/s  
 K.E. of the vessel = Internal energy of the gas  
 $= (1/2) mv^2 = (1/2) \times 20 \times 10^{-3} \times 50 \times 50 = 25$  J  
 $25 = n \frac{3}{2} r(\Delta T) \Rightarrow 25 = 1 \times \frac{3}{2} \times 8.31 \times \Delta T \Rightarrow \Delta T = \frac{50}{3 \times 8.3} \approx 2$  K.
2.  $m = 5$  g,  $\Delta t = 25 - 15 = 10^\circ\text{C}$   
 $C_V = 0.172$  cal/g- $^\circ\text{C}$  =  $4.2$  J/Cal.  
 $dQ = du + dw$   
 Now,  $V = 0$  (for a rigid body)  
 So,  $dw = 0$ .  
 So  $dQ = du$ .  
 $Q = msdt = 5 \times 0.172 \times 10 = 8.6$  cal =  $8.6 \times 4.2 = 36.12$  Joule.
3.  $\gamma = 1.4$ ,  $w$  or piston =  $50$  kg.,  $A$  of piston =  $100$  cm $^2$   
 $P_o = 100$  kpa,  $g = 10$  m/s $^2$ ,  $x = 20$  cm.  
 $dw = pdv = \left( \frac{mg}{A} + P_o \right) Adx = \left( \frac{50 \times 10}{100 \times 10^{-4}} + 10^5 \right) 100 \times 10^{-4} \times 20 \times 10^{-2} = 1.5 \times 10^5 \times 20 \times 10^{-4} = 300$  J.  
 $nRdt = 300 \Rightarrow dT = \frac{300}{nR}$   
 $dQ = nC_p dT = nC_p \times \frac{300}{nR} = \frac{n\gamma R 300}{(\gamma - 1)nR} = \frac{300 \times 1.4}{0.4} = 1050$  J.
4.  $C_{VH_2} = 2.4$  Cal/g $^\circ\text{C}$ ,  $C_{PH_2} = 3.4$  Cal/g $^\circ\text{C}$   
 $M = 2$  g/ Mol,  $R = 8.3 \times 10^7$  erg/mol- $^\circ\text{C}$   
 We know,  $C_p - C_v = 1$  Cal/g $^\circ\text{C}$   
 So, difference of molar specific heats  
 $= C_p \times M - C_v \times M = 1 \times 2 = 2$  Cal/g $^\circ\text{C}$   
 Now,  $2 \times J = R \Rightarrow 2 \times J = 8.3 \times 10^7$  erg/mol- $^\circ\text{C} \Rightarrow J = 4.15 \times 10^7$  erg/cal.
5.  $\frac{C_p}{C_v} = 7.6$ ,  $n = 1$  mole,  $\Delta T = 50$  K  
 (a) Keeping the pressure constant,  $dQ = du + dw$ ,  
 $\Delta T = 50$  K,  $\gamma = 7/6$ ,  $m = 1$  mole,  
 $dQ = du + dw \Rightarrow nC_v dT = du + RdT \Rightarrow du = nC_p dT - RdT$   
 $= 1 \times \frac{R\gamma}{\gamma - 1} \times dT - RdT = \frac{R \times \frac{7}{6}}{\frac{7}{6} - 1} dT - RdT$   
 $= DT - RdT = 7RdT - RdT = 6RdT = 6 \times 8.3 \times 50 = 2490$  J.  
 (b) Kipping Volume constant,  $dv = nC_v dT$   
 $= 1 \times \frac{R}{\gamma - 1} \times dt = \frac{1 \times 8.3}{\frac{7}{6} - 1} \times 50$   
 $= 8.3 \times 50 \times 6 = 2490$  J  
 (c) Adiabatically  $dQ = 0$ ,  $du = -dw$   
 $= \left[ \frac{n \times R}{\gamma - 1} (T_1 - T_2) \right] = \frac{1 \times 8.3}{\frac{7}{6} - 1} (T_2 - T_1) = 8.3 \times 50 \times 6 = 2490$  J

6.  $m = 1.18 \text{ g}$ ,  $V = 1 \times 10^3 \text{ cm}^3 = 1 \text{ L}$   $T = 300 \text{ K}$ ,  $P = 10^5 \text{ Pa}$   
 $PV = nRT$  or  $n = \frac{PV}{RT} = 10^5 = \text{atm.}$   
 $N = \frac{PV}{RT} = \frac{1}{8.2 \times 10^{-2} \times 3 \times 10^2} = \frac{1}{8.2 \times 3} = \frac{1}{24.6}$   
 Now,  $C_v = \frac{1}{n} \times \frac{Q}{dt} = 24.6 \times 2 = 49.2$   
 $C_p = R + C_v = 1.987 + 49.2 = 51.187$   
 $Q = nC_p dT = \frac{1}{24.6} \times 51.187 \times 1 = 2.08 \text{ Cal.}$
7.  $V_1 = 100 \text{ cm}^3$ ,  $V_2 = 200 \text{ cm}^3$   $P = 2 \times 10^5 \text{ Pa}$ ,  $\Delta Q = 50 \text{ J}$   
 (a)  $\Delta Q = du + dw \Rightarrow 50 = du + 20 \times 10^5 (200 - 100 \times 10^{-6}) \Rightarrow 50 = du + 20 \Rightarrow du = 30 \text{ J}$   
 (b)  $30 = n \times \frac{3}{2} \times 8.3 \times 300$  [ $U = \frac{3}{2} nRT$  for monoatomic]  
 $\Rightarrow n = \frac{2}{3 \times 83} = \frac{2}{249} = 0.008$   
 (c)  $du = nC_v dT \Rightarrow C_v = \frac{dndTu}{0.008 \times 300} = \frac{30}{0.008 \times 300} = 12.5$   
 $C_p = C_v + R = 12.5 + 8.3 = 20.3$   
 (d)  $C_v = 12.5$  (Proved above)
8.  $Q = \text{Amt of heat given}$   
 Work done  $= \frac{Q}{2}$ ,  $\Delta Q = W + \Delta U$   
 for monoatomic gas  $\Rightarrow \Delta U = Q - \frac{Q}{2} = \frac{Q}{2}$   
 $V = n \frac{3}{2} RT = \frac{Q}{2} = nT \times \frac{3}{2} R = 3R \times nT$   
 Again  $Q = n C_p dT$  Where  $C_p > \text{Molar heat capacity at const. pressure.}$   
 $3RnT = ndTC_p \Rightarrow C_p = 3R$
9.  $P = KV \Rightarrow \frac{nRT}{V} = KV \Rightarrow RT = KV^2 \Rightarrow R \Delta T = 2KV \Delta U \Rightarrow \frac{R \Delta T}{2KV} = dv$   
 $dQ = du + dw \Rightarrow mcdT = C_v dT + pdv \Rightarrow msdT = C_v dT + \frac{PRdF}{2KV}$   
 $\Rightarrow ms = C_v + \frac{RKV}{2KV} \Rightarrow C_p + \frac{R}{2}$
10.  $\frac{C_p}{C_v} = \gamma$ ,  $C_p - C_v = R$ ,  $C_v = \frac{r}{\gamma - 1}$ ,  $C_p = \frac{\gamma R}{\gamma - 1}$   
 $Pdv = \frac{1}{b+1} (Rdt)$   
 $\Rightarrow 0 = C_v dT + \frac{1}{b+1} (Rdt) \Rightarrow \frac{1}{b+1} = \frac{-C_v}{R}$   
 $\Rightarrow b+1 = \frac{-R}{C_v} = \frac{-(C_p - C_v)}{C_v} = -\gamma + 1 \Rightarrow b = -\gamma$
11. Considering two gases, in Gas(1) we have,  
 $\gamma$ ,  $C_{p1}$  (Sp. Heat at const. 'P'),  $C_{v1}$  (Sp. Heat at const. 'V'),  $n_1$  (No. of moles)  
 $\frac{C_{p1}}{C_{v1}} = \gamma$  &  $C_{p1} - C_{v1} = R$

$$\Rightarrow \gamma C_{v1} - C_{v1} = R \Rightarrow C_{v1} (\gamma - 1) = R$$

$$\Rightarrow C_{v1} = \frac{R}{\gamma - 1} \text{ \& } C_{p1} = \frac{\gamma R}{\gamma - 1}$$

In Gas(2) we have,  $\gamma$ ,  $C_{p2}$  (Sp. Heat at const. 'P'),  $C_{v2}$  (Sp. Heat at const. 'V'),  $n_2$  (No. of moles)

$$\frac{C_{p2}}{C_{v2}} = \gamma \text{ \& } C_{p2} - C_{v2} = R \Rightarrow \gamma C_{v2} - C_{v2} = R \Rightarrow C_{v2} (\gamma - 1) = R \Rightarrow C_{v2} = \frac{R}{\gamma - 1} \text{ \& } C_{p2} = \frac{\gamma R}{\gamma - 1}$$

Given  $n_1 : n_2 = 1 : 2$

$$dU_1 = nC_{v1} dT \text{ \& } dU_2 = 2nC_{v2} dT = 3nC_{v1} dT$$

$$\Rightarrow C_v = \frac{C_{v1} + 2C_{v2}}{3} = \frac{\frac{R}{\gamma-1} + \frac{2R}{\gamma-1}}{3} = \frac{3R}{3(\gamma-1)} = \frac{R}{\gamma-1} \quad \dots(1)$$

$$\text{\& } C_p = \gamma C_v = \frac{\gamma R}{\gamma-1} \quad \dots(2)$$

$$\text{So, } \frac{C_p}{C_v} = \gamma \text{ [from (1) \& (2)]}$$

$$12. C_p' = 2.5 R C_p'' = 3.5 R$$

$$C_v' = 1.5 R \quad C_v'' = 2.5 R$$

$$n_1 = n_2 = 1 \text{ mol} \quad (n_1 + n_2)C_v dT = n_1 C_v dT + n_2 C_v dT$$

$$\Rightarrow C_v = \frac{n_1 C_v' + n_2 C_v''}{n_1 + n_2} = \frac{1.5R + 2.5R}{2} = 2R$$

$$C_p = C_v + R = 2R + R = 3R$$

$$\gamma = \frac{C_p}{C_v} = \frac{3R}{2R} = 1.5$$

$$13. n = \frac{1}{2} \text{ mole, } R = \frac{25}{3} \text{ J/mol-k, } \gamma = \frac{5}{3}$$

$$(a) \text{ Temp at A} = T_a, P_a V_a = nRT_a$$

$$\Rightarrow T_a = \frac{P_a V_a}{nR} = \frac{5000 \times 10^{-6} \times 100 \times 10^3}{\frac{1}{2} \times \frac{25}{3}} = 120 \text{ k.}$$

Similarly temperatures at point b = 240 k at C it is 480 k and at D it is 240 k.

(b) For ab process,

$$dQ = nC_p dT \quad [\text{since ab is isobaric}]$$

$$= \frac{1}{2} \times \frac{R\gamma}{\gamma-1} (T_b - T_a) = \frac{1}{2} \times \frac{\frac{35}{3} \times \frac{5}{3}}{\frac{5}{3}-1} \times (240 - 120) = \frac{1}{2} \times \frac{125}{9} \times \frac{3}{2} \times 120 = 1250 \text{ J}$$

$$\text{For bc, } dQ = du + dw \quad [dq = 0, \text{ Isochoric process}]$$

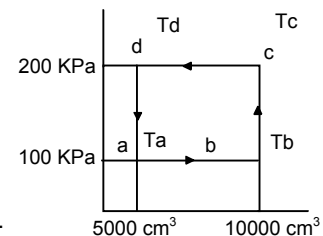
$$\Rightarrow dQ = du = nC_v dT = \frac{nR}{\gamma-1} (T_c - T_a) = \frac{1}{2} \times \frac{\frac{25}{3}}{\left(\frac{5}{3}-1\right)} (240) = \frac{1}{2} \times \frac{25}{3} \times \frac{3}{2} \times 240 = 1500 \text{ J}$$

$$(c) \text{ Heat liberated in cd} = -nC_p dT$$

$$= \frac{-1}{2} \times \frac{nR}{\gamma-1} (T_d - T_c) = \frac{-1}{2} \times \frac{125}{3} \times \frac{3}{2} \times 240 = 2500 \text{ J}$$

$$\text{Heat liberated in da} = -nC_v dT$$

$$= \frac{-1}{2} \times \frac{R}{\gamma-1} (T_a - T_d) = \frac{-1}{2} \times \frac{25}{2} \times (120 - 240) = 750 \text{ J}$$

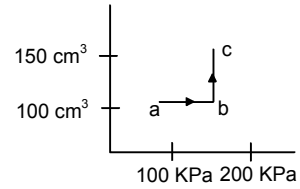


14. (a) For a, b 'V' is constant

$$\text{So, } \frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{100}{300} = \frac{200}{T_2} \Rightarrow T_2 = \frac{200 \times 300}{100} = 600 \text{ K}$$

For b, c 'P' is constant

$$\text{So, } \frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{100}{600} = \frac{150}{T_2} \Rightarrow T_2 = \frac{600 \times 150}{100} = 900 \text{ K}$$



(b) Work done = Area enclosed under the graph  $50 \text{ cc} \times 200 \text{ kPa} = 50 \times 10^{-6} \times 200 \times 10^3 \text{ J} = 10 \text{ J}$

(c) 'Q' Supplied =  $nC_v dT$

Now,  $n = \frac{PV}{RT}$  considering at pt. 'b'

$$C_v = \frac{R}{\gamma - 1} dT = 300 \text{ a, b.}$$

$$Q_{bc} = \frac{PV}{RT} \times \frac{R}{\gamma - 1} dT = \frac{200 \times 10^3 \times 100 \times 10^{-6}}{600 \times 0.67} \times 300 = 14.925 \quad (\because \gamma = 1.67)$$

$$Q \text{ supplied to be } nC_p dT \quad [\because C_p = \frac{\gamma R}{\gamma - 1}]$$

$$= \frac{PV}{RT} \times \frac{\gamma R}{\gamma - 1} dT = \frac{200 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 900} \times \frac{1.67 \times 8.3}{0.67} \times 300 = 24.925$$

(d)  $Q = \Delta U + w$

Now,  $\Delta U = Q - w = \text{Heat supplied} - \text{Work done} = (24.925 + 14.925) - 1 = 29.850$

15. In Joly's differential steam calorimeter

$$C_v = \frac{m_2 L}{m_1 (\theta_2 - \theta_1)}$$

$m_2 = \text{Mass of steam condensed} = 0.095 \text{ g}$ ,  $L = 540 \text{ Cal/g} = 540 \times 4.2 \text{ J/g}$

$m_1 = \text{Mass of gas present} = 3 \text{ g}$ ,  $\theta_1 = 20^\circ\text{C}$ ,  $\theta_2 = 100^\circ\text{C}$

$$\Rightarrow C_v = \frac{0.095 \times 540 \times 4.2}{3(100 - 20)} = 0.89 \approx 0.9 \text{ J/g-K}$$

16.  $\gamma = 1.5$

Since it is an adiabatic process, So  $PV^\gamma = \text{const.}$

$$(a) P_1 V_1^\gamma = P_2 V_2^\gamma \quad \text{Given } V_1 = 4 \text{ L, } V_2 = 3 \text{ L,} \quad \frac{P_2}{P_1} = ?$$

$$\Rightarrow \frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^\gamma = \left( \frac{4}{3} \right)^{1.5} = 1.5396 \approx 1.54$$

(b)  $TV^{\gamma-1} = \text{Const.}$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left( \frac{V_1}{V_2} \right)^{\gamma-1} = \left( \frac{4}{3} \right)^{0.5} = 1.154$$

17.  $P_1 = 2.5 \times 10^5 \text{ Pa}$ ,  $V_1 = 100 \text{ cc}$ ,  $T_1 = 300 \text{ K}$

$$(a) P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\Rightarrow 2.5 \times 10^5 \times V^{1.5} = \left( \frac{V}{2} \right)^{1.5} \times P_2$$

$$\Rightarrow P_2 = 2^{1.5} \times 2.5 \times 10^5 = 7.07 \times 10^5 \approx 7.1 \times 10^5$$

$$(b) T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow 300 \times (100)^{1.5-1} = T_2 \times (50)^{1.5-1}$$

$$\Rightarrow T_2 = \frac{3000}{7.07} = 424.32 \text{ K} \approx 424 \text{ K}$$

(c) Work done by the gas in the process

$$W = \frac{mR}{\gamma - 1} [T_2 - T_1] = \frac{P_1 V_1}{T(\gamma - 1)} [T_2 - T_1]$$

$$= \frac{2.5 \times 10^5 \times 100 \times 10^{-6}}{300(1.5 - 1)} [424 - 300] = \frac{2.5 \times 10}{300 \times 0.5} \times 124 = 20.72 \approx 21 \text{ J}$$

18.  $\gamma = 1.4$ ,  $T_1 = 20^\circ\text{C} = 293 \text{ K}$ ,  $P_1 = 2 \text{ atm}$ ,  $p_2 = 1 \text{ atm}$

We know for adiabatic process,

$$P_1^{1-\gamma} \times T_1^\gamma = P_2^{1-\gamma} \times T_2^\gamma \text{ or } (2)^{1-1.4} \times (293)^{1.4} = (1)^{1-1.4} \times T_2^{1.4}$$

$$\Rightarrow (2)^{0.4} \times (293)^{1.4} = T_2^{1.4} \Rightarrow 2153.78 = T_2^{1.4} \Rightarrow T_2 = (2153.78)^{1/1.4} = 240.3 \text{ K}$$

19.  $P_1 = 100 \text{ KPa} = 10^5 \text{ Pa}$ ,  $V_1 = 400 \text{ cm}^3 = 400 \times 10^{-6} \text{ m}^3$ ,  $T_1 = 300 \text{ K}$ ,

$$\gamma = \frac{C_P}{C_V} = 1.5$$

(a) Suddenly compressed to  $V_2 = 100 \text{ cm}^3$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow 10^5 (400)^{1.5} = P_2 \times (100)^{1.5}$$

$$\Rightarrow P_2 = 10^5 \times (4)^{1.5} = 800 \text{ KPa}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow 300 \times (400)^{1.5-1} = T_2 \times (100)^{1.5-1} \Rightarrow T_2 = \frac{300 \times 20}{10} = 600 \text{ K}$$

(b) Even if the container is slowly compressed the walls are adiabatic so heat transferred is 0.

Thus the values remain,  $P_2 = 800 \text{ KPa}$ ,  $T_2 = 600 \text{ K}$ .

20. Given  $\frac{C_P}{C_V} = \gamma$   $P_0$  (Initial Pressure),  $V_0$  (Initial Volume)

(a) (i) Isothermal compression,  $P_1 V_1 = P_2 V_2$  or,  $P_0 V_0 = \frac{P_2 V_0}{2} \Rightarrow P_2 = 2P_0$

(ii) Adiabatic Compression  $P_1 V_1^\gamma = P_2 V_2^\gamma$  or  $2P_0 \left(\frac{V_0}{2}\right)^\gamma = P_1 \left(\frac{V_0}{4}\right)^\gamma$

$$\Rightarrow P' = \frac{V_0^\gamma}{2^\gamma} \times 2P_0 \times \frac{4^\gamma}{V_0^\gamma} = 2^\gamma \times 2 P_0 \Rightarrow P_0 2^{\gamma+1}$$

(b) (i) Adiabatic compression  $P_1 V_1^\gamma = P_2 V_2^\gamma$  or  $P_0 V_0^\gamma = P' \left(\frac{V_0}{2}\right)^\gamma \Rightarrow P' = P_0 2^\gamma$

(ii) Isothermal compression  $P_1 V_1 = P_2 V_2$  or  $2^\gamma P_0 \times \frac{V_0}{2} = P_2 \times \frac{V_0}{4} \Rightarrow P_2 = P_0 2^{\gamma+1}$

21. Initial pressure =  $P_0$

Initial Volume =  $V_0$

$$\gamma = \frac{C_P}{C_V}$$

(a) Isothermally to pressure  $\frac{P_0}{2}$

$$P_0 V_0 = \frac{P_0}{2} V_1 \Rightarrow V_1 = 2 V_0$$

Adiabatically to pressure =  $\frac{P_0}{4}$

$$\frac{P_0}{2} (V_1)^\gamma = \frac{P_0}{4} (V_2)^\gamma \Rightarrow \frac{P_0}{2} (2V_0)^\gamma = \frac{P_0}{4} (V_2)^\gamma$$

$$\Rightarrow 2^{\gamma+1} V_0^\gamma = V_2^\gamma \Rightarrow V_2 = 2^{(\gamma+1)/\gamma} V_0$$

$$\therefore \text{Final Volume} = 2^{(\gamma+1)/\gamma} V_0$$

(b) Adiabatically to pressure  $\frac{P_0}{2}$  to  $P_0$

$$P_0 \times (2^{\gamma+1} V_0^\gamma) = \frac{P_0}{2} \times (V')^\gamma$$

Isothermal to pressure  $\frac{P_0}{4}$

$$\frac{P_0}{2} \times 2^{1/\gamma} V_0 = \frac{P_0}{4} \times V'' \Rightarrow V'' = 2^{(\gamma+1)/\gamma} V_0$$

$$\therefore \text{Final Volume} = 2^{(\gamma+1)/\gamma} V_0$$

22.  $PV = nRT$

$$\text{Given } P = 150 \text{ KPa} = 150 \times 10^3 \text{ Pa}, \quad V = 150 \text{ cm}^3 = 150 \times 10^{-6} \text{ m}^3, \quad T = 300 \text{ K}$$

$$(a) n = \frac{PV}{RT} = \frac{150 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 300} = 9.036 \times 10^{-3} = 0.009 \text{ moles.}$$

$$(b) \frac{C_P}{C_V} = \gamma \Rightarrow \frac{\gamma R}{(\gamma-1)C_V} = \gamma \quad \left[ \therefore C_P = \frac{\gamma R}{\gamma-1} \right]$$

$$\Rightarrow C_V = \frac{R}{\gamma-1} = \frac{8.3}{1.5-1} = \frac{8.3}{0.5} = 16.6 \text{ J/mole}$$

(c) Given  $P_1 = 150 \text{ KPa} = 150 \times 10^3 \text{ Pa}$ ,  $P_2 = ?$

$$V_1 = 150 \text{ cm}^3 = 150 \times 10^{-6} \text{ m}^3, \quad \gamma = 1.5$$

$$V_2 = 50 \text{ cm}^3 = 50 \times 10^{-6} \text{ m}^3, \quad T_1 = 300 \text{ K}, \quad T_2 = ?$$

Since the process is adiabatic Hence  $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\Rightarrow 150 \times 10^3 (150 \times 10^{-6})^\gamma = P_2 \times (50 \times 10^{-6})^\gamma$$

$$\Rightarrow P_2 = 150 \times 10^3 \times \left( \frac{150 \times 10^{-6}}{50 \times 10^{-6}} \right)^{1.5} = 150000 \times 3^{1.5} = 779.422 \times 10^3 \text{ Pa} \approx 780 \text{ KPa}$$

(d)  $\Delta Q = W + \Delta U$  or  $W = -\Delta U$  [ $\therefore \Delta U = 0$ , in adiabatic]

$$= -nC_V dT = -0.009 \times 16.6 \times (520 - 300) = -0.009 \times 16.6 \times 220 = -32.8 \text{ J} \approx -33 \text{ J}$$

(e)  $\Delta U = nC_V dT = 0.009 \times 16.6 \times 220 \approx 33 \text{ J}$

23.  $V_A = V_B = V_C$

For A, the process is isothermal

$$P_A V_A = P_A' V_A' \Rightarrow P_A' = P_A \frac{V_A}{V_A'} = P_A \times \frac{1}{2}$$

For B, the process is adiabatic,

$$P_A (V_B)^\gamma = P_A' (V_B)^\gamma = P_B' = P_B \left( \frac{V_B}{V_B'} \right)^\gamma = P_B \times \left( \frac{1}{2} \right)^{1.5} = \frac{P_B}{2^{1.5}}$$

For, C, the process is isobaric

$$\frac{V_C}{T_C} = \frac{V_C'}{T_C'} \Rightarrow \frac{V_C}{T_C} = \frac{2V_C'}{T_C'} \Rightarrow T_C' = \frac{2}{T_C}$$

Final pressures are equal.

$$= \frac{P_A}{2} = \frac{P_B}{2^{1.5}} = P_C \Rightarrow P_A : P_B : P_C = 2 : 2^{1.5} : 1 = 2 : 2\sqrt{2} : 1$$

24.  $P_1 = \text{Initial Pressure}$        $V_1 = \text{Initial Volume}$        $P_2 = \text{Final Pressure}$        $V_2 = \text{Final Volume}$

$$\text{Given, } V_2 = 2V_1, \text{ Isothermal workdone} = nRT_1 \ln \left( \frac{V_2}{V_1} \right)$$

$$\text{Adiabatic workdone} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

Given that workdone in both cases is same.

$$\text{Hence } nRT_1 \ln \left( \frac{V_2}{V_1} \right) = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} \Rightarrow (\gamma - 1) \ln \left( \frac{V_2}{V_1} \right) = \frac{P_1 V_1 - P_2 V_2}{nRT_1}$$

$$\Rightarrow (\gamma - 1) \ln \left( \frac{V_2}{V_1} \right) = \frac{nRT_1 - nRT_2}{nRT_1} \Rightarrow (\gamma - 1) \ln 2 = \frac{T_1 - T_2}{T_1} \quad \dots(i) \quad [\because V_2 = 2V_1]$$

We know  $TV^{\gamma-1} = \text{const.}$  in adiabatic Process.

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}, \text{ or } T_1 (V_2)^{\gamma-1} = T_2 \times (2)^{\gamma-1} \times (V_1)^{\gamma-1}$$

$$\text{Or, } T_1 = 2^{\gamma-1} \times T_2 \text{ or } T_2 = T_1^{1-\gamma} \quad \dots(ii)$$

From (i) & (ii)

$$(\gamma - 1) \ln 2 = \frac{T_1 - T_1 \times 2^{1-\gamma}}{T_1} \Rightarrow (\gamma - 1) \ln 2 = 1 - 2^{1-\gamma}$$

25.  $\gamma = 1.5, \quad T = 300 \text{ K}, \quad V = 1 \text{ L} \Rightarrow V = \frac{1}{2} \text{ L}$

(a) The process is adiabatic as it is sudden,

$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow P_1 (V_0)^\gamma = P_2 \left( \frac{V_0}{2} \right)^\gamma \Rightarrow P_2 = P_1 \left( \frac{1}{1/2} \right)^{1.5} = P_1 (2)^{1.5} \Rightarrow \frac{P_2}{P_1} = 2^{1.5} = 2\sqrt{2}$$

(b)  $P_1 = 100 \text{ KPa} = 10^5 \text{ Pa}$   $W = \frac{nR}{\gamma-1} [T_1 - T_2]$

$$T_1 V_1^{\gamma-1} = P_2 V_2^{\gamma-1} \Rightarrow 300 \times (1)^{1.5-1} = T_2 (0.5)^{1.5-1} \Rightarrow 300 \times 1 = T_2 \sqrt{0.5}$$

$$T_2 = 300 \times \sqrt{\frac{1}{0.5}} = 300\sqrt{2} \text{ K}$$

$$P_1 V_1 = nRT_1 \Rightarrow n = \frac{P_1 V_1}{RT_1} = \frac{10^5 \times 10^{-3}}{R \times 300} = \frac{1}{3R} \quad (V \text{ in m}^3)$$

$$w = \frac{nR}{\gamma-1} [T_1 - T_2] = \frac{1R}{3R(1.5-1)} [300 - 300\sqrt{2}] = \frac{300}{3 \times 0.5} (1 - \sqrt{2}) = -82.8 \text{ J} \approx -82 \text{ J}.$$

(c) Internal Energy,

$$dQ = 0, \quad \Rightarrow du = -dw = -(-82.8) \text{ J} = 82.8 \text{ J} \approx 82 \text{ J}.$$

(d) Final Temp =  $300\sqrt{2} = 300 \times 1.414 \times 100 = 424.2 \text{ K} \approx 424 \text{ K}.$

(e) The pressure is kept constant.  $\therefore$  The process is isobaric.

$$\text{Work done} = nRdT = \frac{1}{3R} \times R \times (300 - 300\sqrt{2}) \quad \text{Final Temp} = 300 \text{ K}$$

$$= -\frac{1}{3} \times 300 (0.414) = -41.4 \text{ J. Initial Temp} = 300\sqrt{2}$$

(f) Initial volume  $\Rightarrow \frac{V_1}{T_1} = \frac{V_1'}{T_1'} = V_1' = \frac{V_1}{T_1} \times T_1' = \frac{1}{2 \times 300 \times \sqrt{2}} \times 300 = \frac{1}{2\sqrt{2}} \text{ L}.$

Final volume = 1L

$$\text{Work done in isothermal} = nRT \ln \frac{V_2}{V_1}$$

$$= \frac{1}{3R} \times R \times 300 \ln \left( \frac{1}{1/2\sqrt{2}} \right) = 100 \times \ln(2\sqrt{2}) = 100 \times 1.039 \approx 103$$

(g) Net work done =  $W_A + W_B + W_C = -82 - 41.4 + 103 = -20.4 \text{ J}.$

26. Given  $\gamma = 1.5$

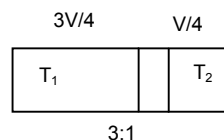
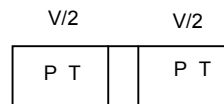
We know for adiabatic process  $TV^{\gamma-1} = \text{Const.}$

$$\text{So, } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad \dots(\text{eq})$$

As, it is an adiabatic process and all the other conditions are same. Hence the above equation can be applied.

$$\text{So, } T_1 \times \left(\frac{3V}{4}\right)^{1.5-1} = T_2 \times \left(\frac{V}{4}\right)^{1.5-1} \Rightarrow T_1 \times \left(\frac{3V}{4}\right)^{0.5} = T_2 \times \left(\frac{V}{4}\right)^{0.5}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V}{4}\right)^{0.5} \times \left(\frac{4}{3V}\right)^{0.5} = \frac{1}{\sqrt{3}} \quad \text{So, } T_1 : T_2 = 1 : \sqrt{3}$$



27.  $V = 200 \text{ cm}^3$ ,  $C = 12.5 \text{ J/mol-k}$ ,  $T = 300 \text{ K}$ ,  $P = 75 \text{ cm}$

(a) No. of moles of gas in each vessel,

$$\frac{PV}{RT} = \frac{75 \times 13.6 \times 980 \times 200}{8.3 \times 10^7 \times 300} = 0.008$$

(b) Heat is supplied to the gas but  $dv = 0$

$$dQ = du \Rightarrow 5 = nC_v dT \Rightarrow 5 = 0.008 \times 12.5 \times dT \Rightarrow dT = \frac{5}{0.008 \times 12.5} \text{ for (A)}$$

$$\text{For (B) } dT = \frac{10}{0.008 \times 12.5} \quad \therefore \frac{P}{T} = \frac{P_A}{T_A} \text{ [For container A]}$$

$$\Rightarrow \frac{75}{300} = \frac{P_A \times 0.008 \times 12.5}{5} \Rightarrow P_A = \frac{75 \times 5}{300 \times 0.008 \times 12.5} = 12.5 \text{ cm of Hg.}$$

$$\therefore \frac{P}{T} = \frac{P_B}{T_B} \text{ [For Container B]} \Rightarrow \frac{75}{300} = \frac{P_B \times 0.008 \times 12.5}{10} \Rightarrow P_B = 2 P_A = 25 \text{ cm of Hg.}$$

Mercury moves by a distance  $P_B - P_A = 25 - 12.5 = 12.5 \text{ cm}$ .

28.  $m_{\text{He}} = 0.1 \text{ g}$ ,  $\gamma = 1.67$ ,  $\mu = 4 \text{ g/mol}$ ,  $m_{\text{H}_2} = ?$

$$\mu = 28/\text{mol } \gamma_2 = 1.4$$

Since it is an adiabatic surrounding

$$\text{He } dQ = nC_v dT = \frac{0.1}{4} \times \frac{R}{\gamma-1} \times dT = \frac{0.1}{4} \times \frac{R}{(1.67-1)} \times dT \quad \dots(\text{i})$$

$$\text{H}_2 = nC_v dT = \frac{m}{2} \times \frac{R}{\gamma-1} \times dT = \frac{m}{2} \times \frac{R}{1.4-1} \times dT \quad [\text{Where } m \text{ is the reqd.}]$$

Mass of  $\text{H}_2$

Since equal amount of heat is given to both and  $\Delta T$  is same in both.

Equating (i) & (ii) we get

$$\frac{0.1}{4} \times \frac{R}{0.67} \times dT = \frac{m}{2} \times \frac{R}{0.4} \times dT \Rightarrow m = \frac{0.1}{2} \times \frac{0.4}{0.67} = 0.0298 \approx 0.03 \text{ g}$$

29. Initial pressure =  $P_0$ , Initial Temperature =  $T_0$

Initial Volume =  $V_0$

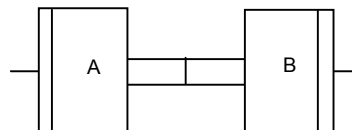
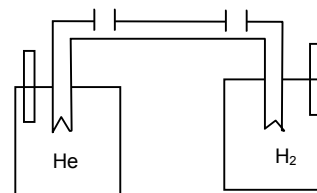
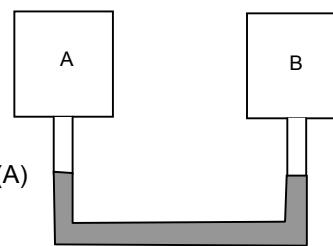
$$\frac{C_P}{C_V} = \gamma$$

(a) For the diathermic vessel the temperature inside remains constant

$$P_1 V_1 - P_2 V_2 \Rightarrow P_0 V_0 = P_2 \times 2V_0 \Rightarrow P_2 = \frac{P_0}{2}, \quad \text{Temperature} = T_0$$

For adiabatic vessel the temperature does not remain constant. The process is adiabatic

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_0 V_0^{\gamma-1} = T_2 \times (2V_0)^{\gamma-1} \Rightarrow T_2 = T_0 \left(\frac{V_0}{2V_0}\right)^{\gamma-1} = T_0 \times \left(\frac{1}{2}\right)^{\gamma-1} = \frac{T_0}{2^{\gamma-1}}$$





$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow P_0 V_0^\gamma = p_1 (2V_0)^\gamma \Rightarrow P_1 = P_0 \left( \frac{V_0}{2V_0} \right)^\gamma = \frac{P_0}{2^\gamma}$$

(b) When the values are opened, the temperature remains  $T_0$  through out

$$P_1 = \frac{n_1 RT_0}{4V_0}, P_2 = \frac{n_2 RT_0}{4V_0} \text{ [Total value after the expt} = 2V_0 + 2V_0 = 4V_0]$$

$$P = P_1 + P_2 = \frac{(n_1 + n_2)RT_0}{4V_0} = \frac{2nRT_0}{4V_0} = \frac{nRT_0}{2V} = \frac{P_0}{2}$$

30. For an adiabatic process,  $Pv^\gamma = \text{Const.}$

There will be a common pressure 'P' when the equilibrium is reached

$$\text{Hence } P_1 \left( \frac{V_0}{2} \right)^\gamma = P(V')^\gamma$$

$$\text{For left } P = P_1 \left( \frac{V_0}{2} \right)^\gamma (V')^\gamma \dots (1)$$

$$\text{For Right } P = P_2 \left( \frac{V_0}{2} \right)^\gamma (V_0 - V')^\gamma \dots (2)$$

Equating 'P' for both left & right

$$= \frac{P_1}{(V')^\gamma} = \frac{P_2}{(V_0 - V')^\gamma} \text{ or } \frac{V_0 - V'}{V'} = \left( \frac{P_2}{P_1} \right)^{1/\gamma}$$

$$\Rightarrow \frac{V_0}{V'} - 1 = \frac{P_2^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow \frac{V_0}{V'} = \frac{P_2^{1/\gamma} + P_1^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow V' = \frac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}} \text{ For left .....(3)}$$

$$\text{Similarly } V_0 - V' = \frac{V_0 P_2^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}} \text{ For right .....(4)}$$

(b) Since the whole process takes place in adiabatic surroundings. The separator is adiabatic. Hence heat given to the gas in the left part = Zero.

$$(c) \text{ From (1) Final pressure } P = \frac{P_1 \left( \frac{V_0}{2} \right)^\gamma}{(V')^\gamma}$$

$$\text{Again from (3) } V' = \frac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}} \text{ or } P = \frac{P_1 \left( \frac{V_0}{2} \right)^\gamma}{\left( \frac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}} \right)^\gamma} = \frac{P_1 (V_0)^\gamma}{2^\gamma} \times \frac{(P_1^{1/\gamma} + P_2^{1/\gamma})^\gamma}{(V_0)^\gamma P_1} = \left( \frac{P_1^{1/\gamma} + P_2^{1/\gamma}}{2} \right)^\gamma$$

31.  $A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$ ,  $M = 0.03 \text{ g} = 0.03 \times 10^{-3} \text{ kg}$ ,  
 $P = 1 \text{ atm} = 10^5 \text{ pascal}$ ,  $L = 40 \text{ cm} = 0.4 \text{ m}$ .  
 $L_1 = 80 \text{ cm} = 0.8 \text{ m}$ ,  $P = 0.355 \text{ atm}$   
 The process is adiabatic

$$P(V)^\gamma = P(V')^\gamma \Rightarrow 1 \times (AL)^\gamma = 0.355 \times (A2L)^\gamma \Rightarrow 1 = 0.355 \cdot 2^\gamma \Rightarrow \frac{1}{0.355} = 2^\gamma$$

$$= \gamma \log 2 = \log \left( \frac{1}{0.355} \right) = 1.4941$$

$$V = \sqrt{\frac{\gamma P}{f}} = \sqrt{\frac{1.4941 \times 10^5}{\text{m/v}}} = \sqrt{\frac{1.4941 \times 10^5}{\left( \frac{0.03 \times 10^{-3}}{10^{-4} \times 1 \times 0.4} \right)}} = \sqrt{\frac{1.441 \times 10^5 \times 4 \times 10^{-5}}{3 \times 10^{-5}}} = 446.33 \approx 447 \text{ m/s}$$

32.  $V = 1280 \text{ m/s}$ ,  $T = 0^\circ\text{C}$ ,  $\rho_{\text{H}_2} = 0.089 \text{ kg/m}^3$ ,  $R = 8.3 \text{ J/mol-k}$ ,  
At STP,  $P = 10^5 \text{ Pa}$ , We know

$$V_{\text{sound}} = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow 1280 = \sqrt{\frac{\gamma \times 10^5}{0.089}} \Rightarrow (1280)^2 = \frac{\gamma \times 10^5}{0.089} \Rightarrow \gamma = \frac{0.089 \times (1280)^2}{10^5} \approx 1.458$$

Again,

$$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.458 - 1} = 18.1 \text{ J/mol-k}$$

Again,  $\frac{C_P}{C_V} = \gamma$  or  $C_P = \gamma C_V = 1.458 \times 18.1 = 26.3 \text{ J/mol-k}$

33.  $\mu = 4g = 4 \times 10^{-3} \text{ kg}$ ,  $V = 22400 \text{ cm}^3 = 22400 \times 10^{-6} \text{ m}^3$   
 $C_P = 5 \text{ cal/mol-k} = 5 \times 4.2 \text{ J/mol-k} = 21 \text{ J/mol-k}$

$$C_P = \frac{\gamma R}{\gamma - 1} = \frac{\gamma \times 8.3}{\gamma - 1}$$

$$\Rightarrow 21(\gamma - 1) = \gamma (8.3) \Rightarrow 21\gamma - 21 = 8.3\gamma \Rightarrow \gamma = \frac{21}{12.7}$$

Since the condition is STP,  $P = 1 \text{ atm} = 10^5 \text{ pa}$

$$V = \sqrt{\frac{\gamma f}{\rho}} = \sqrt{\frac{\frac{21}{12.7} \times 10^5}{\frac{4 \times 10^{-3}}{22400 \times 10^{-6}}}} = \sqrt{\frac{21 \times 10^5 \times 22400 \times 10^{-6}}{12.7 \times 4 \times 10^{-3}}} = 962.28 \text{ m/s}$$

34. Given  $\rho = 1.7 \times 10^{-3} \text{ g/cm}^3 = 1.7 \text{ kg/m}^3$ ,  $P = 1.5 \times 10^5 \text{ Pa}$ ,  $R = 8.3 \text{ J/mol-k}$ ,  
 $f = 3.0 \text{ KHz}$ .

Node separation in a Kundt's tube  $= \frac{\lambda}{2} = 6 \text{ cm}$ ,  $\Rightarrow \lambda = 12 \text{ cm} = 12 \times 10^{-2} \text{ m}$

So,  $V = f\lambda = 3 \times 10^3 \times 12 \times 10^{-2} = 360 \text{ m/s}$

We know, Speed of sound  $= \sqrt{\frac{\gamma P}{\rho}} \Rightarrow (360)^2 = \frac{\gamma \times 1.5 \times 10^5}{1.7} \Rightarrow \gamma = \frac{(360)^2 \times 1.7}{1.5 \times 10^5} = 1.4688$

But  $C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.4688 - 1} = 17.72 \text{ J/mol-k}$

Again  $\frac{C_P}{C_V} = \gamma$  So,  $C_P = \gamma C_V = 17.72 \times 1.4688 = 26.01 \approx 26 \text{ J/mol-k}$

35.  $f = 5 \times 10^3 \text{ Hz}$ ,  $T = 300 \text{ K}$ ,  $\frac{\lambda}{2} = 3.3 \text{ cm} \Rightarrow \lambda = 6.6 \times 10^{-2} \text{ m}$

$V = f\lambda = 5 \times 10^3 \times 6.6 \times 10^{-2} = (66 \times 5) \text{ m/s}$

$V = \frac{\lambda P}{f} [Pv = nRT \Rightarrow P = \frac{m}{mV} \times R T \Rightarrow PM = \rho RT \Rightarrow \frac{P}{\rho} = \frac{RT}{m}]$

$$= \sqrt{\frac{\gamma RT}{m}} (66 \times 5) = \sqrt{\frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}}} \Rightarrow (66 \times 5)^2 = \frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}} \Rightarrow \gamma = \frac{(66 \times 5)^2 \times 32 \times 10^{-3}}{8.3 \times 300} = 1.3995$$

$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{0.3995} = 20.7 \text{ J/mol-k}$ ,

$C_P = C_V + R = 20.7 + 8.3 = 29.07 \text{ J/mol-k}$ .

