SOLUTIONS TO CONCEPTS CHAPTER 11

1. Gravitational force of attraction,

$$F = \frac{GMm}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 10 \times 10}{(0.1)^2} = 6.67 \times 10^{-7} \text{ N}$$

2. To calculate the gravitational force on 'm' at unline due to other mouse.

$$\overrightarrow{F_{OD}} = \frac{G \times m \times 4m}{(a/r^2)^2} = \frac{8Gm^2}{a^2}$$

$$\overrightarrow{F_{OI}} = \frac{G \times m \times 2m}{(a/r^2)^2} = \frac{6Gm^2}{a^2}$$

$$\overrightarrow{F_{OB}} = \frac{G \times m \times 2m}{(a/r^2)^2} = \frac{4Gm^2}{a^2}$$

$$\overrightarrow{F_{OA}} = \frac{G \times m \times m}{(a/r^2)^2} = \frac{2Gm^2}{a^2}$$

Resultant
$$\overrightarrow{F_{OF}} = \sqrt{64 \left(\frac{Gm^2}{a^2}\right)^2 + 36 \left(\frac{Gm^2}{a^2}\right)^2} = 10 \frac{Gm^2}{a^2}$$

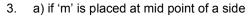
Resultant
$$\overrightarrow{F_{OE}} = \sqrt{64 \left(\frac{Gm^2}{a^2}\right)^2 + 4 \left(\frac{Gm^2}{a^2}\right)^2} = 2\sqrt{5} \frac{Gm^2}{a^2}$$

The net resultant force will be,

$$F = \sqrt{100 \left(\frac{Gm^2}{a^2}\right)^2 + 20 \left(\frac{Gm^2}{a^2}\right)^2 - 2 \left(\frac{Gm^2}{a^2}\right) \times 20\sqrt{5}}$$

$$= \sqrt{\left(\frac{Gm^2}{a^2}\right)^2 \left(120 - 40\sqrt{5}\right)} = \sqrt{\left(\frac{Gm^2}{a^2}\right)^2 (120 - 89.6)}$$

$$= \frac{Gm^2}{a^2} \sqrt{40.4} = 4\sqrt{2} \frac{Gm^2}{a^2}$$

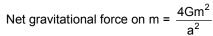


then
$$\overrightarrow{F_{OA}} = \frac{4Gm^2}{a^2}$$
 in OA direction

$$\overrightarrow{F_{OB}} = \frac{4Gm^2}{a^2}$$
 in OB direction

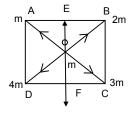
Since equal & opposite cancel each other

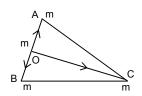
$$\overrightarrow{F_{oc}} = \frac{Gm^2}{\left[(r^3/2)a \right]^2} = \frac{4Gm^2}{3a^2}$$
 in OC direction

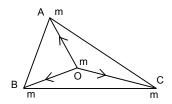


b) If placed at O (centroid)

the
$$\overrightarrow{F_{OA}} = \frac{Gm^2}{(a/r_3)} = \frac{3Gm^2}{a^2}$$







$$\overrightarrow{F_{OB}} = \frac{3Gm^2}{a^2}$$

Resultant
$$\vec{F} = \sqrt{2 \left(\frac{3Gm^2}{a^2}\right)^2 - 2 \left(\frac{3Gm^2}{a^2}\right)^2 \times \frac{1}{2}} = \frac{3Gm^2}{a^2}$$

Since
$$\overrightarrow{F_{OC}} = \frac{3Gm^2}{a^2}$$
, equal & opposite to F, cancel

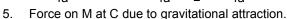
Net gravitational force = 0

4.
$$\overrightarrow{F_{CB}} = \frac{Gm^2}{4a^2}\cos 60\hat{i} - \frac{Gm^2}{4a^2}\sin 60\hat{j}$$

$$\overrightarrow{F_{CA}} = \frac{Gm^2}{-4a^2}\cos 60\hat{i} - \frac{Gm^2}{4a^2}\sin 60\hat{j}$$

$$\vec{F} = \overrightarrow{F_{CB}} + \overrightarrow{F_{CA}}$$

$$= \frac{-2Gm^2}{4a^2} \sin 60\hat{j} = \frac{-2Gm^2}{4a^2} \frac{r_3}{2} = \frac{r_3Gm^2}{4a^2}$$



$$\overrightarrow{F_{CB}} = \frac{Gm^2}{2R^2}\hat{j}$$

$$\overrightarrow{F_{CD}} = \frac{-GM^2}{4R^2}\hat{i}$$

$$\overrightarrow{F_{CA}} = \frac{-GM^2}{4R^2}\cos 45\hat{j} + \frac{GM^2}{4R^2}\sin 45\hat{j}$$

So, resultant force on C,

$$\therefore \overrightarrow{F_C} = \overrightarrow{F_{CA}} + \overrightarrow{F_{CB}} + \overrightarrow{F_{CD}}$$

$$= -\frac{GM^2}{4R^2} \left(2 + \frac{1}{\sqrt{2}}\right)\hat{i} + \frac{GM^2}{4R^2} \left(2 + \frac{1}{\sqrt{2}}\right)\hat{j}$$

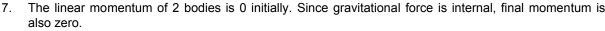
$$\therefore F_{C} = \frac{GM^{2}}{4R^{2}} \left(2\sqrt{2} + 1\right)$$

For moving along the circle, $\vec{F} = \frac{mv^2}{R}$

or
$$\frac{GM^2}{4R^2} (2\sqrt{2} + 1) = \frac{MV^2}{R}$$
 or $V = \sqrt{\frac{GM}{R} (\frac{2\sqrt{2} + 1}{4})}$

6.
$$\frac{GM}{(R+h)^2} = \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{(1740 + 1000)^2 \times 10^6} = \frac{49.358 \times 10^{11}}{2740 \times 2740 \times 10^6}$$

=
$$\frac{49.358 \times 10^{11}}{0.75 \times 10^{13}}$$
 = 65.8 × 10⁻² = 0.65 m/s²



So
$$(10 \text{ kg})v_1 = (20 \text{ kg}) v_2$$

Or
$$v_1 = v_2$$
 ...(1)

Since P.E. is conserved

Initial P.E. =
$$\frac{-6.67 \times 10^{-11} \times 10 \times 20}{1}$$
 = -13.34×10^{-9} J

When separation is 0.5 m,





$$-13.34 \times 10^{-9} + 0 = \frac{-13.34 \times 10^{-9}}{(1/2)} + (1/2) \times 10 \text{ v}_1^2 + (1/2) \times 20 \text{ v}_2^2 \quad \dots (2)$$

$$\Rightarrow$$
 - 13.34 × 10⁻⁹ = -26.68 × 10⁻⁹ + 5 v_1^2 + 10 v_2^2

$$\Rightarrow$$
 - 13.34 × 10⁻⁹ = -26.68 ×10⁻⁹ + 30 v_2^2

$$\Rightarrow$$
 $v_2^2 = \frac{13.34 \times 10^{-9}}{30} = 4.44 \times 10^{-10}$

$$\Rightarrow$$
 v₂ = 2.1 × 10⁻⁵ m/s.

So,
$$v_1 = 4.2 \times 10^{-5}$$
 m/s.

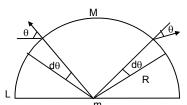
8. In the semicircle, we can consider, a small element of d, then R $d\theta$ = (M/L) R $d\theta$ = dM.

$$F = \frac{GMRd\theta m}{LR^2}$$

$$dF_3 = 2 dF \text{ since} = \frac{2GMm}{LR} \sin \theta d\theta.$$

$$\therefore F = \int_{0}^{\pi/2} \frac{2GMm}{LR} \sin\theta d\theta = \frac{2GMm}{LR} \left[-\cos\theta \right]_{0}^{\pi/2}$$

$$\therefore = -2 \frac{\text{GMm}}{\text{LR}} (-1) = \frac{2\text{GMm}}{\text{LR}} = \frac{2\text{GMm}}{\text{L} \times \text{L/A}} = \frac{2\pi \text{GMm}}{\text{L}^2}$$



9. A small section of rod is considered at 'x' distance mass of the element = (M/L). dx = dm

$$dE_1 = \frac{G(dm) \times 1}{(d^2 + x^2)} = dE_2$$

$$=2\times\frac{G(dm)}{\left(\!d^2+x^2\right)}\times\frac{d}{\sqrt{\!\left(\!d^2+x^2\right)}}\,=\,\frac{2\times GM\times d\;dx}{L\!\left(\!d^2+x^2\right)\!\left(\sqrt{\!\left(\!d^2+x^2\right)}\right)}$$

Total gravitational field

$$E = \int_{0}^{L/2} \frac{2Gmd dx}{L(d^{2} + x^{2})^{3/2}}$$

Integrating the above equation it can be found that,

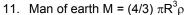
$$E = \frac{2GM}{d\sqrt{L^2 + 4d^2}}$$

10. The gravitational force on 'm' due to the shell of M₂ is 0.

M is at a distance
$$\frac{R_1 + R_2}{2}$$

Then the gravitational force due to M is given by

$$= \frac{GM_1m}{(R_1 + R_{2/2})} = \frac{4GM_1m}{(R_1 + R_2)^2}$$



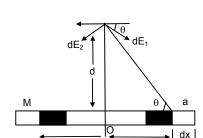
Man of the imaginary sphere, having

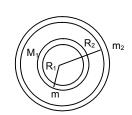
Radius = x, M' =
$$(4/3)\pi x^3 \rho$$

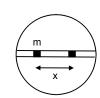
or
$$\frac{M'}{M} = \frac{x^3}{R^3}$$

$$\therefore \text{ Gravitational force on F} = \frac{\text{GM'm}}{\text{m}^2}$$

or F =
$$\frac{GMx^3m}{R^3x^2}$$
 = $\frac{GMmx}{R^3}$







12. Let d be the distance from centre of earth to man 'm' then

$$D = \sqrt{x^2 + \left(\frac{R^2}{4}\right)} = (1/2) \sqrt{4x^2 + R^2}$$

M be the mass of the earth, M' the mass of the sphere of radius d/2.

Then M = $(4/3) \pi R^3 \rho$

$$M' = (4/3)\pi d^3 \tau$$

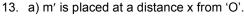
or
$$\frac{M'}{M} = \frac{d^3}{R^3}$$

.. Gravitational force is m,

$$F = \frac{Gm'm}{d^2} = \frac{Gd^3Mm}{R^3d^2} = \frac{GMmd}{R^3}$$

So, Normal force exerted by the wall = $F \cos\theta$.

=
$$\frac{\text{GMmd}}{\text{R}^3} \times \frac{\text{R}}{2\text{d}} = \frac{\text{GMm}}{2\text{R}^2}$$
 (therefore I think normal force does not depend on x)



If r < x, 2r, Let's consider a thin shell of man

dm =
$$\frac{m}{(4/3)\pi r^2} \times \frac{4}{3}\pi x^3 = \frac{mx^3}{r^3}$$

Thus
$$\int dm = \frac{mx^3}{r^3}$$

Then gravitational force F =
$$\frac{Gmdm}{x^2}$$
 = $\frac{Gmx^3/r^3}{x^2}$ = $\frac{Gmx}{r^3}$

b) 2r < x < 2R, then F is due to only the sphere.

$$\therefore F = \frac{Gmm'}{(x-r)^2}$$

c) if x > 2R, then Gravitational force is due to both sphere & shell, then due to shell,

$$F = \frac{GMm'}{(x-R)^2}$$

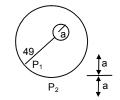
due to the sphere =
$$\frac{Gmm'}{(x-r)^2}$$

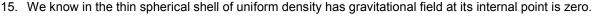
So, Resultant force =
$$\frac{Gmm'}{(x-r)^2} + \frac{GMm'}{(x-R)^2}$$

14. At P₁, Gravitational field due to sphere M =
$$\frac{GM}{(3a+a)^2} = \frac{GM}{16a^2}$$

At P2, Gravitational field is due to sphere & shell

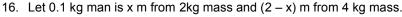
$$= \frac{GM}{(a+4a+a)^2} + \frac{GM}{(4a+a)^2} = \frac{GM}{a^2} \left(\frac{1}{36} + \frac{1}{25} \right) = \left(\frac{61}{900} \right) \frac{GM}{a^2}$$



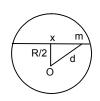


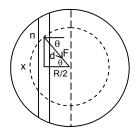
At A and B point, field is equal and opposite and cancel each other so Net field is zero.

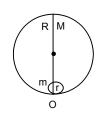




$$\therefore \frac{2 \times 0.1}{x^2} = -\frac{4 \times 0.1}{(2 - x)^2}$$







or
$$\frac{0.2}{x^2} = -\frac{0.4}{(2-x)^2}$$

or
$$\frac{1}{x^2} = \frac{2}{(2-x)^2}$$
 or $(2-x)^2 = 2x^2$

or
$$2 - x = \sqrt{2} x$$
 or $x(r_2 + 1) = 2$

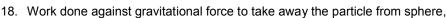
or x =
$$\frac{2}{2.414}$$
 = 0.83 m from 2kg mass.

17. Initially, the ride of Δ is a

To increase it to 2a,

work done =
$$\frac{Gm^2}{2a} + \frac{Gm^2}{a} = \frac{3Gm^2}{2a}$$





=
$$\frac{G \times 10 \times 0.1}{0.1 \times 0.1}$$
 = $\frac{6.67 \times 10^{-11} \times 1}{1 \times 10^{-1}}$ = 6.67×10^{-10} J

19.
$$\vec{E} = (5 \text{ N/kg}) \hat{i} + (12 \text{ N/kg}) \hat{j}$$

a)
$$\vec{F} = \vec{E} m$$

=
$$2kg [(5 \text{ N/kg}) \hat{i} + (12 \text{ N/kg}) \hat{j}] = (10 \text{ N}) \hat{i} + (12 \text{ N}) \hat{j}$$

$$|\vec{F}| = \sqrt{100 + 576} = 26 \text{ N}$$

b)
$$\vec{V} = \vec{E} r$$

At (12 m, 0),
$$\vec{V} = -(60 \text{ J/kg})\hat{i} |\vec{V}| = 60 \text{ J}$$

At (0, 5 m),
$$\vec{V} = -(60 \text{ J/kg})\hat{j} |\vec{V}| = -60 \text{ J}$$

c)
$$\Delta \vec{V} = \int_{(0,0)}^{(1,2,5)} \vec{E} \, mdr = \left[\left[(10N)\hat{i} + (24N)\hat{j} \right] r \right]_{(0,0)}^{(12,5)}$$

$$= - (120 \,\mathrm{J}\,\hat{\mathrm{i}} + 120 \,\mathrm{J}\,\hat{\mathrm{i}}) = 240 \,\mathrm{J}$$

d)
$$\Delta v = -\left[r(10N\hat{i} + 24N\hat{j})\right]_{(12m,0)}^{(0,5m)}$$

$$=-120 \hat{j} + 120 \hat{i} = 0$$

20. a)
$$V = (20 \text{ N/kg}) (x + y)$$

$$\frac{GM}{R} = \frac{MLT^{-2}}{M}L \text{ or } \frac{M^{-1}L^{3}T^{-2}M^{1}}{L} = \frac{ML^{2}T^{-2}}{M}$$

Or
$$M^0 L^2 T^{-2} = M^0 L^2 T^{-2}$$

b)
$$\vec{E}_{(x,y)} = -20(N/kg)\hat{i} - 20(N/kg)\hat{j}$$

c)
$$\vec{F} = \vec{E} m$$

= 0.5kg [- (20 N/kg)
$$\hat{i}$$
 - (20 N/kg) \hat{j} = -10N \hat{i} -10 N \hat{j}

$$|\vec{F}| = \sqrt{100 + 100} = 10\sqrt{2} \text{ N}$$

21.
$$\vec{E} = 2\hat{i} + 3\hat{j}$$

The field is represented as

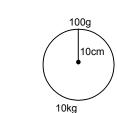
$$\tan \theta_1 = 3/2$$

Again the line
$$3y + 2x = 5$$
 can be represented as

$$\tan \theta_2 = -2/3$$

$$m_1 m_2 = -1$$

Since, the direction of field and the displacement are perpendicular, is done by the particle on the line.



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22. Let the height be h

∴ (1/2)
$$\frac{GM}{R^2} = \frac{GM}{(R+h)^2}$$

Or $2R^2 = (R+h)^2$
Or $\sqrt{2} R = R + h$
Or $h = (r_2 - 1)R$

23. Let g' be the acceleration due to gravity on mount everest.

$$g' = g\left(1 - \frac{2h}{R}\right)$$

$$= 9.8\left(1 - \frac{17696}{6400000}\right) = 9.8 (1 - 0.00276) = 9.773 \text{ m/s}^2$$

24. Let g' be the acceleration due to gravity in mine.

Then g'= g
$$\left(1 - \frac{d}{R}\right)$$

= 9.8 $\left(1 - \frac{640}{6400 \times 10^3}\right)$ = 9.8 × 0.9999 = 9.799 m/s²

25. Let g' be the acceleration due to gravity at equation & that of pole = g

g'=
$$g - \omega^2 R$$

= $9.81 - (7.3 \times 10^{-5})^2 \times 6400 \times 10^3$
= $9.81 - 0.034$
= 9.776 m/s^2
mg' = 1 kg × 9.776 m/s^2
= $9.776 \text{ N or } 0.997 \text{ kg}$

The body will weigh 0.997 kg at equator.

26. At equator, $g' = g - \omega^2 R$...(1)

Let at 'h' height above the south pole, the acceleration due to gravity is same.

Then, here
$$g' = g\left(1 - \frac{2h}{R}\right)$$
 ...(2)

$$\therefore g - \omega^2 R = g\left(1 - \frac{2h}{R}\right)$$
or $1 - \frac{\omega^2 R}{g} = 1 - \frac{2h}{R}$
or $h = \frac{\omega^2 R^2}{2g} = \frac{\left(7.3 \times 10^{-5}\right)^2 \times \left(6400 \times 10^3\right)^2}{2 \times 9.81} = 11125 \text{ N} = 10 \text{Km (approximately)}$

27. The apparent 'q' at equator becomes zero.

i.e.
$$g' = g - \omega^2 R = 0$$

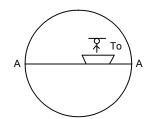
or $g = \omega^2 R$
or $\omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{6400 \times 10^3}} = \sqrt{1.5 \times 10^{-6}} = 1.2 \times 10^{-3} \text{ rad/s}.$
 $T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{1.2 \times 10^{-3}} = 1.5 \times 10^{-6} \text{ sec.} = 1.41 \text{ hour}$

28. a) Speed of the ship due to rotation of earth $v = \omega R$

b)
$$T_0 = mgr = mg - m\omega^2 R$$

 $\therefore T_0 - mg = m\omega^2 R$
c) If the ship shifts at speed 'v'

$$T = mg - m\omega_1^2 R$$



$$= T_0 - \left(\frac{(v - \omega R)^2}{R^2}\right) R$$

$$= T_0 - \left(\frac{v^2 + \omega^2 R^2 - 2\omega Rv}{R}\right) m$$

 $T = T_0 + 2\omega v m$

29. According to Kepler's laws of planetary motion,

$$T^{2} \alpha R^{3}$$

$$\frac{T_{m}^{2}}{T_{e}^{2}} = \frac{R_{ms}^{3}}{R_{es}^{3}}$$

$$\left(\frac{R_{ms}}{R_{es}}\right)^{3} = \left(\frac{1.88}{1}\right)^{2}$$

$$\therefore \frac{R_{ms}}{R_{es}} = (1.88)^{2/3} = 1.52$$

30.
$$T = 2\pi \sqrt{\frac{r^3}{GM}}$$

$$27.3 = 2 \times 3.14 \sqrt{\frac{\left(3.84 \times 10^{5}\right)^{3}}{6.67 \times 10^{-11} \times M}}$$
or $2.73 \times 2.73 = \frac{2 \times 3.14 \times \left(3.84 \times 10^{5}\right)^{3}}{6.67 \times 10^{-11} \times M}$
or $M = \frac{2 \times (3.14)^{2} \times (3.84)^{3} \times 10^{15}}{3.335 \times 10^{-11} (27.3)^{2}} = 6.02 \times 10^{24} \text{ kg}$

 \therefore mass of earth is found to be 6.02×10^{24} kg.

31.
$$T = 2\pi \sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow 27540 = 2 \times 3.14 \sqrt{\frac{(9.4 \times 10^3 \times 10^3)^3}{6.67 \times 10^{-11} \times M}}$$

or
$$(27540)^2 = (6.28)^2 \frac{(9.4 \times 10^6)^2}{6.67 \times 10^{-11} \times M}$$

or M =
$$\frac{(6.28)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (27540)^2} = 6.5 \times 10^{23} \text{ kg}.$$

32. a)
$$V = \sqrt{\frac{GM}{r+h}} = \sqrt{\frac{gr^2}{r+h}}$$

= $\sqrt{\frac{9.8 \times (6400 \times 10^3)^2}{10^6 \times (6.4 + 2)}} = 6.9 \times 10^3 \text{ m/s} = 6.9 \text{ km/s}$

b) K.E. =
$$(1/2) \text{ mv}^2$$

= $(1/2) 1000 \times (47.6 \times 10^6) = 2.38 \times 10^{10} \text{ J}$

c) P.E. =
$$\frac{GMm}{-(R+h)}$$

$$= - \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 10^{3}}{(6400 + 2000) \times 10^{3}} = - \frac{40 \times 10^{13}}{8400} = -4.76 \times 10^{10} \text{ J}$$

d) T =
$$\frac{2\pi(r+h)}{V}$$
 = $\frac{2\times3.14\times8400\times10^3}{6.9\times10^3}$ = 76.6 × 10² sec = 2.1 hour

33. Angular speed f earth & the satellite will be same

$$\frac{2\pi}{T_e} = \frac{2\pi}{T_s}$$

or
$$\frac{1}{24 \times 3600} = \frac{1}{2\pi \sqrt{\frac{(R+h)^3}{gR^2}}}$$
 or 12 I 3600 = 3.14 $\sqrt{\frac{(R+h)^3}{gR^2}}$

or 12 I 3600 = 3.14
$$\sqrt{\frac{(R+h)^3}{gR^2}}$$

or
$$\frac{(R+h)^2}{gR^2} = \frac{(12 \times 3600)^2}{(3.14)^2}$$

or
$$\frac{(R+h)^2}{gR^2} = \frac{(12\times3600)^2}{(3.14)^2}$$
 or $\frac{(6400+h)^3\times10^9}{9.8\times(6400)^2\times10^6} = \frac{(12\times3600)^2}{(3.14)^2}$

or
$$\frac{(6400+h)^3 \times 10^9}{6272 \times 10^9} = 432 \times 10^4$$

or
$$(6400 + h)^3 = 6272 \times 432 \times 10^4$$

or
$$6400 + h = (6272 \times 432 \times 10^4)^{1/3}$$

or h =
$$(6272 \times 432 \times 10^4)^{1/3} - 6400$$

- = 42300 cm.
- b) Time taken from north pole to equator = (1/2) t

=
$$(1/2) \times 6.28 \sqrt{\frac{(43200 + 6400)^3}{10 \times (6400)^2 \times 10^6}} = 3.14 \sqrt{\frac{(497)^3 \times 10^6}{(64)^2 \times 10^{11}}}$$

= 3.14
$$\sqrt{\frac{497 \times 497 \times 497}{64 \times 64 \times 10^5}}$$
 = 6 hour.

34. For geo stationary satellite,

$$r = 4.2 \times 10^4 \text{ km}$$

$$h = 3.6 \times 10^4 \text{ km}$$

$$mgh = mg \left(\frac{R^2}{(R+h)^2} \right)$$

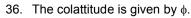
$$= 10 \left[\frac{\left(6400 \times 10^{3}\right)^{2}}{\left(6400 \times 10^{3} + 3600 \times 10^{3}\right)^{2}} \right] = \frac{4096}{17980} = 0.23 \text{ N}$$

35.
$$T = 2\pi \sqrt{\frac{R_2^3}{gR_1^2}}$$

Or
$$T^2 = 4\pi^2 \frac{R_2^3}{gR_1^2}$$

Or g =
$$\frac{4\pi^2}{T^2} \frac{R_2^3}{R_1^2}$$

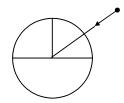
 \therefore Acceleration due to gravity of the planet is = $\frac{4\pi^2}{T^2} \frac{R_2^3}{R_1^2}$

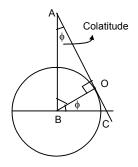


Again
$$\angle OBC = \phi = \angle OAB$$

$$\therefore \sin \phi = \frac{6400}{42000} = \frac{8}{53}$$

$$\therefore \phi = \sin^{-1}\left(\frac{8}{53}\right) = \sin^{-1} 0.15.$$





37. The particle attain maximum height = 6400 km. On earth's surface, its P.E. & K.E.

$$E_e = (1/2) \text{ mv}^2 + \left(\frac{-\text{ GMm}}{\text{R}}\right)$$
 ...(1)

In space, its P.E. & K.E.

$$E_s = \left(-\frac{GMm}{R+h}\right) + 0$$

$$E_s = \left(-\frac{GMm}{2R}\right) \qquad ...(2) \quad (\because h = R)$$

Equating (1) & (2)

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{2R}$$

Or (1/2)
$$\text{mv}^2 = \text{GMm} \left(-\frac{1}{2R} + \frac{1}{R} \right)$$

Or
$$v^2 = \frac{GM}{R}$$

$$=\frac{6.67\times10^{-11}\times6\times10^{24}}{6400\times10^3}$$

$$=\frac{40.02\times10^{13}}{6.4\times10^6}$$

$$= 6.2 \times 10^7 = 0.62 \times 10^8$$

Or v =
$$\sqrt{0.62 \times 10^8}$$
 = 0.79 × 10⁴ m/s = 7.9 km/s.

38. Initial velocity of the particle = 15km/s

Let its speed be 'v' at interstellar space.

:.(1/2) m[(15 × 10³)² – v²] =
$$\int_{R}^{\infty} \frac{GMm}{x^2} dx$$

$$\Rightarrow$$
 (1/2) m[(15 × 10³)² – v²] = GMm $\left[-\frac{1}{x} \right]_{R}^{\infty}$

$$\Rightarrow$$
 (1/2) m[(225 × 10⁶) – v^2] = $\frac{GMm}{R}$

$$\Rightarrow 225 \times 10^6 - v^2 = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 \times 10^3}$$

$$\Rightarrow$$
 v² = 225 × 10⁶ - $\frac{40.02}{32}$ × 10⁸

$$\Rightarrow$$
 v² = 225 × 10⁶ – 1.2 × 10⁸ = 10⁸ (1.05)

Or
$$v = 1.01 \times 10^4$$
 m/s or

= 10 km/s

39. The man of the sphere = 6×10^{24} kg.

Escape velocity = 3×10^8 m/s

$$V_c = \sqrt{\frac{2GM}{R}}$$

Or R =
$$\frac{2GM}{V_0^2}$$

$$= \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{\left(3 \times 10^{8}\right)^{2}} = \frac{80.02}{9} \times 10^{-3} = 8.89 \times 10^{-3} \text{ m} \approx 9 \text{ mm}.$$

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