RCC DESIGN

WORKING STRESS AND LIMIT STATE METHOD OF ANALYSIS

There are three design philosphies relating to reinforced and prestressed concrete:

(*i*) Working stress method,

(*ii*) Ultimate load method, and

(*iii*) Limit state method.

Working Stress Method

The assumptions made are:

(*i*) A section which is plane before bending remains plane after bending,

(*ii*) Bond between steel and concrete is perfect within the elastic limit of steel,

(*iii*) Tensile strength of concrete can be ignored,

(*iv*) Concrete is elastic, i.e. stress in concrete varies linearly from zero at the neutral axis to a maximum at the extreme fibre,

(*v*) The modular ratio has the value $\left(\frac{280}{3\sigma_{cbc}}\right)$, where σ_{cbc} is the permissible compressive stress in bending in N/mm^2.

According to Indian Standard IS:456, 2000 the permissible compressive stress in bending in concrete is one-third the 28-day cube strength of concrete. The corresponding factor of safety is 1.78 for steel, which is applied on the yield strength of steel in tension.

Drawbacks of the working stress method are:

(*i*) Concrete is not elastic, and the stress distribution in the concrete section is not triangular,

(*ii*) The factor of safety is applied on the stresses, and the different degrees of uncertainties associated with different types of loads is not accounted for, and

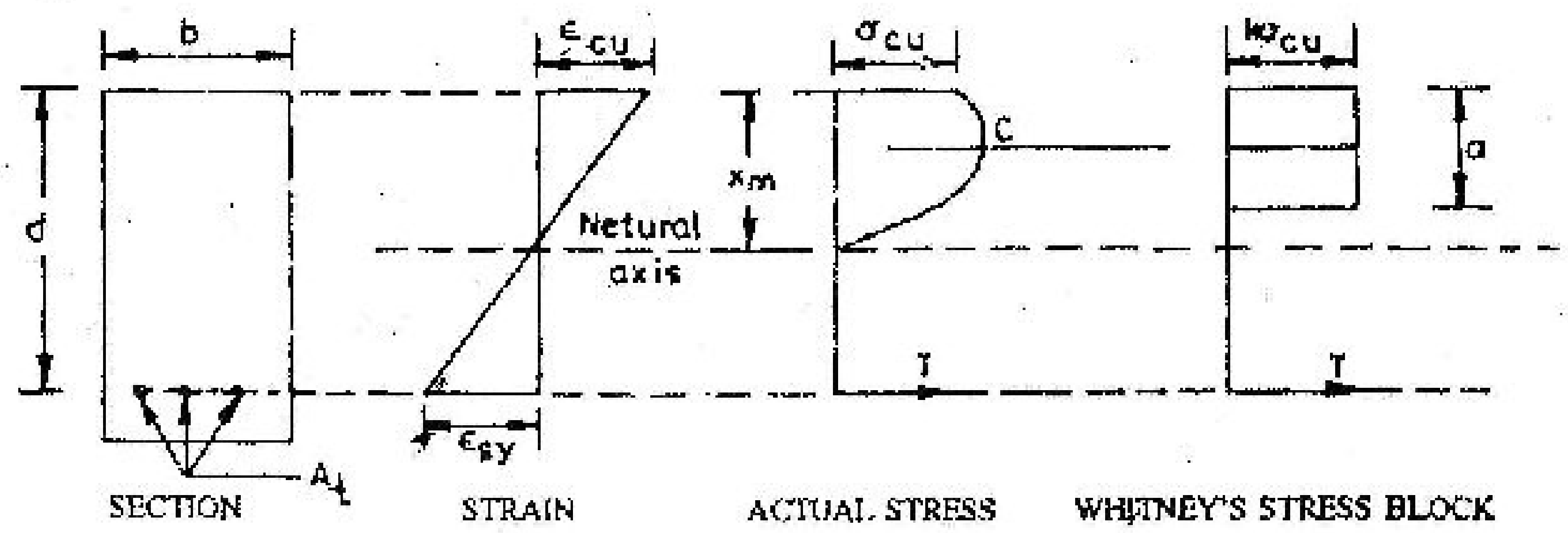
(*iii*) It is difficult to account for shrinkage and creep by calculation of elastic stresses.

Ultimate Load Method

In this method, working loads are increased by suitable factors, known as load factors, to obtain ultimate loads and the structure is designed to resist these loads.

Whitney's Theory

It is based on the assumption that the ultimate strain in concrete is 0.3 per cent and the compressive stress at the extreme edge corresponds to this strain. Whitney replaced the actual parabolic stress diagram by a rectangular stress diagram such that the centre of gravity of both diagrams lies at the same point and their areas are also equal.



where, a = depth of rectangular stress block
= 0.573 d according to Whitney and
= 0.43 d according to IS: 456-1964,

x_m = depth of neutral axis, z = level arm,

σ_{cu} = ultimate 28-day cylinder strength of concrete,

$K\sigma_{cu}$ = average stress
= 0.55 σ_{cu} according to Whitney and
= 0.85 σ_{cu} according to 18:456-1964,

σ_y = yield stress in steel, ε_{cu} = ultimate strain in concrete, σ_{sy} = yield strain in steel.

Assumptions

(*i*) A section which is plane before bending remains plane after bending.

(*ii*) At ultimate strength, stresses and strains are not proportional and the distribution of compressive stresses is non-linear in a section subjected to bending. The stress block may be assumed as a rectangle, parabola or any other shape, which gives ultimate strength in reasonable agreement with test results.

(*iii*) Maximum fibre strength in concrete does not exceed 0.68 σ_{cu} As in Whitney's theory, a rectangular stress block with a = 0.43 d and the average stress = 0.55 σ_{cu} can be assumed for analysis.

(*iv*) Tensile strength of concrete is ignored.

Indian Standard, IS:456-2000 stipulated the following conditions for ultimate design load, u:

(*a*) for structures in which the effects of wind and earthquake loads can be ignored, u: should be equal to 1.5 D + 2.2, and

(*b*) for structures in which earthquake loads should be considered, u should be equal to (1.5 D + 2.2 L + 0.5 W) or (1.5 D + 0.5 L + 2.2 W) whichever gives the critical condition.

where, D is dead load, L is live load, and W is wind or earthquake load.

Drawbacks:

(*i*) Since the load factor is applied on the working loads, there is no provision to account for the uncertainties associated with variations in material stresses.

(*ii*) There is no provision in the method for control against excessive deflections.

Limit State Design

The object of limit state design is to achieve an acceptable probability that a structure will

not become unserviceable in its lifetime. The condition of a structure when it becomes unserviceable is called a limit state. The most important of these limit states which must be examined in design are:

(*i*) *Ultimate limit state:* Neither the whole structure nor any part of the structure should collapse under foreseeable overload,

(*ii*) *Serviceability limit state of deflection :* The deflection of the structure should not adversely affect the appearance of the structure,

(*iii*) *Serviceability limit state of cracking :* The cracking of the concrete should not affect the appearance or durability of the concrete,

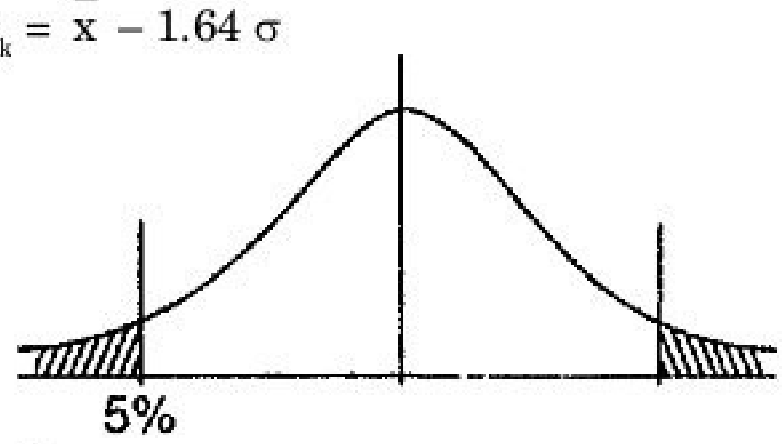
(*iv*) *Serviceability limit state of vibration :* Vibration should not be such as to cause alarm of discomfort to the user.

Therefore, collapse limit state at which the strength must be adequate to carry the loads with due consideration to stability, and serviceability limit state at which the deflection cracking and vibration must not be excessive.

Characteristic Strength

The characteristic strength σ_k of a material means that value of its strength below which not more than 5% of the test results are expected to fall. Hence

$$\sigma_k = \bar{x} - 1.64\,\sigma$$



where $\bar{x}$ = arithmetic mean of sample, and

σ = standard deviation associated with normal distribution in Statistics.

The characteristic strength, σ_k is usually represented by the 28-day cube strength, σ_{ck} of concrete, and the yield or 0.2% proof stress, σ_y , of reinforcement.

Characteristic Load

It has a 95% probability of not being exceeded during the life of a structure.

Design Values

Design strength of a material, $\sigma_d = \dfrac{\sigma_k}{\mu_m}$

where σ_k = characteristic strength, and μ_m = partial factor of safety.

For limit state of collapse, partial factors of safety adopted for concrete: μ_m =1.5, and for steel: μ_m = 1.15.

The design load is given by, and $\omega_d = \lambda\omega_k$.

where ω_k = characteristic load, λ = partial factor of safety.

Partial factor of safety for limit state of collapse

Load combinations	*DL*	*LL*	*WL*
DL + LL	1.5	1.5	-
DL + LL	1.5	-	-
*DL + LL	0.9	-	1.5
DL + LL + WL	1.2	1.2	1.2

* Considered when stability against overturning or stress reversal is critical.

Where, DL - dead load, WL - Wind load, and LL - Live load

Partial safety factors for load under limit state of serviceability area

Load combinations	*DL*	*LL*	*WL*
DL + LL	1.0	1.0	-
DL + WL	1.0	-	1.0
DL + LL + WL	1.0	0.8	0.8

The load obtained by multiplying a characteristic load by an appropriate partial safety factor is known as a factored load.

Value of Elastic Modulii

Concrete: Short-term value 0, $E_c = 5000\sqrt{\sigma_{ck}}$ (used in the calculation of elastic deflection),

Long term value $E_{ce} = \dfrac{E_c}{1+\theta}$

where, 0 = creep coefficient, equal to 1.6 at 28-days and 1.1. at the end of 365-days of concreting,

σ_{ck}, = 28-day characteristic strength of concrete in N/mm^2

Steel: E_s = 200 kN/mm^2 (applicable to all reinforcing steels).

Assumptions made in Limit State Design

(*a*) Plane sections normal to the axis of the member remain plane after bending. i.e., the strain at any pointon the cross-section is directly proportional to the distance from the neutral axis.

(*b*) Maximum strain in concrete at the outermost compression fibre is 0.0035.

(*c*) Design stress-strain relationship for concrete is as shown in the Figure

(*d*) Tensile strength of concrete can be ignored.

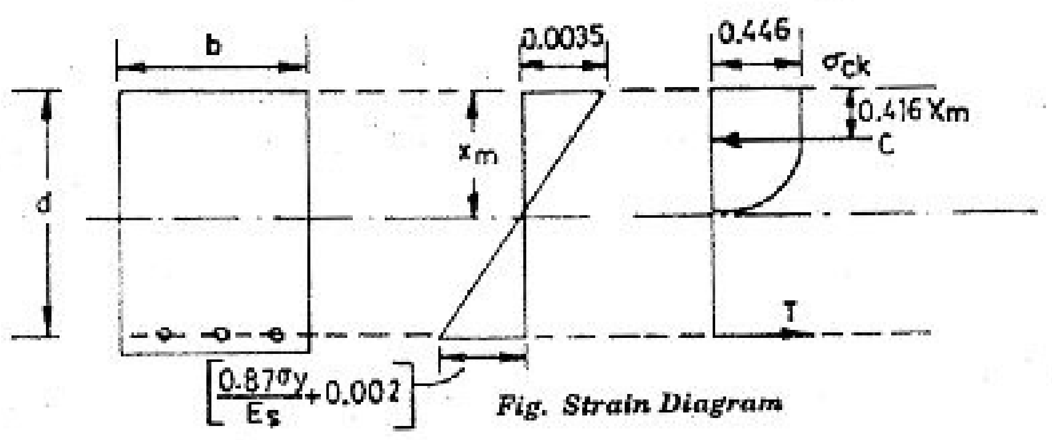


Fig. Strain Diagram

(*e*) Maximum compressive stres in concrete is equal to 0.67 σ_{ck}.

(*f*) Strain in the tension reinforcement is to be not less than $\dfrac{0.87\sigma_y}{E_S} + 0.002$

This assumption is intended to ensure ductile failure, i.e., the tensile reinforcement has to undergo a certain degree of inelastic deformation before the concrete fails in compression.

Recommendations of Indian Standard Codes

(1) Grades of Concrete : The following six grades of concrete can be used for reinforced concrete work: M15, M20, M25, M30, M35 and M40. The number in the grade designation refers to the characteristic strength, σ_{ck}' of 15 cm cubes at 28 days, expressed in N/mm^2.

Generally, grades M15 and M20 are used for flexural members. Grade M35 and M40 are used for prestressed beam.

(2) Reinforcement Bars : For design purposes, the characteristic strength of steel is taken as 250 or 415 N/mm^2, as the case may be.

(3) Stress-Strain Relationship for Concrete : Codes permits the use of stress-strain curve as shown in the Figure below

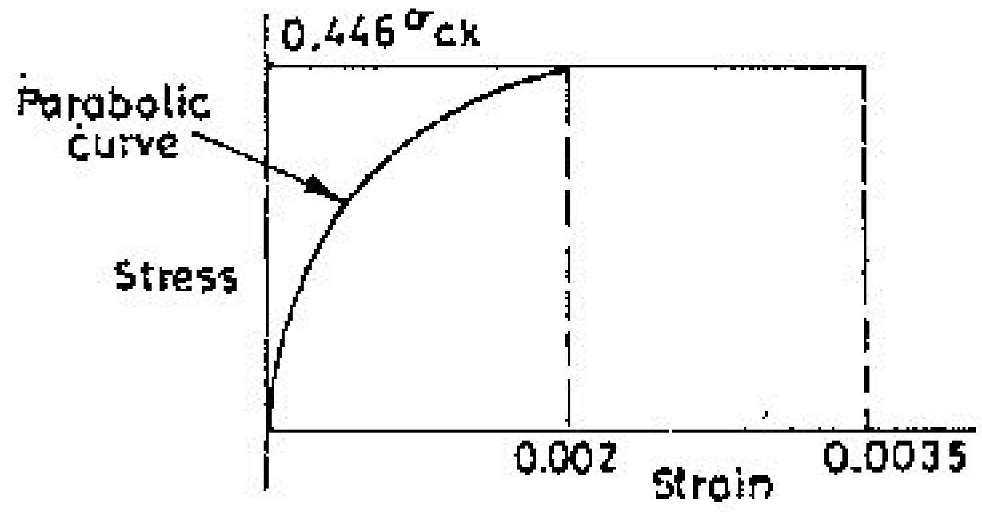


Fig. Design Stress-Strain Curve for Concrete

The compressive strength of concrete is assumed as 0.67 σ_{ck}. With a value of 1.5 for the partial safety factor for material strength, the maximum compressive stress in concrete for design purpose is 0.44 σ_{ck}.

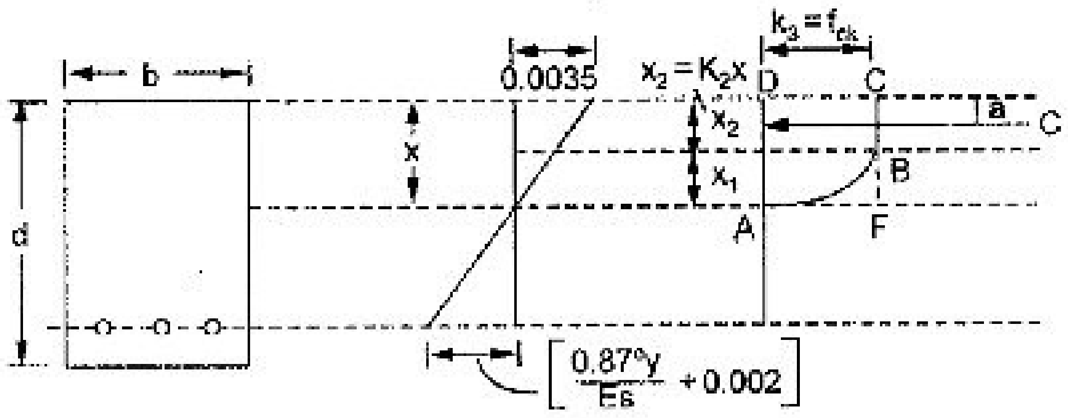
(4) Stress-Strain Relationship for Steel: The design yield stress works out, with an assumed value of 1.15 for steel, to 0.87 σ_y. The stress-strain relationship for steel in tension and compression can be assumed to be the same.

For mild steel, the stress-strain curve, for design purposes, will be as shown in the figure for cold worked bars, the stress is proportional to strain up to a stress equal to 0.87 σ_y.

Limit State of Collapse-Flexure

Consider a simply supported beam subjected to bending under factored loads. At collapse, the factored bending moment is equal to the resisting moment of the section provided by the internal stresses. This is called the ultimate moment of resistance.

From the assumptions made, strain in concrete is proportional to its distance from neutral axis below the neutral axis, concrete is cracked and hence all the tensile forces are borne by steel bars.



$$x_2 = k_2 x = \frac{3}{7} x$$

$$k_3 = 0.446$$

Maximum compressive stress in concrete (without partial factor of safety)

$$= k_3 \sigma_{ck} = 0.67\sigma_{ck}$$

Moment of resistance due to concrete

$$= k_1 k_3 \sigma_{ck} bx[d - k_2 x]$$

$$= k_1 k_3 \sigma_{ck} bd^2 \left(\frac{x}{d}\right)\left[1 - k_2\left(\frac{x}{d}\right)\right]$$

where k_1 is the shape factor for the stress block. Depth of the compressive force from the top fibre can be defined as equal to $k_2 x$ in terms of x.

k_1 = ratio between the area of stress block ABCD to area of rectangle AFCD.

Modes of Failure in Flexure

If the ratio of steel to concrete in a beam is such that maximum strain in the two materials reach simultaneously, a sudden failure would occur with less alarming deflection. Such a beam is referred to as a *balanced reinforced beam.*

When the amount of steel is kept less than that in the balanced section, the neutral axis moves upward to satisfy the equilibrium condition that the force of compression is equal to force of tension. In this process, the centre of gravity of compressive force shifts upward.

Under the increasing bending moment, steel is strained beyond the yield point and the maximum strain in concrete remains less than 0.35 per cent.

If the beam is further loaded, the strain in the section increases. Once the steel has yielded, it does not take any additional stress for the additional strain and the total force of tension remains constant. However, compressive stresses in concrete do increase with the additional strains. Thus, neutral axis and the centre of gravity of compressive forces further shift upward to maintain equilibrium. This process of shift in neutral axis continues until maximum strain in concrete reaches its ultimate value, that is, 0.35 per cent, and concrete is crushed. Such a beam is referred to as under-reinforced beam. The failure is called tension failure because yielding of steel was responsible for continued higher strains in concrete, resulting in its failure.

When the amount of steel is kept more than that in the balanced condition, the neutral axis tends to move downward and strain in steel remains in the elastic region. If the beam is further loaded, the stress and strain in steel keep on increasing and so is the force in tension. The additional increase in the concrete stress is much slower. Thus, to maintain the equilibrium of tension and compression forces, the area of concrete resisting compression has to increase.

In this process the neutral axis shifts further downward until maximum strain in concrete reaches its ultimate value, i.e., 0.35 per cent and concrete is crushed. The steel is still well within the elastic limit. Such a beam is referred to as an over-rainforced beam and the failure, as compression failure.

A compression failure is a brittle failure and is to be avoided. The maximum depth of neutral axis in a design is, therefore, to be limited. Hence, the stipulation in the code that the maximum strain in tension reinforcement in the section at failure should not be less than

$$\frac{\sigma_y}{1.15E_s} + 0.002.$$

The limiting values of the depth of neutral axis, x_{r1}, for different grades of steel can be obtained from the strain diagram.

$$\frac{x_m}{d - x_m} = \frac{0.0035}{\left(\frac{0.87\sigma_y}{E_S} + 0.002\right)},$$

or

$$\frac{x_m}{d} = \frac{0.0035}{\left(0.0055 + \frac{0.87\sigma_y}{E_S}\right)}$$

Values of x_m for the grades of steel

σ_y(N/mm^2)	x_m
250	0.53d
415	0.48d
500	0.46d

where *d* is the effective depth.

Limiting Values of Moment of Resistance

The limiting value of moment of resistance with respect to concrete is obtained by substituting value of x_m for x in the expression

$$0.36\sigma_{ck} bx(d - 0.42x)$$

Limiting moment of resistance

$$= 0.36\sigma_{ck} bx_m(d - 0.42\, x_m)$$

Correspondingly, limiting value of moment of resistance with respect to steel = $0.87\sigma_y At(d - 0.42x_m)$

For a given rectangular section, the limiting value of moment depends on the concrete mix and the grade of steel.

Table: Limiting M.R (N-mm) for various combinations of concrete mix and steel grade.

Grade of concrete	Fe 250 steel	Fe 415 steel	Fe 500 steel
M10	$0.148\sigma_{ck}bd^2$	$0.138\sigma_{ck}bd^2$	$0.133\sigma_{ck}bd^2$
M15	$2.22bd^2$	$2.07bd^2$	$2.00bd^2$
M20	$2.96bd^2$	$2.76bd^2$	$2.66bd^2$
M25	$3.70bd^2$	$3.45bd^2$	$3.33bd^2$

The percentage of tensile reinforcement corresponding to the limiting moment of resistance is obtained by equating $0.87\,\sigma_y A_t$ and $0.36\,\sigma_{ck}\,bx_m$

$$A_t = \left(\frac{0.36}{0.87}\right)\left(\frac{\sigma_{ck}}{\sigma_y}\right)bx_m$$

Putting A_t = pbd, we get

$$P_{limit} = \left(\frac{0.36\sigma_{ck}}{0.87\sigma_y}\right)\left(\frac{x_m}{d}\right)$$

Limit State of Collapse-Shear

In beams of uniform depth, nominal shear stress,

$$\tau_v = \frac{V_u}{bd}$$

where V_u = shear force due to design loads,

d = effective depth, and

b = breadth of the member, restricted to rib width (b_w) for flanged sections,

For beams of varying depth, $\tau_v = \dfrac{V_u \pm \dfrac{M_u}{d}\tan\beta}{bd}$

where M_u = bending moment at the section, and

β = angle between the top and bottom edges of the beam.

The design shear strength of concrete in beams (τ_c) without shear reinforcement is based on the percentage of tensile reinforcement provided in a beam and is given by

$$\tau_c = \frac{0.85}{6\beta}\sqrt{0.80\sigma_{ck}}\left[\sqrt{(1+5\beta)}-1\right]$$

where $\beta = \dfrac{0.8}{6.89p_t} \not< 1$; and $p_t = \dfrac{100A_t}{b_w d}$

The prevent crushing of concrete the maximum shear stress is limited. Their values are 2.5, 2.8 and 3.1 N/mm^2 for, respectively M15, M20 and M25 grades of concrete. For slabs, the nominal shear stress is restricted to 50% of the above values.

For solid slabs (other than flat slabs), the design shear strength of concrete in $k\tau_c$, where k is coefficient with values ranging from 1.00 for slabs 300 mm thick or more too .30 for slabs 15.0 mm thick or less.

For members subjected to axial compression P_u the design strength τ_c given in the above table should be multiplied by the factor δ, given by

$$\delta = 1 + \frac{3P_u}{A_g\sigma_{ck}}$$

where P_u = axial compressive force (N),

A_g = gross area of concrete section (mm^2), and

σ_{ck} = characteristic compressive strength of concrete.

where τ_v exceeds τ_c , shear reinforcement should be provided to carry a shear equal to $V_u - \tau_c bd$

Design basis

Case (1) : where shear force V_u is less than 50% of the shear capacity of the section - no shear reinforcement,

Case (2) : where $\tau_v \le \tau_c$ minimum shear reinforcement to be provided,

Case (3) : where $\tau_v > \tau_c$, but < maximum permissible limiting shear stress - shear reinforcement to take care of the shear force, $V_u - \tau_c$ bd, to be provided, and

Case (4) : where τ_v > maximum limiting shear stress - section to be redesigned.

The nominal shear stress is obtained by dividing the factored shear force by bd.

Limit State of Serviceasbility

Deflection : In all normal cases, the deflection of a flexural member will not be greater in the ratios of span to the effective depth are not greater than the values given below–

(*i*)

	Ratio of span to effective depth
cantilevers	7
simply supported beams	20
continuous beams	26

(*ii*) For spans above 10 m

cantilevers - deflections to be calculated

beams - ratios given to be multiplied by $\dfrac{10}{\text{span(m)}}$

The final deflection, if calculated, should not exceed $\dfrac{\text{span}}{250}$ and should take into account all loads and effects of temperature, creep and shrinkage. Deflection occurring after erection of partitions and the application of finishes should not normally exceed $\dfrac{\text{span}}{350}$ or 20 mm, whichever is less.

Limit State of Collapse : Compression

Following assumptions are made :

(a) Maximum compressive strain in concrete in axial compression is taken as 0.002 :

(b) Maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending, with no tension on the section, is taken as 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

All columns should be designed for a minimum eccentricity equal to the unsupported length of column divided by 500 plus lateral dimension divided by 300, subject to a minimum of 20mm.

A short axially loaded column should be designed by considering the assumptions and the minimum eccentricity. Where the minimum eccentricity does not exceed 0.05 times the lateral dimension, the member may be designed using the following equation :

$$P_u = 0.45\sigma_{ck}.A_C + 0.67\sigma_y.A_{sc}$$

where P_u = axial load on the member,

σ_{ck} = characteristic compressive strength of concrete (cube),

A_c = area of concrete,

σ_y = characteristic strength of compression reinforcement, and

A_{sc} = area of longitudinal reinforcement for columns.

Limit State of Serviceability: Cracking

Following spacing requirements are considered sufficient to control flexural cracking:

(a) **Beams:** The horizontal distance between parallel reinforcement bars, or groups, near the tension of a beam should not be greater than the value given in the table below, depending on the amount of redistribution carried out in analysis and the characteristic strength of reinforcement.

Table: Clear Distance between Bars

σ_y	Percentage redistribution to or from section considered				
	–30	–15	0	+15	+30
	Clear distance between bars				
N/mm²	mm	mm	mm	mm	mm
250	215	260	300	300	300
415	125	155	180	210	235
500	105	130	150	175	195

(b) **Slabs:** The horizontal distance between parallel main reinforcement should not be more than three times the effective depth of a solid slab or 300 mm, whichever is smaller, and the horizontal distance between parallel reinforcement bars provided to take care of shrinkage and temperature shall not be more than five times the effective depth of a solid slab or 450 mm, whichever is smaller.

Cracks due to bending in a compression member subjected to a design axial load greater than 0.2 $\sigma_{ck}A_c$, where σ_{ck} is the characteristic compressive strength of concrete and A_c is the area of gross section of the member, need not be checked. A member subjected to load lesser than $0.2\sigma_{ck}A_C$ may be considered as a flexural member for the purpose of crack control.

DESIGN OF BEAMS

Assumptions:

(a) Plane sections transverse to the centre line of a member before bending remain plane sections after bending.

(b) Elastic modulus of concrete has the same value within the limits of deformation of the member.

(c) Elastic modulus for steel has the same value within the limits of deformation of the member

(d) The reinforcement does not slip from concrete surrounding it.

(e) Tension is borne entirely by steel.

(f) The steel is free from initial stresses when embedded in concrete.

(g) There is no resultant thrust on any transverse section of the member.

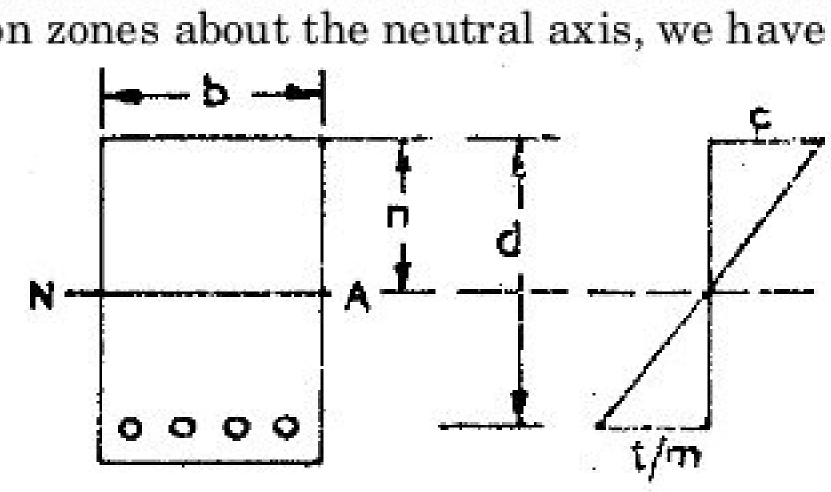
Neutral Axis

The neutral axis for a beam section is the line of intersection of the neutral layer with the beam section.

Let 'b' and 'd' be the breadth and effective depth of the beam section and 'n' be the depth of neutral axis. From the Figure,

$$\frac{c}{t/m} = \frac{n}{d-n}$$

Equating the moments of area of the compression and tension zones about the neutral axis, we have



$$\frac{bn^2}{2} = mAt(d-n)$$

Lever Arm

Distance between the line of action of the resultant compression and the line of action of the resultant tension is called Lever arm.

Lever arm, $a = \left(d - \frac{n}{3}\right)$

Moment of Resistance:

The resisting moment offered by a beam section to resist the bending moment at the section is called moment of resistance.

Moment of Resistance (M.R.)

= (Total Compression (or) Total Tension) × lever arm

$$\text{M.R.} = bn\frac{c}{2}\left(d - \frac{n}{3}\right) \text{ and M.R.} = A_t t\left(d - \frac{n}{3}\right)$$

$$\text{M.R.} = Qbd_2$$

Also

where $Q = \frac{1}{2}n_1\left(1 - \frac{n_1}{3}\right)c$ (where $n = n_1 d$)

Balanced or Critical Section:

This is a section in which the quantity of steel provided is such that, when the most distant concrete fibre in the compression zone reaches the allowable stress in compression, the tensile stress in the reinforcement reaches its allowable stress.

When = $c = 5$ N/mm^2 and $t = 140$ N/mm^2,

For a balanced section:

Lever arm distance = 0.87 d

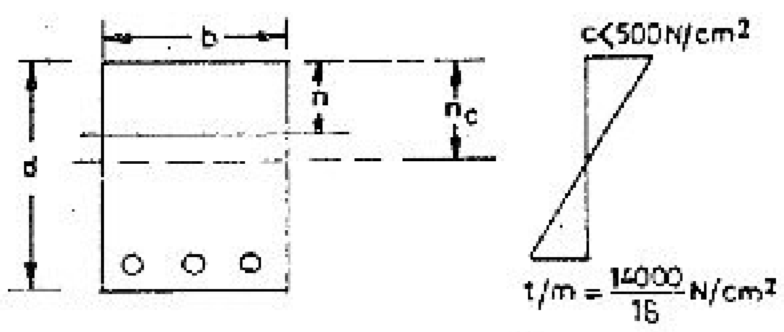
Moment of resistance = 850 bd^2 N mm

Area of steel = 0.699% of bd.

Unbalanced Section

This may be classified as "under-reinforced" and "over-reinforced" sections.

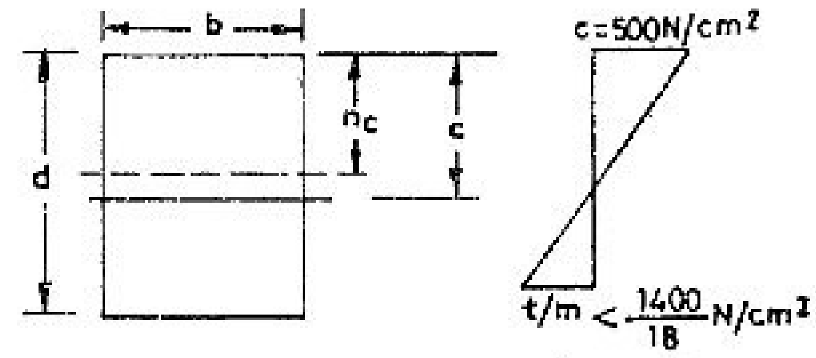
(*i*) **Under-reinforced Section:** This is a section in which the quantity of steel provided is less than for a balanced section where steel reaches its tensile stress while concrete does not attain its limiting permissible stress.



Moment of Resistance = $A_t t\left(d - \frac{n}{3}\right)$

Then stress t is taken as allowable stress in steel.

(*ii*) **Over-reinforced Section:** This is a section in which the quantity of steel provided is more than the quantity required for a balanced section. Here concrete reaches its limiting stress earlier than steel reach its permissible tensile stress.

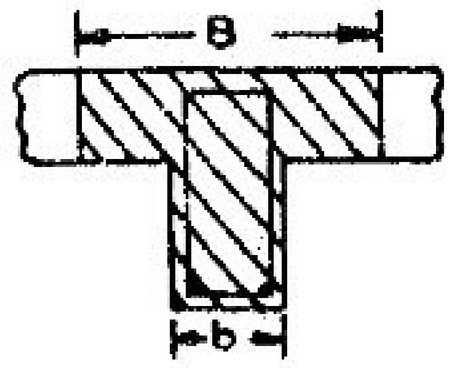


Moment of Resistance = $bn\left(d - \frac{n}{3}\right)\frac{c}{2}$,

where the stress c being taken as allowable compressive stress for concrete as shown in the above figure.

T-Beams

The slab and the rib due to their monolithic nature from a T-beam. The flange of the T-beam provides the necessary resistance to compression while the vertical rib provides the depth and hence the necessary lever arm.



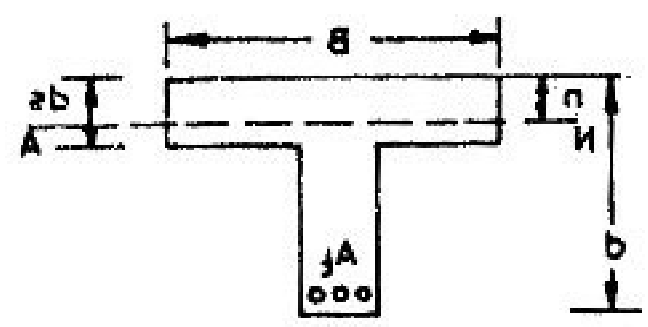
Neutral Axis of a T-beam

The depth of the neutral axis can be determined by equating moments of areas on either side of the neutral axis.

There are two cases:

(1) Neutral axis within the flange

Taking moments about N.A



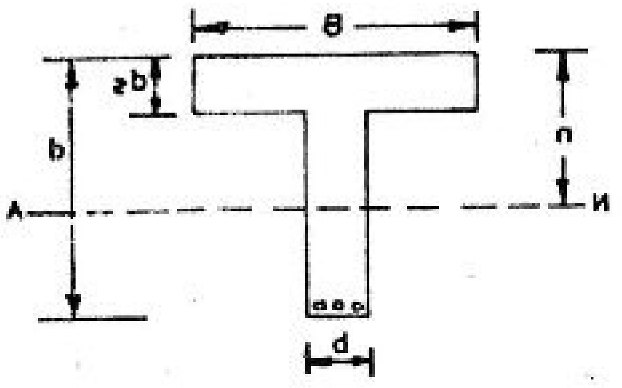
$$B\frac{n^2}{2} = mA_t(d - n)$$

(2) Neutral axis below the flange

Taking moments about N.A.

$$Bd_s\left(n - \frac{d_s}{2}\right) + \frac{b(n - d_s)^2}{2} = mA_t(d - n)$$

Neglecting the term $\frac{b(n-d_s)^2}{2}$, we have



$$Bd_s\left(n-\frac{d_s}{2}\right)=mA_t(d-n)$$

Design of T-beams

(*i*) Fix the overall dimensions of T-beam. The depth may be taken as $\frac{1}{12}$ to $\frac{1}{20}$ of the span. Width of the beam may be taken as $\frac{1}{3}$ to $\frac{2}{3}$ of the depth of rib.

(*ii*) Estimate the maximum bending moment due to the given loading and self-weight.

(*iii*) Approximately take the lever arm (a) as $d-\frac{d_s}{2}$ where d = effective depth and d_s = overall thickness of slab.

(*iv*) Calculate the approximate steel requirement from the relation.

$A_t=\frac{M}{t\times a}$, where t = safe stress in steel.

(*v*) Provide sufficient number of bars of a suitable diameter.

(*vi*) Now the section being fully known, determine the exact position of the neutral axis.

(*vii*) Now determine the stresses in concrete and steel with the known value of actual lever arm. lfthe stresses calculated are within the allowable limits, the design is safe.

(*viii*)The design is now checked for shear stress.

Nominal shear stress at the support, $\tau_V=\frac{S}{bd}$,

where S = maximum shear force,

If $\tau_V>\tau_C$, the shear reinforcement should be provided.

Single Span Beams:

1. **Beams simply supported at their ends**

   Maximum positive bending moment at centre

   $$=\frac{+wl^2}{8}$$

   Bending moment at ends = 0

2. **Beams fixed at both ends**

   Maximum negative B.M. at centre = $\frac{+wl^2}{12}$

   Maximum negative B.M. at support = $\frac{+wl^2}{12}$

3. **Beams continuous for more than two spans**

   Let w_d be the dead load per unit run of the beam, and w_l be the live load per unit run of beam.

B.M.	Near middle of end span	At middle of interior spans	At support next to end support	At other interior support
Moment due to Dead load	$\frac{+w_d l^2}{12}$	$\frac{+w_d l^2}{16}$	$\frac{-w_d l^2}{10}$	$\frac{-w_d l^2}{12}$
Moment due to Live load	$\frac{+w_l l^2}{10}$	$\frac{+w_l l^2}{12}$	$\frac{-w_l l^2}{9}$	$\frac{-w_l l^2}{9}$

Double Reinforced Beams

Beams with reinforcement in compression and tension zones are called doubly reinforced beams. It is used in the following conditions -

1. When due to head room consideration or architectural consideration, the depth of the beam is restricted.
2. External live load may occur on either side of the member.
3. The loading may be eccentric.
4. The member may be subjected to impact.

Analysis of a Doubly Reinforced Section

(*i*) Elastic Theory

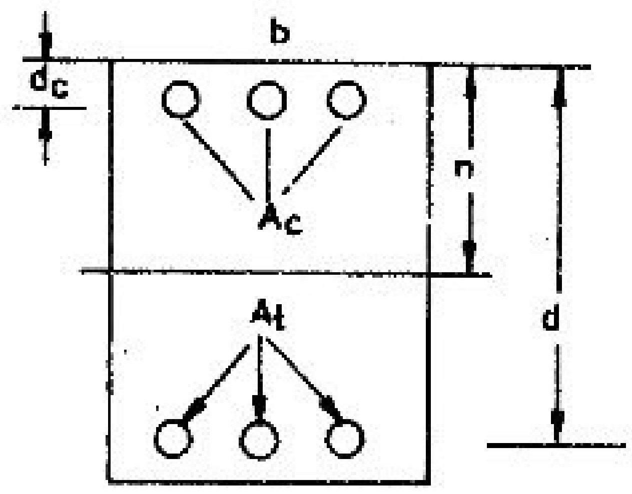
Let width of beam be 'b' mm and depth be 'd' mm A_C and A_t be the areas of reinforcement in cotnpression and tension zones, 'n' be the depth of neutral axis.

$$\frac{bn^2}{2}+mA_C(n-d_C)-A_C(n-d_C)=mA_t(d-n)$$

$$\therefore\ \frac{bn^2}{2}+mA_C(m-d_C)-A_C(n-d_C)=mA_t(d-n)$$

Moment of Resistance

$$= bc\frac{c}{2}\left(d-\frac{n}{3}\right)+(m-1)A_C c'\times(d-d_S)$$



***(ii)* Steel Beam Theory**

Assumptions

(1) Compression is resisted only in compression steel,

(2) Tension is resisted only by tension steel,

(3) Stress in compression steel = stress in tension steel.

Moment of resistance of the beam,

$M = A_C c(d - d_C) = A_t t(d - d_C)$

DESIGN OF COMPONENTS OF BUILDINGS

Design of Beams-I

Effective Span: The effective span of a member that is not built integrally with its supports shall be taken as clear span plus the effective depth of beam or centre to centre of the supports, whichever is less.

In continuous beams, if the width of the support is less than one-twelth of the clear span, the effective span be as for a simply supported beam. If the supports are wider or 600 mm

whichever is less, the effective span shall be taken as follows-

(*i*) For end span with one end fixed and the other continuous, or for intermediate spans, the effective span shall be clear span between the supports, and

(*ii*) For end span with end free and the other continuous, the effective span shall be equal to the clear span plus half the effective depth or the clear span plus half the width of the discontinuous support, whichever is less.

Effective Depth: The effective depth of a beam is the distance between the centroid of the area of tension reinforcement and the maximum compression fibre, excluding the thickness of finishing material not placed monolithically with the member and the thickness of any concrete to allow for wear.

Design Approach

(*a*) For under-reinforced section the value of $\left(\frac{x}{d}\right)$ is less than the $\left(\frac{x_m}{d}\right)$ value, the moment of resistance is calculated by the following equations

$$x=\frac{0.87\sigma_y A_t}{0.36\sigma_{ck}bd}$$

$$=2.41\frac{\sigma_y A_t}{\sigma_{ck}bd}$$

Lever arm, $z=\left(d-241\frac{\sigma_y A_t}{\sigma_{ck}bd}\right)$

and, $M_u = 0.87\,\sigma_y A_t z.$

(*b*) For balanced section, the moment of resistance is calculated by the following equations

$$x = x_m$$

$$z = d - 0.42x_m$$

$$M_u = 0.36\,\sigma_{ck} b x_m (d - 0.42x_m)d$$

(*c*) For over-reinforced section the value of $\left(\frac{x}{d}\right)$ is limited to $\left(\frac{x_m}{d}\right)$ and the moment of resistance is calculated based on concrete.

$$x = x_m$$

$$z = d - 0.042x_m$$

and $M_u = 0.36\sigma_{ck} b x_m (d - 0.42x_m)$

It is possible to generate design tables for singly-reinforced beams by making use of the equation.

$$M_u = 0.87\sigma_y A_t\left(d-\frac{\sigma_y A_t}{\sigma_{ck}b}\right),$$

or $$\frac{M_u}{bd^2}=0.87\frac{\sigma_y A_t}{dbd}\left(d-\frac{\sigma_y}{\sigma_{ck}}\times\frac{A_t}{b}\right)$$

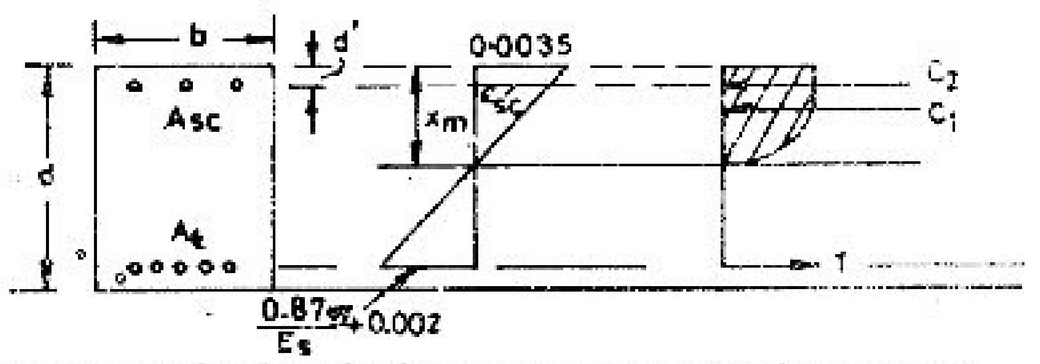
By putting $A_t = pdb$, and $\frac{M_u}{bd^2} = R$ we get,

$$R = 0.87\sigma_y P\left(1-p.\frac{\sigma_y}{\sigma_{ck}}\right)$$

or $0.87\frac{\sigma_y^2}{\sigma_{ck}}p^2 - 0.87\sigma_y p + R = 0$

Design of Beam

Doubly Reinforced Beams tension reinforcement is designed for a stress of 0.87 f_y, but for compression reinforcement of lower stress must be used. Because steel bars may buckle between shear stirrups under compressive stresses. The design stress in compression reinforcement corresponds to the strain in concrete at that level.



Strain at the level of compression reinforcement

$$= \left[1 - \frac{d'}{x_m}\right]$$

where d' = effective cover to compressive reinforcement.

Knowing the strain at the level of compressive steel, the stress in steel can be obtained from the appropriate stress-strain curve.

DESIGN OF COMPRESSION MEMBER

The relationship between stress and strain in concrete is assumed as parabolic. The maximum compressive stress is equal to $\left(\frac{0.67\sigma_{ck}}{1.5}\right)$ or 0.446 σ_{ck}. The tensile strength of concrete is ignored.

It is assumed that the maximum compressive strain at the highly compressed extreme fibre in concrete is 0.0035 minus 0.75 times the strain at the least compressed fibre.

The maximum compressive strain at the highly compressed extreme fibre, when part of the section is in tension, is taken as 0.0035.

Short Column under Axial Compression

Factored axial load, $p_u = 0.4\,\sigma_{ck} A_c + 0.67\,\sigma_y A_{ac}$ where
A_c = area of concrete; and
A_{sc} = area of longitudinal reinforcement of columns

Rearranging, $\frac{P_u}{\sigma_{ck}.bD} = 0.4 + \frac{P}{100\sigma_{ck}}(0.67\sigma_y - 0.40\sigma_{ck})$

Reinforcement: There are two kinds of reinforcement in a column: longitudinal and transverse reinforcement. The pressure of transverse reinforcement is to hold the vertical bars in position providing lateral support so that individual bars cannot buckle outwards and split the concrete.

Longitudinal Reinforcement (as per I.S. Code IS : 456)

(*a*) The cross-sectional area of longitudinal reinforcement shall be not less than 0.8 per cent nor more than 6 percent of the gross cross-sectional area of the column.

(*b*) In any column that has larger cross-sectional area than that required to support the load, the minimum percentage of steel shall be based upon the area of concrete required to resist the direct stress and not upon the actual area.

(*c*) The minimum number of longitudinal bars provided in a column shall be four in rectangular columns and six in circular columns.

(*d*) The bars shall not be less than 12 mm in diameter.

(*e*) A reinforced concrete-column having helical reinforcement shall have atleast six bars in longitudinal reinforcement within the helical reinforcement.

(*f*) In a helically reinforced column, the longitudinal bars shall be in contact with the helical reinforcement and equidistant around its inner circumference.

(*g*) Spacing of longitudinal bars measured along the periphery of the column shall not exceed 300 mm.

(*h*) In case of pedestals in which the longitudinal reinforcement is not taken into account in strength calculations, nominal longitudinal reinforcement not less than 0.15 per cent of the cross-section area shall be provided.

Pedestal is a compression member, the effective length of which does not exceed three times the least lateral dimension.

Transverse Reinforcement (as per I.S. Code IS : 456 : 2000)

(*a*) A reinforced concrete compression member shall have transverse or helical reinforcement so disposed that every longitudinal bar nearest to the compression face have effective lateral support against buckling subject to provisions in (*b*). The effective lateral support is given by transverse reinforcement either in the form of circular ring capable of taking up circumferential tension or by polygonal links (lateral ties) with internal angles not exceeding 135°, The ends of the transverse reinforcement shall be properly anchored.

(*b*) Arrangement of transverse reinforcement

(1) If the longitudinal bars are not spaced more than 75 mm on either side, transverse reinforcement need only to go round corner and alternate bars for the purpose of providing effective lateral supports [Figure (*a*)].

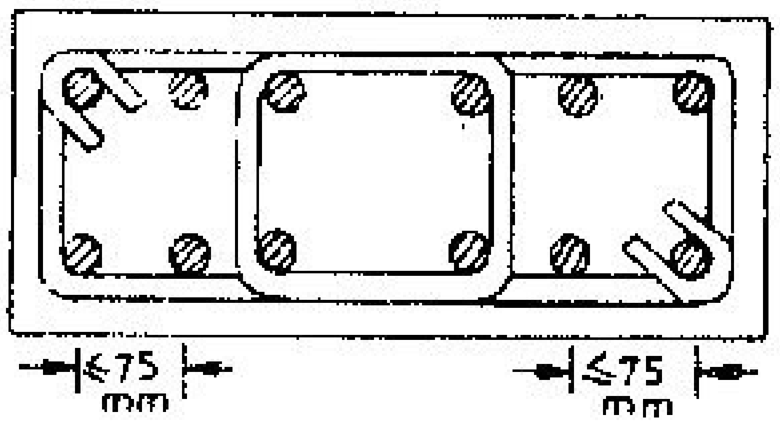
(2) If the longitudinal bars spaced at a distance of not exceeding 48 times the diameter of the tie are effectively tied in two directions, additional longitudinal bars in between these bars need to be tied in one direction, by open ties (Figure b).

(3) Where the longitudinal reinforcing bars in a compression member are placed in more than one row, effective lateral support to the longitudinal bars in the inner rows may be assumed to have been provided if

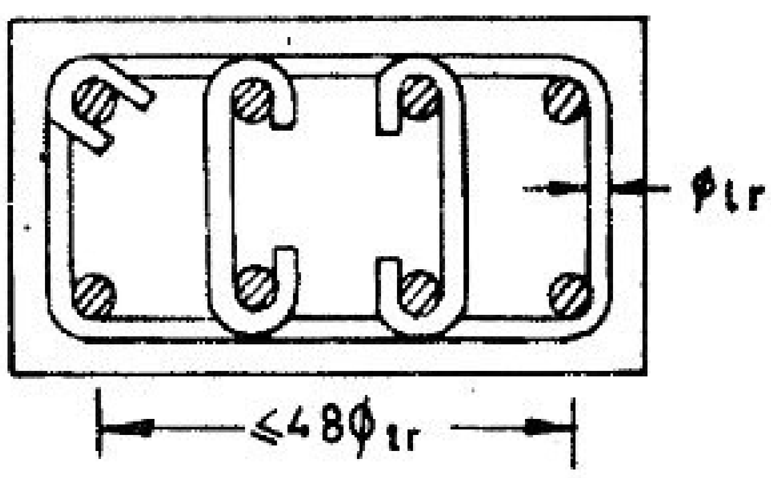
(*i*) transverse reinforcement is provided for the outermost row, and

(*ii*) no bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row (Figure c).

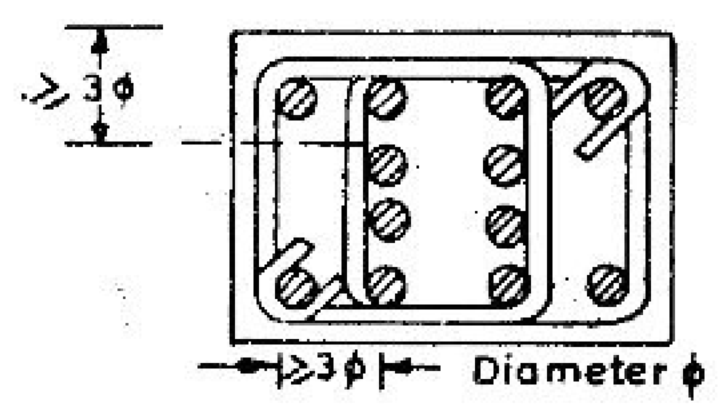
(4) Where the longitudinal bars in a compression member are grouped (not in contact) and each group adequately tied with transverse reinforcement. The transverse reinforcement for the compression member as a whole may be provided on the assumption that each group is a single longitudinal bar for purpose of determining the pitch and diameter of the transverse reinforcement. Diameter of such transverse reinforcement need not, however, exceed 20 mm (Figure d).



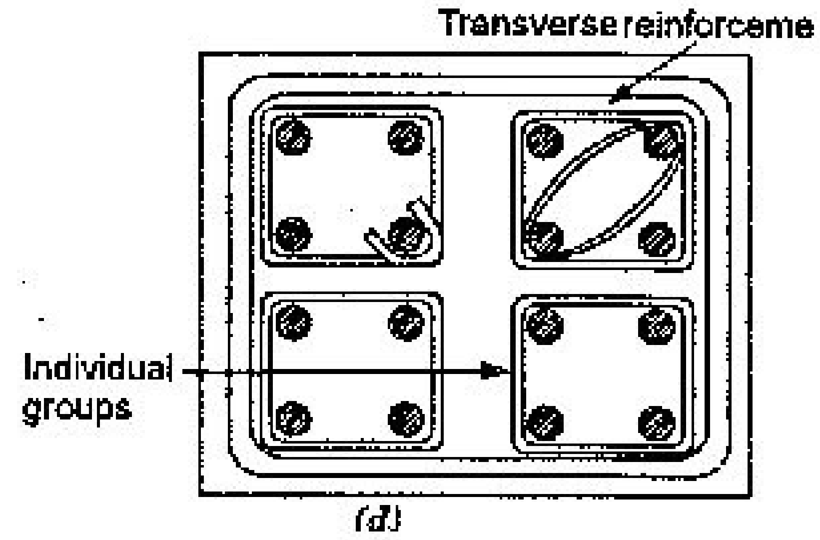
(a)



(b)



(c)



(d)

(*c*) Pitch and diameter of lateral ties (as per I.S. Code IS: 456- 2000)

(1) Pitch. Pitch of transverse reinforcement shall be not more than the least of the following distance:

(*i*) Least lateral dimension of the compression member;

(*ii*) Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied;

(*iii*) 300 mm

(2) Diameter. The diameter of the polygonal links or lateral ties shall be not be less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 16 mm.

(*d*) Helical reinforcement (as per I.S. Code IS: 456–2000)

(1) Pitch. Helical reinforcement shall be of regular formation with the turns of the helix spaced evenly and its ends shall be anchored properly by providing one and half extra turns of the spiral bar. Where an increased load on the column on the strength of the helical reinforcement is allowed for, the pitch of helical turns shall be not more than 75 mm, nor more than one-sixth of the core diameter of the column, nor less than 25 mm, nor less than three times the diameter of the steel bar forming the helix. In other cases, the requirements of transverse reinforcement shall be complied with.

(2) Diameter. Diameter of the helical reinforcement shall be in accordance with para (*c*) above.

Short Column with Helical Reinforcement

The permissible load for columns with helical reinforcement satisfying the following shall be 1.05 times the permissible load for similar member with lateral tie or rings. Provided ratio of the volume of