Exercise 20.1

Q1: A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many are such tiles required to cover a floor of area 1080 m^2?

Answer:

Given :

Base of a flooring tile that is in the shape of a parallelogram = b = 24 cm

Corresponding height = h = 10 cm.

Now, in a parallelogram : Area (A) = Base (b) $\times$ Height (h)

Therefore, Area of a tile = 24 cm $\times$ 10 cm = 240 cm^2

Now, observe that the area of the floor is 1080 m^2.

1080 m^2 = 1080 $\times$ 1m $\times$ 1m

= 1080 $\times$ 100 cm $\times$ 100 cm (Because 1 m = 100 cm)

= 1080 $\times$ 100 $\times$ 100 $\times$ cm $\times$ cm

= 10800000 cm^2

Therefore, Number of required tiles = $\frac{10800000}{240}$ = 45000

Hence, we need 45000 tiles to cover the floor.

2. A plot is in the form of a rectangle ABCD having semi-circle on BC as shown in Fig. 20.23. If AB = 60 m and BC = 28 m, find the area of the plot.

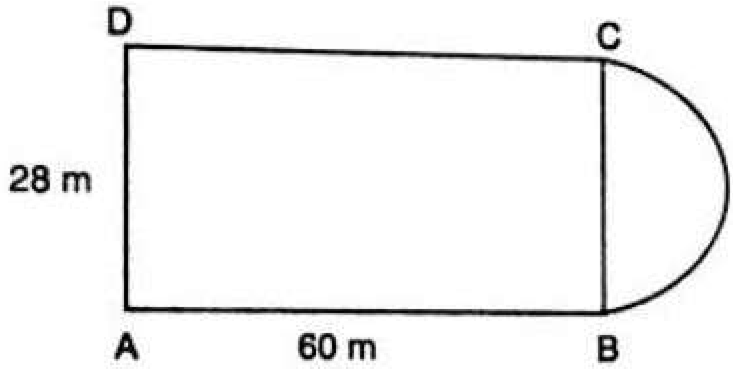
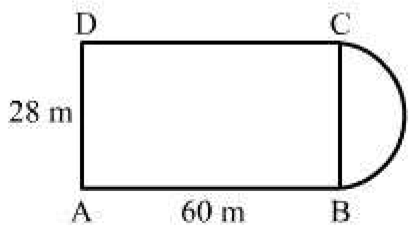


Fig. 20.23

Answer :

The given figure has a rectangle with a semicircle on one of its sides:



Total area of the plot = Area of rectangle ABCD + Area of semicircle with radius ($r = \frac{28}{2} = 14m$)

Therefore, Area of the rectangular plot with sides 60m and 28m $=60 \times 28 = 1680\ m^2$ ....(i)

And area of the semicircle with radius 14m $= \frac{1}{2}\pi \times (14)^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14 = 308\ m^2$ .... (ii)

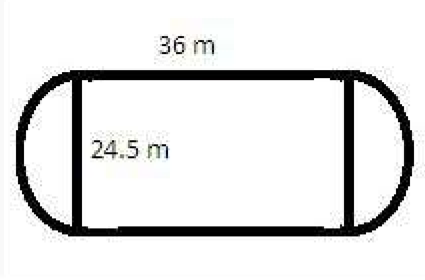
Total area of the plot $=1680 + 308 = 1088\ m^2$ ....(from (i) and (ii)).

3. A playground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 36 m and 24.5 m, find the area of the playground. (Take pi = 22 / 7).

Answer :

It is given that the playground is in the shape of a rectangle with two semicircle on its smaller sides.

Length of the rectangular portion is 36 m and its width is 24.5 m as shown in the figure below.



Thus, the area of the playground will be the sum of the area of a rectangle and the areas of the two semicircles with equal diameter 24.5 m.

Now, area of rectangle with length 36m and width 24.5m:

Area of rectangle = length x width

= 36m $\times$ 24. 5 m =882 m^2

Radius of the semicircle = r = $\frac{diameter}{2}$ = $\frac{24.5}{2}$ = 12.25m

Therefore, Area of the semicircle = $\frac{1}{2}\pi r^2$

= ½ $\times$ 22/7 $\times$ $(12.25)^2$ = 235.8 m^2

Therefore, Area of the complete playground = area of the rectangular ground + 2 $\times$ area of a semicircle

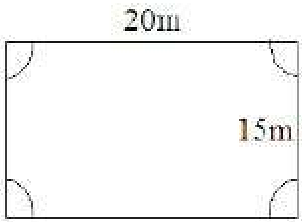
= 882 + 2 $\times$ 235.8 = 1353.6 m^2

4. A rectangular piece is 20 m long and 15 m wide. From its four corners, quadrants of radii 3.5 m have been cut. Find the area of the remaining part.

Answer :

It is given that the length of the rectangular piece is 20 m and its width is 15 m.

And, from each corner a quadrant each of radius 3.5 m has been cut out. A rough figure for this is given below :



Therefore, Area of the remaining part = Area of the rectangular piece $-$ (4 $\times$ Area of a quadrant of radius 3.5m)

Now, area of the rectangular piece = $20 \times 15 = 300$ m^2

And, area of a quadrant with radius 3.5 m = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 3.5^2 = 9.625$ m^2

Therefore, Area of the remaining part = $300 - (4 \times 9.625) = 261.5$ m^2

5. The inside perimeter of a running track (shown in Fig. 20.24) is 400 m. The length of each of the straight portion is 90 m and the ends are semi-circles. If the track is everywhere 14 m wide, find the area of the track. Also, find the length of the outer running track.

Answer:

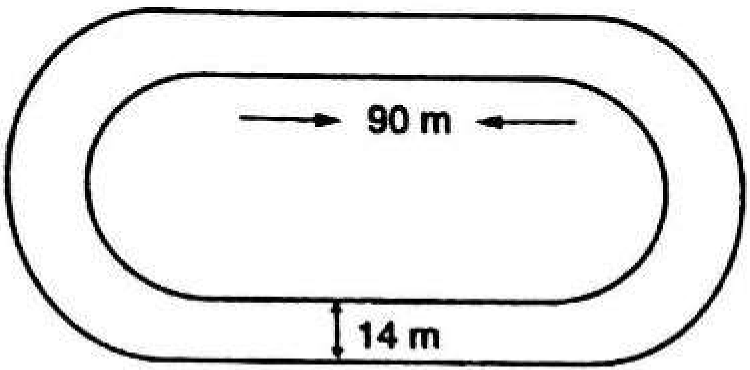


Fig. 20.24

It is given that the inside perimeter of the running track is 400m. It means the length of the inner track is 400 m.

Let r be the radius of the inner semicircles.

Observe : Perimeter of the inner track = Length of two straight portions of 90 m + Length of two semicircles 400

Therefore, 400 =(2 $\times$ 90) +(2 $\times$ Perimeter of a semicircle)

400 = 180 + (2 $\times$ 22/7 $\times$ r)

400 -180 = (44/7 $\times$ r)

44/7 $\times$ r = 220

r = $\frac{220\times7}{44}$ = 35 m

Therefore, Width of the inner track = 2r = 2 $\times$ 35 = 70 m

Since the track is 14 m wide at all places, so the width of the outer track : 70 + (2 $\times$ 14) = 98m

Radius of the outer track semicircles = 98/2 = 49 m

Area of the outer track = (Area of the rectangular portion with sides 90 m and 98 m) + (2 × Area of two semicircles with radius 49 m)

$=(98 \times 90) + (2 \times ½ \times 22/7 \times 49^2)$

$= (8820) + (7546) = 16366\ m^2$

And, area of the inner track = (Area of the rectangular portion with sides 90 m and 70 m) + (2 × Area of the semicircle with radius 35 m)

$= (70 \times 90) + (2 \times ½ \times 22/7 \times 35^2)$

$= (6300) + (3850)$

$=10150\ m^2$

Therefore, Area of the running track = Area of the outer track – Area of the inner track

$= 16366 - 10150$

$= 6216\ m^2$

And, length of the outer track = (2 × length of the straight portion) + (2 × perimeter of the semicircles with radius 49 m)

$= (2 \times 90) -(2 \times 22/7 \times 49)$

$= 180 +308 = 488$ m

6. Find the area of Fig. 20.25, in square cm, correct to one place of decimal. (Take it = 22/7)

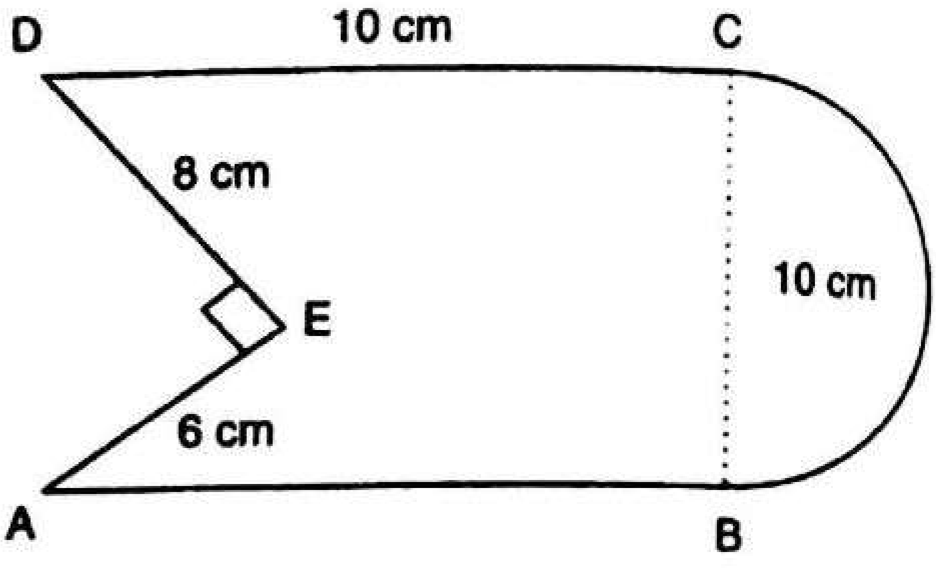
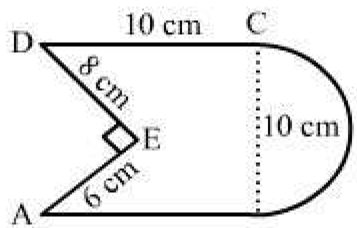


Fig. 20.25

Answer:

The given figure is:



Construction: Connect A to D.

Then, we have Area of the given figure = (Area of rectangle ABCD + Area of the semicircle) − (Area of triangle AED).

Therefore, Total area of the figure = (Area of rectangle with sides 10 cm and 10 cm) + (Area of semicircle with radius = 10/2 = 5 cm) − (Area of triangle AED with base 6 cm and height 8 cm)

$= (10 \times 10) + (1/2 \times 22/7 \times 5^2) - (1/2 \times 6 \times 8)$

$= 100 + 39.3 - 24 = 115.3 \text{ cm}^2$

7. The diameter of a wheel of a bus is 90 cm which makes 315 revolutions per minute. Determine its speed in kilometers per hour. [Use pi = 22 / 7].

Answer :

It is given that the diameter of the wheel is 90 cm.

Therefore, Radius of the circular wheel, r = 90/2 = 45 cm.

Therefore, Perimeter of the wheel = $2 \times \pi \times$ r

= $2 \times 22/7 \times 45$

= 282.857 cm

It means the wheel travels 282.857 cm in a revolution.

Now, it makes 315 revolutions per minute.

Therefore, Distance travelled by the wheel in one minute = 315×282.857 = 89100 cm

Therefore, Speed = 89100 cm per minute = $\frac{89100\ cm}{1\ minute}$

Now, we need to convert it into kilometers per hour.

Therefore, $\frac{89100\ cm}{1\ minute} = \frac{89100 \times \frac{1}{100000}\ kilometer}{\frac{1}{60}\ hour}$

$= \frac{891000}{100000} \times \frac{60}{1} \frac{kilometer}{hour}$ = 53.46 kilometers per hour.

8. The area of a rhombus is 240 cm^2 and one of the diagonal is 16 cm. Find another diagonal.

Answer :

Given :

Area of the rhombus = 240 cm

Length of one of its diagonals = 16 cm

We know that if the diagonals of a rhombus are d_1 and d_2, then the area of the rhombus is given by :

Area = $\frac{1}{2}(d_1 \times d_2)$

Putting the given values :

240 = $\frac{1}{2}(16 \times d_2)$

$240 \times 2 = 16 \times d_2$

This can be written as follows :

$16 \times d_2 = 480$

$d_2 = 480/16 = 30$ cm

Thus, the length of the other diagonal of the rhombus is 30 cm.

9. The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

Answer :

Given :

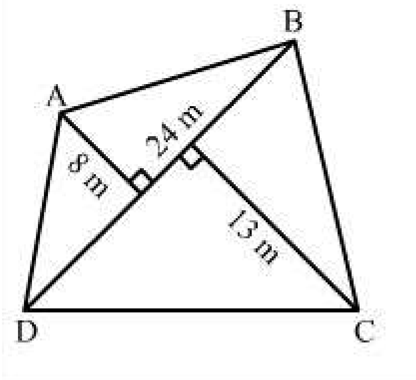
Lengths of the diagonals of a rhombus are 7.5 cm and 12 cm.

Now, we know : Area = $\frac{1}{2}(d_1 \times d_2)$

Area of rhombus = $\frac{1}{2}(7.5 \times 12)$ = 45 cm^2

10. The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. find the area of the field.

Answer:



Given :

Diagonal of a quadrilateral shaped field = 24 m

Perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m.

Now, we know : Area = $\frac{1}{2} \times d \times (h_1 + h_2)$

Therefore, Area of the field = $\frac{1}{2} \times 24 \times (8 + 13)$

= 12 x 21 = 252 m^2

11. Find the area of a rhombus whose side is 6 cm and whose altitude is 4 cm. If one of its diagonals is 8 cm long, find the length of the other diagonal.

Answer :

Given :

Side of the rhombus = 6 cm

Altitude = 4 cm

One of the diagonals = 8 cm

Area of the rhombus = Side $\times$ Altitude = 6 $\times$ 4 = 24 cm^2

We know :

Area of rhombus = $\frac{1}{2}(d_1 \times d_2)$

Using (i) :

24 = $\frac{1}{2}(d_1 \times d_2)$

24= $\frac{1}{2}(8 \times d_2)$

D_2 = 6 cm .

12. The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m^2 is Rs 4.

Answer :

Given :

The floor consists of 3000 rhombus shaped tiles.

The lengths of the diagonals of each tile are 45 cm and 30 cm.

Area of a rhombus shaped the = $\frac{1}{2}(45 \times 30)$ = 675 cm^2

Therefore, Area of the complete floor = 3000 $\times$ 675 = 2025000 cm^2

Now, we need to convert this area into m^2 because the rate of polishing is given as per m^2.

Therefore, 2025000 cm^2 = 2025000 $\times$ cm $\times$ cm

= 2025000 $\times$ 1/100 m $\times$ 1/100 m

= 202.5 m^2

Now, the cost of polishing 1 m^2 is Rs 4.

Therefore, Total cost of polishing the complete floor = 202.5 $\times$ 4 = 810

Thus, the total cost of polishing the floor is Rs 810.

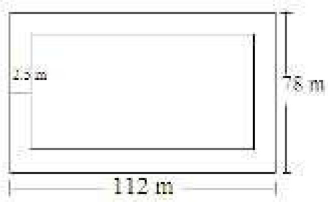
13. A rectangular grassy plot is 112 m long and 78 m broad. It has a gravel path 2.5 m wide all around it on the side. Find the area of the path and the cost of constructing it at Rs 4.50 per square meter.

Answer :

Given: The length of a rectangular grassy plot is 112 m and its width is 78 m.

Also, it has a gravel path of width 2.5 m around it on the sides

Its rough diagram is given below :



Length of the inner rectangular field = $112 - (2 \times 2.5) = 107$ m

The width of the inner rectangular field = $78 - (2 \times 2.5) = 73$ m

Area of the path = (Area of the rectangle with sides 112 m and 78 m) − (Area of the rectangle with sides 107 m and 73 m)

$= (112 \times 78) - (107 \times 73)$

$= 8736 - 7811 = 925\ m^2$

Now, the cost of constructing the path is Rs 4.50 per square meter.

Cost of constructing the complete path = 925×4.50 = Rs 4162.5

Thus, the total cost of constructing the path is Rs 4162.5.

14. Find the area of a rhombus, each side of which measures 20 cm and one of whose diagonals is 24 cm.

Answer :

Given :

Side of the rhombus = 20 cm

Length of a diagonal = 24 cm

We know : If d_1 and d_2 are the lengths of the diagonals of the rhombus, then side of the rhombus = $\frac{1}{2}\sqrt{d_1^2 + d_2^2}$

So, using the given data to find the length of the other diagonal of the rhombus

$20 = \frac{1}{2}\sqrt{24^2 + d_2^2}$

$40 = \sqrt{24^2 + d_2^2}$

Squaring both sides to get rid of the square root sign :

$40^2 = 24^2 - {d_2}^2$

${d_2}^2 = 1600 - 576 = 1024$

$d_2 = \sqrt{1024} = 32$ cm

Therefore, Area of the rhombus = $\frac{1}{2}(24 \times 32) = 384\ cm^2$

15. The length of a side of a square field is 4 m. What will be the altitude of the rhombus, if the area of the rhombus is equal to the square field and one of its diagonal is 2 m?

Answer :

Given:

Length of the square field = 4 m

Area of the square field = 4 $\times$ 4 = 16 m2

Given: Area of the rhombus = Area of the square field

Length of one diagonal of the rhombus = 2 m

Side of the rhombus = $\frac{1}{2}\sqrt{d_1^2 + d_2^2}$

And, area of the rhombus = $\frac{1}{2} \times (d_1 \times d_2)$

Therefore, Area :

16 = $\frac{1}{2} \times (2 \times d_2)$

Now, we need to find the length of the side of the rhombus.

Therefore, Side of the rhombus = $\frac{1}{2}\sqrt{2^2 + 16^2}$ = $\frac{1}{2}\sqrt{260}$ = $\frac{1}{2}\sqrt{4 \times 65}$ = $\frac{1}{2} \times 2\sqrt{65}$ = $\sqrt{65}$

Also, we know: Area of the rhombus = Side $\times$ Altitude

Therefore, 16 = $\sqrt{65}$ $\times$ Altitude

Altitude= $\frac{16}{\sqrt{65}}$ m .

16. Find the area of the field in the form of a rhombus, if the length of each side be 14 on and the altitude is 16 cm.

Answer :

Given:

Length of each side of a field in the shape of a rhombus = 14 cm Altitude = 16 cm

Now, we know: Area of the rhombus = Side $\times$ Altitude

Therefore, Area of the field = 14 $\times$ 16 = 224 cm2

17. The cost of fencing a square field at 60 paise per meter is Rs 1200. Find the cost of reaping the field at the rate of 50 paise per 100 sq. metres.

Answer :

Given:

Cost of fencing 1 metre of a square field = 60 paise

And, the total cost of fencing the entire field = Rs 1200 = 1,20,000 paise

Perimeter of the square field = $\frac{120000}{60}$ = 2000 metres

Now, perimeter of a square = 4 $\times$ side

For the given square field :

4 $\times$ Side = 2000 m

Side = $\frac{2000}{4}$ = 500 metres

Therefore, Area of the square field = 500 $\times$ 500 = 250000 m^2

Again, given: Cost of reaping per 100 m^2 = 50 paise

Therefore, Cost of reaping per 1 m^2 = $\frac{50}{100}$ paise

Therefore, Cost of reaping 250000 m^2 = $\frac{50}{100} \times 250000$ = 125000 paise .

Thus, the total cost of reaping the complete square field is 125000 paise, i.e. RS. 1250.

18. In exchange of a square plot one of whose sides is 84 m, a man wants to buy a rectangular plot 144 m long and of the same area as of the square plot. Find the width of the rectangular plot.

Answer :

Given:

Side of the square plot = 84 m

Now, the man wants to exchange it with a rectangular plot of the same area with length 144.

Area of the square plot = $84 \times 84 = 7056\ m^2$

Therefore, Area of the rectangular plot = Length $\times$ Width

$7056 = 144 \times$ Width

Width = 7056/144 = 49 m

Hence, the width of the rectangular plot is 49 m.

19. The area of a rhombus is 84 m^2. If its perimeter is 40 m, then find its altitude.

Answer :

Given:

Area of the rhombus = 84 m^2

Perimeter = 40 m

Now, we know: Perimeter of the rhombus = 4 $\times$ Side

$40 = 4 \times$ Side

Side = 40/4 = 10 m

Again, we know: Area of the rhombus = Side $\times$ Altitude

$84 = 10 \times$ Altitude

Altitude = 84/10 = 8.4 m

Hence, the altitude of the rhombus is 8.4 m.

20. A garden is in the form of a rhombus whose side is 30 meters and the corresponding altitude is 16 m. Find the cost of leveling the garden at the rate of Rs 2 per m^2

Answer :

Given:

Side of the rhombus-shaped garden = 30 m

Altitude = 16 m

Now, area of a rhombus = side $\times$ Altitude

Area of the given garden = $30 \times 16 = 480\ m^2$

Also, it is given that the rate of leveling the garden is Rs 2 per $1m^2$.

Therefore, Total cost of levelling the complete garden of area $480\ m^2 = 480 \times 2$ = Rs 960.

21. A field in the form of a rhombus has each side of length 64 m and altitude 16 m. What is the side of a square field which has the same area as that of a rhombus?

Answer :

Given:

Each side of a rhombus-shaped field = 64 m

Altitude =16 m

We know: Area of rhombus = Side $\times$ Altitude

Therefore, Area of the field = $64 \times 16 = 1024\ m^2$

Given: Area of the square field = Area of the rhombus

We know: Area of a square = $(Side)^2$

Therefore, $1024 = (Side)^2$

Side = $\sqrt{1024}$ = 32 m

Thus, the side of the square field is 32 m.

22. The area a rhombus is equal to the area of a triangle whose base and the corresponding altitudes are 24.8 cm and 16.5 cm respectively. If one of the diagonals of the rhombus is 22 cm, find the length of the other diagonal.

Answer :

Given:

Area of the rhombus = Area of the triangle with base 24.8 cm and altitude 16.5 cm

Area of the triangle = $1/2 \times$ base $\times$ altitude $= 1/2 \times 24.8 \times 16.5 = 204.6\ cm^2$

Therefore, Area of the rhombus = 204.6 cm2

Also, length of one of the diagonals of the rhombus = 22 cm

We know : Area of rhombus = $\frac{1}{2} \times (d_1 \times d_2)$

$204.6 = \frac{1}{2} \times (22 \times d_2)$

$22 \times d^2 = 409.2$

$d^2 = 409.2/22 = 18.6$ cm

Hence, the length of the other diagonal of the rhombus is 18.6 cm.

Exercise 20.2

1. Find the area, in square metres, of the trapezium whose bases and altitudes are as under:

(i) bases = 12 dm and 20 dm, altitude = 10 dm

(ii) bases = 28 cm and 3 dm, altitude = 25 cm

(iii) bases = 8 m and 60 dm, altitude = 40 dm

(iv) bases = 150 cm and 30 dm, altitude = 9 dm.

(i) Given:

Bases :

12 dm = $\frac{12}{10}$ = 1.2 m

And, 20 dm = $\frac{20}{10}$ m = 2 m

Altitude = 10 dm = $\frac{10}{10}$m = lm

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ bases) \times (Altitude)$

= $\frac{1}{2} \times (1.2 + 2) \times (1)$

=1.6 $\times$ m $\times$ m =1.6 m^2

(ii)

Given:

Bases:

28 cm = $\frac{28}{100}$ m=0.28 m

And, 3 dm = $\frac{3}{10}$ = 0.3 m

Altitude = 25 cm = $\frac{25}{100}$ m = 0.25 m

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ bases) \times (Altitude)$

= $\frac{1}{2} \times (0.28 + 0.3)\, m \times (0.25)\, m$

= 0.0725 m^2

(iii)

Given:

Bases: 8m

And , 60 dm = $\frac{60}{10}$ m = 6m

Altitude = 40 dm = $\frac{40}{10}$ m = 4 m

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ bases) \times (Altitude)$

= $\frac{1}{2} \times (8 + 6) \times (4)$

= 28 × m × m = 28 m^2

(iv) Given:

Bases: 150cm = $\frac{150}{100}$ m= 1.5 m

And, 30 dm = $\frac{3}{10}$ m = 3m

Altitude = 9 dm = $\frac{9}{10}$ m = 0.9 m

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ bases) \times (Altitude)$

= $\frac{1}{2} \times (1.5 + 3) \times (0.9)$

= 2.025 × m × m = 2.025 m^2

2. Find the area of trapezium with base 15 cm and height 8 cm, if the side parallel to the given base is 9 cm long.

Answer :

Given: Lengths of the parallel sides are 15 cm and 9 cm.

Height = 8 cm

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ bases) \times (Sum\ of\ parallel\ sides)$

= $\frac{1}{2} \times (15 + 9) \times (8)$

= 96 cm^2

3. Find the area of a trapezium whose parallel sides are of length 16 dm and 22 dm and whose height is 12 dm.

Answer :

Given:

Lengths of the parallel sides are 16 dm and 22 dm.

And, height between the parallel sides is 12 dm.

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ bases) \times (height)$

= $\frac{1}{2} \times (16 + 22) \times (12)$

= 228 dm^2

= 228 $\times$ dm $\times$ dm

= 228 $\times$ $\frac{1}{10}$m $\times$ $\frac{1}{10}$m

= 2.28 m^2

4. Find the height of a trapezium, the sum of the lengths of whose bases (parallel sides) is 60 cm and whose area is 600 cm^2.

Answer :

Given:

Sum of the parallel sides of a trapezium = 60 cm

Area of the trapezium = 600 cm^2

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ bases) \times (height)$

On putting the values :

600 = $\frac{1}{2} \times (60) \times (height)$

600 = 30 $\times$ (Height)

Height = $\frac{600}{30}$ = 20 cm.

5. Find the altitude of a trapezium whose area is 65 cm^2 and whose bases are 13 cm and 26 cm.

Answer :

Given:

Area of the trapezium = 65 cm^2

The lengths of the opposite parallel sides are 13 cm and 26 cm.

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ bases) \times (Altitude)$

On putting the values :

65 = $\frac{1}{2} \times (13 + 26) \times (Altitude)$

65 $\times$ 2 = 39 $\times$ Altitude

Altitude = $\frac{130}{39}$ = $\frac{10}{3}$ cm.

6. Find the sum of the length of the bases of a trapezium whose area is 4.2 m^2 and whose height is 280 cm.

Answer :

Given:

Area of the trapezium = 4.2 m^2

Height = 280 cm = $\frac{280}{100}$ m = 2.8 m

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ bases) \times (Altitude)$

$4.2 = \frac{1}{2} \times (Sum\ of\ bases) \times (2.8)$

4.2×2 = (Sum of the parallel bases) $\times$ 2.8

Sum of the parallel bases = $\frac{8.4}{2.8}$ = 3 m

7. Find the area of a trapezium whose parallel sides of lengths 10 cm and 15 cm are at a distance of 6 cm from each other. Calculate this area as:

(i) the sum of the areas of two triangles and one rectangle.

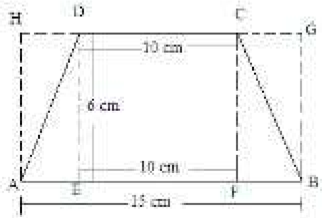
(ii) the difference of the area of a rectangle and the sum of the areas of two triangles.

Answer :

Given: Length of the parallel sides of a trapezium are 10 cm and 15 cm.

The distance between them is 6 cm.

Let us extend the smaller side and then draw perpendiculars from the ends of both sides.



(i) Area of trapezium ABCD =(Area of rectangle EFCD)+(Area of triangle AED + Area of triangle BFC)

= $(10 \times 6) + [(\frac{1}{2} \times AE \times ED) + (\frac{1}{2} \times BF \times FC)]$

= $60 + [(\frac{1}{2} \times AE \times 6) + (\frac{1}{2} \times BF \times 6)]$

= 60 + [3 AE + 3 BF]

= $60 + 3 \times (AE + BF)$

Here, AE + EF + FB = 15 cm

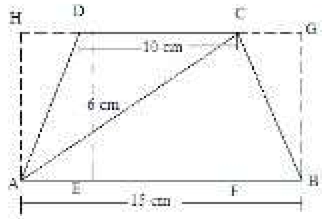
And EF = 10 cm

Therefore, AE + 10 + BF = 15

Or, AE + BF = 15 − 10 = 5 cm

Putting this value in the above formula: Area of the trapezium = $60 + 3 \times (5) = 60 + 15 = 75$ cm^2

(ii) In this case, the figure will look as follows:



Area of trapezium ABCD = (Area of rectangle ABGH) − [(Area of triangle AHD) + (Area of BGC)]

= (15 × 6) − [(! × DH × 6) + (× GC × 6)]

= 90 − [3 × DH + 3 × GC]

= 90 − 3 [DH + GC]

Here, HD + DC + CG = 15 cm

DC = 10 cm

HD + 10 + CG = 15

HD + GC = 15 − 10 = 5 cm

Putting this value in the above equation:

Area of the trapezium = 10 − 3(5) = 90 − 15 = 75 cm^2

8. The area of a trapezium is 960 cm^2. If the parallel sides are 34 cm and 46 cm, find the distance between them.

Answer :

Given:

Area of the trapezium = 960 cm^2

And the length of the parallel sides are 34 cm and 46 cm.

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

960 = $\frac{1}{2} \times (34 + 46) \times (Altitude)$

960 = 40 × (Height)

Height = $\frac{960}{40}$ = 24 cm.

9. Find the area of Fig. 20.35 as the sum of the areas of two trapeziums and a rectangle.

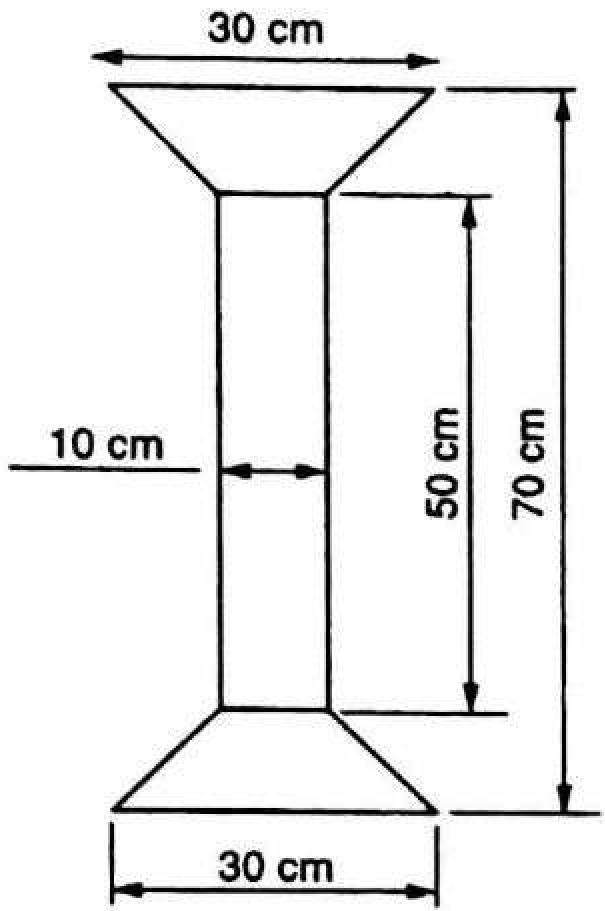


Fig. 20.35

Answer:

In the given figure, we have a rectangle of length 50 cm and width 10 cm and two similar trapeziums with parallel sides as 30 cm and 10 cm at both ends.

Suppose x is the perpendicular distance between the parallel sides in both the trapeziums.

We have:

Total length of the given figure = Length of the rectangle + 2 × Perpendicular distance between the parallel sides in both the trapeziums

$70 = 50 + 2 \times x$

$2 \times x = 70 - 50 = 20$

$x = \frac{20}{2}$ m = 10 cm.

Now, area of the complete figure = (area of the rectangle with sides 50 cm and 10 cm) + 2 × (area of the trapezium with parallel sides 30 cm and 10 cm, and height 10 cm)

$= (50 \times 10) + 2 \times [1/2 \times (30 + 10) \times (10)] = 500 + 2 \times [200] = 900\ cm^2$.

10. Top surface of a table is trapezium in shape. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.

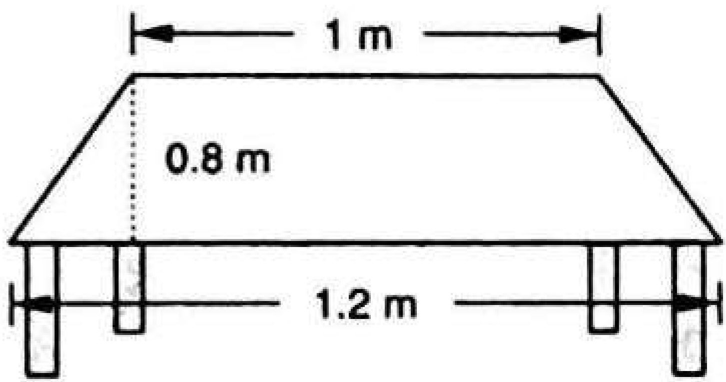
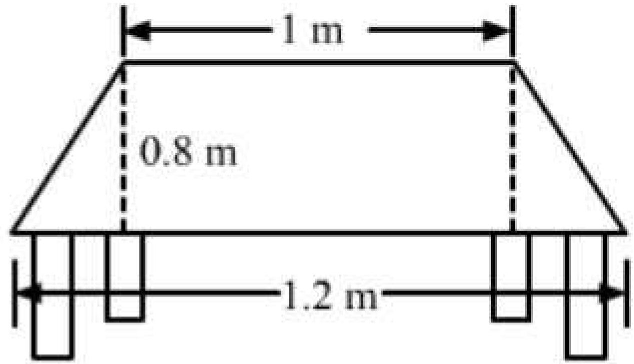


Fig. 20.36

Answer:

The given figure is :



Lengths of the parallel sides are 1.2 m and 1 m and the perpendicular distance between them is 0.8 m.

Area of the trapezium shaped surface = $\frac{1}{2} \times (Sum\ of\ bases) \times (Altitude)$

= $\frac{1}{2} \times (1.2 + 1) \times (0.8)$

=$1/2 \times 2.2 \times 0.8$ =0.88 m^2.

11. The cross-section of a canal is a trapezium in shape. If the canal is 10 m wide at the top 6 m wide at the bottom and the area of cross-section is 72 m^2 determine its depth.

Answer :

Let the depth of canal be d.

Given:

Lengths of the parallel sides of the trapezium shape canal are 10 m and 6 m.

And, the area of the cross-section of the canal is 72 m^2.

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

72 = $\frac{1}{2} \times (10 + 6) \times (d)$

72 = 8 $\times$ d

d = $\frac{72}{8}$ = 9 m.

12. The area of a trapezium is 91 c m^2 and its height is 7 cm. If one of the parallel sides is longer than the other by 8 cm, find the two parallel sides.

Answer :

Given:

Area of the trapezium = 91 cm^2

Height = 7 cm

Let the length of the smaller side be x.

Then, the length of the longer side will be 8 more than the smaller side, i.e. 8 + x.

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

91 = 1/2 × [(8 + x)+x] × (7)

91 = 7/2 × [8 + x + x]

91 × 2 = 7 × [8 + 2x]

We can rewrite it as follows:

7 × [8 +2x] = 182

[8 + 2 x] = $\frac{182}{7}$ = 26

8 + 2x = 26

2x = 26 − 8 =18

x = 18/2 = 9cm

Therefore, Length of the shorter side of the trapezium = 9 cm

And, length of the longer side = 8 + x = 8 + 9 = 17 cm.

13. The area of a trapezium is 384 c m^2. Its parallel sides are in the ratio 3 : 5 and the perpendicular distance between them is 12 cm. Find the length of each one of the parallel sides.

Answer :

Given:

Area of the trapezium = 384 cm^2

The parallel sides are in the ratio 3:5 and the perpendicular height between them is 12 cm.

Suppose that the sides are in x multiples of each other.

Then, length of the shorter side = 3x

Length of the longer side = 5x

Area of a trapezium = $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

384 = $\frac{1}{2} \times (3x + 5x) \times (12)$

384 = $\frac{12}{2} \times$ (8x)

384 = 6 $\times$ (8x)

8x = $\frac{384}{6}$ = 64

x = $\frac{64}{8}$ = 8cm

Therefore, Length of the shorter side = 3 $\times$ x = 3 $\times$ 8 = 24 cm

And, length of the longer side = 5 $\times$ 5 $\times$ 8 = 40 cm.

14. Mohan wants to buy a trapezium-shaped field. Its side along the river is parallel and twice the side of the road. If the area of this field is 10500 m^2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.

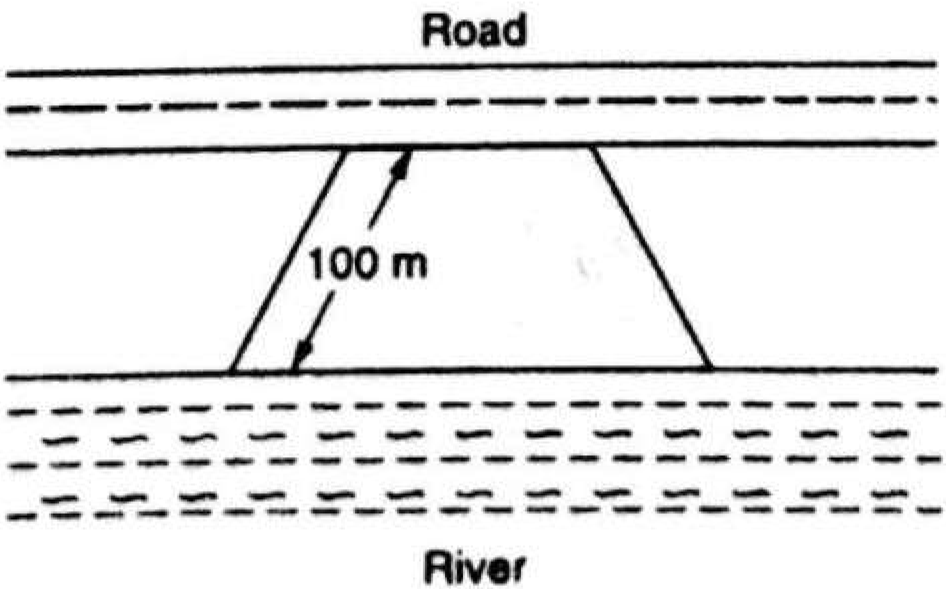


Fig. 20.37

Given:

Area of the trapezium-shaped field = 10500 m^2

It is also given that the length of the side along the river is double the length of the side ;

Let us suppose the length of the side along the road to be x.

Then, the length of the side along the river = 2 $\times$ x = 2x

And, the perpendicular distance between these parallel sides = 100 m

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

10500 = $\frac{1}{2} \times (2x + x) \times (100)$

10500 = 50 $\times$ (3x)

3x = $\frac{10500}{50}$ = 210

x = $\frac{210}{3}$ = 70 m

Therefore, Length of the side along the river = 2 $\times$ x = 2 $\times$ 7 = 140 m.

15. The area of a trapezium is 1586 c m^2 and the distance between the parallel sides is 26 cm. If one of the parallel sides is 38 cm, find the other.

Answer :

Given:

Area of the trapezium = 1586 cm^2

Distance between the parallel sides = 26 cm

And, length of one parallel side = 38 cm

Let us suppose the length of the other side to be x cm.

Now, area of the trapezium= $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

1586 = $\frac{1}{2} \times (38 + x) \times (26)$

1586 = $\frac{26}{2} \times$ (38 + x)

13 $\times$ (38 + x) = 1586

38 + x = $\frac{1586}{13}$ = 122

x = 122 − 38 = 84 cm

Hence, the length of the other parallel side is 84 cm.

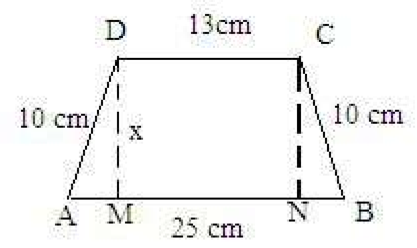
16. The parallel sides of a trapezium are 25 cm and 13 cm; its nonparallel sides are equal, each being 10 cm, find the area of the trapezium.

Answer :

Given: The parallel sides of a trapezium are 25 cm and 13 cm.

Its nonparallel sides are equal in length and each is equal to 10 cm.

A rough sketch for the given trapezium is given below:



In above figure, we observe that both the right angle triangles AMD and BNC are congruent triangles.

AD = BC = 10 cm

D = CN = x cm

$\angle DMA = \angle CNB = 90^\circ$

Hence, the third side of both the triangles will also be equal.

Therefore, AM = BN

Also, MN = 13

Since AB = AM + MN + NB

Therefore, 25 = AM + 13 + BN

AM + BN = 25 − 13 = 12 cm

Or, BN + BN = 12 cm (Because AM=I3N)

2 BN = 12

BN = $\frac{12}{2}$ =6 cm

Therefore, AM = BN = 6cm.

Now, to find the value of x, we will use the Pythagoras theorem in the right angle triangle AMD, whose sides are 11

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Altitude})^2$

$(10)^2 = (6)^2 + (x)^2$

$100 = 36 + x^2$

$x^2 = 100 - 36 = 64$

$x = \sqrt{64} = 8$ cm

Therefore, Distance between the parallel sides = 8 cm

Therefore, Area of trapezium = $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

= $\frac{1}{2} \times (25 + 13) \times (8)$

= 152 cm^2

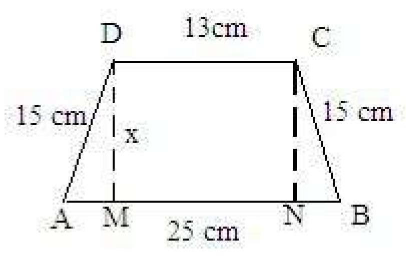
17. Find the area of a trapezium whose parallel sides are 25 cm, 13 cm and the other sides are 15 cm each.

Answer :

Given: Parallel sides of a trapezium are 25 cm and 13 cm.

Its nonparallel sides are equal in length and each is equal to 15 cm.

A rough sketch of the trapezium is given below:



In above figure, we observe that both the right angle triangles AMD and BNC are similar

This is because both have two common sides as 15 cm and the altitude as x and a right angle.

Hence, the remaining side of both the triangles will be equal.

Therefore, AM = BN

Also MN = 13

Now, since AB = AM + MN + NB:

Therefore, 25 = AM + 13 + BN

AM + BN = 25 − 13 = 12 cm

Or, BN + BN = 12 cm (Because AM = BN)

2 BN = 12

BN = 12/2 = 6 cm

AM = BN=6 cm

Now, to find the value of x, we will use the Pythagorian theorem in the right angle triangle AMD whose sides are 15, 6 and x.

$(\text{Hypotenus})^2 = (\text{Base})^2 - (\text{Altitude})^2$

$(15)^2 = (6)^2 + (x)^2$

$225 = 36 + (x)^2$

$(x)^2 = 225 - 36 = 189$

Therefore, $x = \sqrt{189}$

$= \sqrt{9 \times 21}$

$= 3\sqrt{21}$ cm

Therefore, Distance between the parallel sides = $3\sqrt{21}$ cm

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

$= \frac{1}{2} \times (25 + 13) \times \left(3\sqrt{21}\right)$

$=57\sqrt{21}$ cm^2

18. If the area of a trapezium is 28 cm^2 and one of its parallel sides is 6 cm, find the other parallel side if its altitude is 4 cm.

Answer :

Given: Area of the trapezium = 28 cm^2

Length of one of its parallel sides = 6 cm

Altitude = 4 cm

Let the other side be x cm.

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

28 = $\frac{1}{2} \times (6 + x) \times (4)$

28 = 2 × (6 + x)

6 + x = $\frac{28}{2}$ = 14

x = 14 − 6 = 8 cm

Hence, the length of the other parallel side of the trapezium is 8 cm.

19. In Fig. 20.38, a parallelogram is drawn in a trapezium, the area of the parallelogram is 80 cm^2, find the area of the trapezium.

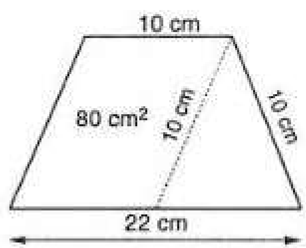


Fig. 20.38

Answer:

The given figure is:

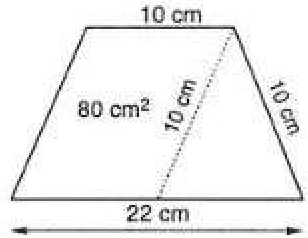


Fig. 20.38

From above figure, it is clear that the length of the parallel sides of the trapezium are 22 cm and 10 cm.

Also, it is given that the area of the parallelogram is 80 cm^2 and its base is 10 cm.

We know:

Area of parallelogram = Base $\times$ Height

80 = 10 $\times$ Height

Height = 80/10 = 8 cm

So, now we have the distance between the parallel sides of trapezium, which is equal to 8 cm.

Area of trapezium = $\frac{1}{2} \times (Sum\ of\ parallel\ sides) \times (Altitude)$

= $\frac{1}{2} \times (22 + 10) \times (8)$ = 128 cm^2

20. Find the area of the field shown in Fig. 20.39 by dividing it into a square, a rectangle and a trapezium.

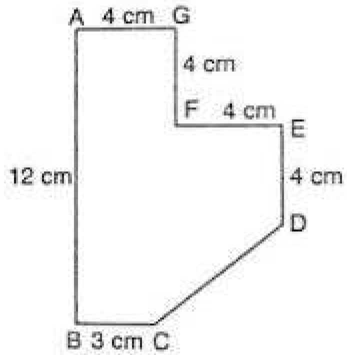
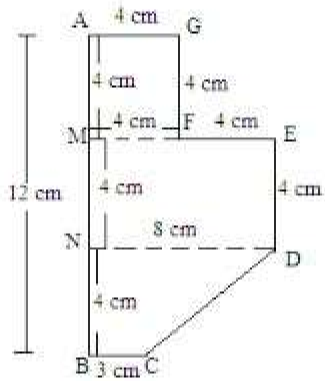


Fig. 20.39

Answer :

The given figure can be divided into a square, a parallelogram and a trapezium as shown in figure.



From the above figure: Area of the figure(Area of square AGFM with sides 4 cm) + (Area of rectangle MEDN with length 8 cm and width 4 cm) + (Area of trapezium NDCB with parallel sides 8 cm and 3 cm and perpendicular height 4 cm)

$= (4 \times 4) + (8 \times 4) + [1/2 \times (8 + 3) \times (4)] = 16 + 32 + 22 = 70 \text{ cm}^2$

Exercise 20.3

1. Find the area of the pentagon shown in fig. 20.48, if AD = 10 cm, AG = 8 cm, AH= 6 cm, AF= 5 cm, BF = 5 cm, CG = 7 cm and EH = 3 cm.

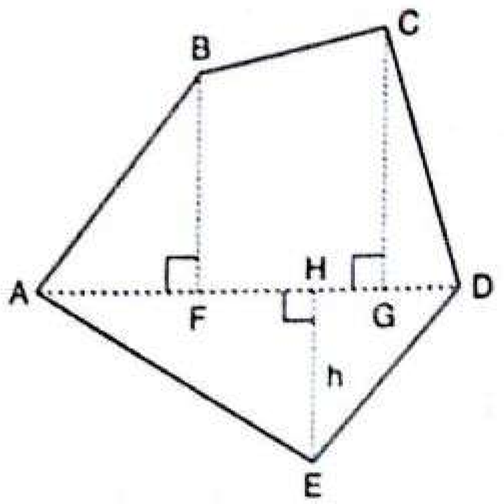


Fig. 20.48

Answer:

Given:

AD = 10 cm, AG = 8 cm, AH = 6 cm, AF = 5 cm

BF = 5 cm, CG = 7 cm, EH = 3 cm

Therefore, FG = AG − AF = 8 − 5 = 3 cm

And, GD = AD − AG = 10 − 8 = 2 cm

From given figure:

Area of Pentagon = (Area of triangle AFB) + (Area of trapezium FBCG) + (Area of triangle CGD) + (Area of triangle ADE)= ($\frac{1}{2}$x AF x BF) + [$\frac{1}{2}$ x (BF + CG) x (FG)] + ($\frac{1}{2}$ x GD x CG) + ($\frac{1}{2}$ x AD x EH).

= ($\frac{1}{2}$ x 5 x 5) + [$\frac{1}{2}$ x (5 + 7) x (3)] + ($\frac{1}{2}$ x 2 x 7) + ($\frac{1}{2}$ x 10 x 3)

= ($\frac{25}{2}$) + ($\frac{36}{2}$) + ($\frac{14}{2}$) + ($\frac{30}{2}$)

= 12.5 + 18 + 7 + 15 = 52.5 cm^2

2. Find the area enclosed by each of the following figures (Fig. 20.49 (i)-(iii)J as the sum of the areas of a rectangle and a trapezium:

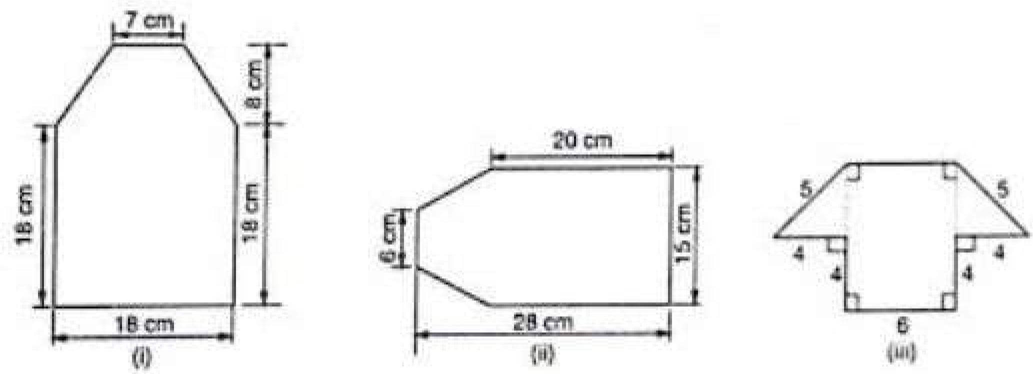


Fig. 20.49

Answer:

(i) From the figure:

Area of the complete figure = (Area of square ABCF) + (Area of trapezium CDEF)

=(AB x BC)+[$\frac{1}{2}$ x (FC +ED) x (Distance between FC and ED)]

=(18 x 18)+[$\frac{1}{2}$ x (18 + 7) x (8)]

= 324 + 100 = 424 cm^2

(ii) From the figure:

AB = AC − BC = 28 − 20 = 8 cm

So that area of the complete figure = (area of rectangle BCDE) + (area of trapezium ABEF)

$=(BC \times CD) + [\frac{1}{2} \times (BE + AF) \times (AB)]$

$=(20 \times 15) + [\frac{1}{2} \times (15 + 6) \times (8)]$

$= 300 + 84 = 384\ cm^2$

(iii) From the figure:

EF = AB = 6 cm

Now, using the Pythagoras theorem in the right angle triangle CDE:

$5^2 = 4^2 + CE^2$

$CE^2 = 25\text{-}16 = 9$

$CE = \sqrt{9} = 3\ cm$

And, GD = GH + HC + CD = 4 + 6 + 4 = 14 cm

Area of the complete figure = (Area of rectangle ABCH)+(Area of trapezium GDEF)

$=(AB \times BC)+[\frac{1}{2} \times (GD + EF) \times (CE)]$

$=(6 \times 4) + [\frac{1}{2} \times (14 + 6) \times (3)]$

$= 24 + 30 = 54\ cm^2$

3. There is a pentagonally shaped park as shown in Fig. 20.50. Jyoti and Kavita divided it in two different ways.

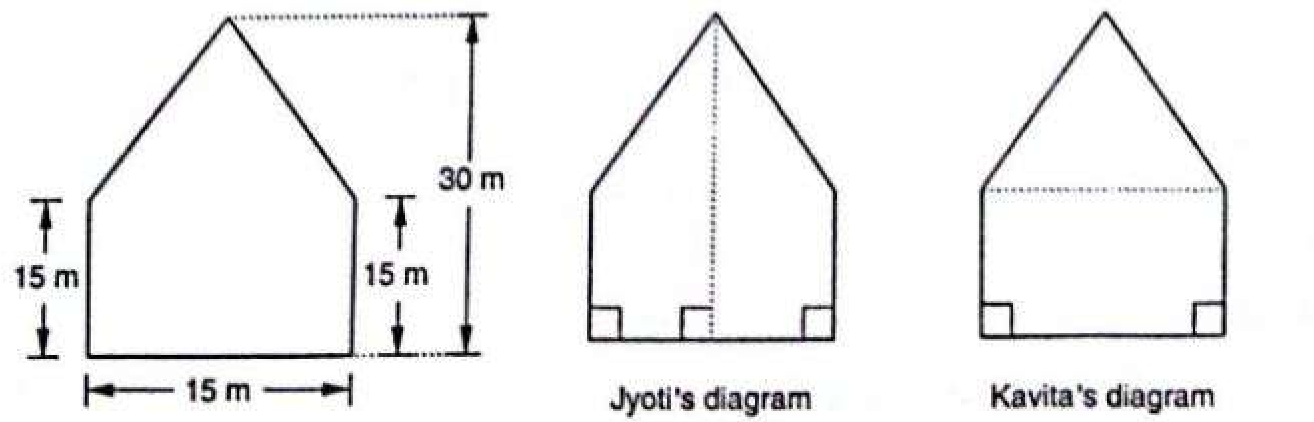
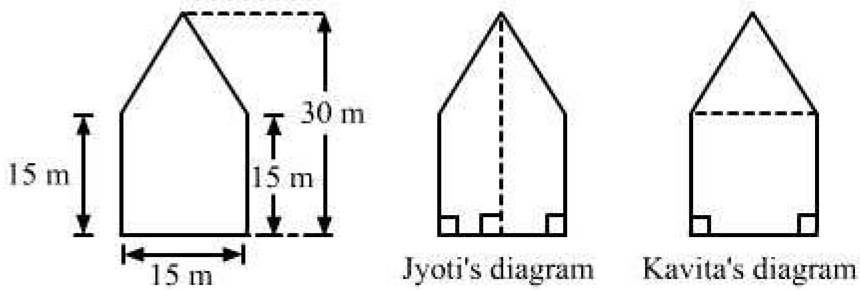


Fig. 20.50

Find the area of this park using both ways. Can you suggest some another way of finding its area?

Answer:

A pentagonal park is given below:



Jyoti and Kavita divided it in two different ways.

(i) Jyoti divided is into two trapeziums. It is clear that the park is divided in two equal trapeziums whose parallel sides are 30 m and 15 m.

And, the distance between the two parallel lines: $\frac{15}{2}$ = 7.5 m

Therefore, Area of the park = 2 x (Area of a trapezium) = 2 x [$\frac{1}{2}$ x (30 + 15) x (7.5)] = 337.5 m^2

(ii) Kavita divided the park into a rectangle and a triangle. Here, the height of the triangle = 30 − 15 = 15 m

Therefore, Area of the park = (Area of square with sides 15 cm)+(Area of triangle with base 15 m and altitude 15 m]

=(15 x 15) + ($\frac{1}{2}$ x 15 x 15)

= 225 + 112. 5 = 337.5 m^2

4. Find the area of the following polygon, if AL =10 cm, AM = 20 cm, AN = 50 cm, AO = 60 cm and AD = 90 cm.

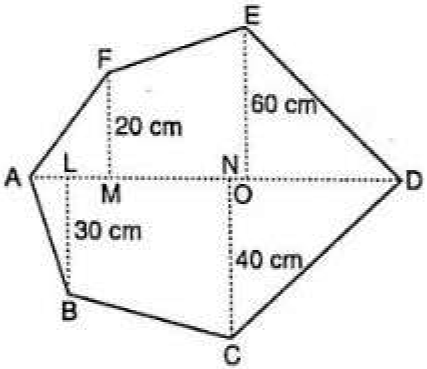


Fig. 20.51

Answer:

Given: AL = 10 cm, AM=20 cm, AN=50 cm

AO = 60 cm, AD = 90 cm

Hence, we have the following: MO = AO − AM = 60 − 20 = 40 cm

OD = AD − A0 = 90 − 60 = 30 cm

ND = AD − AN = 90 − 50 = 40 cm

LN = AN − AL = 50 − 10 = 40 cm

From given figure:

Area of Polygon = (Area of triangle AMF) + (Area of trapezium MOEF) + (Area of triangle DNC) + (Area of trapezium NLBC) + (Area of triangle ALB)

= ($\frac{1}{2}$ x AM x MF) + [$\frac{1}{2}$ x (MF + OE) x OM] + ($\frac{1}{2}$ x OD x OE) + ($\frac{1}{2}$ x DN x NC) + [$\frac{1}{2}$ x (LB + NC) x NL] + ($\frac{1}{2}$ x AL x LB)

$=(\frac{1}{2} \times 20 \times 20) + [\frac{1}{2} \times (20 + 60) \times (40)] + (\frac{1}{2} \times 30 \times 60) + (\frac{1}{2} \times 40 \times 40) + [\frac{1}{2} \times (30 + 40) \times (40)] + (\frac{1}{2} \times 10 \times 30)$

$= 200 + 1600 + 900 + 800 + 1400 + 150 = 5050\ cm^2$

5. Find the area of the following regular hexagon.

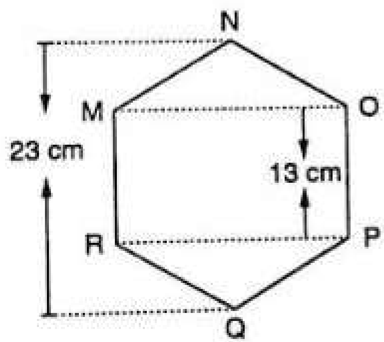
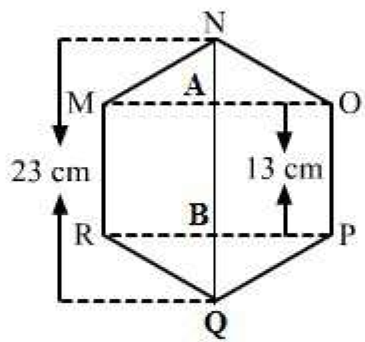


Fig. 20.52

Answer:

The given figure is :

Join QN



It is given that the hexagon is regular. So, all its sides must be equal to 13 cm.

Also, AN = BQ

QB + BA + AN = QN

AN + 13 + AN = 23

2AN = 23 − 13 = 10

AN = $\frac{10}{2}$ = 5 cm

Hence, AN = BQ = 5 cm

Now, in the right angle triangle MAN:

$MN^2 = AN^2 + AM^2$

$13^2 = 5^2 + AM^2$

$AM^2 = 169 - 25 = 144$

AM = $\sqrt{144}$ = 12cm.

Therefore, OM = RP = 2 x AM = 2 x 12 = 24 cm

Hence, area of the regular hexagon = (area of triangle MON)+(area of rectangle MOPR) + (area of triangle RPQ)

= ($\frac{1}{2}$ x OM x AN) + (RP x PO) + ($\frac{1}{2}$ x RP x BQ)

= ($\frac{1}{2}$ x 24 x 5) + (24 x 13) + ($\frac{1}{2}$ x 24 x 5)

= 60 + 312 + 60 = 432 cm^2.