

#### 4.34 Soil Mechanics

corresponding to zero consolidation. The straight line portion of the laboratory curve extended backwards so as to cut zero time. The ordinate corresponding to zero time is the zero dial reading. Through zero dial reading draw another straight line such that its abscissa everywhere are 1.15 times the corresponding abscissa on the straight line portion of the laboratory curve. The second line intersects the laboratory curve at point T' and the point corresponds to 90% consolidation.

Time ( $t_{90}$ ) corresponding to P is known from the graph. By knowing the value of time factor ( $T_v$ )<sub>90</sub>,  $t_{90}$  and the drainage path, d, the value of  $C_v$  can be computed from the relation

$$(T_v)_{90} = \frac{C_v \cdot t_{90}}{d^2}$$

**(ii) Logarithm of time fit method :** In this method laboratory curve between compression and log time is compared with characteristics of theoretical consolidation log time curve. The remarkable characteristic of theoretical curve which is drawn between degree of consolidation, U and  $T_v$  is that the point of intersection of a tangent and the asymptote of 100% consolidation. The same idea is extended to laboratory curve to identify a point which corresponds to 100% consolidation.

The value of  $C_v$  can be computed from

$$C_v = \frac{(T_v)_{50} d^2}{t_{50}}$$

where  $(T_v)_{50}$  = time factor at 50% consolidation  
= 0.197

**Rate of settlement due to consolidation :** Let 'S' be the settlement, at any time 't' after the imposition of load on the clay layer,  $S_t$  be the ultimate settlement of a clay layer due to consolidation, therefore

Degree of consolidation of the layer,

$$U\% = \frac{S}{S_t} \times 100\%$$

Since 'U' is a function of the time factor T, therefore

$$U\% = 100f(T) = \frac{S}{S_t} \times 100$$

**Determining settlements in terms of  $C_e$  (compression index).**

$$\text{Settlement, } \Delta H = \frac{C_e \cdot H_o}{(1 + e_o)} \cdot \log_{10} \frac{p_o + \Delta p}{p_o}$$

where,  $e_o$  and  $p_o$  are the initial void ratio and pressure,  $\Delta p$  is the change in pressure and  $H_o$  is initial thickness.

Compression index ( $C_e$ ) can also be roughly estimated more quickly and easily without a consolidation test, based on the knowledge of liquid limit, as the two are empirically related by Terzaghi as :

(i) For undisturbed clays of normal sensitivity  $> 4$  :

$$C_e = 0.009 [L.L - 10]$$

(ii) For remoulded clays :

$$C_e = 0.007 [L.L - 10]$$

where L.L = Liquid limit moisture-content in per cent.

$C_e$  has also been empirically related to insitu void ratio  $e_o$  by the relation,

$$C_e = 0.54 (e_o - 0.35)$$

**Example.** A stratum of clay is 2 m thick and has an intial overburden pressure of  $0.5 \text{ kg/cm}^2$  at its middle. Determine the final settlement due to an increase in pressure of  $0.4 \text{ kg/cm}^2$  at the middle of the clay layer. The clay is over consolidated with a pre-consolidation pressure of  $0.75 \text{ kg/cm}^2$ . The value of the coefficient of recompression and compression index are 0.05 and 0.25 respectively. Initial void ratio is 1 : 1.

**Solution :** Final settlement

$$\begin{aligned} S_f &= \frac{C_r}{1 + e_o} H_o \log_{10} \left\{ \frac{\sigma_c}{\sigma_o} \right\} + \frac{C_e}{1 + e_o} H_o \log_{10} \left\{ \frac{\sigma_o + \Delta \bar{\sigma}}{\bar{\sigma}_o} \right\} \\ &= \frac{0.05 \times 2}{1 + 1.1} \log_{10} \frac{0.75}{0.50} + \frac{0.25 \times 2}{1 + 1.1} \times \log_{10} \frac{0.5 + 0.4}{0.75} \\ &= 27.24 \times 10^{-3} \text{ m} = \mathbf{27.24 \text{ mm}.} \end{aligned}$$

**Example.** A 2 m thick clay layer beneath a building is overlain by a permeable stratum and is underlain by an impervious rock.  $C_v = 0.025 \text{ cm}^2/\text{min}$ . How much will it take for 80% of the total settlement to take place ? For  $U = 80\%$ ,  $T_v = 0.567$ .

$$\begin{aligned} \text{Solution : From } C_v &= \frac{T_v d^2}{t} \\ 0.025 &= \frac{0.567 \times 200^2}{t} \\ \text{or} \quad t &= 907200 \text{ minutes} \\ &= 630 \text{ days} \\ &= 21 \text{ months.} \\ &= \mathbf{1.75 \text{ years}.} \end{aligned}$$

**Example.** A soft, normally consolidated clay layer is 15 m thick with a natural moisture content of 45 per cent. The clay has a saturated unit weight of  $17.2 \text{ kN/m}^3$ , a particle specific gravity of 2.68 and a liquid limit of 65 percent. A foundation load will subject the centre of the layer to a vertical stress increase of  $10 \text{ kN/m}^2$ . Then find an approximate value for the settlement of the foundation if the ground water level is at the surface of the clay ?

**Solution :**

Initial vertical effective stress at centre of layer

$$= (17.2 - 9.81) \frac{15}{2} = 55.43 \text{ kN/m}^2$$

Final effective vertical stress

$$= 55.43 + 10 = 65.43 \text{ kN/m}^2$$

Initial void ratio,  $e_1 = wG_s = 0.45 \times 2.68 = 1.21$

$$C_c = 0.009 (W_L - 10)$$

$$= 0.009 (65 - 10) = 0.495$$

$$\rho_c = \frac{0.495}{1+1.20} \log_{10} \frac{65.43}{55.43} \times 15$$

$$= 0.242 \text{ m} = 242 \text{ mm}$$

**Example.** A 2.5 cm thick sample of clay was taken from field for predicting the time of settlement for a proposed building which exerts a uniform pressure of 10.5 tonnes/m<sup>2</sup> over the clay stratum. The sample was loaded to 10.5 tonnes/m<sup>2</sup> and proper drainage was allowed from the bottom. It was seen that 50 per cent of the total settlement occurred in 3 minutes. Find the time required for 50 per cent of the total settlement of the building if it is to stand on 6 m thick layer of clay which extends from ground surface and is underlain by sand.

**Solution :** As soil and degree of consolidation are same in both the cases, the time factor T and coefficient of consolidation C<sub>v</sub>, will be same, i.e., in both cases, there is double drainage.

$$\therefore \frac{4C_v t_s}{d_s^2} = \frac{4C_v t_n}{d_n^2}$$

$$\text{or } \frac{t_s}{d_s^2} = \frac{t_n}{d_n^2}$$

$$\text{or } t_n = \frac{d_n^2 t_s}{d_s^2}$$

Given,  $d_n = 6m = 600 \text{ cm}$ ,

$t_s = 3 \text{ minutes}$

$$= \frac{1}{20} \text{ hrs,}$$

$d_s = 2.5 \text{ cm}$

$$\therefore t_n = \frac{600 \times 600 \times \frac{1}{20}}{2.5 \times 2.5}$$

$$= 2880 \text{ hours} = 120 \text{ days}$$

**Example.** A layer of soft saturated clay, 5 m thick, lies under a newly constructed building. The effective pressure due to overlying strata on the clay layer is 300 kN/m<sup>2</sup>, and the new construction increases the effective over-burden by 120 kN/m<sup>2</sup>. If the compression index of the clay is 0.45, then find settlement, assuming the natural water content of the clay layer to be 43%, and the specific gravity of its soil grains as 2.7.

**Solution :** Initial pressure on clay layer,

$$p_o = 300 \text{ kN/m}^2$$

Incremental pressure caused by building,

$$\Delta p = 120 \text{ kN/m}^2$$

For clay compression index,  $C_c = 0.45$ ,

$$w = 0.43,$$

$$S_c = 2.7$$

Initial depth of height,  $H_o = 5 \text{ m}$ ,

Void ratio of clay =  $e_o$

$$\text{For saturated soils, } e_o = w \cdot S_c \\ = 0.43 \times 0.7 = 1.16$$

$$\begin{aligned} \text{Settlement } \Delta H &= \frac{C_c \cdot H_o}{1 + e_o} \log_{10} \frac{p_o + \Delta p}{p_o} \\ &= \frac{0.45 \times 5}{1 + 1.16} \log_{10} \frac{300 + 120}{300} \\ &= 15.22 \text{ m} \end{aligned}$$

**Example.** A consolidation test on a sample of clay having thickness of 2.3 cm indicates that half the ultimate compression occurs in the first 5 minutes. Under similar drainage conditions, how long will be required for a building on a 6m layer of the same clay to experience half of its final settlement ? Neglect secondary time effect.

**Solution :** Given,

On sample  $t_1 = 5 \text{ minutes}$ ,

$$H_1 = 2.3 \text{ cm.}$$

In field :  $H_2 = 6 \text{ m} = 600 \text{ cm.}$

For the two similar soils to achieve the same degree of consolidation, we have

$$\frac{t_1}{t_2} = \left( \frac{H_1}{H_2} \right)^2,$$

$$\text{or } \frac{5}{t_2} = \left( \frac{2.3}{600} \right)^2$$

$$\begin{aligned} \text{or } t_2 &= 5 \times \left( \frac{600}{2.3} \right)^2 \text{ minutes} \\ &= \frac{5 \times (260.87)^2}{60 \times 24} \text{ days} \\ &= 236.29 \text{ days} \end{aligned}$$

**Example.** How many days would be required by a clay stratum 5 m thick, draining at both ends with an average value of co-efficient of consolidation =  $50 \times 10^{-4} \text{ cm}^2/\text{sec}$ , to attain 50% of its ultimate settlement.

**Solution :** For  $U < 0.6$ ,

$$T_v = \frac{\pi}{4} U^2$$

$$(T_v)_{50} = \frac{\pi}{4} (0.5)^2 = 0.196$$

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Now, time taken to obtain 50% consolidation,

$$T_{v(50)} = \frac{C_v}{d^2} t_{50}$$

where  $d$  = maximum length of drainage path

$$= \frac{1}{2} \times 5 = 2.5 \text{ m} = 250 \text{ cm}$$

$$\therefore 0.196 = \frac{50 \times 10^{-4}}{(250)^2} \times t_{50}$$

$$\begin{aligned} \text{or } t_{50} &= \frac{0.196 \times (250)^2}{50 \times 10^{-4}} \text{ sec} \\ &= \frac{0.196 \times (250)^2}{50 \times 10^{-4} \times 60 \times 60 \times 24} \\ &= 28.36 \text{ days} \end{aligned}$$

**Example.** A saturated soil has a compression index  $C_c = 0.27$ . Its void ratio at a stress of  $125 \text{ kN/m}^2$  is 2.04, and its permeability is  $3.5 \times 10^{-8} \text{ cm/sec}$ , then reduction in void ratio if the stress is increased to  $187.5 \text{ kN/m}^2$ .

$$\text{Solution : } e = e_o - C_c \log_{10} \frac{p_o + \Delta p}{p_o}$$

where  $p_o = 125 \text{ kN/m}^2$ ,

$$e_o = 2.04$$

$$\begin{aligned} \Delta p &= 187.5 - 125 \\ &= 62.5 \text{ kN/m}^2, \end{aligned}$$

$$C_c = 0.27$$

$$\therefore e = 2.04 - 0.27 \left[ \log_{10} \frac{187.5}{125} \right] = 1.993$$

$$\begin{aligned} \text{Change in void ratio} &= \Delta e = e - e_o \\ &= 1.993 - 2.04 = (-) 0.047 \text{ (reduced)} \end{aligned}$$

**Example.** A 3 m thick soil stratum has coefficient of permeability of  $3 \times 10^{-7} \text{ m/sec}$ . A separate test gave porosity 40% and bulk unit weight  $21 \text{ kN/m}^3$  at a moisture content of 31%. Determine the head at which upward seepage will cause quick sand condition.

**Solution :** Given,  $n = 40\% = 0.4$ ,

$$\gamma_w = 10 \text{ kN/m}^3,$$

$$w = 0.31,$$

$$\gamma = 21 \text{ kN/m}^3$$

$$e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.67$$

From

$$\gamma = \frac{G(1+w)}{1+e} \gamma_w,$$

$$21 = \frac{G(1+0.31)}{1+0.67} \times 10$$

or

$$G = 2.68$$

$$\text{Critical gradient, } i_c = \frac{h}{L} = \frac{G-1}{1+e}$$

$$\text{or } \frac{h}{3} = \frac{2.68-1}{1+0.67}$$

$$\text{or } h = 3.018 \text{ m}$$

Hence head at which upward seepage will cause quick sand condition is **3.018 m**.

**Example.** The time to reach 60% consolidation is 30 seconds for a sample of 1 cm thick, tested in the laboratory under conditions of double drainage. How many years will the corresponding layer in nature require to reach the same degree of consolidation, if it is 10 m thick and drainage is on one side only?

$$\text{Solution : } T_v = \frac{C_v \cdot t}{d^2} = \frac{C_v \times 30}{\left(\frac{0.01}{2}\right)^2} \quad \dots(i)$$

$$\text{and } T_v = \frac{C_v \cdot t}{(10)^2} \quad \dots(ii)$$

From equations (i) and (ii) we get,

$$\begin{aligned} t &= 120 \times 10^6 \text{ sec} \\ &= \frac{120 \times 10^6}{60 \times 60 \times 24 \times 365} = 3.8 \text{ yrs.} \end{aligned}$$

#### BEARING CAPACITY

The lower part of a structure that transmits the load, of the superstructure and also its own weight to the soil or rock, is called foundation.

#### Types of Foundations :

**1. Footings :** The direct load bearing structure which is constructed as a spread under the base of a wall is called a footing.

**2. Combined footings :** The combination of two or more footings joined together to form a small mat, is called a combined footing.

**3. Strip foundations :** The foundation whose length is considerably greater than its width is called a strip or continuous foundation.

**4. Raft foundations :** The foundation which supports a large number of footing of loads of a single unit and covers the whole loaded area is called a raft foundation.

**5. Pile foundation :** The foundation which is provided in soils incapable to transmit the structural load to suitable stratum by inserting relatively slender structural elements called piles, is known as pile foundation.

The pile foundation may be classified as under :

**(i) End bearing piles.** The piles which act as column and transmit the load through weak soil to a firm stratum at a greater depth, are called end bearing piles.

(ii) **Friction piles.** The piles which carry the structural load by the friction between the surfaces of the piles and the surrounding soil, are called friction piles.

(iii) **Compaction piles.** The friction piles which are driven into cohesionless soil for increasing the shear strength of the soils by compaction, are called compaction piles.

**Shallow and Deep Foundations :** The foundation whose depth is not more than its width, is called a *shallow foundation*. The foundation whose depth is many times more than its width, is called a *deep foundation*.

### Bearing Capacity of soil

The load or pressure developed under the foundation without introducing any damaging movement in foundation and in the supported structure, is called bearing capacity of the soil.

**Ultimate Bearing Capacity.** The maximum pressure that causes sinking of footing into the soil as a result of shear failure is known as ultimate bearing capacity.

### Net ultimate bearing capacity ( $q_{nf}$ )

It is a maximum net pressure causing shear failure.

$$q_{nf} = q_f - \gamma D$$

where  $\gamma$  is unit weight of soil and  $D$  is depth of foundation.

### Safe bearing capacity ( $q_s$ )

The maximum pressure which the soil can carry safely without risk of shear failure is known as *safe bearing capacity*.

$$q_s = \frac{q_{nf}}{F} \gamma D,$$

where  $F$  is factor of safety

**Allowable Bearing Capacity.** The pressure intensity which is considered safe both with respect to shear failure and settlement, is *allowable bearing capacity*.

### Rankine's Minimum Depth of Foundation.

According to Rankine's formula,

$$\text{Depth of foundations, } D = \frac{q}{\gamma} \left[ \frac{1 - \sin \phi'}{1 + \sin \phi'} \right]^2$$

where  $q$  = intensity of loading

$\phi'$  = effective angle of shearing

$\gamma$  = density of soil solids

### Ultimate bearing capacity of shallow foundations :

The ultimate bearing capacity of a footing with centre load depends upon the following factors.

- (i) Unit weight, shear strength and deformation characteristics of soil.

- (ii) Size, shape, depth and roughness of footing, and
- (iii) Depths of water-table and initial stress condition in soil

### Three principal models of shear failure under footing :

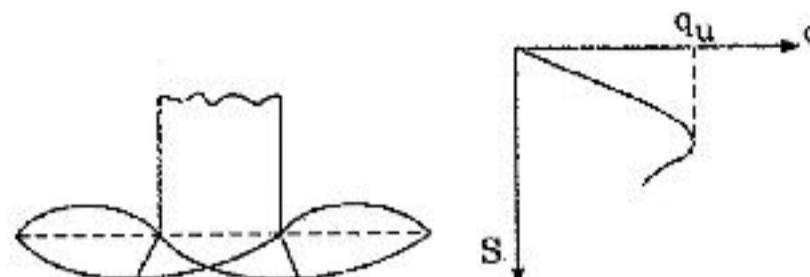


Fig. (a) General shear failure

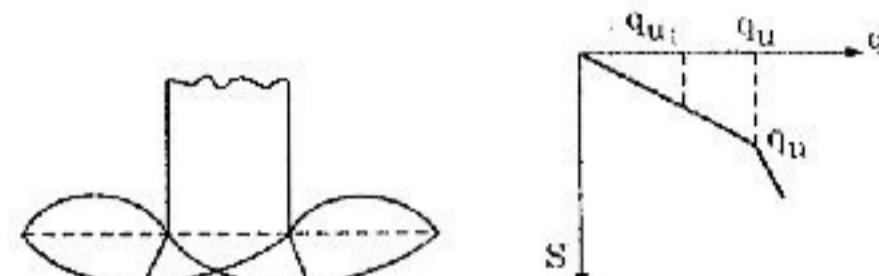


Fig. (b) Local shear failure



Fig. (c) Punching shear failure

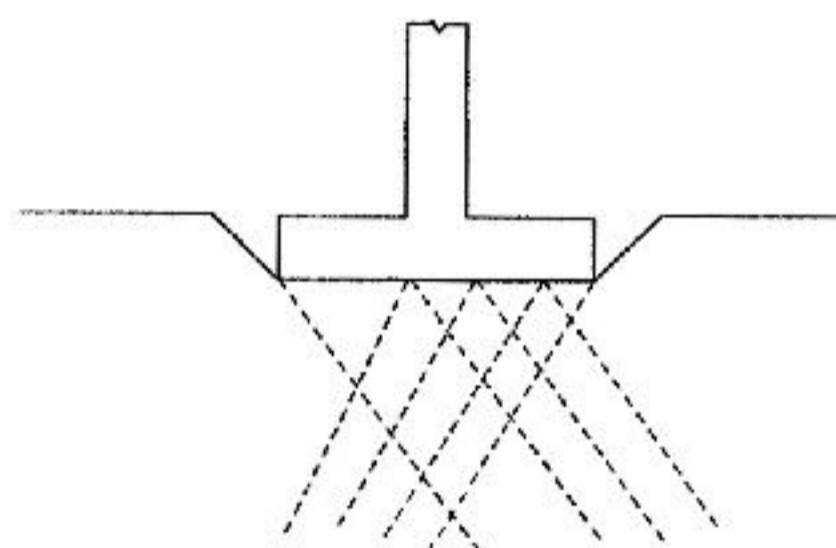


Fig. Punching shear failure

General shear failure is a characteristic of narrow footings of shallow depths resting on strong dense soil that are relatively incompressible.

For weaker, more compressible soil and wider or deeper footings, the failure may be taken as local shear failure or punching shear failure. Local shear failure is characterized by well defined slip lines below the footing but extending only a short distance into the soil mass.

In highly compressible soils the punching shear mode of failure results. This is characterized by lack of a well defined slip line below the footing ; Vertical movement of footing is primarily due to the compression of soil below the footing with the soil on the side not being involved:

**TERZAGHI'S BEARING CAPACITY THEORY****Assumptions.**

- (i) The footing is shallow, i.e.  $D_f \leq B$  to  $2B$
- (ii) The footing is continuous
- (iii) The footing has a rough base
- (iv) The soil above the base of footing can be replaced by equivalent surcharge  $\gamma D_f$  and offer no shear resistance.

According to Terzaghi, the general bearing capacity equation for footings is as under :

$$q_f = c' N_c + \gamma D N_q + 0.5 \gamma B N_\gamma$$

where  $q_f$  = ultimate bearing capacity of soil in kg or tonnes/m<sup>2</sup>  
 $c$  = cohesion kg or tonnes/m<sup>2</sup>  
 $\gamma$  = density of soil, kg or tonnes/m<sup>3</sup>

B and D = width and depth of the footing respectively in metres

$N_c, N_q, N_\gamma$  = Terzaghi's bearing capacity factors

**Ultimate and net ultimate bearing capacities of saturated clays (cohesive soils)**

$$q_f = 5.7 C_u + \gamma D : Q_{nf} = 5.7 C_u$$

where  $C_u$  is shear strength in undrained condition.

**Effect of Water table on Bearing Capacity :** When the soil is cohesionless,

- (i) Water table lies below the failure zone,

$$q_f = \gamma D N_q + 0.5 \gamma' B N_\gamma$$

where  $\gamma$  is bulk density.

- (ii) Water table at the base of footing,

$$q_f = \gamma D N_q + 0.5 [\gamma' + N\gamma]$$

where  $\gamma'$  is submerged density.

- (iii) Water table at the ground surface,

$$q_f = \gamma' D N_q + \gamma_w D + 0.5 \gamma' B N_\gamma$$

According to Meyerhof, the effect of water table on the bearing capacity is as under :

- (i) Water table between the ground surface and footing level,

$$q_f = [\gamma' D + (\gamma - \gamma') Z_w] N_q + \gamma_w (D - Z_m) + 0.5 \gamma' B N_\gamma$$

where  $Z_w$  = depth of water table below ground surface

- (ii) Water table between the base of footing and depth of failure zone

$$q_f = \gamma D N_q + 0.5 [\gamma' + K(\gamma - \gamma')] B N_\gamma$$

where  $K$  = a factor whose value is zero if the water table is at the footing level and unity when it lies below the depth of failure zone.

Net ultimate bearing capacity of cohesionless soils,

$$Q_{nf} = \sigma' (N_q - 1) + 0.5 \gamma' B N_\gamma$$

where  $q_{nf}$  = net ultimate bearing capacity

$\sigma'$  = effective overburden pressure at the footing level

$\gamma'$  = effective unit weight of soil beneath the footing

*There is no effect of water table if its depth is greater than 1.5 B below the footing level.*

**General equation for safe bearing capacity of a C -  $\phi$  soil**

$$q_s = \frac{1}{F} [c' N_c + \sigma' (N_q - 1) + 0.5 \gamma' B N_\gamma] + \sigma$$

where  $q_s$  = safe bearing capacity

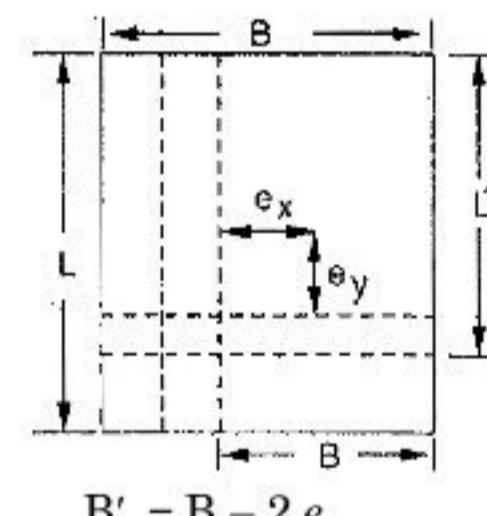
$F$  = factor of safety

$\sigma'$  = effective overburden pressure at the footing with due regard to submergence

$\sigma$  = total overburden pressure ( $\gamma D$ ) at the footing level.

**Footing under eccentric vertical loads**

The vertical load acting on the foundation not passing through the centre of gravity of the footing develops moment. For determining ultimate bearing capacity of eccentrically loaded footing of width and length given by



$$B' = B - 2e_x$$

$$L' = L - 2e_y$$

**Skempton's bearing capacity theory**

Skempton determined that the bearing capacity factor  $N_c$  increases with the ratio  $\frac{D_f}{B}$ . He found that for purely cohesive soil ( $\phi = 0$ ),  $N_c$  has a maximum value of 9 for square or circular footing and 7.5 for strip footing.

*His recommendations are given as follows :*

- (i) When  $D_f = 0$ ,  $N_c = 5.14$  (for strip footing) and  $N_c = 6.20$  (for square or circular footing)

- (ii) At depths  $\frac{D_f}{B} < 2.5$ ,

$$N_c = \left(1 + 0.2 \frac{D_f}{B}\right) (N_c)_{\text{surface}}$$

The value of  $(N_c)_{\text{surface}}$  may be taken as 5 for surface strip footing and 6 for square or circular footings.

(iii) At depths  $\frac{D_f}{B} > 2.5$ ,

$$N_c = 1.5 [N_c]_{\text{surface}}$$

(iv) For rectangular footing,

$$[N_c]_{\text{rectangular}} = \left(1 + 0.2 \frac{B}{L}\right) [N_c]_{\text{strip}}$$

$$\begin{aligned} \text{But } [N_c]_{\text{strip}} &= \left(1 + 0.2 \frac{B}{L}\right) [N_c]_{\text{surface}} \\ &= 5 \left(1 + 0.2 \frac{D_f}{B}\right) \end{aligned}$$

$$[N_c]_{\text{rectangular}} = 5 \left(1 + 0.2 \frac{B}{L}\right) \left(1 + 0.2 \frac{D_f}{B}\right)$$

### Settlement Criteria of Footings

**Total Settlement :** According to Skempton and Bjerrum (1957), the total settlement of a footing in clay be determined by the formula

$$S = S_i + S_c + S_s$$

where  $S_i$  = immediate elastic settlement

$S_c$  = consolidation settlement

$S_s$  = settlement due to secondary consolidation of clay

**Immediate Settlement ( $S_i$ ) :** The immediate settlement can be determined by the formula

$$S_i = q B \left(\frac{1-\mu^2}{E_s}\right) I_w$$

where  $q$  = intensity of contact pressure in  $\text{kg/m}^2$

$B$  = least lateral dimension of footing in m

$E_s$  = Modulus of elasticity of soil in  $\text{kg/m}^2$

$I_w$  = influence factor

= 0.88 for rigid circular footing

= 0.82 for rigid square footing

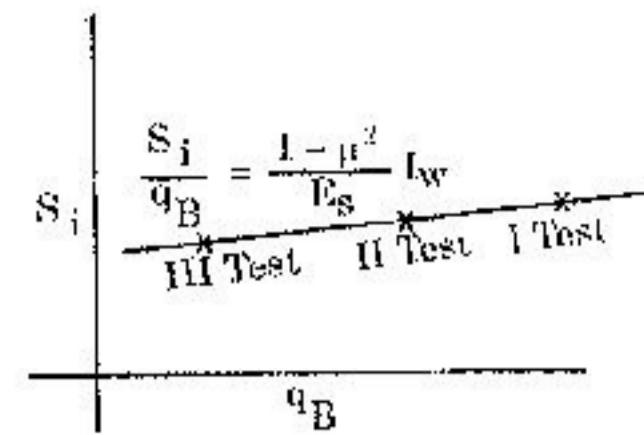
= 1.06 for rigid rectangular footing with  $\frac{L}{B} = 1.5$

= 1.70 for rigid rectangular footing with  $\frac{L}{B} = 5$

$L$  = Length of the footing

$B$  = Breadth of the footing, and

$\mu$  = Poisson's ratio



To find out and  $\left(\frac{1-\mu^2}{E_s}\right) I_w$ , graph is drawn between  $S_i$  and  $q_B$  which gives a straight line, the slope is taken as  $\left(\frac{1-\mu^2}{E_s}\right) I_w$

### Consolidation Settlement ( $S_c$ ) :

$$S_c = C_c \cdot \frac{C}{1+e_o} H \log \frac{\sigma_o + \Delta\sigma}{\sigma_o}$$

where  $C_c$  = compression index

$e_o$  = initial void ratio

$\sigma_o$  = effective overburden pressure due to soil overburden

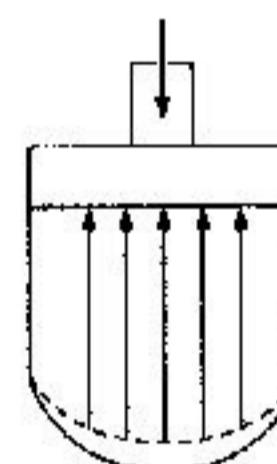
$\Delta\sigma$  = vertical stress on footing

$H$  = thickness of compressible layer

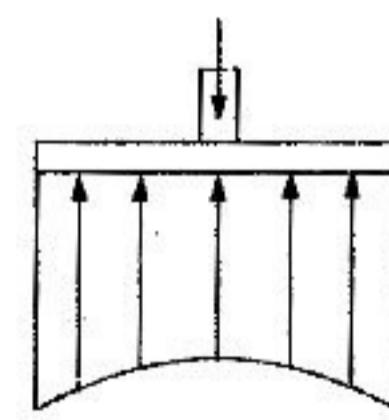
$C$  = a coefficient or correction factor depending upon the geometry of the footing and loading on the clay (i.e., on the pore pressure coefficient  $A$ )

### Pressure Distribution in Footings :

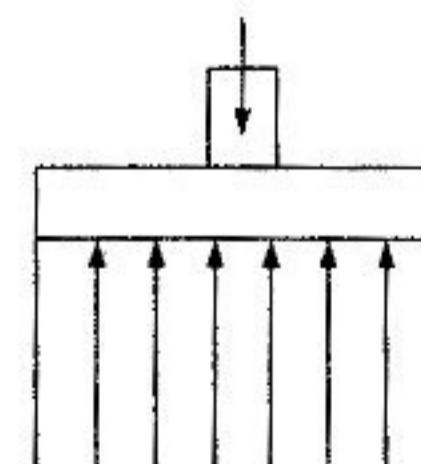
Pressure distribution beneath footings, symmetrically loaded is not uniform but it depends upon the rigidity of footings, type of soil and condition of soil.



(a) Cohesionless soil



(b) Cohesive soil



(c) Uniform pressure distribution

Fig. Pressure distribution beneath a rigid footing.

#### 4.40 Soil Mechanics

##### Design Aspects for Footings :

Two basic criteria used in evaluating loads are

- (i) Determination of the bearing capacity of soil and selection of adequate factor of safety.
- (ii) Estimation of settlement under the expected load and comparison with the permissible settlement.

##### Bearing Capacity of Footings

The allowable bearing capacity should be taken as the smaller of

1. the safe bearing capacity based on ultimate capacity.
2. the allowable bearing pressure on tolerable settlement.

1. **Safe bearing capacity ( $q_s$ ) :** According to Terzaghi's analysis, the SBC is calculated by

For strip footing,

$$q_s = \frac{1}{F} [CN_c + \gamma D(N_q - 1)R_{w1} + 0.5 \gamma B N_\gamma R_{w2}] + \gamma D$$

For square footing,

$$q_s = \frac{1}{F} [1.3 CN_c + \gamma D(N_q - 1)R_{w1} + 0.4 \gamma B N_\gamma R_{w2}] + \gamma D$$

For circular footing,

$$q_s = \frac{1}{F} [1.3 CN_c + \gamma D(N_q - 1)R_{w1} + 0.3 \gamma B N_\gamma R_{w2}] + \gamma D$$

where  $q_s$  = safe bearing capacity of footing,

$D$  = depth of footing

$B$  = width of footing (strip or square) or diameter of circular footing

$N_c, N_q, N_\gamma$  = bearing capacity factors for general shear failure.

For local shear failure,  $N'_c, N'_q$  and  $N'_\gamma$  should be used.

$R_{w1}$  and  $R_{w2}$  = water reduction factor

$F$  = factor of safety (2 to 3)

$$R_{w1} = 0.5 \left( 1 + \frac{z_{w1}}{B} \right);$$

$$R_{w2} = 0.5 \left( 1 + \frac{z_{w2}}{B} \right)$$

where,  $z_{w1}$ , and  $z_{w2}$  are depth of W.T. below ground level and base of footing respectively.

2. **Allowable Bearing Pressure ( $q_p$ ) :** The allowable bearing pressure based on a maximum settlement of individual footing to 2.5 cm, can be computed from the following empirical formula :

$$q_p = 3.5(N - 3) \left( \frac{B + 0.3}{2B} \right)^2 R_{w2} R_d$$

where  $q_p$  = allowable net increase in soil pressure over existing soil pressure for settlement of 2.5 cm in t/m<sup>2</sup>

$N$  = standard penetration number with applicable correction

$B$  = width of footing in metres

$$R_d = \text{depth factors} = \left( 1 + \frac{0.2D}{B} \right) \leq 1.20$$

##### Raft footing or Mat footing

Raft footing is a combined footing that covers the entire area beneath a structure and supports all the walls and columns. If the allowable soil pressure is low, or the building loads are heavy, the use of spread footings would cover more than one-half of the area and it may prove more economical to use raft foundation.

Usually, rafts are designed as reinforced concrete flat slabs. If the centre of gravity of loads coincide with the centroid of the raft, the upward load regarded as a uniform pressure is equal to the downward load divided by the area of the raft. The raft is subdivided into a series of continuous beams (strips) centred on the appropriate column rows. The raft may also be designed as inverted slab, using heavy beams from column to column.

##### Design of raft footings (As per IS : 2950 1965) :

The maximum differential settlement in foundation on clayey soils and sands should not exceed 40 mm and 25 mm respectively. Maximum settlement should generally be limited to the following values.

Raft foundation on clay : 65 to 100 mm

Raft foundation on sand : 40 to 65 mm

Important methods for the design of raft foundations are :

1. Conventional method, and
2. Soil line method

##### Allowable Bearing pressure based on Tolerable settlement :

This is based on the following two criterions :

- (i) Shear failure criterion, and
- (ii) Settlement criterion

**Settlement Criterion :** The allowable pressure based on this criterion is determined from

- (a) Field plate load tests
- (b) Charts prepared by Terzaghi and Peck
- (c) Standard penetration test

- (a) **Plate load tests :** The plate load test is semi-direct method to estimate the allowable bearing pressure of soil to induce a given amount of settlement, Plates round or square varying in size

from 30 to 60 cms and thickness 2.5 cms are employed for the test. The load on the plate is applied by making use of a hydraulic jack. The reaction of the jack load is taken by a cross beam or a steel truss anchored suitably at both the ends. The settlement of the plate is measured by a set of four dial gauges of sensitivity 0.02 mm placed at 90° apart. The dial gauges are fixed to independent supports which do not disturbed during the test.

#### Method :

1. Excavate a pit of size not less than 5 times the size of the plate. The bottom of the pit coincides with the level of the foundation.
2. If water table is above the level of foundation, pump out the water carefully and it should be kept just at the level of foundation.
3. A plate of size 30 cms is used in sandy soil and bigger size is clay soils. The ground should be levelled and the plate is seated over the ground.
4. A seating load of about 700 kg/m<sup>2</sup> or 7 kN/m<sup>2</sup> is first applied and released after some time. A higher load is next placed on the plate and settlements are recorded by means of the dial gauges. Observations on every load increment shall be taken until the rate of settlement is less than 0.25 mm per hour. Load increments shall be approximately one-fifth of the estimated safe bearing capacity of the soil. The average of the settlement recorded by 2 or 3 dial guages is taken as the settlement of the plate for each of the load increment.
5. The test should continue until a total settlement of 2.5 cms or the settlement at which the soil fails whichever is earlier is obtained. After the load is released, the elastic rebound of the soil should be recorded.

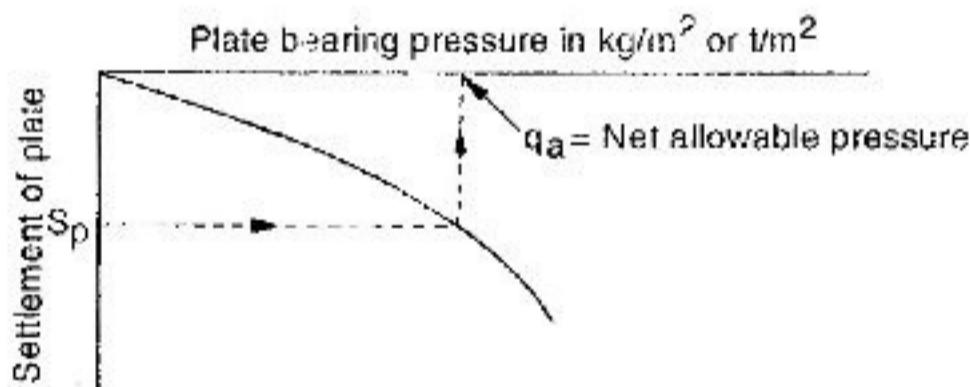


Fig. Load-settlement curve

Since the load test is of short duration, consolidation settlements cannot be predicted. The test gives the value of immediate settlement only. If the underlying soil is sandy in nature, immediate settlement may be taken as the total settlement. If the soil is clayey type, the immediate settlement is only a fraction of the total settlement. Load tests, therefore, do not have much significance in clayey soils to determine allowable pressure on the basis of settlement criterion.

Plate load tests should be used with caution and the present practice is not to rely too much on this test. If the soil is not homogenous to a great depth, plate load tests gives very misleading results.

The allowable pressure on a prototype foundation for an assumed settlement may be calculated by the following equation given by Terzaghi and Peck

$$\text{For granular soils, } S_f = S_p \left[ \frac{B(b_p + 0.3)}{b_p(B + 0.3)} \right]^2$$

$$\text{For clayey soils, } S_f = S_p \times \frac{B}{b_p}$$

where  $S_f$  = permissible settlement of foundation in mm

$S$  = settlement of plate in mm

$B$  = size of foundation in meters

$b_p$  = size of plate in meters

The ultimate bearing capacity of the proposed footing,  $q_u(f)$  can be obtained from the following relations.

(a) For Clayey soils,

$$q_u(f) = q_u(p)$$

(b) For Sandy soils,

$$q_u(f) = q_u(p) \times \frac{B_f}{B_p}$$

#### Effect of water Table on the Allowable Bearing Pressure

If the plate load test is carried out above the water table, the allowable pressure as determined from the load settlement curve will be affected. If the foundation gets submerged at a later date, the effect of submergence is to reduce the allowable bearing pressure by about 30 percent.

## SOLVED EXAMPLES

- 1.** Find safe bearing capacity of a foundation wall, 1.8 m wide resting on a soil with  $\gamma = 17.5 \text{ kN/m}^3$ ,  $c = 25 \text{ kN/m}^2$  and  $\phi = 25^\circ$ , at a depth of 2.5 m.

**Solution :** Assuming local shear failure, by Terzaghi's equation,

$$q_{un} = c \cdot N_c + q_f (N_q - 1) + 0.5 \gamma B \cdot N_\gamma$$

For  $\phi = 25^\circ$ ,  $N_c = 14.8$ ,  $N_q = 5.6$ ,  $N_\gamma = 3.2$

$$\begin{aligned} \therefore q_{un} &= 25 \times 14.8 + 17.5 (2.5) (5.6 - 1) \\ &\quad + 0.5 (17.5) (1.8) (3.2) \\ &= 621.65 \text{ kN/m}^2 \end{aligned}$$

Assume a factor of safety of 3, we have

$$\begin{aligned} q_F &= \frac{q_{un}}{F_s} + \gamma D_f \\ &= \frac{541.15}{3} + 17.5 (2.5) \\ &= 233.71 \text{ kN/m}^2 \end{aligned}$$

- 2.** Find the safe bearing capacities on a medium sand layer for a footing 2 m  $\times$  2 m, located at a depth of 0.8 m, if the sand has  $\gamma = 16.5 \text{ kN/m}^3$ ,  $\phi = 30^\circ$ . (Assume  $F_s = 3.0$ ).

**Solution :** Assume general shear failure,

$$\begin{aligned} N_r &= 22.50, \quad N_g = 19.7 \\ q_u &= q \cdot N_q + 0.4 \gamma B \cdot N_\gamma \\ &= 16.5 (0.8) (22.50) + 0.4 (16.5) 2 (19.7) \\ &= 557.04 \text{ kN/m}^2 \end{aligned}$$

$$\begin{aligned} q_{un} &= q_u - \gamma D_f \\ &= 557.04 - 16.5 (0.8) \\ &= 543.84 \text{ kN/m}^2 \end{aligned}$$

$\therefore$  Safe bearing capacity,

$$\begin{aligned} q_F &= \frac{q_{un}}{F_s} + \gamma \cdot D_f \\ &= \frac{543.84}{3} + (16.5) (0.8) \\ &= 194.48 \text{ kN/m}^2 \end{aligned}$$

- 3.** A wall of 5 m height retains sand having a specific weight  $1.9 \text{ t/m}^3$  and angle of internal friction of  $30^\circ$ . If the surface of the backfill slope upwards  $15^\circ$  to the horizontal, then find active thrust per meter length of the wall. (Use Rankine's conditions)

**Solution :**  $P_a = \frac{1}{2} K_a \gamma H^2$

$$\text{where } K_a = \cos i \times \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i + \cos^2 \phi}}$$

where  $i = 15^\circ$ ,

$$\cos i = 0.9659;$$

$$\cos^2 i = 0.933,$$

$$\phi = 30^\circ;$$

$$\cos^2 \phi = \frac{3}{4} = 0.75$$

$$\begin{aligned} K_a &= 0.9659 \times \frac{0.9659 - \sqrt{0.933 - 0.75}}{0.9659 + \sqrt{0.933 + 0.75}} \\ &= 0.23 \end{aligned}$$

$$P_a = \frac{1}{2} \times 0.23 \times 1.9 \times 5^2$$

= 5.4625 t/m length of wall.

- 4.** A column load of 1200 kN is supported on a round footing (diameter 2m) at a depth of 1.5 m. The soil properties are known as  $\gamma = 18.0 \text{ kN/m}^3$ ,  $c = 30 \text{ kN/m}^2$ ,  $\phi = 20^\circ$ . Then find safety factor.

**Solution :** For a round footing, Terzaghi equation gives,

$$q_{un} = 1.2 c N_c + q (N_q - 1) + 0.3 \gamma D \cdot N_\gamma$$

$$\text{For } \phi = 20^\circ, \quad N_c = 17.7, \quad N_q = 7.4$$

$$\text{and } N_\gamma = 5, \quad C = 30 \text{ kN/m}^2$$

$$\begin{aligned} \therefore q_{un} &= 1.2 (30) 17.7 + 18 (1.5) (7.4 - 1) \\ &\quad + 0.3 (18) 2 (5) \\ &= 864 \text{ kN/m}^2 \end{aligned}$$

$$\text{Area of footing} = \frac{\pi D^2}{4} = 0.785 (2)^2 = 3.14 \text{ m}^2$$

$$\text{Column load} = 1200 \text{ kN}$$

$$\therefore \text{Applied load, } q_f = \frac{1200}{3.14} = 382.17 \text{ kN/m}^2$$

Equating  $q_f$  with safe bearing capacity, we have

$$q_F = \frac{q_{un}}{F_s} + \gamma \cdot D_f$$

$$\text{or } 382.17 = \frac{864}{F_s} + 18(1.5)$$

$$\therefore F_s = 2.43$$

- 5.** A rectangular footing, 1.5 m  $\times$  2.5 m rests on a clayey layer at a depth of 1.5 m. Then find safe bearing capacity, if the soil has  $\gamma = 18 \text{ kN/m}^3$  and  $c_u = 30 \text{ kN/m}^2$ .

**Solution :** Use Terzaghi's equation ( $\phi = 0$ )

$$q_{un} = 5.7 c \left( 1 + 0.2 \frac{B}{L} \right)$$

$$= 5.7(30) \left( 1 + 0.2 \times \frac{1.5}{2.5} \right)$$

$$= 205 \text{ kN/m}^2$$

Assume a factor of safety of 2.5,

$$\begin{aligned} q_F &= \frac{q_{un}}{\text{F.S.}} + \gamma \cdot D_f \\ &= \frac{205}{2.5} + 18(1.5) \\ &= 107 \text{ kN/m}^2 \end{aligned}$$

- 6.** A vertical excavation was made in a clay deposit having weight of  $19 \text{ kN/m}^3$ . It caved in after a depth of digging reached  $3.8 \text{ m}$ . Taking the angle of internal friction to be zero, find the value of cohesion. If the same clay is used as a backfill against a retaining wall, up to a height of  $6 \text{ m}$ . Find total active pressure and total passive pressure.

(Assume that the wall yields far enough to allow Rankine's deformation conditions to be satisfied)

**Solution :** Critical height of an unsupported vertical cut in cohesive soil,

$$H_c = \frac{4c}{\gamma} \tan \alpha$$

$$\begin{aligned} \text{where } \tan \alpha &= \tan \left( 45 + \frac{\phi}{2} \right) \\ &= 1 \dots \dots \text{ (since } \phi = 0 \text{)} \end{aligned}$$

$$\begin{aligned} c &= \frac{H_c \cdot \gamma}{4} = \frac{3.8 \times 19}{4} \\ &= 18.05 \text{ kN/m}^2 \end{aligned}$$

(i) Total active earth pressure,

$$\begin{aligned} P_a &= \frac{1}{2} \gamma H^2 \cot^2 \alpha - 2c H \cot \alpha \\ &= \left( \frac{1}{2} \times 19 \times 6^2 \times 1 \right) - (2 \times 18.05 \times 6 \times 1) \\ &= 125.4 \text{ kN/m} \end{aligned}$$

(ii) Total passive earth pressure,

$$\begin{aligned} P_p &= \frac{1}{2} \gamma H^2 \tan^2 \alpha + 2c H \tan \alpha \\ &= \left( \frac{1}{2} \times 19 \times 6^2 \times 1 \right) + (2 \times 18.05 \times 6 \times 1) \\ &= 558.6 \text{ kN/m} \end{aligned}$$

- 7.** A cantilever retaining wall  $5 \text{ m}$  high retains sand. The properties of sand are

$$e = 0.5; \phi = 30^\circ; \text{ and } G = 2.7$$

Using Rankine's theory, find the active earth pressure in the base when the backfill is

- (i) dry
- (ii) saturated
- (iii) submerged

**Solution :** Dry unit weight,

$$\begin{aligned} \gamma_d &= \left( \frac{G}{1+e} \right) \gamma_w \\ &= \left( \frac{2.7}{1+0.5} \right) \times 1 = 1.8 \text{ t/m}^3 \end{aligned}$$

Saturated unit weight,

$$\begin{aligned} \gamma_{sat} &= \left( \frac{G+e}{1+e} \right) \gamma_w \\ &= \left( \frac{2.7+0.5}{1+0.5} \right) \times 1 = 2.13 \text{ t/m}^3 \end{aligned}$$

Submerged unit weight,

$$\begin{aligned} \gamma_{sub} &= \gamma_{sat} - \gamma_w = 2.13 - 1 \\ &= 1.13 \text{ t/m}^3 \end{aligned}$$

Coefficient of active earth pressure,

$$\begin{aligned} K_a &= \frac{1 - \sin \phi}{1 + \sin \phi} \\ &= \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3} \end{aligned}$$

Active earth pressure at the base

(i) For dry backfill,

$$\begin{aligned} p_a &= K_a \gamma_d H \\ &= \frac{1}{3} \times 1.8 \times 5 = 3 \text{ t/m}^2 \end{aligned}$$

(ii) For saturated backfill,

$$\begin{aligned} p_a &= K_a \gamma_{sat} H \\ &= \frac{1}{3} \times 2.13 \times 5 = 3.55 \text{ t/m}^2 \end{aligned}$$

(iii) For submerged backfill,

$$\begin{aligned} p_a &= K_a \gamma_{sub} H \\ &= \frac{1}{3} \times 1.13 \times 5 = 1.88 \text{ t/m}^2 \end{aligned}$$

- 8.** A plate load test gives the settlement of  $6 \text{ mm}$  at a loading of  $400 \text{ kN/m}^2$ , on a  $0.3 \text{ m} \times 0.3 \text{ m}$  plate. Determine settlement of a footing  $2 \text{ m} \times 2 \text{ m}$  carrying a load of  $1800 \text{ kN}$  on the same soil at the same depth.

**Solution :** For a given intensity of loading,

$$\begin{aligned} \frac{S_f}{S_p} &= \left( \frac{B_f}{B_p} \cdot \frac{(B_p + 0.3)}{(B_f + 0.3)} \right)^2 \\ &= \left( \frac{0.2}{0.5} \cdot \frac{(0.3 + 0.3)}{(2 + 0.3)} \right)^2 = 3.025 \end{aligned}$$

$$\therefore S_f = 3.025 \times 6 = 18.15 \text{ mm}$$

This settlement corresponds to,

$$q = 400 \text{ kN/m}^2$$

#### 4.44 Soil Mechanics

$$\text{Footing load, } q_f = \frac{1800}{2 \times 2} = 450 \text{ kN/m}^2$$

$$\therefore (S_f)_{450} = (S_f)_{400} \times \frac{450}{400}$$

$$= 18.15 \left( \frac{450}{400} \right)$$

$$= 20.42 \text{ mm}$$

- 9.** The base of a retaining wall is 3 m wide and is 1 m below the ground surface in front of the walls, the water table is well below the base level. The vertical and horizontal components of the base reaction are 282 kN/m and 102 kN/m respectively. The eccentricity of the base reaction is 0.36 m. If the appropriate shear strength parameters for the foundation soil are  $c' = 0$  and  $\phi' = 35^\circ$ , the unit weight of the soil is  $18 \text{ kN/m}^3$ , find factor of safety against shear failure.

**Solution :** Effective width of the base,

$$B' = B - 2e$$

$$= 3 - 2 \times 0.36 = 2.28 \text{ m}$$

For  $\phi' = 35^\circ$ ,  $N_\gamma = 41$  and  $N_q = 33$

Angle of inclination (to the vertical) of the resultant load,

$$\alpha = \tan^{-1} \left( \frac{102}{282} \right) = 20^\circ$$

Hence, the inclination factors according to Meyerhof are :

$$i_\gamma = \left( 1 - \frac{20}{35} \right)^2 = 0.18$$

$$\text{and } i_q = \left( 1 - \frac{20}{90} \right)^2 = 0.60$$

Ultimate bearing capacity,

$$q_u = \frac{1}{2} \gamma B N_\gamma i_\gamma + \gamma' D N_q i_q$$

$$= \left( \frac{1}{2} \times 18 \times 2.28 \times 41 \times 0.18 \right) + (18 \times 1 \times 33 \times 0.60)$$

$$= 507.84 \text{ kN/m}^2$$

Net ultimate bearing capacity,

$$q_{nf} = q_u - \gamma D$$

$$= 507.84 - 18 \times 1$$

$$= 489.84 \text{ kN/m}^2$$

$$\approx 490 \text{ kN/m}^2$$

Net foundation pressure,

$$q_n = \frac{282}{2.28} - 18 = 106 \text{ kN/m}^2$$

$$\therefore \text{Factor of safety, } F = \frac{490}{106} = 4.6$$

- 10.** A footing 2.25 m square is located at a depth of 1.5 m in a sand, the shear strength parameters being  $c' = 0$  and  $\phi = 38^\circ$ . Find ultimate bearing capacity

- (i) if the water-table is well below foundation level  
(ii) if water-table is at the surface.

The unit weight of the sand above the water-table is  $18 \text{ kN/m}^3$

**Solution :**

- (i) For a square footing, the ultimate bearing capacity (with  $c = 0$ ) is given by

$$q_u = 0.4 \gamma B N_\gamma + \gamma B N_q$$

For  $\phi = 38^\circ$ ,  $N_\gamma = 67$  and  $N_q = 49$

$$\therefore q_u = (0.4 \times 18 \times 2.25 \times 67) + (18 \times 1.5 \times 49)$$

$$= 5408.4 \text{ kN/m}^2$$

- (ii) When the water-table is at the surface, the ultimate bearing capacity is given by

$$q_u = 0.4 \gamma' B N_\gamma + \gamma' B N_q$$

$$= [0.4 \times (20 - 9.8) \times 2.25 \times 67]$$

$$+ [(20 - 9.8) \times 1.5 \times 49]$$

$$= 1364.76 \text{ kN/m}^2$$

- 11.** A 2.2 m square footing is located at a depth of 4.4 m in a stiff clay of saturated unit weight  $21 \text{ kN/m}^3$ . The undrained strength of clay at a depth of 4.4 m is given by parameter  $c_u = 120 \text{ kN/m}^2$  and  $\phi_u = 0$ . For a factor of safety 3, with respect to shear failure, find the net value of bearing capacity.

**Solution :**

Given,  $D_f = 4.4 \text{ m}$ ,  $B = 2.2 \text{ m}$

$$\therefore \frac{D_f}{B} = \frac{4.4}{2.2} = 2$$

Since, it is a deep footing, therefore, from Skempton's equations (for  $\phi_u = 0$ )

$$q_f = c_u N_c + \gamma D_f$$

where  $c_u = 120 \text{ kN/m}^2$ ;

$\gamma_{sat} = 21 \text{ kN/m}^3$ ,

$D_f = 4.4 \text{ m}$

For a square footing with  $\frac{D_f}{B} = \frac{4.4}{2.2} = 2$ ,

$$N_c = 6 \left( 1 + 0.2 \frac{D_f}{B} \right) = 5(1 + 0.2 \times 2)$$

$$= 8.4 \text{ (to be limited to 9)}$$

$$\therefore q_f = 120 \times 8.4 + 21 \times 4.4$$

$$= 1100.4 \text{ kN/m}^2$$