Exercise – 2.1

1. Assuming that x, y, z are positive real numbers, simplify each of the following:

(i) $\left(\sqrt{x^{-3}}\right)^5$

(ii) $\sqrt{x^3 y^{-2}}$

(iii) $\left(x^{-\frac{2}{3}} y^{-\frac{1}{2}}\right)^2$

(iv) $\left(\sqrt{x}\right)^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}}$

(v) $\sqrt[5]{243 x^{10} y^5 z^{10}}$

(vi) $\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}}$

Sol:

We have,

$$\left(\sqrt{x^{-3}}\right)^5 = \left(\sqrt{\frac{1}{x^3}}\right)^5$$

$$= \left(\frac{1}{x^{\frac{3}{2}}}\right)^5$$

$$= \frac{1}{x^{\frac{3}{2}\times 5}}$$

$$= \frac{1}{x^{\frac{15}{2}}}$$

$$\Rightarrow \left(\sqrt{x^{-3}}\right)^5 = \frac{1}{x^{\frac{15}{2}}}$$

We have,

$$\sqrt{x^3 y^{-2}} = \sqrt{\frac{x^3}{y^2}}$$

$$= \left(\frac{x^3}{y^2}\right)^{\frac{1}{2}}$$

$$= \frac{x^{3x\frac{1}{2}}}{y^{2x\frac{1}{2}}}$$

$$= \frac{x^{\frac{3}{2}}}{y}$$

$$\Rightarrow \sqrt{x^3 y^{-2}} = \frac{x^{\frac{3}{2}}}{y}$$

We have,

$$\left(x^{-\frac{2}{3}} y^{-\frac{1}{2}}\right)^2 = \left(\frac{1}{x^{\frac{2}{3}} y^{\frac{1}{2}}}\right)^2$$

$$= \left(\frac{1}{x^{\frac{2}{3}\times 2}\, y^{2\times\frac{1}{2}}}\right)^2$$

$$= \frac{1}{x^{\frac{4}{3}} y^1}$$

$$= \frac{1}{x^{\frac{4}{3}} y}$$

$$\Rightarrow \left(x^{-\frac{2}{3}} y^{-\frac{1}{2}}\right)^2 = \frac{1}{x^{\frac{4}{3}} y}$$

We have,

$$\left(\sqrt{x}\right)^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}}$$

$$= \left(x^{\frac{1}{2}}\right)^{-\frac{2}{3}} \left(y^2\right) \div \sqrt{xy^{-\frac{1}{2}}}$$

$$= \frac{x^{\frac{1}{2}\times\frac{2}{3}} y^2}{\left(xy^{-\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$= \frac{x^{-\frac{1}{3}} y^2}{x^{\frac{1}{2}} y^{-\frac{1}{2}\times\frac{1}{2}}}$$

$$= \left(x^{-\frac{1}{3}} \times x^{-\frac{1}{2}} \right) \times \left(y^2 \times y^{\frac{1}{4}} \right)$$

$$= \left(x^{-\frac{1}{3}-\frac{1}{2}} \right)\left(y^{2+\frac{1}{4}} \right)$$

$$= \left(x^{\frac{-2-3}{6}} \right)\left(y^{\frac{8+1}{4}} \right)$$

$$= \left(x^{-\frac{5}{6}} \right)\left(y^{\frac{9}{4}} \right)$$

$$= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

$$\Rightarrow \left(\sqrt{x}\right)^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}} = \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

We have,

$$\sqrt[5]{243x^{10}y^5z^{10}} = \left(243x^{10}y^5z^{10}\right)^{\frac{1}{5}}$$

$$= \left(243\right)^{\frac{1}{5}} x^{\frac{10}{5}} y^{\frac{5}{5}} z^{\frac{10}{5}}$$

$$= \left(3^5\right)^{\frac{1}{5}} x^2 y^1 z^2$$

$$= 3^{5\times\frac{1}{5}} x^2 yz^2$$

$$= 3x^2 yz^2$$

$$\Rightarrow \sqrt[5]{243x^{10}y^5z^{10}} = 3x^2 yz^2$$

We have,

$$\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}} = \left(\frac{y^{10}}{x^4}\right)^{\frac{5}{4}}$$

$$= \frac{y^{10\times\frac{5}{4}}}{x^{4\times\frac{5}{4}}}$$

$$= \frac{y^{\frac{25}{2}}}{x^5}$$

$$\Rightarrow \left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}} = \frac{y^{\frac{25}{2}}}{x^5}$$

2. Simplify:

(i) $\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$

(ii) $\sqrt[3]{(342)^{-2}}$

(iii) $(0.001)^{\frac{1}{3}}$

(iv) $\dfrac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$

(v) $\left(\dfrac{\sqrt{2}}{5}\right)^{8}\div\left(\dfrac{\sqrt{2}}{5}\right)^{13}$

(vi) $\left[\dfrac{5^{-1}\times7^{2}}{5^{2}\times7^{-4}}\right]^{\frac{7}{2}}\times\left[\dfrac{5^{-2}\times7^{3}}{5^{3}\times7^{-5}}\right]^{-\frac{5}{2}}$

Sol:

(i) We have

$$\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}=(16)^{-\frac{1}{5}\times\frac{5}{2}}=(16)^{-\frac{1}{2}}=\left(4^{2}\right)^{-\frac{1}{2}}=4^{2x-\frac{1}{2}}=4^{-1}=\frac{1}{4}$$

Hence, $\boxed{\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}=\frac{1}{4}}$

(ii) We have,

$$\sqrt[3]{(342)^{-2}}=\left[(343)^{-2}\right]^{\frac{1}{3}}=(343)^{-2\times\frac{1}{3}}$$

$$=\left(7^{3}\right)^{-\frac{2}{3}}$$

$$=7^{3\times\frac{-2}{3}}$$

$$=7^{-2}=\frac{1}{7^{2}}=\frac{1}{49}$$

Hence, $\boxed{\sqrt[3]{(343)^{-2}}=\frac{1}{49}}$

(iii) We have,

$$(0.001)^{\frac{1}{3}}=\left(\frac{1}{1000}\right)^{\frac{1}{3}}=\left(\frac{1}{10^{3}}\right)^{\frac{1}{3}}$$

$$= \frac{1^{\frac{1}{3}}}{\left(10^3\right)^{\frac{1}{3}}} = \frac{1}{10^{3\times\frac{1}{3}}} = \frac{1}{10} = 0.01$$

Hence, $\boxed{\left(0.001\right)^{\frac{1}{3}} = 0\cdot 1}$

(iv) We have,

$$\frac{\left(25\right)^{\frac{3}{2}}\times\left(243\right)^{\frac{3}{5}}}{\left(16\right)^{\frac{5}{4}}\times\left(8\right)^{\frac{4}{3}}} = \frac{\left(5^2\right)^{\frac{3}{2}}\times\left(3^5\right)^{\frac{3}{5}}}{\left(2^4\right)^{\frac{5}{4}}\times\left(2^3\right)^{\frac{4}{3}}}$$

$$= \frac{5^{2\times\frac{3}{2}}\times 3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}}\times 2^{3\times\frac{4}{3}}}$$

$$= \frac{5^3\times 3^3}{2^5\times 2^4} = \frac{125\times 27}{32\times 16} = \frac{3375}{512}$$

Hence, $\boxed{\frac{\left(25\right)^{\frac{3}{2}}\times\left(243\right)^{\frac{3}{5}}}{\left(16\right)^{\frac{5}{4}}\times\left(8\right)^{\frac{4}{3}}} = \frac{3375}{512}}$

(v) We have,

$$\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} = \frac{\left(\frac{\sqrt{2}}{5}\right)^8}{\left(\frac{\sqrt{2}}{5}\right)^{13}}$$

$$\Rightarrow \left(\frac{\sqrt{2}}{5}\right)^{8-13} = \left(\frac{\sqrt{2}}{5}\right)^{-5} = \frac{\left(2^{\frac{1}{2}}\right)^{-5}}{\left(5\right)^{-5}} = \frac{2^{\frac{1}{2}x-5}}{5^{-5}} = \frac{2^{-\frac{5}{2}}}{5^{-5}}$$

$$\Rightarrow \frac{1}{2^{\frac{5}{2}}}\times\frac{5^5}{1} = \frac{5^5}{2^{\frac{5}{2}}} = \frac{3125}{4\sqrt{2}}$$

Hence $\boxed{\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} = \frac{3125}{4\sqrt{2}}}$

(vi) We have,

$$\left[\frac{5^{-1}\times 7^2}{5^2\times 7^{-4}}\right]^{\frac{7}{2}} \times \left[\frac{5^{-2}\times 7^3}{5^3\times 7^{-5}}\right]^{-\frac{5}{2}}$$

$$\Rightarrow \frac{\left(5^1\times 7^2\right)^{\frac{7}{2}}}{\left(5^2\times 7^{-4}\right)^{\frac{7}{2}}}\times\frac{\left(5^{-2}\times 7^3\right)^{-\frac{5}{2}}}{\left(5^3\times 7^{-5}\right)^{-\frac{5}{2}}}$$

$$\Rightarrow \frac{\left(5^{-1}\right)^{\frac{7}{2}}\times\left(7^2\right)^{\frac{7}{2}}}{\left(5^2\right)^{\frac{7}{2}}\times\left(7^{-4}\right)^{\frac{7}{2}}}\times\frac{\left(5^{-2}\right)^{-\frac{5}{2}}\times\left(7^3\right)^{-\frac{5}{2}}}{\left(5^3\right)^{-\frac{5}{2}}\times\left(7^{-5}\right)^{-\frac{5}{2}}}$$

$$\Rightarrow \frac{5^{-1\times\frac{7}{2}}\times 7^{2\times\frac{7}{2}}}{5^{2\times\frac{7}{2}}\times 7^{-4\times\frac{7}{2}}}\times\frac{5^{-2\times\frac{-5}{2}}\times 7^{3\times\frac{-5}{2}}}{5^{3\times\frac{-5}{2}}\times 7^{-5x-\frac{5}{2}}}$$

$$\Rightarrow \frac{5^{\frac{-7}{2}}\times 7^7}{5^7\times 7^{-14}}\times\frac{5^5\times 7^{\frac{-15}{2}}}{5^{\frac{-15}{2}}\times 7^{\frac{25}{2}}}$$

$$\Rightarrow \frac{7^{7+14}}{5^{7+\frac{7}{2}}}\times\frac{5^{5+\frac{15}{2}}}{7^{\frac{25}{2}+\frac{15}{2}}}$$

$$\Rightarrow \frac{5^{\frac{-7}{2}}\times 7^7}{5^7\times 7^{-14}}\times\frac{5^5\times 7^{\frac{-15}{2}}}{5^{\frac{-15}{2}}\times 7^{\frac{25}{2}}}$$

$$\Rightarrow \frac{7^{7+14}}{5^{7+\frac{7}{2}}}\times\frac{5^{5+\frac{15}{2}}}{7^{\frac{25}{2}+\frac{15}{2}}}$$

$$\Rightarrow \frac{7^{21}}{5^{\frac{21}{2}}}\times\frac{5^{\frac{25}{2}}}{7^{\frac{40}{2}}}$$

$$\Rightarrow \frac{7^{21}}{7^{20}}\times\frac{5^{\frac{25}{2}}}{5^{\frac{21}{2}}}$$

$$\Rightarrow 7^{21-20}\times 5^{\frac{25}{2}-\frac{21}{2}}$$

$$\Rightarrow 7^1\times 5^{\frac{4}{2}}$$

$$\Rightarrow 7^1\times 5^2 \Rightarrow 7\times 25 \Rightarrow 175$$

Hence, $\left[\frac{5^{-1}\times 7^2}{5^2\times 7^{-4}}\right]^{\frac{7}{2}}\times\left[\frac{5^{-2}\times 7^3}{5^3\times 7^{-5}}\right]^{-\frac{5}{2}}=175$

3. Prove that:

(i) $9^{\frac{3}{2}}-3\times5^{0}-\left(\frac{1}{81}\right)^{-\frac{1}{2}}=15$

(ii) $\left(\frac{1}{4}\right)^{-2}-3\times8^{\frac{2}{3}}\times4^{0}+\left(\frac{9}{16}\right)^{-\frac{1}{2}}=\frac{16}{3}$

(iii) $\frac{2^{\frac{1}{2}}\times3^{\frac{1}{3}}\times4^{\frac{1}{4}}}{10^{-\frac{1}{5}}\times5^{\frac{3}{5}}}\div\frac{3^{\frac{4}{3}}\times5^{\frac{-7}{5}}}{4^{\frac{-3}{5}}\times6}=10$

(iv) $\frac{(0.6)^{0}-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^{3}+\left(-\frac{1}{3}\right)^{-1}}=-\frac{3}{2}$

(v) $\sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}=\frac{3}{2}$

(vi) $\frac{2^{n}+2^{n-1}}{2^{n+1}-2^{n}}=\frac{2^{n}+2^{n}\times2^{-1}}{2^{n}\times2^{1}-2^{n}}$

(vii) $\left(\frac{64}{125}\right)^{-\frac{2}{3}}+\frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}}+\left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)$

(viii) $\frac{3^{-3}\times6^{2}\times\sqrt{98}}{5^{2}\times\sqrt[3]{\frac{1}{25}}\times(15)^{-\frac{4}{3}}\times3^{\frac{1}{3}}}$

Sol:

(i) We have,

$$9^{\frac{3}{2}}-3\times5^{0}-\left(\frac{1}{81}\right)^{-\frac{1}{2}}$$

$$=\left(3^{2}\right)^{\frac{3}{2}}-3\times1-\left(\frac{1}{9^{2}}\right)^{-\frac{1}{2}}$$

$$=3^{2\times\frac{3}{2}}-3-\left(9^{-2}\right)^{-\frac{1}{2}}$$

$$=3^{3}-3-9^{-2x\left(-\frac{1}{2}\right)}$$

$$=3^{3}-3-9$$

$$=27-3-9$$

$$=27-12$$

$$=15$$

$$\Rightarrow 9^{\frac{3}{2}} - 3\times 5^{0} - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$$

(ii) We have,

$$\left(\frac{1}{4}\right)^{-2} - 3\times 8^{\frac{2}{3}} \times 4^{0} + \left(\frac{9}{16}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{1}{2^2}\right)^{-2} - 3\times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^2}{4^2}\right)^{-\frac{1}{2}}$$

$$= \left(2^{-2}\right)^{-2} - 3\times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^{2\times\frac{-1}{2}}}{4^{2\times\frac{-1}{2}}}\right)$$

$$= 2^{(-2)\times(-2)} - 3\times 8^{\frac{2}{3}} + \left(\frac{3^{-1}}{4^{-1}}\right)$$

$$= 2^{4} - 3\times 2^{3\times\frac{2}{3}} + \frac{4}{3}$$

$$= 2^{4} - 3\times 2^{2} + \frac{4}{3}$$

$$= 2^{4} - 3\times 4 + \frac{4}{3}$$

$$= 16 - 12 + \frac{4}{3}$$

$$= 4 + \frac{4}{3} = \frac{12+4}{3}$$

$$= \frac{16}{3}$$

$$\Rightarrow \left(\frac{1}{4}\right)^{-2} - 3\times 8^{\frac{2}{3}} \times 4^{0} + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \frac{16}{3}$$

(iii) We have,

$$\frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \times 6}$$

$$= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \times \frac{4^{-\frac{3}{5}} \times 6}{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$$

$$= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times \left(2^2\right)^{\frac{1}{4}} \times \left(2^2\right)^{-\frac{3}{5}} \times (2 \times 3)}{(2 \times 5)^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$$

$$= \frac{\left(2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times 2^{-\frac{6}{5}} \times 2^1\right) \times \left(3^{\frac{1}{3}} \times 3^1\right)}{2^{-\frac{1}{5}} \times 5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$$

$$= \frac{\left(2 \times 2^{\frac{-6}{5}} \times 2\right) \times \left(3^{\frac{1}{3}} \times 3^1 \times 3^{-\frac{4}{3}}\right)}{2^{-\frac{1}{5}} \times \left(5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}\right)}$$

$$= \frac{\left(2 \times 2^{-\frac{6}{5}} \times 2 \times 2^{\frac{1}{5}}\right) \times \left(3^{\frac{1}{3}} \times 3^1 \times 3^{-\frac{4}{3}}\right)}{\left(5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}\right)}$$

$$= \frac{(2)^{1-\frac{6}{5}+1+\frac{1}{5}} \times (3)^{\frac{1}{3}+1-\frac{4}{3}}}{(5)^{-\frac{1}{5}+\frac{3}{5}-\frac{7}{5}}}$$

$$= \frac{(2)^{2-\frac{6}{5}+\frac{1}{5}} \times (3)^{\frac{1+3-4}{3}}}{(5)^{\frac{-1+3-7}{5}}}$$

$$= \frac{(2)^{2-\frac{5}{5}} \times (3)^{\frac{0}{3}}}{(5)^{-\frac{5}{5}}}$$

$$= \frac{(2)^{2-1} \times (3)^0}{(5)^{-1}}$$

$$= 2^1 \times 1 \times 5^1$$

$$= 10$$

$$\Rightarrow \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \times 6} = 10$$

(iv) We have,

$$\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}}$$

$$= \frac{1-\frac{1}{0.1}}{\left(\frac{8}{3}\right)\left(\frac{3}{2}\right)^3+(-3)^1}$$

$$= \frac{1-10}{\frac{8}{3}\times\frac{3^3}{2^3}-3}$$

$$= \frac{-9}{3^2-3}$$

$$= \frac{-9}{9-3} = -\frac{9}{6} = -\frac{3}{2}$$

(v) We have,

$$\sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}$$

$$= \frac{1}{2}+\frac{1}{(0.01)^{\frac{1}{2}}}-\left(3^3\right)^{\frac{2}{3}}$$

$$= \frac{1}{2}+\frac{1}{(0.1)^{2\times\frac{1}{2}}}-3^{3\times\frac{2}{3}}$$

$$= \frac{1}{2}+\frac{1}{0.1}-3^2$$

$$= \frac{1}{2}+10-9$$

$$= \frac{1}{2}+1 = \frac{3}{2}$$

$$\Rightarrow \sqrt{\frac{1}{4}}(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}} = \frac{3}{2}$$

(vi) We have,

$$\frac{2^n+2^{n-1}}{2^{n+1}-2^n} = \frac{2^n+2^n\times 2^{-1}}{2^n\times 2^1-2^n}$$

$$= \frac{2^n\left[1+2^{-1}\right]}{2^n\left[2-1\right]}$$

$$= \frac{1+\frac{1}{2}}{1}$$

$$= 1+\frac{1}{2}$$

$$= \frac{3}{2}$$

$$\Rightarrow \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{3}{2}$$

(vii) We have,

$$\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)$$

$$= \left(\frac{125}{64}\right)^{\frac{2}{3}} + \frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}} + \left(\frac{5}{(64)^{\frac{1}{3}}}\right)$$

$$= \left(\frac{5^3}{4^3}\right)^{\frac{2}{3}} + \frac{1}{\frac{4}{5}} + \left(\frac{5}{4^{3\times\frac{1}{3}}}\right)$$

$$= \frac{5^2}{4^2} + \frac{5}{4} + \frac{5}{4}$$

$$= \frac{25}{16} + \frac{10}{4}$$

$$= \frac{25+40}{16} = \frac{65}{16}$$

(viii) We have,

$$\frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times \sqrt{7 \times 7 \times 2}}{5^2 \times \left(\frac{1}{25}\right)^{\frac{1}{3}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times \left(\frac{1}{5^{2\times\frac{1}{3}}}\right) \times \frac{1}{(15)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times 5^{-\frac{2}{3}} \times \frac{1}{(5\times 3)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{\left(5^2 \times 5^{-\frac{2}{3}} \times 5^{-\frac{4}{3}}\right) \times 3^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2} \times 3^{\frac{4}{3}} \times 3^{-\frac{1}{3}}}{(5)^{2-\frac{2}{3}-\frac{4}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2} \times 3^{\frac{4}{3}} \times 3^{-\frac{1}{3}}}{(5)^{\frac{6-2-4}{3}}}$$

$$= \frac{3^{-3+\frac{4}{3}-\frac{1}{3}} \times 36 \times 7\sqrt{2}}{5^0}$$

$$= 3^{-3+\left(\frac{4-1}{3}\right)} \times 36 \times 7\sqrt{2}$$

$$= 3^{-3+\frac{3}{3}} \times 36 \times 7\sqrt{2}$$

$$= 3^{-3+1} \times 36 \times 7\sqrt{2}$$

$$= 3^{-2} \times 36 \times 7\sqrt{2}$$

$$= \frac{1}{9} \times 36 \times 7\sqrt{2}$$

$$= 4 \times 7\sqrt{2}$$

$$= 28\sqrt{2}$$

$$\Rightarrow \frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}} = 28\sqrt{2}$$

4. If $27^x = \frac{9}{3^x}$, find x

Sol:

We have,

$$\left(27^x\right) = \frac{9}{3^x}$$

$$\Rightarrow \quad \left(3^3\right)^x = \frac{9}{3^x}$$

$$\Rightarrow \quad 3^{3\times x} = \frac{3^2}{3^x}$$

$$\Rightarrow \quad 3^{3x} = 3^{2-x}$$

$$\Rightarrow \quad 3x = 2 - x \qquad \text{[On equating exponents]}$$

$\Rightarrow 3x + x = 2$

$\Rightarrow 4x = 2 \Rightarrow x = \frac{2}{4} \Rightarrow \boxed{x = \frac{1}{2}}$

Hence, value of x is $\frac{1}{2}$

5. Find the values of x in each of the following:

(i) $2^{5x} \div 2^{x} = \sqrt[5]{2^{20}}$

(ii) $\left(2^{3}\right)^{4} = \left(2^{2}\right)^{x}$

(iii) $\left(\frac{3}{5}\right)^{x}\left(\frac{5}{x}\right)^{2x} = \frac{125}{27}$

(iv) $5^{x-2} \times 3^{2x-3} = 135$

(v) $2^{x-5} \times 5^{x-4} = 5$

(vi) $2^{x-7} \times 5^{x-4} = 1250$

Sol:

(i) We have

$2^{5x} \div 2^{x} = \sqrt[5]{2^{20}}$

$\frac{2^{5x}}{2^{x}} = \left(2^{20}\right)^{\frac{1}{5}}$

$2^{5x-x} = 2^{20\times\frac{1}{5}}$

$2^{4x} = 2^{4}$

$\Rightarrow 4x = 4$ [On equating exponents]

$\Rightarrow \boxed{x = 1}$

Hence value of x is 1

(ii) We have,

$\left(2^{3}\right)^{4} = \left(2^{2}\right)^{x}$

$\Rightarrow \quad 2^{3\times4} = 2^{2\times x}$

$\Rightarrow \quad 12 = 2x$ [On equating exponents]

$\Rightarrow \quad 2x = 12$

$\Rightarrow \quad \boxed{x = 6}$

Hence, value of x is 6.

(iii) We have,

$$\left(\frac{3}{5}\right)^x\left(\frac{5}{x}\right)^{2x}=\frac{125}{27}$$

$$\Rightarrow \quad \frac{(3)^x}{(5)^x}\frac{(5)^{2x}}{(3)^{2x}}=\frac{5^3}{3^3}$$

$$\Rightarrow \quad \frac{5^{2x-x}}{3^{2x-x}}=\left(\frac{5}{3}\right)^3$$

$$\Rightarrow \quad \frac{5^x}{3^x}=\left(\frac{5}{3}\right)^3$$

$$\Rightarrow \quad \left(\frac{5}{3}\right)^x=\left(\frac{5}{3}\right)^3 \qquad \text{[On equating exponents]}$$

$$\Rightarrow \quad \boxed{x=3}$$

Hence, value of x is 3

(iv) We have

$$5^{x-2}\times 3^{2x-3}=135$$

$$\Rightarrow \quad 5^{x-2}\times 3^{2x-3}=5\times 27$$

$$\Rightarrow \quad 5^{x-2}\times 3^{2x-3}=5^1\times 3^3$$

$$\Rightarrow \quad x-2=1,\ \ 2x-3=3 \qquad \text{[On equating exponents]}$$

$$\Rightarrow \quad x=2+1,\ \ 2x=3+3$$

$$\Rightarrow \quad x=3,\qquad 2x=6\Rightarrow x=3$$

Hence, the value of x is 3

(v) We have,

$$2^{x-5}\times 5^{x-4}=5$$

$$\Rightarrow \quad 2^{x-5}\times 5^{x-4}=5^1\times 2^0$$

$$\Rightarrow \quad x-5=0,\ \ x-4=1$$

$$\Rightarrow \quad x=5,\qquad x=4+1$$

$$\Rightarrow \quad \boxed{x=5}$$

Hence, the value of x is 5

(vi) We have,

$$2^{x-7}\times 5^{x-4}=1250$$

$$\Rightarrow \quad 2^{x-7}\times 5^{x-4}=2\times 625$$

$$\Rightarrow \quad 2^{x-7}\times 5^{x-4}=2^1\times 5^4$$

$$\Rightarrow \quad x-7=1$$

$$\Rightarrow \quad x=8,\quad x-4=4$$

$$\Rightarrow \quad \boxed{x=8}$$

Hence, the value of x is 8