Exercise 17.1

Q 1. Given below is a parallelogram ABCD. Complete each statement along with the definition or property used.

(i) AD =

(ii) $\angle DCB$ =

(iii) OC =

(iv) $\angle DAB + \angle CDA$ =

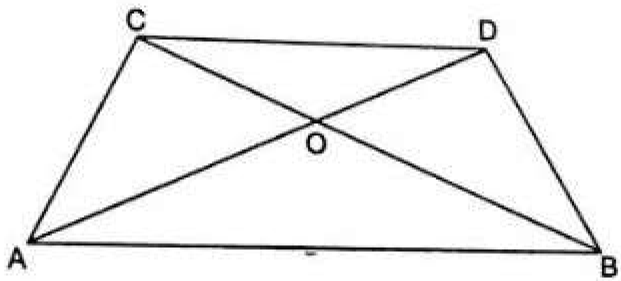
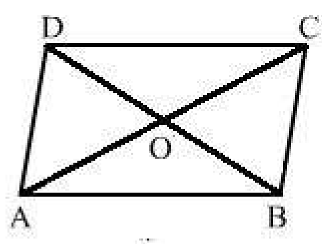


Fig. 17.21

SOLUTION:

The correct figure is



(i) AD = BC (opposite sides of a parallelogram are equal)

(ii) $\angle$DCB = $\angle$BAD (opposite angles are equal)

(iii) OC = OA (diagonals of a parallelogram bisect each other)

(iv) $\angle$DAB +$\angle$CDA = 180° (the sum of two adjacent angles of a parallelogram is 180°)

Q 2. The following figures are parallelograms. Find the degree values of the unknowns x, y and z.

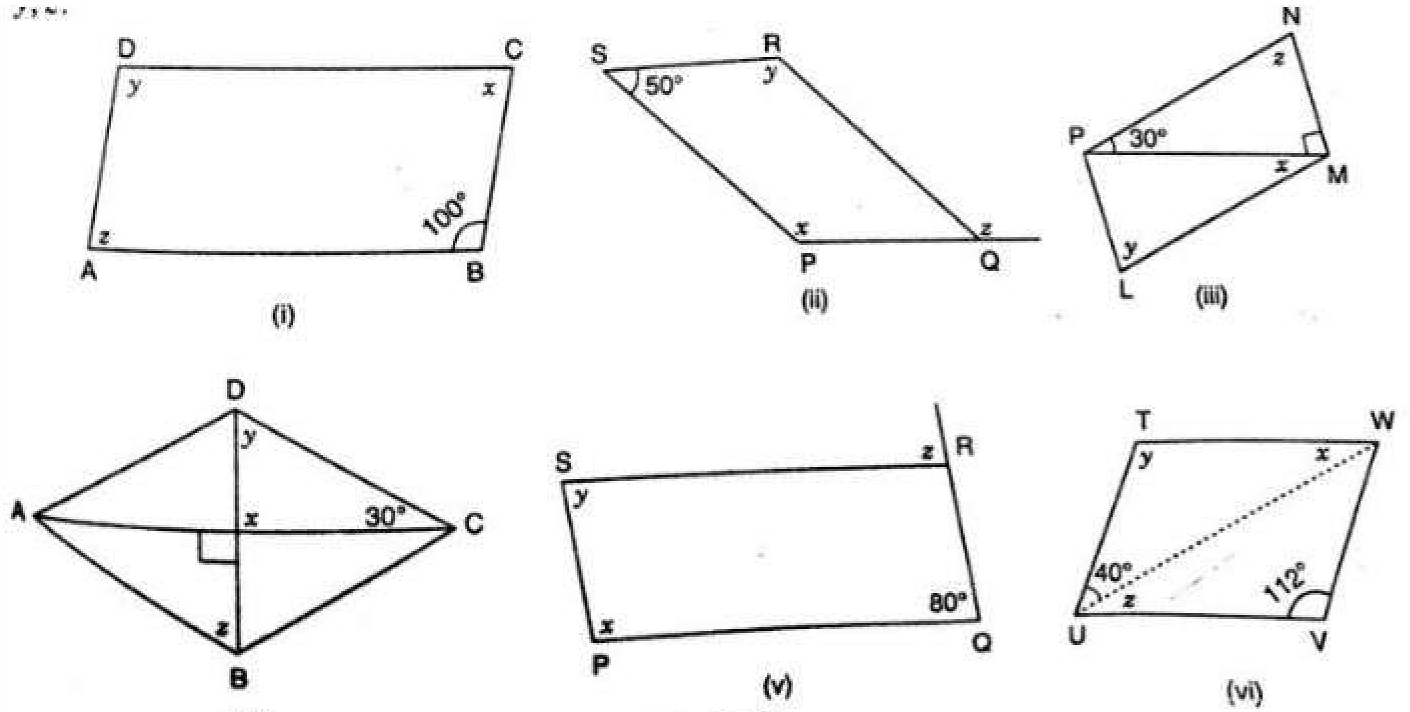


Fig. 17.22

SOLUTION:

(i) Opposite angles of a parallelogram are same.

Therefore, $x = z$ and $y = 100°$

Also, $y + z = 180°$ (sum of adjacent angle of quadrilateral is 180°)

$z + 100° = 180°$

$x = 180° - 100°$

$\Rightarrow x = 80°$

Therfore, $x = 80°$, $y = 100°$ and $z = 80°$

(ii) Opposite angles of a parallelogram are same.

Therefore, $x = y$ and $\angle ROP = 100°$

$\angle PSR + \angle SRQ = 180°$

$\Rightarrow y + 50° = 180°$

$x = 180° - 50°$

$\Rightarrow x = 130°$

Therefore, $x=130°$, $y=130°$

Since y and z are alternate angles, $z = 130°$.

(iii) Sum of all angles in a triangle is 180°

Therefore, $30° + 90° + z = 180°$

$\Rightarrow z = 60°$

Opposite angles are equal in the parallelogram.

Therefore, $y = z = 60°$ and $x=30°$ (alternate angles)

(iv) $x = 90^\circ$ (vertically opposite angle)

Sum of all angles in a triangle is 180°.

Therefore, $y + 90^\circ + 30^\circ = 180^\circ$

$\Rightarrow y = 180^\circ - (90^\circ + 30^\circ)$

$\Rightarrow y = 60^\circ$

$y = z = 60^\circ$ (alternate angles)

(v)Opposite angles are equal in a parallelogram.

Therefore, $y = 80^\circ$

$y + x = 180^\circ$

$\Rightarrow x = 180^\circ - 100^\circ = 80^\circ$

$z = y = 80^\circ$ (alternate angles)

(vi) $y = 112^\circ$ (opposite angles are equal in a parallelogram)

In triangleUTW :

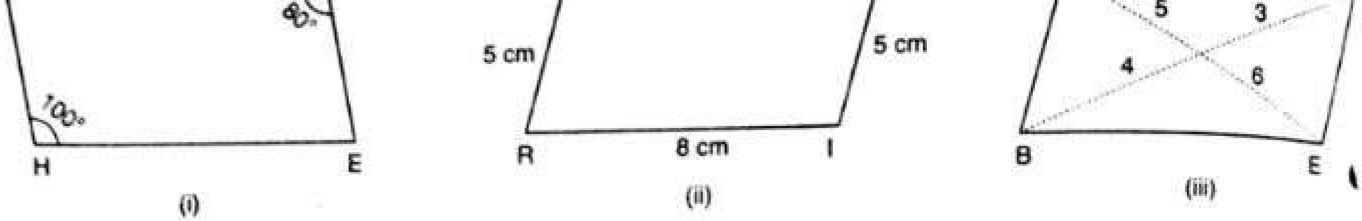
$x + y + 40^\circ = 180^\circ$ (angle sum property of a triangle)

$x = 180^\circ - (112^\circ - 40^\circ) = 28^\circ$

Bottom left vertex $= 180^\circ - 112^\circ = 68^\circ$

Therefore, $z = x = 28^\circ$ (alternate angles)

Q 3. Can the following figures be parallelograms? Justify your answers.



SOLUTION:

(i) No. This is because the opposite angles are not equal.

(ii) Yes. This is because the opposite sides are equal.

(iii) No. This is because the diagonals do not bisect each other.

Q 4. In the adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the geometrical truths you use to find them.

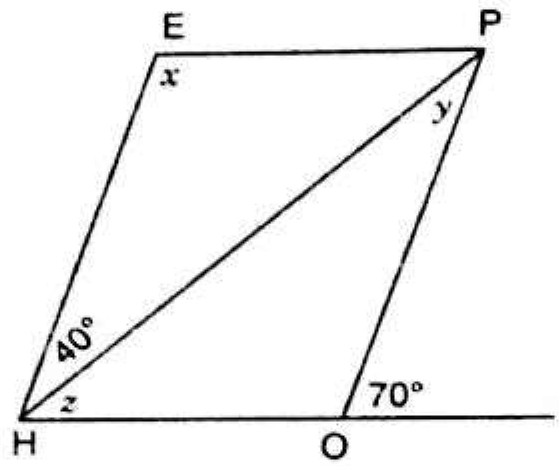


Fig. 17.24

SOLUTION:

$\angle H0P + 70^\circ = 180^\circ$ (linear pair)

$\angle HOP = 180^\circ - 70^\circ = 110^\circ$

$x = \angle HOP = 110^\circ$ (opposite angles of a parallelogram are equal)

$\angle EHP + \angle HEP = 180^\circ$ (sum of adjacent angles of a parallelogram is 180°)

$110^\circ + 40^\circ + z = 180^\circ$

$z = 180^\circ - 150^\circ = 30^\circ$

$y = 40^\circ$ (alternate angles)

Q 5. In the following figures GUNS and RUNS are parallelograms. Find x and y.

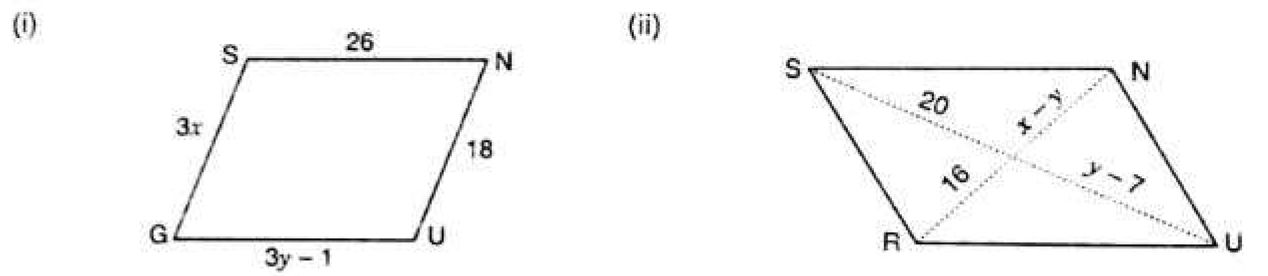


Fig. 17.25

SOLUTION:

(i) Opposite sides are equal in a parallelogram.

Therefore, $3y - 1 = 26$

$\Rightarrow 3y = 27$

$y = 9$.

Similarly, $3x = 18$

$x = 6$.

(ii) Diagonals bisect each other in a parallelogram.

Therefore, $y - 7 = 20$

$y = 27$

$x - y = 16$

$x - 27 = 16$

$x = 43.$

Q 6. In the following figure RISK and CLUE are parallelograms. Find the measure of x.

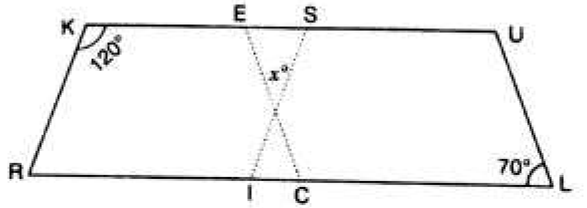


Fig. 17.26

SOLUTION:

In the parallelogram RISK:

$\angle ISK + \angle RKS = 180^\circ$ (sum of adjacent angles of a parallelogram is 180°)

$\angle ISK = 180^\circ - 120^\circ = 60^\circ$

Similarly, in parallelogram CLUE:

$\angle CEU = \angle CLU = 70^\circ$ (opposite angles of a parallelogram are equal)

In the triangle:

$x + \angle ISK + \angle CEU = 180°$

$x = 180° - 70° + 60°$

$x = 50°$.

Q 7. Two opposite angles of a parallelogram are $(3x - 2)°$ and $(50 - x)°$. Find the measure of each angle of the parallelogram.

SOLUTION:

Oppostie angles of a parallelogram are congurent.

Therefore, $3x - 2° = 50 - x°$

$3x° - 2° = 50° - x°$

$3x° + x° = 50° + 2°$

$4x° = 52°$

$x° = 13°$

Putting the value of x in one angle:

$3x° - 2° = 39° - 2° = 37°$

Opposite angles are congruent.

Therefore, $50° - x° = 37°$

Let the remaining two angles be y and z.

Angles y and z are congruent because they are also opposite angles.

Therefore, $y = z$

The sum of adjacent angles of a parallelogram is equal to 180°

Therefore, $37° + y = 180°$

$y = 180° - 37°$

$y = 143°$

So, the anlges measure are: $37°$, $37°$, $143°$ and $143°$.

Q 8. If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

SOLUTION:

Two adjacent angles of a parallelogram add up to $180°$.

Let x be the angle. Therefore, $x + \frac{2x}{3} = 180°$

$\frac{5x}{3} = 180°$

$x = 72°$

$\frac{2x}{3} = \frac{2(72°)}{3} = 108°$

Thus, two of the angles in the parallelogram are $108°$ and the other two are $72°$.

Q 9. The measure of one angle of a parallelogram is 70°. What are the measures of the remaining angles?

SOLUTION:

Given that one angle of the parallelogram is $70°$.

Since opposite angles have same value, if one is $70°$, then the one directly opposite will also be$70°$

So, let one angle be $x°$.

$x° + 70° = 180°$ (the sum of adjacent angles of a parallelogram is $180°$)

$x° = 180° - 70°$

$x^\circ = 110^\circ$

Thus, the remaining angles are 110°, 110° and 70°.

Q 10. Two adjacent angles of a parallelogram are as 1 : 2. Find the measures of all the angles of the parallelogram.

SOLUTION:

Let the angle be A and B.

The angles are in the ratio of 1:2.

Measures of $\angle A$ and $\angle B$ are x° and $2x^\circ$.

Then, As we know that the sum of adjacent angles of a parallelogram is 180°.

Therefore, $\angle A + \angle B = 180^\circ$

$\Rightarrow x^\circ + 2x^\circ = 180^\circ$

$\Rightarrow 3x^\circ = 180^\circ$

$\Rightarrow x^\circ = 60^\circ$

Thus, measure of $\angle A = 60^\circ$, $\angle B = 120^\circ$, $\angle C = 60^\circ$ and $\angle D = 120^\circ$.

Q 11. In a parallelogram ABCD, $\angle D=135^\circ$, determine the measure of $\angle A$ and $\angle B$.

SOLUTION:

In a parallelogram, opposite angles have the same value.

Therefore, $\angle D = \angle B = 135^\circ$

Also, $\angle A + \angle B + \angle C + \angle D = 360^\circ$ and $\angle A + \angle D = 180^\circ$.

$\angle A = 180^\circ - 135^\circ = 45^\circ$.

Q 12. ABCD is a parallelogram in which $\angle A = 70°$. Compute $\angle B$, $\angle C$ and $\angle D$.

SOLUTION:

Opposite angles of a parallelogram are equal.

Therefore, $\angle C = 70° = \angle A$

$\angle B = \angle D$

Also, the sum of the adjacent angles of a parallelogram is 180°

Therefore, $\angle A + \angle B = 180°$

$70° + \angle B = 180°$

$\angle B = 110°$

$\angle C = 70°$

$\angle D = 110°$

Q 13. The sum of two opposite angles of a parallelogram is 130°. Find all the angles of the parallelograms.

SOLUTION:

Let the angles be A, B, C and D.

It is given that the sum of two opposite angles is 130°.

Therefore, $\angle A + \angle C = 130°$

$\angle A + \angle A = 130°$ (opposite angles of a parallelogram are equal)

$\angle A = 65°$and $\angle C = 65°$

The sum of adjacent angles of a parallelogram is 180°.

$\angle A + \angle B = 180^\circ$

$65^\circ + \angle B = 180^\circ$

$\angle B = 180^\circ - 65^\circ$

$\angle B = 115^\circ$

Therefore, $\angle A = 65^\circ$, $\angle B = 115^\circ$, $\angle C = 65^\circ$ and $\angle D = 115^\circ$.

Q 14. All the angles of a quadrilateral are equal to each other. Find the measure of each. Is the quadrilateral a parallelogram? What special type of parallelogram is it?

SOLUTION:

Let the angle be x.

All the angles are equal.

Therefore, $x + x + x + x = 360^\circ$.

$4x = 360^\circ$.

$x = 90^\circ$.

So, each angle is 90° and quadrilateral is a parallelogram. It is a rectangle.

Q 15. Two adjacent sides of a parallelogram are 4 cm and 3 cm respectively. Find its perimeter.

SOLUTION:

We know that the opposite sides of a parallelogram are equal.

Two sides are given, i.e. 4 cm and 3 cm. Therefore, the rest of the sides will also be 4 cm and 3 cm.

Therefore, Perimeter = Sum of all the sides of a parallelogram = 4 + 3 + 4 + 3 = 14 cm

Q 16. The perimeter of a parallelogram is 150 cm. One of its sides is greater than the other by 25 cm. Find the length of the sides of the parallelogram.

SOLUTION:

Opposite sides of a parallelogram are same.

Let two sides of the parallelogram be x and y.

Given: $x = y + 25$

Also, $x + y + x + y = 150$ (Perimeter= Sum of all the sides of a parallelogram)

$y + 25 + y + y + 25 + y = 150$

$4y = 150 - 50$

$4y = 100$

$y = 100/4 = 25$

therefore, $x = y + 25 = 25 + 25 = 50$

Thus, the lengths of the sides of the parallelogram are 50 cm and 25 cm.

Q 17. The shorter side of a parallelogram is 4.8 cm and the longer side is half as much again as the shorter side. Find the perimeter of the parallelogram.

SOLUTION:

Given:

Shorter side = 4.8 cm, Longer side = $\frac{4.8}{2} + 4.8 = 7.2$ cm

Perimeter = Sum of all sides = 4.8 + 4.8 + 7.2 + 7.2 = 24 cm

Perimeter = Sum of all sides = 4.8 + 4.8 + 7.2 + 7.2 = 24 cm

Q 18. Two adjacent angles of a parallelogram are $(3x - 4)^\circ$ and $(3x + 10)^\circ$. Find the angles of the parallelogram.

SOLUTION:

We know that the adjacent angles of a parallelogram are supplementary.

Hence, $3x + 10^\circ$ and $3x - 4^\circ$ are supplementry.

$3x + 10^\circ + 3x - 4^\circ = 180^\circ$

$6x^\circ + 6^\circ = 180^\circ$

$6x^\circ = 174^\circ$

$x = 29^\circ$

First angle = $3x+10^\circ = 3(29^\circ) + 10^\circ = 97^\circ$

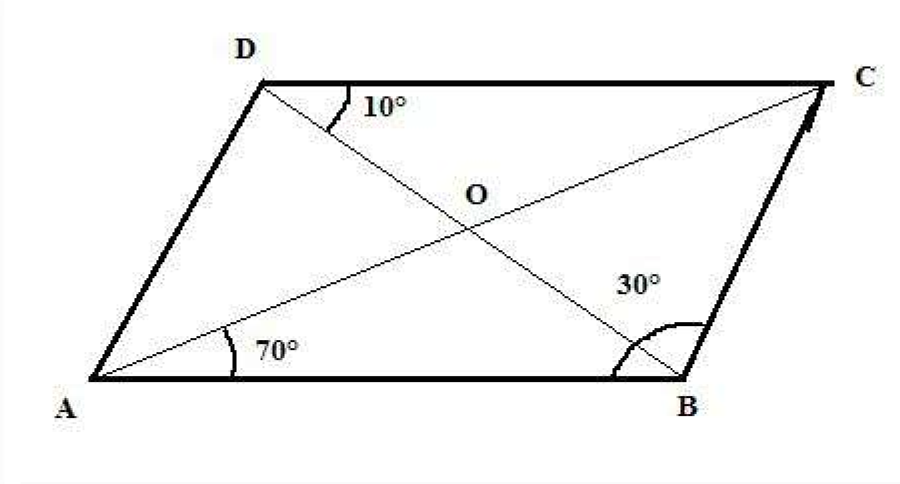
Second angle = $3x - 4^\circ = 83^\circ$

Thus, the angles of the parallelogram are 97°, 83°, 97° and 83°.

Q 19. In a parallelogram ABCD, the diagonals bisect each other at O. If $\angle ABC = 30^\circ$, $\angle BDC = 10^\circ$ and $\angle CAB = 70^\circ$. Find:

$\angle DAB$, $\angle ADC$, $\angle BCD$, $\angle AOD$, $\angle DOC$, $\angle BOC$, $\angle AOB$, $\angle ACD$, $\angle CAB$, $\angle ADB$, $\angle ACB$, $\angle DBC$ and $\angle DBA$.

SOLUTION:



∠ABC=30°

Therefore, ∠ADC = 30° (opposite angle of the parallelogram) and ∠BDA = ∠ADC − ∠BDC = 30°-10° = 20°

∠BAC = ∠ACD = 70° (alternate angle)

In triangle ABC: ∠CAB + ∠ABC + ∠BCA = 180°

70° + 30° + ∠BCA = 180°

Therefore, ∠BCA = 80°

∠DAB = ∠DAC + ∠CAB = 70° + 80° = 150°

∠BCD = 150° (opposite angle of the parallelogram)

∠DCA = ∠CAB = 70°

In triangle DOC: ∠ODC + ∠DOC + ∠OCD = 180°

10° + 70° + ∠DOC = 180°

Therefore, ∠DOC=100°

∠DOC + ∠BOC = 180°

∠BOC = 180° − 100°

$\angle BOC = 80°$

$\angle AOD = \angle BOC = 80°$ (vertically opposite angles)

$\angle AOB = \angle DOC = 100°$(vertically opposite angles)

$\angle CAB = 70°$

Given $\angle ADB = 20°$

$\angle DBA = \angle BDC = 10°$ (alternate angles)

$\angle ADB = \angle DBC = 20°$ (alternate angle).

Q 20. Find the angles marked with a question mark shown in Fig. 17.27.

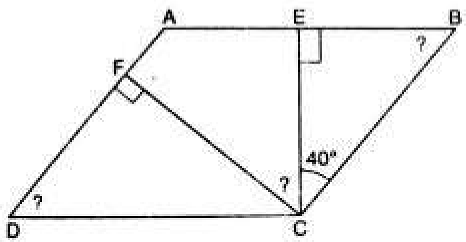


Fig. 17.27

SOLUTION:

In triangleCEB: $\angle ECB + \angle CBE + \angle BEC = 180°$ (angle sum property of a triangle)

$40° + 90° + \angle EBC = 180°$

Therefore, $\angle EBC = 50°$

Also, $\angle EBC = \angle ADC = 50°$(opposite angle of a parallelogram)

In triangleFDC: $\angle FDC + \angle DCF + \angle DCF = 180°$

$50° + 90° + \angle DCF = 180°$

Therefore, $\angle DCF = 40°$

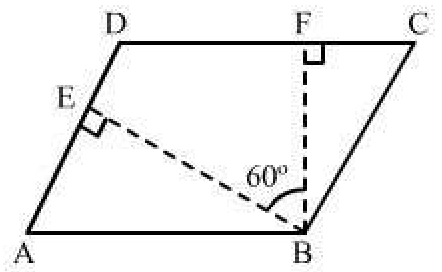
Now, $\angle BCE + \angle ECF + \angle FCD + \angle FDC = 180°$ (in a parallelogram, the sum of alternate angle is 180°

$50° + 40° + \angle ECF + 40° = 180°$

$\angle ECF = 180° - 50° + 40° - 40° = 50°$

Q 21. The angle between the altitudes of a parallelogram, through the same vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.

SOLUTION:



Draw a parallelogram ABCD.

Drop a perpendicular from B to the side AD, at the point E.

Drop a perpendicular from B to the side CD, at the point F.

In the quadrilateral BEDF: $\angle EBF = 60°$, $\angle BED = 90°$, $\angle BFD = 90°$

$\angle EDF = 360° - (60° + 90° + 90°) = 120°$

In a parallelogram, opposite angles are congruent and adjacent angles are supplementary.

In the parallelogram ABCD: $\angle B = \angle D = 120°$

$\angle A = \angle C = 180° - 120° = 60°$

Q 22. In Fig. 17.28, ABCD and AEFG are parallelograms. If $\angle C = 55°$, what is the measure of $\angle F$?

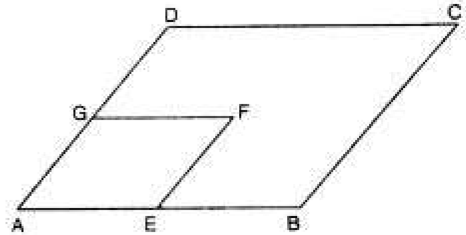


Fig. 17.28

SOLUTION:

Both the parallelograms ABCD and AEFG are similar.

Therefore, $\angle C = \angle A = 55°$ (opposite angles of a parallelogram are equal)

Therefore, $\angle A = \angle F = 55°$ (opposite angles of a parallelogram are equal).

Q 23. In Fig. 17.29, BDEF and DCEF are each a parallelogram. Is it true that BD = DC? Why or why not?

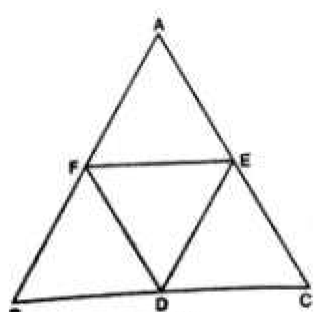


Fig. 17.29

SOLUTION:

In parallelogram BDEF

Therefore, BD = EF ....(i) (opposite sides of a parallelogram are equal)

In parallelogram DCEF

CD = EF ....(ii) (opposite sides of a parallelogram are equal)

From equations (i) and (ii)

BD=CD

Q 24. In Fig. 17.29, suppose it is known that DE = DF. Then, is triangle ABC isosceles? Why why not?

SOLUTION:

In ΔFDE: DE = DF

$\angle$FED = $\angle$DFE .... (i) (angles opposite to equal sides)

In the ||gm BDEF: $\angle$FBD = $\angle$FED .... (ii) (opposite angles of a parallelogram are equal)

In the ||gm DCEF: $\angle$DCE = $\angle$DFE .... (iii) (opposite angles of a parallelogram are equal)

From equations (i), (ii) and (iii): $\angle$FBD = $\angle$DCE

In triangle ABC: if $\angle$FBD = $\angle$DCE, then AB = AC(sides opposite to the equal angles.)

Hence, triangle ABC is isosceles.

Q 25. Diagonals of parallelogram ABCD intersect at O as shown in Fig. 17.30. XY contain, O, and X, Y are points on opposite sides of the parallelogram. Give reasons for each of the following:

(i) OB =OD

(ii) $\angle$OBY = $\angle$ODX

(iii) $\angle$BOY = $\angle$DOX

(iv) $\Delta BOY \cong \Delta DOX$

Now, state if XY is bisected at O.

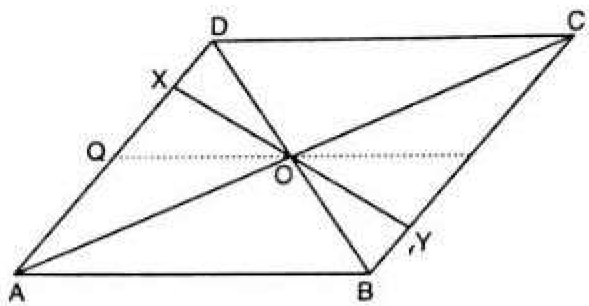


Fig. 17.30

SOLUTION:

(i) Diagonals of a parallelogram bisect each other.

(ii) Alternate angles

(iii) vertically opposite angles

(iv) ΔBOY and ΔDOX: OB=OD (diagonals of a parallelogram bisect each other)

$\angle$OBY = $\angle$ODX (alternate angles)

$\angle$BOY = $\angle$DOX (verticalty opposite angles)

$\angle BOY = \angle DOX$ (vertically opposite angles)

ASA congruence:

XO = YO (c.p.c.t)

So, XY is bisected at O.

Q 26.In fig. 17.31,ABCD is a parallelogram, CE bisects $\angle C$ and AF bisects $\angle A$. In each of the following, if the statement is true, give a reason for the same:

(i) $\angle A = \angle C$

(ii) $\angle FAB = 1/2\angle A$

(iii) $\angle DCE = 1/2\angle C$

(iv) $\angle CEB = \angle FAB$

(v) CE||AF

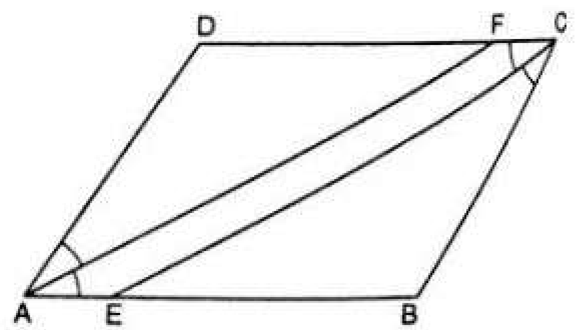


Fig. 17.31

SOLUTION:

(i) True, since opposite angles of a parallelogram are equal.

(ii) True, as AF is the bisector of LA.

(iii) True, as CE is the bisector of zC.

(iv) True

$\angle CEB = \angle DCE$ ....(i) (alternate angles)

$\angle DCE = \angle FAB$ ....(ii) (opposite angles of a parallelogram are equal)

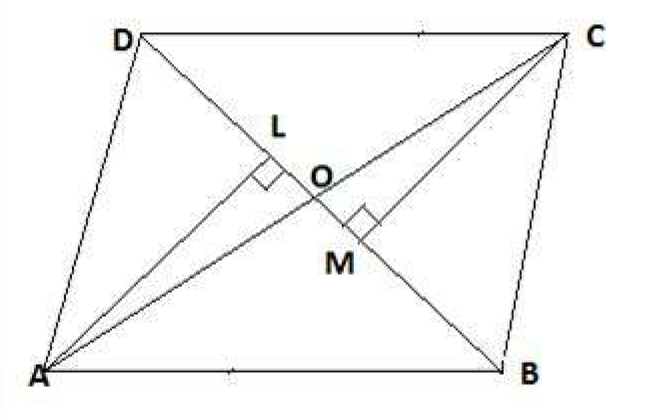
From equations (i) and (ii):

$\angle CEB = \angle FAB$

(v) True, as corresponding angles are equal ($\angle CEB = \angle FAB$).

Q 27. Diagonals of a parallelogram ABCD intersect at O. AL and CM are drawn perpendiculars to BD such that L and M lie on BD. Is AL = CM? Why or why not?

SOLUTION:



In ΔAOL and ΔCMO:

$\angle AOL = \angle COM$(vertically opposite angle).......(i)

$\angle ALO = \angle CM0 = 90°$ (each right angle)........... (ii)

Using angle sum property: $\angle A0L + \angle ALO + \angle LA0 = 180°$ .... (iii)

$\angle COM + \angle CMO + \angle OCM = 180°$.......... (iv)

From equations (iii) and (iv):

$\angle AOL + \angle ALO + \angle LAO = \angle COM + \angle CMO + \angle OCM$

$\angle LAO = \angle OCM$ (from equation (i) and (ii))

In ΔAOL and ΔCMO:

$\angle ALO = \angle CMO$ (each right angle)

AO=OC (diagonals of a parallelogram bisect each)

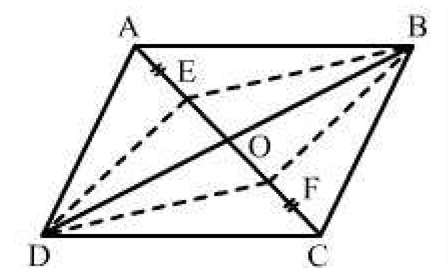
$\angle LAO = \angle OCM$ (proved above)

So, ΔAOL is congruent to ΔCMO (SAS).

AL = CM [cpct]

Q 28. Points E and F lie on diagonal AC of a parallelogram ABCD such that AE = CF. What type of quadrilateral is BFDE ?

SOLUTION:



In the ||gm ABCD:

AO = OC............. (i) (diagonals of a parallelogram bisect each other)

AE = CF..........(ii) (given)

Subtracting (ii) from (i): AO − AE = OC − CF

EO = OF............(iii)

In ΔDOE and ΔBOF: EO = OF (proved above)

DO = OB (diagonals of a parallelogram bisect each other)

$\angle$DOE = $\angle$BOF (vertically opposite angles)

By SAS congruence: $\Delta DOE \cong \Delta BOF$

Therefore, DE = BF (c.p.c.t)

In ΔBOE and ΔDOF:

EO = OF (proved above)

DO = OB (diagonals of a parallelogram bisect each other)

$\angle$DOF = $\angle$BOE (vertically opposite angles)

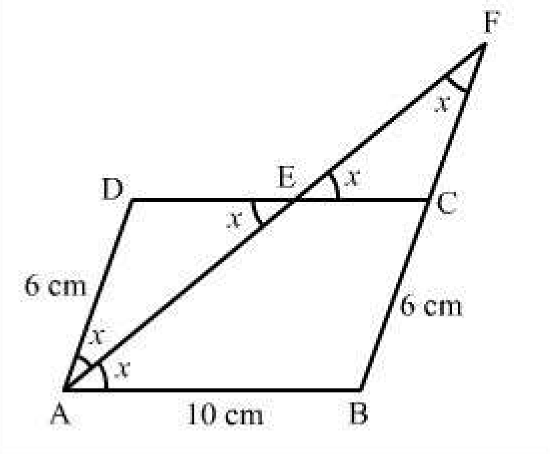
By SAS congruence: $\Delta DOE \cong \Delta BOF$

Therefore, DF = BE (c.p.c.t)

Hence, the pair of opposite sides are equal. Thus, DEBF is a parallelogram.

Q 29. In a parallelogram ABCD, AB =10 cm, AD = 6 cm. The bisector of $\angle$A meets DC in E, AE and BC produced meet at F. Find the length CF.

SOLUTION:



AE is the bisector of $\angle DAE = \angle BAE = x$

$\angle BAE = \angle AED = x$ (alternate angles)

Since opposite angles in triangle ADE are equal, Triangle ADE is an isosceles triangle.

Therefore, AD = DE = 6 cm (sides opposite to equal angles)

AB = CD = 10 cm

CD = DE + EC

EC = CD −DE

EC = 10 − 6 = 4 cm

$\angle DEA = \angle CEF = x$ (vertically opposite angle)

$\angle EAD = \angle EFC = x$ (alternate angles)

Since opposite angle in triangle EFC are equal, Triangle EFC is an isosceles triangle.

Therefore, CF = CE = 4 cm (sides opposite to equal angles)

:.Therefore CF= 4cm.

Exercise 17.2

Q 1. Which of the following statements are true for a rhombus?

(i) It has two pairs of parallel sides.

(ii) It has two pairs of equal sides.

(iii) It has only two pairs of equal sides.

(iv) Two of its angles are at right angles.

(v) its diagonals bisect each other at right angles.

(vi) Its diagonals are equal and perpendicular.

(vii) It has all its sides of equal lengths.

(viii) It is a parallelogram.

(ix) It is a quadrilateral.

(x) It can be a square.

(xi) It is a square.

SOLUTION:

(i) True

(ii) True

(iii) True

(iv) False

(v) True

(vi) False

Diagonals of a rhombus are perpendicular, but not equal.

(vii) True

(viii) True

It is a parallelogram because it has two pairs of parallel sides.

(ix) True

It is a quadrilateral because it has four sides.

(x) True

It can be a square if each of the angle is a right angle.

(xi) False

It is not a square because each of the angles is a right angle in a square.

Q 2. Fill in the blanks, in each of the following, so as to make the statement true:

(i) A rhombus is a parallelogram in which ....

(ii) A square is a rhombus in which ......

(iii) A rhombus has all its sides of .....length.

(iv) The diagonals of a rhombus ....each other at...... angles.

(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a ....

SOLUTION:

(i) A rhombus is a parallelogram in which **adjacent sides are equal.**

(ii) A square is a rhombus in which **all angles are right angled.**

(iii) A rhombus has all its sides of **equal** length.

(iv) The diagonals of a rhombus **bisect** each other at **right** angles.

(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a **rhombus.**

Q 3. The diagonals of a parallelogram are not perpendicular. Is it a rhombus? Why or why not?

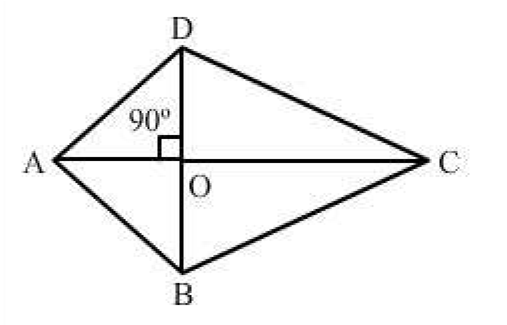
SOLUTION:

No, it is not a rhombus. This is because diagonals of a rhombus must be a perpendicular.

Q 4. The diagonals of a quadrilateral are perpendicular to each other. Is such a quadrilateral always a rhombus ? If your answer is 'No', draw a figure to justify your answer.

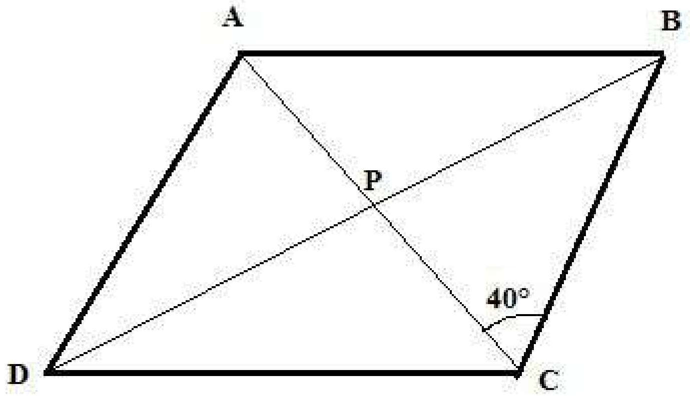
SOLUTION:

No, it is not so. Diagonals of a rhombus are perpendicular and bisect each other. Along with this, all of its sides are equal. In the figure given below, the diagonals are perpendicular to each other, but do not bisect each other.



Q 5. ABCD is a rhombus. If $\angle ACE = 40°$, find $\angle ADB$.

SOLUTION:



In a rhombus, the diagonals are perpendicular.

Therefore, $\angle BPC = 90°$

From Triangle BPC, the sum of angles is 180°.

Therefore, $\angle CBP + \angle BPC + \angle PBC = 180°$

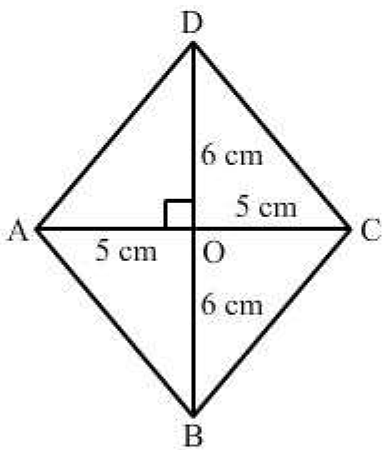
$\angle CBP = 180° - \angle BPC - \angle PBC$

$\angle CBP = 180° - 40° - 90° = 50°$

$\angle ADB = \angle CBP = 50°$ (alternate angle)

Q 6. If the diagonals of a rhombus are 12 cm and 16 cm, find the length of each side.

SOLUTION:



All sides of a rhombus are equal in length.

The diagonals intersect at 90° and the sides of the rhombus form right triangles.

One leg of these right triangles is equal to 8 cm and the other is equal to 6 cm.

The sides of the triangle form the hypotenuse of these right triangles.

So, we get :

$(8^2 + 6^2)\, cm^2$
= $(64 + 36)\, cm^2$

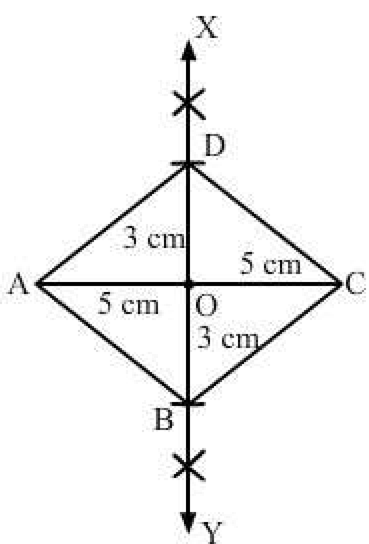
= 100 cm^2

The hypotenuse is the square root of 100 cm^2. This makes the hypotenuse equal to 10.

Thus, the side of the rhombus is equal to 10 cm.

Q 7. Construct a rhombus whose diagonals are of length 10 cm and 6 cm.

SOLUTION:



1. Draw AC equal to 10 cm.

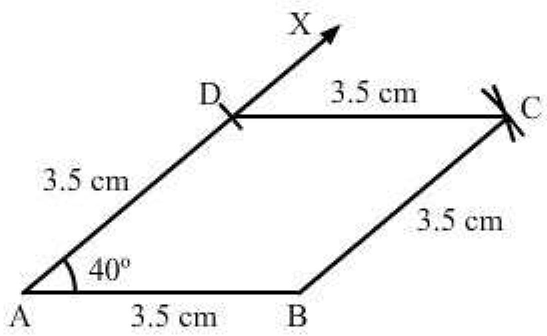
2. Draw XY, the right bisector of AC, meeting it at O.

3. With O as centre and radius equal to half of the length of the other diagonal, i.e. 3 cm, cut OB = OD = 3 cm.

4. Join AB, AD and CB, CD.

Q 8. Draw a rhombus, having each side of length 3.5 cm and one of the angles as 40°.

SOLUTION:



1. Draw a line segment AB of 3.5 cm.

2. Draw $\angle BAX$ equal to 40°.

3. With A as the center and the radius equal to AB, cut AD at 3.5 cm.

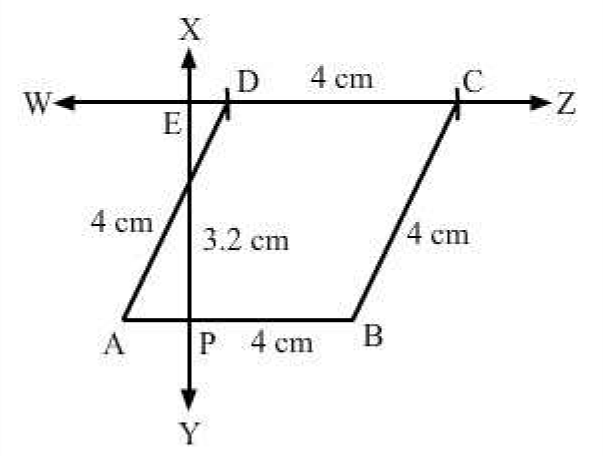
4. With D as the center, cut an arc of radius 3.5 cm.

5. With B as the centre, cut an arc of radius 3.5 cm. This arc cuts the arc of step 4 at C.

6. Join DC and BC.

Q 9. One side of a rhombus is of length 4 cm and the length of an altitude is 3.2 cm. Draw the rhombus.

SOLUTION:



1. Draw a line segment AB of 4 cm.

2. Draw a perpendicular XY on AB, which intersects AB at P.

3. With P as the center, cut PE at 3.2 cm.

4. Draw a line WZ that passes through E. This line should be parallel to AB.

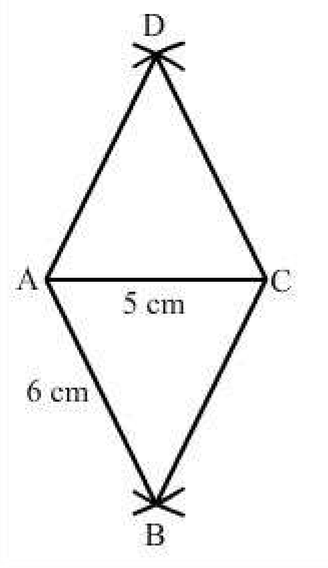
5. With A as the center, draw an arc of radius 4 cm that cuts WZ at D.

6. With D as center and radius 4 cm, cut line DZ. Label it as point C.

7. Join AD and CB.

Q10. Draw a rhombus ABCD, if AB = 6 cm and AC = 5 cm.

SOLUTION:



1. Draw a line segment AC of 5 cm.

2. With A as centre, draw an arc of radius 6 cm on each side of AC.

3. With C as centre, draw an arc of radius 6 cm on each side of AC. These arcs intersect the arcs of step 2 at B and D.

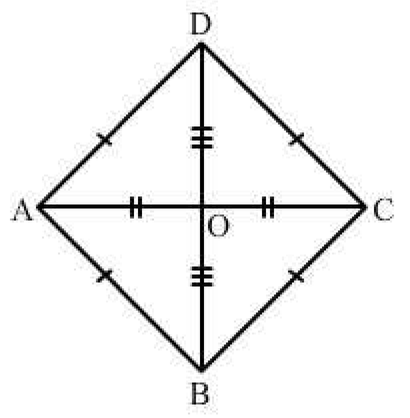
4. Join AB, AD, CD and CB.

Q 11. ABCD is a rhombus and its diagonals intersect at 0.

(i) Is ΔBOC = ΔDOC? State the congruence condition used?

(ii) Also state, if $\angle$ BCO = $\angle$DCO.

SOLUTION:



(i) Yes

In ΔBCO and ΔDCO :

OC = OC (common)

BC = DC (all sides of a rhombus are equal)

BO = OD (diagonals of a rhobmus bisect each other)

By SSS congruence : $\Delta BCO \cong \Delta DCO$

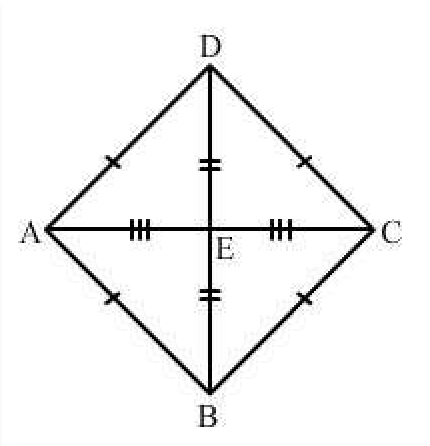
(ii) Yes

By c.p.c.t:

$\angle BCO = \angle DCO$

Q 12. Show that each diagonal of a rhombus bisects the angle through which it passes.

SOLUTION:



In ΔAED and ΔDEC :

AE = EC (diagonals bisect each other)

AD = DC (sides are equal)

DE = DE (common)

By SSS congruence : $\Delta AED \cong \Delta CED$

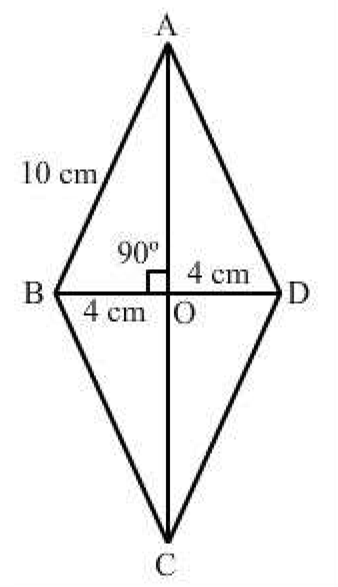
$\angle ADE = \angle CDE$ (c. p. c. t)

Similarly, we can prove ΔAEB and ΔBEC, ΔBEC and ΔDEC, ΔAED and ΔAEB are congruent to each other.

Hence, diagonal of a rhombus bisects the angle through which it passes.

Q 13. ABCD is a rhombus whose diagonals intersect at 0. If AB =10 cm, diagonal BD =16 cm, find the length of diagonal AC.

SOLUTION:



We know that the diagonals of a rhombus bisect each other at right angles.

Therefore, BO = $\frac{1}{2}$ BD = ($\frac{1}{2} \times 16$) cm = 8cm

AB = 10 cm and $\angle$A0B = 90°

From right ΔOAB :

$AB^2 = A0^2 + B0^2$

$A0^2 = (AB^2 - B0^2)$

$A0^2 = (10)^2 - (8)^2\ cm^2$

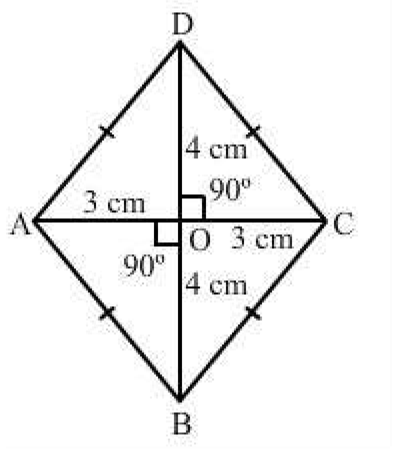
$A0^2 = (100 - 64)\ cm^2 = 36cm^2$

AO = 6cm

Therefore, AC = 2 $\times$ AO = (2 $\times$ 6) cm = 12 cm.

Q 14. The diagonals of a quadrilateral are of lengths 6 cm and 8 cm. If the diagonals bisect each other at right angles, what is the length of each side of the quadrilateral?

SOLUTION:



Let the given quadrilateral be ABCD in which diagonals AC is equal to 6 cm and BD is equal to 8 cm.

Also, it is given that the diagonals bisect each other at right angle, at point O.

Therefore, AO = OC = $\frac{1}{2}$ AC = 3 cm

Also, OB = OD = $\frac{1}{2}$ BD = 4 cm

In right ΔAOB :

$AB^2 = A0^2 + B0^2$

$AB^2 = (9 + 16)\ cm^2$

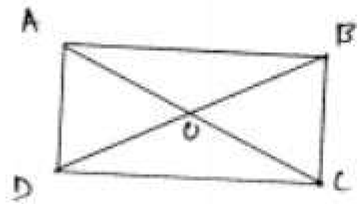
$AB^2 = 25\ cm^2$

AB = 5 cm

Thus, the length of each side of the quadrilateral is 5 cm.

Exercise 17.3

(1)



(i) True $AB = DC$ & $AD = BC$

(ii) False $AD \neq DC$

(iii) True

(iv) True

(v) False [Need not be]

(vi) False

(vii) True [$AC = BD$ & $AO = OC$; $DO = OB$]

(viii) False [They are not $\perp^r$]

(ix) False [They posses different lengths]

(x) True

(xi) True

(xii) False [because all squares are parallelograms].

(2)

(i) True

(ii) True

(iii) True

(iv) False, (Diagonal = $\sqrt{2} \times$ side)

(3)

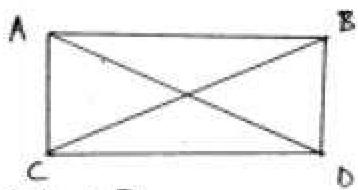
(i) <u>angles are right angles</u>

(ii) <u>angles are right angles</u>

(iii) <u>all sides are equal</u>

(4) No, In rectangle, the length of diagonals are equal and they do bisect each other

(5) Given Rectangle ABCD,



Here AD = BC

[Diagonals are of equal length in Rectangle]

$\angle BAC = \angle ACD = 90°$ [Right angles]

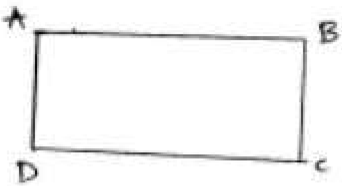
AC = AC = common sides

By S-A-S Congruency.

$\boxed{\Delta ACB \cong \Delta CAD}$

(6) Let the Rectangle be ABCD

Given AD : DC = 2 : 3



let AD = $2x$

DC = $3x$

Perimeter = $2(AD + DC)$

$= 2[2x + 3x]$

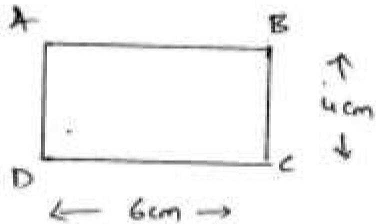
$= 10x$

But, given that perimeter is 20 cm

$\Rightarrow$ $10x = 20$

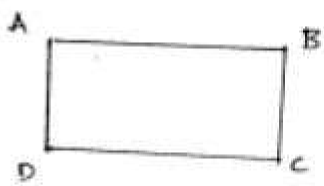
$\boxed{x = 2\text{ cm}}$

$\Rightarrow$ sides of Rectangle are 4 cm and 6 cm.



(7) Let the Rectangle be ABCD

Given length : Breadth = 5:4



let length = $5x$

& breadth = $4x$

Perimeter is given by $p = 2(\text{length} + \text{breadth})$

$$= 2[5x + 4x]$$

$$= 18x$$

But, given that perimeter is 90 cm

$$\Rightarrow 18x = 90$$

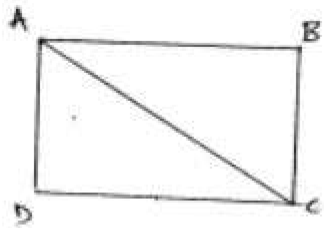
$$x = \frac{90}{18}$$

$$\boxed{x = 5}$$

Sides of Rectangle are given by $= 5x, 4x, 5x, 4x$

$= 25$ cm, 20 cm, 25 cm, 20 cm

(8) Given the Rectangle be ABCD



Given $AD = 5$ cm

$DC = 12$ cm

From Δ^{le} ADC,

$AD^2 + DC^2 = AC^2$ (Hypotenuse theorem)

$$AC = \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$AC = 13$ cm

length of the diagonal $= 13$ cm