SOIL MECHANICS

SOIL CLASSIFICATION

Introduction. In engineering terms, soil is generally softer, weaker and more weathered material overlying rock. All soils consists of solid particles assembled in a relatively loose packing. The voids between the particles may be filled completely with water (fully saturated soils) or may be partly filled with water and partly with air (partly saturated soils).

Formation of Soil. Soil is defined as a natural aggregate of mineral grains, with or without organic constitutents, that can be separated by gentle mechanical means such as agitation in water. Whereas rock is considered to be a natural aggregate of mineral grains connected by strong and permanent cohesive forces. Weathering of the rock decreases the cohesive forces binding the material grains and leads to the disintegration of bigger masses to smaller particles. Soils are formed by the process of weathering of the parent rock. The weathering of the rocks might be mechanical (disintegration) and/or chemical (decomposition).

The disintegrated or weathered material may either be found deposited at its own place of origin, or may get transported by agents like water, wind, ice, etc., before deposition. Moreover depending upon whether the sediments are transported by water, ice or wind, the soils are called as alluvial, glacial, or aeolin, respectively.

Three stages involved in the formation of transported soils are :

(*i*) Weathering

(*ii*) Transportation

(*iii*) Deposition of weathered material

Types of Soils :

(*a*) On the basis of origin, types of soils are

(*i*) Residual Soil : A soil that is formed by weathering of the parent rock and still occupies the position of the rock from which it has been formed, is called a residual soil.

(*ii*) Transported Soil : Any soil that has been transported from its place of origin by wind, water, ice or some other agency, and has been re-deposited, is called a transported soil. Transported soils are classified as follows :

(*a*) Alluvial deposits

(*b*) Lacustrine deposits

(*c*) Marine deposits

(*d*) Aeolin deposits; and

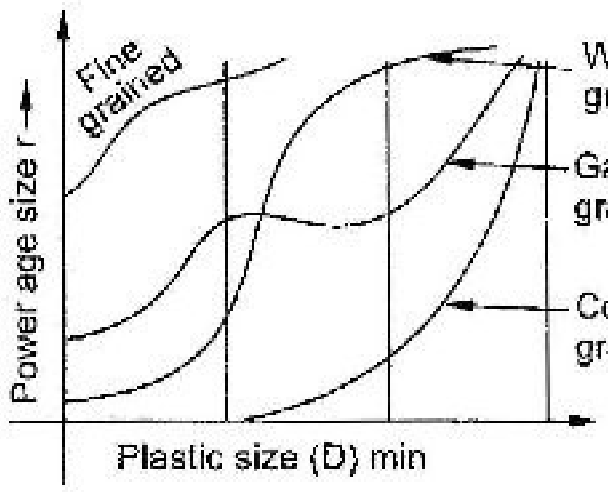
(*e*) Glacial deposits

CLASSIFICATION OF SOIL ON THE BASIS OF THEIR UTILITY

S. No.	Criteria	Utility
(*i*)	Origin	Provides useful information, but too complicated to assess.
(*ii*)	Colour	Knowledge not sufficient to indicate probable engineering behaviour
(*iii*)	Smell	Knowledge not sufficient to indicate probable engineering behaviour
(*iv*)	Porosity	Knowledge not sufficient to indicate probable engineering behaviour
(*v*)	Water content	Knowledge not sufficient to indicate probable engineering behaviour
(*vi*)	Sulphate content	Of little relevance to soil engineer
(*vii*)	Ability to sustain plant life	Of little relevance to soil engineer
(*viii*)	Strength	Too complicated to determine, and once known no need to classify
(*ix*)	Permeability	Too complicated to determine, and once known no need to classify
(*x*)	Compressibility	Too complicated to determine, and once known no need to classify
(*xi*)	Size of particles	Useful criteria for classification
(*xii*)	Stickiness or plasticity	Useful criteria for classification

CLASSIFICATION ON THE BASIS OF GRAIN SIZE

(*i*) ISI Classification : Soil particles below 0.002 mm are categorized as clay-size; those between 0.002 to 0.075 mm as silt-size; those between 0.07 to 4.75 mm as sand-size; and between 4.75 to 80 mm as gravel size and so on.



(*ii*) Coefficient of Uniformity and Coefficient of Curvature : The particle size distribution of a soil is presented as a curve on a semi-logarithmic plot, the ordinate being the percentage by weight of particles smaller than the size given by abscissa. The flatter the distribution curve, the larger the range of particles sizes in the soil. The steeper the curve the smaller the size range. A coarse-grained soil is described as well graded if there is no excess of particles in any size range if no intermediate sizes are lacking. In general a well graded soil is represented by a smooth concave distribution curve. A coarse graind soil is described as poorly graded.

(*a*) If a high proportion of the particles have sizes within narrow limits (it is called a uniform soil).

(*b*) If particles of both large and small sizes are present with a relatively low proportion of particles of intermediate sizes (it is called gap-graded soil). Particle size is represented on a logarithmic scale so that two soils having the same degree of uniformity are represented by curve of the same shape regardless of their position on the particle size distribution. The particle size corresponding to any specified value on the "percentage smaller" scale can be read from the particle size distribution curve. The size such that 10% of the particles are smaller than that size is denoted by D_{10} is defined as the effective size. The general slope and shape of the distribution curve can be described by means of the coefficient of uniformity (Cu) and the coefficient of curvature (Cc) defined as follows :

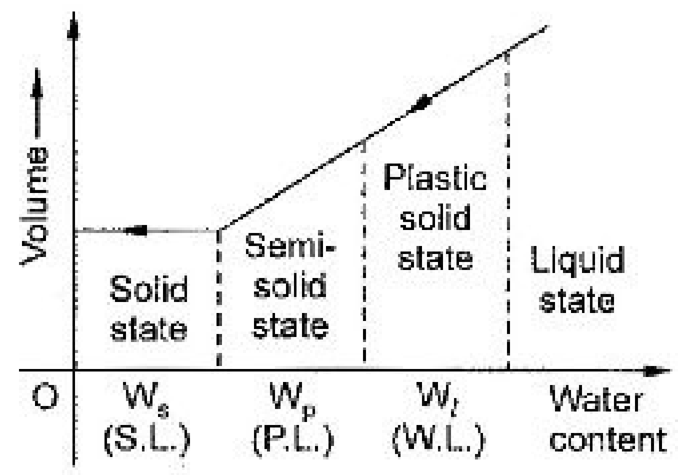
$$Cu = \text{Coefficient of uniformity} = \frac{D_{60}}{D_{10}}$$

$$C_c = \text{Coefficient of curvature} = \frac{D_{30}^2}{D_{60} \cdot D_{10}}$$

A uniformly graded soil will have its coefficient of uniformity (C_u) of less than 2.0. Sands with a value of C_u of 6 or more are well graded. Grave's with a value of C_u of 4 or more are well graded. A well graded soil will have its coefficient of curvature (C_c) lies between 1 and 3.

CLASSIFICATION BASED ON PLASTICITY

Consistency : The consistency of clay and other cohesive soils is greatly influenced by the water content of the soil. The term consistency denotes the degree of stiffness of soil as soft, medium, firm, stiff.



If a clay slurry is allowed to dry slowly, it will pass from liquid state to plastic state and semi-solid and finally it reaches the solid state.

Atterberg divided the range from liquid to solid state in four states of consistency.

(*i*) liquid state

(*ii*) plastic state

(*iii*) semi-solid state; and

(*iv*) solid state.

These limits are known as consistency limits or Atterberg limits.

Liquid Limit : The lowest water content at which the soil is in a liquid state is called the liquid limit (LL). At *liquid limit*, the clay is practically like a liquid but possesses a small shearing strength.

Plastic Limit : Plastic limit is the water content below which the soil stops behaving as a plastic material. The plastic limit is determined by rolling a part of soil into thread, when the thread begins to crumble at a diameter of 3.18 mm or $\frac{1''}{8'}$, the water content at this stage is the *plastic limit*.

Shrinkage Limit : The maximum water content at which a decrease in water content does not cause a reduction in the volume of soil mass. It is the minimum water content at which a soil is still in satuated condition. The water content at this limit is 100%.

Plasticity Index, $I_p = w_L - w_P$

Consistency Index, $I_c = \frac{w_1 - w}{I_p} \times 100$

- I_c indicates the. consistancy (firmness) of a soil.

Liquidity Index, $I_L = \frac{w - w_p}{I_p} \times 100$

- I_L of soild indicates the nearness of its water content to its liquid limit.

Flow Index, $I_f = \frac{w_1 - w_2}{\log_{10} \frac{n_2}{n_1}}$

$$I_c + I_L = 1$$

where, w_L, w_p, w are moisture' contents at liquid limit, plastic limit and natural condition respectively.

w_1 = water content corresponding to n_1 blows.

w_2 = water content corresponding to n_2 blows.

Toughness, $I_t = \frac{I_P}{I_f}$

(It is a measure of the shearing strength of soil at the plastic limit)

Shrinkage Ratio (SR) : It is defined as the ratio of given volume change, expressed as percentage of dry volume to the corresponding change in water content above shrinkage limit.

$$SR = \frac{\frac{V_1 - V_2}{V_d} \times 100}{(w_1 - w_2)}$$

where V_1 = volume of soil mass at water content w_1

V_2 = volume of soil mass at water content w_2

V_d = volume of dry soil mass

Again $w_1 - w_2 = \frac{(V_1 - V_2)\rho_w}{\mu_s}$

$\therefore$ $SR = \frac{M_s}{V_d \rho_w} = \frac{\mu_d}{\mu_s} = G_m$

Thus shrinkage ratio is equal to the mass gravity of the soil in-dry state.

Volumetric Shrinkage (VS) : Volumetric shrinkage is defined as the decrease in volume expressed as a percentage of the dry volume of the soil mass, when the moisture content is decreased from given percentage to shrinkage limit.

$$VS = \frac{V_1 - V_d}{V_d} \times 100$$

$$VS = SR\,(w_1 - w_2)$$

Linear Shrinkage (LS). It is defined as the change in length divided by the initial length, when the water content is reduced to the shrinkage limit.

$$LS = \left(\frac{\text{Initial length} - \text{Final length}}{\text{Initial length}}\right) \times 100$$

Sensitivity (s) : It is defined as the ratio between the unconfined compressive strength of undisturbed clay to the unconfined compressive strength of remoulded clay at the same water content.

$$S = \frac{\text{UCS of undisturbed sample}}{\text{UCS of remoulded sample}}$$

at the same moisture content

Thixotropy : A regaining of portion of strength lost due to remoulding with respect to time is known as thixotropy. It is mainly due to a gradual reorientation of molecules of water in the aisborbed water layer and due to re-establishment of chemical equilibrium.

Activity : The plasticity of clays arises due to the interaction between clay fraction and the water. The ratio of plasticity index to the clay content is called the activity of soil.

$$\text{Activity (A)} = \frac{\text{Plasticity Index, } I_p}{\%\text{ finer than } 2\mu \text{ size}}$$

This is a measure of the water holding capacity of clayey soils. The change in the volume of clayey soil during swelling on shrinkage depend upon the activity.

Activity values are given in the following table.

Activity	type of soil
A < 0.75	Inactive
A = 0.75 to 1.25	Normal
A > 1.25	Active

Example. *A soil has a liquid limit of 28% and a flow index of 13%. If the plastic limit is 18%. If the water content of the soil in its natural condition in the field is 20%. Find the liquidity index, and consistency index*

Solution : Liquidity index,

$$I_l = \frac{w - w_p}{I_p} \times 100$$

$$= \frac{0.20 - 0.18}{0.10} \times 100 = 20\%$$

Consistency index, $I_c = \frac{w_l - w}{I_p} \times 100$

$$= \frac{0.28 - 0.20}{0.10} \times 100 = 80\%$$

Mineral Contents of Soil

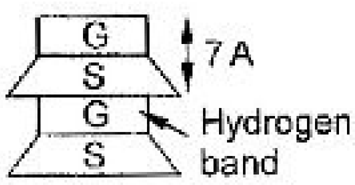
Rock fragments can be reduced by mechanical means to a limiting size of about 0.002 mm so that a soil containing particles above this size has a mineral content similar to the parent rock from which it was created.

For the production of particles smaller than 0.002 mm some form of chemical action is generally necessary before breakdown can be achieved. Such particles, although having a chemical content similar to the parent rock, have a different crystalline structure and are known as clay particles.

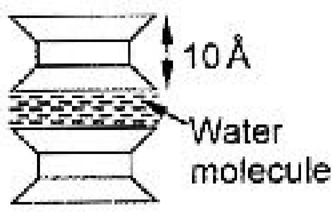
Classes of Clay Minerals

The minerals comprising a clay are invariably the result of the chemical weathering of rock particles and are hydrates of aluminum, iron and magnesium silicate generally combined in such a manner as to create sheet-like structures only a few molecules thick. These sheets are built from two basic units the tetrahedral unit silica and the octahedral unit of the hydroxide of aluminum, iron and magnesium. The main dimensions of clay particles is usually less than 0.002 mm and the different types of minerals have been created from the manner in which these structures were stacked together. The principal clay minerals are

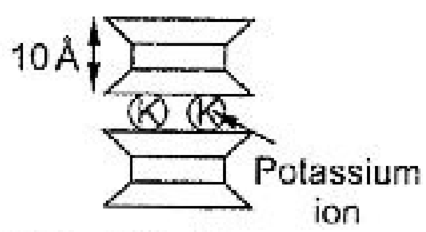
(*i*) Kaolinite



(*ii*) Montmorillonite



(*iii*) Illite



Isomorphous Substitution

The process of replacing one atom in a basic unit by another atom is called isomorphous substitution, e.g. one silicon atom in a tetrahedral unit may be substituted by aluminium atom.

Three Phase System

A soil mass consists of solid soil particles, containing void spaces between them. These voids may be filled either with air, or water, or both. When the void space is filled with either water or air alone, the soil mass will consists of only two phases, i.e. solid and water (liquid), or solid and air (gas) respectively. The three phase system can be easily represented by the Schematic diagram as follows. At saturated condition, the soil may be considered as two phase system.

Total volume of soil mass,

$$V = (V_a + V_w)V_s$$

where, V = total volume of soil

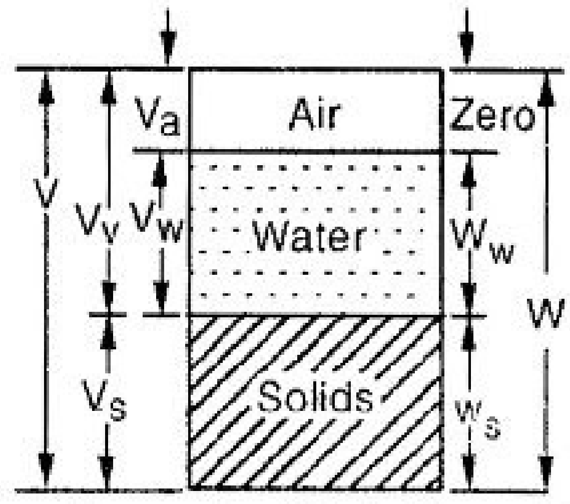
V_a = volume of air in the voids

V_w = volume of water in the voids

and V_s = volume of solids

But $V_v = V_a + V_w$

$\therefore$ $V = V_a + V_s$



where, V_v = volume of voids in the soil

V_a = volume of air in the soil

V_w = volume of water in the soil

Void ratio (*e*). Void ratio of a soil is defined as the ratio of the total volume of voids to the volume of solids, i.e.

$$e = \frac{V_v}{V_s} = \frac{n}{1-n}$$

Porosity (n). Porosity of a soil is defined as the ratio of the total volume of voids to the total volume of soil, i.e.

$$n = \frac{V_v}{V}$$

Also, $$n = \frac{e}{1+e}$$

Degree of Saturation (S_r) : It is the ratio of volume of water (V_w) to the volume of voids (V_v) in the soil mass.

$$S_r = \frac{V_w}{V_v}$$

For fully saturated soil mass, $S_r = 1$

For perfectly dry soil mass, $S_r = 0$

Air content : It is the ratio of volume of air (V_a) to the volume of voids, (V_v)

$$a_0 = \frac{V_a}{V_v} = 1 - S$$

Percentage Air voids (η_a): It is the volume of air (V_a) to the total volume of the soil (V).

$$\eta_a = \frac{V_a}{V} \times 100 = \eta_{ac}$$

Water content or Moisture content (w) : It is the ratio of weight of water to the weight dry soil solids in a given soil sample.

$$W = \frac{W_w}{W_s} \times 100$$

Density of soil mass (Unit weight)

The density of soil mass may be of the following types.

(*i*) **Bulk density (γ) :** The total weight (W) of a given soil mass per unit of its total volume (V), is called the bulk density, i.e.

$$\gamma = \frac{W}{V}$$

(*ii*) **Dry density (γ_d) :** The weight of soil solids of a given soil mass per unit of its total volume (in moist condition) is called dry density, i.e.

$$\gamma_d = \frac{W_d}{V}$$

(*iii*) **Density of solids (γ_s) :** The weight of soil solids (W_d) per unit volume of solid (V_s) in given soil mass, is called density of solids, i.e.

$$\gamma_s = \frac{W_d}{V_s}$$

(*iv*) **Saturated Density of soil mass (γ_{sat}) :** The total weight (W_{sat}) of a fully saturated soil mass per unit total volume is called the saturated density of soil mass.

i.e. $$\gamma_{sat} = \frac{W_{sat}}{V}$$

(*v*) **Submerged density of soil mass (γ_{sub}) :** The submerged weight of soil solids (Wrf) volume of the given soil mass or buoyant unit weight, per unit of its total volume, i.e.

$$\gamma_{sub} = \frac{(W_d)_{sub}}{V}$$

Mathematically, $\gamma_{sub} = \gamma_{sat} - \gamma_w$,

where γ_w is water density

Specific Gravity of Soil Solids (G) : The ratio of the weight of a given volume of soil solids and the weight of an equal volume of distilled water at given temperature (27°C) both weight being taken in air, is called specific gravity of soil solids, i.e.

$$G = \frac{\gamma_s}{\gamma_w}$$

Bulk Mass or Apparent Specific Gravity (G_m) : It is the ratio of bulk unit weight of the soil to the unit weight of water, i.e.

$$G_m = \frac{\gamma}{\gamma_w}$$

Functional Relationships

1.Relationship between *e*, G,*w*, and S_r :

Let e_w = water void ratio (for a fully saturated $e_w = e$)

$$\therefore \quad e = \frac{wG}{S_r}$$

For a fully saturated sample,

$$S_r = 1,$$

and $$w = w_{sat}$$

$$\therefore \quad e = w_{sat} = G$$

Shrinkage Limit

$$w_s = \frac{(V_2 - V_s)\rho_w}{\mu_s}$$

$$w = \left(\frac{V_2}{M_s} - \frac{V_s}{G\,\rho_w V_s}\right)\rho_w$$

$$= \left(\frac{\rho_w}{\rho_d} - \frac{1}{G}\right) \times 100$$

2. Relation between n_a, a_e and n :

$$n_a = \frac{V_a}{V} = n\,a_c,\ a_c = 1 - S_r$$

3. Relation between γ_w, γ_d' G and C :

$$\gamma_d = \frac{\gamma_s \cdot V_s}{V} = \frac{G \cdot \gamma_w}{1+e}$$

Also $$\gamma_d = (1-n)\,G \cdot w$$

5. Relation between γ_{sat} G and *e* :

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$$

Also $$\gamma_{sat} = G\gamma_w(1-n) + \gamma_w \cdot n$$

6. Relation between γ, G, *e* and S_r :

$$\gamma = \frac{(G + e \cdot S_r)\gamma_w}{1+e}$$

7. Relation between γ_{sub} G and *e* :

$$\gamma_{sub} = \frac{(G-1)\gamma_w}{1+e}$$

8. Relation between γ_d, γ and *w* :

$$\gamma_d = \frac{\gamma}{1+w}$$

9. Relation between γ_{sab}, γ_d and *n* :

$$\gamma_{sub} = \gamma_d - (1-n)\,\gamma_w$$

10. Relation between γ_{sat}, γ, γ_d and S_r :

$$\gamma = \gamma_d\, S_r\,[\gamma_{sat} - \gamma_d]$$

11. Relation between γ_d, G, *w* and S_r :

$$\gamma_d = \frac{G_w \cdot w}{1 + \frac{w \cdot G}{S_r}}$$

12. Relation between γ_d, G, *w* and n_a :

$$\gamma_d = \frac{(1-n_a)G \cdot \gamma_w}{1 + w \cdot G}$$

Relative density D_r

It is the ratio of the difference in void ratio of the soil at its loosest state (e_{max}) and its natural state (e) to the difference in void ratio at its loosest (e_{max}) and densest state (e_{min}).

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \times 100$$

$$= \frac{\gamma_{mass}}{\gamma_d}\left[\frac{\gamma_d - \gamma_{\min}}{\gamma_{\max} - \gamma_{\min}}\right]$$

$$e_{max} = \frac{\gamma_w G - \gamma_{\max} d}{\gamma_{\max}, d}$$

$$e = \frac{G \cdot \gamma_w - \gamma_d}{\gamma_d}$$

$$e_{min} = \frac{\gamma_w \cdot G - \gamma_{\min} d}{\gamma_{\min} d}$$

where, $\gamma_{max}\, d$ = dry unit weight of soil when the void ratio is e_{max}

$\gamma_{min,}\, d$ = dry unit weight of soil when the void ratio is e_{min}

γ_d = dry unit weight of soil when the void ratio is e

Example. *Laboratory tests on a sample of soil yields the following results*

Natural moisture content, $w_n = 30\%$,

Plastic limit $P.L. = 32\%$,

Liquid limit, $L.L. = 60\%$,

Flow index, $F.I. = 27\%$,

then find Plasticity index and Liquidity index.

Solution :

Plasticity index, $P.I. = L.L - P.L. = 60 - 32 = \mathbf{28\%}$

Liquidity index, $L.L. = \frac{w_n - P.L.}{L.L. - P.L.} = \frac{w_n - P.L.}{P.I.}$

$$= \frac{0.3 - 0.32}{0.28} = \mathbf{-0.0714}$$

Example. *In a bulk density determination a sample of clay with a mass of 683 g was coated with paraffin wax. The combined mass of the clay and the wax was 690.6 g. The volume of the clay and the wax was found by immersion in water, to be 350 ml.*

The sample was then broken open and moisture content and particle specific gravity tests gave respectively 17 per cent and 2.73. The specific gravity of the wax was 0.98. Then find the degree of saturation

Solution : Given,

Mass of soil = 683 g

Mass of wax = 690.6 – 683 = 7.6 g

$\therefore$ Volume of wax $= \frac{7.6}{0.89} = 8.4$ ml

and Volume of soil = 350 – 8.4 = 341.6 ml.

$$\rho_b = \frac{683}{341.6} = 2 \text{ mg/ml} = 2 \text{ mg/m}^3$$

$$\gamma_b = 2 \times 9.81 = 19.6 \text{ kN/m}^3$$

$$\rho_d = \frac{2}{1.17} \text{ mg/m}^3$$

From $\frac{\rho_w \cdot G_s}{1+e} = 1.71$

$$e = \frac{2.73 \times 1.17}{1.71} - 1 = 0.596$$

Now $\rho_b = 2 = \frac{w \cdot (G_s + e \cdot S_r)}{1+e}$

or $1.596 \times 2 = 2.73 + 0.596 \times S_r$

$\therefore$ $S_r = 0.77 = 77\%$

Example. *A saturated soil sample has bulk density of 2 gm / cc with 7% water content. Calculate the amount of water required to be added to one cubic metre of soil to raise the water content to 15% while the voids ratio remains constant. What is then the degree of saturation. [G = 2.67.] ?*

Solution : Bulk density,

$$\rho = 2.0 \text{ gm/cc} [1 \text{ gm/cc} = 9.81 \text{ kN/m}^3]$$

$$\gamma_{\text{bulk}} = 2 \times 9.81 = 19.62 \text{ kN/m}^3$$

$$\gamma_{\text{dry}} = \frac{\gamma_{\text{bulk}}}{1+\omega} = \frac{19.62}{1+0.07}$$

$$= 18.336 \text{ kN/m}^3$$

$$e = \left(\frac{G\gamma_w}{\gamma_d}\right) - 1$$

$$= \left(\frac{2.67 \times 9.81}{18.336}\right) - 1 = 0.428$$

Degree of saturation at 7% water content

$$= \frac{\omega G}{e} = \frac{0.07 \times 2.67}{0.428}$$

$$= 0.436 = 43.6\%$$

Degree of saturation at 15% water content

$$= \frac{\omega G}{e} = \frac{0.15 \times 2.67}{0.428}$$

$$= 0.935 = 93.5\%$$

For 1 m³ of soil, $V = 1 \text{ m}^3$

$\therefore$ $W_d = \gamma_d V = 18.336 \times 1 = 18.336$ kN

and $W_w = 0.07 \times 18.336 = 1.2835$ kN

$$V_\omega = \frac{W_\omega}{\gamma_\omega} = \frac{1.2835}{9.81} = 0.131 \text{ m}^3$$

when $\omega = 15\%$, $W_{\omega} = 0.15 \times 18.336 = 2.7504$ kN

and $$V_{\omega} = \frac{W_{\omega}}{\gamma_{\omega}} = \frac{2.7504}{9.81} = 0.280 \text{ m}^3$$

Hence additional water required to raise the water content from 6% to 15%

$$= 0.280 - 0.131$$
$$= 0.149\text{m}^3 = \textbf{149 litres}$$

Example. *A sample of saturated soil has a water content of 25 per cent and a bulk unit weight of 18 kN/ m³. Then find dry unit weight*

Solution :

From $$\gamma_{sat} = \frac{G\gamma_{\omega}}{1+wG}(1+w)$$

$$18 = \frac{G \times 10(1+0.25)}{1+(0.25 \times G)}$$

or $$G = 2.25$$

From $$S = \frac{wG}{e},$$

taking $$S = 1$$

$$e = wG = 0.25 \times 2.25 = 0.56$$

$$\therefore \quad \gamma_d = \frac{G\gamma_{\omega}}{1+e} = \frac{2.25 \times 10}{1+0.56} = \mathbf{14.42\ kN/m^3}$$

Example. *A natural soil deposit has a bulk unit weight of 18 kN/m³ and water content of 5 per cent. Calculate the amount of water required to be added to 1 cubic metre of soil to raise the water content to 15 per cent. Assume the void ratio to remain constant. What will then be the degree of saturation ? Take G = 2.7.*

Solution :

Given, $$\gamma = 18 \text{ kN/m}^3$$

and $$w = 5\%$$

$$\gamma_d = \frac{\gamma}{1+w} = \frac{18}{1+0.05} = 17.14 \text{ kN/m}^3$$

When $$w = 5\%,$$

then $$w = 0.05 = \frac{W_w}{W_d}$$

For one cubic meter of soil,

$$V = 1 \text{ m}$$

$$\therefore \quad W_d = \gamma_d \cdot V = 17.14 \times 1 = 17.14 \text{ kN}$$

$$W_w = 0.05 \times 17.14 = 0.86 \text{ kN}$$

and $$V_w = \frac{W_w}{\gamma_w} = \frac{0.86}{9.81} = 0.088 \text{ m}^3$$

When $$w = 15\%,$$

then $$W_w = w\,W_d = 0.15 \times 17.14 = 2.57 \text{ kN}$$

$$\therefore \quad V_w = \frac{W_w}{\gamma_w} = \frac{2.57}{9.81} = 0.262 \text{ m}^3$$

Hence additional water required to raise the water content from 5% to 15 %

$$= 0.262 - 0.088$$
$$= 0.174 \text{ m}^3 = 174 \text{ litres}$$

Void ratio, $$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.6 \times 9.81}{17.14} - 1 = 0.49$$

After the water has been passed, e remains the same

$$\therefore \quad S_r = \frac{wG}{e} = \frac{0.15 \times 2.7}{0.49} = 0.826 = \mathbf{82.6\%}$$

Example. *A fully saturated clay sample weighs 1.30 gm and has a volume of 65 cm2. The clay weighs 105 gm after oven drying. Assuming that the volume does not change during drying. Then find*

(i) Specific gravity of soil solids

(ii) Void ratio

(iii) Porosity

(iv) Dry density

Solution :

Given, $S_r = 1$; W= 130 gms; $W_d = 105$ gms; V = 64 cm³

$$W_w = 130 - 105 = 25 \text{ gms.}$$

Water content $$(w) = \frac{W_w}{W_d} = \frac{25}{102} = 24\%$$

$$\gamma = \frac{W}{V} = \frac{130}{64} = 2.30 \text{ gm/cm}^3$$

(i) $$G = \frac{W_d}{V_s} = \frac{105}{40} = \mathbf{2.62}$$

(ii) $$G = \frac{V_r}{V_s} = \frac{24}{40} = \mathbf{0.60}$$

(iii) $$n = \frac{e}{1+e} = \frac{6}{1+6} = \mathbf{0.375 \text{ or } 37.5\%}$$

(iv) $$\gamma_d = \frac{\gamma}{1+w} = \frac{130}{64(1+.24)} = \mathbf{164\ gm/cm^3}$$

Example. *The consistency limits of a soil sample are : Liquid limit 52%, Plastic limit 32%, Shrinkage limit 17%*

If the specimen of this soil shrinks from a volume of 10 cm³ at a liquid limit to 6.01 cm³ at the shrinkage limit, find the specific gravity of solids

Solution :

Given, $W_1 = 52\%$, $V_L = 10$ cm³,
$W_p = 32\%$,
$W_s = 17\%$, $V_t = 6.01$ cm³

Difference in volume in

$$a, b = 10 - 6.01 = 3.99 \text{ cm}^3$$

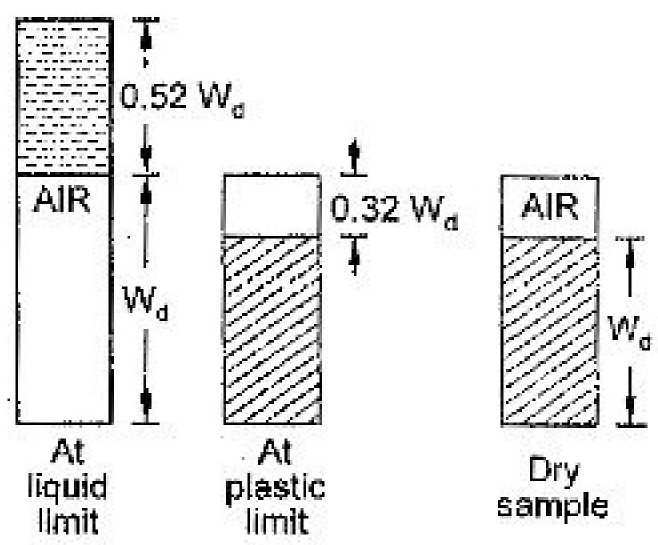
Difference in weight of water in

$$a_f b = 3.99 \text{ gms.}$$

But from the figure, this difference = (0.52 – 0.18) W_d

$\therefore$ $(0.52 - 0.18)\, W_d = 3.99$

or $$W_d = \frac{3.99}{0.34} = 11.73 \text{ gms.}$$



(a) (b) (c)

Weight of water in (*b*) = 0.17 × 11.73 = 1.99

Volume of water (V_d) in (*b*) = 1.99 cm^3

Volume of solid in (*b*) = 6.01 – 1.99 = 4.02 cm^3

$$\gamma_d = \frac{W_d}{V_d} = \frac{11.73}{4.2} = 2.91 \text{ gm/cm}^3$$

$$G = \frac{\gamma}{\gamma_w} = 2.91 \text{ gm/cm}^3$$

Example. *The decrease in volume to be expected in the sample when moisture content is reduced by evaporation to 20 p.c. Find its volume at L.L. is 10 c.c.*

Solution : LL = 66%;

V_1 = 10 cc at m.c. 20%;

G = 2.7;

Shrinkage limit (known) = 25%

From $$w_s = \left(\frac{\gamma_w}{\gamma_d} - \frac{1}{G}\right) \times 100,$$

$$G = \frac{1}{\dfrac{\gamma_w}{\gamma_d} - \dfrac{w_s}{100}} = \frac{1}{\dfrac{1}{\gamma_d} - \dfrac{0.25}{100}} = 2.7$$

or $$2.7\left[\frac{1}{\gamma_d} - \frac{25}{100}\right] = 1$$

or $$\frac{1}{\gamma_d} = \frac{1}{2.7} + \frac{1}{4} = \frac{16.7}{10.8}$$

or $$\gamma_d = 1.611 \text{ gm/cc}$$

$\therefore$ Shrinkage ratio, $$S = \frac{\gamma_d}{\gamma_w} = \frac{1.611}{1} = 1.611$$

Volumetric shrinkage,

$$\begin{aligned} VS &= (W_1 - W_2)\, S_r \\ &= (60 - 20) \times 1.611 \\ &= 64.44\% \end{aligned}$$

Volumetric change expected = $\dfrac{V_1 - V_2}{V_2} \times 100$

or $$\frac{10 - V_2}{V_2} \times 100 = 64.44$$

or $$10 - V_2 = 0.644\, V_2$$

$\therefore$ $$V_2 = \frac{10}{1.664} = \mathbf{6.082\ cc}$$

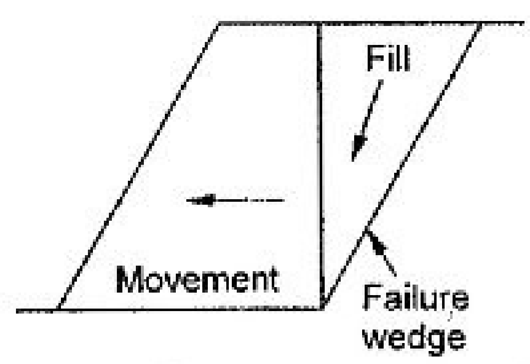
EARTH PRESSURE

Soil exerts a lateral pressure on any structure with which it is in contact. In the design of retailing walls, sheet piles or other earth retaining structures, it is necessary to determine; the lateral earth pressure exerted by the retained mass of soil. The magnitude of the lateral earth pressure depends upon the displacement of retaining structure, nature of soil and boundary conditions.

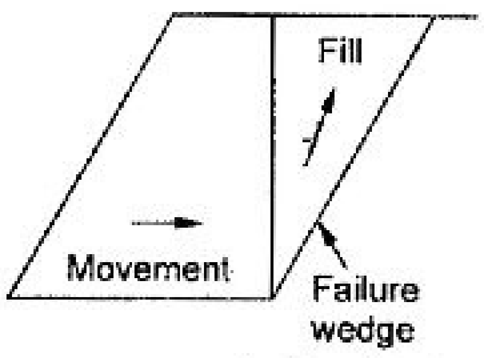
If the position of the back fill lies above a horizontal plane at the elevation of top of the structure, it is *called surcharge*. The inclination of the surcharge to the horizontal is called surcharge angle.

Based on the deformation of retaining wall, lateral pressure can be one of the following types.

(*i*) **Active Earth Pressure.** Due to excessive pressure of the retained soil, the retaining wall tends to move away from the back fill. Consequently a certain portion of the back fill located immediately behind the retaining wall, gets separated from the rest of the soil mass and hence the earth pressure on the retaining wall decreases. The wedged shaped portion of the back fill tending to move with the wall, is called a failure wedge. The retaining wall is kept in equilibrium by the resisting force developed due to shear strength of the soil along the plane of the failure wedge in a direction away from the retaining wall. There is a limit with . which the retaining wall may move away from the back fill, thereby limiting the pressure. The minimum pressure exerted by the soil on the retaining wall, is called Active Earth Pressure.

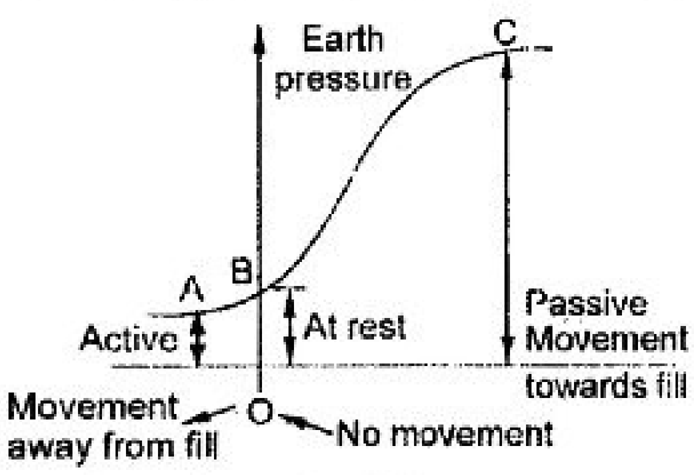


(*ii*) **Passive Earth Pressure.** Whenever the retaining wall moves towards the back fill due to any natural cause, the earth pressure increases because the retaining soil gets compressed and the resulting shearing strength develops along the plane of the failure wedge in the direction towards retaining wall. The pressure reaches a maximum limit when the shearing resistance of the soil has been fully mobilised. The maximum earth pressure due to maximum shear stress on the retaining wall, is called Passive Earth Pressure.



(*iii*) Earth Pressure at Rest. As active earth pressure is accompanied by the movement of the retaining wall away from the back fill and passive earth pressure is accompanied by the movement of the retaining wall towards the back fill, thus, there occurs an intermediate situation when the retaining wall does not move due to earth pressure but remains perfectly stationary. The pressure which develops due to back fill at zero movement, is called earth pressure at rest. Its value is higher than limiting active pressure but less than the passive pressure.

Rankine's Theory of Active Earth Pressure



The theory is based on the following assumptions.

(*i*) The soil mass is homogeneous, dry, cohesionless and semi-infinite.

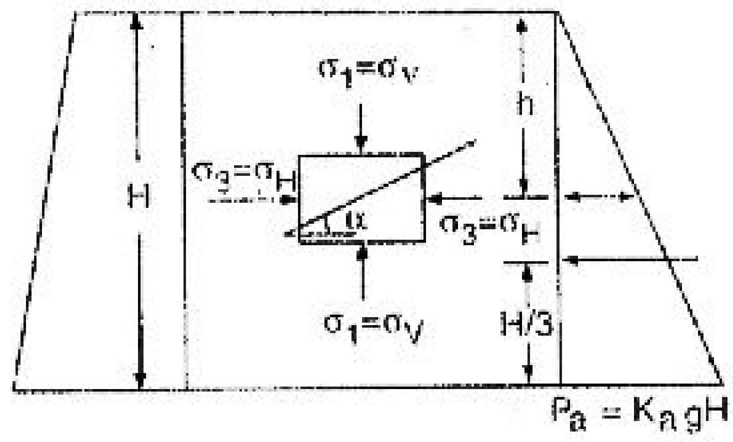
(*ii*) The ground surface is a plane which may be horizontal or inclined.

(*iii*) The back of the wall is smooth and vertical.

(*iv*) The wall yields about the base.

Various Cases of Cohesionless Back Fill

1. Dry or moist back fill with no surcharge : Consider an element of the soil mass at a depth h below the ground level. When the wall moves away from the back fill, the active state of plastic equilibrium is established. In this state, the vertical pressure σ_v is the major principal stress which is equal to σ_1. The horizontal pressure (σ_h) is minor principal stress which is equal to σ_3.



In plastic equilibrium,

$$\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2c\tan\left(45° + \frac{\phi}{2}\right)$$

where ϕ = angle of shearing resistance

c = cohesion

= 0, (soil being cohesionless)

$$\therefore \quad \sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right)$$

or

$$\frac{\sigma_3}{\sigma_1} = \frac{\sigma_h}{\sigma_v} = \frac{1}{\tan^2\left(45^2 + \frac{\phi}{2}\right)}$$

$$= \cot^2\left(45° + \frac{\phi}{2}\right)$$

But $\sigma_h = P_a$ = lateral earth pressure

and $\sigma_v = \gamma H$ = vertical earth pressure on the element

$$\therefore \quad P_a = \gamma h \cdot \cot^2\left(45° + \frac{\phi}{2}\right) = K_a \gamma H$$

where K_a = coefficient of active earth pressure

$$= \cot^2\left(45° + \frac{\phi}{2}\right)$$

$$= \left[\frac{1 - \frac{\tan\phi}{2}}{1 + \frac{\tan\phi}{2}}\right]^2 = \left[\frac{\frac{\cos\phi}{2} - \frac{\sin\phi}{2}}{\frac{\cos\phi}{2} + \frac{\sin\phi}{2}}\right]^2$$

$$= \frac{\frac{\cos^2\phi}{2} + \frac{\sin^2\phi}{2} - \frac{2\sin\phi}{2}\cdot\frac{\cos\phi}{2}}{\frac{\cos^2\phi}{2} + \frac{\sin^2\phi}{2} + \frac{2\sin\phi}{2}\cdot\frac{\cos\phi}{2}}$$

$$= \frac{\frac{1-2\sin\phi}{2}\cdot\frac{\cos\phi}{2}}{\frac{1+2\sin\phi}{2}\cdot\frac{\cos\phi}{2}} = \left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

Active earth pressure on the top of the wall,

$$P_{ah} = K_a \cdot \gamma \cdot H$$

and Active earth pressure on the top of the wall,

$$P_{av} = 0 \; (H = 0)$$

Hence, the pressure distribution is triangular, varying from zero at the top to $K_a \cdot \gamma$ H at the bottom.

∴ Resultant active earth pressure,

$$P_a = \frac{1}{2}K_a\gamma H^2,$$

acting at $\frac{H}{3}$ above the base of the wall.

2. **Submerged Backfill :** In this case, the sand fill behind the retaining wall is saturated with water. The lateral pressure is made up of two components.

   (*i*) Lateral pressure due to submerged weight of the soil, and

   (*ii*) Lateral pressure due to water.

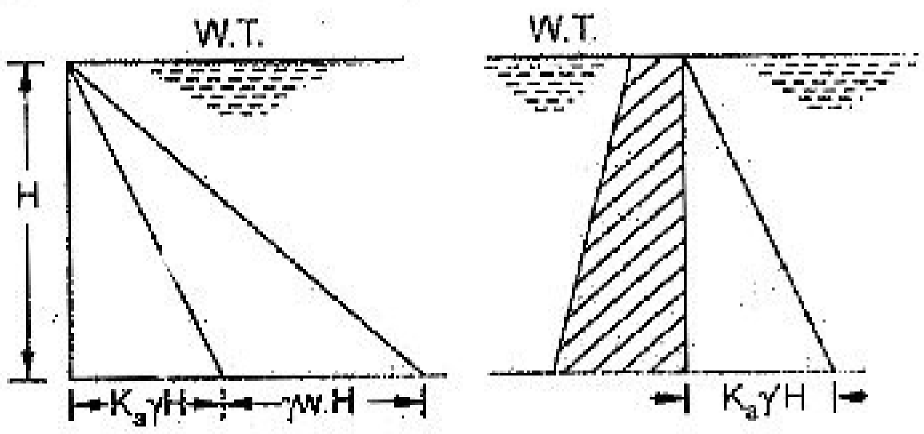
   Thus at any depth z below the top,

$$P_a = K_a \gamma' z + \gamma_w z$$

   Pressure at the base of the retaining wall (z = H) is given by,

$$P_a = K_a \gamma' H + \gamma_w H$$

   (*a*) If the free water stands on both sides of the wall, as shown in figure the water pressure need not be considered, and the net lateral pressure is given by,



$$P_a = K_a \gamma H$$

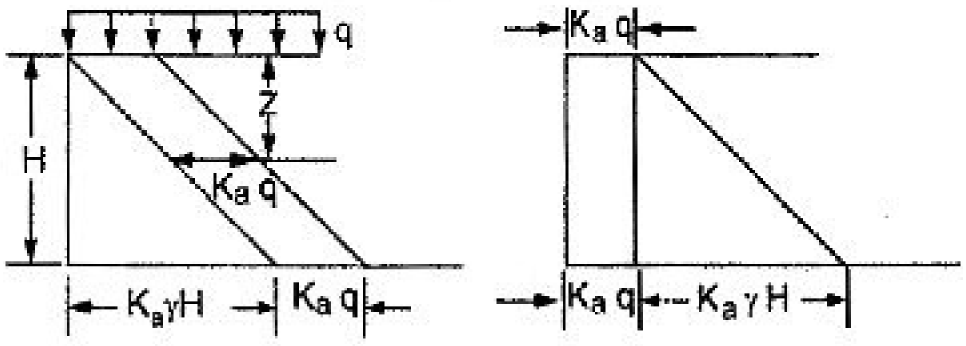
   (*b*) If the backfill is partly submerged, i.e. the backfill moist to a depth H_2 below the ground level, and then it is submerged, the lateral pressure intensity at the base of the wall is given by

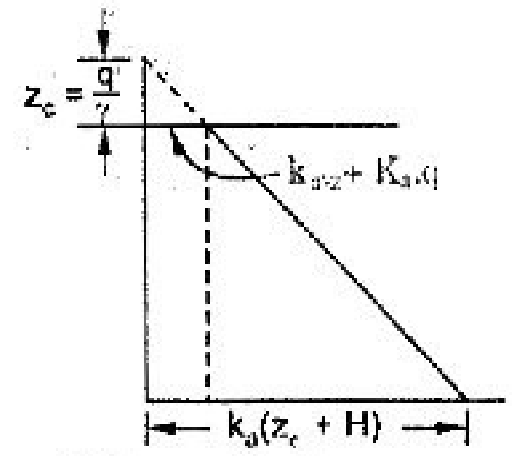
$$P_a = K_a \gamma H' + K_a \gamma_1 H_2 + \gamma_w H_2$$

3. **Backfill with Uniform Surcharge :** If the backfill is horizontal and carries a surcharge of uniform intensity q per unit area, the vertical pressure increment, at any depth z, will increase by q.

   The increase in the lateral pressure due to this will be $K_a q$. Hence the lateral pressure at any depth z is given by

$$P_a = K_a \gamma z + K_a q$$





A the base of the wall, the pressure intensity,

$$P_a = K_a \gamma H + K_a q$$

4. **Backfill with a sloping surface :** In this case by Rankine's theory, an additional assumption that the vertical and lateral stresses are conjugate is made.

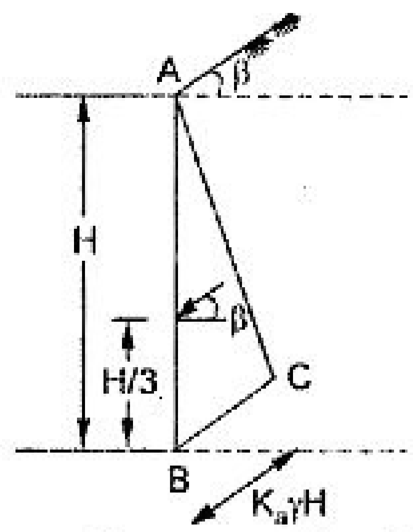


Fig. Lateral pressure distribution for slopping curvature

Earth pressure,

$$P_a = \gamma z \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos\beta - \cos^2\phi}}$$

where, $$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

When β = 0 (i.e. horizontal ground surface), then

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

The pressure distribution given by the above equation will be triangular. The total active pressure P_a for the wall of height H is given by,

$$P_a = \frac{1}{2}K_a\gamma H^2$$

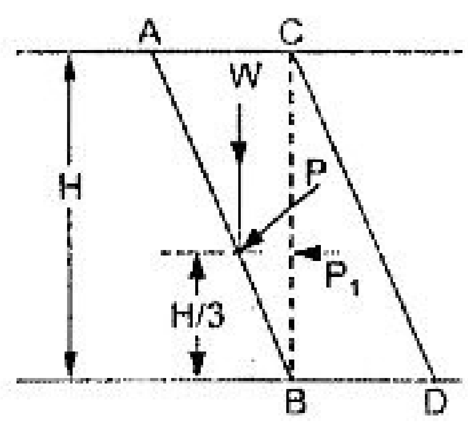


Fig. Inclined Backfill

The resultant acts at H/3 above the base in direction parallel to surface, as shown in the figure.

If the backfill is submerged, the lateral pressure due to submerged weight of the soil will act at β with horizontal, while the lateral pressure due to water will act normal to the wall.

5. Inclined back and surcharge : Figure below shows a retaining wall with an inclined back supporting a backfill with horizontal ground surface. The total active pressure P_1 is first calculated on the vertical plane BC passing through the heel BC. The total pressure P is the resultant of the horizontal pressure P_1 and the weight of the wedge ABC.

$$P = \sqrt{P_1^2 + W^2}$$

where, $$P_1 = \frac{1}{2} K_a \gamma H^2$$

RANKINE'S THEORY OF PASSIVE EARTH PRESSURE

(a) Cohesionless backfill : In the case of passive state of plastic equilibrium, the lateral pressure is the major principal stress while the vertical pressure is the minor principal stress. Thus,

$$\sigma_h = p_p = \sigma_1$$

and $$\sigma_v = \sigma_3 = \gamma z$$

Substituting this in the principal stress relationship, (i.e. $\sigma_1 = \sigma_3 \tan^2 \alpha$) we get

$$p_p = \gamma z \tan^2 \alpha = K_p \gamma z$$

where p_p = passive earth pressure intensity

K_p = Rankine's coefficient of passive earth pressure

$$= \tan^2 \alpha = N_\phi$$

$$= \frac{1 + \sin\phi}{1 - \sin\phi} = \frac{1}{K_a}$$

The distribution of passive earth presure is triangular, with a maximum value of $K_p \gamma H$ at the base of the retaining wall of height H. The total pressure for a depth H is given by

$$p_p = \int_o^H K_p \gamma z \cdot dz$$

$$= K_p \cdot \frac{1}{2} \gamma H^2$$

In a uniform surcharge intensity q per unit acts over the surface of the backfill, the increase in the passive pressure will be equal to $K_p\, q$. The passive pressure intensity at a depth z is then given by

$$p_p = K_p (\gamma z + q)$$

If the backfill is having its top surface inclined at an angle β, the passive pressure is given by

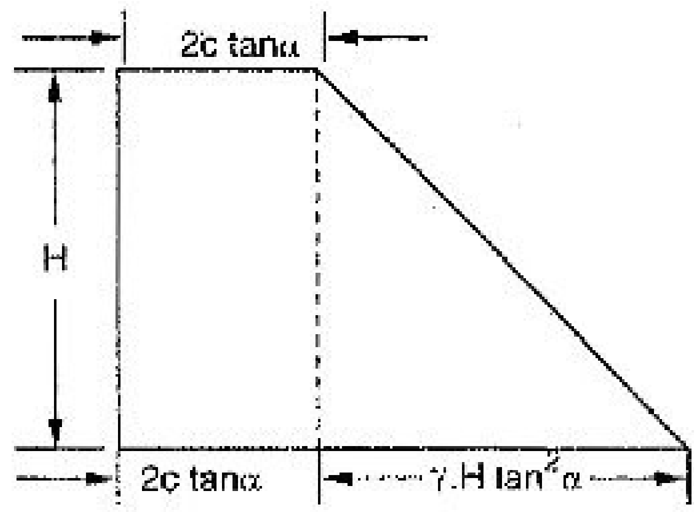
$$p_p = \gamma z \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$p_p = K_p \gamma \cdot z$$

where, $$K_p = \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

(b) Cohesive backfill : For the case of cohesive soil, the principal stress relationship at failure is given by

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$$



For the case of passive pressure,

$$\sigma_1 = \sigma_h = p_p, \sigma_1 = \gamma_2$$

Substituting these values of σ_1 and σ_3, we get

$$p_p = \gamma z \tan^2 \alpha + 2c \tan \alpha$$

$$= \gamma z N_\phi + 2c\sqrt{N_\phi}$$

At $z = 0$, $p_p = 2c \tan \alpha$

At$_z$ = H, $p_p = \gamma H \tan^2 \alpha + 2c \tan \alpha$

Figure shows the pressure distribution diagram.

Total pressure, $$p_p = \int_0^H \cdot p_p \cdot dz$$

$$= \frac{1}{2} \gamma H^2 \tan^2 \alpha + 2cH \tan\alpha$$

$$= \frac{1}{2} \gamma H^2 N_\phi + 2cH \sqrt{N_\phi}$$

Limitation of Rankine's Theory

1. As the retaining walls are usually constructed of masonry or cement concrete, the back of the wall is never smooth, and hence frictional forces develop.
2. Due to assumption that wall back is smooth, the resultant pressure must act parallel to the surface but due to frictional forces, the active earth pressure gets inclined to the wall at an angle equal to angle of friction.
3. The wall back may not always be vertical. In practice a batter is given to the wall back.
4. The retained soil may not always be cohesionless.