THEORY OF STRUCTURES

DIRECT AND BENDING STRESSES :

When a structure is loaded, work is done on the structure. The work done is transformed either partially or completely in the form of potential energy referred to as strain energy of internal energy. If the strain produced remains within elastic limit, the potential energy of strain can be recovered on unloading of the structure. The strain energy in a structure may be due to direct stresses, shear stresses, and bending stresses. In care of beams and frames, the strain energy due to direct and shear stress is very small in comparision to strain energy due to bending stresses. Therefore the strain energy due to direct and shear stresses, is generally neglected while finding the indeterminate reactions, and moments in beams and frames. However in the find checks the effect of direct and shear stresses may be taken into account.

Strain energy due to bending in a member of length 'l' is given by $\int U \int_0^l \frac{M^2 dS}{2EI} \int$ where 'M' is the bending moment at a section having moment of inertia 'I' and "ds" is the elementary length at the section.

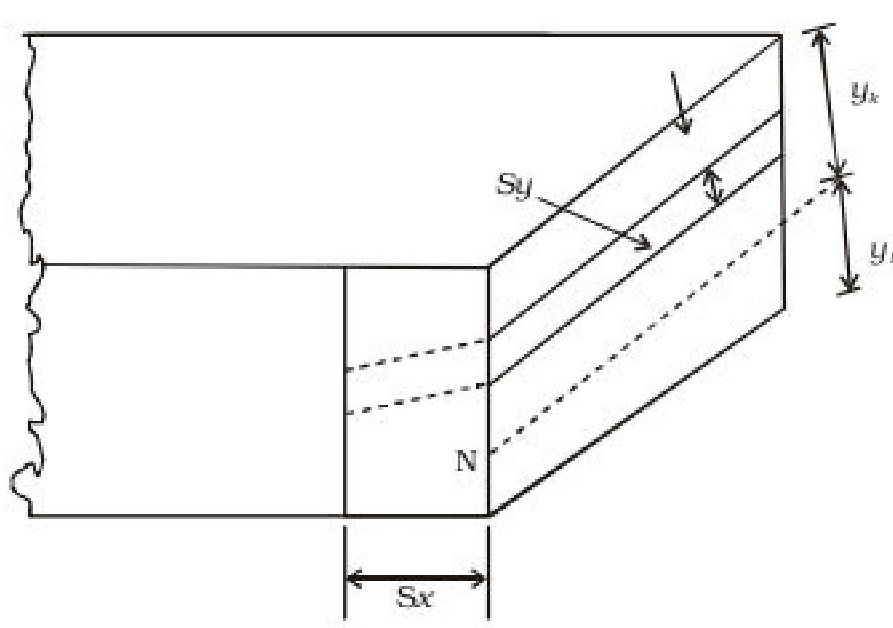


Fig: 1

Strain energy due to direct stresses is given by $\int U \sum \frac{P^2 Sl}{2AE}$ where 'P' is the direct force action on length 'Sl' and 'A' is the area of cross-section.

Strain energy due to shear stresses is given by:

$$U = \int_0^l \int_{-y_p}^{y_k} \frac{qy^2}{2N} dy dx$$

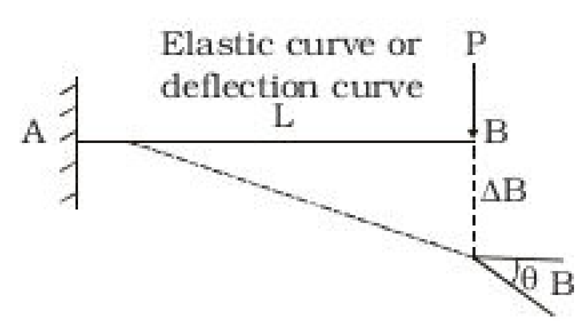
where 'q_y' is the shear stress at distance 'y' from N.A (Fig.1) 'b' is the breadth of section and 'N' is the Modulus of Rigidity.

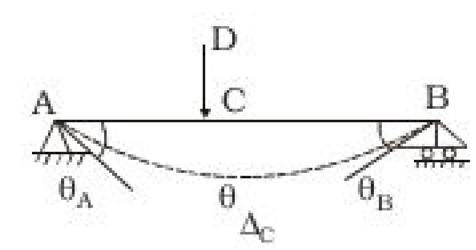
In reality, the method of strain energy is a force method, wherein redundant reaction forces or moments are determined at the very beginning. The method use the theorem of least work which states that for any statically indeterminate structure, the redundants should be such as to make the total strain energy within the structure a minimum. The validity of the theorem of least works comes directly from costigiliano's second theorem which may be stated as follows.

The partial derivative of the total internal energy in a beam, with respect to the load applied at any point, is equal to the deflection at that point.

For externally redundant structures, if the support do not yield, the work done by the loads is stored up as strain energy will be minimum. However if the supports yield some work is also done on supports. In such cases total work done will consist of strain energy stored in the structure plus the work done is to be minimum, the sum of strain energy stored in the structure plus work done on supports will be minimum.

DEFLECTION OF BEAMS





At fixed end – shope is zero

$(\theta) \approx \tan\theta$

Deflection is zero (Δ)

At free end – slope and deflection both will exist.

A simple supports (hinge/roller) slope will exist but diflection is zero.

The slope at any point w.r.t. x – direction is dy/dx = $\tan\theta$. If θ is small, $\tan\theta = \theta$.

Transverse deflection is only calculated for Bending

Note : The transverse deflection in Beams may be caused by (i) B.M, (ii) S.f.

The deflection caused by B.M is much greater than deflection caused by S.F.

∴ For all Practical purposes shear deflection is neglected.

$$\frac{\Delta \text{ Bending}}{\Delta \text{ Shear}} = K. \left(\frac{l}{d}\right)^2$$

where, l = centre to centre span of beam

d = depth of beam

The shear deflection is neglected except in deep beams.

The computation of slope and deflection is require to satisfy serviceability condition.

A/C to "BIS" maximum permissible deflection is $\frac{L}{325}$ (L = Centre to centre span).

STANDARD RESULT OF SLOPE AND DEFLECTION FOR DIFFERENT BEAMS :

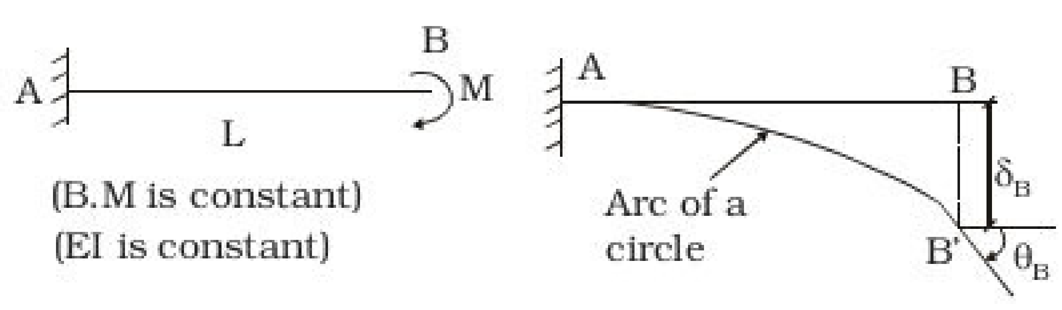
EI = Constant

EI = Flexural Rigidity

$\frac{EI}{L}$ = Flexural stiffness.

1. Cantilever Beam

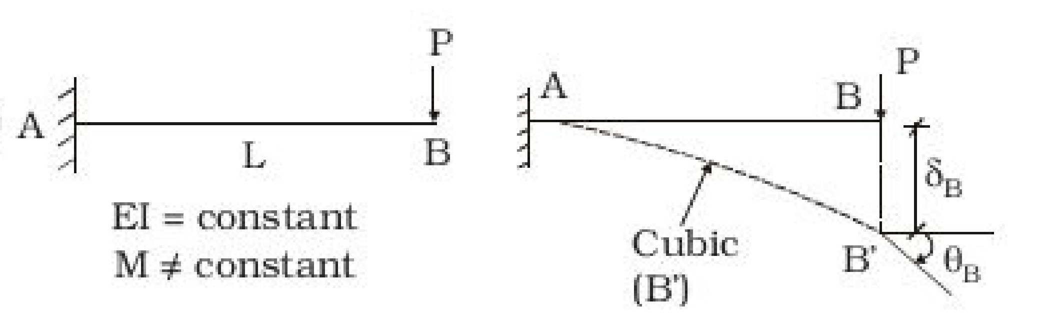
(i)



$$\delta_B = \frac{ML^2}{2EI}$$

$$\theta_B = \frac{ML}{EI}$$

(ii)



$$\delta_B = \frac{PL^3}{3EI}$$

$$\theta_B = \frac{PL^2}{2EI}$$

Note : $y = \delta \rightarrow$ deflection

$\frac{dy}{dx} = \tan\theta \approx \theta \rightarrow$ Slope

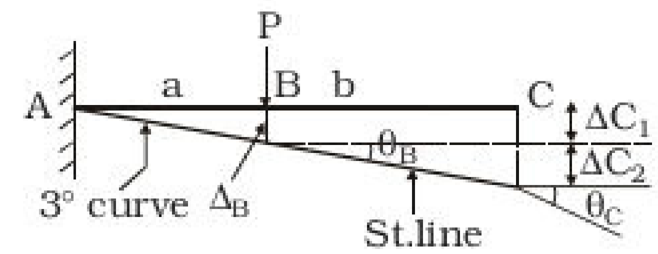
$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{R} \rightarrow$ Curvature.

$\frac{d^3y}{dx^3} = \frac{\frac{dM}{dx}}{EI} = \frac{d}{dx}\left(\frac{1}{R}\right)$ = Rate of change of curvature it express S.F

$\frac{d4y}{dx^4} = \frac{\frac{d}{dx}(\text{Shear force})}{EI} = \frac{d^2\left(\frac{1}{R}\right)}{dx^2} \rightarrow$ represents Loading Rate.

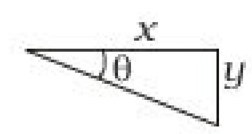
It means deflection curve is too greater than B.M. Curve.

(iii)



$$\theta_B = \frac{P.a^2}{2EI}$$

$$\Delta_b = \frac{P.a^3}{3EI}$$



$$\tan\theta = \frac{y}{x}$$

$$\theta \,.\, x \,.\, y$$

$$\theta_C = \theta_B = \frac{P.a^2}{2EI}$$

$$\Delta_C = \Delta_{C1} + \Delta_{C2} = \Delta_B + \theta_B \,.\, L_{BC}$$

$$\Delta_C = \frac{Pa^3}{3EI} + \frac{P.a^2}{3EI} \,.\, b$$

Special case :

If $a = b = \frac{l}{2}$;

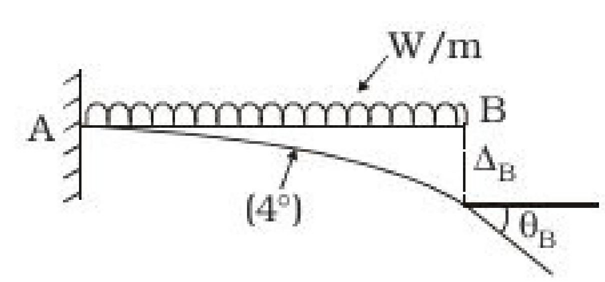
then;

$$\theta_B = \theta_C = \frac{pl^2}{8.EI};$$

$$\Delta_C = \frac{pl^3}{24\ EI} + \frac{pl^2}{24\ EI} \,.\, L$$

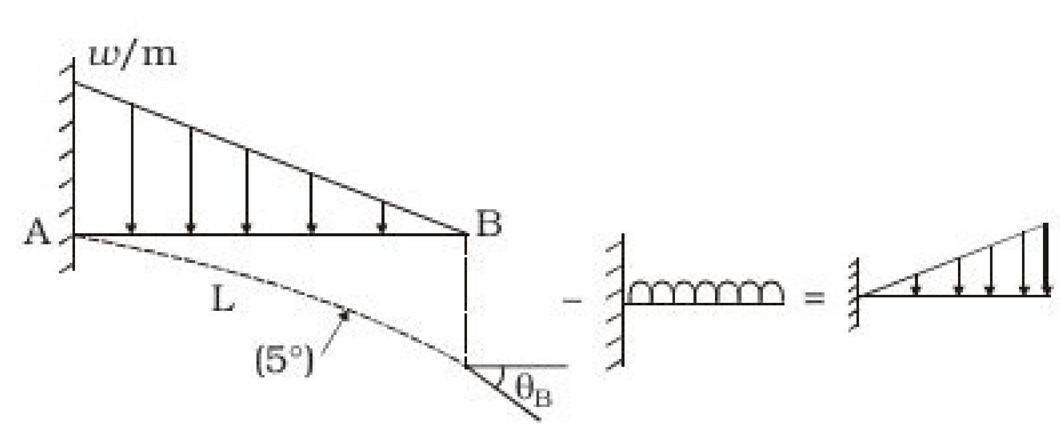
$$\Delta_C = \frac{pl^3}{48\ EI}$$

(iv)



$\theta_B = \frac{WL^3}{6EI}$; $\Delta_B = \frac{WL^4}{8EI}$

(v)



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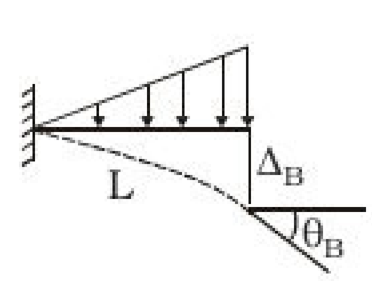


$$\theta_B = \frac{WL^3}{24EI}$$

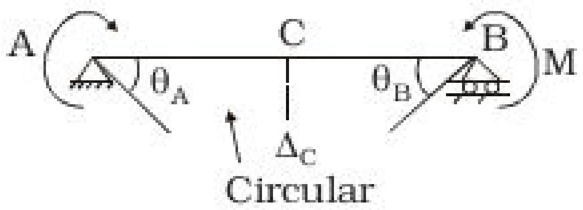
$$\Delta_B = \frac{WL^4}{30EI}$$

$$\theta_B = \frac{WL^3}{6EI} - \frac{WL^3}{24EI}$$

$$\Delta_B = \frac{WL^4}{8EI} - \frac{WL^4}{30EI}$$

2. Simply Supported Beam

(i)

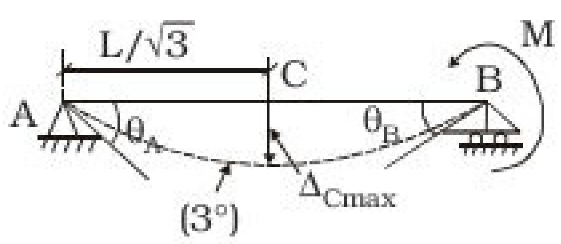


$$\theta_A = \theta_B = \frac{ML}{2EI}$$

$$\Delta_C = \Delta_{max} = \frac{ML^2}{8EI}$$

M = Constant

(ii)

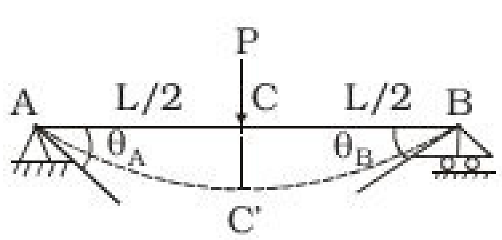


$$\theta_A = \frac{ML}{6EI}\ ;\ \theta_B = \frac{ML}{3EI}$$

$$\Delta_{max} = \frac{ML^2}{9\sqrt{3}\,EI}$$

Which occurs at $x = \frac{L}{\sqrt{3}}$ from A

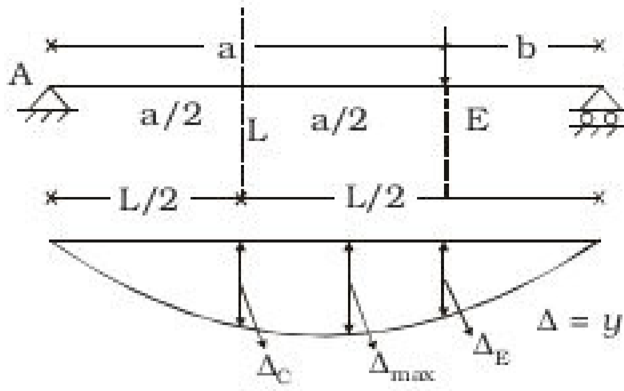
(iii)



$$\theta_A = \theta_B = \frac{PL^2}{6EI}$$

$$\Delta_{max} = \Delta_C = \frac{PL^3}{48EI}$$

(iv)



(Δ_{max}) = always between centre of span and load

At Δ max

$$\frac{dy}{dx} = \tan\theta = \theta = 0$$

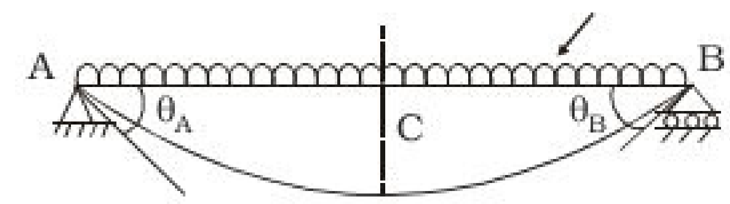
Note : (learn) Important for objectives

1. In eccentric loading case, maximum deflection occurs between center of span and position of load.
2. When load reaches close to the support, then Δ_{max} occurs at $\frac{L}{13}$ from the centre. It means the zone of Δ_{max} in S.S.B due to a concern load is from $\left(\frac{-L}{13} \text{ to } \frac{L}{13}\right)$ if centre is considered to zero. i.e., $\frac{2L}{13}$. in central part.

   When load is close to the support, then Δ_{max} is 2.5% greater than Δ_C (at centre).

$\therefore$ For all practical purpose, $\Delta_{max} \approx \Delta_C$.

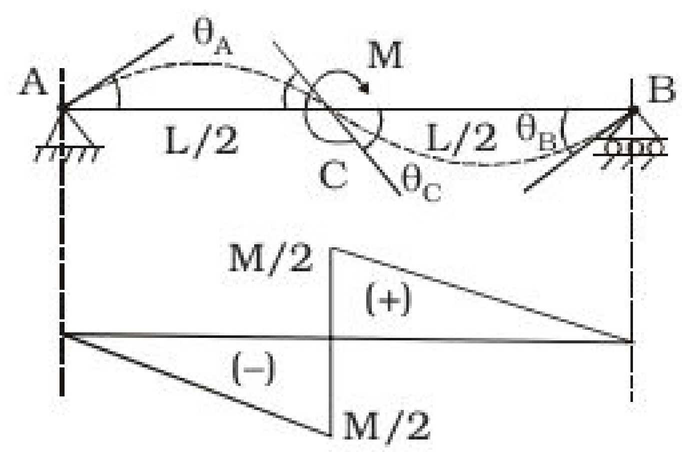
(v)



$$\theta_A = \theta_B = \frac{WL^3}{24EI}$$

$$\Delta_C = \Delta_{max} = \frac{5}{384}\cdot\frac{WL^4}{EI}$$

(vi)

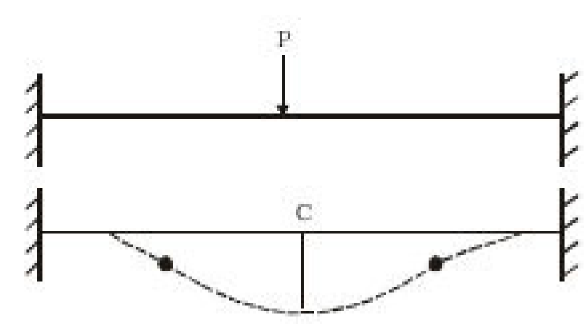


$$\theta_A = \theta_B = \frac{ML}{24EI}$$

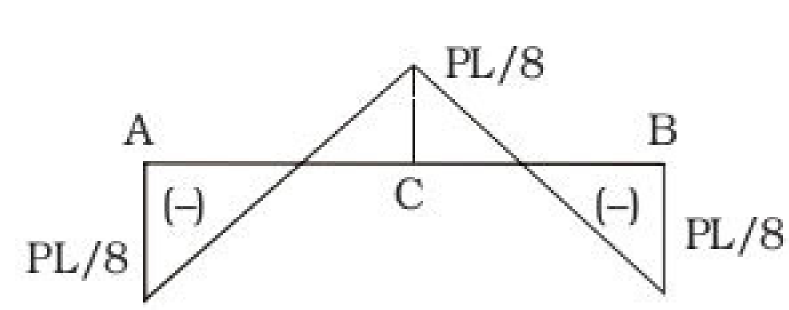
$$\theta_A = \frac{ML}{12EI}$$

$$\Delta_A = \Delta_B = \Delta_C = 0$$

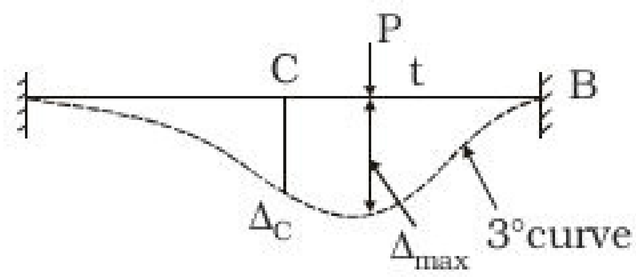
(vii)



$$\Delta_C = \frac{PL^3}{192EI}\ ;\ \theta_A = \theta_B = \theta_C = 0$$



(viii)

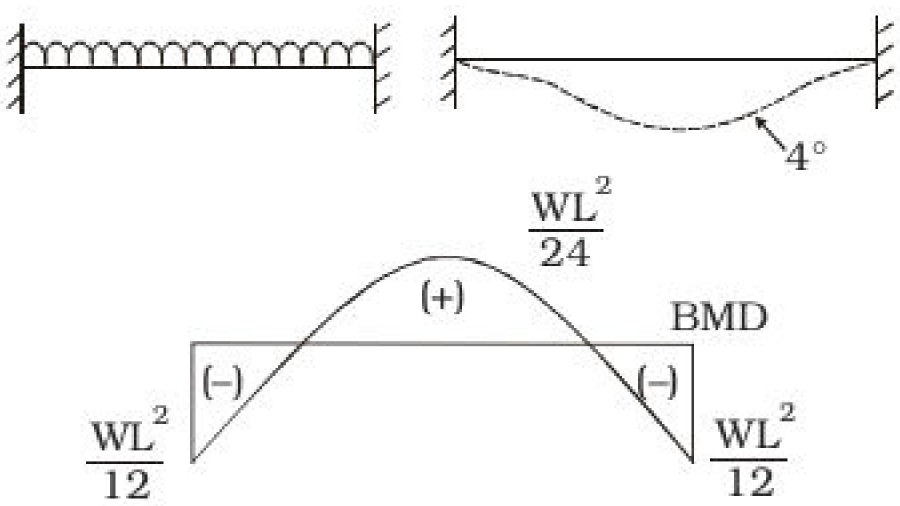


Δ_{max} occurs between centre of span and position of load

Δ_E = deflection below load

$$= \frac{P.a^3.b^3}{3EIL^3}$$

(ix)



$$\Delta_C = \frac{WL^4}{384EI}$$

$$= \frac{1}{5} \text{ th of deflection in S.S. Beam.}$$

● **METHOD TO FIND SLOPE AND DEFLECTION :**

1. Double Integration Method.
2. Macaulay's Method.
3. Area-moment theorem (Mohr's theorem).
4. Conjugate Beam Method (Mohr's theorem).
5. Strain Energy Method (Costigliano's 2nd theorem).
6. Super Position theorem.
7. Unit load Method (Maxwell's theorem).
8. Virtual work.
9. Williot Mohr Method (Graphical Method) It is applicable to find deflection of truss joints only.

1. Double Integration Method

$$\frac{EId^2y}{dx^2} = M \qquad \text{.........(i)}$$

$$y = \text{deflection}; \ \frac{dy}{dx} = \text{Slope} ;$$

$$\frac{d^2y}{dx^2} = \text{Curvature}$$

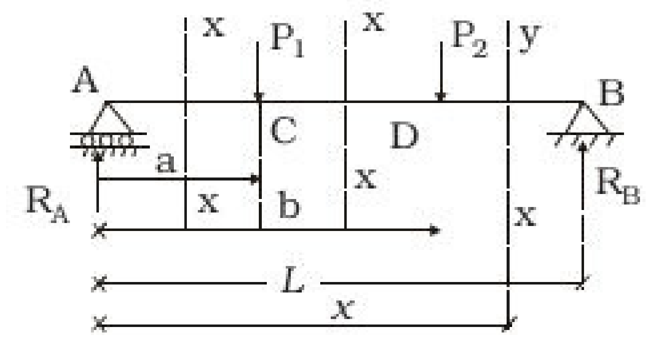
◆ This method is suitable for prismatic sections in which equation of B.M remains valid for entire length/span.
UDL and UVL
S.S.B with UDL and UVL

2. Macaulay's Method :

◆ This method is suitable when multiple concern load/moment are acting on the beam.

◆ It is an improvement over double integration method. If equation of BM change from I Part to the other part of span then double integration method is not suitable but this method can be applied.

◆ It is suitable for prismatic sections.



$$\frac{\text{B.M at} \times \text{in AC}}{M_x \ (x \text{ from A}) = RA.x}$$

$$0 \le x \le a$$

B.M in DB

M_x (x from A) (most generalised equation)
$= R_A \cdot x - P_1 (x - a)$
$- P_2 (x - b)$
$b \le x \le L$

B.M. in CD

M_x (x from A)
$= R_A \cdot x - [P_1 (x - a)]$
$a \le x \le b$

◆ If Equation of B.M changes, then most generalised of B.M is written in specific format by macaulay as shown below;

$$EI.\frac{d^2y}{dx^2} = \underset{I}{RA \cdot x} \ / \underset{II}{- P_1 (x - a)} \ / \underset{III}{-P_2 (x - b)}$$

◆ oblique means the remaining terms may be or may not be valid.
For equation if $0 \le x \le a$; then only term is valid & soon.

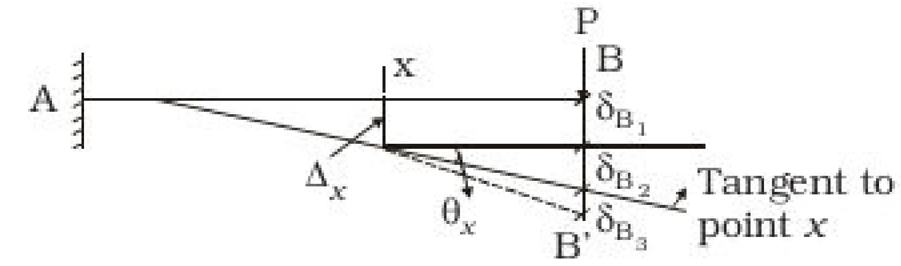
◆ When end conditions are applied to find C_1 & C_2, then only valids terms are accounted.

3. Area – Moment theorem :

This method is applicable for prismatic & non-prismatic sections both. In this method continuity of slope is considered, therefore, it is not suitable when, slope suddenly changes, such as presence of internal hinge.

This method is found suitable when area of curvature diagram ($\frac{M}{ET}$-diagram) & C.G of curvature diagram can be easily computed.

Theorem 1 :



S_{B_3} = Deflection of B.w,r to tangent at $x = S_B/x$

$S_{B_2} = \theta x . L_{x_B}$

$S_{B_1} = \delta_x$

Total deflection of B w.r. to horizontal

$\Delta_B = S_{B_1} + S_{B_2} + S_{B_3}$

$= Sx + \theta x . L_{x_B} + S_B/x$

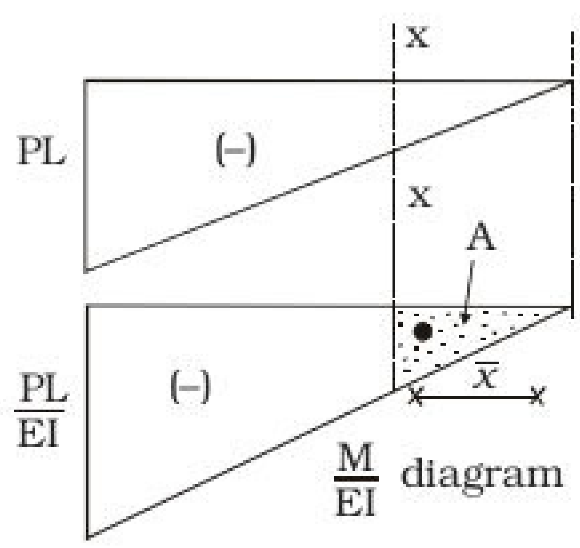
Statement ; (theorem 1 :)

BMD

"The change in slope from any point 'x' to 'B' is equal to area of $\frac{M}{EI}$ diagram between 'x' & 'B'

$\therefore \theta_{x_B} = \theta_B - \theta_x$

= Area of $\frac{M}{EI}$ diagram between x & B



Special case ; If Reflection point is taken

at fixed support A = A $\begin{cases} \text{(–Ve) as B.M is hoggin} \\ \text{+Ve as B.M is sagging} \end{cases}$

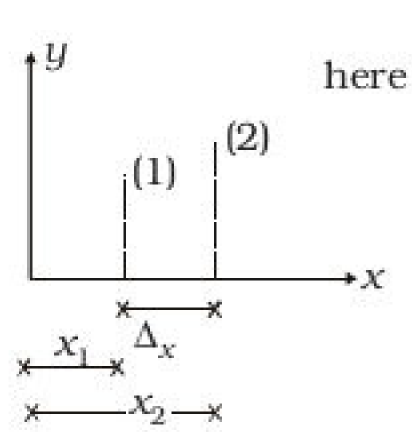
Then ;

Change in slope from A to B is equal to Area of $\frac{M}{EI}$ diagram between A & B.

$\therefore \quad \theta_{AB} = \theta_B - \cancel{\theta_A}^{O}$

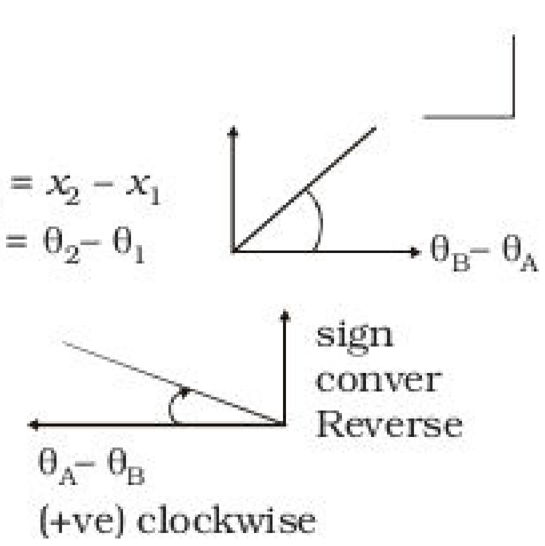
$= \frac{-1}{2} \times L \times \frac{PL}{EI} \Rightarrow \theta_B = \frac{-PL^2}{2EI}$

Note : This method is suitable when point of zero slope & zero deflection is known such as fixed support



here ; $\Delta_x = x_2 - x_1$

$\theta = \theta_2 - \theta_1$



(Theorem 2 :) Statement :

The deflection of any point B w.r. to tangent point x is equal to moment of area of diagram between x & B about B.

$S_B/x = S_{B_3}$ = Moment of area of $\frac{M}{EI}$ between $x \times$ B about B

$= A.\ \bar{x}$

S = (–Ve) ↓ w.r to tangent

Total deflection of B w.r to horizontal is

$\Delta_B = S_{B_1} + S_{B_2} + S_{B_3}$

$= \delta_x + \theta_x . L_{x_B} + A.\ \bar{x}$

Special case; Deflection of B w.r to tangent at A is equal to moment of area of $\frac{M}{EI}$ between A & B about B.

$S_B/A = A.\ \bar{x}$

$= \frac{-1}{2} \times L \times \frac{PL}{EI} \times \frac{2L}{3} \Rightarrow \frac{-PL^3}{3EI} = \frac{S_B}{A}$

Since, tangent at A is horizontal, hence, S_B/A is total deflection.

Total $\Delta_B = S_{B_1} + S_{B_2} + S_{B_3}$

$\underset{0}{\cancel{S_B}} + \underset{0}{\cancel{\theta_A}} . L_{AB} + A.\ \bar{x}$

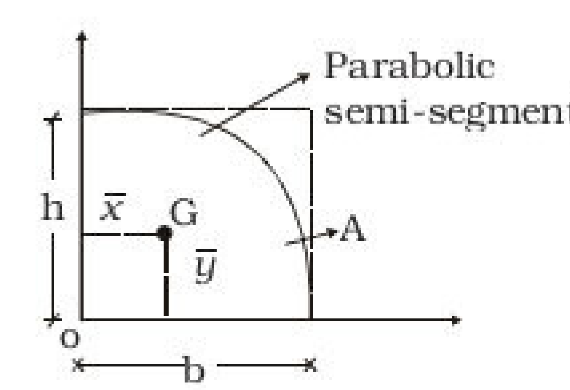
$\Delta_B = A.\ \bar{x}$

PROPERTIES OF AREA :

(a) Parabolic; (Semi-segment)

$y = h\left(1 - \frac{x^2}{b^2}\right)$

Equation :



$A = \frac{2}{3}$ bh

$\bar{x} = \frac{3}{8}$ b

$\bar{y} = \frac{2}{5}$ h

$\therefore$ For n^{th} degree semi-segment

$y = h\left(1 - \frac{x^n}{b^n}\right)$

$A = \frac{n}{n+1}$ b.h

$\bar{x} = \frac{n+1}{2(n+2)}$ .b ; $\bar{y} = \frac{n}{2(n+2)}$ .h

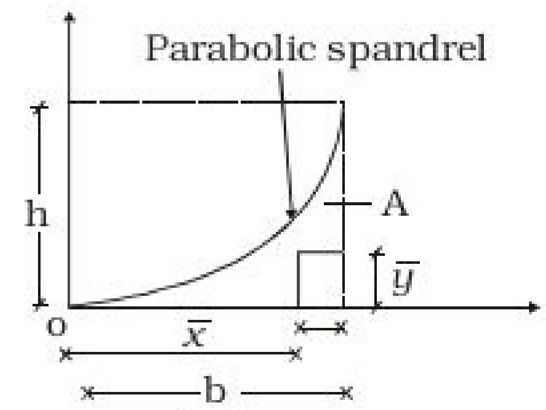
(b) Parabolic spandrel

$$A = \frac{1}{3}bh$$

$$\bar{x} = \frac{3}{4}b$$

$$\bar{y} = \frac{3}{10}h$$

$$y = \frac{h.x^2}{b^2}$$



For n^{th} degree spandrel

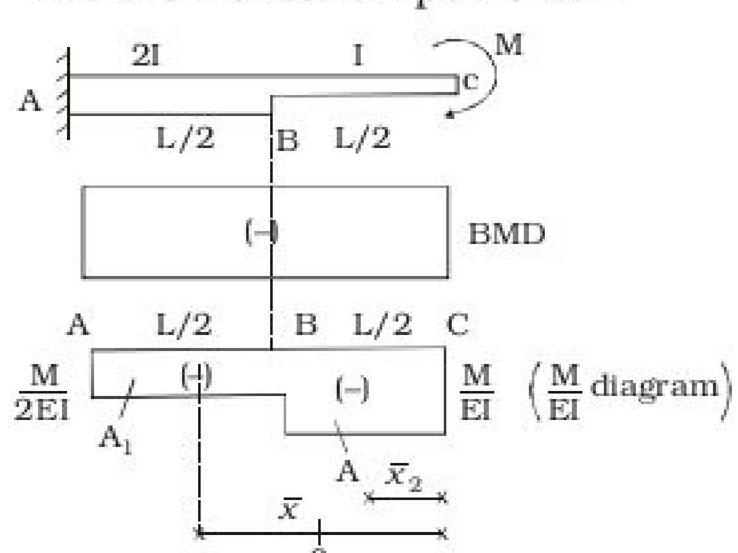
$$y = \frac{h.x^n}{b^n}$$

$$A = \frac{1}{n+1}bh \; ; \; \bar{x} = \frac{n+1}{n+2}b$$

$$\bar{y} = \frac{n+1}{2(2n+1)} . h$$

Using A-M Theorem find slope & deflection free end

Let the Reflection point at A



Theorem 1;

$$\theta_{AC} = \theta_C - \cancel{\theta_A} = \frac{-L}{2} \times \frac{M}{EI} - \frac{M}{2EI} \times \frac{L}{2}$$

$$\theta_C = \frac{-3M}{4EI}$$

Theorem 2;

$$\delta_C /A = -\frac{M}{EI} \times \frac{L}{2} \times \frac{L}{4} - \frac{M}{2EI} \times \frac{L}{2} \times \frac{3L}{4}$$

$$= \frac{-ML^2}{8EI} - \frac{3ML^2}{16EI} = \frac{-5}{16}\frac{ML^2}{EI} = \Delta_C$$

4. Conjucate Beam Method : (Given by Mohr)

It is applicable for prismatic & non prismatic both. It can be applied for beams containing internal hinges and also Conjugate Beam is an imaginary beam for which loading diagram is $\frac{M}{EI}$ diagram (curvature diagram) of given beam. The end condition & support conditions are modified such that slope & deflection in given beam is represented by shear force & B.M respective in conjugate Beam.

If given real beam is stable & determine then conjugate beam is also stable & determine, but if given beam is indeterminate then conjugate beam is unstable & if given beam is unstable then conjugate beam is indeterminate.

Theorem 1 :

" The slope at any point in the given beam is equal to shear force at that point in the conjugate beam. It Means S.F.D of conjugate beam represents slope curve of given beam.

Theorem 2 :

" The deflection at any point in the given beam is equal to B.M point in the conjugate beam. It means MD of conjugate Beam represents deflection curve / elastic curve of given beam.

◆ **GUIDELINES TO DRAW CONJUGATE BEAM :**

1. Diagram of given beam is loading diagram of conjugate beam. If B.M.D of given beam is (+ve) (sagging) then loading in conjugate Beam will be upward & if BMD in given beam (–ve) hogging then loading in conjugate beam will be downward.
2. If S.F at any beam in conjugate beam is (+ve) then slope at that point is given beam is also (+ve) (anticlockwise) & vice-versa.
3. If B.M at any point in conjugate Beam is (+ve) (sagging) then deflection an actual beam at that point is (+ve) upward & if BM is hogging (–ve) then deflection in given beam is downwards.
4. The support condition will be modified in such a way if given beam has slope then at that point conjugate beam should have S.F & if given beam has deflection then conjugate Beam should have Given Real Beam conditions.

MOMENTS DISTRIBUTION METHOD

Analysis of indeterminate beams and frames by the methods of strain energy and slope deflection involves solving a number of simultaneous equation which is tedious and time consuming. The moment-distribution method is a displacement method of analysis that is easy to apply once certain elastic constants have been determined. Essentially it is a method to solve the simultaneous equations in the slope-deflection method by successive approximations, accurate to as many significant figure as desired.

In fact this method sidesteps the calculation of the displacements and instead make it possible to apply a series of converging corrections that allow direct calculation of the end moments.

Basic Concept and Definition:

The deformation response of a continuous beam or a rigid frame without unknown joint translation is completely defined by the unknown joint rotations, such as θ_B, θ_C and θ_D in fig. (a.c.) physically, it is conceivable, that locking moment can be applied to joint 'B', 'C' and 'D' (fig. 1 (b)(d)). So the method begins by assuming each joint of structure is fixed (locked joint). Then by unlocking and locking each joint in succession, the internal moment at joints are distributed and balanced untill the joints have routed to their final or nearly final value. In fact, the magnitude of these locking moments are known in advance in terms of the applied loads or the support settlements. When the locking moment at one of the joint is released, that joint will rotate. This rotation induces changes not only in the movments at the member ends entering the relased joint, but also in the locking moments at the immediately adjacent joints on both sides of the released joint.

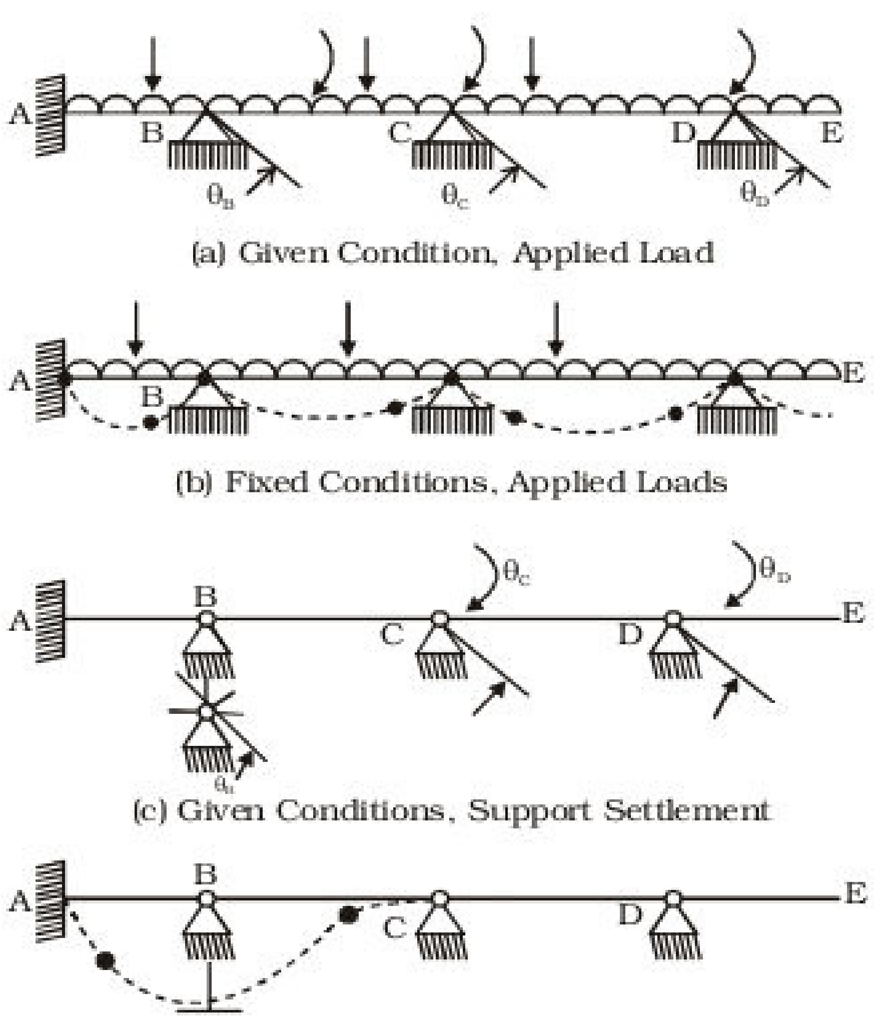
If each joint is successively released and locked back and then this process is repeated, a time will be reached at which every point joint has attained its full needed value in the final deformation response. Then the locking moment would have beed dissipated, or distributed throughout the structure by means of successive amount of joint rotations.

Sign Convention:

Clockwise moments at the fixed joints of a loaded and clockwise moments acting at the member ends are considered positive

Fixed end Moments: (FEMs)

The moments at the fixed joints of a loaded member are called fixed and moments.



(a) Given Condition, Applied Load

(b) Fixed Conditions, Applied Loads

(c) Given Conditions, Support Settlement

(d) Fixed Condition, Support Settlement

Fig. 1 : Fixed Conditions in Moment Distribution Method

SHORT COLUMN

Stress at any point in a short column, loaded eccentrically, varies linearly with the position of the point. At any point 'A' fig (2) having co-ordinates (x, y) the stress is given by:

$$f = a + bx + cy$$

where 'a', 'b' and 'c' are constants consider small area δA at point 'A' Force on elementary area will be

$$\delta P = f\delta A$$

Total force on the section

$$P = \int f dA = \int (a + bx + cy) dA$$

$$= a\int dA + b\int x dA + C\int y dA$$

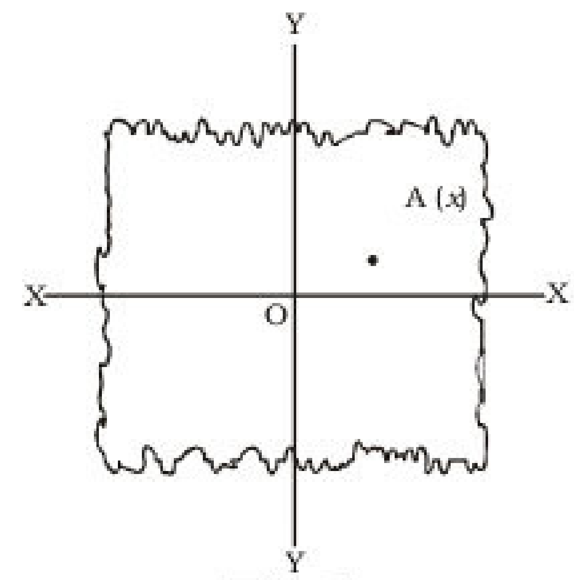


Fig: 1

In case axes of reference pass through the centre of gravity of section, i.e., point 'O' coincides with centre of gravity.

$$\int x dA = 0 \quad and \int y dA = 0$$

$$\therefore P = a\int dA = aA,$$

where 'A' is the total area, of section

$$\therefore a = \frac{P}{A} \ldots (1)$$

The moment of elementry force δP about x-axis is

$$\delta M_x = y\delta P$$

$$\therefore \text{ Total moment } Mx = \int y dp = \int y f dA = \int y(a + bx + cy) dA$$

$$= a\int y dA + b\int xy dA + C\int y^2 dA$$

$$= 0 + bI_y + C\, I_{xy} \ldots (2)$$

where I_{xy} is the product moment of Inertia and I_x is the moment of inertia alcout x-axis.

moment of elementory force δP about y-axis

$$\delta M_y = x\,\delta P = x f \delta A$$

$$\therefore \text{ Total moment, } M_y = \int x\,(a + bx + cy)$$

$$= a\int x dA + b\int x^2 dA + c\int xy dA$$

$$= 0 + bI_y + CI_{xy} \ldots (3)$$

Solving equations (2) and (3) for 'b' and 'c'

$$b = \frac{M_y I_x - M_x I_{xy}}{I_y I_x - I^2_{xy}}$$

$$c = \frac{M_x I_y - M_y I_{xy}}{I_y I_x - I^2_{xy}}$$

$$f = \frac{P}{A} + \frac{M_y I_x - M_x I_{xy}}{I_y I_x - I^2_{xy}} \times x + \frac{M_x I_y - M_y I_{xy}}{I_y I_x - I^2_{xy}} \times y$$

where P = Sum of vertical forces acting on the section

= $P_1 + P_2 + P_3 + \ldots$

M_x = Sum of the moments of vertical forces about x-axis

= $P_1Y_1 + P_2Y_2 + P_3+Y_3 + \ldots$

M_y = Sum of the moments of vertical forces about y-axis

= $P_1Y_1 + P_2Y_2 + P_3+Y_3 + \ldots$

and (x_1, y_1), (x_2, y_2), (x_3, y_3)...are the co-ordinates of P_1, P_2, P_3...

If the refrence axes are principal axes then,

$$I_{xy} = 0$$

$$\therefore f = \frac{P}{A} + \frac{M_y}{I_y} x + \frac{M_x}{I_x} y.$$

THEORY OF COLUMN ANALOGY

Indeterminate structures of a single span or single leay or closed rings can be analysed by this method. The bending moment at any point in such structures will consist of bending moment Ms, considering the structures as a statically determinate one, i.e., by removing the redundancy of the structure, and M_i bending moment due to redundancy.

B.M at any section = $M_s + M_i$

Let θ be the relative rotation of the ends of the structure, 'H' be the relative horizontal displacement of the ends, 'V' be the relative vertical displacement of the ends.

$$\therefore \theta = \int \frac{Mds}{EI}, H = \frac{M_{yds}}{E_I}, V = \int \frac{M_{xds}}{EI}$$

In case relatives rotation and displacements of the ends are zero, $\theta = 0 = \int \frac{Mds}{EI} = \int \frac{(M_s + M_i)d_s}{EI}$

$$\therefore \int \frac{Mids}{EI} = \int - \frac{M_{sds}}{EI}$$

$$H = 0 = \int \frac{Myds}{EI} = \int \frac{(M_s - M_i)yds}{EI}$$

$$\therefore \int \frac{M_{yds}}{EI} = \int - \frac{M_{sds}}{EI} \times y$$

$$v = 0 = \int \frac{M_{xds}}{EI} = \int \frac{(Es + Mi)xds}{EI}$$

$$\therefore \int \frac{M_{ixds}}{EI} = \int - \frac{M_{sds}}{EI} \times x$$

consider a short column of width at any section equal to $\frac{1}{EI}$ and the load intensity as –Ms.

P = Total load = $\int - \frac{M_{sds}}{EI}$

$$\therefore \int - \frac{M_{ids}}{EI} = P \therefore dP = \frac{M_{ids}}{EI}$$

$$\int \frac{M_{iyds}}{EI} = \int ydp \ and \int \frac{M_{ixds}}{EI} = \int xdp$$

compare these expressions with the expressions for stress in columns with ecentric load,

$\int fda = P$, $\int fydA = Mx = \int ydP$, $\int fxdA = \int My = \int xdp$

In the analogous column, $\frac{ds}{EI}$ will be the elementory area, 'f' will be the stress at any point due to the load of intensity $-M_s$ or total load $\int - \frac{M_{sds}}{EI}$.

$$\therefore M_i = \frac{P}{A} + \frac{M_y I_x - M_x I_{xy}}{I_y I_x + I^2_{xy}} \times x + \frac{M_x I_y - M_y I_{xy}}{I_y I_x + I^2_{xy}} \times y$$

$$\therefore M_i = \frac{P}{A} + \frac{M_y - M_x \frac{I_{xy}}{I_x}}{I_y\left(1 - \frac{I^2 xy}{I_x I_y}\right)} \times x + \frac{M_x - M_y \frac{I_{xy}}{I_x}}{I_x\left(1 - \frac{I^2_{xy}}{I_x I_y}\right)} \times y$$

putting, $M'_y = M_y - M_x \frac{I_{xy}}{I_x}$

$$M'_x = M_x - M_y \frac{I_{xy}}{I_y}$$

$$I'_y = I_y\left(1 - \frac{I^2_{xy}}{I_x I_y}\right)$$

$$I'_x = I_x\left(1 - \frac{I^2_{xy}}{I_x I_y}\right)$$

$$\therefore Mi = \frac{P}{A} + \frac{M_y}{I_y} \times x + \frac{M'x}{I'x} \times y$$

where, M_s will be positive if it induces tension in the inside fibres.

M_s will be negative if it induces tension in the outer fibres.

P will be tensile for positive M_s and will be considered negative.

'p' will be compressive for negative M_s and will be considered positive.

Mi will be positive if 'f' is compressive

Mi will be negative if 'f' is tensile

Final B.M. = M = $M_s + M_i$

In a structure, a hinge is considered as having zero flexural rigidity as there can be any rotation and therefore $\frac{1}{EI}$ is taken as infinity. A fixed end is considered as having infinite flexural rigidity as there can be no rotation and therefore, $\frac{1}{EI}$ is taken as zero.

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