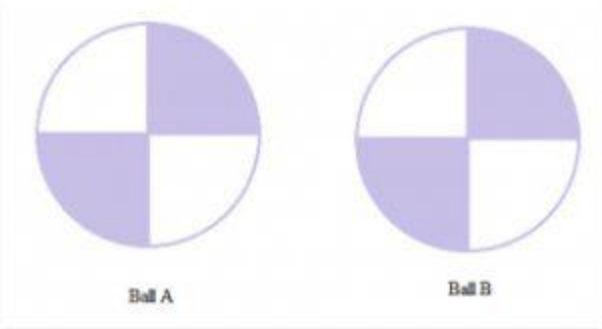
Exercise 16.1

Q1. Explain the concept of congruence of figures with the help of certain examples.

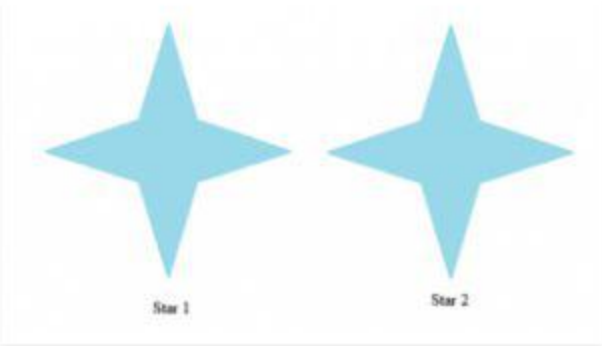
Answer:

Congruent objects or figures are exact copies of each other or we can say mirror images of each other. The relation of two objects being congruent is called congruence.

Consider Ball A and Ball B. These two balls are congruent.



Now consider the two stars below. Star A and Star B are exactly the same in size, colour and shape. These are congruent stars.



Q2. Fill in the blanks:

(i) Two line segments are congruent if ___

(ii) Two angles are congruent if ___

(iii) Two square are congruent if ___

(iv) Two rectangles are congruent if ___

(v) Two circles are congruent if ___

Answer:

(i) They have the same length, since they can superpose on each other.

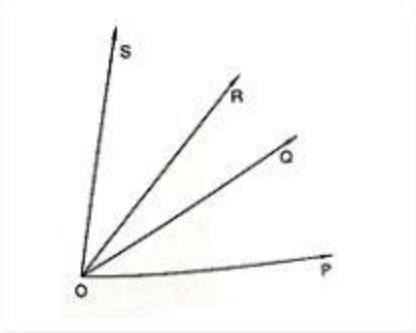
(ii) Their measures are the same. On superposition, we can see that the angles are equal.

(iii) Their sides are equal. All the sides of a square are equal and if two squares have equal sides, then all their sides are of the same length. Also angles of a square are 90° which is also the same for both the squares.

(iv) Their lengths are equal and their breadths are also equal. The opposite sides of a rectangle are equal. So if two rectangles have lengths of the same size and breadths of the same size, then they are congruent to each other.

(v) Their radii are of the same length. Then the circles will have the same diameter and thus will be congruent to each other.

Q3. In Figure, $\angle POQ \cong \angle ROS$, can we say that $\angle POR \cong \angle QOS$



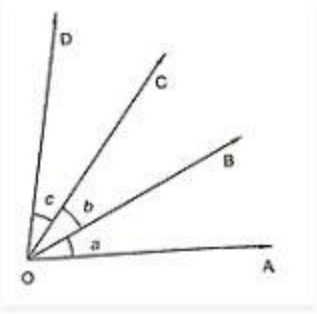
Answer:

We have,

$\angle POQ \cong \angle ROS \ \ (1)$
$Also, \angle ROQ \cong \angle ROQ$
$Therefore\ adding\ \angle ROQ to\ both\ sides\ of\ (1), We\ get,$
$\angle POQ + \angle ROQ \cong \angle ROQ + \angle ROS$
$Therefore,\ \angle PQR = \angle QOS$

Q4. In figure, a = b = c, name the angle which is congruent to $\angle AOC$

Answer:



We have,

$\angle$ AOB = $\angle$ BOC = $\angle$ COD

Therefore, $\angle$ AOB = $\angle$ COD

Also, $\angle$ AOB + $\angle$ BOC = $\angle$ BOC + $\angle$ COD

$\angle$ AOC = $\angle$ BOD

Hence, $\angle$ BOD is congruent to $\angle$ AOC

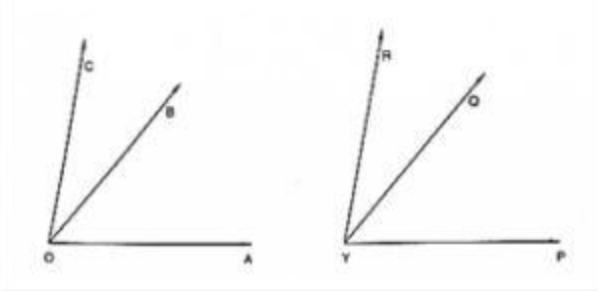
Q5. Is it correct to say that any two right angles are congruent? Give reasons to justify your answer.

Answer:

Two right angles are congruent to each other because they both measure 90 degrees.

We know that two angles are congruent if they have the same measure.

Q6. In figure, $\angle AOC \cong \angle PYR$ and $\angle BOC \cong \angle QYR$. Name the angle which is congruent to $\angle AOB$.



Answer:

$\angle AOC \cong \angle PYR \ldots . (i)$

$Also, \angle BOC \cong \angle QYR$

$Now,\ \angle AOC = \angle AOB + \angle BOC\ \angle PYR = \angle PYQ + \angle QYR$

$By\ putting\ the\ value\ of\ \angle AOC\ and\ \angle PYR\ in\ equation (i),\ we\ get,$

$\angle AOB + \angle BOC \cong \angle PYQ + \angle QYR$

$\angle AOB \cong \angle PYQ (\angle BOC \cong \angle QYR)$

$Hence,\ \angle AOB \cong \angle PYQ$

Q7. Which of the following statements are true and which are false;

(i) All squares are congruent.

(ii) If two squares have equal areas, they are congruent.

(iii) If two rectangles have equal areas, they are congruent.

(iv) If two triangles have equal areas, they are congruent.

Answer:

(i) False.

All the sides of a square are of equal length.

However, different squares can have sides of different lengths. Hence all squares are not congruent.

(ii) True.

Area of a square = side x side

Therefore, two squares that have the same area will have sides of the same lengths. Hence they will be congruent.

(iii) False Area of a rectangle = length x breadth

Two rectangles can have the same area. However, the lengths of their sides can vary and hence they are not congruent.

Example: Suppose rectangle 1 has sides 8 m and 8 m and area 64 meter square. Rectangle 2 has sides 16 m and 4 m and area 64 meter square. Then rectangle 1 and 2 are not congruent.

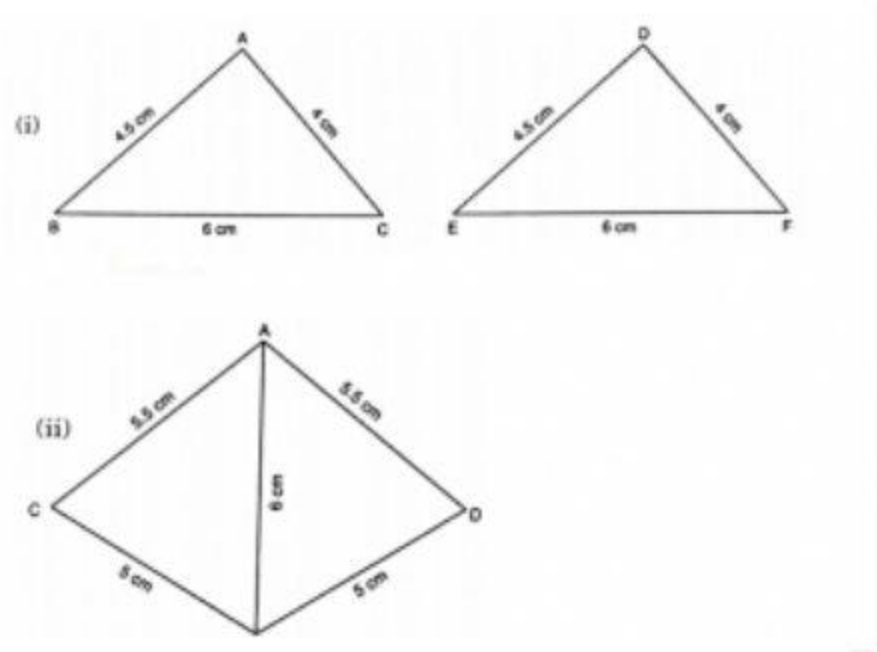
(iv) False

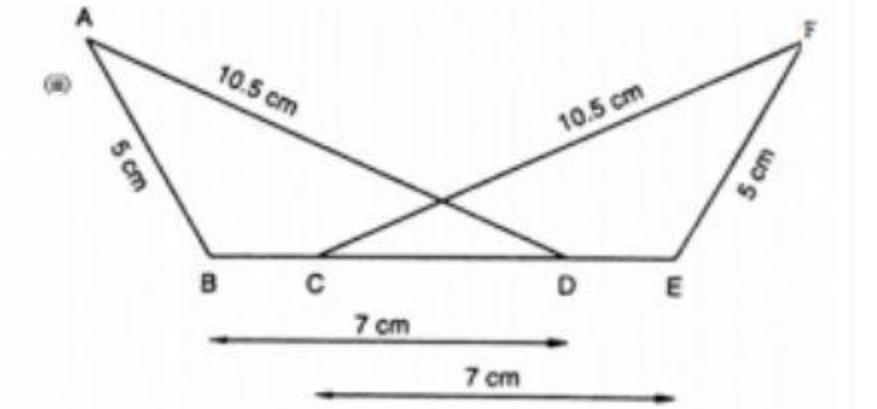
Area of a triangle = 12 x base x height

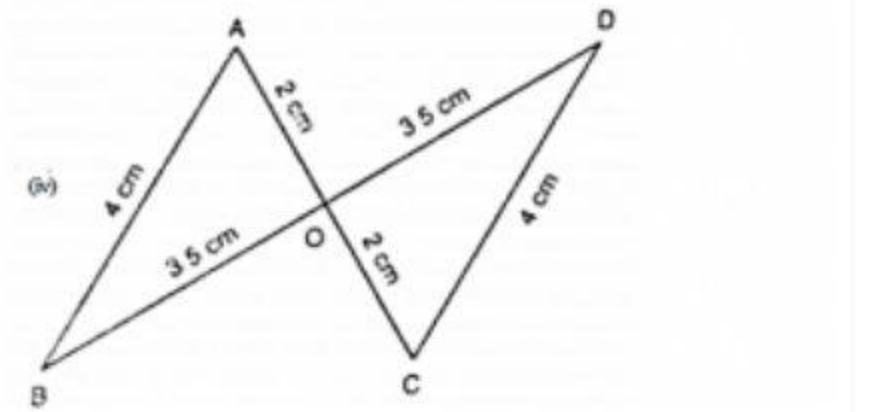
Two triangles can have the same area but the lengths of their sides can vary and hence they cannot be congruent.

Exercise 16.2

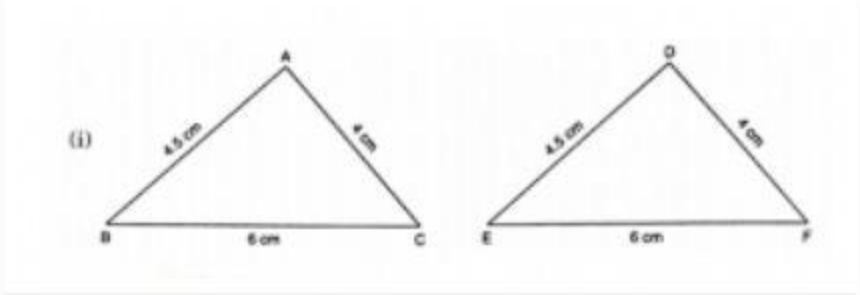
Q1. In the following pairs of triangle (Fig. 12 to 15), the lengths of the sides are indicated along sides. By applying SSS condition, determine which are congruent. State the result in symbolic.







Answer:



1) In Δ ABC and Δ DEF

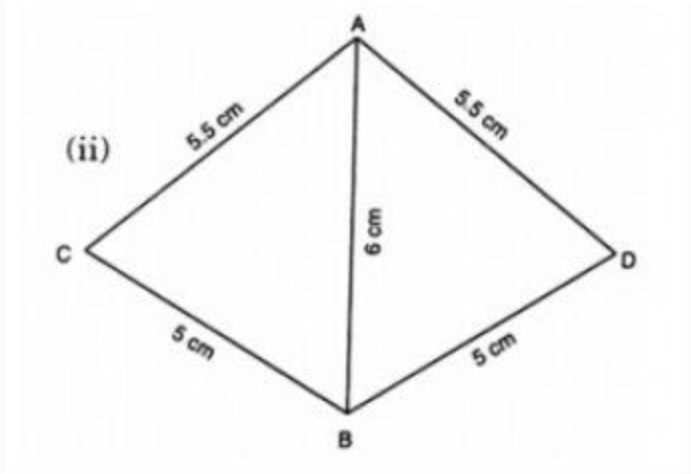
AB = DE = 4.5 cm (Side)

BC = EF = 6 cm (Side) and

AC = DF = 4 cm (Side)

Therefore, by SSS criterion of congruence, $\Delta ABC \cong \Delta DEF$

2)



In Δ ACB and Δ ADB

AC = AD (Side)

BC = BD (Side) and

AB = AB (Side)

Therefore, by SSS criterion of congruence, $\Delta ACB \cong \Delta ADB$

3) In Δ ABD and Δ FEC,

AB = FE (Side)

AD = FC (Side)

BD = CE (Side)

Therefore, by SSS criterion of congruence, $\Delta ABD \cong \Delta FEC$

4) In Δ ABO and Δ DOC,

AB = DC (Side)

AO = OC (Side)

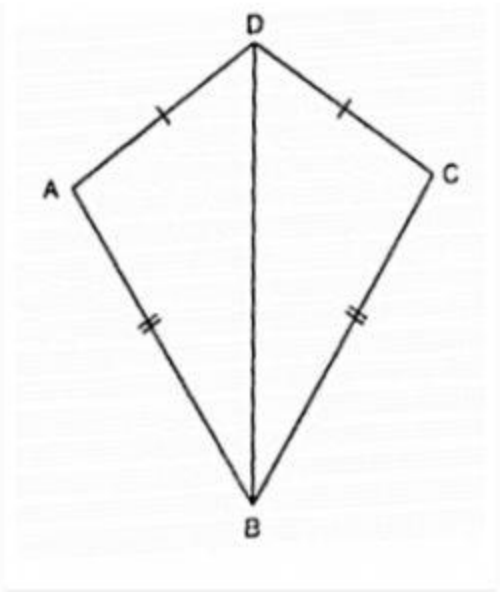
BO = OD (Side)

Therefore, by SSS criterion of congruence, $\Delta ABO \cong \Delta ODC$

Q2. In figure, AD = DC and AB = BC

(i) Is $\Delta ABD \cong \Delta CBD$?

(ii) State the three parts of matching pairs you have used to answer (i).



Answer:

Yes Δ ABD = Δ CBD by the SSS criterion. We have used the three conditions in the SSS criterion as follows:

AD = DC

AB = BC and

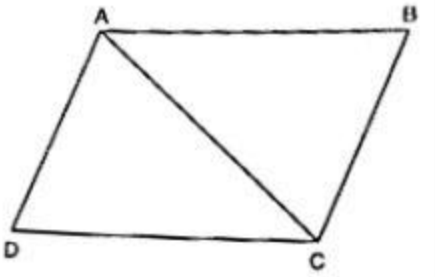
DB = BD

Q3. In Figure, AB = DC and BC = AD.

(i) Is $\Delta ABC \cong \Delta CDA$?

(ii) What congruence condition have you used?

(iii) You have used some fact, not given in the question, what is that?



Answer:

We have AB = DC

BC = AD

and AC = AC

Therefore by SSS $\Delta ABC \cong \Delta CDA$

We have used Side Side Side congruence condition with one side common in both the triangles.

Yes, have used the fact that AC = CA.

Q4. In $\Delta PQR \cong \Delta EFD$,

(i) Which side of ΔPQR equals ED?

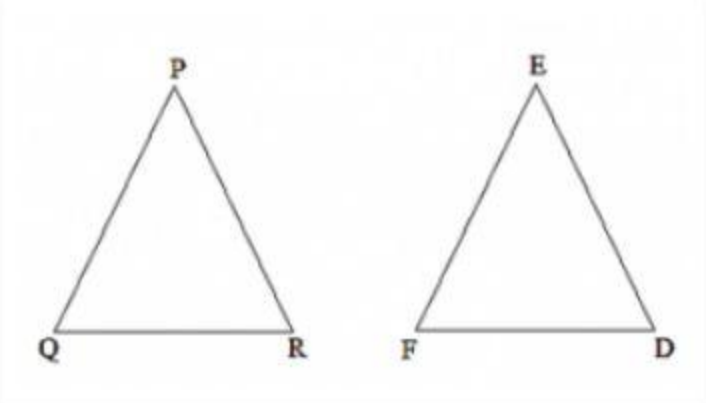
(ii) Which angle of ΔPQR equals angle E?

Answer:

$\Delta PQR \cong \Delta EFD$

(i) Therefore PR = ED since the corresponding sides of congruent triangles are equal.

(ii) $\angle$ QPR = $\angle$ FED since the corresponding angles of congruent triangles are equal.



Q5. Triangles ABC and PQR are both isosceles with AB = AC and PO = PR respectively. If also, AB = PQ and BC = QR, are the two triangles congruent? Which condition do you use?

It $\angle$ B= 50°, what is the measure of $\angle$ R?

Answer:

We have AB = AC in isosceles Δ ABC

And PQ = PR in isosceles Δ PQR.

Also, we are given that AB = PQ and QR = BC.

Therefore, AC = PR (AB = AC, PQ = PR and AB = PQ)

Hence, $\Delta ABC \cong \Delta PQR$

Now

$\angle$ ABC = $\angle$ PQR (Since triangles are congruent)However, Δ PQR is isosceles.

Therefore, $\angle$ PRQ = $\angle$ PQR = $\angle$ ABC = 50°

Q6. ABC and DBC are both isosceles triangles on a common base BC such that A and D lie on the same side of BC. Are triangles ADB and ADC congruent? Which condition do you use? If $\angle$ BAC= 40° and $\angle$ BDC = 100°, then find $\angle$ ADB.

Answer,

Yes, triangles ADB and ADC are congruent (By SSS)

AB=AC, DB = DC and AD = DA

$\angle$ BAD = $\angle$ CAD (c.p.c.t)

$\angle BAD + \angle CAD = 40°/ 2 \angle BAD = 40°$

$\angle BAD = 40°/2 = 20°$

$\angle ABC + \angle BCA + \angle BAC = 180°$ (Angle sum property)

Since Δ ABC is an isosceles triangle,

$\angle ABC = \angle BCA \angle ABC + \angle ABC + 40° = 180°$

$2 \angle ABC = 180° - 40° = 140° \angle ABC = 140°/2 = 70°$

$\angle DBC + \angle BCD + \angle BDC = 180°$ (Angle sum property)

Since Δ ABC is an isosceles triangle, $\angle DBC = \angle BCD \angle DBC + \angle DBC + 100^{o} = 180°$

$2 \angle DBC = 180° - 100^{o} = 80°$

$\angle DBC = 80°/2 = 40°$

In Δ BAD,

$\angle ABD + \angle BAD + \angle ADB = 180°$(Angle sum property)

$30° + 20° + \angle ADB = 180° (\angle ADB = \angle ABC - \angle DBC), \angle ADB = 180° - 20° - 30°$

$\angle ADB = 130°$

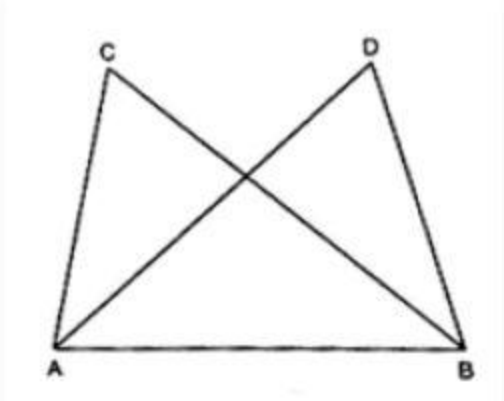
$\angle ADB = 130°$

Q7. Δ ABC and Δ ABD are on a common base AB, and AC = BD and BC = AD as shown in Fig. 18. Which of the following statements is true?

(i) $\Delta ABC \cong \Delta ABD$

(ii) $\Delta ABC \cong \Delta ADB$

(iii) $\Delta ABC \cong \Delta BAD$



Answer:

In Δ ABC and Δ BAD we have,

AC = BD (given)

BC = AD (given)

and AB = BA (common)

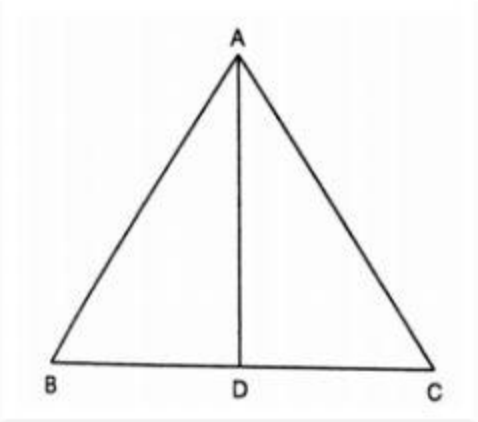
Therefore by SSS criterion of congruency, $\Delta ABC \cong \Delta BAD$

There option (iii) is true.

Q8. In Figure, Δ ABC is isosceles with AB = AC, D is the mid-point of base BC.

(i) Is $\Delta ADB \cong \Delta ADC$?

(ii) State the three pairs of matching parts you use to arrive at your answer.



Answer:

We have AB = AC.

Also since D is the midpoint of BC, BD = DC

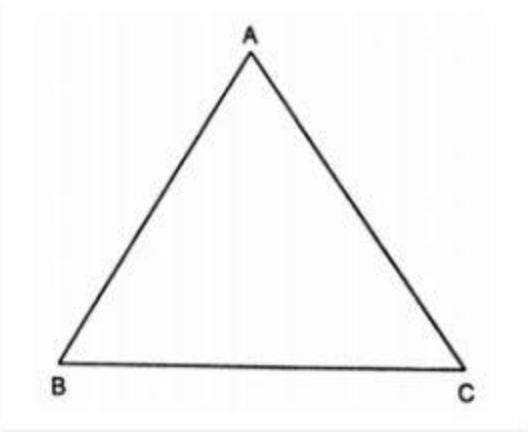
Also, AD = DA

Therefore by SSS condition,

$\Delta ADB \cong \Delta ADC$

We have used AB, AC : BD, DC AND AD, DA

Q9. In figure, Δ ABC is isosceles with AB = AC. State if $\Delta ABC \cong \Delta ACB$. If yes, state three relations that you use to arrive at your answer.



Answer:

Yes, $\Delta ABC \cong \Delta ACB$ by SSS condition.

Since, ABC is an isosceles triangle, AB = BC, BC = CB and AC = AB

Q10. Triangles ABC and DBC have side BC common, AB = BD and AC = CD. Are the two triangles congruent? State in symbolic form, which congruence do you use? Does $\angle$ ABD equal $\angle$ ACD? Why or why not?

Answer:

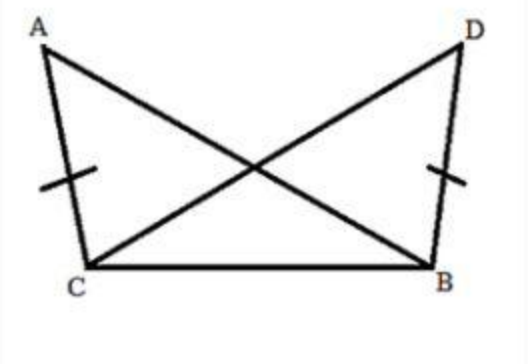
Yes,

Given,

Δ ABC and Δ DBC have side BC common, AB = BD and AC = CD

By SSS criterion of congruency, $\Delta ABC \cong \Delta DBC$

No, $\angle$ ABD and $\angle$ ACD are not equal because AB $\neq$ AC

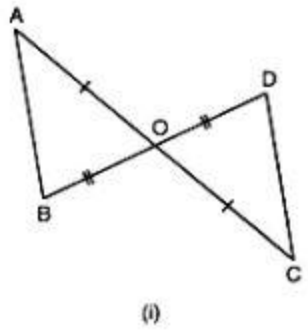


Exercise 16.3

Q1. By applying SAS congruence condition, state which of the following pairs of triangle are congruent. State the result in symbolic form

Answer:

(i)



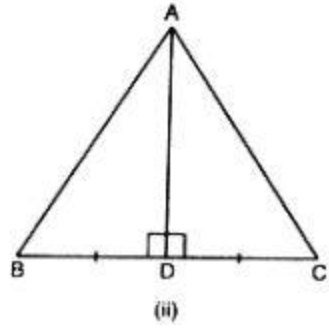
(i)

We have OA = OC and OB = OD and

$\angle$ AOB = $\angle$ COD which are vertically opposite angles. Therefore by SAS condition,

$\Delta AOC \cong \Delta BOD$

(ii)



(ii)

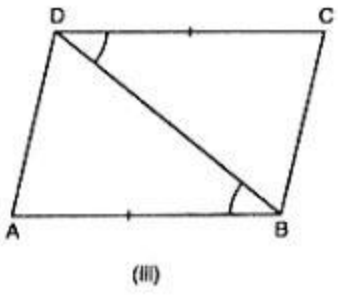
We have BD = DC

$\angle$ ADB = $\angle$ ADC = 90° and

AD = AD

Therefore, by SAS condition, $\Delta ADB \cong \Delta ADC$.

(iii)



(iii)

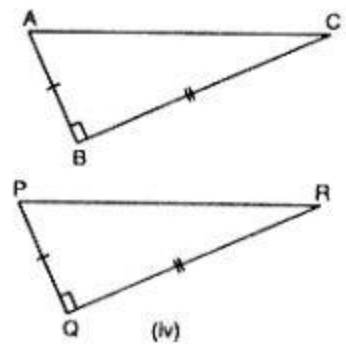
We have AB = DC

$\angle$ ABD = $\angle$ CDB and

BD = DB

Therefore, by SAS condition, $\Delta ABD \cong \Delta CBD$

(iv)



(iv)

We have BC = QR

ABC = PQR = 90°

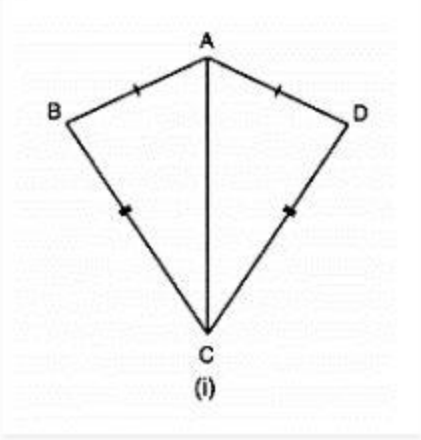
And AB = PQ

Therefore, by SAS condition, $\Delta ABC \cong \Delta PQR$.

Q2. State the condition by which the following pairs of triangles are congruent.

Answer:

(i)

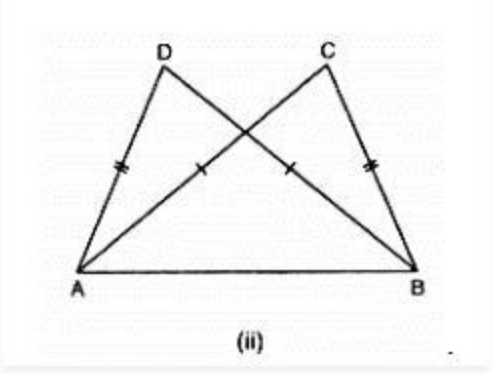


AB = AD

BC = CD and AC = CA

Therefore by SSS condition, $\Delta ABC \cong \Delta ADC$

(ii)

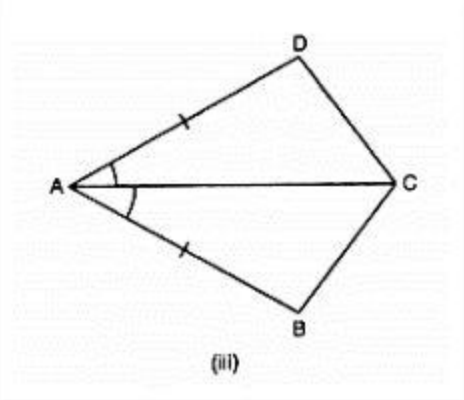


AC = BD

AD = BC and AB = BA

Therefore, by SSS condition, $\Delta ABD \cong \Delta ADC$

(iii)



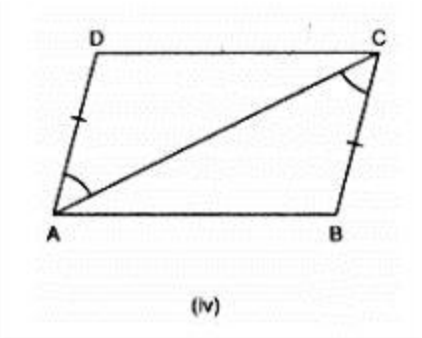
AB = AD

$\angle$ BAC = $\angle$ DAC and

AC = AC

Therefore by SAS condition, $\Delta BAC \cong \Delta BAC$

(iv)



AD = BC

$\angle$ DAC = $\angle$ BCA and

AC = CA

Therefore, by SAS condition, $\Delta ABC \cong \Delta ADC$

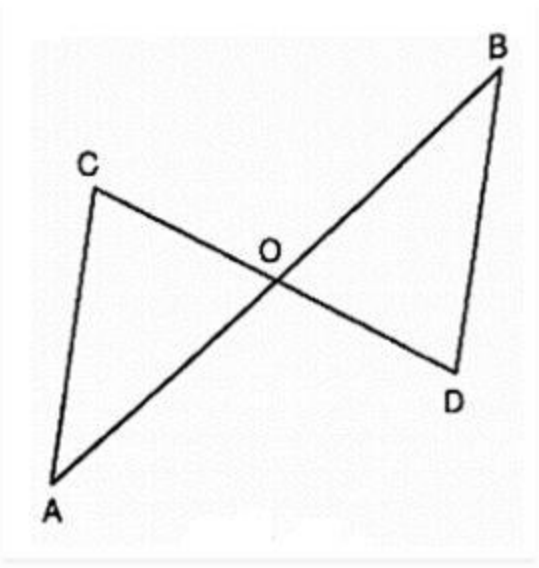
Q3. In figure, line segments AB and CD bisect each other at O. Which of the following statements is true?

(i) $\Delta AOC \cong \Delta DOB$

(ii) $\Delta AOC \cong \Delta BOD$

(iii) $\Delta AOC \cong \Delta ODB$

State the three pairs of matching parts, you have used to arrive at the answer.



Answer:

We have,

And, CO = OD

Also, AOC = BOD

Therefore, by SAS condition, $\Delta AOC \cong \Delta BOD$

Q4. Line-segments AB and CD bisect each other at O. AC and BD are joined forming triangles AOC and BOD. State the three equality relations between the parts of the two triangles that are given or otherwise known. Are the two triangles congruent? State in symbolic form, which congruence condition do you use?

Answer:

We have AO = OB and CO = OD since AB and CD bisect each other at 0.

Also $\angle$ AOC = $\angle$ BOD since they are opposite angles on the same vertex.

Therefore by SAS congruence condition, $\Delta AOC \cong \Delta BOD$

Q5. Δ ABC is isosceles with AB = AC. Line segment AD bisects $\angle$ A and meets the base BC in D.

(i) Is $\Delta ADB \cong \Delta ADC$?

(ii) State the three pairs of matching parts used to answer (i).

(iii) Is it true to say that BD = DC?

Answer:

(i) We have AB = AC (Given)

$\angle$BAD = $\angle$CAD (AD bisects $\angle$ BAC)

And AD = AD (common)

Therefore by SAS condition of congruence, $\Delta ABD \cong \Delta ACD$

(ii) We have used AB, AC; $\angle$ BAD = $\angle$ CAD; AD, DA.

(iii) Now, $\Delta ABD \cong \Delta ACD$ therefore by c.p.c.t BD = DC.

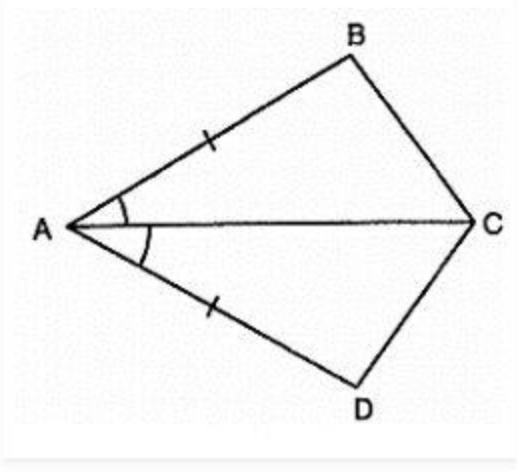
Q6. In Figure, AB = AD and $\angle$ BAC = $\angle$ DAC. (i) State in symbolic form the congruence of two triangles ABC and ADC that is true.

(ii) Complete each of the following, so as to make it true:

(a) $\angle$ ABC =

(b) $\angle$ ACD =

(c) Line segment AC bisects ___ and ___



Answer:

i) AB = AD (given)

$\angle$ BAC = $\angle$ DAC (given)

AC = CA (common)

Therefore by SAS condition of congruency, $\Delta ABC \cong \Delta ADC$

ii) $\angle$ ABC = $\angle$ ADC (c.p.c.t)

$\angle$ ACD = $\angle$ ACB (c.p.c.t)

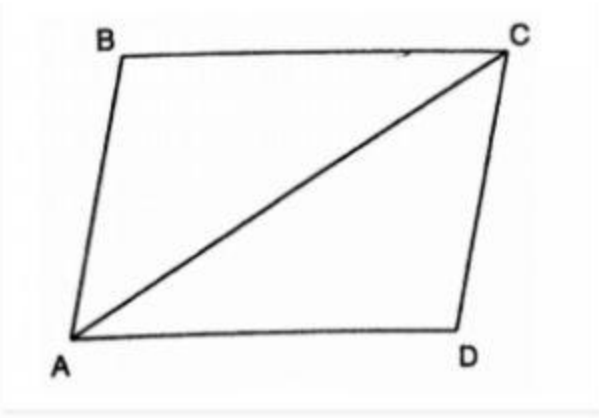
Q7. In figure, AB || DC and AB = DC.

(i) Is $\Delta ACD \cong \Delta CAB$?

(ii) State the three pairs of matching parts used to answer (i).

(iii) Which angle is equal to $\angle$ CAD ?

(iv) Does it follow from (iii) that AD || BC?



Answer:

(i) Yes by SAS condition of congruency, $\Delta DCA \cong \Delta BAC$

(ii) We have used AB = DC, AC = CA and $\angle$ DCA = $\angle$ BAC.

(iii) $\angle$ CAD = $\angle$ ACB since the two triangles are congruent.

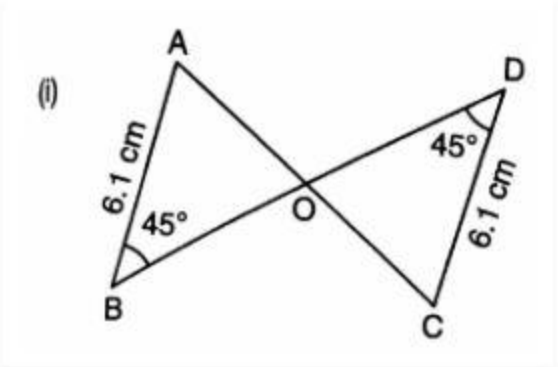
(iv) Yes this follows from AD // BC as alternate angles are equal. If alternate angles are equal the lines are parallel

Exercise 16.4

Q1. Which of the following pairs of triangle are congruent by ASA condition?

Answer:

i)



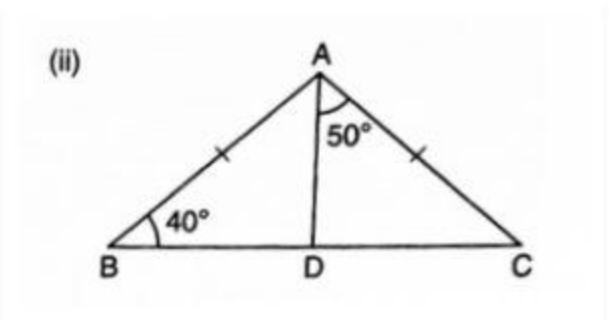
We have,

Since $\angle$ ABO = $\angle$ CDO = 45° and both are alternate angles, AB // DC, $\angle$ BAO = $\angle$ DCO (alternate angle, AB // CD and AC is a transversal line)

$\angle$ ABO = $\angle$ CDO = 45° (given in the figure) Also, AB = DC (Given in the figure)

Therefore, by ASA $\Delta AOB \cong \Delta DOC$

ii)



In ABC ,

Now AB =AC (Given)

$\angle$ ABD = $\angle$ ACD = 40° (Angles opposite to equal sides)

$\angle$ ABD + $\angle$ ACD + $\angle$ BAC = 180° (Angle sum property)

40° + 40° + $\angle$ BAC=180°

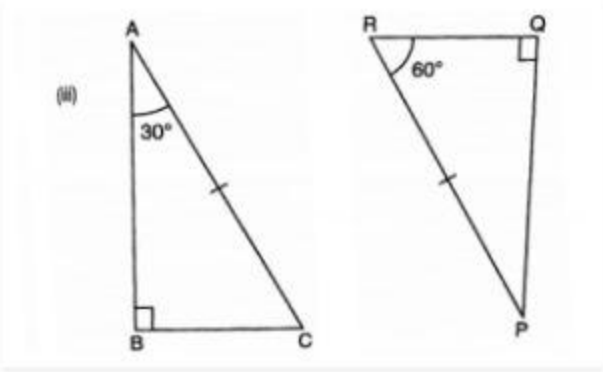
$\angle$ BAC =180°- 80° =100°

$\angle$ BAD + $\angle$ DAC = $\angle$ BAC $\angle$ BAD = $\angle$ BAC − $\angle$ DAC = 100° − 50° = 50°

$\angle$ BAD = $\angle$ CAD = 50°

Therefore, by ASA, $\Delta ABD \cong \Delta ADC$

iii)



In Δ ABC,

$\angle A + \angle B + \angle C = 180°$(Angle sum property)

$\angle C = 180° - \angle A - \angle B \angle C = 180° - 30° - 90° = 60°$

In PQR,

$\angle P + \angle Q + \angle R = 180°$(Angle sum property)

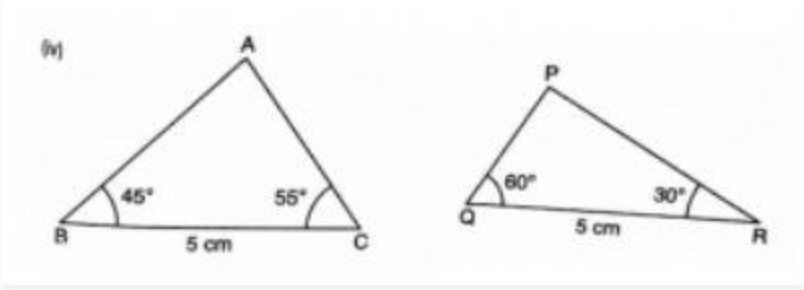
$\angle P = 180° - \angle Q - \angle R \angle P = 180° - 60° - 90° = 30°$

$\angle BAC = \angle QPR = 30°$

$\angle BCA = \angle PRQ = 60°$and AC = PR (Given)

Therefore, by ASA, $\Delta ABC \cong \Delta PQR$

iv)



We have only

BC = QR but none of the angles of Δ ABC and Δ PQR are equal.

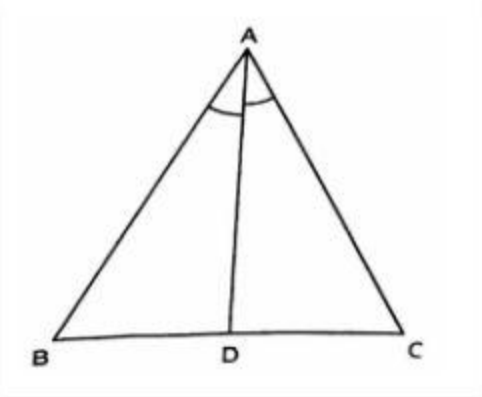
Therefore, $\Delta ABCandcong\Delta PRQ$

Q2. In figure, AD bisects A and AD and AD $\perp$ BC.

(i) Is $\Delta ADB \cong \Delta ADC$?

(ii) State the three pairs of matching parts you have used in (i)

(iii) Is it true to say that BD = DC?



Answer:

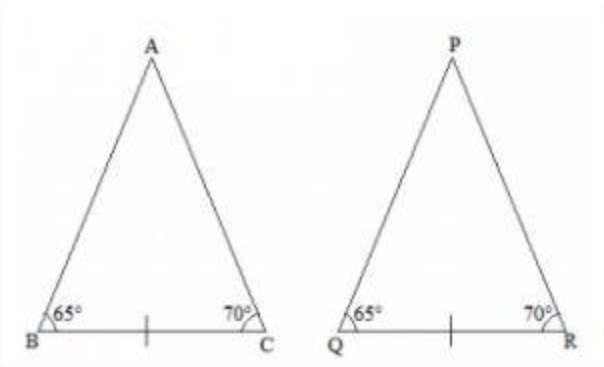
(i) Yes, $\Delta ADB \cong \Delta ADC$, by ASA criterion of congruency.

(ii) We have used $\angle$ BAD = $\angle$ CAD $\angle$ ADB = $\angle$ ADC = 90°

Since, AD $\perp$ BC and AD = DA

(iii) Yes, BD = DC since, $\Delta ADB \cong \Delta ADC$

Q3. Draw any triangle ABC. Use ASA condition to construct other triangle congruent to it.



Answer:

We have drawn

Δ ABC with $\angle$ ABC = 65° and $\angle$ ACB = 70°

We now construct $\Delta PQR \cong \Delta ABC$ has $\angle$ PQR = 65° and $\angle$ PRQ = 70°

Also we construct Δ PQR such that BC = QR

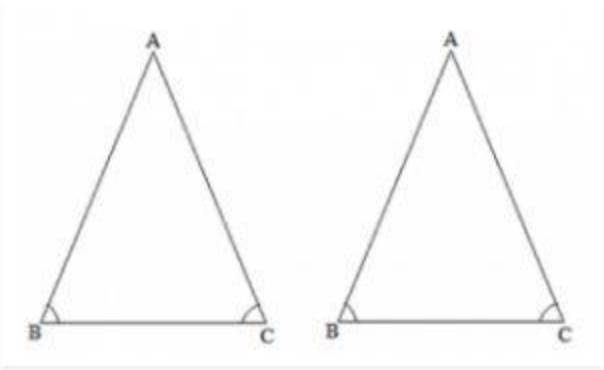
Therefore by ASA the two triangles are congruent

Q4. In Δ ABC, it is known that $\angle$ B = C. Imagine you have another copy of Δ ABC

(i) Is $\Delta ABC \cong \Delta ACB$

(ii) State the three pairs of matching parts you have used to answer (i).

(iii) Is it true to say that AB = AC?



Answer:

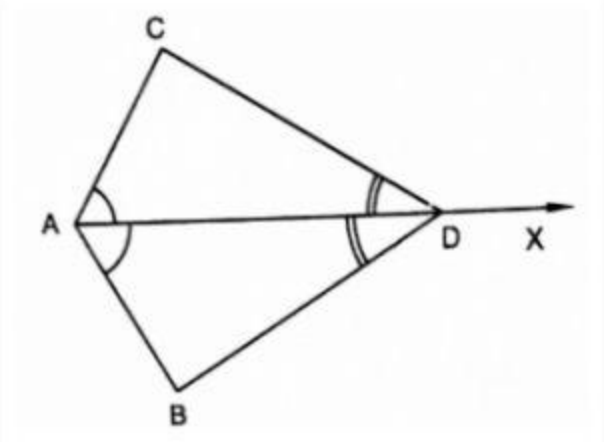
(i) Yes $\Delta ABC \cong \Delta ACB$

(ii) We have used $\angle$ ABC = $\angle$ ACB and $\angle$ ACB = $\angle$ ABC again.

Also BC = CB

(iii) Yes it is true to say that AB = AC since $\angle$ ABC = $\angle$ ACB.

Q5. In Figure, AX bisects $\angle$ BAC as well as $\angle$ BDC. State the three facts needed to ensure that $\Delta ACD \cong \Delta ABD$



Answer:

As per the given conditions, $\angle$ CAD = $\angle$ BAD and $\angle$ CDA = $\angle$ BDA (because AX bisects $\angle$ BAC)

AD = DA (common)

Therefore, by ASA, $\Delta ACD \cong \Delta ABD$

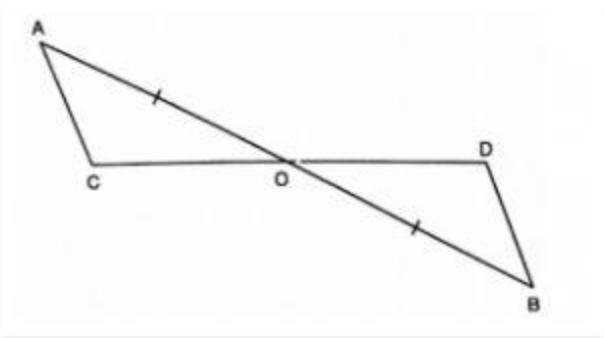
Q6. In Figure, AO = OB and $\angle$ A = $\angle$ B.

(i) Is $\Delta AOC \cong \Delta BOD$

(ii) State the matching pair you have used, which is not given in the question.

(iii) Is it true to say that $\angle$ ACO = $\angle$

BDO?



Answer:

We have

$\angle$ OAC = $\angle$ OBD,

AO = OB

Also, $\angle$ AOC = $\angle$ BOD (Opposite angles on same vertex)

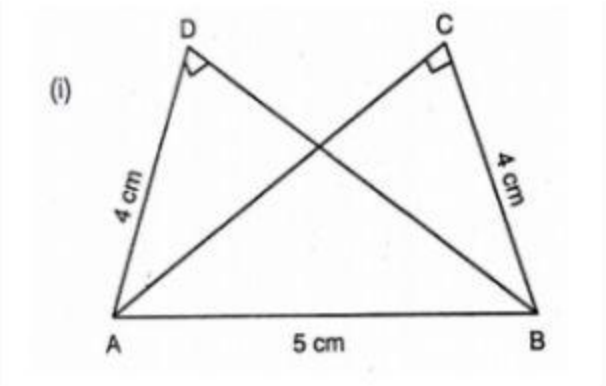
Therefore, by ASA $\Delta AOC \cong \Delta BOD$

Exercise 16.5

Q1. In each of the following pairs of right triangles, the measures of some part are indicated along side. State by the application of RHS congruence conditions which are congruent, and also state each result in symbolic form.

Answer:

i)

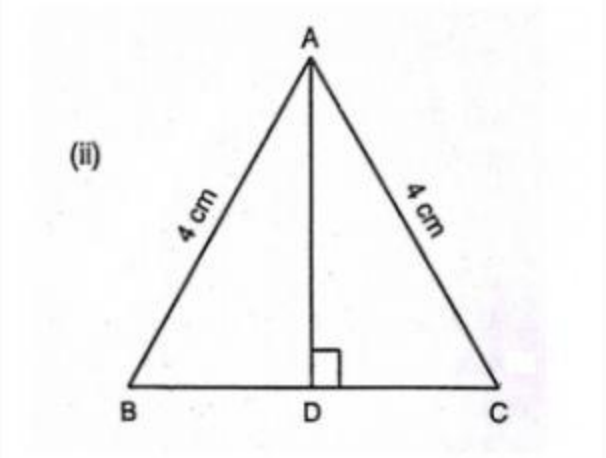


$\angle ADC = \angle BCA = 90°$

AD = BC and hyp AB = hyp AB

Therefore, by RHS $\Delta ADB \cong \Delta ACB$

ii)



AD = AD (Common)

hyp AC = hyp AB (Given)

$\angle$ ADB + $\angle$ ADC = 180° (Linear pair)

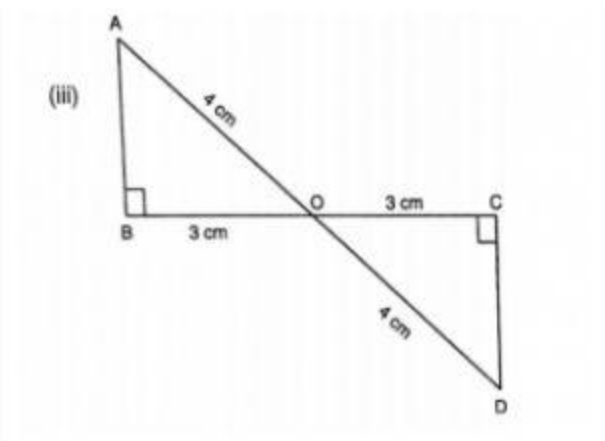
$\angle$ ADB + 90° = 180°

$\angle$ ADB = 180° − 90° = 90°

$\angle$ ADB = $\angle$ ADC = 90°

Therefore, by RHS Δ ADB = Δ ADC

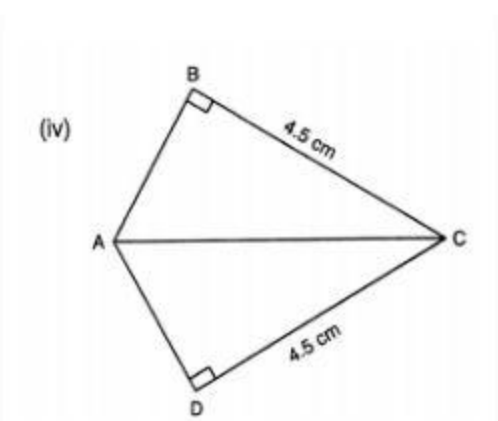
iii)



hyp AO = hyp DOBO = CO $\angle$ B = $\angle$ C = 90°

Therefore, by RHS, $\Delta AOB \cong \Delta DOC$

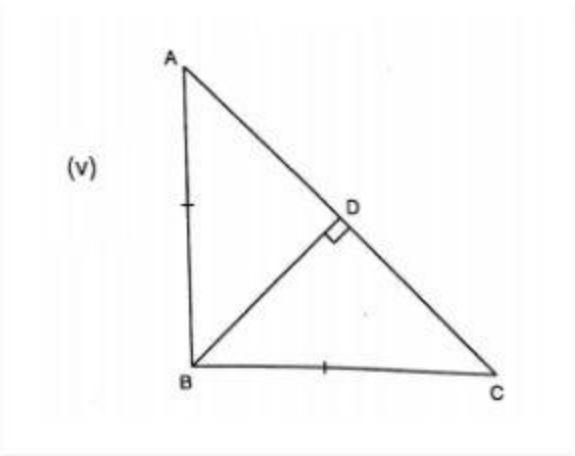
iv)



Hyp A = Hyp CABC = DC $\angle$ ABC = $\angle$ ADC = 90°

Therefore, by RHS, $\Delta ABC \cong \Delta ADC$

v)



BD = DB Hyp AB = Hyp BC, as per the given figure,

$\angle$ BDA + $\angle$ BDC = 180°

$\angle$ BDA + 90° = 180°

$\angle$ BDA= 180°- 90° = 90°

$\angle$ BDA = $\angle$ BDC = 90°

Therefore, by RHS, $\Delta ABD \cong \Delta CBD$

Q2. Δ ABC is isosceles with AB = AC. AD is the altitude from A on BC.

i) Is$\Delta ABD \cong \Delta ACD$?

(ii) State the pairs of matching parts you have used to answer (i).

(iii) Is it true to say that BD= DC?

Answer:

(i)Yes, $\Delta ABD \cong \Delta ACD$ by RHS congruence condition.

(ii) We have used Hyp AB = Hyp AC

AD = DA

$\angle$ ADB = $\angle$ ADC = 90° (AD $\perp$ BC at point D)

(iii)Yes, it is true to say that BD = DC (c.p.c.t) since we have already proved that the two triangles are congruent.

Q3. Δ ABC is isosceles with AB = AC. Also. AD $\perp$ BC meeting BC in D. Are the two triangles ABD and ACD congruent? State in symbolic form. Which congruence condition do you use? Which side of ADC equals BD? Which angle of Δ ADC equals $\angle$ B?

Answer:

We have AB = AC ..... (i)

AD = DA (common) .....(ii)

And, $\angle$ ADC = $\angle$ ADB (AD $\perp$ BC at point D) .....(iii)

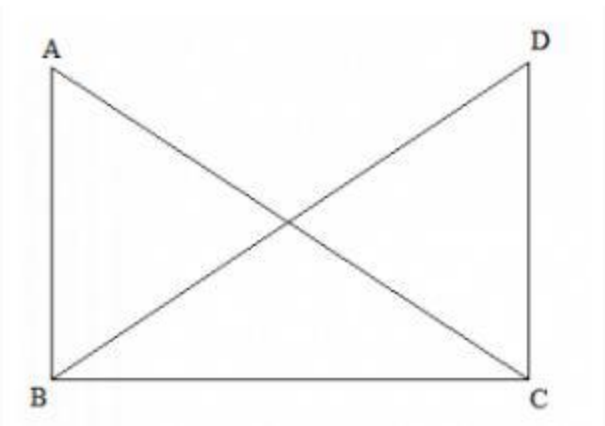
Therefore, from (i), (ii) and (iii), by RHS congruence condition, $\Delta ABD \cong \Delta ACD$, the triangles are congruent.

Therefore, BD = CD.

And $\angle$ ABD = $\angle$ ACD (c.p.c.t)

Q4. Draw a right triangle ABC. Use RHS condition to construct another triangle congruent to it.

Answer:



Consider

Δ ABC with $\angle$ B as right angle.

We now construct another triangle on base BC, such that $\angle$ C is a right angle and AB = DC

Also, BC = CB

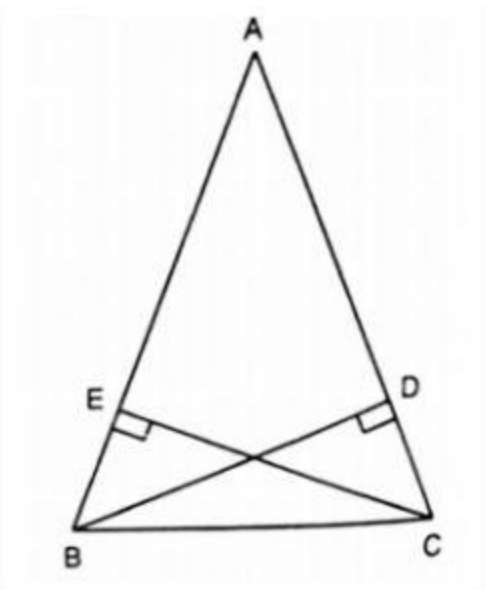
Therefore, BC = CB

Therefore by RHS, $\Delta ABC \cong \Delta DCB$

Q5. In figure, BD and CE are altitudes of Δ ABC and BD = CE.

(i) Is $\Delta BCD \cong \Delta CBE$?

(ii) State the three pairs or matching parts you have used to answer (i)



Answer:

(i) Yes, $\Delta BCD \cong \Delta CBE$ by RHS congruence condition.

(ii) We have used hyp BC = hyp CB

BD = CE (Given in question)

And $\angle$ BDC = $\angle$ CBE = 90^{o}