# HYDRAULICS

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**Fluid Pressure :** It is the pressure transmitted with equal intensity in all directions and acts normal to any plane. It is defined by limiting equation,

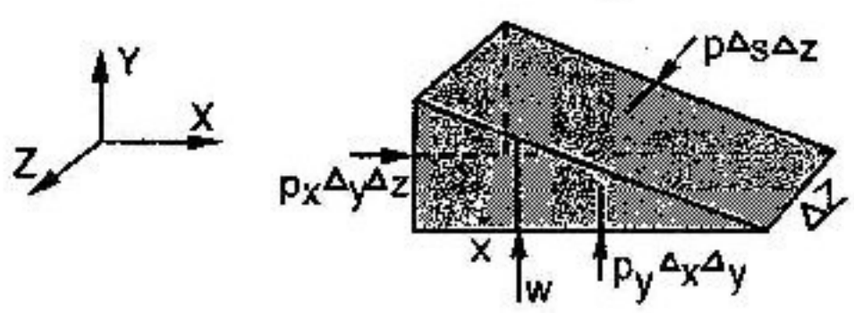
$$p = \lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A}$$

and measured in $N/m^2$ of $kN/m^2$ or bar and 1 bar = $10^5$ $N/m^2$ = atm pressure at sea-level.

Also 1 Pascal = 1 $N/m^2$

**Pressure at a Point (Pascal's Law) :** Pressure intensity at a point is defined as the *ratio of force acting on an elemental area and the elemental area as the area tends to zero at the point.* This pressure is always normal to the surface, as fluid at rest cannot sustain any shear stress. If an elemental area is allowed to rotate freely in a fluid at rest, the pressure on the area will be of constant magnitude and is independent of the orientation of the area at the point. Then the pressure being scalar quantity must be same in all directions at a point. This is known as Pascal's Law.

Pressure at a point can be considered at the average of any three mutually perpendicular normal compressive stresses at a point, *i.e.* $\frac{1}{3}(p_x + p_y + p_z)$




**Variation of Static Pressure :**

(*i*) $\frac{dp}{dx} = 0$, *i.e.* pressure does not vary in the x-direction or horizontal direction.

(*ii*) In $z$-direction, $-\frac{dp}{dz} = \gamma$, where $\gamma$ is density of fluid *i.e.* the static pressure is a function of $z$ only.

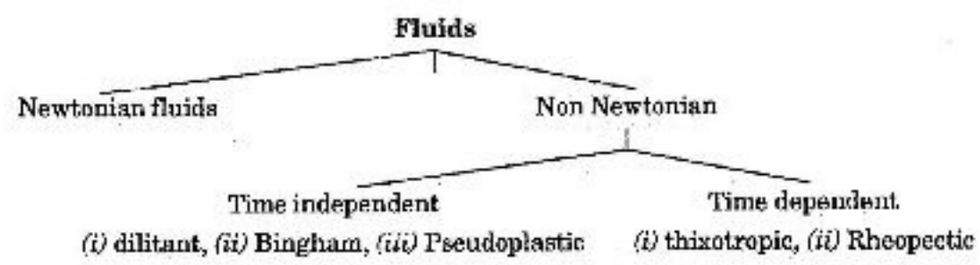
For incompressible fluid $\gamma$ is constant, therefore piezometric head = $\frac{p}{\gamma} + z$ = constant, *i.e.* for incompressible fluid the piezometric head is constant throughout the fluid.

**Hydrostatic Law of Pressure Variation :**

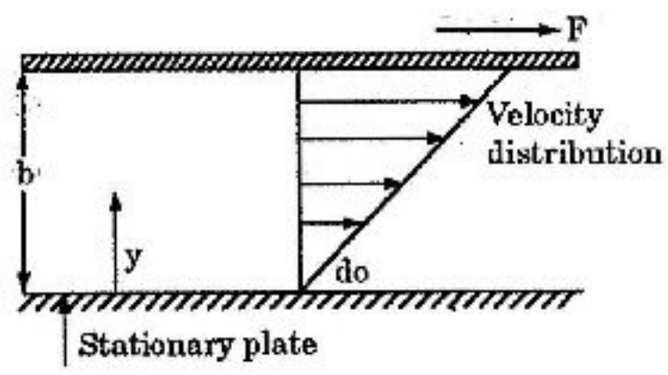
$$p = \gamma(H - z)$$

If distances are measured from free surface, the pressure at a distance $h$ below the free surface will $\gamma h$. This equation is called hydrostatic law of pressure variation.

## FLUIDS




(*i*) **Newtonian Fluids :** Newtonian fluids tend to have a certain constant viscosity. For instance, many common fluids air, water, light oils gasoline or shear stress is linearly dependent upon velocity gradient.




$$\frac{F}{A} \propto \frac{d\theta}{dt}$$

or $$\frac{F}{A} = \mu \frac{d\theta}{dt}$$

$$= \mu \frac{U}{b}$$

where $\mu$, is proportionality constant and called coefficient of viscosity.

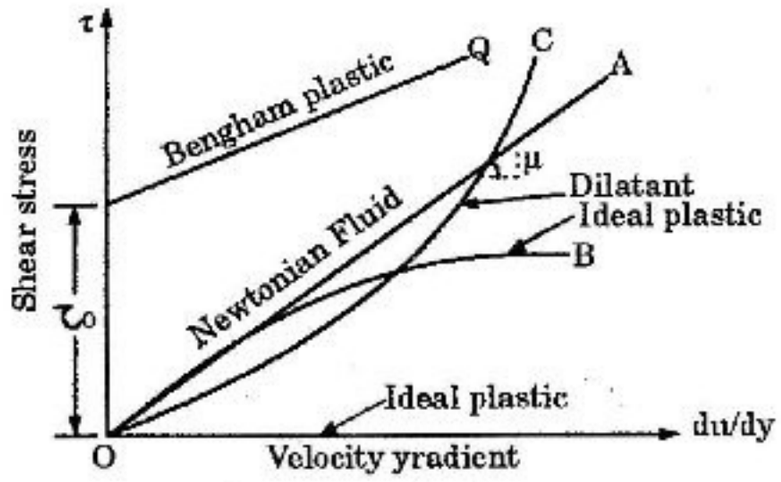
If shear stress is denoted by,

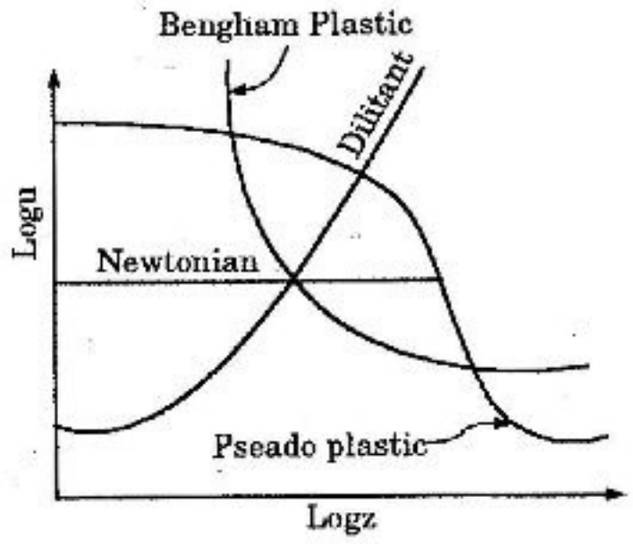
$$\tau = \frac{F}{A}$$

then $$\tau = \mu \frac{U}{b} = \mu \frac{u}{y}$$

or $$\tau = \mu \frac{du}{dy}$$

The rate change of angular deformation is proportional to the shear stress, *i.e.*




A Newtonain fluid is a special case of power law fluid having $n = 1$ and B = 0 and the constant A varying only with the type of fluid.

**Ideal Fluid :** It has been utilised in the analytical treatment of flow phenomenon. An imaginary flow of fluid non viscous and incompressible (inelastic) is known as *ideal fluid*. In ideal fluid shear stress is linearly dependent on velocity gradient.

(*ii*) **Non Newtonian Fluid :** Certain fluids which do not obey Newtonian fluid, *i.e.* shear stress is not linearly dependent upon velocity gradient. Common examples are, human blood, lubricating oils, clay suspension in water etc.

A general relationship of non newtonian fluid is :

$$\tau = A\left(\frac{dx}{dy}\right) + B$$

where A and B are constants which depends on the type of fluid and conditions imposed on the shear stress. These are also known as power fluids.

B is zero for all fluids except Bingham.

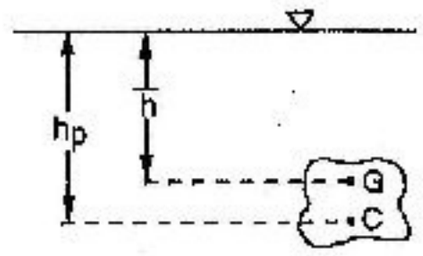
(*a*) **Dilatant :** If $n > 1$ (*e.g.* quick sand, butter printing inks)

(*b*) **Bingham plastic :** If $n = 1$ (*e.g.* sewage sludge drilling muds)

(*c*) **Pseudoplastic :** if $n < 1$ (*e.g.* paper pulp, polymeric solutions, rubber suspensions and points.)

**Pressure on Fully Immersed Bodies :**

(*a*) **Plane lamina immersed vertically :** On a plane lamina immersed vertically in a fluid of specific weight $\gamma$, the total pressure force




$$p = \gamma A\bar{h} \qquad ...(i)$$

where A = area of the lamina and

$\bar{h}$ = depth of its center of gravity, G from the free surface.

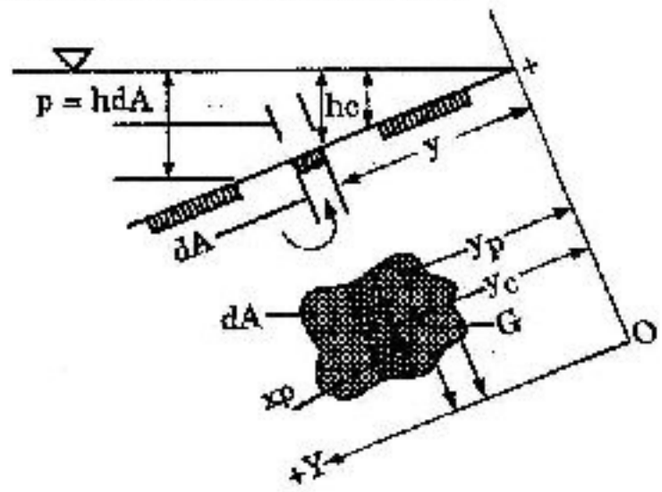
This total pressure force P acts at a point C which is called Center of Pressure. It lies on the same vertical line as 'G' does. The depth $h_p$ of center of pressure of vertical lamina from free surface is given by,

$$h_p = \frac{I_G}{A\bar{h}} + \bar{h} \qquad ...(ii)$$

where $I_G$ = moments of inertia of the lamina about an axis passing through the center of gravity of the lamina and parallel to the free surface.

In equation (*ii*), the value of term $\frac{I_G}{A\bar{h}}$ decreases with increasing value of $\bar{h}$, *i.e.* depth of center of gravity from free surface, and therefore $h_p \to \bar{h}$ ($h_p$ tends to $\bar{h}$) as $\bar{h} \to \infty$. This also concludes that center of pressure is always below the center of gravity.

(*b*) **Inclined Immersed Bodies :**




If θ is the inclination of the body with the free surface, $\bar{h}$ is the vertical depth of C.G. of the body inclined position from the free surface, then vertical distance of center of pressure is given by

$$h_p = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

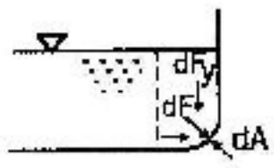
The depth of center of pressure from the free surface is given by,

$$h_p = y_p \sin\theta$$

**Forces on Curved Surface :**

Total forces on a curved surface,

$$F = \sqrt{F_x^2 + F_y^2}$$




where, $F_x$ = total pressure force on the projected area of the curved surface on the vertical plane

$= \gamma A\bar{h}$

$F_y$ = weight of liquid actually or imaginary supported by the curved surface.

Inclination of the resultant force on curved surface with horizontal,

$$\tan\theta = \frac{F_y}{F_x}$$

**Buoyancy and Floatation :**

**Archimedes' Principle:** When a body is totally or partially immersed in a fluid, it is lifted up by a force which is equal to the weight of the fluid displaced by the body. This vertical upward force is called the center of buoyancy (C.B) and it is at the center of gravity of the displaced fluid.

**Stability of Submerged Bodies:** For stability of submerged body, the center of gravity of the body must lie directly below the center of buoyancy (gravity) of the displaced fluid. The submerged body is in neutral equilibrium of all positions as long as the w.f. of the body and C.B. coincides.

**Floatation:** A body immersed in any fluid is acted upon by weight of body W acting vertically downward and buoyant force $F_b$ acting vertically upwards.

If $W > F_B$, the body will sink.

If $W = F_B$, the body will remain in equilibrium at any level.

If $W < F_B$, the buoyant force will move the body upwards and the body will come to rest, when the buoyant force becomes equal to its weight. Then the body is said to be *floating*.

**Principal of Floation:** The weight of a floating body is equal to the weight of the fluid displaced by the body.

**Stability of Floating Bodies:** For stability of floating cylinder or spheres the center of buoyancy (C.B.) should lie considerably below the w.f. (G) the body.

Stability of other floating bodies will depend upon whether a righting or overturning moment is developed when the w.f. and C.B. move out of the vertical alignment due to the shifting of the position of C.B. The C.B. will shift because of the floating body tilts, the shape of displaced liquid changes and hence its w.f. shifts.

**Metacenter:** It is the point of intersection of line joining previous C.B. and w.f. (G) and vertical through the new C.B.

Larger is the metacentric height (GM), more stable is the body.

*The body is in*

(*a*) stable equilibrium, if M is above G, or GM is positive,

(*b*) unstable equilibrium, if M is below G,

(*c*) neutral equilibrium if M coincides with G.

The metacentric height is given by,

$$GM = \frac{I_{yy}}{V} - BG$$

Experimentaly, $GM = \dfrac{w.l}{W\tan\theta}$

where, $w$ is the moving weight through a distance $l$ causing a vessel of weight W to bit through an angle $\theta$.

## KINEMATICS OF FLOW

A particular quantity of matter or specified region in space is designated as a '*System*'. All matter external to system is called its 'Surroundings'.

A closed system refers to a specified mass limited by the boundaries, *e.g.* a kg mass of air contained in a cylinder.

**An open system** or control volume refers to definite fixed region in space through which the matter moves *e.g.*, flow of air through a pipe.

**Process** is a path of succession of state through which the systems passes e.g., change in pressure, velocity, elevation, density, temperature.

**Types of Flow:**

If F is a fluid or flow property such as velocity, pressure, mass, density or temperature, then the types of flows can be defined as follows :

(*i*) **Steady Flow:** $\left(\dfrac{\partial F}{\partial t}\right)_{\text{at a point or section}} = 0$

**Unsteady Flow:** $\left(\dfrac{\partial F}{\partial t}\right)_{\text{at a point or section}} \neq 0$

(*ii*) **Uniform Flow:** $\left(\dfrac{\partial F}{\partial S}\right)_{t=t_0} = 0$

*i.e.*, the magnitude and direction of the velocity do not change from point to point in the fluid space.

**Non-uniform Flow:** $\left(\dfrac{\partial F}{\partial S}\right)_{t=t_0} \neq d$

(*iii*) **One-dimensional Flow :** $F = f(x, t)$, *i.e.* direction and magnitude of velocity at all points are identical.

**Two-dimensional Flow:** F = $f(x, y, z, t)$, i.e. flow parameters are functions of time and two rectangular space coordinates say $x$ and $y$ and fluid particles move in planes or parallel planes.

**Three-dimensional Flow:** F = $f(x, y, z, t)$ flow parameters are function of time and three mutually perpendicular directions.

(*iv*) **Irrotational Flow:** If the fluid elements do not rotate about their mass axis which are perpendicular to the plane of motion. For irrotational flow the angular velocity should be zero everywhere in the fluids, which gives

For 3-D flow : $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}, \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial x} = \frac{\partial v}{\partial z}$

**Rotational Flow:** If at every point in the flow the above condition is not satisfied, the flow is known as rotational flow, i.e. the net rotation of the fluid element is not zero.

**Stream Lines, Path Lines, Streak Lines and Stream Tube**

Stream lines are imaginary lines drawn through a flow field such that tangent to the line at any point on the line indicates the direction of motion at that instant.

Equation of stream line is,

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Path Line is the path followed by a fluid particle during its travel.

Streak line is the path followed by all the fluid particles passing through a given point in space.

Stream Tube is an cementary portion of a flowing fluid bounded by a group of stream lines which confine the flow.

**CONTINUITY EQUATIONS**

**For Incompressible Fluid:**

$$Q = A_1V_1 = A_2V_2,$$

where $V_1$ and $V_2$ are velocities at sections (1) and (2) respectively and Q is the volume rate of flow. ,

**For Compressible Fluids**

(*i*) For unsteady 3-D flow:

$$\frac{\partial \rho}{\partial t} = \left[\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right]$$

(*ii*) For steady flow :

$$\left[\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right] = 0$$

**Fore Steady Incompressible Flow:** ($\rho$ = constant)

Three dimensional equation become,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

For Two-dimensional flow since,

$$\frac{\partial w}{\partial z} = 0$$

$$\therefore \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

**For One-dimensional Flow:**

Since $\frac{\partial v}{\partial y} = 0$, and $\frac{\partial w}{\partial z} = 0$, therefore $\frac{\partial u}{\partial x} = 0$

**FLUID DISPLACEMENTS**

Consider all possible motion of a cubical fluid element.

(*a*) **Translations:** When this element is bodily moved from one position to another position Fig. (*a*), such that the median lines do not change in length and they are parallel to their respective former positions, then the movement is called translation.

(*b*) **Linear Deformation:** When there is change in the length of median line without change in the orientation, such a type of motion is called linear deformation as shown in Fig. (*b*). Two-dimensional fluid element of constant density will have both side lengths altered due to linear deformation.

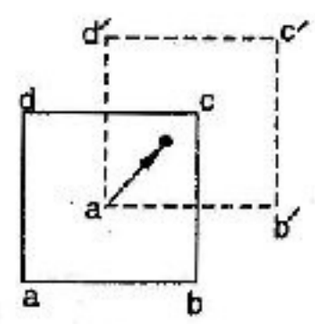
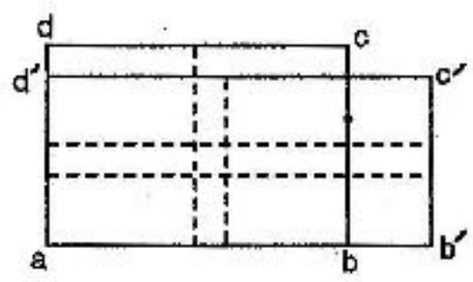



*Fig. (a) Translation*




*(b) Linear Deformation*

(*c*) **Angular Deformation:** When the angle between two adjacent sides of a fluid element change as shown in Fig. (*c*) such a type of motion is called angular deformation. The angular deformation results in an incompressible fluid, the volume of the fluid element remains constant, while the deformation takes place.

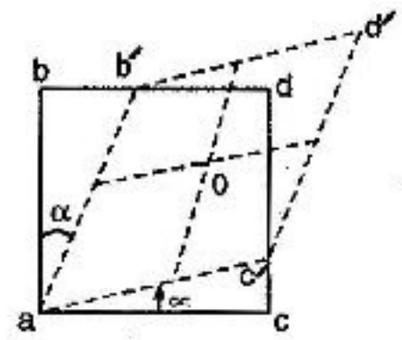
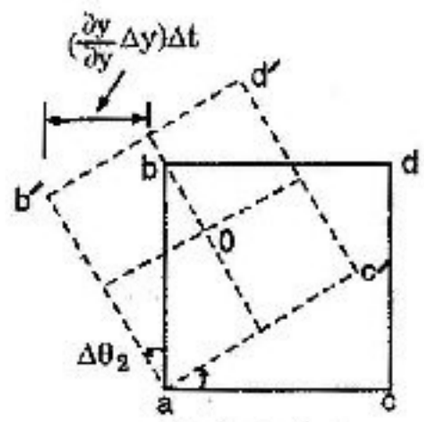



*Fig. (c) Angular Deformation*




*(d) Rotation*

(*d*) **Rotation:** In the Fig. (*d*), the fluid element bodily rotates about the *z*-axis, i.e. perpendicular to the paper at point *a*. The average of the sum of the angular velocities of sides *ab* and *ad* is the net rotation of the fluid element about *z*-axis.

$$\text{Average angular velocity} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}\right)$$

The above value is called rotation of the fluid element about z-axis. Twice the value is called vorticity.

For irrotational flow, rotation would be zero and hence $\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y}$

For a three-dimensional flow, the following equations represents rotation about *x*, *y* and *z*-axis.

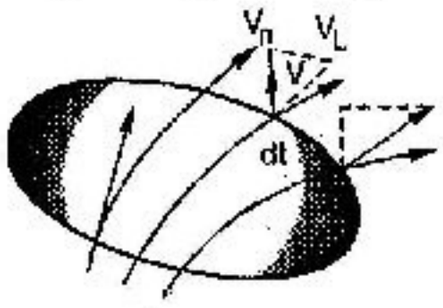
$$2\omega_x = \frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial z} = 0,$$

$$2\omega_y = \frac{\partial v}{\partial z} - \frac{\partial \omega}{\partial x} = 0,$$

$$2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial v}{\partial y} = 0$$

## CIRCULATION

Circulation (τ) around a closed curved C is defined as the line integral of $V_L$, $d_L$ along the curve. C is taken positive in the clockwise direction. The velocity vector normal to the length is $V_n$ and tangential to it is $V_L$.




$$\therefore \qquad T = \int V_L . d_L \ .$$

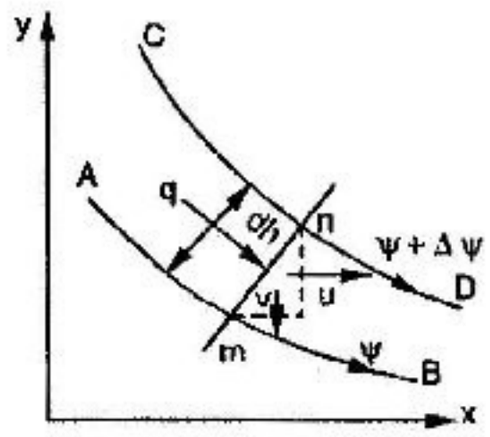
Vorticity : Ratio of the differential circulation around an infinitesimal closed curve enclosing the point to the area of the closed curve.

$$\frac{\tau}{\text{area enclosed by curve C}} = 2 \times \text{rotation perpendicular to area} = 2\omega$$

The quantity 2ω is known as velocity, which is also a vector quantity.

## STREAM FUNCTION (Ψ)

Flow rate or mass per unit time across any line between the two stream lines and per unit thickness is known as stream function.




$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0$$

This equation shows that for an irrotational flow, the stream function must be also satisfy the Laplace's equation.

- It may be emphasized that stream function finds its application in two dimensional flow.
- Lines of constant stream functions represent stream lines and that the numerical difference between two stream lines is equal to the flow rate per unit width passing between them.

### Properties of Stream Function (Ψ)

(*i*) Ψ is constant everywhere on anyone stream line.

(*ii*) If the flow is continuous, the flow around any path in the fluid is zero.

(*iii*) Rate of change of Ψ with distance in any arbitrary direction is proportional to the component of velocity normal to that direction.

(*iv*) Velocity vectors may be found by differentiating the stream function.

(*v*) Algebraic sum of stream functions for two incompressible flow pattern is the stream function for the flow resulting from the super-imposition of these patterns, *i.e.*

$$\frac{\partial \Psi_1}{\partial s} + \frac{\partial \Psi_2}{\partial s} = \frac{\partial(\Psi_1 + \Psi_2)}{\partial s}$$

### Velocity Potential (ϕ) :

It is a scalar function of *x*, *y*, *z* and such that its negative derivative with respect to any of *x*, *y*, *z* gives the velocity component in that direction.

Therefore, $u = -\frac{\partial\phi}{\partial x}, v = -\frac{\partial\phi}{\partial y}$ and $w = -\frac{\partial\phi}{\partial z}$

Velocity potential $\phi$ is given by,

$$\phi = \int v_z ds$$

$$d\phi = -v_s ds$$

or $$v_s = -\frac{-d\phi}{ds}$$

The velocity potential is a scalar quantity dependent upon space and time. Its negative derivative with respect to any direction gives velocity in that direction.

$$u = -\frac{\partial\phi}{\partial e}, v = -\frac{\partial\phi}{\partial y}, w = -\frac{\partial\phi}{\partial z}$$

**In polar coordinates:**

$$V_r = -\frac{d\phi}{\partial r}, V_\theta = -\frac{1}{r}\frac{\partial\phi}{\partial\theta}, V_z = -\frac{\partial\phi}{\partial z}$$

When velocity potential function or $\phi$ exists it must satisfy the continuity equation or the Laplace equation.

*i.e.* $$-\frac{\partial\mu}{\partial e}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0; \quad \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2} = 0$$

If velocity potential exists, its vorticity is zero or the fluid is irrotational if $w_x$ and $w_y$ are angular velocity, then

$$w_x = 0$$

$$w_y = 0$$

{since vorticity = 2 × angular velocity}

*i.e.* $$w_x = \frac{1}{2}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right) = 0$$

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$$

$$w_y = \frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)$$

**Relation between Stream Function and Velocity Potential**

The stream function $\Psi$ is applicable both for rotational and irrotational flows. But potential function is aplicable only for irrotational flow. So for an irrotational steady, incompressible flow

$$u = -\frac{\partial\phi}{\partial x} = \frac{\partial\Psi}{\partial y}$$

and $$V = -\frac{\partial\phi}{\partial y} = \frac{\partial\Psi}{\partial x}$$

Therefore, $$\frac{\partial\Psi}{\partial y} = -\frac{\partial\phi}{\partial x}$$

and $$\frac{\partial\Psi}{\partial x} = \frac{\partial\phi}{\partial x}$$

This equation is known as Cauchy-Riemann Equation.

**Laminar and Turbulent Flow and Flow in Pipes (Viscous Flow)**

**Laminar Flow:** In laminar flow the fluid particles move along straight, parallel paths in layers or laminae. The magnitudes of the velocities of adjacent laminae are not same.

**Characteristics of Laminar Flow**

(*i*) The flow is goverened by the law relating to rate of angular deformation, *i.e.*

$$\tau = \mu\frac{dv}{dy} \text{ for flow in x direction.}$$

(*ii*) There is practically no interference of fluid particles of one layer over those of adjacent layer.

(*iii*) There is "*no slip*" condition occurs at the boundary.

(*iv*) The flow is rotational.

(*v*) There is continuous dissipation of energy due to viscous shear and energy must be supplied externally to maintain the flow.

(*vi*) Energy loss is proportional to first power of velocity and viscosity.

(*vii*) Flow remains laminar as long as $\frac{V.l\,\rho}{\mu} < R_{critical}$.

**Turbulent Flow :** In turbulent flow, the particles of the fluid move in a haphazard manner in all directions. It is impossible to trace the motion of an individual particle. The shear stress for turbulent flow is given by

$$\tau = (\mu + 1),$$

where $\eta$ is a factor depending upon the density of the fluid and the fluid motion.

**Critical Velocity:** The velocity at which the flow changes from laminar to turbulent is called critical velocity. It is the velocity at lower critical Reynolds number. It is different for different fluids.

**Reynolds Number:** Reynolds showed that the frictional resistance in

(*i*) Laminar flow depends upon velocity of flow, area of surface in contact, temperature of fluid and is independent of nature of surface in contact and pressure of flow.

(*ii*) Turbulent flow depends upon square of velocity of flow, area of surface in contact, temperature, density of fluid and nature of contact and is independent of pressure of flow.

Reynolds determined a non-dimensional quantity called Reynolds number (R).

For circular pipe flowing full,

$$R = \frac{\rho V d}{\mu} = \frac{Vd}{\nu}$$

where V is mean velocity, $d$ is diameter of pipe, $\nu$ is kinematic viscosity of the fluid, $\rho$ is mass density of fluid and $\mu$ is absolute viscosity.

For non-circular cross-section, the ratio of the cross-sectional area to the wetted perimeter is called the hydraulic radius (R). It is used in the Reynolds number, *viz.*

$$R = \frac{V(4R)}{\nu}$$

**Upper and Lower Critical Reynolds Number:** The lower critical Reynolds number is defined as that value of Reynolds number at which the flow essentially remains laminar and all turbulences entering through whatever sources are damped down. The upper critical Reynolds number is that value at and above which the flow can never remain laminar. In between these two values the flow character is transitory.

| Case | Characteristic length Reynolds Number | Upper Critical |
|---|---|---|
| 1. Pipe Flow | | 4000 |
| 2. Open Channel Flow | (*a*) Hydraulic Mean Depth R<br>(*b*) Flow depth '*y*' | 2000<br>300 |
| 3. Parallel Plates | Spaced '*a*' apart | 3000 |
| 4. Movement of small sphere through stationary | Sphere diameter | |

Discharge velocity, $u = k \cdot \frac{h}{l} = (-)\, k \frac{d(p/r+z)}{dl}$

This law is valid for velocities from 3 to 4.5 mm/sec.

**Equation of Motion (Navier Stokes Equation):**

Navier Stoke's equation are the dynamic equation governing the flow when body force, pressure force and force due to viscosity affect the motion.

For incompressible fluid flow,

$$a_x = X - \frac{1}{\rho}\cdot\frac{\partial p}{\partial x} + \frac{\mu}{\rho}(\nabla^2 u)$$

$$a_y = Y - \frac{1}{\rho}\cdot\frac{\partial p}{\partial y} + \frac{\mu}{\rho}(\nabla^2 v)$$

and $$a_z = Z - \frac{1}{\rho}\cdot\frac{\partial p}{\partial y} + \frac{\mu}{\rho}(\nabla^2 w)$$

where $$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

These equations are known as Navier-Stokes Equation of motion for viscous fluid flow.

**Relation between Shear Stress and Pressure Gradient**

For steady two-dimensional uniform flows,

$$-\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} = 0$$

But $\tau = \mu\frac{\partial u}{\partial y}$, therefore $\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$

Therefore the pressure gradient in the direction of flow is equal to the shear gradient normal to the direction of motion.

**STOKE'S LAW :**

Resisting force or Drag on a small sphere moving under laminar condition in a viscous fluid was first calcuated by Stokes and is known as Stoke's solution.

Total drag, $F_D = 3\pi\mu Vd$

Out of which $\pi\mu Vd$ is contributed by the pressure and remaining $2\pi\mu Vd$ is contributed by the shear stress on the body.

The sphere after a short distance of fall in the viscous fluid attains a constant velocity called terminal velocity. To attain this equilibrium of the sphere under the steady state condition, the drag force and buoyant force must add up to balance the weight of the falling sphere.

At equilibrium, $F_D = 3\pi \cdot \mu \cdot V \cdot d \cdot = \frac{\pi d^3}{6}(\gamma_s - \gamma)$

and terminal velocity, $V = \frac{d^2}{18\mu}(\gamma_s - \gamma)$

Above expressions are valid for R < 1 and is known as Stokes' law.

**The Drag Force** exprienced by the sphere is generally expressed as

$$F_D = \frac{\rho V^2}{C_D 2} A$$

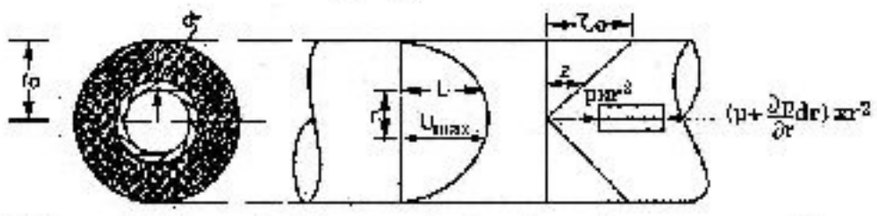
where A is area of cross-section of the moving body normal to the direction of motion and $C_D$ is known as the coefficient of drag.

**Steady Laminar Flow in Horizontal Circular Pipe (Hagen-Poisuile Equation)**

For steady laminar flow to occur in the pipe, a pressure gradient is maintained in the direction of flow which overcomes the frictional forces (or shear forces) opposing the flow. It is also assumed that the pressure is distributed unformly across the pipe cross-section.

(*a*) **Shear Stress Disrtibution :**

$$\tau = -\frac{r}{2}.\frac{dp}{dl}$$



This relation holds good for laminar as well as turbulent flow. The negative sign indicates that the pressure is decreasing in the direction of flow;

At $r = r_0$, Max. $\tau_0 = (-)\frac{r_0}{2}.\frac{dp}{dl}$

and at $r = 0$, $\tau = 0$

As shear stress is directly proportional to distance from centreline, shear stress distribution is linear.

(*b*) **Velocity Distribution :** Velocity distribution for laminar flow is obtained as,

$$u = (-)\frac{1}{4\mu}\left(\frac{\partial p}{\partial l}\right)\left(r_0^2 - r^2\right)$$

This is equation of a parabola, so the maixmum velocity varies parabolically along the diameter.

(*c*) **Maximum Velocity :** The maximum velocity will be obtained if $r = 0$, *i.e.* at the centre.

It is given by, $U_{max} = (-)\frac{1}{4\mu}\left(\frac{\partial p}{\partial l}\right).r_0^2$

(*d*) **Discharge :** $Q = (-)\frac{1}{8\mu}\left(\frac{\partial p}{\partial l}\right).\pi r_0^4$

(*e*) **Average or Mean Velocity :** The mean velocity of flow V through the pipe is obtained from continuity equation

$$V = \frac{Q}{\pi r_0^2} = (-)\frac{1}{8\mu}\left(\frac{\partial p}{\partial l}\right).r_0^2$$

Let average velocity V occurs at radius $r_1$, therefore equating u to V, we get

$$(-)\frac{1}{8\mu}\left(\frac{\partial p}{\partial l}\right).\pi r_0^2 = (-)\frac{1}{4\mu}\frac{\partial p}{\partial l}.\left(r_0^2 - r_1^2\right)$$

On simplification, $r_1 = 0.707\ r_0$, *i.e.* the average velocity occurs at a radius of $0.707\ r_0$.

(*f*) **Head Loss :** From Hagen-Poiseulli's equation for laminar flow

Head less, $h_l = \frac{p_1 - p_2}{\gamma} = \frac{8\mu\ VL}{\gamma\ r_0^2} = \frac{32\mu\ VL}{\gamma\ d^2}$

(g) **Relation between Darcy's friction coefficient *f* and R :** For laminar flow through circular pipe (*i.e.* for $R \le 2000$),

$$f = \frac{64}{R}$$

(*h*) **Local Velocity in terms of maximum velocity :**

$$u = U_{max}\left(1 - \frac{r^2}{r_0^2}\right)$$

**CORRECTION FACTORS**

**Kinetic Energy Correction Factor (α) :** Due to viscosity, the velocity distribution across any cross-section in real fluid flow is non-uniform and **boundary resistance** and so the kinetic energy per unit weight given by $\frac{V^2}{2g}$ does not represent the true kinetic energy across the section. To compensate the discrepancy, a coefficient known as kinetic energy correction factor ($\alpha$) is used.

Consider the velocity distribution as shown in the figure.

The total kinetic energy per unit time through any cross-section in a fluid flow

$$= \int_A \frac{v^2}{2}\rho v\ .\ dA$$

$$= \int_A \frac{1}{2}\rho v^3 dA$$

But actual kinetic energy based on average velocity V, passing the section

$$= \alpha\ .\frac{1}{2}\rho V^3 A$$

Equating, we get $\alpha = \frac{1}{A}\int_A\left(\frac{v}{V}\right)dA$

where $v$ = velocity at any point in the cross-section.

$\alpha = 1.0$ for uniform velocity distribution

$\alpha = 1.02$ to $1.05$ for turbulent flows

and $\alpha = 2.$ for laminar flow.

**Momentum Correction factor (β) :** Due to non-uniform distribution of velocity, the actual momentum transferred across the section is different than that computed using the average velocity. It is given by

$$\beta = \frac{1}{A}\int\left(\frac{v}{V}\right)^2 dA$$

For laminar flow in pipe $\beta = 1.33$ and for turbulent flow in pipe, $\beta$ varies from 1.01 to 1.07.

**TURBULENT FLOW**

The flow establishment length is given by,

$L = 0.057\ R\ d$..... for laminar flow

and $L = 0.7\ R\ d$..... for turbulent flow

where '*d*' is the diameter of the pipe and R is Reynolds Number of flow.

**Darcy's Friction Factor**

In smooth pipes (due to Blasius):

$$f = \frac{64}{R} \text{ for laminar flow}$$

and $$f = \frac{0.3164}{(R^{1/4})} \text{ for turbulent flow}$$

**Head Losses**

(*i*) **Major Head Loss :** Head loss due to friction falls in this category and is given by Darcy-Weisbach's equation,

$$h_l = \frac{fV^2}{2gd} = \frac{32\mu Vl}{\lambda d^2}$$

where $l$ is length of pipe, V is average velocity of flow, $\lambda$ is the specific weight of fluid and $\mu$ is coefficient of dynamic viscosity.

(*ii*) **Minor Head Losses**

(*a*) At the entrance,

$$h = 0.5\frac{V^2}{2g}$$

(*b*) Due to sudden contraction: If $V_2$ is the velocity in the contracted portion, we have

$$h_l = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g},$$

where coefficient of contraction '$C_c$' is normally taken as 0.62 or, $h_l = 0.5\frac{V_2^2}{2g}$

(*c*) Sudden Expansion : If $V_1$ and $V_2$ are the velocities in the pre-enlargement and enlarged section respectively, then the head loss is given by,

$$h_l = \frac{(V_1 - V_2)^2}{2g}$$

**Other Head Losses**

The other head losses are those in bends, pipe fitting and joineries such as valves, reducers, T-junctions, etc. If V is the velocity of flow in the pipe, these losses are expressed in terms of velocity head $\frac{V^2}{2g}$ as K times velocity head

$$h_l = \frac{KV^2}{2g}$$

where, K is called 'loss coefficient'.

**Equivalent Length**

It is the length of an unobstructed straight pipe in which an equal loss of energy due to friction will occur. If K is loss coefficient for a pipe fitting, in a pipe of diameter $d$, then pipe length equivalent to this fitting,

$$L_e = K\frac{d}{f}$$

*Equivalent lengths can be determined as follows*

All such lengths corresponding to various pipe fittings and transition referred to a pipe of diameter '$d$', and friction factor '$f$' are calculated and added so that head loss calculated by frictional head loss formula for the total equivalent length is the same as the sum of the individual losses in the pipe fittings and transitions.

(*i*) **Loss of head at the entrance to a pipe** of diameter $d_1$ having a loss coefficient K (normally its value for the entrance to the pipe is 0.5) can be reduced to the frictional head loss in a pipe of length $L_e$ of diameter $d$ and friction factor $f$ in the following manner:

Equivalent loss coefficient,

$$K_e = \frac{d^4}{d_1^4}$$

and equivalent length $L_e = K_e\frac{d}{f}$

(*ii*) **Loss of head in sudden contraction:** If the diameter of the contracted section is $d_2$ and the loss coefficient referred to the velocity $V_2$ in this section is K, then equivalent loss coefficient,

$$K_e = \frac{d^4}{d_1^4} \text{ such that } K\frac{V_2^2}{2g} = K_e\frac{V^2}{2g}$$

where V refers *tg* the velocity corresponding to diameter $d$ of pipe and corresponding equivalent length, $L_e = K_e\frac{g}{f}$.

(*iii*) **Loss of head in sudden enlargement:** If $d_1$ and $d_2$ are the diameters of pre-enlargement section and enlarged section respectively, then equivalent loss coefficient is given by

$$K_e = \left(1 - \frac{d_1^2}{d_2^2}\right)\frac{d^4}{d_1^4}$$

$$= \left(\frac{d^2}{d_1^2} - \frac{d^2}{d_s^2}\right)$$

$$= d^4\left(\frac{d_2^2}{d_1^2} - \frac{d_1^2}{d_2^2}\right);$$

and $$L_e = K_e\frac{d}{f}$$

(*iv*) **Velocity head at the pipe exit:** It can be reduced to an equivalent length given by

$$L_e = \frac{d^4}{d_3^4}\left(\frac{d}{f}\right),$$

where $d_3$ refers to diameter at exit end

and $$K_e = \frac{d^4}{d_3^4}$$

(*v*) **Frictional head loss:** The frictional head loss in a pipe of diameter $d_2$ length $L_2$ and friction factor $f_2$ can be reduced to an equivalent pipe of diameter $d$ length $L_e$ and friction factor $f$ by the relation,

$$K_e = \frac{f_2 L_2}{d_2}\left(\frac{d}{d_2}\right)^4 \text{ and equivalent length,}$$

$$L_e = K_e \frac{d}{f}$$

If discharge in the pipe is given, the frictional head loss,

$$h_f = \frac{fLV^2}{2gd} = \frac{8fLQ^2}{\pi^2 g d^5}$$

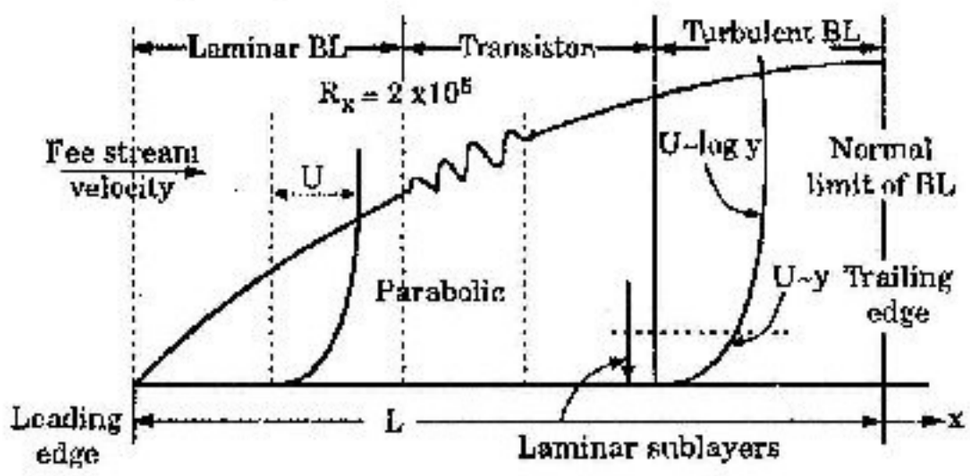
## BOUNDARY LAYER THEORY

Prandtl gave a hypothesis, the fluid flowing along a solid wall may be divided into two independent regions:

(*i*) Fluid within a thin region adjacent to the boundary where the viscous effects are predominant and in know as boundary layer.

(*ii*) Fluid outside this region, where ideal fluid flow exits. This region is known as main flow and velocity in this region approaches the free stream or potential velocity U.

**Boundary Layer Flow over a Flat Plate**




Consider a flat plate at zero incidence in an infinite extent of fluid. The fluid approaches the plate with uniform velocity U (or free stream velocity). As soon as it touches the plate at its leading edege, its velocity reduces to zero in accordance with the no slip condition. The velocity changes rapidly from zero at the boundary to its full value U within a short distance normal to the plate causing a large velocity gradient to developed in the boundary layer. Therefore even for a less viscous fluid (*e.g.* air or water) large shear resistance is developed over the plate and there by a sufficiently thick boundary layer is formed along the plate. As the flow proceeds downstream more fluid is retarded in the lateral direction thereby cursing boundary layer growth in the direction of flow.

**Boundary Layer Thickness**

Velocity of fluid u becomes zero at the stationary wall and becomes equal to free stream velocity U at certain distance normal to the plate. This variation becomes asymptotic in the vertical direction and hence this distance required will be too large to acquire free stream velocity U. Thus bondary layer thickness is defined in many ways as follows :

**Nominal Thickness (δ):** Nominal thickness of boundary layer measures loss of velocity due to boundary layer formation. It is defined as that distance measured normal to the boundary, where 99% of the ambient velocity is restored. The thickness increases in the flow direction and this process is known as *growth of boundary layer.*

Thus, at $y = \delta$, $u\delta = 0.99\ U$

**Displacement Thickness (δ*) :** It indicated the loss of mass rate of flow per meter width of the plate due to boundary layer formation. It can be defined in any one of the following ways:

(*i*) It is the distance measured perpendicular to the boundary, by which free stream is displaced due to boundary layer formation, or

(*ii*) An additional wall thickness that would have to be added to compensate for the reduction in flow rate on account of boudary layer formation.

Mathematically, $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) d\,y$, when loss of mass rate of flow $= \rho U \delta_x$

**Momentum Thickness (θ):** In indicates loss of momentum per meter width per second due to boundary layer formation

Mathematically, $\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) d\,y$, when loss of momentum $= \rho U^2 \theta$

**Energy Thickness (δ**):** It indicates loss of energy per meter width of plate per second due to boundary layer formation.

Mathematically $\delta^{**} = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)$

when loss of energy per unit width of plate per second $= \frac{1}{2} U^3 \delta^{**}$

## OPEN CHANNEL FLOW

Open channel is a conduit in which the liquid flows with a free surface subjected to atmospheric pressure. The flow is caused by the slope of the channel and of liquid surface.

**Types of Channel:** The open channel may be either natural or artificial. The natural channels are irregular in shape and their size varies from hilly rivulets to

large rivers. Artificial channels are constructed and developed by human efforts. Their shape and size depend upon the requirement of the project.

**Geometric Elements of Open Channel**

(*i*) Prismatic and Non-prismatic Channel: A channel of constant slope and cross-section for sufficient length is called a prismatic channel. Artificial channels are generally prismatic channel. In natural streams, slope at various cross-sections do not remain the same throughout a reach. Such channels are called non-prismatic channels.

(*ii*) Depth of Flow ($y$) : It is the vertical distance from the free surface to the bed of the channel.

(*iii*) Depth of Flow-section ($d$) : It is the depth of flow normal to the channel bed. It is given by

$$d = y \cos \theta$$

where, $\theta$ is angle for bed slope of the channel.

(*iv*) Channel Top Width (T) : It is the width of water surface exposed to the atmospheric pressure. It is given by,

$$T = \frac{dA}{dy}.$$

(*v*) Free Board : It is the vertical distance from the top of channel to the water surface. It is provided to prevent overflowing of water due to waves and fluctuations.

(*vi*) Cross-sectional Area of Flow (A) : It is the area of flow through which flow takes place.

(*vii*) Wetted perimeter (P) : It is the length of boundary in contact with the liquid flowing in this cross-section plane $x$-$x$.

(*viii*) Hydraulic Radius (R) : It is the ratio of cross-sectional area of flow to the wetted perimeter,

*i.e.* $$R = \frac{A}{P}$$

(*ix*) Hydraulic Depth (D) : It is the ratio of cross-sectional area (A) to the top width (T),

*i.e.* $$D = \frac{A}{T}$$

(*x*) Bed Slope ($S_0$) : It is the slope of the bed of the channel. It is given by

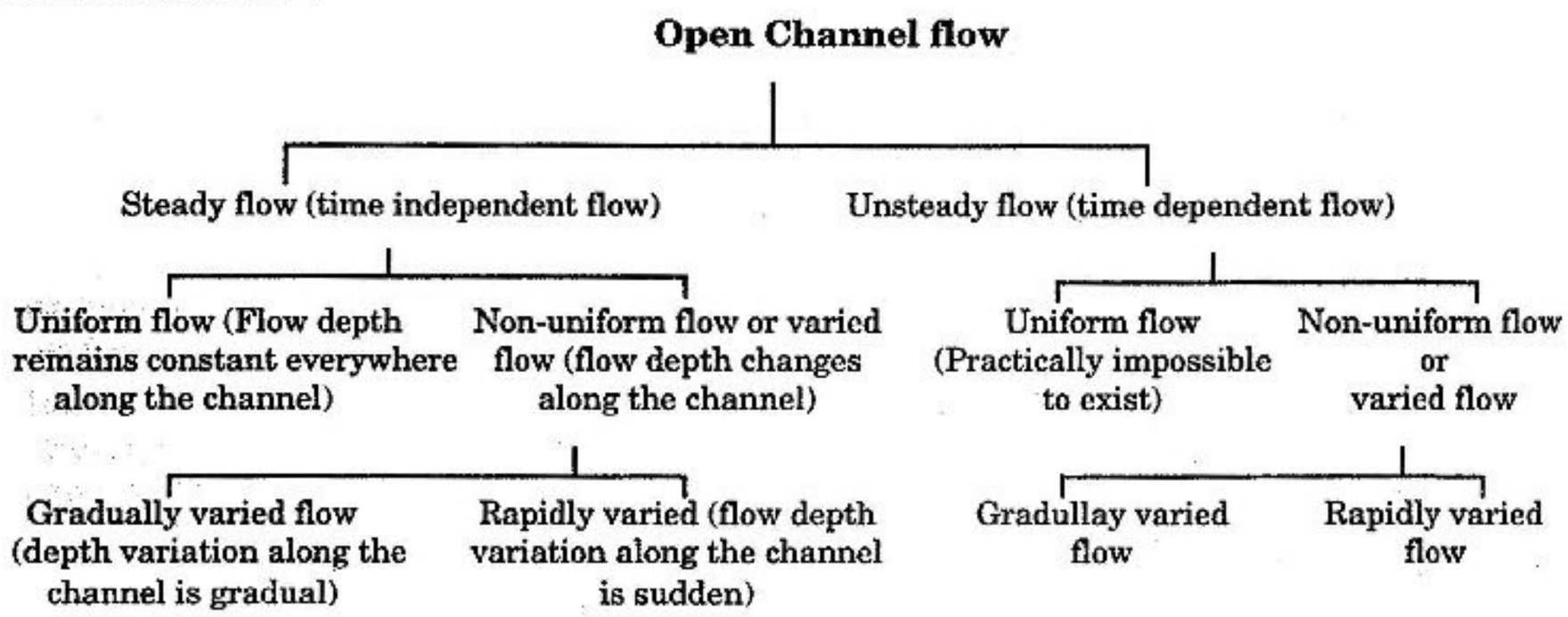
$$S_0 = \sin \theta \text{ or } \tan \theta$$

(*xi*) Hydraulic Slope ($S_f$) : It is the slope of energy line,

$$S_f = \frac{\text{head lost due to friction } (h_f) \text{ in length (L)}}{\text{length (L)}}$$

(*xii*) Water Surface Slope ($S_w$) : It is the slope of water surface line at any section. In a channel it represents the hydraulic graident.

**Classification of Flow :**




**1. Open channel flow can be classified into many types based on**

**Change in Flow depth with respect to time and space**

(*a*) *Steady and Unsteady flows :* In steady flow, conditions of flow *e.g.* depth of flow and velocity does not vary with time and if conditions vary with time, it is known as unsteady flow.

For steady flows,

$$\frac{\partial V}{\partial t} = 0 \text{ and } \frac{\partial y}{\partial t} = 0$$

For unsteady flows,

$$\frac{\partial V}{\partial t} = 0 \text{ and } \frac{\partial y}{\partial t} = 0$$

(*b*) *Uniform and Non-uniform Flow :* Space is the basis. In uniform flow conditions of flow, *e.g.* depth, slope, velocity and cross-section remains constant over given length of channel

*i.e.* $$\frac{\partial y}{\partial L} = 0, \ \frac{\partial y}{\partial L} = 0 \text{ etc.}$$