
Exercise – 16A

1. Find the distance between the points

- (i) $A(9,3)$ and $B(15,11)$
- (ii) $A(7,-4)$ and $B(-5,1)$
- (iii) $A(-6,-4)$ and $B(9,-12)$
- (iv) $A(1,-3)$ and $B(4,-6)$
- (v) $P(a+b, a-b)$ and $Q(a-b, a+b)$
- (vi) $P(a \sin \alpha, a \cos \alpha)$ and $Q(a \cos \alpha, -a \sin \alpha)$

Sol:

- (i) $A(9,3)$ and $B(15,11)$

The given points are $A(9,3)$ and $B(15,11)$.

Then $(x_1 = 9, y_1 = 3)$ and $(x_2 = 15, y_2 = 11)$

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(15 - 9)^2 + (11 - 3)^2} \\ &= \sqrt{(15 - 9)^2 + (11 - 3)^2} \\ &= \sqrt{(6)^2 + (8)^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \text{ units} \end{aligned}$$

- (ii) $A(7,-4)$ and $B(-5,1)$

The given points are $A(7,-4)$ and $B(-5,1)$.

Then, $(x_1 = 7, y_1 = -4)$ and $(x_2 = -5, y_2 = 1)$

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5 - 7)^2 + \{1 - (-4)\}^2} \\ &= \sqrt{(-5 - 7)^2 + (1 + 4)^2} \\ &= \sqrt{(-12)^2 + (5)^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \end{aligned}$$

$$= 13 \text{ units}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

- (iii) $A(-6, -4)$ and $B(9, -12)$

The given points are $A(-6, -4)$ and $B(9, -12)$

Then $(x_1 = -6, y_1 = -4)$ and $(x_2 = 9, y_2 = -12)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(9 - (-6))^2 + \{-12 - (-4)\}^2}$$

$$= \sqrt{(9 + 6)^2 + (-12 + 4)^2}$$

$$= \sqrt{(15)^2 + (-8)^2}$$

$$= \sqrt{225 + 64}$$

$$= \sqrt{289}$$

$$= 17 \text{ units}$$

- (iv) $A(1, -3)$ and $B(4, -6)$

The given points are $A(1, -3)$ and $B(4, -6)$

Then $(x_1 = 1, y_1 = -3)$ and $(x_2 = 4, y_2 = -6)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - 1)^2 + \{-6 - (-3)\}^2}$$

$$= \sqrt{(4 - 1)^2 + (-6 + 3)^2}$$

$$= \sqrt{(3)^2 + (-3)^2}$$

$$= \sqrt{9 + 9}$$

$$= \sqrt{18}$$

$$= \sqrt{9 \times 2}$$

$$= 3\sqrt{2} \text{ units}$$

- (v) $P(a+b, a-b)$ and $Q(a-b, a+b)$

The given points are $P(a+b, a-b)$ and $Q(a-b, a+b)$

Then $(x_1 = a+b, y_1 = a-b)$ and $(x_2 = a-b, y_2 = a+b)$

$$\begin{aligned}
 PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\{(a-b) - (a+b)\}^2 + \{(a+b) - (a-b)\}^2} \\
 &= \sqrt{(a-b-a-b)^2 + (a+b-a+b)^2} \\
 &= \sqrt{(-2b)^2 + (2b)^2} \\
 &= \sqrt{4b^2 + 4b^2} \\
 &= \sqrt{8b^2} \\
 &= \sqrt{4 \times 2b^2} \\
 &= 2\sqrt{2}b \text{ units}
 \end{aligned}$$

(vi) $P(a \sin \alpha, a \cos \alpha)$ and $Q(a \cos \alpha, -a \sin \alpha)$

The given points are $P(a \sin \alpha, a \cos \alpha)$ and $Q(a \cos \alpha, -a \sin \alpha)$

Then $(x_1 = a \sin \alpha, y_1 = a \cos \alpha)$ and $(x_2 = a \cos \alpha, y_2 = -a \sin \alpha)$

$$\begin{aligned}
 PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(a \cos \alpha - a \sin \alpha)^2 + (-a \sin \alpha - a \cos \alpha)^2} \\
 &= \sqrt{(a^2 \cos^2 \alpha + a^2 \sin^2 \alpha - 2a^2 \cos \alpha \times \sin \alpha) + (a^2 \sin^2 \alpha + a^2 \cos^2 \alpha + 2a^2 \cos \alpha \times \sin \alpha)} \\
 &= \sqrt{2a^2 \cos^2 \alpha + 2a^2 \sin^2 \alpha} \\
 &= \sqrt{2a^2 (\cos^2 \alpha + \sin^2 \alpha)} \\
 &= \sqrt{2a^2 (1)} \quad (\text{From the identity } \cos^2 \alpha + \sin^2 \alpha = 1) \\
 &= \sqrt{2a^2} \\
 &= \sqrt{2}a \text{ units}
 \end{aligned}$$

2. Find the distance of each of the following points from the origin:

(i) $A(5, -12)$ (ii) $B(-5, 5)$ (iii) $C(-4, -6)$

Sol:

(i) $A(5, -12)$

Let $O(0, 0)$ be the origin

$$OA = \sqrt{(5-0)^2 + (-12-0)^2}$$

$$\begin{aligned} &= \sqrt{(5)^2 + (-12)^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \text{ units} \end{aligned}$$

(ii) $B(-5, 5)$

Let $O(0, 0)$ be the origin.

$$\begin{aligned} OB &= \sqrt{(-5-0)^2 + (5-0)^2} \\ &= \sqrt{(-5)^2 + (5)^2} \\ &= \sqrt{25 + 25} \\ &= \sqrt{50} \\ &= \sqrt{25 \times 2} \\ &= 5\sqrt{2} \text{ units} \end{aligned}$$

(iii) $C(-4, -6)$

Let $O(0, 0)$ be the origin

$$\begin{aligned} OC &= \sqrt{(-4-0)^2 + (-6-0)^2} \\ &= \sqrt{(-4)^2 + (-6)^2} \\ &= \sqrt{16 + 36} \\ &= \sqrt{52} \\ &= \sqrt{4 \times 13} \\ &= 2\sqrt{13} \text{ units} \end{aligned}$$

3. Find all possible values of x for which the distance between the points $A(x, -1)$ and $B(5, 3)$ is 5 units.

Sol:

Given $AB = 5 \text{ units}$

Therefore, $(AB)^2 = 25 \text{ units}$

$$\Rightarrow (5-a)^2 + \{3-(-1)\}^2 = 25$$

$$\Rightarrow (5-a)^2 + (3+1)^2 = 25$$

$$\Rightarrow (5-a)^2 + (4)^2 = 25$$

$$\Rightarrow (5-a)^2 + 16 = 25$$

$$\Rightarrow (5-a)^2 = 25 - 16$$

$$\Rightarrow (5-a)^2 = 9$$

$$\Rightarrow (5-a) = \pm\sqrt{9}$$

$$\Rightarrow 5-a = \pm 3$$

$$\Rightarrow 5-a = 3 \text{ or } 5-a = -3$$

$$\Rightarrow a = 2 \text{ or } 8$$

Therefore, $a = 2$ or 8 .

4. Find all possible values of y for which distance between the points $A(2, -3)$ and $B(10, y)$ is 10 units.

Sol:

The given points are $A(2, -3)$ and $B(10, y)$

$$\therefore AB = \sqrt{(2-10)^2 + (-3-y)^2}$$

$$= \sqrt{(-8)^2 + (-3-y)^2}$$

$$= \sqrt{64 + 9 + y^2 + 6y}$$

$$\because AB = 10$$

$$\therefore \sqrt{64 + 9 + y^2 + 6y} = 10$$

$$\Rightarrow 73 + y^2 + 6y = 100 \quad (\text{Squaring both sides})$$

$$\Rightarrow y^2 + 6y - 27 = 0$$

$$\Rightarrow y^2 + 9y - 3y - 27 = 0$$

$$\Rightarrow y(y+9) - 3(y+9) = 0$$

$$\Rightarrow (y+9)(y-3) = 0$$

$$\Rightarrow y+9 = 0 \text{ or } y-3 = 0$$

$$\Rightarrow y = -9 \text{ or } y = 3$$

Hence, the possible values of y are -9 and 3 .

5. Find value of x for which the distance between the points $P(x, 4)$ and $Q(9, 10)$ is 10 units.

Sol:

The given points are $P(x, 4)$ and $Q(9, 10)$.

$$\therefore PQ = \sqrt{(x-9)^2 + (4-10)^2}$$

$$= \sqrt{(x-9)^2 + (-6)^2}$$

$$= \sqrt{x^2 - 18x + 81 + 36}$$

$$= \sqrt{x^2 - 18x + 117}$$

$$\because PQ = 10$$

$$\therefore \sqrt{x^2 - 18x + 117} = 10$$

$$\Rightarrow x^2 - 18x + 117 = 100 \quad (\text{Squaring both sides})$$

$$\Rightarrow x^2 - 18x + 17 =$$

$$\Rightarrow x^2 - 17x - x + 17 = 0$$

$$\Rightarrow x(x-17) - 1(x-17) = 0$$

$$\Rightarrow (x-17)(x-1) = 0$$

$$\Rightarrow x-17 = 0 \text{ or } x-1 = 0$$

$$\Rightarrow x = 17 \text{ or } x = 1$$

Hence, the values of x are 1 and 17.

6. If the point $A(x, 2)$ is equidistant from the points $B(8, -2)$ and $C(2, -2)$, find the value of x . Also, find the value of x . Also, find the length of AB .

Sol:

As per the question

$$AB = AC$$

$$\Rightarrow \sqrt{(x-8)^2 + (2+2)^2} = \sqrt{(x-2)^2 + (2+2)^2}$$

Squaring both sides, we get

$$(x-8)^2 + 4^2 = (x-2)^2 + 4^2$$

$$\Rightarrow x^2 - 16x + 64 + 16 = x^2 + 4 - 4x + 16$$

$$\Rightarrow 16x - 4x = 64 - 4$$

$$\Rightarrow x = \frac{60}{12} = 5$$

Now,

$$AB = \sqrt{(x-8)^2 + (2+2)^2}$$

$$= \sqrt{(5-8)^2 + (2+2)^2} \quad (\because x = 5)$$

$$= \sqrt{(-3)^2 + (4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

Hence, $x = 5$ and $AB = 5$ units.

7. If the point $A(0, 2)$ is equidistant from the points $B(3, p)$ and $C(p, 5)$ find the value of p . Also, find the length of AB .

Sol:

As per the question

$$AB = AC$$

$$\Rightarrow \sqrt{(0-3)^2 + (2-p)^2} = \sqrt{(0-p)^2 + (2-5)^2}$$

$$\Rightarrow \sqrt{(-3)^2 + (2-p)^2} = \sqrt{(-p)^2 + (-3)^2}$$

Squaring both sides, we get

$$(-3)^2 + (2-p)^2 = (-p)^2 + (-3)^2$$

$$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 9$$

$$\Rightarrow 4p = 4$$

$$\Rightarrow p = 1$$

Now,

$$AB = \sqrt{(0-3)^2 + (2-p)^2}$$

$$= \sqrt{(-3)^2 + (2-1)^2} \quad (\because p = 1)$$

$$= \sqrt{9+1}$$

$$= \sqrt{10} \text{ units}$$

Hence, $p = 1$ and $AB = \sqrt{10} \text{ units}$

8. Find the point on the x -axis which is equidistant from the points $(2, -5)$ and $(-2, 9)$.

Sol:

Let $(x, 0)$ be the point on the x axis. Then as per the question, we have

$$\sqrt{(x-2)^2 + (0+5)^2} = \sqrt{(x+2)^2 + (0-9)^2}$$

$$\Rightarrow \sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (-9)^2}$$

$$\Rightarrow (x-2)^2 + (5)^2 = (x+2)^2 + (-9)^2 \quad (\text{Squaring both sides})$$

$$\Rightarrow x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$$

$$\Rightarrow 8x = 25 - 81$$

$$\Rightarrow x = -\frac{56}{8} = -7$$

Hence, the point on the x -axis is $(-7, 0)$.

9. Find the points on the x-axis, each of which is at a distance of 10 units from the point A(11, -8).

Sol:

Let $P(x, 0)$ be the point on the x-axis. Then as per the question we have

$$AP = 10$$

$$\Rightarrow \sqrt{(x-11)^2 + (0+8)^2} = 10$$

$$\Rightarrow (x-11)^2 + 8^2 = 100 \quad (\text{Squaring both sides})$$

$$\Rightarrow (x-11)^2 = 100 - 64 = 36$$

$$\Rightarrow x-11 = \pm 6$$

$$\Rightarrow x = 11 \pm 6$$

$$\Rightarrow x = 11-6, 11+6$$

$$\Rightarrow x = 5, 17$$

Hence, the points on the x-axis are (5,0) and (17,0).

10. Find the points on the y-axis which is equidistant from the points A(6,5) and B(-4,3)

Sol:

Let P (0, y) be a point on the y-axis. Then as per the question, we have

$$AP = BP$$

$$\Rightarrow \sqrt{(0-6)^2 + (y-5)^2} = \sqrt{(0+4)^2 + (y-3)^2}$$

$$\Rightarrow \sqrt{(6)^2 + (y-5)^2} = \sqrt{(4)^2 + (y-3)^2}$$

$$\Rightarrow (6)^2 + (y-5)^2 = (4)^2 + (y-3)^2 \quad (\text{Squaring both sides})$$

$$\Rightarrow 36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$$

$$\Rightarrow 4y = 36$$

$$\Rightarrow y = 9$$

Hence, the point on the y-axis is (0,9).

11. If the points $P(x, y)$ is point equidistant from the points A(5,1) and B(-1,5), Prove that

$3x=2y$. **Sol:**

As per the question, we have

$$AP = BP$$

$$\Rightarrow \sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y-5)^2}$$

$$\Rightarrow (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2 \quad (\text{Squaring both sides})$$

$$\Rightarrow x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 + 2x + 1 + y^2 - 10y + 25$$

$$\Rightarrow -10x - 2y = 2x - 10y$$

$$\Rightarrow 8y = 12x$$

$$\Rightarrow 3x = 2y$$

$$\text{Hence, } 3x = 2y$$

12. If $P(x, y)$ is point equidistant from the points $A(6, -1)$ and $B(2, 3)$, show that $x - y = 3$

Sol:

The given points are $A(6, -1)$ and $B(2, 3)$. The point $P(x, y)$ equidistant from the points A and B . So, $PA = PB$

$$\text{Also, } (PA)^2 = (PB)^2$$

$$\Rightarrow (6-x)^2 + (-1-y)^2 = (2-x)^2 + (3-y)^2$$

$$\Rightarrow x^2 - 12x + 36 + y^2 + 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$$

$$\Rightarrow x^2 + y^2 - 12x + 2y + 37 = x^2 + y^2 - 4x - 6y + 13$$

$$\Rightarrow x^2 + y^2 - 12x + 2y - x^2 - y^2 + 4x + 6y = 13 - 37$$

$$\Rightarrow -8x + 8y = -24$$

$$\Rightarrow -8(x - y) = -24$$

$$\Rightarrow x - y = \frac{-24}{-8}$$

$$\Rightarrow x - y = 3$$

Hence proved.

13. Find the co-ordinates of the point equidistant from three given points $A(5, 3)$, $B(5, -5)$ and $C(1, -5)$

Sol:

Let the required point be $P(x, y)$. Then $AP = BP = CP$

$$\text{That is, } (AP)^2 = (BP)^2 = (CP)^2$$

$$\text{This means } (AP)^2 = (BP)^2$$

$$\Rightarrow (x-5)^2 + (y-3)^2 = (x-5)^2 + (y+5)^2$$

$$\Rightarrow x^2 - 10x + 25 + y^2 - 6y + 9 = x^2 - 10x + 25 + y^2 + 10y + 25$$

$$\Rightarrow x^2 - 10x + y^2 - 6y + 34 = x^2 - 10x + y^2 + 10y + 50$$

$$\Rightarrow x^2 - 10x + y^2 - 6y - x^2 + 10x - y^2 - 10y = 50 - 34$$

$$\Rightarrow -16y = 16$$

$$\Rightarrow y = -\frac{16}{16} = -1$$

$$\text{And } (BP)^2 = (CP)^2$$

$$\Rightarrow (x-5)^2 + (y+5)^2 = (x-1)^2 + (y+5)^2$$

$$\Rightarrow x^2 - 10x + 25 + y^2 + 10y + 25 = x^2 - 2x + 1 + y^2 + 10y + 25$$

$$\Rightarrow x^2 - 10x + y^2 + 10y + 50 = x^2 - 2x + y^2 + 10y + 26$$

$$\Rightarrow x^2 - 10x + y^2 + 10y - x^2 + 2x - y^2 - 10y = 26 - 50$$

$$\Rightarrow -8x = -24$$

$$\Rightarrow x = \frac{-24}{-8} = 3$$

Hence, the required point is $(3, -1)$.

14. If the points $A(4, 3)$ and $B(x, 5)$ lie on a circle with the centre $O(2, 3)$. Find the value of x .

Sol:

Given, the points $A(4, 3)$ and $B(x, 5)$ lie on a circle with center $O(2, 3)$.

Then $OA = OB$

$$\text{Also } (OA)^2 = (OB)^2$$

$$\Rightarrow (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$$

$$\Rightarrow (2)^2 + (0)^2 = (x-2)^2 + (2)^2$$

$$\Rightarrow 4 = (x-2)^2 + 4$$

$$\Rightarrow (x-2)^2 = 0$$

$$\Rightarrow x-2 = 0$$

$$\Rightarrow x = 2$$

Therefore, $x = 2$

15. If the point $C(-2, 3)$ is equidistant from the points $A(3, -1)$ and $B(x, 8)$, find the value of x .

Also, find the distance between BC

Sol:

As per the question, we have

$$AC = BC$$

$$\Rightarrow \sqrt{(-2-3)^2 + (3+1)^2} = \sqrt{(-2-x)^2 + (3-8)^2}$$

$$\Rightarrow \sqrt{(5)^2 + (4)^2} = \sqrt{(x+2)^2 + (-5)^2}$$

$$\Rightarrow 25 + 16 = (x+2)^2 + 25 \quad (\text{Squaring both sides})$$

$$\Rightarrow 25 + 16 = (x+2)^2 + 25$$

$$\Rightarrow (x+2)^2 = 16$$

$$\Rightarrow x+2 = \pm 4$$

$$\Rightarrow x = -2 \pm 4 = -2-4, -2+4 = -6, 2$$

Now

$$BC = \sqrt{(-2-x)^2 + (3-8)^2}$$

$$= \sqrt{(-2-2)^2 + (-5)^2}$$

$$= \sqrt{16+25} = \sqrt{41} \text{ units}$$

Hence, $x = 2$ or -6 and $BC = \sqrt{41} \text{ units}$

- 16.** If the point $P(2, 2)$ is equidistant from the points $A(-2, k)$ and $B(-2k, -3)$, find k . Also, find the length of AP .

Sol:

As per the question, we have

$$AP = BP$$

$$\Rightarrow \sqrt{(2+2)^2 + (2+k)^2} = \sqrt{(2+2k)^2 + (2+3)^2}$$

$$\Rightarrow \sqrt{(4)^2 + (2-k)^2} = \sqrt{(2+2k)^2 + (5)^2}$$

$$\Rightarrow 16+4+k^2-4k = 4+4k^2+8k+25$$

(Squaring both sides)

$$\Rightarrow k^2+4k+3=0$$

$$\Rightarrow (k+1)(k+3)=0$$

$$\Rightarrow k = -3, -1$$

Now for $k = -1$

$$AP = \sqrt{(2+2)^2 + (2-k)^2}$$

$$= \sqrt{(4)^2 + (2+1)^2}$$

$$= \sqrt{16+9} = 5 \text{ units}$$

For $k = -3$

$$AP = \sqrt{(2+2)^2 + (2-k)^2}$$

$$= \sqrt{(4)^2 + (2+3)^2}$$

$$= \sqrt{16+25} = \sqrt{41} \text{ units}$$

Hence, $k = -1, -3$; $AP = 5 \text{ units}$ for $k = -1$ and $AP = \sqrt{41} \text{ units}$ for $k = -3$.

17. If the point (x, y) is equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$, prove that $bx = ay$.

Sol:

As per the question, we have

$$\begin{aligned}\sqrt{(x-a-b)^2 + (y-b+a)^2} &= \sqrt{(x-a+b)^2 + (y-a-b)^2} \\ \Rightarrow (x-a-b)^2 + (y-b+a)^2 &= (x-a+b)^2 + (y-a-b)^2 \quad (\text{Squaring both sides}) \\ \Rightarrow x^2 + (a+b)^2 - 2x(a+b) + y^2 + (a-b)^2 - 2y(a-b) &= x^2 + (a-b)^2 - 2x(a-b) + y^2 \\ &+ (a+b)^2 - 2y(a+b) \\ \Rightarrow -x(a+b) - y(a-b) &= -x(a-b) - y(a+b) \\ \Rightarrow -xa - xb - ay + by &= -xa + bx - ya - by \\ \Rightarrow by &= bx\end{aligned}$$

Hence, $bx = ay$.

18. Using the distance formula, show that the given points are collinear:

- (i) $(1, -1)$, $(5, 2)$ and $(9, 5)$ (ii) $(6, 9)$, $(0, 1)$ and $(-6, -7)$
(iii) $(-1, -1)$, $(2, 3)$ and $(8, 11)$ (iv) $(-2, 5)$, $(0, 1)$ and $(2, -3)$

Sol:

- (i) Let $A(1, -1)$, $B(5, 2)$ and $C(9, 5)$ be the given points. Then

$$\begin{aligned}AB &= \sqrt{(5-1)^2 + (2+1)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ units} \\ BC &= \sqrt{(9-5)^2 + (5-2)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ units} \\ AC &= \sqrt{(9-1)^2 + (5+1)^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10 \text{ units} \\ \therefore AB + BC &= (5+5) \text{ units} = 10 \text{ units} = AC\end{aligned}$$

Hence, the given points are collinear

- (ii) Let $A(6, 9)$, $B(0, 1)$ and $C(-6, -7)$ be the given points. Then

$$\begin{aligned}AB &= \sqrt{(0-6)^2 + (1-9)^2} = \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10 \text{ units} \\ BC &= \sqrt{(-6-0)^2 + (-7-1)^2} = \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10 \text{ units} \\ AC &= \sqrt{(-6-6)^2 + (-7-9)^2} = \sqrt{(-12)^2 + (-16)^2} = \sqrt{400} = 20 \text{ units} \\ \therefore AB + BC &= (10+10) \text{ units} = 20 \text{ units} = AC\end{aligned}$$

Hence, the given points are collinear

- (iii) Let $A(-1, -1)$, $B(2, 3)$ and $C(8, 11)$ be the given points. Then

$$AB = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5 \text{ units}$$

$$BC = \sqrt{(8-2)^2 + (11-3)^2} = \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10 \text{ units}$$

$$AC = \sqrt{(8+1)^2 + (11+1)^2} = \sqrt{(9)^2 + (12)^2} = \sqrt{225} = 15 \text{ units}$$

$$\therefore AB + BC = (5+10) \text{ units} = 15 \text{ units} = AC$$

Hence, the given points are collinear

- (iv) Let $A(-2, 5)$, $B(0, 1)$ and $C(2, -3)$ be the give points. Then

$$AB = \sqrt{(0+2)^2 + (1-5)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$BC = \sqrt{(2-0)^2 + (-3-1)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$AC = \sqrt{(2+2)^2 + (-3-5)^2} = \sqrt{(4)^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5} \text{ units}$$

$$\therefore AB + BC = (2\sqrt{5} + 2\sqrt{5}) \text{ units} = 4\sqrt{5} \text{ units} = AC$$

Hence, the given points are collinear

- 19.** Show that the points A (7, 10), B(-2, 5) and C(3, -4) are the vertices of an isosceles right triangle.

Sol:

The given points are $A(7, 10)$, $B(-2, 5)$ and $C(3, -4)$.

$$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{(-9)^2 + (-5)^2} = \sqrt{81+25} = \sqrt{106}$$

$$BC = \sqrt{(3-(-2))^2 + (-4-5)^2} = \sqrt{(5)^2 + (-9)^2} = \sqrt{25+81} = \sqrt{106}$$

$$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$$

Since, AB and BC are equal, they form the vertices of an isosceles triangle

$$\text{Also, } (AB)^2 + (BC)^2 = (\sqrt{106})^2 + (\sqrt{106})^2 = 212$$

$$\text{and } (AC)^2 = (\sqrt{212})^2 = 212.$$

$$\text{Thus, } (AB)^2 + (BC)^2 = (AC)^2$$

This show that $\triangle ABC$ is right- angled at B.

Therefore, the points $A(7, 10)$, $B(-2, 5)$ and $C(3, -4)$ are the vertices of an isosceles right-angled triangle.

20. Show that the points A (3, 0), B(6, 4) and C(-1, 3) are the vertices of an isosceles right triangle.

Sol:

The given points are A(3,0), B(6,4) and C(-1,3). Now,

$$AB = \sqrt{(3-6)^2 + (0-4)^2} = \sqrt{(-3)^2 + (-4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

$$BC = \sqrt{(6+1)^2 + (4-3)^2} = \sqrt{(7)^2 + (1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

$$AC = \sqrt{(3+1)^2 + (0-3)^2} = \sqrt{(4)^2 + (-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

$$\therefore AB = AC \text{ and } AB^2 + AC^2 = BC^2$$

Therefore, A(3,0), B(6,4) and C(-1,3) are the vertices of an isosceles right triangle

21. If A(5,2), B(2, -2) and C(-2, t) are the vertices of a right triangle with $\angle B=90^\circ$, then find the value of t.

Sol:

$$\therefore \angle B = 90^\circ$$

$$\therefore AC^2 = AB^2 + BC^2$$

$$\Rightarrow (5+2)^2 + (2-t)^2 = (5-2)^2 + (2+2)^2 + (2+2)^2 + (-2-t)^2$$

$$\Rightarrow (7)^2 + (t-2)^2 = (3)^2 + (4)^2 + (4)^2 + (t+2)^2$$

$$\Rightarrow 49 + t^2 - 4t + 4 = 9 + 16 + 16 + t^2 + 4t + 4$$

$$\Rightarrow 8 - 4t = 4t$$

$$\Rightarrow 8t = 8$$

$$\Rightarrow t = 1$$

Hence, $t = 1$.

22. Prove that the points A(2, 4), B(2, 6) and $C(2 + \sqrt{3}, 5)$ are the vertices of an equilateral triangle.

Sol:

The given points are A(2,4), B(2,6) and $C(2 + \sqrt{3}, 5)$. Now

$$AB = \sqrt{(2-2)^2 + (4-6)^2} = \sqrt{(0)^2 + (-2)^2}$$

$$= \sqrt{0+4} = 2$$

$$BC = \sqrt{(2-2-\sqrt{3})^2 + (6-5)^2} = \sqrt{(-\sqrt{3})^2 + (1)^2}$$

$$= \sqrt{3+1} = 2$$

$$AC = \sqrt{(2-2-\sqrt{3})^2 + (4-5)^2} = \sqrt{(-\sqrt{3})^2 + (-1)^2}$$

$$= \sqrt{3+1} = 2$$

Hence, the points $A(2,4)$, $B(2,6)$ and $C(2+\sqrt{3},5)$ are the vertices of an equilateral triangle.

- 23.** Show that the points $(-3, -3)$, $(3,3)$ and $C(-3\sqrt{3}, 3\sqrt{3})$ are the vertices of an equilateral triangle.

Sol:

Let the given points be $A(-3, -3)$, $B(3,3)$ and $C(-3\sqrt{3}, 3\sqrt{3})$. Now

$$AB = \sqrt{(-3-3)^2 + (-3-3)^2} = \sqrt{(-6)^2 + (-6)^2}$$

$$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$$BC = \sqrt{(3+3\sqrt{3})^2 + (3-3\sqrt{3})^2}$$

$$= \sqrt{9+27+18\sqrt{3}+9+27-18\sqrt{3}} = \sqrt{72} = 6\sqrt{2}$$

$$AC = \sqrt{(-3+3\sqrt{3})^2 + (-3-3\sqrt{3})^2} = \sqrt{(3-3\sqrt{3})^2 + (3+3\sqrt{3})^2}$$

$$= \sqrt{9+27-18\sqrt{3}+9+27+18\sqrt{3}}$$

$$= \sqrt{72} = 6\sqrt{2}$$

Hence, the given points are the vertices of an equilateral triangle.

- 24.** Show that the points $A(-5,6)$, $B(3,0)$ and $C(9,8)$ are the vertices of an isosceles right-angled triangle. Calculate its area.

Sol:

Let the given points be $A(-5,6)$, $B(3,0)$ and $C(9,8)$.

$$AB = \sqrt{(3-(-5))^2 + (0-6)^2} = \sqrt{(8)^2 + (-6)^2} = \sqrt{64+36} = \sqrt{100} = 10 \text{ units}$$

$$BC = \sqrt{(9-3)^2 + (8-0)^2} = \sqrt{(6)^2 + (8)^2} = \sqrt{36+64} = \sqrt{100} = 10 \text{ units}$$

$$AC = \sqrt{(9-(-5))^2 + (8-6)^2} = \sqrt{(14)^2 + (2)^2} = \sqrt{196+4} = \sqrt{200} = 10\sqrt{2} \text{ units}$$

Therefore, $AB = BC = 10 \text{ units}$

$$\text{Also, } (AB)^2 + (BC)^2 = (10)^2 + (10)^2 = 200$$

$$\text{and } (AC)^2 = (10\sqrt{2})^2 = 200$$

$$\text{Thus, } (AB)^2 + (BC)^2 = (AC)^2$$

This show that $\triangle ABC$ is right angled at B .

Therefore, the points $A(-5, 6)$, $B(3, 0)$ and $C(9, 8)$ are the vertices of an isosceles right-angled triangle

$$\text{Also, area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

If AB is the height and BC is the base,

$$\text{Area} = \frac{1}{2} \times 10 \times 10$$

$$= 50 \text{ square units}$$

25. Show that the points $O(0, 0)$, $A(3, \sqrt{3})$ and $B(3, -\sqrt{3})$ are the vertices of an equilateral triangle. Find the area of this triangle.

Sol:

The given points are $O(0, 0)$, $A(3, \sqrt{3})$ and $B(3, -\sqrt{3})$.

$$OA = \sqrt{(3-0)^2 + \{(\sqrt{3})-0\}^2} = \sqrt{(3)^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

$$AB = \sqrt{(3-3)^2 + (-\sqrt{3}-\sqrt{3})^2} = \sqrt{(0)^2 + (2\sqrt{3})^2} = \sqrt{4(3)} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

$$OB = \sqrt{(3-0)^2 + (-\sqrt{3}-0)^2} = \sqrt{(3)^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

Therefore, $OA = AB = OB = 2\sqrt{3} \text{ units}$

Thus, the points $O(0, 0)$, $A(3, \sqrt{3})$ and $B(3, -\sqrt{3})$ are the vertices of an equilateral triangle

$$\text{Also, the area of the triangle } OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (2\sqrt{3})^2$$

$$= \frac{\sqrt{3}}{4} \times 12$$

$$= 3\sqrt{3} \text{ square units.}$$

26. Show that the following points are the vertices of a square:

- (i) A (3,2), B(0,5), C(-3,2) and D(0,-1)
 (ii) A (6,2), B(2,1), C(1,5) and D(5,6)
 (iii) A (0,-2), B(3,1), C(0,4) and D(-3,1)

Sol:

- (i) The given points are A(3,2), B(0,5), C(-3,2) and D(0,-1).

$$AB = \sqrt{(0-3)^2 + (5-2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(-3-0)^2 + (2-5)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$CD = \sqrt{(0+3)^2 + (-1-2)^2} = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$DA = \sqrt{(0-3)^2 + (-1-2)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$\text{Therefore, } AB = BC = CD = DA = 3\sqrt{2} \text{ units}$$

$$\text{Also, } AC = \sqrt{(-3-3)^2 + (2-2)^2} = \sqrt{(-6)^2 + (0)^2} = \sqrt{36} = 6 \text{ units}$$

$$BD = \sqrt{(0-0)^2 + (-1-5)^2} = \sqrt{(0)^2 + (-6)^2} = \sqrt{36} = 6 \text{ units}$$

Thus, diagonal AC = diagonal BD

Therefore, the given points form a square.

- (ii) The given points are A(6,2), B(2,1), C(1,5) and D(5,6)

$$AB = \sqrt{(2-6)^2 + (1-2)^2} = \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

$$BC = \sqrt{(1-2)^2 + (5-1)^2} = \sqrt{(-1)^2 + (4)^2} = \sqrt{1+16} = \sqrt{17} \text{ units}$$

$$CD = \sqrt{(5-1)^2 + (6-5)^2} = \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

$$DA = \sqrt{(5-6)^2 + (6-2)^2} = \sqrt{(1)^2 + (4)^2} = \sqrt{1+16} = \sqrt{17} \text{ units}$$

$$\text{Therefore, } AB = BC = CD = DA = \sqrt{17} \text{ units}$$

$$\text{Also, } AC = \sqrt{(1-6)^2 + (5-2)^2} = \sqrt{(-5)^2 + (3)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

$$BD = \sqrt{(5-2)^2 + (6-1)^2} = \sqrt{(3)^2 + (5)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

Thus, diagonal AC = diagonal BD

Therefore, the given points form a square.

- (iii) The given points are P(0,-2), Q(3,1), R(0,4) and S(-3,1)

$$PQ = \sqrt{(3-0)^2 + (1+2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$QR = \sqrt{(0-3)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$RS = \sqrt{(-3-0)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$SP = \sqrt{(-3-0)^2 + (1+2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$\text{Therefore, } PQ = QS = RS = SP = 3\sqrt{2} \text{ units}$$

$$\text{Also, } PR = \sqrt{(0-0)^2 + (4+2)^2} = \sqrt{(0)^2 + (6)^2} = \sqrt{36} = 6 \text{ units}$$

$$QS = \sqrt{(-3-3)^2 + (1-1)^2} = \sqrt{(-6)^2 + (0)^2} = \sqrt{36} = 6 \text{ units}$$

Thus, diagonal PR = diagonal QS

Therefore, the given points form a square.

27. Show that the points $A(-3, 2)$, $B(-5, -5)$, $C(2, -3)$ and $D(4, 4)$ are the vertices of a rhombus. Find the area of this rhombus

Sol:

The given points are $A(-3, 2)$, $B(-5, -5)$, $C(2, -3)$ and $D(4, 4)$.

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2} = \sqrt{(-2)^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53} \text{ units}$$

$$BC = \sqrt{(2+5)^2 + (-3+5)^2} = \sqrt{(7)^2 + (2)^2} = \sqrt{49+4} = \sqrt{53} \text{ units}$$

$$CD = \sqrt{(4-2)^2 + (4+3)^2} = \sqrt{(2)^2 + (7)^2} = \sqrt{4+49} = \sqrt{53} \text{ units}$$

$$DA = \sqrt{(4+3)^2 + (4-2)^2} = \sqrt{(7)^2 + (2)^2} = \sqrt{49+4} = \sqrt{53} \text{ units}$$

$$\text{Therefore, } AB = BC = CD = DA = \sqrt{53} \text{ units}$$

$$\text{Also, } AC = \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{(5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2} \text{ units}$$

$$BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{(9)^2 + (9)^2} = \sqrt{81+81} = \sqrt{162} = \sqrt{81 \times 2} = 9\sqrt{2} \text{ units}$$

Thus, diagonal AC is not equal to diagonal BD .

Therefore ABCD is a quadrilateral with equal sides and unequal diagonals

Hence, ABCD a rhombus

$$\text{Area of a rhombus} = \frac{1}{2} \times (\text{product of diagonals})$$

$$= \frac{1}{2} \times (5\sqrt{2}) \times (9\sqrt{2})$$

$$= \frac{45(2)}{2}$$

$$= 45 \text{ square units.}$$

28. Show that the points A(3,0), B(4,5), C(-1,4) and D(-2,-1) are the vertices of a rhombus. Find its area.

Sol:

The given points are A(3,0), B(4,5), C(-1,4) and D(-2,-1)

$$AB = \sqrt{(3-4)^2 + (0-5)^2} = \sqrt{(-1)^2 + (-5)^2} \\ = \sqrt{1+25} = \sqrt{26}$$

$$BC = \sqrt{(4+1)^2 + (5-4)^2} = \sqrt{(5)^2 + (1)^2} \\ = \sqrt{25+1} = \sqrt{26}$$

$$CD = \sqrt{(-1+2)^2 + (4+1)^2} = \sqrt{(1)^2 + (5)^2} \\ = \sqrt{1+25} = \sqrt{26}$$

$$AD = \sqrt{(3+2)^2 + (0+1)^2} = \sqrt{(5)^2 + (1)^2} \\ = \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(3+1)^2 + (0-4)^2} = \sqrt{(4)^2 + (-4)^2} \\ = \sqrt{16+16} = 4\sqrt{2}$$

$$BD = \sqrt{(4+2)^2 + (5+1)^2} = \sqrt{(6)^2 + (6)^2} \\ = \sqrt{36+36} = 6\sqrt{2}$$

$$\therefore AB = BC = CD = AD = 6\sqrt{2} \text{ and } AC \neq BD$$

Therefore, the given points are the vertices of a rhombus

$$\text{Area } (\Delta ABCD) = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24 \text{ sq. units}$$

Hence, the area of the rhombus is 24 sq. units.

29. Show that the points A(6,1), B(8,2), C(9,4) and D(7,3) are the vertices of a rhombus. Find its area.

Sol:

The given points are A(6,1), B(8,2), C(9,4) and D(7,3).

$$AB = \sqrt{(6-8)^2 + (1-2)^2} = \sqrt{(-2)^2 + (-1)^2} \\ = \sqrt{4+1} = \sqrt{5}$$

$$BC = \sqrt{(8-9)^2 + (2-4)^2} = \sqrt{(-1)^2 + (-2)^2}$$

$$= \sqrt{1+4} = \sqrt{5}$$

$$CD = \sqrt{(9-7)^2 + (4-3)^2} = \sqrt{(2)^2 + (1)^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

$$AD = \sqrt{(7-6)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2}$$

$$= \sqrt{1+4} = \sqrt{5}$$

$$AC = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2}$$

$$= \sqrt{9+9} = 3\sqrt{2}$$

$$BD = \sqrt{(8-7)^2 + (2-3)^2} = \sqrt{(1)^2 + (-1)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$$\therefore AB = BC = CD = AD = \sqrt{5} \text{ and } AC \neq BD$$

Therefore, the given points are the vertices of a rhombus. Now

$$\text{Area } (\Delta ABCD) = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 3\sqrt{2} \times \sqrt{2} = 3 \text{ sq. units}$$

Hence, the area of the rhombus is 3 sq. units.

- 30.** Show that the points A(2,1), B(5,2), C(6,4) and D(3,3) are the angular points of a parallelogram. Is this figure a rectangle?

Sol:

The given points are A(2,1), B(5,2), C(6,4) and D(3,3)

$$AB = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$$BC = \sqrt{(6-5)^2 + (4-2)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5} \text{ units}$$

$$CD = \sqrt{(3-6)^2 + (3-4)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$$AD = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5} \text{ units}$$

Thus, $AB = CD = \sqrt{10}$ units and $BC = AD = \sqrt{5}$ units

So, quadrilateral ABCD is a parallelogram

$$\text{Also, } AC = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

$$BD = \sqrt{(3-5)^2 + (3-2)^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$

But diagonal AC is not equal to diagonal BD.

Hence, the given points do not form a rectangle.

- 31.** Show that $A(1,2)$, $B(4,3)$, $C(6,6)$ and $D(3,5)$ are the vertices of a parallelogram. Show that $ABCD$ is not rectangle.

Sol:

The given vertices are $A(1,2)$, $B(4,3)$, $C(6,6)$ and $D(3,5)$.

$$AB = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{(-3)^2 + (-1)^2} \\ = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(4-6)^2 + (3-6)^2} = \sqrt{(-2)^2 + (-3)^2} \\ = \sqrt{4+9} = \sqrt{13}$$

$$CD = \sqrt{(6-3)^2 + (6-5)^2} = \sqrt{(3)^2 + (1)^2} \\ = \sqrt{9+1} = \sqrt{10}$$

$$AD = \sqrt{(1-3)^2 + (2-5)^2} = \sqrt{(-2)^2 + (-3)^2} \\ = \sqrt{4+9} = \sqrt{13}$$

$$\therefore AB = CD = \sqrt{10} \text{ units and } BC = AD = \sqrt{13} \text{ units}$$

Therefore, $ABCD$ is a parallelogram

$$AC = \sqrt{(1-6)^2 + (2-6)^2} = \sqrt{(-5)^2 + (-4)^2} \\ = \sqrt{25+16} = \sqrt{41}$$

$$BD = \sqrt{(4-3)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2} \\ = \sqrt{1+4} = \sqrt{5}$$

Thus, the diagonal AC and BD are not equal and hence $ABCD$ is not a rectangle

- 32.** Show that the following points are the vertices of a rectangle.

- (i) $A(-4,-1)$, $B(-2,-4)$, $C(4,0)$ and $D(2,3)$
 (ii) $A(2,-2)$, $B(14,10)$, $C(11,13)$ and $D(-1,1)$
 (iii) $A(0,-4)$, $B(6,2)$, $C(3,5)$ and $D(-3,-1)$

Sol:

- (i) The given points are $A(-4,-1)$, $B(-2,-4)$, $C(4,0)$ and $D(2,3)$

$$AB = \sqrt{\{-2 - (-4)\}^2 + \{-4 - (-1)\}^2} = \sqrt{(2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13} \text{ units}$$

$$BC = \sqrt{\{4 - (-2)\}^2 + \{0 - (-4)\}^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13} \text{ units}$$

$$CD = \sqrt{(2-4)^2 + (3-0)^2} = \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13} \text{ units}$$

$$AD = \sqrt{\{2 - (-4)\}^2 + \{3 - (-1)\}^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13} \text{ units}$$

Thus, $AB = CD = \sqrt{13}$ units and $BC = AD = 2\sqrt{13}$ units

$$\text{Also, } AC = \sqrt{\{4 - (-4)\}^2 + \{0 - (-1)\}^2} = \sqrt{(8)^2 + (1)^2} = \sqrt{64 + 1} = \sqrt{65} \text{ units}$$

$$BD = \sqrt{\{2 - (-2)\}^2 + \{3 - (-4)\}^2} = \sqrt{(4)^2 + (7)^2} = \sqrt{16 + 49} = \sqrt{65} \text{ units}$$

Also, diagonal $AC =$ diagonal BD

Hence, the given points form a rectangle

- (ii) The given points are $A(2, -2), B(14, 10), C(11, 13)$ and $D(-1, 1)$

$$AB = \sqrt{(14 - 2)^2 + \{10 - (-2)\}^2} = \sqrt{(12)^2 + (12)^2} = \sqrt{144 + 144} = \sqrt{288} = 12\sqrt{2} \text{ units}$$

$$BC = \sqrt{(11 - 14)^2 + (13 - 10)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$CD = \sqrt{(-1 - 11)^2 + (1 - 13)^2} = \sqrt{(-12)^2 + (-12)^2} = \sqrt{144 + 144} = \sqrt{288} = 12\sqrt{2} \text{ units}$$

$$AD = \sqrt{(-1 - 2)^2 + \{1 - (-2)\}^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

Thus, $AB = CD = 12\sqrt{2}$ units and $BC = AD = 3\sqrt{2}$ units

Also,

$$AC = \sqrt{(11 - 2)^2 + \{13 - (-2)\}^2} = \sqrt{(9)^2 + (15)^2} = \sqrt{81 + 225} = \sqrt{306} = 3\sqrt{34} \text{ units}$$

$$BD = \sqrt{(-1 - 14)^2 + (1 - 10)^2} = \sqrt{(-15)^2 + (-9)^2} = \sqrt{81 + 225} = \sqrt{306} = 3\sqrt{34} \text{ units}$$

Also, diagonal $AC =$ diagonal BD

Hence, the given points form a rectangle

- (iii) The given points are $A(0, -4), B(6, 2), C(3, 5)$ and $D(-3, -1)$.

$$AB = \sqrt{(6 - 0)^2 + \{2 - (-4)\}^2} = \sqrt{(6)^2 + (6)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2} \text{ units}$$

$$BC = \sqrt{(3 - 6)^2 + (5 - 2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$CD = \sqrt{(-3 - 3)^2 + (-1 - 5)^2} = \sqrt{(-6)^2 + (-6)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2} \text{ units}$$

$$AD = \sqrt{(-3 - 0)^2 + \{-1 - (-4)\}^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

Thus, $AB = CD = \sqrt{10}$ units and $BC = AD = \sqrt{5}$ units

$$\text{Also, } AC = \sqrt{(3 - 0)^2 + \{5 - (-4)\}^2} = \sqrt{(3)^2 + (9)^2} = \sqrt{9 + 81} = \sqrt{90} = 3\sqrt{10} \text{ units}$$

$$BD = \sqrt{(-3 - 6)^2 + (-1 - 2)^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{81 + 9} = \sqrt{90} = 3\sqrt{10} \text{ units}$$

Also, diagonal $AC =$ diagonal BD

Hence, the given points form a rectangle

Exercise – 16B

1. Find the coordinates of the point which divides the join of $A(-1, 7)$ and $B(4, -3)$ in the ratio $2:3$

Sol:

The end points of AB are $A(-1, 7)$ and $B(4, -3)$.

Therefore, $(x_1 = -1, y_1 = 7)$ and $(x_2 = 4, y_2 = -3)$

Also, $m = 2$ and $n = 3$

Let the required point be $P(x, y)$.

By section formula, we get

$$\begin{aligned}x &= \frac{(mx_2 + nx_1)}{(m+n)}, y = \frac{(my_2 + ny_1)}{(m+n)} \\ \Rightarrow x &= \frac{\{2 \times 4 + 3 \times (-1)\}}{2+3}, y = \frac{\{2 \times (-3) + 3 \times 7\}}{2+3} \\ \Rightarrow x &= \frac{8-3}{5}, y = \frac{-6+21}{5} \\ \Rightarrow x &= \frac{5}{5}, y = \frac{15}{5}\end{aligned}$$

Therefore, $x = 1$ and $y = 3$

Hence, the coordinates of the required point are $(1, 3)$.

2. Find the co-ordinates of the point which divides the join of $A(-5, 11)$ and $B(4, -7)$ in the ratio $7:2$

Sol:

The end points of AB are $A(-5, 11)$ and $B(4, -7)$.

Therefore, $(x_1 = -5, y_1 = 11)$ and $(x_2 = 4, y_2 = -7)$

Also, $m = 7$ and $n = 2$

Let the required point be $P(x, y)$.

By section formula, we get

$$\begin{aligned}x &= \frac{(mx_2 + nx_1)}{(m+n)}, y = \frac{(my_2 + ny_1)}{(m+n)} \\ \Rightarrow x &= \frac{\{7 \times 4 + 2 \times (-5)\}}{7+2}, y = \frac{\{7 \times (-7) + 2 \times 11\}}{7+2} \\ \Rightarrow x &= \frac{28-10}{9}, y = \frac{-49+22}{9}\end{aligned}$$

$$\Rightarrow x = \frac{18}{9}, y = -\frac{27}{9}$$

Therefore, $x = 2$ and $y = -3$

Hence, the required point are $P(2, -3)$.

3. If the coordinates of points A and B are $(-2, -2)$ and $(2, -4)$ respectively. Find the coordinates of the point P such that $AP = \frac{3}{7} AB$, where P lies on the segment AB.

Sol:

The coordinates of the points A and B are $(-2, -2)$ and $(2, -4)$ respectively, where

$AP = \frac{3}{7} AB$ and P lies on the line segment AB. So

$$AP + BP = AB$$

$$\Rightarrow AP + BP = \frac{7AP}{3} \quad \because AP = \frac{3}{7} AB$$

$$\Rightarrow BP = \frac{7AP}{3} - AP$$

$$\Rightarrow \frac{AP}{BP} = \frac{3}{4}$$

Let (x, y) be the coordinates of P which divides AB in the ratio 3 : 4 internally Then

$$x = \frac{3 \times 2 + 4 \times (-2)}{3 + 4} = \frac{6 - 8}{7} = -\frac{2}{7}$$

$$y = \frac{3 \times (-4) + 4 \times (-2)}{3 + 4} = \frac{-12 - 8}{7} = -\frac{20}{7}$$

Hence, the coordinates of point P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

4. Point A lies on the line segment PQ joining P(6, -6) and Q(-4, -1) in such a way that $\frac{PA}{PQ} = \frac{2}{5}$. If that point A also lies on the line $3x + k(y + 1) = 0$, find the value of k.

Sol:

Let the coordinates of A be (x, y) . Here $\frac{PA}{PQ} = \frac{2}{5}$. So,

$$PA + AQ = PQ$$

$$\Rightarrow PA + AQ = \frac{5PA}{2} \quad \left[\because PA = \frac{2}{5} PQ \right]$$

$$\Rightarrow AQ = \frac{5PA}{2} - PA$$

$$\Rightarrow \frac{AQ}{PA} = \frac{3}{2}$$

$$\Rightarrow \frac{PA}{AQ} = \frac{2}{3}$$

Let (x, y) be the coordinates of A , which divides PQ in the ratio 2 : 3 internally. Then using section formula, we get

$$x = \frac{2 \times (-4) + 3 \times (6)}{2 + 3} = \frac{-8 + 18}{5} = \frac{10}{5} = 2$$

$$y = \frac{2 \times (-1) + 3 \times (-6)}{2 + 3} = \frac{-2 - 18}{5} = \frac{-20}{5} = -4$$

Now, the point $(2, -4)$ lies on the line $3x + k(y + 1) = 0$, therefore

$$3 \times 2 + k(-4 + 1) = 0$$

$$\Rightarrow 3k = 6$$

$$\Rightarrow k = \frac{6}{3} = 2$$

Hence, $k = 2$.

5. Points P, Q, R and S divide the line segment joining the points $A(1, 2)$ and $B(6, 7)$ in five equal parts. Find the coordinates of the points P, Q and R

Sol:

Since, the points P, Q, R and S divide the line segment joining the points

$A(1, 2)$ and $B(6, 7)$ in five equal parts, so

$$AP = PQ = QR = RS = SB$$

Here, point P divides AB in the ratio of 1 : 4 internally. So using section formula, we get

$$\begin{aligned} \text{Coordinates of } P &= \left(\frac{1 \times (6) + 4 \times (1)}{1 + 4}, \frac{1 \times (7) + 4 \times (2)}{1 + 4} \right) \\ &= \left(\frac{6 + 4}{5}, \frac{7 + 8}{5} \right) = (2, 3) \end{aligned}$$

The point Q divides AB in the ratio of 2 : 3 internally. So using section formula, we get

$$\begin{aligned} \text{Coordinates of } Q &= \left(\frac{2 \times (6) + 3 \times (1)}{2 + 3}, \frac{2 \times (7) + 3 \times (2)}{2 + 3} \right) \\ &= \left(\frac{12 + 3}{5}, \frac{14 + 6}{5} \right) = (3, 4) \end{aligned}$$

The point R divides AB in the ratio of 3 : 2 internally. So using section formula, we get

$$\begin{aligned}\text{Coordinates of } R &= \left(\frac{3 \times (6) + 2 \times (1)}{3+2}, \frac{3 \times (7) + 2 \times (2)}{3+2} \right) \\ &= \left(\frac{18+2}{5}, \frac{21+4}{5} \right) = (4, 5)\end{aligned}$$

Hence, the coordinates of the points P , Q and R are $(2, 3)$, $(3, 4)$ and $(4, 5)$ respectively

6. Points P , Q , and R in that order are dividing line segment joining $A(1, 6)$ and $B(5, -2)$ in four equal parts. Find the coordinates of P , Q and R .

Sol:

The given points are $A(1, 6)$ and $B(5, -2)$.

Then, $P(x, y)$ is a point that divides the line AB in the ratio $1:3$

By the section formula:

$$\begin{aligned}x &= \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n} \\ \Rightarrow x &= \frac{(1 \times 5 + 3 \times 1)}{1+3}, y = \frac{(1 \times (-2) + 3 \times 6)}{1+3} \\ \Rightarrow x &= \frac{5+3}{4}, y = \frac{-2+18}{4} \\ \Rightarrow x &= \frac{8}{4}, y = \frac{16}{4} \\ \Rightarrow x &= 2 \text{ and } y = 4\end{aligned}$$

Therefore, the coordinates of point P are $(2, 4)$

Let Q be the mid-point of AB

Then, $Q(x, y)$

$$\begin{aligned}x &= \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2} \\ \Rightarrow x &= \frac{1+5}{2}, y = \frac{6+(-2)}{2} \\ \Rightarrow x &= \frac{6}{2}, y = \frac{4}{2} \\ \Rightarrow x &= 3, y = 2\end{aligned}$$

Therefore, the coordinates of Q are $(3, 2)$

Let $R(x, y)$ be a point that divides AB in the ratio $3:1$

Then, by the section formula:

$$\begin{aligned}
 x &= \frac{(mx_2 + nx_1)}{(m+n)}, y = \frac{(my_2 + ny_1)}{(m+n)} \\
 \Rightarrow x &= \frac{(3 \times 5 + 1 \times 1)}{3+1}, y = \frac{(3 \times (-2) + 1 \times 6)}{3+1} \\
 \Rightarrow x &= \frac{15+1}{4}, y = \frac{-6+6}{4} \\
 \Rightarrow x &= \frac{16}{4}, y = \frac{0}{4} \\
 \Rightarrow x &= 4 \text{ and } y = 0
 \end{aligned}$$

Therefore, the coordinates of R are $(4, 0)$.

Hence, the coordinates of point P , Q and R are $(2, 4)$, $(3, 2)$ and $(4, 0)$ respectively.

7. The line segment joining the points $A(3, -4)$ and $B(1, 2)$ is trisected at the points $P(p, -2)$ and $Q\left(\frac{5}{3}, q\right)$. Find the values of p and q .

Sol:

Let P and Q be the points of trisection of AB .

Then, P divides AB in the ratio $1:2$

So, the coordinates of P are

$$\begin{aligned}
 x &= \frac{(mx_2 + nx_1)}{(m+n)}, y = \frac{(my_2 + ny_1)}{(m+n)} \\
 \Rightarrow x &= \frac{\{1 \times 1 + 2 \times (3)\}}{1+2}, y = \frac{\{1 \times 2 + 2 \times (-4)\}}{1+2} \\
 \Rightarrow x &= \frac{1+6}{3}, y = \frac{2-8}{3} \\
 \Rightarrow x &= \frac{7}{3}, y = -\frac{6}{3} \\
 \Rightarrow x &= \frac{7}{3}, y = -2
 \end{aligned}$$

Hence, the coordinates of P are $\left(\frac{7}{3}, -2\right)$

But $(p, -2)$ are the coordinates of P .

$$\text{So, } p = \frac{7}{3}$$

Also, Q divides the line AB in the ratio $2:1$

So, the coordinates of Q are

$$x = \frac{(mx_2 + mx_1)}{(m+n)}, y = \frac{(my_2 + my_1)}{(m+n)}$$

$$\Rightarrow x = \frac{(2 \times 1 + 1 \times 3)}{2+1}, y = \frac{\{2 \times 2 + 1 \times (-4)\}}{2+1}$$

$$\Rightarrow x = \frac{2+3}{3}, y = \frac{4-4}{3}$$

$$\Rightarrow x = \frac{5}{3}, y = 0$$

Hence, coordinates of Q are $\left(\frac{5}{3}, 0\right)$.

But the given coordinates of Q are $\left(\frac{5}{3}, q\right)$.

So, $q = 0$

Thus, $p = \frac{7}{3}$ and $q = 0$

8. Find the coordinates of the midpoints of the line segment joining
 (i) $A(3,0)$ and $B(-5, 4)$ (ii) $P(-11,-8)$ and $Q(8,-2)$

Sol:

- (i) The given points are $A(3,0)$ and $B(-5,4)$.

Let (x, y) be the midpoint of AB . Then:

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow x = \frac{3 + (-5)}{2}, y = \frac{0 + 4}{2}$$

$$\Rightarrow x = \frac{-2}{2}, y = \frac{4}{2}$$

$$\Rightarrow x = -1, y = 2$$

Therefore, $(-1, 2)$ are the coordinates of midpoint of AB .

- (ii) The given points are $P(-11,-8)$ and $Q(8,-2)$.

Let (x, y) be the midpoint of PQ . Then:

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow x = \frac{-11 + 8}{2}, y = \frac{-8 - 2}{2}$$

$$\Rightarrow x = -\frac{3}{2}, y = -\frac{10}{2}$$

$$\Rightarrow x = -\frac{3}{2}, y = -5$$

Therefore, $\left(-\frac{3}{2}, -5\right)$ are the coordinates of midpoint of PQ .

9. If $(2, p)$ is the midpoint of the line segment joining the points $A(6, -5)$ and $B(-2, 11)$ find the value of p .

Sol:

The given points are $A(6, -5)$ and $B(-2, 11)$.

Let (x, y) be the midpoint of AB . Then,

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow x = \frac{6 + (-2)}{2}, y = \frac{-5 + 11}{2}$$

$$\Rightarrow x = \frac{6 - 2}{2}, y = \frac{-5 + 11}{2}$$

$$\Rightarrow x = \frac{4}{2}, y = \frac{6}{2}$$

$$\Rightarrow x = 2, y = 3$$

So, the midpoint of AB is $(2, 3)$.

But it is given that midpoint of AB is $(2, p)$.

Therefore, the value of $p = 3$.

10. The midpoint of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $C(1, 2a+1)$. Find the values of a and b .

Sol:

The points are $A(2a, 4)$ and $B(-2, 3b)$.

Let $C(1, 2a+1)$ be the mid-point of AB . Then:

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow 1 = \frac{2a + (-2)}{2}, 2a + 1 = \frac{4 + 3b}{2}$$

$$\Rightarrow 2 = 2a - 2, 4a + 2 = 4 + 3b$$

$$\Rightarrow 2a = 2 + 2, 4a - 3b = 4 - 2$$

$$\Rightarrow a = \frac{4}{2}, 4a - 3b = 2$$

$$\Rightarrow a = 2, 4a - 3b = 2$$

Putting the value of a in the equation $4a + 3b = 2$, we get:

$$4(2) - 3b = 2$$

$$\Rightarrow -3b = 2 - 8 = -6$$

$$\Rightarrow b = \frac{6}{3} = 2$$

Therefore, $a = 2$ and $b = 2$.

11. The line segment joining $A(-2, 9)$ and $B(6, 3)$ is a diameter of a circle with center C . Find the coordinates of C .

Sol:

The given points are $A(-2, 9)$ and $B(6, 3)$

Then, $C(x, y)$ is the midpoint of AB .

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow x = \frac{-2 + 6}{2}, y = \frac{9 + 3}{2}$$

$$\Rightarrow x = \frac{4}{2}, y = \frac{12}{2}$$

$$\Rightarrow x = 2, y = 6$$

Therefore, the coordinates of point C are $(2, 6)$.

12. Find the coordinates of a point A , where AB is a diameter of a circle with center $C(2, -3)$ and the other end of the diameter is $B(1, 4)$.

Sol:

$C(2, -3)$ is the center of the given circle. Let $A(a, b)$ and $B(1, 4)$ be the two end-points of the given diameter AB . Then, the coordinates of C are

$$x = \frac{a + 1}{2}, y = \frac{b + 4}{2}$$

It is given that $x = 2$ and $y = -3$.

$$\Rightarrow 2 = \frac{a + 1}{2}, -3 = \frac{b + 4}{2}$$

$$\Rightarrow 4 = a + 1, -6 = b + 4$$

$$\Rightarrow a = 4 - 1, b = -6 - 4$$

$$\Rightarrow a = 3, b = -10$$

Therefore, the coordinates of point A are $(3, -10)$.

- 13.** In what ratio does the point $P(2, 5)$ divide the join of A $(8, 2)$ and B $(-6, 9)$?

Sol:

Let the point $P(2, 5)$ divide AB in the ratio $k : 1$.

Then, by section formula, the coordinates of P are

$$x = \frac{-6k + 8}{k + 1}, y = \frac{9k + 2}{k + 1}$$

It is given that the coordinates of P are $(2, 5)$.

$$\Rightarrow 2 = \frac{-6k + 8}{k + 1}, 5 = \frac{9k + 2}{k + 1}$$

$$\Rightarrow 2k + 2 = -6k + 8, 5k + 5 = 9k + 2$$

$$\Rightarrow 2k + 6k = 8 - 2, 5 - 2 = 9k - 5k$$

$$\Rightarrow 8k = 6, 4k = 3$$

$$\Rightarrow k = \frac{6}{8}, k = \frac{3}{4}$$

$$\Rightarrow k = \frac{3}{4} \text{ in each case.}$$

Therefore, the point $P(2, 5)$ divides AB in the ratio $3 : 4$

- 14.** Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points

$$A\left(\frac{1}{2}, \frac{3}{2}\right) \text{ and } B(2, -5).$$

Sol:

Let $k : 1$ be the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the

points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and $(2, -5)$. Then

$$\left(\frac{3}{4}, \frac{5}{12}\right) = \left(\frac{k(2) + \frac{1}{2}}{k + 1}, \frac{k(-5) + \frac{3}{2}}{k + 1}\right)$$

$$\Rightarrow \frac{k(2) + \frac{1}{2}}{k + 1} = \frac{3}{4} \text{ and } \frac{k(-5) + \frac{3}{2}}{k + 1} = \frac{5}{12}$$

$$\Rightarrow 8k + 2 = 3k + 3 \text{ and } -60k + 18 = 5k + 5$$

$$\Rightarrow k = \frac{1}{5} \text{ and } k = \frac{1}{5}$$

Hence, the required ratio is 1 : 5.

- 15.** Find the ratio in which the point P(m, 6) divides the join of A(-4, 3) and B(2, 8) Also, find the value of m.

Sol:

Let the point $P(m, 6)$ divide the line AB in the ratio $k : 1$.

Then, by the section formula:

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

The coordinates of P are $(m, 6)$.

$$m = \frac{2k - 4}{k + 1}, 6 = \frac{8k + 3}{k + 1}$$

$$\Rightarrow m(k + 1) = 2k - 4, 6k + 6 = 8k + 3$$

$$\Rightarrow m(k + 1) = 2k - 4, 6 - 3 = 8k - 6k$$

$$\Rightarrow m(k + 1) = 2k - 4, 2k = 3$$

$$\Rightarrow m(k + 1) = 2k - 4, k = \frac{3}{2}$$

Therefore, the point P divides the line AB in the ratio 3 : 2

Now, putting the value of k in the equation $m(k + 1) = 2k - 4$, we get:

$$m\left(\frac{3}{2} + 1\right) = 2\left(\frac{3}{2}\right) - 4$$

$$\Rightarrow m\left(\frac{3+2}{2}\right) = 3 - 4$$

$$\Rightarrow \frac{5m}{2} = -1 \Rightarrow 5m = -2 \Rightarrow m = -\frac{2}{5}$$

Therefore, the value of $m = -\frac{2}{5}$

So, the coordinates of P are $\left(-\frac{2}{5}, 6\right)$.

- 16.** Find the ratio in which the point $(-3, k)$ divides the join of A(-5, -4) and B(-2, 3), Also, find the value of k .

Sol:

Let the point $P(-3, k)$ divide the line AB in the ratio $s : 1$

Then, by the section formula:

$$x = \frac{mx_1 + nx_2}{m+n}, y = \frac{my_1 + ny_2}{m+n}$$

The coordinates of P are $(-3, k)$.

$$-3 = \frac{-2s-5}{s+1}, k = \frac{3s-4}{s+1}$$

$$\Rightarrow -3s-3 = -2s-5, k(s+1) = 3s-4$$

$$\Rightarrow -3s+2s = -5+3, k(s+1) = 3s-4$$

$$\Rightarrow -s = -2, k(s+1) = 3s-4$$

$$\Rightarrow s = 2, k(s+1) = 3s-4$$

Therefore, the point P divides the line AB in the ratio $2 : 1$.

Now, putting the value of s in the equation $k(s+1) = 3s-4$, we get:

$$k(2+1) = 3(2)-4$$

$$\Rightarrow 3k = 6-4$$

$$\Rightarrow 3k = 2 \Rightarrow k = \frac{2}{3}$$

Therefore, the value of $k = \frac{2}{3}$

That is, the coordinates of P are $\left(-3, \frac{2}{3}\right)$.

- 17.** In what ratio is the line segment joining $A(2, -3)$ and $B(5, 6)$ divided by the x -axis? Also, find the coordinates of the point of division.

Sol:

Let AB be divided by the x -axis in the ratio $k : 1$ at the point P .

Then, by section formula the coordinates of P are

$$P = \left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1} \right)$$

But P lies on the x -axis; so, its ordinate is 0.

$$\text{Therefore, } \frac{6k-3}{k+1} = 0$$

$$\Rightarrow 6k-3 = 0 \Rightarrow 6k = 3 \Rightarrow k = \frac{3}{6} \Rightarrow k = \frac{1}{2}$$

Therefore, the required ratio is $\frac{1}{2} : 1$, which is same as $1 : 2$

Thus, the x -axis divides the line AB in the ratio $1 : 2$ at the point P .

Applying $k = \frac{1}{2}$, we get the coordinates of point.

$$\begin{aligned} P & \left(\frac{5k+1}{k+1}, 0 \right) \\ & = P \left(\frac{5 \times \frac{1}{2} + 1}{\frac{1}{2} + 1}, 0 \right) \\ & = P \left(\frac{\frac{5+2}{2}}{\frac{1+2}{2}}, 0 \right) \\ & = P \left(\frac{7}{3}, 0 \right) \\ & = P(3, 0) \end{aligned}$$

Hence, the point of intersection of AB and the x -axis is $P(3, 0)$

- 18.** In what ratio is the line segment joining the points $A(-2, -3)$ and $B(3, 7)$ divided by the y -axis? Also, find the coordinates of the point of division.

Sol:

Let AB be divided by the y -axis in the ratio $k : 1$ at the point P .

Then, by section formula the coordinates of P are

$$P = \left(\frac{3k-2}{k+1}, \frac{7k-3}{k+1} \right)$$

But P lies on the y -axis; so, its abscissa is 0.

$$\text{Therefore, } \frac{3k-2}{k+1} = 0$$

$$\Rightarrow 3k - 2 = 0 \Rightarrow 3k = 2 \Rightarrow k = \frac{2}{3} \Rightarrow k = \frac{2}{3}$$

Therefore, the required ratio is $\frac{2}{3} : 1$, which is same as $2 : 3$

Thus, the y -axis divides the line AB in the ratio $2 : 3$ at the point P .

Applying $k = \frac{2}{3}$, we get the coordinates of point.

$$P \left(0, \frac{7k-3}{k+1} \right)$$

$$= P \left(0, \frac{7 \times \frac{2}{3} - 3}{\frac{2}{3} + 1} \right)$$

$$= P \left(0, \frac{\frac{14-9}{3}}{\frac{2+3}{3}} \right)$$

$$= P \left(0, \frac{5}{5} \right)$$

$$= P(0,1)$$

Hence, the point of intersection of AB and the x -axis is $P(0,1)$.

- 19.** In what ratio does the line $x - y - 2 = 0$ divide the line segment joining the points $A(3, -1)$ and $B(8, 9)$?

Sol:

Let the line $x - y - 2 = 0$ divide the line segment joining the points $A(3, -1)$ and $B(8, 9)$ in the ratio $k : 1$ at P .

Then, the coordinates of P are

$$P \left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1} \right)$$

Since, P lies on the line $x - y - 2 = 0$, we have:

$$\left(\frac{8k+3}{k+1} \right) - \left(\frac{9k-1}{k+1} \right) - 2 = 0$$

$$\Rightarrow 8k + 3 - 9k + 1 - 2k - 2 = 0$$

$$\Rightarrow 8k - 9k - 2k + 3 + 1 - 2 = 0$$

$$\Rightarrow -3k + 2 = 0$$

$$\Rightarrow -3k = -2$$

$$\Rightarrow k = \frac{2}{3}$$

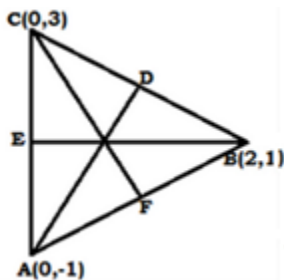
So, the required ratio is $\frac{2}{3} : 1$, which is equal to $2 : 3$.

- 20.** Find the lengths of the medians of a $\triangle ABC$ whose vertices are $A(0, -1)$, $B(2, 1)$ and $C(0, 3)$.

Sol:

The vertices of $\triangle ABC$ are $A(0,-1)$, $B(2,1)$ and $C(0,3)$.

Let AD, BE and CF be the medians of $\triangle ABC$.



Let D be the midpoint of BC . So, the coordinates of D are

$$D\left(\frac{2+0}{2}, \frac{1+3}{2}\right) \text{ i.e. } D\left(\frac{2}{2}, \frac{4}{2}\right) \text{ i.e. } D(1, 2)$$

Let E be the midpoint of AC . So the coordinate of E are

$$E\left(\frac{0+0}{2}, \frac{-1+3}{2}\right) \text{ i.e. } E\left(\frac{0}{2}, \frac{0}{2}\right) \text{ i.e. } E(0, 1)$$

Let F be the midpoint of AB . So, the coordinates of F are

$$F\left(\frac{0+2}{2}, \frac{-1+1}{2}\right) \text{ i.e. } F\left(\frac{2}{2}, \frac{0}{2}\right) \text{ i.e. } F(1, 0)$$

$$AD = \sqrt{(1-0)^2 + (2-(-1))^2} = \sqrt{(1)^2 + (3)^2} = \sqrt{1+9} = \sqrt{10} \text{ units.}$$

$$BE = \sqrt{(0-2)^2 + (1-1)^2} = \sqrt{(-2)^2 + (0)^2} = \sqrt{4+0} = \sqrt{4} = 2 \text{ units.}$$

$$CF = \sqrt{(1-0)^2 + (0-3)^2} = \sqrt{(1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10} \text{ units.}$$

Therefore, the lengths of the medians: $AD = \sqrt{10}$ units, $BE = 2$ units and $CF = \sqrt{10}$ units.

21. Find the centroid of $\triangle ABC$ whose vertices are $A(-1, 0)$, $B(5, -2)$ and $C(8, 2)$

Sol:

Here, $(x_1 = -1, y_1 = 0)$, $(x_2 = 5, y_2 = -2)$ and $(x_3 = 8, y_3 = 2)$

Let $G(x, y)$ be the centroid of the $\triangle ABC$. Then,

$$x = \frac{1}{3}(x_1 + x_2 + x_3) = \frac{1}{3}(-1 + 5 + 8) = \frac{1}{3}(12) = 4$$

$$y = \frac{1}{3}(y_1 + y_2 + y_3) = \frac{1}{3}(0 - 2 + 2) = \frac{1}{3}(0) = 0$$

Hence, the centroid of $\triangle ABC$ is $G(4, 0)$.

22. If $G(-2, 1)$ is the centroid of a $\triangle ABC$ and two of its vertices are $A(1, -6)$ and $B(-5, 2)$, find the third vertex of the triangle.

Sol:

Two vertices of $\triangle ABC$ are $A(1, -6)$ and $B(-5, 2)$. Let the third vertex be $C(a, b)$.

Then the coordinates of its centroid are

$$G\left(\frac{1-5+a}{3}, \frac{-6+2+b}{3}\right)$$

$$G\left(\frac{-4+a}{3}, \frac{-4+b}{3}\right)$$

But it is given that $G(-2, 1)$ is the centroid. Therefore,

$$-2 = \frac{-4+a}{3}, 1 = \frac{-4+b}{3}$$

$$\Rightarrow -6 = -4 + a, 3 = -4 + b$$

$$\Rightarrow -6 + 4 = a, 3 + 4 = b$$

$$\Rightarrow a = -2, b = 7$$

Therefore, the third vertex of $\triangle ABC$ is $C(-2, 7)$.

23. Find the third vertex of a $\triangle ABC$ if two of its vertices are $B(-3, 1)$ and $C(0, -2)$, and its centroid is at the origin

Sol:

Two vertices of $\triangle ABC$ are $B(-3, 1)$ and $C(0, -2)$. Let the third vertex be $A(a, b)$.

Then, the coordinates of its centroid are

$$G\left(\frac{-3+0+a}{3}, \frac{1-2+b}{3}\right)$$

$$\text{i.e., } \left(\frac{-3+a}{3}, \frac{-1+b}{3}\right)$$

But it is given that the centroid is at the origin, that is $G(0, 0)$. Therefore

$$0 = \frac{-3+a}{3}, 0 = \frac{-1+b}{3}$$

$$\Rightarrow 0 = -3 + a, 0 = -1 + b$$

$$\Rightarrow 3 = a, 1 = b$$

$$\Rightarrow a = 3, b = 1$$

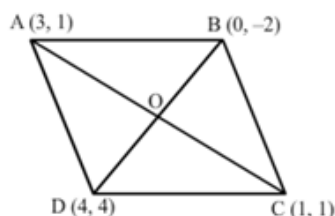
Therefore, the third vertex of $\triangle ABC$ is $A(3, 1)$.

24. Show that the points $A(3,1)$, $B(0,-2)$, $C(1,1)$ and $D(4,4)$ are the vertices of parallelogram ABCD.

Sol:

The points are $A(3,1)$, $B(0,-2)$, $C(1,1)$ and $D(4,4)$

Join AC and BD , intersecting at O .



We know that the diagonals of a parallelogram bisect each other.

$$\text{Midpoint of } AC = \left(\frac{3+1}{2}, \frac{1+1}{2} \right) = \left(\frac{4}{2}, \frac{2}{2} \right) = (2, 1)$$

$$\text{Midpoint of } BD = \left(\frac{0+4}{2}, \frac{-2+4}{2} \right) = \left(\frac{4}{2}, \frac{2}{2} \right) = (2, 1)$$

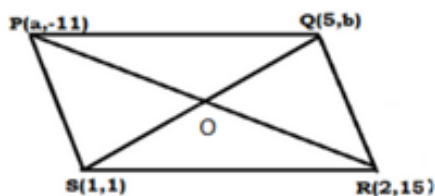
Thus, the diagonals AC and BD have the same midpoint

Therefore, $ABCD$ is a parallelogram.

25. If the points $P(a, -11)$, $Q(5, b)$, $R(2, 15)$ and $S(1, 1)$ are the vertices of a parallelogram PQRS, find the values of a and b .

Sol:

The points are $P(a, -11)$, $Q(5, b)$, $R(2, 15)$ and $S(1, 1)$.



Join PR and QS , intersecting at O .

We know that the diagonals of a parallelogram bisect each other

Therefore, O is the midpoint of PR as well as QS .

$$\text{Midpoint of } PR = \left(\frac{a+2}{2}, \frac{-11+15}{2} \right) = \left(\frac{a+2}{2}, \frac{4}{2} \right) = \left(\frac{a+2}{2}, 2 \right)$$

$$\text{Midpoint of } QS = \left(\frac{5+1}{2}, \frac{b+1}{2} \right) = \left(\frac{6}{2}, \frac{b+1}{2} \right) = \left(3, \frac{b+1}{2} \right)$$

$$\text{Therefore, } \frac{a+2}{2} = 3, \frac{b+1}{2} = 2$$

$$\Rightarrow a + 2 = 6, b + 1 = 4$$

$$\Rightarrow a = 6 - 2, b = 4 - 1$$

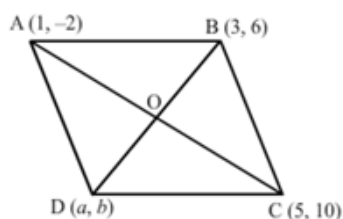
$$\Rightarrow a = 4 \text{ and } b = 3$$

26. If three consecutive vertices of a parallelogram $ABCD$ are $A(1, -2)$, $B(3, 6)$ and $C(5, 10)$, find its fourth vertex D .

Sol:

Let $A(1, -2)$, $B(3, 6)$ and $C(5, 10)$ be the three vertices of a parallelogram $ABCD$ and the fourth vertex be $D(a, b)$.

Join AC and BD intersecting at O .



We know that the diagonals of a parallelogram bisect each other

Therefore, O is the midpoint of AC as well as BD .

$$\text{Midpoint of } AC = \left(\frac{1+5}{2}, \frac{-2+10}{2} \right) = \left(\frac{6}{2}, \frac{8}{2} \right) = (3, 4)$$

$$\text{Midpoint of } BD = \left(\frac{3+a}{2}, \frac{6+b}{2} \right)$$

$$\text{Therefore, } \frac{3+a}{2} = 3 \text{ and } \frac{6+b}{2} = 4$$

$$\Rightarrow 3 + a = 6 \text{ and } 6 + b = 8$$

$$\Rightarrow a = 6 - 3 \text{ and } b = 8 - 6$$

$$\Rightarrow a = 3 \text{ and } b = 2$$

Therefore, the fourth vertex is $D(3, 2)$.

27. In what ratio does y -axis divide the line segment joining the points $(-4, 7)$ and $(3, -7)$?

Sol:

Let y -axis divides the line segment joining the points $(-4, 7)$ and $(3, -7)$ in the ratio $k : 1$. Then

$$0 = \frac{3k - 4}{k + 1}$$

$$\Rightarrow 3k = 4$$

$$\Rightarrow k = \frac{4}{3}$$

Hence, the required ratio is 4 : 3

28. If the point $P\left(\frac{1}{2}, y\right)$ lies on the line segment joining the points A(3, -5) and B(-7, 9) then find the ratio in which P divides AB. Also, find the value of y.

Sol:

Let the point $P\left(\frac{1}{2}, y\right)$ divides the line segment joining the points A(3, -5) and B(-7, 9)

in the ratio $k : 1$. Then

$$\left(\frac{1}{2}, y\right) = \left(\frac{k(-7)+3}{k+1}, \frac{k(9)-5}{k+1}\right)$$

$$\Rightarrow \frac{-7k+3}{k+1} = \frac{1}{2} \text{ and } \frac{9k-5}{k+1} = y$$

$$\Rightarrow k+1 = -14k+6 \Rightarrow k = \frac{1}{3}$$

Now, substituting $k = \frac{1}{3}$ in $\frac{9k-5}{k+1} = y$, we get

$$\frac{\frac{9}{3}-5}{\frac{1}{3}+1} = y \Rightarrow y = \frac{9-15}{1+3} = -\frac{3}{2}$$

Hence, required ratio is 1 : 3 and $y = -\frac{3}{2}$.

29. Find the ratio which the line segment joining the points A(3, -3) and B(-2, 7) is divided by x-axis. Also, find the point of division.

Sol:

The line segment joining the points A(3, -3) and B(-2, 7) is divided by x-axis. Let the required ratio be $k : 1$. So,

$$0 = \frac{k(7)-3}{k+1} \Rightarrow k = \frac{3}{7}$$

Now,

$$\begin{aligned} \text{Point of division} &= \left(\frac{k(-2)+3}{k+1}, \frac{k(7)-3}{k+1}\right) \\ &= \left(\frac{\frac{3}{7} \times (-2) + 3}{\frac{3}{7} + 1}, \frac{\frac{3}{7} \times (7) - 3}{\frac{3}{7} + 1}\right) \quad \left(\because k = \frac{3}{7}\right) \end{aligned}$$

$$= \left(\frac{-6+21}{3+7}, \frac{21-21}{3+7} \right)$$

$$= \left(\frac{3}{2}, 0 \right)$$

Hence, the required ratio is 3 : 7 and the point of division is $\left(\frac{3}{2}, 0 \right)$

- 30.** The base QR of an equilateral triangle PQR lies on x-axis. The coordinates of the point Q are (-4, 0) and origin is the midpoint of the base. Find the coordinates of the points P and R.

Sol:

Let $(x, 0)$ be the coordinates of R. Then

$$0 = \frac{-4+x}{2} \Rightarrow x = 4$$

Thus, the coordinates of R are $(4, 0)$.

Here, $PQ = QR = PR$ and the coordinates of P lies on y-axis. Let the coordinates of P be $(0, y)$. Then,

$$PQ = QR \Rightarrow PQ^2 = QR^2$$

$$\Rightarrow (0+4)^2 + (y-0)^2 = 8^2$$

$$\Rightarrow y^2 = 64 - 16 = 48$$

$$\Rightarrow y = \pm 4\sqrt{3}$$

Hence, the required coordinates are $R(4, 0)$ and $P(0, 4\sqrt{3})$ or $P(0, -4\sqrt{3})$.

- 31.** The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are $(0, -3)$. The origin is the midpoint of the base. Find the coordinates of the points A and B. Also, find the coordinates of another point D such that ABCD is a rhombus.

Sol:

Let $(0, y)$ be the coordinates of B. Then

$$0 = \frac{-3+y}{2} \Rightarrow y = 3$$

Thus, the coordinates of B are $(0, 3)$

Here, $AB = BC = AC$ and by symmetry the coordinates of A lies on x-axis Let the coordinates of A be $(x, 0)$. Then

$$AB = BC \Rightarrow AB^2 = BC^2$$

$$\Rightarrow (x-0)^2 + (0-3)^2 = 6^2$$

$$\Rightarrow x^2 = 36 - 9 = 27$$

$$\Rightarrow x = \pm 3\sqrt{3}$$

If the coordinates of point A are $(3, \sqrt{3}, 0)$. then the coordinates of D are $(-3\sqrt{3}, 0)$.

If the coordinates of point A are $(-3\sqrt{3}, 0)$. then the coordinates of D are $(3\sqrt{3}, 0)$.

Hence the required coordinates are $A(3\sqrt{3}, 0), B(0, 3)$ and $D(-3\sqrt{3}, 0)$ or

$A(-3\sqrt{3}, 0), B(0, 3)$ and $D(3\sqrt{3}, 0)$.

- 32.** Find the ratio in which the point $(-1, y)$ lying on the line segment joining points $A(-3, 10)$ and $B(6, -8)$ divides it. Also, find the value of y .

Sol:

Let k be the ratio in which $P(-1, y)$ divides the line segment joining the points

$A(-3, 10)$ and $B(6, -8)$

Then,

$$(-1, y) = \left(\frac{k(6) - 3}{k + 1}, \frac{k(-8) + 10}{k + 1} \right)$$

$$\Rightarrow \frac{k(6) - 3}{k + 1} = -1 \text{ and } y = \frac{k(-8) + 10}{k + 1}$$

$$\Rightarrow k = \frac{2}{7}$$

Substituting $k = \frac{2}{7}$ in $y = \frac{k(-8) + 10}{k + 1}$, we get

$$y = \frac{\frac{-8 \times 2}{7} + 10}{\frac{2}{7} + 1} = \frac{-16 + 70}{9} = 6$$

Hence, the required ratio is $2:7$ and $y = 6$.

- 33.** ABCD is rectangle formed by the points $A(-1, -1)$, $B(-1, 4)$, $C(5, 4)$ and $D(5, -1)$. If P, Q, R and S be the midpoints of AB, BC, CD and DA respectively, Show that PQRS is a rhombus.

Sol:

Here, the points P, Q, R and S are the midpoint of AB, BC, CD and DA respectively. Then

$$\text{Coordinates of } P = \left(\frac{-1-1}{2}, \frac{-1+4}{2} \right) = \left(-1, \frac{3}{2} \right)$$

Coordinates of $Q = \left(\frac{-1+5}{2}, \frac{4+4}{2} \right) = (2, 4)$

Coordinates of $R = \left(\frac{5+5}{2}, \frac{4-1}{2} \right) = \left(5, \frac{3}{2} \right)$

Coordinates of $S = \left(\frac{-1+5}{2}, \frac{-1-1}{2} \right) = (2, -1)$

Now,

$$PQ = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$QR = \sqrt{(5-2)^2 + \left(\frac{3}{2} - 4\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$RS = \sqrt{(5-2)^2 + \left(\frac{3}{2} + 1\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$SP = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$PR = \sqrt{(5-1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} = \sqrt{36} = 6$$

$$QS = \sqrt{(2-2)^2 + (-1-4)^2} = \sqrt{25} = 5$$

Thus, $PQ = QR = RS = SP$ and $PR \neq QS$ therefore $PQRS$ is a rhombus.

- 34.** The midpoint P of the line segment joining points $A(-10, 4)$ and $B(-2, 0)$ lies on the line segment joining the points $C(-9, -4)$ and $D(-4, y)$. Find the ratio in which P divides CD . Also, find the value of y .

Sol:

The midpoint of AB is $\left(\frac{-10-2}{2}, \frac{4+0}{2} \right) = P(-6, 2)$.

Let k be the ratio in which P divides CD . So

$$(-6, 2) = \left(\frac{k(-4) - 9}{k+1}, \frac{k(y) - 4}{k+1} \right)$$

$$\Rightarrow \frac{k(-4) - 9}{k+1} = -6 \text{ and } \frac{k(y) - 4}{k+1} = 2$$

$$\Rightarrow k = \frac{3}{2}$$

Now, substituting $k = \frac{3}{2}$ in $\frac{k(y)-4}{k+1} = 2$, we get

$$\frac{y \times \frac{3}{2} - 4}{\frac{3}{2} + 1} = 2$$

$$\Rightarrow \frac{3y-8}{5} = 2$$

$$\Rightarrow y = \frac{10+8}{3} = 6$$

Hence, the required ratio is 3 : 2 and $y = 6$.

Exercise – 16C

1. Find the area of $\triangle ABC$ whose vertices are:

(i) $A(1,2), B(-2,3)$ and $C(-3,-4)$

(ii) $A(-5,7), B(-4,-5)$ and $C(4,5)$

(iii) $A(3,8), B(-4,2)$ and $C(5,-1)$

(iv) $A(10,-6), B(2,5)$ and $C(-1,-3)$

Sol:

(i) $A(1,2), B(-2,3)$ and $C(-3,-4)$ are the vertices of $\triangle ABC$. Then,

$$(x_1 = 1, y_1 = 2), (x_2 = -2, y_2 = 3) \text{ and } (x_3 = -3, y_3 = -4)$$

Area of triangle ABC

$$= \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$= \frac{1}{2} \{1(3 - (-4)) + (-2)(-4 - 2) + (-3)(2 - 3)\}$$

$$= \frac{1}{2} \{1(3 + 4) - 2(-6) - 3(-1)\}$$

$$= \frac{1}{2} \{7 + 12 + 3\}$$

$$= \frac{1}{2} (22)$$

$$= 11 \text{ sq. units}$$

(ii) $A(-5,7), B(-4,-5)$ and $C(4,5)$ are the vertices of $\triangle ABC$. Then,

$$(x_1 = -5, y_1 = 7), (x_2 = -4, y_2 = -5) \text{ and } (x_3 = 4, y_3 = 5)$$

Area of triangle ABC

$$\begin{aligned}&= \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}\\&= \frac{1}{2} \{(-5)(-5 - 5) + (-4)(5 - 7) + 4(7 - (5))\}\\&= \frac{1}{2} \{(-5)(-10) - 4(-2) + 4(12)\}\\&= \frac{1}{2} \{50 + 8 + 48\}\\&= \frac{1}{2}(106)\\&= 53 \text{ sq. units}\end{aligned}$$

- (iii) $A(3, 8)$, $B(-4, 2)$ and $C(5, -1)$ are vertices of ΔABC . Then,

$$(x_1 = 3, y_1 = 8), (x_2 = -4, y_2 = 2) \text{ and } (x_3 = 5, y_3 = -1)$$

Area of triangle ABC

$$\begin{aligned}&= \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}\\&= \frac{1}{2} \{3(2 - (-1)) + (-4)(-1 - 8) + 5(8 - 2)\}\\&= \frac{1}{2} \{3(2 + 1) - 4(-9) + 5(6)\}\\&= \frac{1}{2} \{9 + 36 + 30\}\\&\Rightarrow \frac{1}{2}(75)\\&= 37.5 \text{ sq. units}\end{aligned}$$

- (iv) $A(10, -6)$, $B(2, 5)$ and $C(-1, -3)$ are the vertices of ΔABC . Then,

$$(x_1 = 10, y_1 = -6), (x_2 = 2, y_2 = 5) \text{ and } (x_3 = -1, y_3 = -3)$$

Area of triangle ABC

$$\begin{aligned}&= \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}\\&= \frac{1}{2} \{10(5 - (-3)) + 2(-3 - (-6)) + (-1)(-6 - 5)\}\\&= \frac{1}{2} \{10(8) + 2(3) - 1(-11)\}\\&= \frac{1}{2} \{80 + 6 + 11\}\end{aligned}$$

$$= \frac{1}{2}(49)$$

$$= 24.5 \text{ sq. units}$$

2. Find the area of a quadrilateral ABCD whose vertices are A(3, -1), B(9, -5), C(14, 0) and D(9, 19).

Sol:

By joining A and C, we get two triangles ABC and ACD .

Let

$$A(x_1, y_1) = A(3, -1), B(x_2, y_2) = B(9, -5), C(x_3, y_3) = C(14, 0) \text{ and } D(x_4, y_4) = D(9, 19)$$

Then,

$$\text{Area of } \triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [3(-5 - 0) + 9(0 + 1) + 14(-1 + 5)]$$

$$= \frac{1}{2} [-15 + 9 + 56] = 25 \text{ sq. units}$$

$$\text{Area of } \triangle ACD = \frac{1}{2} [x_1(y_3 - y_4) + x_3(y_4 - y_1) + x_4(y_1 - y_3)]$$

$$= \frac{1}{2} [3(0 - 19) + 14(19 + 1) + 9(-1 - 0)]$$

$$= \frac{1}{2} [-57 + 280 - 9] = 107 \text{ sq. units}$$

So, the area of the quadrilateral is $25 + 107 = 132 \text{ sq. units}$.

3. Find the area of quadrilateral PQRS whose vertices are P(-5, -3), Q(-4, -6), R(2, -3) and S(1, 2).

Sol:

By joining P and R, we get two triangles PQR and PRS .

Let $P(x_1, y_1) = P(-5, -3), Q(x_2, y_2) = Q(-4, -6), R(x_3, y_3) = R(2, -3)$ and. Then

$$S(x_4, y_4) = S(1, 2)$$

$$\text{Area of } \triangle PQR = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [-5(-6 + 3) - 4(-3 + 3) + 2(-3 + 6)]$$

$$= \frac{1}{2} [15 - 0 + 6] = \frac{21}{2} \text{ sq. units}$$

$$\begin{aligned}
 \text{Area of } \triangle PRS &= \frac{1}{2} [x_1(y_3 - y_4) + x_3(y_4 - y_1) + x_4(y_1 - y_3)] \\
 &= \frac{1}{2} [-5(-3 - 2) + 2(2 + 3) + 1(-3 + 3)] \\
 &= \frac{1}{2} [25 + 10 + 0] = \frac{35}{2} \text{ sq. units}
 \end{aligned}$$

So, the area of the quadrilateral $PQRS$ is $\frac{21}{2} + \frac{35}{2} = 28 \text{ sq. units}$

4. Find the area of quadrilateral $ABCD$ whose vertices are $A(-3, -1)$, $B(-2, -4)$, $C(4, -1)$ and $D(3, 4)$

Sol:

By joining A and C , we get two triangles ABC and ACD .

Let $A(x_1, y_1) = A(-3, -1)$, $B(x_2, y_2) = B(-2, -4)$, $C(x_3, y_3) = C(4, -1)$ and. Then

$$D(x_4, y_4) = D(3, 4)$$

$$\begin{aligned}
 \text{Area of } \triangle ABC &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\
 &= \frac{1}{2} [-3(-4 + 1) - 2(-1 + 1) + 4(-1 + 4)] \\
 &= \frac{1}{2} [9 - 0 + 12] = \frac{21}{2} \text{ sq. units}
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of } \triangle ACD &= \frac{1}{2} [x_1(y_3 - y_4) + x_3(y_4 - y_1) + x_4(y_1 - y_3)] \\
 &= \frac{1}{2} [-3(-1 - 4) + 4(4 + 1) + 3(-1 + 1)] \\
 &= \frac{1}{2} [15 + 20 + 0] = \frac{35}{2} \text{ sq. units}
 \end{aligned}$$

So, the area of the quadrilateral $ABCD$ is $\frac{21}{2} + \frac{35}{2} = 28 \text{ sq. units}$

5. Find the area of quadrilateral $ABCD$ whose vertices are $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$

Sol:

By joining A and C , we get two triangles ABC and ACD .

Let $A(x_1, y_1) = A(-5, 7)$, $B(x_2, y_2) = B(-4, -5)$, $C(x_3, y_3) = C(-1, -6)$ and.

$$D(x_4, y_4) = D(4, 5)$$

Then

$$\begin{aligned}
 \text{Area of } \triangle ABC &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\
 &= \frac{1}{2} [-5(-5 + 6) - 4(-6 - 7) - 1(7 + 5)] \\
 &= \frac{1}{2} [-5 + 52 - 12] = \frac{35}{2} \text{ sq. units}
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of } \triangle ACD &= \frac{1}{2} [x_1(y_3 - y_4) + x_3(y_4 - y_1) + x_4(y_1 - y_3)] \\
 &= \frac{1}{2} [-5(-6 - 5) - 1(5 - 7) + 4(7 + 6)] \\
 &= \frac{1}{2} [55 + 2 + 52] = \frac{109}{2} \text{ sq. units}
 \end{aligned}$$

So, the area of the quadrilateral $ABCD$ is $\frac{35}{2} + \frac{109}{2} = 72 \text{ sq. units}$

6. Find the area of the triangle formed by joining the midpoints of the sides of the triangle whose vertices are $A(2,1)$, $B(4,3)$ and $C(2,5)$

Sol:

The vertices of the triangle are $A(2,1)$, $B(4,3)$ and $C(2,5)$.

$$\text{Coordinates of midpoint of } AB = P(x_1, y_1) = \left(\frac{2+4}{2}, \frac{1+3}{2} \right) = (3, 2)$$

$$\text{Coordinates of midpoint of } BC = Q(x_2, y_2) = \left(\frac{4+2}{2}, \frac{3+5}{2} \right) = (3, 4)$$

$$\text{Coordinates of midpoint of } AC = R(x_3, y_3) = \left(\frac{2+2}{2}, \frac{1+5}{2} \right) = (2, 3)$$

Now,

$$\begin{aligned}
 \text{Area of } \triangle PQR &= \frac{1}{2} [x_2(y_2 - y_3) + x_3(y_3 - y_1) + x_1(y_1 - y_2)] \\
 &= \frac{1}{2} [3(4 - 3) + 3(3 - 2) + 2(2 - 4)] \\
 &= \frac{1}{2} [3 + 3 - 4] = 1 \text{ sq. unit}
 \end{aligned}$$

Hence, the area of the quadrilateral triangle is 1 sq. unit.

7. $A(7, -3)$, $B(5,3)$ and $C(3,-1)$ are the vertices of a $\triangle ABC$ and AD is its median. Prove that the median AD divides $\triangle ABC$ into two triangles of equal areas.

Sol:

The vertices of the triangle are $A(7, -3)$, $B(5,3)$, $C(3, -1)$.

Coordinates of $D = \left(\frac{5+3}{2}, \frac{3-1}{2} \right) = (4, 1)$

For the area of the triangle ADC , let

$A(x_1, y_1) = A(7, -3), D(x_2, y_2) = D(4, 1)$ and $C(x_3, y_3) = C(3, -1)$. Then

$$\text{Area of } \triangle ADC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [7(1+1) + 4(-1+3) + 3(-3-1)]$$

$$= \frac{1}{2} [14 + 8 - 12] = 5 \text{ sq. unit}$$

Now, for the area of triangle ABD , let

$A(x_1, y_1) = A(7, -3), B(x_2, y_2) = B(5, 3)$ and $D(x_3, y_3) = D(4, 1)$. Then

$$\text{Area of } \triangle ADC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [7(3-1) + 5(1+3) + 4(-3-3)]$$

$$= \frac{1}{2} [14 + 20 - 24] = 5 \text{ sq. unit}$$

Thus, $\text{Area}(\triangle ADC) = \text{Area}(\triangle ABD) = 5 \text{ sq. units}$

Hence, AD divides $\triangle ABC$ into two triangles of equal areas.

8. Find the area of $\triangle ABC$ with $A(1, -4)$ and midpoints of sides through A being $(2, -1)$ and $(0, -1)$.

Sol:

Let (x_2, y_2) and (x_3, y_3) be the coordinates of B and C respectively. Since, the coordinates of A are $(1, -4)$, therefore

$$\frac{1+x_2}{2} = 2 \Rightarrow x_2 = 3$$

$$\frac{-4+y_2}{2} = -1 \Rightarrow y_2 = 2$$

$$\frac{1+x_3}{2} = 0 \Rightarrow x_3 = -1$$

$$\frac{-4+y_3}{2} = -1 \Rightarrow y_3 = 2$$

Let $A(x_1, y_1) = A(1, -4), B(x_2, y_2) = B(3, 2)$ and $C(x_3, y_3) = C(-1, 2)$ Now

$$\text{Area}(\triangle ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [1(2-2) + 3(2+4) - 1(-4-2)]$$

$$= \frac{1}{2} [0 + 18 + 6]$$

$$= 12 \text{ sq. units}$$

Hence, the area of the triangle $\triangle ABC$ is 12 sq. units

9. A(6,1), B(8,2) and C(9,4) are the vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of $\triangle ADE$

Sol:

Let (x, y) be the coordinates of D and (x', y') be the coordinates of E . Since, the diagonals of a parallelogram bisect each other at the same point, therefore

$$\frac{x+8}{2} = \frac{6+9}{2} \Rightarrow x = 7$$

$$\frac{y+2}{2} = \frac{1+4}{2} \Rightarrow y = 3$$

Thus, the coordinates of D are $(7, 3)$

E is the midpoint of DC , therefore

$$x' = \frac{7+9}{2} \Rightarrow x' = 8$$

$$y' = \frac{3+4}{2} \Rightarrow y' = \frac{7}{2}$$

Thus, the coordinates of E are $\left(8, \frac{7}{2}\right)$

Let $A(x_1, y_1) = A(6, 1)$, $E(x_2, y_2) = E\left(8, \frac{7}{2}\right)$ and $D(x_3, y_3) = D(7, 3)$ Now

$$\text{Area } (\triangle ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} \left[6\left(\frac{7}{2} - 3\right) + 8(3 - 1) + 7\left(1 - \frac{7}{2}\right) \right]$$

$$= \frac{1}{2} \left[\frac{3}{2} \right]$$

$$= \frac{3}{4} \text{ sq. unit}$$

Hence, the area of the triangle $\triangle ADE$ is $\frac{3}{4}$ sq. units

10. If the vertices of $\triangle ABC$ be $A(1, -3)$, $B(4, p)$ and $C(-9, 7)$ and its area is 15 square units, find the values of p .

Sol:

Let $A(x_1, y_1) = A(1, -3)$, $B(x_2, y_2) = B(4, p)$ and $C(x_3, y_3) = C(-9, 7)$ Now

$$\text{Area}(\triangle ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow 15 = \frac{1}{2} [1(p - 7) + 4(7 + 3) - 9(-3 - p)]$$

$$\Rightarrow 15 = \frac{1}{2} [10p + 60]$$

$$\Rightarrow |10p + 60| = 30$$

Therefore

$$\Rightarrow 10p + 60 = -30 \text{ or } 30$$

$$\Rightarrow 10p = -90 \text{ or } -30$$

$$\Rightarrow p = -9 \text{ or } -3$$

Hence, $p = -9$ or $p = -3$.

11. Find the value of k so that the area of the triangle with vertices $A(k+1, 1)$, $B(4, -3)$ and $C(7, -k)$ is 6 square units.

Sol:

Let $A(x_1, y_1) = A(k+1, 1)$, $B(x_2, y_2) = B(4, -3)$ and $C(x_3, y_3) = C(7, -k)$ now

$$\text{Area}(\triangle ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow 6 = \frac{1}{2} [(k+1)(-3+k) + 4(-k-1) + 7(1+3)]$$

$$\Rightarrow 6 = \frac{1}{2} [k^2 - 2k - 3 - 4k - 4 + 28]$$

$$\Rightarrow k^2 - 6k + 9 = 0$$

$$\Rightarrow (k-3)^2 = 0 \Rightarrow k = 3$$

Hence, $k = 3$.

12. For what value of k ($k > 0$) is the area of the triangle with vertices $(-2, 5)$, $(k, -4)$ and $(2k+1, 10)$ equal to 53 square units?

Sol:

Let $A(x_1, y_1) = (-2, 5)$, $B(x_2, y_2) = (k, -4)$ and $C(x_3, y_3) = (2k+1, 10)$ be the vertices of the triangle, So

$$\text{Area } (\triangle ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow 53 = \frac{1}{2} [(-2)(-4 - 10) + k(10 - 5) + (2k + 1)(5 + 4)]$$

$$\Rightarrow 53 = \frac{1}{2} [28 + 5k + 9(2k + 1)]$$

$$\Rightarrow 28 + 5k + 18k + 9 = 106$$

$$\Rightarrow 37 + 23k = 106$$

$$\Rightarrow 23k = 106 - 37 = 69$$

$$\Rightarrow k = \frac{69}{23} = 3$$

Hence, $k = 3$.

13. Show that the following points are collinear:

(i) A(2, -2), B(-3, 8) and C(-1, 4)

(ii) A(-5, 1), B(5, 5) and C(10, 7)

(iii) A(5, 1), B(1, -1) and C(11, 4)

(iv) A(8, 1), B(3, -4) and C(2, -5)

Sol:

(i) Let $A(x_1 = 2, y_1 = -2)$, $B(x_2 = -3, y_2 = 8)$ and $C(x_3 = -1, y_3 = 4)$ be the given points.

$$\text{Now } x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$$

$$= 2(8 - 4) + (-3)(4 + 2) + (-1)(-2 - 8)$$

$$= 8 - 18 + 10$$

$$= 0$$

Hence, the given points are collinear.

(ii) Let $A(x_1 = -5, y_1 = 1)$, $B(x_2 = 5, y_2 = 5)$ and $C(x_3 = 10, y_3 = 7)$ be the given points.

$$\text{Now } x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$$

$$= (-5)(5 - 7) + 5(7 - 1) + 10(1 - 5)$$

$$= -5(-2) + 5(6) + 10(-4)$$

$$= 10 + 30 - 40$$

$$= 0$$

Hence, the given points are collinear.

(iii) Let $A(x_1 = 5, y_1 = 1)$, $B(x_2 = 1, y_2 = -1)$ and $C(x_3 = 11, y_3 = 4)$ be the given points.

$$\text{Now } x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$$

$$\begin{aligned} &= 5(-1-4) + 1(4-1) + 11(1+1) \\ &= -25 + 3 + 22 \\ &= 0 \end{aligned}$$

Hence, the given points are collinear.

- (iv) Let $A(x_1 = 8, y_1 = 1)$, $B(x_2 = 3, y_2 = -4)$ and $C(x_3 = 2, y_3 = -5)$ be the given points.

$$\begin{aligned} &\text{Now } x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \\ &= 8(-4 + 5) + 3(-5 - 1) + 2(1 + 4) \\ &= 8 - 18 + 10 \\ &= 0 \end{aligned}$$

Hence, the given points are collinear.

14. Find the value of x for which points $A(x, 2)$, $B(-3, -4)$ and $C(7, -5)$ are collinear.

Sol:

Let $A(x_1, y_1) = A(x, 2)$, $B(x_2, y_2) = B(-3, -4)$ and $C(x_3, y_3) = C(7, -5)$. So the condition for three collinear points is

$$\begin{aligned} &x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0 \\ \Rightarrow &x(-4 + 5) - 3(-5 - 2) + 7(2 + 4) = 0 \\ \Rightarrow &x + 21 + 42 = 0 \\ \Rightarrow &x = -63 \end{aligned}$$

Hence, $x = -63$.

15. For what value of x are the points $A(-3, 12)$, $B(7, 6)$ and $C(x, 9)$ collinear.

Sol:

$A(-3, 12)$, $B(7, 6)$ and $C(x, 9)$ are the given points. Then:

$$(x_1 = -3, y_1 = 12), (x_2 = 7, y_2 = 6) \text{ and } (x_3 = x, y_3 = 9)$$

It is given that points A , B and C are collinear. Therefore,

$$\begin{aligned} &x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0 \\ \Rightarrow &(-3)(6 - 9) + 7(9 - 12) + x(12 - 6) = 0 \\ \Rightarrow &(-3)(-3) + 7(-3) + x(6) = 0 \\ \Rightarrow &9 - 21 + 6x = 0 \\ \Rightarrow &6x - 12 = 0 \\ \Rightarrow &6x = 12 \\ \Rightarrow &x = \frac{12}{6} = 2 \end{aligned}$$

Therefore, when $x = 2$, the given points are collinear

16. For what value of y , are the points $P(1, 4)$, $Q(3, y)$ and $R(-3, 16)$ are collinear?

Sol:

$P(1, 4)$, $Q(3, y)$ and $R(-3, 16)$ are the given points. Then:

$$(x_1 = 1, y_1 = 4), (x_2 = 3, y_2 = y) \text{ and } (x_3 = -3, y_3 = 16)$$

It is given that the points P , Q and R are collinear.

Therefore,

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow 1(y - 16) + 3(16 - 4) + (-3)(4 - y) = 0$$

$$\Rightarrow 1(y - 16) + 3(12) - 3(4 - y) = 0$$

$$\Rightarrow y - 16 + 36 - 12 + 3y = 0$$

$$\Rightarrow 8 + 4y = 0$$

$$\Rightarrow 4y = -8$$

$$\Rightarrow y = -\frac{8}{4} = -2$$

When, $y = -2$, the given points are collinear.

17. Find the value of y for which the points $A(-3, 9)$, $B(2, y)$ and $C(4, -5)$ are collinear.

Sol:

Let $A(x_1 = -3, y_1 = 9)$, $B(x_2 = 2, y_2 = y)$ and $C(x_3 = 4, y_3 = -5)$ be the given points

The given points are collinear if

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow (-3)(y + 5) + 2(-5 - 9) + 4(9 - y) = 0$$

$$\Rightarrow -3y - 15 - 28 + 36 - 4y = 0$$

$$\Rightarrow 7y = 36 - 43$$

$$\Rightarrow y = -1$$

18. For what values of k are the points $A(8, 1)$, $B(3, -2k)$ and $C(k, -5)$ collinear.

Sol:

Let $A(x_1 = 8, y_1 = 1)$, $B(x_2 = 3, y_2 = -2k)$ and $C(x_3 = k, y_3 = -5)$ be the given points

The given points are collinear if

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow 8(-2k + 5) + 3(-5 - 1) + k(1 + 2k) = 0$$

$$\Rightarrow -16k + 40 - 18 + k + 2k^2 = 0$$

$$\Rightarrow 2k^2 - 15k + 22 = 0$$

$$\Rightarrow 2k^2 - 11k - 4k + 22 = 0$$

$$\Rightarrow k(2k - 11) - 2(2k - 11) = 0$$

$$\Rightarrow (k-2)(2k-11)=0$$

$$\Rightarrow k=2 \text{ or } k=\frac{11}{22}$$

$$\text{Hence, } k=2 \text{ or } k=\frac{11}{22}.$$

19. Find a relation between x and y, if the points A(2, 1), B(x, y) and C(7,5) are collinear.

Sol:

Let $A(x_1=2, y_1=1)$, $B(x_2=x, y_2=y)$ and $C(x_3=7, y_3=5)$ be the given points

The given points are collinear if

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow 2(y-5) + x(5-1) + 7(1-y) = 0$$

$$\Rightarrow 2y - 10 + 4x + 7 - 7y = 0$$

$$\Rightarrow 4x - 5y - 3 = 0$$

Hence, the required relation is $4x - 5y - 3 = 0$.

20. Find a relation between x and y, if the points A(x, y), B(-5, 7) and C(-4, 5) are collinear.

Sol:

Let $A(x_1=x, y_1=y)$, $B(x_2=-5, y_2=7)$ and $C(x_3=-4, y_3=5)$ be the given points

The given points are collinear if

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow x(7-5) + (-5)(5-y) + (-4)(y-7) = 0$$

$$\Rightarrow 7x - 5x - 25 + 5y - 4y + 28 = 0$$

$$\Rightarrow 2x + y + 3 = 0$$

Hence, the required relation is $2x + y + 3 = 0$

21. Prove that the points $A(a,0)$, $B(0,b)$ and $C(1,1)$ are collinear, if $\left(\frac{1}{a} + \frac{1}{b}\right) = 1$.

Sol:

Consider the points $A(a,0)$, $B(0,b)$ and $C(1,1)$.

Here, $(x_1=a, y_1=0)$, $(x_2=0, y_2=b)$ and $(x_3=1, y_3=1)$.

It is given that the points are collinear. So,

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow a(b-1) + 0(1-0) + 1(0-b) = 0$$

$$\Rightarrow ab - a - b = 0$$

Dividing the equation by ab :

$$\Rightarrow 1 - \frac{1}{b} - \frac{1}{a} = 0$$

$$\Rightarrow 1 - \left(\frac{1}{a} + \frac{1}{b} \right) = 0$$

$$\Rightarrow \left(\frac{1}{a} + \frac{1}{b} \right) = 1$$

Therefore, the given points are collinear if $\left(\frac{1}{a} + \frac{1}{b} \right) = 1$.

- 22.** If the points P(-3, 9), Q(a, b) and R(4, -5) are collinear and $a+b=1$, find the value of a and b.

Sol:

Let $A(x_1 = -3, y_1 = 9)$, $B(x_2 = a, y_2 = b)$ and $C(x_3 = 4, y_3 = -5)$ be the given points.

The given points are collinear if

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow (-3)(b + 5) + a(-5 - 9) + 4(9 - b) = 0$$

$$\Rightarrow -3b - 15 - 14a + 36 - 4b = 0$$

$$\Rightarrow 2a + b = 3$$

Now solving $a + b = 1$ and $2a + b = 3$, we get $a = 2$ and $b = -1$.

Hence, $a = 2$ and $b = -1$.

- 23.** Find the area of $\triangle ABC$ with vertices A(0, -1), B(2, 1) and C(0, 3). Also, find the area of the triangle formed by joining the midpoints of its sides. Show that the ratio of the areas of two triangles is 4:1.

Sol:

Let $A(x_1 = 0, y_1 = -1)$, $B(x_2 = 2, y_2 = 1)$ and $C(x_3 = 0, y_3 = 3)$ be the given points. Then

$$\text{Area}(\triangle ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [0(1 - 3) + 2(3 + 1) + 0(-1 - 1)]$$

$$= \frac{1}{2} \times 8 = 4 \text{ sq. units}$$

So, the area of the triangle $\triangle ABC$ is 4 sq. units

Let $D(a_1, b_1)$, $E(a_2, b_2)$ and $F(a_3, b_3)$ be the midpoints of AB, BC and AC respectively

Then

$$a_1 = \frac{0+2}{2} = 1 \quad b_1 = \frac{-1+1}{2} = 0$$

$$a_2 = \frac{2+0}{2} = 1 \quad b_2 = \frac{1+3}{2} = 2$$

$$a_3 = \frac{0+0}{2} = 0 \quad b_3 = \frac{-1+3}{2} = 1$$

Thus, the coordinates of D , E and F are $D(a_1 = 1, b_1 = 0)$, $E(a_2 = 1, b_2 = 2)$ and $F(a_3 = 0, b_3 = 1)$. Now

$$\text{Area}(\triangle DEF) = \frac{1}{2} [a_1(b_2 - b_3) + a_2(b_3 - b_1) + a_3(b_1 - b_2)]$$

$$= \frac{1}{2} [1(2 - 1) + 1(1 - 0) + 0(0 - 2)]$$

$$= \frac{1}{2} [1 + 1 + 0] = 1 \text{ sq. unit}$$

So, the area of the triangle $\triangle DEF$ is 1 sq. unit.

Hence, $\triangle ABC : \triangle DEF = 4 : 1$.

Exercise – 16D

1. Points $A(-1, y)$ and $B(5, 7)$ lie on the circle with centre $O(2, -3y)$. Find the value of y .

Sol:

The given points are $A(-1, y)$, $B(5, 7)$ and $O(2, -3y)$.

Here, AO and BO are the radii of the circle. So

$$AO = BO \Rightarrow AO^2 = BO^2$$

$$\Rightarrow (2+1)^2 + (-3y-y)^2 = (2-5)^2 + (-3y-7)^2$$

$$\Rightarrow 9 + (4y)^2 = (-3)^2 + (3y+7)^2$$

$$\Rightarrow 9 + 16y^2 = 9 + 9y^2 + 49 + 42y$$

$$\Rightarrow 7y^2 - 42y - 49 = 0$$

$$\Rightarrow y^2 - 6y - 7 = 0$$

$$\Rightarrow y^2 - 7y + y - 7 = 0$$

$$\Rightarrow y(y-7) + 1(y-7) = 0$$

$$\Rightarrow (y-7)(y+1) = 0$$

$$\Rightarrow y = -1 \text{ or } y = 7$$

Hence, $y = 7$ or $y = -1$.

2. If the point $A(0, 2)$ is equidistant from the points $B(3, p)$ and $C(p, 5)$, find p .

Sol:

The given points are $A(0, 2)$, $B(3, p)$ and $C(p, 5)$.

$$AB = AC \Rightarrow AB^2 = AC^2$$

$$\Rightarrow (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$$

$$\Rightarrow 9 + p^2 - 4p + 4 = p^2 + 9$$

$$\Rightarrow 4p = 4 \Rightarrow p = 1$$

Hence, $p = 1$.

3. ABCD is a rectangle whose three vertices are A(4,0), C(4,3) and D(0,3). Find the length of one its diagonal.

Sol:

The given vertices are B(4, 0), C(4, 3) and D(0, 3) Here, BD one of the diagonals So

$$BD = \sqrt{(4-0)^2 + (0-3)^2}$$

$$= \sqrt{(4)^2 + (-3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

$$= 5$$

Hence, the length of the diagonal is 5 units.

4. If the point P(k-1, 2) is equidistant from the points A(3,k) and B(k,5), find the value of k.

Sol:

The given points are $P(k-1, 2)$, $A(3, k)$ and $B(k, 5)$.

$$\because AP = BP$$

$$\therefore AP^2 = BP^2$$

$$\Rightarrow (k-1-3)^2 + (2-k)^2 = (k-1-k)^2 + (2-5)^2$$

$$\Rightarrow (k-4)^2 + (2-k)^2 = (-1)^2 + (-3)^2$$

$$\Rightarrow k^2 - 8k + 16 + 4 + k^2 - 4k = 1 + 9$$

$$\Rightarrow k^2 - 6k + 5 = 0$$

$$\Rightarrow (k-1)(k-5) = 0$$

$$\Rightarrow k = 1 \text{ or } k = 5$$

Hence, $k = 1$ or $k = 5$

5. Find the ratio in which the point P(x,2) divides the join of A(12, 5) and B(4, -3).

Sol:

Let k be the ratio in which the point $P(x, 2)$ divides the line joining the points

$A(x_1 = 12, y_1 = 5)$ and $B(x_2 = 4, y_2 = -3)$. Then

$$x = \frac{k \times 4 + 12}{k + 1} \text{ and } 2 = \frac{k \times (-3) + 5}{k + 1}$$

Now,

$$2 = \frac{k \times (-3) + 5}{k + 1} \Rightarrow 2k + 2 = -3k + 5 \Rightarrow k = \frac{3}{5}$$

Hence, the required ratio is 3 : 5.

6. Prove that the diagonals of a rectangle ABCD with vertices A(2,-1), B(5,-1) C(5,6) and D(2,6) are equal and bisect each other.

Sol:

The vertices of the rectangle ABCD are A(2,-1), B(5,-1), C(5,6) and D(2,6). Now

$$\text{Coordinates of midpoint of } AC = \left(\frac{2+5}{2}, \frac{-1+6}{2} \right) = \left(\frac{7}{2}, \frac{5}{2} \right)$$

$$\text{Coordinates of midpoint of } BD = \left(\frac{5+2}{2}, \frac{-1+6}{2} \right) = \left(\frac{7}{2}, \frac{5}{2} \right)$$

Since, the midpoints of AC and BD coincide, therefore the diagonals of rectangle ABCD bisect each other

7. Find the lengths of the medians AD and BE of $\triangle ABC$ whose vertices are A(7,-3), B(5,3) and C(3,-1)

Sol:

The given vertices are A(7,-3), B(5,3) and C(3,-1).

Since D and E are the midpoints of BC and AC respectively. therefore

$$\text{Coordinates of } D = \left(\frac{5+3}{2}, \frac{3-1}{2} \right) = (4, 1)$$

$$\text{Coordinates of } E = \left(\frac{7+3}{2}, \frac{-3-1}{2} \right) = (5, -2)$$

Now

$$AD = \sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{9+16} = 5$$

$$BE = \sqrt{(5-5)^2 + (3+2)^2} = \sqrt{0+25} = 5$$

Hence, $AD = BE = 5$ units.

8. If the point C(k,4) divides the join of A(2,6) and B(5,1) in the ratio 2:3 then find the value of k.

Sol:

Here, the point C(k,4) divides the join of A(2,6) and B(5,1) in ratio 2 : 3. So

$$\begin{aligned}k &= \frac{2 \times 5 + 3 \times 2}{2 + 3} \\&= \frac{10 + 6}{5} \\&= \frac{16}{5}\end{aligned}$$

$$\text{Hence, } k = \frac{16}{5}.$$

9. Find the point on x-axis which is equidistant from points A(-1,0) and B(5,0)

Sol:

Let $P(x, 0)$ be the point on x -axis. Then

$$AP = BP \Rightarrow AP^2 = BP^2$$

$$\Rightarrow (x+1)^2 + (0-0)^2 = (x-5)^2 + (0-0)^2$$

$$\Rightarrow x^2 + 2x + 1 = x^2 - 10x + 25$$

$$\Rightarrow 12x = 24 \Rightarrow x = 2$$

Hence, $x = 2$

10. Find the distance between the points $A\left(\frac{-8}{5}, 2\right)$ and $B\left(\frac{2}{5}, 2\right)$

Sol:

The given points are $A\left(\frac{-8}{5}, 2\right)$ and $B\left(\frac{2}{5}, 2\right)$

Then, $\left(x_1 = \frac{-8}{5}, y_1 = 2\right)$ and $\left(x_2 = \frac{2}{5}, y_2 = 2\right)$

Therefore,

$$\begin{aligned}AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{\left\{\frac{2}{5} - \left(\frac{-8}{5}\right)\right\}^2 + (2 - 2)^2} \\&= \sqrt{(2)^2 + (0)^2} \\&= \sqrt{4 + 0} \\&= \sqrt{4} \\&= 2 \text{ units.}\end{aligned}$$

11. Find the value of a , so that the point $(3, a)$ lies on the line represented by $2x - 3y = 5$.

Sol:

The points $(3, a)$ lies on the line $2x - 3y = 5$.

If point $(3, a)$ lies on the line $2x - 3y = 5$, then $2x - 3y = 5$

$$\Rightarrow (2 \times 3) - (3 \times a) = 5$$

$$\Rightarrow 6 - 3a = 5$$

$$\Rightarrow 3a = 1$$

$$\Rightarrow a = \frac{1}{3}$$

Hence, the value of a is $\frac{1}{3}$.

12. If the points $A(4, 3)$ and $B(x, 5)$ lie on the circle with center $O(2, 3)$, find the value of x .

Sol:

The given points $A(4, 3)$ and $B(x, 5)$ lie on the circle with center $O(2, 3)$.

Then, $OA = OB$

$$\Rightarrow \sqrt{(x-2)^2 + (5-3)^2} = \sqrt{(4-2)^2 + (3-3)^2}$$

$$\Rightarrow (x-2)^2 + 2^2 = 2^2 + 0^2$$

$$\Rightarrow (x-2)^2 = (2^2 - 2^2)$$

$$\Rightarrow (x-2)^2 = 0$$

$$\Rightarrow x - 2 = 0$$

$$\Rightarrow x = 2$$

Hence, the value of $x = 2$

13. If $P(x, y)$ is equidistant from the points $A(7, 1)$ and $B(3, 5)$, find the relation between x and y .

Sol:

Let the point $P(x, y)$ be equidistant from the points $A(7, 1)$ and $B(3, 5)$

Then,

$$PA = PB$$

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$$

$$\Rightarrow x^2 + y^2 - 14x - 2y + 50 = x^2 + y^2 - 6x - 10y + 34$$

$$\Rightarrow 8x - 8y = 16$$

$$\Rightarrow x - y = 2$$

14. If the centroid of $\triangle ABC$ having vertices $A(a, b)$, $B(b, c)$ and $C(c, a)$ is the origin, then find the value of $(a + b + c)$.

Sol:

The given points are $A(a, b)$, $B(b, c)$ and $C(c, a)$

Here,

$(x_1 = a, y_1 = b)$, $(x_2 = b, y_2 = c)$ and $(x_3 = c, y_3 = a)$

Let the centroid be (x, y) .

Then,

$$x = \frac{1}{3}(x_1 + x_2 + x_3)$$

$$= \frac{1}{3}(a + b + c)$$

$$= \frac{a + b + c}{3}$$

$$y = \frac{1}{3}(y_1 + y_2 + y_3)$$

$$= \frac{1}{3}(b + c + a)$$

$$= \frac{a + b + c}{3}$$

But it is given that the centroid of the triangle is the origin.

Then, we have

$$\frac{a + b + c}{3} = 0$$

$$\Rightarrow a + b + c = 0$$

15. Find the centroid of $\triangle ABC$ whose vertices are $A(2, 2)$, $B(-4, -4)$ and $C(5, -8)$.

Sol:

The given points are $A(2, 2)$, $B(-4, -4)$ and $C(5, -8)$.

Here, $(x_1 = 2, y_1 = 2)$, $(x_2 = -4, y_2 = -4)$ and $(x_3 = 5, y_3 = -8)$

Let $G(x, y)$ be the centroid of $\triangle ABC$ Then,

$$x = \frac{1}{3}(x_1 + x_2 + x_3)$$

$$= \frac{1}{3}(2 - 4 + 5)$$

$$= 1$$

$$\begin{aligned}y &= \frac{1}{3}(y_1 + y_2 + y_3) \\&= \frac{1}{3}(2 - 4 - 8) \\&= \frac{-10}{3}\end{aligned}$$

Hence, the centroid of $\triangle ABC$ is $G\left(1, \frac{-10}{3}\right)$.

16. In what ratio does the point $C(4,5)$ divides the join of $A(2,3)$ and $B(7,8)$?

Sol:

Let the required ratio be $k : 1$

Then, by section formula, the coordinates of C are

$$C\left(\frac{7k+2}{k+1}, \frac{8k+3}{k+1}\right)$$

Therefore,

$$\frac{7k+2}{k+1} = 4 \text{ and } \frac{8k+3}{k+1} = 5 \quad [\because C(4,5) \text{ is given}]$$

$$\Rightarrow 7k+2 = 4k+4 \text{ and } 8k+3 = 5k+5 \Rightarrow 3k = 2$$

$$\Rightarrow k = \frac{2}{3} \text{ in each case}$$

So, the required ratio is $\frac{2}{3} : 1$, which is same as $2 : 3$.

17. If the points $A(2,3)$, $B(4,k)$ and $C(6,-3)$ are collinear, find the value of k .

Sol:

The given points are $A(2,3)$, $B(4,k)$ and $C(6,-3)$

Here, $(x_1 = 2, y_1 = 3)$, $(x_2 = 4, y_2 = k)$ and $(x_3 = 6, y_3 = -3)$

It is given that the points A , B and C are collinear. Then,

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow 2(k+3) + 4(-3-3) + 6(3-k) = 0$$

$$\Rightarrow 2k + 6 - 24 + 18 - 6k = 0$$

$$\Rightarrow -4k = 0$$

$$\Rightarrow k = 0$$

Exercise – Multiple Choice Questions

1. The distance of the point P(-6,8) from the origin is

(a) 8 (b) $2\sqrt{7}$ (c) 6 (d) 10

Answer: (d) 10

Sol:

The distance of a point (x, y) from the origin $O(0,0)$ is $\sqrt{x^2 + y^2}$

Let $P(x = -6, y = 8)$ be the gen point. Then

$$\begin{aligned} OP &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-6)^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} = 10 \end{aligned}$$

2. The distance of the point (-3, 4) from x-axis is

(a) 3 (b) -3 (c) 4 (d) 5

Answer: (c) 4

Sol:

The distance of a point (x, y) from x -axis is $|y|$.

Here, the point is $(-3, 4)$. So, its distance from x -axis is $|4| = 4$

3. The point on x-axis which is equidistant from the points A(-1, 0) and B(5,0) is

(a) (0,2) (b) (2,0) (c) (3,0) (d) (0,3)

Answer: (b) (2,0)

Sol:

Let $P(x, 0)$ the point on x -axis, then

$$\begin{aligned} AP &= BP \Rightarrow AP^2 = BP^2 \\ \Rightarrow (x+1)^2 + (0-0)^2 &= (x-5)^2 + (0-0)^2 \\ \Rightarrow x^2 + 2x + 1 &= x^2 - 10x + 25 \\ \Rightarrow 12x &= 24 \Rightarrow x = 2 \end{aligned}$$

Thus, the required point is (2, 0).

4. If R(5,6) is the midpoint of the line segment AB joining the points A(6,5) and B(4,4) then y equals

(a) 5 (b) 7 (c) 12 (d) 6

Answer: (b) 7

Sol:

Since $R(5,6)$ is the midpoint of the line segment AB joining the points

$A(6,5)$ and $B(4,y)$, therefore

$$\frac{5+y}{2} = 6$$

$$\Rightarrow 5+y = 12$$

$$\Rightarrow y = 12 - 5 = 7$$

5. If the point $C(k,4)$ divides the join of the points $A(2,6)$ and $B(5,1)$ in the ratio 2:3 then the value of k is

(a) 16 (b) $\frac{28}{5}$ (c) $\frac{16}{5}$ (d) $\frac{8}{5}$

Answer: (c) $\frac{16}{5}$

Sol:

The point $C(k,4)$ divides the join of the points $A(2,6)$ and $B(5,1)$ in the ratio 2:3. So

$$k = \frac{2 \times 5 + 3 \times 2}{2 + 3} = \frac{10 + 6}{5} = \frac{16}{5}$$

6. The perimeter of the triangle with vertices $(0,4)$, $(0,0)$ and $(3,0)$ is

(a) $(7 + \sqrt{5})$ (b) 5 (c) 10 (d) 12

Answer: (d) 12

Sol:

Let $A(0,4)$, $B(0,0)$ and $C(3,0)$ be the given vertices. So

$$AB = \sqrt{(0-0)^2 + (4-0)^2} = \sqrt{16} = 4$$

$$BC = \sqrt{(0-3)^2 + (0-0)^2} = \sqrt{9} = 3$$

$$AC = \sqrt{(0-3)^2 + (4-0)^2} = \sqrt{9+16} = 5$$

Therefore

$$AB + BC + AC = 4 + 3 + 5 = 12.$$

7. If $A(1,3)$, $B(-1,2)$, $C(2,5)$ and $D(x,4)$ are the vertices of a ||gm ABCD then the value of x is

(a) 3 (b) 4 (c) 0 (d) $\frac{3}{2}$

Answer: (b) 4

Sol:

The diagonals of a parallelogram bisect each other. The vertices of the $\square ABCD$ are $A(1,3), B(-1,2)$ and $C(2,5)$ and $D(x,4)$

Here, AC and BD are the diagonals. So

$$\frac{1+2}{2} = \frac{-1+x}{2}$$

$$\Rightarrow x-1=3$$

$$\Rightarrow x=1+3=4$$

8. If the points $A(x,2), B(-3, -4)$ and $C(7, -5)$ are collinear then the value of x is

(a) -63 (b) 63 (c) 60 (d) -60

Answer: (a) -63

Sol:

Let $A(x_1=x, y_1=2), B(x_2=-3, y_2=-4)$ and $C(x_3=7, y_3=-5)$ be collinear points. Then

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow x(-4+5) + (-3)(-5-2) + 7(2+4) = 0$$

$$\Rightarrow x+21+42=0$$

$$\Rightarrow x=-63$$

9. The area of a triangle with vertices $A(5,0), B(8,0)$ and $C(8,4)$ in square units is

(a) 20 (b) 12 (c) 6 (d) 16

Answer: (c) 6

Sol:

Let $A(x_1=5, y_1=0), B(x_2=8, y_2=0)$ and $C(x_3=8, y_3=4)$ be the vertices of the triangle.

Then,

$$\text{Area}(\triangle ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [5(0-4) + 8(4-0) + 8(0-0)]$$

$$= \frac{1}{2} [-20 + 32 + 0]$$

$$= 6 \text{ sq. units}$$

10. The area of $\triangle ABC$ with vertices $A(a,0), O(0,0)$ and $B(0,b)$ in square units is

(a) ab (b) $\frac{1}{2}ab$ (c) $\frac{1}{2}a^2b^2$ (d) $\frac{1}{2}b^2$

Answer: (b) $\frac{1}{2}ab$

Sol:

Let $A(x_1 = a, y_1 = 0)$, $O(x_2 = 0, y_2 = 0)$ and $B(x_3 = 0, y_3 = b)$ be the given vertices. So

$$\begin{aligned} \text{Area}(\Delta ABO) &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} |a(0 - b) + 0(b - 0) + 0(0 - 0)| \\ &= \frac{1}{2} |-ab| \\ &= \frac{1}{2} ab \end{aligned}$$

11. If $P\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points $A(-6, 5)$ and $B(-2, 3)$ then

the value of a is

- (a) -8 (b) 3 (c) -4 (d) 4

Answer: (a) -8

Sol:

The point $P\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points $A(-6, 5)$ and $B(-2, 3)$.

$$\text{So } \frac{a}{2} = \frac{-6 - 2}{2}$$

$$\Rightarrow \frac{a}{2} = -4$$

$$\Rightarrow a = -8$$

12. ABCD is a rectangle whose three vertices are $B(4, 0)$, $C(4, 3)$ and $D(0, 3)$. The length of one of its diagonals is

- (a) 5 (b) 4 (c) 3 (d) 245

Answer: (a) 5

Sol:

Here, AC and BD are two diagonals of the rectangle $ABCD$. So

$$\begin{aligned} BD &= \sqrt{(4 - 0)^2 + (0 - 3)^2} \\ &= \sqrt{(4)^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \text{ units} \end{aligned}$$

13. The coordinates of the point P dividing the line segment joining the points A(1,3), and B(4,6) in the ratio 2:1 is
 (a) (2,4) (b) (3,5) (c) (4,2) (d) (5,3)

Answer: (b) (3,5)

Sol:

Here, the point P divides the line segment joining the points A(1,3) and B(4,6) in the ratio 2:1. Then,

$$\begin{aligned}\text{Coordinates of } P &= \left(\frac{2 \times 4 + 1 \times 1}{2+1}, \frac{2 \times 6 + 1 \times 3}{2+1} \right) \\ &= \left(\frac{8+1}{3}, \frac{12+3}{3} \right) \\ &= \left(\frac{9}{3}, \frac{15}{3} \right) \\ &= (3,5)\end{aligned}$$

14. If the coordinates of one end of a diameter of a circle are (2,3) and the coordinates of its centre are (-2,5), then the coordinates of the other end of the diameter are
 (a) (-6,7) (b) (6,-7) (c) (4,2) (d) (5,3)

Answer: (a) (-6,7)

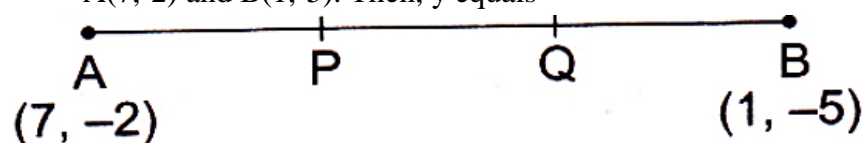
Sol:

Let (x, y) be the coordinates of the other end of the diameter. Then

$$-2 = \frac{2+x}{2} \Rightarrow x = -6$$

$$5 = \frac{3+y}{2} \Rightarrow y = 7$$

15. In the given figure P(5,-3) and Q(3,y) are the points of trisection of the line segment joining A(7,-2) and B(1,-5). Then, y equals



- (a) 2 (b) 4 (c) -4 (d) $-\frac{5}{2}$

Answer: (c) -4

Sol:

Here, $AQ : BQ = 2 : 1$. Then,

$$y = \frac{2 \times (-5) + 1 \times (-2)}{2+1}$$

$$\begin{aligned} &= \frac{-10-2}{3} \\ &= -4 \end{aligned}$$

16. The midpoint of segment AB is P(0,4). If the coordinates of B are (-2, 3), then the coordinates of A are

(a) (2,5) (b) (-2,-5) (c) (2,9) (d) (-2,11)

Answer: (a) (2,5)

Sol:

Let (x, y) be the coordinates of A. then,

$$0 = \frac{-2+x}{2} \Rightarrow x = 2$$

$$4 = \frac{3+y}{2} \Rightarrow y = 8-3 = 5$$

Thus, the coordinates of A are $(2, 5)$.

17. The point P which divides the line segment joining the points A(2,-5) and B(5,2) in the ratio 2:3 lies in the quadrant

(a) I (b) II (c) III (d) IV

Answer: (d) IV

Sol:

Let (x, y) be the coordinates of P. Then,

$$x = \frac{2 \times 5 + 3 \times 2}{2+3} = \frac{10+6}{5} = \frac{16}{5}$$

$$y = \frac{2 \times 2 + 3 \times (-5)}{2+3} = \frac{4-15}{5} = \frac{-11}{5}$$

Thus, the coordinates of point P are $\left(\frac{16}{5}, \frac{-11}{5}\right)$ and so it lies in the fourth quadrant

18. If A(-6,7) and B(-1,-5) are two given points then the distance 2AB is

(a) 13 (b) 26 (c) 169 (d) 238

Answer: (b) 26

Sol:

The given points are $A(-6, 7)$ and $B(-1, -5)$. So

$$AB = \sqrt{(-6+1)^2 + (7+5)^2}$$

$$= \sqrt{(-5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

Thus, $2AB = 26$.

19. Which point on x-axis is equidistant from the points A(7,6) and B(-3,4)

(a) (0,4) (b) (-4,0) (c) (3,0) (d) (0,3)

Answer: (c) (3,0)

Sol:

Let $P(x, 0)$ be the point on x -axis. Then as per the question

$$AP = BP \Rightarrow AP^2 = BP^2$$

$$\Rightarrow (x-7)^2 + (0-6)^2 = (x-3)^2 + (0-4)^2$$

$$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$$

$$\Rightarrow 60 = 20x$$

$$\Rightarrow x = \frac{60}{20} = 3$$

Thus, the required point is $(3, 0)$.

20. The distance of P(3,4) from the x-axis is

(a) 3 units (b) 4 units (c) 5 units (d) 1 unit

Answer: (b) 4 units

Sol:

The y-coordinate the distance of the point from the x-axis

Here, the y-coordinate is 4.

21. In what ratio does the x-axis divide the join of A(2, -3) and B(5,6)?

(a) 2:3 (b) 3:5 (c) 1:2 (d) 2:1

Answer: (c) 1 :2

Sol:

Let AB be divided by the x -axis in the ratio $k : 1$ at the point P .

Then, by section formula, the coordinates of P are

$$P\left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$$

But P lies on the x -axes so, its ordinate is 0.

$$\frac{6k-3}{k+1} = 0$$

$$\Rightarrow 6k - 3 = 0$$

$$\Rightarrow 6k = 3$$

$$\Rightarrow k = \frac{1}{2}$$

Hence, the required ratio is $\frac{1}{2}:1$ which is same as $1:2$.

22. In what ratio does the y-axis divide the join of P(-4,2) and Q(8,3)?

(a) 3:1 (b) 1:3 (c) 2:1 (d) 1:2

Answer: (d) 1:2

Sol:

Let AB be divided by the y-axis in the ratio $k:1$ at the point P .

Then, by section formula, the coordinates of P are

$$P\left(\frac{8k-4}{k+1}, \frac{3k+2}{k+1}\right)$$

But, P lies on the y-axis, so, its abscissa is 0.

$$\Rightarrow \frac{8k-4}{k+1} = 0$$

$$\Rightarrow 8k-4=0$$

$$\Rightarrow 8k=4$$

$$\Rightarrow k = \frac{1}{2}$$

Hence, the required ratio is $\frac{1}{2}:1$, which is same as $1:2$.

23. If P(-1,1) is the midpoint of the line segment joining A(-3,b) and B(1, b+4) then b=?

(a) 1 (b) -1 (c) 2 (d) 0

Answer: (b) -1

Sol:

The given points are $A(-3,b)$ and $B(1,b+4)$.

Then, $(x_1 = -3, y_1 = b)$ and $(x_2 = 1, y_2 = b+4)$

Therefore,

$$x = \frac{[(-3)+1]}{2}$$

$$= \frac{-2}{2}$$

$$= -1$$

And

$$y = \frac{[b + (b + 4)]}{2}$$

$$= \frac{2b + 4}{2}$$

$$= b + 2$$

But the midpoint is $P(-1, 1)$.

Therefore,

$$b + 2 = 1$$

$$\Rightarrow b = -1$$

- 24.** The line $2x + y - 4 = 0$ divide the line segment joining $A(2, -2)$ and $B(3, 7)$ in the ratio
(a) 2:5 (b) 2:9 (c) 2:7 (d) 2:3

Answer: (b) 2:9

Sol:

Let the line $2x + y - 4 = 0$ divide the line segment in the ratio $k : 1$ at the point P .

Then, by section formula the coordinates of P are

$$P\left(\frac{3k + 2}{k + 1}, \frac{7k - 2}{k + 1}\right)$$

Since P lies on the line $2x + y - 4 = 0$, we have

$$\frac{2(3k + 2)}{k + 1} + \frac{7k - 2}{k + 1} - 4 = 0$$

$$\Rightarrow (6k + 4) + (7k - 2) - (4k + 4) = 0$$

$$\Rightarrow 9k = 2$$

$$\Rightarrow k = \frac{2}{9}$$

Hence, the required ratio is $\frac{2}{9} : 1$ which is same as 2:9.

- 25.** If $A(4, 2)$, $B(6, 5)$ and $C(1, 4)$ be the vertices of $\triangle ABC$ and AD is a median, then the coordinates of D are

- (a) $\left(\frac{5}{2}, 3\right)$ (b) $\left(5, \frac{7}{2}\right)$ (c) $\left(\frac{7}{2}, \frac{9}{2}\right)$ (d) none of these

Answer: (c) $\left(\frac{7}{2}, \frac{9}{2}\right)$

Sol:

D is the midpoint of BC

So, the coordinates of D are

$$D\left(\frac{6+1}{2}, \frac{5+4}{2}\right) [B(6,5) \text{ and } C(1,4) \Rightarrow (x_1 = 6, y_1 = 5) \text{ and } (x_2 = 1, y_2 = 4)]$$
$$\text{i.e., } D\left(\frac{7}{2}, \frac{9}{2}\right)$$

26. If A(-1,0), B(5,-2) and C(8,2) are the vertices of $\triangle ABC$ then its centroid is
(a) (12,0) (b) (6,0) (c) (0,6) (d) (4,0)

Answer: (d) (4,0)

Sol:

The given point are $A(-1,0)$, $B(5,-2)$ and $C(8,2)$.

Here, $(x_1 = -1, y = 0)$, $(x_2 = 5, y = -2)$ and $(x_3 = 8, y_3 = 2)$

Let $G(x, y)$ be the centroid of $\triangle ABC$. Then,

$$x = \frac{1}{3}(x_1 + x_2 + x_3)$$

$$= \frac{1}{3}(-1 + 5 + 8)$$

$$= 4$$

and

$$y = \frac{1}{3}(y_1 + y_2 + y_3)$$

$$= \frac{1}{3}(0 - 2 + 2)$$

$$= 0$$

Hence, the centroid of $\triangle ABC$ is $G(4,0)$.

27. Two vertices of $\triangle ABC$ are A(-1,4) and B(5,2) and its centroid is G(0,-3). Then the coordinates of C are
(a) (4,3) (b) (4,15) (c) (-4,-15) (d) (-15, -4)

Answer: (c) (-4,-15)

Sol:

Two vertices of $\triangle ABC$ are $A(-1,4)$ and $B(5,2)$.

Let the third vertex be $C(a,b)$.

Then, the coordinates of its centroid are

$$G\left(\frac{-1+5+a}{3}, \frac{4+2+b}{3}\right)$$

$$\text{i.e., } G\left(\frac{4+a}{3}, \frac{6+b}{3}\right)$$

But it is given that the centroid is $G(0, -3)$.

Therefore,

$$\frac{4+a}{3} = 0 \text{ and } \frac{6+b}{3} = -3$$

$$\Rightarrow 4+a = 0 \text{ and } 6+b = -9$$

$$\Rightarrow a = -4 \text{ and } b = -15$$

Hence, the third vertex of $\triangle ABC$ is $C(-4, -15)$.

28. The points $A(-4,0)$, $B(4,0)$ and $C(0,3)$ are the vertices of a triangle, which is
(a) isosceles (b) equilateral (c) scalene (d) right-angled

Answer: (a) isosceles

Sol:

Let $A(-4,0)$, $B(4,0)$ and $C(0,3)$ be the given points. Then,

$$AB = \sqrt{(4+4)^2 + (0-0)^2}$$

$$= \sqrt{(8)^2 + (0)^2}$$

$$= \sqrt{64+0}$$

$$= \sqrt{64}$$

$$= 8 \text{ units}$$

$$BC = \sqrt{(0-4)^2 + (3-0)^2}$$

$$= \sqrt{(-4)^2 + (3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

$$AC = \sqrt{(0+4)^2 + (3-0)^2}$$

$$= \sqrt{(4)^2 + (3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

$$BC = AC = 5 \text{ units}$$

Therefore, $\triangle ABC$ is isosceles

29. The points P(0,6), Q(-5,3) and R(3,1) are the vertices of a triangle, which is
(a) equilateral (b) isosceles (c) scalene (d) right-angled

Ans: (d) right - angled

Sol:

Let $P(0,6)$, $Q(-5,3)$ and $R(3,1)$ be the given points. Then,

$$PQ = \sqrt{(-5-0)^2 + (3-6)^2}$$

$$= \sqrt{(-5)^2 + (-3)^2}$$

$$= \sqrt{25+9}$$

$$= \sqrt{34} \text{ units}$$

$$QR = \sqrt{(3+5)^2 + (1-3)^2}$$

$$= \sqrt{(8)^2 + (-2)^2}$$

$$= \sqrt{64+4}$$

$$= \sqrt{68}$$

$$= 2\sqrt{17} \text{ units}$$

$$PR = \sqrt{(3-0)^2 + (1-6)^2}$$

$$= \sqrt{(3)^2 + (-5)^2}$$

$$= \sqrt{9+25}$$

$$= \sqrt{34} \text{ units}$$

$$PQ^2 + PR^2 \Rightarrow \left\{ (\sqrt{34})^2 + (\sqrt{34})^2 \right\} = 68$$

$$QR^2 \Rightarrow (2\sqrt{17})^2 = 68$$

$$\text{Thus, } PQ^2 + PR^2 = QR^2$$

Therefore, ΔPQR is right-angled.

30. If the points A(2,3), B(5,k) and C(6,7) are collinear then

(a) $k=4$ (b) $k=6$ (c) $k=\frac{-3}{2}$ (d) $k=\frac{11}{4}$

Ans: (b) $k=6$

Sol:

The given points are $A(2,3)$, $B(5,k)$ and $C(6,7)$.

Here, $(x_1=2, y_1=3)$, $(x_2=5, y_2=k)$ and $(x_3=6, y_3=7)$.

Points A , B and C are collinear. Then,

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow 2(k - 7) + 5(7 - 3) + 6(3 - k) = 0$$

$$\Rightarrow 2k - 14 + 20 + 18 - 6k = 0$$

$$\Rightarrow -4k = -24$$

$$\Rightarrow k = 6$$

31. If the point $A(1,2)$, $O(0,0)$ and $C(a,b)$ are collinear, then

(a) $a = b$ (b) $a = 2b$ (c) $2a = b$ (d) $a + b = 0$

Ans: (c) $2a = b$

Sol:

The given points are $A(1,2)$, $O(0,0)$ and $C(a,b)$

Here, $(x_1 = 1, y_1 = 2)$, $(x_2 = 0, y_2 = 0)$ and $(x_3 = a, y_3 = b)$.

Point A , O and C are collinear

$$\Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow 1(0 - b) + 0(b - 2) + a(2 - 0) = 0$$

$$\Rightarrow -b + 2a = 0$$

$$\Rightarrow 2a = b$$

32. The area of $\triangle ABC$ with vertices $A(3,0)$, $B(7,0)$ and $C(8,4)$ is

(a) 14 sq units (b) 28 sq units (c) 8 sq units (d) 6 sq units

Ans: (c) 8 sq units

Sol:

The given points are $A(3,0)$, $B(7,0)$ and $C(8,4)$.

Here, $(x_1 = 3, y_1 = 0)$, $(x_2 = 7, y_2 = 0)$ and $(x_3 = 8, y_3 = 4)$

Therefore,

$$\text{Area of } \triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [3(0 - 4) + 7(4 - 0) + 8(0 - 0)]$$

$$= \frac{1}{2} [-12 + 28 + 0]$$

$$= \left(\frac{1}{2} \times 16 \right)$$

$$= 8 \text{ sq. units}$$

33. AOBC is rectangle whose three vertices are A(0,3), O(0,0) and B(5,0). The length of each of its diagonals is

(a) 5 units (b) 3 units (c) 4 units (d) $\sqrt{34}$ units

Ans: (c) 4 units

Sol:

A(0,3), O(0,0) and B(5,0) are the three vertices of a rectangle; let C be the fourth vertex

Then, the length of the diagonal,

$$\begin{aligned} AB &= \sqrt{(5-0)^2 + (0-3)^2} \\ &= \sqrt{(5)^2 + (-3)^2} \\ &= \sqrt{25+9} \\ &= \sqrt{34} \text{ units} \end{aligned}$$

Since, the diagonals of rectangle are equal.

Hence, the length of its diagonals is $\sqrt{34}$ units.

34. If the distance between the points A(4,p) and B(1,0) is 5 then

(a) p = 4 only (b) p = -4 only (c) p = ± 4 (d) p = 0

Ans: (c) p = ± 4

Sol:

The given points are A(4, p) and B(1,0) and AB = 5.

Then, $(x_1 = 4, y_1 = p)$ and $(x_2 = 1, y_2 = 0)$

Therefore,

$$\begin{aligned} AB &= 5 \\ \Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} &= 5 \\ \Rightarrow \sqrt{(1-4)^2 + (0-p)^2} &= 5 \\ \Rightarrow (-3)^2 + (-p)^2 &= 25 \\ \Rightarrow 9 + p^2 &= 25 \\ \Rightarrow p^2 &= 16 \\ \Rightarrow p &= \pm\sqrt{16} \\ \Rightarrow p &= \pm 4 \end{aligned}$$