Exercise 7.1

Q1) Identify the monomials, binomials, trinomials and quadrinomials from the following expressions:

(i) a^2

(ii) $a^2 - b^2$

(iii) $x^3 + y^3 + z^3$

(iv) $x^3 + y^3 + z^3 + 3xyz$

(v) 7 + 5

(vi) abc + 1

(vii) 3x − 2 + 5

(viii) 2x − 3y + 4

(ix) xy + yz + zx

(x) $ax^3 + bx^2 + cx + d$

Solution:

The monomials, binomials, trinomials and quadrinomials are as follows.

(i) a^2 is a monomial expression as it contains one term only.

(ii) $a^2 - b^2$ is a binomial expression as it contains two terms.

(iii) $x^3 + y^3 + z^3$ is a trinomial expression as it contains three terms.

(iv) $x^3 + y^3 + z^3 + 3xyz$ is a quadrinomial expression as it contains four terms.

(v) 7 + 5 = 12 is a monomial expression as it contains one term only.

(vi) abc + 1 is a binomial expression as it contains two terms.

(vii) 3x − 2 + 5 = 3x + 3 is a binomial expression as it contains two terms.

(viii) 2x − 3y + 4 is a trinomial expression as it contains three terms.

(ix) xy + yz + zx is a trinomial expression as it contains three terms.

(x) $ax^3 + bx^2 + cx + d$ is a quadrinomial expression as it contains four terms.

Q2) Write all the terms of each of the following algebraic expressions:

(i) 3x (ii) 2x – 3 (iii) $2x^2 - 7$ *(iv)* $2x^2 + y^2 - 3xy + 4$

Solution:

The terms of each of the given algebraic expressions are as follows.

(i) 3x is the onty term of the given algebraic expression.

(ii) 2x and -3 are the terms of the given algebraic expression.

(iii) $2x^2 \; and \; -7$ are the terms of the given algebraic expression.

(iv) $2x^2,\ y^2,\ -3xy \; and \; 4$ are the terms of the given algebraic expression.

Q3) Identify the terms and also mention the numerical coefficients of those terms:

(i) $4xy,\ -5x^2y,\ -3yx,\ 2xy^2$

(ii) $7a^2bc,\ -3ca^2b, -\frac{5}{2}abc^2,\ \frac{3}{2}abc^2,\ -\frac{4}{3}cba^2$

Solution:

Like terms	Numerical coefficients
(i) 4xy, -3yx	4, -3
(ii) $\{7a^2bc,\ -3ca^2b\}$	{7, -3}
$\{-\frac{5}{2}abc^2\}$	$\{-\frac{5}{2}\}$
$\{\frac{3}{2}abc^2\}$	$\{\frac{3}{2}\}$
$\{-\frac{4}{3}cba^2\}$	$\{-\frac{4}{3}\}$

Q4) Identify the like terms in the following algebraic expressions:

(i) $a^2 + b^2 - 2a^2 + c^2 + 4a$

(ii) $3x + 4xy - 2yz + \frac{5}{2}zy$

(iii) $abc + ab^2c + 2acb^2 + 3c^2ab + b^2ac - 2a^2bc + 3cab^2$

Solution:

The like terms in the given algebraic expressions are as follows.

(i) The like terms in the given algebraic expressions are $a^2 \; and \; -2a^2$.

(ii) The like terms in the given algebraic expressions are -2yz and $\frac{5}{2}zy$.

(iii) The like terms in the given algebraic expressions are $ab^2c,\ 2acb^2,\ b^2ac \; and \; 3cab^2$.

Q5) Write the coefficient of x in the following:

(i) −12x *(ii) −7xy* *(iii) xyz* *(iv) −7ax*

Solution:

The coefficients of x are as follows.

(i) The numerical coefficient of x is -12.

(ii) The numerical coefficient of x is -7y.

(iii) The numerical coefficient of x is yz.

(iv) The numerical coefficient of x is -7a.

Q6) Write the coefficient of x^2 in the following:

(i) $-3x^2$

(ii) $5x^2yz$

(iii) $\frac{5}{7}x^2z$

(iv) $-\frac{3}{2}ax^2 + yx$

Solution:

The coefficient of x^2 are as follows.

(i) The numerical coefficient of x^2 is -3.

(ii) The numerical coefficient of x^2 is 5yz.

(iii) The numerical coefficient of x^2 is $\frac{5}{7}z$.

(iv) The numerical coefficient of x^2 is $-\frac{3}{2}a$.

Q7) Write the coefficient of:

(i) y in −3y

(ii) a in 2ab

(iii) z in −7xyz

(iv) p in −3pqr

(v) $y^2\ in\ 9xy^2z$

(vi) $x^3\ in\ x^3+1$

(vii) $x^2\ in\ -x^2$

Solution:

The coefficients are as follows.

(i) The coefficient of y is -3.

(ii) The coefficient of a is 2b.

(iii) The coefficient of z is -7xy.

(iv) The coefficient of p is -3qr.

(v) The coefficient of y^2 is 9xz.

(vi) The coefficient of x^3 is 1.

(vii) The coefficient of $-x^2$ is -1.

Q8) Write the numerical coefficient of each in the following

(i) xy

(ii) -6yz

(iii) 7abc

(iv) $-2x^3y^2z$

Solution:

The numerical coefficient of each of the given terms is as follows.

(i) The numerical coefficient in the term xy is 1.

(ii) The numerical coefficient in the term -6yz is -6.

(iii) The numerical coefficient in the term 7abc is 7.

(iv) The numerical coefficient in the term $-2x^3y^2z$ is -2.

Q9) Write the numerical coefficient of each term in the following algebraic expressions:

(i) $4x^2y - \frac{3}{2}xy + \frac{5}{2}xy^2$

(ii) $-\frac{5}{3}x^2y + \frac{7}{4}xyz + 3$

Solution:

The numerical coefficient of each term in the given algebraic expression is as follows.

	Term	Coefficient
(i)	$4x^2y$	4
	$-\frac{3}{2}xy$	$-\frac{3}{2}$
	$\frac{5}{2}xy^2$	$\frac{5}{2}$
(ii)	$-\frac{5}{3}x^2y$	$-\frac{5}{3}$
	$\frac{7}{4}xyz$	$\frac{7}{4}$
	3	3

Q10) Write the constant term of each of the following algebraic expressions:

(i) $x^2y - xy^2 + 7xy - 3$

(ii) $a^3 - 3a^2 + 7a + 5$

Solution:

The constant term of each of the given algebraic expressions is as follows.

(i) The constant term in the given algebraic expressions is -3.

(ii) The constant term in the given algebraic expressions is 5.

Q11) Evaluate each of the following expressions for x = -2, y = -1, z = 3:

(i) $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$

(ii) $x^2 + y^2 + z^2 - xy - yz - zx$

Solution:

We have x = -2, y = -1 and z = 3

Thus,

(i)

$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = \frac{-2}{-1} + \frac{-1}{3} + \frac{3}{-2} = \frac{12-2-9}{6} = \frac{1}{6}$

(ii) $x^2 + y^2 + z^2 - xy - yz - zx$

$= (-2)^2 + (-1)^2 + (3)^2 - (-2)(-1) - (-1)(3) - (3)(-2)$

= 4 + 1 + 9 − 2 + 3 + 6

= (4 + 1 + 9 + 3 + 6) − 2

= 23 − 2

= 21

Q12) Evaluate each of the following algebraic expressions for x = 1, y = -1, z = 2, a = -2, b = 1, c = -2:

(i) ax + by + cz

(ii) $ax^2 + by^2 - cz^2$

(iii) axy + byz + cxy

Solution:

We have x = 1, y = -1, z = 2, a = -2, b = 1 and c = -2.

Thus,

(i) ax + by + cz

= (-2)(1) + (1)(-1) + (-2)(2)

= −2 − 1 − 4

= −7

(ii) $ax^2 + by^2 - cz^2$

$= (-2)(1)^2 + (1)(-1)^2 - (-2)(2)^2$

= −2 + 1 − (−8)

= −2 + 1 + 8

= −2 + 9

= 7

(iii) axy + byz + cxy

= (−2)(1)(−1) + (1)(−1)(2) + (−2)(1)(−1)

= 2 + (−2) + 2

= 2 − 2 + 2

= 4 − 2

= 2

Exercise 7.2

Q1) Add the following:

(i) 3x and 7x *(ii) -5xy and 9xy*

Solution:

We have

(i) 3x + 7x = (3 + 7)x = 10x

(ii) -5xy + 9xy = (-5 + 9)xy = 4xy

Q2) Simplify each of the following:

(i) $7x^3y + 9yx^3$ *(ii)* $12a^2b + 3ba^2$

Solution:

Simplifying the given expressions, we have

(i) $7x^3y + 9yx^3 = (7 + 9)x^3y = 16x^3y$

(ii) $12a^2b + 3ba^2 = (12 + 3)a^2b = 15a^2b$

Q3) Add the following:

(i) 7abc, -5abc, 9abc, -8abc *(ii)* $2x^2y,\ -4x^2y,\ 6x^2y,\ -5x^2y$

Solution:

Adding the given terms, we have

(i) 7abc + (-5abc) + (9abc) + (-8abc)

= 7abc − 5abc + 9abc − 8abc

= (7 − 5 + 9 − 8)abc

= (16 − 13)abc

= 3abc

(ii) $2x^2y + (-4x^2y) + (6x^2y) + (-5x^2y)$

$= 2x^2y - 4x^2y + 6x^2y - 5x^2y$

$= (2 - 4 + 6 - 5)x^2y$

$= (8 - 9)x^2y$

$= -x^2y$

Q4) Add the following expressions:

(i) $x^3 - 2x^2y + 3xy^2 - y^3,\ 2x^3 - 5xy^2 + 3x^2y - 4y^3$

(ii) $a^4 - 2a^3b + 3ab^3 + 4a^2b^2 + 3b^4,\ -2a^4 - 5ab^3 + 7a^3b - 6a^2b^2 + b^4$

Solution:

Adding the given expressions, we have

(i) $x^3 - 2x^2y + 3xy^2 - y^3 + 2x^3 - 5xy^2 + 3x^2y - 4y^3$

Collecting positive and negative like terms together, we get

$x^3 + 2x^3 - 2x^2y + 3x^2y + 3xy^2 - 5xy^2 - y^3 - 4y^3$
$= 3x^3 + x^2y - 2xy^2 - 5y^3$

(ii) $(a^4 - 2a^3b + 3ab^3 + 4a^2b^2 + 3b^4) + (-2a^4 - 5ab^3 + 7a^3b - 6a^2b^2 + b^4)$

$a^4 - 2a^3b + 3ab^3 + 4a^2b^2 + 3b^4 - 2a^4 - 5ab^3 + 7a^3b - 6a^2b^2 + b^4$

Collecting positive and negative like terms together, we get

$a^4 - 2a^4 - 2a^3b + 7a^3b + 3ab^3 - 5ab^3 + 4a^2b^2 - 6a^2b^2 + 3b^4 + b^4$
$= -a^4 + 5a^3b - 2ab^3 - 2a^2b^2 + 4b^4$

Q5) Add the following expressions:

(i) 8a − 6ab + 5b, −6a − ab − 8b and −4a + 2ab + 3b

(ii)

$5x^3 + 7 + 6x - 5x^2,\ 2x^2 - 8 - 9x,\ 4x - 2x^2 + 3x^3,\ 3x^3 - 9x - x^2\ and\ x - x^2 - x^3 - 4$

Solution:

(i) Required expression = (8a − 6ab + 5b) + (−6a − ab − 8b) + (−4a + 2ab + 3b)

Collecting positive and negative like terms together, we get

8a − 6a − 4a − 6ab − ab + 2ab + 5b − 8b + 3b

= 8a − 10a − 7ab + 2ab + 8b − 8b

= −2a − 5ab

(ii) Required expression =

$(5x^3 + 7 + 6x - 5x^2) + (2x^2 - 8 - 9x) + (4x - 2x^2 + 3x^3) + (3x^3 - 9x - x^2)$
$+ (x - x^2 - x^3 - 4)$

Collecting positive and negative like terms together, we get

$5x^3 + 3x^3 + 3x^3 - x^3 - 5x^2 + 2x^2 - 2x^2 - x^2 - x^2 + 6x - 9x + 4x - 9x + x + 7 - 8$
$- 4$
$= 10x^3 - 7x^2 - 7x - 5$

Q6) Add the following:

(i) x − 3y − 2z
5x + 7y − 8z
3x − 2y + 5z

(ii) 4ab − 5bc + 7ca
−3ab + 2bc − 3ca
5ab − 3bc + 4ca

Solution:

(i) Required expression = (x − 3y − 2z) + (5x + 7y − 8z) + (3x − 2y + 5z)

Collecting positive and negative like terms together, we get

x + 5x + 3x − 3y + 7y − 2y − 2z − 8z + 5z

= 9x − 5y + 7y − 10z + 5z

= 9x + 2y − 5z

(ii) Required expression = (4ab − 5bc + 7ca) + (−3ab + 2bc − 3ca) + (5ab − 3bc + 4ca)

Collecting positive and negative like terms together, we get

4ab − 3ab + 5ab − 5bc + 2bc − 3bc + 7ca − 3ca + 4ca

= 9ab − 3ab − 8bc + 2bc + 11ca − 3ca

= 6ab − 6bc + 8ca

Q7) Add $2x^2 - 3x + 1$ *to the sum of* $3x^2 - 2x$ *and 3x + 7.*

Solution:

Sum of $3x^2 - 2x$ and 3x + 7

= $(3x^2 - 2x)$ + (3x + 7)

= $3x^2 - 2x + 3x + 7$

= $(3x^2 + x + 7)$

Now, required expression = $2x^2 - 3x + 1 + (3x^2 + x + 7)$

= $2x^2 + 3x^2 - 3x + x + 1 + 7$

= $5x^2 - 2x + 8$

Q8) Add $x^2 + 2xy + y^2$ *to the sum of* $x^2 - 3y^2$ *and* $2x^2 - y^2 + 9$.

Solution:

Sum of $x^2 - 3y^2$ and $2x^2 - y^2 + 9$

= $(x^2 - 3y^2) + (2x^2 - y^2 + 9)$

= $x^2 + 2x^2 - 3y^2 - y^2 + 9$

= $3x^2 - 4y^2 + 9$

Now, required expression = $(x^2 + 2xy + y^2) + 3x^2 - 4y^2 + 9$

= $x^2 + 3x^2 + 2xy + y^2 - 4y^2 + 9$

= $4x^2 + 2xy - 3y^2 + 9$

Q9) Add $a^3 + b^3 - 3$ *to the sum of* $2a^3 - 3b^3 - 3ab + 7$ *and* $-a^3 + b^3 + 3ab - 9$.

Solution:

First, we need to find the sum of $2a^3 - 3b^3 - 3ab + 7$ and $-a^3 + b^3 + 3ab - 9$

$= (2a^3 - 3b^3 - 3ab + 7) + (-a^3 + b^3 + 3ab - 9)$

Collecting positive and negative like terms together, we get

$= 2a^3 - a^3 - 3b^3 + b^3 - 3ab + 3ab + 7 - 9$

$= a^3 - 2b^3 - 2$

Now, the required expression = $(a^3 + b^3 - 3) + (a^3 - 2b^3 - 2)$

$= a^3 + a^3 + b^3 - 2b^3 - 3 - 2$

$= 2a^3 - b^3 - 5$

Q10) Subtract:

(i) $7a^2b$ *from* $3a^2b$

(ii) 4xy from -3xy

Solution:

(i) Required expression = $3a^2b - 7a^2b$

$= (3 - 7)a^2b$

$= -4a^2b$

(ii) Required expression = −3xy − 4xy

= −7xy

Q11) Subtract:

(i) -4x from 3y

(ii) -2x from -5y

Solution:

(i) Required expression = (3y) − (−4x)

= 3y + 4x

(ii) Required expression = (-5y) − (−2x)

= −5y + 2x

Q12) Subtract:

(i) $6x^3 - 7x^2 + 5x - 3$ *from* $4 - 5x + 6x^2 - 8x^3$

(ii) $-x^2 - 3z$ *from* $5x^2 - y + z + 7$

(iii) $x^3 + 2x^2y + 6xy^2 - y^3$ *from* $y^3 - 3xy^2 - 4x^2y$

Solution:

(i) Required expression = $(4 - 5x + 6x^2 - 8x^3) - (6x^3 - 7x^2 + 5x - 3)$

= $4 - 5x + 6x^2 - 8x^3 - 6x^3 + 7x^2 - 5x + 3$

= $-8x^3 - 6x^3 + 7x^2 + 6x^2 - 5x - 5x + 3 + 4$

= $-14x^3 + 13x^2 - 10x + 7$

(ii) Required expression = $(5x^2 - y + z + 7) - (-x^2 - 3z)$

= $5x^2 - y + z + 7 + x^2 + 3z$

= $5x^2 + x^2 - y + z + 3z + 7$

= $6x^2 - y + 4z + 7$

(iii) Required expression = $(y^3 - 3xy^2 - 4x^2y) - (x^3 + 2x^2y + 6xy^2 - y^3)$

= $y^3 - 3xy^2 - 4x^2y - x^3 - 2x^2y - 6xy^2 + y^3$

= $y^3 + y^3 - 3xy^2 - 6xy^2 - 4x^2y - 2x^2y - x^3$

= $2y^3 - 9xy^2 - 6x^2y - x^3$

Q13) From

(i) $p^3 - 4 + 3p^2$, *take away* $5p^2 - 3p^3 + p - 6$

(ii) $7 + x - x^2$, *take away* $9 + x + 3x^2 + 7x^3$

(iii) $1 - 5y^2$, *take away* $y^3 + 7y^2 + y + 1$

(iv) $x^3 - 5x^2 + 3x + 1$, *take away* $6x^2 - 4x^3 + 5 + 3x$

Solution:

(i) Required expression = $(p^3 - 4 + 3p^2) - (5p^2 - 3p^3 + p - 6)$

= $p^3 - 4 + 3p^2 - 5p^2 + 3p^3 - p + 6$

= $p^3 + 3p^3 + 3p^2 - 5p^2 - p - 4 + 6$

= $4p^3 - 2p^2 - p + 2$

(ii) Required expression = $(7 + x - x^2) - (9 + x + 3x^2 + 7x^3)$

= $7 + x - x^2 - 9 - x - 3x^2 - 7x^3$

= $-7x^3 - x^2 - 3x^2 + 7 - 9$

= $-7x^3 - 4x^2 - 2$

(iii) Required expression = $(1 - 5y^2) - (y^3 + 7y^2 + y + 1)$

= $1 - 5y^2 - y^3 - 7y^2 - y - 1$

= $-y^3 - 5y^2 - 7y^2 - y$

= $-y^3 - 12y^2 - y$

(iv) Required expression = $(x^3 - 5x^2 + 3x + 1) - (6x^2 - 4x^3 + 5 + 3x)$

= $x^3 - 5x^2 + 3x + 1 - 6x^2 + 4x^3 - 5 - 3x$

= $x^3 + 4x^3 - 5x^2 - 6x^2 + 1 - 5$

= $5x^3 - 11x^2 - 4$

Q14) From the sum of $3x^2 - 5x + 2$ *and* $-5x^2 - 8x + 9$ *subtract* $4x^2 - 7x + 9$.

Solution:

Required expression = $[(3x^2 - 5x + 2) + (-5x^2 - 8x + 9)] - (4x^2 - 7x + 9)$

= $[3x^2 - 5x + 2 - 5x^2 - 8x + 9] - (4x^2 - 7x + 9)$

= $[3x^2 - 5x^2 - 5x - 8x + 2 + 9] - (4x^2 - 7x + 9)$

= $[-2x^2 - 13x + 11] - (4x^2 - 7x + 9)$

= $-2x^2 - 13x + 11 - 4x^2 + 7x - 9$

= $-2x^2 - 4x^2 - 13x + 7x + 11 - 9$

= $-6x^2 - 6x + 2$

Q15) Subtract the sum of 13x − 4y + 7z and −6z + 6x + 3y from the sum of 6x − 4y − 4z and 2x + 4y − 7.

Solution:

Sum of (13x − 4y + 7z) and (−6z + 6x + 3y)

= (13x − 4y + 7z) + (−6z + 6x + 3y)

= (13x − 4y + 7z − 6z + 6x + 3y)

= (13x + 6x − 4y + 3y + 7z − 6z)

= (19x − y + z)

Sum of (6x − 4y − 4z) and (2x + 4y − 7)

= (6x − 4y − 4z) + (2x + 4y − 7)

= (6x − 4y − 4z + 2x + 4y − 7)

= (6x + 2x − 4z − 7)

= (8x − 4z − 7)

Now, required expression = (8x − 4z − 7) − (19x − y + z)

= 8x − 4z − 7 − 19x + y − z

= 8x − 19x + y − 4z − z − 7

= −11x + y − 5z − 7

Q16) From the sum of $x^2 + 3y^2 - 6xy,\ 2x^2 - y^2 + 8xy,\ y^2 + 8$ ***and*** $x^2 - 3xy$ ***subtract*** $-3x^2 + 4y^2 - xy + x - y + 3.$

Solution:

Sum of $(x^2 + 3y^2 - 6xy),\ (2x^2 - y^2 + 8xy),\ (y^2 + 8)$ *and* $(x^2 - 3xy)$

$= (x^2 + 3y^2 - 6xy) + (2x^2 - y^2 + 8xy) + (y^2 + 8) + (x^2 - 3xy)$

$= (x^2 + 3y^2 - 6xy + 2x^2 - y^2 + 8xy + y^2 + 8 + x^2 - 3xy)$

$= (x^2 + 2x^2 + x^2 + 3y^2 - y^2 + y^2 - 6xy + 8xy - 3xy + 8)$

$= (4x^2 + 3y^2 - xy + 8)$

Now, required expression = $(4x^2 + 3y^2 - xy + 8) - (-3x^2 + 4y^2 - xy + x - y + 3)$

$= 4x^2 + 3y^2 - xy + 8 + 3x^2 - 4y^2 + xy - x + y - 3$

$= 4x^2 + 3x^2 + 3y^2 - 4y^2 - xy + xy - x + y - 3 + 8$

$= 7x^2 - y^2 - x + y + 5$

Q17) What should be added to xy − 3yz + 4zx to get 4xy − 3zx + 4yz + 7?

Solution:

The required expression can be got by subtracting xy − 3yz + 4zx from 4xy − 3zx + 4yz + 7.

Therefore, required expression = (4xy − 3zx + 4yz + 7) − (xy − 3yz + 4zx)

= 4xy − 3zx + 4yz + 7 − xy + 3yz − 4zx

= 4xy − xy − 3zx − 4zx + 4yz + 3yz + 7

= 3xy − 7zx + 7yz + 7

Q18) What should be subtracted from $x^2 - xy + y^2 - x + y + 3$ *to obtain* $-x^2 + 3y^2 - 4xy + 1$*?*

Solution:

Let 'M' be the required expression. Then, we have

$x^2 - xy + y^2 - x + y + 3 - M = -x^2 + 3y^2 - 4xy + 1$

Therefore,

M = $(x^2 - xy + y^2 - x + y + 3) - (-x^2 + 3y^2 - 4xy + 1)$

= $x^2 - xy + y^2 - x + y + 3 + x^2 - 3y^2 + 4xy - 1$

Collecting positive and negative like terms together, we get

$x^2 + x^2 - xy + 4xy + y^2 - 3y^2 - x + y + 3 - 1$

= $2x^2 + 3xy - 2y^2 - x + y + 2$

Q19) How much is x − 2y + 3z greater than 3x + 5y − 7?

Solution:

Required expression = (x − 2y + 3z) − (3x + 5y − 7)

= x − 2y + 3z − 3x − 5y + 7

Collecting positive and negative like terms together, we get

x − 3x − 2y + 5y + 3z + 7

= −2x − 7y + 3z + 7

Q20) How much is $x^2 - 2xy + 3y^2$ less than $2x^2 - 3y^2 + xy$?

Solution:

Required expression = $(2x^2 - 3y^2 + xy) - (x^2 - 2xy + 3y^2)$

= $2x^2 - 3y^2 + xy - x^2 + 2xy - 3y^2$

Collecting positive and negative like terms together, we get

$2x^2 - x^2 - 3y^2 - 3y^2 + xy + 2xy\, x^2 - 6y^2 + 3xy$

Q21) How much does $a^2 - 3ab + 2b^2$ exceed $2a^2 - 7ab + 9b^2$?

Solution:

Required expression = $(a^2 - 3ab + 2b^2) - (2a^2 - 7ab + 9b^2)$

= $a^2 - 3ab + 2b^2 - 2a^2 + 7ab - 9b^2$

Collecting positive and negative like terms together, we get

= $a^2 - 2a^2 - 3ab + 7ab + 2b^2 - 9b^2$

= $-a^2 + 4ab - 7b^2$

Q22) What must be added to $12x^3 - 4x^2 + 3x - 7$ to make the sum $x^3 + 2x^2 - 3x + 2$?

Solution:

Let 'M' be the required expression. Thus, we have

$12x^3 - 4x^2 + 3x - 7 + M = x^3 + 2x^2 - 3x + 2$

Therefore,

M = $(x^3 + 2x^2 - 3x + 2) - (12x^3 - 4x^2 + 3x - 7)$

M = $x^3 + 2x^2 - 3x + 2 - 12x^3 + 4x^2 - 3x + 7$

Collecting positive and negative like terms together, we get

$x^3 - 12x^3 + 2x^2 + 4x^2 - 3x - 3x + 7 + 2\, x^3 - 12x^3 + 2x^2 + 4x^2 - 3x - 3x + 7 + 2$

= $-11x^3 + 6x^2 - 6x + 9$

Q23) If P = $7x^2 + 5xy - 9y^2$, Q = $4y^2 - 3x^2 - 6xy$ and R = $-4x^2 + xy + 5y^2$, show that P + Q + R = 0.

Solution:

We have

P + Q + R = $(7x^2 + 5xy - 9y^2) + (4y^2 - 3x^2 - 6xy) + (-4x^2 + xy + 5y^2)$

= $7x^2 + 5xy - 9y^2 + 4y^2 - 3x^2 - 6xy - 4x^2 + xy + 5y^2$

Collecting positive and negative like terms together, we get

$7x^2 - 3x^2 - 4x^2 + 5xy - 6xy + xy - 9y^2 + 4y^2 + 5y^2$
= $7x^2 - 7x^2 + 6xy - 6xy - 9y^2 + 9y^2$

= 0

Q24) If P = $a^2 - b^2 + 2ab$, Q = $a^2 + 4b^2 - 6ab$, R = $b^2 + b$, S = $a^2 - 4ab$ and T = $-2a^2 + b^2 - ab + a$. Find P + Q + R + S − T.

Solution:

We have

P + Q + R + S − T =
$[(a^2 - b^2 + 2ab) + (a^2 + 4b^2 - 6ab) + (b^2 + b) + (a^2 - 4ab)] - (-2a^2 + b^2 - ab + a)$

= $[a^2 - b^2 + 2ab + a^2 + 4b^2 - 6ab + b^2 + b + a^2 - 4ab] - (-2a^2 + b^2 - ab + a)$

= $[3a^2 + 4b^2 - 8ab + b] - (-2a^2 + b^2 - ab + a)$

= $3a^2 + 4b^2 - 8ab + b + 2a^2 - b^2 + ab - a$

Collecting positive and negative like terms together, we get

$3a^2 + 2a^2 + 4b^2 - b^2 - 8ab + ab - a + b$
= $5a^2 + 3b^2 - 7ab - a + b$

Exercise 7.3

Q1) Place the last two terms of the following expressions in parentheses preceded by a minus sign:

(i) x + y − 3z + y

(ii) 3x − 2y − 5z − 4

(iii) 3a − 2b + 4c − 5

(iv) 7a + 3b + 2c + 4

(v) $2a^2 - b^2 - 3ab + 6$

(vi) $a^2 + b^2 - c^2 + ab - 3ac$

Solution:

We have

(i) x + y − 3z + y = x + y − (3z − y)

(ii) 3x − 2y − 5z − 4 = 3x − 2y − (5z + 4)

(iii) 3a − 2b + 4c − 5 = 3a − 2b − (−4c + 5)

(iv) 7a + 3b + 2c + 4 = 7a + 3b − (−2c − 4)

(v) $2a^2 - b^2 - 3ab + 6 = 2a^2 - b^2 - (3ab - 6)$

(vi) $a^2 + b^2 - c^2 + ab - 3ac = a^2 + b^2 - c^2 - (-ab + 3ac)$

Q2) Write each of the following statements by using appropriate grouping symbols:

(i) The sum of a − b and 3a − 2b + 5 is subtracted from 4a + 2b − 7.

(ii) Three times the sum of 2x + y − [5 − (x − 3y)] and 7x − 4y + 3 is subtracted from 3x − 4y + 7

(iii) The subtraction of $x^2 - y^2 + 4xy$ *from* $2x^2 + y^2 - 3xy$ *is added to* $9x^2 - 3y^2 - xy$.

Solution:

(i) The sum of a − b and 3a − 2b + 5 = [(a − b) + (3a − 2b + 5)].

This is subtracted from 4a + 2b − 7.

Thus, the required expression is (4a + 2b − 7) − [(a − b) + (3a − 2b + 5)]

(ii) Three times the sum of 2x + y − {5 − (x − 3y)} and 7x − 4y + 3 = 3[(2x + y − {5 − (x − 3y)}) + (7x − 4y + 3)]

This is subtracted from 3x − 4y + 7.

Thus, the required expression is (3x − 4y + 7) − 3[(2x + y − {5 − (x − 3y)}) + (7x − 4y + 3)]

(iii) The product of subtraction of $x^2 - y^2 + 4xy$ from $2x^2 + y^2 - 3xy$ is given by {($2x^2 + y^2 - 3xy$) − ($x^2 - y^2 + 4xy$)}

When the above equation is added to $9x^2 - 3y^2 - xy$, we get

{($2x^2 + y^2 - 3xy$) − ($x^2 - y^2 + 4xy$)}+($9x^2 - 3y^2 - xy$)

Exercise 7.4

Simplify each of the following algebraic expressions by removing grouping symbols.

Q1) 2x + (5x − 3y)

Solution:

We have

2x + (5x − 3y)

Since the '+' sign precedes the parentheses, we have to retain the sign of each term in the parentheses when we remove them.

= 2x + 5x − 3y

= 7x − 3y

Q2) 3x − (y − 2x)

Solution:

We have

3x − (y − 2x)

Since the '−' sign precedes the parentheses, we have to change the sign of each term in the parentheses when we remove them. Therefore, we have

3x − y + 2x

= 5x − y

Q3) 5a – (3b – 2a + 4c)

Solution:

We have

5a – (3b – 2a + 4c)

Since the '-' sign precedes the parentheses, we have to change the sign of each term in the parentheses when we remove them.

= 5a – 3b + 2a – 4c

= 7a – 3b – 4c

Q4) $-2(x^2 - y^2 + xy) - 3(x^2 + y^2 - xy)$

Solution:

We have

$-2(x^2 - y^2 + xy) - 3(x^2 + y^2 - xy)$

Since the '–' sign precedes the parentheses, we have to change the sign of each term in the parentheses when we remove them. Therefore, we have

= $-2x^2 + 2y^2 - 2xy - 3x^2 - 3y^2 + 3xy$

= $-2x^2 - 3x^2 + 2y^2 - 3y^2 - 2xy + 3xy$

= $-5x^2 - y^2 + xy$

Q5) 3x + 2y − {x − (2y − 3)}

Solution:

We have

3x + 2y − {x − (2y − 3)}

First, we have to remove the small brackets (or parentheses): ( ). Then, we have to remove the curly brackets (or braces): { }.

Therefore,

= 3x + 2y − {x − 2y + 3}

= 3x + 2y − x + 2y − 3

= 2x + 4y − 3

Q6) 5a − {3a − (2 − a) + 4}

Solution:

We have

5a − {3a − (2 − a) + 4}

First, we have to remove the small brackets (or parentheses): ( ). Then, we have to remove the curly brackets (or braces): { }.

Therefore,

= 5a − {3a − 2 + a + 4}

= 5a − 3a + 2 − a − 4

= 5a − 4a − 2

= a − 2

Q7) a − [b − {a − (b − 1) + 3a}]

Solution:

First we have to remove the parentheses, or small brackets, ( ), then the curly brackets, { }, and then the square brackets [ ].

Therefore, we have

a − [b − {a − (b − 1) + 3a}]

= a − [b − {a − b + 1 + 3a}]

= a − [b − {4a − b + 1}]

= a − [b − 4a + b − 1]

= a − [2b − 4a − 1]

= a − 2b + 4a + 1

= 5a − 2b + 1

Q8) a − [2b − {3a − (2b − 3c)}]

Solution:

First we have to remove the small brackets, or parentheses, ( ), then the curly brackets, { }, and then the square brackets, [ ].

Therefore, we have

a − [2b − {3a − (2b − 3c)}]

= a − [2b − {3a − 2b + 3c}]

= a − [2b − 3a + 2b − 3c]

= a − [4b − 3a − 3c]

= a − 4b + 3a + 3c

= 4a − 4b + 3c

Q9) -x + [5y −{2x − (3y − 5x)}]

Solution:

First we have to remove the small brackets, or parentheses, ( ), then the curly brackets { }, and then the square brackets, [ ].

Therefore, we have

$- x + [5y - \{2x - (3y - 5x)\}]$

$= - x + [5y - \{2x - 3y + 5x)]$

$= - x + [5y - \{7x - 3y\}]$

$= - x + [5y - 7x + 3y]$

$= - x + [8y - 7x]$

$= - x + 8y - 7x$

$= - 8x + 8y$

Q10) 2a – [4b – {4a – 3(2a – b)}]

Solution:

First we have to remove the small brackets, or parentheses, ( ), then the curly brackets, { }, and then the square brackets, [ ].

Therefore, we have

2a – [4b – {4a – 3(2a – b)}]

= 2a – [4b – {4a – 6a + 3b}]

= 2a – [4b – {- 2a + 3b}]

= 2a – [4b + 2a – 3b]

= 2a – [b + 2a]

= 2a – b – 2a

= – b

Q11) -a – [a +{a+b – 2a – (a – 2b)}- b]

Solution:

First we have to remove the small brackets, or parentheses, ( ), then the curly brackets,{ }, and then the square brackets, [ ].

Therefore, we have

– a – [a + {a + b – 2a – (a – 2b)} – b]

= – a – [a + {a + b – 2a – a + 2b} – b]

= – a – [a + {- 2a + 3b} – b]

= – a – [a – 2a + 3b – b]

= – a – [- a + 2b]

= – a + a – 2b

= – 2b

Q12) 2x − 3y − [3x − 2y -{x − z − (x − 2y)}]

Solution:

First we have to remove the small brackets, or parentheses, ( ), then the curly brackets, { }, and then the square brackets, [ ].

Therefore, we have

$2x - 3y - [3x - 2y - \{x - z - (x - 2y)\})$

$= 2x - 3y - [3x - 2y - \{x - z - x + 2y\}]$

$= 2x - 3y - [3x - 2y - \{- z + 2y\}]$

$= 2x - 3y - [3x - 2y + z - 2y]$

$= 2x - 3y - [3x - 4y + z]$

$= 2x - 3y - 3x + 4y - z$

$=-x + y - z$

Q13) 5+ [x – {2y – (6x + y – 4) + 2x} – {x – (y – 2)}]

Solution:

First we have to remove the small brackets, or parentheses, ( ), then the curly brackets, { }, and then the square brackets, [ ].

Therefore, we have

5 + [x – {2y – (6x + y – 4) + 2x} – {x – (y – 2)}]

= 5 + [x – {2y – 6x – y + 4 + 2x} – {x – y + 2}]

= 5 + [x – {y – 4x + 4} – {x – y + 2}]

= 5 + [x – y + 4x – 4 – x + y – 2]

= 5 + [4x – 6]

= 5 + 4x – 6

= 4x – 1

Q14) $x^2 - [3x + [2x - (x^2 - 1)] + 2]$

Solution:

First we have to remove the small brackets, or parentheses, ( ), then the curly brackets, { }, and then the square brackets, [ ].

Therefore, we have

$x^2 - [3x + [2x - (x^2 - 1)] + 2]$
$= x^2 - [3x + [2x - x^2 + 1] + 2]$

$= x^2 - [3x + 2x - x^2 + 1 + 2]$

$= x^2 - [5x - x^2 + 3]$

$= x^2 - 5x + x^2 - 3$

$= 2x^2 - 5x - 3$

Q15) $20 - [5xy + 3[x^2 - (xy - y) - (x - y)]]$

Solution:

$20 - [5xy + 3[x^2 - (xy - y) - (x - y)]]$
$= 20 - [5xy + 3[x^2 - xy + y - x + y]]$

$= 20 - [5xy + 3[x^2 - xy + 2y - x]]$

$= 20 - [5xy + 3x^2 - 3xy + 6y - 3x]$

$= 20 - [2xy + 3x^2 + 6y - 3x]$

$= 20 - 2xy - 3x^2 - 6y + 3x$

$= -3x^2 - 2xy - 6y + 3x + 20$

Q16) 85 – [12x – 7(8x – 3) – 2{10x – 5(2 – 4x)}]

Solution:

First we have to remove the small brackets, or parentheses, ( ), then the curly brackets, { }, and then the square brackets, [ ].

Therefore, we have

85 – [12x – 7(8x – 3) – 2{10x – 5(2 – 4x)}]

= 85 – [12x – 56x + 21 – 2{10x – 10 + 20x}]

= 85 – [12x – 56x + 21 – 2{30x – 10}]

= 85 – [12x – 56x + 21 – 60x + 20]

= 85 – [12x – 116x + 41]

= 85 – [- 104x + 41]

= 85 + 104x – 41

= 44 + 104x

Q17) xy[yz − zx − {yx − (3y − xz) − (xy − zy)}]

Solution:

First we have to remove the small brackets, or parentheses, ( ), then the curly brackets, { }, and then the square brackets, [ ].

Therefore, we have

xy − [yz − zx − {yx − (3y − xz) − (xy − zy)}]

= xy − [yz − zx − {yx − 3y + xz − xy + zy}]

= xy − [yz − zx − {- 3y + xz + zy}]

= xy − [yz − zx + 3y − xz − zy]

= xy − [- zx + 3y − xz]

= xy − [- 2zx + 3y]

= xy + 2xz − 3y