In non-uniform or varied flow, the depth of flow changes along the length of the open channel,

i.e. $\frac{\partial y}{\partial L} \neq 0,\ \frac{\partial y}{\partial L} = 0$ etc.

A uniform flow may be steady or unsteady depending on whether depth changes with time or not.

(*i*) *Steady Uniform Flow:* In this flow, the depth of flow does not change along the length of channel during the given interval if time, i.e. the depth of flow remains the same at all the cross-section of the channel.

(*ii*) *Unsteady Uniform Flow :* In this flow, water surface fluctuates from time while remaining parallel to the channel bottom. Obviously it is practically impossible condition.

(*iii*) *Steady Non-uniform Flow:* In this flow, the depth of flow at a section remains constant with time, but varies from section to section.

(*iv*) *Unsteady Non-uniform Flow:* In this flow, the depth of flow must vary from section to section as well as with time also e.g. flood waves travelling in a natural stream.

Varied or non-uniform flow : It can further classified into two types :

(*i*) *Rapidly Varied Flow (RVF) :* In rapidly varied flow, the flow condition changes significantly in a relatively short distance of the channel, *e.g.* hydraulic jump.

(*ii*) *Gradually Varied Flow (GVF) :* In gradually varied flow, the flow conditions changes gradually over a long distance the channel, *e.g.* flow behind a dam and at a channel transition.

2. Forces

Flow in an open channel is governed by the effects of viscous, gravity and inertial forces.

(*a*) *Effects of viscous forces :* In open channels, the Reynolds number of flow which is a measure of the effect of viscosity is defined as,

$$R = \frac{VR}{\upsilon}$$

where υ = kinematic viscosity and R = hydraulic radius, which is ratio of flow area to wetted perimeter.

Depending on value of Reynolds number, flow can be classified as

(*i*) Laminar flow, $R < 500$

(*ii*) Transition, $500 < R < 2000$

(*iii*) Turbulent, $R < 2000$

(*b*) *Effect of gravity :* Effect of gravity on flow is represented by Froude number, which is the ratio of square root of $\left(i.e., \frac{\rho U^2}{2}\right)\left(i.e., \frac{\rho U^2}{2}\right)$ inertial forces to gravity forces.

$$\text{Froude number, } F = \frac{V}{\sqrt{gD}}$$

where V is the average velocity of flow and D is the hydraulic depth (= A/P) *Depending on Froude number*, the flow can be classified as

(*i*) Critical flow, $F = 1$

(*ii*) Subcritical or tranquil flow, $F < 1$

(*iii*) Super critical or rapid flow, $F > 1$

3. Celerity

In shallow water, the momentary change in the local depth of water causes small gravity waves. The velocity of these small waves is known as **celerity**, given by

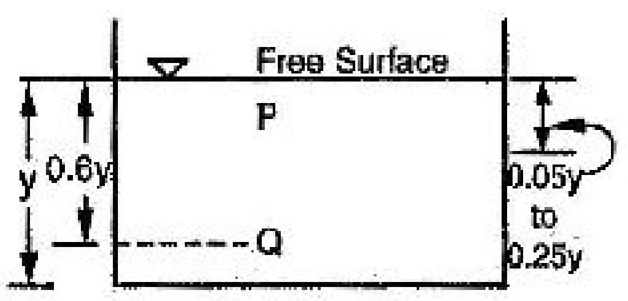
$$C = \sqrt{gD}$$

Celerily can be more than the velocity of flow in subcritical flow thus a gravity wave can travel upstream. In case of supercritical flow, the wave cannot travel upstream.

4. Velocity Distribution over Channel Cross-section

The velocity distribution in an open channel is not uniform because of the effect of friction at the bed, banks and free surface. Theoretically the velocity of flow should be maximum at the topmost point on the vertical center-line. However due to surface tension effect and resistance offered by the air, the velocity is reduced at the water surface.

The maximum velocity occurs at a distance of 0.05 to 0.25 times the flow depth y from the free surface.

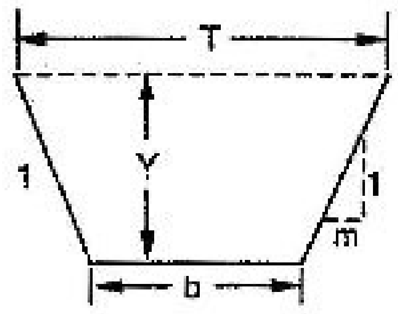


The average velocity in an open channel cross-section is usually the local velocity occuring at 0.6 y from the free surface. Another practice is to measure local velocities at 0.2 y and 0.8 y from the free surface velocity whereas that of average velocity is about 0.7 to 0.8 times.

5. Best Hydraulic Section or Most Economical Section

The best hydraulic section is the most efficient section for conveying a given discharge for given slope and the channel roughness factor, when the area of cross-section is miuimum. Thus for best hydraulic section, the cost of lining and excavation is minimum and it is the most economical section.

(*i*) **Trapezoidal channel section :**



Wetted perimeter $\quad P = b + 2y\sqrt{m^2+1}$

Cross-sectional area, $A = by + my^2$

$$\therefore \quad p = \left(\frac{A}{y} - my\right) + 2y\sqrt{1+m^2}$$

Now for a given cross-sectional area and slope, the rate of flow through a channel of given roughness will be maximum, when the hydraulic radius will be maximum, i.e. wetted perimeter will be minimum.

(*a*) First codition of minimum P is that,

$$\frac{dP}{dy} = 0$$

on differentiating P with respect to y keeping m constant, we get

$$\frac{b+2my}{2} = y\sqrt{m^2+1}$$

*i.e.*Half the top width = sloping side

(*b*) Second condition for minimum P may be obtained by differentiating P with respect to m keeping y constant,

$$R = \frac{\left[2y\sqrt{1+m^2} - my\right]y}{\left[4y\sqrt{1+m^2} - 2my\right]} = \frac{y}{2}$$

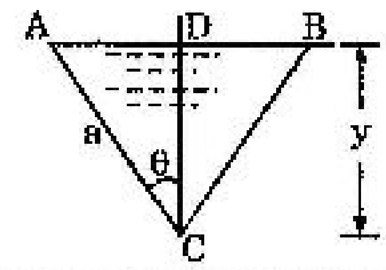
Thus for maximum hydraulic efficiency, the channel should be so designed that, hydraulic radius is equal to half the depth of flow and $m = \frac{1}{\sqrt{3}} = 60°$.

The above two conditions show that the hydraulically economical trapezoidal section is half of a regular hexagon.

(*ii*) **Rectangular channel:**

For a rectangular channel section to be most economical, $b = 2y$. Hence width of channel should be equal to twice the flow depth.

(*iii*)**Triangular section:**

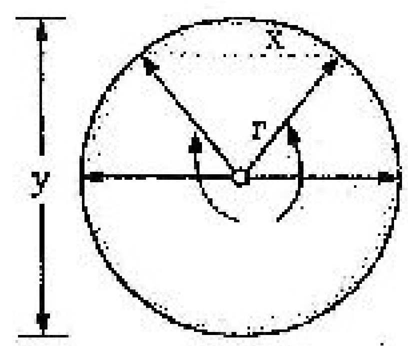


The most economical triangular section have the angle between the two sides as 90°. Each side making an angle of 45° with the vertical,

$$\theta = 45°, \text{ or } m = 1$$

$$R = \frac{y}{2\sqrt{2}}$$

(*iv*) **Semicircular section:**



For circular pipes, the area of flow corresponding to depth y is

$$A = \frac{r^2}{2}(\theta - \sin\theta).$$

where r is the radius of the conduit and θ is given by,

$$\cos\theta = \frac{r-y}{r} = \left(1 - \frac{y}{R}\right)$$

wetted perimeter, $P = r\theta$

(*a*) For Maximum Discharge based on Chezy's Formula :

Discharge, $Q = A.V. = A.C.\ \sqrt{RS_0}$

$$= \frac{r^2}{2}(\theta - \sin\theta).C.\sqrt{\frac{\frac{1}{2}(\theta-\sin\theta)}{\theta}}o$$

$$= \frac{(\theta-\sin\theta)^{\frac{3}{2}}}{\theta^{\frac{1}{2}}}.\frac{r^2C}{2}\sqrt{\frac{r}{2}.S_0}$$

Keeping r, C and S_0 constant, for maximum discharge $\frac{dQ}{d\theta} = 0$

which for a channel reduces to $\frac{d(AR^{\frac{2}{3}})}{d\theta} = 0$

This on solving gives, $\theta = 308°$

For this value of θ and diameter d, we have,

$y = 0.05\,d$, $A = \frac{d^2}{8}(\theta - \sin\theta)$

$$P = r\theta = 5.376\,r \text{ and } R = 0.573\,r$$

(*b*) Based on Manning's formula *i.e.* for maximum discharge,

$$Q = \frac{1}{\pi}AR^{\frac{2}{3}}S^{\frac{1}{2}}$$

we have $\theta = 302°\,20'$, $y = 0.938\,d$, $A = 3.061\,r^2$, $P = 5.277\,r$ and $R = 0.58\,r$

PUMPS AND TURBINES.

Velocity triangle.

Figure shows the changes in velocity as the fluid enters and leaves the impeller of a centrifugal pump.

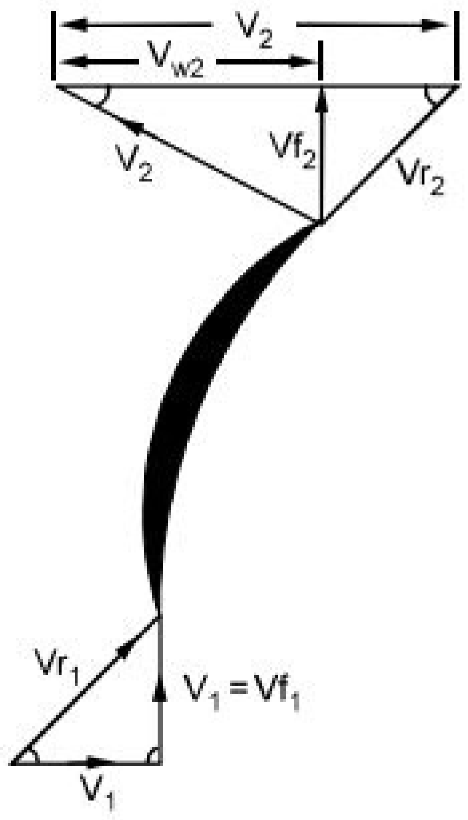


Fig. Velocity triangles for centrifugal pumps.

Following are the notations for velocity.

v_1 & v_2 – tengential veliocity at the inlet and outlet

V_1 & V_2 – absolute velocity with which water enters and leaves the impeller

V_{w_1} & V_{w_2} – velocity of swirl at inlet and outlet

D_1 & D_2 – inner and outer diameter of the impeller

V_{r_1} & V_{r_2} – relative velocity of flow at inlet and outlet.

V_{f_1} & V_{f_2} – velocity of flow at inlet and outlet.

α – angle at which water enters the impeller

β – angle at which water leaves the impeller

θ – vane angle at inlet

ϕ – vane angle at outlet

Work done per second

= torgue developed (T) × angular velocity (ω)

$= Q \cdot \rho (V_{w2}\, v_2 - V_{w1}\, v_1)$

For radial entry, $\alpha = 90°$, and $V_{w1} = 0$

Theoretical head developed

$$= \frac{1}{g}(V_{w2}\, v_2 - V_{w1}\, v_1) = \frac{V_{w2}\, v_2}{g}$$

Different efficiencies for the pumps.

Overall efficiency, $\eta = \eta_{mo}\, \eta_m\, \eta_v$

where

Monomeltric efficiency,

$$\eta_{mo} = \frac{\text{Manometric head}}{\text{Theoretical head}} \times 100$$

$$= \frac{gH_m}{V_{w2}\, v_2} \times 100$$

Mechanical efficiency,

$$\eta_m = \frac{\text{Available head at impeller}}{\text{Energy given by prime mover}} \times 100$$

Volumetric efficiency, $\eta_v = \dfrac{Q}{Q + q} \times 100$

where, Q = actual discharge

q = leakage discharge.

η_v varies from 95 to 98 percent, while η varies from 70 to 85 percent.

SPECIFIC SPEED.

Specific speed of a pump is defined as the speed of rpm of a geometrically similar pump which delivers 1 liter per second under 1 and m head. The specific speed all pumps similar in shape regardless of their size.

Specific speed, $N_s = NQ^{1/2}/H^{3/4}$

where, N = rotational speed in rpm

Q = discharge in m^3/s

H = head in m.

Selection type of pump based on the range of specific speed.

Type of pump	Specific speed of pump		
	N_s	$\dfrac{N\sqrt{Q}}{gH^{3/4}}$	N_s^*
Centrifugal (radial flow)	10 – 100	2 – 18	0.2 – 2
Mixed flow	80 – 200	14 – 36	1.6 – 4
Axial flow	200 – 300	36 – 54	4 – 6

TURBINES.

Turbine specific speed is defined as the speed of that geometrically similar turbine which when working under a head of one meter develops one metric horse power. Thus

Turbine specific speed, $N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$

where 'P' = metric horse power of the turbine on the brake.

Classification based on specific speed.

Turbine	Specific speed range	Type of runner Slow	Medium	Fast
Pleton	10-35	10-20	20-28	28-35
Francis	60-300	60-120	120-180	180-300
Kaplan	300-1100	300-450	450-700	700-1100

CAVITATION.

The process of formation of vapour bubbles (or presence of air bubbles) in low pressure zones, their transit to high pressure zones and growing in size, in the direction of flow and ultimately their collapse is called *cavitation*. Very high localized pressures are created at the time of collapse of bubbles. This causes damage to the surface called *pitting*. This phenomena is accompained by lowered efficiency, noise and vibrations.

Cavitation parameter, $\sigma = \dfrac{(p - pv)}{\left(\dfrac{\rho V_2}{2}\right)}$

The cavitation parameter suggested by Dr. Thoma is

$$\sigma = \frac{(p_a - p_{\min}) / \gamma - H_s}{H}$$

where, H_s = height of the turbine runner above the tail water surface on the suctions - setting in case of a pump

p_a = atmospheric pressure

$p_{\min}$ = minimum pressure occuring in the machine

H = net head across the machine

DIMENSIONAL ANALYSIS.

In dimensional analysis, the dimensions o f physical quantity is expressed in terms of fundamental dimensions M, L and T.

Fundamental and Derived quantites.

All physical quantities can be expressed in terms of *fundamental quantities (mass, length and time* generally denoted by M. L and T). All other quantities such as area, volume, velocity, force, energy etc are called derived quantities and they can be exspressed in terms of above fundamental quantities.

Dimensional homogeneity.

An equation is said to be dimentially homogeneous if dimensions of the various terms on the left and right sides of the equation are identical. Such an equation is also called *rational equation*.

Methods of dimensional analysis.

Rayleigh's method :

This method is suitable for 3 to 5 variables. It comprises of

(*i*) Writing down the equation describing the physical process such that dependent quantities are on the left side and functional expression of independent quantities taking part in the procnesns are on the right side.

(*ii*) Replace two sides by the appropriate dimensional expression of quantities with right side quantities raise to unknown indices.

(*iii*) Principle of dimensional homogeneity is then applied so as to determine the unknown indices.

(*iv*) The substitution of values of indices results in the development of a relationship between the variables of the process.

SHIP MODEL TESTING.

In testing models of ships, both frictional as well as surface wave resistance are required to be considered. In practice the ship model is tested in the loaboratory at a speed corresponding to that of the prototype and based on equality of Froude Number. The total resistance is thus determined. Let it be r.

Method :

(1) Total resistance of the model is computed at the towing speed determined by equating Froude Number. The frictional resistance will depend on Reynolds number and this number will be different for the model and its prototype. Let the frictional resistance of the model thus calculated be r_f.

(2) Surface wave resistance for the model is then determined from

$$r_w = r - r_f$$

(3) Surface wave resistance for the prototype is then determined from the relation

$$R_w = \rho_r L_r^2 V_r^2 \times r_w$$

(4) Frictional resistance of the ship is obtained by applying law appropriate to the crusing speed of the ship.

(5) Total resistance of the ship is then determined by

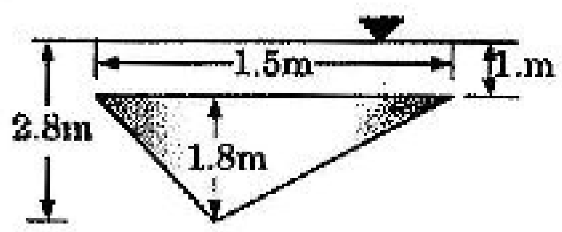
$$R = R_f + R_w$$

SOLVED EXAMPLES

HYDRAULICS

1. A triangular lamina is immersed in water with its apex downwards and base 1 meter below water surface. If its base width and height are 1.5 m and 1.8 m respectively, find the total pressure on the triangle.

Solution:



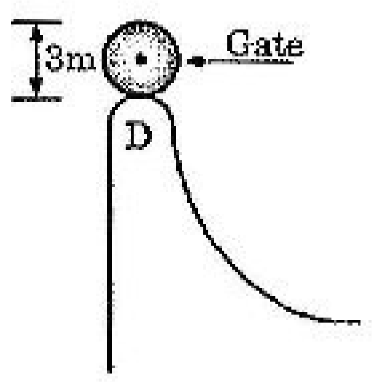
Total Pressure $s = \gamma A h$

$$= 9810 \times \left(\frac{1}{2} \times 1.5 \times 1.8\right)\left(\frac{1}{3} \times 1.8 + 1\right)$$

$$= 21189.6 \text{ N}$$

2. A cylindrical gate 3 m in diameter rests on the crest of a dam. Find the vertical and horizontal components of water pressure acting per meter length of gate, assuming water level upto the top of gate.

Solution:



Intensity of water pressure at point D

$$= \gamma H$$

$$= 9810 \times 3 = 29430 \text{ N/m}^2$$

$$\frac{\text{Horizontal pressure}}{\text{Unit length of gate}} = \frac{29430}{2} \times (\text{dia} \times 1)$$

$$= \frac{29430}{2} \times 3 \text{ N} = 44145 \text{ N}$$

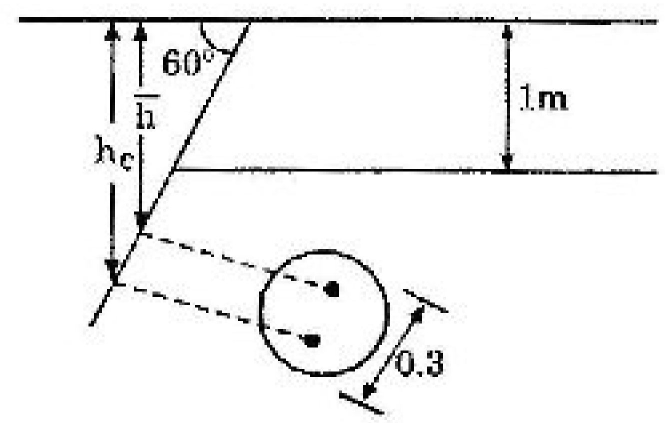
Since, veretical pressure will be the weight of volume of water on half the cylinderical area of cross-section, therefore

$$Pv = g \times \frac{\pi D^2}{4} \times \frac{1}{2}$$

$$= 9810 \times \frac{\pi \times 3^2}{8} = 34671.4 \text{ N}$$

3. A circular plate of 0.3 m diameter is immersed in water at an water at an inclination of 60° to the free surface with its top edge 1m below the water surface. Find the total pressure and centre of pressure on the plate.

Solution:



The diagram shows is the exact configuration of the system

h = distance of centre gravity 'G' from free surface

$$= 1 + \left(\frac{0.3}{2}\right) \sin 60° = 1.13$$

h = depth of centre of pressure 'C' from the free surface

Total water pressure on the plate $= \gamma A \bar{h}$

$$= 9810 \times \frac{\pi}{4} \times 0.3^2 \times 1.13$$

$$= 783.5 \text{ Newtons}$$

Depth of Centre of Pressure 'C' from free surface,

$$h_c = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

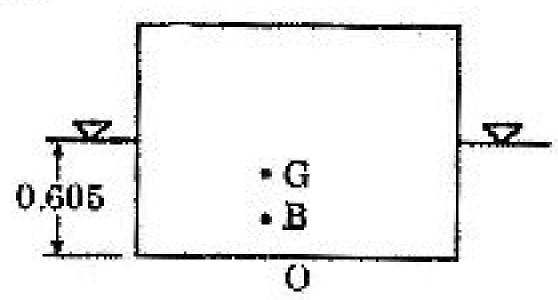
$$= \frac{\pi}{64} d^4 \times \frac{\sin^2 60°}{\frac{\pi}{4} d^2 \bar{h}} - \bar{h}$$

$$= \frac{0.30^2}{16} \times \frac{3}{4} \times \frac{1}{1 + \frac{0.3 \sin 60}{2}} + 1.13$$

$$= 1.337 \text{ m}$$

4. A closed cubical box 1 m on edge is made of uniform sheet and weighs 5.34 kN. What will be its metacentric height when placed in an oil of sp. gravity 0.90 with sides vertical ?

Solution:



Weight of box $= 5.34 \times 10^3$ N

$$\text{Depth of immersion} = \frac{5.34 \times 10^3}{1 \times 1 \times 0.90 \times 9810} = 0.605 \text{ m}$$

$$OB = \frac{0.605}{2}$$

$$= 0.3025 \text{ m}$$

$$OG = OG - OB$$
$$= 0.5 - 0.3025$$
$$= 0.1975\text{m}$$

$$I = \frac{bd^3}{12} = \frac{1 \times 1^3}{12} = \frac{1}{12}$$
$$V = 1 \times 1 \times 0.605 = 0.605$$

$$BM = \frac{1}{V} = \frac{1}{12} \times \frac{1}{0.605} = 0.138 \text{ m}$$

$\therefore$ Melacentre height GM = BM – BG

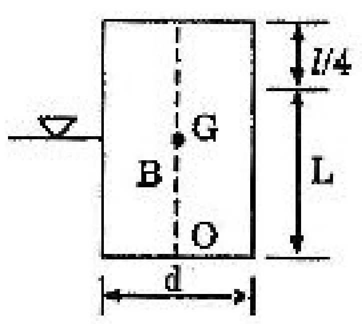
$$= 0.138 - 0.1975$$

= (–) 0.0595 or (–) 0.06 m

Hence, the body is unstable

5. A cylindrical block of length of 'l' diameter 'd' floats in a liquid such that its $\frac{1}{4}$th portion is above the free surface. The specific gravity of the block w.r.t. the liquid is $\frac{3}{4}$. Show that the block will be in stable equilibrium if $d > l\sqrt{1.5}$.

Solution:



Height of immersion = $\frac{1}{4}l$, OB = $\frac{3}{8}l$, OG = $\frac{l}{2}$

$\therefore$ $$BG = \frac{l}{2} - \frac{3}{2}l = \frac{l}{8}, I = \frac{\pi d^4}{64}$$

and $$V = \frac{\pi d^4}{4} \cdot \frac{3}{4}l = \frac{3}{16}\pi d^2 l$$

$$BM = \frac{I}{V} = \frac{d^2}{12l} \text{ and}$$

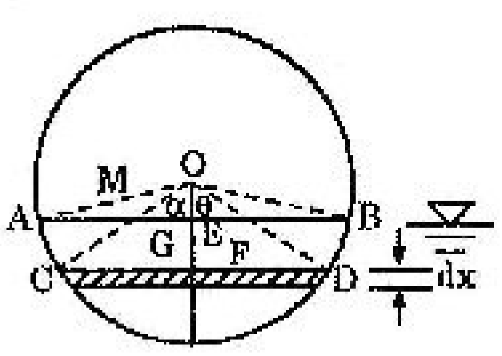
For stable equilibrium, BG < BM

or $$\frac{l}{8} < \frac{d^2}{12l}, \text{ or } d > l\sqrt{1.5}$$

Superfluous data is specific gravity ratio.

6. A solid cylinder of heigher h and radius is floating in a homogeneous liquid with its axis horizontal, at a height C above the surface. Show that the equilibrium would be stable if $h^2 > 4(r^2 - c^2)$

Solution:



Let O be the centre of the cylinder. The angle subtended by a water line AB at the centre of the cylinder

$$= \angle AOB = 2\theta$$

Considering a horizontal elementary strip CD, we have

$$AB = 2AE = 2r \sin \alpha$$

Area of immersion = $r^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

Volume of immersion = $r^2\left(\theta - \frac{\sin 2\theta}{2}\right)h$

$$BM = \frac{1}{V} = \frac{\frac{AB}{12}h^2}{r^2\left(\theta - \frac{\sin 2\theta}{2}\right)h}$$

$$= \frac{h^2 \sin\theta}{6r(\theta - \sin\theta\cos\theta)}$$

Let CD = y and OF = $x = r\cos\alpha$, then BG = $\frac{\int y.xdx}{\int y.dx}x$

where $x = r\cos\alpha$

$\Rightarrow$ $dx = -r\sin\alpha \,.\, d\alpha$

and $y = 2.\,CP = 2r\sin\alpha$

$$\therefore \quad \int y.xdx = \int_0^\theta 2r\sin\alpha \,.\, r\cos\alpha(-r\sin\alpha\, d\alpha)$$

$$= 2r^2\int_0^\theta 2r\sin\alpha \,.\, \cos\alpha\, d\alpha$$

$$= 2r^3 \sin^3\theta$$

Since, $\int y.dx$ = Immersed area = $r^2(\theta - \sin\theta\cos\theta)$

$$\therefore \quad BG = \frac{\frac{2}{3}r^3\sin^3\theta}{r^2(\theta - \sin\theta\cos\theta)}$$

$$= \frac{2}{3}r.\frac{\sin^3\theta}{(\theta - \sin\theta\cos\theta)}$$

The condition of stabilit is BM > BG,

or $$\frac{h^2\sin\theta}{6r(\theta - \sin\theta\cos\theta)} > \frac{2}{3}r.\frac{\sin\theta}{(\theta - \sin\theta\cos\theta)}$$

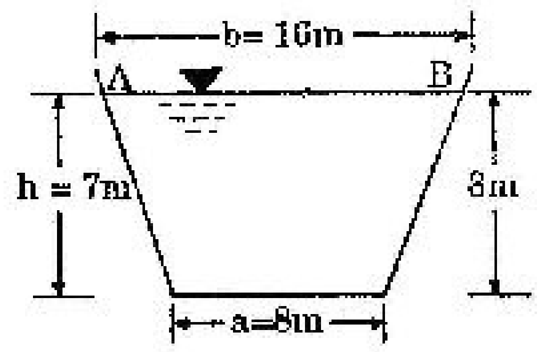
Since $$\sin^2\theta = \frac{OA^2 - OE^2}{OA^2}$$

$$= \frac{r^2 - c^2}{r^2}$$

$$\therefore \quad h^2 > 4r^2\frac{(r^2 - c^2)}{r^2},$$

or $$h^2 > 4(r^2 - c^2)$$

7. A caisson for closing the entrance to a day dock is of trapezoidal form 16m wide at its top and 12 wide at the bottom and 8 cm deep. If the water level outside is 1m below the top level of caisson and the dock is empty, find the totalwater pressure on it and the depth of centre of pressure.



Solution: Since water level stand 1 m below the top level, therefore width of caisson of this level,

$$b = \left[\frac{(16-12)}{8}\times 7+12\right] = 15.5 \text{ m}$$

Area of caisson (upto 7 m depth)

$$= \frac{a+b}{2}\times h$$

$$= \left(\frac{12+15.5}{2}\right)\times 77 = 96.25 \text{ m}$$

Depth of centre of gravity below water surface,

$$\bar{h} = \frac{(2a+b)}{(a+b)}\cdot\frac{h}{3}$$

$$= \left[\frac{2\times 12\times 15.5}{12+15.5}\right]\frac{7}{3}$$

$= 3.35$ m below water surface

M. I., about an is passing through *e.g.*,

$$I_G = \left[\frac{a^2\times 4ab\times b^2}{36(a+b)}\right]h^3$$

$$= \left[\frac{(12)^2+4\times 12\times 15.5+(15.5)^2}{36(12+15.5)}\right](7)^3$$

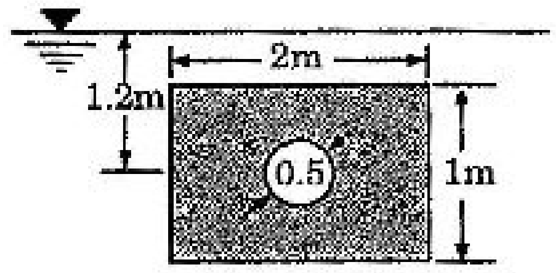
$$= 390.9 \text{ m}^4$$

$$\therefore \quad h_p = \bar{h}+\frac{I_a}{A\bar{h}}$$

$$= 3.35 + \frac{390.9}{96.25\times 3.25}$$

$= 4.56$ m below water surface

8. A rectangular gate 2 m long and 1m high lies in a vertical plane with its central 1.2m below the water surface. Calculate magnitude, direction and location of the total force on the gate. If a circular hole is drilled at the centre of the plate of 0.5 m diameter, calculate the magnitude and location of the total force.



Solution:

$$P = wAh$$

$$= 9.81\times(2\times 1)\times 1.2$$

$$= 23.554 \text{ kN},$$

$$I_G = \frac{2(1)^3}{12} = 0.1667$$

Now,

$$h_p = \bar{h}+\frac{I_G.\sin^2\theta}{A\bar{h}}$$

$$= 1.2+\frac{0.1667}{2\times 1\times 1.2} = 1.27 \text{ m}$$

$$\text{Area of gate} = \left(2\times 1-\frac{\pi}{4}\times 0.5^2\right) = 1.804 \text{ m},$$

$$P = 9.81\times 1.804\times 1.2 = 21.236 \text{ kN}$$

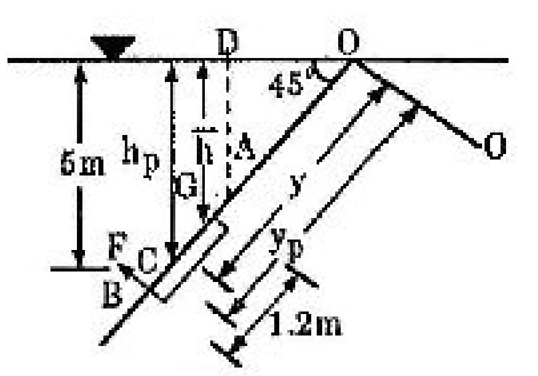
$$I_G \text{ of hole} = \frac{\pi}{64}\times 0.5^4 = 0.00307 \text{ m}^4,$$

$$\therefore \quad h_p = \left[1.2+\frac{(1.667-0.00307)}{1.804\times 1.2}\right]$$

$$= 1.968 \text{ m}$$

9. An inclined rectangular sluice gate AB (1.2 m × 5 m size) is installed to control the discharge of water. End A is hinged. Determine the force normal to the gate applied at B to open it. The angle of inclination of plate is 45°.

Solution:



$$\bar{y} = \left(5\sqrt{2}-0.6\right) = 6.471 \text{ m}$$

$$y_p = \left[6.471+\frac{1}{12}\times\frac{5\times 1.2^3}{5\times 1.2\times 6.47}\right]$$

$$= 6.4895 \text{ M}$$

$$P = wA\bar{h}$$

$$= 9.81\times(1.2\times 5)\times(6.471\sin 45°)$$

$$= 269.325 \text{ kN}$$

Take moment about hinge A, $F\times 1.2 = p[y_p-(\bar{y}-0.6)]$

$$\therefore \quad F = \frac{269.325\times[16.4895-(6.471-0.6)]}{1.2}$$

$$= 138.81 \text{ kN}$$

10. Cylinder of radius R and length L holds the water behind it as shown in the figure. Assuming the contact between the cylinder and wall to be smooth, determine the weight of the cylinder and horizontal force exerted against the wall. What will be the relative density of cylinder ?

Solution: For equilibrium, the vertically upward force on the cylinder will be balanced by its weight.

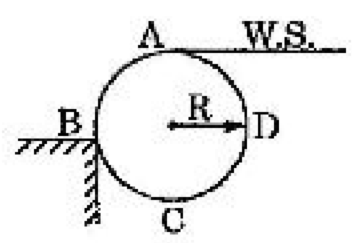
$$\therefore \qquad R_V = R_{BCD} + R_{AD}$$

Since the vertical force on the curved surface is equal to weight of liquid vertically above it and extending up to actual or imaginary free surface, therefore

$$R_V = \left(\frac{\pi R^2 L}{2} + 2R^2L\right)\gamma - \left(R^2L - \frac{\pi R^3 L}{4}\right)\gamma$$

$$= \frac{R^2L\gamma}{4}(2\pi + 8 - 4 + \pi)$$

$$= \frac{R^2L\gamma}{4}(3\pi + 4)$$



Since this must be equal to the weight of cylinder, thefore

$$R_h = R_{ADC} - R_{BC}$$

$$= \gamma \times 2R \times L \times R - \gamma RL \times \frac{3R}{2}$$

$$\therefore \qquad = 2\,R^2L\gamma - \frac{3}{2}R^2L\gamma = \frac{1}{2}R^3L\gamma$$

Weight of cylinder,

$$W = R_V = \frac{R^2L\gamma}{4}(3\pi + 4)$$

Relative density of cylinder

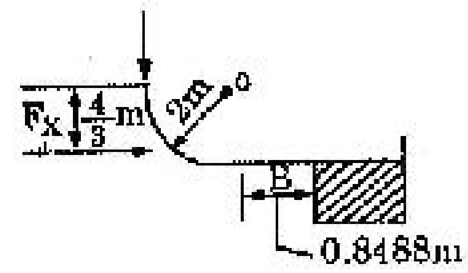
$$= \frac{R^2L\gamma(3\pi+4)}{4\pi r^2 L\gamma}$$

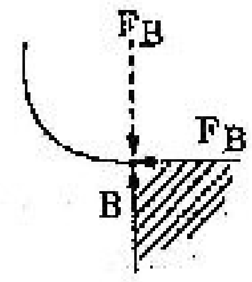
$$= \frac{(3\pi+4)}{4\pi}$$

$$= \left(\frac{3}{4} + \frac{1}{\pi}\right)$$

11. Figure shows a gate having a quadrant shape of radius 2 m. Find the resultant pressure force of water per meter length of gate and its angle with the horizontal. If the gate is hinged at A, find the force required at B to keep the gate closed.

Solution:





$$F_x = \gamma A\bar{h}$$

$$= 9810 \times (2 \times 1) \times \left(\frac{2}{2}\right)$$

$$= 19620 \text{ N}$$

acting at $\left(\frac{2}{3} \times 2\right)$ from free surface

$$F_y = \gamma \times (\text{area of AOB} \times 1 \text{ m})$$

$$= 9810 \times \frac{\pi \times 2^2}{4}$$

$$= 30819 \text{ N}$$

acting at $\frac{4r}{3\pi}$ or $\frac{4 \times 2}{3 \times \pi}$, *i.e.* 0.8488 m from OB

Resultout force, $R = \sqrt{(F_x^2 + F_y^2)}$

$$= \sqrt{19620 + 30819} = 365.34 \text{ N}$$

$$\tan\theta = \frac{F_y}{F_x} = \frac{30819}{19620} = 1.571,$$

$$\therefore \qquad \theta = 57^\circ\, 31'\, 06''$$

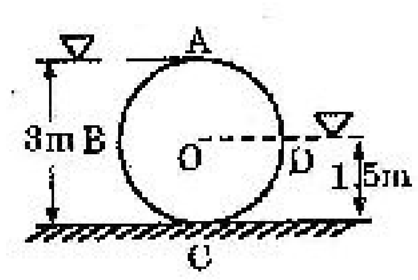
$$F_B = \frac{19620 \times \frac{4}{3} + 30819 \times (1 - 0.8488)}{2}$$

$$= 30918 \text{ N}$$

We can apply F_B vertically downwards also at B↓

12. A steel cylinder of 3 m diameter and 2 m length lies across the full width of an open channed of 2 m width. The water levels on both sides of the steel cylinder are shown in the figure. Determine the magnitude, direction and location of the resulting force on the cylinder.

Solution:



Consider unit length of cylinder

W_1 = weight of water that would fill ABCD

$$= \frac{\pi}{4}d^2 \times \frac{1}{2} \times 1 \times \gamma$$

W_2 = weight of water that would fill CDO

$$= \frac{\pi}{4}d^2 \times \frac{1}{4} \times 1 \times \gamma$$

$$\therefore \quad W = W_1 + W_2 = \frac{3}{4}.\frac{\pi}{4}d^2 \times \gamma$$

$$= \frac{3}{4}.\frac{\pi}{4}3^2 \times 9.810 = 52 \text{ kN}$$

Horizontal thrust, $P_1 = \frac{\gamma h_1^2}{2}$

$$= \frac{9.81 \times 3^3}{2}$$

$$= 44.145 \text{ kN}$$

and $P_2 = \frac{\gamma h_2^2}{2}$

$$= \frac{9.81 \times 1.5^3}{2}$$

$$= 11.063 \text{ kN}$$

$$\therefore \quad P = P_1 - P_2$$

$$= 44.145 - 11.063$$

$$= 33.109 \text{ kN}$$

and $R = \sqrt{(W^2 + P^2)}$

$$= \sqrt{52^2 + 33.109^2}$$

$$= 61.64 \text{ kN}$$

$$\text{Direction} = \tan^{-1}\frac{W}{P}$$

$$= \tan^{-1}\frac{52}{33.109}$$

$$= 57° \, 30' \, 33''$$

MOMENTUM EQUATION AND ITS APPLICATIONS

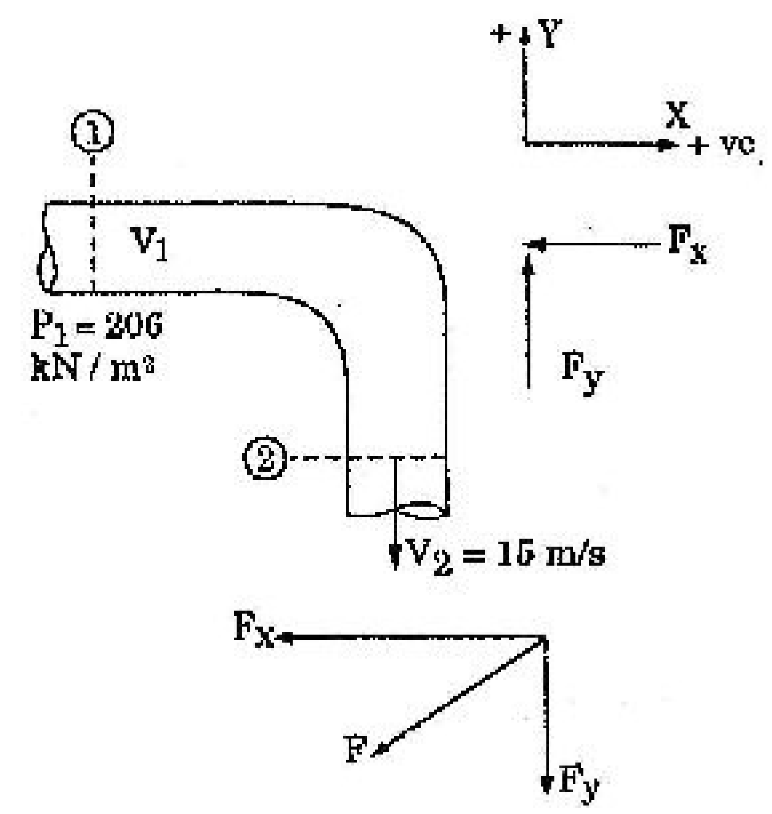
1. Water flows through a 90° reducer bend. The pressure at the inlet is 206 kN/m² (guage) where the cross sectional area is 0.01 m². At the exit section, the area is 0.0025 m² and the velocity is 15 m/s. The 0.0025 m² section continues.

Given: head loss in the reducer bend, $h_L = k_b \frac{V^2}{2g}$,

where $k_b = 1$ and $V = \frac{V_1 + V_2}{2}$.

Determine the force exerted by flow on the elbow [$g = 9.806$ m/s², $\rho = 998$ kg/m³]

Solution:



$$Q = A_2 V_2$$

$$= 15 \times 0.0025$$

$$= 0.0375 \text{ m}^3/\text{s}.$$

Also $Q = A_1 V_1$

$$V_1 = \frac{Q}{A_1}$$

$$= \frac{0.0375}{0.01} = 3.75 \text{ m/s}$$

Assume bend to be horizontal in the x-y plane.

$$\gamma = \rho g$$

$$= 998 \times 9.806$$

$$= 9787 \text{ N/m}^3$$

$$V = \frac{V_1 + V_2}{2}$$

$$= \frac{15 + 3.75}{2} = 9.375 \text{ m/s}$$

$$h_L = 1 \times \frac{(9.375)^2}{2 \times 9.81} = 4.481 \text{ m}$$

Applying Bernoulli's equation to sections (1) and (2), we have

$$\frac{P_1}{\gamma} + \frac{V_2^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

or

$$\frac{P_2}{\gamma} = \frac{206000}{9787} + \frac{3.75^2}{2 \times 9.806} - 4.481$$

$$= 5.811 \text{ m}$$

$$\therefore \quad P_2 = 5.811 \times \gamma$$

$$= 5.811 \times 9787$$

$$= 5687.257 \text{ N/m}^2$$

Momentum equation in re-direction is given by,

$$0.0375 \times 998\,(0 - 3.75) = 0.001 \times 206000 - F_x$$

$$\therefore \quad F_x = 2200.344 \text{ N}$$

Momentum equation in the y-direction is given by,

$$0.0375 \times 998\,(-15 - 0) = 56872.257 \times 0.0025 + F_y$$

$$\therefore \quad F_y = -561.375 - 142.18$$

$$= -703.555 \text{ N (downwards)}$$

$\therefore$ Force on water by bend, $F = \sqrt{F_x^2 + F_y^2}$

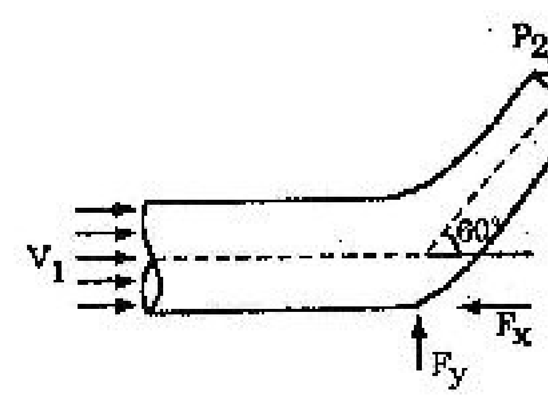
$$= \sqrt{(2200.344)^2 + (703.555)^2}$$

$$= 3210.013 \text{ N}$$

Now, $\tan\theta = \dfrac{703.555}{2200.344} = 0.3195$ or $\theta = 17.72°$

2. The force components on a reducing elbow, making a 60 turn in a horizontal plane are to be determined. At the entering section, D_1 = 6 m, V_1 = 15 m/s, p_1 = 282 kPa and at the exit section D_2 = 4.8 m. Flowing fluid is oil of sp. gr. 0.9 and elbow losses are to be neglected.

Solution:



$$V_1 = 15 \text{ m/s},$$

$$V_2 = \frac{\frac{\pi}{4} \times 6^2 \times 15}{\frac{\pi}{4} \times 4.8^2} = 23.4375 \text{ m/s}$$

and $\quad Q = \dfrac{\pi}{4} \times 6^2 \times 15 = 424 \text{ m}^3\text{/s}$

Assumed directions of forces F are as shown in the figure.

$$\therefore \quad \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

But elbow is in horizontal plane, therefore $z_1 = z_2$

$$\text{or} \quad \frac{282 \times 10^3}{9810} + \frac{15^2}{19.62} = \frac{p_2}{\gamma} + \frac{23.4375^2}{19.62}.$$

Solving , we get p_2 = 136 kPa

Apply Momentum equation between sections 1 and 2 in the flow direction (x-direction)

$$-F + p_1A_1 - p_2A_2 \cos 60° = \frac{\gamma Q}{g}(V_2 \cos 60° - V_1)$$

$$\text{or} \quad -F + 282 \times 10^3 \times \frac{\pi}{4} \times 6^2 - 136 \times 10^3 \times \frac{\pi}{4} \times 4.8^2 \times 0.5$$

$$= \frac{(0.9 \times 9810) \times 424}{9.81}(23.4375 \times 0.5 - 15)$$

F_x = 7995 kN and assumed direction is correct.

Applying Momentum equation in Y-direction, we get

$$p_1 A_1 \sin 0° - p_2 A_2 \sin 60° + F_Y$$

$$= \frac{\gamma Q}{g}(V_2 \sin 60° - 0)$$

$$\text{or} \quad F_y - 136 \times 10^3 \times \frac{\pi}{4} \times 4.8^2 \times \frac{\sqrt{3}}{2}$$

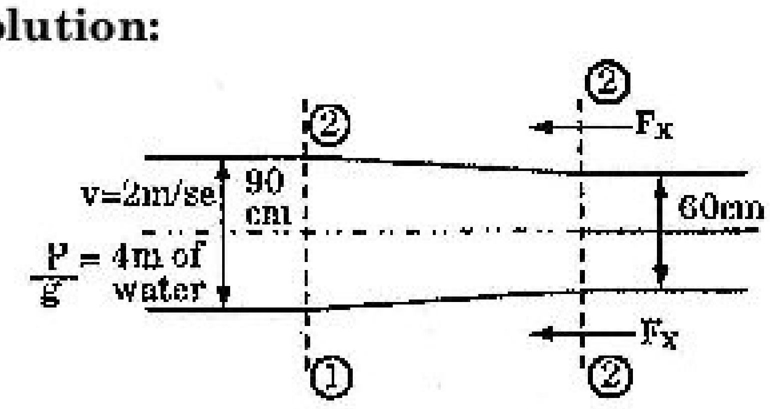
$$= \frac{(0.9 \times 9810) \times 424}{9.81}\left(23.4375 \times \frac{\sqrt{3}}{2}\right)$$

$\therefore$ F_y = 9877 kN and assumed direction is correct.

The force components on the elbow are equal and opposite to F_x and F_y.

3. Water flows at a velocity of 2 m/sec through a 90 cm diameter pipe at the rate of which there is a reducer connected it to a 60 cm diameter pipe. If the gauge pressure at the entrance to the reducer is 4.0 kg/cm². Determine the resultant thrust assuming that the loss of head in the reducer is 1.5 m of water.

Solution:



$$\frac{p_1}{\gamma_w} + Z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma_w} + Z_2 + \frac{v_2^2}{2g}$$

$$\text{or,} \quad 40 + 0 + \frac{4^2}{19.62} = \frac{p_2}{\gamma_w} + 0 + \frac{45^2}{19.62} + 1.5$$

$$\text{or,} \quad \frac{p_2}{\gamma_w} = 37.67 \text{m of water,}$$

$$\therefore \quad p_2 = 3.767 \text{ kgf/cm}^2$$

$$\Sigma\vec{F} = Qr(v_2 - v_1)$$

Quantity of water flowing,

$$Q = a_1 V_1 = \frac{\pi}{4} \times 0.9^2 \times 2 = 1.272 \text{ m}^3\text{/sec}$$

$$p_1 A_1 - p_2 A_2 - F_x = 1.272 \times \frac{10^3}{9.81}(4.5 - 2)$$

where $A_1 = \dfrac{\pi}{4} \times 90^2 = 6361.73 \text{ cm}^3,$

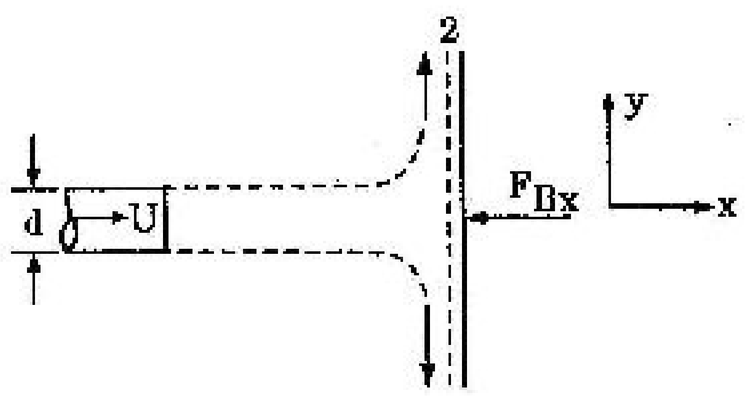
$$A_2 = \frac{\pi}{4} \times 60^2 = 2872.43 \text{ cm}^2$$

$$p_1 = 4 \text{ kg/cm}^2 \text{ and } p_2 = 3.717 \text{ kg/cm}^2$$

$$\therefore \quad F_x = 14471.83 \text{ kgf}$$

4. A circular jet of a fluid of mass density ρ and diameter d, strikes stationary plate held perpendicular to flow with velocity U, as shown in the figure below. Determine the force F_{Bx} exerted by the plate on the fluid

Solution: Assume there is no friction. The pressure everywhere is atmospheric. Hence, the only force in x direction to be considered is F_{Bx}. Assume it to be acting in the direction shown.



$$\Sigma F_x = -F_{Bx} = \frac{\pi d^2}{4} U\rho\,(0 - U)$$

or $$F_{Bx} = \frac{\pi d^2}{4}\rho U^2$$

and it will act in the negative x direction.

Force exerted by the fluid on the plate will be in positive x direction and of the magnitude, viz. $\frac{\pi d^2}{4}\rho U^2$. Since there is no friction and pressure at sections 1 and 2 is atmospheric, velocity along the plate at section at section 2 is also U; further there is no force acting on the plate in y direction.

5. A 40 mm diameter jet of water strikes normally against a plate held in position and causes a force of 900 N on the plate. Determine the jet velocity and the discharge.

Solution:

From $$F = \frac{\pi d^2}{4}\rho U^2$$

$$900 = \frac{\pi \times 0.04^2}{4} \times 998\, U^2$$

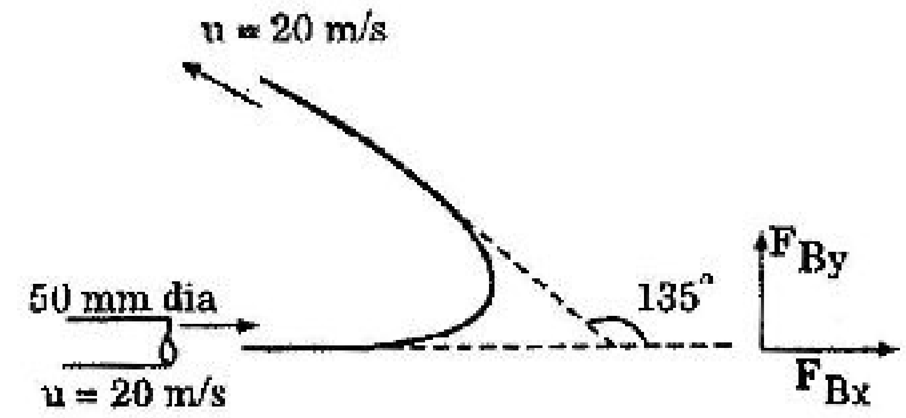
or $$U = 26.765 \text{ m/sec}$$

Also $$Q = \frac{\pi d^2 U^2}{4}$$

$$= \frac{\pi \times 0.04^2 \times 26.795}{4}$$

$$= 0.0337 \text{ m}^3/\text{s}$$

6. Determine the resultant force (and its direction) on the vane shown in figure, if a water jet of 50 mm. dia and 20.0 m/s velocity strikes the vane tangential and is deflected without friction.



Solution: $$Q = \frac{\pi d^2}{4}$$

$$= \frac{\pi}{4} \times 0.5^2 \times 20 = 0.03925 \text{ m}^3/\text{s}$$

Since pressure is atmospheric everywhere, the only external forces to be considered are F_{Bx} and F_{By} the components of force exerted by vane on the fluid. Further, since there is no friction, using Bernoulli's equation it can be shown that U remains constant along the vane.

$$F_{Bx} = Q\rho\,(-U\cos 45° - U)$$
$$= 0.03925 \times 998\,(-20 \times 0.7071 - 20)$$
$$= -1337.393 \text{ N}$$
$$\therefore \quad F_{fx} = 1337.393 \text{ N}$$
$$F_{By} = Q\rho - (U \sin 45° - 0)$$
$$= 0.03925 \times 988 \times 20 \times 0.7071$$
$$= 553.963 \text{ N}$$
$$\therefore \quad F_{fy} = -553.963 \text{ N}$$

where F_{fx} and F_{fy} are components of force exerted by the fluid on the vane

$$F_f = \sqrt{1337.393^2 + 553.963^2}$$
$$= 1447.583 \text{ N}$$

$$\tan\theta = \frac{553.963}{1337.393} = -0.4142$$

$$\therefore \quad \theta = -22.5° \text{ with } x\text{-axis}$$

7. A jet of water of 50 mm diameter strikes a curved van and is deflected through an angle of 170° while the vane moves in the same direction as the jet at a velocity of 10 m/s, as shown in the figure. For a jet discharge of 0.075 m³/s, determine components of the force on the vane, power developed, and absolute velocity of the jet as it leaves the vane.

Solution: