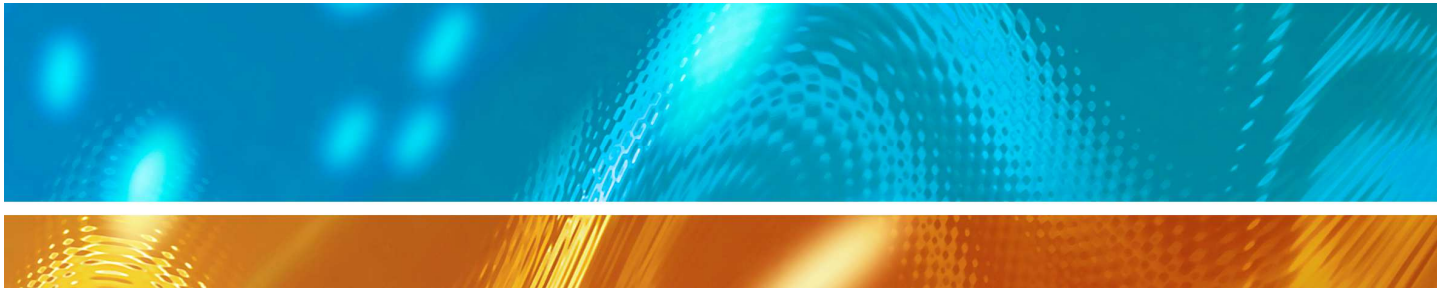
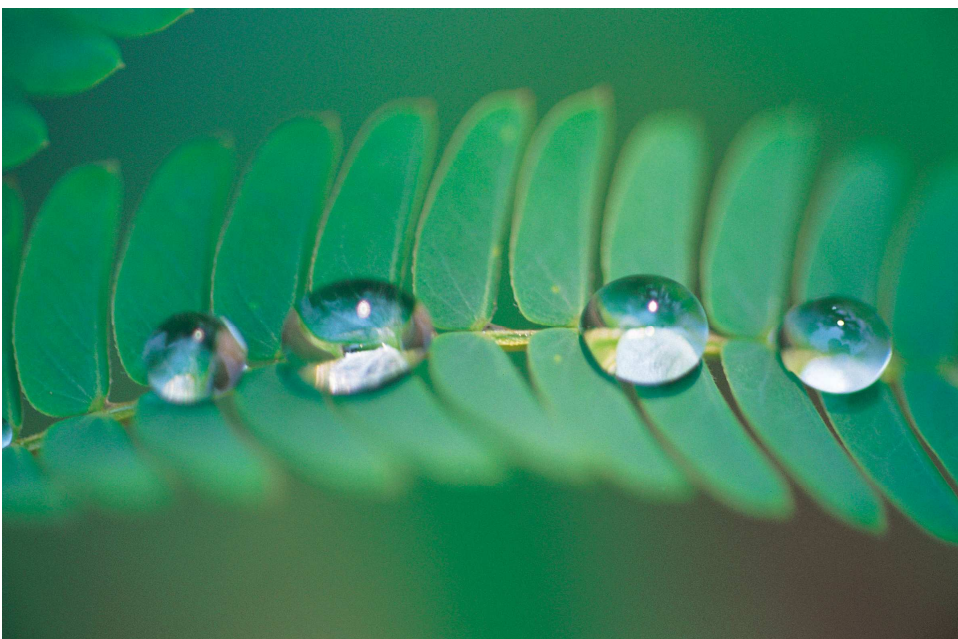


HUMIDITY CONVERSION FORMULAS



Calculation formulas for humidity



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Table of Contents

1. Relative humidity definitions	3
2. Water vapour saturation pressure.....	4
3. Calculation of dewpoint from RH.....	7
3.1 Calculation of dewpoint at different pressure.....	7
3.2 Calculation of RH from dewpoint and ambient temperature	8
4. Using psychrometers	9
5. Mixing ratio	10
6. Enthalpy	11
7. Absolute humidity.....	12
8. Parts per million (ppm)	13
9. Enhancement factor.....	15

1. Relative humidity definitions

Relative humidity is at all temperatures and pressures defined as the ratio of the water vapour pressure to the saturation water vapour pressure (over water) at the gas temperature:

$$RH = P_w / P_{ws} \cdot 100\% \quad (1)$$

The total pressure does not enter the definition. Above 100°C the same definition is valid. But as the saturation vapour pressure P_{ws} is greater than 1 013 hPa (normal ambient pressure) the RH can't reach 100% in an unpressurised system.

Below 0°C the definition is also valid. Here 100%RH is also impossible because condensation will occur at a lower humidity than 100% (when the vapour is saturated against ice).

2. Water vapour saturation pressure

The following formula gives the water vapour saturation pressure to sufficient accuracy between 0°C and 373°C (W. Wagner and A. Pruß:" The IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use ", Journal of Physical and Chemical Reference Data, June 2002 ,Volume 31, Issue 2, pp. 387535):

$$\vartheta = 1 - \frac{T}{T_c} \quad (2)$$

$$\ln\left(\frac{P_{ws}}{P_c}\right) = \frac{T_c}{T} \left(C_1\vartheta + C_2\vartheta^{1.5} + C_3\vartheta^3 + C_4\vartheta^{3.5} + C_5\vartheta^4 + C_6\vartheta^{7.5} \right) \quad (3)$$

T = Temperature in K

P_{ws} = Saturation vapor pressure (hPa)

T_c = Critical temperature, 647.096 K

P_c = Critical pressure 220 640 hPa

C_i = Coefficients,

C₁ = -7.85951783

C₂ = 1.84408259

C₃ = -11.7866497

C₄ = 22.6807411

C₅ = -15.9618719

C₆ = 1.80122502

Correspondingly the saturation water vapour pressure over ice at temperatures between -100 and 0.01°C can be calculated using:

$$\theta = \frac{T}{T_n} \quad (4)$$

$$\ln\left(\frac{P_{wi}}{P_n}\right) = a_0(1 - \theta^{-1.5}) + a_1(1 - \theta^{-1.25}) \quad (5)$$

T = Temperature in K

P_n = Vapor pressure at triple point temperature, 6.11657 hPa

T_n = Triple point temperature 273.16 K

P_{wi} = Saturation vapor pressure (hPa)

a_i = Coefficients,

a₀ = -13.928169

a₁ = 34.707823

If lower accuracy or a limited temperature range can be tolerated a simpler formula can be used for the water vapour saturation pressure over water (and over ice):

$$P_{ws} = A \cdot 10^{\left(\frac{mT}{T+T_n}\right)} \quad (\text{hPa}) \quad , \text{ where} \quad (6)$$

A, m, T_n = constants see Table 1

T = Temperature (°C)

	A	m	Tn	max error	Temperature range
water	6.116441	7.591386	240.7263	0.083%	-20...+50°C
	6.004918	7.337936	229.3975	0.017%	+50...+100°C
	5.856548	7.27731	225.1033	0.003%	+100...+150°C
	6.002859	7.290361	227.1704	0.007%	+150...+200°C
	9.980622	7.388931	263.1239	0.395%	+200...+350°C
	6.089613	7.33502	230.3921	0.368%	0...+200°C
ice	6.114742	9.778707	273.1466	0.052%	-70...0°C

Table 1. Constants for formula 6.

Note: Constants in table 1 are fitted by Vaisala against formula (3) for water vapor saturation pressure and against formula (5) for saturation water vapor pressure over ice.

3. Calculation of dewpoint from RH

Calculate P_{ws} using formula (3) or (6)

Calculate $P_w = P_{ws} \cdot RH/100$ (in hPa!)

Calculate dewpoint using formula (7):

$$Td = \frac{T_n}{\left[\frac{m}{10 \log\left(\frac{P_w}{A}\right)} - 1 \right]} \quad (7)$$

The constants in formula (7) are the same as for formula (6).

Example:

The ambient temperature is 40°C and the RH is 50%. Calculate T_d :

$$P_w = P_{ws}(40^\circ\text{C}) \cdot 50/100 = 36.88 \text{ hPa}$$

$$T_d = 240.7263 / (7.591386 / 10 \log(36.88 / 6.116441) - 1) = \underline{27.6^\circ\text{C}}$$

3.1 Calculation of dewpoint at different pressure

1. Calculate P_{ws} using formula (3) or (6)

2. Calculate measured vapor pressure (at measurement pressure)

$$P_{w_{meas}} = P_{ws} \cdot \frac{RH}{100} \quad (8)$$

3. Multiply with pressure ratio:

$$P_{w_{pres}} = \frac{P_{pres}}{P_{meas}} \cdot P_{w_{meas}} \quad (9)$$

4. Calculate dewpoint at process pressure:

$$Td_{pres} = \left[\frac{T_n}{\frac{m}{10 \log \left(\frac{Pw_{pres}}{A} \right)} - 1} \right] \quad (10)$$

3.2 Calculation of RH from dewpoint and ambient temperature

$$RH = 100\% \cdot \frac{Pws(Td)}{Pws(Tambient)} \quad (11)$$

P_{ws} may be calculated using formulas (3) or (6). If the simplified formula (6) is used the expression for RH may be further simplified to:

$$RH = 100\% \cdot 10^{m \left[\frac{Td}{Td+Tn} - \frac{Tambient}{Tambient+Tn} \right]} \quad (12)$$

Values for the constants M and Tn for the appropriate temperature range can be found in Table 1.

4. Using psychrometers

The drybulb-temperature T_{dry} and the wetbulb-temperature T_{wet} can be converted into P_w using formula (13):

$$P_w = P_{ws}(T_{\text{wet}}) - P_{\text{tot}} \cdot K \cdot (T_{\text{dry}} - T_{\text{wet}}), \text{ where} \quad (13)$$

P_{ws} = Water vapour saturation pressure from formula (6)

P_{tot} = Total ambient pressure

K = Psychrometer constant $0.000662^\circ\text{C}^{-1}$

When P_w is known RH can be calculated using (1) or T_d can be calculated using (7)

Example:

T_{wet} is 38.5°C , $T_{\text{dry}} = 40.0^\circ\text{C}$ and the ambient pressure is 1 013 hPa. Calculate RH and T_d :

$$P_{ws}(38.5^\circ\text{C}) = 68.05 \text{ hPa}$$

$$P_{ws}(40.0^\circ\text{C}) = 73.75 \text{ hPa}$$

$$P_w = 68.05 - 1013 \cdot 0.000662 \cdot (40.0 - 38.5) = 67.04 \text{ (hPa)}$$

$$\text{RH} = 67.04 / 73.75 \cdot 100 = 90.9\%$$

$$T_d = 240.7263 / (7.591386 / {}^{10}\log(67.04 / 6.116441) - 1) = \underline{38.21^\circ\text{C}}$$

5. Mixing ratio

The mixing ratio (mass of water vapour/mass of dry gas) is calculated using (14):

$$X = B \cdot P_w / (P_{\text{tot}} - P_w) \quad [\text{g/kg}] \quad , \text{ where } \quad (14)$$

$$B = 621.9907 \text{ g/kg}$$

The value of B depends on the gas. 621.9907 g/kg is valid for air.

In general the constant can be calculated using:

$$B = M(\text{H}_2\text{O}) / M(\text{gas}) \cdot 1000 \quad [\text{g/kg}] \quad , \text{ where } \quad (15)$$

$M(\text{H}_2\text{O})$ = Molecular weight of water

$M(\text{gas})$ = Molecular weight of gas

For instance for hydrogen we get

$$B = 18.015 / 2.016 \cdot 1000 = 8\,936 \text{ g/kg}$$

Example:

The dewpoint T_d is 40°C and the total ambient pressure P_{tot} is 998 hPa. Calculate mixing ratio:

$$P_w = P_{\text{ws}}(40^\circ\text{C}) = 73.75 \text{ hPa}$$

$$X = 621.9907 \cdot 73.75 / (998 - 73.75) = \underline{49.63 \text{ g/kg}}$$

(To obtain the mixing ratio in units of grains/pound use $B = 4\,354$ [grains/pound])

6. Enthalpy

Enthalpy can be calculated from mixing ratio using (16):

$$h = T \cdot (1.01 + 0.00189X) + 2.5X \quad (\text{kJ/kg}) \quad , \text{ where} \quad (16)$$

T = Temperature (°C)

X = Mixing ratio (g/kg)

To convert to Btu/lb divide by 2.324

Note that a different base temperature for zero enthalpy, 0°F is often used together with the Btu/lb unit.

Example:

The ambient temperature is 20°C and the relative humidity is 50%. Calculate enthalpy:

$$P_w = P_{ws}(20^\circ\text{C}) \cdot 50/100 = 11.69 \text{ hPa}$$

$$X = 621.9907 \cdot 11.69 / (1013 - 11.69) = 7.26 \text{ g/kg}$$

$$h = 20 \cdot (1.01 + 0.00189 \cdot 7.26) + 2.5 \cdot 7.26 = \underline{38.62 \text{ kJ/kg}}$$

7. Absolute humidity

Absolute humidity is defined as the mass of water vapour in a certain volume. If ideal gas behaviour is assumed the absolute humidity can be calculated using (17):

$$A = C \cdot P_w / T \quad (\text{g/m}^3) \quad , \text{ where } \quad (17)$$

C = Constant 2.16679 gK/J

P_w = Vapour pressure in Pa

T = Temperature in K

Example:

The ambient temperature is 20°C and the relative humidity is 80%. Calculate absolute humidity:

$$P_w = P_{ws}(20^\circ\text{C}) \cdot 80/100 = 18.7 \text{ hPa}$$

$$A = 2.16679 \cdot 1870 / (273.15 + 20) = \underline{13.82 \text{ g/m}^3}$$

8. Parts per million (ppm)

Parts per million values are usually given vs. the amount of dry air:

I: Volume/volume PPM_v(dry):

$$PPM_v = \frac{P_w}{P_{tot} - P_w} 10^6 \quad (18)$$

Where

P_w = Water vapour pressure

P_{tot} = Total pressure

II: Mass/mass PPM_m(dry)

$$PPM_m = \frac{M_w P_w}{M_d (P_{tot} - P_w)} 10^6$$
$$\frac{M_w}{M_d} = 0.62199 \quad (19)$$

Where

P_w = Water vapour pressure

P_{tot} = Total pressure

M_w = Molecular mass of water

M_d = Molecular mass of dry air

From wet air:

III: Volume/volume PPM_v(wet):

$$PPM_v = \frac{P_w}{P_{tot}} 10^6 \quad (20)$$

IV: Mass/mass PPM_m(wet)

$$PPM_m = \frac{M_w P_w}{M_d P_{tot}} 10^6$$

$$\frac{M_w}{M_d} = 0.62199 \quad (21)$$

Example:

The dewpoint is 7°C and the total pressure is 998 hPa. Calculate PPM_v(dry).

By using formula (6) with the dewpoint temperature, $P_w = P_{ws}(T_d)$, we get $P_{ws}(7^\circ\text{C}) = 10.02$ hPa. Now:

$$PPM_v = \frac{10.02}{(998 - 10.02)} 10^6 = 10142$$

9. Enhancement factor

The water vapour saturation pressures described in Paragraph 2 are exactly valid only in vacuum (water vapour is the only gas present). If other gases are present the real saturation vapour pressure P_{ws} will increase. For ambient pressure and lower pressures this effect is typically ignored. But at pressures significantly above ambient it has to be taken into account. The enhancement factor f has been defined as follows:

$$f = \frac{X_w \cdot P}{P_{ws}}, \text{ where} \quad (22)$$

X_w = The mole fraction of water vapour in the gas phase

P = Total pressure

P_{ws} = The saturation water vapour pressure (from Paragraph 2)

$x_w \cdot P$ is in effect the water vapour pressure at saturation at pressure P

For CO₂-free air the following equation for f has been proposed (Greenspan: J. of Research of the NBS vol 80A, No. 1 p 41-44)

$$f = e^{\left[\alpha \left(1 - \frac{P_{ws}}{P} \right) + \beta \left(\frac{P}{P_{ws}} - 1 \right) \right]} \quad (23)$$

The parameters α and β depend on the temperature as follows:

$$\alpha = \sum_{i=1}^4 A_i t^{(i-1)} \quad (24)$$

$$\beta = e^{\sum_{i=1}^4 B_i t^{(i-1)}} \quad (25)$$

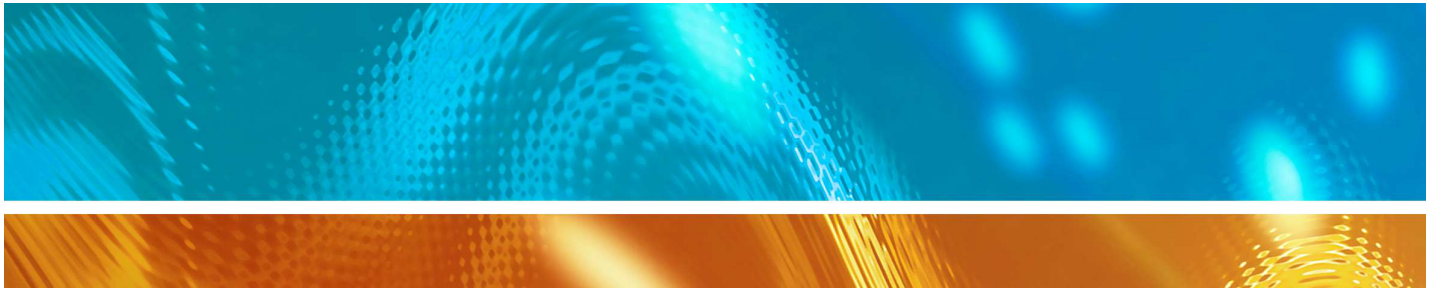
T = Temperature (°C)

The parameters A_i and B_i are listed below:

	water -50 to 0°C	water 0 to 100°C	ice -100 to 0°C
A1	$3.62183 \cdot 10^{-4}$	$3.53624 \cdot 10^{-4}$	$3.64449 \cdot 10^{-4}$
A2	$2.60553 \cdot 10^{-5}$	$2.93228 \cdot 10^{-5}$	$2.93631 \cdot 10^{-5}$
A3	$3.86501 \cdot 10^{-7}$	$2.61474 \cdot 10^{-7}$	$4.88635 \cdot 10^{-7}$
A4	$3.82449 \cdot 10^{-9}$	$8.57538 \cdot 10^{-9}$	$4.36543 \cdot 10^{-9}$
B1	-10.7604	-10.7588	-10.7271
B2	$6.39725 \cdot 10^{-2}$	$6.32529 \cdot 10^{-2}$	$7.61989 \cdot 10^{-2}$
B3	$-2.63416 \cdot 10^{-4}$	$-2.53591 \cdot 10^{-4}$	$-1.74771 \cdot 10^{-4}$
B4	$1.67254 \cdot 10^{-6}$	$6.33784 \cdot 10^{-7}$	$2.46721 \cdot 10^{-6}$

The formulas above are fitted for the pressure range 1 ... 20 atm.

for instance at 20°C ad 10 bars $f = 1.031$



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